Chapter 5: FLUID DYNAMICS

5.1 INTRODUCTION

The science which deals with the geometry of motion of fluids without reference to the forces causing the motion is known as ***"hydrokinematics"*** (or simply kinematics). Thus, kinematics involves merely the description of the motion of fluids in terms of space-time relationship. The science which deals with the action of the forces in producing or changing motion of fluids is known as ***"hydrokinetics"*** (or simply kinetics). Thus, the study of fluids in motion involves the consideration of both the kinematics and kinetics. The dynamic equation of fluid motion is obtained by applying Newton's second law of motion to a fluid element considered as a free body. The fluid is assumed to be incompressible and non-viscous.

In fluid mechanics the basic equations are: (i) Continuity equation, (ii) Energy equation, and (iii) Impulse momentum equation.

6.2. DIFFERENT TYPES OF HEADS (OR ENERGIES) OF A LIQUID IN MOTION

There are three types of energies or heads of flowing liquids:

1. Potential head or potential energy:
   This is due to configuration or position above some suitable datum line. It is denoted by z.
2. Velocity head or kinetic energy:
   This is due to velocity of flowing liquid and is measured as ($V^2/2g$) where, V is the velocity of flow and g is the acceleration due to gravity (g = 9.81).
3. Pressure head or pressure energy:
   This is due to the pressure of liquid and reckoned as (p/γ)
   where, p is the pressure, and γ is the weight density of the liquid.

Total head/energy:

Total head of a liquid particle in motion is the sum of its potential head, kinetic head and pressure head. Mathematically,

$$\text{Total head, } H = z + \frac{V^2}{2g} + \frac{p}{} \text{ m of liquid}$$

$$\text{Total energy, } E = z + \frac{V^2}{2g} + \frac{p}{} \text{ Nm/kg of liquid}$$

Example 5.1. In a pipe of 90 mm diameter water is flowing with a mean velocity of 2 m/s and at a gauge pressure of 350 kN/m^2. Determine the total head, if the pipe is 8 metres above the datum line. Neglect friction.

6.3. BERNOULLI'S EQUATION

Bernoulli's equation states as follows:

"In an ideal incompressible fluid when the flow is steady and continuous, the sum of pressure energy, kinetic energy and potential (or datum) energy is constant along a stream line." Mathematically,

$$\frac{p}{\,} + \frac{V^2}{2g} + z = \text{constant}$$

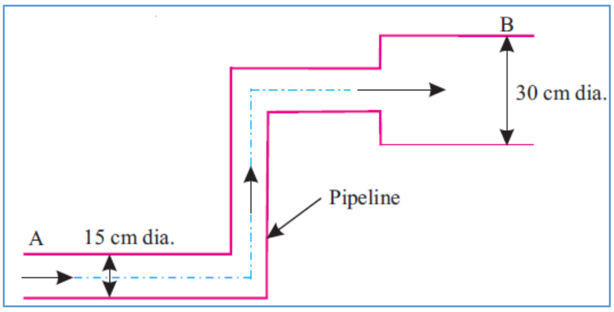
where,

$$\frac{p}{\,} = \text{Pressure energy,}$$

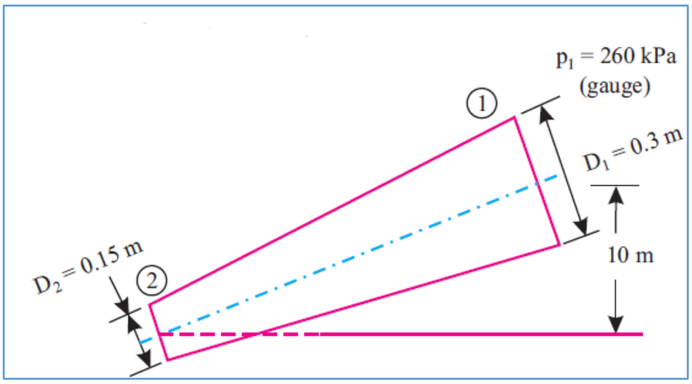
$$\frac{V^2}{2g} = \text{Kinetic energy, and}$$

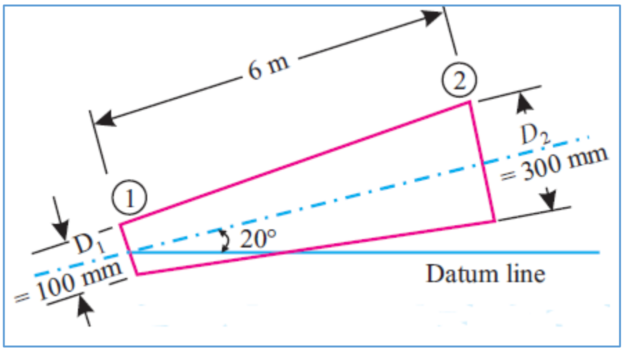
$$z = \text{Datum (or elevation) energy.}$$

Example 5.2. A pipeline is 15 cm in diameter and it is at an elevation of 100 m at section A. At section B it is at an elevation of 107 m and has diameter of 30 cm. When a discharge of 50 litre/sec of water is passed through this pipeline, pressure at A is 35 kPa. The energy loss in pipe is 2m of water. Calculate pressure at B if flow is from A to B.



Example 5.3. Water flows in a circular pipe. At one section the diameter is 0.3 m, the static pressure is 260 kPa gauge, the velocity is 3 m/s and the elevation is 10 m above ground level. The elevation at a section downstream is 0 m, and the pipe diameter is 0.15 m. Find out the gauge pressure at the downstream section. Frictional effects may be neglected. Assume density of water to be 999 kg/m^3.





Example 5.4. A 6m long pipe is inclined at an angle of 20° with the horizontal. The smaller section of the pipe which is at lower level is of 100 mm diameter and the larger section of the pipe is of 300 mm diameter. If the pipe is uniformly tapering and the velocity of water at the smaller section is 1.8 m/s, determine the difference of pressures between the two sections.

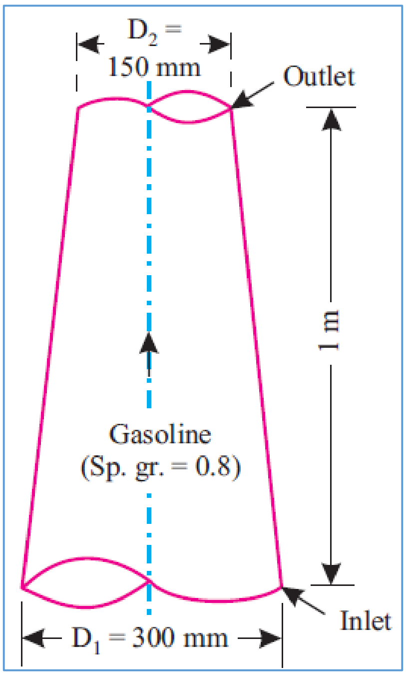
ASSIGNMENT NO.5

1.

Gasoline (sp. gr. 0.8) is flowing upwards a vertical pipeline which tapers from 300 mm to 150 mm diameter. A gasoline mercury differential manometer is connected between 300 mm and 150 mm pipe section to measure the rate of flow. The distance between the manometer tappings is 1 metre and gauge reading is 500 mm of mercury. Find:

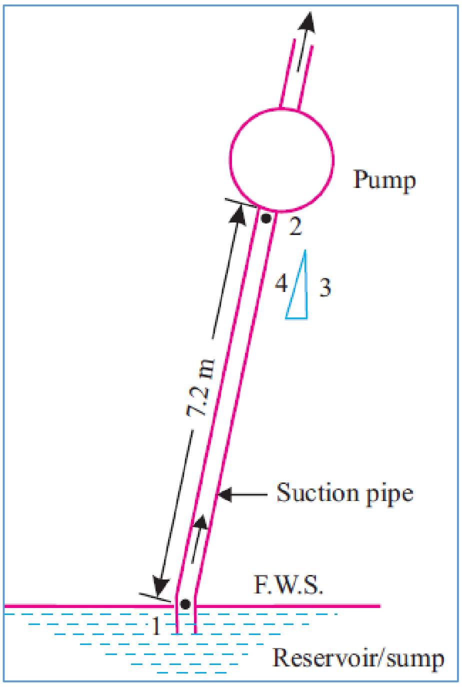
1) Differential gauge reading in terms of gasoline head;
2) Rate of flow.

Neglect friction and other losses between tappings.



2.

The suction pipe of a pump rises at a slope of 3 vertical in 4 along the pipe which is 12 cm in diameter. The pipe is 7.2 m long; its lower end being just below the water surface in the reservoir. For design reasons, it is desirable that pressure at inlet to the pump shall fall to more than 75 kPa below atmospheric pressure. Neglecting friction, determine the maximum discharge that the pump may deliver. Take atmospheric pressure as 101.32 kPa.



5.4. PRACTICAL APPLICATIONS OF BERNOULLI'S EQUATION

Bernoulli equation is one of the most useful equations in fluid mechanics and hydraulics. In addition, it is a statement of the principle of conservation of energy along a streamline.
Bernoulli Equation can be written as following:

$$\frac{P}{\rho g}+\frac{v^2}{2g}+z=H_T=\text{constant}$$

All these terms have a unit of length (m)

- $P/\rho g$ =pressure energy per unit weight=pressure head

$$\text{We know that } P=\rho g h_{pressure} \rightarrow\rightarrow h_{pressure}=\frac{P}{\rho g}\ (m).$$

- $\frac{v^2}{2g}$ = kinetic energy per unit weight = velocity head

$$\text{We know that K.E}=\frac{1}{2}mv^2 \rightarrow \text{divided by weight} \rightarrow \frac{\frac{1}{2}mv^2}{mg}=\frac{v^2}{2g}\ (m).$$

- z = potential energy per unit weight = (potential elevation head)

$$\text{We know that P.E}=mgz \rightarrow \text{divided by weight} \rightarrow \frac{mgz}{mg}=z\ (m).$$

- H_T = total energy per unit weight = total head (m).

By using principle of conservation of energy, we can apply Bernoulli equation between two points (1 and 2) on the streamline:

Total head at (1)=Total head at (2)

$$\frac{P_1}{\rho g}+\frac{v_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{v_2^2}{2g}+z_2$$

But!!, this equation no energy losses (e.g. from friction) or energy gains (e.g. from a pump) along a stream line, so the final form for Bernoulli equation is:

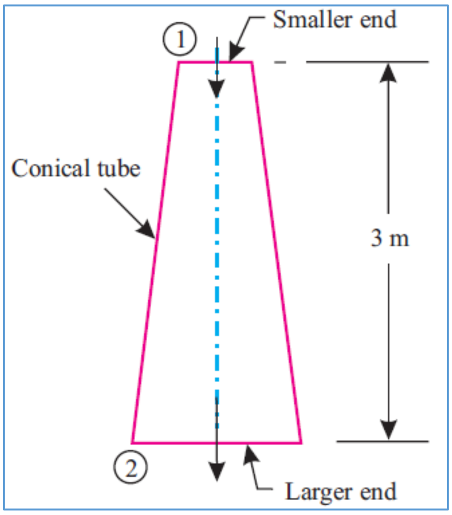
$$\frac{P_1}{\rho g}+\frac{v_1^2}{2g}+z_1+h_P=\frac{P_2}{\rho g}+\frac{v_2^2}{2g}+z_2+h_L+h_T$$

hP=q=Energy supplied by pump per unit weight (m)
hT=w=work done by turbine per unit weight (m)
hL=Total friction losses per unit weight (m)

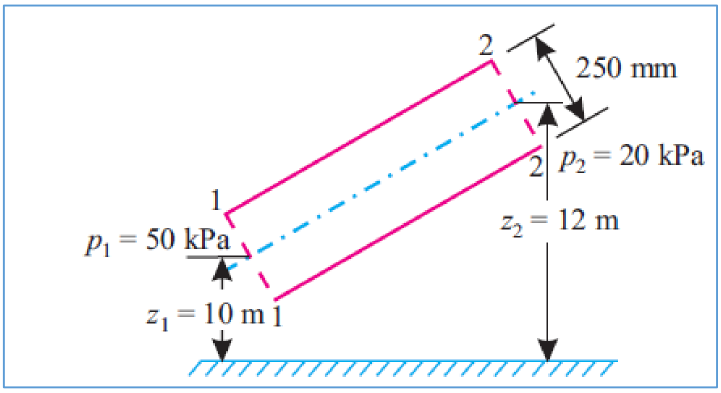
Example 5.5. The following data relate to a conical tube of length 3.0 m fixed vertically with its smaller end upwards and carrying fluid in the downward direction. The velocity of flow at the smaller end = 10 m/s. The velocity of flow at the larger end = 4 m/s.

$$\text{The loss of head in the tube} = \frac{0.4(V_1 - V_2)^2}{2g}$$

where, V_1 and V_2 are velocities at the smaller and larger ends respectively. Pressure head at the smaller end = 4 m of liquid. Determine the pressure head at the larger end.



Example 5.6. In a smooth inclined pipe of uniform diameter 250 mm, a pressure of 50 kPa was observed at section 1, which was at elevation 10 m. At another section 2 at elevation 12 m, the pressure was 20 kPa and the velocity was 1.25 m/s. Determine the direction of flow and the head loss between these two sections. The fluid in the pipe is water. The density of water at 20° C and 760 mm Hg is 998 kg/m^3.



(5.5)

Sol.:

$V_1 = 10\ m/sec$

$V_2 = 4\ m/sec$

$\frac{P_1}{\gamma} = 4^m$ of liquid

loss of head $h_L = \frac{0.4(V_1 - V_2)^2}{2g}$

$= \frac{0.4(10-4)^2}{2\times 9.81} = 0.73\ m$

$\frac{P_2}{\gamma} = ?$

Applying B.E at section ① and ②

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 + h_L$$

$$4 + \frac{10^2}{2\times 9.81} + 3.0 = \frac{P_2}{\gamma} + \frac{4^2}{2\times 9.81} + 0 + 0.73$$

$\therefore \frac{P_2}{\gamma} = 10.55$ m of Liquid.

<u>Sol.</u> <u>(5.6)</u>

$D = 250^{mm}, 0.250\ m$

$P_1 = 50\ kpa = 50 \times 10^3\ pa$

$z_1 = 10^m, \quad z_2 = 12^m$

$P_2 = 20\ kpa, \quad 20 \times 10^3\ pa$

$V_1 = V_2 = 1.25\ m/sec, \quad \rho = 998\ kg/m^3$

Total energy at section 1-1

$$E_1 = \frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = 15.187^m$$

Total energy at section 2-2

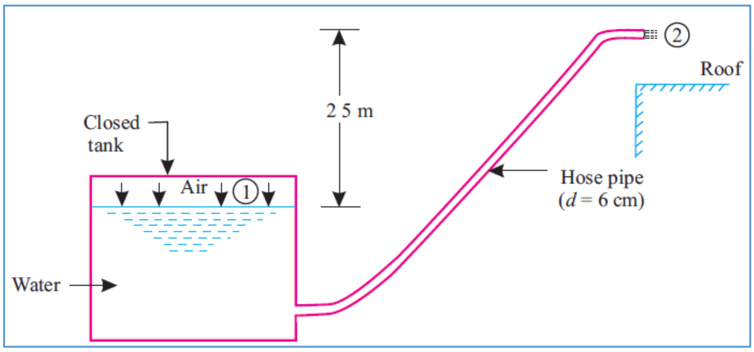
$$E_2 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2 = 14.122\,m$$

$$h_L = E_1 - E_2 = 15.187 - 14.122 = 1.065^m$$

<u>direction</u>

since $E_1 > E_2$ $\therefore$ the direction of flow is from section 1-1 to section 2-2.

Example 5.7. The closed tank of a fire engine is partly filled with water, the air space above being under pressure. A (6 cm) bore connected to the tank discharges on the roof of a building 2.5 m above the level of water in the tank. The friction losses are 45 cm of water. Determine the air pressure, which must be maintained in the tank to deliver 20 litres/sec on the roof.



Sol. (5-7)

diameter of hose pipe = 6 cm = 0.06 m

friction h_f = 45 cm or 0.45 m of water

discharge, Q = 20 l/sec or 0.02 m^3/sec

$$V = \frac{Q}{A} = \frac{0.02}{\frac{\pi}{4}(0.06)^2} = 7.07 \text{ m/sec}$$

Applying B.E between 1 and 2 :-

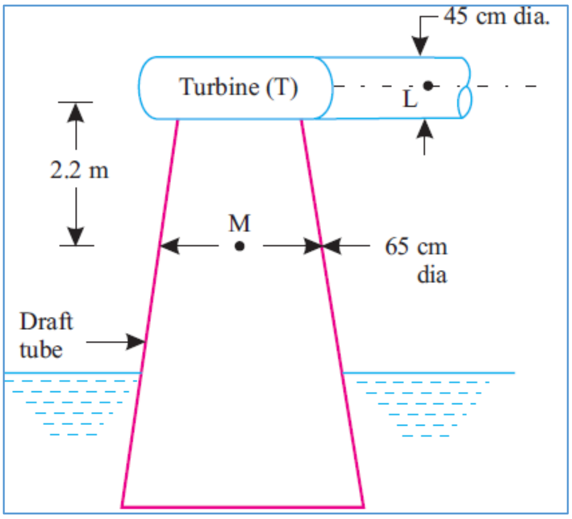
$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 + h_f$$

$$\frac{P_1}{\gamma} + 0 + 0 = 0 + \frac{(7.07)^2}{2 \times 9.81} + 2.5 + 0.45$$

$$\therefore P_1 = 5.497 * 9.81$$

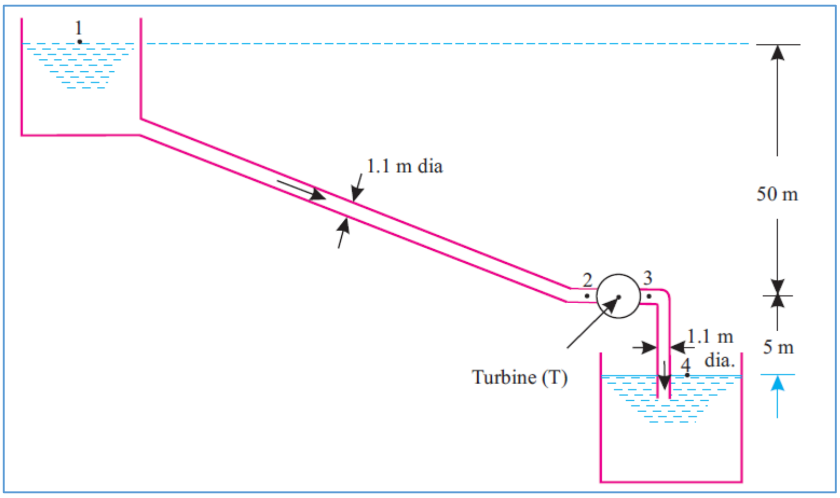
$$= 53.93 \text{ kN/m}^2$$

Example 5.8. A turbine has a supply line of diameter 45 cm and a tapering draft tube. When the flow in the pipe is 0.6 m^3/s the pressure head at point L upstream of the turbine is 35 m and at a point M in the draft tube, where the diameter is 65 cm, the pressure head is – 4.1 m. Point M is 2.2 m below the point L. Determine the power output of the turbine by assuming 92% efficiency.



Example 5.9. This figure shows a pipe connecting a reservoir to a turbine which discharges water to the tail race through another pipe. The head loss between the reservoir and the turbine is 8 times the kinetic head in the pipe and that from the turbine to the tail race is 0.4 times the kinetic head in the pipe. The rate of flow is 1.2 m^3/s and the pipe diameter in both cases is 1.1 m. Determine:

1. The pressure at the inlet and exit of the turbine.
2. The power generated by the turbine.



Sol: (5.8)

$$V_L = \frac{Q}{A_L} = \frac{0.6}{\frac{\pi}{4}(0.45)^2} = 3.77 \text{ m/sec}$$

$$V_M = \frac{Q}{A_M} = \frac{0.6}{\frac{\pi}{4}(0.65)^2} = 1.81 \text{ m/sec}$$

Applying B.E to point L and M

$$\frac{P_L}{\gamma} + \frac{V_L^2}{2g} + Z_L = \frac{P_M}{\gamma} + \frac{V_M^2}{2g} + Z_M + H_T$$

$$35 + \frac{(3.77)^2}{2*9.81} + 2.2 = -4.1 + \frac{(1.81)^2}{2*9.81} + 0 + H_T$$

$$H_T = 41.86 \text{ m}$$

power output of the turbine

$$P = \gamma Q H_T * \eta$$

$$= 9.81 * 0.6 * 41.86 * 0.92$$

$$= 226.68 \text{ kW}$$

Sol. (5-9)

Diameter = 1.1 m

$Q = 1.2\ m^3/sec$

$h_f(1-2) = 8 * \frac{v^2}{2g}$, $h_f(3-4) = 0.4 * \frac{v^2}{2g}$

$V = \frac{Q}{A} = \frac{1.2}{\frac{\pi}{4}(1.1)^2} = 1.263\ m/sec$

$V_2 = V_3$ (Same diameter of pipe).

$V_1 = V_4 = 0$, $P_1 = P_4 = 0$ (atmospheric pressure).

B.E between 1 and 2

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 + 8\frac{V_2^2}{2g}$$

$$0 + 0 + 50 = \frac{P_2}{\gamma} + \frac{(1.263)^2}{2 \times 9.81} + 0 + 8\frac{(1.263)^2}{2 \times 9.81}$$

$\frac{P_2}{\gamma} = 49.27$ m of water

$= 483.34\ KN/m^2$ (Kpa)

B.E between 3 and 4

$$\frac{P_3}{\gamma} + \frac{V_3^2}{2g} + Z_3 = \frac{P_4}{\gamma} + \frac{V_4^2}{2g} + Z_4 + 0.4\frac{V_3^2}{2g}$$

$$\frac{P_3}{\gamma} + \frac{(1.263)^2}{2 \times 9.81} + 5 = 0 + 0 + 0 + 0.4\frac{(1.263)^2}{2 \times 9.81}$$

$\therefore \frac{P_3}{\gamma} = -5.049$ m of water $= -49.53$ KPa.

B.E between 2 and 3

$\frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 = \frac{P_3}{\gamma} + \frac{V_3^2}{2g} + Z_3 + H_T \Rightarrow H_T = 54.32$ m of water

$P = \gamma \cdot Q \cdot H_T = 9.81 \times 1.2 \times 54.32 = 639.46$ KW

5.4. PRACTICAL APPLICATIONS OF BERNOULLI'S EQUATION

Although Bernoulli's equation is applicable in all problems of* incompressible flow where there is involvement of energy considerations but here we shall discuss its applications in the following measuring devices:

1. Venturimeter
2. Orificemeter
3. Pitot tube.

5.4.1. Venturimeter

A venturimeter is one of the most important practical applications of Bernoulli's theorem. *It is an instrument used to measure the rate of discharge in a pipeline and is often fixed permanently at different sections of the pipeline to know the discharges there.*

Types of venturimeters:

Venturimeters may be classified as follows:
1. Horizontal venturimeters.
2. Vertical venturimeters.
3. Inclined venturimeters.

5.5.1.1. Horizontal venturimeters

A venturimeter consists of the following three parts:

i. A short converging part,
ii. Throat, and
iii. Diverging part.

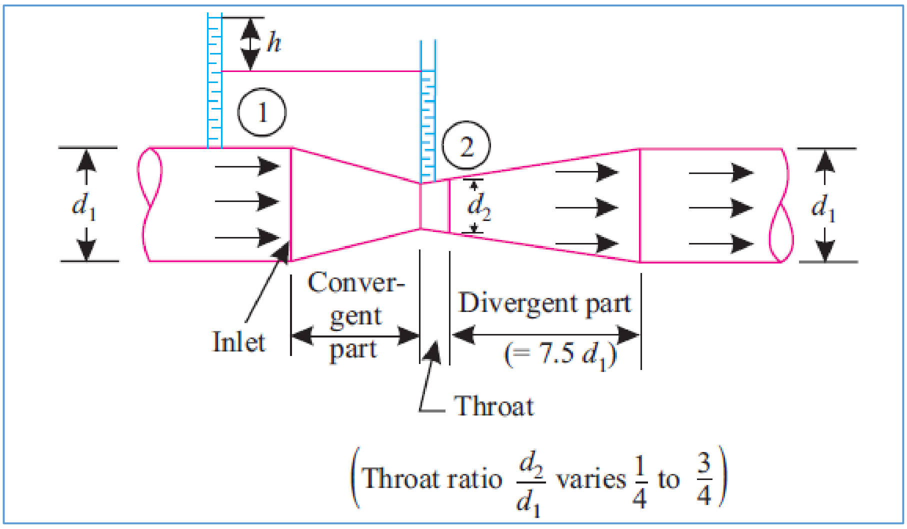
This figure shows a venturimeter fitted in horizontal pipe through which a fluid is flowing.

Let, D_1 = Diameter at inlet or at section 1,

A_1 = Area at inlet $\left(\frac{\pi}{4} d_1^2\right)$

p_1 = Pressure at section 1,

V_1 = Velocity of fluid at section 1, and D_2, A_2, p_2, and V_2 are the corresponding values at section 2.



Applying Bernoulli's equation at sections 1 and 2, we get:

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2 \qquad \ldots(i)$$

Here, $z_1 = z_2$ ... since the pipe is horizontal.

$$\therefore \quad \frac{p_1}{w} + \frac{V_1^2}{2g} = \frac{p_2}{w} + \frac{V_2^2}{2g}$$

or,

$$\frac{p_1 - p_2}{w} = \frac{V_2^2}{2g} - \frac{V_1^2}{2g} \qquad \ldots(ii)$$

But, $\frac{p_1 - p_2}{w}$ = Difference of pressure heads at sections 1 and 2 and is equal to h.

i.e., $$\frac{p_1 - p_2}{w} = h$$

Substituting this value of $\frac{p_1 - p_2}{w}$ in eqn. (*ii*), we get:

$$h = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$$

Applying continuity equation at sections 1 and 2, we have:

$$A_1V_1 = A_2V_2 \quad \text{or} \quad V_1 = \frac{A_2V_2}{A_1}$$

Substituting the value of V_1 in eqn. (*iii*), we get:

$$h = \frac{V_2^2}{2g} - \frac{\left(\frac{A_2V_2}{A_1}\right)^2}{2g} = \frac{V_2^2}{2g}\left(1 - \frac{A_2^2}{A_1^2}\right)$$

or, $$h = \frac{V_2^2}{2g}\left(\frac{A_1^2 - A_2^2}{A_1^2}\right) \quad \text{or} \quad V_2^2 = 2gh\left(\frac{A_1^2}{A_1^2 - A_2^2}\right)$$

or, $$V_2 = \sqrt{2gh\left(\frac{A_1^2}{A_1^2 - A_2^2}\right)} = \frac{A_1}{\sqrt{A_1^2 - A_2^2}}\sqrt{2gh}$$

$\therefore$ $$\text{Discharge, } Q = A_2V_2 = A_2\frac{A_1}{\sqrt{A_1^2 - A_2^2}} \times \sqrt{2gh}$$

or, $$Q = \frac{A_1A_2}{\sqrt{A_1^2 - A_2^2}} \times \sqrt{2gh}$$

or, $$Q = C\sqrt{h}$$

where, C = constant of venturimeter

$$= \frac{A_1A_2}{\sqrt{A_1^2 - A_2^2}}\sqrt{2g}$$

The discharge equation gives the discharge under ideal conditions and is called *theoretical discharge*. Actual discharger (Q_{act}) which is less than the theoretical discharge ($Q_{th.}$) is given by:

$$\boxed{Q_{act} = C_d \times \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \times \sqrt{2gh}}$$

C_d = Co-efficient of venturimeter (or co-efficient of discharge) and its value is less than unity (varies between 0.96 and 0.98)

* Due to variation of C_d venturimeters are not suitable for very low velocities.

Value of '*h*' by differential U-tube manometer:

Case. I. *Differential manometer containing a liquid heavier than the liquid flowing through the pipe.*

Let, S_{hl} = Sp. gravity of heavier liquid,
S_p = Sp. gravity of liquid flowing through pipe, and
y = Difference of the heavier liquid column in *U*-tube.

$$h = y\left[\frac{S_{hl}}{S_p} - 1\right]$$

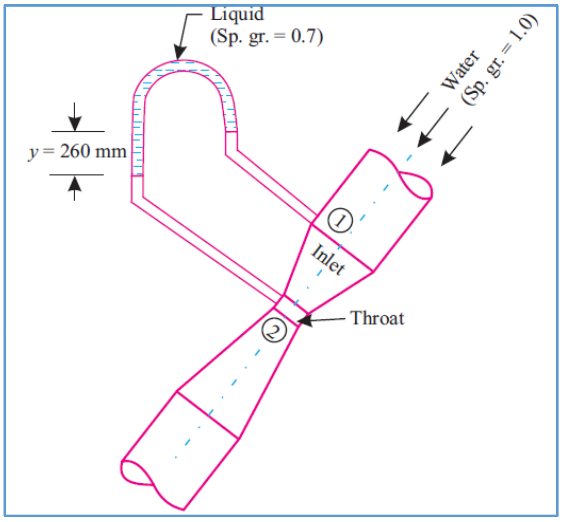
Case. II. *Differential manometer containing a liquid lighter than the liquid flowing through the pipe.*

Let, S_{ll} = Sp. gravity of lighter liquid,
S_p = Sp. gravity of liquid flowing through pipe, and
y = Difference of lighter liquid column in *U*-tube.

$$h = y\left[1 - \frac{S_{ll}}{S_p}\right]$$

Example 5.10. *A horizontal venturimeter with inlet diameter 200 mm and throat diameter 100 mm is used to measure the flow of water. The pressure at inlet is 0.18 N/mm^2 and the vacuum pressure at the throat is 280 mm of mercury. Find the rate of flow. The value of Cd may be taken as 0.98.*

Example 5.12. *Determine the rate of flow of water through a pipe of 300 mm diameter placed in an inclined position where a venturimeter is inserted, having a throat diameter of 150 mm. The difference of pressure between the main and throat is measured by a liquid of sp. gravity 0.7 in an inverted U- tube which gives a*



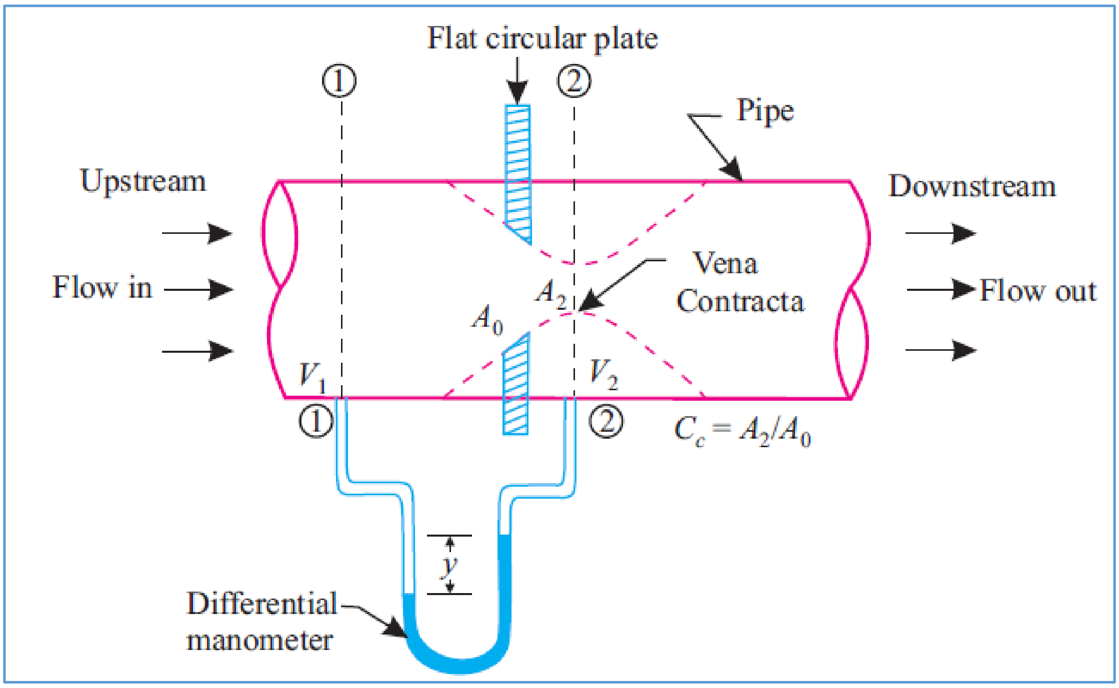
reading of 260 mm. The loss of head between the main and throat is 0.3 times the kinetic head of the pipe.

5.4.2. Orificemeter

Orificemeter or orifice plate is *a device* (*cheaper* than a venturimeter) *employed for measuring the discharge of fluid through a pipe*. It also *works on the same principle of a venturimeter*.

It consists of a flat circular plate having a circular sharp edged hole (called orifice) concentric with the pipe. The diameter of the orifice may vary from 0.4 to 0.8 times the diameter of the pipe but its value is generally chosen as 0.5. A differential manometer is connected at section (1) which is at a distance of 1.5 to 2 times the pipe diameter upstream from the orifice plate, and at section (2) which is at a distance of about half the diameter of the orifice from the orifice plate on the downstream side.

Let, $A1$ = Area of pipe at section (1),
$V1$ = Velocity at section (1),
$p1$ = Pressure at section (1), and
$A2$ $V2$ and $p2$ are corresponding values at section (2).



Applying Bernoulli's equation at sections (1) and (2), we get:

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2$$

or, $$\left(\frac{p_1}{w} + z_1\right) - \left(\frac{p_2}{w} + z_2\right) = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$$

or, $$h = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$$

$$\left[\because h = \left(\frac{p_1}{w} + z_1\right) - \left(\frac{p_2}{w} + z_2\right) = \text{differential head}\right]$$

or, $$\frac{V_2^2}{2g} = h + \frac{V_1^2}{2g} \qquad ...(i)$$

or, $$V_2 = \sqrt{2g\left(h + \frac{V_1^2}{2g}\right)} = \sqrt{2gh + V_1^2}$$

Now, section (2) is at *vena contracta* and A_2 represents the area at vena contracta. If A_0 is the area of orifice, then we have:

$$C_c = \frac{A_2}{A_0}$$

(where, C_c = co-efficient of contraction)

$$\therefore \quad A_2 = A_0 C_c \qquad ...(ii)$$

Using continuity equation, we get:

$$A_1 V_1 = A_2 V_2 \quad \text{or} \quad V_1 = \frac{A_2 V_2}{A_1}$$

or, $$V_1 = \frac{A_0 C_c V_2}{A_1} \qquad ...(iii)$$

Substituting the value of V_1 in eqn. (*i*), we get:

$$V_2 = \sqrt{2gh + \frac{A_0^2 . C_c^2 . V_2^2}{A_1^2}}$$

or, $$V_2^2 = 2gh + \left(\frac{A_0}{A_1}\right)^2 \cdot C_2^2 \cdot V_2^2$$

or, $$V_2^2\left[1-\left(\frac{A_0}{A_1}\right)^2 C_2^2\right] = 2gh$$

$$\therefore \quad V_2 = \frac{\sqrt{2gh}}{\sqrt{1-(A_0/A_1)^2 C_c^2}}$$

$\therefore$ The discharge, $Q = A_2V_2 = A_0.C_c.V_2$

[$\because A_2 = A_0 . C_c$... as above {eqn. (*ii*)}]

$$= A_0C_c\frac{\sqrt{2gh}}{\sqrt{1-(A_0/A_1)^2 C_c^2}} \qquad \ldots(iv)$$

The above expression is simplified by using,

$$C_d = C_c\frac{\sqrt{1-(A_0/A_1)^2}}{\sqrt{1-(A_0/A_1)^2 C_c^2}}$$

(where, C_d = co-efficient of discharge)

$$C_c = C_d\frac{\sqrt{1-(A_0/A_1)^2 C_c^2}}{\sqrt{1-(A_0/A_1)^2}}$$

Substituting this value of C_c in eqn. (*iv*), we get:

$$Q = A_0 \cdot C_d\frac{\sqrt{1-(A_0/A_1)^2 C_c^2}}{\sqrt{1-(A_0/A_1)^2}} \times \frac{\sqrt{2gh}}{\sqrt{1-(A_0/A_1)^2 C_c^2}}$$

$$= \frac{C_d \cdot A_0\sqrt{2gh}}{\sqrt{1-(A_0/A_1)^2}} = \frac{C_d \cdot A_0 \cdot A_1\sqrt{2gh}}{\sqrt{A_1^2 - A_0^2}}$$

i.e., $$Q = C_d\frac{A_0 \cdot A_1\sqrt{2gh}}{\sqrt{A_1^2 - A_0^2}}$$

It may be noted that *Cd* (co-efficient of discharge) of an orifice is *much smaller than that of a venturimeter.*

Example 5.13. *Water flows at the rate of 0.015 m^3/s through a 100 mm diameter orifice used in a 200 mm pipe. What is the difference of pressure head between the upstream section and the vena contracta section? Take co-efficient of contraction Cc = 0.60 and Cv = 1.0.*

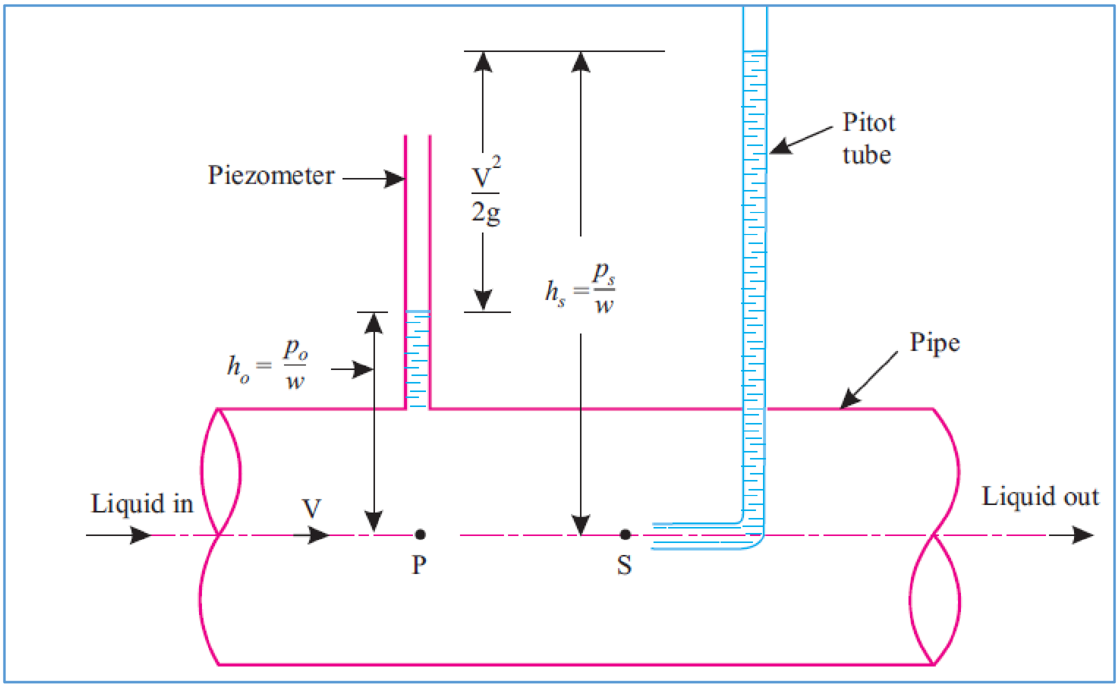
Assignment:

1. *A 200 mm × 100 mm venturimeter is provided in a vertical pipe carrying water, flowing in the upward direction. A differential mercury manometer connected to the inlet and throat gives a reading of 220 mm. Find the rate of flow. Assume Cd = 0.98.*
2. *Write down the advantages and disadvantages of using orificemeter over a venturimeter.*

5.4.3. Pitot Tube

Pitot tube *is one of the most accurate devices for velocity measurement.* It works on the *principle that if the velocity of flow at a point becomes zero, the pressure there is increased due to conversion of kinetic energy into pressure.*

It consists of a glass tube in the form of a 90° bend of short length open at both its ends. It is placed in the flow with its bent leg directed upstream so that a ***stagnation point*** is created immediately in front of the opening. The kinetic energy at this point gets converted into pressure energy causing the liquid to rise in the vertical limb, to a height equal to the stagnation pressure.



Applying Bernoulli's equation between stagnation point (S) and point (P) in the undisturbed flow at the same horizontal plane, we get:

$$\frac{p_0}{w} + \frac{V^2}{2g} = \frac{p_s}{w} \quad \text{or} \quad h_0 + \frac{V^2}{2g} = h_s$$

or,

$$V = \sqrt{2g\,(h_s - h_0)} \quad \text{or} \quad \sqrt{2g\,\Delta h} \qquad \text{...(1)}$$

where,
p_0 = Pressure at point 'P', *i.e.* static pressure,
V = Velocity at point 'P', *i.e.* free flow velocity,
p_s = Stagnation pressure at point 'S', and
Δh = Dynamic pressure
= Difference between stagnation pressure head (h_s) and static pressure head (h_0).

The *height of liquid* rise in the Pitot tube indicates *the stagnation head*. The static pressure head may be measured separately with a piezometer.

If a differential manometer is connected to the tubes of a Pitot static tube it will measure the dynamic pressure head ($v^2/2g$).
If y is the manometric difference, then

$$\Delta h = y\left(\frac{S_m}{S} - 1\right)$$

where,
S_m = Specific gravity of manometric liquid, and
S = Specific gravity of the liquid flowing through the pipe.

When a Pitot tube is placed in the fluid-stream the flow along its outer surface gets accelerated and causes the static pressure to decrease. Also the stem, which is perpendicular to the flow direction, tends to produce an excess pressure head. In order to take these effects into account the above equation (eq.1) is modified to give the actual velocities as:

$$V = C\sqrt{2g\Delta h}$$

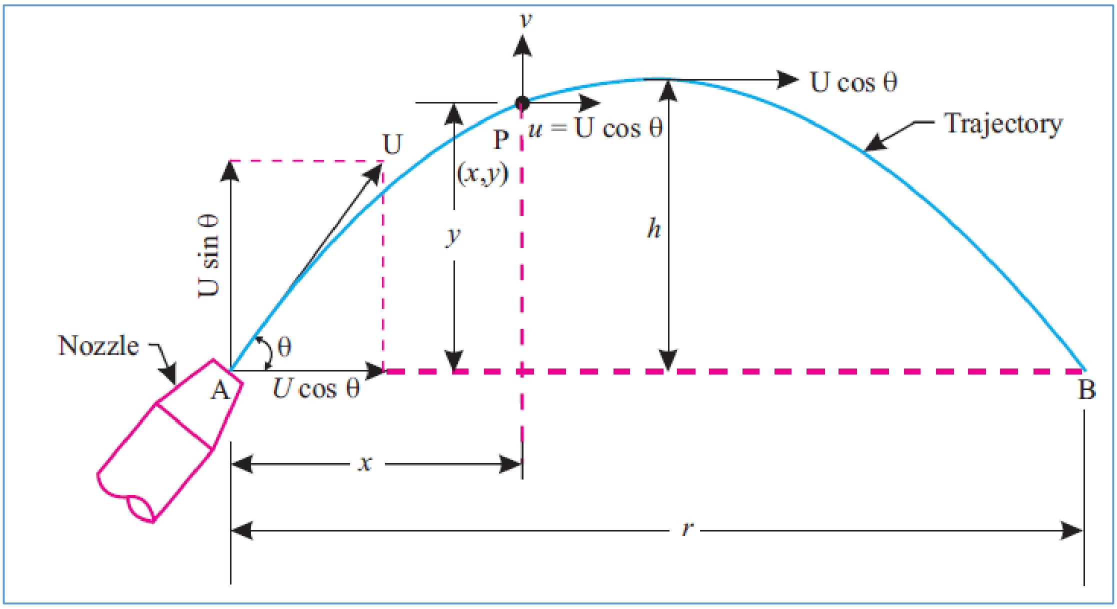
where, C = A connective coefficient which takes into account the effect of stem and bent leg.
The most commonly used form of Pitot static tube known as the Prandle-Pitot-tube is so designed that the effect of stem and bent leg cancel each other, i.e., C = 1.

Example 5.14. *A submarine fitted with a Pitot tube moves horizontally in sea. Its axis is 12 m below the surface of water. The Pitot tube fixed in front of the submarine and along its axis is connected to the two limbs of a U-tube containing mercury, the reading of which is found to be 200 mm. Find the speed of the submarine. Take the specific gravity of sea water = 1.025 times fresh water.* effect of stem and bent leg cancel each other, *i.e.*, $C = 1$.

5.5 FREE LIQUID JET

Refer to figure below. A jet of liquid issuing from the nozzle in atmosphere is called a **free liquid jet**. The parabolic path traversed by the liquid jet under the action of gravity is known as **trajectory**.

Let the jet A make an angle θ with the horizontal direction. If U is the velocity of the water jet, then U cos θ and and U sin θ are the horizontal and vertical components of this velocity respectively. Consider another point P(x, y) on the centre line of the jet.



Let, u = Velocity of the jet at point P in X- direction,
v = Velocity of the jet at point P in Y-direction, and
t = Time taken by a liquid particle to reach from A to P.

Then, $x = u \times t = U\cos\theta \times t$ (where, $u = U\cos\theta$) ...(*i*)

$$y = U\sin\theta \times t - \frac{1}{2}gt^2 \quad ...(ii)$$

(It may be noted that horizontal component of velocity U is $U\cos\theta$ which *remains constant* whereas the vertical component $U\sin\theta$ is *affected by gravity*.)

From eqn. (*i*) we have, $t = \dfrac{x}{U\cos\theta}$

Substituting the value of t in eqn. (*ii*), we get:

$$y = U\sin\theta \times \frac{x}{U\cos\theta} - \frac{1}{2}g \times \frac{x^2}{U^2\cos^2\theta}$$

$$= x\tan\theta - \frac{gx^2}{2U^2\cos^2\theta}$$

$$y = x\tan\theta - \frac{gx^2\sec^2\theta}{2U^2} \qquad \left(\because \frac{1}{\cos^2\theta} = \sec^2\theta\right)$$

This is the equation of *a parabola.*

(*i*) Maximum height attained by the jet, h:

Using the relation:

$$V_2^2 - V_1^2 = -2gh \text{ (}- ve\text{ sign is used as the particle is moving } upward\text{)}$$

where, V_1 = Initial vertical component = U sin θ, and

V_2 = 0 at the highest point.

$$\therefore \quad 0 - (U\sin\theta)^2 = -2gh$$

or

$$h = \frac{U^2\sin^2\theta}{2g}$$

(*ii*) Time of flight, T:

Time of flight is the time taken by the fluid particle in reaching from *A* to *B* (Fig. 6.41). From eqn. (*ii*), we have:

$$y = U\sin\theta \times t - \frac{1}{2}gt^2$$

When the particle reaches the point B, y = 0, t = T
Putting these values in the above equation, we get:

$$0 = U\sin\theta \times T - \frac{1}{2}g \times T^2$$

or,

$$T = \frac{2U\sin\theta}{g}$$

Time taken to reach the highest point, $T' = \frac{T}{2} = \frac{2U\sin\theta}{2g} = \frac{U\sin\theta}{g}$

i.e.,

$$T' = \frac{U\sin\theta}{g}$$

(*iii*) Horizontal range of the jet, *r*:

The range (*r*) of the jet is the total *horizontal distance travelled by the fluid particle.*

Then *r*, (*i.e.*, distance AB) = Velocity component in direction × time taken by the particle to reach from *A* to *B*

$$= U\cos\theta \times T = U\cos\theta \times \frac{2U\sin\theta}{g}$$

$$= \frac{U^2 \times 2\sin\theta \times \cos\theta}{g} = \frac{U^2\sin 2\theta}{g}$$

i.e., $$r = \frac{U^2\sin 2\theta}{g}$$

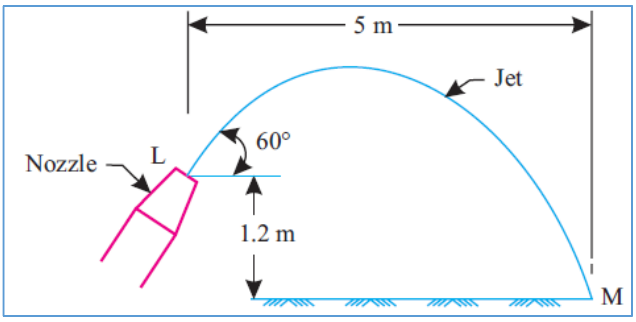
The range will be maximum, when $\sin 2\theta = 1$

i.e., $$2\theta = 90° \text{ or } \theta = 45°$$

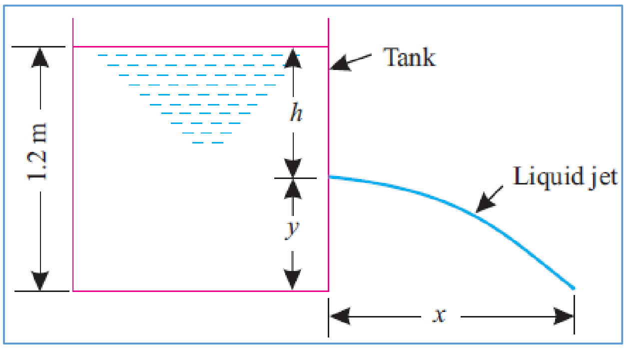
Then maximum range, $$r_{max} = \frac{U^2\sin(2\times 45°)}{g} = \frac{U^2}{g}$$

i.e., $$r_{max} = \frac{U^2}{g}$$

Example 5.15. A nozzle is situated at a distance of 1.2 m above the ground level and is inclined at 60° to the horizontal. The diameter of the nozzle is 40 mm and the jet of water from the nozzle strikes the ground at a horizontal distance of 5 m. Find the flow rate.



Example 5.16. It is required to place an orifice in the side of a tank at such an elevation that the jet will attain a maximum horizontal distance from the tank at the level of its base. What is the proper distance from the orifice to the free surface when the depth of liquid in the tank is maintained at 1.2 m?



5.5. IMPULSE-MOMENTUM EQUATION

The **impulse-momentum equation** is *one of the basic tools* (other being continuity and Bernoulli's equations) *for the solution of flow problems*. Its application leads to the solution of problems in fluid mechanics which *cannot* be solved by energy principles alone. Sometimes it is used in conjunction with the energy equation to obtain complete solution of engineering problems.
The momentum equation is based on the *law of conservation of momentum* or *momentum principle* which states as follows:
"*The net force acting on a mass of fluid is equal to change in momentum of flow per unit time in that direction*".

As per Newton's second law of motion,

$$F = ma$$

where, m = Mass of fluid,
F = Force acting on the fluid, and
a = Acceleration (acting in the same direction as F).

But acceleration, $$a = \frac{dv}{dt}$$

$$\therefore \qquad F = m.\frac{dv}{dt} = \frac{d(mv)}{dt}$$

('m' is taken inside the differential, being constant)
This equation is known as **momentum principle.** It can also be written as:

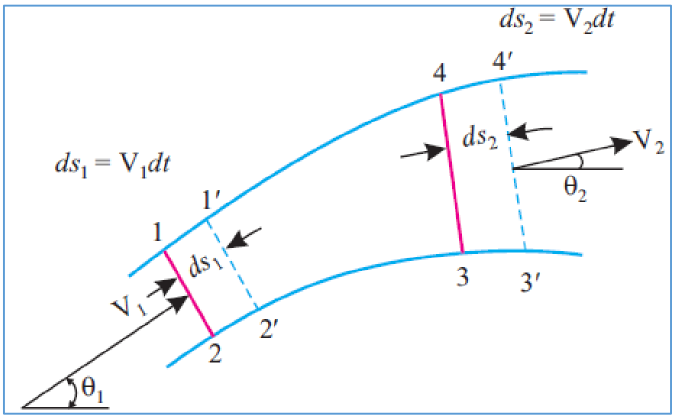
$$F.dt = d\,(mv)$$

This equation is known as **Impulse-momentum equation.** It may be stated as follows:
"*The impulse of a force F acting on a fluid mass 'm' in a short interval of time dt is equal to the change of momentum d(mv) in direction of force*".
The impulse-momentum equations are often called simply *momentum equations*.

Steady flow momentum equation:

The entire flow space may be considered to be made up of innumerable stream tubes. Let us consider one such stream tube lying in the *X*-*Y* plane and having steady flow of fluid. Flow can be assumed to be uniform and normal to the inlet and outlet areas.

Let, V_1, ρ_1 = Average velocity and density (of fluid mass) respectively at the entrance, and V_2, ρ_2 = Average velocity and density respectively at the exit.



Further let the mass of fluid in the region 1 2 3 4 shifts to new position 1′ 2′ 3′ 4′ due to the effect of external forces on the stream after a short interval. Due to gradual increase in the flow area in the direction of flow, velocity of fluid mass and hence the momentum is gradually reduced. Since the area 1′ 2′ 3 4 is common to both the regions 1 2 3 4 and 1′ 2′ 3′ 4′, therefore, it will not experience any change in momentum. Obviously, then the changes in momentum of the fluid masses in the sections 1 2 2′ 1′ and 4 3 3′ 4′ will have to be considered

According to the *principle of mass conservation*,

Fluid mass with the region 1 2 2′ 1′ = Fluid mass within the region 4 3 3′ 4′

$$\rho_1 A_1 ds_1 = \rho_2 A_2 ds_2$$

∴ Momentum of fluid mass contained in the region 1 2 2′ 1′

$$= (\rho_1 A_1 ds_1) V_1 = (\rho_1 A_1 V_1.\, dt) V_1$$

Momentum of fluid mass contained in the region 4 3 3′ 4′

$$= (\rho_2 A_2 ds_2) V_2 = (\rho_2 A_2 V_2.\, dt) V_2$$

∴ Change in momentum = $(\rho_2 A_2 V_2.\, dt) V_2 - (\rho_1 A_1 V_1.\, dt) V_1$

But, $\rho_1 = \rho_2 = \rho$ ...for steady incompressible flow

and, $A_1 V_1 = A_2 V_2 = Q$ ...from continuity considerations

∴ Change in momentum = $\rho Q(V_2 - V_1)\, dt$

Using impulse-momentum principle, we have:

$$F dt = \rho Q\,(V_2 - V_1) dt$$

Or,
$$F = (\gamma\, Q / g)\,(V_2 - V_1)$$

The quantity $\gamma Q/g = \rho Q$ is the mass flow per second and is called **mass flux.**

Resolving V_1 and V_2 along X-axis and Y-axis, we get:

Components along X-axis: $V_1 \cos\theta_1$ and $V_2 \cos\theta_2$
Components along Y- axis : $V_1 \sin\theta_1$ and $V_2 \sin\theta_2$
(where, θ_1 and θ_2 are the inclinations with the horizontal of the centre line of the pipe at 1-2 and 3-4).
∴ Components of force *F* along *X*-axis and *Y*-axis are:

$$F_x = (\gamma\, Q/g)(V_2 \cos\theta_2 - V_1 \cos\theta_1)$$
$$F_y = (\gamma\, Q/g)(V_2 \sin\theta_2 - V_1 \sin\theta_1) \quad \ldots(**)$$

The equation (**) represents the components of the force exerted by the *pipe bend on the fluid mass.*
Usually, we are interested in the forces by the fluid on the pipe bend. Since action and reaction are equal and opposite (Newton's third law of motion), the fluid mass would exert the same force on the pipe bend but in *opposite direction* and as such the force components exerted by the fluid on the pipe bend are given as follows:

$$F_x = (\gamma\, Q/g)(V_1 \cos\theta_1 - V_2 \cos\theta_2)$$
$$F_y = (\gamma\, Q/g)(V_1 \sin\theta_1 - V_2 \sin\theta_2) \quad \ldots(**)$$

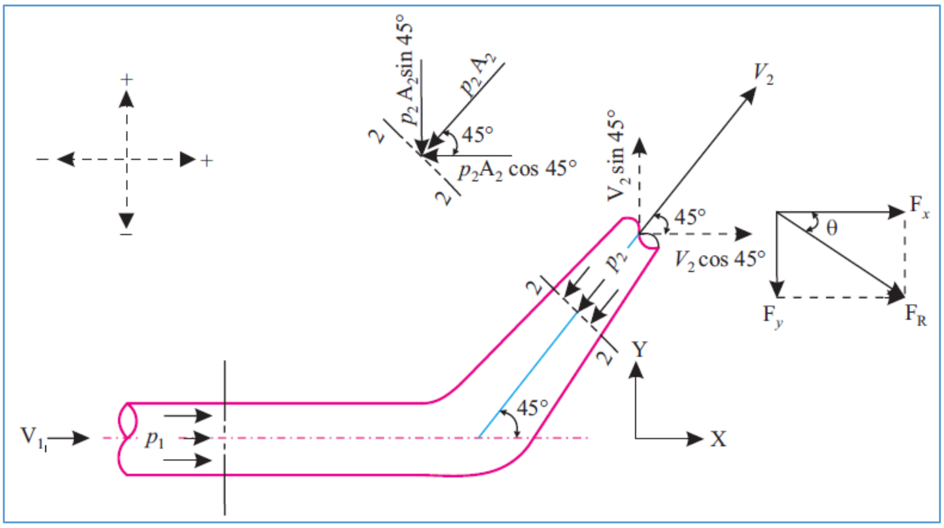
The magnitude of the resultant force acting on the pipe bend,

$$F_R = \sqrt{F_x^2 + F_y^2}$$

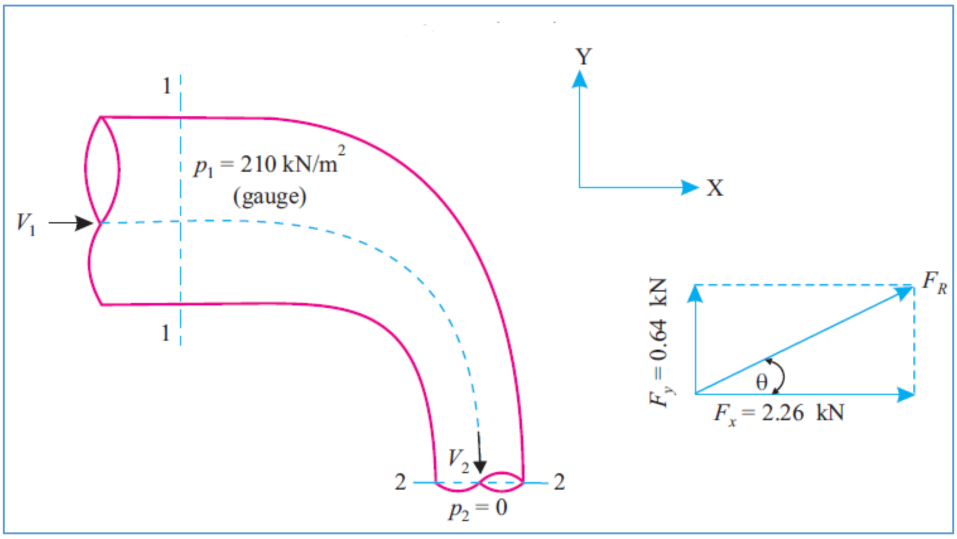
and, the direction of the resultant force with *X*-axis,

$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right)$$

Example 5.17. In a 45° bend a rectangular air duct of 1 m^2 cross-sectional area is gradually reduced to 0.5 m^2 area. Find the magnitude and direction of force required to hold the duct in position if the velocity of flow at 1 m^2 section is 10 m/s, and pressure is 30 kN/m^2. Take the specific weight of air as 0.0116 kN/m^3.


p_2 A_2sin 45°
p_2A_2
45°
p_2A_2 cos 45°
2
2
V_2
V_2 sin 45°
45°
V_2 cos 45°
p_2
F_x
θ
F_y
F_R
Y
X
45°
V_1
p_1

Example 5.18. This figure shows a 90° reducer-bend through which water flows. The pressure at the inlet is 210 kN/m^2 (gauge) where the cross-sectional area is 0.01 m^2. At the exit section, the area is 0.0025 m^2 and the velocity is 16 m/s. The pressure at the exit is atmospheric. Determine the magnitude and direction of the resultant force on the bend.

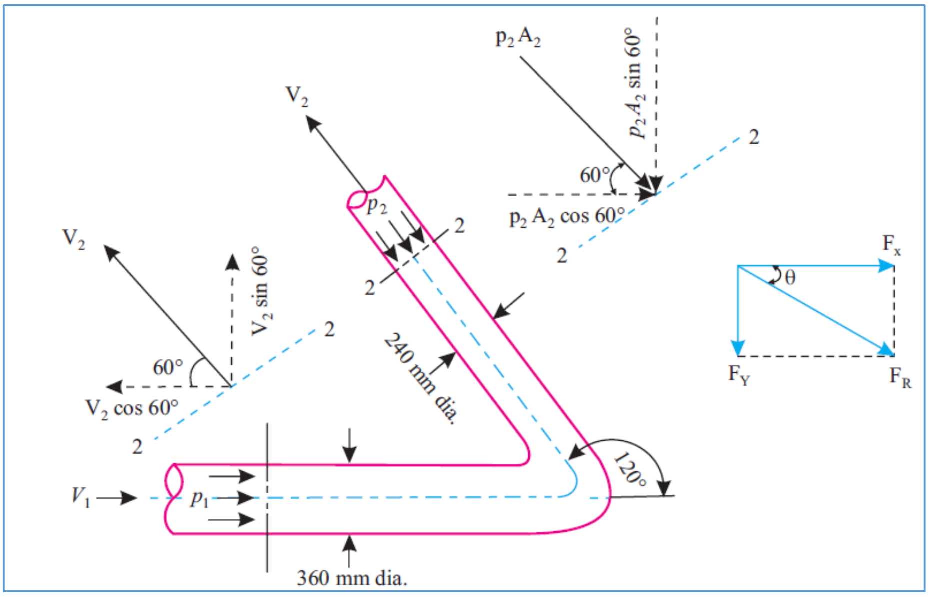


5.5.1. Applications of impulse-momentum equation:

The impulse-momentum equation is used in the following types of problems:

1. *To determine the resultant force acting on the boundary of flow passage by a stream of fluid as the stream changes its direction, magnitude or both.* Problems of this type are: (*i*) Pipe bends, (*ii*) Reducers, (*iii*) Moving vanes, (*iv*) Jet propulsion, etc.
2. *To determine the characteristic of flow when there is an abrupt change of flow section.* Problems of this type are: (*i*) Sudden enlargement in a pipe, (*ii*) Hydraulic jump in a channel, etc.

Example 5.19. 360 litres per second of water is flowing in a pipe. The pipe is bent by 120° .The pipe bend measures 360 mm $\times$ 240 mm and volume of the bend is 0.14 m^3. The pressure at the entrance is 73 kN/m^2 and the exit is 2.4 m above the entrance section. Find the force exerted on the bend.



Chapter 6 Flow through Pipes

6.1. INTRODUCTION

A **pipe** *is a closed conduit* (generally of circular section) *which is used for carrying fluids under pressure.*
The flow in a pipe is termed *pipe flow* only when the fluid *completely fills the cross-section and there is no free surface of fluid. The pipe running partially full* (in such a case *atmospheric pressure exists* inside the pipe) *behaves like an* ***open channel***.

6.2. LOSS OF ENERGY (OR HEAD) IN PIPES

When water flows in a pipe, it experiences some resistance to its motion, due to which its velocity and ultimately the head of water available is reduced. This loss of energy (or head) is classified as follows :

A. Major Energy Losses

This loss is due to *friction.*

B. Minor Energy Losses

These losses are due to :

1. Sudden enlargement of pipe,
2. Sudden contraction of pipe,
3. Bend of pipe,
4. An obstruction in pipe,
5. Pipe fittings, etc.

6.3. MAJOR ENERGY LOSSES

These losses which are *due to friction* are calculated by :

1. Darcy-Weisbach formula
2. Chezy's formula.

6·3·1 Darcy-Weisbach Formula

The loss of head (or energy) in pipes due to friction is calculated from Darcy-Weisbach formula, which is given by:

$$\boxed{h_f = \frac{4fLV^2}{D \times 2g}}$$

where, h_f = Loss of head due to friction,
f = Co-efficient of friction, (a function of Reynolds number, Re)

$$= \frac{0.0791}{(Re)^{1/4}} \text{ for } Re \text{ varying from 4000 to } 10^6$$

$$= \frac{16}{Re} \text{ for } Re < 2000 \text{ (laminar/viscous flow)}$$

L= Length of the pipe,
V = Mean velocity of flow, and
D = Diameter of the pipe.

6·3·2 Chezy's Formula for Loss of Head due to Friction

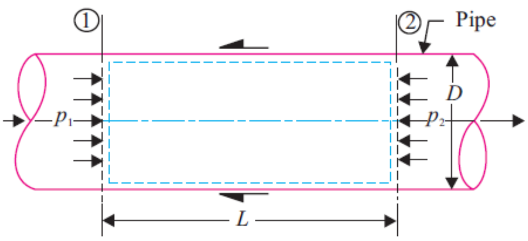
Refer to the figure below An equilibrium between the propelling force due to pressure difference and the frictional resistance gives :

$$(p_1 - p_2)\, A = f'\, PLV^2$$

or $$\frac{(p_1 - p_2)}{w} \cdot A = \frac{f'}{w} PLV^2$$

or $$h_f = \frac{f'}{w} \frac{P}{A} LV^2$$

$\therefore$ $$\text{Mean velocity, } V = \sqrt{\frac{w}{f'}} \times \sqrt{\frac{A}{P} \times \frac{h_f}{L}}$$



where, the factor $\sqrt{\frac{w}{f}}$, is called the Chezy's constant, C;

the ratio $\frac{A}{P} \left(= \frac{\text{area of flow}}{\text{wetted perimeter}} \right)$ is called the ***hydraulic mean depth*** or ***hydraulic radius*** and lenoted by m (or R);

the ratio $\frac{h_f}{L}$ prescribes the *loss of head per unit length of pipe* and is denoted by **i or S** (slope).

$\therefore$ $$\text{Mean velocity, } V = C\sqrt{m\, i}$$

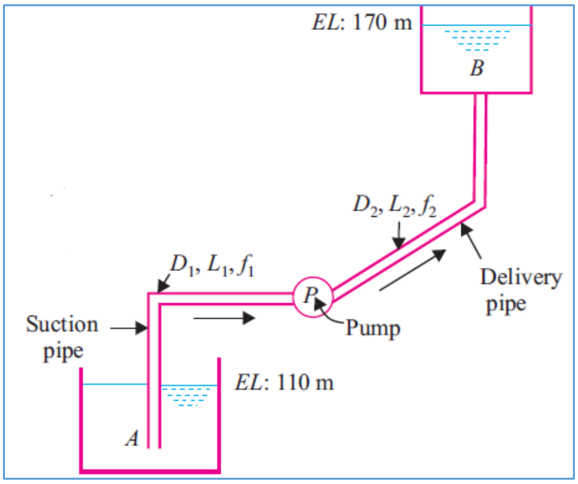
The last equation is *known as* **Chezy's formula.** This formula helps to find the head loss due to friction if the mean flow velocity through the pipe and the value of Chezy's constant C are known.

i. Darcy-Weisbach formula (for loss of head) is generally used for the flow through *pipes.*
ii. Chezy's formula (for loss of head) is generally used for the flow through *open channels.*
iii. The values of hydraulic mean depth for a *circular pipe,*

$$\boxed{m = \frac{D}{4} \left[\because m = \frac{\text{Area}}{\text{Perimeter}} = \frac{\frac{\pi}{4} \times D^2}{\pi D} = \frac{D}{4} \right]}$$

Example 6.1. *In a pipe of diameter 350 mm and length 75 m water is flowing at a velocity of 2·8 m/s. Find the head lost due to friction using :* **(*i*)** *Darcy-Weisbach formula;* **(*ii*)** *Chezy's formula for which C = 55.* Assume kinematic viscosity of water as 0·012 stoke.

Example 6.2. *Water flows through a pipe of diameter 300 mm with a velocity of 5 m/s. If the co-efficient of friction is given by $f = 0.015 + (0.08/Re^{0.3})$ where Re is the Reynolds number, find the head lost due to friction for a length of 10 m. Take kinematic viscosity of water as 0·01 stoke.*



Example 6.3. *A pump delivers water from a tank A (water surface elevation = 110 m) to tank B (water surface elevation = 170 m). The suction pipe is 45 m long (friction factor, f = 0·024) and 35 cm in diameter. The delivery pipe is 950 m long (f = 0·022) and 25 cm in diameter. The head discharge relationship for the pump is given by $H_p = (90 - 8000\,Q^2)$, where Hp is in metres and Q in m^3/s. Calculate:* **(i)** *The discharge in the pipeline.* **(ii)** *The power delivered by the pump.*

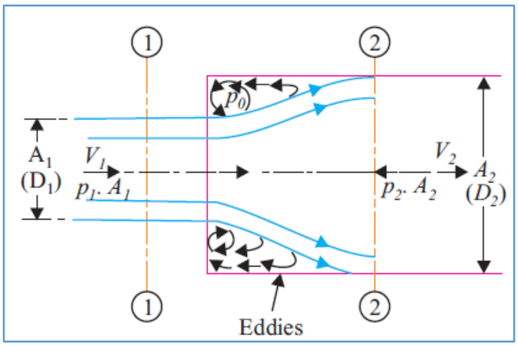
6.4. MINOR ENERGY LOSSES

Whereas the major loss of energy or head is due to friction, the minor loss of energy (or head) includes the following cases:

1. Loss of head due to sudden enlargement,
2. Loss of head due to sudden contraction,
3. Loss of head due to an obstruction in the pipe,
4. Loss of head at the entrance to a pipe,
5. Loss of head at the exit of a pipe,
6. Loss of head due to bend in the pipe, and
7. Loss of head in various pipe fittings.

6·4·1 Loss of Head due to Sudden Enlargement

Figure below shows a liquid flowing through a pipe, which has *sudden enlargement*. Due to sudden enlargement, the flow is decelerated abruptly and eddies are developed resulting in loss of energy (or head).



$$h_e = \frac{(V_1 - V_2)^2}{2g}$$

Example 6.4. *The diameter of a horizontal pipe, which is 300 mm, is suddenly enlarged to 600 mm. The rate of flow of water through this pipe is 0·4 m³/s. If the intensity of pressure in the smaller pipe is 125 kN/m², determine.*

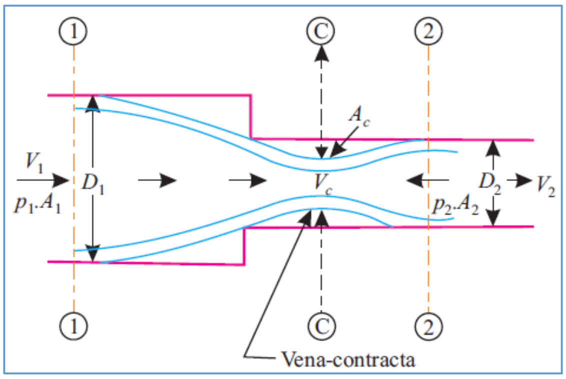
(i) *Loss of head, due to sudden enlargement,*

(ii) *Intensity of pressure in the larger pipe, and*

(iii) *Power lost due to enlargement.*

6·4·2. Loss of Head due to Sudden Contraction

Due to sudden contraction, the stream lines converge to a minimum cross-section called the *vena-contracta* and then expand to fill the downstream pipe.



In general, $$h_c = k\frac{V_2^2}{2g}$$

where, $$k = \left(\frac{1}{C_c} - 1\right)^2$$

From experiments : $C_c = 0.62 + 0.38\left(\frac{A_2}{A_1}\right)^3$

and thus the loss co-efficient k is a function of ratio

$$\frac{A_1}{A_2} \text{ or } \frac{D_2}{D_1}$$

and, $k = 0{\cdot}375$ for $C_c = 0{\cdot}62$.

For gradual contraction (conical reducers) k is a function of cone angle and $\simeq 0{\cdot}1$.

If the value of Cc is not given then loss of head due to contraction may be taken as 0.5 $v^2/2g$

$$\boxed{h_e = 0.5\frac{V_2^2}{2g}}$$

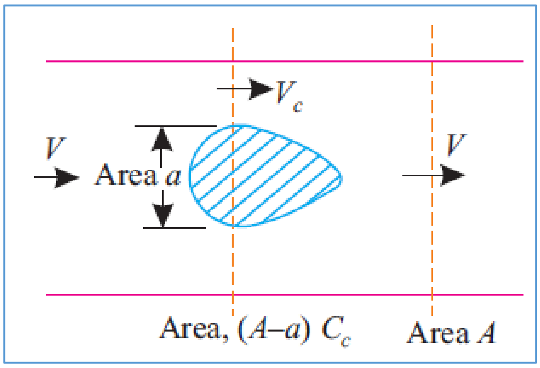
Example 6.5. *A horizontal pipe carries water at the rate of 0·04 m^3/s. Its diameter, which is 300 mm reduces abruptly to 150 mm. Calculate the pressure loss across the contraction. Take the co-efficient of contraction = 0·62.*

6·4·3 Loss of Head due to Obstruction in Pipe

Refer to figure below The loss of energy due to an obstruction in pipe takes place on account of the reduction in the cross-sectional area of the pipe by the presence of obstruction which is followed by an abrupt enlargement of the stream beyond the obstruction.

Head loss due to obstruction ($h_{obs.}$) is given by the relation :

$$\boxed{h_{obs.} = \left[\frac{A}{C_c\,(A-a)}\right]^2 \frac{V^2}{2g}}$$



where, A = Area of the pipe,
a = Maximum area of obstruction, and
V = Velocity of liquid in pipe.

6·4·4 Loss of Head at the Entrance to Pipe

Loss of head at the entrance to pipe (*hi*) is given by the relation :

$$\boxed{h_i = 0.5\frac{V^2}{2g}}$$

where, V = Velocity of liquid in pipe.

6·4·5 Loss of Dead at the Exit of a Pipe

Loss of head at the exit of a pipe is denoted by *h*0 and is given by the relation:

$$\boxed{h_0 = \frac{V^2}{2g}}$$

where, V = Velocity at outlet of pipe.

6·4·6 Loss of Head due to Bend in the Pipe

In general the loss of head in bends (*hb*) provided in pipes may be expressed as :

$$\boxed{h_0 = k\frac{V^2}{2g}}$$

where, V = Mean velocity of flow of fluid, and
k = Co-efficient of bend; it depends upon *angle of bend, radius of curvature of bend* and *diameter of pipe.*

6·4·7 Loss of Head in Various Pipe Fittings

The loss of head in the various pipe fittings (such as valves, couplings, etc.) may also be represented as:

$$\boxed{h_{\text{fittings}} = k\frac{V^2}{2g}}$$

where, V = Mean velocity flow in the pipe, and
k = value of the co-efficient; it depends on the type of the pipe fitting.

6.5. HYDRAULIC GRADIENT AND TOTAL ENERGY LINES

The concept of hydraulic gradient line and total energy line is quite useful in the study of flow of fluid in pipes. These lines may be obtained as indicated below.

Total Energy Line (T.E.L. or E.G.L.):

It is known that the *total head* (which is also total energy per unit weight) with respect to any arbitrary datum, is the sum of the elevation (potential) head, pressure head and velocity head, *i.e.*,

$$\boxed{\text{Total head} = \frac{p}{w} + z + \frac{V^2}{2g}}$$

When the fluid flows along the pipe, there is loss of head (energy) and the total energy decreases in the direction of flow. If the total energy at various points along the axis of the pipe is plotted and joined by a line, the line so obtained is called the **'Energy gradient line' (E.G.L.).**

In literature, energy gradient line (E.G.L.) is also known as '**Total energy line' (T.E.L.).**

Hydraulic Gradient Line (H.G.L.):

The sum of potential (or elevation) head and the pressure head at any point is called the piezometric head. If a line is drawn joining the piezometric levels at various points, the line so obtained is called the **'Hydraulic gradient line.'**

The following points are worth noting:

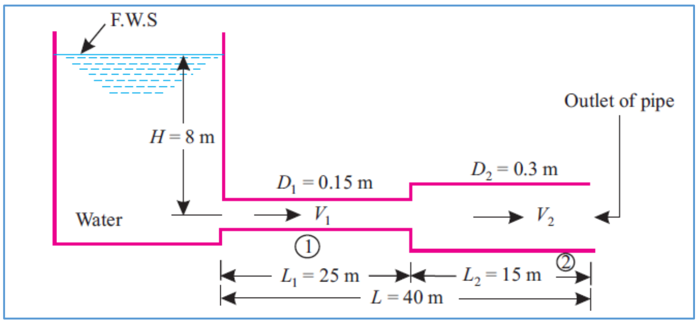
1. Energy gradient line **(E.G.L.)** always drops in the direction of flow because of loss of head.
2. Hydraulic gradient line **(H.G.L.)** may rise or fall depending on the pressure changes.
3. Hydraulic gradient line (H.G.L.) is always below the energy gradient line (E.G.L.) and the vertical intercept between the two is equal to the ***velocity head***
4. For a pipe of uniform cross-section, the slope of the hydraulic gradient line is equal to ***the slope of energy gradient line.***
5. There is no relation whatsoever between the slope of energy gradient line and the slope of the axis of the pipe.

Example 6.6. A horizontal pipeline 40 m long is connected to a water tank at one end and discharges freely into the atmosphere at the other end. For the first 25 m of its length from the tank, the pipe is 150 mm diameter and its diameter is suddenly enlarged to 300 mm. The height of water level in the tank is 8 m above the center of the pipe. Considering all losses of head which occur,

i. Determine the rate of flow.
ii. Draw the hydraulic gradient and energy gradient lines.

Take f = 0.01 for both sections of the pipe.

Sol.



To draw E.G.L. and H.G.L. the following procedure is followed.

E.G.L. (Energy gradient line):

The point L lies on F.W.S. (free water surface).

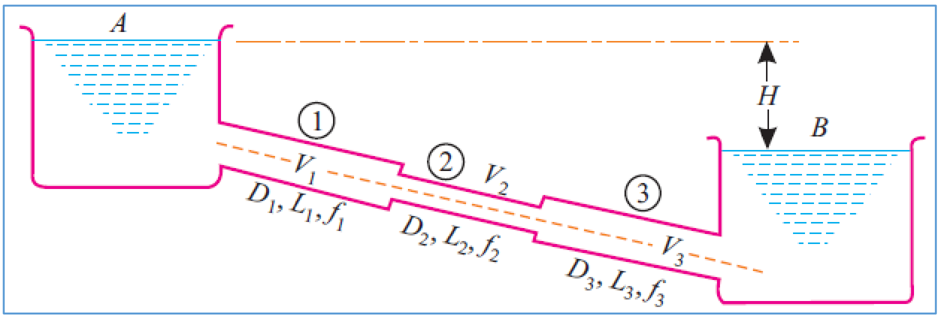
- Take LM = h_i = 0·5 m
- From M draw a horizontal line. Taking MA equal to the length of the pipe (i.e., L1) draw a vertical line downward from the point A. Cut AN = h_{f1} = 6·7 m
- Join MN
- From N, draw a line NS vertically downward equal to h_e (= 0·56 m)
- From S, draw SB horizontal and from point U (which is lying on the center of the pipe) draw a vertical line in the upward direction, meeting at B. From B take BT = h_{f2} = 0·126 m.
- Join ST
- The line LMNST represents the energy gradient line (E.G.L.)

H.G.L. (Hydraulic gradient line) :

- From *M*, take MP=$V^2/2g = (4*1.11)^2 / 2*9.81=1$m
- Draw the line *PQ* parallel to the line *MN*
- From the point *U*, draw a line *UV* parallel to the line *TS*
- Join *QV*
- The line *PQVU* represents the hydraulic gradient line (H.G.L.).

6.6. PIPES IN SERIES OR COMPOUND PIPES

Figure below shows a system of pipes in series.



Let, D_1, D_2, D_3 = Diameters of pipes 1, 2 and 3 respectively,
L_1, L_2, L_3 = Lengths of pipes 1, 2 and 3 respectively,
V_1, V_2, V_3 = Velocities of flow through pipes 1, 2 and 3 respectively
f_1, f_2, f_3 = Co-efficients of friction for pipes 1, 2 and 3 respectively, and
H = Difference of water level in the two tanks.

As the rate of flow (Q) of water through each pipe is same, therefore,

$$Q = A_1V_1 = A_2V_2 = A_3V_3$$

Also, The difference in liquid surface levels = Sum of the various head losses in the pipes

$$H = h_i + h_{f_1} + h_c + h_{f_2} + h_e + h_{f_3} + \frac{V_3^3}{2g}$$

where,

$$h_i = \text{Head loss at entrance} = \frac{0.5V_1^2}{2g}$$

$$h_{f_1} = \text{Head loss due to friction in pipe 1} = \frac{4f_1L_1V_1^2}{D_1 \times 2g}$$

$$h_c = \text{Head loss at contraction} = \frac{0.5V_2^2}{2g}$$

$$h_{f_2} = \text{Head loss due to friction in pipe 2} = \frac{4f_2L_2V_2^2}{D_2 \times 2g}$$

$$h_e = \text{Head loss due to enlargement} = \frac{(V_2 - V_3)^2}{2g}$$

$$h_{f_3} = \text{Head loss due to friction in pipe 3} = \frac{4f_3L_3V_3^2}{D_3 \times 2g}$$

$$H = h_i + h_{f_1} + h_c + h_{f_2} + h_e + h_{f_3} + \frac{V_3^2}{2g}$$

$$= \frac{0.5V_1^2}{2g} + \frac{4f_1L_1V_1^2}{D_1 \times 2g} + \frac{0.5V_2^2}{2g} + \frac{4f_2L_2V_2^2}{D_2 \times 2g} + \frac{(V_2 - V_3)^2}{2g} + \frac{4f_3L_3V_3^2}{D_3 \times 2g} + \frac{V_3^2}{2g}$$

If *minor losses are neglected*, then above equation becomes:

$$H = \frac{4f_1L_1V_1^2}{D_1 \times 2g} + \frac{4f_2L_2V_2^2}{D_2 \times 2g} + \frac{4f_3L_3V_3^2}{D_3 \times 2g}$$

If, $f_1 = f_2 = f_3 = f$, then:

$$H = \frac{4fL_1V_1^2}{D_1 \times 2g} + \frac{4fL_2V_2^2}{D_2 \times 2g} + \frac{4fL_3V_3^2}{D_3 \times 2g}$$

$$= \frac{4f}{2g}\left[\frac{L_1V_1^2}{D_1} + \frac{L_2V_2^2}{D_2} + \frac{L_3V_3^2}{D_3}\right]$$

Example 6.7. *Three pipes of diameters 300 mm, 200 mm and 400 mm and lengths 450 m, 255 m and 315 m respectively are connected in series. The difference in water surface levels in two tanks is 18 m. Determine the rate of flow of water if co-efficients of friction are 0·0075, 0·0078 and 0·0072 respectively considering:*

(*i*) *Minor losses also, and*

(*ii*) *Neglecting minor losses.*

6.7. EQUIVALENT PIPE

An **equivalent pipe** *is defined as the pipe of uniform diameter having loss of head and discharge equal to the loss of head and discharge of a compound pipe consisting of several pipes of different lengths and diameters*. The uniform diameter of the equivalent pipe is known as the *equivalent diameter* of the series or compound pipe.

Head loss in the equivalent pipe,

$$H = \frac{4fL\left(\frac{4Q}{\pi D^2}\right)^2}{D \times 2g} = \frac{4 \times 16 fQ^2 f}{\pi^2 \times 2g}\left[\frac{L}{D^5}\right]$$

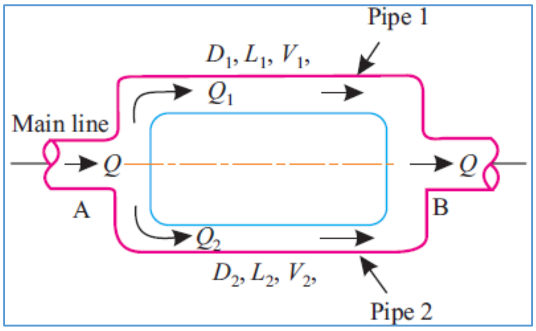
$$\frac{L}{D^5} = \frac{L_1}{D_1^5} + \frac{L_2}{D_2^5} + \frac{L_3}{D_3^5} + \ldots$$

The last equation is known as **Dupit's equation.** If the length of the equivalent pipe is equal to the length of the compound pipe *i.e.*, $L = (L_1 + L_2 + L_3 + \ldots)$, the diameter D of the equivalent pipe may be determined by using this equation. Sometimes a pipe of a given diameter D which is available may be required to be used as equivalent pipe to replace a compound pipe; in this case the length of the equivalent pipe may be required to be determined and the same may also be determined by using last equation.

Example 6.8. *A piping system consists of three pipes arranged in series; the lengths of the pipes are 1200 m, 750 m and 600 m and diameters 750 mm, 600 mm and 450 mm respectively.*
(*i*) *Transform the system to an equivalent 450 mm diameter pipe, and*
(*ii*) *Determine an equivalent diameter for the pipe, 2550 m long.*

6.8. PIPES IN PARALLEL

The pipes are said to be in *parallel*, when a main line divides into two or more parallel pipes which again join together downstream and continues as a main line.



It may be seen from this figure that the rate of discharge in the main line is equal to the pipes.

Thus,

$$Q = Q1 + Q2$$

When the pipes are arranged in parallel, the loss of head in each pipe (branch) is same.

∴ Loss of head in pipe 1 = Loss of head in pipe 2.

or, $$h_f = \frac{4f_1L_1V_1^2}{D_1 \times 2g} = \frac{4f_2L_2V_2^2}{D_2 \times 2g}$$

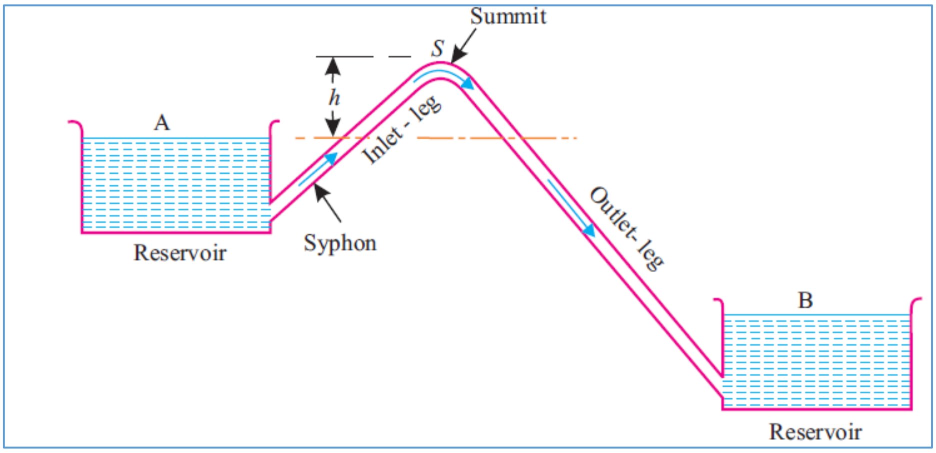
When, $f_1 = f_2$, then:

$$\frac{L_1V_1^2}{D_1 \times 2g} = \frac{L_2V_2^2}{D_2 \times 2g}$$

Example 6.9. *The main pipe divides into two parallel pipes, which again forms one pipe as shown in the*
figure above. The data is as follows:
First parallel pipe: Length = 1000 m, diameter = 0·8 m Second parallel pipe: Length = 1000 m, diameter = 0·6 m
Co-efficient of friction for each parallel pipe = 0·005
If the total rate of flow in the main is 2 m^3/s find the rate of flow in each parallel pipe.

6.9. SYPHON

A **syphon** *is a long bent pipe employed for carrying water from a reservoir at a higher elevation to another reservoir at a lower elevation when the two reservoirs are separated by a hill or high level ground in between* as shown in figure below.



Example 6.10. *Two reservoirs, having a difference in elevation of 15 m, are connected by a 200 mm diameter syphon. The length of the syphon is 400 m and the summit is 3 m above the water level in the upper reservoir. The length of the pipe from upper reservoir to the summit is 120 m. If the co-efficient of friction is 0.005, determine:*

(*i*) *Discharge through the syphon, and*

(*ii*) *Pressure at the summit.*

Neglect minor losses.

6.10. WATER HAMMER IN PIPES

In a long pipe, when the *flowing water is suddenly brought to rest by closing the valve or by any similar cause*, there will be a *sudden rise in pressure* due to the momentum of water being destroyed.

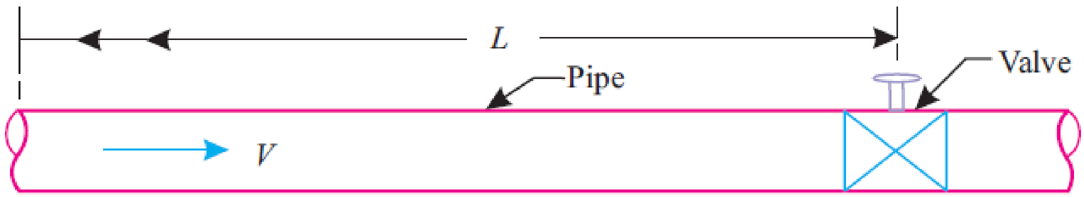
A pressure wave is transmitted along the pipe. A sudden rise in pressure has the effect of hammering action on the walls of the pipe. This *phenomenon of sudden rise in pressure is known as* **water hammer** or **hammer blow.** The magnitude of pressure rise depends on:

i. *The speed at which valve is closed,*
ii. *The velocity of flow,*
iii. *The length of pipe, and*
iv. *The elastic properties of the pipe material as well as that of the flowing fluid.*

The rise in pressure in some cases may be so large that the pipe may even burst and therefore it is essential to take into account this pressure rise in the design of the pipes.

6·10·1 Gradual Closure of Valve

Consider a long pipe carrying liquid (Figure below) and provided with a valve which is closed gradually.



Let, A = Area of cross-section of the pipe,
L = Length of the pipe,
V = Velocity of flow of water in the pipe,
t = Time required to close the valve (in seconds), and
p = Intensity of pressure wave produced.

The mass of liquid contained in the pipe is = ρAL

Assuming that the rate of closure of the valve is so adjusted that the liquid column in the pipe is brought to rest with a uniform retardation; from an initial velocity V to zero in time t seconds, we have:

$$\text{Retardation of water } = \frac{V-0}{t} = \frac{V}{t}$$

The axial force available for producing retardation = Mass $\times$ retardation

$$= \rho AL \times \frac{V}{t} \qquad ...(i)$$

Also, force due to pressure wave is $= p.A$ ...(*ii*)

Equating the two forces given by eqns. (*i*) and (*ii*), we have:

$$\rho AL \times \frac{V}{t} = p \times A$$

or,
$$p = \frac{\rho LV}{t}$$

$\therefore$ Head of pressure, $H = \frac{p}{w} = \frac{\rho LV}{w \times t} = \frac{\rho LV}{\rho \cdot g \cdot t} = \frac{LV}{gt}$

i.e.,
$$H = \frac{LV}{gt}$$

(*i*) The closure of valve is said to be *gradual* when $t > \frac{2L}{C}$

(*ii*) The closure of valve is said to be *instantaneous* when $t < \frac{2L}{C}$

where, C = velocity of the pressure wave.

6·10·2 Instantaneous Closure of Valve in Rigid Pipes

Consider a pipe of length L and area of cross-section A (Figure above) carrying water which is flowing through it at a velocity V. When the valve is closed instantaneously the K.E. of the flowing water is converted into strain energy of water (neglecting effect of friction and assuming the pipe wall to be perfectly rigid).

$$\text{Loss of } K.E. = \frac{1}{2} mV^2 = \frac{1}{2} \rho AL \times V^2 \qquad (\because m = \rho \times A \times L)$$

$$\text{Gain of strain energy} = \frac{1}{2}\left(\frac{p^2}{K}\right) \times \text{volume} = \frac{1}{2}\frac{p^2}{K} \times AL$$

$$\begin{bmatrix} \text{where, } k = \text{Bulk modulus of elasticity of water, and} \\ p = \text{Intensity of pressure wave produced.} \end{bmatrix}$$

Equating the loss of *K.E.* to the gain of strain energy, we get:

$$\frac{1}{2}\rho AL \times V^2 = \frac{1}{2}\frac{p^2}{K} \times AL$$

or,
$$p^2 = \frac{1}{2}\rho ALV^2 \times \frac{2K}{AL} = \rho KV^2$$

$$\therefore \qquad p = \sqrt{\rho K V^2} = V\sqrt{\rho K} = V\sqrt{\frac{K\rho^2}{\rho}}$$

or,

$$p = V\rho C$$

$$\left(\text{where, } C = \sqrt{\frac{K}{\rho}}, C \text{ being the } \textit{velocity of pressure wave.}\right)$$

12·12·3 Instantaneous Closure of Valve in Elastic Pipes

As shown in Figure above, consider a pipe of length L, diameter D, thickness t (small compared to diameter).

Let, p = Increase of pressure due to water hammer,
E = Modulus of elasticity of pipe material, and 1 m = Poisson's ratio for pipe material.

When the valve is closed instantaneously, rise of pressure takes place due to which circumferential and longitudinal stresses are produced in the pipe wall; these stresses are given as (from knowledge of strength of materials):

$$\sigma_c = \frac{pD}{2t} \quad \text{and} \quad \sigma_l = \frac{pD}{4t}$$

where, σ_c = Circumferential stress, and
σ_l = Longitudinal stress.

Also, strain energy stored in the pipe material per unit volume is

$$= \frac{1}{2E}\left(\sigma_c^2 + \sigma_l^2 - \frac{2\sigma_c\sigma_l}{m}\right)$$

$$= \frac{1}{2E}\left[\left(\frac{pD}{2t}\right)^2 + \left(\frac{pD}{4t}\right)^2 - \frac{2 \times \frac{pD}{2t} \times \frac{pD}{4t}}{m}\right]$$

$$= \frac{1}{2E}\left[\frac{p^2D^2}{4t^2} + \frac{p^2D^2}{16t^2} - \frac{p^2D^2}{4mt^2}\right]$$

Assuming, $\frac{1}{m} = 1/4$, we have:

$$\text{Strain energy per unit volume} = \frac{1}{2E}\left[\frac{p^2D^2}{4t^2} + \frac{p^2D^2}{16t^2} - \frac{p^2D^2}{16t^2}\right] = \frac{p^2D^2}{8Et^2}$$

Total strain energy stored in pipe material

$$= \frac{p^2D^2}{8Et^2} \times \text{total volume of pipe material}$$

$$= \frac{p^2 D^2}{8Et^2} \times \pi\, Dt \times L = \frac{p^2 \times D^3 L}{8Et}$$

$$= \frac{p^2 \times \pi D^2 \times DL}{8Et} = \frac{p^2 ADL}{2Et} \qquad [\because A \text{ (area of the pipe)} = \frac{\pi}{4} \times D^2]$$

$$\text{Loss of K.E. of water} = \frac{1}{2} mV^2 = \frac{1}{2} \rho AL \times V^2$$

$$\text{Gain of strain energy in water} = \frac{1}{2}\left(\frac{p^2}{K}\right) \times \text{volume} = \frac{1}{2}\frac{p^2}{K} \times AL$$

Also, The loss of K.E. of water = Gain of strain energy in water + strain energy stored in material.

$$\therefore \qquad \frac{1}{2}\rho AL \times V^2 = \frac{1}{2}\frac{p^2}{K} \times AL + \frac{p^2 ADL}{2Et}$$

Dividing both sides by $\frac{AL}{2}$, we get:

$$\rho V^2 = \frac{p^2}{K} + \frac{p^2 D}{Et} = p^2\left(\frac{1}{K} + \frac{D}{Et}\right)$$

$$\therefore \qquad p^2 = \frac{\rho V^2}{\left(\frac{1}{K} + \frac{D}{Et}\right)}$$

or,

$$p = \sqrt{\frac{\rho V^2}{\left(\frac{1}{K} + \frac{D}{Et}\right)}} = V \times \sqrt{\frac{\rho}{\left(\frac{1}{K} + \frac{D}{Et}\right)}}$$

6·10·4 Time required by Pressure Wave to travel from the Valve to the Tank and from Tank to Valve

$$\text{Time taken, } t = \frac{\text{Distance travelled from valve to tank and back}}{\text{Velocity of pressure wave}}$$

$$= \frac{L+L}{C} = \frac{2L}{C} \qquad i.e., \qquad t = \frac{2L}{C}$$

where, L = Length of the pipe, and

C = Velocity of pressure wave.

Example 6.11. *Water is flowing in a pipe of 150 mm diameter with a velocity of 2.5 m/s when it is suddenly brought to rest by closing the valve. Find the pressure rise assuming pipe is elastic, E = 206 GN/m^2, Poisson's ratio = 0.25 and K for water = 2.06 GN/m^2. Pipe wall is 5 mm thick.*

Example 6.12. *In a pipe 600 mm diameter and 3000 m length, provided with a valve at its end, water is flowing with a velocity of 2 m/s. Assuming velocity of pressure wave C = 1500 m/s, find :*

i. *The rise in pressure if the valve is closed in 20 seconds, and*
ii. *The rise in pressure if the valve is closed in 2.5 seconds. Assume the pipe to be rigid one and take bulk modulus of water as 2 GN/m^2.*

Chapter 4: Motion of Fluid Particles and Streams

Fluid kinematics is a branch of 'Fluid mechanics' which deals with the study of velocity and acceleration of the particles of fluids in motion and their distribution in space without considering any force or energy involved.

DESCRIPTION OF FLUID MOTION

The motion of fluid particles may be described by the following methods:
1. Langrangian method.
2. Eulerian method.

1. Langrangian Method

In this method, the observer *concentrates on the movement of a single particle*. The path taken by the particle and the changes in its velocity and acceleration are studied.

2. Eulerian Method

In Eulerian method, the observer *concentrates on a point in the fluid system*. Velocity, acceleration and other characteristics of the fluid at that particular point are studied.

TYPES OF FLUID FLOW

Fluids may be *classified* as follows:
1. Steady and unsteady flows
2. Uniform and non-uniform flows
3. One, two and three dimensional flows
4. Rotational and irrotational flows
5. Laminar and turbulent flows
6. Compressible and incompressible flows.

1. Steady and Unsteady Flows

Steady flow. The type of flow in which the fluid characteristics like velocity, pressure, density, etc. at a point *do not change* with time is called *steady flow*. Mathematically, we have:

$$\left(\frac{\partial u}{\partial t}\right)_{x_0,y_0,z_0} = 0; \left(\frac{\partial v}{\partial t}\right)_{x_0,y_0,z_0} = 0; \left(\frac{\partial w}{\partial t}\right)_{x_0,y_0,z_0} = 0$$

$$\left(\frac{\partial p}{\partial t}\right)_{x_0,y_0,z_0} = 0; \left(\frac{\partial \rho}{\partial t}\right)_{x_0,y_0,z_0} = 0; \text{ and so on}$$

where (x_0, y_0, z_0) is a fixed point in a fluid field where these variables are being measured *w.r.t.* time.
Example. *Flow through a prismatic or non-prismatic conduit at a constant flow rate Q m^3/s is steady.*
(A prismatic conduit has a constant size shape and has a velocity equation in the form $u = ax2 + bx + c$, which is independent of time t).

Unsteady flow. It is that type of flow in which the velocity, pressure or density at a point *change* w.r.t. time. Mathematically, we have:

$$\left(\frac{\partial u}{\partial t}\right)_{x_0, y_0, z_0} \neq 0; \left(\frac{\partial v}{\partial t}\right)_{x_0, y_0, z_0} \neq 0; \left(\frac{\partial w}{\partial t}\right)_{x_0, y_0, z_0} \neq 0$$

$$\left(\frac{\partial p}{\partial t}\right)_{x_0, y_0, z_0} \neq 0; \left(\frac{\partial \rho}{\partial t}\right)_{x_0, y_0, z_0} \neq 0; \text{ and so on}$$

Example. *The* flow *in a pipe whose valve is being opened or closed gradually* (*velocity equation is in the form* $u = ax^2 + bxt$).

2. Uniform and Non-uniform Flows

Uniform flow. The type of flow, in which the velocity at any given time *does not change* with respect to space is called *uniform flow*. Mathematically, we have:

$$\left(\frac{\partial V}{\partial s}\right)_{t\,=\,\text{constant}} = 0$$

where, ∂V = Change in velocity, and
∂s = Displacement in any direction.
Example. *Flow through a straight prismatic conduit* (*i.e. flow through a straight pipe of constant diameter*).

Non-uniform flow. It is that type of flow in which the velocity at any given time *changes* with respect to space. Mathematically,

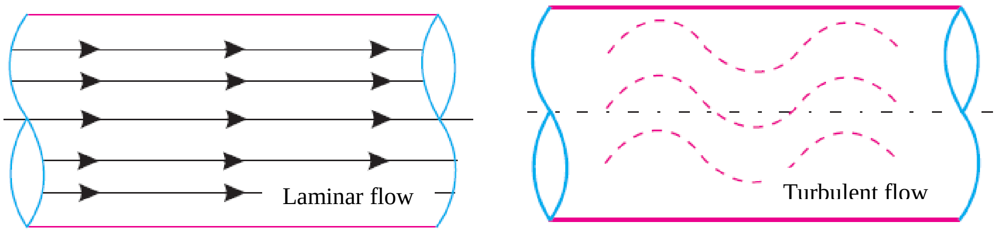
$$\left(\frac{\partial V}{\partial s}\right)_{t\,=\,\text{constant}} \neq 0$$

Example. (*i*) *Flow through a non-prismatic conduit.*
(*ii*) *Flow around a uniform diameter pipe-bend or a canal bend.*

3. Laminar and Turbulent Flows

Laminar flow. A laminar flow is one in which *paths taken by the individual particles do not cross one another and move along well defined paths*. This type of flow is also called *stream-line flow or viscous flow*.
Examples. (*i*) *Flow through a capillary tube.*
(*ii*) *Flow of blood in veins and arteries.*
(*iii*) *Ground water flow.*

Turbulent flow. A turbulent flow is that flow in which fluid *particles move in a zig zag way*.
Example. *High velocity flow in a conduit of large size. Nearly all fluid flow problems encountered in engineering practice have a turbulent character*.



Patterns of Flow

Reynolds Number (R_e):
A dimensionless number used to identify the type of flow.

$$R_e = \frac{\text{Inertia Forces}}{\text{Viscous Forces}} = \frac{\rho \times V \times D}{\mu} = \frac{V \times D}{\nu}$$

V=mean velocity (m/s) , D=pipe diameter (m), ρ=fluid density (Kg/m^3)
μ=Dynamic viscosity (Pa.s) , ν=kinematic viscosity (m^2/s)
For flow in pipe: If ($R_e \leq 2000$)→→The flow is laminar If ($2000 < R_e \leq 4000$)→→The flow is transitional If ($R_e > 2000$)→→The flow is turbulent

4. Compressible and Incompressible Flows

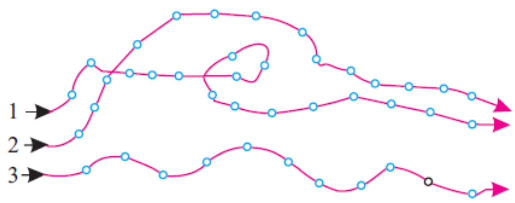
Compressible flow. It is that type of flow in which the *density* (ρ) *of the fluid changes* from point to point (or in other words *density is not constant for this flow*). Mathematically: $\rho \neq$ constant.
Example. *Flow of gases through orifices, nozzles, gas turbines, etc*.

Incompressible flow. *It is that type of flow in which density is constant for the fluid flow. Liquids are generally considered flowing incompressibly*. Mathematically: ρ = constant. ***Example.*** *Subsonic aerodynamics*.

TYPES OF FLOW LINES

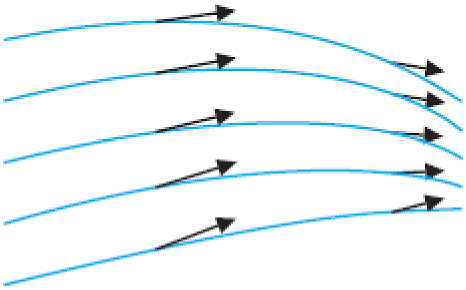
1. Path line

A path line is the *path followed by a fluid particle in motion*. A path line shows the direction of particular particle as it moves ahead.



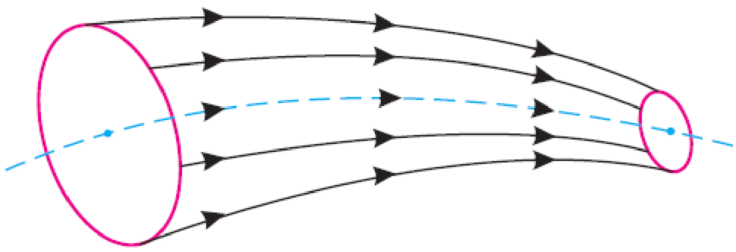
2. Stream line

A ***stream line*** way be defined as *an imaginary line within the flow so that the tangent at any point on it indicates the velocity at that point*



3. Stream Tube

A ***stream tube*** is a *fluid mass bounded by a group of streamlines*. The contents of a stream tube are known as '*current filament*'.



RATE OF FLOW OR DISCHARGE AND MEAN VELOCITY:

Rate of flow (or **discharge**) *is defined as* the *quantity of a liquid flowing per second through a section of pipe or a channel.*

Flow Rate can be measured by one of the following two methods:

1. In terms of mass (Mass Flow Rate, $\dot{m}$):

$$\dot{m} = \frac{\text{Mass of fluid}}{\text{time taken to collect the fluid}} = \frac{dm}{dt} = \rho \times Q \ (\text{Kg/s}).$$

2. In terms of volume (Volume Flow Rate or discharge, Q,):

$$Q = \frac{\text{Volume of Fluid}}{\text{Time}} = \frac{V}{t} \ (m^3/s).$$

This method is the most commonly used method to represents discharge. There is another important way to represents Q:

$$Q = \frac{V}{t} = \frac{\text{Area} \times L}{t} = \text{Area} \times \text{Speed} \rightarrow Q = A \times v \ (m^3/s).$$

CONTINUITY OF FLOW

Matter cannot be created or destroyed (principle of conservation of mass)

Mass entering per unit time = Mass leaving per unit time + Increasing of mass in the control volume per unit time

If the flow is steady, no increase in the mass within the control volume. So,

Mass entering per unit time = Mass leaving per unit time

Continuity Equation for Steady Flow and Incompressible Flow:

$$A_1 v_1 = A_2 v_2 = Q = \text{constant}$$

This equation is a very powerful tool in fluid mechanics and will be used repeatedly throughout the rest of this course.

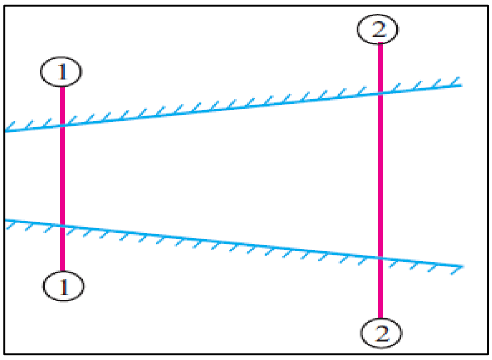
The following problems clarify the concept of continuity of flow

Example 1.

The diameters of a pipe at the sections 1-1and 2-2 are 200 mm and 300 mm respectively. If the velocity of water flowing through the pipe at section 1-1 is 4 m/s, find:

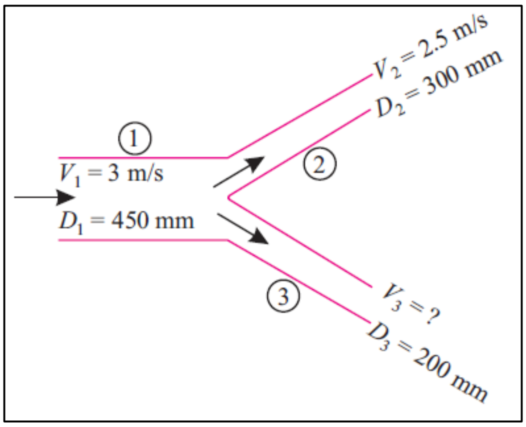
(i) Discharge through the pipe, and

(ii) Velocity of water at section 2-2



Sol.

Example 2.



A pipe (1) 450 mm in diameter branches into two pipes (2 and 3) of diameters 300 mm and 200 mm respectively. If the average velocity in 450 mm diameter pipe is 3 m/s find:

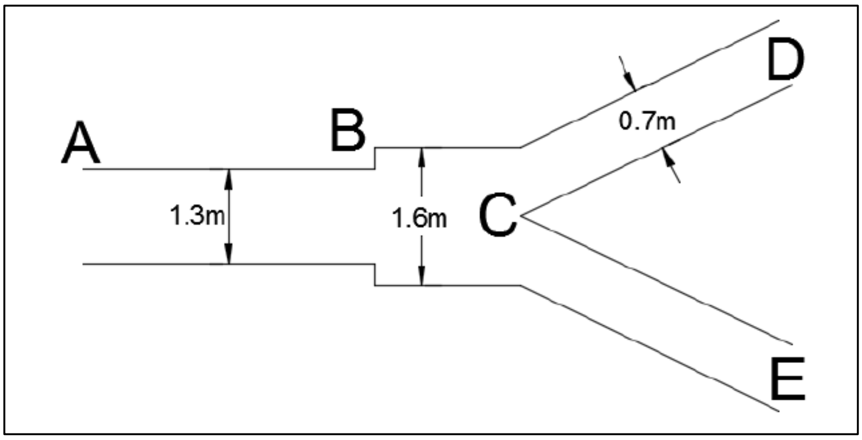
(i) Discharge through 450 mm diameter pipe;

(ii) Velocity in 200 mm diameter pipe if the average velocity in 300 mm pipe is 2.5 m/s.

ASSIGNMENT NO.4

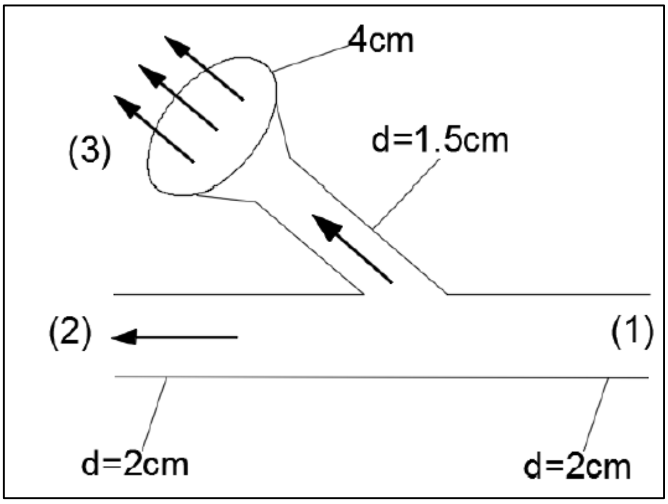
1.

Water flows through pipe AB 1.3m diameter at speed of 3m/s and passes through a pipe BC of 1.6m diameter. At C the pipe branches. Branch CD is 0.7m in diameter and carries one third of the flow in AB. The velocity in branch CE is 2.7m/s. **Find** the flow rate in AB, the velocity in BC, the velocity in CD and the diameter of CE.



2.

Pipe flow steadily through the piping junction (as shown in the figure) entering section **(1)** at a flow rate of 4.5m^3/hr. The average velocity at section **(2)** is 2.5 m/s. A portion of the flow is diverted through the showerhead 100 holes of 1-mm diameter. Assuming uniform shower flow, estimate the exit velocity from the showerhead holes.



Chapter 5: FLUID DYNAMICS

5.1 INTRODUCTION

The science which deals with the geometry of motion of fluids without reference to the forces causing the motion is known as ***"hydrokinematics"*** (or simply kinematics). Thus, kinematics involves merely the description of the motion of fluids in terms of space-time relationship. The science which deals with the action of the forces in producing or changing motion of fluids is known as ***"hydrokinetics"*** (or simply kinetics). Thus, the study of fluids in motion involves the consideration of both the kinematics and kinetics. The dynamic equation of fluid motion is obtained by applying Newton's second law of motion to a fluid element considered as a free body. The fluid is assumed to be incompressible and non-viscous.

In fluid mechanics the basic equations are: (i) Continuity equation, (ii) Energy equation, and (iii) Impulse momentum equation.

6.2. DIFFERENT TYPES OF HEADS (OR ENERGIES) OF A LIQUID IN MOTION

There are three types of energies or heads of flowing liquids:

1. Potential head or potential energy:
   This is due to configuration or position above some suitable datum line. It is denoted by *z*.
2. Velocity head or kinetic energy:
   This is due to velocity of flowing liquid and is measured as ($V^2/2g$) where, V is the velocity of flow and g is the acceleration due to gravity (g = 9.81).
3. Pressure head or pressure energy:
   This is due to the pressure of liquid and reckoned as (p/ɣ)
   where, p is the pressure, and ɣ is the weight density of the liquid.

Total head/energy:

Total head of a liquid particle in motion is the sum of its potential head, kinetic head and pressure head. Mathematically,

$$\text{Total head, } H = z + \frac{V^2}{2g} + \frac{p}{\gamma} \text{ m of liquid}$$

$$\text{Total energy, } E = z + \frac{V^2}{2g} + \frac{p}{\rho} \text{ Nm/kg of liquid}$$

Example 5.1. In a pipe of 90 mm diameter water is flowing with a mean velocity of 2 m/s and at a gauge pressure of 350 kN/m^2. Determine the total head, if the pipe is 8 metres above the datum line. Neglect friction.

6.3. BERNOULLI'S EQUATION

Bernoulli's equation states as follows:

"In an ideal incompressible fluid when the flow is steady and continuous, the sum of pressure energy, kinetic energy and potential (or datum) energy is constant along a stream line." Mathematically,

$$\frac{p}{} + \frac{V^2}{2g} + z = \text{constant}$$

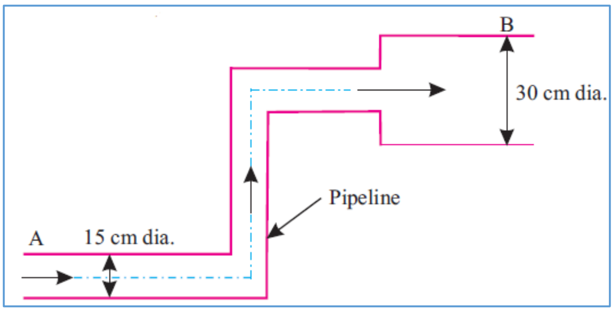
where,

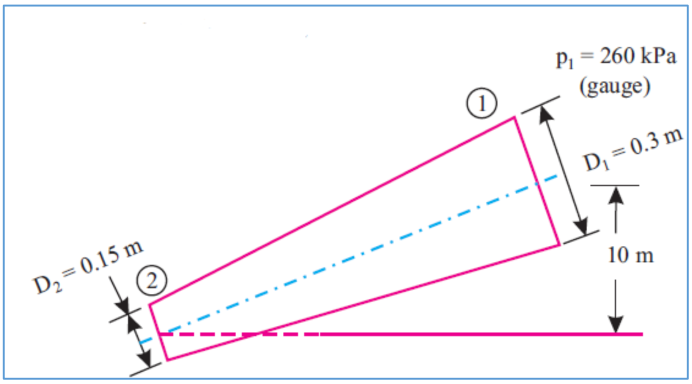
$$\frac{p}{} = \text{Pressure energy,}$$

$$\frac{V^2}{2g} = \text{Kinetic energy, and}$$

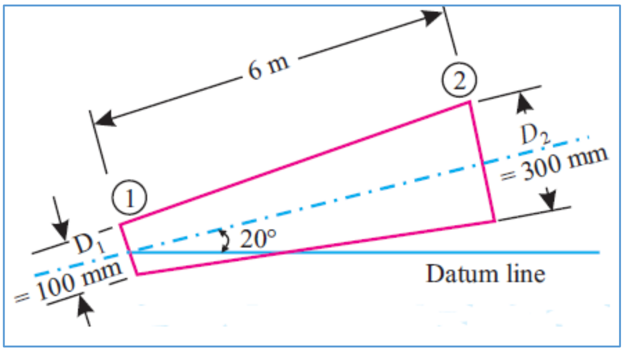
$$z = \text{Datum (or elevation) energy.}$$

Example 5.2. A pipeline is 15 cm in diameter and it is at an elevation of 100 m at section A. At section B it is at an elevation of 107 m and has diameter of 30 cm. When a discharge of 50 litre/sec of water is passed through this pipeline, pressure at A is 35 kPa. The energy loss in pipe is 2m of water. Calculate pressure at B if flow is from A to B.





Example 5.3. Water flows in a circular pipe. At one section the diameter is 0.3 m, the static pressure is 260 kPa gauge, the velocity is 3 m/s and the elevation is 10 m above ground level. The elevation at a section downstream is 0 m, and the pipe diameter is 0.15 m. Find out the gauge pressure at the downstream section. Frictional effects may be neglected. Assume density of water to be 999 kg/m^3.



Example 5.4. A 6m long pipe is inclined at an angle of 20° with the horizontal. The smaller section of the pipe which is at lower level is of 100 mm diameter and the larger section of the pipe is of 300 mm diameter. If the pipe is uniformly tapering and the velocity of water at the smaller section is 1.8 m/s, determine the difference of pressures between the two sections.

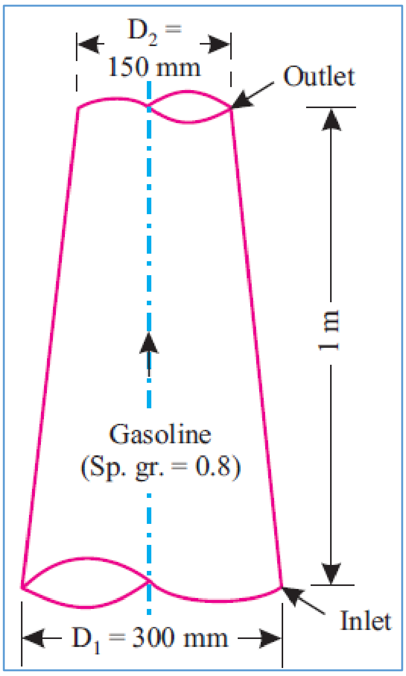
ASSIGNMENT NO.5

1.

Gasoline (sp. gr. 0.8) is flowing upwards a vertical pipeline which tapers from 300 mm to 150 mm diameter. A gasoline mercury differential manometer is connected between 300 mm and 150 mm pipe section to measure the rate of flow. The distance between the manometer tappings is 1 metre and gauge reading is 500 mm of mercury. Find:

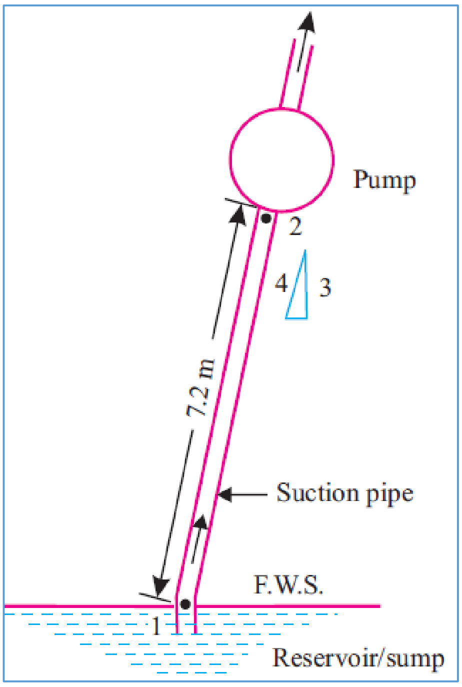
1) Differential gauge reading in terms of gasoline head;
2) Rate of flow.

Neglect friction and other losses between tappings.



2.

The suction pipe of a pump rises at a slope of 3 vertical in 4 along the pipe which is 12 cm in diameter. The pipe is 7.2 m long; its lower end being just below the water surface in the reservoir. For design reasons, it is desirable that pressure at inlet to the pump shall fall to more than 75 kPa below atmospheric pressure. Neglecting friction, determine the maximum discharge that the pump may deliver. Take atmospheric pressure as 101.32 kPa.



5.4. PRACTICAL APPLICATIONS OF BERNOULLI'S EQUATION

Bernoulli equation is one of the most useful equations in fluid mechanics and hydraulics. In addition, it is a statement of the principle of conservation of energy along a streamline.
Bernoulli Equation can be written as following:

$$\frac{P}{\rho g}+\frac{v^2}{2g}+z=H_T=\text{constant}$$

All these terms have a unit of length (m)

- $P/\rho g$ =pressure energy per unit weight=pressure head

We know that $P=\rho g h_{\text{pressure}} \rightarrow\rightarrow h_{\text{pressure}}=\frac{P}{\rho g}$ (m).

- $\frac{v^2}{2g}$ = kinetic energy per unit weight = velocity head

We know that $K.E=\frac{1}{2}mv^2 \rightarrow$ divided by weight $\rightarrow \frac{\frac{1}{2}mv^2}{mg}=\frac{v^2}{2g}$ (m).

- z = potential energy per unit weight = (potential elevation head)

We know that $P.E=mgz \rightarrow$ divided by weight $\rightarrow \frac{mgz}{mg}=z$ (m).

- H_T = total energy per unit weight = total head (m).

By using principle of conservation of energy, we can apply Bernoulli equation between two points (1 and 2) on the streamline:

Total head at (1)=Total head at (2)

$$\frac{P_1}{\rho g}+\frac{v_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{v_2^2}{2g}+z_2$$

But!!, this equation no energy losses (e.g. from friction) or energy gains (e.g. from a pump) along a stream line, so the final form for Bernoulli equation is:

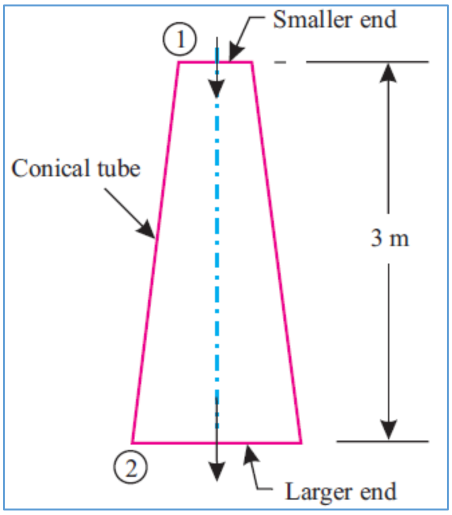
$$\frac{P_1}{\rho g}+\frac{v_1^2}{2g}+z_1+h_P=\frac{P_2}{\rho g}+\frac{v_2^2}{2g}+z_2+h_L+h_T$$

hP=q=Energy supplied by pump per unit weight (m)
hT=w=work done by turbine per unit weight (m)
hL=Total friction losses per unit weight (m)

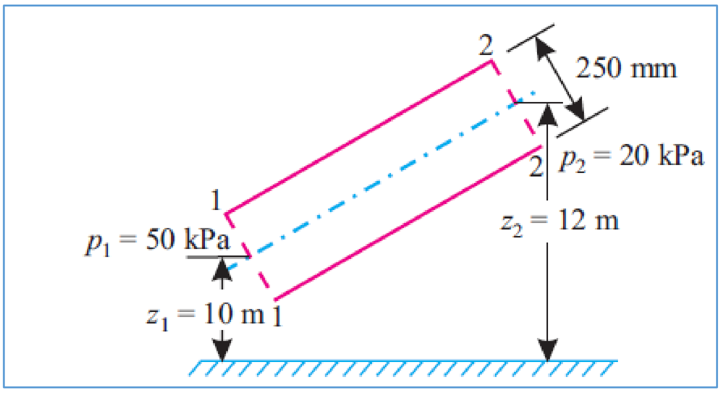
Example 5.5. The following data relate to a conical tube of length 3.0 m fixed vertically with its smaller end upwards and carrying fluid in the downward direction. The velocity of flow at the smaller end = 10 m/s. The velocity of flow at the larger end = 4 m/s.

$$\text{The loss of head in the tube} = \frac{0.4(V_1 - V_2)^2}{2g}$$

where, V_1 and V_2 are velocities at the smaller and larger ends respectively. Pressure head at the smaller end = 4 m of liquid. Determine the pressure head at the larger end.



Example 5.6. In a smooth inclined pipe of uniform diameter 250 mm, a pressure of 50 kPa was observed at section 1, which was at elevation 10 m. At another section 2 at elevation 12 m, the pressure was 20 kPa and the velocity was 1.25 m/s. Determine the direction of flow and the head loss between these two sections. The fluid in the pipe is water. The density of water at 20° C and 760 mm Hg is 998 kg/m^3.



(5.5)

Sol.:

$V_1 = 10\ m/sec$

$V_2 = 4\ m/sec$

$\frac{P_1}{\gamma} = 4^m$ of liquid

loss of head $h_L = \frac{0.4(V_1 - V_2)^2}{2g}$

$= \frac{0.4(10-4)^2}{2 \times 9.81} = 0.73\ m$

$\frac{P_2}{\gamma} = ?$

Applying B.E at section ① and ②

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 + h_L$$

$$4 + \frac{10^2}{2 \times 9.81} + 3.0 = \frac{P_2}{\gamma} + \frac{4^2}{2 \times 9.81} + 0 + 0.73$$

$\therefore \frac{P_2}{\gamma} = 10.55\ m$ of Liquid.

Sol. (5.6)

$D = 250^{mm}, 0.250\ m$

$P_1 = 50\ kpa = 50 \times 10^3\ pa$

$z_1 = 10^m\ ,\ z_2 = 12^m$

$P_2 = 20\ kpa\ ,\ 20 \times 10^3\ pa$

$v_1 = v_2 = 1.25\ m/sec\ ,\ \rho = 998\ kg/m^3$

Total energy at section 1-1

$$E_1 = \frac{P_1}{\gamma} + \frac{v_1^2}{2g} + z_1 = 15.187^m$$

Total energy at section 2-2

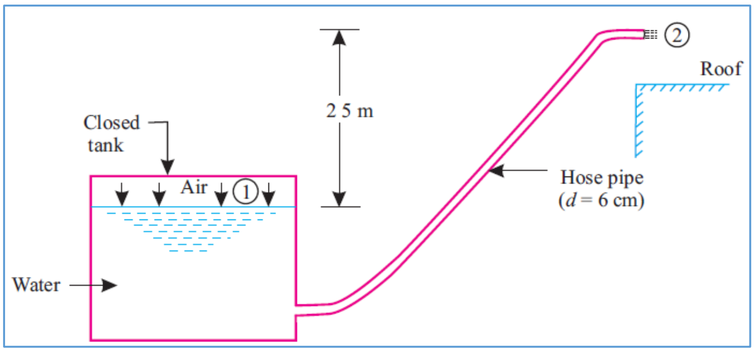
$$E_2 = \frac{P_2}{\gamma} + \frac{v_2^2}{2g} + z_2 = 14.122m$$

$$hL = E_1 - E_2 = 15.187 - 14.122 = 1.065^m$$

direction

since $E_1 > E_2$ $\therefore$ the direction of flow is from section 1-1 to section 2-2.

Example 5.7. The closed tank of a fire engine is partly filled with water, the air space above being under pressure. A (6 cm) bore connected to the tank discharges on the roof of a building 2.5 m above the level of water in the tank. The friction losses are 45 cm of water. Determine the air pressure, which must be maintained in the tank to deliver 20 litres/sec on the roof.



Sol. (5-7)

diameter of hose pipe = 6 cm = 0.06 m

friction h_f = 45 cm or 0.45 m of water

discharge, Q = 20 l/sec or 0.02 m^3/sec

$$V = \frac{Q}{A} = \frac{0.02}{\frac{\pi}{4}(0.06)^2} = 7.07 \text{ m/sec}$$

Applying B.E between 1 and 2 :-

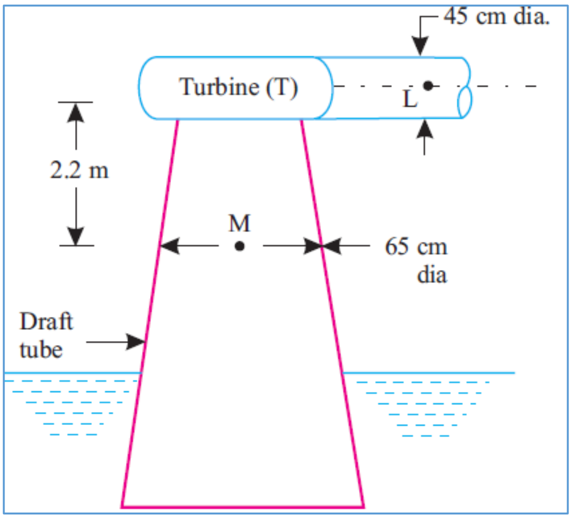
$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 + h_f$$

$$\frac{P_1}{\gamma} + 0 + 0 = 0 + \frac{(7.07)^2}{2 \times 9.81} + 2.5 + 0.45$$

$$\therefore P_1 = 5.497 * 9.81$$

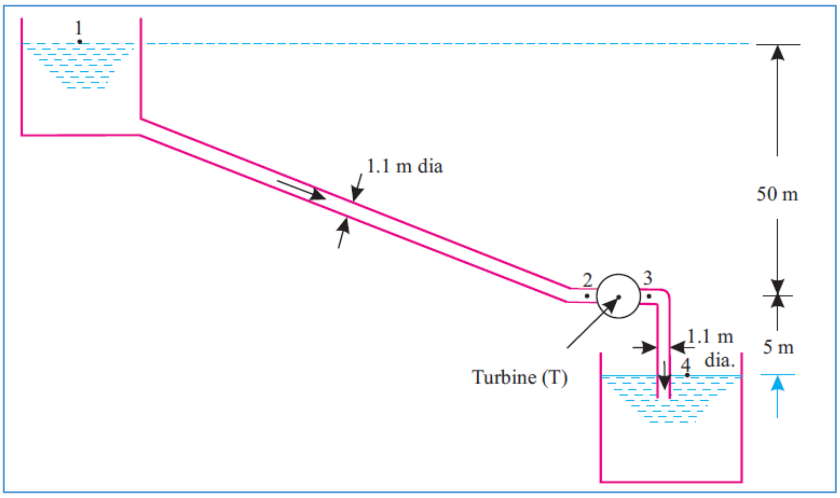
$$= 53.93 \text{ kN/m}^2$$

Example 5.8. A turbine has a supply line of diameter 45 cm and a tapering draft tube. When the flow in the pipe is 0.6 m^3/s the pressure head at point L upstream of the turbine is 35 m and at a point M in the draft tube, where the diameter is 65 cm, the pressure head is – 4.1 m. Point M is 2.2 m below the point L. Determine the power output of the turbine by assuming 92% efficiency.



Example 5.9. This figure shows a pipe connecting a reservoir to a turbine which discharges water to the tail race through another pipe. The head loss between the reservoir and the turbine is 8 times the kinetic head in the pipe and that from the turbine to the tail race is 0.4 times the kinetic head in the pipe. The rate of flow is 1.2 m^3/s and the pipe diameter in both cases is 1.1 m. Determine:

1. The pressure at the inlet and exit of the turbine.
2. The power generated by the turbine.



Sol.: (5.8)

$$V_L = \frac{Q}{A_L} = \frac{0.6}{\frac{\pi}{4}(0.45)^2} = 3.77 \ m/sec$$

$$V_M = \frac{Q}{A_M} = \frac{0.6}{\frac{\pi}{4}(0.65)^2} = 1.81 \ m/sec$$

Applying B.E to point L and M

$$\frac{P_L}{\gamma} + \frac{V_L^2}{2g} + Z_L = \frac{P_M}{\gamma} + \frac{V_M^2}{2g} + Z_M + H_T$$

$$35 + \frac{(3.77)^2}{2 \times 9.81} + 2.2 = -4.1 + \frac{(1.81)^2}{2 \times 9.81} + 0 + H_T$$

$$H_T = 41.86 \ m$$

power output of the turbine

$$P = \gamma Q H_T * \eta$$

$$= 9.81 * 0.6 * 41.86 * 0.92$$

$$= 226.68 \ kW$$

Sol. (5-9)

Diameter = 1.1 m

$Q = 1.2\ m^3/sec$

$h_{f(1-2)} = 8 * \frac{v^2}{2g}$, $h_{f(3-4)} = 0.4 * \frac{v^2}{2g}$

$v = \frac{Q}{A} = \frac{1.2}{\frac{\pi}{4}(1.1)^2} = 1.263\ m/sec$

$v_2 = v_3$ (Same diameter of pipe).

$v_1 = v_4 = 0$, $P_1 = P_4 = 0$ (atmospheric pressure).

B.E between ① and ②

$$\frac{P_1}{\gamma} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{v_2^2}{2g} + z_2 + 8\frac{v_2^2}{2g}$$

$$0 + 0 + 50 = \frac{P_2}{\gamma} + \frac{(1.263)^2}{2 \times 9.81} + 0 + 8\frac{(1.263)^2}{2 \times 9.81}$$

$\frac{P_2}{\gamma} = 49.27$ m of water
$= 483.34\ KN/m^2$ (Kpa)

B.E between ③ and ④

$$\frac{P_3}{\gamma} + \frac{v_3^2}{2g} + z_3 = \frac{P_4}{\gamma} + \frac{v_4^2}{2g} + z_4 + 0.4\frac{v_3^2}{2g}$$

$$\frac{P_3}{\gamma} + \frac{(1.263)^2}{2 \times 9.81} + 5 = 0 + 0 + 0 + 0.4\frac{(1.263)^2}{2 \times 9.81}$$

$\therefore \frac{P_3}{\gamma} = -5.049$ m of water $= -49.53$ Kpa.

B.E between ② and ③

$\frac{P_2}{\gamma} + \frac{v_2^2}{2g} + z_2 = \frac{P_3}{\gamma} + \frac{v_3^2}{2g} + z_3 + H_T \Rightarrow H_T = 54.32$ m of water

$P = \gamma \cdot Q \cdot H_T = 9.81 * 1.2 * 54.32 = \underline{639.46\ KW}$

Although Bernoulli's equation is applicable in all problems of incompressible flow where there is involvement of energy considerations but here we shall discuss its applications in the following measuring devices:

1. Venturimeter
2. Orificemeter
3. Pitot tube.

5.4.1. Venturimeter

A venturimeter is one of the most important practical applications of Bernoulli's theorem. *It is an instrument used to measure the rate of discharge in a pipeline and is often fixed permanently at different sections of the pipeline to know the discharges there.*

Types of venturimeters:

Venturimeters may be classified as follows:
1. Horizontal venturimeters.
2. Vertical venturimeters.
3. Inclined venturimeters.

5.5.1.1. Horizontal venturimeters

A venturimeter consists of the following three parts:

i. A short converging part,
ii. Throat, and
iii. Diverging part.

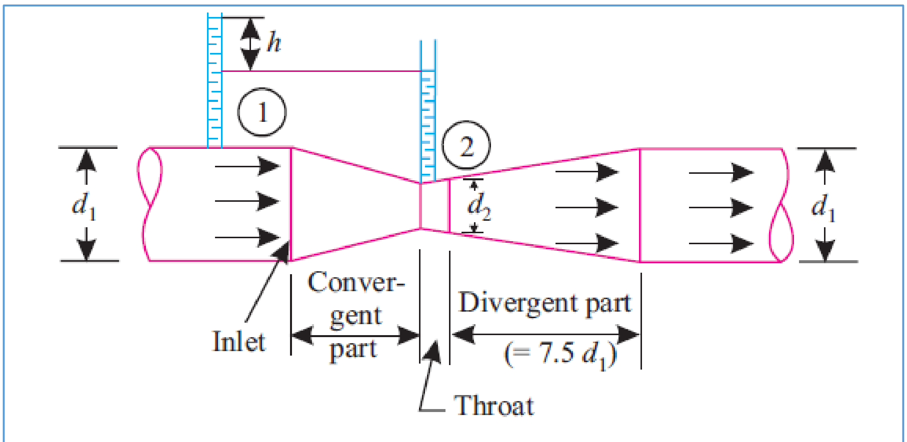
This figure shows a venturimeter fitted in horizontal pipe through which a fluid is flowing.

Let, D_1 = Diameter at inlet or at section 1,

A_1 = Area at inlet $\left(\frac{\pi}{4}d_1^2\right)$

p_1 = Pressure at section 1,

V_1 = Velocity of fluid at section 1, and D_2, A_2, p_2, and V_2 are the corresponding values at section 2.



$\left(\text{Throat ratio } \frac{d_2}{d_1} \text{ varies } \frac{1}{4} \text{ to } \frac{3}{4}\right)$

Applying Bernoulli's equation at sections 1 and 2, we get:

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2 \qquad \text{...}(i)$$

Here, $z_1 = z_2$ ... since the pipe is horizontal.

$$\therefore \qquad \frac{p_1}{w} + \frac{V_1^2}{2g} = \frac{p_2}{w} + \frac{V_2^2}{2g}$$

or,
$$\frac{p_1 - p_2}{w} = \frac{V_2^2}{2g} - \frac{V_1^2}{2g} \qquad \text{...}(ii)$$

But, $\frac{p_1 - p_2}{w}$ = Difference of pressure heads at sections 1 and 2 and is equal to h.

i.e., $$\frac{p_1 - p_2}{w} = h$$

Substituting this value of $\frac{p_1 - p_2}{w}$ in eqn. (*ii*), we get:

$$h = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$$

Applying continuity equation at sections 1 and 2, we have:

$$A_1V_1 = A_2V_2 \quad \text{or} \quad V_1 = \frac{A_2V_2}{A_1}$$

Substituting the value of V_1 in eqn. (*iii*), we get:

$$h = \frac{V_2^2}{2g} - \frac{\left(\frac{A_2V_2}{A_1}\right)^2}{2g} = \frac{V_2^2}{2g}\left(1 - \frac{A_2^2}{A_1^2}\right)$$

or, $$h = \frac{V_2^2}{2g}\left(\frac{A_1^2 - A_2^2}{A_1^2}\right) \quad \text{or} \quad V_2^2 = 2gh\left(\frac{A_1^2}{A_1^2 - A_2^2}\right)$$

or, $$V_2 = \sqrt{2gh\left(\frac{A_1^2}{A_1^2 - A_2^2}\right)} = \frac{A_1}{\sqrt{A_1^2 - A_2^2}}\sqrt{2gh}$$

$\therefore$ $$\text{Discharge, } Q = A_2V_2 = A_2\frac{A_1}{\sqrt{A_1^2 - A_2^2}} \times \sqrt{2gh}$$

or, $$Q = \frac{A_1A_2}{\sqrt{A_1^2 - A_2^2}} \times \sqrt{2gh}$$

or, $$Q = C\sqrt{h}$$

where, C = constant of venturimeter

$$= \frac{A_1A_2}{\sqrt{A_1^2 - A_2^2}}\sqrt{2g}$$

The discharge equation gives the discharge under ideal conditions and is called *theoretical discharge.* Actual discharger (Q_{act}) which is less than the theoretical discharge ($Q_{th.}$) is given by:

$$\boxed{Q_{act} = C_d \times \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \times \sqrt{2gh}}$$

C_d = Co-efficient of venturimeter (or co-efficient of discharge) and its value is less than unity (varies between 0.96 and 0.98)

* Due to variation of C_d venturimeters are not suitable for very low velocities.

Value of '*h*' by differential U-tube manometer:

Case. I. *Differential manometer containing a liquid heavier than the liquid flowing through the pipe.*

Let, S_{hl} = Sp. gravity of heavier liquid,
S_p = Sp. gravity of liquid flowing through pipe, and
y = Difference of the heavier liquid column in *U*-tube.

$$h = y\left[\frac{S_{hl}}{S_p} - 1\right]$$

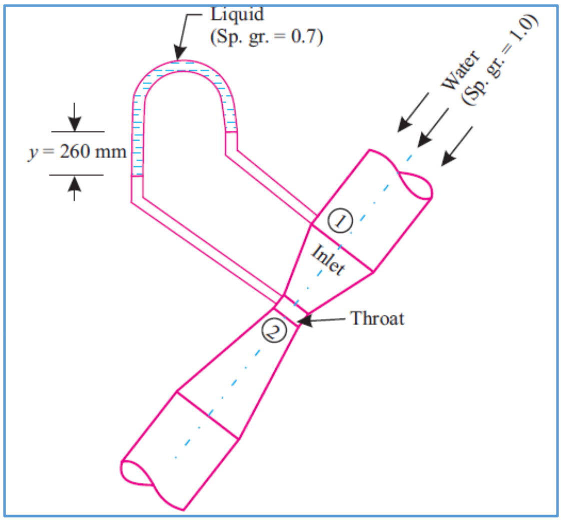
Case. II. *Differential manometer containing a liquid lighter than the liquid flowing through the pipe.*

Let, S_{ll} = Sp. gravity of lighter liquid,
S_p = Sp. gravity of liquid flowing through pipe, and
y = Difference of lighter liquid column in *U*-tube.

$$h = y\left[1 - \frac{S_{ll}}{S_p}\right]$$

Example 5.10. *A horizontal venturimeter with inlet diameter 200 mm and throat diameter 100 mm is used to measure the flow of water. The pressure at inlet is 0.18 N/mm^2 and the vacuum pressure at the throat is 280 mm of mercury. Find the rate of flow. The value of Cd may be taken as 0.98.*

Example 5.12. *Determine the rate of flow of water through a pipe of 300 mm diameter placed in an inclined position where a venturimeter is inserted, having a throat diameter of 150 mm. The difference of pressure between the main and throat is measured by a liquid of sp. gravity 0.7 in an inverted U- tube which gives a reading of 260 mm. The loss of head between the main and throat is 0.3 times the kinetic head of the pipe.*

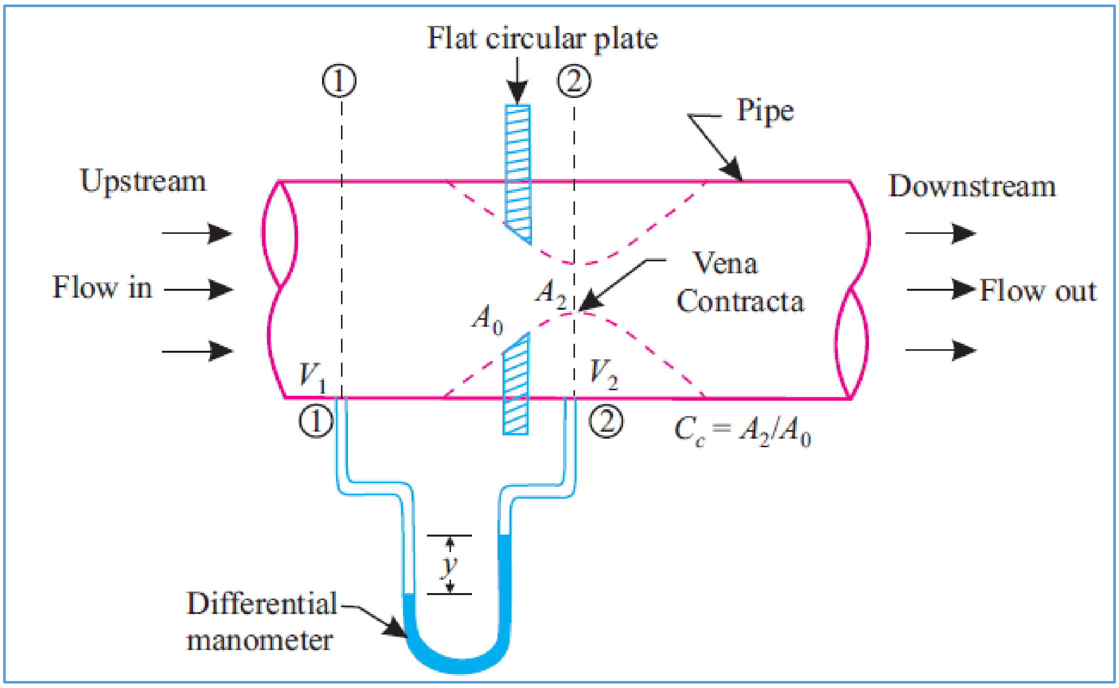


5.4.2. Orificemeter

Orificemeter or orifice plate is *a device* (*cheaper* than a venturimeter) *employed for measuring the discharge of fluid through a pipe*. It also *works on the same principle of a venturimeter*.

It consists of a flat circular plate having a circular sharp edged hole (called orifice) concentric with the pipe. The diameter of the orifice may vary from 0.4 to 0.8 times the diameter of the pipe but its value is generally chosen as 0.5. A differential manometer is connected at section (1) which is at a distance of 1.5 to 2 times the pipe diameter upstream from the orifice plate, and at section (2) which is at a distance of about half the diameter of the orifice from the orifice plate on the downstream side.

Let, $A1$ = Area of pipe at section (1),
$V1$ = Velocity at section (1),
$p1$ = Pressure at section (1), and
$A2$ $V2$ and $p2$ are corresponding values at section (2).



Applying Bernoulli's equation at sections (1) and (2), we get:

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2$$

or, $$\left(\frac{p_1}{w} + z_1\right) - \left(\frac{p_2}{w} + z_2\right) = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$$

or, $$h = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$$

$$\left[\because h = \left(\frac{p_1}{w} + z_1\right) - \left(\frac{p_2}{w} + z_2\right) = \text{differential head}\right]$$

or, $$\frac{V_2^2}{2g} = h + \frac{V_1^2}{2g} \qquad ...(i)$$

or, $$V_2 = \sqrt{2g\left(h + \frac{V_1^2}{2g}\right)} = \sqrt{2gh + V_1^2}$$

Now, section (2) is at *vena contracta* and A_2 represents the area at vena contracta. If A_0 is the area of orifice, then we have:

$$C_c = \frac{A_2}{A_0}$$

(where, C_c = co-efficient of contraction)

$$\therefore \quad A_2 = A_0 C_c \qquad ...(ii)$$

Using continuity equation, we get:

$$A_1V_1 = A_2V_2 \quad \text{or} \quad V_1 = \frac{A_2V_2}{A_1}$$

or, $$V_1 = \frac{A_0 C_c V_2}{A_1} \qquad ...(iii)$$

Substituting the value of V_1 in eqn. (*i*), we get:

$$V_2 = \sqrt{2gh + \frac{A_0^2 . C_c^2 . V_2^2}{A_1^2}}$$

or, $$V_2^2 = 2gh + \left(\frac{A_0}{A_1}\right)^2 \cdot C_2^2 \cdot V_2^2$$

or, $$V_2^2\left[1 - \left(\frac{A_0}{A_1}\right)^2 C_2^2\right] = 2gh$$

$$\therefore \quad V_2 = \frac{\sqrt{2gh}}{\sqrt{1-(A_0/A_1)^2 C_c^2}}$$

$\therefore$ The discharge, $Q = A_2V_2 = A_0.C_c.V_2$

$[\because A_2 = A_0 \,.\, C_c ...$ as above {eqn. (ii)}]

$$= A_0C_c \frac{\sqrt{2gh}}{\sqrt{1-(A_0/A_1)^2 C_c^2}} \qquad ...(iv)$$

The above expression is simplified by using,

$$C_d = C_c \frac{\sqrt{1-(A_0/A_1)^2}}{\sqrt{1-(A_0/A_1)^2 C_c^2}}$$

(where, C_d = co-efficient of discharge)

$$C_c = C_d \frac{\sqrt{1-(A_0/A_1)^2 C_c^2}}{\sqrt{1-(A_0/A_1)^2}}$$

Substituting this value of C_c in eqn. (iv), we get:

$$Q = A_0 \cdot C_d \frac{\sqrt{1-(A_0/A_1)^2 C_c^2}}{\sqrt{1-(A_0/A_1)^2}} \times \frac{\sqrt{2gh}}{\sqrt{1-(A_0/A_1)^2 C_c^2}}$$

$$= \frac{C_d \cdot A_0\sqrt{2gh}}{\sqrt{1-(A_0/A_1)^2}} = \frac{C_d \cdot A_0 \cdot A_1\sqrt{2gh}}{\sqrt{A_1^2 - A_0^2}}$$

i.e., $$Q = C_d \frac{A_0 \cdot A_1\sqrt{2gh}}{\sqrt{A_1^2 - A_0^2}}$$

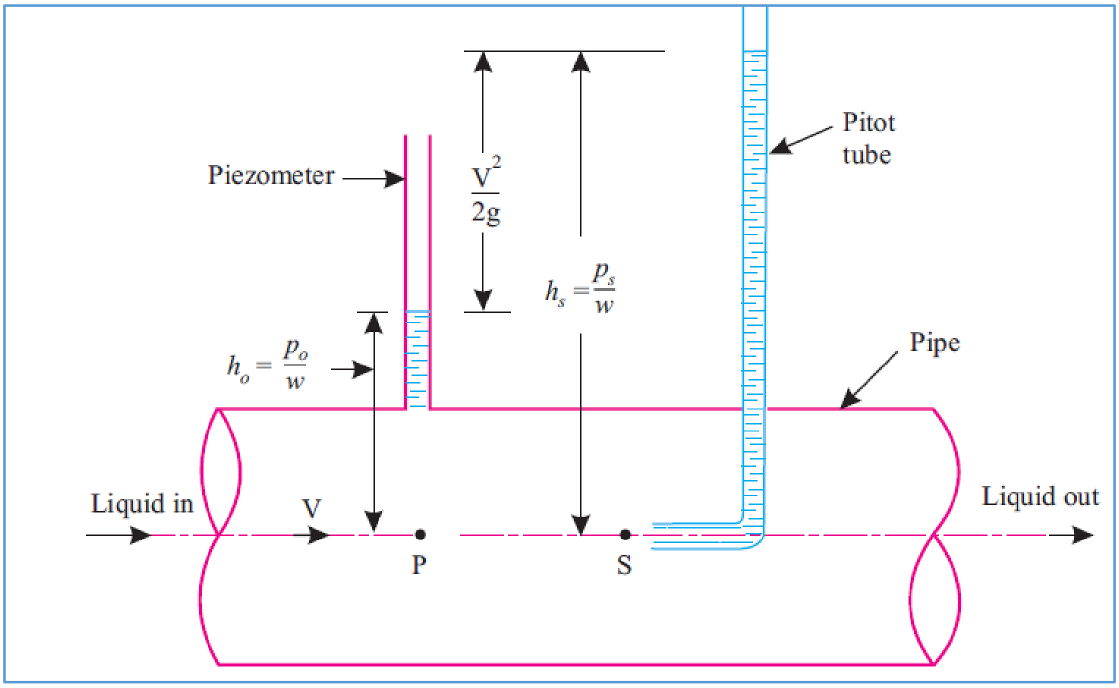
It may be noted that *Cd* (co-efficient of discharge) of an orifice is *much smaller than that of a venturimeter.*

Example 5.13. *Water flows at the rate of 0.015 m^3/s through a 100 mm diameter orifice used in a 200 mm pipe. What is the difference of pressure head between the upstream section and the vena contracta section? Take co-efficient of contraction C_c = 0.60 and C_v = 1.0.*

5.4.3. Pitot Tube

Pitot tube *is one of the most accurate devices for velocity measurement.* It works on the *principle that if the velocity of flow at a point becomes zero, the pressure there is increased due to conversion of kinetic energy into pressure.*

It consists of a glass tube in the form of a 90° bend of short length open at both its ends. It is placed in the flow with its bent leg directed upstream so that a ***stagnation point*** is created immediately in front of the opening. The kinetic energy at this point gets converted into pressure energy causing the liquid to rise in the vertical limb, to a height equal to the stagnation pressure.



Applying Bernoulli's equation between stagnation point (S) and point (P) in the undisturbed flow at the same horizontal plane, we get:

$$\frac{p_0}{w} + \frac{V^2}{2g} = \frac{p_s}{w} \quad \text{or} \quad h_0 + \frac{V^2}{2g} = h_s$$

or,

$$V = \sqrt{2g\,(h_s - h_0)} \quad \text{or} \quad \sqrt{2g\,\Delta h} \qquad \ldots(1)$$

where,
p_0 = Pressure at point 'P', *i.e.* static pressure,
V = Velocity at point 'P', *i.e.* free flow velocity,
p_s = Stagnation pressure at point 'S', and
Δh = Dynamic pressure
= Difference between stagnation pressure head (h_s) and static pressure head (h_0).

The *height of liquid* rise in the Pitot tube indicates *the stagnation head*. The static pressure head may be measured separately with a piezometer.

If a differential manometer is connected to the tubes of a Pitot static tube it will measure the dynamic pressure head ($v^2/2g$).
If y is the manometric difference, then

$$\Delta h = y\left(\frac{S_m}{S} - 1\right)$$

where,
S_m = Specific gravity of manometric liquid, and
S = Specific gravity of the liquid flowing through the pipe.

When a Pitot tube is placed in the fluid-stream the flow along its outer surface gets accelerated and causes the static pressure to decrease. Also the stem, which is perpendicular to the flow direction, tends to produce an excess pressure head. In order to take these effects into account the above equation (eq.1) is modified to give the actual velocities as:

$$V = C\sqrt{2g\Delta h}$$

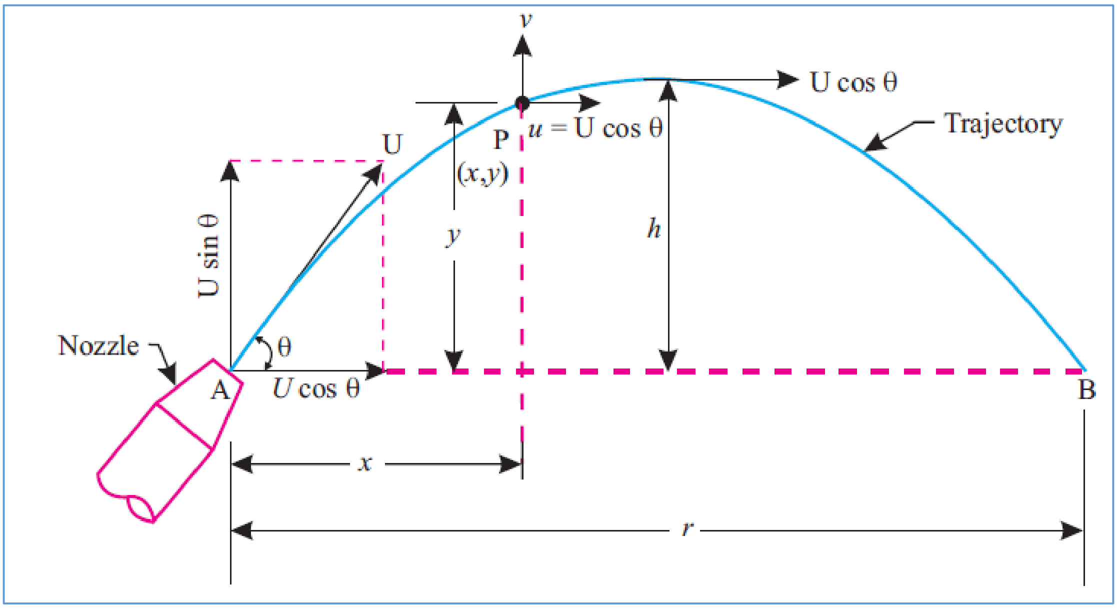
where, C = A connective coefficient which takes into account the effect of stem and bent leg.
The most commonly used form of Pitot static tube known as the Prandle-Pitot-tube is so designed that the effect of stem and bent leg cancel each other, i.e., C = 1.

Example 5.14. *A submarine fitted with a Pitot tube moves horizontally in sea. Its axis is 12 m below the surface of water. The Pitot tube fixed in front of the submarine and along its axis is connected to the two limbs of a U-tube containing mercury, the reading of which is found to be 200 mm. Find the speed of the submarine. Take the specific gravity of sea water = 1.025 times fresh water.* effect of stem and bent leg cancel each other, *i.e.*, $C = 1$.

5.5 FREE LIQUID JET

Refer to figure below. A jet of liquid issuing from the nozzle in atmosphere is called a **free liquid jet**. The parabolic path traversed by the liquid jet under the action of gravity is known as **trajectory**.

Let the jet A make an angle θ with the horizontal direction. If U is the velocity of the water jet, then U cos θ and and U sin θ are the horizontal and vertical components of this velocity respectively. Consider another point P(x, y) on the centre line of the jet.



Let, u = Velocity of the jet at point P in X- direction,
v = Velocity of the jet at point P in Y-direction, and
t = Time taken by a liquid particle to reach from A to P.

Then, $x = u \times t = U\cos\theta \times t$ (where, $u = U\cos\theta$) ...(*i*)

$$y = U\sin\theta \times t - \frac{1}{2}gt^2 \qquad ...(ii)$$

(It may be noted that horizontal component of velocity U is $U\cos\theta$ which *remains constant* whereas the vertical component $U\sin\theta$ is *affected by gravity*.)

From eqn. (*i*) we have, $t = \dfrac{x}{U\cos\theta}$

Substituting the value of t in eqn. (*ii*), we get:

$$y = U\sin\theta \times \frac{x}{U\cos\theta} - \frac{1}{2}g \times \frac{x^2}{U^2\cos^2\theta}$$

$$= x\tan\theta - \frac{gx^2}{2U^2\cos^2\theta}$$

$$y = x\tan\theta - \frac{gx^2\sec^2\theta}{2U^2} \qquad \left(\because \frac{1}{\cos^2\theta} = \sec^2\theta\right)$$

This is the equation of *a parabola.*

(*i*) Maximum height attained by the jet, h:

Using the relation:

$$V_2^2 - V_1^2 = -2gh \text{ (}-ve\text{ sign is used as the particle is moving } upward\text{)}$$

where, V_1 = Initial vertical component = U sin θ, and

$V_2 = 0$ at the highest point.

$$\therefore \quad 0 - (U\sin\theta)^2 = -2gh$$

or
$$h = \frac{U^2\sin^2\theta}{2g}$$

(*ii*) Time of flight, T:

Time of flight is the time taken by the fluid particle in reaching from *A* to *B* (Fig. 6.41). From eqn. (*ii*), we have:

$$y = U\sin\theta \times t - \frac{1}{2}gt^2$$

When the particle reaches the point B, y = 0, t = T
Putting these values in the above equation, we get:

$$0 = U\sin\theta \times T - \frac{1}{2}g \times T^2$$

or,
$$T = \frac{2U\sin\theta}{g}$$

Time taken to reach the highest point, $T' = \frac{T}{2} = \frac{2U\sin\theta}{2g} = \frac{U\sin\theta}{g}$

i.e.,
$$T' = \frac{U\sin\theta}{g}$$

(*iii*) Horizontal range of the jet, *r*:

The range (*r*) of the jet is the total *horizontal distance travelled by the fluid particle.*

Then *r*, (*i.e.*, distance AB) = Velocity component in direction × time taken by the particle to reach from *A* to *B*

$$= U\cos\theta \times T = U\cos\theta \times \frac{2U\sin\theta}{g}$$

$$= \frac{U^2 \times 2\sin\theta \times \cos\theta}{g} = \frac{U^2 \sin 2\theta}{g}$$

i.e., $$r = \frac{U^2 \sin 2\theta}{g}$$

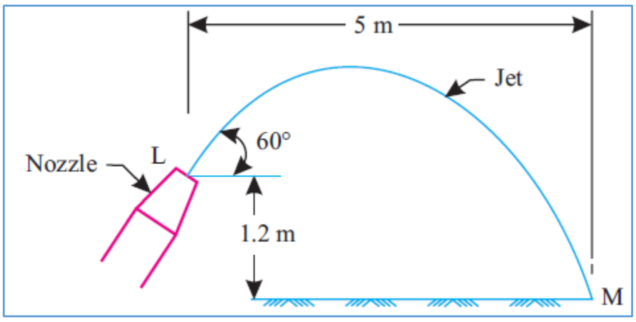
The range will be maximum, when $\sin 2\theta = 1$

i.e., $$2\theta = 90° \text{ or } \theta = 45°$$

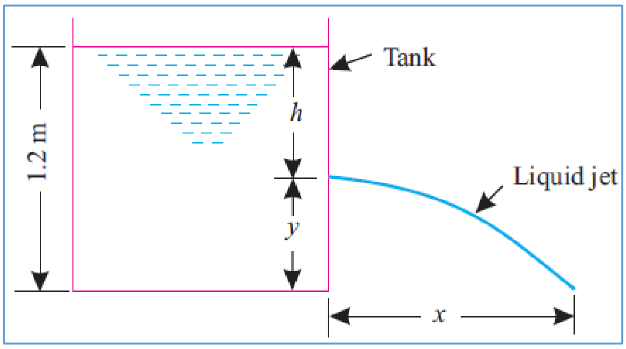
Then maximum range, $$r_{max} = \frac{U^2 \sin(2 \times 45°)}{g} = \frac{U^2}{g}$$

i.e., $$r_{max} = \frac{U^2}{g}$$

Example 5.15. A nozzle is situated at a distance of 1.2 m above the ground level and is inclined at 60° to the horizontal. The diameter of the nozzle is 40 mm and the jet of water from the nozzle strikes the ground at a horizontal distance of 5 m. Find the flow rate.



Example 5.16. It is required to place an orifice in the side of a tank at such an elevation that the jet will attain a maximum horizontal distance from the tank at the level of its base. What is the proper distance from the orifice to the free surface when the depth of liquid in the tank is maintained at 1.2 m?



5.5. IMPULSE-MOMENTUM EQUATION

The **impulse-momentum equation** is *one of the basic tools* (other being continuity and Bernoulli's equations) *for the solution of flow problems*. Its application leads to the solution of problems in fluid mechanics which *cannot* be solved by energy principles alone. Sometimes it is used in conjunction with the energy equation to obtain complete solution of engineering problems.
The momentum equation is based on the *law of conservation of momentum* or *momentum principle* which states as follows:
"*The net force acting on a mass of fluid is equal to change in momentum of flow per unit time in that direction*".

As per Newton's second law of motion,

$$F = ma$$

where, m = Mass of fluid,
F = Force acting on the fluid, and
a = Acceleration (acting in the same direction as F).

But acceleration, $$a = \frac{dv}{dt}$$

$$\therefore \quad F = m.\frac{dv}{dt} = \frac{d(mv)}{dt}$$

('m' is taken inside the differential, being constant)
This equation is known as **momentum principle.** It can also be written as:

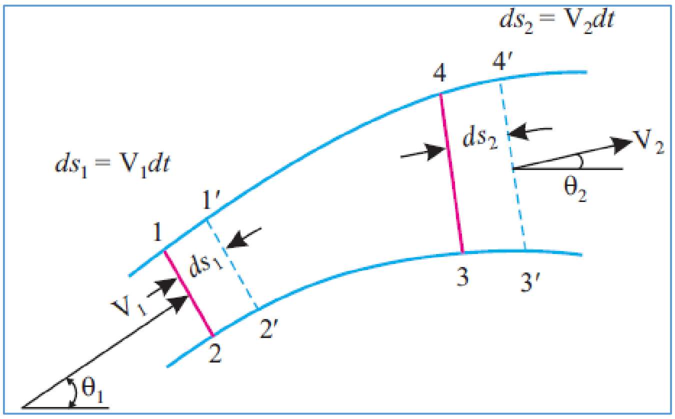
$$F.dt = d\,(mv)$$

This equation is known as **Impulse-momentum equation.** It may be stated as follows:
"*The impulse of a force F acting on a fluid mass 'm' in a short interval of time dt is equal to the change of momentum d(mv) in direction of force*".
The impulse-momentum equations are often called simply *momentum equations*.

Steady flow momentum equation:

The entire flow space may be considered to be made up of innumerable stream tubes. Let us consider one such stream tube lying in the *X*-*Y* plane and having steady flow of fluid. Flow can be assumed to be uniform and normal to the inlet and outlet areas.

Let, V_1, ρ_1 = Average velocity and density (of fluid mass) respectively at the entrance, and V_2, ρ_2 = Average velocity and density respectively at the exit.



Further let the mass of fluid in the region 1 2 3 4 shifts to new position 1′ 2′ 3′ 4′ due to the effect of external forces on the stream after a short interval. Due to gradual increase in the flow area in the direction of flow, velocity of fluid mass and hence the momentum is gradually reduced. Since the area 1′ 2′ 3 4 is common to both the regions 1 2 3 4 and 1′ 2′ 3′ 4′, therefore, it will not experience any change in momentum. Obviously, then the changes in momentum of the fluid masses in the sections 1 2 2′ 1′ and 4 3 3′ 4′ will have to be considered

According to the *principle of mass conservation*,

Fluid mass with the region 1 2 2′ 1′ = Fluid mass within the region 4 3 3′ 4′

$$\rho_1 A_1 ds_1 = \rho_2 A_2 ds_2$$

∴ Momentum of fluid mass contained in the region 1 2 2′ 1′

$$= (\rho_1 A_1 ds_1) V_1 = (\rho_1 A_1 V_1 . dt) V_1$$

Momentum of fluid mass contained in the region 4 3 3′ 4′

$$= (\rho_2 A_2 ds_2) V_2 = (\rho_2 A_2 V_2 . dt) V_2$$

∴ Change in momentum = $(\rho_2 A_2 V_2 . dt) V_2 - (\rho_1 A_1 V_1 . dt) V_1$

But, $\rho_1 = \rho_2 = \rho$ ...for steady incompressible flow

and, $A_1 V_1 = A_2 V_2 = Q$ ...from continuity considerations

∴ Change in momentum = $\rho Q(V_2 - V_1)\, dt$

Using impulse-momentum principle, we have:

$$F dt = \rho Q (V_2 - V_1) dt$$

Or,
$$F = (\gamma Q / g) (V_2 - V_1)$$

The quantity $\gamma Q/g = \rho Q$ is the mass flow per second and is called **mass flux.**

Resolving V_1 and V_2 along X-axis and Y-axis, we get:

Components along X-axis: $V_1 \cos \theta_1$ and $V_2 \cos \theta_2$
Components along Y- axis : $V_1 \sin \theta_1$ and $V_2 \sin \theta_2$
(where, θ_1 and θ_2 are the inclinations with the horizontal of the centre line of the pipe at 1-2 and 3-4).
∴ Components of force *F* along *X*-axis and *Y*-axis are:

$$F_x = (\gamma\ Q\ /g)\ (V_2 \cos \theta_2 - V_1 \cos \theta_1)$$
$$F_y = (\gamma\ Q\ /g)\ (V_2 \sin \theta_2 - V_1 \sin \theta_1) \quad \ldots .(**)$$

The equation (**) represents the components of the force exerted by the *pipe bend on the fluid mass.*
Usually, we are interested in the forces by the fluid on the pipe bend. Since action and reaction are equal and opposite (Newton's third law of motion), the fluid mass would exert the same force on the pipe bend but in *opposite direction* and as such the force components exerted by the fluid on the pipe bend are given as follows:

$$F_x = (\gamma\ Q\ /g)\ (V_1 \cos \theta_1 - V_2 \cos \theta_2)$$
$$F_y = (\gamma\ Q\ /g)\ (V_1 \sin \theta_1 - V_2 \sin \theta_2) \quad \ldots .(**)$$

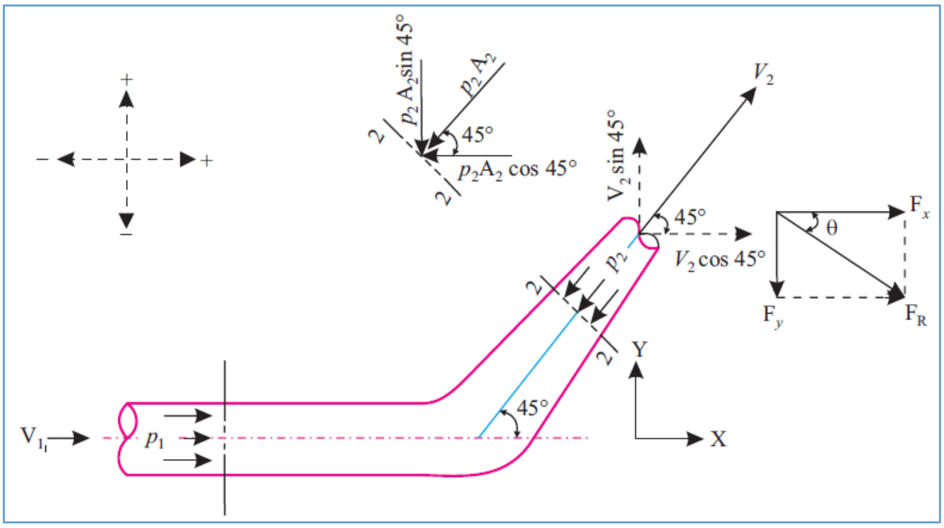
The magnitude of the resultant force acting on the pipe bend,

$$F_R = \sqrt{F_x^2 + F_y^2}$$

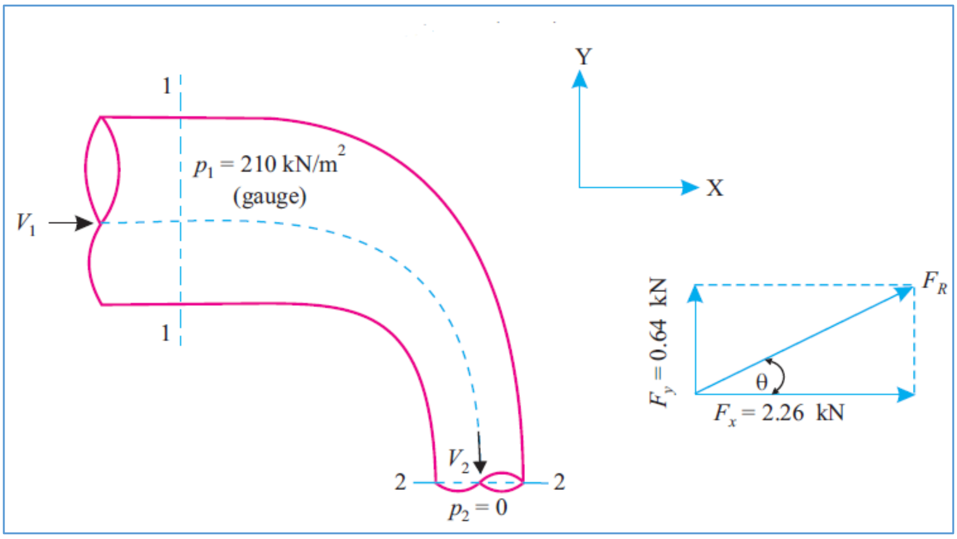
and, the direction of the resultant force with *X*-axis,

$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right)$$

Example 5.17. In a 45° bend a rectangular air duct of 1 m^2 cross-sectional area is gradually reduced to 0.5 m^2 area. Find the magnitude and direction of force required to hold the duct in position if the velocity of flow at 1 m^2 section is 10 m/s, and pressure is 30 kN/m^2. Take the specific weight of air as 0.0116 kN/m^3.


$p_2 A_2 \sin 45°$
$p_2 A_2$
45°
$p_2 A_2 \cos 45°$
V_2
$V_2 \sin 45°$
45°
$V_2 \cos 45°$
F_x
θ
F_y
F_R
p_2
2
2
2
2
Y
X
45°
V_1
p_1

Example 5.18. This figure shows a 90° reducer-bend through which water flows. The pressure at the inlet is 210 kN/m^2 (gauge) where the cross-sectional area is 0.01 m^2. At the exit section, the area is 0.0025 m^2 and the velocity is 16 m/s. The pressure at the exit is atmospheric. Determine the magnitude and direction of the resultant force on the bend.



5.5.1. Applications of impulse-momentum equation:

The impulse-momentum equation is used in the following types of problems:

1. *To determine the resultant force acting on the boundary of flow passage by a stream of fluid as the stream changes its direction, magnitude or both.* Problems of this type are: (*i*) Pipe bends, (*ii*) Reducers, (*iii*) Moving vanes, (*iv*) Jet propulsion, etc.
2. *To determine the characteristic of flow when there is an abrupt change of flow section.* Problems of this type are: (*i*) Sudden enlargement in a pipe, (*ii*) Hydraulic jump in a channel, etc.

Example 5.19. 360 litres per second of water is flowing in a pipe. The pipe is bent by 120° .The pipe bend measures 360 mm $\times$ 240 mm and volume of the bend is 0.14 m^3. The pressure at the entrance is 73 kN/m^2 and the exit is 2.4 m above the entrance section. Find the force exerted on the bend.