- Shear and moment Diagrams

Members that are slender and support loadings that are applied perpendicular to their longitudinal axis are called beams. In general، beams are long, straight bars having a constant cross-sectional area. Often they are classified as to how they are supported. For example, a simply supported beam is pinned at one end and roller supported at the other، Fig. 6–1, a cantilevered beam is fixed at one end and free at the other, and an overhanging beam has one or both of its ends freely extended over the supports. Beams are considered among the most important of all structural elements. They are used to support the floor of a building, the deck of a bridge, or the wing of an aircraft. Also, the axle of an automobile, the boom of a crane, even many of the bones of the body act as beams.

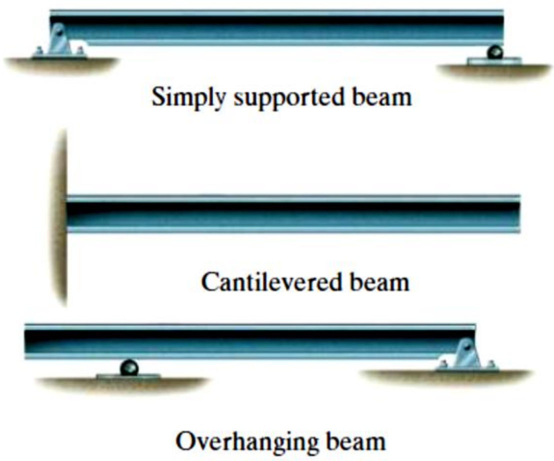
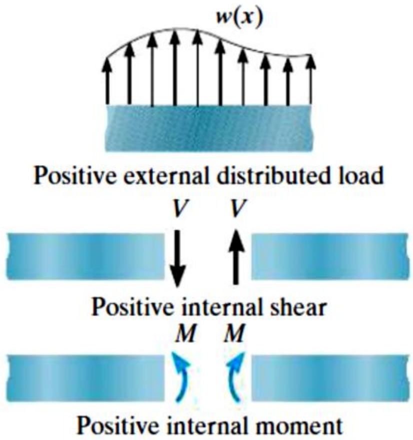


Fig. 6–1

Beam Sign Convention. Before presenting a method for determining the shear and moment as functions of x and later plotting these functions (shear and moment diagrams), it is first necessary to establish a sign convention so as to define "positive" and "negative" values for V and M . Although the choice of a sign convention is arbitrary, here we will use the one often used in engineering practice and shown in Fig. 6–3 . The positive directions are as follows: the distributed load acts upward on the beam; the internal shear force causes a clockwise rotation of the beam segment on which it acts; and the internal moment causes compression in the top



Beam sign convention

Fig. 6–3

fibers of the segment such that it bends the segment so that it "holds water". Loadings that are opposite to these are considered negative.

Important point

- Beams are long straight members that are subjected to loads perpendicular to their longitudinal axis. They are classified according to the way they are supported, e.g., simply supported, cantilevered, or overhanging.
- In order to properly design a beam, it is important to know the variation of the internal shear and moment along its axis in order to find the points where these values are a maximum.
- Using an established sign convention for positive shear and moment, the shear and moment in the beam can be determined as a function of its position x on the beam, and then these functions can be plotted to form the shear and moment diagrams.

• Procedure for Analysis

The shear and moment diagrams for a beam can be constructed using the following procedure.

Support Reactions.

• Determine all the reactive forces and couple moments acting on the beam, and resolve all the forces into components acting perpendicular and parallel to the beam's axis.

Shear and Moment Functions.

• Specify separate coordinates x having an origin at the beam's left end and extending to regions of the beam between concentrated forces and/or couple moments, or where there is no discontinuity of distributed loading.

• Section the beam at each distance x, and draw the free-body diagram of one of the segments. Be sure V and M are shown acting in their positive sense, in accordance with the sign convention given in Fig. 6–3.

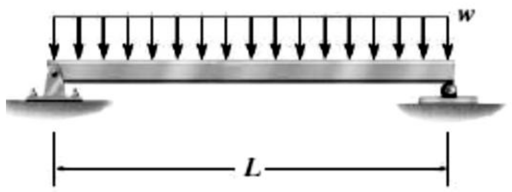
• The shear is obtained by summing forces perpendicular to the beam's axis.

• To eliminate V , the moment is obtained directly by summing moments about the sectioned end of the segment.

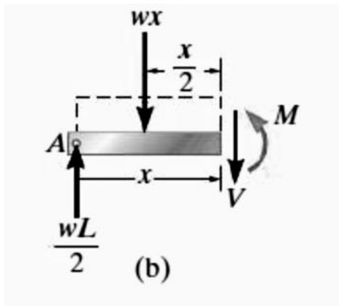
Shear and Moment Diagrams.

- Plot the shear diagram (V versus x) and the moment diagram (M versus x). If numerical values of the functions describing V and M are positive, the values are plotted above the x axis, whereas negative values are plotted below the axis.
- Generally it is convenient to show the shear and moment diagrams below the free-body diagram of the beam.

Example 01: Draw the shear and moment diagrams for the beam shown in Fig. 6–4 *a*.



(a)



Solution:

1. Support Reactions
2. Shear and Moment Functions.

$$+\uparrow \Sigma F_y = 0; \qquad \frac{wL}{2} - wx - V = 0$$

$$V = w\left(\frac{L}{2} - x\right)$$

$$\circlearrowleft + \Sigma M = 0; \qquad -\left(\frac{wL}{2}\right)x + (wx)\left(\frac{x}{2}\right) + M = 0$$

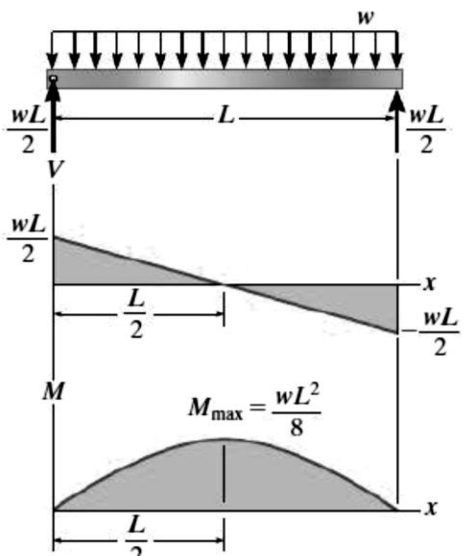
$$M = \frac{w}{2}(Lx - x^2)$$

3. Shear and Moment Diagrams.

The shear and moment diagrams shown in Fig. 6–4 *c* are obtained by plotting moment and shear obtained above the point of *zero shear* can be found from:

$$V = w\left(\frac{L}{2} - x\right) = 0$$

$$x = \frac{L}{2}$$



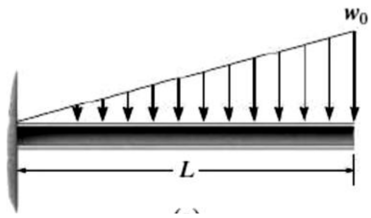
(c)

NOTE: From the moment diagram, this value of *x* represents the point on the beam where the *maximum moment* occurs, so from moment Eq., we have

$$M_{max} = \frac{w}{2}\left[L\left(\frac{L}{2}\right) - \left(\frac{L}{2}\right)^2\right]$$

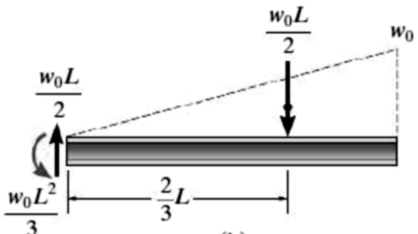
$$= \frac{wL^2}{8}$$

Example 02: Draw the shear and moment diagrams for the beam shown in Fig. 6–5 *a*.



Solution:

Support Reactions. The distributed load is replaced by its Resultant force and the reactions have been determined as shown in Fig. 6–5 *b.*



(b)

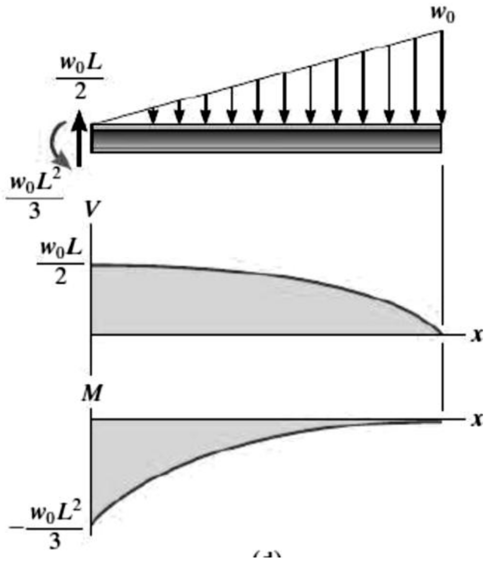
Shear and Moment Functions. A free-body diagram of a beam segment of length x is shown in Fig. 6–5 *c.* Note that the intensity of the triangular load at the section is found by proportion, that is, $w/x = w_0/L$ or $w = w_0x/L$. With the load intensity known, the resultant of the distributed loading is determined from the area under the diagram. Thus,

$$+\uparrow\Sigma F_y = 0; \qquad \frac{w_0L}{2} - \frac{1}{2}\left(\frac{w_0x}{L}\right)x - V = 0$$

$$V = \frac{w_0}{2L}(L^2 - x^2) \qquad (1)$$

$$\circlearrowleft +\Sigma M = 0; \qquad \frac{w_0L^2}{3} - \frac{w_0L}{2}(x) + \frac{1}{2}\left(\frac{w_0x}{L}\right)x\left(\frac{1}{3}x\right) + M = 0$$

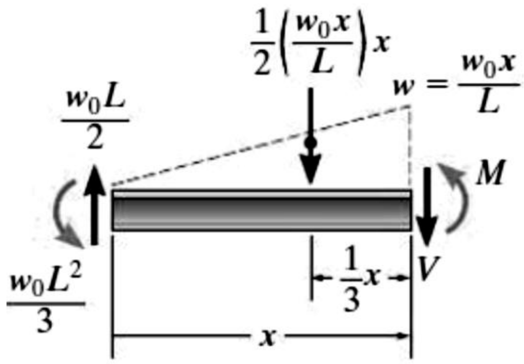
$$M = \frac{w_0}{6L}(-2L^3 + 3L^2x - x^3) \qquad (2)$$



These results can be checked by,

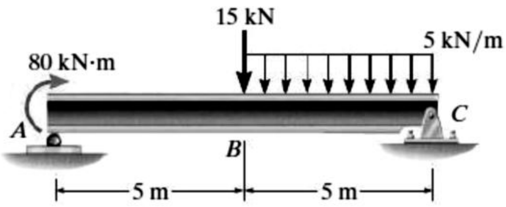
$$w = \frac{dV}{dx} = \frac{w_0}{2L}(0 - 2x) = -\frac{w_0x}{L} \qquad \text{OK}$$

$$V = \frac{dM}{dx} = \frac{w_0}{6L}(0 + 3L^2 - 3x^2) = \frac{w_0}{2L}(L^2 - x^2) \qquad \text{OK}$$



(c)

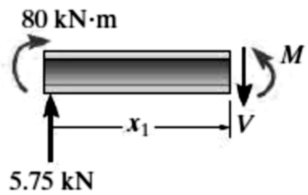
Example 03: Draw the shear and moment diagrams for the beam shown in Fig. below



(a)

Solution:

Support Reactions. The reactions at the supports have been determined and are shown on the free-body diagram of the beam, Fig. *d.*
Shear and Moment Functions. Since there is a discontinuity of distributed load and also a concentrated load at the beam's center, two regions of *x* must be considered in order to describe the shear and moment functions for the entire beam.



(b)

0 ∠X1<5 m, fig. b

$$+\uparrow\Sigma F_y = 0; \qquad 5.75\text{ kN} - V = 0$$

$$V = 5.75\text{ kN} \qquad (1)$$

$$\circlearrowleft + \Sigma M = 0; \qquad -80\text{ kN}\cdot\text{m} - 5.75\text{ kN}\,x_1 + M = 0$$

$$M = (5.75x_1 + 80)\text{ kN}\cdot\text{m} \qquad (2)$$

$5\text{ m} < x_2 \leq 10\text{ m}$, Fig. 6–7*c*:

$$+\uparrow\Sigma F_y = 0; \qquad 5.75\text{ kN} - 15\text{ kN} - 5\text{ kN/m}(x_2 - 5\text{ m}) - V = 0$$

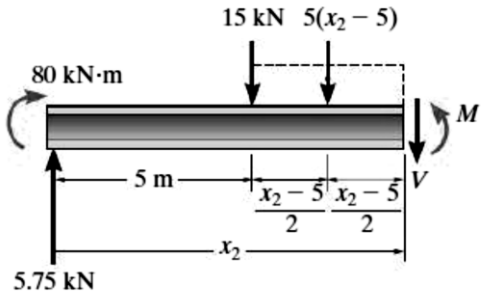
$$V = (15.75 - 5x_2)\text{ kN} \qquad (3)$$

$$\circlearrowleft + \Sigma M = 0; \qquad -80\text{ kN}\cdot\text{m} - 5.75\text{ kN}\,x_2 + 15\text{ kN}(x_2 - 5\text{ m})$$

$$+ 5\text{ kN/m}(x_2 - 5\text{ m})\left(\frac{x_2 - 5\text{ m}}{2}\right) + M = 0$$

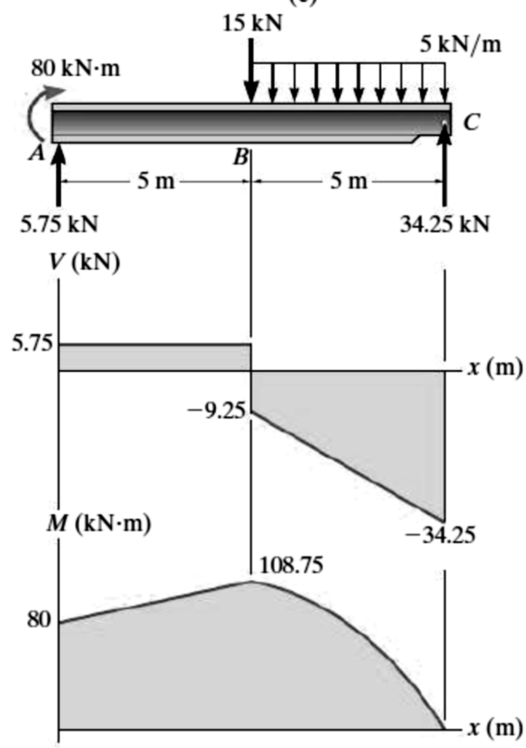
$$M = (-2.5x_2^2 + 15.75x_2 + 92.5)\text{ kN}\cdot\text{m} \qquad (4)$$

These results



(c)

These results can be checked in part by noting that $w = dV/dx$ and $V = dM/dx$. Also, when $x1 = 0$, Eqs. 1 and 2 give $V = 5.75$ kN and $M = 80$ kN.m; when $x2 = 10$ m, Eqs. 3 and 4 give $V = -34.25$ kN and $M = 0$. These values check with the support reactions shown on the free-body diagram, Fig. *d* .
Shear and Moment Diagrams. Equations 1 through 4 are plotted in Fig. *d* .

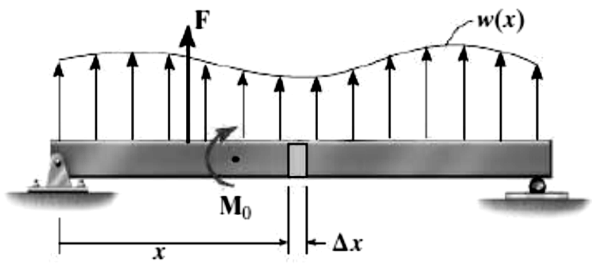


(d)

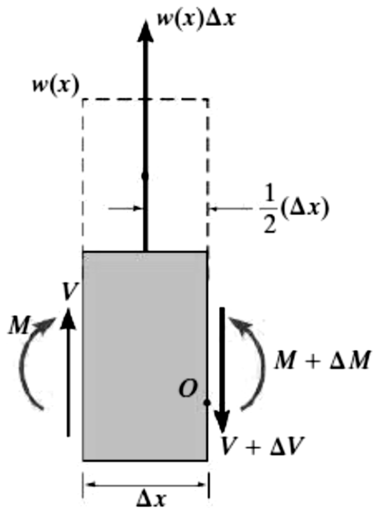
❖ Graphical Method for Constructing Shear and moment diagrams

In cases where a beam is subjected to *several* different loadings, determining V and M as functions of x and then plotting these equations can become quite tedious. In this section a simpler method for constructing the shear and moment diagrams is discussed—a method based on two differential relations, one that exists between distributed load and shear, and the other between shear and moment.

Regions of Distributed Load. For purposes of generality, consider the beam shown in Fig. 6–8*a,* which is subjected to an arbitrary loading. A free-body diagram for a very small segment Δx of the beam is shown in Fig. 6–8*b.* Since this segment has been chosen at a position x where there is no concentrated force or couple moment, the results to be obtained will *not* apply at these points of concentrated loading. Notice that all the loadings shown on the segment act in their positive directions according to the established sign convention, Fig. 6–3. Also, both the internal resultant shear and moment, acting on the right face of the segment, must be changed by a small amount in order to keep the segment in equilibrium. The distributed load, which is approximately constant over Δx, has been replaced by a resultant force $w(x)\ \Delta x$ that acts at $1/2(\ \Delta x\ )$ from the right side. Applying the equations of equilibrium to the segment, we have.



(a)



(b)

$$+\uparrow\Sigma F_y = 0; \qquad V + w(x)\,\Delta x - (V + \Delta V) = 0$$

$$\Delta V = w(x)\,\Delta x$$

$$\circlearrowleft +\Sigma M_O = 0; \qquad -V\,\Delta x - M - w(x)\,\Delta x\left[\tfrac{1}{2}(\Delta x)\right] + (M + \Delta M) = 0$$

$$\Delta M = V\,\Delta x + w(x)\,\tfrac{1}{2}(\Delta x)^2$$

Dividing by Δx and taking the limit as $\Delta x \rightarrow 0$, the above two equations become

$$\frac{dV}{dx} = w(x)$$

slope of shear diagram at each point	=	distributed load intensity at each point

$$\frac{dM}{dx} = V(x)$$

slope of moment diagram at each point	=	shear at each point

These two equations provide a convenient means for quickly obtaining the shear and moment diagrams for a beam. Equation 6–1 states that at a point the *slope* of the shear diagram equals the intensity of the distributed loading. For example, consider the beam in Fig. 6–9 *a.* The distributed loading is negative and increases from zero to w_B . Therefore, the shear diagram will be a curve that has a *negative slope,* increasing from zero to $-w_B$. Specific slopes $w_A = 0$, $-w_C$, $-w_D$, and $-w_B$ are shown in Fig. 6–9 *b.* In a similar manner, Eq. 6–2 states that at a point the *slope* of the moment diagram is equal to the shear. Notice that the shear diagram in Fig. 6–9 *b* starts at $+V_A$, decreases to zero, and then becomes negative and decreases to $-V_B$. The moment diagram will then have an initial slope of $+V_A$ which decreases to zero, then the slope becomes negative and decreases to $-V_B$. Specific slopes V_A, V_C, V_D, 0, and $-V_B$ are shown in Fig. 6–9c.

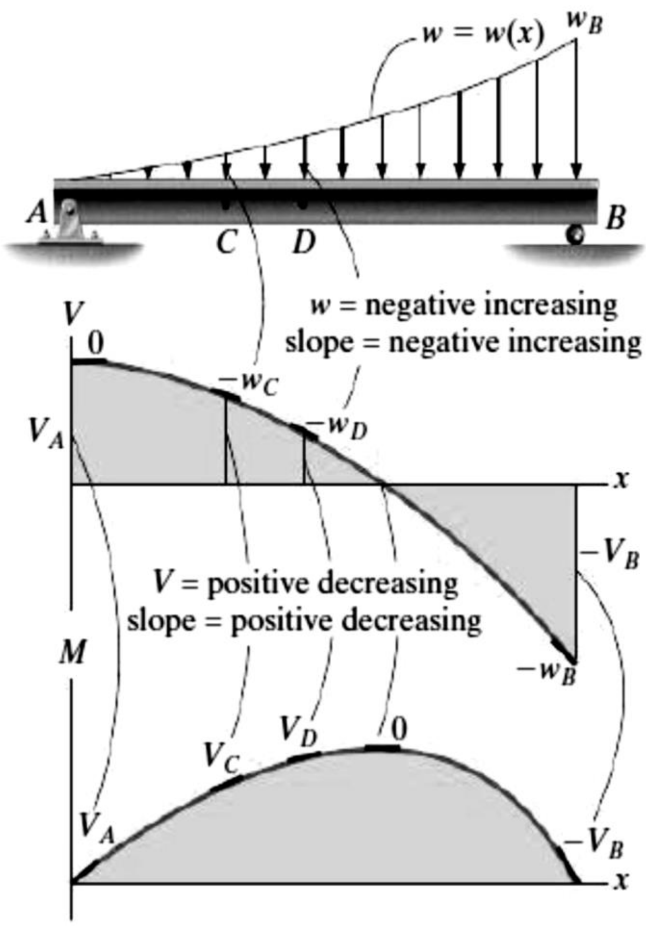


Fig. 6–9

Equations 6–1 and 6–2 may also be rewritten in the form $dV = w(x)dx$ and $dM = Vdx$. Noting that $w(x)\,dx$ and $V\,dx$ represent differential areas under the distributed loading and shear diagram, respectively, we can integrate these areas between any two points C and D on the beam, Fig. 6–9 *d,* and write

$$\Delta V = \int w(x)dx \qquad (6\text{–}3)$$

change in shear = area under distributed loading

$$\Delta M = \int V(x)dx \qquad (6\text{–}4)$$

change in moment = area under shear diagram

Equation 6–3 states that the *change in shear* between *C* and *D* is equal to the *area* under the distributed-loading curve between these two points, Fig. 6–9 *d.* In this case the change is negative since the distributed load acts downward. Similarly, from Eq. 6–4, the change in moment between *C* and *D,* Fig. 6–9 *f* , is equal to the area under the shear diagram within
the region from *C* to *D.* Here the change is positive. Since the above equations do not apply at points where a concentrated force or couple moment acts, we will now consider each of these cases.

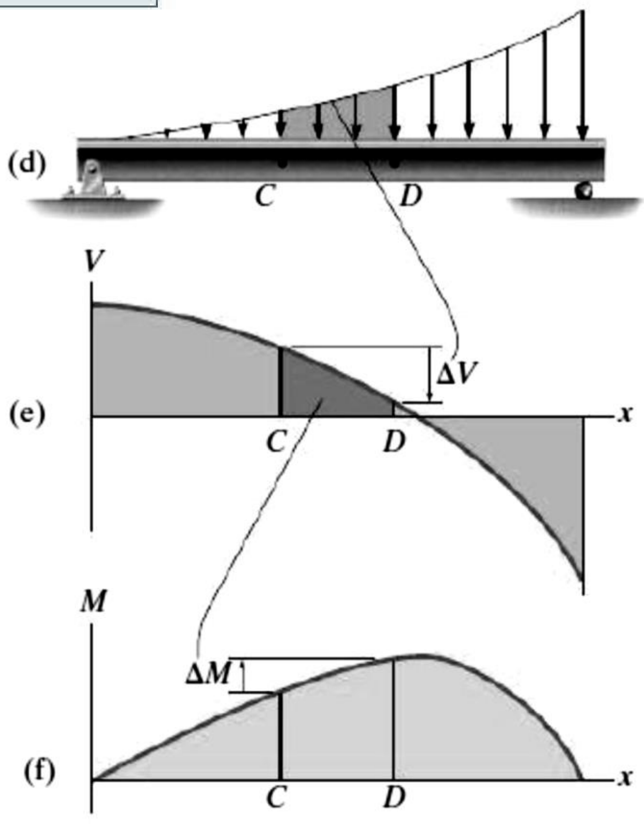


Fig. 6–9 (cont.)

Regions of Concentrated Force and Moment. A freebody diagram of a small segment of the beam in Fig. 6–8 *a* taken from under the force is shown in Fig. 6–10 *a* . Here it can be seen that force equilibrium requires

$$+\uparrow \Sigma F_y = 0; \qquad V + F - (V + \Delta V) = 0$$

$$\Delta V = F \qquad (6\text{–}5)$$

Thus, when **F** acts *upward* on the beam, !*V* is *positive* so the shear will "jump" *upward.* Likewise, if **F** acts *downward,* the jump (!*V*) will be *downward.*

When the beam segment includes the couple moment *M*0 , Fig. 6–10 *b* , then moment equilibrium requires the change in moment to be

$$\circlearrowleft + \Sigma M_O = 0; \qquad M + \Delta M - M_0 - V\,\Delta x - M = 0$$

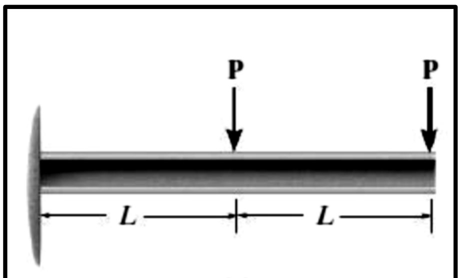
Letting $\Delta x \rightarrow 0$, we get

$$\Delta M = M_0 \qquad (6\text{–}6)$$

In this case, if **M**0 is applied *clockwise,* !*M* is *positive* so the moment diagram will "jump" *upward.* Likewise, when **M**0 acts *counterclockwise,* the jump (Δ*M*) will be *downward.*

Example 01

Draw the shear and moment diagrams for the beam shown

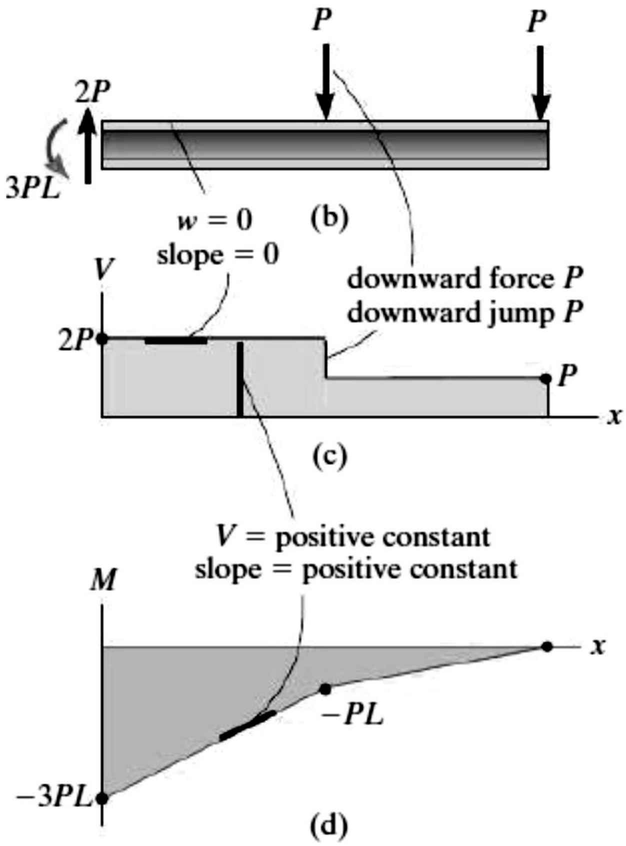


(a)

Solution

$$M|_{x=L} = M|_{x=0} + \Delta M$$

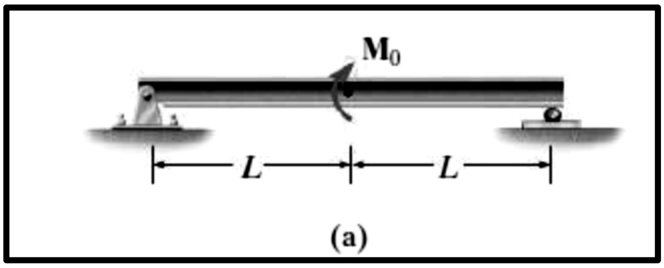
$$M|_{x=L} = -3PL + (2P)(L) = -PL$$



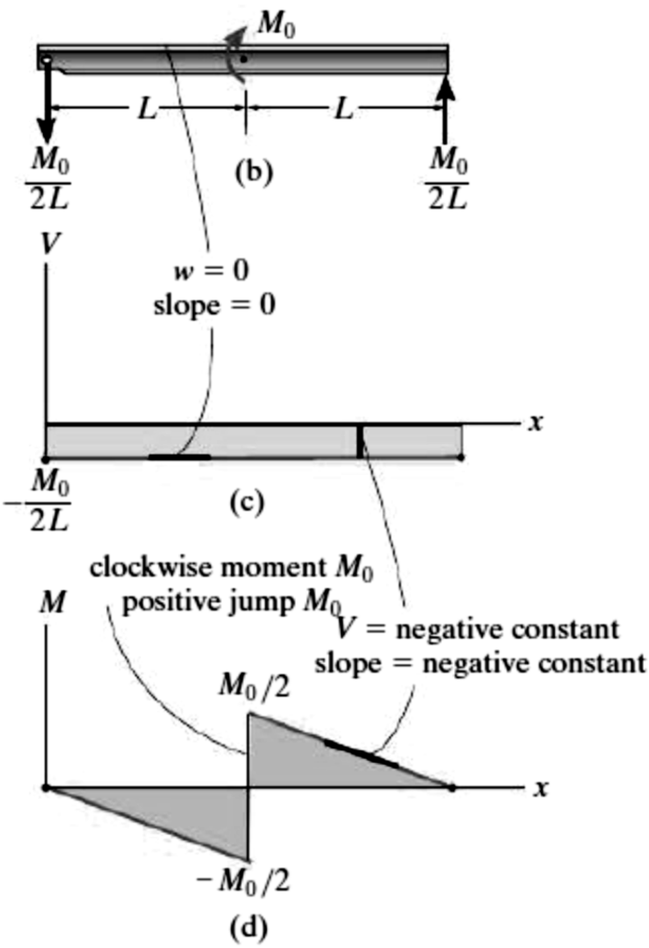
(b) (c) (d)

Example 02

Draw the shear and moment diagrams for the beam shown



Solution



Stresses in Beams

✓ Symmetrical beams

1. INTRODUCTION

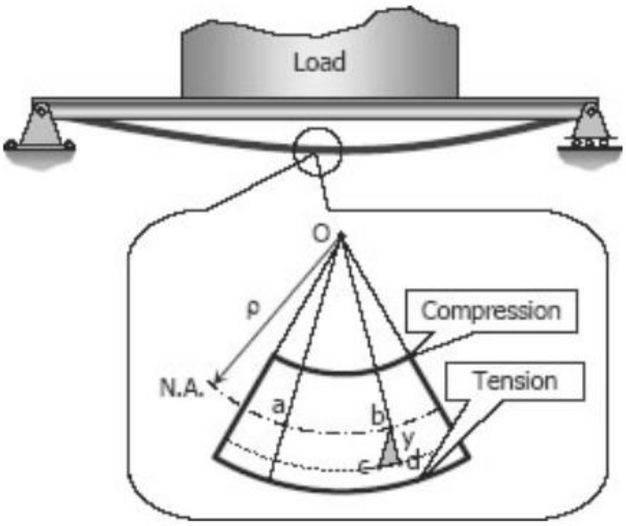
Forces and couples acting on the beam cause bending (flexural stresses) and shearing stresses on any cross section of the beam and deflection perpendicular to the longitudinal axis of the beam. If couples are applied to the ends of the beam and no forces act on it, the bending is said to be pure bending. If forces produce the bending, the bending is called ordinary bending.

2. ASSUMPTIONS

In using the following formulas for flexural and shearing stresses, it is assumed that a plane section of the beam normal to its longitudinal axis prior to loading remains plane after the forces and couples have been applied, and that the beam is initially straight and of uniform cross section and that the moduli of elasticity in tension and compression are equal.

3. FLEXURE FORMULA

Stresses caused by the bending moment are known as flexural or bending stresses. Consider a beam to be loaded as shown.



Consider a fiber at a distance y from the neutral axis, because of the beam's curvature, as the effect of bending moment, the fiber is stretched by an amount of cd. Since the curvature of the beam is very small, bcd and Oba are considered as similar triangles. The strain on this fiber is

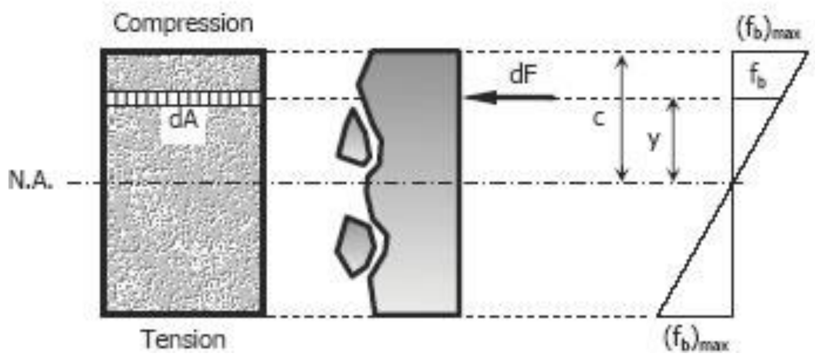
$$\varepsilon = \frac{cd}{ab} = \frac{y}{\rho}$$

By Hooke's law, $\varepsilon = \sigma / E$, then

$$\frac{\sigma}{E} = \frac{y}{\rho}; \ \sigma = \frac{y}{\rho} E$$

which means that the stress is proportional to the distance y from the neutral axis.

For this chapter, the notation f_b will be used instead of σ, to denote flexural stresses.



Considering a differential area dA at a distance y from N.A., the force acting over the area is

$$dF = f_b\,dA = \frac{y}{\rho}E\,dA = \frac{E}{\rho}y\,dA$$

The resultant of all the elemental moment about N.A. must be equal to the bending moment on the section.

$$M = \int y\,dF = \int y\frac{E}{\rho}y\,dA$$

$$M = \frac{E}{\rho}\int y^2\,dA$$

But $\int y^2\,dA = I,$

Then $$M = \frac{EI}{\rho} \text{ or } \rho = \frac{EI}{M}$$

substituting $\rho = Ey / f_b$

$$\frac{Ey}{f_b} = \frac{EI}{M}$$

then

$$f_b = \frac{My}{I}$$

and

$$(f_b)_{max} = \frac{Mc}{I}$$

The bending stress due to beams curvature is

$$f_b = \frac{Mc}{I} = \frac{\frac{EI}{\rho}c}{I}$$

$$f_b = \frac{Ec}{\rho}$$

The beam curvature is:

$$k = 1 / \rho$$

where ρ *is the radius of curvature of the beam in mm (in), M is the bending moment in N·mm (lb·in),* f_b *is the flexural stress in MPa (psi), I is the centroidal moment of inertia in* mm^4 *(*in^4*), and c is the distance from the neutral axis to the outermost fiber in mm (in).*

4. SECTION MODULUS

In the formula

$$(f_b)_{max} = \frac{Mc}{I} = \frac{M}{I/c},$$

the ratio I/c is called the section modulus and is usually denoted by S with units of mm^3 *(*in^3*). The maximum bending stress may then be written as*

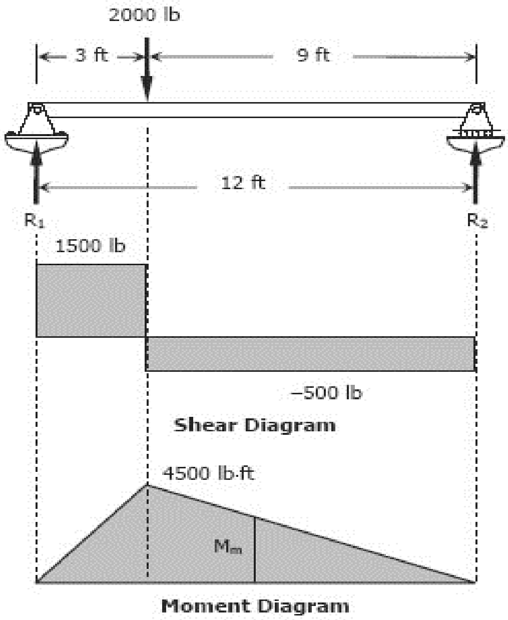
$$(f_b)_{max} = \frac{M}{S}$$

This form is convenient because the values of S are available in handbooks for a wide range of standard structural shapes.

5. SOLVED PROBLEMS

***Problem 01**. A simply supported beam, 2 in wide by 4 in high and 12 ft long is subjected to concentrated load of 2000 lb at a point 3 ft from one of the supports. Determine the maximum fiber stress and the stress in a fiber located 0.5 in from the top of the beam at mid span.*

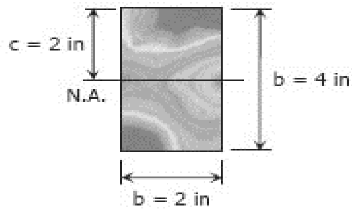
Solution



$$\Sigma M_{R2} = 0$$
$$12R_1 = 9(2000)$$
$$R_1 = 1500 \text{ lb}$$

$$\Sigma M_{R1} = 0$$
$$12R_2 = 3(2000)$$
$$R_2 = 500 \text{ lb}$$

Maximum fiber stress:



$$(f_b)_{max} = \frac{Mc}{I} = \frac{4500(12)(2)}{\frac{2(4)^3}{12}}$$

$$(f_b)_{max} = \mathbf{10{,}125 \text{ psi}}$$

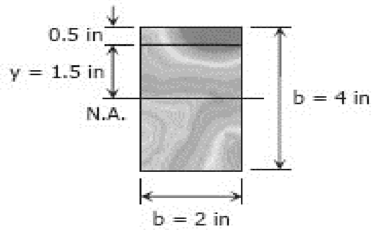
Stress in a fiber located 0.5 in from the top of the beam at midspan:

$$\frac{M_m}{6} = \frac{4500}{9}$$
$$M_m = 3000 \text{ lb·ft}$$

$$f_b = \frac{My}{I}$$

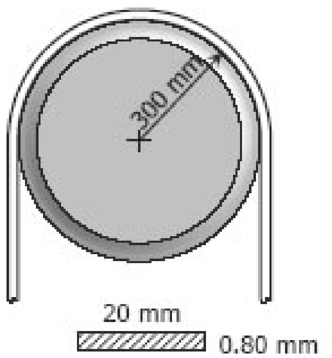
$$f_b = \frac{3000(12)(1.5)}{\frac{2(4^3)}{12}}$$

$$f_b = \mathbf{5{,}062.5 \text{ psi}}$$



Problem 01. *A high strength steel band saw, 20 mm wide by 0.80 mm thick, runs over pulleys 600mm in diameter. What maximum flexural stress is developed? What minimum diameter pulleys can be used without exceeding a flexural stress of 400 MPa? Assume E = 200 GPa*

Solution



Flexural stress developed:

$$M = \frac{EI}{\rho}$$

$$f_b = \frac{Mc}{I} = \frac{(EI/\rho)c}{I}$$

$$f_b = \frac{Ec}{\rho} = \frac{200000(0.80/2)}{300}$$

$$f_b = \mathbf{266.67\ MPa}$$

Minimum diameter of pulley:

$$f_b = \frac{Ec}{\rho}$$

$$400 = \frac{200000(0.80/2)}{\rho}$$

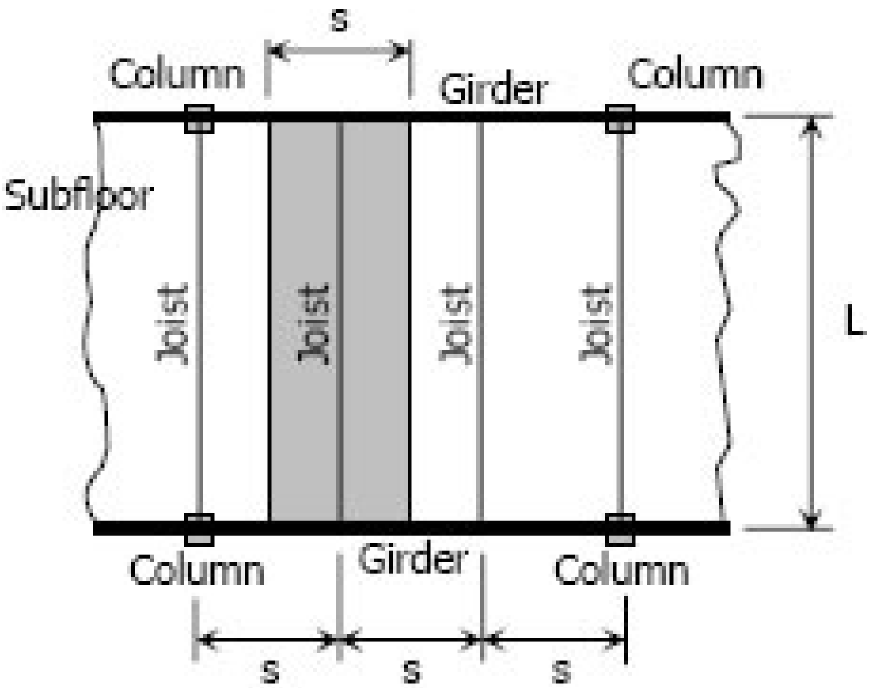
$$\rho = 200\ \text{mm}$$

diameter, d = **400 mm**

6. Floor Framing

In floor framing, the subfloor is supported by light beams called floor joists or simply joists which in turn supported by heavier beams called girders then girders pass the load to columns. Typically, joist act as simply supported beam carrying a uniform load of magnitude p over an area of sL,

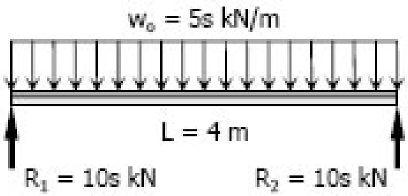
where
p = floor load per unit area
L = length (or span) of joist
s = center to center spacing of joists and
wo = sp = intensity of distributed load in joist.



Typical Floor Framing Plan

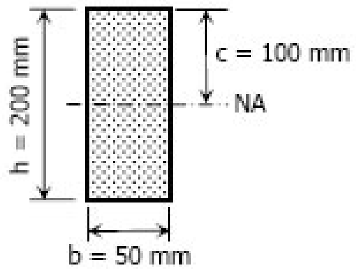
Example. *Floor joists 50 mm wide by 200 mm high, simply supported on a 4-m span, carry a floor loaded at 5 kN/m2. Compute the center-line spacing between joists to develop abending stress of 8 MPa. What safe floor load could be carried on a center-line spacing of 0.40 m?*

Solution



Load Diagram

$M_{max} = 1/8\, w_o L^2$



Cross-Section

Part 1:

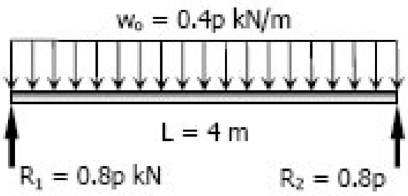
$$(f_b)_{max} = \frac{Mc}{I}$$

where: $(f_b)_{max} = 8$ MPa

$$M = \tfrac{1}{8}(5s)(4^2)$$
$$= 10s \text{ kN·m}$$
$$c = h/2 = 200/2 = 100 \text{ mm}$$
$$I = \frac{bh^3}{12} = \frac{50(200^3)}{12} = 33.33 \times 10^6 \text{ mm}^4$$

$$8 = \frac{10s(100)(1000^2)}{33.33 \times 10^6}$$

$s = \mathbf{0.267\ m}$



Load Diagram

$M_{max} = 1/8\, w_o L^2$

Part 2:

$$(f_b)_{max} = \frac{Mc}{I}$$

where: $M = \tfrac{1}{8} w_o L^2$

$$= \tfrac{1}{8}(0.4p)(4^2)$$
$$= 0.8p$$

$$8 = \frac{0.8p(100)(1000^2)}{33.33 \times 10^6}$$

$p = \mathbf{3.33\ kN/m^2}$

Composite Beams

1. INTRODUCTION

- *Beams constructed of two or more different materials are referred to as* ***composite beams****. For example, a beam can be made of wood with straps of steel at its top and bottom, Fig. 6–35. Engineers purposely design beams in this manner in order to develop a more efficient means for supporting loads. Since the flexure formula was developed only for beams having homogeneous material, this formula cannot be applied directly to determine the normal stress in a composite beam. In this section, however, we will develop a method for modifying or "transforming" a composite beam's cross section into one made of a single material. Once this has been done, the flexure formula can then be used for the stress analysis.*

للضرورة الانشائية يتم استخدام عتبات ذات مقاطع مركبة اي تتكون من اكثر من مادة واحدة

ولأيجاد الاجهادات لتلك الانواع من المقاطع يجب ان تحول الى مادة واحدة. عادتا يتم التحويل الى المادة الاضعف والتي تملك معامل مرونة اكبر.

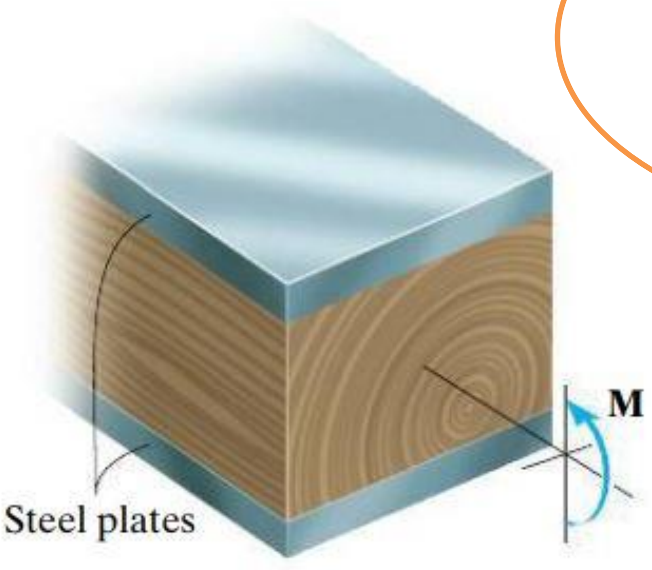


Fig. 6–35

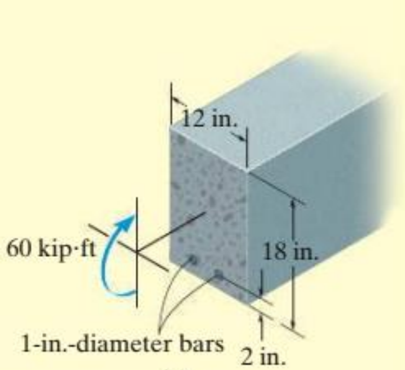
2. Procedure for analysis

- *Calculate transformation factor (n),* $\left(n = \frac{E_1}{E_2}\right)$ *where E_1 represented greater stiffness material and E_2 smaller stiffness materials*
- *Determine the new width of strong material* $b_{new} = b * n$
- *Find the location of the centroid* $, \bar{y} = \frac{\sum \bar{y} A}{\sum A}$
- *Determine the equivalent moment of inertia* $I_{NA} = \frac{bh^3}{12} + +Ad^2$ *where d represented the distance between center of mass to the* $, \bar{y}$
- *Calculate normal stresses in each segment* $\sigma = \frac{M\,c}{I_{NA}}$ *, C represented the distance of required point from neutral axis in a section*
- *Finally, the stress of strong materials must be multiplied with n*

1. نجد مقدار (n)
2. نضرب عرض المادة المطلوب تحويلها * n
3. اجهاد المادة المحولة يجب ان يضرب في n

Example

The reinforced concrete beam has the cross-sectional area shown in Fig. 6–39a. If it is subjected to a bending moment of M = 60 kip.ft determine the normal stress in each of the steel reinforcing rods and the maximum normal stress in the concrete. Take $E_{st.} = 29(10^3)$ ksi and $E_{conc.} = 3.6(10^3)$ ksi



(a)

Solution

$$\boldsymbol{n} = \frac{\boldsymbol{E}_{st}}{\boldsymbol{E}_w} = \frac{29000}{3600} = 8$$

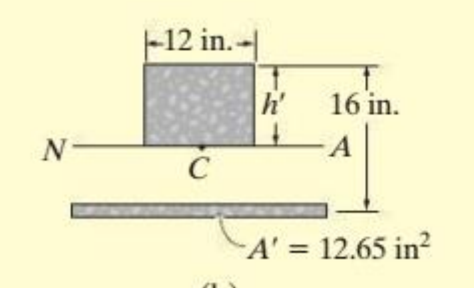
$$\boldsymbol{As' = n.As = 8 * 2 * \pi * 0.5^2 = 12.57\ in^2}$$

We require the centroid to lie on the neutral axis. Thus $\sum \bar{y}A = 0$

$$12\text{ in.}\,(h')\frac{h'}{2} - 12.65\text{ in}^2(16\text{ in.} - h') = 0$$

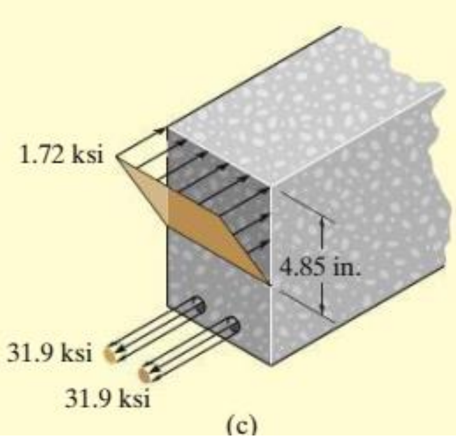
$$h'^2 + 2.11h' - 33.7 = 0$$

$$h' = 4.85\text{ in.}$$



(b)

$$I = \left[\frac{1}{12}(12\text{ in.})(4.85\text{ in.})^3 + 12\text{ in.}\,(4.85\text{ in.})\left(\frac{4.85\text{ in.}}{2}\right)^2\right] + 12.65\text{ in}^2(16\text{ in.} - 4.85\text{ in.})^2 = 2029\text{ in}^4$$



(c)

$$(\sigma_{\text{conc}})_{\max} = \frac{[60\text{ kip}\cdot\text{ft}(12\text{ in./ft})](4.85\text{ in.})}{2029\text{ in}^4} = 1.72\text{ ksi}$$

$$\sigma'_{\text{conc}} = \frac{[60\text{ kip}\cdot\text{ft}(12\text{ in./ft})](16\text{ in.} - 4.85\text{ in.})}{2029\text{ in}^4} = 3.96\text{ ksi}$$

$$\sigma_{\text{st}} = n\sigma'_{\text{conc}} = \left(\frac{29(10^3)\text{ ksi}}{3.6(10^3)\text{ ksi}}\right)3.96\text{ ksi} = 31.9\text{ ksi}$$

Example

A composite beam is made of wood and reinforced with a steel strap located on its bottom side. It has the cross-sectional area shown in Fig. 6–38a. If the beam is subjected to a bending moment of M = 2 kN.m, determine the normal stress at points B and C. Take E_w = 12 GPa and E_{st} = 200 GPa.

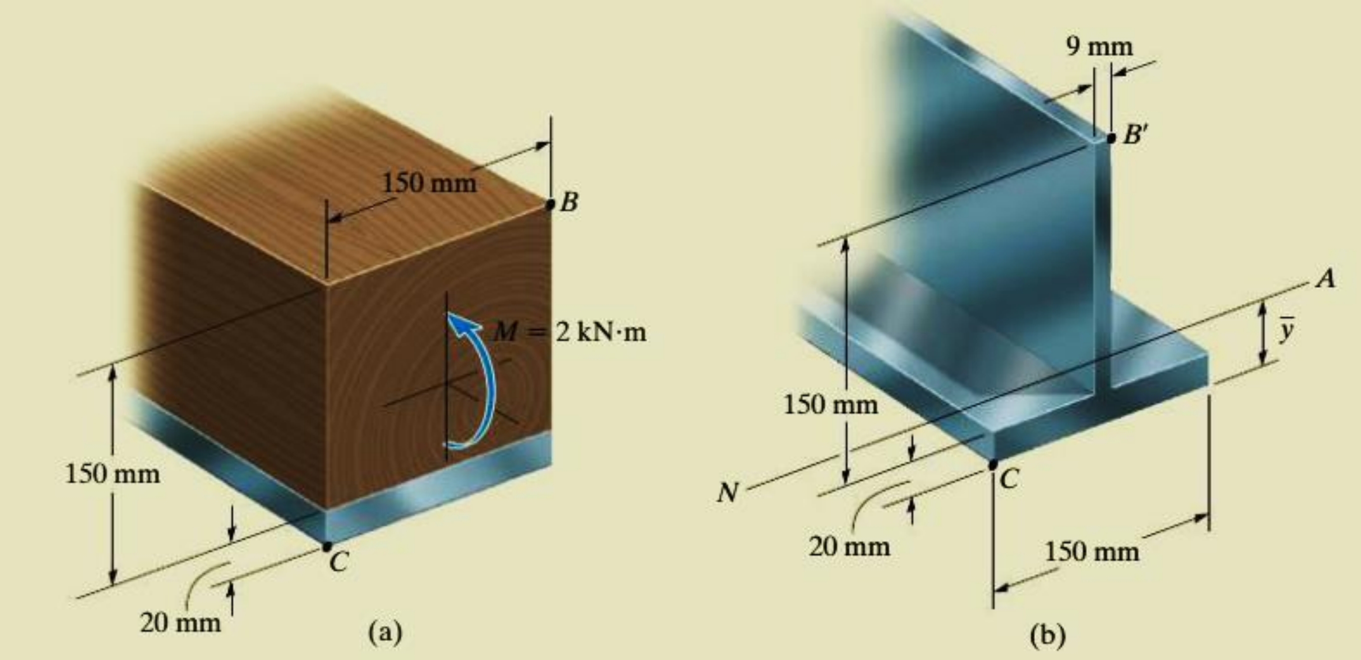


Fig. 6–38

Solution

$$\boldsymbol{n} = \frac{\boldsymbol{E}_{st}}{\boldsymbol{E}_w} = \frac{200}{12} = 16.667$$

$$\boldsymbol{b}_{wood} = \frac{150}{16.667} = 9$$

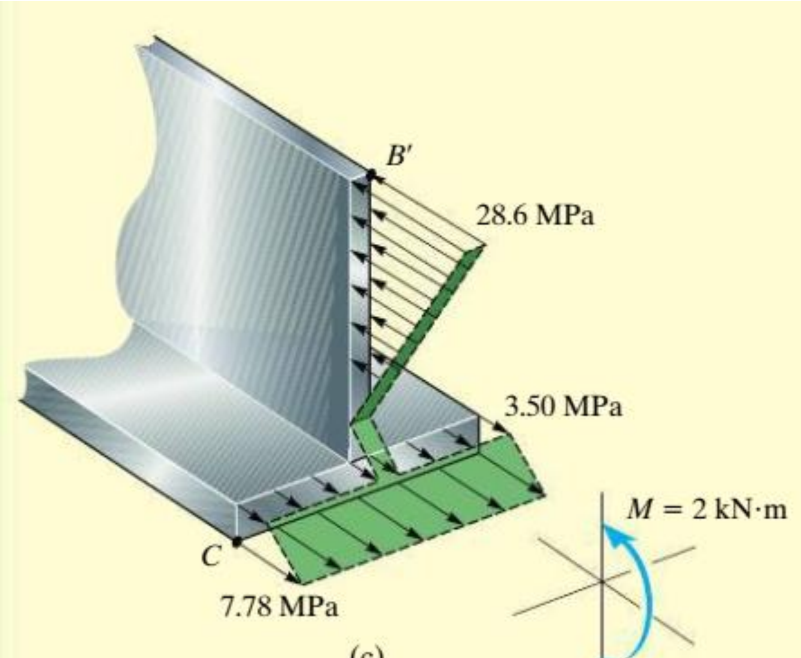
$$\bar{y} = \frac{\sum \bar{y}\,A}{\sum A} = \frac{0.01 * 0.02 * 0.150 + 0.095 * 0.009 * 0.150}{0.02 * 0.150 + 0.009 * 0.150} = 0.03638\,m$$

$$I_{NA} = \frac{bh^3}{12} + +AC^2$$

$$= \frac{1}{12}[0.150 * 0.02^3 + 0.150 * 0.02 * (0.03638 - 0.01)^2 + 0.009 * 0.150^3 * (0.095 - 0.03638)^2] = 9.358 * 10^{-6} m^4$$

$$\sigma_{B\prime} = \frac{M\,\bar{y}}{I_{NA}} = \frac{2 * 10^3 * (0.170 - 0.03638)}{9.358 * 10^{-6}} = 28.6\ MPa$$

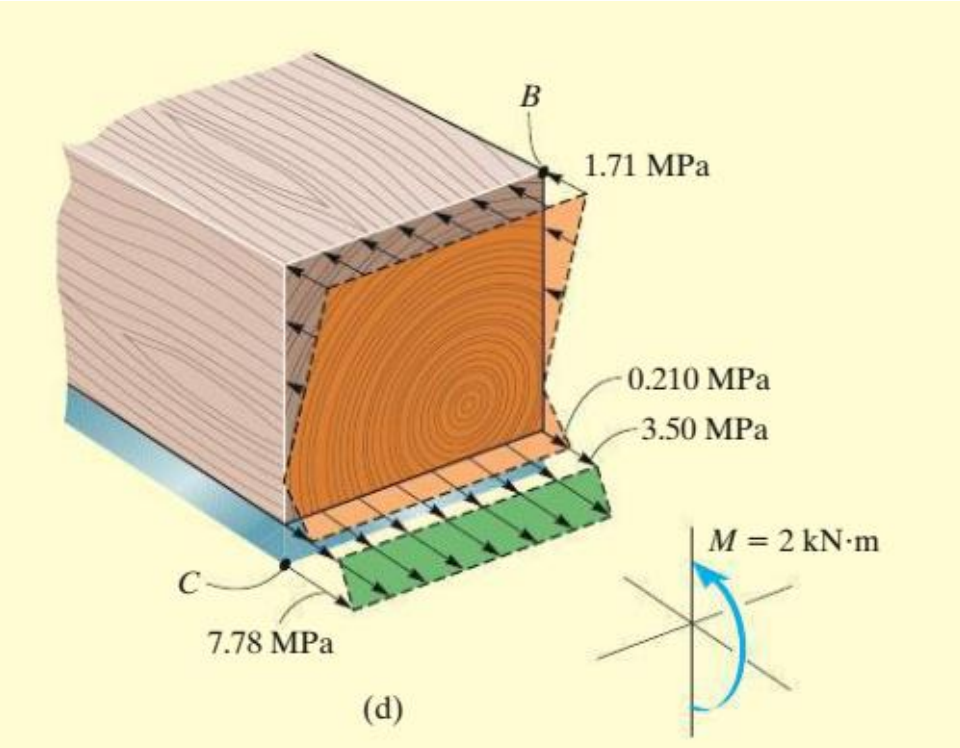
$$\sigma_C = \frac{M\,\bar{y}}{I_{NA}} = \frac{2 * 10^3 * (0.03638)}{9.358 * 10^{-6}} = 7.78\ MPa$$



(c)

The normal-stress distribution on the transformed (all steel) section is shown in Fig. 6–38c. The normal stress in the wood at B in Fig. 6–38a, is determined from Eq. 6–21; that is,

$$\sigma_B = n\sigma_{B\prime} = \frac{12}{200} * 28.56 = 1.71\ MPa$$



(d)

Using these concepts, show that the normal stress in the steel and the wood at the point where they are in contact is s_{st} = 3.50 MPa and s_w = 0.210 MPa, respectively. The normal-stress distribution in the actual beam is shown in Fig. 6–38d.

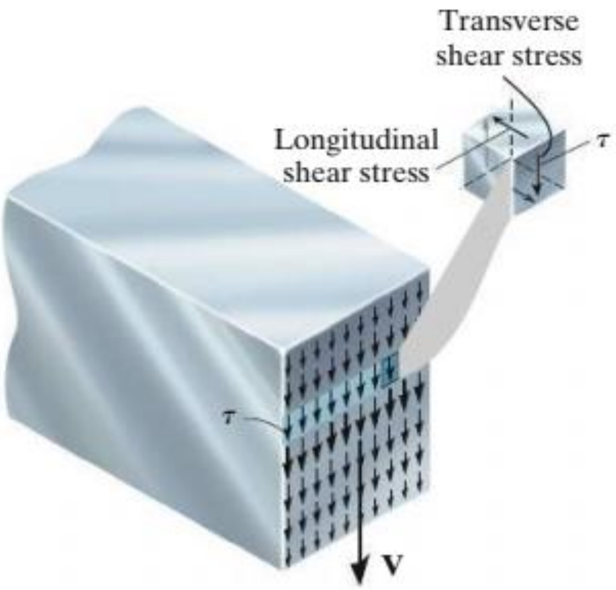
Shear stresses in beams

1. INTRODUCTION

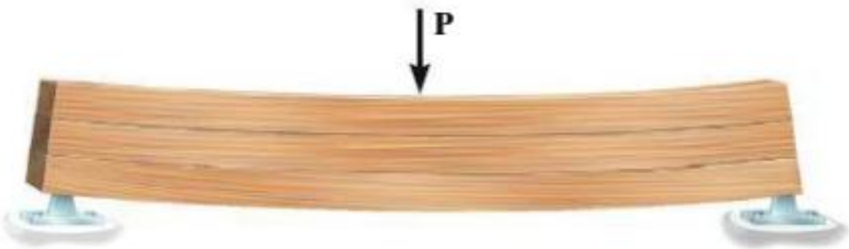
- *In this chapter, we will develop a method for finding the shear stress in a beam having a prismatic cross section and made from homogeneous material that behaves in a linear-elastic manner. The method of analysis to be developed will be somewhat limited to special cases of cross-sectional geometry. Although this is the case, it has many wide-range applications in engineering design and analysis. The concept of shear flow, along with shear stress, will be discussed for beams. The chapter ends with a discussion of the shear center.*

في هذا الموضوع سوف نتعرف على طريقة تمكننا من حساب اجهادات القص في الاعتاب ذات المقاطع المتجانسة والتي تتصرف بشكل خطي.

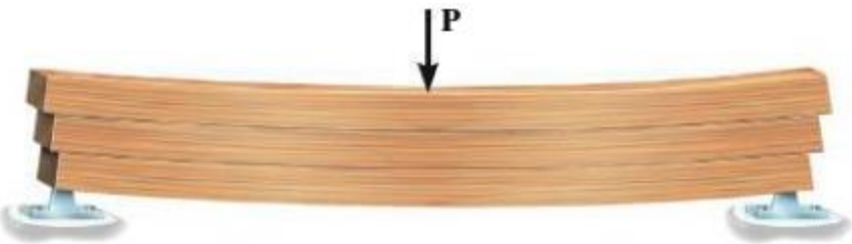
اجهادات القص لها تطبيقات عديدة في مجال الهندسة والتي سوف نتعرف عليها بالتفصيل من خلال هذا الفصل.



- *To illustrate this effect, consider the beam to be made from three boards, Fig. a. If the top and bottom surfaces of each board are smooth, and the boards are not bonded together, then application of the load **P** will cause the boards to slide relative to one another when the beam deflects. However, if the boards are bonded together, then the longitudinal shear stresses acting between the boards will prevent their relative sliding, and consequently the beam will act as a single unit, Fig. b .*

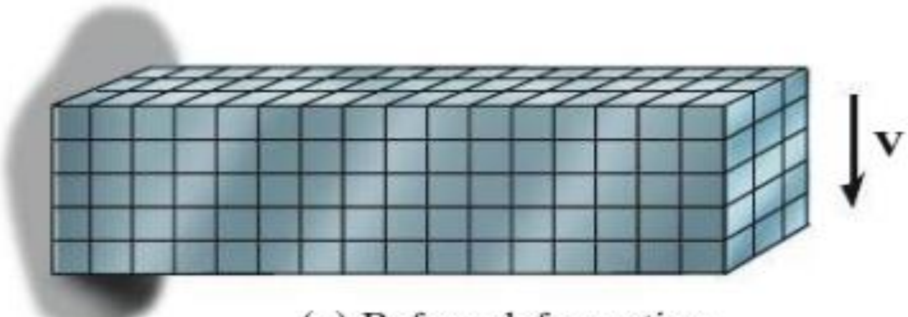


Boards bonded together

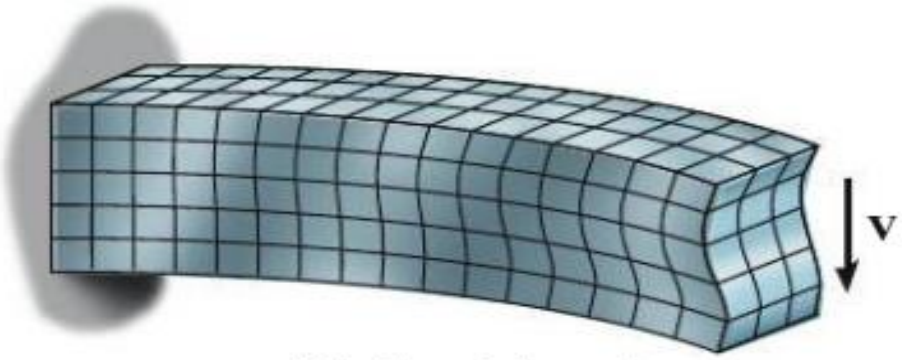


Boards not bonded together

- *As a result of the shear stress, shear strains will be developed and these will tend to distort the cross section in a rather complex manner. For example, consider the short bar in Fig.* a *made of a highly deformable material and marked with horizontal and vertical grid lines. When a shear* ***V*** *is applied, it tends to deform these lines into the pattern shown in Fig.* b. *This nonuniform shear-strain distribution will cause the cross section to* warp.



(a) Before deformation



(b) After deformation

2. The Shear Formula

$$\tau = \frac{V.Q}{I.t}$$

$\boldsymbol{\tau}$ = *the shear stress in the member at the point located a distance* ***y'*** *from the neutral axis. This stress is assumed to be constant and therefore averaged across the width* ***t*** *of the member*

V = *the internal resultant shear force, determined from the method of sections and the equations of equilibrium*

I = *the moment of inertia of the entire cross-sectional area calculated about the neutral axis*

t = *the width of the member's cross-sectional area, measured at the point where* ***t*** *is to be determined*

Q = ***y'A'****, where* ***A'*** *is the area of the top (or bottom) portion of the member's cross-sectional area, above (or below) the section plane where* ***t*** *is measured, and* ***y'*** *is the distance from the neutral axis to the centroid of* ***A***

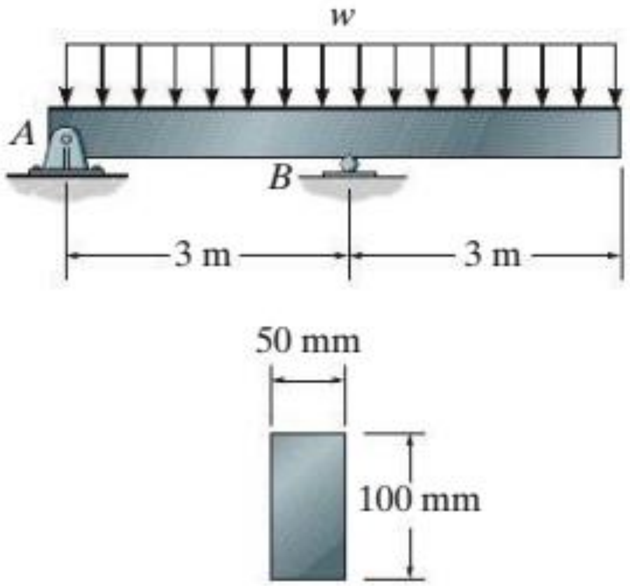
3. Procedure for analysis

- *Section the member perpendicular to its axis at the point where the shear stress is to be determined, and obtain the internal shear **V** at the section*

  *Determine the location of the neutral axis, and determine the moment of inertia **I** of the entire cross sectional area about the neutral axis.*
- *Pass an imaginary horizontal section through the point where the shear stress is to be determined. Measure the width **t** of the cross-sectional area at this section.*
- *The portion of the area lying either above or below this width is **A'**. Determine **Q** by using Q = **y'A'** Here **y'** is the distance to the centroid of **A'**, measured from the neutral axis*
- *Using a consistent set of units, substitute the data into the shear formula and calculate the shear stress t.*

Example

The overhang beam is subjected to the uniform distributed load having an intensity of w = 50 kN/m. Determine the maximum shear stress developed in the beam.

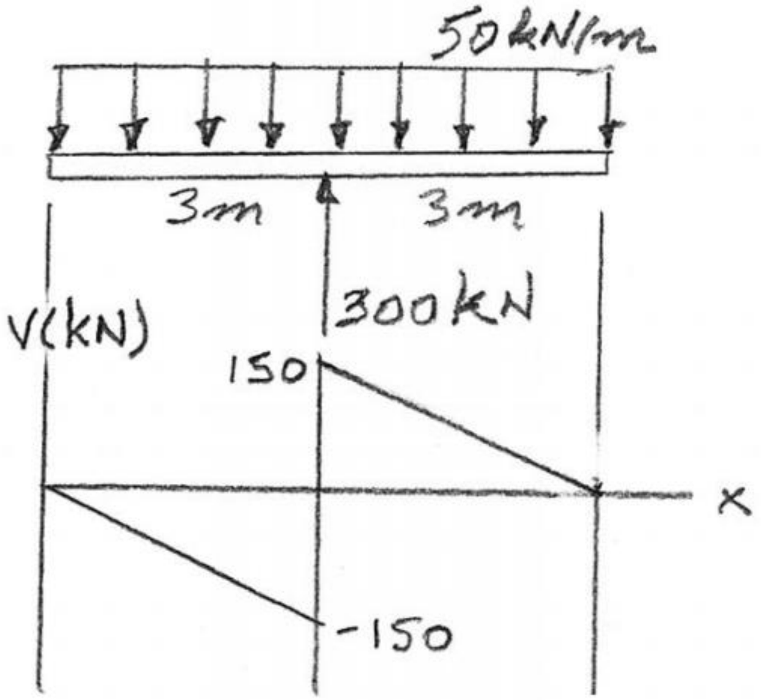


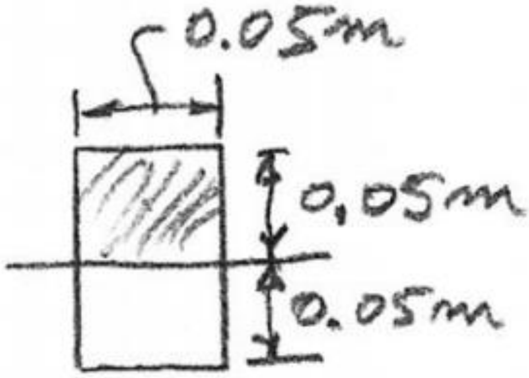
Solution

$$Q = 0.05 * 0.05 * 0.025 = 6.25 * 10^{-5}\, m^3$$

$$I = \frac{0.05 * 0.1^3}{12} = 4.1667 * 10^{-6}\, m^4$$

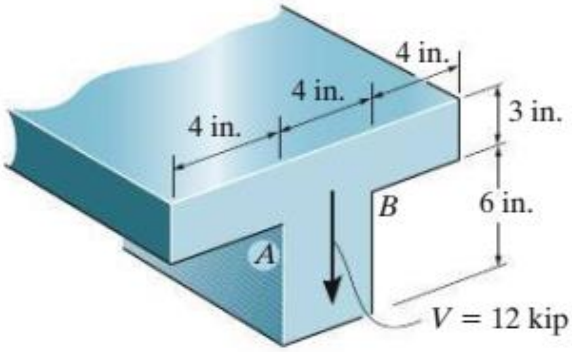
$$\tau = \frac{6.25 * 10^{-5} * 150 * 10^3}{4.1667 * 10^{-6} * 0.05} = 45\, MPa$$





Example

If the T-beam is subjected to a vertical shear of V = 12 kip, determine the maximum shear stress in the beam. Also, compute the shear-stress jump at the flange web junction AB. Sketch the variation of the shear-stress intensity over the entire cross section



Solution

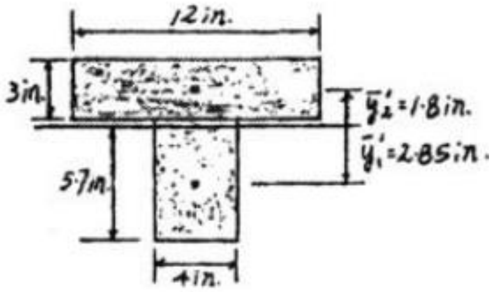
$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{1.5(12)(3) + 6(4)(6)}{12(3) + 4(6)} = 3.30 \text{ in.}$$

$$I_{NA} = \frac{1}{12}(12)\left(3^3\right) + 12(3)(3.30 - 1.5)^2 + \frac{1}{12}(4)\left(6^3\right) + 4(6)(6 - 3.30)^2$$

$$= 390.60 \text{ in}^4$$

$$Q_{max} = \bar{y}_1' A' = 2.85(5.7)(4) = 64.98 \text{ in}^3$$

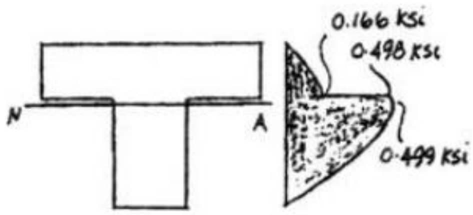
$$Q_{AB} = \bar{y}_2' A' = 1.8(3)(12) = 64.8 \text{ in}^3$$



$$\tau_{max} = \frac{VQ_{max}}{It} = \frac{12(64.98)}{390.60(4)} = 0.499 \text{ ksi}$$

$$(\tau_{AB})_f = \frac{VQ_{AB}}{It_f} = \frac{12(64.8)}{390.60(12)} = 0.166 \text{ ksi}$$

$$(\tau_{AB})_w = \frac{VQ_{AB}}{I\,t_W} = \frac{12(64.8)}{390.60(4)} = 0.498 \text{ ksi}$$



Combined Stresses

1. INTRODUCTION

In previous topics we developed methods for determining the stress distributions in a member subjected to either an internal axial force, a shear force, a bending moment, or a torsional moment. Most often, however, the cross section of a member is subjected to several of these loadings simultaneously. When this occurs, the method of upper position can be used to determine the resultant stress distribution. That the principle of superposition can be used for this purpose provided a linear relationship exists between the stress and the loads. Also, the geometry of the member should not undergo significant change when the loads are applied. These conditions are necessary in order to ensure that the stress produced by one load is not related to the stress produced by any other load.

2. NORMAL FORCE.

The internal normal force is developed by a uniform normal-stress distribution determined from $\boldsymbol{\sigma = P/A}$.

3. SHEAR FORCE.

The internal shear force in a member is developed by a shear stress distribution determined from the shear formula, $\boldsymbol{\tau = VQ/It}$.

4. BENDING MOMENT.

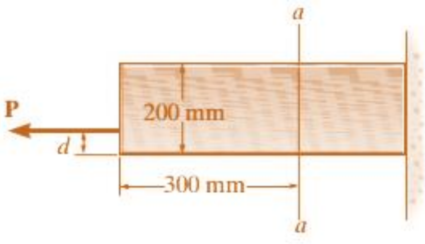
For straight members the internal bending moment is developed by a normal-stress distribution that varies linearly from zero at the neutral axis to a maximum at the outer boundary of the member. This stress distribution is determined from the flexure formula, $\boldsymbol{\sigma = -My/I}$.

5. SUPERPOSITION

$$\boldsymbol{\sigma = \mp \frac{P}{A} \mp \frac{MC}{I}}$$

Example

*Determine the smallest distance d to the edge of the plate at which the force **P** can be applied so that it produces no compressive stresses in the plate at section a–a. The plate has a thickness of 20 mm and **P** acts along the centerline of this thickness.*



Solution

$$\xrightarrow{+} \Sigma F_x = 0; \qquad N - P = 0 \qquad N = P$$

$$\circlearrowleft + \Sigma M_C = 0; \qquad M - P(0.1 - d) = 0 \qquad M = P(0.1 - d)$$

$$A = 0.2\,(0.02) = 0.004 \text{ m}^4 \qquad I = \frac{1}{12}(0.02)(0.2^3) = 13.3333(10^{-6}) \text{ m}^4$$

$$\sigma = \frac{N}{A} \pm \frac{My}{I}$$

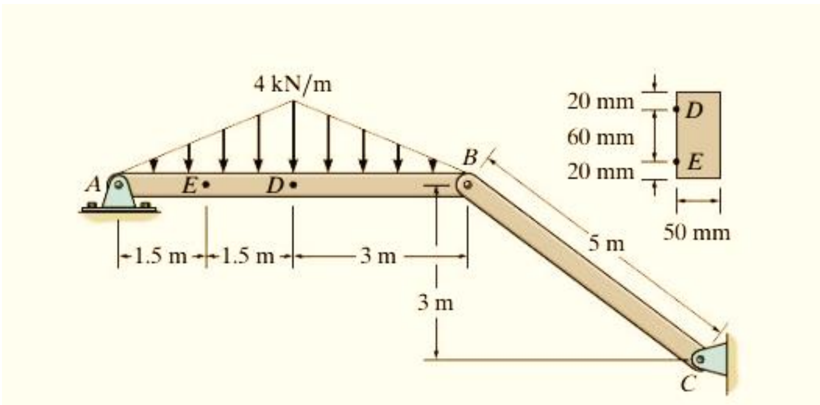
$$0 = \frac{P}{0.004} \pm \frac{P(0.1 - d)(0.1)}{13.3333\,(10^{-6})}$$

$$0 = 250\,P - 7500\,P\,(0.1 - d)$$

$$d = 0.06667 \text{ m} = 66.7 \text{ mm}$$

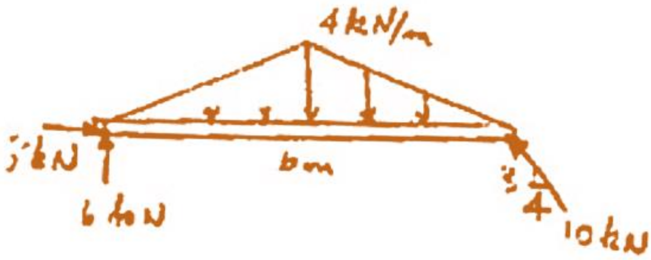
Example

The frame supports the distributed load shown. Determine the state of stress acting at point E. Show the results on a differential element at this point.



Solution

$$\sum M@A = 0 \;\rightarrow BC * \frac{3}{5} * 6 = 4 * 6 * 3 * 0.5$$
$$\therefore Bc = 10kN$$



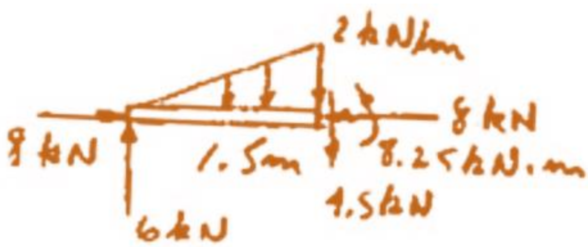
$$\sum Fy = 0 \rightarrow Ay + 10 * \frac{3}{5} - 4 * 6 * 0.5 = 0$$
$$\therefore Ay = 6kN$$

$$\sum Fx = 0 \rightarrow Ax - 10 * \frac{4}{5} = 0$$
$$\therefore Ax = 8kN$$

From Sec@E we can get the following

$$\frac{4}{3} = \frac{x}{1.5}$$

$$\therefore x = 2kN$$



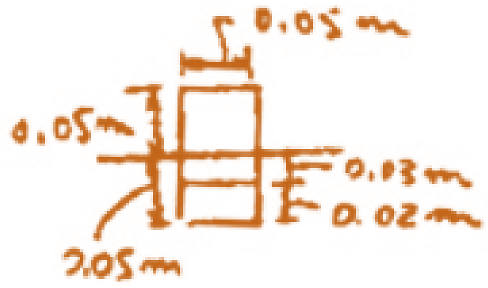
$$\sum M@E = 0 \;\rightarrow 6 * 1.5 - 2 * 1.5 * 0.5^2 = M@E$$
$$\therefore M@E = 8.25\; kN.m$$

$$\sum Fy = 0 \;\rightarrow 6 - 2 * 1.5 * 0.5 - V = 0$$
$$\therefore V = 4.5\; kN$$

$$\sum Fx = 0 \;\rightarrow Ex = 8kN$$

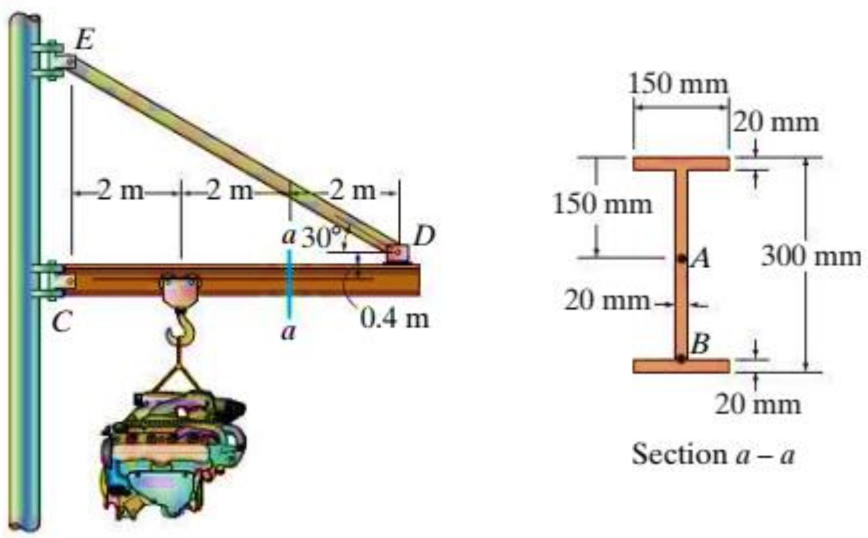
$$\sigma_E = -\frac{P}{A} + \frac{My}{I} = \frac{-8(10^3)}{(0.1)(0.05)} + \frac{8.25(10^3)(0.03)}{\frac{1}{12}(0.05)(0.1)^3} = 57.8 \text{ MPa}$$

$$\tau_E = \frac{VQ}{It} = \frac{4.5(10^3)(0.04)(0.02)(0.05)}{\frac{1}{12}(0.05)(0.1)^3(0.05)} = 864 \text{ kPa}$$



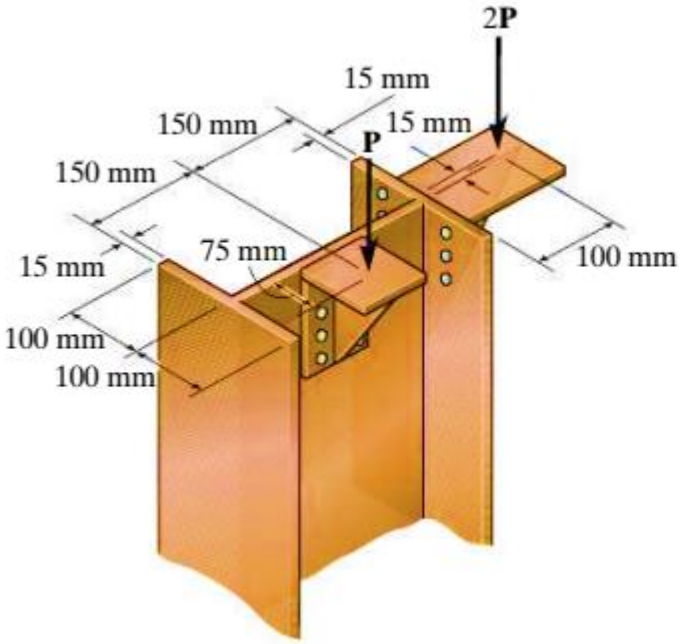
HW

1. *The 500-kg engine is suspended from the jib crane at the position shown. Determine the state of stress at point B on the cross section of the boom at section a–a. Point B is just above the bottom flange.*



Section *a* – *a*

2. *If P = 60 kN, determine the maximum normal stress developed on the cross section of the column.*



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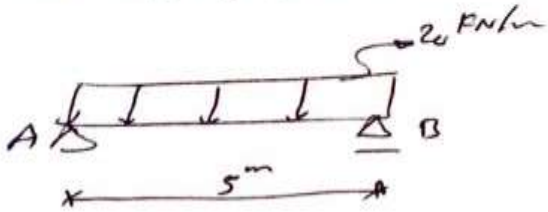
30/6/2020

Ex1 Find Max Deflection and θ_A, θ_B for Beam Below?



Solution

From FBD

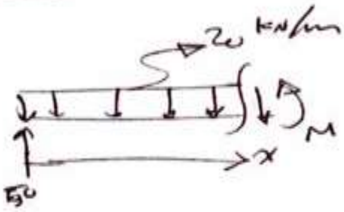
$\Sigma M_B = 0 \rightarrow$

$A_y * 5 - 20 * 5 * 2.5 = 0$

$\rightarrow A_y = 50 \text{ kN} \uparrow$

$\Sigma F_y = 0 \rightarrow B_y = 50 \text{ kN} \uparrow$

segment (1) $\quad 0 \le x \le 5^m$



$\Sigma M_o = 0.$

$50(x) - 20 * x * \frac{x}{2} - M = 0$

$\rightarrow M = 50x - 10x^2$

From Eq (1) we get.

$EI \frac{d^2v}{dx^2} = M_x \rightarrow$

$EI \frac{d^2v}{dx^2} = 50x - 10x^2$ (Both side Integral)

$EI\theta = EI \frac{dv}{dx} = \frac{50x^2}{2} - \frac{10x^3}{3} + C_1$ (Both side Integral)

$EI\Delta = EIy = \frac{25}{3}x^3 - \frac{10x^4}{12} + C_1 x + C_2$

From B.C

(1) $x = 0 \rightarrow y = 0$

(2) $x = 5 \rightarrow y = 0 \rightarrow$

e

① $0 = \frac{25}{3}(0)^3 - \frac{10}{12}(0)^4 + C_1(0) + C_2$

$\therefore C_2 = 0$

② $0 = \frac{25}{3}(5)^3 - \frac{10}{12}(5)^4 + C_1(5)$

$5(C_1) = -520.8334. \rightarrow C_1 = -104.167.$

$\therefore \theta = \frac{1}{EI}(25x^2 - \frac{10}{3}x^3 - 104.167) \longrightarrow (1)$

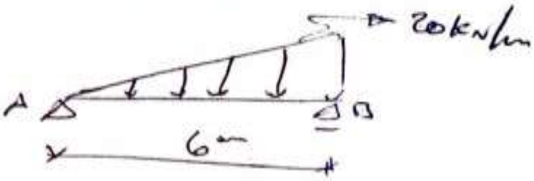
$v = \frac{1}{EI}\left(\frac{25}{3}x^3 - \frac{10}{12}x^4 - 104.167x\right) \longrightarrow (2)$

$\theta_A = \theta_B = \boxed{\frac{-104.167}{EI}}$

$v_{max} @ x = 2.5^m = \boxed{\frac{97.656}{EI}}$

3

Ex2 Find the Max deflection and θ_A, θ_B for Beam shown ?



Solution

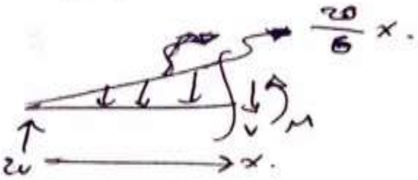
From F.B.D

$$\Sigma M_B = 0 \rightarrow$$

$$A_y(6) - 20 \times 6 \times 0.5 \times \left(\frac{6}{3}\right) = 0 \rightarrow A_y = 20\ kN \uparrow$$

$$\Sigma F_y = 0 \rightarrow B_y = 40\ kN \uparrow$$

Segment ① $0 \le x \le 6\ m$



$$\Sigma M_o = 0 \rightarrow$$

$$0 = 20(x) - \frac{20}{6}x(x \times 0.5) \times \frac{x}{3} - M$$

$$\therefore M = 20x - \frac{20}{36}x^3$$

From Eq ① $\rightarrow$

$$EI\frac{d^2v}{dx^2} = M_x \rightarrow EI\frac{d^2v}{dx^2} = 20x - \frac{20}{36}x^3$$

$$1^{st}\int \rightarrow EI\frac{dv}{dx} = \frac{20}{2}x^2 - \frac{20}{144}x^4 + C_1 = EI\,\theta$$

$$2^{nd}\int \rightarrow EI\,v = \frac{10}{3}x^3 - \frac{20}{720}x^5 + C_1 x + C_2 = EI\,\Delta$$

From B.C $\rightarrow$ $x = 0 \rightarrow v = 0$ — ①

$x = 6 \rightarrow v = 0$ — ②

$$① \quad 0 = \frac{10}{3}(0)^3 - \frac{20}{720}(0)^5 + C_1(0) + C_2 \rightarrow C_2 = 0$$

$$② \quad 0 = \frac{10}{3}(6)^3 - \frac{20}{720}(6)^5 + C_1(6)$$

$$\therefore C_1 = -84$$

a

So $\theta = \frac{1}{EI}\left(10x^2 - \frac{20}{144}x^4 - 84\right)$

$v = \frac{1}{EI}\left(\frac{10}{3}x^3 - \frac{20}{720}x^5 - 84x\right)$

$\because \theta_A \ @ \ x = 0 \longrightarrow$

$\therefore \theta_A = \boxed{\frac{-84}{EI}}$

$\because \theta_B \ @ \ x = 6^m \longrightarrow$

$\therefore \theta_B = \boxed{\frac{96}{EI}}$

$v_{max} \ @ \ \theta = 0$ so

$0 = 10x^2 - \frac{20}{144}x^4 - 84$

$\therefore x = 3.116^m.$

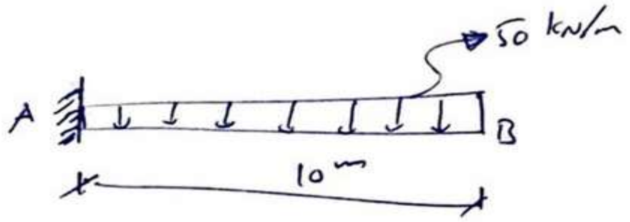
So $v_{max} \ @ \ x = 3.116^m$

$$\boxed{\therefore v_{max} = \frac{-169.055}{EI}}$$

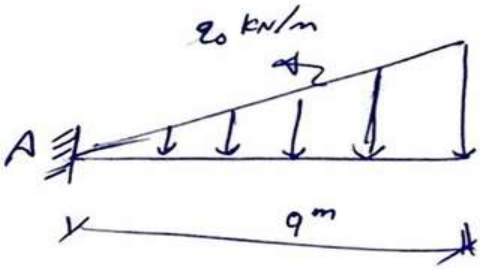
H.W

Find Max Deflection & slope for Beam shown Below?

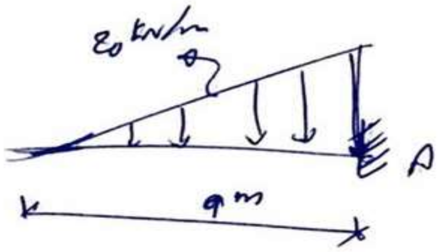
(1)



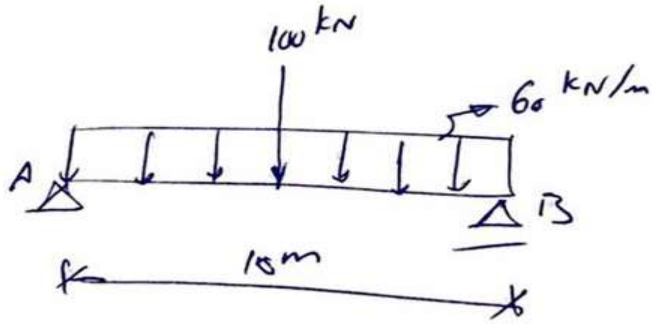
(2)



(3)



(4)



Defection of Beam

1. Introduction

Often limits must be placed on the amount of deflection a beam may undergo when it is subjected to a load, and so in this chapter we will discuss various methods for determining the deflection and slope at specific points on beams. The analytical methods include the integration method, the use of discontinuity functions, and the method of superposition. Also, a semi graphical technique, called the moment-area method, will be presented. At the end of the chapter, we will use these methods to solve for the support reactions on a beam that is statically indeterminate.

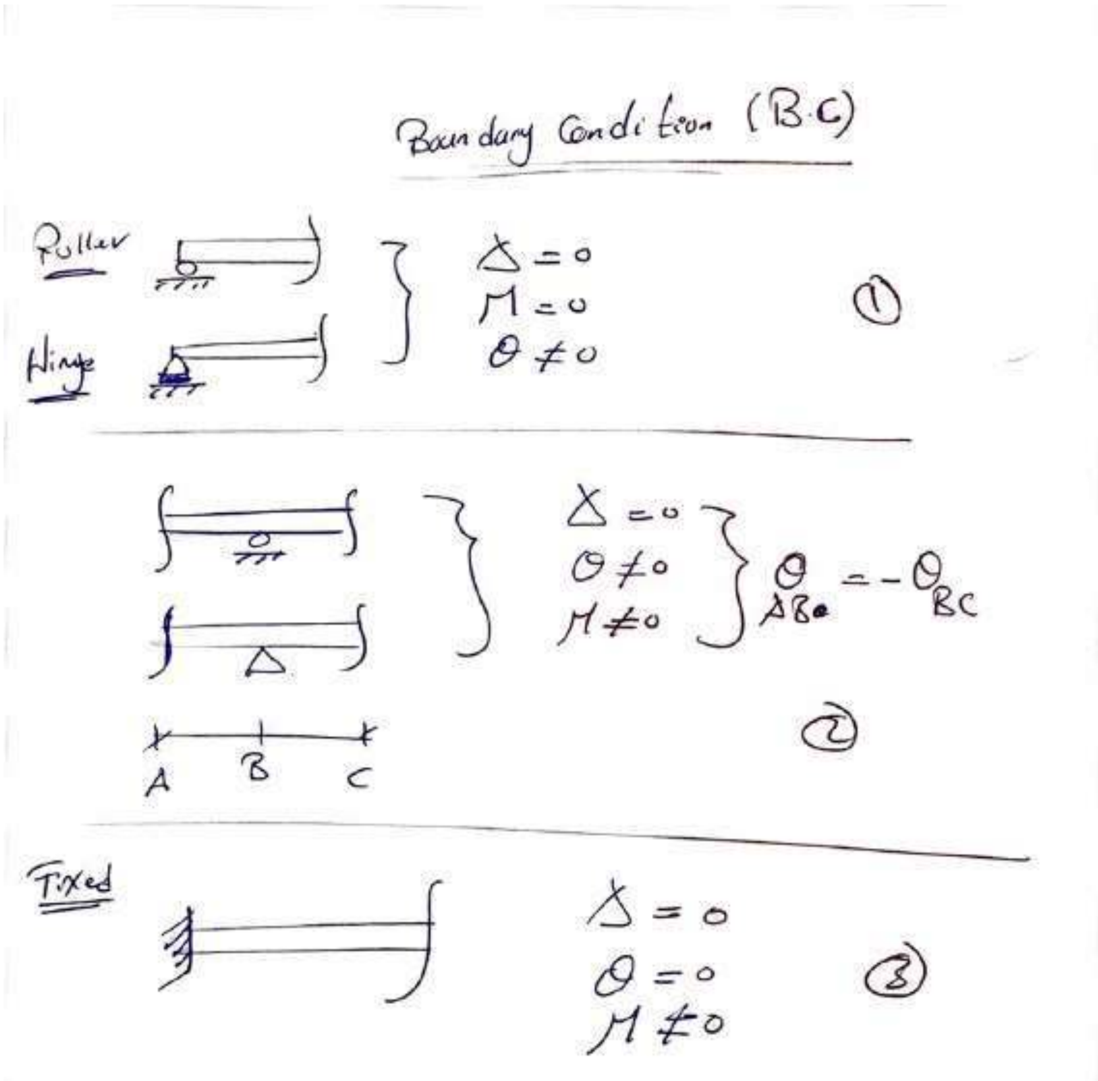
2. Slope and Displacement by Integration

The equation of the elastic curve for a beam can be expressed mathematically as $\vartheta = f(x)$. To obtain this equation, we must first represent the curvature $(1/\rho)$ in terms of ϑ and x.

$$EI\frac{d^4v}{dx^4} = w_x \;\ldots..(1)$$

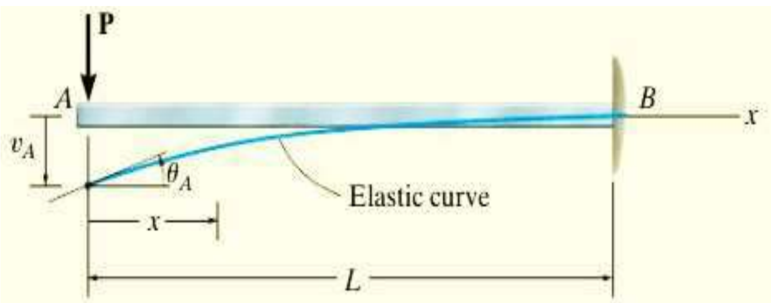
$$EI\frac{d^3v}{dx^3} = v_x \;\ldots\ldots(2)$$

$$EI\frac{d^2v}{dx^2} = M_x \;\ldots\ldots.(3)$$



Example

*The cantilevered beam shown below is subjected to a vertical load **P** at its end. Determine the equation of the elastic curve. EI is constant.*

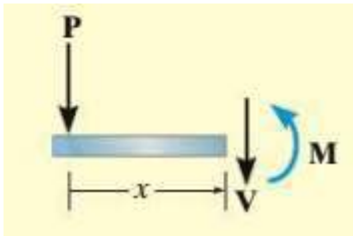


Solution

1. نعمل قطع على مسافة x ونجد مقدار العزم M
2. نعوض معادلة رقم 3 محل مقدار M
3. نكامل الطرفين لكل نجد مقدار الميل (θ)
4. نكامل الطرفين لكي نجد مقدار الهطول (υ)

ملاحظة/

عند كل تكامل نحصل على معامل التكامل (c) حيث نستطيع معرفة مقدار هذا المعامل من خلال نوع وشروط الاسناد في الاعتاب (Boundary Conditions)



1. $M = -Px$

2. $EI\frac{d^2v}{dx^2} = -Px \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$

3. $EI\frac{dv}{dx} = -\frac{Px^2}{2} + C_1$ .... (2)

4. $EIv = -\frac{Px^3}{6} + C_1x + C_2$ .... (3)

Using the boundary condition $\frac{dv}{dx} = 0$ *at x = L and v = 0 at x =L, Eq.2 and Eq.3 become.*

$$0 = -\frac{PL^2}{2} + C_1$$

$$0 = -\frac{PL^3}{6} + C_1L + C_2$$

Thus, $C_1 = PL^2/2$ and $C_2 = -PL^3/3$. Substituting these results into Eqs. 2 and 3 with $\theta = dv/dx$, we get.

$$\theta = \frac{P}{2EI}(L^2 - x^2)$$

$$v = \frac{P}{6EI}(-x^3 + 3L^2x - 2L^3)$$

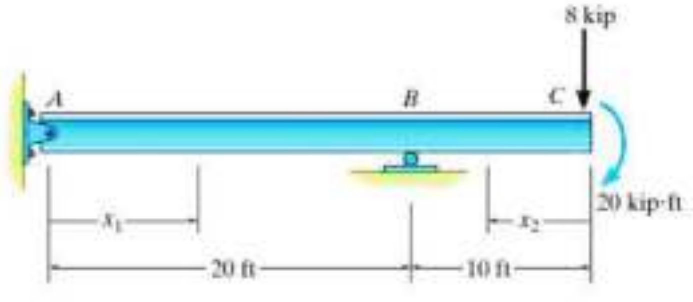
Maximum slope and displacement occur at A(x = 0), for which

$$\theta_A = \frac{PL^2}{2EI}$$

$$v_A = -\frac{PL^3}{3EI}$$

Prob. 12-20 _ Hibbeler

Determine the equations of the elastic curve using the x_1 and x_2 coordinates, and specify the slope at A and the deflection at C. EI is constant.



1

Solution Prob. 12-20

المطلوب ايجاد معادلة المطل (Deflection) وكذلك مقدار الميل (θ) عند النقاط (A,B)

①

خطوة رقم ① نجد مقادير القوى عند كل مسند

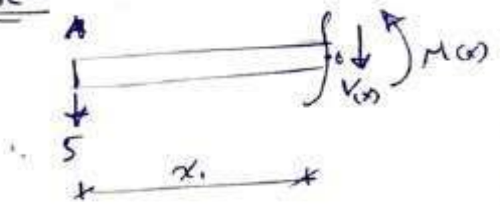
$$\Sigma M@A=0 \longrightarrow 8(30)+20 = B_y(20) \longrightarrow B_y = 13 \uparrow$$

$$\Sigma F_y = 0 \longrightarrow A_y = 5 \downarrow$$

②

خطوة رقم ② ثم نجد معادلات المطل نأخذ مقطع على مسافة (x) عن كل مسند او نقطة مطلوبة ونحسب مقدار العزم ومن ثم تطبيق شروط الحدود (B.C) عند المسند ثم نحسب مقدار ثابت التكامل (C)

المقطع الاول



$$\Sigma M_o = 0 \rightarrow$$

$$M_{(x)} + 5(x_1) = 0$$

$$\therefore M_{(x)} = -5x_1$$

From Eq.③

$$EI\frac{d^2v}{dx^2} = M_{(x)} \rightarrow EI\frac{d^2v}{dx^2} = -5x_1 .$$

الآن نكامل الطرفين

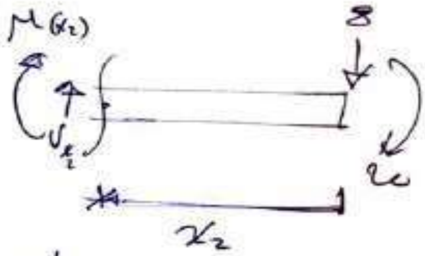
$$EI\frac{dv}{dx} = -\frac{5x_1^2}{2} + C_1$$

حيث ان مقدار $\frac{dv}{dx}$ يمثل قيمة (θ) الان نكامل الطرفين.

2

$$EIv_1 = -\frac{5x_1^3}{6} + C_1 x_1 + C_2$$ ← معادلة المطال للمقطع الاول

المقطع الثاني



$\sum M_o = 0 \rightarrow$

$$8(x_2) + 20 + M_{x_2} = 0$$

$$\therefore M_{x_2} = -8x_2 - 20$$

From Eq (3) $\rightarrow EI\frac{d^2v}{dx^2} = -8x_2 - 20$

* تكامل الطرفين

$$EI\frac{dv_2}{dx} = -\frac{8x_2^2}{2} - 20x_2 + C_3$$

* تكامل الطرفين

معادلة المطال للمقطع الثاني $\rightarrow$ $$EIv_2 = -\frac{8x^3}{6} - 20\frac{x_2^2}{2} + C_3 x_2 + C_4$$

* لكي نحسب مقادير الثوابت (C_4, C_3, C_2, C_1) يجب ان نطبق شروط الحدود (BC)

(Boundary Condition)

① $At\ x_1 = 0 \rightarrow v = 0 \rightarrow 0 = -\frac{5(0)}{6} + C_1(0) + C_2$

$\therefore C_2 = 0 \rightarrow \boxed{EIv = -\frac{5x_1^3}{6} + C_1 x_1}$

② $At\ x_1 = 20 \rightarrow v = 0 \rightarrow 0 = -\frac{5(20)^3}{6} + C_1(20$

3

$$\therefore C_1 = \frac{5\times(20)^{\cancel{3}\,2}}{6\times\cancel{x}} = 333.334$$

$$\therefore EI\,v_1 = -\frac{5x_1^3}{6} + 333.334\,x_1$$

* الآن نطبق شروط الحد للمقطع الثاني

$$At\ x_2 = 10 \rightarrow v_2 = 0$$

$$\therefore 0 = -\frac{8}{6}(10)^3 - 10(10)^2 + C_3(10) + C_4$$

$$\therefore 10C_3 + C_4 = 2333.334 \quad —(1)$$

* لكي نحسب مقادير (C_3 , C_4) نحتاج الى معادلات ثانية
او شرط حدي جديد عند المسند (B) اذا

$$\theta_{BA} = -\theta_{BC} \rightarrow -\frac{5}{2}(20)^2 + 333.334 = -\left[4(10)^2 - 20(10) + C_3\right]$$

- نلاحظ عند θ_{BA} نعوض مقدار $x_1 = 20$
وعند θ_{BC} " " $x_2 = 10$

$$\therefore C_3 = 1266.67 \quad \text{Sub @ (1)}$$

$$C_4 = -10333.33$$

$$\text{So}\ EI\,v_2 = -\frac{8}{6}x_2^3 - 10x_2^2 + 1266.67x_2 + 10333.33$$

(4)

الآن يجب المطلوب الثاني وهو مقدار (θ) عند A و B

$$\theta_A = \frac{dv_1}{dx} = \frac{1}{EI}\left(-\frac{5}{2}x_1^2 + 333.33\right)$$

$$\because x_1 = 0 \rightarrow \theta_A = \frac{333.33}{EI}$$

$$\theta_B = \frac{dv_2}{dx} = \frac{1}{EI}\left(-\frac{8}{6}x_2^2 - 20x_2 + 1266.67\right.$$

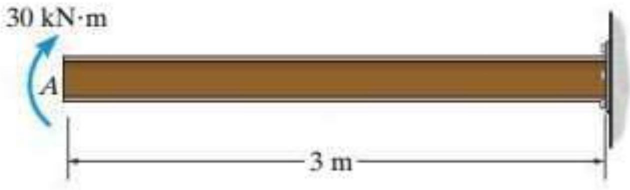
$$\because x = 10 \rightarrow$$

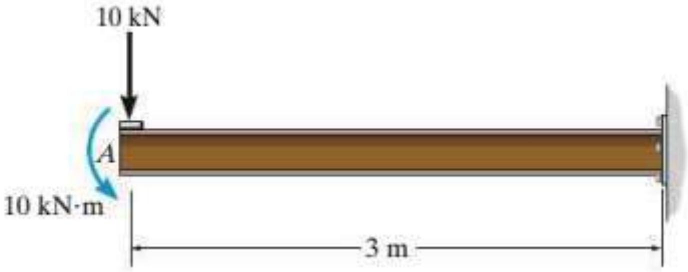
$$\therefore \theta_B = \frac{1}{EI}\left(-\frac{8}{6}(10)^2 - 20(10) + 1266.67\right.$$

$$\theta_B = \frac{933.336}{EI}$$

HW

Determine the slope and deflection of end A of the cantilevered beams below. E = 200 GPa and I = 65.0(10[6]*) mm*[4]*.*

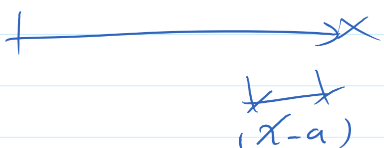




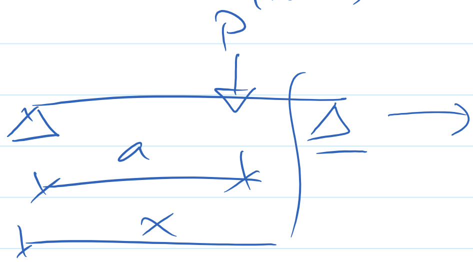
Macaulay's Method to calculate Deflection

①

$$\boxed{M = M_0 \langle x-a \rangle^0}$$

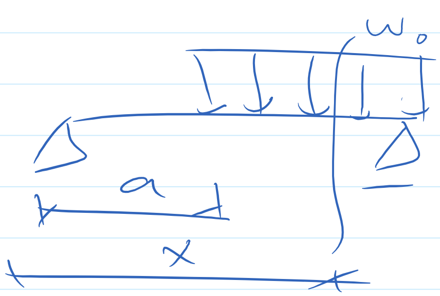


②



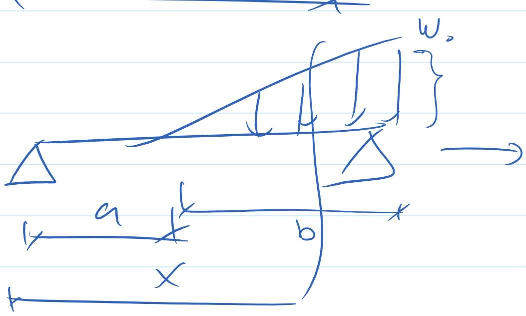
$$\boxed{M = P(x-a)^1}$$

③



$$M = \frac{w_0}{2}\langle x-a \rangle^2$$

④



Slope

$$M = \frac{m}{6}\langle x-a \rangle^3$$

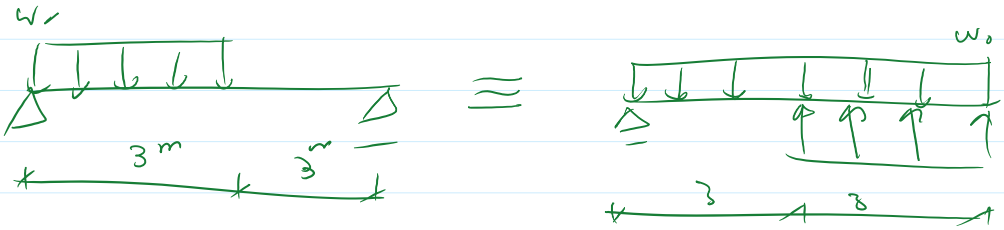
$$m = \text{slope} = \frac{w_0}{b}$$

B.C

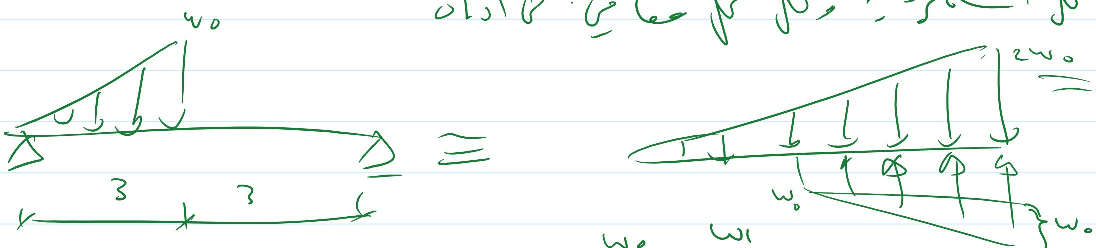
$$\langle x-a \rangle^n = \begin{cases} 0 & x < a \\ (x-a) & x \geq a \end{cases}$$

ملاحظات

① اذا كان الحمل المنتشر عند بداية السند (Support) وعلم تحديد النهاية ال (Beam) . يجب عمل استمرارية للحمل كما ادناه



② اذا كان حمل منتشر على شكل مثلث ولم تحديد النهاية (Beam) . يجب عمل استمرارية وعمل حمل معاكس كما ادناه

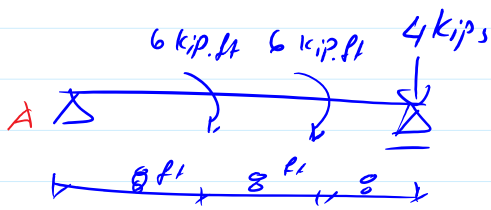


$$\frac{w_o}{3} = \frac{w_1}{6} \rightarrow w_1 = 2w_o$$

$$slope_1 = \frac{2w_o}{6} = \frac{1}{3} w_o$$

$$slope_2 = \frac{w_o}{3}$$

Ex 1



Find slope & elastic curve Equation?

Sol

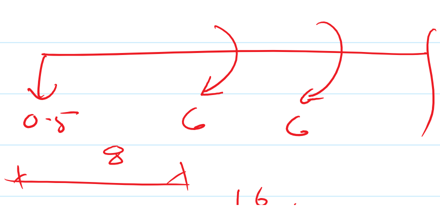
① Reaction

$$\Sigma M_B = 0 \rightarrow A_y(24) + 6 + 6 = 0$$

$$\therefore A_y = \frac{-12}{24} = -0.5 \text{ kips.}$$

② Find Moment formula

$$M = -0.5(x-0)^1 + 6\langle x-8\rangle^0 + 6\langle x-16\rangle^0$$



$$6\langle x-8\rangle^0 + 6\langle x-16\rangle^0$$

$$M = -0.5x + 6\langle x-8\rangle^0 + 6\langle x-16\rangle^0 = EI\frac{d^2y}{dx^2}$$

$$\theta EI = \frac{-0.5}{2}x^2 + 6\langle x-8\rangle + 6\langle x-16\rangle + C_1$$

$$y EI = \frac{-0.5}{6}x^3 + \frac{6}{2}\langle x-8\rangle^2 + \frac{6}{2}\langle x-16\rangle^2 + C_1 x + C_2$$

③ B.C

$x = 0 \rightarrow y = 0 \rightarrow$

$$0 = \cancel{\frac{-0.5}{6}(0)^3} + \cancel{\frac{6}{2}\langle 0-8\rangle^2} + \cancel{\frac{6}{2}\langle 0-16\rangle^2} + C_1(0) + C_2$$

$(x < a)$

$\therefore C_2 = 0$

$x = 24 \rightarrow y = 0$

$$0 = \frac{-0.5}{6}(24)^3 + 3(24-8)^2 + 3(24-16)^2 + C_1(24)$$

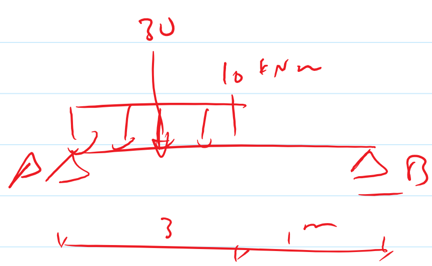
$$0 = \frac{-0.5}{6}(24)^3 + 3(16)^2 + 3(8)^2 + C_1(24)$$

$\therefore C_1 = 8$

$$\therefore \theta = \frac{1}{EI}\left(\frac{-0.5}{2}x^2 + 6(x-8) + 6(x-16) + 8\right)$$

$$y = \frac{1}{EI}\left(\frac{-0.5}{6}x^3 + 3(x-8)^2 + 3(x-16)^2 + 8x\right)$$

Ex 2

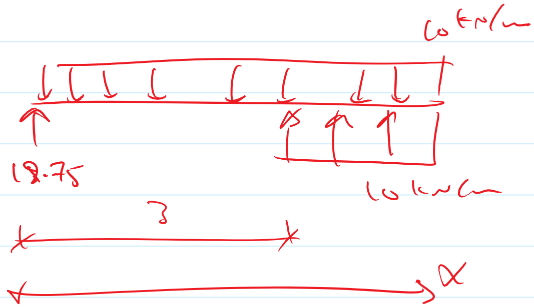


Find slope & elastic curve Equation?

(1) Reactions. $\sum M_B = 0 \rightarrow A_y(4) - 30(3.5) = 0$

$\therefore A_y = \frac{75}{4} = 18.75$ kN

① Redraw the Beam



$$M = 18.75\langle x-0\rangle^{1} - \frac{10}{2}(x-0)^{2} + \frac{10}{2}(x-3)^{2} = EI\frac{d^2y}{dx^2}$$

$$M = 18.75x - 5x^2 + 5(x-3)^2$$

② $$\theta EI = \frac{18.75}{2}x^2 - \frac{5}{3}x^3 + \frac{5}{3}(x-3)^3 + C_1$$

$$yEI = \frac{18.75}{6}x^3 - \frac{5}{12}x^4 + \frac{5}{12}(x-3)^4 + C_1x + C_2$$

③ B.C

① $x = 0 \rightarrow y = 0 \rightarrow C_2 = 0$

② $x = 4 \rightarrow y = 0 \rightarrow$

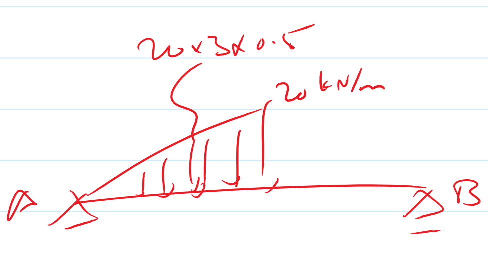
$$0 = \frac{18.75}{6}(4)^3 - \frac{5}{12}(4)^4 + \frac{5}{12}(4-3)^4 + C_1(4)$$

$$C_1 = -23.4375$$

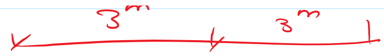
$$\therefore \theta = \frac{1}{EI}\left(\frac{18.75}{2}x^2 - \frac{5}{3}x^3 + \frac{5}{3}(x-3)^3 - 23.4375\right)$$

$$y = \frac{1}{EI}\left(\frac{18.75}{6}x^4 - \frac{5}{12}x^4 + \frac{5}{12}\langle x-3\rangle^4 - 23.4375x\right)$$

Ex3



Find slope & elastic curve Equation?



1. Calculate all Reactions

$\sum M_B = 0 \rightarrow A_y(6) - 20 \times 3 \times 0.5\,(4) = 0$

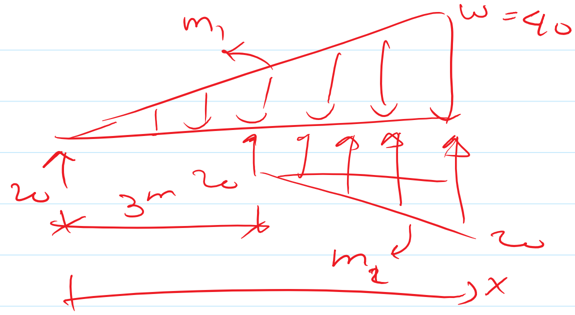
$\therefore A_y = \frac{120}{6} = 20\ kN.$

2. Re draw the structure and write the Moment Equation

$\frac{w_1}{6} = \frac{20}{3} \rightarrow w_1 = 40$

$m_1 = \frac{40}{6} = 6.667$

$m_2 = \frac{20}{3} = 6.6667$



$M = 20(x-0)^1 - \frac{6.667}{6}\langle x-0\rangle^3 + \frac{20}{2}(x-3)^2 + \frac{6.667}{6}\langle x-3\rangle^3$

$M = 20x - \frac{6.667}{6}x^3 + 10\langle x-3\rangle^2 + \frac{6.667}{6}\langle x-3\rangle^3$

$\theta EI = \frac{20}{2}x^2 - \frac{6.667}{24}x^4 + \frac{10}{3}\langle x-3\rangle^3 + \frac{6.667}{24}\langle x-3\rangle^4 + C_1$

$y EI = \frac{20}{6}x^3 - \frac{6.667}{120}x^5 + \frac{10}{12}(x-3)^4 + \frac{6.667}{120}(x-3)^5 + C_1 x + C_2$

3) B.C

1) $x = 0 \rightarrow y = 0 \rightarrow C_2 = 0$

2) $x = 6 \rightarrow y = 0 \rightarrow$

$0 = \frac{20}{6}(6)^3 - \frac{6.667}{120}6^5 + \frac{10}{12}(3)^4 + \frac{6.667}{120}(3)^5 + C_1(6)$

$\therefore C_1 = -61.5$

$\theta = \frac{1}{EI}\left(10x^2 - \frac{6.667}{24}x^4 + \frac{10}{3}(x-3)^3 + \frac{6.667}{24}\langle x-3\rangle^4 - 61.5\right)$

$y = \frac{1}{EI}\left(\frac{10}{3}x^3 - \frac{6.667}{120}x^5 + \frac{10}{12}(x-3)^4 + \frac{6.667}{120}(x-3)^5 - 61.5x\right)$

H-W