▪ Stress

- Introduction

Mechanics of materials is a branch of mechanics that studies the internal effects of stress and strain in a solid body that is subjected to an external loading. Stress is associated with the strength of the material from which the body is made, while strain is a measure of the deformation of the body. In addition to this, mechanics of materials includes the study of the body's stability when a body such as a column is subjected to compressive Loading. A thorough understanding of the fundamentals of this subject is of vital importance because many of the formulas and rules of design cited in engineering codes are based upon the principles of this subject.

- Equilibrium of a Deformable Body

Since statics has an important role in both the development and application of mechanics of materials, it is very important to have a good grasp of its fundamentals. For this reason we will review some of the main principles of statics that will be used throughout the text.

External Loads. A body is subjected to only two types of external loads; namely, Surface forces and body forces, Fig. 1-1.

Surface Forces. Surface forces are caused by the direct contact of one body with the surface of another. In all cases these forces are distributed over the area of contact between the bodies. If this area is small in comparison with the total surface area of the body, then the surface force can be idealized as a single concentrated force, which is applied to a point on the body. For example, the force of the ground on the wheels of a bicycle can be considered as a concentrated force. If the surface loading is applied along a narrow strip of area, the loading can be idealized as a linear distributed load, w(s).

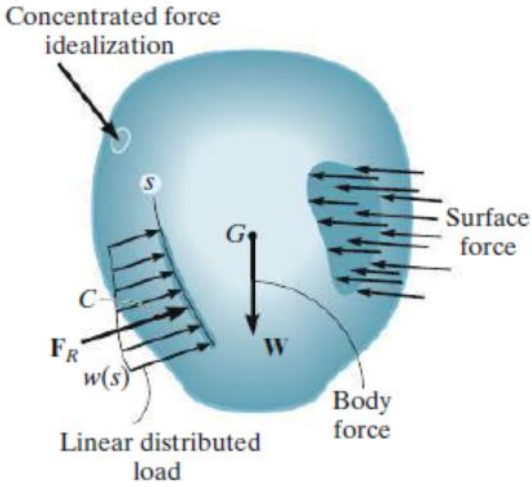


Fig. 1–1

Here the loading is measured as having an intensity of force/length along the strip and is represented graphically by a series of arrows along the lines. The resultant force F_R of w(s) is equivalent to the area under the distributed loading curve، and this resultant acts through the centroid C or geometric center of this area. The loading along the length of a beam is a typical example of where this idealization is often applied.

Body Forces. A both' force is developed when one body exerts a force on another body without direct physical contact between the bodies. Examples include the effects caused by the earth's gravitation or its electromagnetic field. Although body forces affect each of the particles composing the body, these forces are normally represented by a single concentrated force acting on the body. In the case of gravitation this force is called the weight of the body and acts through the body's center of gravity.

Internal Resultant Loadings. In mechanics of materials, statics is primarily used to determine the resultant loadings that act within a body. For example, consider the body shown in Fig. 1-2a, which is held in equilibrium by the four external forces.* In order to obtain the internal loadings acting on a specific region within the body, it is necessary to pass an imaginary section or "cut" through the region where the internal loadings are to be determined. The two parts of the body are then separated, and a free-body diagram of one of the parts is drawn, Fig. 1-2b. Notice that there is actually a distribution of internal force acting on the "exposed" area of the section. These forces represent the effects of the material of the top part of the body acting on the adjacent material of the bottom part.

Although the exact distribution of this internal loading may he unknown, we can use the equations of equilibrium to relate the external forces on the bottom part of the body to the distribution's resultant force And moment, F_R and M_{R0}, at anyispecint O on the sectioned area, Fig. 1 -2c. It will be shown in later portions of the text that point O is most often chosen at the centroid of the sectioned area, and so we will always choose this location for O, unless otherwise stated. Also, if a member is long and slender, as in the case of a rod or beam, the section to be considered is generally taken perpendicular to the longitudinal axis of The member. This section is referred to as the cross section.

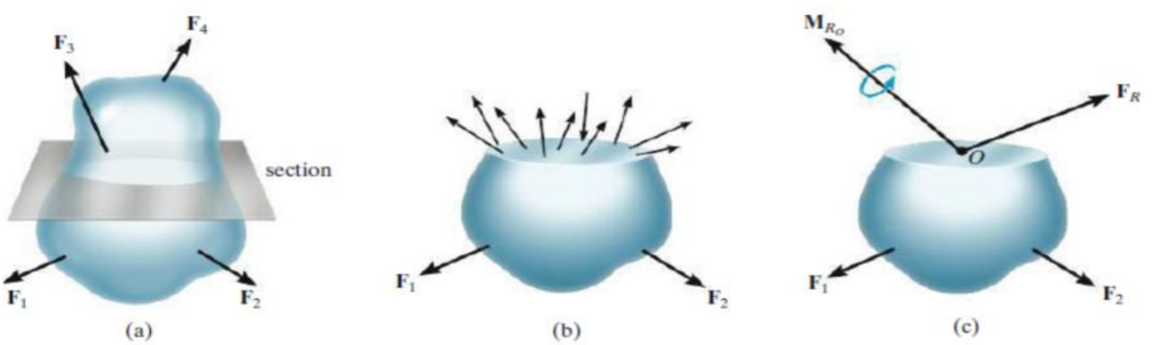
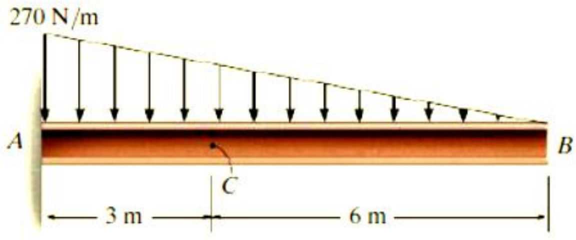


Fig. 1–2

Example:

Determine the resultant internal loadings acting on the cross section at C of the cantilevered beam shown in Fig. 1-4a.



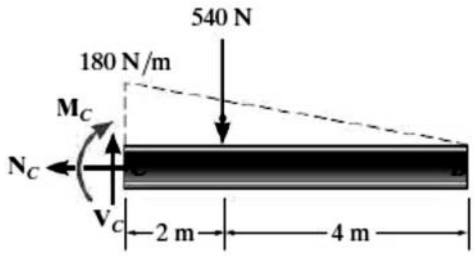
(a)

Fig. 1–4

Solution:

Support Reactions. The support reactions at A do not have to be determined if segment CB is considered.

Free-Body Diagram. The free-body diagram of segment CB is shown in Fig. 1–4*b*. It is important to keep the distributed loading on the segment until *after* the section is made. Only then should this loading be replaced by a single resultant force. Notice that the intensity of the distributed loading at C is found by proportion, i.e., from Fig. 1–4*a*, $w/6\text{ m} = (270\text{ N/m})/9\text{ m}$, $w = 180\text{ N/m}$. The magnitude of the resultant of the distributed load is equal to the area under the loading curve (triangle) and acts through the centroid of this area. Thus, $F = \frac{1}{2}(180\text{ N/m})(6\text{ m}) = 540\text{ N}$, which acts $\frac{1}{3}(6\text{ m}) = 2\text{ m}$ from C as shown in Fig. 1–4*b*.



(b)

Equations of Equilibrium. Applying the equations of equilibrium we have

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad -N_C = 0$$

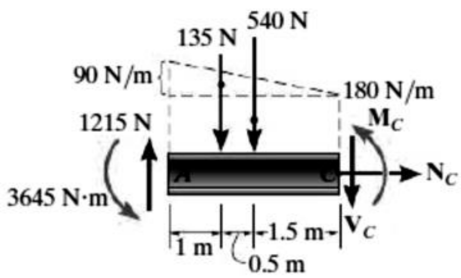
$$N_C = 0 \qquad \textit{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad V_C - 540\text{ N} = 0$$

$$V_C = 540\text{ N} \qquad \textit{Ans.}$$

$$\circlearrowleft + \Sigma M_C = 0; \qquad -M_C - 540\text{ N}(2\text{ m}) = 0$$

$$M_C = -1080\text{ N}\cdot\text{m} \qquad \textit{Ans.}$$

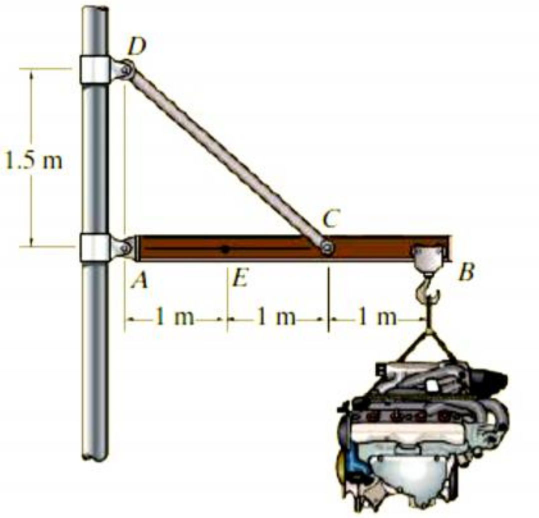


(c)

NOTE: The negative sign indicates that $\mathbf{M}_C$ acts in the opposite direction to that shown on the free-body diagram. Try solving this problem using segment AC, by first obtaining the support reactions at A, which are given in Fig. 1–4*c*.

Example:

The 500-kg engine is suspended from the crane boom in Fig. 1-5a. Determine the resultant internal loadings acting on the cross section of the boom at point E.



Solution:

Support Reactions. We will consider segment AE of the boom, so we must first determine the pin reactions at A. Notice that member CD is a two-force member. The free-body diagram of the boom is shown in Fig. 1–5*b*. Applying the equations of equilibrium,

$$\circlearrowleft +\Sigma M_A = 0; \qquad F_{CD}\left(\tfrac{3}{5}\right)(2\text{ m}) - [500(9.81)\text{ N}](3\text{ m}) = 0$$

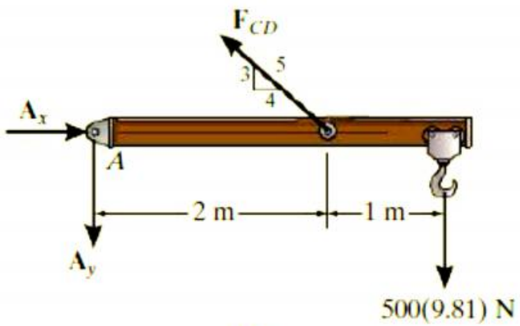
$$F_{CD} = 12\,262.5\text{ N}$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad A_x - (12\,262.5\text{ N})\left(\tfrac{4}{5}\right) = 0$$

$$A_x = 9810\text{ N}$$

$$+\uparrow \Sigma F_y = 0; \qquad -A_y + (12\,262.5\text{ N})\left(\tfrac{3}{5}\right) - 500(9.81)\text{ N} = 0$$

$$A_y = 2452.5\text{ N}$$



(b)

Free-Body Diagram. The free-body diagram of segment AE is shown in Fig. 1–5*c*.

Equations of Equilibrium.

$$\xrightarrow{+} \Sigma F_x = 0; \qquad N_E + 9810\text{ N} = 0$$

$$N_E = -9810\text{ N} = -9.81\text{ kN} \qquad \textit{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad -V_E - 2452.5\text{ N} = 0$$

$$V_E = -2452.5\text{ N} = -2.45\text{ kN} \qquad \textit{Ans.}$$

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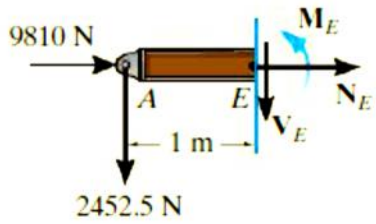

(c)

Fig. 1–5

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Stress

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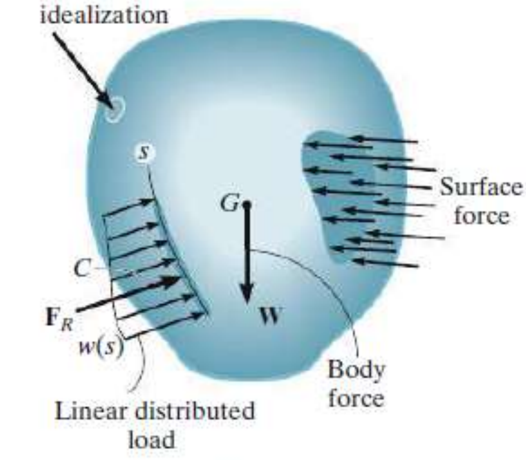

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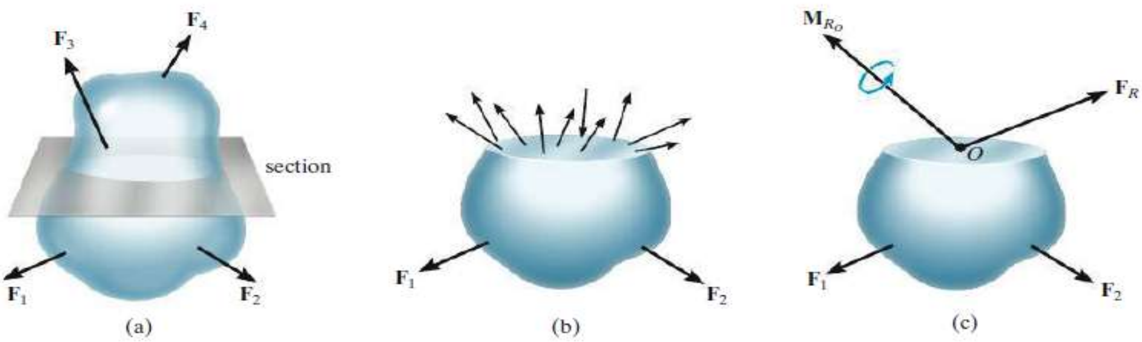

Fig. 1–2

Example:

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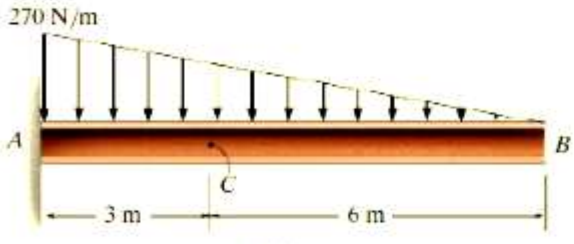

(a)

Fig. 1–4

Solution:

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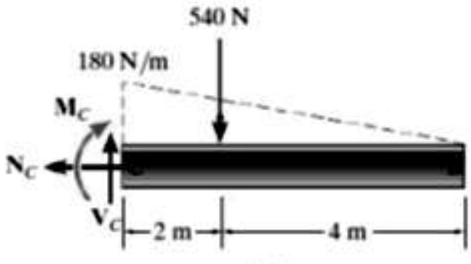

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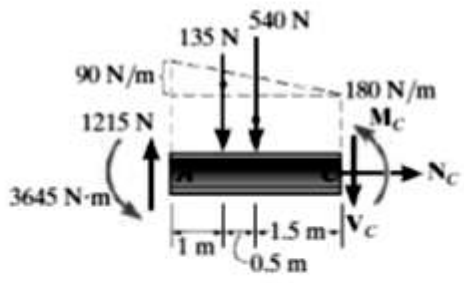

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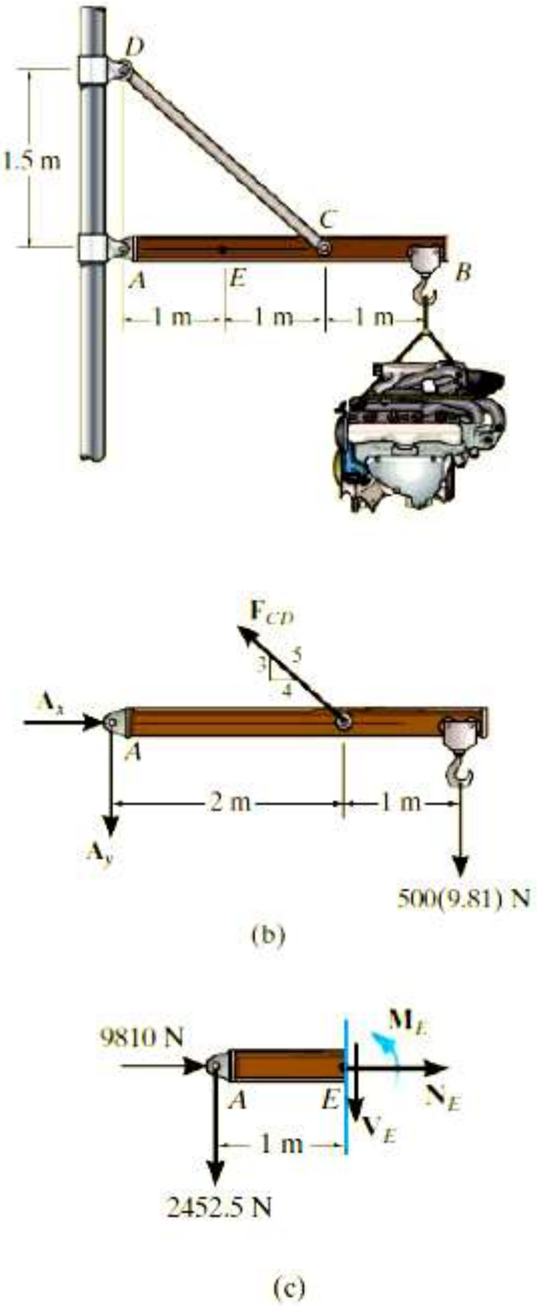

Fig. 1–5

Solution:

Support Reactions. We will consider segment AE of the boom, so we must first determine the pin reactions at A. Notice that member CD is a two-force member. The free-body diagram of the boom is shown in Fig. 1–5b. Applying the equations of equilibrium,

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Free-Body Diagram. The free-body diagram of segment AE is shown in Fig. 1–5c.

Equations of Equilibrium.

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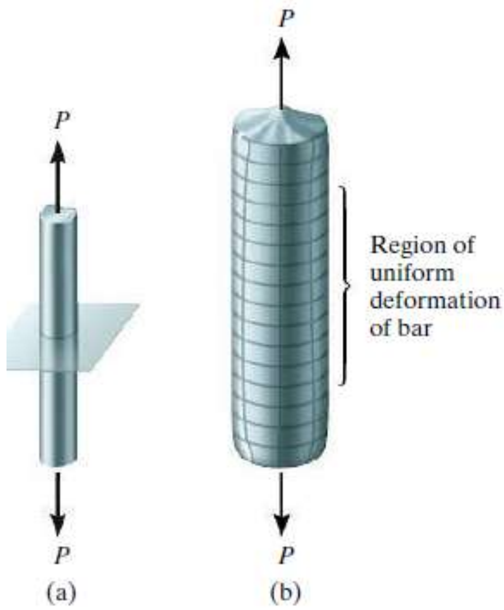
- Average Normal Stress in an Axially Loaded Bar

zIn this section we will determine the average stress distribution acting on the cross-sectional area of an axially loaded bar such as the one shown in Fig. l-12a. This bar is **prismatic** since all cross sections are the same throughout its length. When the load P is applied to the bar through the centroid of its cross-sectional area, then the bar will deform uniformly throughout the central region of its length, as shown in Fig. 1-12b, provided the material of the bar is both homogeneous and isotropic. **Homogeneous material** has the same physical and mechanical properties throughout its volume, and **isotropic material** has these same properties in all directions. Many engineering materials may be approximated as being both homogeneous and isotropic as assumed here. Steel, for example, contains thousands of randomly oriented crystals in each cubic millimeter of its volume, and since most problems involving this material have a physical size that is very much larger than a single crystal, the above assumption regarding its material composition is quite realistic. Note that anisotropic materials such as wood have different properties in different directions, and although this is the case, if the anisotropy is oriented along the bar's axis (as for instance in a typical wood rod), then the bar will also deform uniformly when subjected to the axial load P.



Average Normal Stress Distribution.

If we pass a section through the bar, and separate it into two parts, then equilibrium requires the resultant normal force at the section to be P, Fig. 1-12c. Due to the uniform deformation of the material, it is necessary that the cross section be subjected to a constant normal stress distribution, Fig. 1-12d.

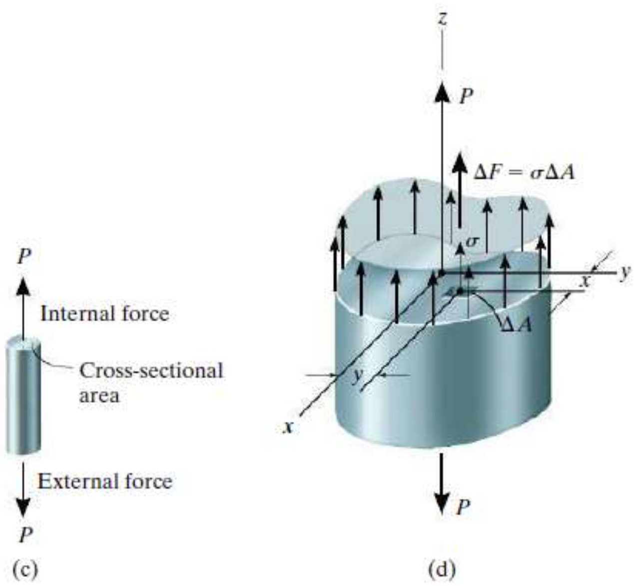


Fig. 1–12

As a result, each small area ΔA on the cross section is subjected to a force $\Delta F = \sigma \, \Delta A$, and the *sum* of these forces acting over the entire cross-sectional area must be equivalent to the internal resultant force **P** at the section. If we let $\Delta A \rightarrow dA$ and therefore $\Delta F \rightarrow dF$, then, recognizing σ is *constant*, we have

$$+\uparrow F_{Rz} = \Sigma F_z; \qquad \int dF = \int_A \sigma \, dA$$

$$P = \sigma A$$

$$\boxed{\sigma = \frac{P}{A}}$$

Here

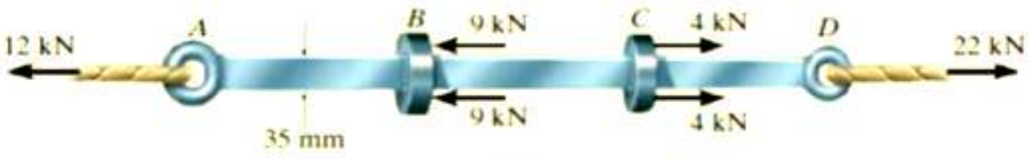
σ = average normal stress at any point on the cross-sectional area

P = *internal resultant normal force*, which acts through the *centroid* of the cross-sectional area. P is determined using the method of sections and the equations of equilibrium

A = cross-sectional area of the bar where σ is determined

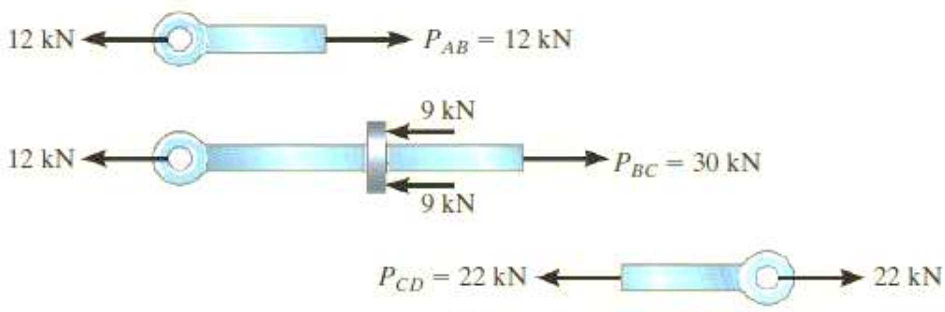
Example:

The bar in Fig. 1-15a has a constant width of 35 mm and a thickness of 10 mm. Determine the maximum average normal stress in the bar when it is subjected to the loading shown.



(a)

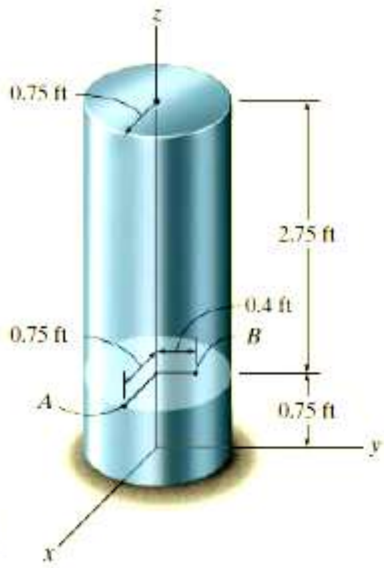
Solution:



(b)

Example:

The casting shown in Fig. 1–17*a* is made of steel having a specific weight of $\gamma_{st} = 490\ \text{lb/ft}^3$. Determine the average compressive stress acting at points A and B.



(a)

Solution:

$$+\uparrow \Sigma F_z = 0: \qquad P - W_{st} = 0$$

$$P - (490\ \text{lb/ft}^3)(2.75\ \text{ft})\left[\pi(0.75\ \text{ft})^2\right] = 0$$

$$P = 2381\ \text{lb}$$

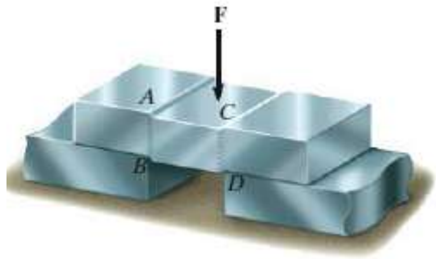
Average Compressive Stress. The cross-sectional area at the section is $A = \pi(0.75\ \text{ft})^2$, and so the average compressive stress becomes

$$\sigma = \frac{P}{A} = \frac{2381\ \text{lb}}{\pi(0.75\ \text{ft})^2} = 1347.5\ \text{lb/ft}^2$$

$$\sigma = 1347.5\ \text{lb/ft}^2(1\ \text{ft}^2/144\ \text{in}^2) = 9.36\ \text{psi} \qquad \textit{Ans.}$$

- Average Shear stress

Shear stress has been defined as the stress component that acts in the plane of the sectioned area. To show how this stress can develop, consider the effect of applying a force F to the bar in Fig. 1-19a. If the supports are considered rigid, and F is large enough, it will cause the material of the bar to deform and fail along the planes identified by AB and CD. A free-body diagram of the unsupported center segment of the bar, Fig. 1-19b, indicates that the shear force V = F/2 must be applied at each section to hold the segment in equilibrium. The average shear stress distributed over each sectioned area that develops this shear force is defined by;



(a)

$$\tau = \frac{V}{A}$$

(b)

Here

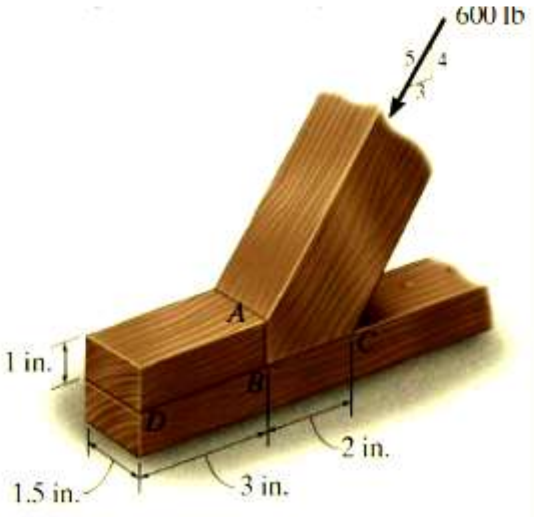
τ = average shear stress at the section, which is assumed to be the same at each point located on the section

V = internal resultant shear force on the section determined from the equations of equilibrium

A = area at the section

Example:

The inclined member in Fig. 1–23 a is subjected to a compressive force of 600 lb. Determine the average compressive stress along the smooth areas of contact defined by AB and BC , and the average shear stress along the horizontal plane defined by DB



(a)

Solution:

Internal Loadings. The free-body diagram of the inclined member is shown in Fig. 1–23*b*. The compressive forces acting on the areas of contact are

$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_{AB} - 600\text{ lb}\left(\tfrac{3}{5}\right) = 0 \qquad F_{AB} = 360\text{ lb}$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{BC} - 600\text{ lb}\left(\tfrac{4}{5}\right) = 0 \qquad F_{BC} = 480\text{ lb}$$

Also, from the free-body diagram of the top segment *ABD* of the bottom member, Fig. 1–23*c*, the shear force acting on the sectioned horizontal plane *DB* is

$$\xrightarrow{+} \Sigma F_x = 0; \qquad V - 360\text{ lb} = 0 \qquad V = 360\text{ lb}$$

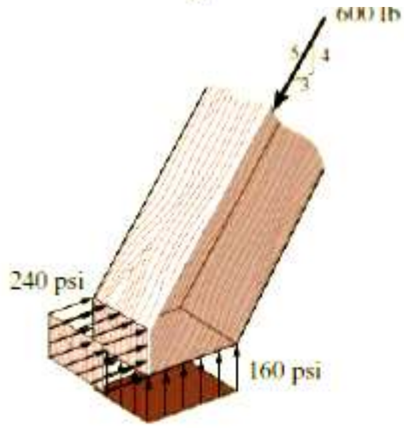
(c)

Average Stress. The average compressive stresses along the horizontal and vertical planes of the inclined member are

$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{360\text{ lb}}{(1\text{ in.})(1.5\text{ in.})} = 240\text{ psi} \qquad \textit{Ans.}$$

$$\sigma_{BC} = \frac{F_{BC}}{A_{BC}} = \frac{480\text{ lb}}{(2\text{ in.})(1.5\text{ in.})} = 160\text{ psi} \qquad \textit{Ans.}$$

These stress distributions are shown in Fig. 1–23*d*.

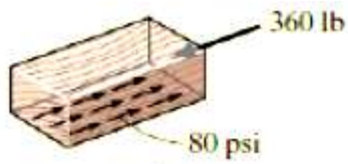


(d)

The average shear stress acting on the horizontal plane defined by *DB* is

$$\tau_{avg} = \frac{360\text{ lb}}{(3\text{ in.})(1.5\text{ in.})} = 80\text{ psi} \qquad \textit{Ans.}$$

This stress is shown uniformly distributed over the sectioned area in Fig. 1–23*e*.



(e)

- STRESS ON AN OBLIQUE PLANE UNDER AXIAL LOADING

In the preceding sections, axial forces exerted on a two-force member (Fig. 1 .26a) were found to cause normal stresses in that member (Fig. 1 .26b), while transverse forces exerted on bolts and pins (Fig. 1 .27a) were fo und to cause shearing stresses in those connections (Fig. 1 .27b). The reason such a relation was observed between axial forces and normal stresses on one hand, and trans verse forces and shearing stresses on the other, was because stresses were being determined only on planes perpendicular to the axis of the member or connection. As you will see in this section, axial forces cause both normal and shearing stresses on planes which are not perpendicular to the axis of the member. Similarly, transverse forces exerted on a bolt or a pin cause both normal and shearing stresses on planes which are not perpendicular to the axis of the bolt or pin.

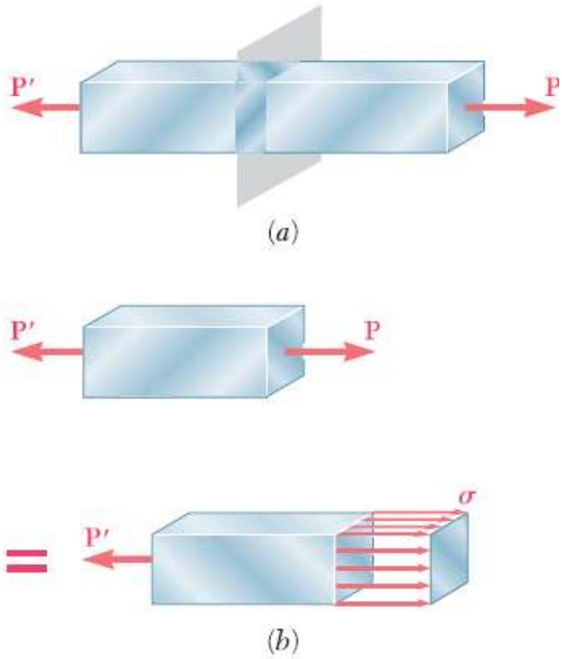


Fig. 1.26 Axial forces.

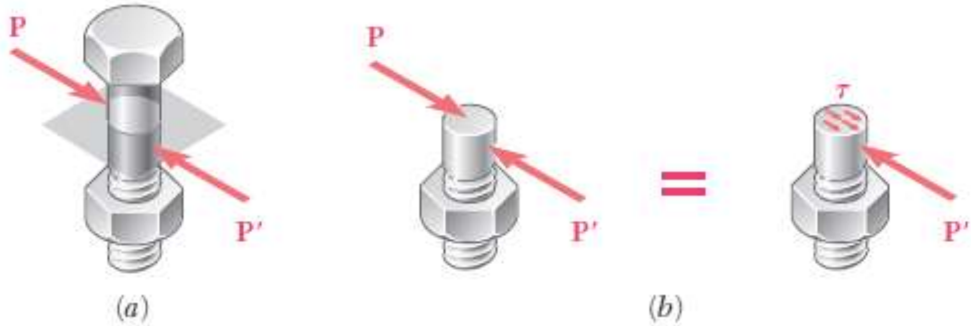


Fig. 1.27 Transverse forces.

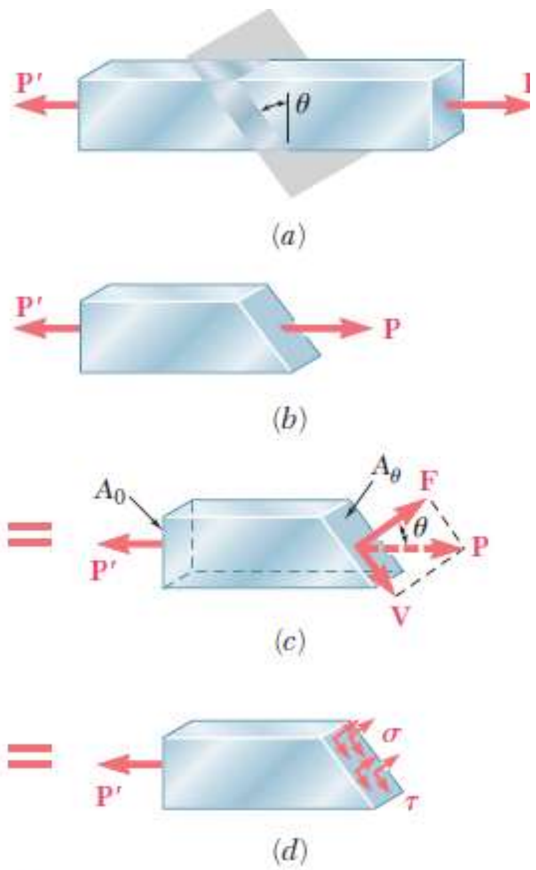


Fig. 1.28

Consider the two-force member of Fig. 1.26, which is subjected to axial forces **P** and **P**'. If we pass a section forming an angle u with a normal plane (Fig. 1.28*a*) and draw the free-body diagram of the portion of member located to the left of that section (Fig. 1.28*b*), we find from the equilibrium conditions of the free body that the distributed forces acting on the section must be equivalent to the force **P.**

Resolving P into components F and V, respectively normal and tangential to the section (Fig. 1.28c), we have

$$F = P\cos\theta \qquad V = P\sin\theta$$

The force **F** represents the resultant of normal forces distributed over the section, and the force **V** the resultant of shearing forces (Fig. 1.28*d*). The average values of the corresponding normal and shearing stresses are obtained by dividing, respectively, *F* and *V* by the area $A\theta$ of the section:

$$\sigma = \frac{F}{A_\theta} \qquad \tau = \frac{V}{A_\theta}$$

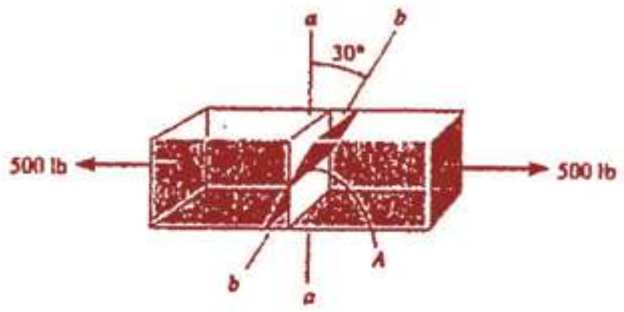
Substituting for *F* and *V* from (1.12) into (1.13), and observing from Fig. 1.28*c* that $A_0 = A\theta \cos\theta$, or $A\theta = A_0/\cos\theta$, where *A*0 denotes the area of a section perpendicular to the axis of the member, we obtain.

$$\sigma = \frac{P\cos\theta}{A_0/\cos\theta} \qquad \tau = \frac{P\sin\theta}{A_0/\cos\theta}$$

$$\sigma = \frac{P}{A_0}\cos^2\theta \qquad \tau = \frac{P}{A_0}\sin\theta\cos\theta$$

Example:

Determine the resultant internal normal and shear stresses in the member at **(a)** section a-a **(b)** section b-b, each of which passes through point A. The 500-1b load is applied along the centroid of the member. Take cross-sec. of a-a sec. = 64in2



Solution:

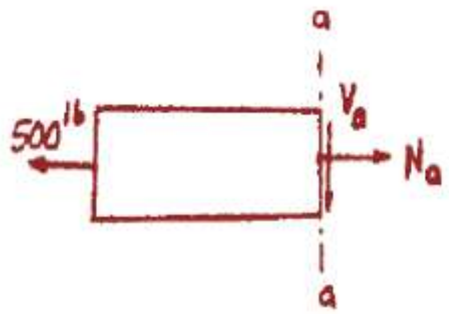
Section a-a

$\sum Fx = 0$;

Na – 500 = 0; Na = 500 lb

$\sum Fy = 0$; Va = 0

σa = P/Aa = 500/64 = 7.81 lb/in2



Section b-b

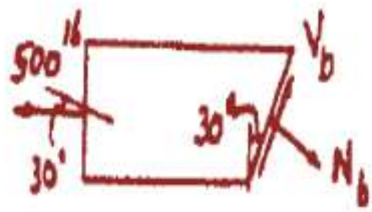
$\sum Fx = 0$; Nb – 500 Cos 30 = 0; Nb = 433 lb

$\sum Fy = 0$; Vb – 500 Sin 30 = 0; Vb = 250 lb

Ab = Aa / Cos 30 = (64 / Cos 30) = 73.9 in2

$\sigma_a = N_b / A_b = (433/73.9) = 5.85\ lb/in^2$

$\tau = V_b/A_b = (250/73.9) = 3.38\ lb/in^2$



- BEARING STRESS IN CONNECTIONS

Bolts, pins, and rivets create stresses in the members they connect, along the bearing surface, or surface of contact. For example, consider again the two plates **A** and **B** connected by a bolt **CD** that we have discussed in the preceding section (Fig. 1.16). The bolt exerts on plate **A** a force **P** equal and opposite to the force F exerted by the plate on the bolt (Fig. 1.20). The force **P** represents the resultant of elementary forces distributed on the inside surface of a half-cylinder of diameter **d** and of length **t** equal to the thickness of the plate. Since the distribution of these forces and of the corresponding stresses is quite complicated, one uses in practice an average nominal value **b** of the stress, called the bearing stress, obtained by dividing the load **P** by the area of the rectangle representing the projection of the bolt on the plate section (Fig. 1 .21). Since this area is equal to td, where t is the plate thickness and d the diameter of the bolt, we have.

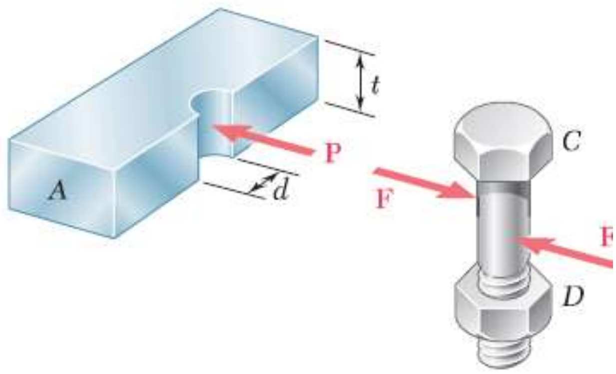


Fig. 1.20

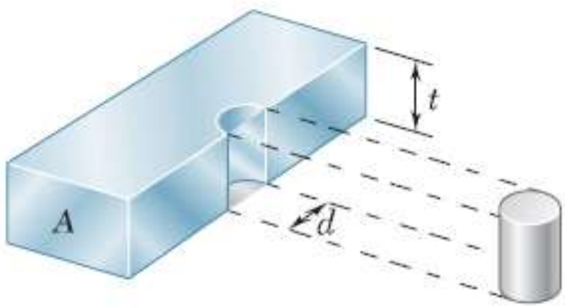


Fig. 1.21

$$\sigma_b = \frac{P}{A} = \frac{P}{td}$$

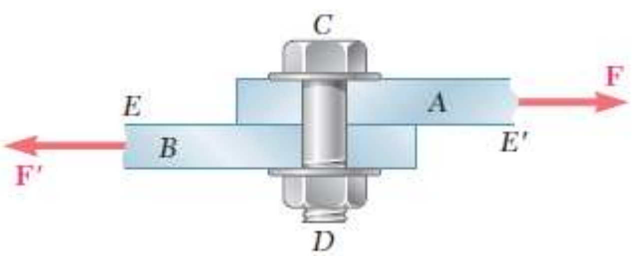


Fig. 1.16 Bolt subject to single shear.

Example:

In Fig. 1-12, assume that a 20-mm-diameter rivet joins the plates that are each 110 mm wide. **The allowable stresses are 120 MPa for bearing** in the plate material and **60 MPa for shearing** of rivet. Determine (a) the minimum thickness of each plate; and (b) the largest average tensile stress in the plates.

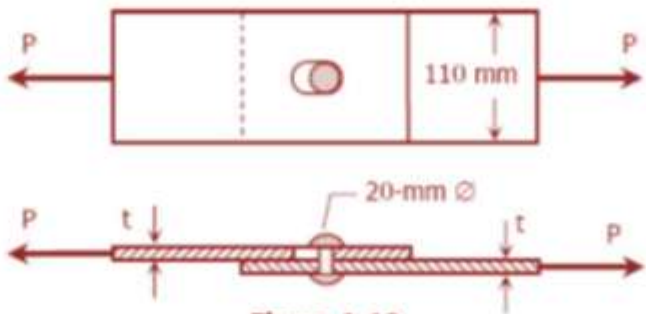


Figure 1-12

Solution:

Part (a):

From shearing of rivet:

$$P = \tau A_{rivets}$$
$$P = 60[\,\tfrac{1}{4}\pi(20^2)\,]$$
$$P = 6000\pi\, text N$$

From bearing of plate material:

$$P = \sigma_b A_b$$
$$6000\pi = 120(20t)$$
$$\text{answer} t = 7.85\text{ mm} \rightarrow$$

Part (b): Largest average tensile stress in the plate:

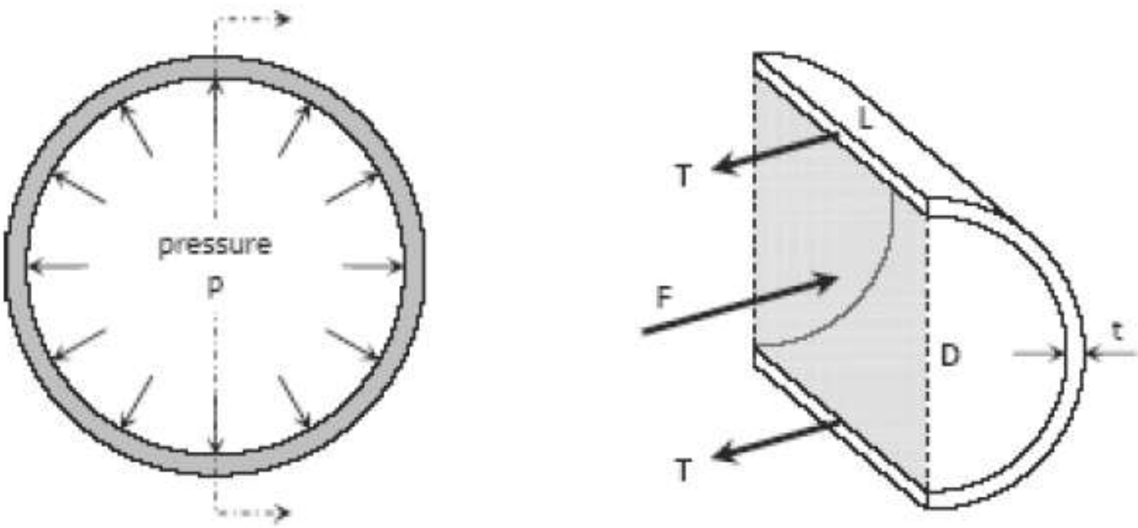
$$P = \sigma A$$
$$6000\pi = \sigma[\,7.85(110 - 20)\,]$$
$$\text{answer} \sigma = 26.67\text{ MPa} \rightarrow$$

- **Thin-Walled Pressure Vessels**

A tank or pipe carrying a fluid or gas under a pressure is subjected to tensile forces, which resist bursting, developed across longitudinal and transverse sections.

TANGENTIAL STRESS

Consider the tank shown being subjected to an internal pressure p. The length of the tank is L and the wall thickness is t. Isolating the right half of the tank:



$$F = pA = pDL$$
$$T = \sigma_t A_{wall} = \sigma_t tL$$
$$[\ \Sigma F_H = 0\ ]$$
$$F = 2T$$
$$pDL = 2(\sigma_t tL)$$
$$\sigma_t = \frac{pD}{2t}$$

If there exist an external pressure po and an internal pressure pi, the formula may be expressed as:

$$\sigma_t = \frac{(p_i - p_o)D}{2t}$$

- LONGITUDINAL STRESS, σ_L

The total force acting at the rear of the tank F must equal to the total longitudinal stress on the wall PT = $\sigma_L A_{wall}$. Since t is so small compared to D, the area of the wall is close to πDt

$$F = pA = p\frac{\pi}{4}D^2$$

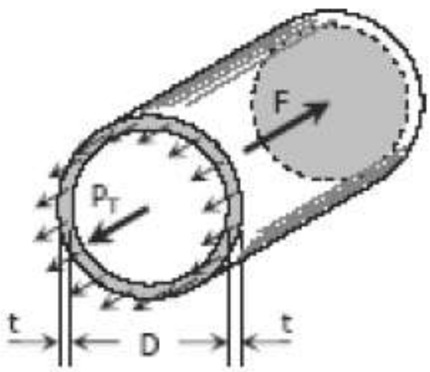
$$P_T = \sigma_L \pi Dt$$

$$[\Sigma F_H = 0]$$

$$P_T = F$$

$$\sigma_L \pi Dt = p\frac{\pi}{4}D^2$$

$$\sigma_L = \frac{pD}{4t}$$



If there exist an external pressure po and an internal pressure pi, the formula may be expressed as:

$$\sigma_L = \frac{(p_i - p_o)D}{4t}$$

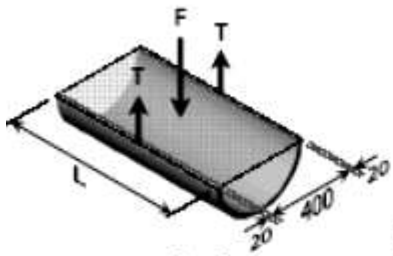
It can be observed that the tangential stress is twice that of the longitudinal stress.

$$\sigma_t = 2\,\sigma_L$$

Example:

A cylindrical steel pressure vessel 400 mm in diameter with a wall thickness of 20 mm, is subjected to an internal pressure of 4.5 MN/m2. (a) Calculate the tangential and longitudinal stresses in the steel. (b) To what value may the internal pressure be increased if the stress in the steel is limited to 120 MN/m2? (c) If the internal pressure were increased until the vessel burst, sketch the type of fracture that would occur.

Solution:



Longitudinal Section

(a) Tangential stress (longitudinal section):

$$F = 2T$$

$$pDL = 2(\sigma_t\, tL)$$

$$\sigma_t = \frac{pD}{2t} = \frac{4.5(400)}{2(20)}$$

$$\sigma_t = 45 \text{ MPa}$$

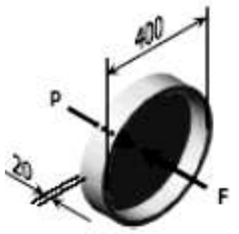
Longitudinal Stress (transverse section)::

$$F = P$$

$$\tfrac{1}{4}\pi D^2 p = \sigma_l\,(\pi D t)$$

$$\sigma_l = \frac{pD}{4t} = \frac{4.5(400)}{4(20)}$$

$$\sigma_l = 22.5 \text{ MPa}$$



Transverse Section

(b) From (a), $\sigma_t = \frac{pD}{2t}$ and $\sigma_l = \frac{pD}{4t}$ thus, $\sigma_t = 2\sigma_l$,

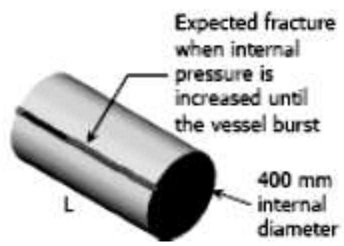
this shows that tangential stress is the critical.

$$\sigma_t = \frac{pD}{2t}$$

$$120 = \frac{p(400)}{2(20)}$$

$$P = 12 \text{ MPa}$$

(c) The bursting force will cause a stress on the longitudinal section that is twice to that of the transverse section. Thus, fracture is expected as shown.



Example:

The strength of longitudinal joint in Fig. 1-17 is 33 kips/ft, whereas for the girth is 16 kips/ft. Calculate the maximum diameter of the cylinder tank if the internal pressure is 150 psi.

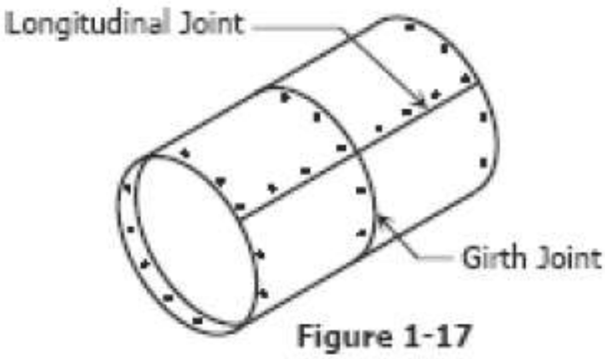


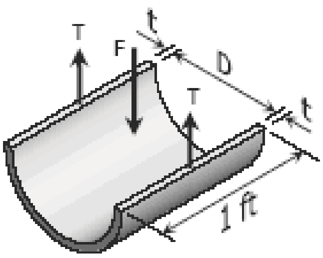
Figure 1-17

Solution:

Internal pressure, p:

$$p = 150 \text{ psi} = \frac{150 \text{ lb}}{\text{in}^2}\left(\frac{12 \text{ in}}{\text{ft}}\right)^2$$

$$p = 21\,600 \text{ lb/ft}^2$$



For longitudinal joint (tangential stress):

Consider 1 ft length

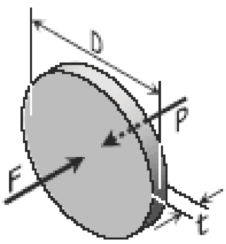
$$F = 2T$$

$$pD = 2\sigma_t t$$

$$\sigma_t = \frac{pD}{2t}$$

$$\frac{33000}{t} = \frac{21\,600\,D}{2t}$$

$$D = 3.06 \text{ ft} = 36.67 \text{ in}$$



For girth joint (longitudinal stress):

$$F = P$$

$$p\left(\tfrac{1}{4}\pi D^2\right) = \sigma_l\,(\pi D t)$$

$$\sigma_l = \frac{pD}{4t}$$

$$\frac{16000}{t} = \frac{21\,600\,D}{4t}$$

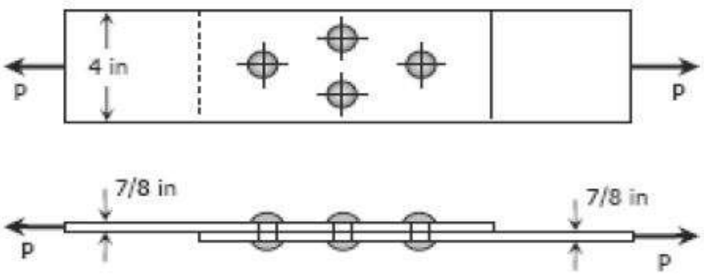
$$D = 2.96 \text{ ft} = 35.56 \text{ in.}$$

Use the smaller diameter, $D = 35.56$ in.

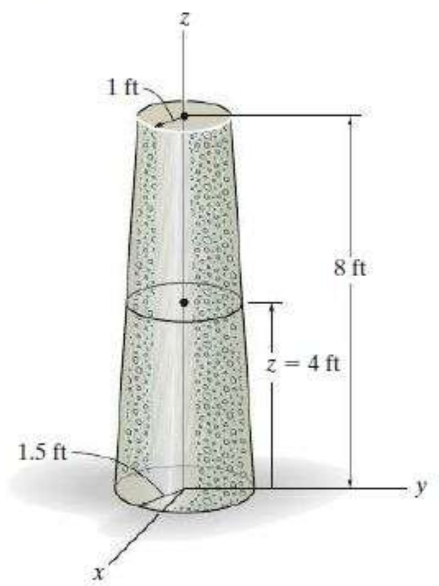
HWs

Problem 126

The lap joint shown in Fig. P-126 is fastened by four ¾-in.-diameter rivets. Calculate the maximum safe load P that can be applied if the shearing stress in the rivets is limited to 14 ksi and the bearing stress in the plates is limited to 18 ksi. Assume the applied load is uniformly distributed among the four rivets.



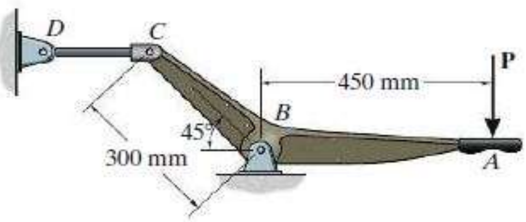
*1–68. The pedestal in the shape of a frustum of a cone is made of concrete having a specific weight of 150 lb/ft^3. Determine the average normal stress acting in the pedestal at its midheight, $z = 4$ ft. *Hint:* The volume of a cone of radius r and height h is $V = \frac{1}{3}\pi r^2 h$.



Probs. 1–67/68

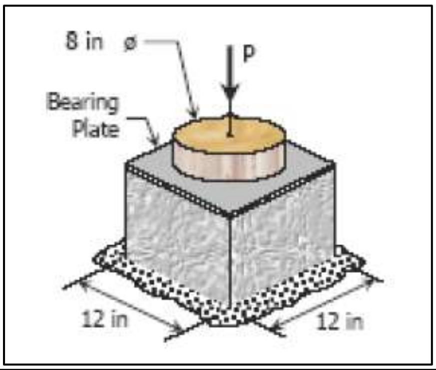
*1–64. A vertical force of $P = 1500$ N is applied to the bell crank. Determine the average normal stress developed in the 10-mm diamater rod CD, and the average shear stress developed in the 6-mm diameter pin B that is subjected to double shear.

1–65. Determine the maximum vertical force **P** that can be applied to the bell crank so that the average normal stress developed in the 10-mm diameter rod CD, and the average shear strees developed in the 6-mm diameter double sheared pin B not exceed 175 MPa and 75 MPa respectively.

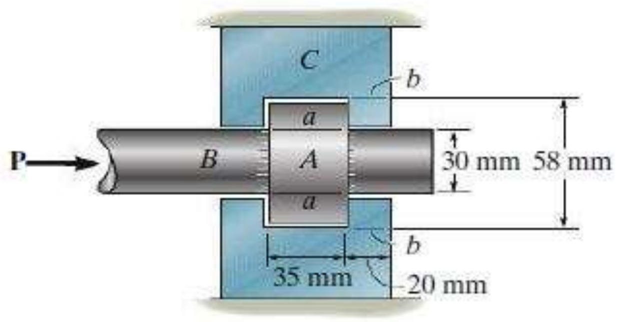


Probs. 1–64/65

1. 12-inches square steel bearing plate lies between an 8-inches diameter wooden post and a concrete footing as shown in Fig. P-110. Determine the maximum value of the load P if the stress in wood is limited to 1800 psi and that in concrete to 650 psi.



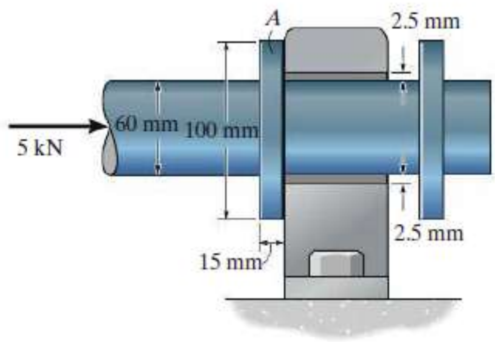
*1–80. The thrust bearing consists of a circular collar A fixed to the shaft B. Determine the maximum axial force P that can be applied to the shaft so that it does not cause the shear stress along a cylindrical surface a or b to exceed an allowable shear stress of $\tau_{allow} = 170$ MPa.



Prob. 1–80

*1–60. If the shaft is subjected to an axial force of 5 kN, determine the bearing stress acting on the collar A.

1–61. If the 60-mm diameter shaft is subjected to an axial force of 5 kN, determine the average shear stress developed in the shear plane where the collar A and shaft are connected.



Probs. 1–60/61

2. Determine the largest weight W that can be supported by two wires shown in Fig. P-109. The stress in either wire is not to exceed 30 ksi. The cross-sectional areas of wires AB and AC are 0.4 in2 and 0.5 in2, respectively.

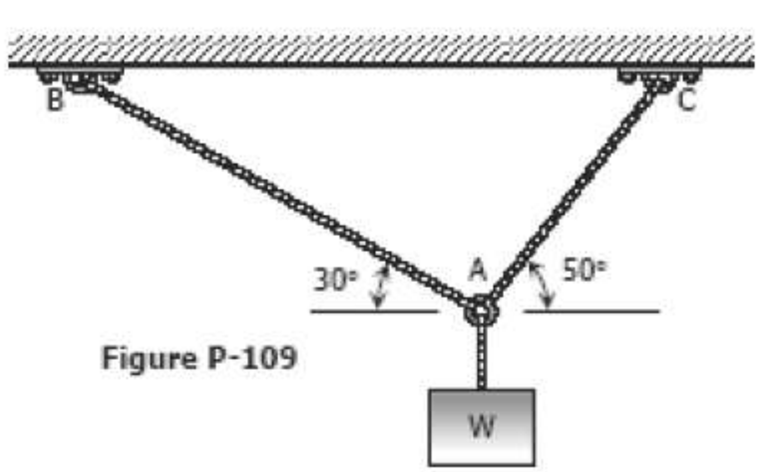
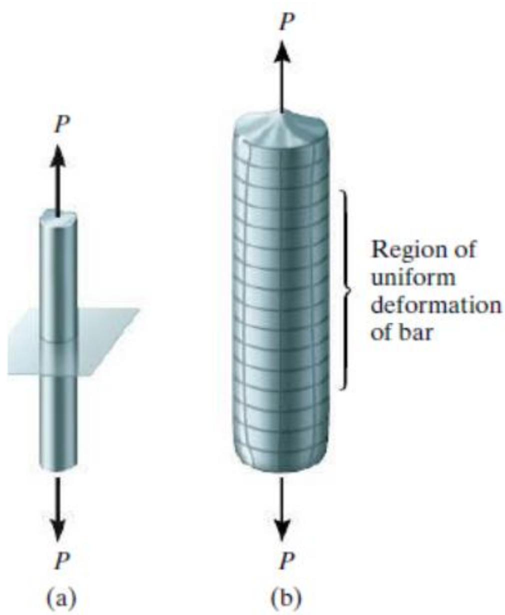


Figure P-109

- Average Normal Stress in an Axially Loaded Bar

zIn this section we will determine the average stress distribution acting on the cross-sectional area of an axially loaded bar such as the one shown in Fig. l-12a. This bar is **prismatic** since all cross sections are the same throughout its length. When the load P is applied to the bar through the centroid of its cross-sectional area, then the bar will deform uniformly throughout the central region of its length, as shown in Fig. 1-12b, provided the material of the bar is both homogeneous and isotropic. **Homogeneous material** has the same physical and mechanical properties throughout its volume, and **isotropic material** has these same properties in all directions. Many engineering materials may be approximated as being both homogeneous and isotropic as assumed here. Steel, for example, contains thousands of randomly oriented crystals in each cubic millimeter of its volume, and since most problems involving this material have a physical size that is very much larger than a single crystal, the above assumption regarding its material composition is quite realistic. Note that anisotropic materials such as wood have different properties in different directions, and although this is the case, if the anisotropy is oriented along the bar's axis (as for instance in a typical wood rod), then the bar will also deform uniformly when subjected to the axial load P.



Average Normal Stress Distribution.

If we pass a section through the bar, and separate it into two parts, then equilibrium requires the resultant normal force at the section to be P, Fig. 1-12c. Due to the uniform deformation of the material, it is necessary that the cross section be subjected to a constant normal stress distribution, Fig. 1-12d.

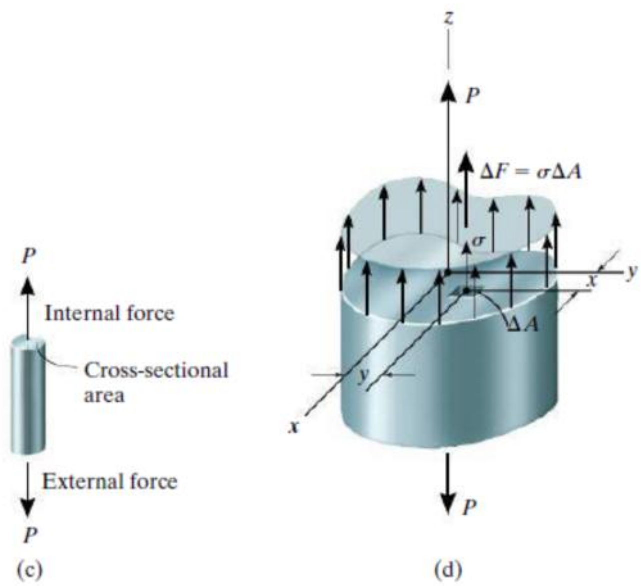


Fig. 1–12

As a result, each small area ΔA on the cross section is subjected to a force $\Delta F = \sigma\,\Delta A$, and the *sum* of these forces acting over the entire cross-sectional area must be equivalent to the internal resultant force **P** at the section. If we let $\Delta A \rightarrow dA$ and therefore $\Delta F \rightarrow dF$, then, recognizing σ is *constant*, we have

$$+\uparrow F_{Rz} = \Sigma F_z; \qquad \int dF = \int_A \sigma\, dA$$

$$P = \sigma A$$

$$\boxed{\sigma = \frac{P}{A}}$$

Here

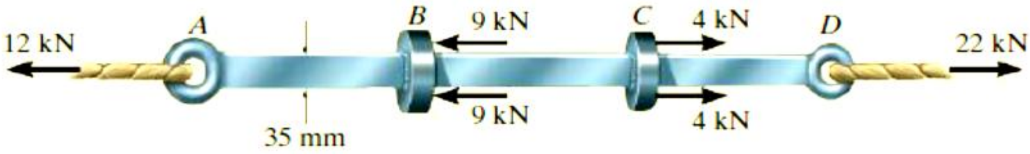
σ = average normal stress at any point on the cross-sectional area

P = *internal resultant normal force*, which acts through the *centroid* of the cross-sectional area. P is determined using the method of sections and the equations of equilibrium

A = cross-sectional area of the bar where σ is determined

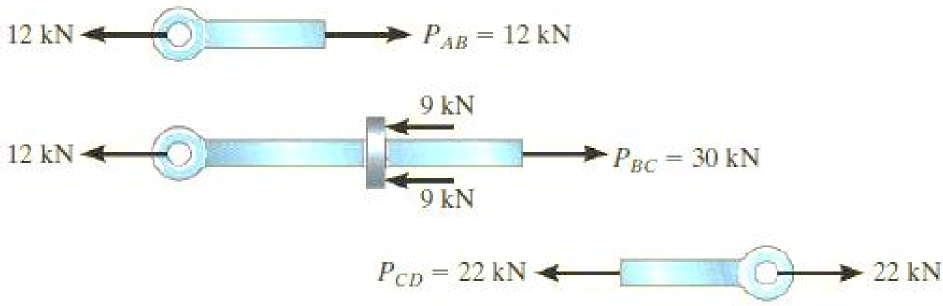
Example:

The bar in Fig. 1-15a has a constant width of 35 mm and a thickness of 10 mm. Determine the maximum average normal stress in the bar when it is subjected to the loading shown.



(a)

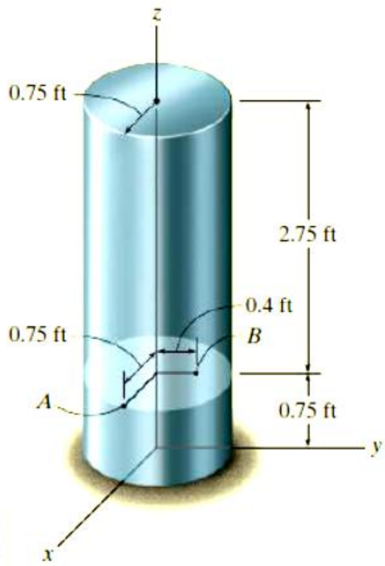
Solution:



(b)

Example:

The casting shown in Fig. 1–17*a* is made of steel having a specific weight of $\gamma_{st} = 490\ \text{lb/ft}^3$. Determine the average compressive stress acting at points *A* and *B*.



(a)

Solution:

$$+\uparrow \Sigma F_z = 0; \qquad P - W_{st} = 0$$

$$P - (490\ \text{lb/ft}^3)(2.75\ \text{ft})\left[\pi(0.75\ \text{ft})^2\right] = 0$$

$$P = 2381\ \text{lb}$$

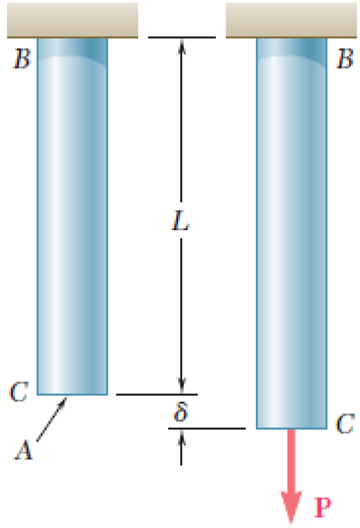
Average Compressive Stress. The cross-sectional area at the section is $A = \pi(0.75\ \text{ft})^2$, and so the average compressive stress becomes

$$\sigma = \frac{P}{A} = \frac{2381\ \text{lb}}{\pi(0.75\ \text{ft})^2} = 1347.5\ \text{lb/ft}^2$$

$$\sigma = 1347.5\ \text{lb/ft}^2(1\ \text{ft}^2/144\ \text{in}^2) = 9.36\ \text{psi} \qquad \textit{Ans.}$$

Strain

- Simple strain

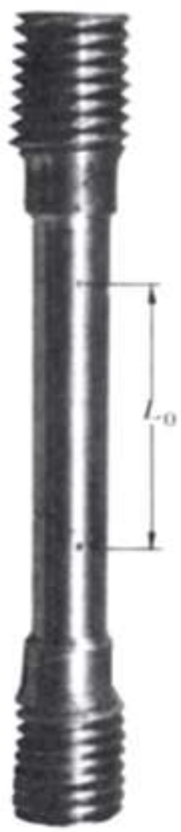


Strain is the ratio of the change in length caused by the applied force, to the original length.

$$\epsilon = \frac{\delta}{L}$$

Where δ is the deformation and L is the original length, thus ε is dimensionless.

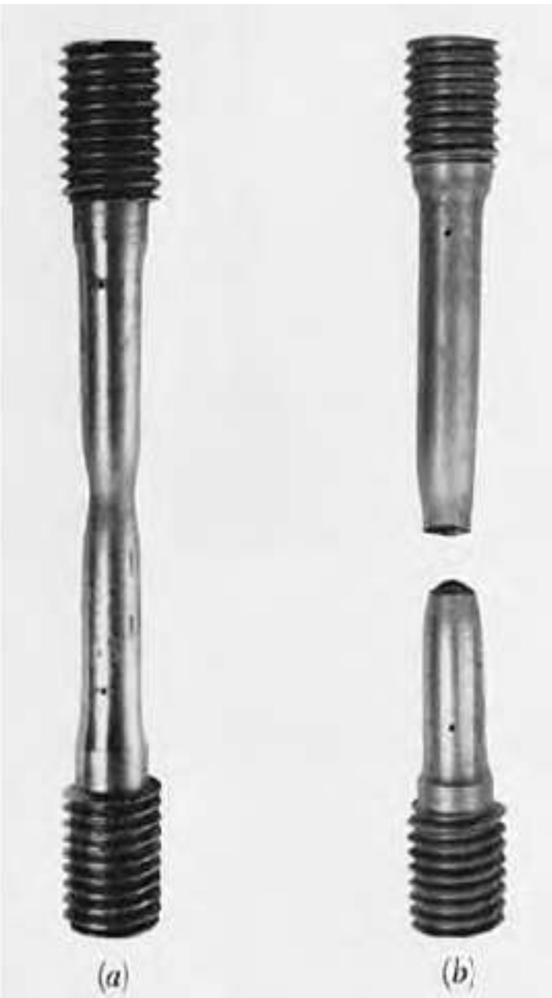
- Stress-Strain Diagram



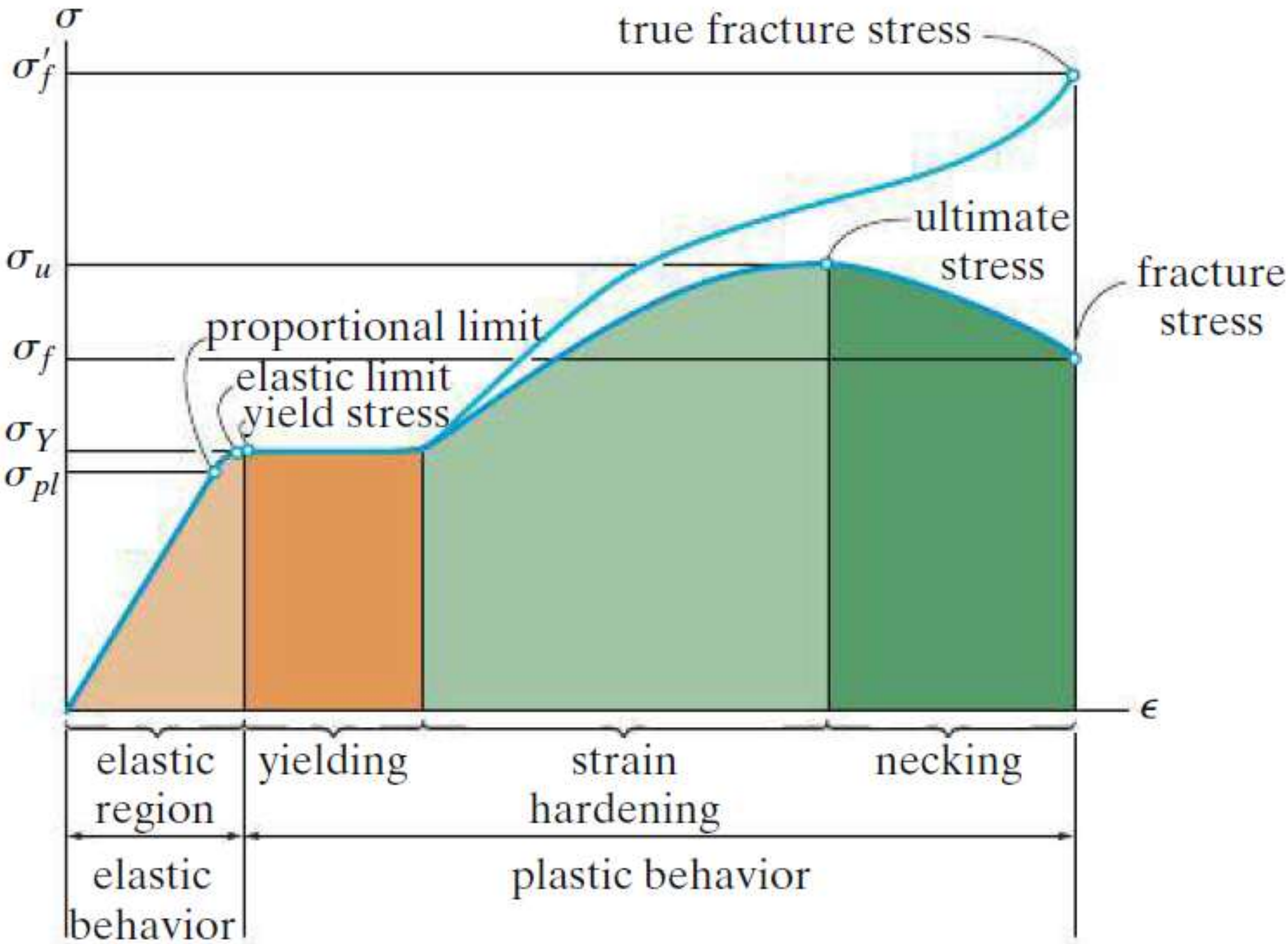
Suppose that a metal specimen be placed in tension-compression testing machine. As the axial load is gradually increased in increments, the total elongation over the gage length is measured at each increment of the load and this is continued until failure of the specimen takes place. Knowing the original cross-sectional area and length of the specimen, the normal stress σ and the strain ε can be obtained. The graph of these quantities with the stress σ along the y-axis and the strain ε along the x-axis is called the stress-strain diagram. The stress-strain diagram differs in form for various materials. The diagram shown below is that for a medium carbon structural steel.

Metallic engineering materials are classified as either ductile or brittle materials. A ductile material is one having relatively large tensile strains up to the point of rupture like structural steel and aluminum, whereas brittle materials has a relatively small strain up to the point of rupture like cast iron and concrete. An arbitrary strain of 0.05 mm/mm is frequently taken as the dividing line between these two classes.

If the corresponding values of s and P are plotted so that the vertical axis is the stress and the horizontal axis is the strain, the resulting curve is called a conventional stress–strain diagram. Realize, however, that two stress–strain diagrams for a particular material will be quite similar, but will never be exactly the same. This is because the results actually depend on variables such as the material's composition, microscopic imperfections, the way it is manufactured, the rate of loading, and the temperature during the time of the test.



We will now discuss the characteristics of the conventional stress–strain curve as it pertains to steel, a commonly used material for fabricating both structural members and mechanical elements. Using the method described above, the characteristic stress–strain diagram for a steel specimen is shown in Fig. 3–4. From this curve we can identify four different ways in which the material behaves, depending on the amount of strain induced in the material.



Conventional and true stress-strain diagrams

Elastic Behavior. Elastic behavior of the material occurs when the strains in the specimen are within the light orange region shown in Fig. 3–4. Here the curve is actually a straight line throughout most of this region, so that the stress is proportional to the strain. The material in this region is said to be linear elastic. The upper stress limit to this linear relationship is called the proportional limit, σ_{pl}. If the stress slightly exceeds the proportional limit, the curve tends to bend and flatten out as shown. This continues until the stress reaches the elastic limit. Upon reaching this point, if the load is removed the specimen will still return back to its original shape. Normally for steel, however, the elastic limit is seldom determined, since it is very close to the proportional limit and therefore rather difficult to detect.

Yielding. A slight increase in stress above the elastic limit will result in a breakdown of the material and cause it to deform permanently. This behavior is called yielding, and it is indicated by the rectangular dark orange region of the curve. The stress that causes yielding is called the yield stress or yield point, σ_Y , and the deformation that occurs is called plastic deformation . Although not shown in Fig. 3–4و for low-carbon steels or those that are hot rolled, the yield point is often distinguished by two values. The upper yield point occurs first، followed by a sudden decrease in load-carrying capacity to a lower yield point. Notice that once the yield point is reached, then as shown in Fig. 3–4 , the specimen will continue to elongate (strain) without any increase in load . When the material is in this state, it is often referred to as being perfectly plastic.

Strain Hardening. When yielding has ended, an increase in load can be supported by the specimen, resulting in a curve that rises continuously but becomes flatter until it reaches a maximum stress referred to as the ultimate stress, σ_u. The rise in the curve in this manner is called strain hardening, and it is identified in Fig. 3–4 as the region in light green.

Necking. Up to the ultimate stress, as the specimen elongates, its cross-sectional area will decrease. This decrease is fairly uniform over the specimen's entire gauge length; however, just after, at the ultimate stress, the cross-sectional area will begin to decrease in a localized region of the specimen. As a result, a constriction or "neck" tends to form in this region as the specimen elongates further, Fig. 3–5 a . This region of the curve due to necking is indicated in dark green in Fig. 3–4. Here the stress–strain diagram tends to curve downward until the specimen breaks at the fracture stress, σ_f, Fig. 3–5 b.

- Hooke's Law

As noted in the previous section, the stress–strain diagrams for most engineering materials exhibit a linear relationship between stress and strain within the elastic region. Consequently, an increase in stress causes a proportionate increase in strain. This fact was discovered by Robert Hooke in 1676 using springs and is known as Hooke's law . It may be expressed mathematically as

$$\boxed{\sigma = E\epsilon}$$

Here E represents the constant of proportionality, which is called the modulus of elasticity or Young's modulus, named after Thomas Young، who published an account of it in 1807.

- AXIAL DEFORMATION

In the linear portion of the stress-strain diagram, the tress is proportional to strain and is given by

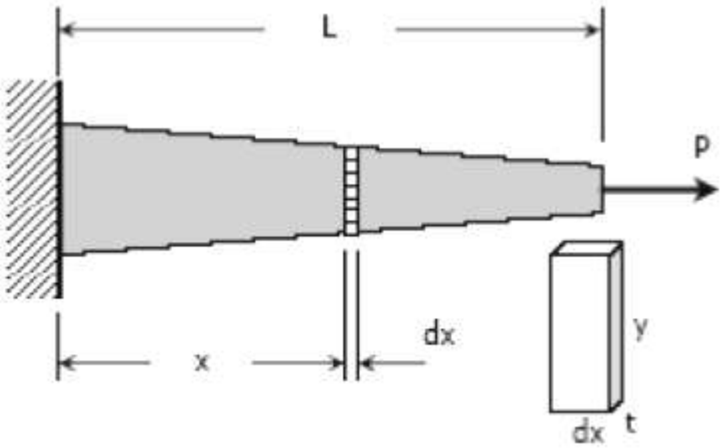
$$\sigma = E\varepsilon$$

Since $\sigma = P / A$ and $\varepsilon e = \delta / L$, then $P / A = E\,\delta / L$. Solving for δ,

$$\delta = \frac{PL}{AE} = \frac{\sigma L}{E}$$

To use this formula, the load must be axial, the bar must have a uniform cross-sectional area, and the stress must not exceed the proportional limit. If however, the cross-sectional area is not uniform, the axial deformation can be determined by considering a differential length and applying integration.

If however, the cross-sectional area is not uniform, the axial deformation can be determined by considering a differential length and applying integration.



$$\delta = \frac{P}{E}\int_0^L \frac{dx}{A}$$

Where A = ty and y and t, if variable, must be expressed in terms of x.

For a rod of unit mass ρ suspended vertically from one end, the total elongation due to its own weight is

$$\delta = \frac{\rho g L^2}{2E} = \frac{MgL}{2AE}$$

Where ρ is in kg/m3, L is the length of the rod in mm, M is the total mass of the rod in kg, A is the cross-sectional area of the rod in mm2, and g = 9.81 m/s^2.

- STIFFNESS, k

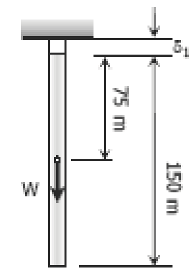
Stiffness is the ratio of the steady force acting on an elastic body to the resulting displacement. It has the unit of N/mm.

$$k = P / \delta$$

Solved problems:

1. A steel rod having a cross-sectional area of 300 mm2 and a length of 150 m is suspended vertically from one end. It supports a tensile load of 20 kN at the lower end. If the unit mass of steel is 7850 kg/m3 and E = 200 × 103 MN/m2, find the total elongation of the rod.

Solution:



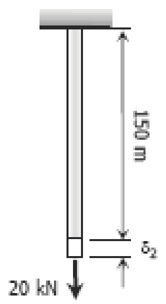
Let δ = total elongation
δ_1 = elongation due to its own weight
δ_2 = elongation due to applied load

$$\delta = \delta_1 + \delta_2$$

$$\delta_1 = \frac{PL}{AE}$$

Where: P = W = 7850(1/1000)3(9.81)[300(150)(1000)]
P = 3465.3825 N
L = 75(1000) = 75 000 mm
A = 300 mm^2
E = 200 000 MPa

$$\delta_1 = \frac{3465.3825\,(75000)}{300\,(200\,000)} = 4.33\text{ mm}$$



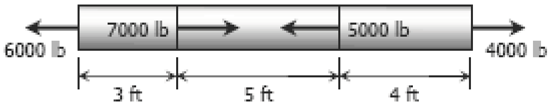
$$\delta_2 = \frac{PL}{AE}$$

Where: P = 20 kN = 20 000 N
L = 150 m = 150 000 mm
A = 300 mm^2
E = 200 000 MPa

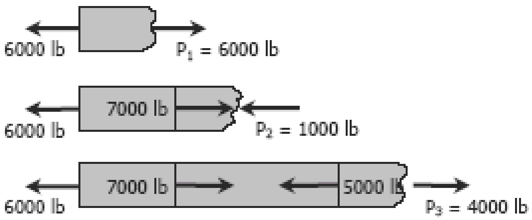
$$\delta_2 = \frac{20000(150000)}{300(200000)} = 50\text{ mm}$$

Total elongation:
δ = 4.33 + 50 = 54.33 mm

2. An aluminum bar having a cross-sectional area of 0.5 in2 carries the axial loads applied at the positions shown below Compute the total change in length of the bar if E = 10 × 106 psi. Assume the bar is suitably braced to prevent lateral buckling.



Solution:



$P_1 = 6000$ lb tension
$P_2 = 1000$ lb compression
$P_3 = 4000$ lb tension

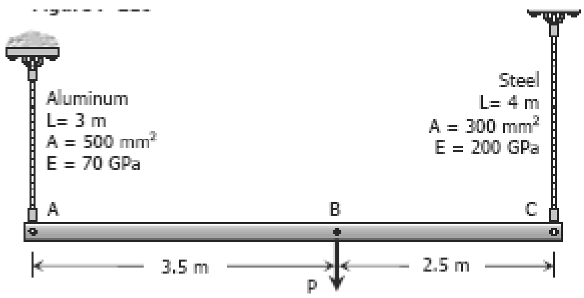
$$\delta = \frac{PL}{AE}$$

$$\delta = \delta_1 - \delta_2 + \delta_3$$

$$\delta = \frac{6000(3\times12)}{0.5(10\times10^6)} - \frac{1000(5\times12)}{0.5(10\times10^6)} + \frac{4000(4\times12)}{0.5(10\times10^6)}$$

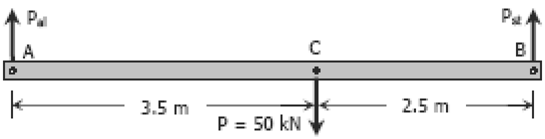
δ = **0.0696 in (lengthening)**

3. The rigid bar AB, attached to two vertical rods as shown below, is horizontal before the load P is applied. Determine the vertical movement of P if its magnitude is 50 kN.



Solution:

Free body diagram:



For aluminum:

$[\Sigma M_B = 0]$ $\quad 6P_{al} = 2.5(50)$
$P_{al} = 20.83$ kN

$$\left[\delta = \frac{PL}{AE}\right]_{al} \qquad \delta_{al} = \frac{20.83(3)1000^2}{500(70000)}$$

$\delta_{al} = 1.78$ mm

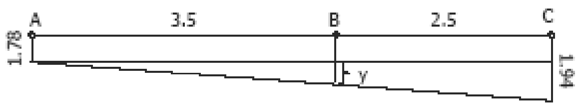
For steel:

$[\Sigma M_A = 0]$ $\quad 6P_{st} = 3.5(50)$
$P_{st} = 29.17$ kN

$$\left[\delta = \frac{PL}{AE}\right]_{st} \qquad \delta_{st} = \frac{29.17(4)1000^2}{300(200000)}$$

$\delta_{st} = 1.94$ mm

Movement diagram:



$$\frac{y}{3.5} = \frac{1.94 - 1.78}{6}$$

$y = 0.09$ mm
δ_B = vertical movement of P
$\delta_B = 1.78 + y = 1.78 + 0.09$
δ_B = **1.87 mm**

4. The following data were recorded during the tensile test of a 14-mm-diameter mild steel rod. The gage length was 50 mm.

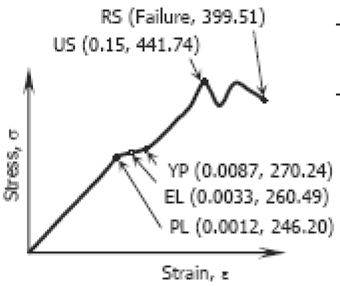
Load (N)	Elongation (mm)	Load (N)	Elongation (mm)
0	0	46200	1.25
6310	0.010	52400	2.50
12600	0.020	58500	4.50
18800	0.030	68000	7.50
25100	0.040	59000	12.50
31300	0.050	67800	15.50
37900	0.060	65000	20.00
40100	0.163	61500	Fracture
41600	0.433		

Plot the stress-strain diagram and determine the following mechanical properties: (a) proportional limits; (b) modulus of elasticity; (c) yield point; (d) ultimate strength; and (e) rupture strength.

Solution:

Area, $A = \frac{1}{4}\pi(14)^2 = 49\pi$ mm²; Length, $L = 50$ mm

Strain = Elongation/Length; Stress = Load/Area



Stress-Strain Diagram
(not drawn to scale)

PL = Proportional Limit
EL = Elastic Limit
YP = Yield Point
US = Ultimate Strength
RS = Rupture Strength

Load (N)	Elongation (mm)	Strain (mm/mm)	Stress (MPa)
0	0	0	0
6310	0.010	0.0002	40.99
12600	0.020	0.0004	81.85
18800	0.030	0.0006	122.13
25100	0.040	0.0008	163.05
31300	0.050	0.001	203.33
37900	0.060	0.0012	246.20
40100	0.163	0.0033	260.49
41600	0.433	0.0087	270.24
46200	1.250	0.025	300.12
52400	2.500	0.05	340.40
58500	4.500	0.09	380.02
68000	7.500	0.15	441.74
59000	12.500	0.25	383.27
67800	15.500	0.31	440.44
65000	20.000	0.4	422.25
61500	Failure		399.51

From stress-strain diagram:

(a) Proportional Limit = **246.20 MPa**

(b) Modulus of Elasticity

E = slope of stress-strain diagram within proportional limit

$$E = \frac{246.20}{0.0012} = 205\,166.67 \text{ MPa}$$

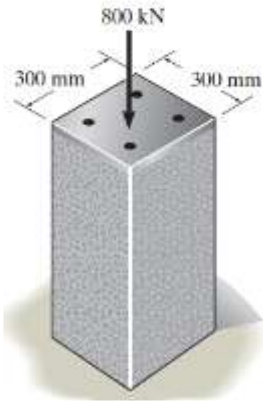
E = 205.2 GPa

(c) Yield Point = **270.24 MPa**

(d) Ultimate Strength = **441.74 MPa**

(e) Rupture Strength = **399.51 MPa**

5. The column is constructed from high-strength concrete and four A-36 steel reinforcing rods. If it is subjected to an axial force of 800 kN, determine the required diameter of each rod so that one-fourth of the load is carried by the steel and three-fourths by the concrete. Est = 200 GPa, Ec = 25 GPa.



Solution:

Equilibrium: Require $P_{st} = \frac{1}{4}(800) = 200$ kN and

$P_{con} = \frac{3}{4}(800) = 600$ kN.

Compatibility:

$$\delta_{con} = \delta_{st}$$

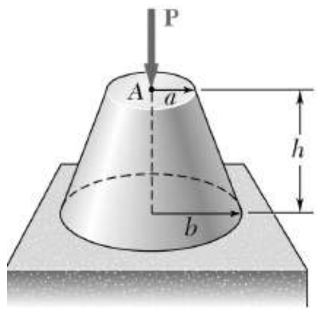
$$\frac{P_{con}L}{(0.3^2 - A_{st})(25.0)(10^9)} = \frac{P_{st}L}{A_{st}(200)(10^9)}$$

$$A_{st} = \frac{0.09P_{st}}{8P_{con} + P_{st}}$$

$$4\left[\left(\frac{\pi}{4}\right)d^2\right] = \frac{0.09(200)}{8(600) + 200}$$

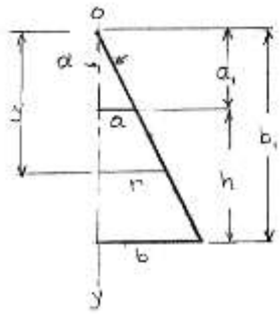
$$d = 0.03385 \text{ m} = 33.9 \text{ mm}$$

6. A vertical load **P** is applied at the center A of the upper section of a homogeneous frustum of a circular cone of height h, minimum radius a, and maximum radius b. Denoting by E the modulus of elasticity of the material and neglecting the effect of its weight, determine the deflection of point A.



Solution:

Extend the slant sides of the cone to meet at a point O and place the origin of the coordinate system there.



From geometry, $\tan\alpha = \dfrac{b-a}{h}$

$$a_1 = \frac{a}{\tan\alpha}, \quad b_1 = \frac{b}{\tan\alpha}, \quad r = y\tan\alpha$$

At coordinate point y, $A = \pi r^2$

Deformation of element of height dy: $d\delta = \dfrac{P\,dy}{AE}$

$$d\delta = \frac{P}{E\pi}\frac{dy}{r^2} = \frac{P}{\pi E\tan^2\alpha}\frac{dy}{y^2}$$

Total deformation.

$$\delta_A = \frac{P}{\pi E\tan^2\alpha}\int_{a_1}^{b_1}\frac{dy}{y^2} = \frac{P}{\pi E\tan^2\alpha}\left(-\frac{1}{y}\right)\Bigg|_{a_1}^{b_1} = \frac{P}{\pi E\tan^2\alpha}\left(\frac{1}{a_1} - \frac{1}{b_1}\right)$$

$$= \frac{P}{\pi E\tan^2\alpha}\frac{b_1 - a_1}{a_1 b_1} = \frac{P(b_1 - a_1)}{\pi E ab} \qquad \delta_A = \frac{Ph}{\pi E ab}\downarrow$$

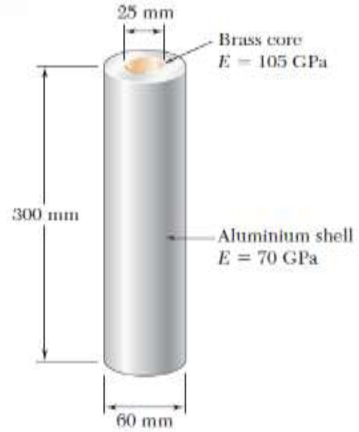
HWs.

B.J 2.30

A homogeneous cable of length L and uniform cross section is suspended from one end. (a) Denoting by ρ the density (mass per unit volume) of the cable and by E its modulus of elasticity, determine the elongation of the cable due to its own weight. (b) Show that the same elongation would be obtained if the cable were horizontal and if a force equal to half of its weight were applied at each end.

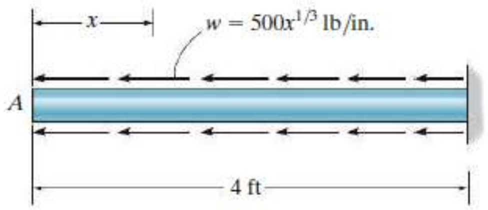
B. 2.34

The length of the assembly shown decreases by 0.40 mm when an axial force is applied by means of rigid end plates. Determine (a) the magnitude of the applied force, (b) the corresponding stress in the brass core.



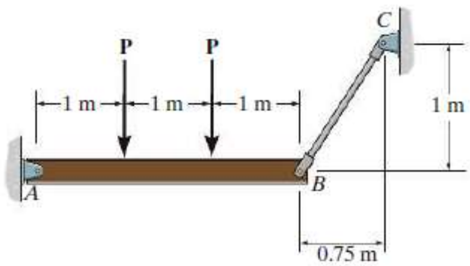
H.4-6

The bar has a cross-sectional area of 3 in2, and E = 35 (103) Ksi. Determine the displacement of its end A when it is subjected to the distributed loading.



H.3–28.

If P = 150 kN, determine the elastic elongation of rod BC and the decrease in its diameter. Rod BC is made of A-36 streel and has a diameter of 40 mm.



- *Thermal Stress*

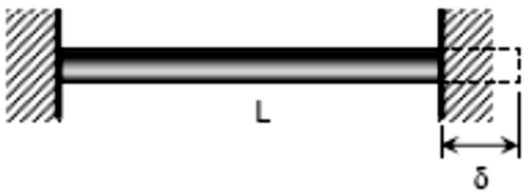
Temperature changes cause the body to expand or contract. The amount δ_T, is given by

$$\delta_T = \alpha L(T_f - T_i) = \alpha L\,\Delta T$$

Where α is the coefficient of thermal expansion in m/m°C, L is the length in meter, and T_i and T_f are the initial and final temperatures, respectively in °C.

If temperature deformation is permitted to occur freely, no load or stress will be induced in the structure. In some cases where temperature deformation is not permitted, an internal stress is created. The internal stress created is termed as thermal stress.

For a homogeneous rod mounted between unyielding supports as shown, the thermal stress is computed as:



Deformation due to temperature changes;

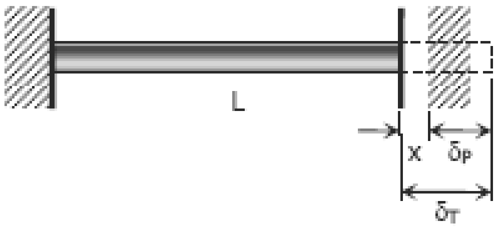
$$\delta_T = \alpha L\,\Delta T$$

Deformation due to equivalent axial stress;

$$\delta_P = \frac{PL}{AE} = \frac{\sigma L}{E}$$
$$\delta_T = \delta_P$$
$$\alpha L\,\Delta T = \frac{\sigma L}{E}$$
$$\sigma = E\,\alpha\,\Delta T$$

Where σ is the thermal stress in MPa and E is the modulus of elasticity of the rod in MPa.

If the wall yields a distance of x as shown, the following calculations will be made:



$$\delta_T = x + \delta_P$$
$$\alpha L\,\Delta T = x\;\frac{\sigma L}{E}$$

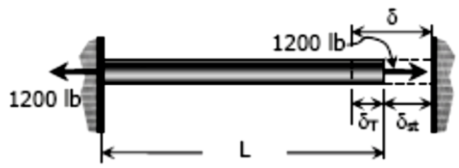
Where σ is represents the thermal stress.

Take note that as the temperature rises above the normal, the rod will be in compression, and if the temperature drops below the normal, the rod is in tension.

Solved Problems in Thermal Stress

Q1. A steel rod with a cross-sectional area of 0.25 in2 is stretched between two fixed points. The tensile load at 70°F is 1200 lb. What will be the stress at 0°F? At what temperature will the stress be zero? Assume α = 6.5 × 10-6 in / (in·°F) and E = 29 × 106 psi.

Solution



For the stress at 0°C:

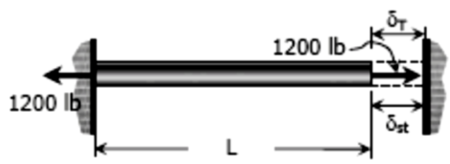
$$\delta = \delta_T + \delta_{st}$$

$$\frac{\sigma \cancel{L}}{E} = \alpha \cancel{L}(\Delta T) + \frac{P\cancel{L}}{AE}$$

$$\sigma = \alpha E(\Delta T) + \frac{P}{A}$$

$$\sigma = (6.5 \times 10^{-6})(29 \times 10^{6})(70) + \frac{1200}{0.25}$$

$$\sigma = 17\,995 \text{ psi} = \mathbf{18\ ksi}$$



For the temperature that causes zero stress:

$$\delta_T = \delta_{st}$$

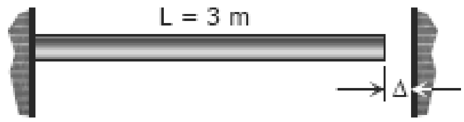
$$\alpha \cancel{L}(\Delta T) = \frac{P\cancel{L}}{AE}$$

$$(6.5 \times 10^{-6})(T - 70) = \frac{1200}{0.25(29 \times 10^{6})}$$

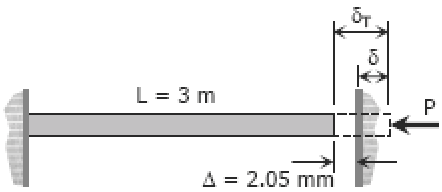
$$T = \mathbf{95.46°C}$$

Q2. A bronze bar 3 m long with a cross sectional area of 320 mm2 is placed between two rigid walls as shown in Fig. P-265. At a temperature of -20°C, the gap Δ = 25 mm. Find the temperature at which the compressive stress in the bar will be 35 MPa. Use α = 18.0 × 10-6 m/(m·°C) and E = 80 GPa.

Figure P-265



Solution



$$\delta_T = \delta + \Delta$$

$$\alpha L(\Delta T) = \frac{\sigma L}{E} + 2.5$$

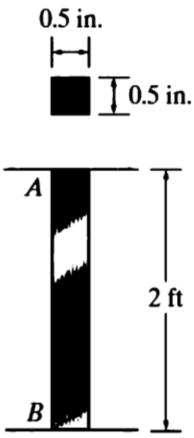
$$(18 \times 10^{-6})(3000)(\Delta T) = \frac{35(3000)}{80000} + 2.5$$

$$\Delta T = 70.6°C$$

$$T = 70.6 - 20$$

$$T = 50.6°C$$

Q3. The A-36 steel bar shown below is constrained to just fit between two fixed supports when T 1 = 60°F. If the temperature is raised to T 2 = 120°F, determine the average normal thermal stress developed in the bar.



Solution

$$\delta_{A/B} = 0 = \delta_T - \delta_F$$

$$0 = \alpha \Delta T L - \frac{FL}{AE}$$

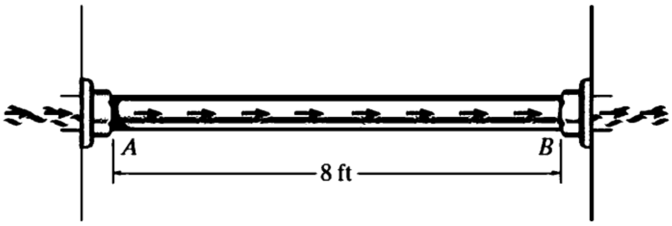
$$F = \alpha \Delta T A E$$

$$= [6.60(10^{-6})/°F](120°F - 60°F)(0.5\text{ in.})^2\,[29(10^3)\text{ kip/in}^2]$$

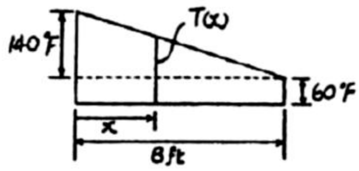
$$= 2.871\text{ kip}$$

$$\sigma = \frac{F}{A} = \frac{2.871\text{ kip}}{(0.5\text{ in.})^2} = 11.5\text{ ksi}$$

Q4. The bronze C86100 pipe has an inner radius of 0.5 in. and a wall thickness of 0.2 in. If the gas flowing through it changes the temperature of the pipe uniformly from T A = 200°F at A to T B = 60°F at B, determine the axial force it exerts on the walls. The pipe was fitted between the walls when T = 60°F.



Solution



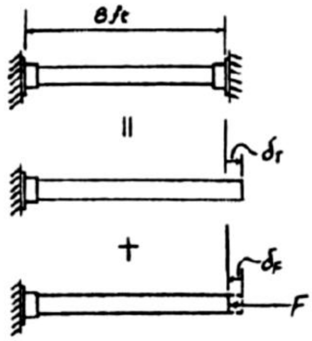
$$T(x) = 60 + \left(\frac{8-x}{8}\right)140 = 200 - 17.5x$$

$$0 = \delta_T - \delta_F \qquad \text{Where} \qquad \delta_T = \int \alpha \Delta T dx$$

$$0 = 9.60\left(10^{-6}\right)\int_0^{8\text{ ft}} [(200 - 17.5x) - 60]\, dx - \frac{F(8)}{\frac{\pi}{4}(1.4^2 - 1^2)15.0(10^3)}$$

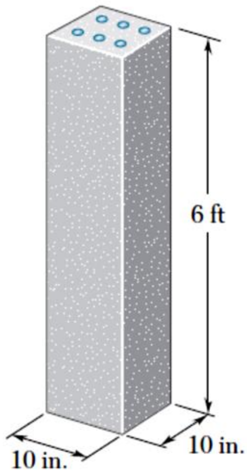
$$0 = 9.60\left(10^{-6}\right)\int_0^{8\text{ ft}} (140 - 17.5x)\, dx - \frac{F(8)}{\frac{\pi}{4}(1.4^2 - 1^2)\,15.0(10^3)}$$

$$F = 7.60 \text{ kip}$$

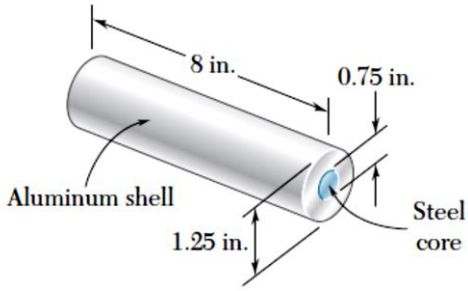


HW

1. The concrete post (E_c = 3.6 x 10^6 psi and α_c = 5.5 x 10^{-6}/ ^{0}F) is reinforced with six steel bars, each of 78 -in diameter (E_s = 29 x 10^6 psi and α_s = 6.5 x 10^{-6}/ ^{0}F). Determine the normal stresses induced in the steel and in the concrete by a temperature rise of 65^{0}F.

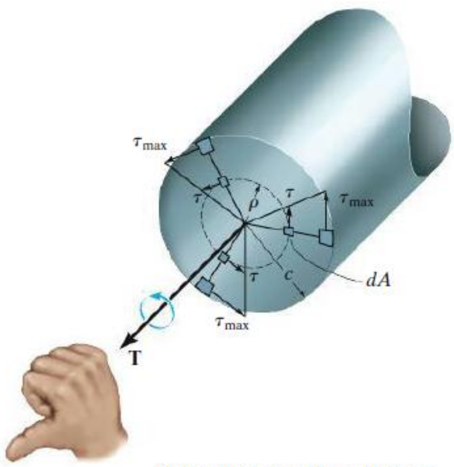


2. The assembly shown consists of an aluminum shell (E_a = 10.6 x 10^6 psi, α_a = 12.9 x 10^{-6}/ ^{0}F) fully bonded to a steel core (E_s =29x10^6 psi, α_s = 6.5 x 10^{-6}/ ^{0}F) and is unstressed. Determine a) the largest allowable change in temperature if the stress in the aluminum shell is not to exceed 6 ksi, (b) the corresponding change in length of the assembly.



- *Torsion*

When an external torque is applied to a shaft, it creates a corresponding internal torque within the shaft. In this section, we will develop an equation that relates this internal torque to the shear stress distribution on the cross section of a circular shaft or tube. If the material is linear-elastic, then Hooke's law applies, $\tau = G\gamma$, and consequently a linear variation in shear strain, as noted in the previous section, leads to a corresponding linear variation in shear stress along any radial line on the cross section. Hence, τ will vary from zero at the shaft's longitudinal axis to a maximum value, τ , at its outer surface .This variation is shown in Fig. 5-5 on the front faces of a selected number of elements, located at an intermediate radial position p and at the outer radius c. Due to the proportionality of triangles, we can write



Shear stress varies linearly along each radial line of the cross section.

Fig. 5–5

$$\tau = \left(\frac{\rho}{c}\right)\tau_{max} \qquad (5\text{–}3)$$

Equation expresses the shear-stress distribution over the cross section in terms of the radial position p of the element. Using it, we can now apply the condition that requires the torque produced by the stress distribution over the entire cross section to be equivalent to the resultant internal torque T at the section, which holds the shaft in equilibrium, Fig. 5-5.

Specifically, each element of area dA, located at p, is subjected to a force of dF= τ dA. The torque produced by this force is dT =(τ dA). We therefore have for the entire cross section

$$T = \int_A \rho(\tau\, dA) = \int_A \rho\left(\frac{\rho}{c}\right)\tau_{max}\, dA \qquad (5\text{–}4)$$

Since τ_{max}/c is constant,

$$T = \frac{\tau_{max}}{c}\int_A \rho^2\, dA \qquad (5\text{–}5)$$

The integral depends only on the geometry of the shaft. It represents the polar moment of inertia of the shaft's cross-sectional area about the shaft's longitudinal axis. We will symbolize its value as J, and therefore the above equation can be rearranged and written in a more compact form, namely

$$\boxed{\tau_{max} = \frac{Tc}{J}} \qquad (5\text{–}6)$$

Here

τ_{max} = the maximum shear stress in the shaft, which occurs at the outer surface

T = the resultant *internal torque* acting at the cross section. Its value is determined from the method of sections and the equation of moment equilibrium applied about the shaft's longitudinal axis

J = the polar moment of inertia of the cross-sectional area

c = the outer radius of the shaft

Combining Eqs. 5–3 and 5–6, the shear stress at the intermediate distance ρ can be determined from

$$\boxed{\tau = \frac{T\rho}{J}} \qquad (5\text{–}7)$$

- ***Solid Shaft***

If the shaft has a solid circular cross section, the polar moment of inertia J can be determined using an area element in the form of a differential ring or annulus having a thickness dρ and circumference 2πρ, Fig. 5-6. For this ring, dA = 2πρdρ, and so

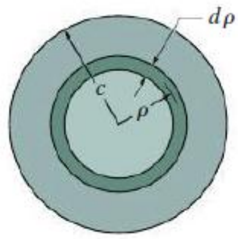


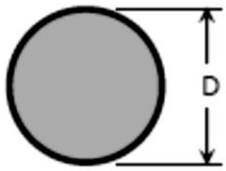
Fig. 5–6

$$J = \int_A \rho^2\,dA = \int_0^c \rho^2(2\pi\rho\,d\rho) = 2\pi\int_0^c \rho^3\,d\rho = 2\pi\left(\frac{1}{4}\right)\rho^4\Big|_0^c$$

$$\boxed{J = \frac{\pi}{2}c^4} \qquad (5\text{–}8)$$

$$J = \frac{\pi}{32}D^4$$

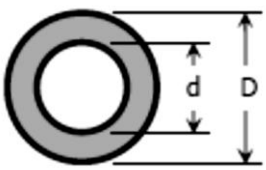
$$\tau_{max} = \frac{16T}{\pi D^3}$$



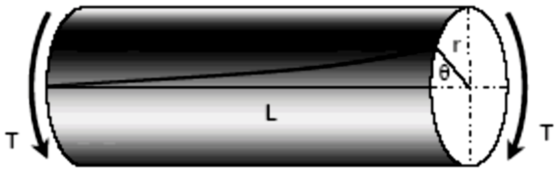
- **For hollow cylindrical shaft:**

$$J = \frac{\pi}{32}(D^4 - d^4)$$

$$\tau_{max} = \frac{16TD}{\pi(D^4 - d^4)}$$



- Angle of Twist



The angle θ through which the bar length L will twist is

$$\theta = \frac{TL}{JG} \text{ in radians}$$

Where T is the torque in N·mm, L is the length of shaft in mm, G is shear modulus in MPa, J is the polar moment of inertia in mm4, D and d are diameter in mm, and r is the radius in mm.

Pr. 01 A steel shaft 3 ft long that has a diameter of 4 in. is subjected to a torque of 15 kip·ft. Determine the maximum shearing stress and the angle of twist. Use G = 12 × 106 psi.

Solution:

$$\tau_{max} = \frac{16T}{\pi D^3} = \frac{16(15)(1000)(12)}{\pi(4^3)}$$

$$\tau_{max} = 14\,324 \text{ psi}$$

$$\tau_{max} = 14.3 \text{ ksi}$$

$$\theta = \frac{TL}{JG} = \frac{15(3)(1000)(12^2)}{\frac{1}{32}\pi(4^4)(12\times10^6)}$$

$$\theta = 0.0215 \text{ rad}$$

$$\theta = 1.23°$$

Pr. 02 What is the minimum diameter of a solid steel shaft that will not twist through more than 3° in a 6-m length when subjected to a torque of 12 kN·m? What maximum shearing stress is developed? Use G = 83 GPa.

Solution:

$$\theta = \frac{TL}{JG}$$

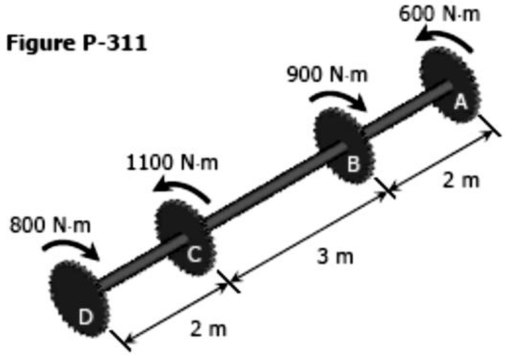
$$3^\circ\left(\frac{\pi}{180^\circ}\right) = \frac{12(6)(1000^3)}{\frac{1}{32}\pi d^4(83000)}$$

$$d = 113.98 \text{ mm}$$

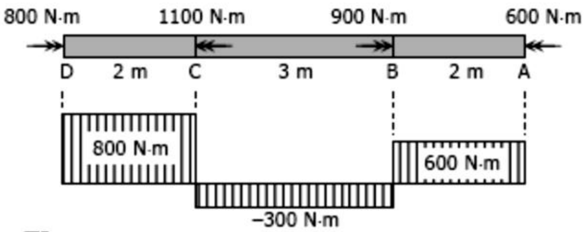
$$\tau_{max} = \frac{16T}{\pi d^3} = \frac{16(12)(1000^2)}{\pi(113.98^3)}$$

$$\tau_{max} = 41.27 \text{ MPa}$$

Pr. 03 An aluminum shaft with a constant diameter of 50 mm is loaded by torques applied to gears attached to it as shown in Fig. P-311. Using G = 28 GPa, determine the relative angle of twist of gear D relative to gear A.



Solution:



$$\theta = \frac{TL}{JG}$$

Rotation of D relative to A:

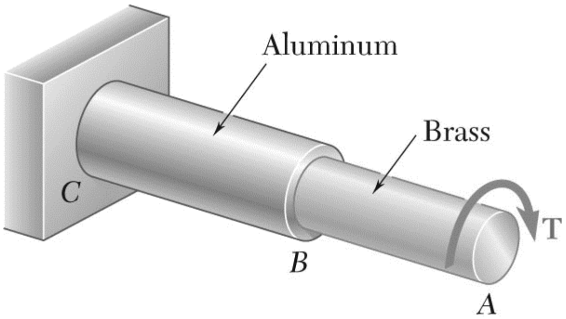
$$\theta_{D/A} = \frac{1}{JG}\sum TL$$

$$\theta_{D/A} = \frac{1}{\frac{1}{32}\pi(50^4)(28000)}[800(2) - 300(3) + 600(2)]\,(100^2)$$

$$\theta_{D/A} = 0.1106 \text{ rad}$$

$$\theta_{D/A} = 6.34^\circ$$

Pr. 04 The solid rod BC has a diameter of 30 mm and is made of an aluminum for which the allowable shearing stress is 25 MPa. Rod AB is hollow and has an outer diameter of 25 mm; it is made of a brass for which the allowable shearing stress is 50 MPa. Determine (a) the largest inner diameter of rod AB for which the factor of safety is the same for each rod, (b) the largest torque that can be applied at A.



Solution

Solid rod *BC*:

$$\tau = \frac{Tc}{J} \qquad J = \frac{\pi}{2}c^4$$

$$\tau_{all} = 25 \times 10^6 \text{ Pa}$$

$$c = \frac{1}{2}d = 0.015 \text{ m}$$

$$T_{all} = \frac{\pi}{2}c^3\tau_{all} = \frac{\pi}{2}(0.015)^3(25 \times 10^6) = 132.536 \text{ N} \cdot \text{m}$$

Hollow rod AB:

$$\tau_{all} = 50 \times 10^6 \text{ Pa}$$

$$T_{all} = 132.536 \text{ N} \cdot \text{m}$$

$$c_2 = \frac{1}{2}d_2 = \frac{1}{2}(0.025) = 0.0125 \text{ m}$$

$$c_1 = ?$$

$$T_{all} = \frac{J\tau_{all}}{c_2} = \frac{\pi}{2}\left(c_2^4 - c_1^4\right)\frac{\tau_{all}}{c_2}$$

$$c_1^4 = c_2^4 - \frac{2T_{all}c_2}{\pi\tau_{all}}$$

$$= 0.0125^4 - \frac{(2)(132.536)(0.0125)}{\pi(50 \times 10^6)} = 3.3203 \times 10^{-9} \text{ m}^4$$

(*a*) $c_1 = 7.59 \times 10^{-3} \text{ m} = 7.59 \text{ mm}$ $\qquad d_1 = 2c_1 = 15.18 \text{ mm}$ ◄

(*b*) Allowable torque. $\qquad T_{all} = 132.5 \text{ N} \cdot \text{m}$ ◄

- Plane-Stress Transformation

the general state of stress at a point is characterized by six independent normal and shear stress components، which act on the faces of an element of material located at the point، Fig. 9–1 a . This state of stress, however, is not often encountered in engineering practice. Instead, engineers frequently make approximations or simplifications of the loadings on a body in order that the stress produced in a structural member or mechanical element can be analyzed in a single plane. When this is the case, the material is said to be subjected to plane stress, Fig. 9–1 b. For example, if there is no load on the surface of a body, then the normal and shear stress components will be zero on the face of an element that lies on this surface. Consequently, the corresponding stress components on the opposite face will also be zero، and so the material at the point will be subjected to plane stress.

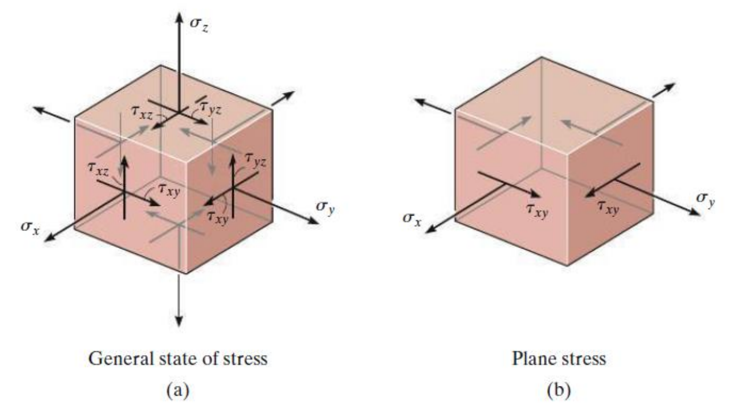


Fig. 9–1

The general state of plane stress at a point is therefore represented by a combination of two normal-stress components, σ_x , σ_y, and one shear stress component, τ_{xy}, which act on four faces of the element. For convenience, in this text we will view this state of stress in the x–y plane, with the x , y axes as shown in Fig. 9–2 a . If this state of stress is defined on an element having a different orientation u as in Fig. 9–2 b , then it will be subjected to three different stress components defined as $\sigma_{x'}$, $\sigma_{y'}$ $\tau_{x'y'}$, relative to the x' , y', axes. In other words*, the state of plane stress at the point is uniquely represented by two normal stress components and one shear stress component acting on an element. These three Components will be different for each specific orientation U of the element at the point.*

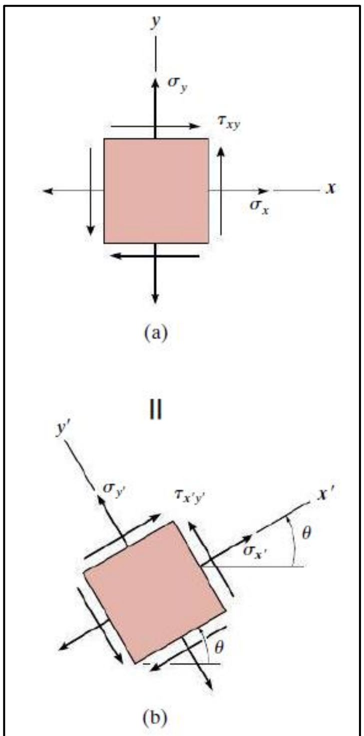


Fig. 9–2

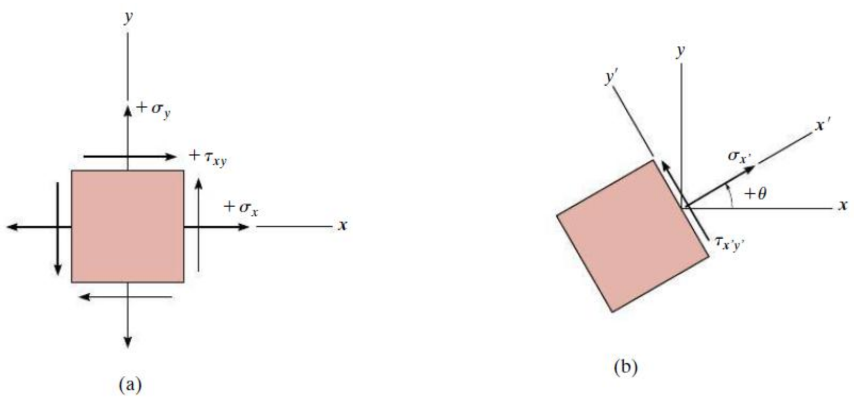
- **General Equations of Plane-Stress Transformation**

The method of transforming the normal and shear stress components from the x , y to the x' , y' coordinate axes, as discussed in the previous section, can be developed in a general manner and expressed as a set of stress-transformation equations.

Sign Convention. First we must establish a sign convention for the stress component. To do this the +x and +x' axes are used to define the outward normal from a side of the element. Then σ_x, and $\sigma_{x'}$ Are positive when they act in the positive x and x' directions, and τ_{xy} and $\tau_{x'y'}$ are positive when they act in the positive y and y' directions, Fig. 9–5 The orientation of the plane on which the normal and shear stress components are to be determined will be defined by the angle $\boldsymbol{\theta}$, which is measured from the +x axis to the +x' axis, Fig. 9–5 b . Notice that the unprimed and primed sets of axes in this figure both form right-handed coordinate systems; that is, the positive z (or z') axis is established by the right-hand rule. Curling the fingers from x (or x') toward y (or y') gives the direction for the positive z (or z') axis that points outward, along the thumb. The angle $\boldsymbol{\theta}$ will be positive provided it follows the curl of the right-hand fingers, i.e., counterclockwise as shown in Fig. 9–5 b.

Normal and Shear Stress components.

Using the established sign convention, the element in Fig. 9–6 a is sectioned along the inclined plane and the segment shown in Fig. 9–6 b is isolated. Assuming the sectioned area is ΔA, then the horizontal and vertical faces of the segment have an area of ΔA Sin $\boldsymbol{\theta}$ and ΔA Cos $\boldsymbol{\theta}$, respectively.



Positive Sign Convention

Fig. 9-5

The resulting *free-body diagram* of the segment is shown in Fig. 9–6 *c.* Applying the equations of equilibrium to determine the unknown normal and shear stress components $\sigma_{x'}$ and $\tau_{x'y'}$, we have

$$\Sigma F_{x'} = 0; \quad \sigma_{x'}\,\Delta A - (\tau_{xy}\,\Delta A \sin\boldsymbol{\theta}) \cos\boldsymbol{\theta} - (\sigma_y\,\Delta A \sin\boldsymbol{\theta}) \sin\boldsymbol{\theta}$$

$$- (\tau_{xy}\,\Delta A \cos\boldsymbol{\theta}) \sin\boldsymbol{\theta} - (\sigma_x\,\Delta A \cos\boldsymbol{\theta}) \cos\boldsymbol{\theta} = 0$$

$$\sigma_{x'} = \sigma_x \cos^2\boldsymbol{\theta} + \sigma_y \sin^2\boldsymbol{\theta} + \tau_{xy}(2\sin\boldsymbol{\theta}\cos\boldsymbol{\theta})$$

$\Sigma F_{y'} = 0;$ $\tau_{x'y'}\,\Delta A + (\tau_{xy}\,\Delta A \sin\boldsymbol{\theta})\sin\boldsymbol{\theta} - (\sigma_y\,\Delta A \sin\boldsymbol{\theta})\cos\boldsymbol{\theta}$

$-\tau_{xy}\,\Delta A \cos\boldsymbol{\theta})\cos\boldsymbol{\theta} + (\sigma_x\,\Delta A\cos\boldsymbol{\theta})\sin\boldsymbol{\theta} = 0$

$$\tau_{x'y'} = (\sigma_y - \sigma_x)\sin\boldsymbol{\theta}\cos\boldsymbol{\theta} + \tau_x\,\boldsymbol{\theta}\,(\cos^2\boldsymbol{\theta} - \sin^2\boldsymbol{\theta})$$

These two equations may be simplified by using the trigonometric identities

$\sin(2\boldsymbol{\theta}) = 2\sin\boldsymbol{\theta}\cos\boldsymbol{\theta}$, $\sin^2\boldsymbol{\theta} = (1 - \cos 2\boldsymbol{\theta})/2$, and $\cos^2\boldsymbol{\theta} = (1 + \cos 2\boldsymbol{\theta})/2$, in which case,

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta \qquad (9\text{–}1)$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta \qquad (9\text{–}2)$$

If the normal stress acting in the *y*' direction is needed, it can be obtained by simply substituting $\boldsymbol{\theta} + 90^o$ for $\boldsymbol{\theta}$ into Eq. 9–1, Fig. 9–6 *d*. This yield

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta \qquad (9\text{–}3)$$

If σ_y' is calculated as a positive quantity, it indicates that it acts in the Positive *y*' direction as shown in Fig. 9–6 *d* .

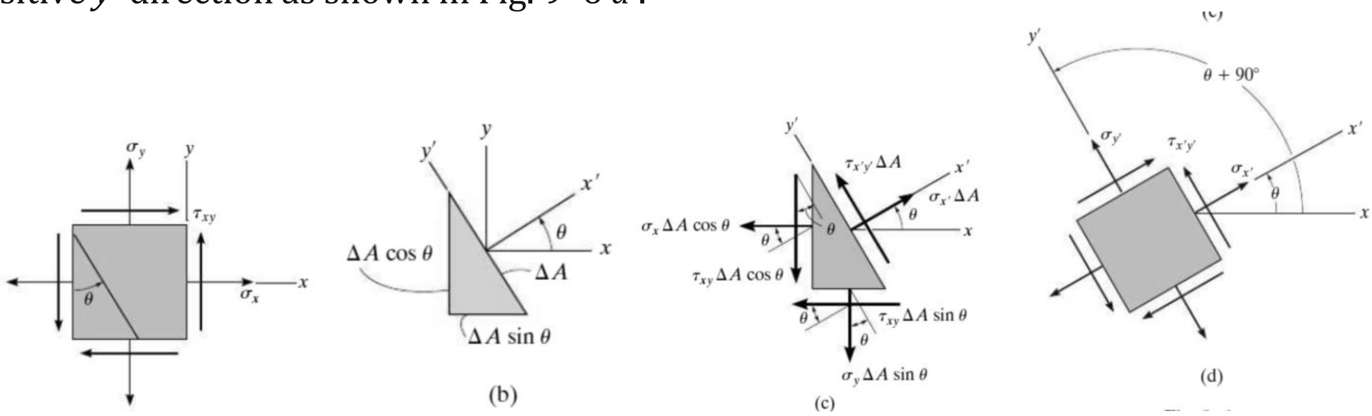
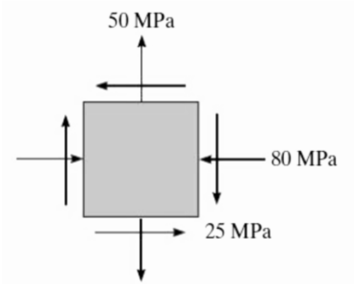


Fig. 9–6

Example: The state of plane stress at a point is represented by the element shown in Fig. 9–7 *a*. Determine the state of stress at the point on another element oriented 30° clockwise from the position shown.



SOLUTION

σ_x = -80 MPa , σ_y = 50 MPa, τ_{xy} =25 MPa

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

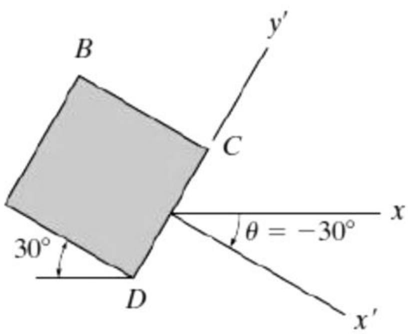
$$= \frac{-80 + 50}{2} + \frac{-80 - 50}{2}\cos 2(-30^\circ) + (-25)\sin 2(-30^\circ)$$

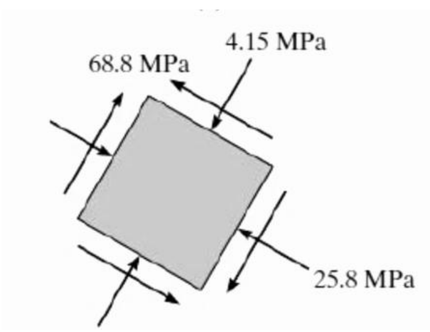
$$= -25.8 \text{ MPa}$$ Ar

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

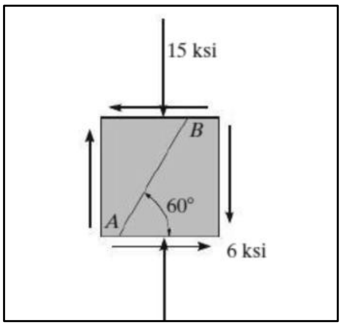
$$= -\frac{-80 - 50}{2}\sin 2(-30^\circ) + (-25)\cos 2(-30^\circ)$$

$$= -68.8 \text{ MPa}$$





Problem 01: Determine the normal stress and shear stress acting on the inclined plane *AB.* Solve the problem using the stress transformation equations. Show the results on the sectional element.



SOLUTION

Stress Transformation Equations:

$\theta = +150°$ (Fig. *a*) $\quad \sigma_x = 0 \quad \sigma_y = -15\text{ ksi} \quad \tau_{xy} = -6\text{ ksi}$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

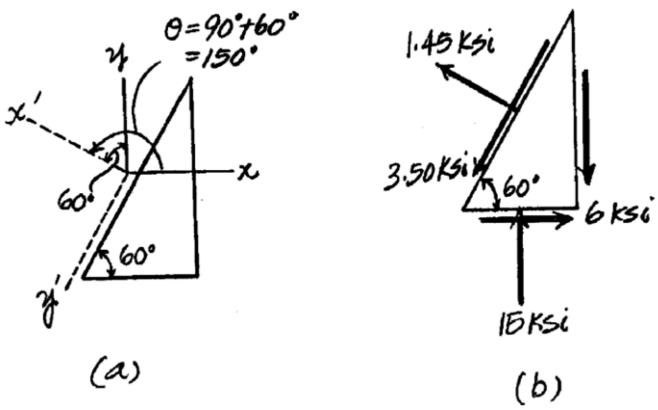
$$= \frac{0 + (-15)}{2} + \frac{0 - (-15)}{2}\cos 300° + (-6)\sin 300°$$

$$= 1.45\text{ ksi}$$

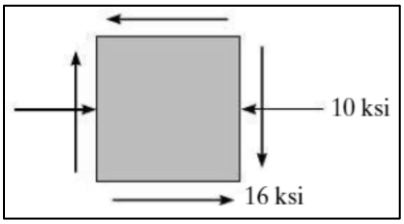
$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\frac{0 - (-15)}{2}\sin 300° + (-6)\cos 300°$$

$$= 3.50\text{ ksi}$$



Problem 02: Determine the equivalent state of stress on an element if it is oriented 50° counterclockwise from the element shown. Use the stress-transformation equations.



SOLUTION

Stress Transformation Equations:

$$\sigma_x = -10 \text{ ksi} \qquad \sigma_y = 0 \qquad \tau_{xy} = -16 \text{ ksi} \qquad \theta = +50^\circ$$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

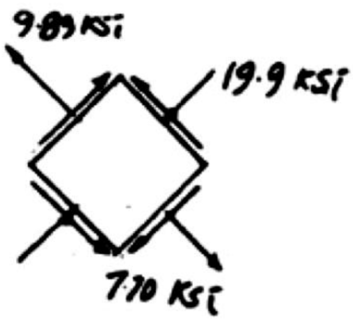
$$= \frac{-10 + 0}{2} + \frac{-10 - 0}{2}\cos 100^\circ + (-16)\sin 100^\circ = -19.9 \text{ ksi}$$

$$\tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\left(\frac{-10 - 0}{2}\right)\sin 100^\circ + (-16)\cos 100^\circ = 7.70 \text{ ksi}$$

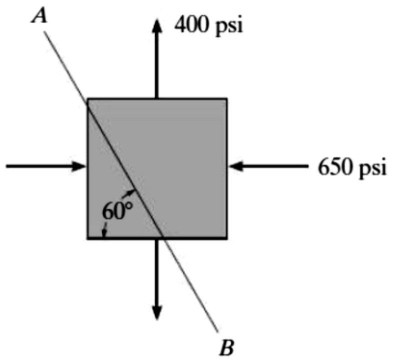
$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$= \frac{-10 + 0}{2} - \left(\frac{-10 - 0}{2}\right)\cos 100^\circ - (-16)\sin 100^\circ = 9.89 \text{ ksi}$$



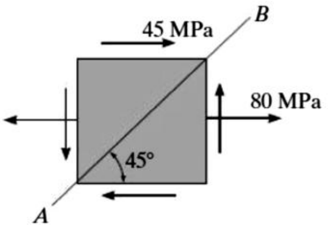
HW.

9–3. The state of stress at a point in a member is shown on the element. Determine the stress components acting on the inclined plane AB. Solve the problem using the method of equilibrium described in Sec. 9.1.



9–6. Determine the normal stress and shear stress acting on the inclined plane AB. Solve the problem using the method of equilibrium described in Sec. 9.1.

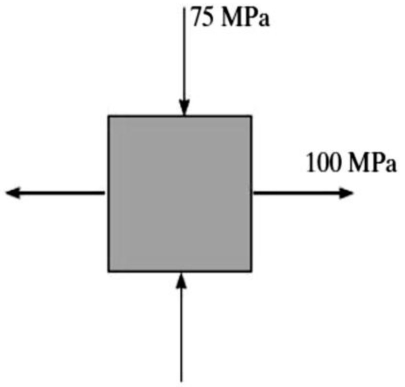
9–7. Determine the normal stress and shear stress acting on the inclined plane AB. Solve the problem using the stress transformation equations. Show the result on the sectioned element.



Probs. 9–6/7

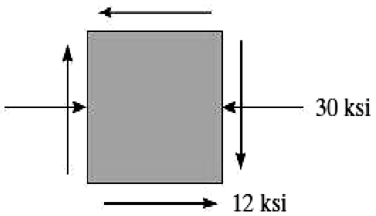
***9–8.** Determine the equivalent state of stress on an element at the same point oriented 30° clockwise with respect to the element shown. Sketch the results on the element.

9–9. Determine the equivalent state of stress on an element at the same point oriented 30° counterclockwise with respect to the element shown. Sketch the results on the element.



Probs. 9–8/9

9–14. The state of stress at a point is shown on the element. Determine (a) the principal stress and (b) the maximum in-plane shear stress and average normal stress at the point. Specify the orientation of the element in each case. Show the results on each element.



Prob. 9–14

- Principal Stresses & Maximum in Plane shear Stress

From Eqs. 9–1 and 9–2, it can be seen that the magnitudes of $\sigma_{x'}$ and $\tau_{x'y'}$ depend on the angle of inclination $\boldsymbol{\theta}$ of the planes on which these stresses act. In engineering practice it is often important to determine the orientation of the element that causes the normal stress to be a maximum and a minimum and the orientation that causes the shear stress to be a maximum. In this section each of these problems will be considered.

In-Plane Principal Stresses. To determine the maximum and minimum normal stress, we must differentiate Eq. 9–1 with respect to $\boldsymbol{\theta}$ and set the result equal to zero. This gives

$$\frac{d\sigma_{x'}}{d\theta} = -\frac{\sigma_x - \sigma_y}{2}(2\sin 2\theta) + 2\tau_{xy}\cos 2\theta = 0$$

Solving this equation we obtain the orientation $\boldsymbol{\theta} = \boldsymbol{\theta}$p of the planes of maximum and minimum normal stress.

$$\boxed{\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2}}$$

The solution has two roots, $\boldsymbol{\theta}_{p1}$, and $\boldsymbol{\theta}_{p2}$. Specifically, the values of $2\boldsymbol{\theta}_{p1}$ and $2\boldsymbol{\theta}_{p2}$ are 180° apart, so $\boldsymbol{\theta}_{p1}$ and $\boldsymbol{\theta}_{p2}$ will be 90° apart. The values of $\boldsymbol{\theta}_{p1}$ and $\boldsymbol{\theta}_{p2}$ must be substituted into Eq. 9–1 if we are to obtain the required normal stresses. To do this we can obtain the necessary sine and cosine of $2\boldsymbol{\theta}_{p1}$ and $2\boldsymbol{\theta}_{p2}$ from the shaded triangles shown in Fig. 9–8. The construction of these triangles is based on Eq. 9– 4, assuming that τ_{xy} and $(\sigma_x - \sigma_y)$ are either positive or both negative quantities.

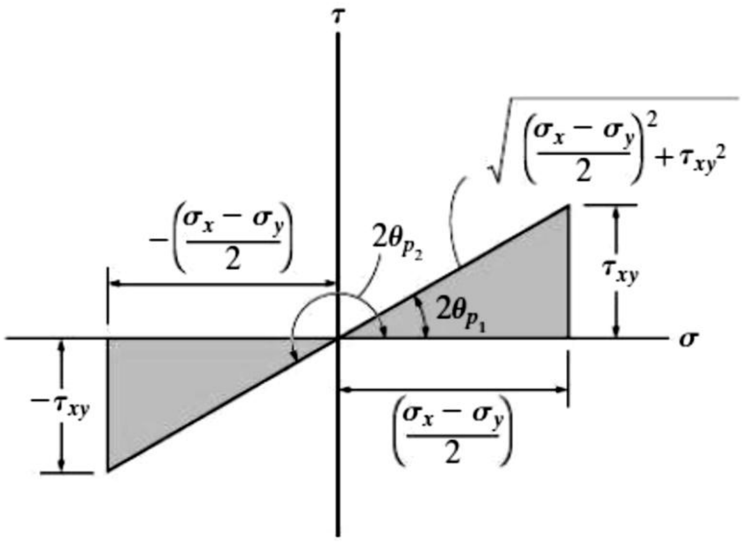


Fig. 9–8

The cracks in this concrete beam were caused by tension stress, even though the beam was subjected to both an internal moment and shear. The stress transformation equations can be used to predict the direction of the cracks, and the principal normal stresses that caused them.

Substituting these trigonometric values into Eq. 9–1 and simplifying, we obtain

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

Depending upon the sign chosen, this result gives the maximum or minimum in-plane normal stress acting at a point, where $\sigma_1 \geq \sigma_2$. These particular sets of values are called the in-plane ***principal stresses,*** and the corresponding planes on which they act are called the ***principal planes*** of stress, Fig. 9–9. Furthermore, if the trigonometric relations for $\boldsymbol{\theta}_{p1}$ or $\boldsymbol{\theta}_{p2}$ are substituted into Eq. 9–2, it can be seen that $\tau_{x'y'} = 0$; in other words, ***no shear stress acts on the principal planes.***

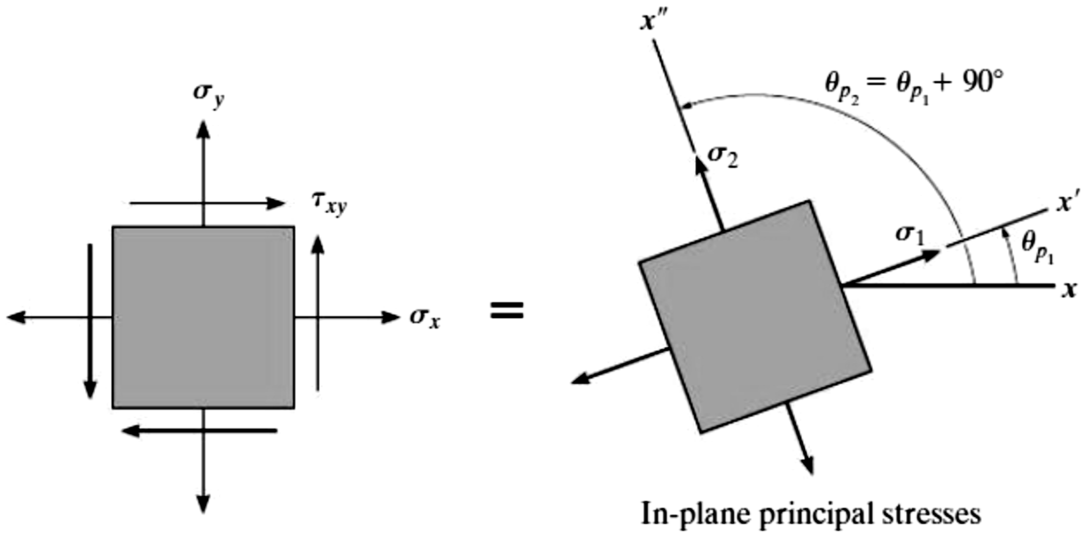


Fig. 9–9

Maximum In-Plane Shear Stress. The orientation of an element that is subjected to maximum shear stress on its sides can be determined by taking the derivative of Eq. 9–2 with respect to u and setting the result equal to zero. This gives

$$\boxed{\tan 2\theta_s = \frac{-(\sigma_x - \sigma_y)/2}{\tau_{xy}}} \qquad (9\text{–}6)$$

The two roots of this equation, θ_{s1} and θ_{s2}, can be determined from the shaded triangles shown in Fig. 9–10. By comparison with Eq. 9–4, tan $2\theta_s$ is the negative reciprocal of tan $2\theta_p$ and so each root $2\theta_s$ is 90° from $2\theta_p$, and the roots θ_s and θ_p are 45° apart. Therefore, an element subjected to ***maximum shear stress will be 45° from the position of an element that is subjected to the principal stress.*** Using either one of the roots θ_{s1} or θ_{s2}, the maximum shear stress can be found by taking the trigonometric values of sin $2\theta_s$ and Cos $2\theta_s$ from Fig. 9–10 and substituting them into Eq. 9–2. The result is

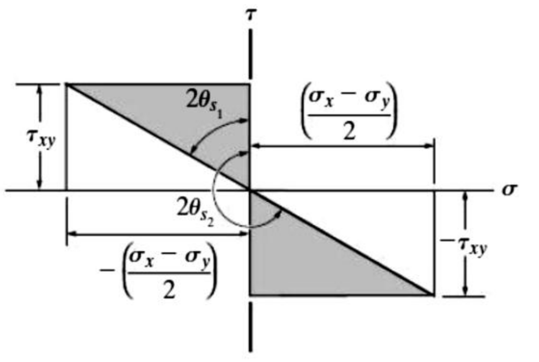


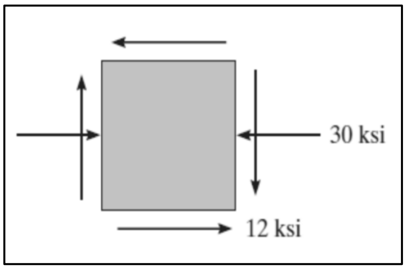
Fig. 9–10

$$\boxed{\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^{\;2}}} \qquad (9\text{–}7)$$

The value of $\tau_{max/in\text{-}plane}$ as calculated from this equation is referred to as the ***maximum in-plane shear stress*** because it acts on the element in the *x* – *y* plane.
Substituting the values for sin $2\theta_s$ and Cos $2\theta_s$ into Eq. 9–1, we see that there is *also* an average normal stress on the planes of maximum in-plane shear stress. We get

$$\boxed{\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2}} \qquad (9\text{–}8)$$

Example: The state of stress at a point is shown on the element. Determine (a) the principal stress and (b) the maximum in-plane shear stress and average normal stress at the point. Specify the orientation of the element in each case. Show the results on each element.



SOLUTION

$$\sigma_x = -30 \text{ ksi} \qquad \sigma_y = 0 \qquad \tau_{xy} = -12 \text{ ksi}$$

a)

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2} = \frac{-30 + 0}{2} \pm \sqrt{\left(\frac{-30 - 0}{2}\right)^2 + (-12)^2}$$

$$\sigma_1 = 4.21 \text{ ksi}$$

$$\sigma_2 = -34.2 \text{ ksi}$$

Orientation of principal stress:

$$\tan 2\theta_P = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{-12}{(-30-0)/2} = 0.8$$

$$\theta_P = 19.33^\circ \quad \text{and} \quad -70.67^\circ$$

Use Eq. 9-1 to determine the principal plane of σ_1 and σ_2.

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\theta = 19.33^\circ$$

$$\sigma_{x'} = \frac{-30 + 0}{2} + \frac{-30 - 0}{2}\cos 2(19.33^\circ) + (-12)\sin 2(19.33^\circ) = -34.2 \text{ ksi}$$

Therefore $\theta_{P_2} = 19.3^\circ$

and $\theta_{P_1} = -70.7^\circ$

b)

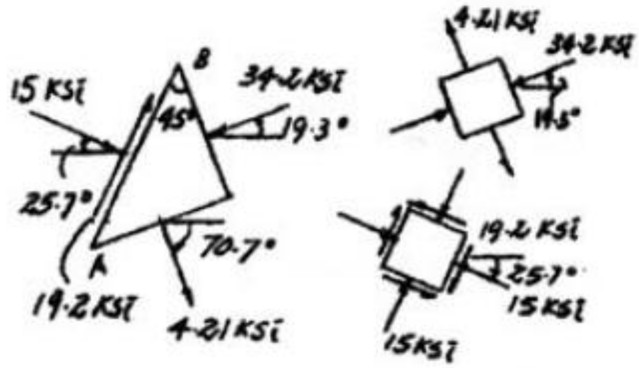
$$\tau_{\max_{\text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2} = \sqrt{\left(\frac{-30 - 0}{2}\right)^2 + (-12)^2} = 19.2 \text{ ksi}$$

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{-30 + 0}{2} = -15 \text{ ksi}$$

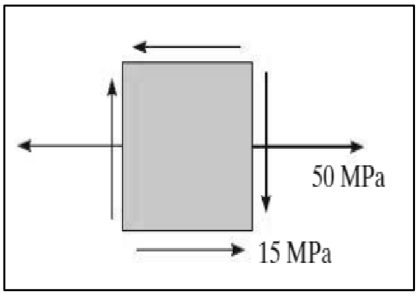
Orientation of max, in - plane shear stress:

$$\tan 2\theta_s = \frac{-(\sigma_x - \sigma_y)/2}{\tau_{xy}} = \frac{-(-30 - 0)/2}{-12} = -1.25$$

$$\theta_s = -25.7° \qquad \text{and} \qquad 64.3°$$



Prob. 01: Determine the equivalent state of stress on an element at the point which represents (a) the principal Stresses and (b) the maximum in-plane shear stress and the associated average normal stress. Also, for each case, determine the corresponding orientation of the element with respect to the element shown and sketch the results on the element.



SOLUTION

Normal and Shear Stress:

$$\sigma_x = 50 \text{ MPa} \qquad \sigma_y = 0 \qquad \tau_{xy} = -15 \text{ MPa}$$

In-Plane Principal Stresses:

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= \frac{50 + 0}{2} \pm \sqrt{\left(\frac{50 - 0}{2}\right)^2 + (-15)^2}$$

$$= 25 \pm \sqrt{850}$$

$$\sigma_1 = 54.2 \text{ MPa} \qquad \sigma_2 = -4.15 \text{ MPa} \qquad \textbf{Ans.}$$

Orientation of Principal Plane:

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{-15}{(50 - 0)/2} = -0.6$$

$$\theta_p = -15.48^\circ \text{ and } 74.52^\circ$$

Substitute $\theta = -15.48^\circ$ into

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$= \frac{50 + 0}{2} + \frac{50 - 0}{2} \cos(-30.96^\circ) + (-15) \sin(-30.96^\circ)$$

$$= 54.2 \text{ MPa} = \sigma_1$$

Thus,

$$(\theta_p)_1 = -15.5^\circ \text{ and } (\theta_p)_2 = 74.5^\circ$$

The element that represents the state of principal stress is shown in Fig. *a*.

Maximum In-Plane Shear Stress:

$$\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2} = \sqrt{\left(\frac{50 - 0}{2}\right)^2 + (-15)^2} = 29.2 \text{ MPa} \qquad \textbf{Ans.}$$

Orientation of the Plane of Maximum In-Plane Shear Stress:

$$\tan 2\theta_s = \frac{-(\sigma_x - \sigma_y)/2}{\tau_{xy}} = \frac{-(50 - 0)/2}{-15} = 1.667$$

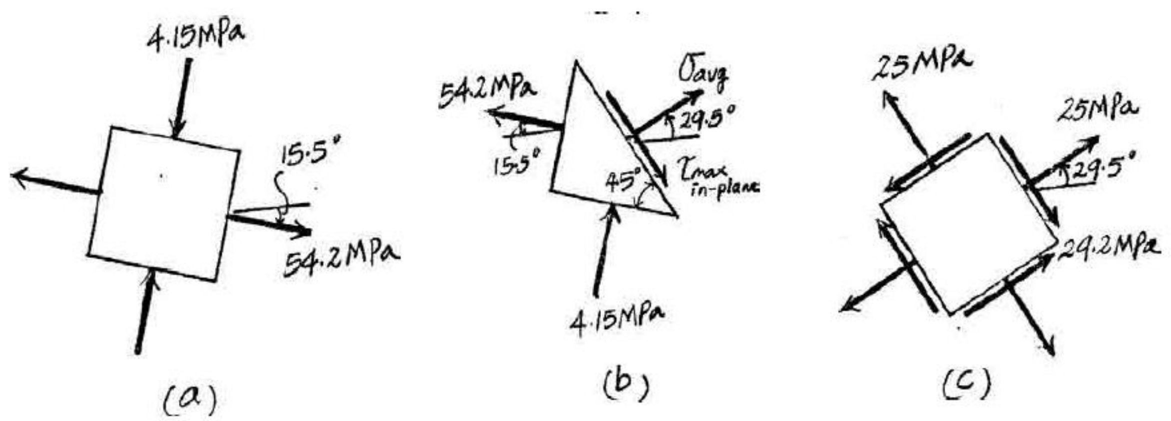
$$\theta_s = 29.5° \text{ and } 120° \qquad \textbf{Ans.}$$

By inspection, $\tau_{\substack{\max \\ \text{in-plane}}}$ has to act in the same sense shown in Fig. *b* to maintain equilibrium.

Average Normal Stress:

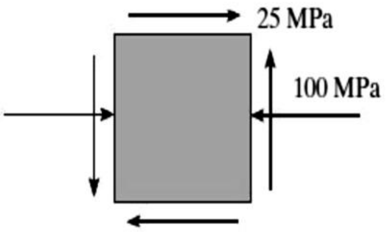
$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{50 + 0}{2} = 25 \text{ MPa} \qquad \textbf{Ans.}$$

The element that represents the state of maximum in-plane shear stress is shown in Fig. *c*.

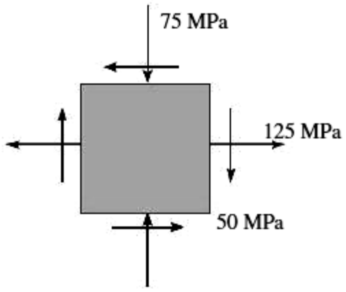


Homework:

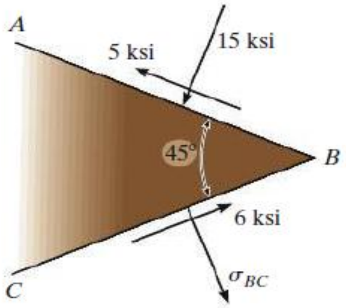
9–19. Determine the equivalent state of stress on an element at the same point which represents (a) the principal stress, and (b) the maximum in-plane shear stress and the associated average normal stress. Also, for each case, determine the corresponding orientation of the element with respect to the element shown and sketch the results on the element.



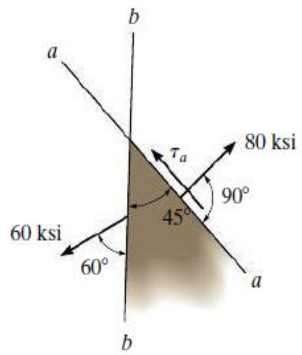
9–17. Determine the equivalent state of stress on an element at the same point which represents (a) the principal stress, and (b) the maximum in-plane shear stress and the associated average normal stress. Also, for each case, determine the corresponding orientation of the element with respect to the element shown. Sketch the results on each element.



***9–20.** Planes *AB* and *BC* at a point are subjected to the stresses shown. Determine the principal stresses acting at this point and find σ_{BC}.

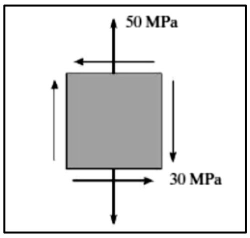


9–21. The stress acting on two planes at a point is indicated. Determine the shear stress on plane *a–a* and the principal stresses at the point.



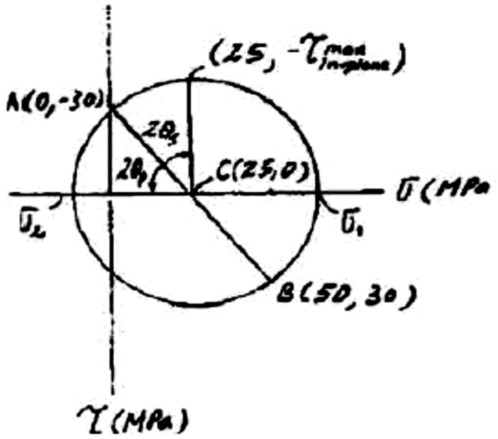
❖ MOHR'S CIRCLE – Solved Problems & Homeworks

Prob. 01 Determine (a) the principal stresses and (b) the maximum in-plane shear stress and average normal stress. Specify the orientation of the element in each case.



Solution.

$A(0, -30) \qquad B(50, 30) \qquad C(25, 0)$



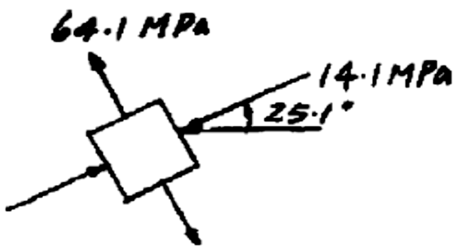
$$R = CA = \sqrt{(25-0)^2 + 30^2} = 39.05$$

$$\sigma_1 = 25 + 39.05 = 64.1 \text{ MPa}$$

$$\sigma_2 = 25 - 39.05 = -14.1 \text{ MPa}$$

$$\tan 2\theta_p = \frac{30}{25-0} = 1.2$$

$$\theta_{p2} = 25.1^\circ$$

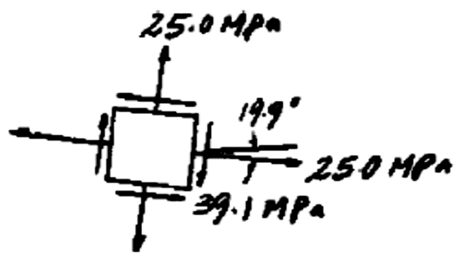


$$\sigma_{avg} = 25.0 \text{ MPa}$$

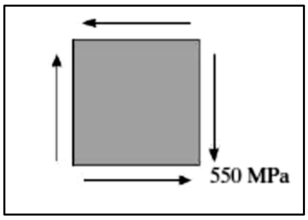
$$\tau_{\substack{\max \\ \text{in-plane}}} = R = 39.1 \text{ MPa}$$

$$\tan 2\theta_s = \frac{25-0}{30} = 0.8333$$

$$\theta_s = -19.9^\circ$$



Prob. 02 Determine the equivalent state of stress if an element is oriented 25° counterclockwise from the element shown.



Solution.

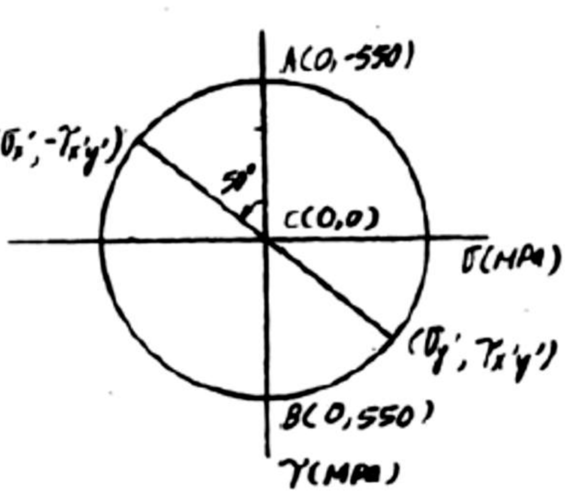
$A(0, -550) \qquad B(0, 550) \qquad C(0, 0)$

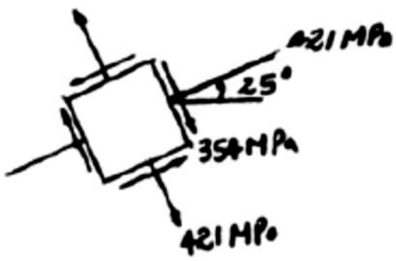
$R = CA = CB = 550$

$\sigma_{x'} = -550 \sin 50° = -421\text{ MPa}$

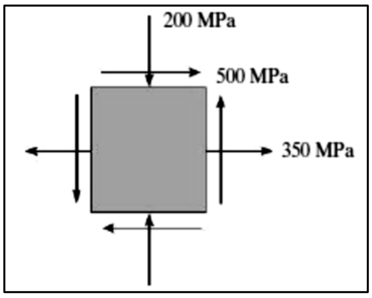
$\tau_{x'y'} = -550 \cos 50° = -354\text{ MPa}$

$\sigma_{y'} = 550 \sin 50° = 421\text{ MPa}$





Prob. 03 Determine the principal stresses, the maximum in-plane shear stress, and average normal stress. Specify the orientation of the element in each case.



Solution.

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{350 + (-200)}{2} = 75.0 \text{ MPa}$$

$A(350, 500) \qquad C(75.0, 0)$

$$R = \sqrt{(350 - 75.0)^2 + 500^2} = 570.64 \text{ MPa}$$

$$\sigma_1 = 75.0 + 570.64 = 646 \text{ MPa}$$

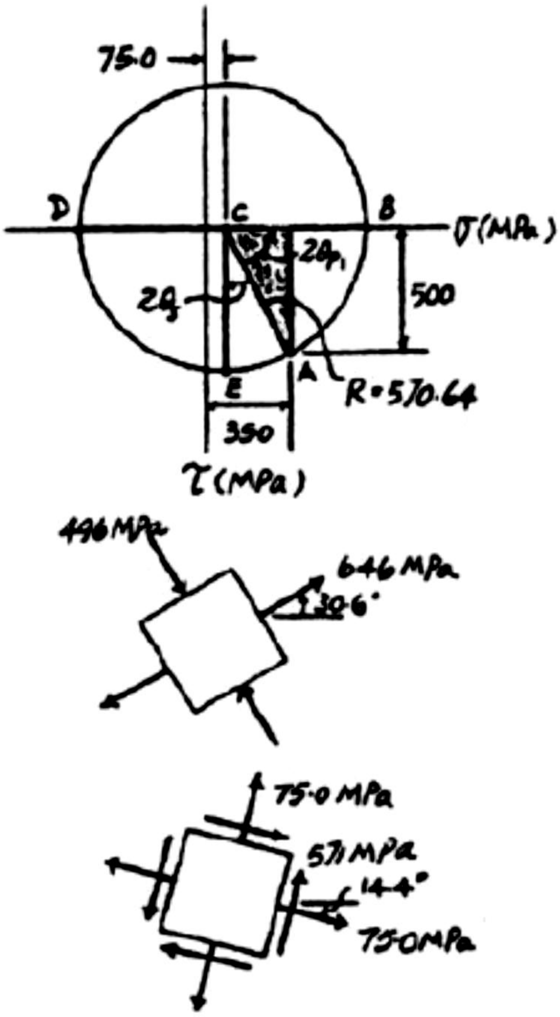
$$\sigma_2 = 75.0 - 570.64 = -496 \text{ MPa}$$

$$\tan 2\theta_{P1} = \frac{500}{350 - 75.0} = 1.82$$

$\theta_{P1} = 30.6°$ (*Counterclockwise*)

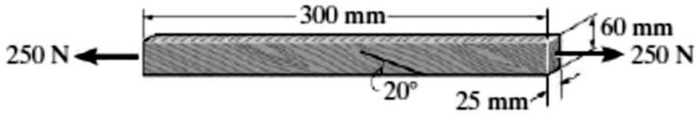
$$\tan 2\theta_s = \frac{350 - 75.0}{500} = 0.55$$

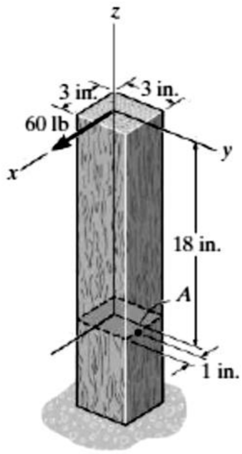
$\theta_s = 14.4°$ (*Clockwise*)



HWs.

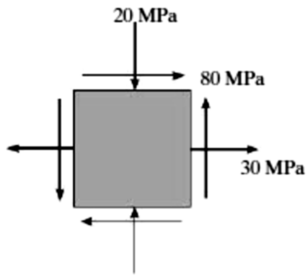
1. The grains of wood in the board make an angle of 20° with the horizontal as shown. Using Mohr's circle, determine the normal and shear stresses that act perpendicular and parallel to the grains if the board is subjected to an axial load of 250 N.





2. The post has a square cross-sectional area. If it is fixed supported at its base and a horizontal force is applied at its end as shown, determine (a) the maximum in-plane shear stress developed at A and (b) the principal stresses at A

3. Determine the principal stress, the maximum in-plane shear stress, and average normal stress. Specify the orientation of the element in each case.



- MOHR'S CIRCLE - PLANE STRESS

In this section, we will show how to apply the equations for plane stress transformation using a graphical solution that is often convenient to use and easy to remember. Furthermore, this approach will allow us to "visualize" how the normal and shear stress components $\sigma_{x'}$ and $\tau_{x'y'}$ vary as the plane on which they act is oriented in different directions, Fig. 9–15 *a.* If we write Eqs. 9–1 and 9–2 in the form

$$\sigma_{x'} - \left(\frac{\sigma_x + \sigma_y}{2}\right) = \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta \qquad (9\text{–}9)$$

$$\tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta \qquad (9\text{–}10)$$

Then the parameter $\boldsymbol{\theta}$ can be eliminated by squaring each equation and adding the equations together. The result is

$$\left[\sigma_{x'} - \left(\frac{\sigma_x + \sigma_y}{2}\right)\right]^2 + \tau_{x'y'}^2 = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2$$

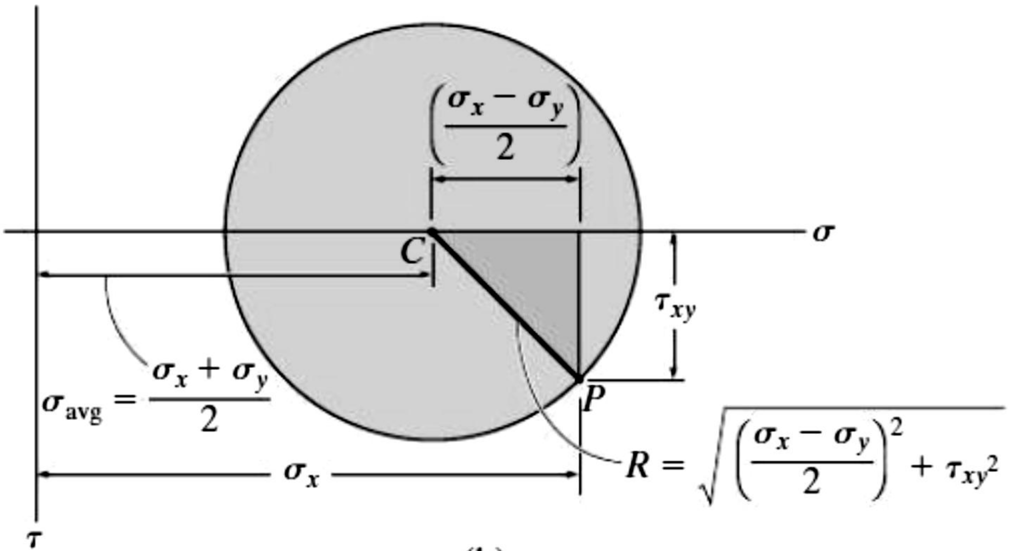
For a specific problem, σ_x, σ_y, τ_{xy} are *known constants.* Thus the above equation can be written in a more compact form as

$$(\sigma_{x'} - \sigma_{avg})^2 + \tau_{x'y'}^2 = R^2 \qquad (9\text{–}11)$$

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \qquad (9\text{–}12)$$

If we establish coordinate axes, σ *positive to the right* and τ *positive downward* , and then plot Eq. 9–11 , it will be seen that this equation represents a *circle* having a radius R and center on the σ axis at point C $(\sigma_{avg}, 0)$, Fig. 9–15 *b* . This circle is called *Mohr's circle,* because it was developed by the German engineer Otto Mohr.



(b)

Each point on Mohr's circle represents the two stress components $\sigma_{x'}$ and $\tau_{x'y'}$ acting on the side of the element defined by the x' axis, when the axis is in a specific direction $\boldsymbol{\theta}$. For example, when x' is coincident with the x axis as shown in Fig. 9–16 *a,* then $\boldsymbol{\theta} = 0^o$ and $\sigma_{x'} = \sigma_x$, $\tau_{x'y'} = \tau_{xy}$ .We will refer to this as the "reference point" A and plot its coordinates A (σ_x, τ_{xy}), Fig. 9–16 *c.* Now consider rotating the x' axis 90° counterclockwise, Fig. 9–16 *b.* Then $\sigma_{x'} = \sigma_y$, $\tau_{x'y'} = -\tau_{xy}$. These values are the coordinates of point G $(\sigma_y, -\tau_{x'y'})$ on the circle, Fig. 9–16 *c.* Hence, the radial line CG is 180° counterclockwise from the "reference line" CA. In other words, a rotation $\boldsymbol{\theta}$ of the x' axis on the element will correspond to a rotation $2\boldsymbol{\theta}$ on the circle in the *same direction.* (If instead the t axis were constructed *positive upwards*, then the angle $2\boldsymbol{\theta}$ on the circle would be measured in the *opposite direction* to the orientation $\boldsymbol{\theta}$ of the x' axis.) Once constructed, Mohr's circle can be used to determine the principal stresses the maximum in-plane shear stress and associated average normal stress, or the stress on any arbitrary plane.

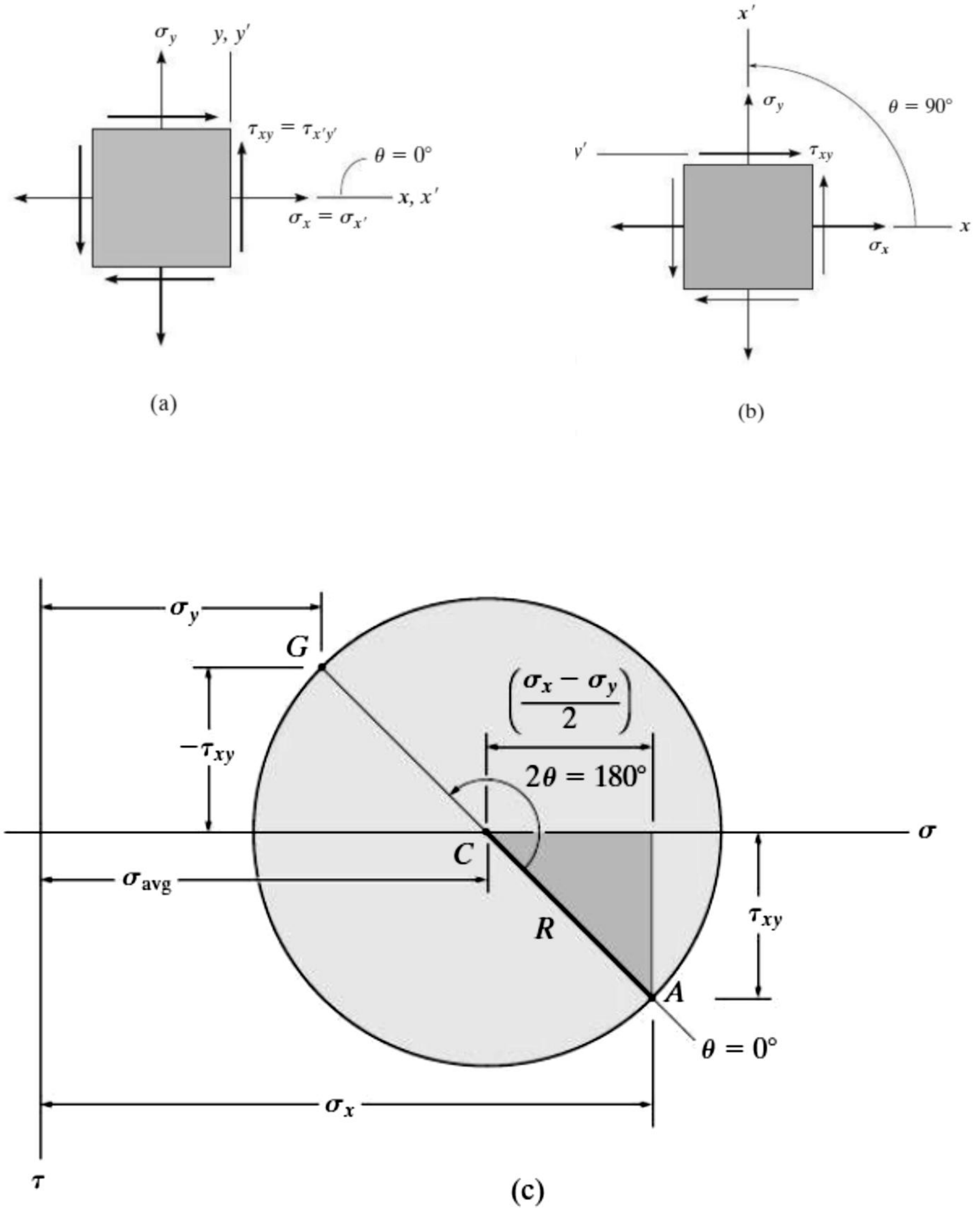
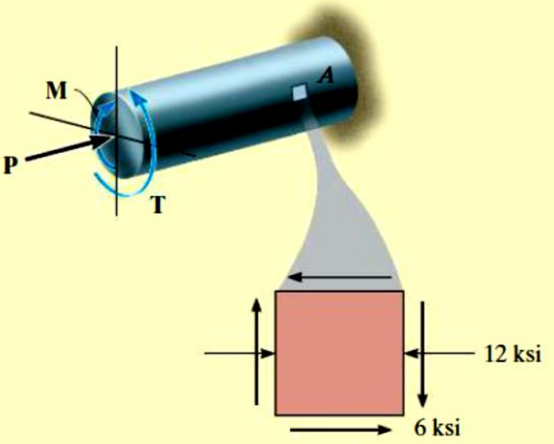

σ_y
y, y'
$\tau_{xy} = \tau_{x'y'}$
$\theta = 0°$
x, x'
$\sigma_x = \sigma_{x'}$
(a)
x'
σ_y
$\theta = 90°$
y'
τ_{xy}
x
σ_x
(b)
σ_y
G
$-\tau_{xy}$
$\left(\frac{\sigma_x - \sigma_y}{2}\right)$
$2\theta = 180°$
σ
C
σ_{avg}
R
τ_{xy}
A
$\theta = 0°$
σ_x
τ
(c)

Fig. 9–16

EXAMPLE:

Due to the applied loading, the element at point A on the solid shaft in Fig. 9–18*a* is subjected to the state of stress shown. Determine the principal stresses acting at this point.



(a)

SOLUTION

Construction of the Circle. From Fig. 9–18*a*,

$$\sigma_x = -12 \text{ ksi} \qquad \sigma_y = 0 \qquad \tau_{xy} = -6 \text{ ksi}$$

The center of the circle is at

$$\sigma_{avg} = \frac{-12 + 0}{2} = -6 \text{ ksi}$$

The reference point $A(-12, -6)$ and the center $C(-6, 0)$ are plotted in Fig. 9–18*b*. The circle is constructed having a radius of

$$R = \sqrt{(12 - 6)^2 + (6)^2} = 8.49 \text{ ksi}$$

Principal Stress. The principal stresses are indicated by the coordinates of points B and D. We have, for $\sigma_1 > \sigma_2$,

$$\sigma_1 = 8.49 - 6 = 2.49 \text{ ksi} \qquad \textit{Ans.}$$

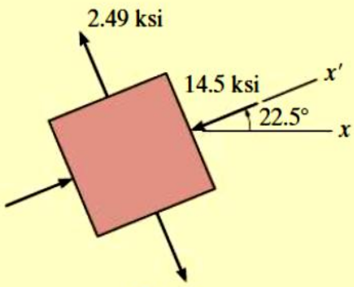
$$\sigma_2 = -6 - 8.49 = -14.5 \text{ ksi} \qquad \textit{Ans.}$$

The orientation of the element can be determined by calculating the angle $2\theta_{p_2}$ in Fig. 9–18*b*, which here is measured *counterclockwise* from CA to CD. It defines the direction θ_{p_2} of σ_2 and its associated principal plane. We have

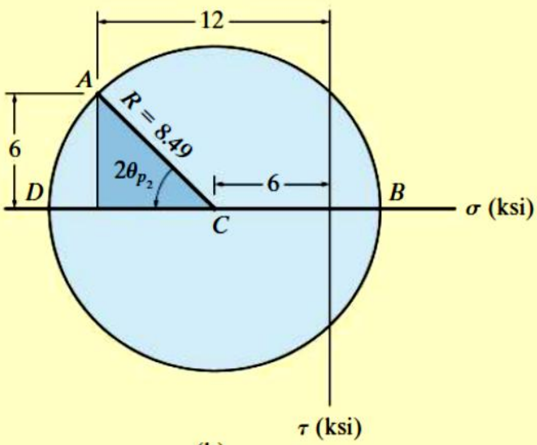
$$2\theta_{p_2} = \tan^{-1}\frac{6}{12 - 6} = 45.0^\circ$$

$$\theta_{p_2} = 22.5^\circ$$

The element is oriented such that the x' axis or σ_2 is directed 22.5° *counterclockwise* from the horizontal (x axis) as shown in Fig. 9–18*c*.



(c)



(b)

Fig. 9–18

EXAMPLE 02:

The state of plane stress at a point is shown on the element in Fig. 9–19*a*. Determine the maximum in-plane shear stress at this point.

SOLUTION

Construction of the Circle. From the problem data,

$$\sigma_x = -20 \text{ MPa} \qquad \sigma_y = 90 \text{ MPa} \qquad \tau_{xy} = 60 \text{ MPa}$$

The σ, τ axes are established in Fig. 9–19*b*. The center of the circle C is located on the σ axis, at the point

$$\sigma_{avg} = \frac{-20 + 90}{2} = 35 \text{ MPa}$$

Point C and the reference point $A(-20, 60)$ are plotted. Applying the Pythagorean theorem to the shaded triangle to determine the circle's radius CA, we have

$$R = \sqrt{(60)^2 + (55)^2} = 81.4 \text{ MPa}$$

Maximum In-Plane Shear Stress. The maximum in-plane shear stress and the average normal stress are identified by point E (or F) on the circle. The coordinates of point $E(35, 81.4)$ give

$$\tau_{\substack{max \\ in\text{-}plane}} = 81.4 \text{ MPa} \qquad \textit{Ans.}$$

$$\sigma_{avg} = 35 \text{ MPa} \qquad \textit{Ans.}$$

The angle θ_{s_1}, measured *counterclockwise* from CA to CE, can be found from the circle, identified as $2\theta_{s_1}$. We have

$$2\theta_{s_1} = \tan^{-1}\left(\frac{20 + 35}{60}\right) = 42.5^\circ$$

$$\theta_{s_1} = 21.3^\circ \qquad \textit{Ans.}$$

This *counterclockwise* angle defines the direction of the x' axis, Fig. 9–19*c*. Since point E has *positive* coordinates, then the average normal stress and the maximum in-plane shear stress both act in the *positive* x' and y' directions as shown.

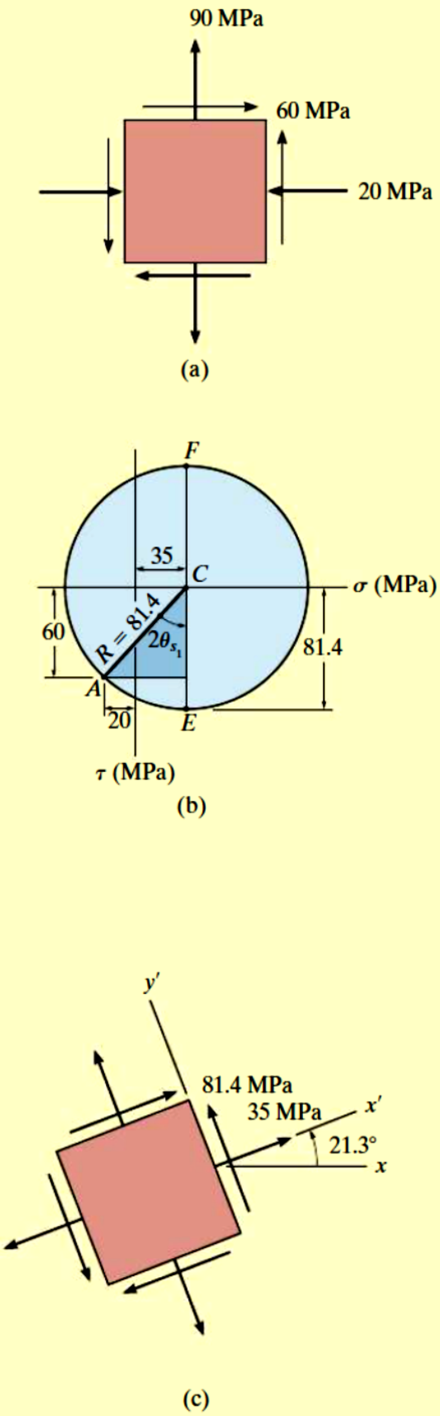


Fig. 9–19

Mohr's circle

Procedure for Analysis

1. Find A, B, C coordinates (*A and B in the same straight line and pass through the center of circle*).
2. Determine $\sigma_{av.}$ Where $\sigma_{av.} = \frac{\sigma x+\sigma y}{2}$
3. Calculate R value, where R = $\sqrt{\left(\frac{\sigma x-\sigma y}{2}\right)^2 + (\tau xy)^2}$, *Note that* $R = \tau_{max\ in\_plane}$
4. Calculate Principal stresses, $\sigma_{1,2} = \sigma_{av.} \pm R$
5. Determine Q coordinate *where Q represented* ***max in plane shear stress*** *coordinate*

Point	X coordinate	Y coordinate
A	σ_x	τ_{xy}
B	σ_y	$-\tau_{xy}$
C	$\sigma_{av.}$	0
Q	$\sigma_{av.}$	R

6. Drawing Mohr's circle by using all the coordinated above