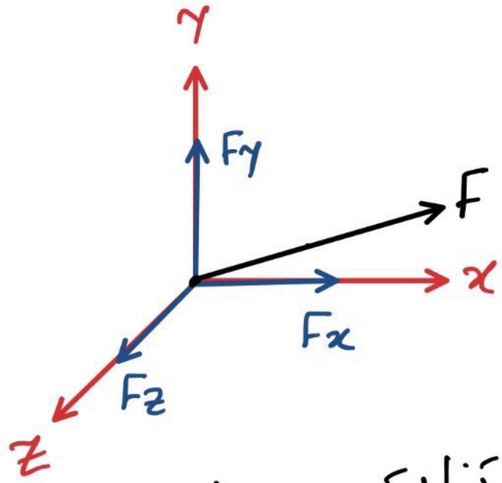
In three dimensions

Sunday, April 26, 2020 4:15 PM

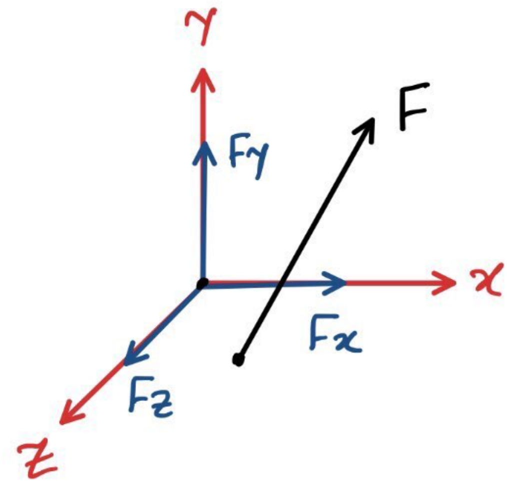
$$F_x = F * \frac{x}{\sqrt{x^2+y^2+z^2}}$$



* القوة تنطلق من نقطة الاصل.

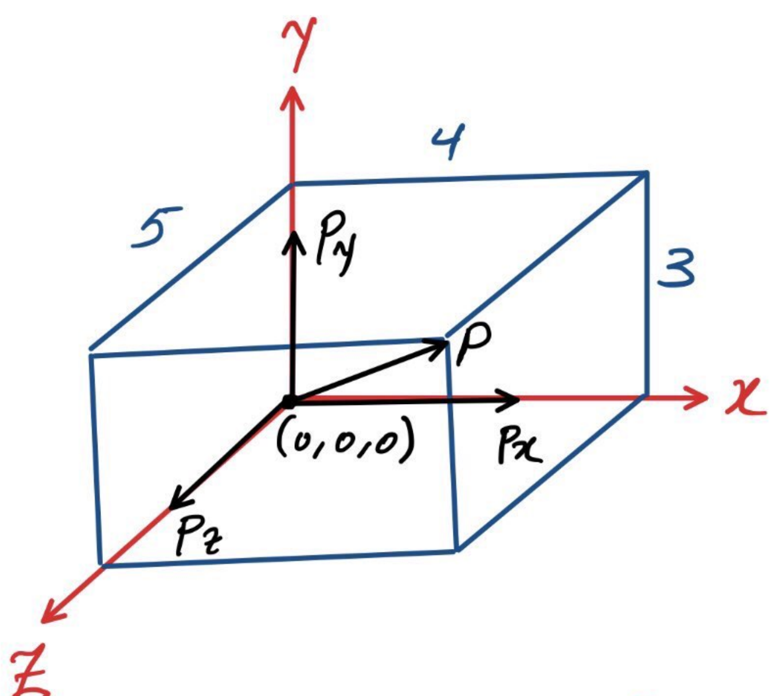
$$F_y = F * \frac{y}{\sqrt{x^2+y^2+z^2}}$$

$$F_z = F * \frac{z}{\sqrt{x^2+y^2+z^2}}$$



* القوة لا تنطلق من نقطة الاصل.

$$\overrightarrow{P_x} = P * \frac{4}{\sqrt{3^2+4^2+5^2}}$$



$$P_y \uparrow = P * \frac{3}{\sqrt{3^2+4^2+5^2}}$$

$$P_z \swarrow = P * \frac{5}{\sqrt{3^2+4^2+5^2}}$$

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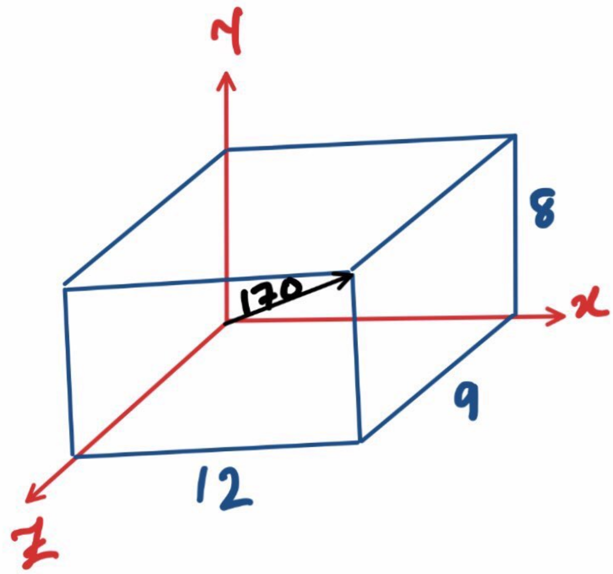
Example: determine a set of three rectangular components of 190 Ib force



Solution :

$$F_y = 170 * \frac{8}{\sqrt{8^2+9^2+12^2}}$$

$$= \uparrow 80 \text{ Ib}$$

$$F_z = 170 * \frac{9}{\sqrt{8^2+9^2+12^2}}$$

$$= \nearrow 90 \text{ Ib}$$

$$F_x = 170 * \frac{12}{\sqrt{8^2+9^2+12^2}}$$

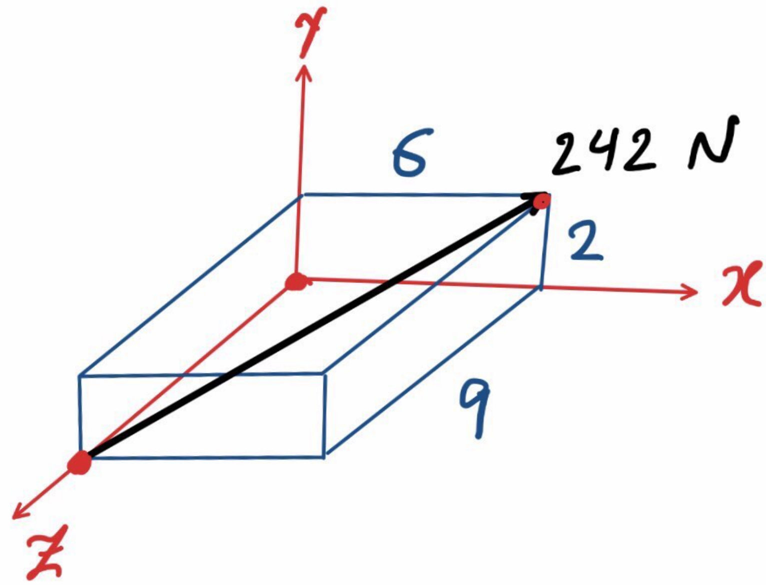
$$= \overrightarrow{120} \text{ Ib}$$

Example:

$\Delta x = 6$

$\Delta y = 2$

$\Delta z = 9$



$$F_x = 242 * \frac{6}{\sqrt{6^2+2^2+9^2}}$$

$$= \overrightarrow{132}\ N$$

$$F_y = 242 * \frac{2}{\sqrt{6^2+2^2+9^2}} = \uparrow 44\ N$$

$$F_z = 242 * \frac{9}{\sqrt{6^2+2^2+9^2}} = \nearrow 198\ N$$

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R.R.

In two dimentions

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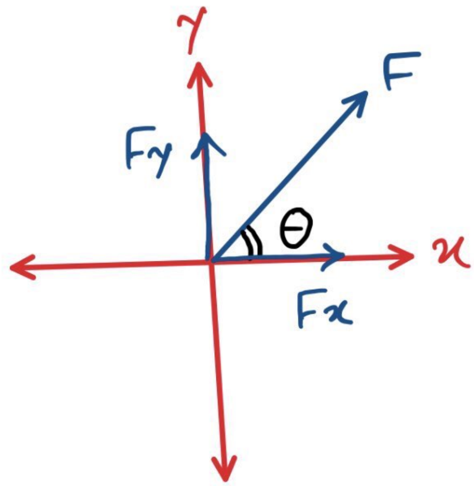
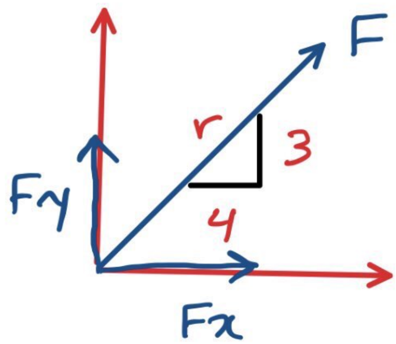
$$\overset{+}{\rightarrow} F_x = F\cos\theta$$

$$\overset{+}{\uparrow} F_y = F\sin\theta$$



$$r = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$$



$$\frac{F_x}{4} = \frac{F}{5}$$

$$\boxed{F_x = F * \frac{4}{5}}$$

$$\frac{F_y}{3} = \frac{F}{5}$$

$$\boxed{F_y = F * \frac{3}{5}}$$

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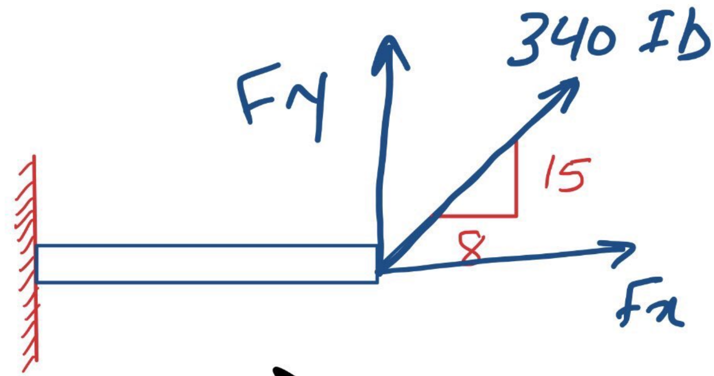
Ex: determine the pair of horizontal and vertical components of the 340 Ib force

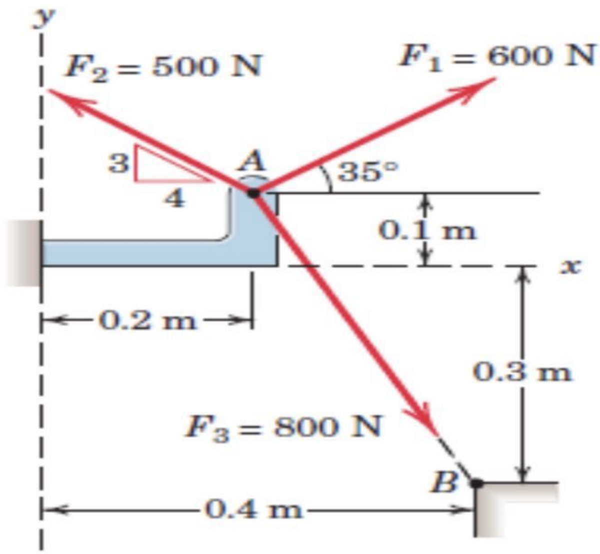


Solution :

$$F_x = 340 * \frac{8}{\sqrt{8^2+15^2}} = \overrightarrow{160\ lb}$$

$$F_y = 340 * \frac{15}{\sqrt{8^2+15^2}} = \uparrow 300\ Ib$$

EX1: The forces **F**1, **F**2, and **F**3, all of which act on point *A* of the bracket, are specified in three different ways. Determine the *x* and *y* scalar components of each of the three force



Solution. The scalar components of $\mathbf{F}_1$, from Fig. *a*, are

$$F_{1_x} = 600 \cos 35° = 491 \text{ N} \qquad \textit{Ans.}$$

$$F_{1_y} = 600 \sin 35° = 344 \text{ N} \qquad \textit{Ans.}$$

The scalar components of $\mathbf{F}_2$, from Fig. *b*, are

$$F_{2_x} = -500\left(\frac{4}{5}\right) = -400 \text{ N} \qquad \textit{Ans.}$$

$$F_{2_y} = 500\left(\frac{3}{5}\right) = 300 \text{ N} \qquad \textit{Ans.}$$

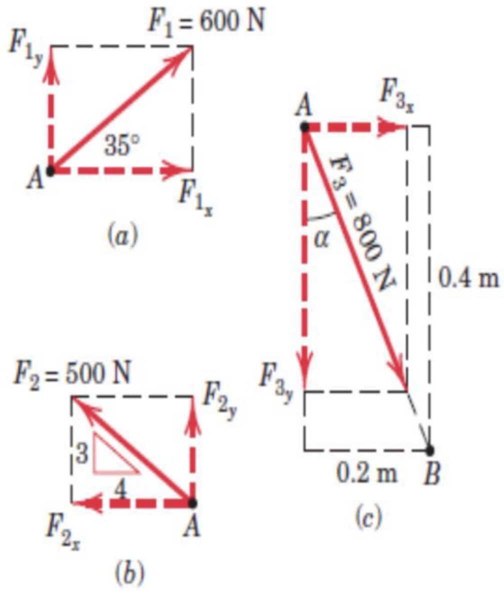
Note that the angle which orients $\mathbf{F}_2$ to the *x*-axis is never calculated. The cosine and sine of the angle are available by inspection of the 3-4-5 triangle. Also note that the *x* scalar component of $\mathbf{F}_2$ is negative by inspection.

The scalar components of $\mathbf{F}_3$ can be obtained by first computing the angle α of Fig. *c*.

$$\alpha = \tan^{-1}\left[\frac{0.2}{0.4}\right] = 26.6°$$

Then $F_{3_x} = F_3 \sin \alpha = 800 \sin 26.6° = 358$ N *Ans.*

$$F_{3_y} = -F_3 \cos \alpha = -800 \cos 26.6° = -716 \text{ N} \qquad \textit{Ans.}$$

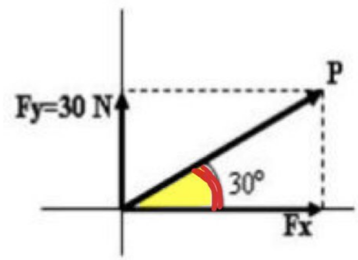


A.R.R

م.م. أحمد

Example 8

The direction of the force (P) is (30^{o}) , Find the horizontal component if the vertical component is (30 N) ?



<u>Solution :</u>

From the diagram shown :

Fy = 30 N

$\boxed{Fy = F \cdot \sin\theta}$

30 = P * sin 30

30 = P * 0.5

P = 30 / 0.5 = 60 N

$\boxed{Fx = F \cdot \cos\theta}$

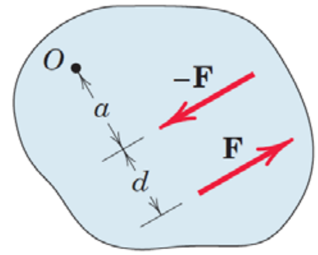
$= 60 * \cos 30 = 60 * \frac{\sqrt{3}}{2} = 30\sqrt{3}$ N

Moment of Couples

The moment produced by two equal, opposite, and non collinear forces is called a *couple*. Couples have certain unique properties and have important applications in mechanics.

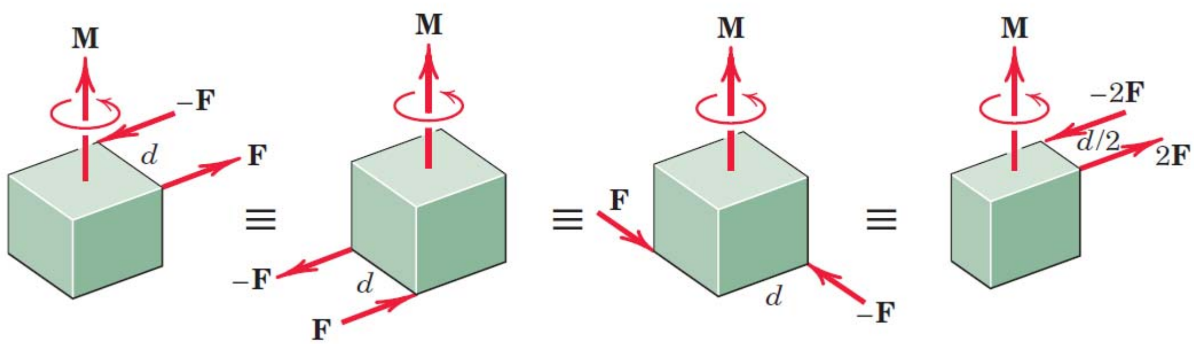
$$M = F(a + d) - Fa$$

$$M = Fd$$



Equivalent Couples

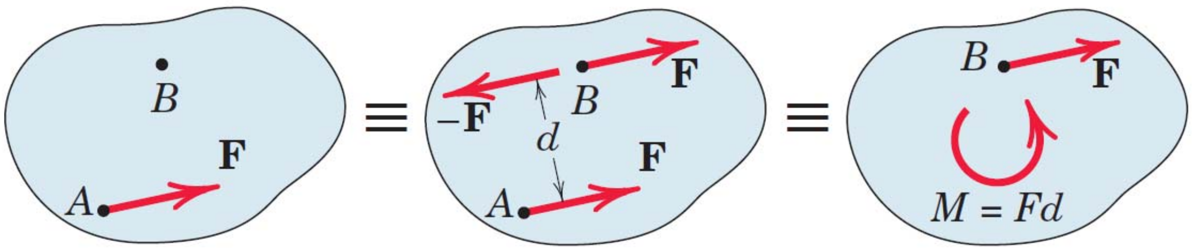
Changing the values of F and d does not change a given couple as long as the product Fd remains the same. Likewise, a couple is not affected if the forces act in a different but parallel plane. The figure below shows four different configurations of the same couple M.



Force–Couple Systems

The effect of a force acting on a body is the tendency to push or pull the body in the direction of the force, and to rotate the body about any fixed axis which does not intersect the line of the force. We can represent this dual effect more easily by replacing the given force by an equal parallel force and a couple to compensate for the change in the moment of the force.

The replacement of a force by a force and a couple is illustrated in the figure below..



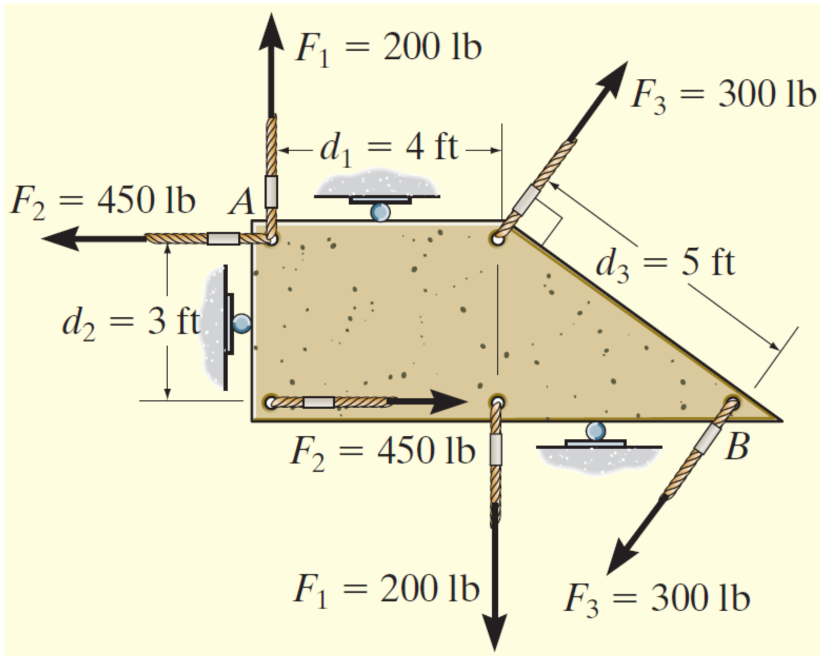
The given force **F** acting at point *A* is replaced by an equal force **F** at some point *B* and the counterclockwise couple $M = Fd$. Thus, we have replaced the original force at *A* by the same force acting at a different point *B* and a couple, without altering the external effects of the original force on the body. The combination of the force and couple in the right-hand part of Fig. is referred to as a ***force–couple system*.** By reversing this process, we can combine a given couple and a force which lies in the plane of the couple (normal to the couple vector) to produce a single, equivalent force.

Important Points:

1. A couple moment is produced by two noncollinear forces that are equal in magnitude but opposite in direction. Its effect is to produce pure rotation, or tendency for rotation in a specified direction.

2. The moment of the two couple forces can be determined about any point. For convenience, this point is often chosen on the line of action of one of the forces in order to eliminate the moment of this force about the point.

Example: Determine the resultant couple moment of the three couples acting on the plate in the figure.



Solution:

As shown the perpendicular distances between each pair of couple forces are

$d1 = 4$ ft, $d2 = 3$ ft, and $d3 = 5$ ft.

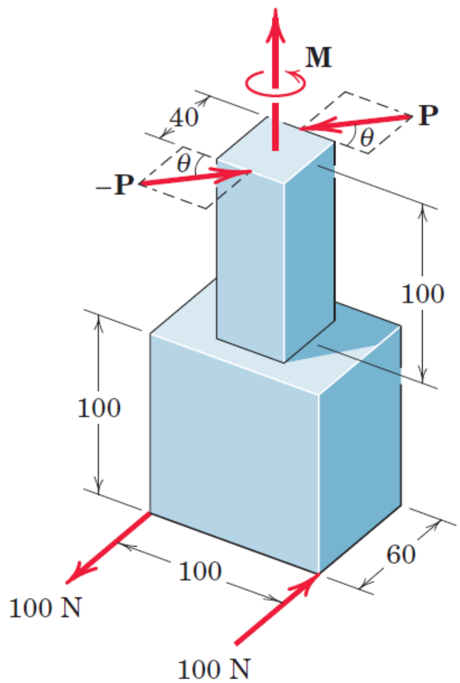
Considering counterclockwise couple moments as positive, we have

$$\curvearrowleft + M_R = \Sigma M; \quad M_R = -F1d1 + F2d2 - F3d3$$

$$= -(200 \text{ lb})(4 \text{ ft}) + (450 \text{ lb})(3 \text{ ft}) - (300 \text{ lb})(5 \text{ ft})$$

$$= -950 \text{ lb.ft} = 950 \text{ lb.ft} \curvearrowright$$

Example: The rigid structural member is subjected to a couple consisting of the two 100-N forces. Replace this couple by an equivalent couple consisting of the two forces **P** and _**P**, each of which has a magnitude of 400 N. Determine the proper angle.



Dimensions in millimeters

Solution:

The original couple is counterclockwise when the plane of the forces is viewed from above, and its magnitude is

[$M = Fd$] $M = 100(0.1) = 10$ N_m

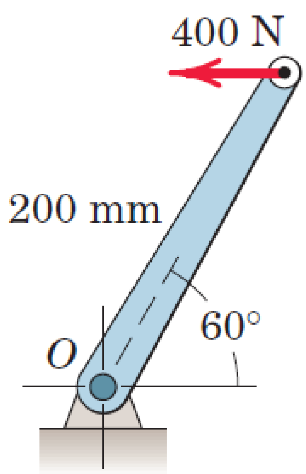
The forces **P** and _**P** produce a counterclockwise couple

$M = 400(0.040) \cos \theta$

$10 = (400)(0.040) \cos \theta$

$\cos^{-1}(\frac{10}{16}) = 51.3°$

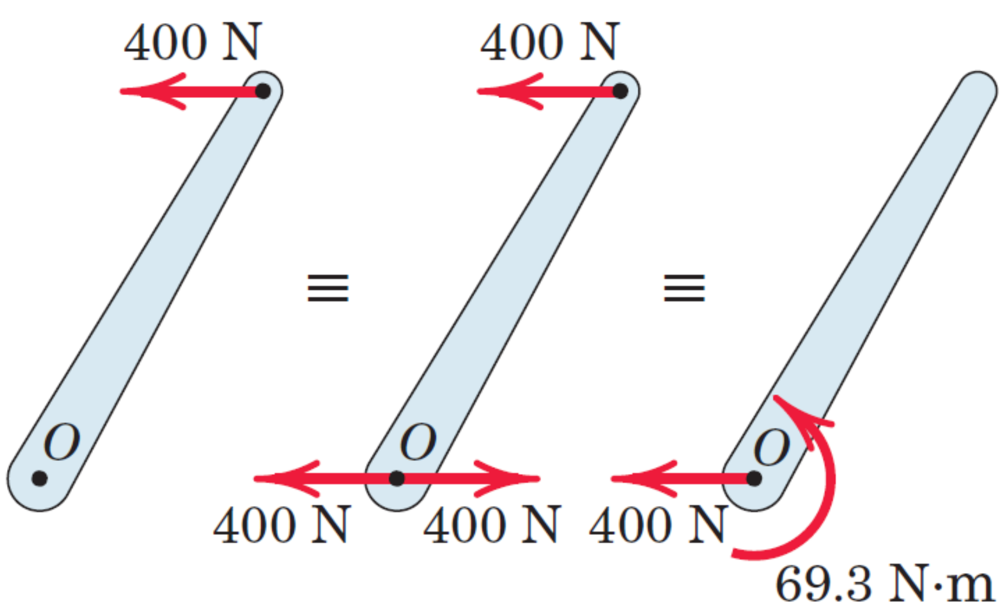
Example: Replace the horizontal 400-N force acting on the lever by a force couple system consisting of a force at O and a couple.



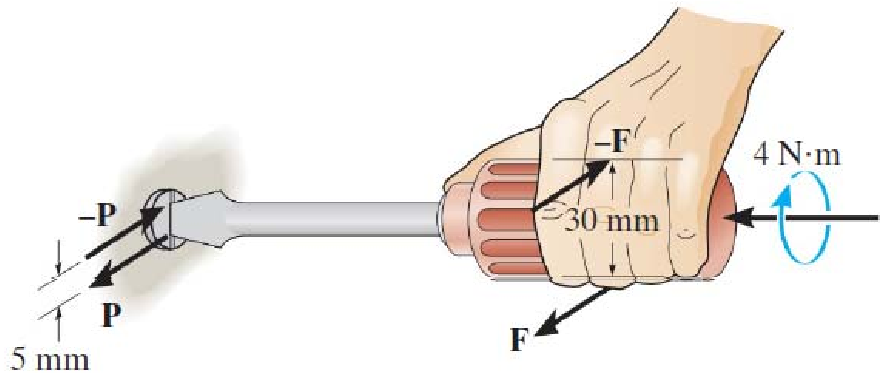
Solution:
We apply two equal and opposite 400-N forces at O and identify the counterclockwise couple

[*M* _ *Fd*] *M* = 400(0.200 sin 60°) = 69.3 N_m

Thus, the original force is equivalent to the 400-N force at O and the couple as shown in the third of the three equivalent figures.



Example: A twist of 4 N . m is applied to the handle of the screwdriver. Resolve this couple moment into a pair of couple forces F exerted on the handle and P exerted on the blade.



Solution:

For the handle

$M_C = \Sigma M_x$; $F(0.03) = 4$

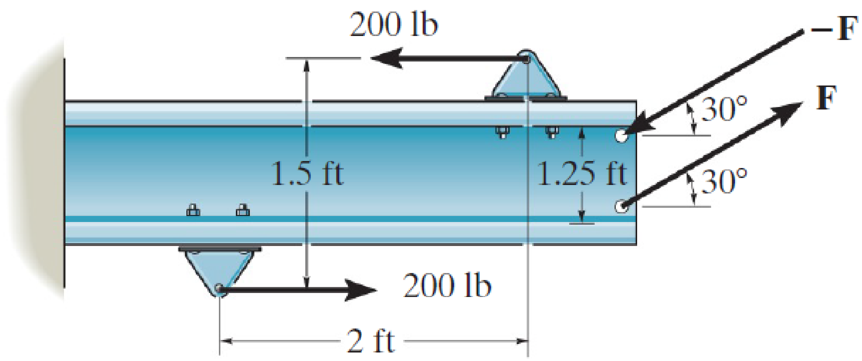
$F = 133$ N

For the blade

$M_C = \Sigma M_x$; $P(0.005) = 4$

$P = 800$ N

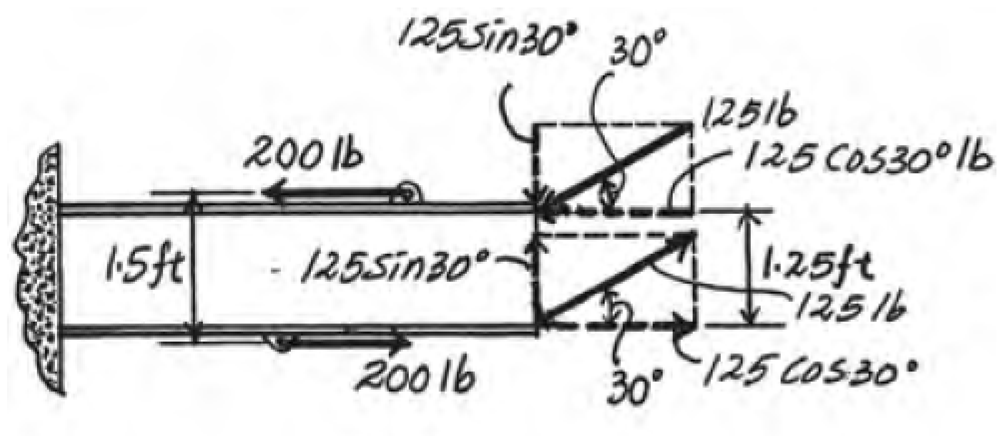
Example: Two couples act on the beam. If F = 125 lb , determine the resultant couple moment.



Solution:

$\circlearrowleft + (M_R)_C = 200(1.5) + 125 \cos 30° (1.25)$

$= 435.32 \text{ lb.ft} = 435 \text{ lb.ft} \circlearrowleft$



Equilibrium

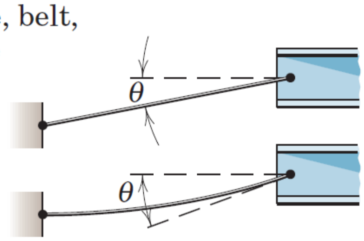
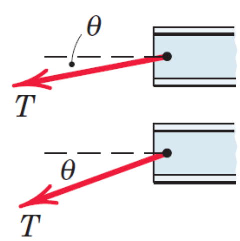
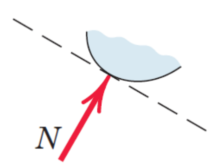
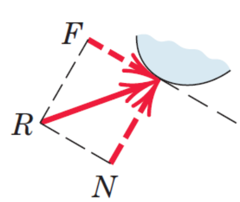
When a body is in equilibrium, the resultant of *all* forces acting on it is zero. Thus, the resultant force **R** and the resultant couple **M** are both zero, and we have the equilibrium equations

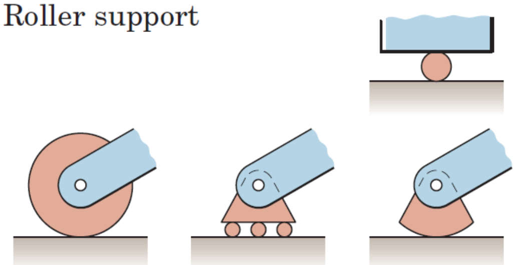
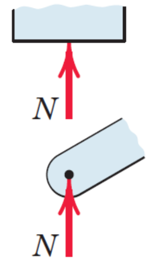
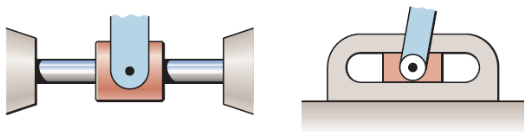
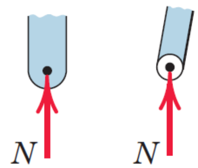
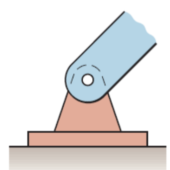
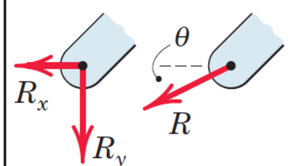
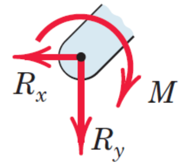
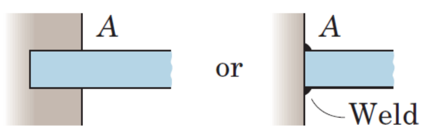
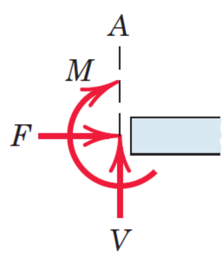
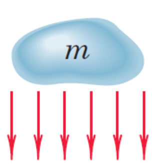
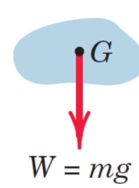
$$R = \Sigma F = 0 \qquad M = \Sigma M = 0$$

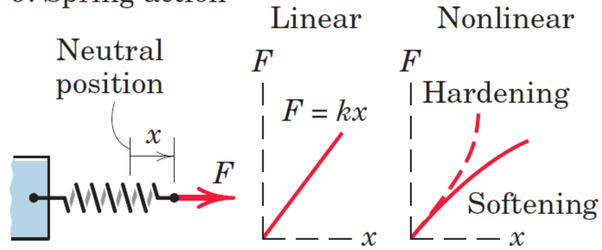
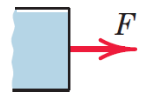
All physical bodies are three-dimensional, but we can treat many of them as two-dimensional when the forces to which they are subjected act in a single plane or can be projected onto a single plane. When this simplification is not possible, the problem must be treated as three-dimensional.

Equilibrium in Two Dimensions:

Before we apply the equation, we must define, analyzed and represent clearly and completely *all* forces acting *on* the body. Once we decide which body or combination of bodies to analyze, we then treat this body or combination as a single body *isolated* from all surrounding bodies. This isolation is accomplished by means of the ***free-body diagram***. Figure bellow shows the common types of force application on mechanical systems for analysis in two dimensions.

MODELING THE ACTION OF FORCES IN TWO-DIMENSIONAL ANALYSIS	
Type of Contact and Force Origin	**Action on Body to Be Isolated**
1. Flexible cable, belt, chain, or rope Weight of cable negligible Weight of cable not negligible 	 Force exerted by a flexible cable is always a tension away from the body in the direction of the cable.
2. Smooth surfaces 	 Contact force is compressive and is normal to the surface.
3. Rough surfaces 	 Rough surfaces are capable of supporting a tangential compo-nent F (frictional force) as well as a normal component N of the resultant

4. Roller support 	 Roller, rocker, or ball support transmits a compressive force normal to the supporting surface.
5. Freely sliding guide 	 Collar or slider free to move along smooth guides; can support force normal to guide only.
6. Pin connection 	Pin free to turn  Pin not free to turn  A freely hinged pin connection is capable of supporting a force in any direction in the plane normal to the pin axis. We may either show two components R_x and R_y or a magnitude R and direction θ. A pin not free to turn also supports a couple M.
7. Built-in or fixed support 	 A built-in or fixed support is capable of supporting an axial force F, a transverse force V (shear force), and a couple M (bending moment) to prevent rotation.
8. Gravitational attraction 	 The resultant of gravitational attraction on all elements of a body of mass m is the weight $W = mg$ and acts toward the center of the earth through the center mass G.

9. Spring action 	 Spring force is tensile if spring is stretched and compressive if compressed. For a linearly elastic spring the stiffness k is the force required to deform the spring a unit distance.

Typical examples of actual supports

The cable exerts a force on the bracket in the direction of the cable.	
The rocker support for this bridge girder allows horizontal movement so the bridge is free to expand and contract due to a change in temperature.	
Typical pin support for a beam.	
This concrete girder rests on the ledge that is assumed to act as a smooth contacting surface.	
The floor beams of this building are welded together and thus form fixed connections.	

Notes:

1. The force exerted *by* the cable *on* the body to which it is attached is always *away* from the body.

2. When the smooth surfaces of two bodies are in contact, the force exerted by one on the other is *normal* to the tangent to the surfaces and is compressive.

3. Example 3 when mating surfaces of contacting bodies are rough the force of contact is not necessarily normal to the tangent to the surfaces, but may be resolved into a *tangential* or *frictional component F* and a *normal component N*.

4. Example 4 illustrates a number of forms of mechanical support, which effectively eliminate tangential friction forces. In these cases, the net reaction is normal to the supporting surface.

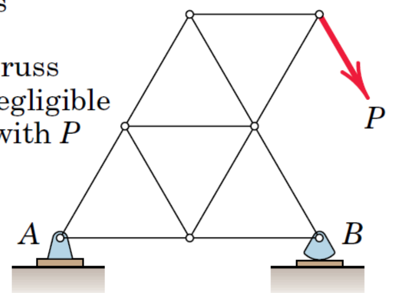
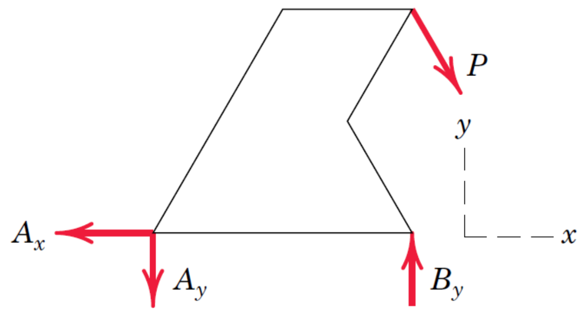
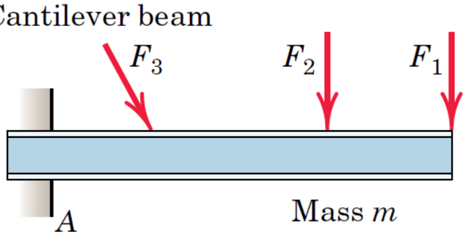
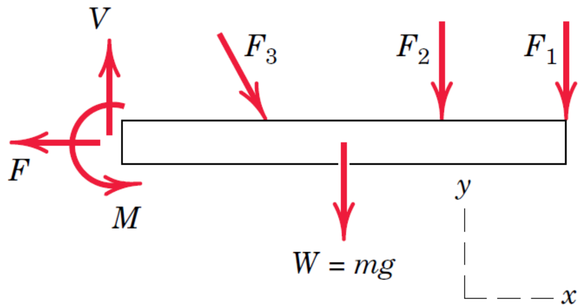
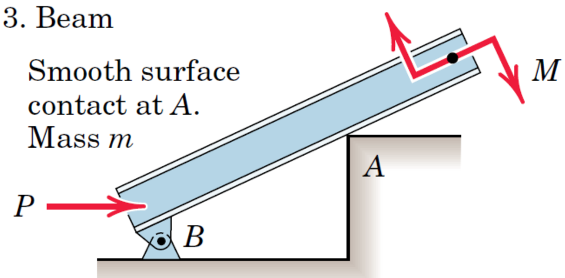
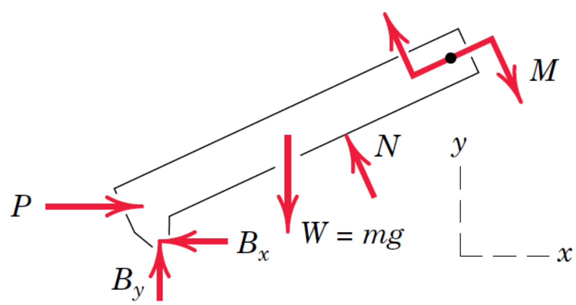
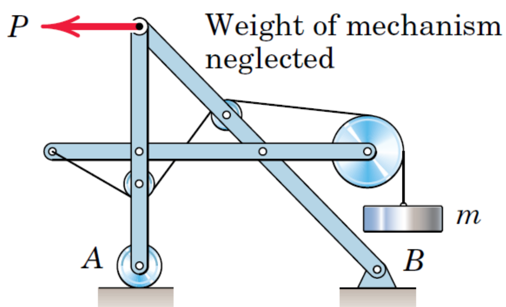
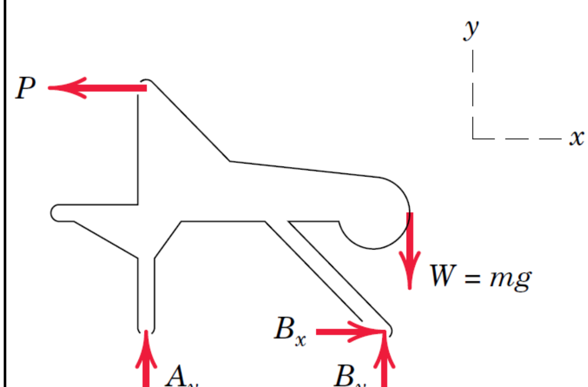
5. Example 5 shows the action of a smooth guide on the body it supports. There cannot be any resistance parallel to the guide.

6. Example 6 illustrates the action of a pin connection. Such a connection can support force in any direction normal to the axis of the pin. We usually represent this action in terms of two rectangular components. If the joint is free to turn about the pin, the connection can support only the force *R*. If the joint is not free to turn, the connection can also support a resisting couple *M*.

7. Example 7 shows the resultants of the rather complex distribution of force over the cross section of a slender bar or beam at a build-in or fixed support. The sense of the reactions *F* and *V* and the bending couple *M* in a given problem depends, of course, on how the member is loaded.

Examples of Free-Body Diagrams

The figures below give four examples of mechanisms and structures together with their correct free-body diagrams. Dimensions and magnitudes are omitted for clarity. In each case, we treat the entire system as a single body, so that the internal forces are not shown.

SAMPLE FREE-BODY DIAGRAMS	
Mechanical System	**Free-Body Diagram of Isolated Body**
1. Plane truss Weight of truss assumed negligible compared with P 	
2. Cantilever beam 	
3. Beam Smooth surface contact at A. Mass m 	
4. Rigid system of interconnected bodies analyzed as a single unit Weight of mechanism neglected 	

EQUILIBRIUM CONDITIONS

Previously, we defined equilibrium as the condition in which the resultant of all forces and moments acting on a body is zero. Stated in another way, a body is in equilibrium if all forces and moments applied to it are in balance.

$$\Sigma Fx=0 \qquad \Sigma Fy=0 \qquad \Sigma M=0$$

Category of Free-Body Diagram

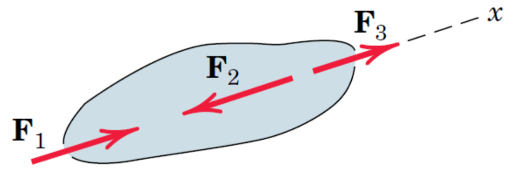
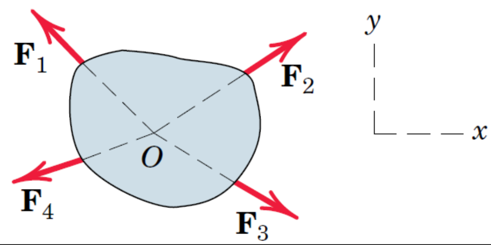
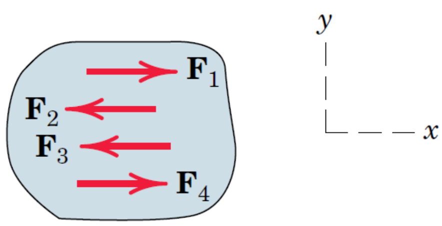
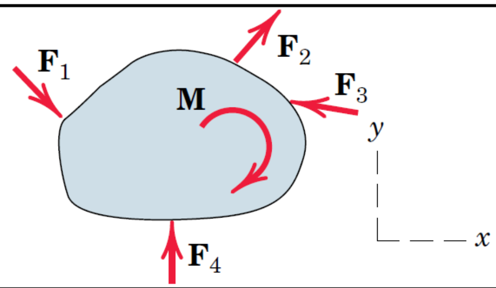
The categories of force systems acting on bodies in two-dimensional equilibrium are summarized in the figure below and are explained further as follows.

- **Category 1** equilibrium of collinear forces clearly requires only the one force equation in the direction of the forces (x-direction), since all other equations are automatically satisfied.

- **Category 2** equilibrium of forces which lie in a plane (x-y plane) and are concurrent at a point O, requires the two force equations only, since the moment sum about O, that is, about a z-axis through O, is necessarily zero.

- **Category 3** equilibrium of parallel forces in a plane requires the one force equation in the direction of the forces (x-direction) and one moment equation about an axis (z-axis) normal to the plane of the forces.

- **Category 4** equilibrium of a general system of forces in a plane (*x*-*y*) requires the two force equations in the plane and one moment equation about an axis (*z*-axis) normal to the plane.

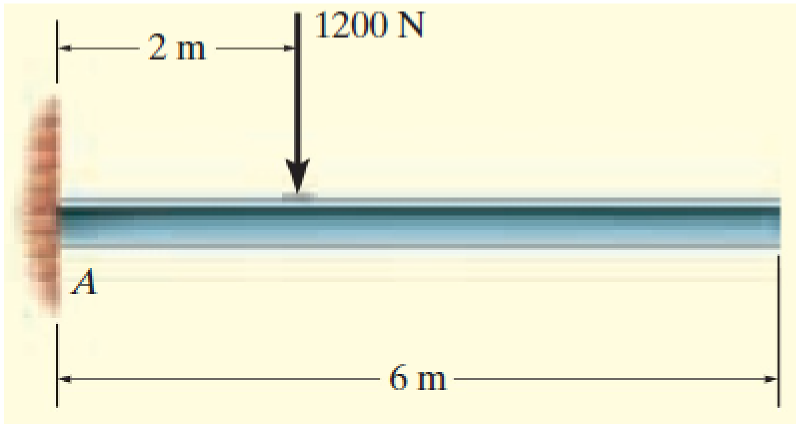
CATEGORIES OF EQUILIBRIUM IN TWO DIMENSIONS		
Force System	Free-Body Diagram	Independent Equations
1. Collinear		$\Sigma F_x = 0$
2. Concurrent at a point		$\Sigma F_x = 0$ $\Sigma F_y = 0$
3. Parallel		$\Sigma F_x = 0$ $\Sigma M_z = 0$
4. General		$\Sigma F_x = 0$ $\Sigma M_z = 0$ $\Sigma F_y = 0$

To construct a free-body diagram for a rigid body or any group of bodies considered as a single system, the following steps should be performed:

1. Draw Outlined Shape.
2. Show All Forces and Couple Moments.
3. Identify Each Loading and Give Dimensions.

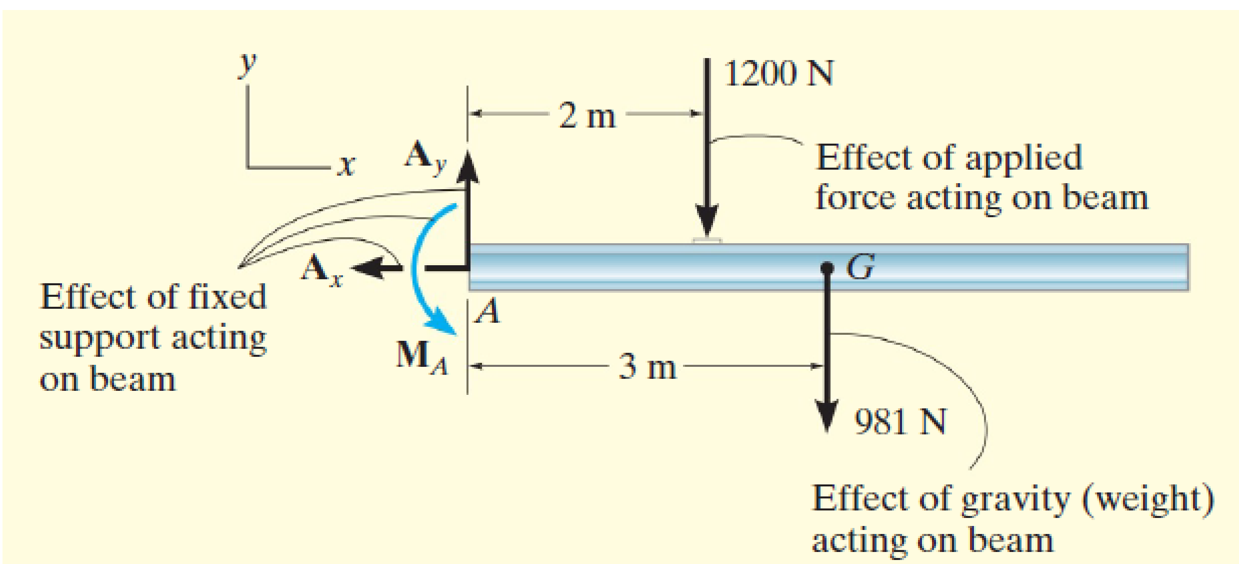
Note: the weight W of the body locates at the center of gravity.

Example: Draw the free-body diagram of the uniform beam shown in the figure below. The beam has a mass of 100 kg.

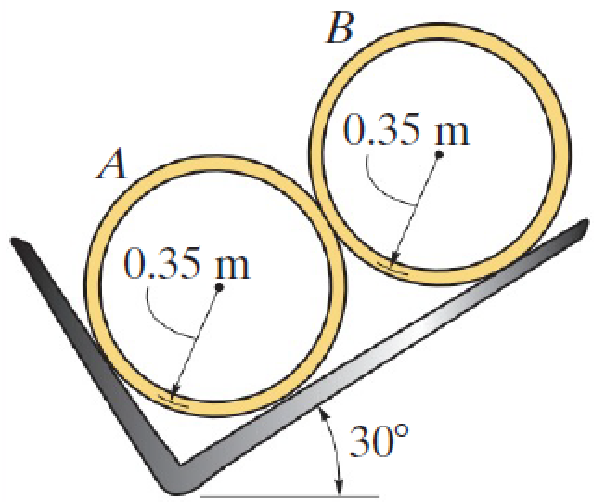


Solution:

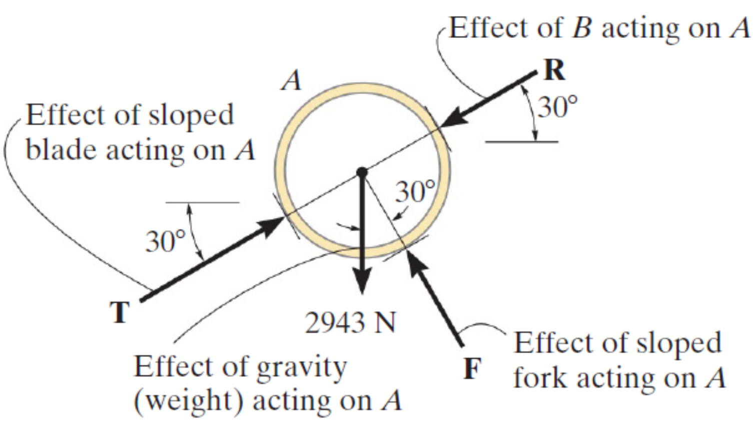
The free-body diagram of the beam is shown in the figure below. Since the support at A is fixed, the wall exerts three reactions on the beam, denoted as Ax, Ay, and MA. The magnitudes of these reactions are unknown, and their sense has been assumed. The weight of the beam, W = 100(9.81) N = 981 N, acts through the beam's center of gravity G, which is 3 m from A since the beam is uniform.

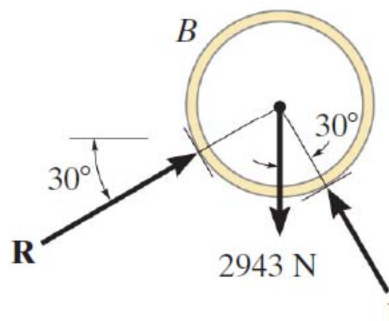


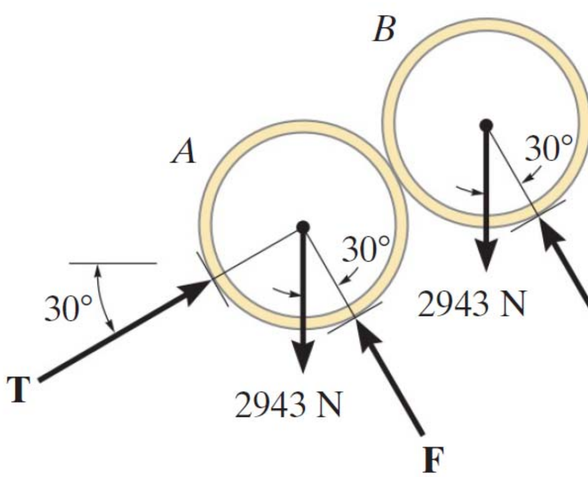
Example: Two smooth pipes, each having a mass of 300 kg, are supported by the forked tines of the tractor. Draw the free-body diagrams for each pipe and both pipes together.



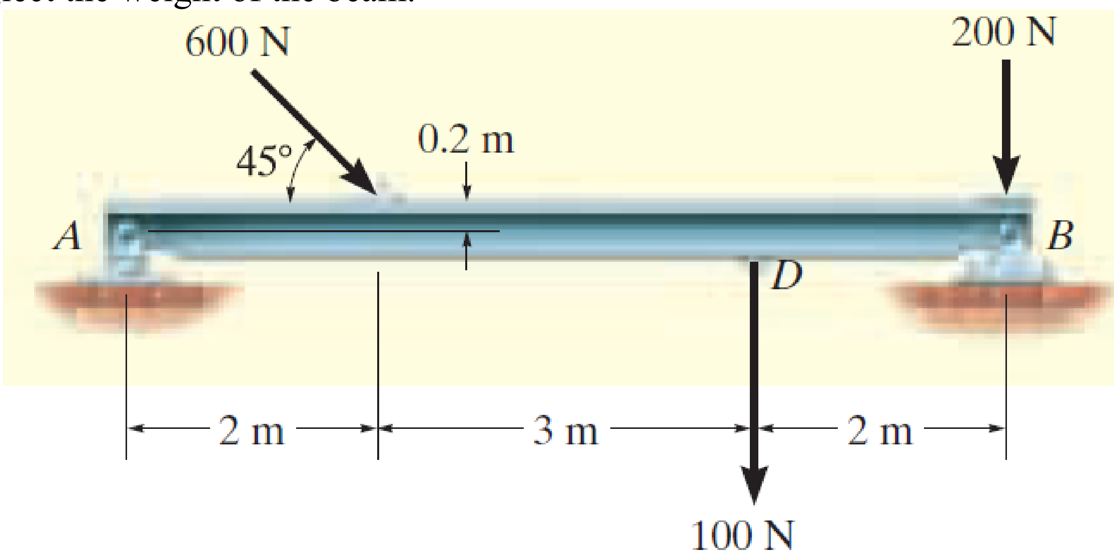
Solution:





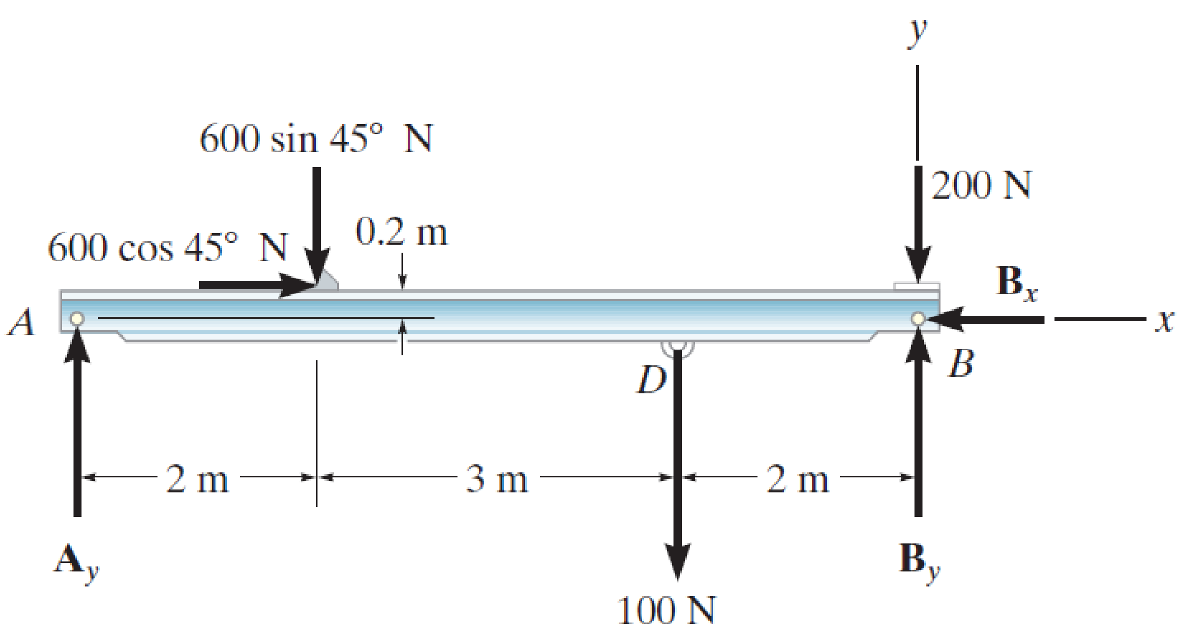


Example: Determine the horizontal and vertical components of reaction on the beam caused by the pin at B and the roller at A as shown in the figure below. Neglect the weight of the beam.



Solution:

Free-Body Diagram. The supports are removed, and the free-body diagram of the beam is shown in the figure below. For simplicity, the 600-N force is represented by its x and y components.



Equations of Equilibrium. Summing forces in the x direction yields

$$\xrightarrow{+} \Sigma F_x = 0; \qquad 600 \cos 45^\circ \text{ N} - B_x = 0$$

$$B_x = 424 \text{ N} \qquad \textit{Ans.}$$

A direct solution for $\mathbf{A}_y$ can be obtained by applying the moment equation $\Sigma M_B = 0$ about point B.

$$\circlearrowleft + \Sigma M_B = 0; \qquad 100 \text{ N}(2 \text{ m}) + (600 \sin 45^\circ \text{ N})(5 \text{ m})$$

$$- (600 \cos 45^\circ \text{ N})(0.2 \text{ m}) - A_y(7 \text{ m}) = 0$$

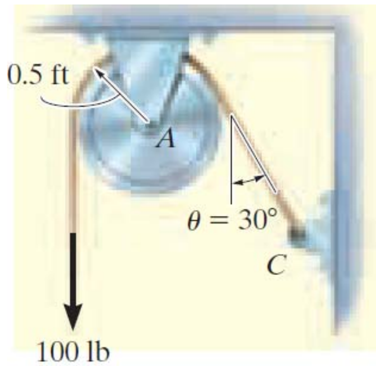
$$A_y = 319 \text{ N} \qquad \textit{Ans.}$$

Summing forces in the y direction, using this result, gives

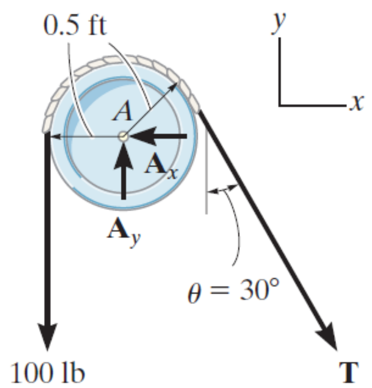
$$+\uparrow \Sigma F_y = 0; \qquad 319 \text{ N} - 600 \sin 45^\circ \text{ N} - 100 \text{ N} - 200 \text{ N} + B_y = 0$$

$$B_y = 405 \text{ N} \qquad \textit{Ans.}$$

Example: The cord shown in the figure below supports a force of 100 lb and wraps over the frictionless pulley. Determine the tension in the cord at C and the horizontal and vertical components of reaction at pin A.



Solution:



$$\circlearrowleft + \Sigma M_A = 0; \qquad 100 \text{ lb } (0.5 \text{ ft}) - T(0.5 \text{ ft}) = 0$$

$$T = 100 \text{ lb}$$

Using this result,

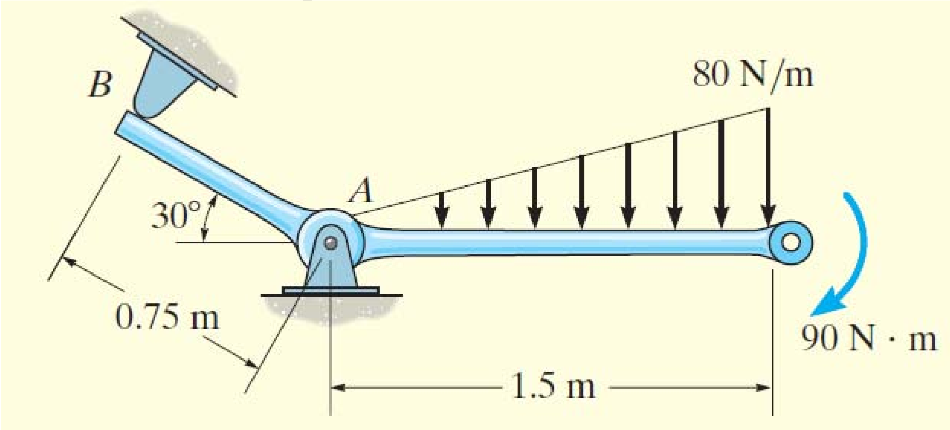
$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad -A_x + 100 \sin 30° \text{ lb} = 0$$

$$A_x = 50.0 \text{ lb}$$

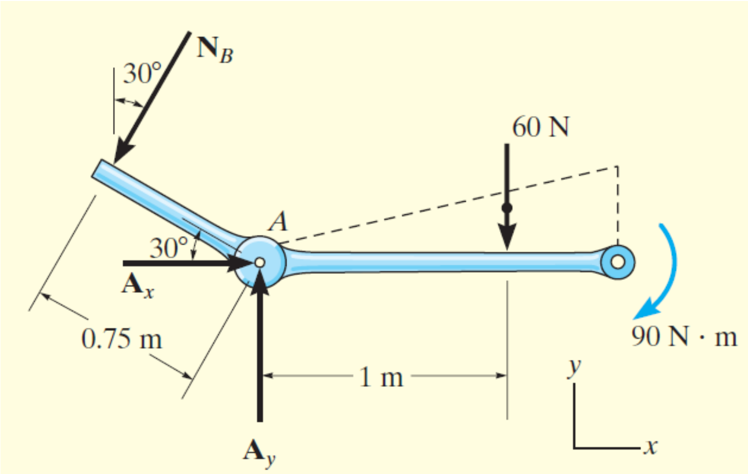
$$+\uparrow \Sigma F_y = 0; \qquad A_y - 100 \text{ lb} - 100 \cos 30° \text{ lb} = 0$$

$$A_y = 187 \text{ lb}$$

Example: The member shown in the figure below. Below is pin connected at A and rests against a smooth support at B. Determine the horizontal and vertical components of reaction at the pin A.



Solution:



As shown in the figure above, the supports are removed and the reaction NB is perpendicular to the member at B. Also, horizontal and vertical components of reaction are represented at A. The resultant of the distributed loading is:
1/2 (1.5 m) (80 N/m) = 60 N. It acts through the centroid of the triangle, 1 m from A as shown.

$$\circlearrowleft + \Sigma M_A = 0; \quad -90\text{ N}\cdot\text{m} - 60\text{ N}(1\text{ m}) + N_B(0.75\text{ m}) = 0$$

$$N_B = 200\text{ N}$$

Using this result,

$$\xrightarrow{+} \Sigma F_x = 0; \quad A_x - 200\sin 30^\circ\text{ N} = 0$$

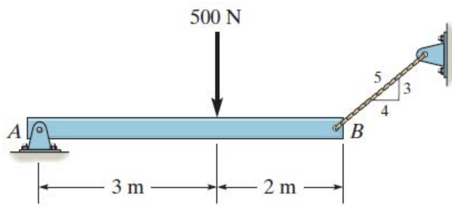
$$A_x = 100\text{ N}$$

$$+\uparrow \Sigma F_y = 0; \quad A_y - 200\cos 30^\circ\text{ N} - 60\text{ N} = 0$$

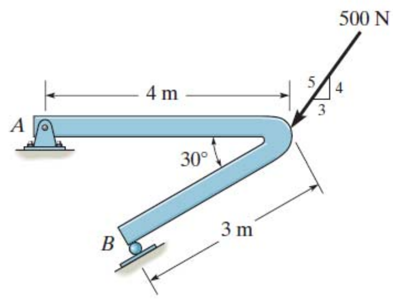
$$A_y = 233\text{ N}$$

Homework:

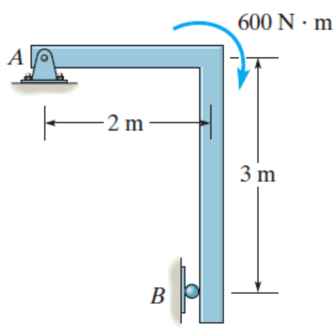
1. Draw the free-body diagram of each object.



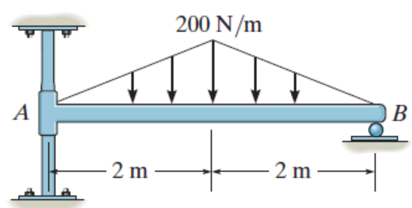
(a)



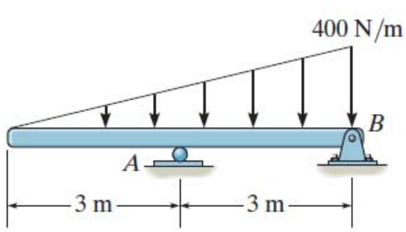
(d)



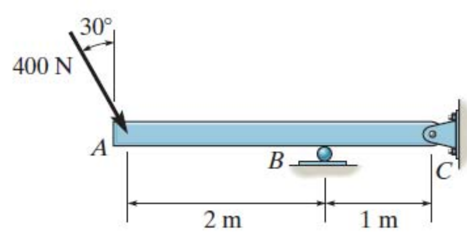
(b)



(e)

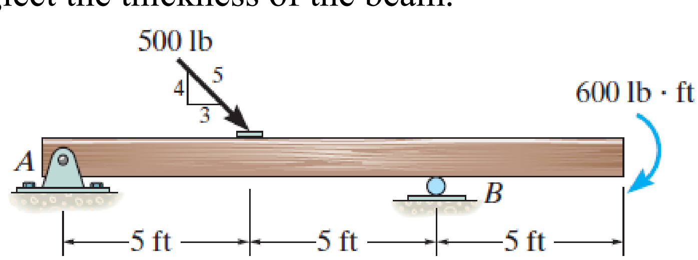


(c)

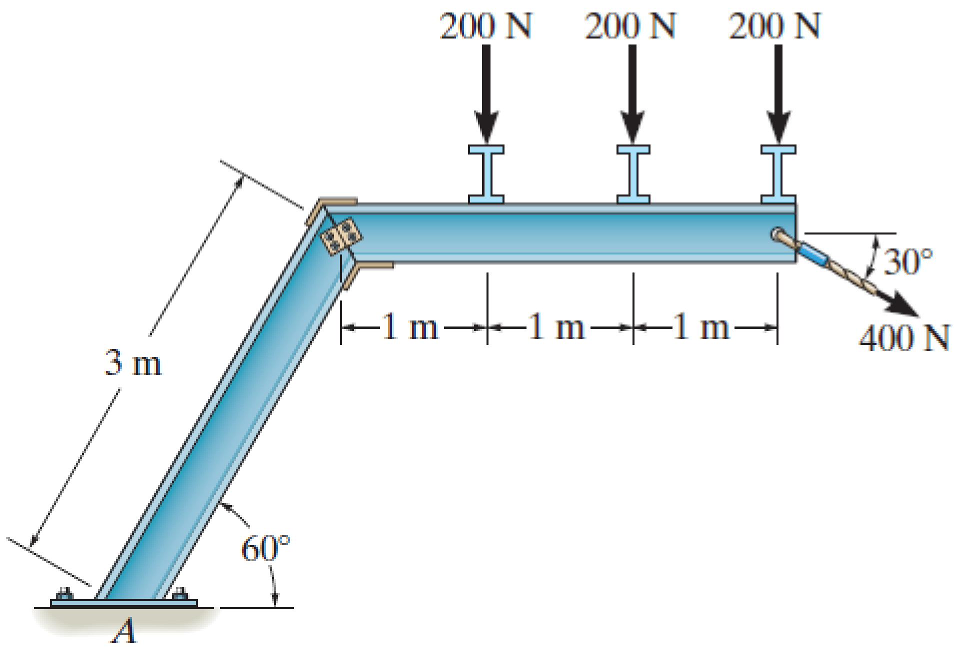


(f)

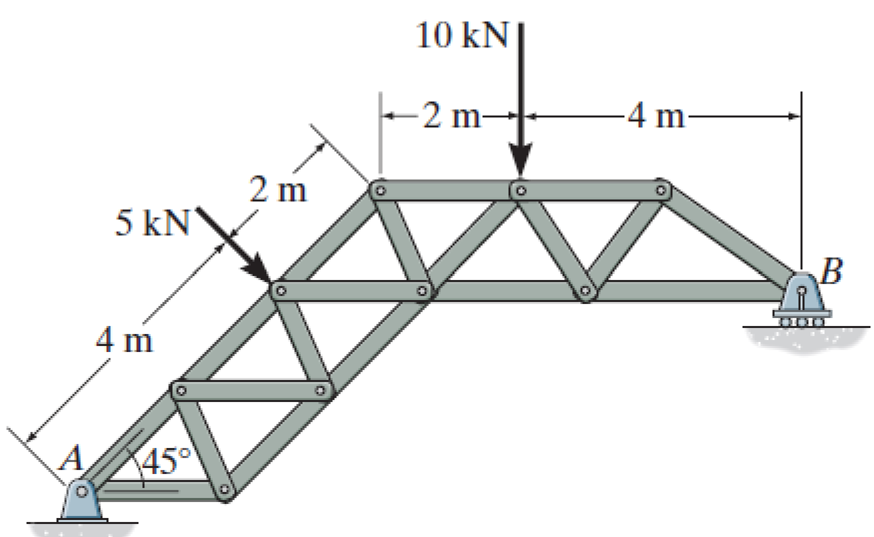
2. Determine the horizontal and vertical components of reaction at the supports. Neglect the thickness of the beam.



3. Determine the components of reaction at the fixed support A. Neglect the thickness of the beam.

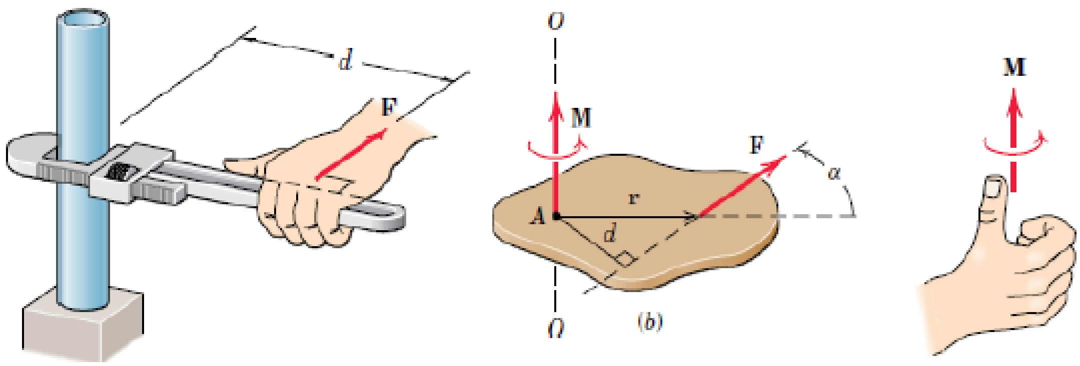


4. The truss is supported by a pin at A and a roller at B. Determine the support reactions.



Moment of Force

In addition to the tendency to move a body in the direction of its application, a force can also tend to rotate a body about an axis. The axis may be any line which neither intersects nor is parallel to the line of action of the force. This rotational tendency is known as the *moment* **M** of the force. Moment is also referred to as *torque*.



The magnitude of the moment or tendency of the force to rotate the body about the axis *O*-*O* perpendicular to the plane of the body is proportional both to the magnitude of the force and to the *moment arm d*, which is the perpendicular distance from the axis to the line of action of the force. Therefore, the magnitude of the moment is defined as

$$M = Fd$$

We represent the moment of **F** about *O*-*O* as a vector pointing in the direction of the thumb, with the fingers curled in the direction of the rotational tendency. Moment directions may be accounted for by using a stated sign convention, such as a plus sign (+) for counterclockwise moments and a minus sign (-) for clockwise moments.

The Cross Product

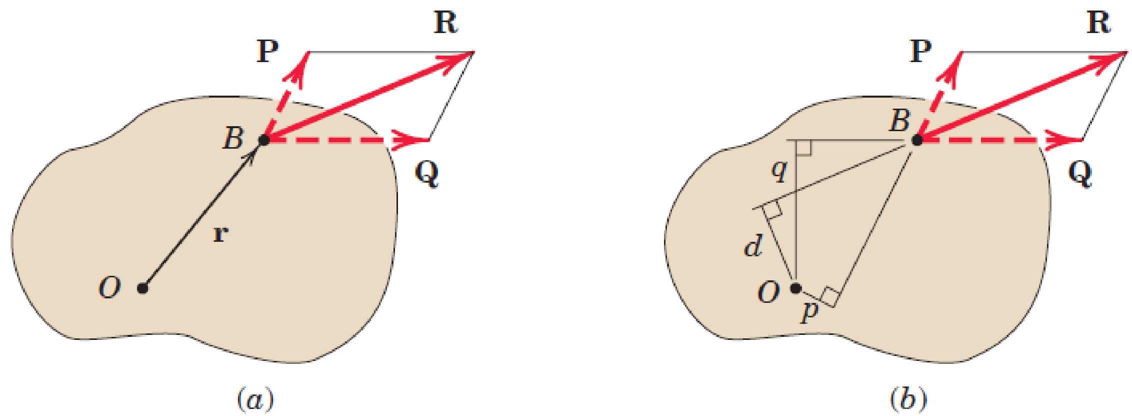
We can calculate the magnitude of the moment by:

$$M = Fr \sin\alpha = Fd$$

Varignon's Theorem (Principle of Moments)

One of the most useful principles of mechanics is *Varignon's theorem*, which states that the moment of a force about any point is equal to the sum of the moments of the components of the force about the same point.

The figure below illustrates the usefulness of Varignon's theorem. The moment of **R** about point O is Rd. However, if **d** is more difficult to determine than **p** and **q,** we can resolve **R** into the components **P** and **Q**, and compute the moment as



Example: For each case illustrated in the figure below, determine the moment of the force about point O.

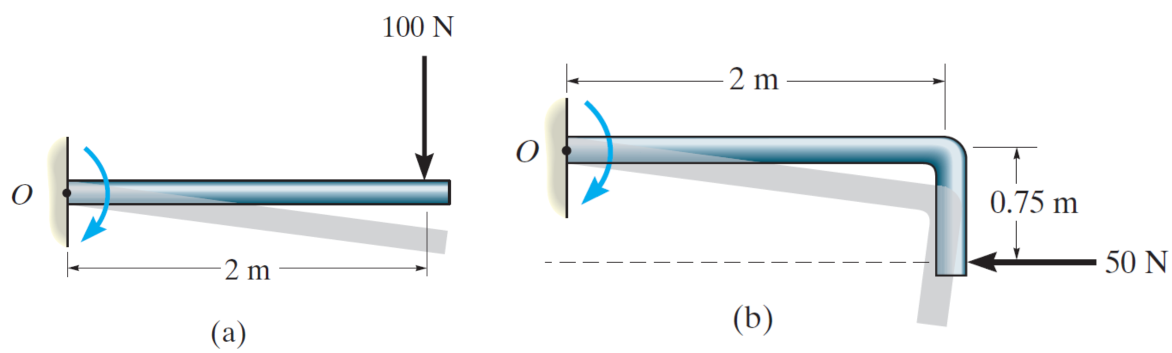
Fig. a *Mo* = (100 N) (2 m) = 200 N.m↷

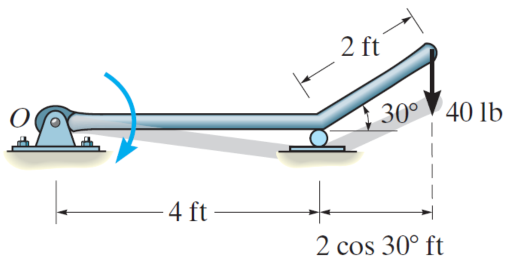
Fig.*b Mo* = (50 N) (0.75 m) = 37.5 N.m↷

Fig. *c Mo* = (40 lb) (4 ft + 2 cos 30- ft) = 229 lb.ft↷

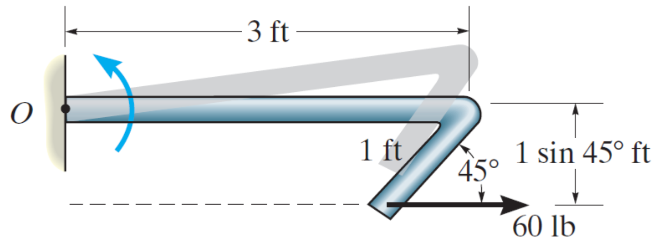
Fig.*d Mo* = (60 lb) (1 sin 45 - ft) = 42.4 lb.ft↶

Fig.*e Mo* = (7 kN) (4 m - 1 m) = 21.0 kN.m↶

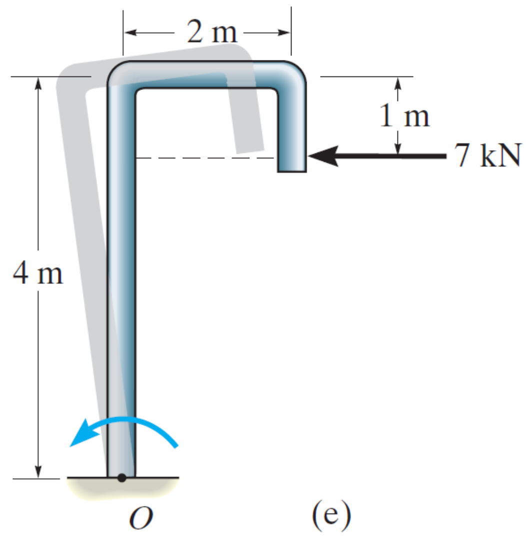




(c)

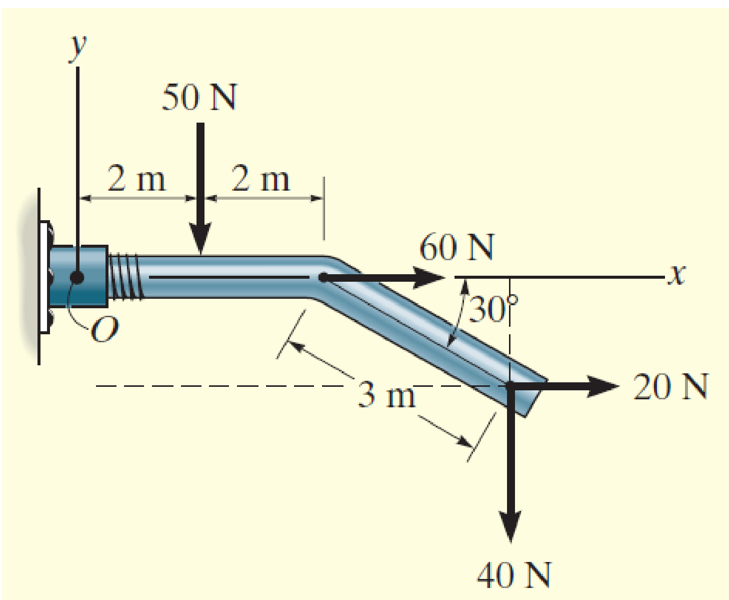


(d)



(e)

Example: Determine the resultant moment of the four forces acting on the rod shown in the figure below about point *O*.



Solution:

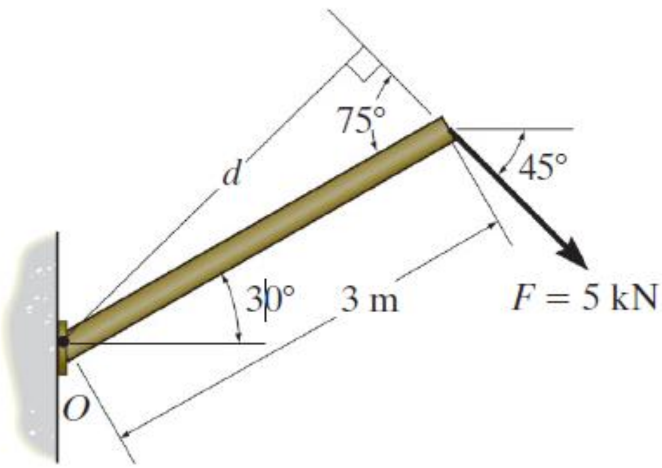
Assuming that positive moments act in the + k direction, i.e., counterclockwise, we have

↺+ $(M_R)_o = \_Fd$

$(M_R)_o$ = -50 N (2 m) + 60 N (0) + 20 N (3 sin 30°m) -40 N (4 m + 3 cos 30°m)

$(M_R)_o$ = -334 N.m = 334 N.m↻

Example: Determine the moment of the force in the figure below about point O.



(a)

Solution:

The moment arm *d* in the figure a *a* can be found from trigonometry.
d = (3 m) sin 75° = 2.898 m
Thus,
$M_O = Fd$ = (5 kN)(2.898 m) = 14.5 kN.m↻

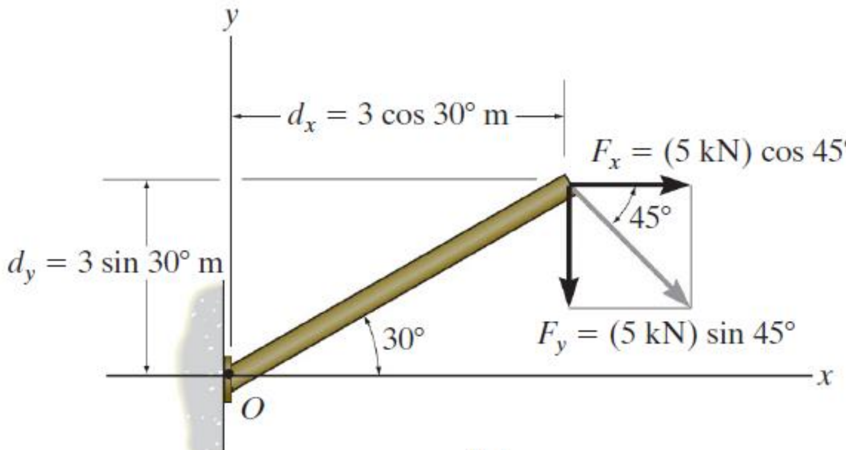
Another Solution:

The *x* and *y* components of the force are indicated in Fig.*b*. Considering counterclockwise moments as positive, and applying the principle of moments, we have:

↺+ $M_O = -F_x d_y - F_y d_x$

= -(5 cos 45° kN)(3 sin 30°m) - (5 sin 45° kN)(3 cos 30° m)

= -14.5 kN.m = 14.5 kN.m↻



(b)

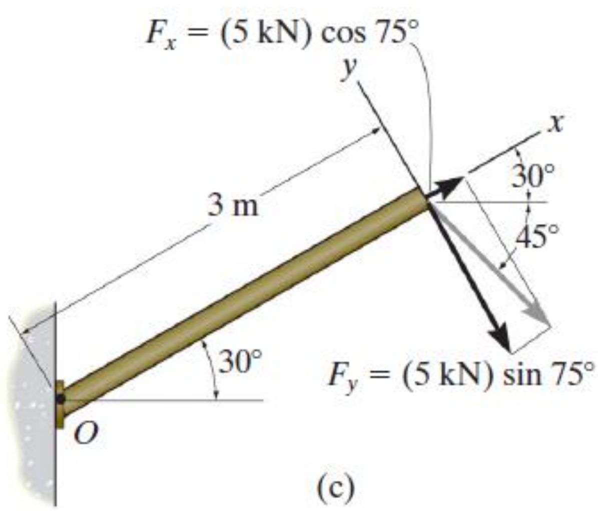
Another Solution:

The *x* and *y* axes can be set parallel and perpendicular to the rod's axis as shown in Fig.*c*. Here **F***x* produces no moment about point *O* since its line of action passes through this point. Therefore,

↻+ $M_O = -F_y d_x$

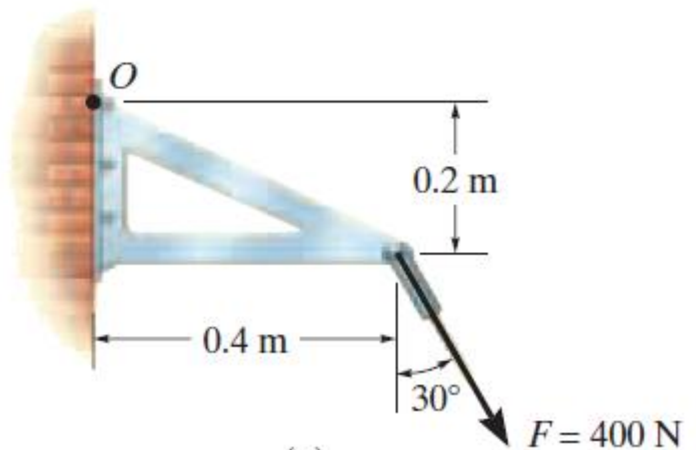
= - (5 sin 75°kN)(3 m)

= -14.5 kN.m = 14.5 kN.m↺



(c)

Example: Force **F** acts at the end of the angle bracket in the figure below. Determine the moment of the force about point *O*.

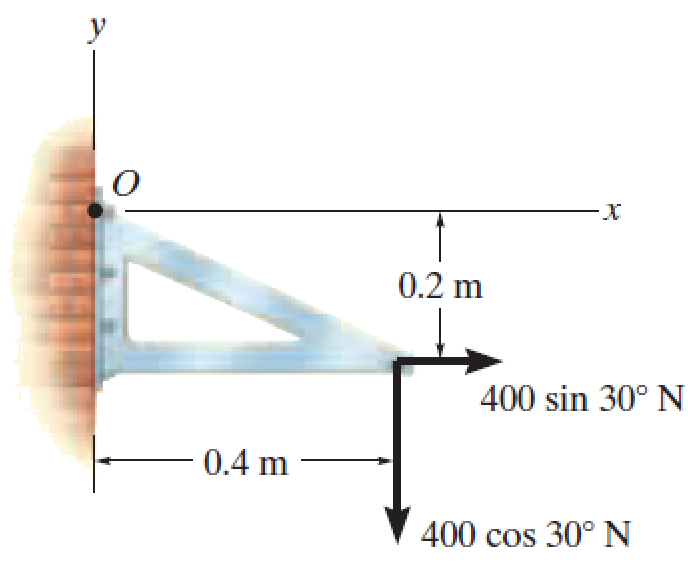


(a)

The force is resolved into its *x* and *y* components, Fig.*b*, then

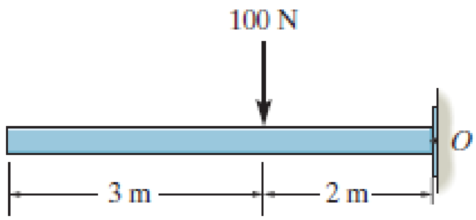
$\curvearrowleft + M_O = 400 \sin 30°\text{N} (0.2 \text{ m}) - 400 \cos 30°\text{N} (0.4 \text{ m})$

$= -98.6 \text{ N.m} = 98.6 \text{ N.m} \curvearrowright$

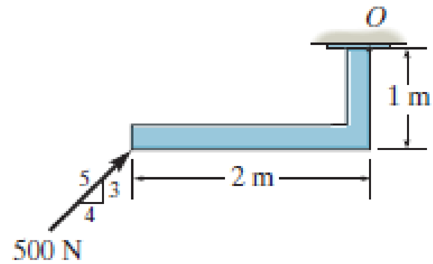


(b)

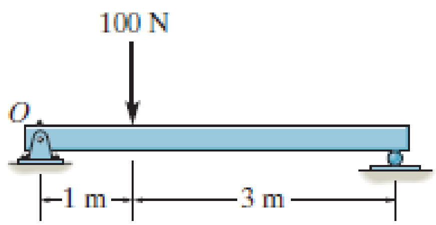
Practice: In each case, determine the moment of the force about point *O*.



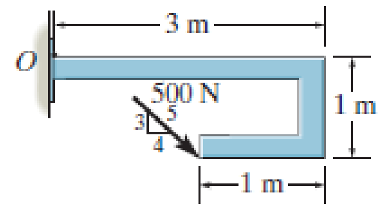
(a)



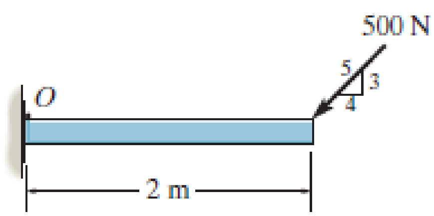
(g)



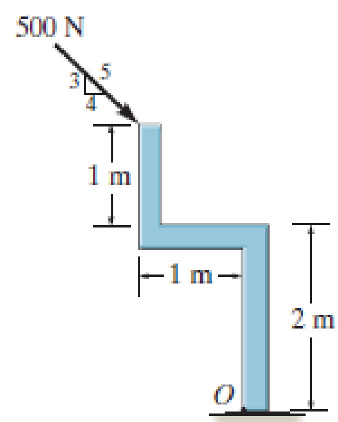
(b)



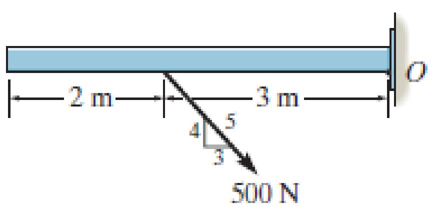
(h)



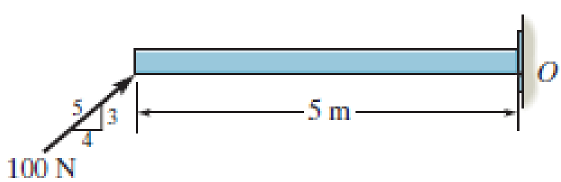
(c)



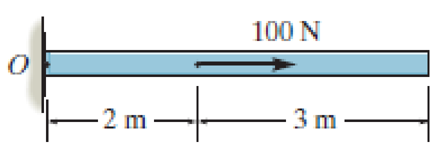
(i)



(d)

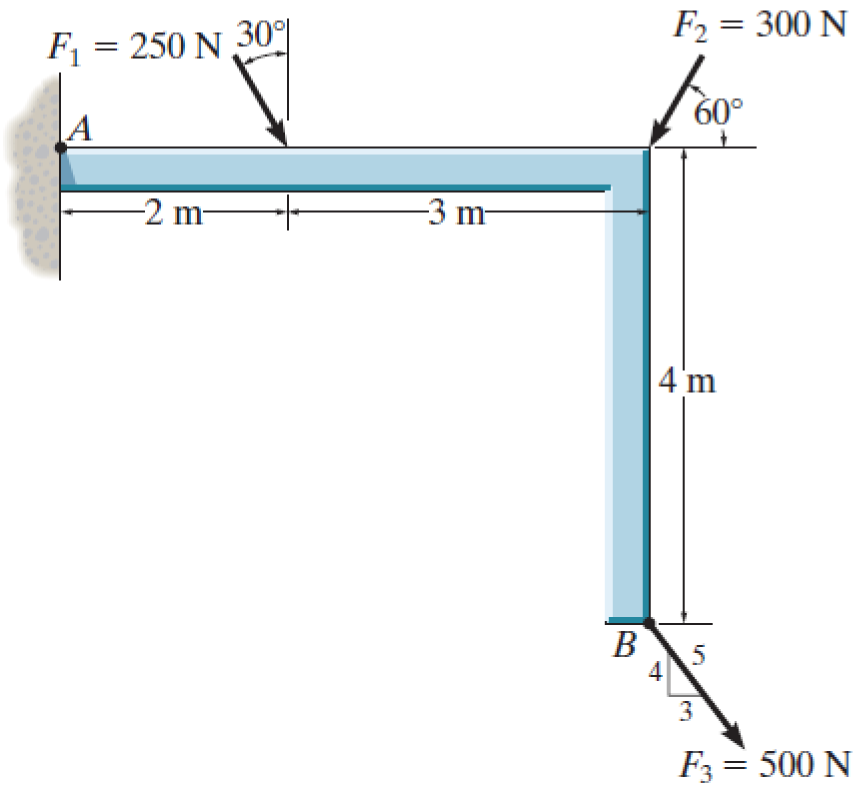


(e)



(f)

Example: Determine the moment of each of the three forces about point *A*.



For **F**1,

↺+M_B = 250 cos 30° (3) - 250 sin 30° (4)

= 149.51 N.m = 150 N.m↺

For **F**2,

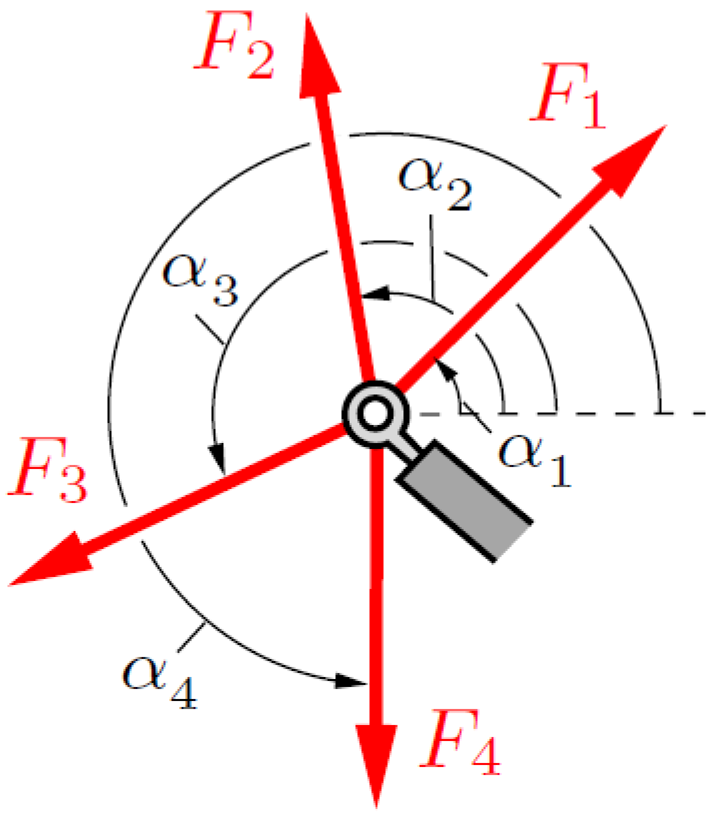
↺+M_B = 300 sin 60° (0) + 300 cos 60° (4)

600 N.m↺

Since the line of action of **F**3 passes through *B*, its moment arm about point *B* is zero. Thus
$M_B = 0$

Force analysis in two dimensions

Quiz: An eyebolt is subjected to four forces (F1 = 12kN, F2 = 8kN, F3 = 18kN, F4 = 4 kN) that act under given angles (α1 = 45◦, α2 = 100◦, α3 = 205◦, α4 = 270◦) with respect to the horizontal as shown in the figure below. Determine the magnitude and direction of the resultant.



Solution:

Rx = F1x + F2x + F3x + F4x

= F1 cos α1 + F2 cos α2 + F3 cos α3 + F4 cos α4

= 12 cos 45° + 8 cos 100° + 18 cos 205° + 4 cos 270°

= −9.22 kN

Ry = F1y + F2y + F3y + F4y

= F1 sin α1+F2 sin α2+F3 sinα3+F4 sin α4 = 4.76 kN

$$\underline{\underline{R}} = \sqrt{R_x^2 + R_y^2} = \sqrt{9.22^2 + 4.76^2} = \underline{\underline{10.4\,\mathrm{kN}}},$$

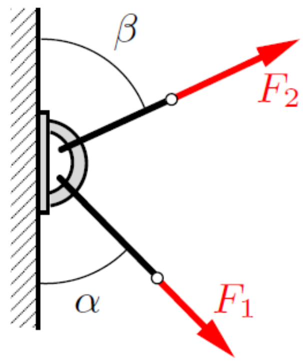
$$\tan\alpha_R = \frac{R_y}{R_x} = -\frac{4.76}{9.22} = -0.52 \quad \rightarrow \quad \underline{\underline{\alpha_R = 152.5^\circ}}.$$

Homework: force analysis in three dimentions and non-rectangular analysis

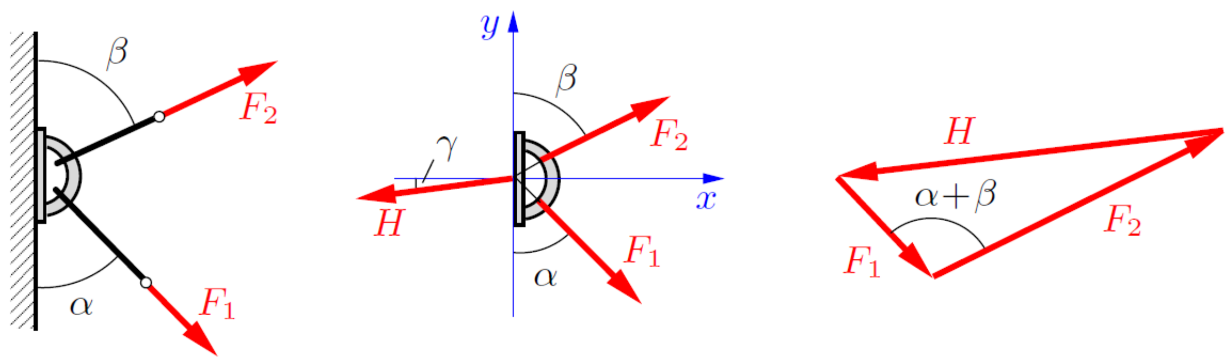
Two cables are attached to an eye as shown in the figure. The directions of the forces F1 and F2 in the cables are given by the angles α and β.

Determine the magnitude of the force H exerted from the wall onto the eye.

F1=8 KN, F2=12KN, α=40°, β=35°



Solution:



The free-body diagram shown in the figure above is drawn as the first step. To this end, the eye is separated from the wall by an imaginary cut. Then all of the forces acting on the eye are drawn into the figure: the two given forces F_1 and F_2 and the force H. These three forces are in equilibrium. The free-body diagram contains two unknown quantities, namely, the magnitude of the force H and the angle γ.

$$H = \sqrt{F_1^2 + F_2^2 - 2F_1F_2\cos(\alpha+\beta)}$$

OR

$$\sum F_{ix} = 0 : \quad F_1 \sin\alpha + F_2 \sin\beta - H\cos\gamma = 0$$

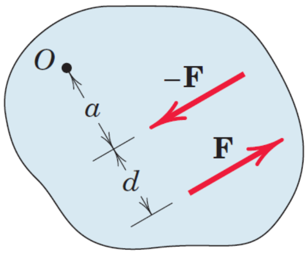
$$\rightarrow \quad H\cos\gamma = F_1 \sin\alpha + F_2 \sin\beta \,,$$

$$\sum F_{iy} = 0 : \quad -F_1 \cos\alpha + F_2 \cos\beta - H\sin\gamma = 0$$

$$\rightarrow \quad H\sin\gamma = -F_1 \cos\alpha + F_2 \cos\beta \,.$$

Moment of Couples

The moment produced by two equal, opposite, and non collinear forces is called a *couple*. Couples have certain unique properties and have important applications in mechanics.

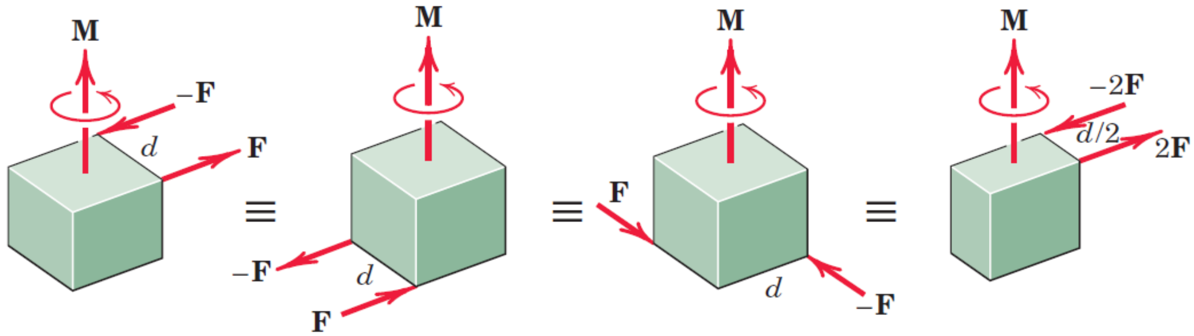


$$M = F(a + d) – Fa$$

$$M = Fd$$

Equivalent Couples

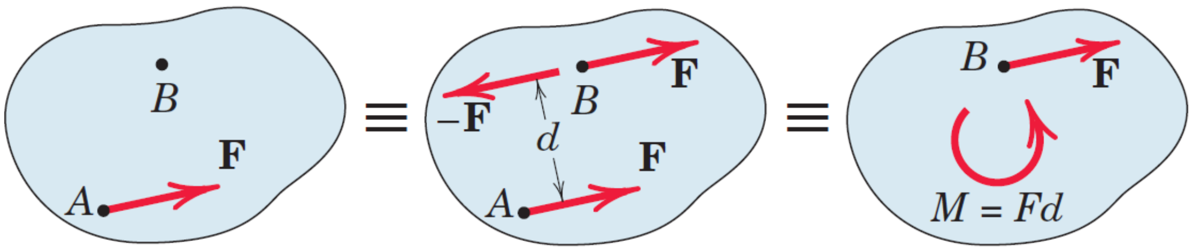
Changing the values of F and d does not change a given couple as long as the product Fd remains the same. Likewise, a couple is not affected if the forces act in a different but parallel plane. The figure below shows four different configurations of the same couple M.



Force–Couple Systems

The effect of a force acting on a body is the tendency to push or pull the body in the direction of the force, and to rotate the body about any fixed axis which does not intersect the line of the force. We can represent this dual effect more easily by replacing the given force by an equal parallel force and a couple to compensate for the change in the moment of the force.

The replacement of a force by a force and a couple is illustrated in the figure below..



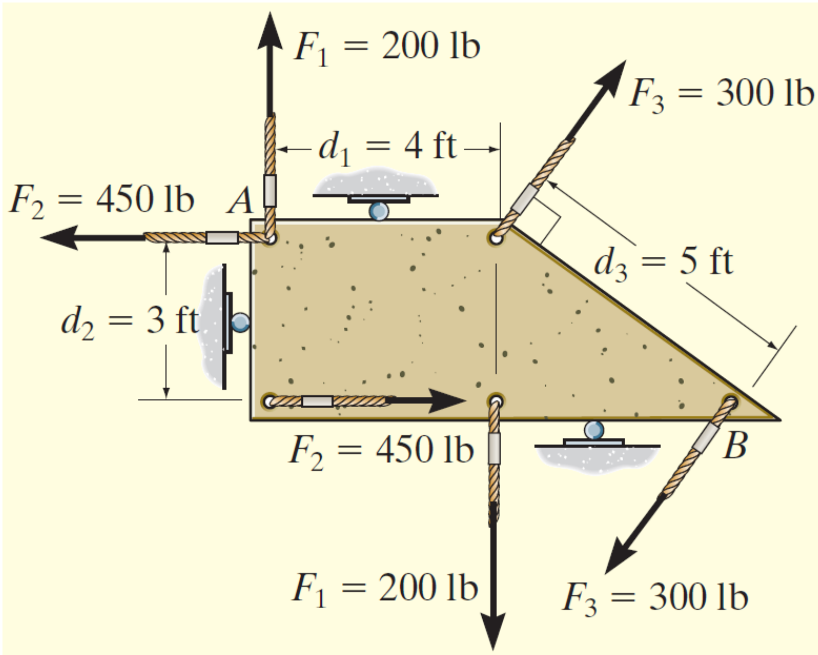
The given force **F** acting at point *A* is replaced by an equal force **F** at some point *B* and the counterclockwise couple $M = Fd$. Thus, we have replaced the original force at *A* by the same force acting at a different point *B* and a couple, without altering the external effects of the original force on the body. The combination of the force and couple in the right-hand part of Fig. is referred to as a ***force–couple system*.** By reversing this process, we can combine a given couple and a force which lies in the plane of the couple (normal to the couple vector) to produce a single, equivalent force.

Important Points:

1. A couple moment is produced by two noncollinear forces that are equal in magnitude but opposite in direction. Its effect is to produce pure rotation, or tendency for rotation in a specified direction.

2. The moment of the two couple forces can be determined about any point. For convenience, this point is often chosen on the line of action of one of the forces in order to eliminate the moment of this force about the point.

Example: Determine the resultant couple moment of the three couples acting on the plate in the figure.



Solution:

As shown the perpendicular distances between each pair of couple forces are

d1 = 4 ft, d2 = 3 ft, and d3 = 5 ft.

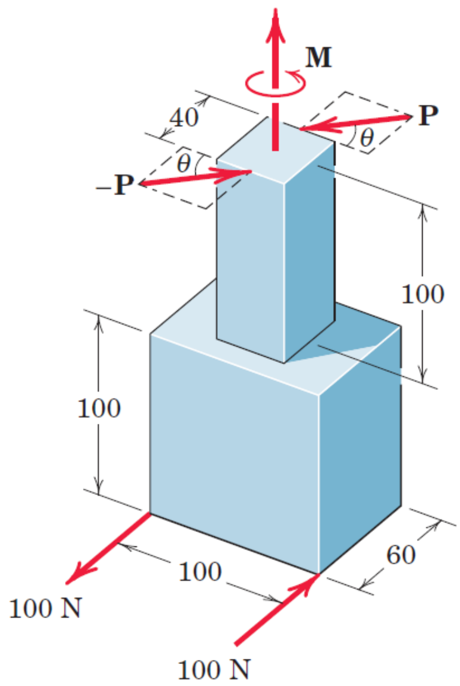
Considering counterclockwise couple moments as positive, we have

$$+M_R = \Sigma M;\ M_R = -F_1 d_1 + F_2 d_2 - F_3 d_3$$

$$= -(200\ \text{lb})(4\ \text{ft}) + (450\ \text{lb})(3\ \text{ft}) - (300\ \text{lb})(5\ \text{ft})$$

$$= -950\ \text{lb.ft} = 950\ \text{lb.ft}$$

Example: The rigid structural member is subjected to a couple consisting of the two 100-N forces. Replace this couple by an equivalent couple consisting of the two forces **P** and _**P**, each of which has a magnitude of 400 N. Determine the proper angle.



Dimensions in millimeters

Solution:

The original couple is counterclockwise when the plane of the forces is viewed from above, and its magnitude is

$[M = Fd]$ $\qquad M = 100(0.1) = 10$ N_m

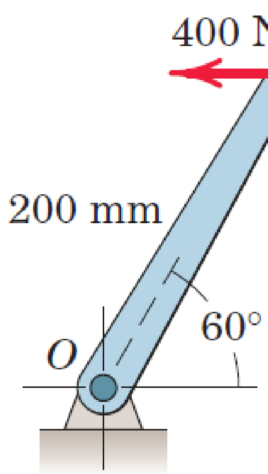
The forces **P** and _**P** produce a counterclockwise couple

$M = 400(0.040)\cos\theta$

$10 = (400)(0.040)\cos\theta$

$\tan^{-1}(\frac{10}{16}) = 51.3°$

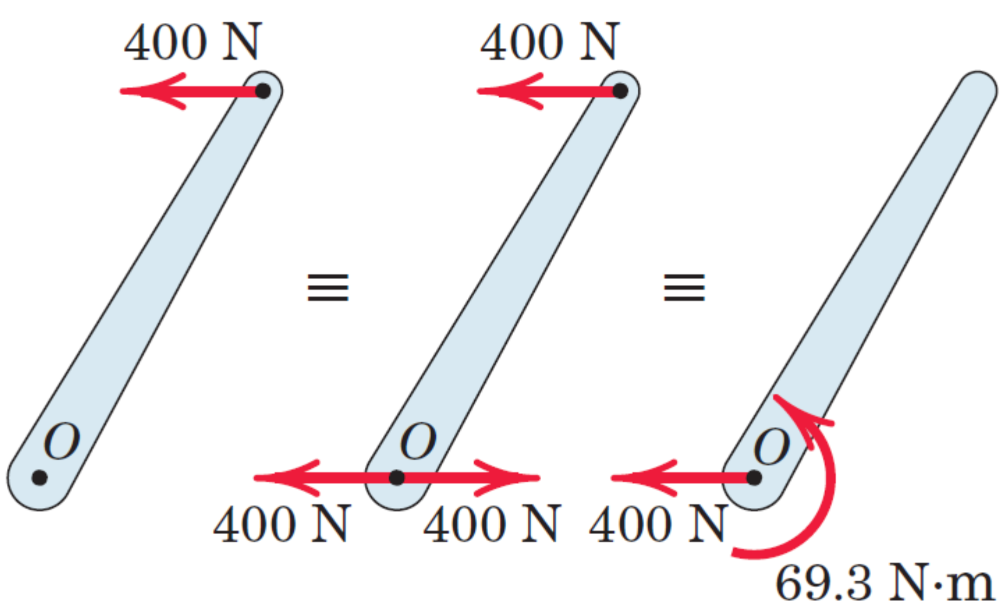
Example: Replace the horizontal 400-N force acting on the lever by an equivalent system consisting of a force at O and a couple.



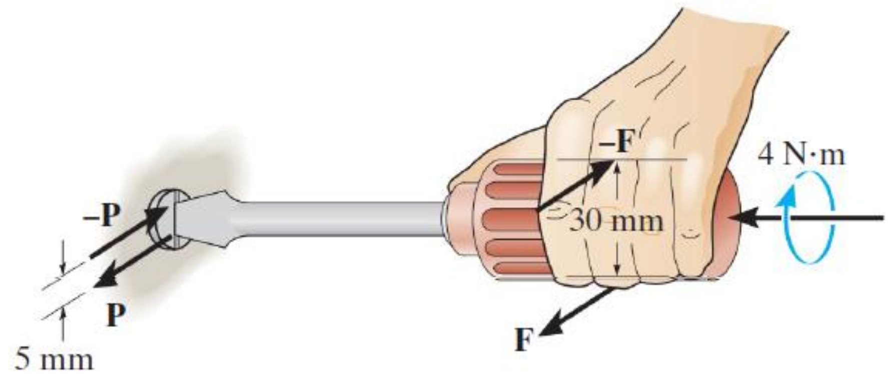
Solution:
We apply two equal and opposite 400-N forces at O and identify the counterclockwise couple

[*M* _ *Fd*] *M* = 400(0.200 sin 60°) = 69.3 N_m

Thus, the original force is equivalent to the 400-N force at O and the couple as shown in the third of the three equivalent figures.



Example: A twist of 4 N # m is applied to the handle of the screwdriver. Resolve this couple moment into a pair of couple forces F exerted on the handle and P exerted on the blade.



Solution:
For the handle

MC = ΣMx ; F10.032 = 4

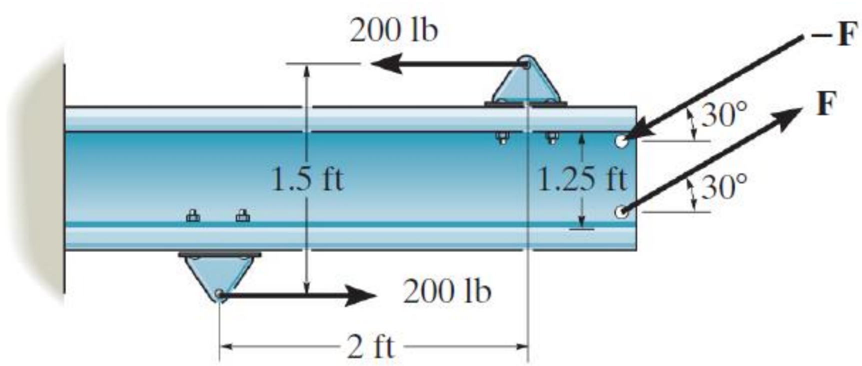
F= 133 N

For the blade

MC = ΣMx ; P10.0052 = 4

P = 800 N

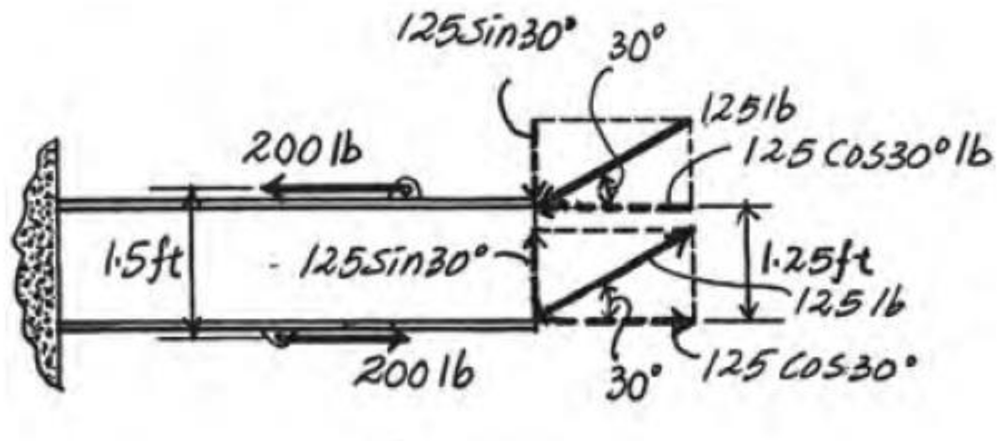
Example: Two couples act on the beam. If F = 125 lb , determine the resultant couple moment.



Solution:

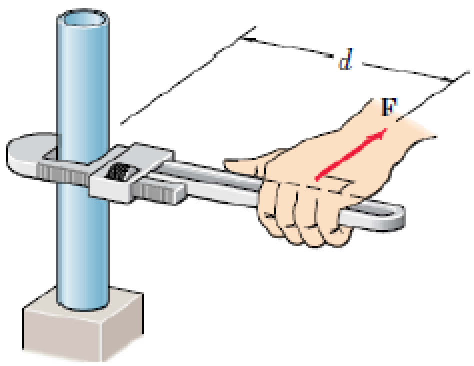
$\circlearrowleft +(M_R)_C = 200(1.5) + 125 \cos 30° (1.25)$

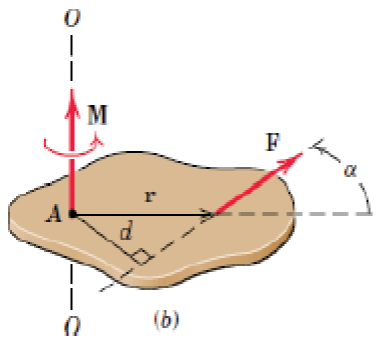
$= 435.32$ lb.ft $= 435$ lb.ft $\circlearrowleft$

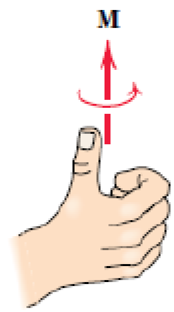

125 Sin30°
30°
125 lb
200 lb
125 Cos30° lb
1.5ft
125 Sin30°
1.25ft
125 lb
200 lb
30°
125 Cos30°

Moment of Force

In addition to the tendency to move a body in the direction of its application, a force can also tend to rotate a body about an axis. The axis may be any line which neither intersects nor is parallel to the line of action of the force. This rotational tendency is known as the *moment* **M** of the force. Moment is also referred to as *torque*.







The magnitude of the moment or tendency of the force to rotate the body about the axis *O-O* perpendicular to the plane of the body is proportional both to the magnitude of the force and to the *moment arm d*, which is the perpendicular distance from the axis to the line of action of the force. Therefore, the magnitude of the moment is defined as

$$M = Fd$$

We represent the moment of **F** about *O-O* as a vector pointing in the direction of the thumb, with the fingers curled in the direction of the rotational tendency. Moment directions may be accounted for by using a stated sign convention, such as a plus sign (+) for counterclockwise moments and a minus sign (-) for clockwise moments.

The Cross Product

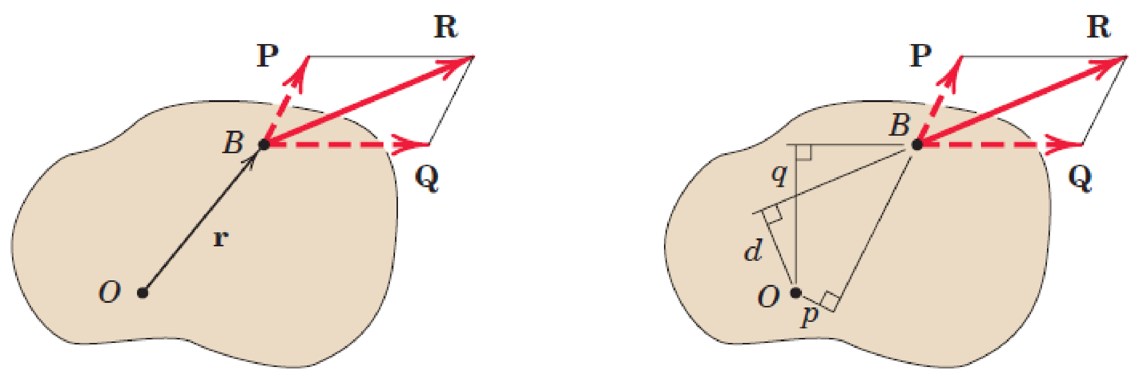
We can calculate the magnitude of the moment by:

$$M = Fr \sin\alpha = Fd$$

Varignon's Theorem (Principle of Moments)

One of the most useful principles of mechanics is *Varignon's theorem*, which states that the moment of a force about any point is equal to the sum of the moments of the components of the force about the same point.

The figure below illustrates the usefulness of Varignon's theorem. The moment of **R** about point O is Rd. However, if **d** is more difficult to determine than **p** and **q,** we can resolve **R** into the components **P** and **Q**, and compute the moment as



(*a*) (*b*)

Example: For each case illustrated in the figure below, determine the moment of the force about point O.

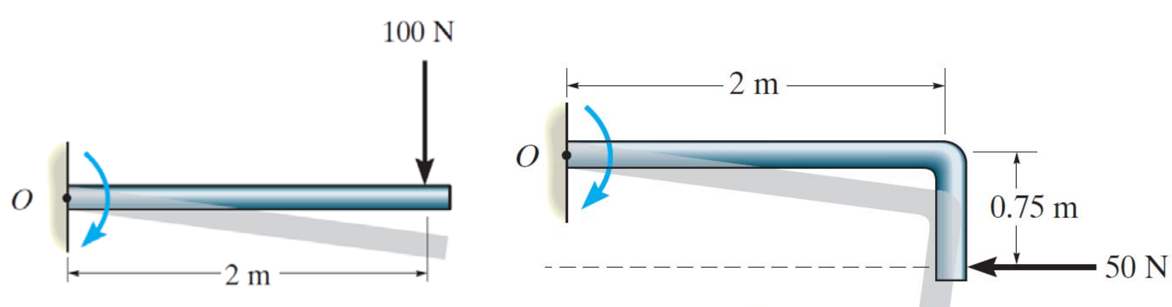
Fig. a *Mo* = (100 N) (2 m) = 200 N.m↷

Fig.*b Mo* = (50 N) (0.75 m) = 37.5 N.m↷

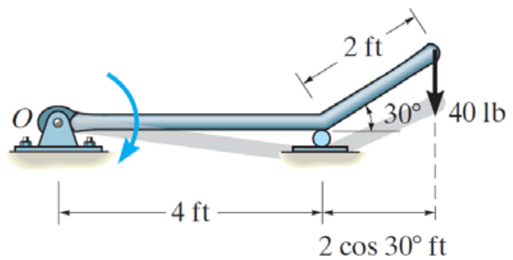
Fig. *c Mo* = (40 lb) (4 ft + 2 cos 30- ft) = 229 lb.ft↷

Fig.*d Mo* = (60 lb) (1 sin 45 - ft) = 42.4 lb.ft↶

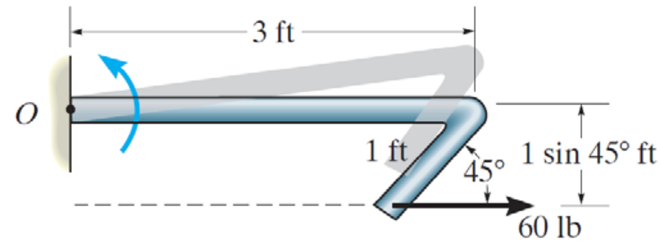
Fig.*e Mo* = (7 kN) (4 m - 1 m) = 21.0 kN.m↶



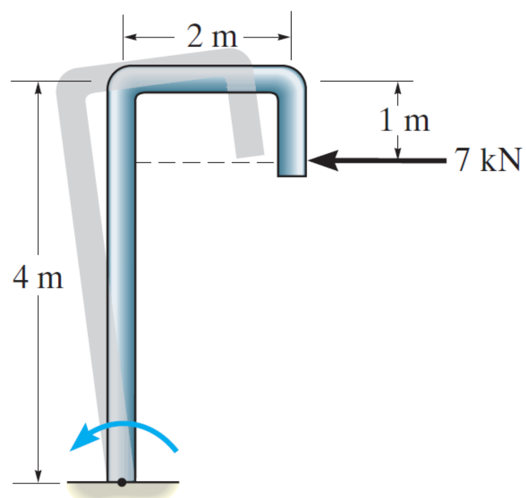
(a) (b)



(c)

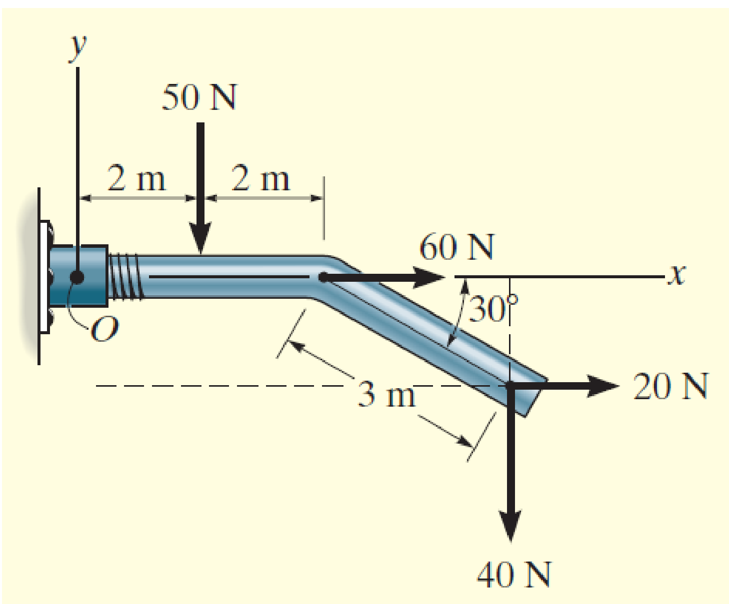


(d)



(e)

Example: Determine the resultant moment of the four forces acting on the rod shown in the figure below about point *O*.



Solution:

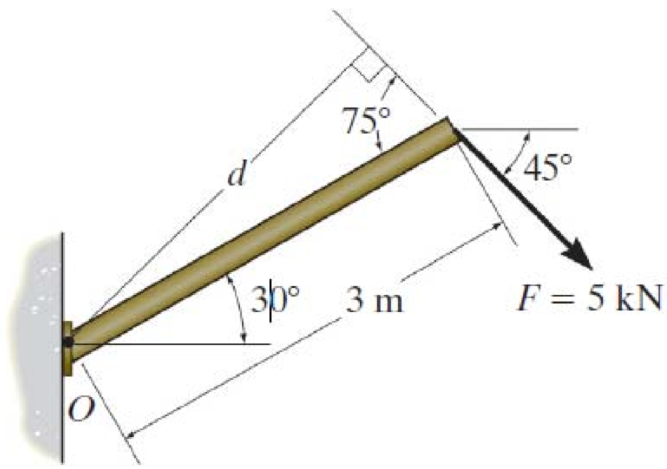
Assuming that positive moments act in the + k direction, i.e., counterclockwise, we have

$\circlearrowleft + (MR)o = \_Fd$

$(MR)o = -50\text{ N }(2\text{ m}) + 60\text{ N }(0) + 20\text{ N }(3 \sin 30°\text{m}) - 40\text{ N }(4\text{ m} + 3 \cos 30°\text{m})$

$(MR)o = -334\text{ N.m} = 334\text{ N.m} \curvearrowright$

Example: Determine the moment of the force in the figure below about point O.



(a)

Solution:

The moment arm *d* in the figure a *a* can be found from trigonometry.
$d = (3\text{ m}) \sin 75° = 2.898\text{ m}$
Thus,
$MO = Fd = (5\text{ kN})(2.898\text{ m}) = 14.5\text{ kN.m} \curvearrowright$

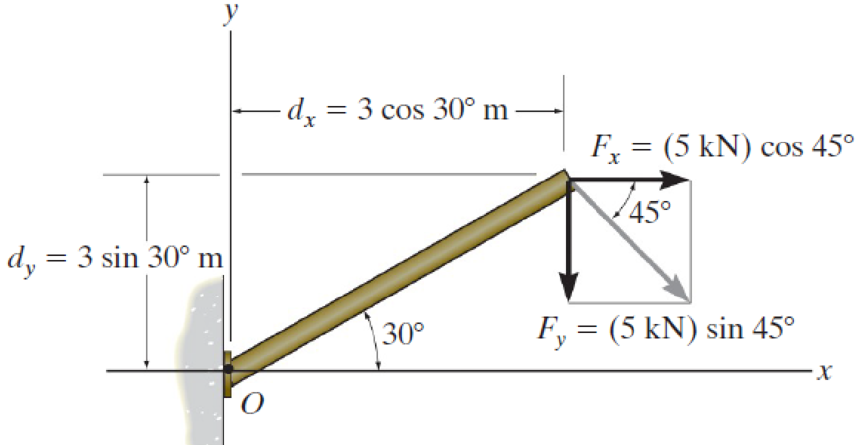
Another Solution:

The x and y components of the force are indicated in Fig.*b*. Considering counterclockwise moments as positive, and applying the principle of moments, we have:

$\curvearrowleft + M_O = -F_x d_y - F_y d_x$

$= -(5 \cos 45° \text{ kN})(3 \sin 30° \text{m}) - (5 \sin 45° \text{ kN})(3 \cos 30° \text{ m})$

$= -14.5 \text{ kN.m} = 14.5 \text{ kN.m} \curvearrowright$



(b)

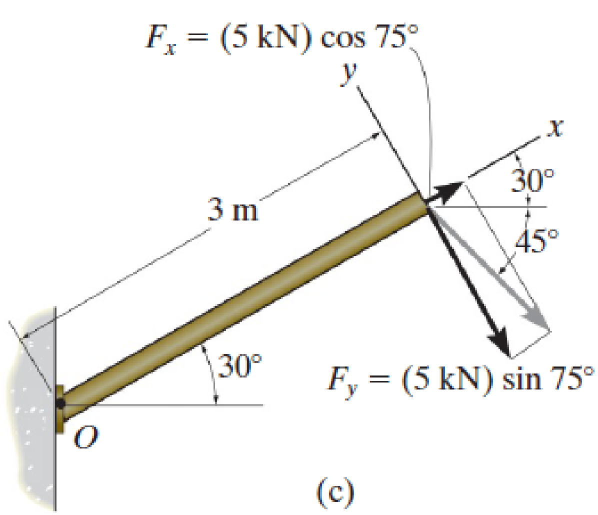
Another Solution:

The x and y axes can be set parallel and perpendicular to the rod's axis as shown in Fig.*c*. Here **F**x produces no moment about point O since its line of action passes through this point. Therefore,

$\curvearrowright + M_O = -F_y d_x$

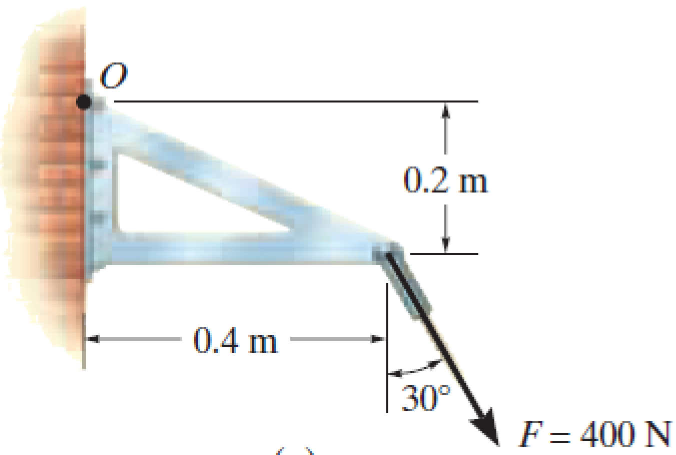
$= -(5 \sin 75° \text{kN})(3 \text{ m})$

$= -14.5 \text{ kN.m} = 14.5 \text{ kN.m} \curvearrowleft$



(c)

Example: Force **F** acts at the end of the angle bracket in the figure below. Determine the moment of the force about point *O*.

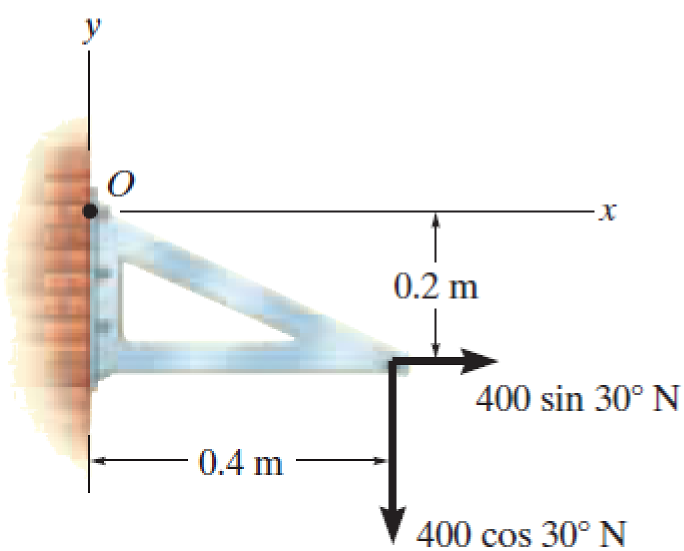


(a)

The force is resolved into its *x* and *y* components, Fig.*b*, then

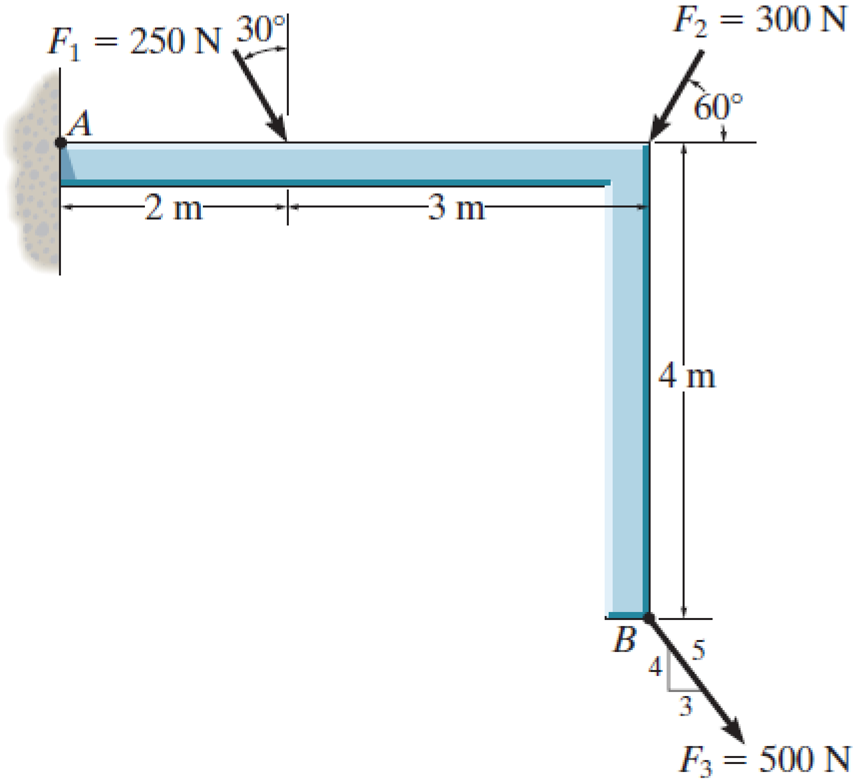
$\circlearrowleft + M_O = 400 \sin 30°\text{N}\ (0.2\text{ m}) - 400 \cos 30°\text{N}\ (0.4\text{ m})$

$= -98.6\text{ N.m} = 98.6\text{ N.m} \circlearrowright$



(b)

Example: Determine the moment of each of the three forces about point *A*.



For **F**1,

↺+M_B = 250 cos 30° (3) - 250 sin 30° (4)

= 149.51 N.m = 150 N.m↺

For **F**2,

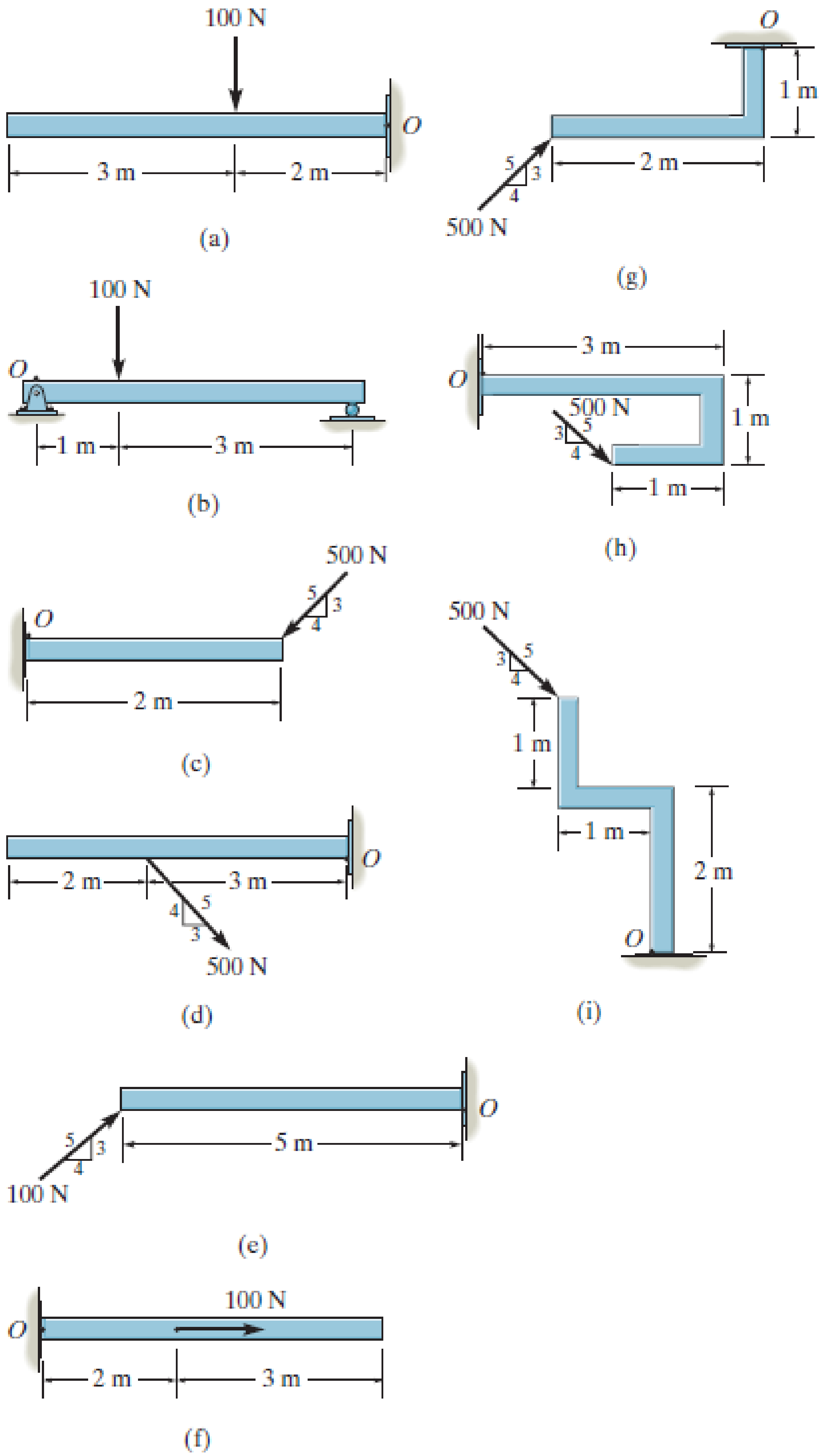
↺+M_B = 300 sin 60° (0) + 300 cos 60° (4)

600 N.m↺

Since the line of action of **F**3 passes through *B*, its moment arm about point *B* is zero. Thus

M_B = 0

Practice: In each case, determine the moment of the force about point *O*.



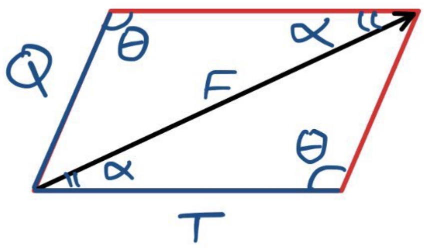
التحليل غير المتعامد

11:41 PM Sunday, April 26, 2020

المركبة — القوة

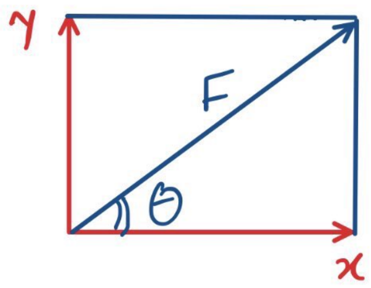
$$\frac{T}{\sin(180-\alpha-\theta)} = \frac{F}{\sin\theta}$$

الزاوية المقابلة — الزاوية المقابلة



$$\frac{Q}{\sin\alpha} = \frac{F}{\sin\theta}$$

$$F = (T^2 + Q^2 - 2TQ\cos\theta)^{\frac{1}{2}}$$



$$\frac{F_x}{\sin(90-\theta)} = \frac{F}{\sin 90}$$

$$F_x = F\sin(90-\theta)$$

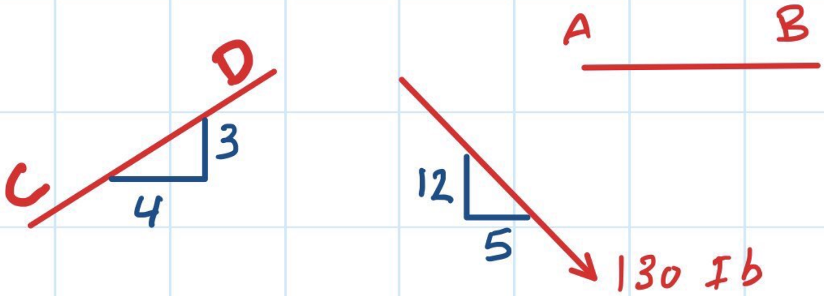
$$F_x = F\cos\theta$$

$$F_y = F\sin\theta$$

AHMED R.R

Example: determine the two non rectangular components of the force 130 Ib as shown in figure. One of them has a line of action along AB and the other is parallel to CD.

Solution :



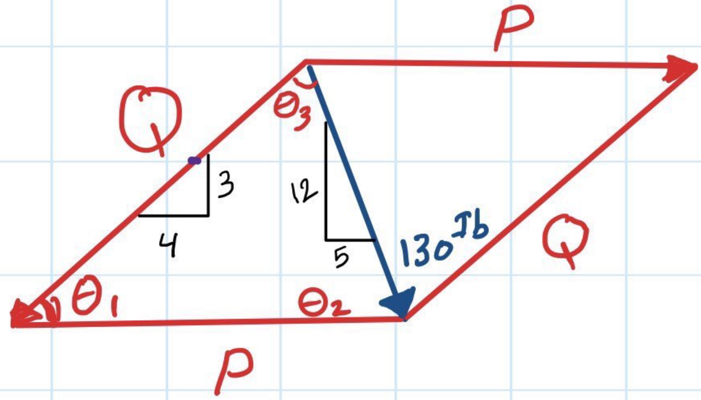
$$\theta_1 = \tan^{-1}\left(\frac{3}{4}\right) = 36.86^\circ$$

$$\theta_2 = \tan^{-1}\left(\frac{12}{5}\right) = 67.38^\circ$$

$$\theta_3 = 180 - \theta_1 - \theta_2$$

$$= 180 - 36.86 - 67.38$$

$$= 75.75^\circ$$



$$\frac{130}{\sin 36.86} = \frac{Q}{\sin 67.38}$$

$$Q = 200 \text{ Ib}$$

$$\frac{130}{\sin 36.86} = \frac{P}{\sin 75.75}$$

$$P = 210.05 \text{ Ib}$$

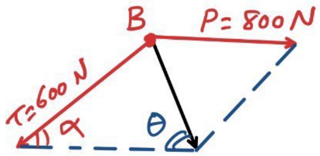
AHMED RAAD, MSCE

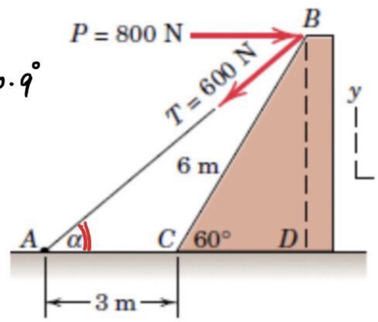
AHMED R.

Example: Combine the two forces P and T, which act on the fixed structure at B, into a single equivalent force R.

Solution :-

$$\tan^{-1}\theta = \frac{BD}{AD} = \frac{6\sin 60}{3+6\cos 60} = 40.9^\circ$$





Cosine law:

$$R^2 = 600^2 + 800^2 - 2(600)(800)\cos(40.9) = 274300$$

$$R = 524\ N$$

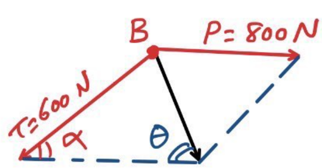
Sines law:-

$$\frac{600}{\sin\theta} = \frac{524}{\sin 40.9} \Rightarrow \sin\theta = 0.75\ ,\ \theta = 48.6^\circ$$

Algebraic Solution:

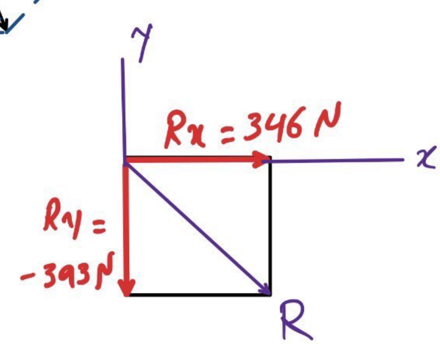
$$R_x = \Sigma F_x = 800 - 600\cos 40.9$$

$$= 346\ N$$



$$R_y = -600\sin 40.9$$

$$= -393\ N$$



$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(346)^2 + (-392)^2}$$

$$= 524\ N$$

$$\theta = \tan^{-1}\frac{|R_y|}{|R_x|} = \tan^{-1}\frac{392}{346} = 48.6^\circ$$

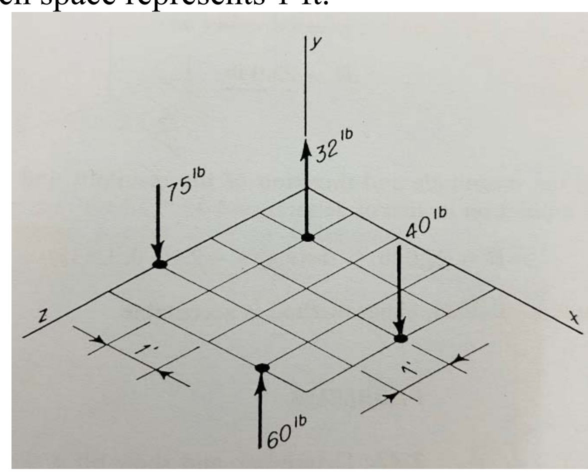
Resultant of a parallel Noncoplaner Force System

The resultant of two parallel forces is either a single force parallel to the given forces or a couple in the plane determined by the action lines of the two forces. If the resultant is a single force, its magnitude is equal to the algebraic sum of the forces of the system, and its position in space can be determined by the coordinates of the intersection- of its action line with any plane perpendicular to the forces of the system.

$$R = \Sigma F_{x,y,z} \qquad R.\bar{X} = \Sigma M_z \qquad R.\bar{Z} = \Sigma M_x$$

Where $\bar{X}$ and $\bar{Z}$ are the coordinates of the intersection of the action line of the resultant, R, with the xz plane.

Example: Determine the resultant of the four parallel forces in the figure below and show it on a sketch. Each space represents 1 ft.



Solution:

↑R=60-75+32-40=-23 Ib = 23 Ib ↓

ΣM_x= -60(40) +75(3) -32(1) +40(2)

=33 Ib.ft

R$\bar{z}$ =33 Ib.ft

$\bar{z}$ = 33/23 = 1.435 ft

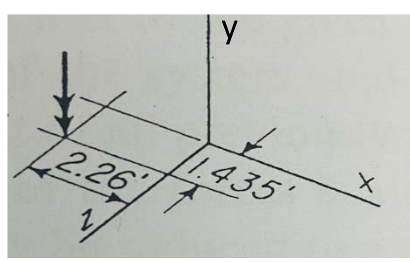
ΣM_z= 60(3) +75(0) + 32(1) -40(4)

=52 Ib.ft

R$\bar{x}$ =52 Ib.ft

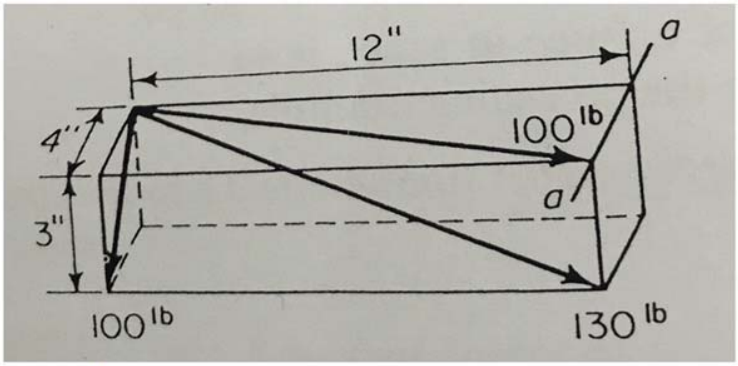
$\bar{x}$=52/23 =2.26 ft

R= 23 Ib through (-2.26, 0, 1.435)



Resultant of concurrent non coplanar force system

Example
1. Determine the resultant of forces system shown.
2. Determine the moment of the resultant with respect to a-a



① $\overset{+}{\rightarrow} R_x = 100\left(\frac{12}{\sqrt{12^2+4^2}}\right) + 130\left(\frac{12}{\sqrt{12^2+4^2+3^2}}\right)$

$= 214.86 \text{ Ib} \rightarrow$

$R_y \overset{+}{\uparrow} = -130\left(\frac{3}{\sqrt{12^2+4^2+3^2}}\right) - 100\left(\frac{3}{\sqrt{4^2+3^2}}\right)$

$= 90 \downarrow \text{Ib}$

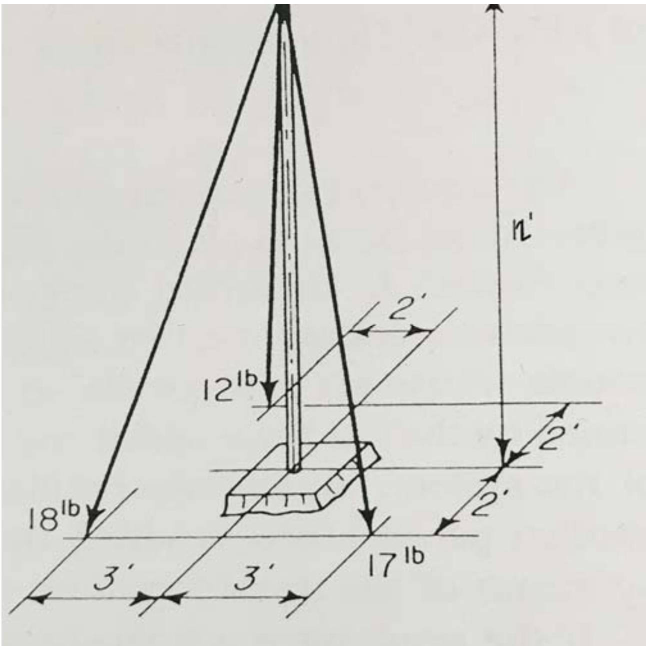
$R_z \overset{+}{\swarrow} = 100\left(\frac{4}{\sqrt{12^2+4^2}}\right) + 130\left(\frac{4}{\sqrt{12^2+3^2+4^2}}\right) + 100\left(\frac{4}{\sqrt{3^2+4^2}}\right)$

$= 151.6 \text{ Ib} \swarrow$

$R = \sqrt{214.86^2 + 90^2 + 151.6^2} = 278 \text{ Ib}$

② $M@a\text{-}a = 90(12) = 108 \text{ Ib}\cdot\text{in} \circlearrowleft$

Example: Determine the resultant of the tensions of the wires shown in figure below.



Solution:

F1=12 Ib   F2=18 Ib   F3=17 Ib

L1=$\sqrt{12^2+2^2+2^2} = \sqrt{152}$ ft

L2=$\sqrt{12^2+3^2+2^2} = \sqrt{157}$ ft

L1=$\sqrt{12^2+2^2+2^2} = \sqrt{157}$ ft

F_{1x}= 12*2/$\sqrt{152}$ = -1.95 Ib

F_{2x}= 18*3/$\sqrt{157}$ = -4.31 Ib

F_{3x}= 17*3/$\sqrt{157}$ = +4.1 Ib

F_{1y}= 12*12/$\sqrt{152}$ = -11.68 Ib

F_{2y}= 18*12/$\sqrt{157}$ = -17.24 Ib

F_{3y}= 17*12/$\sqrt{157}$ = -16.3 Ib

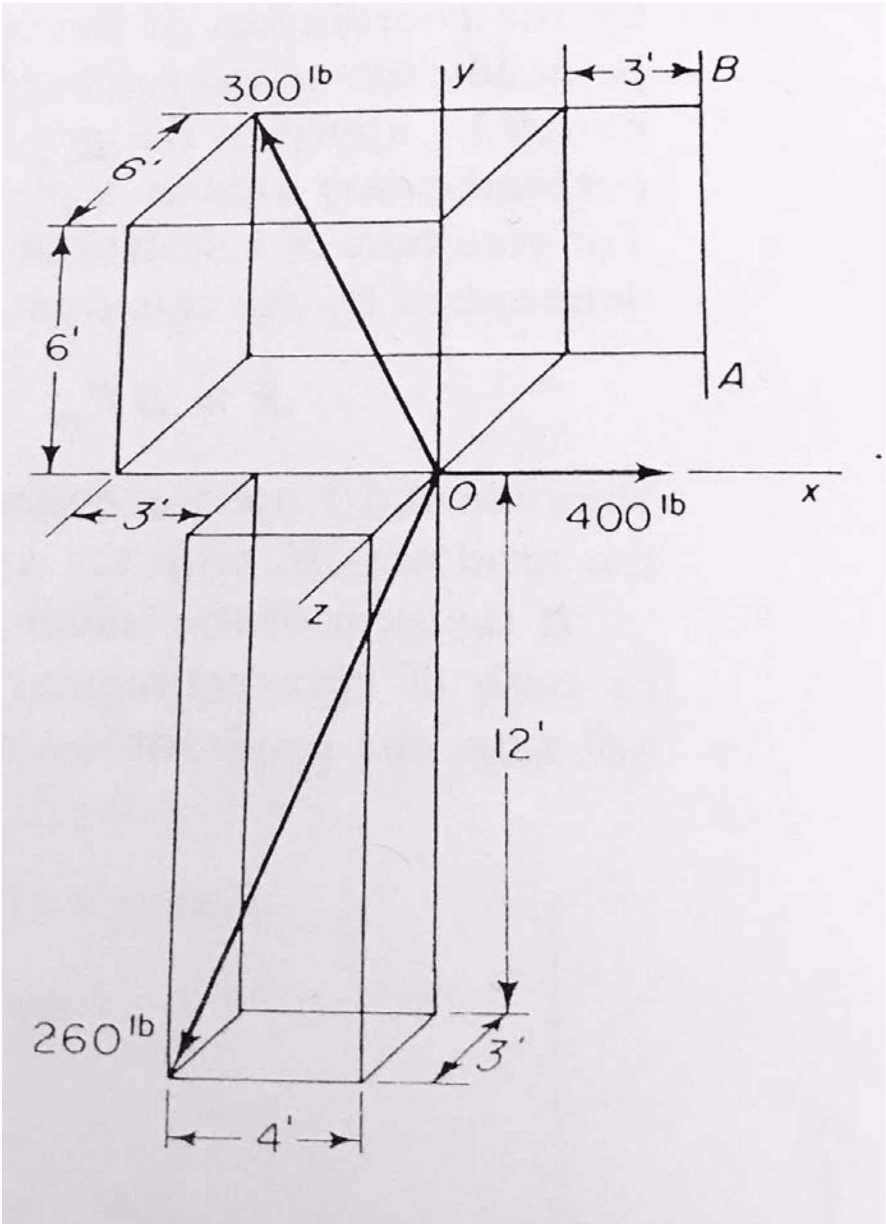
F_{1z}= 12*2/$\sqrt{152}$ = -1.95 Ib

F_{2z}= 18*2/$\sqrt{157}$ = +2.87 Ib

F_{3z}= 17*2/$\sqrt{157}$ = +2.71 Ib

R= $\sqrt{2.16^2+45.22^2+3.63^2}$ = 45.4 Ib

Example 1. Determine the resultant of the three forces shown in the figure below.
2. Determine the moment of the resultant with respect to AB axis



Solution:

F1=300 F2=260 F3=400

L1=$\sqrt{6^2+6^2+7^2}$=11 ft

L1=$\sqrt{4^2+3^2+12^2}$=13 ft

F_{1x}=300*7/11=-191 Ib

F_{2x}=260*4/13=-80 Ib

F_{3x}=400 Ib

R_x=400-80-191= 129 Ib

F_{1y}=300*6/11=163.6 Ib
F_{2y}=260*12/13=-240 Ib
F_{3y}=0

R_y= 163.6-240= -76.4 Ib

F_{1z}=300*6/11=-163.6 Ib
F_{2z}=260*3/13=60 Ib
F_{3z}=0

R_z= 60-163.6 = -103.6 Ib

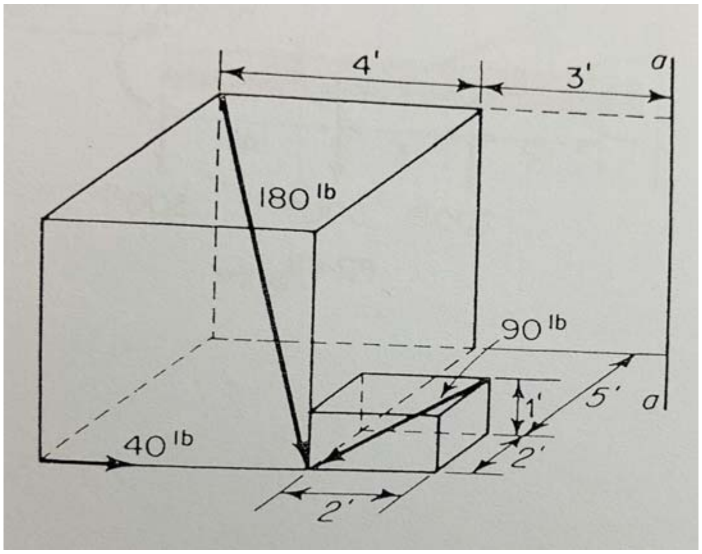
R=$\sqrt{129^2 + 76.4^2 + 103.6^2}$
= 182.23 Ib

2. MAB=103.6*3-129*6
=-463.2 Ib.ft

Resultant of concurrent non coplanar force system

Example: for the figure below:

1. Determine the resultant of the force system.
2. Determine the moment of the resultant with respect to the a-a axis.



Solution:

F1=180 Ib   F2=90 Ib   F3=40 Ib

L1=$\sqrt{4^2+4^2+7^2}$=9 ft

L2=$\sqrt{2^2+2^2+1^2}$=3 ft

F_{1x}=180*4/9=80 Ib
F_{2x}=90*2/3=-60 Ib
F_{3x}=40 Ib
R_x= 60 Ib

F_{1y}=180*4/9=-80 Ib
F_{2y}=90*1/3=-30 Ib
F_{3y}=0
Ry= -110 Ib

F_{1z}=180*7/9=140 Ib
F_{2z}=90*2/3=60 Ib
F_{3z}=0
R_z= 200 Ib
R= $\sqrt{60^2+110^2+200^2}$ = 236 Ib
$M_{a\text{-}a}$=200*7=1400 Ib.ft

Resultant of Forces

The *resultant* of a system of forces is the simplest force combination that can replace the original forces without altering the external effect on the rigid body to which the forces are applied.

Equilibrium of a body is the condition in which the resultant of all forces acting on the body is zero. This condition is studied in statics. When the resultant of all forces on a body is not zero, the acceleration of the body is obtained by equating the force resultant to the product of the mass and acceleration of the body. This condition is studied in dynamics. Thus, the determination of resultants is basic to both statics and dynamics.

For any system of coplanar forces, we may write

$$\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \cdots = \Sigma\mathbf{F}$$

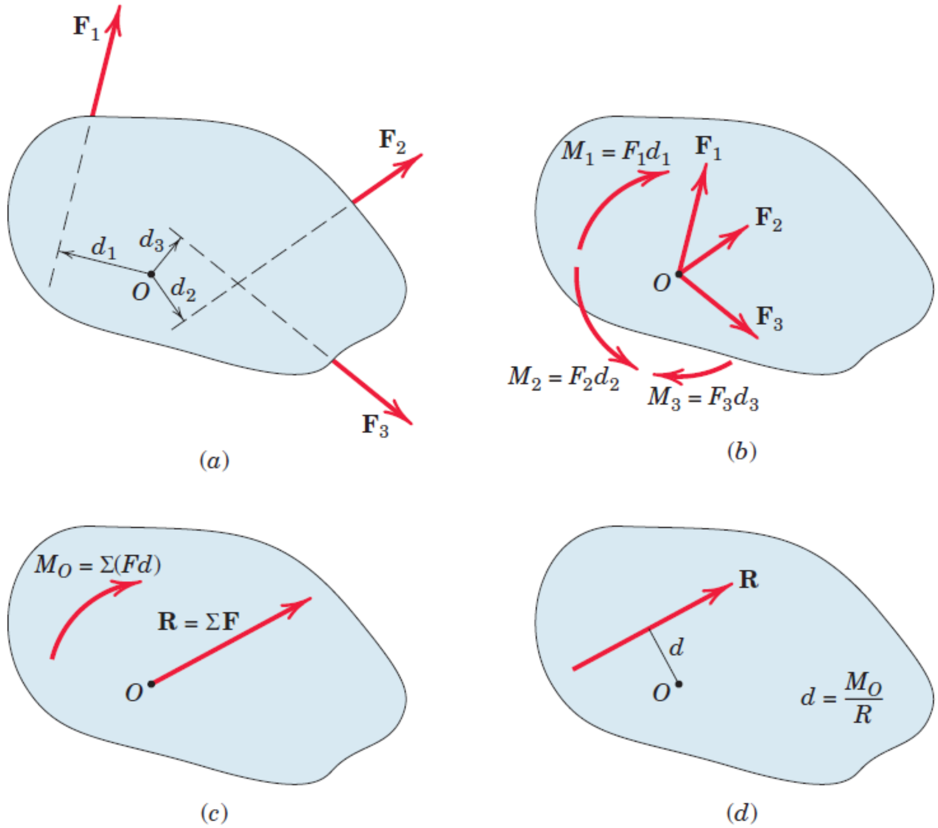
$$R_x = \Sigma F_x \qquad R_y = \Sigma F_y \qquad R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

$$\theta = \tan^{-1}\frac{R_y}{R_x} = \tan^{-1}\frac{\Sigma F_y}{\Sigma F_x}$$

Algebraic Method

We can use algebra to obtain the resultant force and its line of action as follows:

1. Choose a convenient reference point and move all forces to that point. This process is depicted for a three-force system in the figure below a and b, where M1, M2, and M3 are the couples resulting from the transfer of forces F1, F2, and F3 from their respective original lines of action to lines of action through point O.
2. Add all forces at O to form the resultant force R, and add all couples to form the resultant couple MO. We now have the single force–couple system, as shown in the figure c.
3. In the figure d, find the line of action of R by requiring R to have a moment of MO about point O. Note that the force systems of the figures a and d are equivalent, and that Σ (Fd) in figure a is equal to Rd in figure d.



Principle of Moments

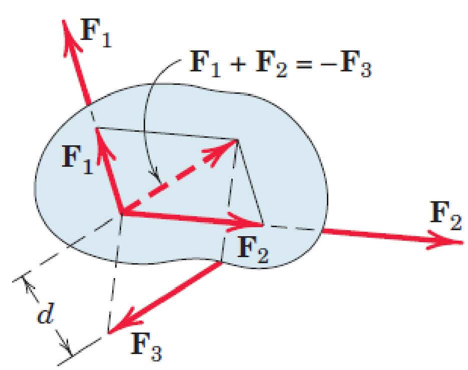
This process is summarized in equation form by

$$\mathbf{R} = \Sigma\mathbf{F}$$

$$M_O = \Sigma M = \Sigma(Fd)$$

$$Rd = M_O$$

The first two of the equations above reduce a given system of forces to a force–couple system at an arbitrarily chosen but convenient point O. The last equation specifies the distance d from point O to the line of action of R, and states that the moment of the resultant force about any point O equals the sum of the moments of the original forces of the system about the same point. This extends Varignon's theorem to the case of nonconcurrent force systems; we call this extension the principle of moments. For a concurrent system of forces where the lines of action of all forces pass through a common point O, the moment sum ΣM_O about that point is zero.



Resultants of Concurrent-coplanar Force Systems

The resultant of a concurrent, coplanar force system is a single force passing through the point of concurrence and that the force will be completely determined when its magnitude, slope are known. In order to determine the resultant algebraically, each force is first resolved into a pair of rectangular components. The components form two perpendicular collinear force systems whose resultants are given by the equations

$$R_x = \Sigma F_x \qquad R_y = \Sigma F_y$$

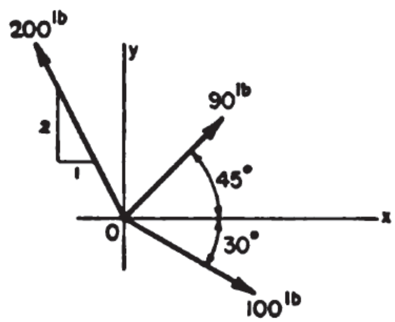
The magnitude of the resultant of the system now composed of the forces Rx and Ry is

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

and the slope of the resultant is

$$\theta = \tan^{-1}\frac{R_y}{R_x} = \tan^{-1}\frac{\Sigma F_y}{\Sigma F_x}$$

Example: Determine the resultant of the concurrent, coplanar force system for the system shown in figure



Solution

$$\begin{array}{rlcrl} & \rightarrow & & & \leftarrow \\ 100\cos 30^\circ = & 86.6 & & 200\,\dfrac{1}{\sqrt{5}} = & 89.4 \\ 90\cos 45^\circ = & 63.6 & & & \\ & \overline{150.2} & & & \overline{89.4}, \end{array}$$

$$R_x = 150.2 - 89.4 = 60.8 \text{ lb} \rightarrow.$$

$$\begin{array}{rlcrl} & \uparrow & & & \downarrow \\ 90\sin 45^\circ = & 63.6 & & 100\sin 30^\circ = & 50.0 \\ 200\,\dfrac{2}{\sqrt{5}} = & 178.8 & & & \\ & \overline{242.4} & & & \overline{50.0} \end{array}$$

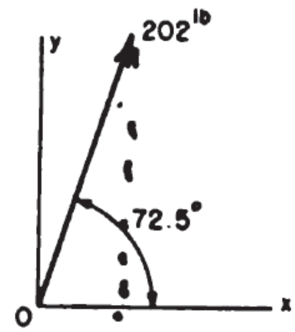
$$R_y = 242.4 - 50.0 = 192.4 \text{ lb} \uparrow.$$

The magnitude of the resultant is

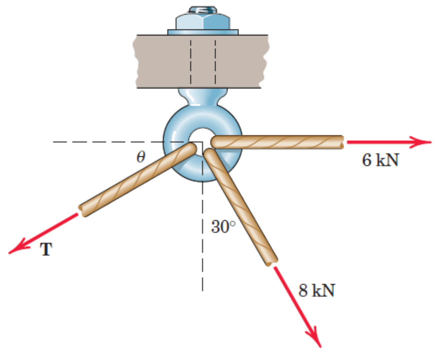
$$R = \sqrt{R_x^2 + R_y^2} = 202 \text{ lb} \;\; \frac{192.4}{60.8} \text{ through } O.$$

The angle the resultant makes with the x axis can be used in place of the slope to indicate the direction. In this case the angle is

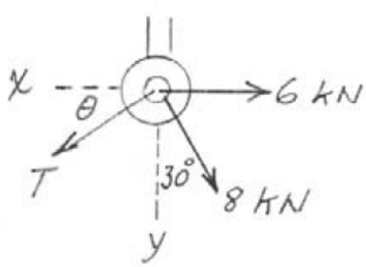
$$\theta_x = \tan^{-1}\frac{192.4}{60.8} = 72.5°$$



Example: Calculate the magnitude of the tension T and the angle θ for which the eye bolt will be under a resultant downward force of 15 kN.



Solution



$$R = R_y = 15 = T\sin\theta + 8\cos 30°$$

$$R_x = 0 = T\cos\theta - 6 - 8\sin 30°$$

So $T\sin\theta = 8.07$

$T\cos\theta = 10$

Divide & get $\theta = \tan \dfrac{8.07}{10}$

$\underline{\theta = 38.9°}$

$$T = \frac{10}{\cos 38.9°} = \underline{12.85 \text{ kN}}$$

<u>Resultant of non-concurrent coplanar force system</u>

From the graphical solution the resultant of a parallel, coplanar force system is known to be either a single force or a couple. If the algebraic sum of the forces is different from zero, the resultant is a force, and the equation

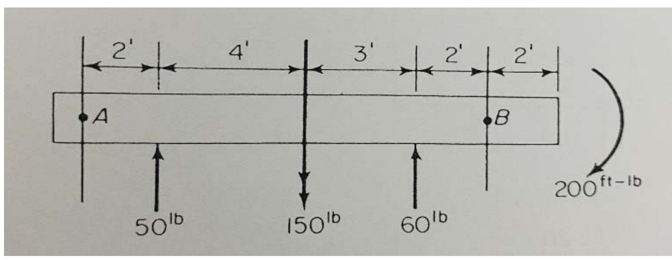
$$R = \Sigma F$$

gives the magnitude and sense of the resultant. The position of a point on the action line is determined by the principle of moments (Art. 1-7), which states that the moment of the resultant about any axis equals the algebraic sum of the moments of the forces of the system about the same axis. Thus

$$Rq = \Sigma M_o = \Sigma Fd$$

Where q is the perpendicular distance from the resultant, R , to the moment center 0 and d is the perpendicular distance from 0 to any force F.

Example: The 150 Ib force is the resultant of the two forces shown in figure and two other vertical forces, one acting through point A and the other through point B. Determine these two unknown forces.



Solution

$Rq = \Sigma M_A = \Sigma Fd$

$-150(6) = 50(2) + 60(9) - 200 + F_B\ (11)$

$F_B = -121.82$ Ib

$Rq = \Sigma M_B = \Sigma Fd$

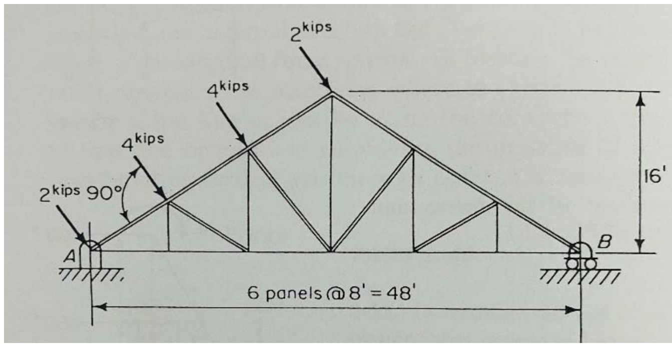
$150(5) = -50(9) - 60(2) - 200 - F_A\ (11)$

$F_A = -138.2$ Ib

ملاحظة/ بعد نهاية الحل.. يمكنك تجبيك صحة الحل بجمع القوى يجب ان تكون المحصلة تساوي 150 باوند اي بمعنى:

$$50 + 60 - 121.82 - 138.2 = 150$$

Example: Determine the resultant of the wind load acting on the roof truss of the figure and locate it with respect to the support A.



Solution:

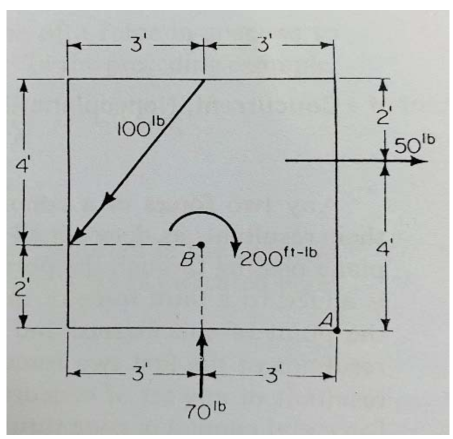
$\Sigma F_y = 2+4+4+2 = 12$ Kips

$Rq = \Sigma M_A$

$12(q) = 4(9.61) + 4(19.22) + 2(28.84)$

$q = 14.41$ feet

Example: The 100 Ib force of the figure is the resultant of the couple and three forces, two of which are shown. Determine the third force and locate it with respect to point A.



Solution:

$R_x = 100*3/5 = -60$ Ib

$R_y = 100*4/5 = -80$ Ib

$R_x = \Sigma F_x$

$-60 = 50 + F_x$

$F_x = -110$ Ib

$R_y = \Sigma F y$
$-80=70+F_y$
$F_y=-150$ Ib

$F=\sqrt{110^2+150^2} = 186$ Ib
$R_q = \Sigma M_A$
$80(3) +60(6) =-50(9) -70(3) -186(d) -200$
d=6.5 feet