FLUID MECHANICS

REFERENCES:

1. **A TEXTBOOK OF FLUID MECHANICS AND HYDRAULIC MACHINES (Er. R.K. RAJPUT)**
2. **2500 SOLVED PROBLEM IN FLUID MECHANICS AND HYDRAULIC**
3. **FLUID MECHANICS, FOURTH EDITION FRANK, M. WHITE**

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Chapter 1

PROPERTIES OF FLUIDS

1.1. INTRODUCTION

Fluid Mechanics: Fluid mechanics may be defined as *that branch of Engineering science, which deals with the behavior of fluid under the conditions of rest and motion*. The fluid mechanics may be divided into three parts: *Statics*, *kinematics* and *dynamics*.

Statics. The study of incompressible fluids under static conditions is called *hydrostatics* and that dealing with the compressible static gases is termed as *aerostatics*.

Kinematics. It deals with the velocities, accelerations and the patterns of flow only. Forces or energy causing velocity and acceleration are *not* dealt under this heading.

Dynamics. It deals with the relations between velocities, accelerations of fluid *with the forces or energy causing them*.

Properties of Fluids–General Aspects: The matter can be classified on the basis of the *spacing between the molecules* of the matter as follows:

1. Solid state, and

2. Fluid state,

(*i*) Liquid state, and (*ii*) Gaseous state.

The differences between liquid, solid and gas state are:

1.2. FLUID

A fluid may be defined as follows:

"A fluid is a substance which is capable of flowing".

or

"A fluid is a substance which deforms continuously when subjected to external shearing force"

A fluid has the following *characteristics*:

1. It has no definite shape of its own, but conforms to the shape of the containing vessel.
2. Even a small amount of shear force exerted on a liquid/fluid will cause it to undergo a deformation, which continues as long as the force continues to be applied. A fluid may be *classified* as follows:

A. (*i*) *Liquid*, (*ii*) *Gas*, (*iii*) *Vapor*.

B. (*i*) *Ideal fluids* (*ii*) *Real fluids*.

Liquid

- A liquid is a fluid, which possesses a *definite volume* (which varies only slightly with temperature and pressure).
- Liquids have bulk elastic modulus when under compression and will store up energy in the same manner as a solid. As the contraction of volume of a liquid under compression is extremely small, it is usually ignored and the *liquid is assumed incompressible*.
- A liquid will withstand slight amount of tension due to molecular attraction between the particles, which will cause an apparent shear

resistance, between two adjacent layers. His phenomenon is known as **viscosity**.

- All known liquids vaporize at narrow pressures above zero, depending on the temperature.

Gas. It possesses *no definite volume* and is *compressible*.

Vapor. It is a gas whose temperature and pressure are such that it is very near the liquid state (*e.g.*, steam).

Ideal fluids. An ideal fluid is one, which has *no viscosity* and *surface tension* and is *incompressible*. In true sense, no such fluid exists in nature. However, fluids, which have low viscosities such as, water and air can be treated as ideal fluids under certain conditions. The assumption of ideal fluids helps in simplifying the mathematical analysis.

Real fluids. *A real practical fluid is one, which has viscosity, surface tension and compressibility in addition to the density*. The real fluids are actually available in nature.

1.3. LIQUIDS AND THEIR PROPERTIES

- Liquid can be easily distinguished from a solid or a gas.
- Solid has a definite shape.
- A liquid takes the shape of vessel into which it is poured.
- A gas completely fills the vessel, which contains it.

Some important properties of water, which will be considered, are:
(*i*) Density (*iv*) Vapor pressure (*vii*) Surface tension,
(*ii*) Specific gravity (*v*) Cohesion (*viii*) Capillarity
(*iii*) Viscosity (*vi*) Adhesion, and (*ix*) Compressibility.

1.4. DENSITY

1.4.1 Mass Density

The *density* (also known as *mass density or specific mass*) of a liquid may be defined as *the mass per unit volume* $\left[\frac{m}{V}\right]$ *at a standard temperature and pressure*. It is usually denoted by ρ (rho). Its units are kg/m^3,

$$\left[\rho = \frac{m}{V}\right] \qquad ...(1.1)$$

1.4.2 Weight Density

The weight density (also known as specific weight) is defined as the *weight per unit volume at the standard temperature and pressure*. It is usually denoted by γ.

$$[\gamma = \rho g] \qquad ...(1.2)$$

For the purposes of all calculations, relating to Hydraulics and hydraulic machines, the specific weight of water is taken as follows:

In S.I. Units: γ = 9.81 kN/m^3 (or 9.81× 10^{-6} N/mm^3)

1.4.3 Specific volume

It is defined as *volume per unit mass of fluid*. It is denoted by *v* *Mathematically*,

$$\left[v = \frac{V}{m} = \frac{1}{\rho}\right] \qquad ...(1.3)$$

1.5. SPECIFIC GRAVITY

Specific gravity is the ratio of the specific weight of the liquid to the specific weight of a standard fluid. It is dimensionless and has no units. It is represented by *S*. Fluid Mechanics For liquids, the standard fluid is pure water at 4°C.

$$\therefore Specific\ gravity = \frac{Specific\ weight\ of\ liquid}{Specific\ weight\ of\ pure\ water} = \frac{\gamma\ liquid}{\gamma\ water} \qquad ...(1.4)$$

Example 1.1. *Calculate the specific weight, specific mass, specific volume and specific gravity of a liquid having a volume of 6 m^3 and weight of 44 kN.*

Solution: Volume of the liquid = 6 m^3
Weight of the liquid = 44 kN

Specific weight, γ =Weight of liquid /Volume of liquid
= 44/6
= 7.333 kN/m^3

Specific mass or mass density, $\rho = \gamma/g$
= (7.333* 1000)/ 9.81
= 747.5 kg/m^3

Specific volume, v =1/ ρ
=1/747.5
= 0.00134 m3/kg

Specific gravity, S = γ *liquid* / γ *water*
=7.333/9.81
= 0.747

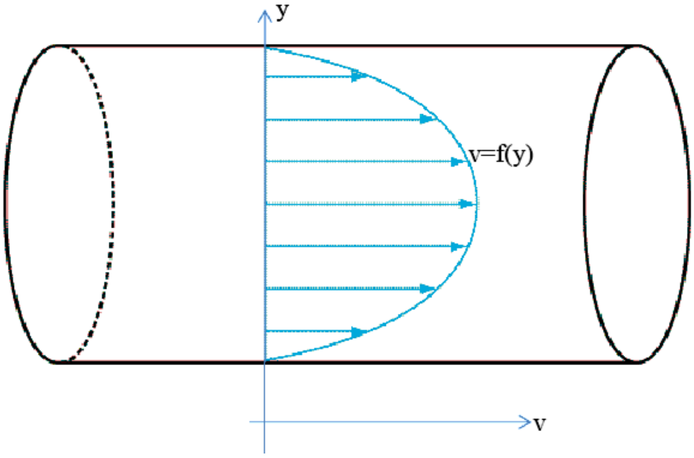
1.6. VISCOSITY

Viscosity may be defined as the *property of a fluid which determines its resistance to shearing stresses*. It is a measure of the internal fluid friction, which causes resistance to flow. It is primarily *due to cohesion and molecular momentum exchange between fluid layers*, and as flow occurs, these effects appear as shearing stresses between the moving layers of fluid. An *ideal fluid has no viscosity*.

If fluid is in motion, shear stress are developed → this occur **if the fluid particles move relative to each other** with different velocities.

However, if the fluid velocity is the same at every point (fluid particles are at rest relative to each other), no shear stress will be produced.

The following figure exhibit the **velocity profile** in a circular pipe:



Note that fluid next to the pipe wall has **zero** velocity (fluid **sticks** to wall), But if the fluid moved away from the wall, velocity increases to **maximum**. Change in velocity (v) with distance(y) is **(velocity gradient):**

$$\text{Velocity gradient} = \frac{dv}{dy} \qquad ...(1.5)$$

This also called **(rate of shear strain)**

Newton's Law of Viscosity:

$$\tau = \mu \frac{du}{dy} \qquad ...(1.6)$$

τ=shear stress (pa=N/m²).

μ=dynamic viscosity

Units of Viscosity:

In *S.I. Units*: N.s/m^2

One poise =10^{-1} N.s/m^2

The viscosity of water at 20°C is 10^{-2} poise or one centipoise.

$$\left[\because \quad \mu = \frac{\text{force/area}}{(\text{length/time}) \times \dfrac{1}{\text{length}}} = \frac{\text{force/length}^2}{\dfrac{1}{\text{length}}} = \frac{\text{force} \times \text{time}}{(\text{length})^2} \right]$$

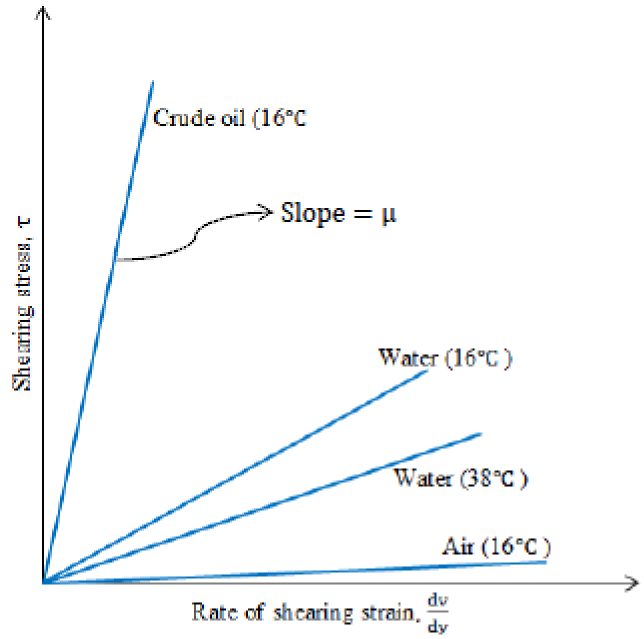
Newtonian Fluids:

✓ Fluids obey Newton's law of viscosity are Newtonian fluids.

✓ For this type of fluids, there is a **linear relationship** between shear stress and the velocity gradient.

- ✓ Dynamic viscosity (μ) is the slope of the line.
- ✓ Dynamic viscosity (μ) is constant for a fluid at the same temperature.
- ✓ As temperature increase →(μ) decreases →slope decreases.
- ✓ Most common fluids are Newtonian, for example: Air, Water, Oil, etc…

The following graph explains the linear variation of shear stress with rate of shear strain (velocity gradient) for common fluids:



Kinematic Viscosity :

Kinematic viscosity is defined as the *ratio between the dynamic viscosity and density of fluid*. It is denoted by ν (called nu).

Mathematically,

$$\nu = \frac{Viscosity}{Density} = \frac{\mu}{\rho} \qquad ...(1.7)$$

Units of kinematic viscosity:

In SI units: m^2/s

the kinematic viscosity is also known as stoke (= cm^2/sec.)

One stoke = 10^{-4} m^2/s

Centistoke means10^{-2} stoke.

1.6. VISCOSITY

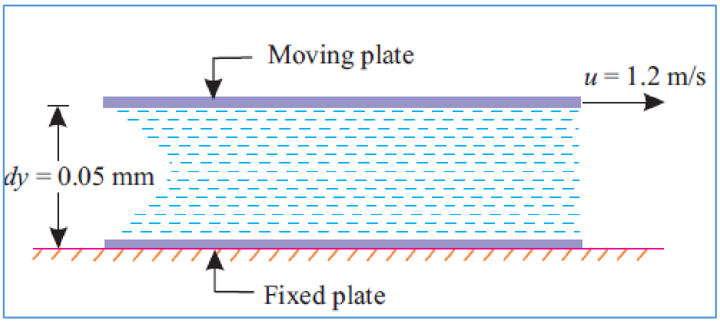
Effect of Temperature on Viscosity

Viscosity is effected by temperature. The viscosity of *liquids decreases* but that of *gases* increases with *increase in temperature*. This is due to the reason that in *liquids* the shear stress is due to the inter-molecular cohesion which *decreases* with increase of temperature. In gases the inter-molecular cohesion is negligible and the shear stress is due to exchange of momentum of the molecules, normal to the direction of motion.

Effect of Pressure on Viscosity

The viscosity under ordinary conditions is not appreciably affected by the changes in pressure. However, the viscosity of some oils has been found to increase with increase in pressure.

Example *A plate 0.05 mm distant from a fixed plate moves at 1.2 m/s and requires a force of 2.2 N/m² to maintain this speed. Find the viscosity of the fluid between the plates.*



Solution: Velocity of the moving plate, $u = 1.2$ m/s

Distance between the plates, $dy = 0.05 \text{ mm} = 0.05 \times 10^{-3}$ m

Force on the moving plate, $F = 2.2 \text{ N/m}^2$

Viscosity of the fluid, μ:

We know, $\tau = \mu . \dfrac{du}{dy}$

where τ = shear stress or force per unit area = 2.2 N/m²,

du = change of velocity = $u - 0 = 1.2$ m/s and

dy = change of distance = 0.05×10^{-3} m.

$$\therefore \quad 2.2 = \mu \times \frac{1.2}{0.05 \times 10^{-3}}$$

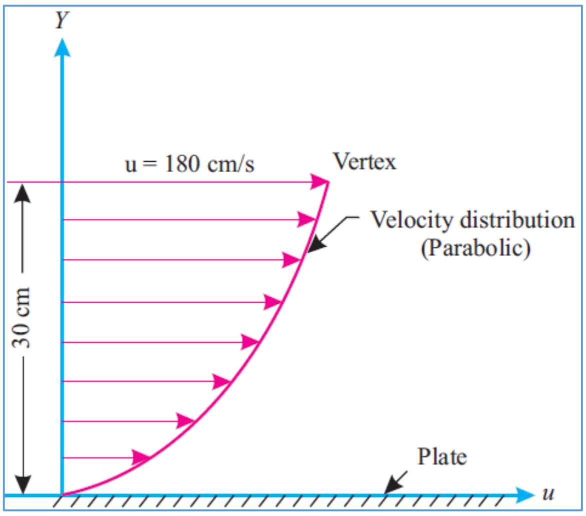
$$\text{or,} \quad \mu = \frac{2.2 \times 0.05 \times 10^{-3}}{1.2} = 9.16 \times 10^{-5} \text{N.s/m}^2$$

$$= \mathbf{9.16 \times 10^{-4} \text{ poise}} \text{ (Ans.)}$$

H.W.:

plate having an area of 0.6 m^2 is sliding down the inclined plane at 30° to the horizontal with a velocity of 0.36 m/s. There is a cushion of fluid 1.8 mm thick between the plane and the plate. Find the viscosity of the fluid if the weight of the plate is 280 N.

Example *The velocity distribution of flow over a plate is parabolic with vertex 30 cm from the plate, where the velocity is 180 cm/s. If the viscosity of the fluid is 0.9 N.s/m^2 find the velocity gradients and shear stresses at distances of 0, 15 cm and 30 cm from the plate.*



Solution. Distance of the vertex from the plate = 30 cm.

Velocity at vertex, $u = 180$ cm/s

Viscosity of the fluid = 0.9 N.s/m^2

The equation of velocity profile, which is parabolic, is given by

$$u = ly^2 + my + n \qquad ...(1)$$

where l, m and n are constants. The values of these constants are found from the following boundary conditions:

(*i*) At $y = 0$, $u = 0$,

(*ii*) At $y = 30$ cm,

$u = 180$ cm/s and

(*iii*) At $y = 30$ cm, $\frac{du}{dy} = 0$.

Substituting boundary conditions (*i*) in eqn. (1), we get

$$0 = 0 + 0 + n \quad \therefore n = 0$$

Substituting boundary conditions (*ii*) in eqn. (1), we get

$$180 = l \times (30)^2 + m \times 30 \quad \text{or} \quad 180 = 900\,l + 30\,m \qquad ...(2)$$

Substituting boundary conditions (*iii*) in eqn. (1), we get

$$\frac{du}{dy} = 2ly + m \quad \therefore \quad 0 = 2l \times 30 + m \quad \text{or} \quad 0 = 60l + m \qquad ...(3)$$

Solving eqns. (2) and (3), we have $l = -0.2$ and $m = 12$.

Substituting the values of l, m and n in eqn. (1), we get $u = -0.2\,y^2 + 12y$

Velocity gradients, $\frac{du}{dy}$:

$$\frac{du}{dy} = -0.2 \times 2y + 12 = -0.4y + 12$$

At $y = 0, \left(\frac{du}{dy}\right)_{y=0} = \mathbf{12/s}$ **(Ans.)**

At $y = 15$ cm, $\left(\frac{du}{dy}\right)_{y=15} = -0.4 \times 15 + 12 = \mathbf{6/s}$ **(Ans.)**

At $y = 30$ cm, $\left(\frac{du}{dy}\right)_{y=30} = -0.4 \times 30 + \mathbf{12} = \mathbf{0}$ **(Ans.)**

Shear stresses, τ:

We know, $\tau = \mu\frac{du}{dy}$

At $y = 0, (\tau)_{y=0} = \mu.\left(\frac{du}{dy}\right)_{y=0} = 0.9 \times 12 = \mathbf{10.8\ N/m^2}$ **(Ans.)**

At $y = 15, (\tau)_{y=15} = \mu.\left(\frac{du}{dy}\right)_{y=15} = 0.9 \times 6 = \mathbf{5.4\ N/m^2}$ **(Ans.)**

At $y = 30, (\tau)_{y=30} = \mu.\left(\frac{du}{dy}\right)_{y=30} = 0.9 \times 0 = \mathbf{0}$ **(Ans.)**

H.W.

Two large fixed parallel planes are 12 mm apart. The space between the surfaces is filled with oil of viscosity 0.972 N.s/m^2. A flat thin plate 0.25 m^2 area moves through the oil at a velocity of 0.3 m/s. Calculate the drag force:

(*i*) *When the plate is equidistant from both the planes, and*

(*ii*) *When the thin plate is at a distance of 4 mm from one of the plane surfaces*

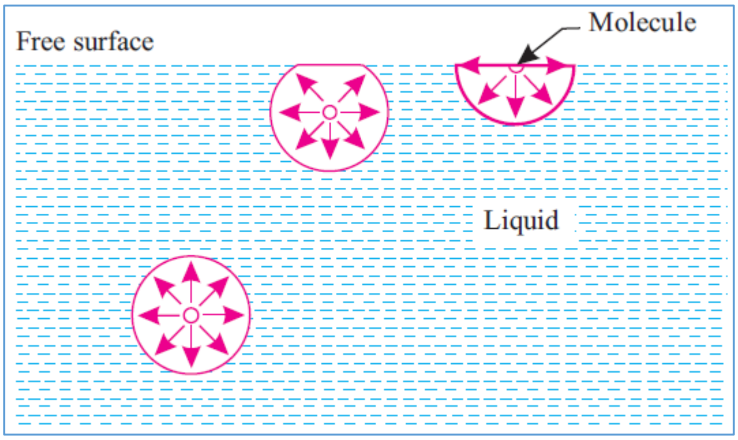
1.7. SURFACE TENSION AND CAPILLARITY

1.7.1. Surface Tension

Cohesion. Cohesion means intermolecular attraction between *molecules of the same liquid*. It enables a liquid to resist small amount of tensile stresses. Cohesion is a tendency of the liquid to remain as one *assemblage of particles*. "*Surface tension*" *is due to cohesion between particles at the free surface.*

Adhesion. *Adhesion* means *attraction between the molecules of a liquid and the molecules of a solid boundary surface in contact with the liquid*. This property enables a liquid to stick to *another body*. *Capillary action* is due to both *cohesion* and *adhesion.*

Surface tension is caused by the *force of cohesion at the free surface.*



Effect of Surface tension

1.7.2. Capillarity

Capillarity *is a phenomenon by which a liquid* (*depending upon its specific gravity*) *rises into a thin glass tube above or below its general level*. This phenomenon is due to the *combined effect of cohesion and adhesion of liquid particles.*

This Figure shows the phenomenon of rising water in the tube of *smaller* diameters

Let, d = Diameter of the capillary tube,

θ = Angle of contact of the water surface

σ = Surface tension force for unit length, and

γ = Weight density (ρg).

Now, upward surface tension force (lifting force) = weight of the water column in the tube (*gravity force*)

$$\pi d.\sigma \cos\theta = \frac{\pi}{4} d^2 \times h \times \gamma$$

$$\therefore \qquad h = \frac{4\sigma \cos\theta}{\gamma\ d}$$

For water and glass: $\theta \simeq 0$.

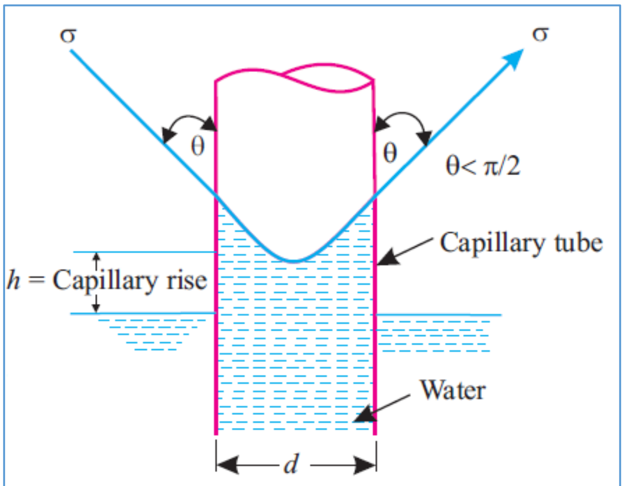
Hence the capillary rise of water in the glass tube,

$$h = \frac{4\sigma}{\gamma\, d}$$

In case of mercury there is a capillary depression as shown in Figure, and the angle of depression is $\theta \sim 140°$.
(It may be noted that here $\cos\theta = \cos 140° = \cos(180 - 40°) = -\cos 40°$, therefore, h is *negative* indicating capillary depression).

Following points are worth noting:

- ✓ Smaller the diameter of the capillary tube, greater is the capillary rise or depression.
- ✓ The measurement of liquid level in laboratory capillary (glass) tubes should not be smaller than 8 mm.
- ✓ Capillary effects are negligible for tubes longer than 12 mm.
- ✓ For *wetting liquid* (water): $\theta < \pi/2$. For water: $\theta = 0$ when pure water is in contact with clean glass. But θ becomes as high as 25° when water is slightly contaminated. For *non-wetting* liquid (mercury): $\theta > \pi/2$. (For mercury: θ varies between 130° to 150°)



Adhesion > Cohesion
(Miniscus concave)

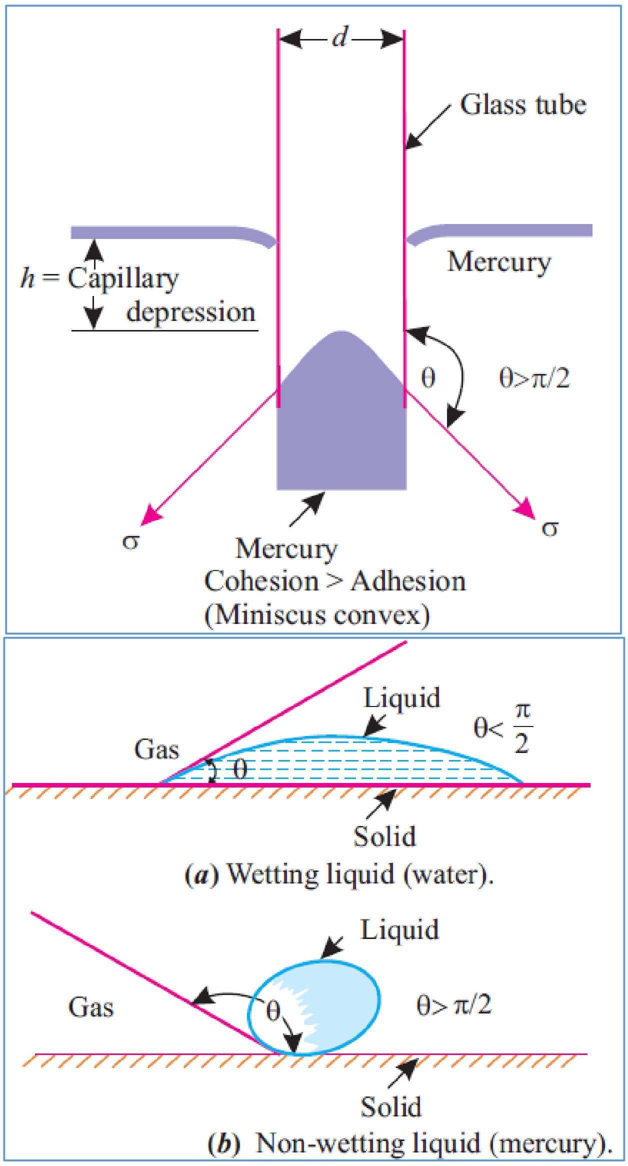


Figure illustrates the capillary effect

Example *A clean tube of diameter 2.5 mm is immersed in a liquid with a coefficient of surface tension = 0.4 N/m. The angle of contact of the liquid with the glass can be assumed to be 135°. The density of the liquid = 13600 kg/m3. What would be the level of the liquid in the tube relative to the free surface of the liquid inside the tube.*

Solution. *Given:* $d = 2.5$ mm ; $\sigma = 4$ N/m, $\theta = 135°$; $\rho = 13600$ kg/m^3

Level of the liquid in the tube, *h*:

The liquid in the tube rises (or falls) due to capillarity. The capillary rise (or fall),

$$h = \frac{4\sigma\cos\theta}{wd}$$

$$= \frac{4 \times 0.4 \times \cos 135°}{(9.81 \times 13600) \times 2.5 \times 10^{-3}} \qquad (\because w = \rho g)$$

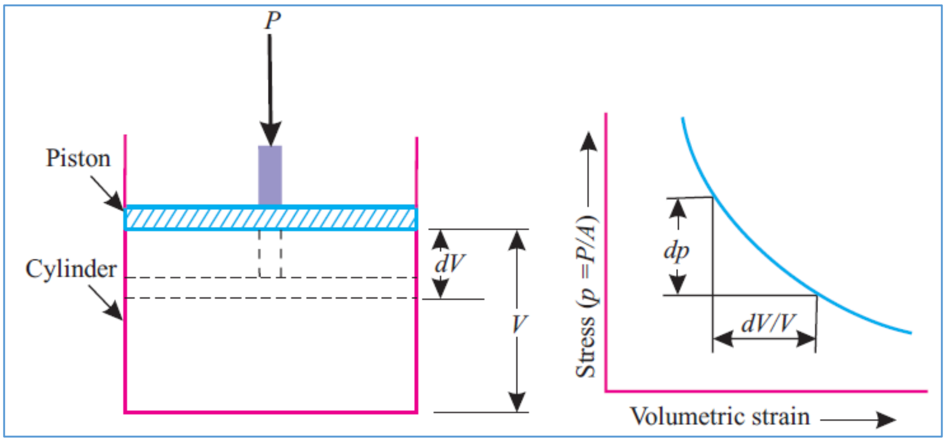
$$= -3.39 \times 10^{-3}\text{m or } -3.39 \text{ mm}$$

Negative sign indicates that there is a capillary **depression (fall) of 3.39 mm. (Ans.)**

1.9. COMPRESSIBILITY AND BULK MODULUS

The property by virtue of which fluids undergo a change in volume under the action of external pressure is known as ***compressibility.*** It decreases with the increases in pressure of fluid as the volume modulus increases with the increase of pressure.

Elasticity of fluids is measured in terms of ***bulk modulus of elasticity*** (K) which is defined as the *ratio of compressive stress to volumetric strain.* Compressibility is the reciprocal of bulk modulus of elasticity.



Let the pressure is increased to $p + dp$, the volume of gas decreases from V to $V - dV$.

Then increase in pressure $= dp$; Decrease in volume $= dV$

$$\therefore \qquad \text{Volumetric strain} = -\frac{dV}{V}$$

(Negative sign indicates *decrease in volume with increase of pressure*)

$$\therefore \qquad \text{Bulk modulus, } K = \frac{dp\,(\text{increase of pressure})}{-dV/V\,(\text{volumetric strain})}$$

i.e.,

$$K = \frac{dp}{-dV/V}$$

$$\left(\text{Compressibility} = \frac{1}{K}\right)$$

Example *When the pressure of liquid is increased from 3.5 MN/m2 to MN/m2 its volume is found to decrease by 0.08 percent. What is the bulk modulus of elasticity of the liquid?*

Solution. Initial pressure = $3.5\ \text{MN/m}^2$

Final pressure = $6.5\ \text{MN/m}^2$

$\therefore$ Increase in pressure, $dp = 6.5 - 3.5 = 3.0\ \text{MN/m}^2$

Decrease in volume = 0.08 percent $\therefore\ -\dfrac{dV}{V} = \dfrac{0.08}{100}$

Bulk modulus (K) is given by:

$$K = \frac{dp}{-\frac{dV}{V}} = \frac{3 \times 10^6}{\frac{0.08}{100}} = 3.75 \times 10^9\ \text{N/m}^2 \text{ or } 3.75\ \text{GN/m}^2$$

Hence, $K = \mathbf{3.75\ GN/m^2}$ **(Ans.)**

1.9 Important Laws

1. **Law of conservation of mass**

   " *The mass can neither be created nor destroyed, and it can not be created from nothing"*

2. **Law of conservation of energy**

   " *The energy can neither be created nor destroyed, though it can be transformed from one form into another"*

   Newton's Laws of Motion

   Newton has formulated three law of motion, which are the basic postulates or assumption on which the whole system of dynamics is based.

3. **Newton's first laws of motion**

   "Every body continues in its state of rest or of uniform motion in a straight line, unless it is acted upon by some external forces"

4. **Newton's second laws of motion**

   "The rate of change in momentum is directly proportional to the impressed force and takes place in the same direction in which the force acts"[momentum = mass × velocity]

5. **Newton's third laws of motion**

   "To every action, there is always an equal and opposite reaction"

1.5 Flow Patterns

The nature of fluid flow is a function of the fluid physical properties, the geometry of the container, and the fluid flow rate. The flow can be characterized either as **Laminar** or as **Turbulent** flow.

Laminar flow is also called "viscous or streamline flow". In this type of flow layers of fluid move relative to each other without any intermixing.

Turbulent flow in this flow, there is irregular random movement of fluid in directions transverse to the main flow.

Example *One liter of certain oil weighs 0.8 kg, calculate the specific weight, density, specific volume, and specific gravity of it.*

Solution:

$$sp.wt. = \frac{\text{Weight of fluid}}{\text{Volume of fluid}} = \frac{(0.8kg)(9.81m/s^2)}{1x10^{-3}m^3} = 7848\frac{N}{m^3}$$

$$\rho = \frac{(0.8kg)}{1x10^{-3}m^3} = 800\frac{kg}{m^3} \qquad \upsilon = \frac{1}{\rho} = 1.25x10^{-3}\frac{m^3}{kg}$$

$$sp.gr. = \frac{\rho_{\text{liquid}}}{\rho_{\text{water}}} = \frac{800kg/m^3}{1000kg/m^3} = 0.8$$

Example *Determine the specific gravity of a fluid having viscosity of 4.0 c.poice and kinematic viscosity of 3.6 c.stokes.*

Solution:

$$\mu = 4c.p\frac{poice}{100c.p} = 0.04poise = 0.04\frac{g}{cm.s} \qquad \nu = 3.6c.s\frac{stoke}{100c.s} = 0.036stoke = \frac{cm^2}{s}$$

$$\nu = \frac{\mu}{\rho} \Rightarrow \rho = \frac{\mu}{\nu} = \frac{0.04\frac{g}{cm.s}}{0.036cm^2/s} = 1.1111\frac{g}{cc} \qquad \Rightarrow \rho = 1111.1\frac{kg}{m^3} \qquad \Rightarrow sp.gr. = 1.1111$$

Example *A plate having an area of 0.6 m^2 is sliding down the inclined plane at 30° to the horizontal with a velocity of 0.36 m/s. There is a cushion of fluid 1.8 mm thick between the plane and the plate. Find the viscosity of the fluid if the weight of the plate is 280 N.*

Solution: Area of plate, $A = 0.6\ \text{m}^2$

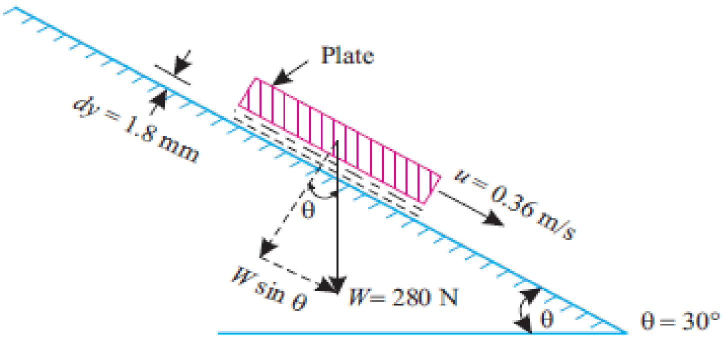
Weight of plate, $W = 280$ N

Velocity of plate, $u = 0.36$ m/s

Thickness of film, $t = dy = 1.8\ \text{mm} = 1.8 \times 10^{-3}$ m

Viscosity of the fluid, μ:

Component of W along the plate $= W \sin\theta = 280 \sin 30° = 140$ N



∴ Shear force on the bottom surface of the plate, $F = 140$ N and shear stress,

$$\tau = \frac{F}{A} = \frac{140}{0.6} = 233.33 \text{N/m}^2$$

We know, $\tau = \mu \cdot \dfrac{du}{dy}$

Where, du = change of velocity = $u - 0 = 0.36$ m/s

$dy = t = 1.8 \times 10^{-3}$ m

$$\therefore \quad 233.33 = \mu \times \frac{0.36}{1.8 \times 10^{-3}}$$

or, $$\mu = \frac{233.33 \times 1.8 \times 10^{-3}}{0.36} = 1.166 \text{N.s/m}^2 = \textbf{11.66 poise} \text{ (Ans.)}$$

Example *Two large fixed parallel planes are 12 mm apart. The space between the surfaces is filled with oil of viscosity 0.972 N.s/m². A flat thin plate 0.25 m² area moves through the oil at a velocity of 0.3 m/s. Calculate the drag force:*

(i) When the plate is equidistant from both the planes, and

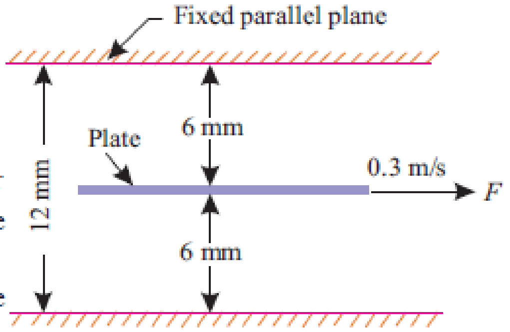
(ii) When the thin plate is at a distance of 4 mm from one of the plane surfaces.

Solution. Given: Distance between the fixed parallel planes = 12 mm = 0.012 m

Area of *thin* plate, $A = 0.25$ m²

Velocity of plate, $u = 0.3$ m/s

Viscosity of oil = 0.972 N.s/m²



Drag force, F:

(*i*) *When the plate is equidistant from both the planes:*

Let, F_1 = Shear force on the upper side of the thin plate,

F_2 = Shear force on the lower side of the thin plate,

F = Total force required to drag the plate $(= F_1 + F_2)$.

The shear τ_1, on the upper side of the thin plate is given by:

$$t_1 = \mu \cdot \left(\frac{du}{dy}\right)_1$$

where, $du = 0.3$ m/s (relative velocity between upper fixed plane and the plate), and dy = 6 mm = 0.006 m (distance between the upper fixed plane and the plate)

(Thickness of the plate neglected).

$$\therefore \quad \tau_1 = 0.972 \times \frac{0.3}{0.006} = 48.6 \text{N/m}^2$$

∴ Shear force, $F_1 = \tau_1 \cdot A = 48.6 \times 0.25 = 12.15$ N

Similarly shear stress (τ_2) on the lower side of the thin plate is given by

$$\tau_2 = u \cdot \left(\frac{du}{dy}\right)_2 = 0.972 \times \frac{0.3}{0.06} = 48.6 \text{ N/m}^2$$

and $\qquad F_2 = \tau_2 . A = 48.6 \times 0.25 = 12.15 \text{ N}$

$\therefore \qquad F = F_1 + F_2 = 12.15 + 12.15 = \mathbf{24.30 \text{ N}}$ **(Ans.)**

(ii) When the thin plate is at a distance of 4 mm from one of the plane surfaces:

The shear force on the upper side of the thin plate,

$$F_1 = \tau_1 . A = \mu . \left(\frac{du}{dy}\right)_1 \times A$$

$$= 0.972 \times \frac{0.3}{0.008} \times 0.25 = 9.11 \text{N}$$

The shear force on the lower side of the thin plate,

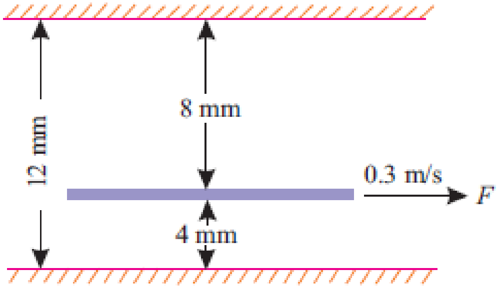


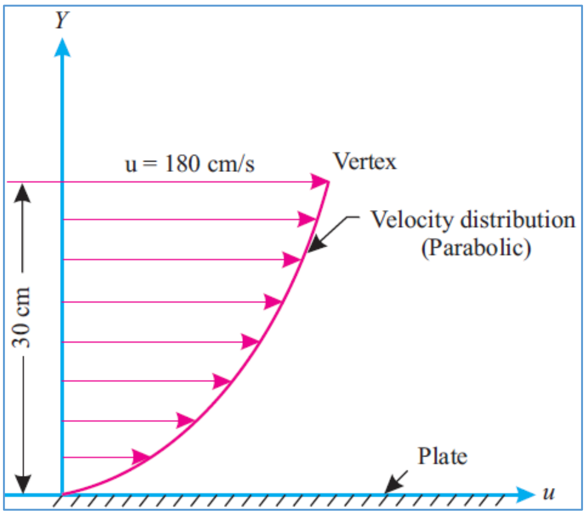
Fig. 1.13

$$F_2 = \tau_2 \times A = \mu . \left(\frac{du}{dy}\right)_2 \times A$$

$$= 0.972 \times \left(\frac{0.3}{0.004}\right) \times 0.25 = 18.22 \text{N}$$

$\therefore$ Total force $F = F_1 + F_2 = 9.11 + 18.22 = \mathbf{27.33 \text{ N}}$ **(Ans.)**

Example *The velocity distribution of flow over a plate is parabolic with vertex 30 cm from the plate, where the velocity is 180 cm/s. If the viscosity of the fluid is 0.9 N.s/m^2 find the velocity gradients and shear stresses at distances of 0, 15 cm and 30 cm from the plate.*



Solution. Distance of the vertex from the plate = 30 cm.

Velocity at vertex, $u = 180$ cm/s

Viscosity of the fluid = 0.9 N.s/m^2

The equation of velocity profile, which is parabolic, is given by

$$u = ly^2 + my + n \qquad \text{...(1)}$$

where l, m and n are constants. The values of these constants are found from the following boundary conditions:

(i) At $y = 0$, $u = 0$,

(ii) At $y = 30$ cm,

$u = 180$ cm/s and

(*iii*) At $y = 30$ cm, $\frac{du}{dy} = 0$.

Substituting boundary conditions (*i*) in eqn. (1), we get

$$0 = 0 + 0 + n \quad \therefore n = 0$$

Substituting boundary conditions (*ii*) in eqn. (1), we get

$$180 = l \times (30)^2 + m \times 30 \quad \text{or} \quad 180 = 900\,l + 30\,m \qquad ...(2)$$

Substituting boundary conditions (*iii*) in eqn. (1), we get

$$\frac{du}{dy} = 2ly + m \quad \therefore \quad 0 = 2l \times 30 + m \quad \text{or} \quad 0 = 60l + m \qquad ...(3)$$

Solving eqns. (2) and (3), we have $l = -0.2$ and $m = 12$.

Substituting the values of l, m and n in eqn. (1), we get $u = -0.2\,y^2 + 12y$

Velocity gradients, $\frac{du}{dy}$:

$$\frac{du}{dy} = -0.2 \times 2y + 12 = -0.4y + 12$$

At $y = 0, \left(\frac{du}{dy}\right)_{y=0} = \mathbf{12/s}$ **(Ans.)**

At $y = 15$ cm, $\left(\frac{du}{dy}\right)_{y=15} = -0.4 \times 15 + 12 = \mathbf{6/s}$ **(Ans.)**

At $y = 30$ cm, $\left(\frac{du}{dy}\right)_{y=30} = -0.4 \times 30 + \mathbf{12} = \mathbf{0}$ **(Ans.)**

Shear stresses, τ:

We know, $\tau = \mu \frac{du}{dy}$

At $y = 0, (\tau)_{y=0} = \mu.\left(\frac{du}{dy}\right)_{y=0} = 0.9 \times 12 = \mathbf{10.8\ N/m^2}$ **(Ans.)**

At $y = 15, (\tau)_{y=15} = \mu.\left(\frac{du}{dy}\right)_{y=15} = 0.9 \times 6 = \mathbf{5.4\ N/m^2}$ **(Ans.)**

At $y = 30, (\tau)_{y=30} = \mu.\left(\frac{du}{dy}\right)_{y=30} = 0.9 \times 0 = \mathbf{0}$ **(Ans.)**

Home Work

P.1.1

Two plates are kept separated by a film of oil of 0.025 mm. the top plate moves with a velocity of 50 cm/s while the bottom plate is kept fixed. Find the fluid viscosity of oil if the force required to move the plate is 0.2 kg/m^2. Ans. $\mu = 9.81\times10^{-5}$ Pa.s ?

P.1.2

If the equation of a velocity profile over a plate is $u=3\,y^{(2/3)}$ in which the velocity in m/s at a distance y meters above the plate, determine the shear stress at y=0 and y=5 cm. Take μ= 8.4 poise Ans. $\tau_{y=0} = \infty$, $\tau_{y=5} = 4.56$ Pa.s

P.1.3

The equation of a velocity distribution over a plate is $u=1/3\ y - y^2$ in which the velocity in m/s at a distance y meters above the plate, determine the shear stress at y=0 and y=0.1 m. Take μ= 8.35 poise Ans. $\tau_{y=0}$ =2.78, $\tau_{y=0.1}$= 4.56 $dyne/cm^2$

CHPTER TWO

Dimensional Analysis

2.1 Introduction

Any phenomenon is physical sciences and engineering can be described by the *fundamentals dimensions* mass, length, time, and temperature. Till the rapid development of science and technology the engineers and scientists depend upon the experimental data. But the rapid development of science and technology has created new mathematical methods of solving complicated problems, which could not have been solved completely by analytical methods and would have consumed enormous time. This mathematical method of obtaining the equations governing certain natural phenomenon by balancing the fundamental dimensions is called *(Dimensional Analysis)*. Of course, the equation obtained by this method is known as *(Empirical Equation).*

2.2 Fundamentals Dimensions

The various physical quantities used by engineer and scientists can be expressed in terms of fundamentals dimensions are: Mass (M), Length (L), Time (T), and Temperature (θ). All other quantities such as area, volume, acceleration, force, energy, etc., are termed as " derived quantities".

2.3 Dimensional Homogeneity

An equation is called *"dimensionally homogeneous"* if the fundamentals dimensions have identical powers of [L T M] (i.e. length, time, and mass) on both sides. Such an equation be independent of the system of measurement (i.e. metric, English, or S.I.). Let consider the common equation of volumetric flow rate,

$$Q = A\,u$$

$$L^3T^{-1} = L^2\,LT^{-1} = L^3T^{-1}.$$

We see, from the above equation that both right and left hand sides of the equation have the same dimensions, and the equation is therefore dimensionally homogeneous.

Example -2.1-

a) Determine the dimensions of the following quantities in M-L-T system 1- force 2-pressure 3- work 4- power 5- surface tension 6- discharge 7- torque 8- momentum.
b) Check the dimensional homogeneity of the following equations

1- $u = \sqrt{\dfrac{2\,g(\rho_m - \rho)\Delta z}{\rho}}$ 2- $Q = \dfrac{8}{15} cd \tan\dfrac{\theta}{2}\sqrt{2\,g}Z_{\circ}^{\frac{5}{2}}$

Solution:

a)

1- F = m.g (kg.m/s^2) ≡ [MLT^{-2}]
2- P=F/A ≡ [(MLT^{-2}) (L^{-2})] (Pa) ≡ [$ML^{-1}T^{-2}$]
3- Work = F.L ≡ [(MLT^{-2}) (L)] (N.m) ≡ [$ML^{2}T^{-2}$]
4- Power = Work/time ≡ [($ML^{2}T^{-2}$) (T^{-1})] (W) ≡ [$ML^{-1}T^{-3}$]
5- Surface tension = F/L ≡ [(MLT^{-2}) (L^{-1})] (N/m) ≡ [$M\ \ T^{-2}$]

6- Discharge (Q) m^3/s ≡ [L^3T^{-1}]
7- Torque (Γ) = F.L ≡ [(MLT^{-2}) (L)] N.m ≡ [ML^2T^{-2}]
8- Moment = m.u L)] N.m ≡ [ML^2T^{-1}]

b) 1- $u = \sqrt{\dfrac{2\,g(\rho_m - \rho)\Delta z}{\rho}}$

L.H.S. u = [LT^{-1}]

R.H.S. u = $\left[\dfrac{LT^{-2}(ML^3)}{ML^{-3}}\right]^{1/2}$ = [LT^{-1}]

Since the dimensions on both sides of the equation are same, therefore the equation is dimensionally homogenous.

2- $Q = \dfrac{8}{15} cd \tan\dfrac{\theta}{2} \sqrt{2\,g} Z_c^{\frac{5}{2}}$

L.H.S. Q= [L^3T^{-1}]

R.H.S. (LT^{-2}) $(L)^{5/2}$ = [L^3T^{-1}]

This equation is dimensionally homogenous.

2.4 Methods of Dimensional Analysis

Dimensional analysis, which enables the variables in a problem to be grouped into form of *dimensionless groups*. Thus reducing the effective number of variables. The method of dimensional analysis by providing a smaller number of independent groups is most helpful to experimenter.

Many methods of dimensional analysis are available; two of these methods are given here, which are:

1- Rayleigh's method (or Power series)

2- Buckingham's method (or Π-Theorem)

2.4.1 Rayleigh's method (or Power series)

In this method, the functional relationship of some variable is expressed in the form of an exponential equation, which must be dimensionally homogeneous. If (y) is some function of independent variables (x_1, x_2, x_3, ………etc.), then functional relationship may be written as;

y = f(x_1, x_2, x_3, ………etc.)

The dependent variable (y) is one about which information is required; whereas the independent variables are those, which govern the variation of dependent variables.

The Rayleigh's method is based on the following steps:-

1. First of all, write the functional relationship with the given data.
2. Now write the equation in terms of a constant with exponents i.e. powers a, b, c,...
3. With the help of the principle of dimensional homogeneity, find out the values of a, b, c, … by obtaining simultaneous equation and simplify it.
4. Now substitute the values of these exponents in the main equation, and simplify it.

Example -2.2-

If the capillary rise (h) depends upon the specific weight (sp.wt) surface tension (σ) of the liquid and tube radius (r) show that:

$$h = r\phi\left(\frac{\sigma}{(sp.wt.)\, r^2}\right), \text{ where } \phi \text{ is any function.}$$

Solution:

Capillary rise (h) m $\equiv [L]$

Specific weight (sp.wt) N/m^3 $(MLT^{-2}\, L^{-3}) \equiv [ML^{-2}T^{-2}]$

Surface tension (σ) N/m $(MLT^{-2}\, L^{-1}) \equiv [MT^{-2}]$

Tube radius (r) m $\equiv [L]$

$$h = f\,(sp.wt., \sigma, r)$$

$$h = k\,(sp.wt.^a, \sigma^b, r^c)$$

$$[L] = [ML^{-2}T^{-2}]^a\,[MT^{-2}]^b[L]^c$$

Now by the principle of dimensional homogeneity, equating the power of M, L, T on both sides of the equation

For M $0 = a + b \Rightarrow a = -b$

For L $1 = -2a + c \Rightarrow a = -b$

For T $0 = -2a - 2b \Rightarrow a = -b$

$$h = k\,(sp.wt.^{-b}, \sigma^b, r^{1-2b})$$

$$h = k\,r\left(\frac{\sigma}{sp.wt.\, r^2}\right)^b \qquad \therefore\ h = r\phi\left(\frac{\sigma}{(sp.wt.)\, r^2}\right)$$

Example -2.3-

Prove that the resistance (F) of a sphere of diameter (d) moving at a constant speed (u) through a fluid density (ρ) and dynamic viscosity (μ) may be expressed as:

$$F = \frac{\mu^2}{\rho}\phi\left(\frac{\rho u d}{\mu}\right), \text{ where } \phi \text{ is any function.}$$

Solution:

Resistance (F) N	$\equiv [MLT^{-2}]$
Diameter (d) m	$\equiv [L]$
Speed (u) m/s	$\equiv [LT^{-1}]$
Density (ρ) kg/m^3	$\equiv [ML^{-3}]$
Viscosity (μ) kg/m.s	$\equiv [ML^{-1}T^{-1}]$

$$F = f(d, u, \rho, \mu)$$

$$F = k(d^a, u^b, \rho^c, \mu^d)$$

$$[MLT^{-2}] = [L]^a [LT^{-1}]^b [ML^{-3}]^c [ML^{-1}T^{-1}]^d$$

For M $1 = c + d \Rightarrow c = 1 - b$ ----------------(1)

For L $1 = a + b - 3c - d$ ----------------(2)

For T $-2 = -b - d \Rightarrow b = 2 - b$ ----------------(3)

By substituting equations (1) and (3) in equation (2) give

$a = 1 - b + 3c + d = 1 - (2 - d) + 3(1 - d) + d = 2 - d$

$F = k(d^{2-d}, u^{2-d}, \rho^{1-d}, \mu^d) = k\{(d^2 u^2 \rho)(\mu / \rho u d)^d\}$ ---------x $\{(\rho/\mu^2)/(\rho/\mu^2)\}$

$F = k\{(d^2 u^2 \rho^2 / \mu^2)(\mu / \rho u d)^d (\mu^2 / \rho)\}$

$$\therefore F = \frac{\mu^2}{\rho}\phi\left(\frac{\rho u d}{\mu}\right)$$

Example -2.4-

The thrust (P) of a propeller depends upon diameter (D); speed (u) through a fluid density (ρ); revolution per minute (N); and dynamic viscosity (μ) Show that:

Solution:

Thrust (P) N	$\equiv [MLT^{-2}]$
Diameter (D) m	$\equiv [L]$
Speed (u) m/s	$\equiv [LT^{-1}]$
Density (ρ) kg/m^3	$\equiv [ML^{-3}]$
Rev. per min. (N) min^{-1}	$\equiv [T^{-1}]$
Viscosity (μ) kg/m.s	$\equiv [ML^{-1}\,T^{-1}]$

$$P = f\,(D, u, \rho, N, \mu)$$

$$P = k\,(D^a, u^b, \rho^c, N^d, \mu^e)$$

$$[MLT^{-2}] = [L]^a\,[LT^{-1}]^b[ML^{-3}]^c[T^{-1}]^d[ML^{-1}T^{-1}]^e$$

For M $1 = c + e \Rightarrow c = 1 - e$ --------------(1)

For L $1 = a + b - 3c - e \Rightarrow a = 1 - b + 3c + e$ --------------(2)

For T $-2 = -b - d - e \Rightarrow b = 2 - e - d$ --------------(3)

By substituting equations (1) and (3) in equation (2) give

$a = 1 - (2 - e - d) + 3\,(1 - e) + e = 2 - e + d$

$$P = k\,(D^{2-e+d}, u^{2-e+d}, \rho^{1-e}, N^d, \mu^e)$$

$$P = (\rho\,D^2\,u^2)\,k\left[\left(\frac{\mu}{\rho\,D\,u}\right)^e, \left(\frac{D\,N}{u}\right)\right]$$

$$\therefore\; P = (\rho\,D^2\,u^2)\,f\left(\left(\frac{\mu}{\rho\,D\,u}\right), \left(\frac{D\,N}{u}\right)\right)$$

Home Work

P.2.1

Show, by dimensional analysis, that the power (P) developed by a hydraulic turbine is given by; $P = (\rho\ N^3\ D^5)\ f\left((\frac{N^2\ D^2}{g\ H}) \right)$ where (ρ) is the fluid density, (N) is speed of rotation in r.p.m., (D) is the diameter of runner, (H) is the working head, and (g) is the gravitational acceleration.

P.2.2

The resistance (R) experienced by a partially submerged body depends upon the velocity (u), length of the body (L), dynamic viscosity (μ) and density (ρ) of the fluid, and gravitational acceleration (g). Obtain a dimensionless expression for (R).

Ans. $R = (u^2 L^2 \rho)\ f\left((\frac{\mu}{u\ L\ g}), (\frac{L\ g}{u^2}) \right)$

P.2.3

Using Rayleigh's method to determine the rational formula for discharge (Q) through a sharp-edged orifice freely into the atmosphere in terms of head (h), diameter (d), density (ρ), dynamic viscosity (μ), and gravitational acceleration (g).

Ans. $Q = (d\sqrt{g\ h})\, f\left[(\frac{\mu}{\rho\, d^{\frac{3}{2}}\ g^{\frac{1}{2}}}), (\frac{h}{d}) \right]$

2.4.2 Buckingham's method (or Π-Theorem)

It has been observed that the Rayleigh's method of dimensional analysis becomes cumbersome, when a large number of variables are involved. In order to overcome this difficulty, the Buckingham's method may be convenient used. It states that " If there are **(n)** variables in a dimensionally homogeneous equation, and if these variables contain **(m)** fundamental dimensions such as (MLT) they may be grouped into **(n-m)** non-dimensional independent Π-terms".

Mathematically, if a dependent variable X_1 depends upon independent variables (X_2, X_3, X_4, ………. X_n), the functional equation may be written as:

$X_1 = k\,(X_2, X_3, X_4, \ldots\ldots\ldots. X_n)$

This equation may be written in its general form as;

$f\,(X_1, X_2, X_3, \ldots\ldots\ldots. X_n) = 0$

In this equation, there are n variables. If there are m fundamental dimensions, then according to Buckingham's Π-theorem;

$f_1\,(\Pi_1, \Pi_2, \Pi_3, \ldots\ldots\ldots. \Pi_{n-m}) = 0$

The Buckingham's Π-theorem is based on the following steps:

1. First of all, write the functional relationship with the given data.
2. Then write the equation in its general form.
3. Now choose **m** *repeating variables (or recurring set)* and write separate expressions for each Π-term. Every Π-term will contain the repeating variables and one of the remaining variables. **Just** the repeating variables are written in exponential form.
4. With help of the principle of dimensional homogeneity find out the values of powers a, b, c, …… by obtaining simultaneous equations.
5. Now substitute the values of these exponents in the Π-terms.
6. After the Π-terms are determined, write the functional relation in the required form.

Note:-

Any Π-term may be replaced by any power of it, because the power of a non-dimensional term is also non-dimensional e.g. Π_1 may be replaced by Π_1^2, Π_1^3, $\Pi_1^{0.5}$,……. or by $2\Pi_1$, $3\Pi_1$, $\Pi_1/2$, ……etc.

2.4.2.1 Selection of repeating variables

In the previous section, we have mentioned that we should choose **(m)** repeating variables and write separate expressions for each Π-term. Though there is no hard or fast rule for the selection of repeating variables, yet the following points should be borne in mind while selecting the repeating variables:

1. The variables should be such that none of them is dimensionless.
2. No two variables should have the same dimensions.
3. Independent variables should, as far as possible, be selected as repeating variables.
4. Each of the fundamental dimensions must appear in at least one of the **m** variables.
5. It must not possible to form a dimensionless group from some or all the variables within the repeating variables. If it were so possible, this dimensionless group would, of course, be one of the Π-term.
6. In general the selected repeating variables should be expressed as the following: **(1)** representing the flow characteristics, **(2)**, representing the geometry and **(3)** representing the physical properties of fluid.
7. In case of that the example is held up, then one of the repeating variables should be changed.

Example -2.5-

By dimensional analysis, obtain an expression for the drag force (F) on a partially submerged body moving with a relative velocity (u) in a fluid; the other variables being the linear dimension (L), surface roughness (e), fluid density (ρ), and gravitational acceleration (g).

Solution:

Drag force (F) N	$\equiv [MLT^{-2}]$
Relative velocity (u) m/s	$\equiv [LT^{-1}]$
Linear dimension (L) m	$\equiv [L]$
Surface roughness (e) m	$\equiv [L]$
Density (ρ) kg/m^3	$\equiv [ML^{-3}]$
Acceleration of gravity (g) m/s^2	$\equiv [L\ T^{-2}]$

$$F = k\,(u, L, e, \rho, g)$$

$$f\,(F, u, L, e, \rho, g) = 0$$

$$n = 6,\ m = 3,\ \Rightarrow \Pi = n - m = 6 - 3 = 3$$

No. of repeating variables = m = 3

The selected repeating variables is (u, L, ρ)

$\Pi_1 = u^{a1}\, L^{b1}\, \rho^{c1}\, F$ --------------(1)

$\Pi_2 = u^{a2}\, L^{b2}\, \rho^{c2}\, e$ --------------(2)

$\Pi_3 = u^{a3}\, L^{b3}\, \rho^{c3}\, g$ --------------(3)

For Π_1 equation (1)

$$[M^0\, L^0\, T^0] = [L\, T^{-1}]^{a1}\, [L]^{b1} [ML^{-3}]^{c1} [MLT^{-2}]$$

Now applied dimensional homogeneity

For M	$0 = c1 + 1$	$\Rightarrow$	$c1 = -1$
For T	$0 = -a1 - 2$	$\Rightarrow$	$a1 = -2$
For L	$0 = a1 + b1 - 3c1 + 1$	$\Rightarrow$	$b1 = -2$

$$\Pi_1 = u^{-2}\, L^{-2}\, \rho^{-1}\, F \quad \Rightarrow \quad \Pi_1 = \frac{F}{u^2\, L^2\, \rho}$$

For Π_2 equation (2)

$$[M^0\, L^0\, T^0] = [L\, T^{-1}]^{a2}\, [L]^{b2} [ML^{-3}]^{c2} [L]$$

For M	$0 = c2$	$\Rightarrow$	$c2 = 0$
For T	$0 = -a2$	$\Rightarrow$	$a2 = 0$
For L	$0 = a2 + b2 - 3c2 + 1$	$\Rightarrow$	$b2 = -1$

$$\Pi_2 = L^{-1}\, e \quad \Rightarrow \quad \Pi_2 = \frac{e}{L}$$

For Π_3 equation (3)

$$[M^0\, L^0\, T^0] = [L\, T^{-1}]^{a3}\, [L]^{b3} [ML^{-3}]^{c3} [L\, T^{-2}]$$

For M	$0 = c3$	$\Rightarrow$	$c3 = 0$
For T	$0 = -a3 - 2$	$\Rightarrow$	$a3 = -2$
For L	$0 = a3 + b3 - 3c3 + 1$	$\Rightarrow$	$b3 = 1$

$\Pi_3 = u^{-2}\, L\, g \qquad \Rightarrow \quad \Pi_3 = \dfrac{L\,g}{u^2}$

$f_1\,(\Pi_1, \Pi_2, \Pi_3) = 0 \qquad \Rightarrow \quad f_1(\dfrac{F}{u^2\; L^2\;\rho}, \dfrac{e}{L}, \dfrac{L\,g}{u^2}) = 0$

$\therefore\; F = u^2 L^2\;\rho\, f(\dfrac{e}{L}, \dfrac{L\,g}{u^2})$

Example -2.6-

Prove that the discharge (Q) over a spillway is given by the relation

$$Q = u\,D^2\;f(\frac{\sqrt{g\,D}}{u}, \frac{H}{D})$$

where (u) velocity of flow (D) depth at the throat, (H), head of water, and (g) gravitational acceleration.

Solution:

Discharge (Q) m^3/s $\equiv [L^3T^{-1}]$
Velocity (u) m/s $\equiv [LT^{-1}]$
Depth (D) m $\equiv [L]$
Head of water (H) m $\equiv [L]$
Acceleration of gravity (g) m/s^2 $\equiv [\; L \;\; T^{-2}]$

$Q = k\,(u, D, H, g)$

$f\,(Q, u, D, H, g) = 0$

$n = 5,\; m = 2, \;\; \Rightarrow \;\Pi = n - m = 5 - 2 = 3$

No. of repeating variables = m = 2
The selected repeating variables is (u, D)

$\Pi_1 = u^{a1}\, D^{b1}\, Q$ --------------(1)

$\Pi_2 = u^{a2}\, D^{b2}\, H$ --------------(2)

$\Pi_3 = u^{a3}\, D^{b3}\, g$ --------------(3)

For Π_1 equation (1)

$[M^0\, L^0\, T^0] = [L\; T^{-1}]^{a1}\; [L]^{b1} [L^3 T^{-1}]$

For T $\quad 0 = -a1 - 1 \quad \Rightarrow \quad a1 = -1$

For L $\quad 0 = a1 + b1 + 3 \quad \Rightarrow \quad b1 = -2$

$$\Pi_1 = u^{-1} D^{-2} Q \quad \Rightarrow \quad \Pi_1 = \frac{Q}{u\,D^2}$$

For Π_2 equation (2)

$$[M^0 L^0 T^0] = [L\,T^{-1}]^{a2}\,[L]^{b2}[L]$$

For T $\quad 0 = -a2 \quad \Rightarrow \quad a2 = 0$

For L $\quad 0 = a2 + b2 + 1 \quad \Rightarrow \quad b2 = -1$

$$\Pi_2 = D^{-1} H \quad \Rightarrow \quad \Pi_2 = \frac{D}{H}$$

For Π_3 equation (3)

$$[M^0 L^0 T^0] = [L\,T^{-1}]^{a3}\,[L]^{b3}\,[L\,T^{-2}]$$

For T $\quad 0 = -a3 - 2 \quad \Rightarrow \quad a3 = -2$

For L $\quad 0 = a3 + b3 + 1 \quad \Rightarrow \quad b3 = 1$

$$\Pi_3 = u^{-2} D\,g \quad \Rightarrow \quad \Pi_3 = \frac{D\,g}{u^2} = \frac{\sqrt{g\,D}}{u}$$

$$f_1(\Pi_1, \Pi_2, \Pi_3) = 0 \quad \Rightarrow \quad f_1\left(\frac{Q}{u\,D^2}, \frac{D}{H}, \frac{\sqrt{D\,g}}{u}\right)$$

$$\therefore\; Q = u\,D^2\, f\left(\frac{\sqrt{g\,D}}{u}, \frac{H}{D}\right)$$

Example -2.7-

Show that the discharge of a centrifugal pump is given by $Q = N\,D^2\, f\left(\frac{g\,H}{N^2 D^2}, \frac{\mu}{ND^2\,\rho}\right)$ where (N) is the speed of the pump in r.p.m., (D) the diameter of impeller, (g) gravitational acceleration, (H) manometric head, (μ), (ρ) are the dynamic viscosity and the density of the fluid.

Solution:

Discharge (Q) m^3/s	$\equiv [L^3T^{-1}]$
Pump speed (N) r.p.m.	$\equiv [T^{-1}]$
Diameter of impeller (D) m	$\equiv [L]$
Acceleration of gravity (g) m/s^2	$\equiv [L\ T^{-2}]$
Head of manometer (H) m	$\equiv [L]$
Viscosity (μ) kg/m.s	$\equiv [ML^{-1}\ T^{-1}]$
Density (ρ) kg/m^3	$\equiv [ML^{-3}]$

$$Q = k\,(N, D, g, H, \mu, \rho)$$

$$f\,(Q, N, D, g, H, \mu, \rho) = 0$$

$$n = 7,\ m = 3,\ \Rightarrow\ \Pi = n - m = 7 - 3 = 4$$

No. of repeating variables = m = 3
The selected repeating variables is (N, D, ρ)

$$\Pi_1 = N^{a1}\,D^{b1}\,\rho^{c1}\,Q \qquad \text{--------------(1)}$$

$$\Pi_2 = N^{a2}\,D^{b2}\,\rho^{c2}\,g \qquad \text{--------------(2)}$$

$$\Pi_3 = N^{a3}\,D^{b3}\,\rho^{c3}\,H \qquad \text{--------------(3)}$$

$$\Pi_4 = N^{a4}\,D^{b4}\,\rho^{c4}\,\mu \qquad \text{--------------(4)}$$

For Π_1 equation (1)

$$[M^0\,L^0\,T^0] = [T^{-1}]^{a1}\,[L]^{b1}[ML^{-3}]^{c1}[L^3\,T^{-1}]$$

For M	$0 = c1$	$\Rightarrow$	$c1 = 0$
For T	$0 = -a1 - 1$	$\Rightarrow$	$a1 = -1$
For L	$0 = b1 - 3c1 + 3$	$\Rightarrow$	$b1 = -3$

$$\Pi_1 = N^{-1}\,D^{-3}\,Q \qquad \Rightarrow \quad \Pi_1 = \frac{Q}{N\,D^3}$$

For Π_2 equation (2)

$$[M^0\,L^0\,T^0] = [T^{-1}]^{a2}\,[L]^{b2}[ML^{-3}]^{c2}[LT^{-2}]$$

For M $\quad 0 = c2 \quad \Rightarrow \quad c2 = 0$

For T $\quad 0 = -a2 - 2 \quad \Rightarrow \quad a2 = -2$

For L $\quad 0 = b2 - 3c2 + 1 \quad \Rightarrow \quad b2 = -1$

$\Pi_2 = N^{-2} D^{-1} g \quad \Rightarrow \quad \Pi_2 = \dfrac{g}{N^2 D}$

For Π_3 equation (3)

$$[M^0 L^0 T^0] = [T^{-1}]^{a3} [L]^{b3} [ML^{-3}]^{c3} [L]$$

For M $\quad 0 = c3 \quad \Rightarrow \quad c3 = 0$

For T $\quad 0 = -a3 \quad \Rightarrow \quad a3 = 0$

For L $\quad 0 = b3 - 3c3 + 1 \quad \Rightarrow \quad b3 = -1$

$\Pi_3 = D^{-1} H \quad \Rightarrow \quad \Pi_3 = \dfrac{H}{D}$

For Π_3 equation (4)

$$[M^0 L^0 T^0] = [T^{-1}]^{a4} [L]^{b4} [ML^{-3}]^{c4} [ML^{-1}T^{-1}]$$

For M $\quad 0 = c4 + 1 \quad \Rightarrow \quad c4 = -1$

For T $\quad 0 = -a4 - 1 \quad \Rightarrow \quad a4 = -1$

For L $\quad 0 = b4 - 3c4 - 1 \quad \Rightarrow \quad b4 = -2$

$\Pi_4 = N^{-1} D^{-2} \rho^{-1} \mu \quad \Rightarrow \quad \Pi 4 = \dfrac{\mu}{ND^2 \rho}$

$$f_1 (\Pi_1, \Pi_2, \Pi_3, \Pi_4) = 0 \quad \Rightarrow \quad f_1\left(\frac{Q}{N D^3}, \frac{g}{N^2 D}, \frac{H}{D}, \frac{\mu}{ND^2 \rho}\right) = 0$$

Since the product of two Π-terms is dimensionless, therefore replace the term Π_2 and Π_3 by $\dfrac{gH}{N^2 D^2}$

$$\Rightarrow \quad f\left(\frac{Q}{ND^3}, \frac{gH}{N^2 D^2}, \frac{\mu}{ND^2 \rho}\right) \qquad \therefore \; Q = ND^3 f\left(\frac{gH}{N^2 D^2}, \frac{\mu}{ND^2 \rho}\right)$$

Note:

The expression outside the bracket may be multiplied or divided by any amount, whereas the expression inside the bracket should not be multiplied or divided. e.g. $\pi/4$, sin θ , tan $\theta/2$, ….c.

2.5 **Dimensions of some important variables**

Item	Property	Symbol	SI Units	M.L.T.
1-	Velocity	u	m/s	LT^{-1}
2-	Angular velocity	ω	Rad/s, Deg/s	T^{-1}
3-	Rotational velocity	N	Rev/s	T^{-1}
4-	Acceleration	a, g	m/s^2	LT^{-2}
5-	Angular acceleration	α	s^{-2}	T^{-2}
6-	Volumetric flow rate	Q	m^3/s	L^3T^{-1}
7-	Discharge	Q	m^3/s	L^3T^{-1}
8-	Mass flow rate	$\dot{m}$	kg/s	MT^{-1}
9-	Mass (flux) velocity	G	$kg/m^2.s$	$ML^{-2}T^{-1}$
10-	Density	ρ	kg/m^3	ML^{-3}
11-	Specific volume	υ	m^3/kg	L^3M
12-	Specific weight	sp.wt	N/m^3	$ML^{-2}T^{-2}$
13-	Specific gravity	sp.gr	[-]	[-]
14-	Dynamic viscosity	μ	kg/m.s, Pa.s	$ML^{-1}T^{-1}$
15-	Kinematic viscosity	ν	m^2/s	L^2T^{-1}
16-	Force	F	N	MLT^{-2}
17-	Pressure	P	N/m^2≡Pa	$ML^{-1}T^{-2}$
18-	Pressure gradient	**ΔP/L**	**Pa/m**	$ML^{-2}T^{-2}$
19-	Shear stress	τ	N/m^2	$ML^{-1}T^{-2}$
20-	Shear rate	$\dot{\gamma}$	s^{-1}	T^{-1}
21-	Momentum	M	kg.m/s	MLT^{-1}
22-	Work	W	N.m≡J	ML^2T^{-2}
23-	Moment	M	N.m≡J	ML^2T^{-2}
24-	Torque	Γ	N.m≡J	ML^2T^{-2}
25-	Energy	E	J	ML^2T^{-2}
26-	Power	P	J/s≡W	ML^2T^{-3}
27-	Surface tension	σ	N/m	MT^{-2}
28-	Efficiency	η	[-]	[-]
29-	Head	h	m	L
30-	Modulus of elasticity	ε, K	Pa	$ML^{-1}T^{-2}$

English Units	SI Units
$g = 32.741\ ft/s^2$	$g = 9.81\ m/s^2$
$g_c = 32.741\ lb_m.ft/lb_f.s^2$	$g_c = 1.0\ kg.m/N.s^2$

psi ≡ lb_f/in^2 Pa ≡ Pascal = N/m^2 bar = 10^5 Pa

1.0 atm = 1.01325 bar = $1.01325*10^5$ Pa = 101.325 kPa = 14.7 psi = 760 torr (mmHg) ≈ 1.0 kg/cm^2

Home Work

P.2.4

The resisting force (F) of a supersonic plane during flight can be considered as dependent upon the length of the air craft (L), Velocity (u), air dynamic viscosity (μ), air density (ρ), and bulk modulus of elasticity of air (ε). Express, by dimensional analysis, the functional relationship between these variables and the resisting force.

Ans. $F = (\rho \;\; L^2 \, u^2) \; f\left((\frac{\mu}{Lu \;\, \rho}) \, , (\frac{\varepsilon}{u^2 \;\; \rho} \right)$

Note: Expressing bulk modulus of elasticity in the form of an equation $\boxed{\varepsilon = -V\frac{dP}{dV}}$ where P is pressure, and V is volume. This mean (ε) is a measure of the increment change in pressure (dP) which takes place when a volume of fluid (V) is changed by an incremental amount (dV). Since arise in pressure always causes a decrease in volume, i.e. (dV) is always negative and so the minus sign comes in the equation to give a positive value of (ε).

where (ρ) is the fluid density, (N) is speed of rotation in r.p.m., (D) is the diameter of runner, (H) is the working head, and (g) is the gravitational acceleration.

P.2.5

The efficiency (η) of a fan depends upon density (ρ), and dynamic viscosity (μ), of the fluid, angular velocity (ω), diameter of the rotator (D), and discharge (Q). Express (η) in terms of dimensionless groups. Ans. $\eta = f\left((\frac{\mu}{\rho \, \omega \, D^2}), (\frac{Q}{\omega \, D3}) \right)$

P.2.6

The pressure drop (ΔP) in a pipe depends upon the mean velocity of flow (u), length of pipe (L), diameter of pipe (d), the fluid density (ρ), and dynamic viscosity (μ), average height of roughness on inside pipe surface (e). By using Buckingham's Π-theorem obtain a dimensionless expression for (ΔP). And show that $h_f = 4f\frac{L}{d}\frac{u^2}{2g}$ where (h_f) is the head loss due to friction ($\frac{\Delta P}{\rho \, g}$) and ($f$) is the dimensionless fanning friction factor.

P.2.7

The Power (P) required to drive the pump depends upon the diameter (D), the angular velocity (ω), the discharge (Q), and the fluid density (ρ). Drive expression for (P) by dimensional analysis. Ans. $P = \omega^3 \, \rho \, D^5 \;\; f\left(, (\frac{\omega \, D}{Q}) \right)$

CHPTER THREE
Fluid Static and Its Applications

3.1 Introduction

Static fluids means that the fluids are at rest.

The pressure in a static fluid is familiar as a surface force exerted by the fluid ageist a unit area of the wall of its container. Pressure also exists at every point within a volume of fluid. It is a scalar quantity; at any given point its magnitude is the same in all directions.

3.2 Pressure in a Fluid

In Figure (1) a stationary column of fluid of height (h_2) and cross-sectional area A, where $A=A_o=A_1=A_2$, is shown. The pressure above the fluid is P_o, it could be the pressure of atmosphere above the fluid. The fluid at any point, say h_1, must support all the fluid above it. It can be shown that the forces at any point in a nonmoving or static fluid must be the same in all directions. Also, for a fluid at rest, the pressure or (force / unit area) in the same at all points with the same elevation. For example, at h_1 from the top, the pressure is the same at all points on the cross-sectional area A_1.

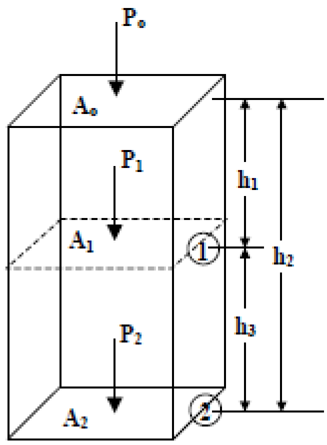


Figure (1): Pressure in a static fluid.

The total mass of fluid for h_2, height and ρ density is: - ($h_2 A \rho$) (kg)

But from Newton's 2nd law in motion the total force of the fluid on area (A) due to the fluid only is: - ($h_2 A \rho g$) (N) i.e. $F = h_2 A \rho g$ (N)

The pressure is defined as ($P = F/A = h_2 \rho g$) (N/m^2 or Pa)

This is the pressure on A_2 due to the weight of the fluid column above it. However to get the total pressure P_2 on A_2, the pressure P_o on the top of the fluid must be added,

i.e. $P_2 = h_2 \rho g + P_o$ (N/m^2 or Pa)

Thus to calculate P_1, $P_1 = h_1 \rho g + P_o$ (N/m_2 or Pa)

The pressure difference between points □ and □ is: -

$P_2 – P_1 = (h_2 \rho g + P_o) – (h_1 \rho g + P_o)$

$\Rightarrow P_2 – P_1 = (h_2 – h_1) \rho g$ SI units

$P_2 – P_1 = (h_2 – h_1) \rho g / g_c$ English units

Since it is vertical height of a fluid that determines the pressure in a fluid, the shape of the vessel does not affect the pressure. For example in Figure (2) the pressure P_1 at the bottom of all three vessels is the same and equal to ($h_1 \rho g + P_o$).

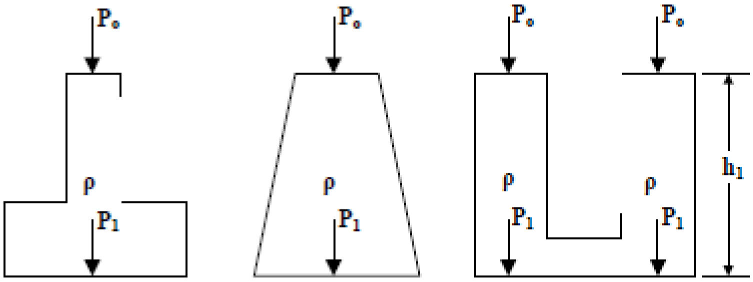


Figure (2): Pressure in vessel of various shapes.

3.3 Absolute and Relative Pressure

The term pressure is sometimes associated with different terms such as *atmospheric*, *gauge*, *absolute*, and *vacuum*. The meanings of these terms have to be understood well before solving problems in hydraulic and fluid mechanics.

1. Atmospheric Pressure

It is the pressure exerted by atmospheric air on the earth due to its weight. This pressure is change as the density of air varies according to the altitudes. Greater the height lesser the density. Also it may vary because of the temperature and humidity of air. Hence for all purposes of calculations the pressure exerted by air at sea level is taken as standard and that is equal to: -

1 atm = 1.01325 bar = 101.325 kPa = 10.328 m H_2o = 760 torr (mm Hg) = 14.7 psi

2. Gauge Pressure or Positive Pressure

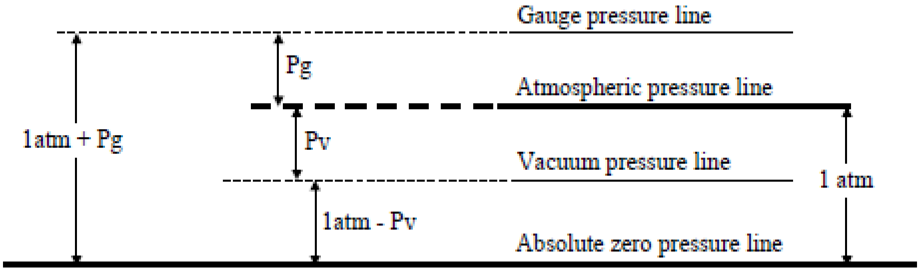
It is the pressure recorded by an instrument. This is always above atmospheric. The zero mark of the dial will have been adjusted to atmospheric pressure.

3. Vacuum Pressure or Negative Pressure

This pressure is caused either artificially or by flow conditions. The pressure intensity will be less than the atmospheric pressure whenever vacuum is formed.

4. Absolute Pressure

Absolute pressure is the algebraic sum of atmospheric pressure and gauge pressure. Atmospheric pressure is usually considered as the datum line and all other pressures are recorded either above or below it.



Absolute Pressure = Atmospheric Pressure + Gauge Pressure
Absolute Pressure = Atmospheric Pressure – Vacuum Pressure
For example if the vacuum pressure is 0.3 atm $\Rightarrow$ absolute pressure = 1.0 – 0.3 = 0.7 atm

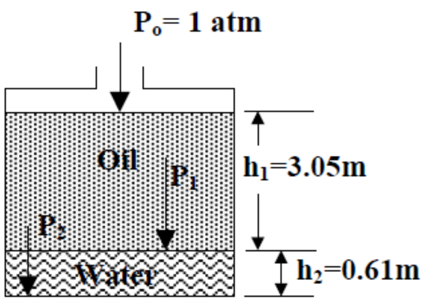
Note: -
Barometric pressure is the pressure that recorded from the barometer (apparatus used to measure atmospheric pressure).

3.4 Head of Fluid

Pressures are given in many different sets of units, such as N/m^2, or Pa, $dyne/cm^2$, psi, lb_f/ft^2. However a common method of expressing pressures is in terms of head (m, cm, mm, in, or ft) of a particular fluid. This height or head of the given fluid will exert the same pressure as the pressures it represents. $P = h \rho g$.

Example -3.1-
A large storage tank contains oil having a density of 917 kg/m^3. The tank is 3.66 m tall and vented (open) to the atmosphere of 1 atm at the top. The tank is filled with oil to a depth of 3.05 m and also contains 0.61 m of water in the bottom of the tank. Calculate the pressure in Pa at 3.05 m from the top of the tank and at the bottom. In addition, calculate the gauge pressure at the bottom of the tank.

Solution:



$P_o = 1 \text{ atm} = 14.696 \text{ psia} = 1.01325 \times 10^5 \text{ Pa}$

$P_1 = h_1 \rho_{oil} g + P_o$
$= 3.05 \text{ m} (917 \text{ kg/m}^3) 9.81 \text{ m/s}^2 + 1.01325 \times 10^5 \text{ Pa}$
$= 1.28762 \times 10^5 \text{ Pa}$

$P_1 = 1.28762 \times 10^5 \text{ Pa} (14.696 \text{ psia}/1.01325 \times 10^5 \text{ Pa})$
$= 18.675 \text{ psia}$

$P_2 = P_1 + h_2 \rho_{water} g$
$= 1.28762 \times 10^5 \text{ Pa} + 0.61 \text{ m} (1000 \text{ kg/m}^3) 9.81 \text{ m/s}^2$
$= 1.347461 \times 10^5 \text{ Pa}$

Example -3.2-
Convert the pressure of [1 atm =101.325 kPa] to
a- head of water in (m) at 4°C
b- head of Hg in (m) at 0°C

Solution:

a- The density of water at 4°C is approximatly1000 kg/m^3

$h = P / \rho_{water}\, g = 1.01325 \times 10^5$ Pa/(1000 kg/m^3 x 9.81m/s^2) =10.33 m H_2o

b- The density of mercury at 0°C is approximatly13595.5 kg/m^3

$h = P / \rho_{mercury}\, g = 1.01325 \times 10^5$ Pa/(13595.5 kg/m^3 x 9.81m/s^2) =0.76 m Hg

or

$P = (h\,\rho\,g)_{water} = (h\,\rho\,g)_{mercury} \Rightarrow h_{Hg} = h_{water}\,(\rho_{water} / \rho_{Hg})$

$h_{Hg} = 10.33\ (1000 / 13595.5) = 0.76$ m Hg

Example -3.3-

Find the static head of a liquid of sp.gr. 0.8 and pressure equivalent to 5×10^4 Pa.

Solution:

$\rho = 0.8\ (1000) = 800$ kg/m^3

$h = P / \rho\, g = 5\text{x}10^4/(800 \times 9.81) = 6.37$ m H_2o

3.5 Measurement of Fluid Pressure

In chemical and other industrial processing plants it is often to measure and control the pressure in vessel or process and/or the liquid level vessel.
The pressure measuring devices are: -
The pressure of a fluid may be measured by the following devices:

1. Manometers:
Manometers *are defined as the devices used for measuring the pressure at a point in a fluid by balancing the column of fluid by the same or another column of liquid.* These are classified as follows:

(*a*) Simple manometers:
(*i*) Piezometer, (*ii*) U-tube manometer, and (*iii*) Single column manometer.
(*b*) Differential manometers.

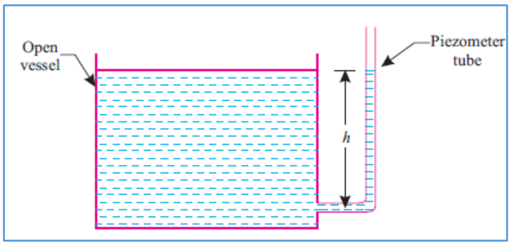
2. Mechanical gauges:
The pressure is measured by balancing the fluid column by spring (elastic element) or dead weight in these devices. Generally these gauges are used for measuring *high pressure* and where *high precision is not required.* Some commonly used mechanical gauges are:
(*i*) Bourdon tube pressure gauge, (*ii*) Diaphragm pressure gauge,
(*iii*) Bellow pressure gauge, and (*iv*) Dead-weight pressure gauge.

3.5.1 Manometers

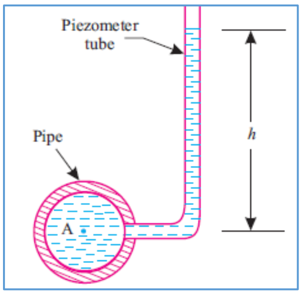
3.5.1.1. Simple manometers

A "simple manometer" is one which consists of a glass tube whose one end is connected to a point where pressure is to be measured and the other end remains open to atmosphere. Common types of simple manometers are discussed below:

1. Piezometer tube

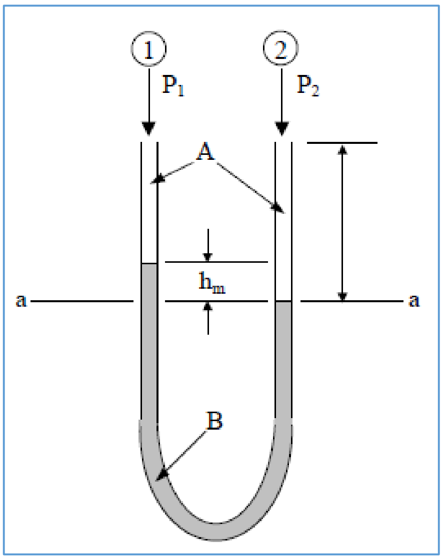


The piezometer consists a tube open at one end to atmosphere, the other end is capable of being inserted into vessel or pipe of which pressure is to be measured. The height to which liquid rises up in the vertical tube gives the pressure head directly.
i.e. $P = h \rho g$



Piezometer is used for measuring moderate pressures. It is meant for measuring *gauge pressure* **only** as the end is open to atmosphere. It cannot be used for *vacuum pressures*.

2. U-tube manometer:



Piezometers cannot be employed when large pressures in the *lighter liquid*s are to be measured, since this would require *very long tubes*, which cannot be handled conveniently. Furthermore, gas pressures cannot be measured by the piezometers because a *gas forms no free atmospheric surface*. These limitations can be overcome by the use of U-tube manometers.

It consists of a transparent U-tube containing the fluid A of density (ρ_A) whose pressure is to be measured and an immiscible fluid (B) of higher density (ρ_B). The limbs are connected to the two points between which the pressure difference ($P_2 - P_1$) is required; the connecting leads should be completely full of fluid A. If P_2 is greater than P_1, the interface between the two liquids in limb 2 will be depressed a distance (h_m) (say) below that in limb 1.

The pressure at the level a — a must be the same in each of the limbs and, therefore:

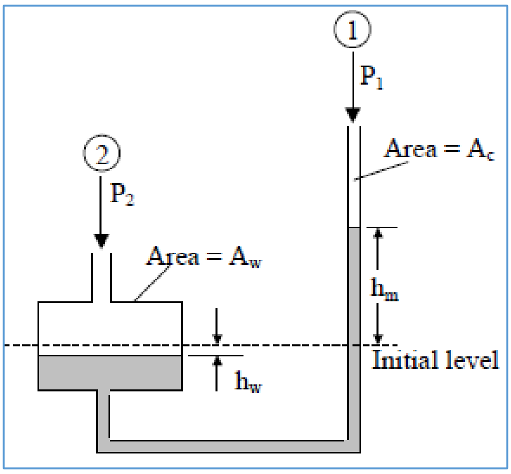
$$P_2 + Z_m \rho_A g = P_1 + (Z_m - h_m) \rho_A g + h_m \rho_B g$$

$$\Rightarrow \quad \Delta p = P_2 - P_1 = h_m (\rho_B - \rho_A) g$$

If fluid A is a gas, the density ρ_A will normally be small compared with the Figure below: The simple manometer density of the manometer fluid pm so that:

$$\Delta p = P_2 - P_1 = h_m \rho_B g$$

3. The well-type manometer



In order to avoid the inconvenience of having to read two limbs, and in order to measure low pressures, where accuracy id of much importance, the well-type manometer shown in the figure below can be used. If A_w and A_c are the cross-sectional areas of the well and the column and h_m is the increase in the level of the column and h_w the decrease in the level of the well, then:

$$P_2 = P1 + (h_m + h_w) \rho g$$

or: $$\Delta p = P_2 - P_1 = (h_m + h_w) \rho g$$

The quantity of liquid expelled from the well is equal to the quantity pushed into the column so that:

$$A_w\, h_w = A_c\, h_m \;\Rightarrow\; h_w = (A_c/A_w)\, h_m$$

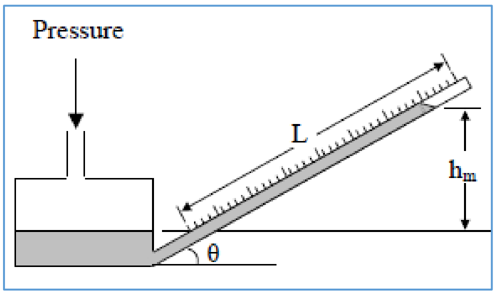
$$\Rightarrow\; \Delta p = P_2 - P_1 = \rho\, g\, h_m\,(1 + A_c/A_w)$$

If the well is large in comparison to the column then:

i.e. $(A_c/A_w) \rightarrow\approx 0 \;\Rightarrow\; \Delta p = P_2 - P_1 = \rho\, g\, h_m$

4. The inclined manometer

This figure enables the sensitivity of the manometers described previously to be increased by measuring the length of the column of liquid. If θ is the angle of inclination of the manometer (typically about 10-20°) and L is the movement of the column of liquid along the limb, then:



$$h_m = L \sin\theta$$

If θ = 10°, the manometer reading L is increased by about 5.7 times compared with the reading h_m which would have been obtained from a simple manometer.

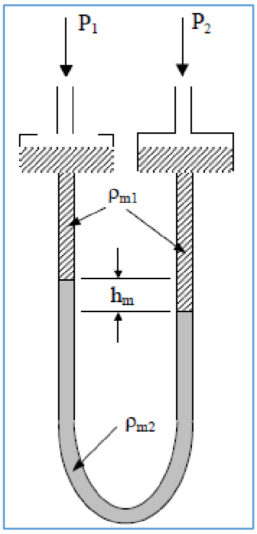
3.5.1.2. Differential manometer

A differential manometer *is used to measure the difference in pressures between two points in a pipe, or in two different pipes.* In its simplest form a differential manometer consists of a U-tube, containing a heavy liquid, whose two ends are connected to the points, whose difference of pressures is required to be found out. Following are the most commonly used types of differential manometers:

1. U-tube differential manometer (Two-liquid manometer).
2. Inverted U-tube differential manometer.

1. The two-liquid manometer

Small differences in pressure in gases are often measured with a manometer of the form shown in this figure. The reservoir at the top of each limb is of a sufficiently large cross-section for the liquid level to remain approximately the on each side of the manometer.



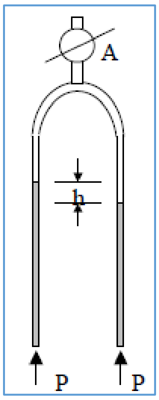
The difference in pressure is then given by:

$$\Delta p = P_2 - P_1 = h_m\,(\rho_{m1} - \rho_{m2})\, g$$

where ρ_{m1} and ρ_{m2} are the densities of the two manometer liquids. The sensitivity of the instrument is very high if the densities of the two liquids are nearly the same. To obtain accurate readings it is necessary to choose liquids, which give sharp interfaces: paraffin oil and industrial alcohol are commonly used.

2. The inverted U-tube differential manometer

This figure is used for measuring *pressure differences in liquids*. The space above the liquid in the manometer is filled with air, which can be admitted or expelled through the tap A in order to adjust the level of the liquid in the manometer.



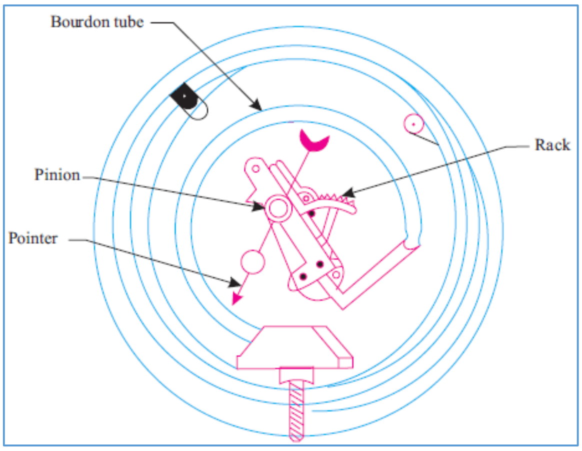
3.5.2 Mechanical Gauges

Whenever a *very high fluid pressure* is to be measured, and a *very great sensitivity* a mechanical gauge is best suited for these purposes. They are also designed to read vacuum pressure. A mechanical gauge is also used for measurement of pressure in boilers or other pipes, where tube manometer cannot be conveniently used.

The Bourdon gauge

The pressure to be measured is applied to a curved tube, oval in cross-section, and the deflection of the end of the tube is communicated through a system of levers to a recording needle. This gauge is widely used for *steam* and *compressed gases*, and frequently *forms the indicating element on flow controllers*. The simple form of the gauge is illustrated in figure below, this figure shows a Bourdon type gauge with the sensing element in the form of a helix; this instrument has a very much greater sensitivity and is suitable for very high pressures.

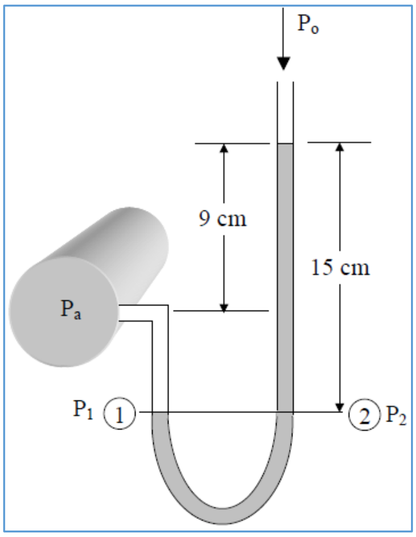
It may be noted that the pressure measuring devices of category (2) all measure a pressure difference ($\Delta p = P_2 - P_1$). In the case of the Bourdon gauge (1) of category (3), the pressure indicated is the difference between that communicated by the system to the tube and the external (ambient) pressure, and this is usually referred to as *the gauge pressure*. It is then necessary to add on the ambient pressure in order to obtain the *(absolute) pressure*.



Gauge pressures are not, however, used in the SI System of units.

Example -3.4-

A simple manometer is used to measure the pressure of oil sp.gr. 0.8 flowing in a pipeline. Its right limb is open to atmosphere and the left limb is connected to the pipe. The center of the pipe is 9.0 cm below the level of the mercury in the right limb. If the difference of the mercury level in the two limbs is 15 cm, determine the absolute and the gauge pressures of the oil in the pipe.



Solution:

$\rho = 0.8\ (1000) = 800\ kg/m^3$

$P_1 = P_2$

$P_1 = (0.15 - 0.0\ 9)m(800\ kg/m^3)9.81m/s^2 + P_a$

$P_2 = (0.15)\ m\ (13600\ kg/m^3)\ 9.81\ m/s^2 + P_o$

$P_a = 15\ (13600)\ 9.81 + P_o + [(15 - 9)cm\ (800\ kg/m^3)\ 9.81\ m/s^2]$

$= 1.20866 \times 10^5$ Pa (Absolute pressure)

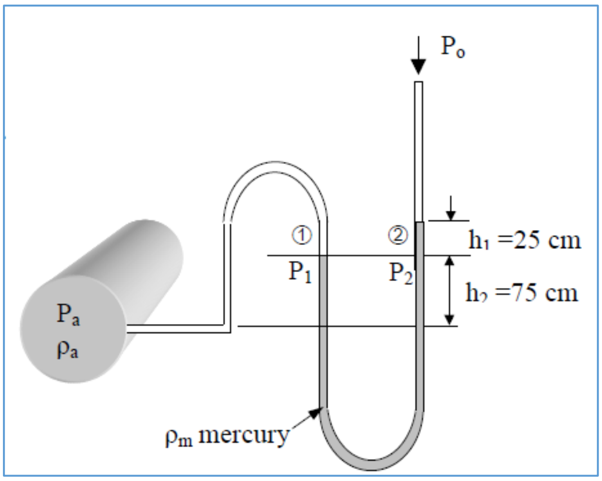
The gauge press. = Abs. press. – Atm. Press.

$= 1.20866 \times 10^5 - 1.0325 \times 10^5$

$= 1.9541 \times 10^4$ Pa

Example -3.5-

The following Figure shows a manometer connected to the pipeline containing oil of sp.gr. 0.8. Determine the absolute pressure of the oil in the pipe, and the gauge pressure.



$\rho_a = 0.8\ (1000) = 800\ kg/m^3$

$P_1 = P_2$

$P_1 = P_a - h_2\ \rho_a\ g$

$P_2 = P_o + h_1\ \rho_m\ g$

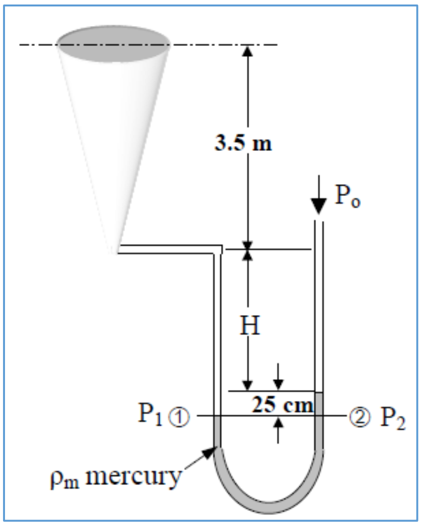
$\Rightarrow P_a = P_o + h_1\ \rho_m\ g + h_2\ \rho_a\ g$

$= 1.0325 \times 10^5 + (0.25)\ m\ (13600\ kg/m^3)\ 9.81\ m/s^2 + (0.75)\ m\ (800\ kg/m^3)\ 9.81\ m/s^2$

$= 1.40565 \times 10^5$ Pa

Example -3.6-

A conical vessel is connected to a U-tube having mercury and water as shown in the Figure. When the vessel is empty the manometer reads 0.25 m. find the reading in manometer, when the vessel is full of water.



Solution:

$$P_1 = P_2$$
$$P_1 = (0.25 + H)\ \rho_w\ g + P_o$$
$$P_2 = 0.25\ \rho_m\ g + P_o$$
$$\Rightarrow (0.25 + H)\ \rho_w\ g + P_o = 0.25\ \rho_m\ g + P_o$$
$$\Rightarrow H = 0.25\ (\rho_m - \rho_w)/\ \rho_w$$
$$= 0.25\ (12600\ /1000) = 3.15\ m$$

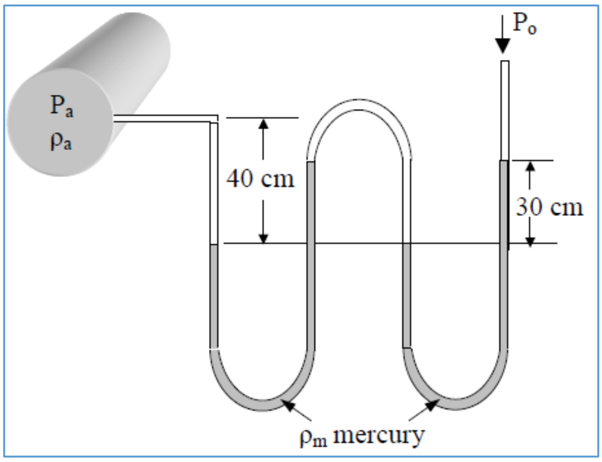
When the vessel is full of water, let the mercury level in the left limp go down by (x) meter and the mercury level in the right limp go to up by the same amount (x) meter. i.e. the reading manometer = (0.25 + 2x)

$$P_1 = P_2$$
$$P_1 = (0.25 + x + H + 3.5)\ \rho_w\ g + P_o$$
$$P_2 = (0.25 + 2x)\ \rho_m\ g + P_o$$
$$\Rightarrow (0.25 + x + H + 3.5)\ \rho_w\ g + P_o = (0.25 + 2x)\ \rho_m\ g + P_o$$
$$\Rightarrow 6.9 + x = (0.25 + 2x)\ (\rho_m/\ \rho_w) \Rightarrow x = 0.1431\ m$$

The manometer reading = 0.25 + 2 (0.1431) = 0.536 m

Example -3.7-

The following Figure shows a compound manometer connected to the pipeline containing oil of sp.gr. 0.8. Calculate P_a.



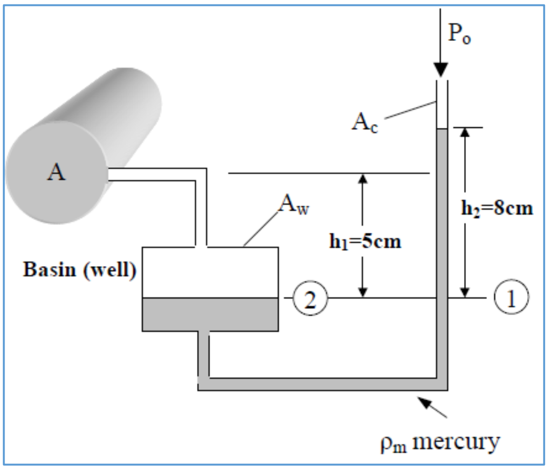
<u>Solution:</u>

$$P_1 = P_2$$
$$P_1 = P_{air} + 0.5\ \rho_w\ g$$
$$P_2 = P_A + 0.1\ \rho_a\ g + 0.05\ \rho_m\ g$$
$$\Rightarrow P_A = P_{air} + 0.5\ \rho_w\ g - 0.1\ \rho_a\ g - 0.05\ \rho_m\ g$$
$$\Rightarrow P_{air} = (1.0\ kg/cm^2\ P_B)\ (9.81 m/s^2)\ (10^4\ cm^2/\ m^2)$$
$$= 9.81\ x10^4\ Pa$$

$\therefore P_A = 9.81\ x10^4\ Pa + 0.5\ (1000)\ 9.81 - 0.1\ (900)\ 9.81 - 0.05\ (13600)\ 9.81$
$= 9.54513\ x10^4\ Pa$

Example -3.8-

A Micromanometer, having ratio of basin to limb areas as 40, was used to determine the pressure in a pipe containing water. Determine the pressure in the pipe for the manometer reading shown in Figure.



Solution:

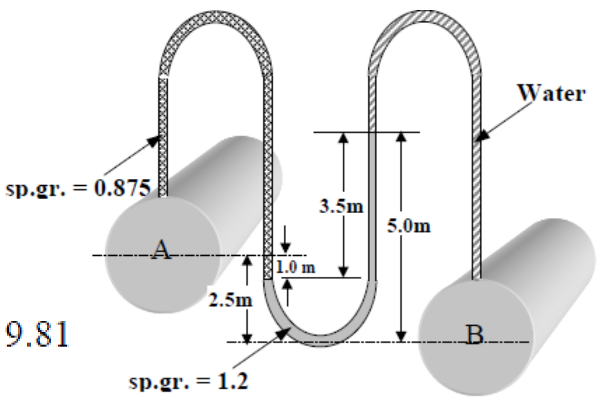
$P_1 = P_2$
$P_1 = P_o + h_2\ \rho_m\ g$
$P_2 = P_A + h_1\ \rho_w\ g$

$\Rightarrow\ P_A = P_o + h_2\ \rho_m\ g - h_1\ \rho_w\ g$
$= 1.01325x10^5 + 0.08\ (13600)\ 9.81 -$
$0.05\ (1000)\ 9.81$
$= 1.11507\ x10^5\ Pa$

Note:
If h_2 and h_1 are the heights from initial level, the ratio (A_w/A_c) will enter in calculation.

Example -3.9-

Two pipes, one carrying toluene of sp.gr. = 0.875, and the other carrying water are placed at a difference of level of 2.5 m. the pipes are connected by a U-tube manometer carrying liquid of sp.gr. = 1.2. The level of the liquid in the manometer is 3.5 m higher in the right limb than the lower level of toluene in the limb of the manometer. Find the difference of pressure in the two pipes. **Solution:**



T $\equiv$ Toluene, W $\equiv$ Water, L $\equiv$ Liquid

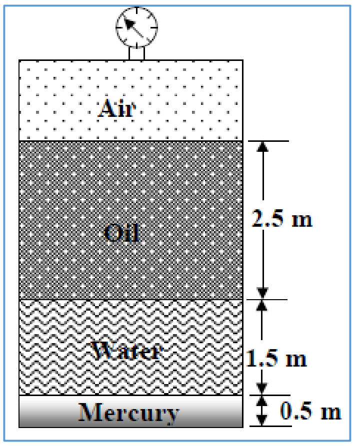
$P_A + 3.5\ \rho_T\ g - 3.5\ \rho_L\ g + 5\ \rho_W\ g - P_B = 0$

$\Rightarrow\ P_A - P_B = [3.5\ (1200) - 3.5\ (875) - 5\ (1000)]\ 9.81$
$= -\ 3862.5\ Pa$

$\Rightarrow\ P_B - P_A = 3862.5\ Pa$

Example -3.10-

A closed tank contains 0.5 m of mercury, 1.5 m of water, 2.5 m of oil of sp.gr. = 0.8 and air space above the oil. If the pressure at the bottom of the tank is 2.943 bar gauge, what should be the reading of mechanical gauge at the top of the tank.



Solution:

Solution:

Pressure due to 0.5 m of mercury

$$P_m = 0.5\ (13600)\ 9.81 = 0.66708 \text{ bar}$$

Pressure due to 1.5 m of water

$$P_w = 1.5\ (1000)\ 9.81 = 0.14715 \text{ bar}$$

Pressure due to 2.5 m of oil

$$P_O = 2.5\ (800)\ 9.81 = 0.19620 \text{ bar}$$

Pressure at the bottom of the tank = $P_m + P_w + P_O + P_{Air}$

$$\Rightarrow\ 2.943 = 0.66708 \text{ bar} + 0.14715 \text{ bar} + 0.19620 \text{ bar} + P_{Air}$$

$$\Rightarrow\ P_{Air} = 1.93257 \text{ bar}$$

Home Work

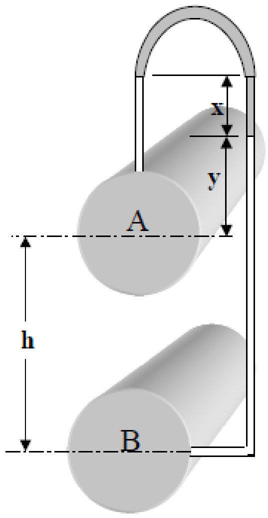
P.3.1

Two pipes A and B carrying water are connected by a connecting tube as shown in Figure,

a- If the manometric liquid is oil of sp.gr. = 0.8, find the difference in pressure intensity at A and B when the difference in level between the two pipes be (h = 2 m) and (x = 40 cm).

b- If mercury is used instead of water in the pipes A and B and the oil used in the manometer has sp.gr. = 1.5, find the difference in pressure intensity at A and B when (h = 50 cm) and (x = 100 cm).

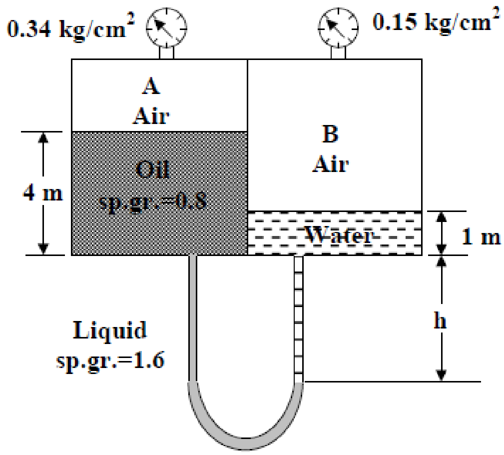
Ans. a- P_B-P_A =18835.2 Pa, b- P_B-P_A =51993 Pa



P.3.2

A closed vessel is divided into two compartments. These compartments contain oil and water as shown in Figure. Determine the value of (h).

Ans. h = 4.5 m



P.3.3

Oil of sp.gr. = 0.9 flows through a vertical pipe (upwards). Two points A and B one above the other 40 cm apart in a pipe are connected by a U-tube carrying mercury. If the difference of pressure between A and B is 0.2 kg/cm^2,

1- Find the reading of the manometer.

2- If the oil flows through a horizontal pipe, find the reading in manometer for the same difference in pressure between A and B.

Ans. 1- R = 0.12913 m, 2- R = 0.1575 m,

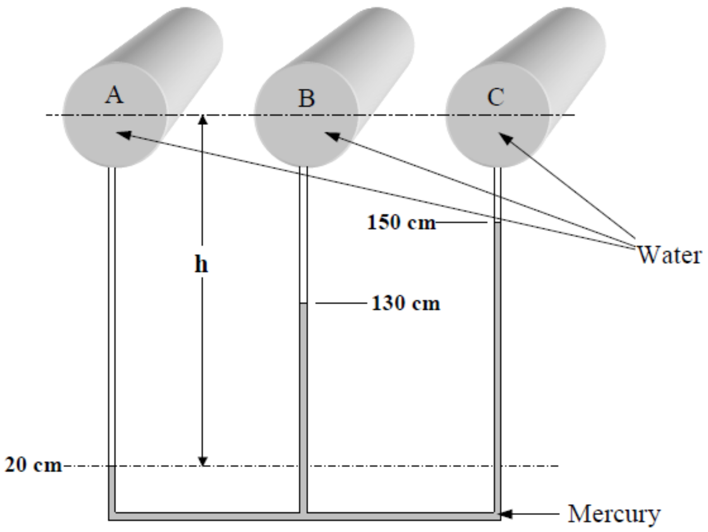
P.3.4

A mercury U-tube manometer is used to measure the pressure drop across an orifice in pipe. If the liquid that flowing through the orifice is brine of sp.gr. 1.26 and upstream pressure is 2 psig and the downstream pressure is (10 in Hg) vacuum, find the reading of manometer.

Ans. R = 394 mm Hg

P.3.5

Three pipes A, B, and C at the same level connected by a multiple differential manometer shows the readings as show in Figure. Find the differential of pressure heads in terms of water column between A and B, between A and C, and between B and C.



Ans. P_A-P_B =1.359666 bar = 13.86 m H_2o
P_A-P_C =1.606878 bar = 16.38 m H_2o
P_B-P_C =0.247212 bar = 2.52 m H_2o

3.5. Total Pressure and Centre of Pressure

Total pressure. It is defined as the *force exerted by static fluid on a surface* (*either plane or curved*) *when the fluid comes in contact with the surface. This force is always at right angle* (*or normal*) *to the surface.*

Centre of pressure. It is defined as the *point of application of the total pressure on the surface.*

Now we shall discuss the total pressure exerted by a liquid on the immersed surface. The immersed surfaces may be:

1. Horizontal plane surface;
2. Vertical plane surface;
3. Inclined plane surface;
4. Curved surface.

3.6. Forces on Plane Surfaces

Table 3.1. **The Centre of Gravity (G) and Moment of Inertia (I) of Some Important Geometrical Figures:**

S.No.	Name of figure	C.G. from the base	Area	I about an axis passing through C.G. and parallel to the base	I about base
1.	Triangle Fig. 3.3	$x = \frac{h}{3}$	$\frac{bh}{2}$	$\frac{bh^3}{36}$	$\frac{bh^3}{12}$
2.	Rectangle Fig. 3.4	$x = \frac{d}{2}$	bd	$\frac{bd^3}{12}$	$\frac{bd^3}{3}$
3.	Circle Fig. 3.5	$x = \frac{d}{2}$	$\frac{\pi d^2}{4}$	$\frac{\pi d^4}{64}$	—
4.	Trapezium Fig. 3.6	$x = \left[\frac{2a+b}{a+b}\right]\frac{h}{3}$	$\left(\frac{a+b}{2}\right)h$	$\left(\frac{a^2+4ab+b^2}{3b\,(a+b)}\right) \times h^2$	—

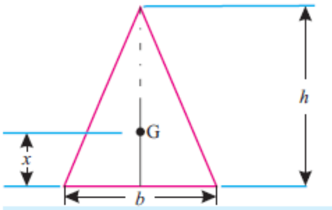


Fig. 3.3

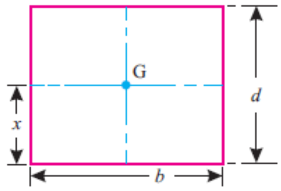


Fig. 3.4

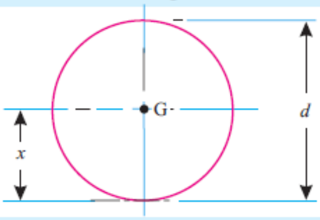


Fig. 3.5

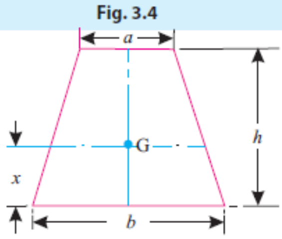
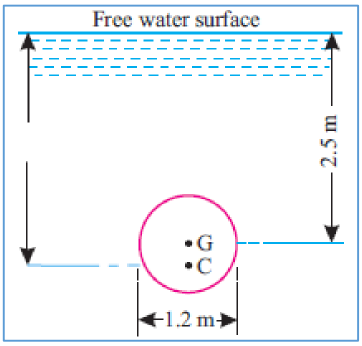


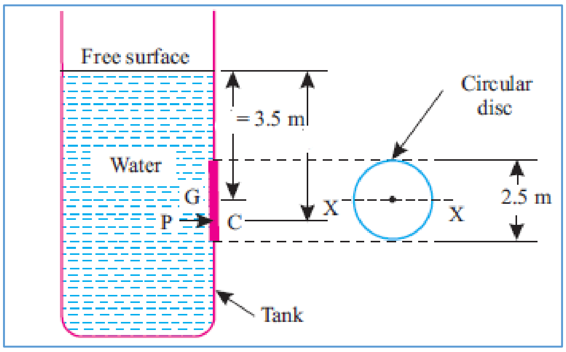
Fig. 3.6

Example 3.1. This figure shows a circular plate of diameter 1.2 m placed vertically in water in such a way that the center of the place is 2.5 m below the free surface of water. Determine: (i) Total pressure on the plate. (ii) Position of center of pressure.



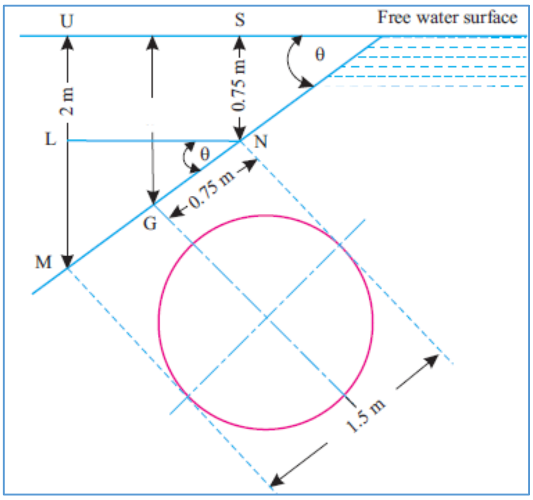
Example 3.2. A circular opening, 2.5 m diameter, in a vertical side of tank is closed by a disc of 2.5 m diameter that can rotate about a horizontal diameter. Determine:

i. The force on the disc;
ii. The torque required to maintain the disc in equilibrium in vertical position when the head of water above horizontal diameter is 3.5 m.

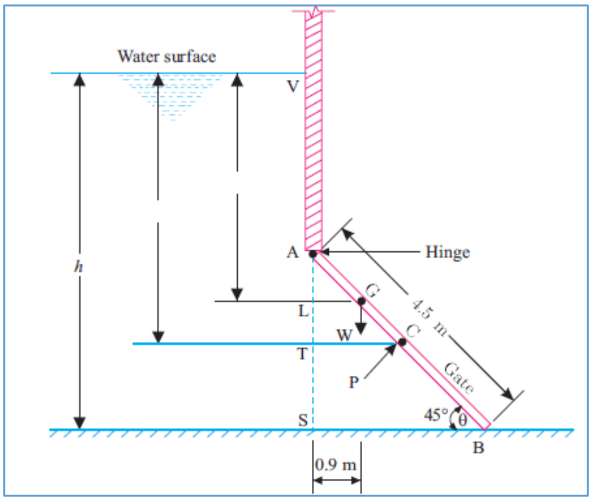


Example 3.3. A circular plate 1.5 m diameter is submerged in water, with its greatest and least depths below the surface being 2 m and 0.75 m respectively. Determine:

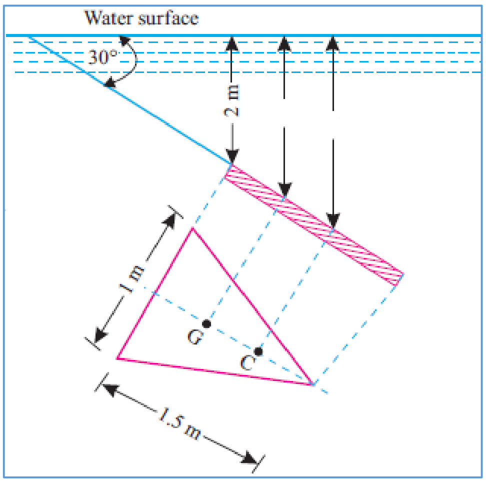
(i) The total pressure on one face of the plate, and
(ii) The position of the center of pressure.



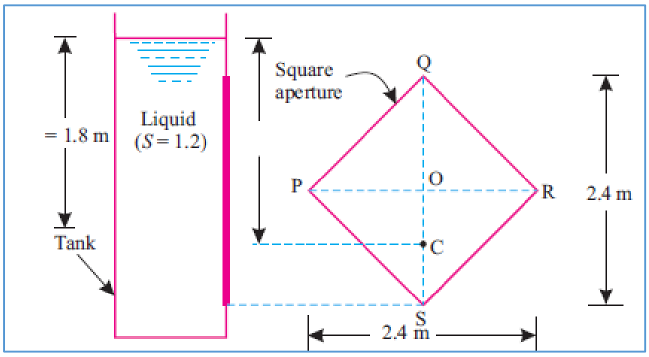
Example 3.4. This figure shows a rectangular sluice gate AB, 3 m wide and 4.5 m long hinged at A. It is kept closed by a weight fixed to the gate. The total weight of the gate and weight fixed to the gate is 515 kN. The center of gravity of the weight and gate is at G. Find the height of the water h, which will first cause the gate to open.



H.W1 A triangular plate of 1 meter base and 1.5 meter altitude is immersed in water. The plane of the plate is inclined at 30° with free water surface and the base is parallel to and at a depth of 2 meters from water surface. Find the total pressure on the plate and the position of center of pressure.



H.W2 A square aperture in the vertical side of a tank has one diagonal vertical and is completely covered by a plane plate hinged along one of the upper sides of the aperture. The diagonals of the aperture are 2.4 m long and the tank contains a liquid of specific gravity 1.2. The centre of aperture is 1.8 m below the free surface. Calculate:



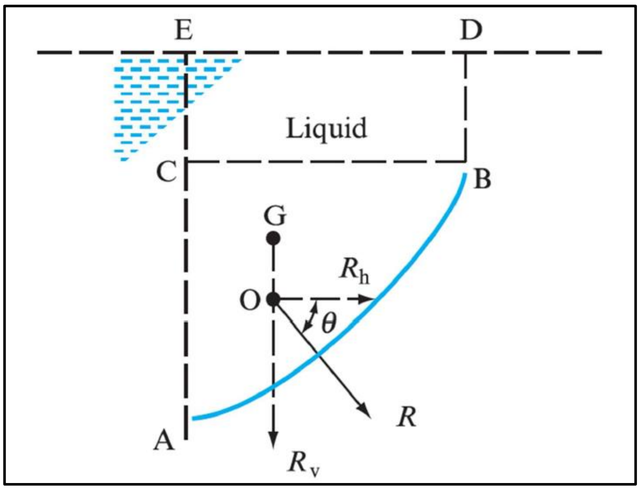
(i) The thrust exerted on the plate by the liquid;
(ii) The position of its center of pressure. (Ans. 61. 026 kN, 1.933 m)

3.7. Force on a Curved Surface due to Hydrostatic Pressure

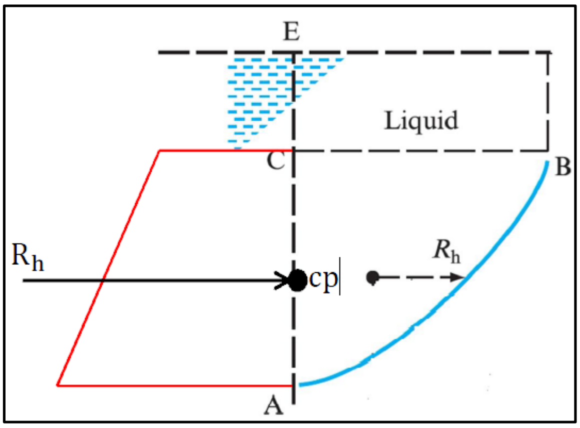
If the surface is curved, the forces on each element of the surface will not be parallel (normal to the surface at each point) and must be combined using some vectorial method.
The most significant method to solve these types of problems is to calculate the **vertical and horizontal** components, and then combine these two forces to obtain the resultant force and its direction.

There are two cases:

Case I: if the fluid is above the curved surface:



Horizontal Component (R_h):
The resultant horizontal force of a fluid above a curved surface is:
R_h = Resultant force on the projection of the curved surface onto a vertical plane (i.e. along line AC in the above figure).
We know that the force must be normal to the plane, so if we take the vertical plane, **the force will act horizontally through the center of pressure of the projected vertical plane** as shown in figure below, **and we can use pressure diagram method.**



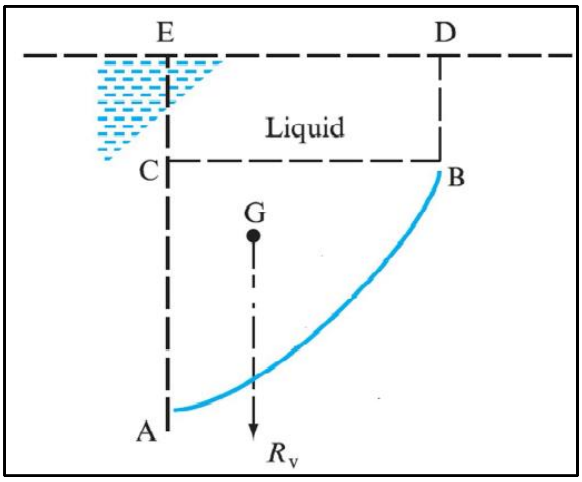
Vertical Component (R_v):

Because the fluid is at rest, there are no shear forces on the vertical edges, so the vertical component can only be due to the weight of the fluid.

The resultant vertical force of a fluid above a curved surface is:

R_v = Weight of fluid directly above the curved surface and will act vertically downward through the **center of gravity of the mass of fluid** as shown in figure below.

R_v = Weight of fluid above the curved surface = $\rho gV = \gamma V$



Resultant Force (R):

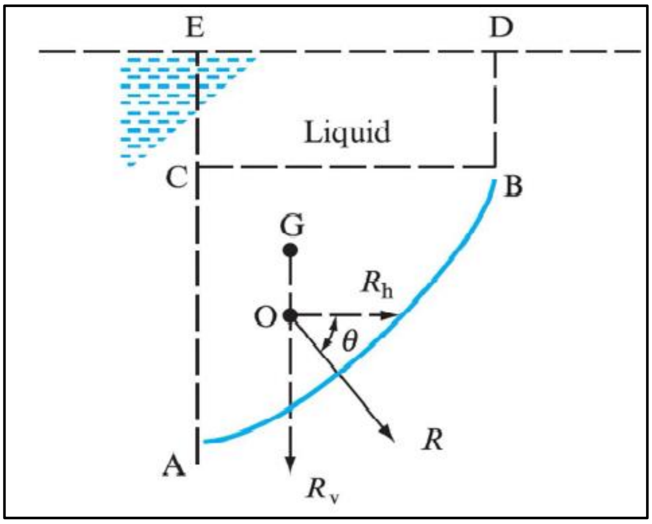
The overall resultant force is found by combining the vertical and horizontal components vectorialy:

$$\boxed{R = \sqrt{R_h^2 + R_v^2}}$$

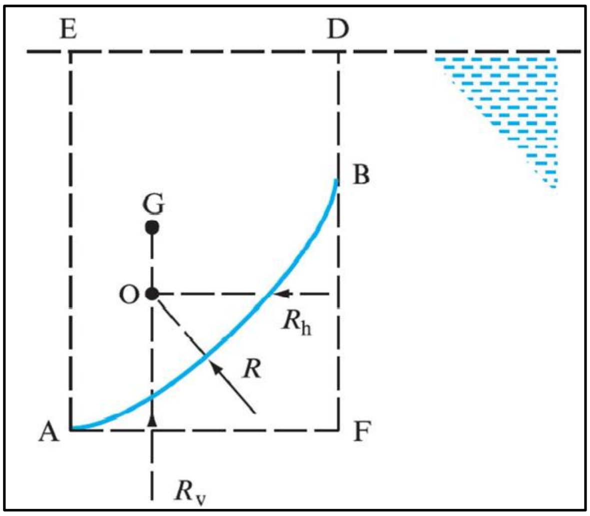
This resultant force acts through point O at an angle (θ) with R_h

The position of O is the point of intersection of the horizontal line of action of R_h and the vertical line of action of R_v as shown in figure below.

$$\boxed{\theta = \tan^{-1}\left(\frac{R_v}{R_h}\right)}$$



Case II: if the fluid is below the curved surface:



The calculation of horizontal force R_h is the same as case I, but calculation of vertical force R_V will differ from case I.

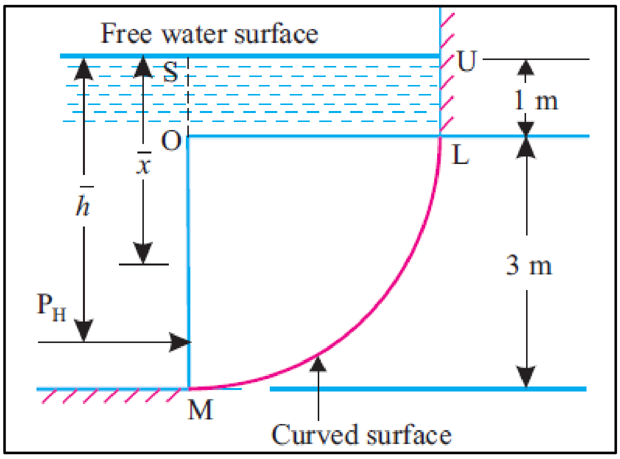
Vertical force component in case of fluid below curved surface:

If the curved surface AB is removed, the area ABDE will replaced by the fluid and the **whole system would be in equilibrium.**

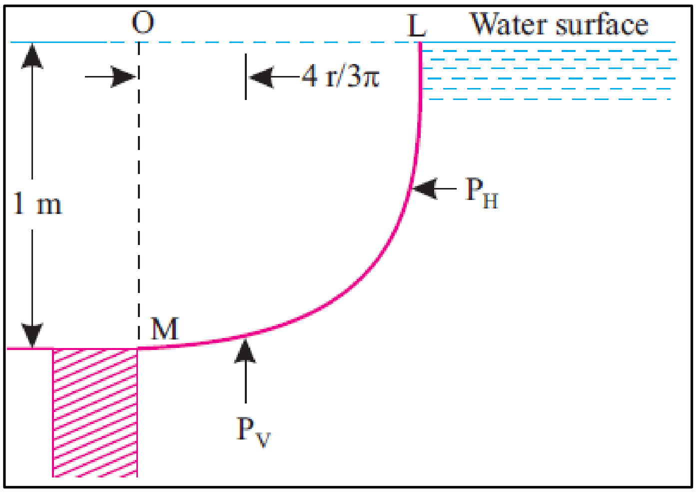
Thus, the force required by the curved surface to maintain equilibrium is equal to that force which the **fluid above the surface would exert (weight of fluid above the curved surface). I.e.** The resultant vertical force of a fluid below a curved surface is:

R_V = Weight of the **imaginary volume** of fluid vertically above the curved surface.

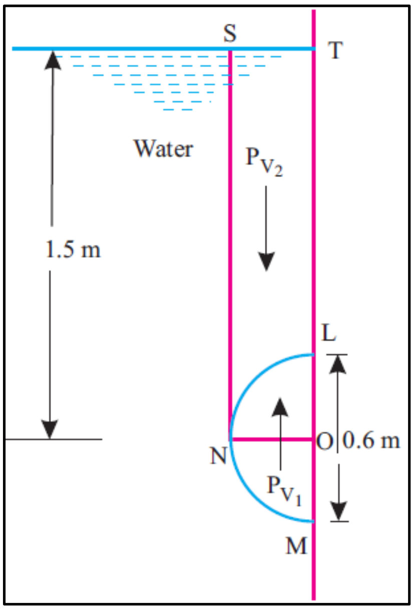
Example 1. *This figure shows a curved surface LM, which is in the form of a quadrant of a circle of radius 3 m, immersed in the water. If the width of the gate is unity,* ***calculate*** *the horizontal and vertical components of the total force acting on the curved surface.*



Example 2. *This figure shows a gate having a quadrant shape of radius of 1 m subjected to water pressure.* ***Find*** *the resultant force and its inclination with the horizontal. Take the length of the gate as 2 m.*

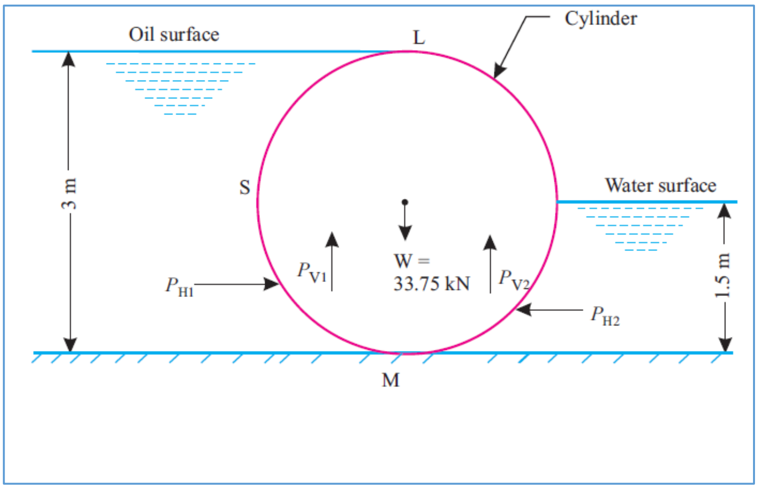


Example 3. *A hemisphere projection of diameter 0.6 m exists on one of the vertical sides of a tank. If the tank contains water to an elevation of 1.5 m above the center of the hemisphere,* ***calculate*** *the vertical and horizontal forces acting on the projection.*

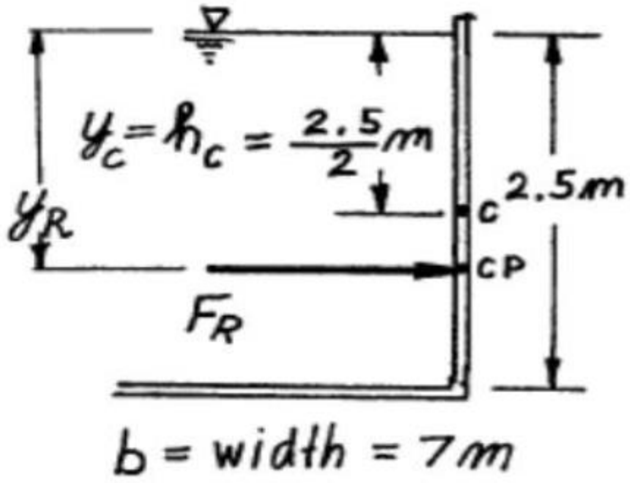


H.W:

A cylinder having 3 m diameter and 1.5 m length is resting on the floor. On one side, water is filled upto half the depth while on the other side oil of relative density 0.8 filled upto the top .If the weight of the cylinder is 33.75 kN, determine the magnitudes of the horizontal and vertical components of the force which will keep the cylinder just touching the floor.

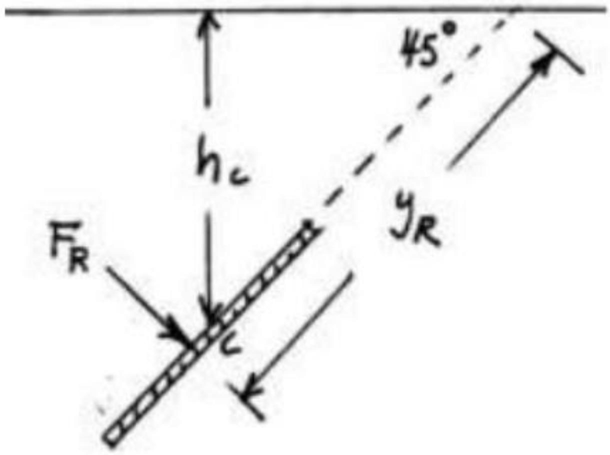


Example 1: A swimming pool is 18m long and 7m wide. Determine the magnitude and the location of the resultant force of the water on the vertical end of the pool where the depth is 2.5m.

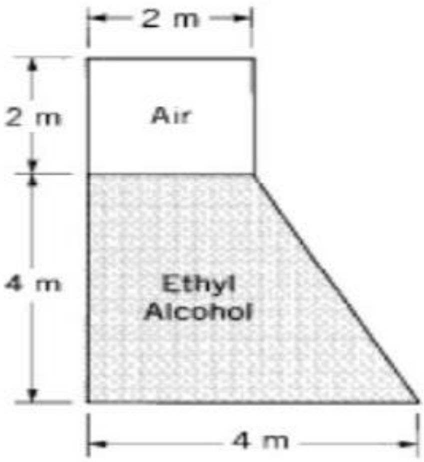


Example 2: A square 3m*3m gate is located in 45° sloping side of a dam. Some measurements indicate that the resultant force of the water on the gate is 500 kN.

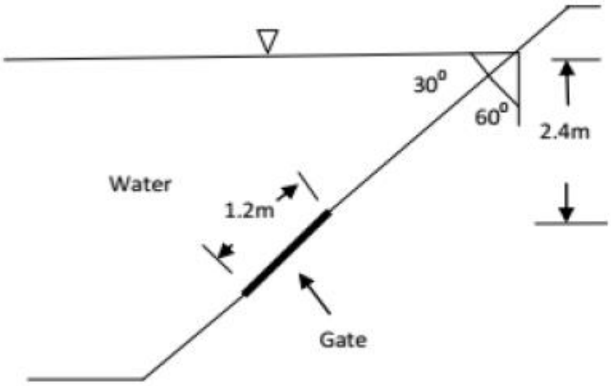
1- Determine the pressure at the bottom of the gate.
2- Show where this force acts.



Example 3: the vertical cross section of a 7m long closed storage tank is shown below. The tank contains ethyl alcohol and air pressure is 40 kpa. Determine the magnitude of the resultant fluid force acting on one end of the tank.



Example ٤: An inclined rectangular gate (1.5m wide) contains water on one side. Determine the total resultant force acting on the gate and the location.

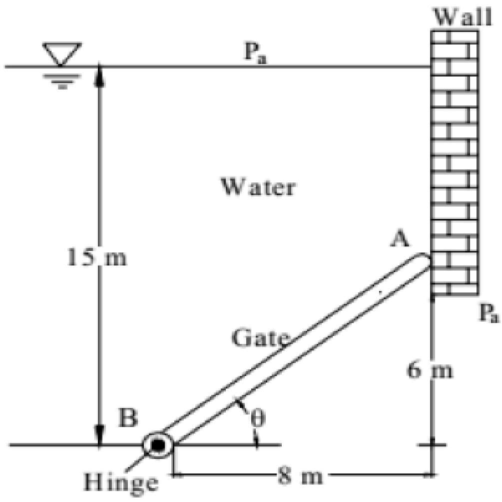


Example 5: The gate in figure below is 5 m wide, is hinged at point B, and rests against a smooth wall at point A. Compute:

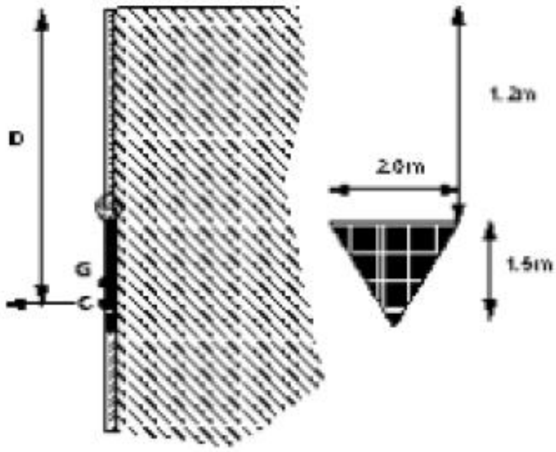
a) The force on the gate due to the water pressure.

b) The distance between the centre of gate and location of force act.

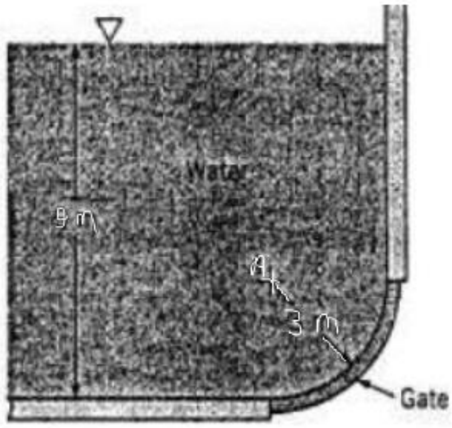
c) The reactions at hinge B.



Example 6 : A tank holding water has a triangular gate, hinged at the top, in one wall. Find the moment at the hinge required to keep this triangular gate closed.

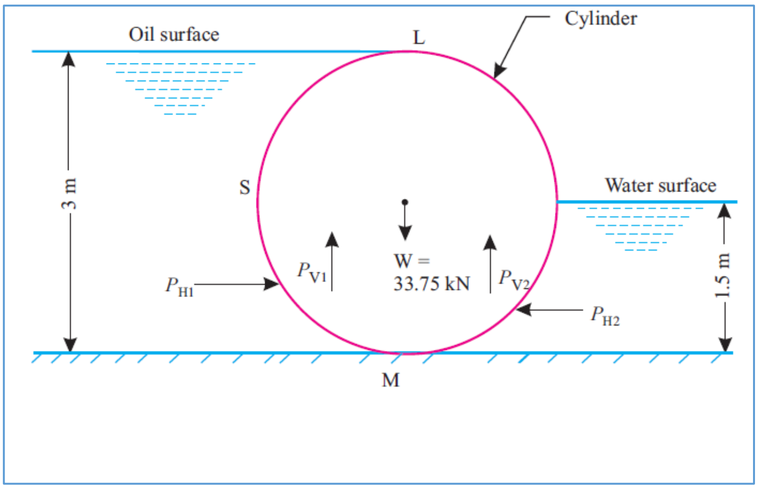


Example 7: A 4m long curved gate is located in the side of a reservoir containing water as shown in figure below. Determine the magnitude of the horizontal and vertical components of the force of the water on the gate.



H.W:

A cylinder having 3 m diameter and 1.5 m length is resting on the floor. On one side, water is filled upto half the depth while on the other side oil of relative density 0.8 filled upto the top .If the weight of the cylinder is 33.75 kN, determine the magnitudes of the horizontal and vertical components of the force which will keep the cylinder just touching the floor.



3.8. Buoyancy

When a body is submerged or floating in a static fluid, the resultant force exerted on it by the fluid is called the *buoyancy force*.

Buoyancy Force= weight of fluid displaced by the body and this force will act vertically upward through the centroid of the volume of fluid displaced, known as the center of buoyancy.

Archimedes' principle

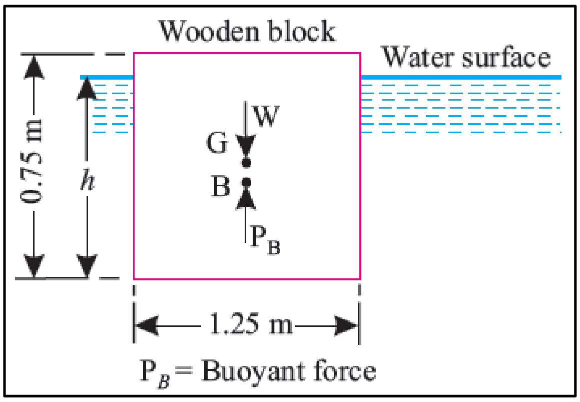
Archimedes' Principle states that the buoyant force has a **magnitude equal to the weight of the fluid displaced** by the body and is directed vertically upward.

$$F_b = \rho_{fluid} \times g \times V_{displaced\ by\ body} \quad (\text{Upward} \uparrow)$$

Problems

1. *A wooden block of width 1.25* m, *depth 0.75 m and length 3.0 m is floating in water. Specific weight of the wood is 6.4 kN/m*^{3}. ***Find:*** **(i)** *Volume of water displaced, and* **(ii)** *Position of centre of buoyancy.*

Note: $P_B=F_b$ *, G:center of gravity, B: center of buoncy*



Solution:

problem ①:

① weight of water displaced = weight of block
وزن السائل المزاح = وزن ال block الخشبي

$W_{block} = \gamma \cdot \forall$
$= 6.4 * (0.75 * 1.25 * 3) = 18\ kN$

$\forall$ (حجم) of water displaced $= \dfrac{\text{weight of water displaced}}{\gamma}$

$= \dfrac{18}{9.81} = 1.835\ m^3$

② $\forall$ (حجم) of block = $\forall$ (حجم) of water displaced

$h * 1.25 * 3 = 1.835$

$\therefore h = 0.482^m$

center of buoyancy $= \dfrac{h}{2} = \dfrac{0.482}{2}$
$= 0.244^m$ from the base

2. *A wooden block of specific gravity 0.7 and having a size of 2 m × 0.5m × 0.25 m is floating in water.* ***Determine*** *the volume of concrete of specific weight 25 kN/m*^{3}*, that may be placed which will immerse (i) the block completely in water, and (ii) the block and concrete completely in water.*

Solution:

Problem ②:

$V_{(block)} = 2 * 0.5 * 0.25 = 0.25\ m^3$

specific gravity (block) $= 0.7 * 9.81 = 6.867\ kN/m^3$

W_c: weight of concrete

V_c: volume of concrete

Total weight of block $= W_c +$ weight of block

الوزن = s.g. * volume

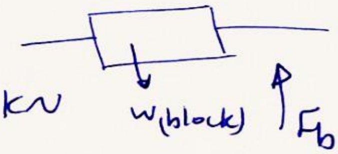
$W_{block} = 6.867 * 0.25 = 1.716\ kN$

① immersion of block only:

volume of water displaced $= 0.25\ m^3$

$W = F_b \Rightarrow$ force $= \gamma \cdot \underset{\text{مغمور}}{V}$

$= 9.81 * 0.25 = 2.45\ kN$



Tot. weight of block $= W_c +$ Weight (block)

$2.45 = W_c + 1.716$

$\therefore W_c = 0.734\ kN$

$V_c = \frac{\text{weight}}{\text{specific weight}} = \frac{0.734}{25} = 0.0294\ m^4$

② immersion of block and concrete

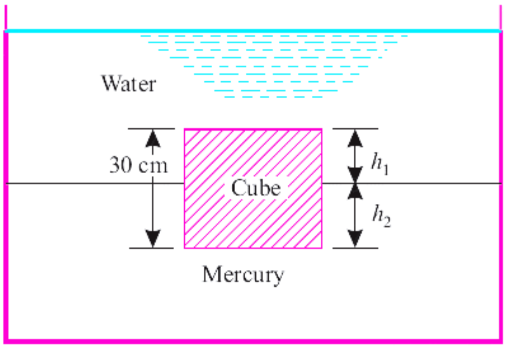
total weight of the block $= 25 V_c + 1.716$

upward thrust $= (V_c + 0.25) \times 9.81$

$25 V_c + 1.716 = (V_c + 0.25) * 9.81$

$\therefore V_c = 0.0483\ m^3$

H.W. *A metallic cube 30 cm side and weighing 450 N is lowered into a tank containing a two-fluid layer of water and mercury.* ***Determine*** *the position of block at mercury-water interface when it has reached equilibrium.*



Equilibrium of Floating Bodies:

To be the floating body in equilibrium, two conditions must be satisfied:

- ✓ The buoyant Force (F_b) must equal the weight of the floating body (W).
- ✓ F_b and W must act in the same straight line. i.e. the center of gravity and the center of buoyancy in the same straight line

So, for equilibrium: $F_b = W_{object}$

The equilibrium of a body may be:

- ✓ Stable.
- ✓ Unstable.
- ✓ Neutral (could be considered **stable**)

Stability of *a* *Submerge* Bodies

ibrium: if when displaced, it returns to its original equilibrium position.

ibrium: if when displaced, it returns to a new equilibrium position

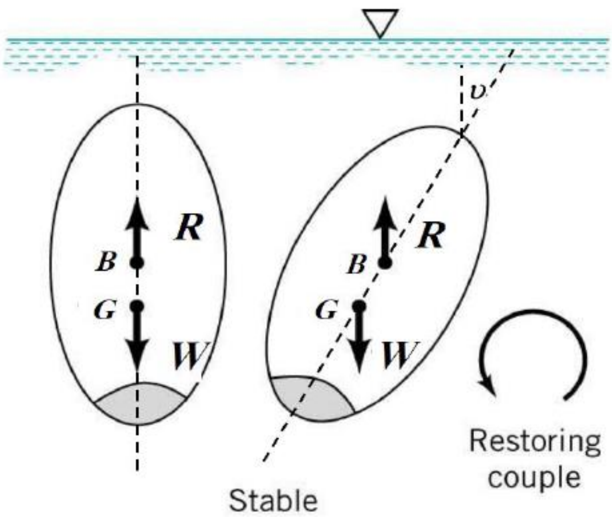
Notes:

- In this case (body is fully immersed in water) when the body is tilted, the shape of the displaced fluid doesn't change, so the center of buoyancy remains unchanged relative to the body.
- The weight of the body is located at the center of gravity of the body (G) and the buoyant force located at the center of buoyancy (B).

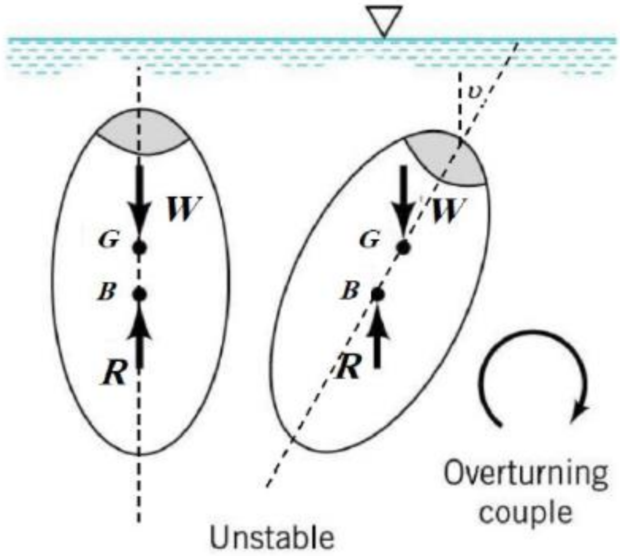
Stable Equilibrium:

A small angular displacement υ or θ from the equilibrium position will generate a moment equals: (W x BG x υ).

The immersed body is considered **Stable** if G is below B, this will generate **righting moment** and the body will tend to return to its **original equilibrium position**.



Unstable Equilibrium:
The immersed body is considered **Unstable** if G is above B, this will generate an **overturning moment** and the body will tend to be in **new equilibrium position**.

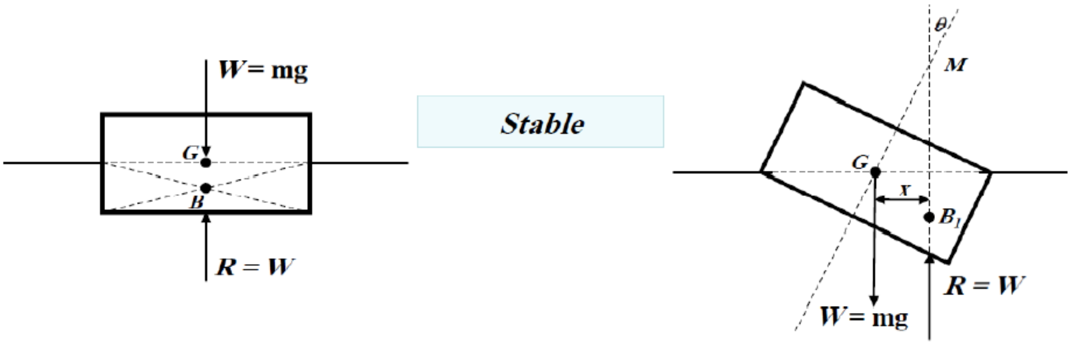


Stability of *Floating* Bodies

Here, the volume of the liquid remains unchanged since F_b=W, but the **shape of this volume changes** and thereby its center of buoyancy will differ.
When the body is displaced through an angle υ or θ the center of buoyancy move from B to B_1 and a turning moment is produced.

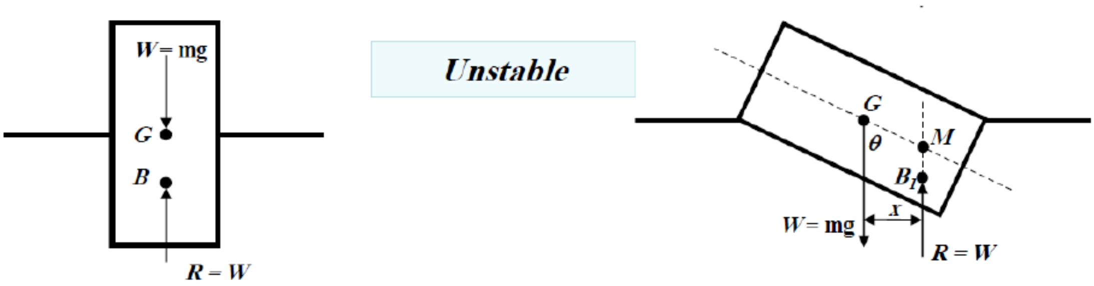
Metacenter (M):
The point at which the line of action of the buoyant force (F_b) **intersects** the original vertical line through G.
So, **Moment Generated is (W x GM x υ).**
GM is known as a **metacentric height**.

Stability:
Stable
If M lies above G, a righting moment is produced, equilibrium is stable and GM is regarded as positive. (GM=+VE)



Unstable

If M lies below G, an overturning moment is produced, equilibrium is unstable and GM is regarded as negative. (GM= -VE).



Neutral:
If M coincides with G, the body is in neutral equilibrium.
Determination of the Position of Metacenter Relative to Centre of Buoyancy:

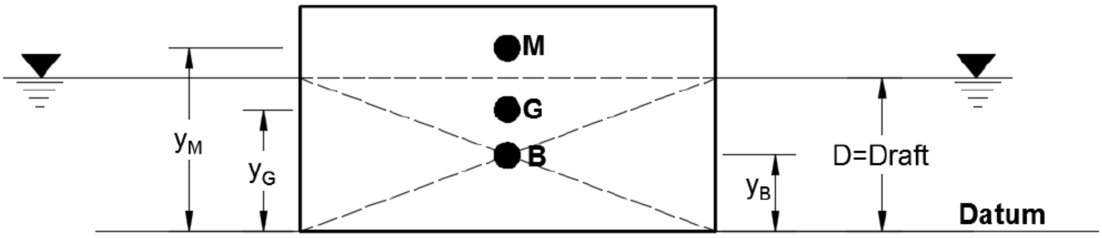
$$BM = \frac{I}{V_{displaced}}$$

I=the smallest moment of inertia of the waterline plane

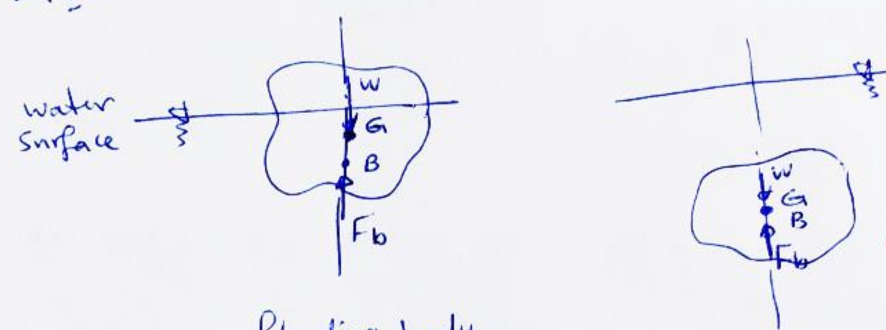
Procedures for Evaluating the Stability of Floating Bodies

1. Determine the position of the floating body (Draft) using the principles of buoyancy (Total Weights = Buoyant Force).
2. Locate the center of buoyance B and compute the distance from some datum to point B (y_B). The bottom of the object is usually taken as a datum.
3. Locate the center of gravity G and compute (y_G) measured from the **same datum**.
4. Determine the shape of the area at the fluid surface (plane view) and compute I for that shape.
5. Compute the displace volume (V_d)
6. Compute BM distance ($BM = I / V_d$).
7. Compute ($y_M = y_B + BM$)
8. If ($y_M > y_G$) >> the body is stable.(GM = +VE)
9. If ($y_M < y_G$) >> the body is unstable.(GM = VE)

Important Note:
If $y_M = y_G$ (GM = 0), this case is called neutral and the object could be **considered stable.**


M
G
B
y_M
y_G
y_B
D=Draft
Datum

* عندما يغمر جسم كلياً أو جزئياً بالماء، فلكي يتزن يجب ان يكون G (center of gravity) على نفس الخط لل (B) center of buoyancy كما في الشكل التالي التوضيحي:-



Floating body Submerged body

* اذا كان هناك تجانس في توزيع الكتلة تماماً فأن G و B تنطبقان

حسب موقع G و B يقسم اتزان الجسم الغاطس إلى ثلاث حالات

① Stable G اسفل B لايوجد تغير بالموقع والجسم يعود الى وضعه الاصلي

② unstable G اعلى B يوجد تغير بالموقع والجسم لايعود إلى وضعه الاصلي

③ Neutral G و B منطبقان يدور الجسم حول نفسه.

الجسم الطافي ايضاً له ثلاث حالات (لان شكل حجم الجزء المغمور يتغير أي مركز الطفو يتغير بتزحزحه عن الوضع الافقي بزاوية θ

meta center: هي النقطة التي يتقاطع فيها خط قوة الطفو قبل وبعد الميلان (M)

meta center height: هي المسافة بين G و M ← ونرمز لها (GM)

meta center radius: هي (BM) تسمى نصف قطر (BM)

① M above G

تتولد عزم يحاول ان يعيد الجسم الى وضعه الاصلي والجسم الطافي
يكون في حالة اتزان (stable)

$GM = +ve$, $GM > 0$

② M below G

الزم يحاول ان يقلب الجسم والجسم الطافي هنا يكون غير متزن
unstable

$GM = -ve$, $GM < 0$

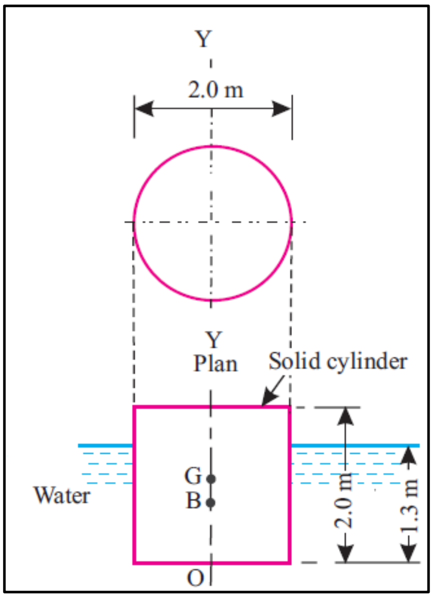
③ M coincide with G
منطبق

الجسم سيكون في موقع جديد بدون ان يتغير شكله لذلك يكون في
حالة اتزان طبيعي Neutral.

$GM = 0$

Problems

1. A solid cylinder 2 m in diameter and 2m high is floating in water with its axis vertical. If the specific gravity of the material of cylinder is 0.65 **find** its metacentric height. **State** also whether the equilibrium is stable or unstable.

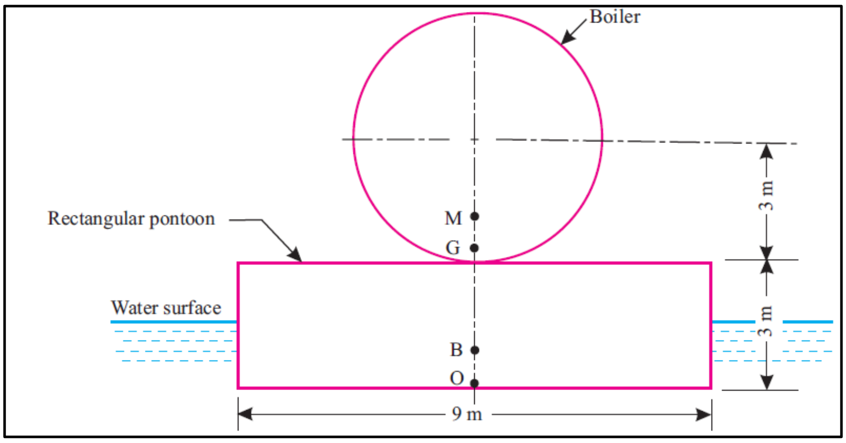


2. Rectangular pontoon 12 m long 9 m wide and 3 m deep weighs 1380 kN and floats in sea water. The pontoon carries on its upper deck a boiler 6 m diameter and weighing 864 kN. The centre of gravity of each unit coincides with geometrical centre of the arrangement and lies in the same vertical line.

(i) What is the metacentric height?

(ii) Is the arrangement stable?

Take specific weight of sea water = 10 kN/m^3



problem (1)

depth of cylinder in water = specific gravity $\times h$

$= 0.65 \times h^{=2}$

$= 1.3^{m}$

distance of center of buoyancy B from O [نصف الجزء المغمور
أو center
[immersion part

$OB = \frac{1.3}{2} = 0.65^{m}$

distance of center of gravity G from O [نصف ارتفاع الجسم
الكلي الطافي والمغمور]

$OG = \frac{2}{2} = 1^{m}$

$BG = OG - OB = 1 - 0.65 = 0.35^{m}$

$BM = \frac{I}{V}$ — I → moment of inertia of the plan of the body about X-X; V → volume of cylinder in water

$$BM = \frac{\frac{\pi}{64} \times 2^{4}}{\frac{\pi}{4} \times 2^{2} \times 1.3} = \frac{0.785\ m^{4}}{4.084\ m^{3}} = 0.192^{m}$$

$GM = BM - BG$

$= 0.192 - 0.35 = -0.158^{m} < 0$

$\therefore$ M is below G and the cyliner is unstable equilibrium

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Problem (2) عوامة مستطيلة تحمل على سطحها غلاية

total weight = 1380 + 864 = 2244 kN
= weight of water displaced.

$$V \text{ of water displaced} = \frac{\text{weight of water displaced}}{\text{specific weight of water}}$$

$$= \frac{2244}{10} = 224.4\ m^3$$

= volume of arrangement under water

$$h = \text{depth of immersion (الغاطس)} = \frac{\text{volume of arrangement under water}}{\text{cross sectional area of pontoon}}$$

$$= \frac{224.4}{9 \times 12} = 2.077\ m$$

$$OB = \frac{2.077}{2} = 1.0385\ m$$

$$BM = \frac{I}{V} = \frac{\frac{1}{12} \times 12 \times 9^3}{224.4} = 3.248\ m$$

$$OM = OB + BM$$
$$= 1.0385 + 3.248 = 4.286\ m$$

$\sum M_O = 0$ [center of gravity لنجد الموقع المشترك لـ (G) بين العوامة والغلاية
arrangement = [boiler & Pontoon]]

$$1380 \times 1.5 + 864 \times 6 = 2244 \times OG \Rightarrow OG = 3.232\ m$$

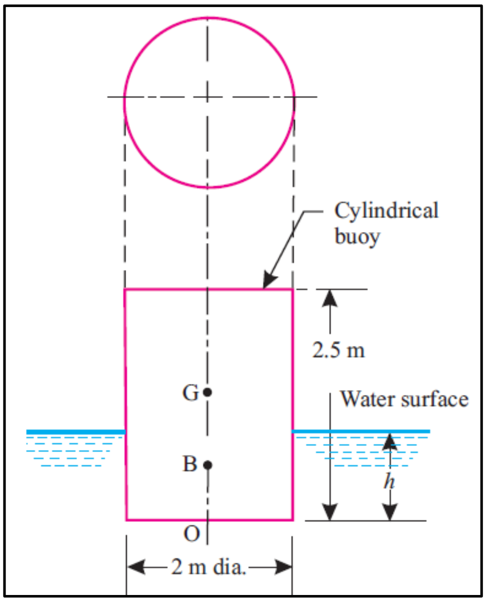
$$GM = OM - OG$$
$$= 4.286 - 3.232 = 1.054\ m$$

$OM > OG$ ∴ M is at a heigher Level than G
∴ arrangement is stable.

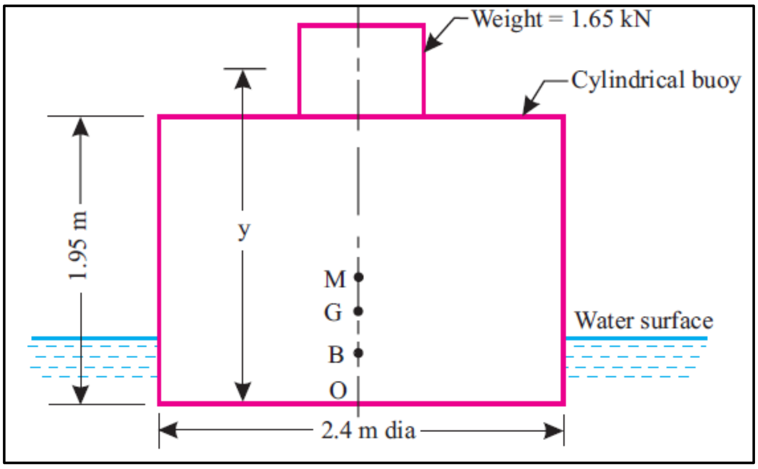
H.W1.

A cylindrical buoy is 2 m in diameter and 2.5m long and weighs 22 kN. The specific weight of sea water is 10.25 kN/m^3. Show that the buoy does not float with its axis vertical? What minimum pull should be applied to a chain attached to the centre of the base to keep the buoy vertical?

Ans. 13.39 kN



H.W2. A buoy having a diameter of 2.4 m and length 1.95 m is floating with its axis vertical in sea water (specific weight = 10 kN/m^3). Its weight is 16.5 kN and a load of 1.65 kN is placed centrally at its top. If the buoy is to remain in stable equilibrium, **find** the maximum permissible height of the centre of gravity of the load above the top of the buoy. **(Ans.0.368 m)**


Weight = 1.65 kN
Cylindrical buoy
1.95 m
y
M
G
B
O
Water surface
2.4 m dia

Buoyancy

When a body is submerged or floating in a static fluid, the resultant force exerted on it by the fluid is called the *buoyancy force*.
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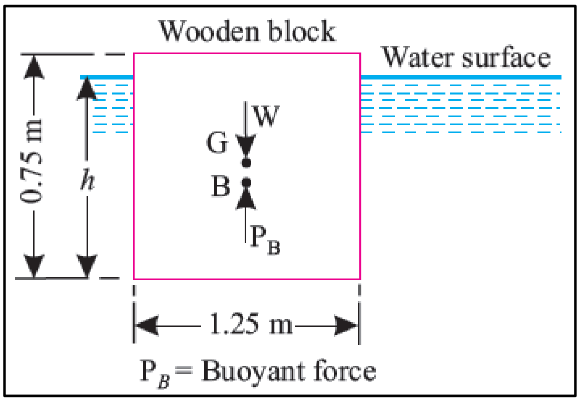
Problems

1. *A wooden block of width 1.25 m, depth 0.75 m and length 3.0 m is floating in water. Specific weight of the wood is 6.4 kN/m*^{3}. ***Find:*** **(*i*)** *Volume of water displaced, and* **(*ii*)** *Position of centre of buoyancy.*
Note: $P_B = F_b$, *G:center of gravity, B: center of buoncy*



Solution:

2. *A wooden block of specific gravity 0.7 and having a size of 2 m $\times$ 0.5m $\times$ 0.25 m is floating in water.* ***Determine*** *the volume of concrete of specific weight 25 kN/m³, that may be placed which will immerse (i) the block completely in water, and (ii) the block and concrete completely in water.*

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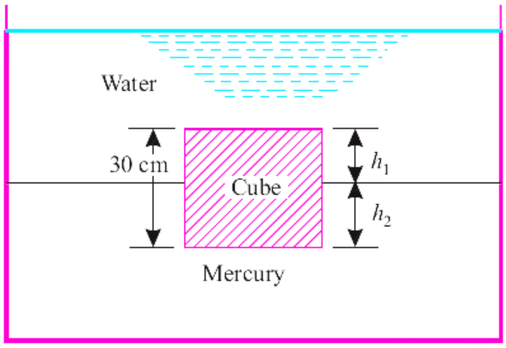
H.W. *A metallic cube 30 cm side and weighing 450 N is lowered into a tank containing a two-fluid layer of water and mercury.* ***Determine*** *the position of block at mercury-water interface when it has reached equilibrium.*



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So, for equilibrium: $F_b = W_{object}$

The equilibrium of a body may be:

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Stability of *a Submerge* Bodies

Stable equilibrium: if when displaced, it returns to its original equilibrium position.

Unstable equilibrium: if when displaced, it returns to a new equilibrium position

Notes:

- In this case (body is fully immersed in water) when the body is tilted, the shape of the displaced fluid doesn't change, so the center of buoyancy remains unchanged relative to the body.
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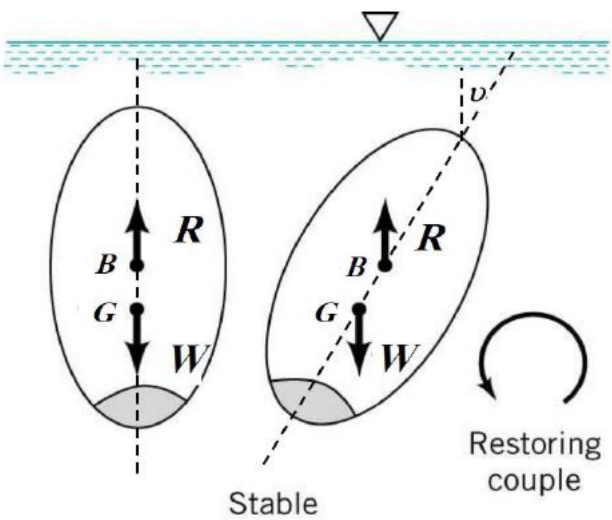
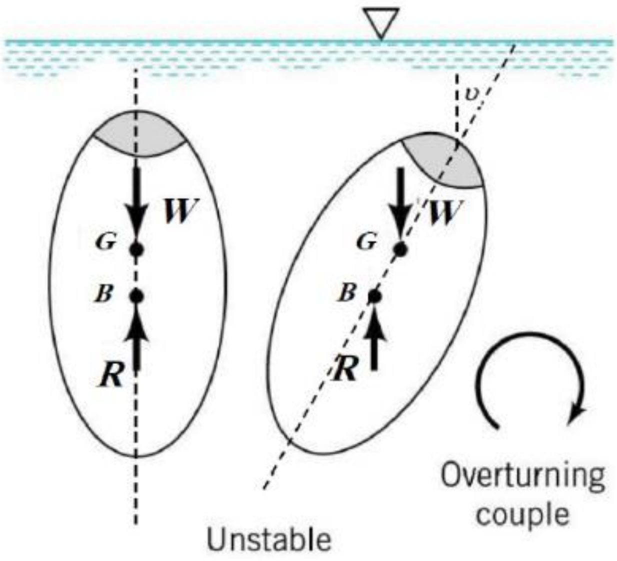


Unstable Equilibrium:
The immersed body is considered **Unstable** if G is above B, this will generate an **overturning moment** and the body will tend to be in **new equilibrium position**.



Stability of *Floating* Bodies

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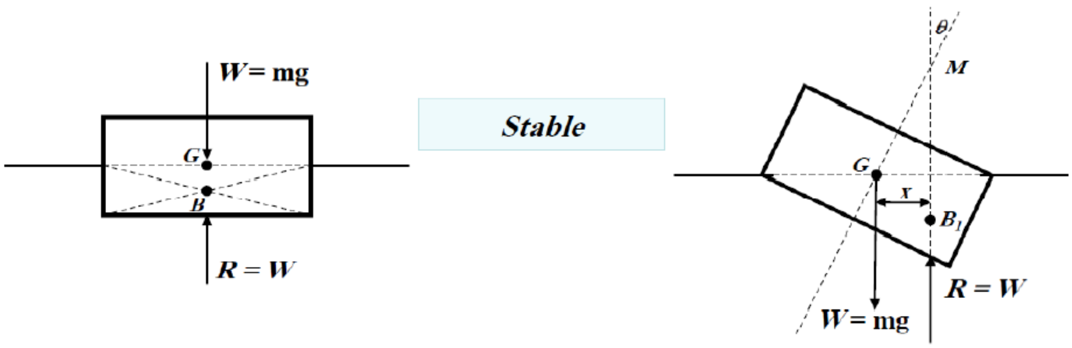
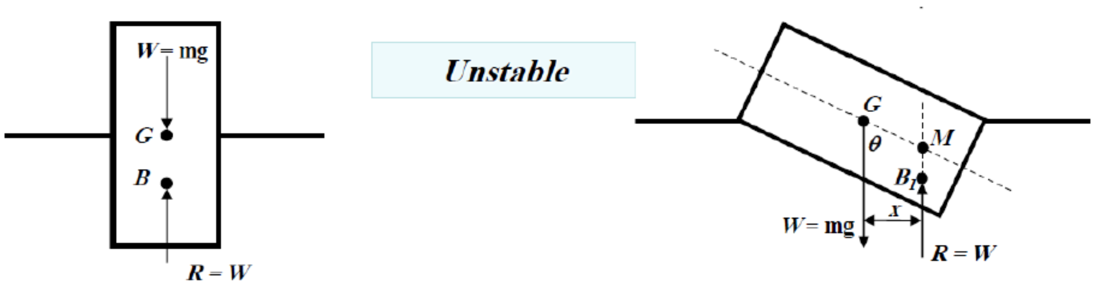


Unstable

If M lies below G, an overturning moment is produced, equilibrium is unstable and GM is regarded as negative. (GM= -VE).



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7. Compute ($y_M = y_B + BM$)
8. If ($y_M > y_G$) >> the body is stable.(GM = +VE)
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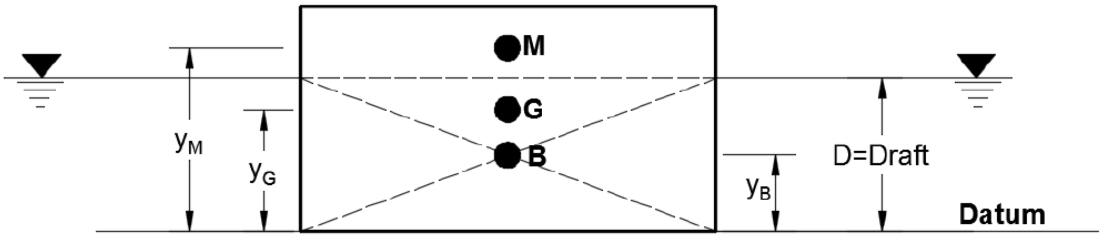
If $y_M = y_G$ (GM = 0), this case is called neutral and the object could be **considered stable.**



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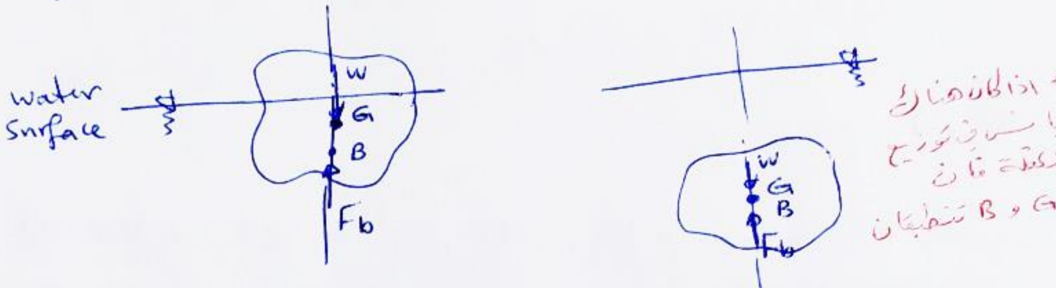

floating body   submerged body

حسب موقع G و B ينقسم اتزان الجسم الغاطس إلى ثلاث حالات

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meta center height: هي المسافة بين G و M ← ورمزها (GM)

meta center radius: هي (BM) تسمى نصف قطر الـ (BM)

① M above G

تتولد عزم يحاول ان يعيد الجسم الى وضعه الاصلي والجسم الطافي
يكون في حالة اتزان (stable)

GM = +ve , GM > 0

② M below G

الزم يحاول ان يقلب الجسم والجسم الطافي هنا يكون غير متزن
unstable

GM = −ve , GM < 0

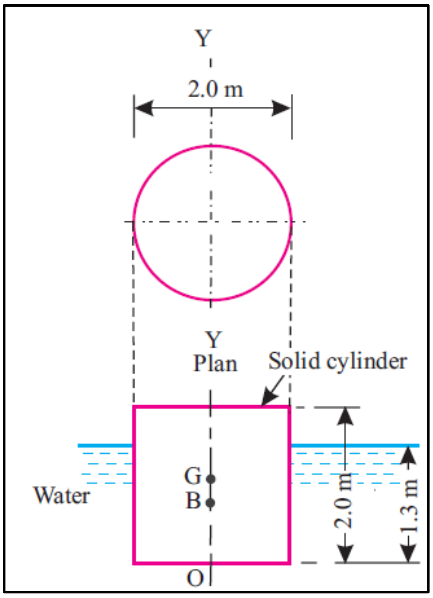
③ M coincide with G
منطبق

الجسم سيكون في موقع جديد بدون ان يتغير شكله لذلك يكون في
حالة اتزان طبيعي
Neutral.

GM = 0

Problems

1. A solid cylinder 2 m in diameter and 2m high is floating in water with its axis vertical. If the specific gravity of the material of cylinder is 0.65 **find** its metacentric height. **State** also whether the equilibrium is stable or unstable.

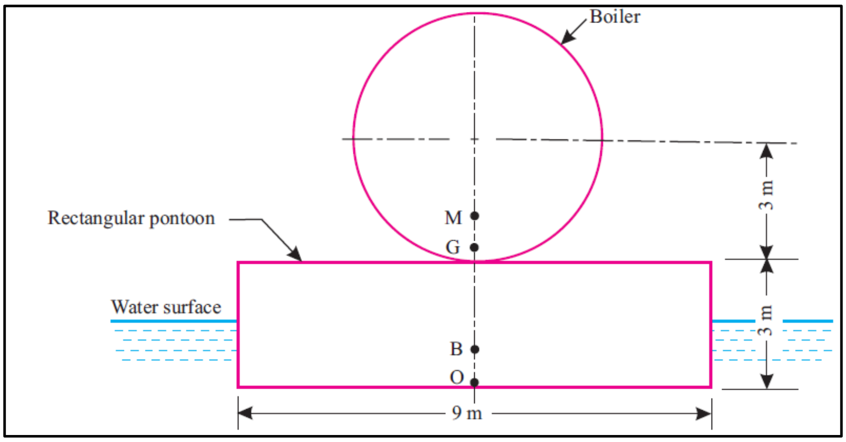


2. Rectangular pontoon 12 m long 9 m wide and 3 m deep weighs 1380 kN and floats in sea water. The pontoon carries on its upper deck a boiler 6 m diameter and weighing 864 kN. The centre of gravity of each unit coincides with geometrical centre of the arrangement and lies in the same vertical line.

(i) What is the metacentric height?

(ii) Is the arrangement stable?

Take specific weight of sea water = 10 kN/m^3



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problem (1)

depth of cylinder in water = specific gravity * h

$= 0.65 \times h$; $h = 2$

$= 1.3^{m}$

distance of center of buoyancy B from O [نصف الجزء المغمور] أو [center of immersion part]

$OB = \frac{1.3}{2} = 0.65^{m}$

distance of center of gravity G from O [نصف ارتفاع الجسم الكلي الطافي المغمور]

$OG = \frac{2}{2} = 1^{m}$

$BG = OG - OB = 1 - 0.65 = 0.35^{m}$

$BM = \frac{I}{V}$; $I \rightarrow$ moment of inertia of the plan of the body about X-Y ; $V \rightarrow$ volume of cylinder in water

$$BM = \frac{\frac{\pi}{64} \times 2^4}{\frac{\pi}{4} \times 2^2 \times 1.3} = \frac{0.785\ m^4}{4.084\ m^3} = 0.192^{m}$$

$GM = BM - BG$

$= 0.192 - 0.35 = -0.158^{m} < 0$

$\therefore$ M is below G and the cyliner is unstable equilibr

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عوامة مستطيلة تحمل على سطحها غلاية

Problem (2)

total weight $= 1380 + 864 = 2244$ kN
$=$ weight of water displaced.

$\forall$ of water displaced $= \dfrac{\text{weight of water displaced}}{\text{specific weight of water}}$

$= \dfrac{2244}{10} = 224.4\ m^3$

$=$ volume of arrangement under water

$h =$ depth of immersion (الغاطس) $= \dfrac{\text{volume of arrangement under water}}{\text{cross. sectional area of pontoon}}$

$= \dfrac{224.4}{9 \times 12} = 2.077^m$

$OB = \dfrac{2.077}{2} = 1.0385^m$

$BM = \dfrac{I}{V} = \dfrac{\frac{1}{12} * 12 * 9^3}{224.4} = 3.248^m$

$OM = OB + BM$
$= 1.0385 + 3.248 = 4.286^m$

$\curvearrowright \Sigma M_O = 0$ [center of gravity لنجد الموقع المشترك لل (G) بين العوامة والغلاية
arrangement = [boiler & Pontoon]]

$1380 \times 1.5 + 864 \times 6 = 2244 * OG \Rightarrow OG = 3.232^m$

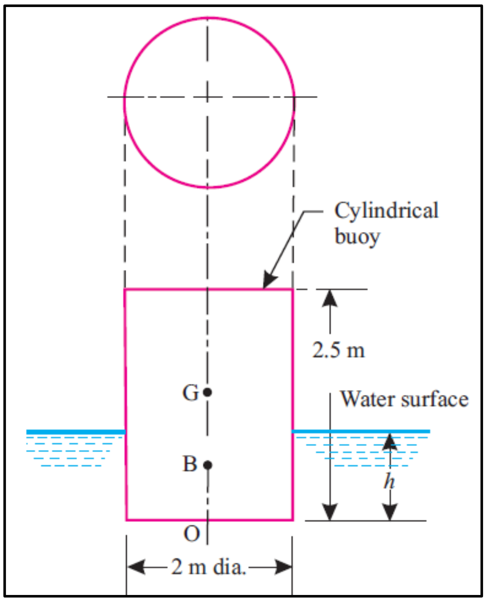
$GM = OM - OG$
$= 4.286 - 3.232 = 1.054^m$

$OM > OG$ $\therefore$ M is at a heigher Level than G
$\therefore$ arrangement is stable.

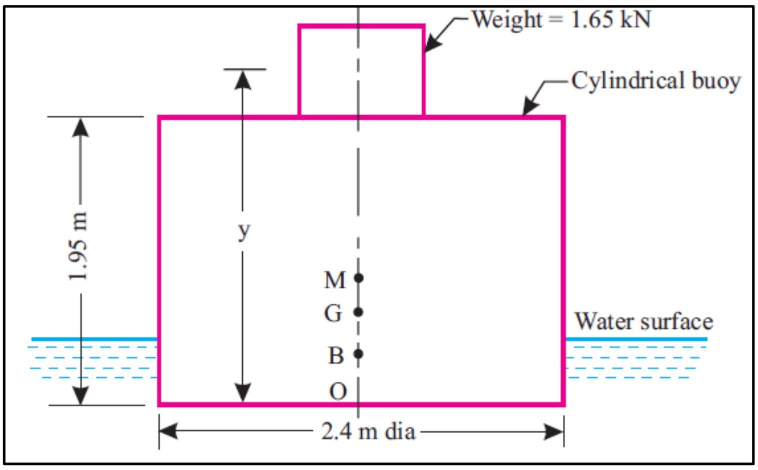
H.W1.

A cylindrical buoy is 2 m in diameter and 2.5m long and weighs 22 kN. The specific weight of sea water is 10.25 kN/m^3. Show that the buoy does not float with its axis vertical? What minimum pull should be applied to a chain attached to the centre of the base to keep the buoy vertical?

Ans. 13.39 kN



H.W2. A buoy having a diameter of 2.4 m and length 1.95 m is floating with its axis vertical in sea water (specific weight = 10 kN/m^3). Its weight is 16.5 kN and a load of 1.65 kN is placed centrally at its top. If the buoy is to remain in stable equilibrium, **find** the maximum permissible height of the centre of gravity of the load above the top of the buoy. **(Ans.0.368 m)**



Chapter 4: Motion of Fluid Particles and Streams

Fluid kinematics is a branch of 'Fluid mechanics' which deals with the study of velocity and acceleration of the particles of fluids in motion and their distribution in space without considering any force or energy involved.

DESCRIPTION OF FLUID MOTION

The motion of fluid particles may be described by the following methods:
1. Langrangian method.
2. Eulerian method.

1. Langrangian Method

In this method, the observer *concentrates on the movement of a single particle*. The path taken by the particle and the changes in its velocity and acceleration are studied.

2. Eulerian Method

In Eulerian method, the observer *concentrates on a point in the fluid system*. Velocity, acceleration and other characteristics of the fluid at that particular point are studied.

TYPES OF FLUID FLOW

Fluids may be *classified* as follows:
1. Steady and unsteady flows
2. Uniform and non-uniform flows
3. One, two and three dimensional flows
4. Rotational and irrotational flows
5. Laminar and turbulent flows
6. Compressible and incompressible flows.

1. Steady and Unsteady Flows

Steady flow. The type of flow in which the fluid characteristics like velocity, pressure, density, etc. at a point *do not change* with time is called *steady flow*. Mathematically, we have:

$$\left(\frac{\partial u}{\partial t}\right)_{x_0, y_0, z_0} = 0; \left(\frac{\partial v}{\partial t}\right)_{x_0, y_0, z_0} = 0; \left(\frac{\partial w}{\partial t}\right)_{x_0, y_0, z_0} = 0$$

$$\left(\frac{\partial p}{\partial t}\right)_{x_0, y_0, z_0} = 0; \left(\frac{\partial \rho}{\partial t}\right)_{x_0, y_0, z_0} = 0; \text{ and so on}$$

where (x_0, y_0, z_0) is a fixed point in a fluid field where these variables are being measured *w.r.t.* time.
Example. *Flow through a prismatic or non-prismatic conduit at a constant flow rate Q m^3/s is steady.*
(A prismatic conduit has a constant size shape and has a velocity equation in the form $u = ax2 + bx + c$, which is independent of time t).

Unsteady flow. It is that type of flow in which the velocity, pressure or density at a point *change* w.r.t. time. Mathematically, we have:

$$\left(\frac{\partial u}{\partial t}\right)_{x_0, y_0, z_0} \neq 0; \left(\frac{\partial v}{\partial t}\right)_{x_0, y_0, z_0} \neq 0; \left(\frac{\partial w}{\partial t}\right)_{x_0, y_0, z_0} \neq 0$$

$$\left(\frac{\partial p}{\partial t}\right)_{x_0, y_0, z_0} \neq 0; \left(\frac{\partial \rho}{\partial t}\right)_{x_0, y_0, z_0} \neq 0; \text{ and so on}$$

Example. *The* flow *in a pipe whose valve is being opened or closed gradually* (*velocity equation is in the form $u = ax^2 + bxt$*).

2. Uniform and Non-uniform Flows

Uniform flow. The type of flow, in which the velocity at any given time *does not change* with respect to space is called *uniform flow*. Mathematically, we have:

$$\left(\frac{\partial V}{\partial s}\right)_{t\,=\,\text{constant}} = 0$$

where, ∂V = Change in velocity, and
∂s = Displacement in any direction.
Example. *Flow through a straight prismatic conduit* (*i.e. flow through a straight pipe of constant diameter*).

Non-uniform flow. It is that type of flow in which the velocity at any given time *changes* with respect to space. Mathematically,

$$\left(\frac{\partial V}{\partial s}\right)_{t\,=\,\text{constant}} \neq 0$$

Example. (*i*) *Flow through a non-prismatic conduit.*
(*ii*) *Flow around a uniform diameter pipe-bend or a canal bend.*

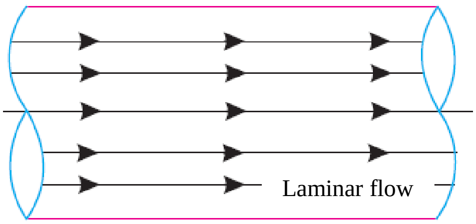
3. Laminar and Turbulent Flows

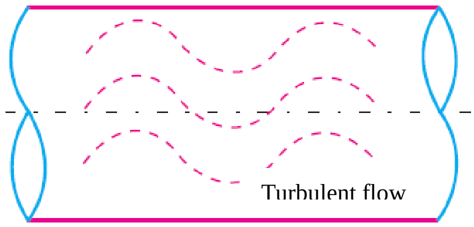
Laminar flow. A laminar flow is one in which *paths taken by the individual particles do not cross one another and move along well defined paths*. This type of flow is also called *stream-line flow or viscous flow*.

Examples. (*i*) *Flow through a capillary tube.*
(*ii*) *Flow of blood in veins and arteries.*
(*iii*) *Ground water flow.*

Turbulent flow. A turbulent flow is that flow in which fluid *particles move in a zig zag way*.

Example. *High velocity flow in a conduit of large size. Nearly all fluid flow problems encountered in engineering practice have a turbulent character*.





Patterns of Flow

Reynolds Number (R_e):
A dimensionless number used to identify the type of flow.

$$R_e = \frac{\text{Inertia Forces}}{\text{Viscous Forces}} = \frac{\rho \times V \times D}{\mu} = \frac{V \times D}{\nu}$$

V=mean velocity (m/s) , D=pipe diameter (m), ρ=fluid density (Kg/m^3)
μ=Dynamic viscosity (Pa.s) , ν=kinematic viscosity (m^2/s)
For flow in pipe: If ($R_e \le 2000$)→→The flow is laminar If ($2000 < R_e \le 4000$)→→The flow is transitional If ($R_e > 2000$)→→The flow is turbulent

4. Compressible and Incompressible Flows

Compressible flow. It is that type of flow in which the *density* (ρ) *of the fluid changes* from point to point (or in other words *density is not constant for this flow*). Mathematically: $\rho \neq$ constant.

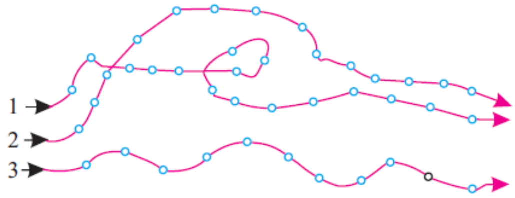
Example. *Flow of gases through orifices, nozzles, gas turbines, etc*.

Incompressible flow. *It is that type of flow in which density is constant for the fluid flow. Liquids are generally considered flowing incompressibly*. Mathematically: ρ = constant. ***Example.*** *Subsonic aerodynamics*.

TYPES OF FLOW LINES

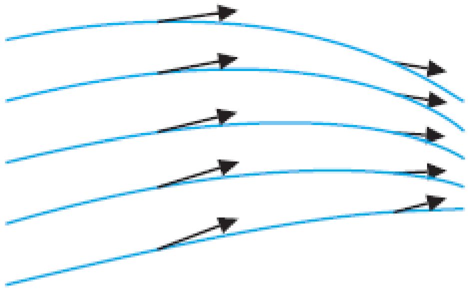
1. Path line

A path line is the *path followed by a fluid particle in motion*. A path line shows the direction of particular particle as it moves ahead.



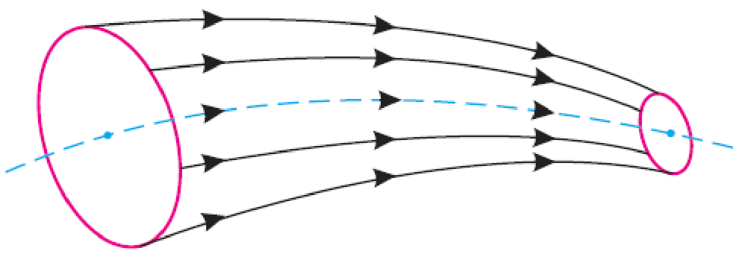
2. Stream line

A ***stream line*** way be defined as *an imaginary line within the flow so that the tangent at any point on it indicates the velocity at that point*



3. Stream Tube

A ***stream tube*** is a *fluid mass bounded by a group of streamlines*. The contents of a stream tube are known as '*current filament*'.



RATE OF FLOW OR DISCHARGE AND MEAN VELOCITY:

Rate of flow (or **discharge**) *is defined as* the *quantity of a liquid flowing per second through a section of pipe or a channel.*

Flow Rate can be measured by one of the following two methods:

1. In terms of mass (Mass Flow Rate, $\dot{m}$):

$$\dot{m} = \frac{\text{Mass of fluid}}{\text{time taken to collect the fluid}} = \frac{dm}{dt} = \rho \times Q \ (\text{Kg/s}).$$

2. In terms of volume (Volume Flow Rate or discharge, Q,):

$$Q = \frac{\text{Volume of Fluid}}{\text{Time}} = \frac{V}{t} \ (m^3/s).$$

This method is the most commonly used method to represents discharge. There is another important way to represents Q:

$$Q = \frac{V}{t} = \frac{\text{Area} \times L}{t} = \text{Area} \times \text{Speed} \rightarrow Q = A \times v \ (m^3/s).$$

CONTINUITY OF FLOW

Matter cannot be created or destroyed (principle of conservation of mass)

Mass entering per unit time = Mass leaving per unit time + Increasing of mass in the control volume per unit time

If the flow is steady, no increase in the mass within the control volume. So,

Mass entering per unit time = Mass leaving per unit time

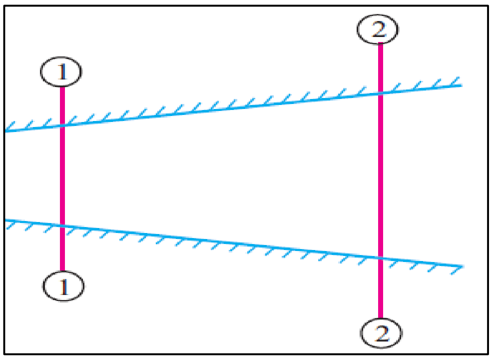
Continuity Equation for Steady Flow and Incompressible Flow:

$$A_1v_1=A_2v_2=Q=constant$$

This equation is a very powerful tool in fluid mechanics and will be used repeatedly throughout the rest of this course.
The following problems clarify the concept of continuity of flow

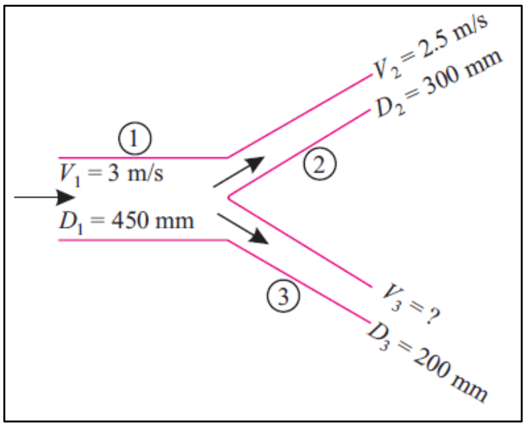
Example 1.

The diameters of a pipe at the sections 1-1and 2-2 are 200 mm and 300 mm respectively. If the velocity of water flowing through the pipe at section 1-1 is 4 m/s, find:
(i) Discharge through the pipe, and
(ii) Velocity of water at section 2-2



Sol.

Example 2.



A pipe (1) 450 mm in diameter branches into two pipes (2 and 3) of diameters 300 mm and 200 mm respectively. If the average velocity in 450 mm diameter pipe is 3 m/s find:

(i) Discharge through 450 mm diameter pipe;

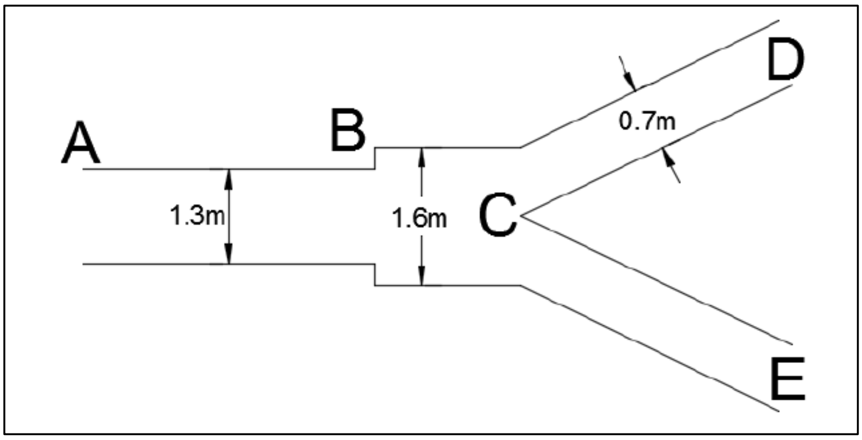
(ii) Velocity in 200 mm diameter pipe if the average velocity in 300 mm pipe is 2.5 m/s.

Sol.

H.W

1.

Water flows through pipe AB 1.3m diameter at speed of 3m/s and passes through a pipe BC of 1.6m diameter. At C the pipe branches. Branch CD is 0.7m in diameter and carries one third of the flow in AB. The velocity in branch CE is 2.7m/s. **Find** the flow rate in AB, the velocity in BC, the velocity in CD and the diameter of CE.



2.

Pipe flow steadily through the piping junction (as shown in the figure) entering section **(1)** at a flow rate of 4.5m^3/hr. The average velocity at section **(2)** is 2.5 m/s. A portion of the flow is diverted through the showerhead 100 holes of 1-mm diameter. Assuming uniform shower flow, estimate the exit velocity from the showerhead holes.