COMPACTION OF SOIL

'Compaction' of soil may be defined as the process by which the soil particles are artificially rearranged and packed together into a state of closer contact by mechanical means in order to decrease its porosity and thereby increase its dry density. This is usually achieved by dynamic means such as tamping, rolling, or vibration. The process of compaction involves the expulsion of air only.

- **COMPACTION PHENOMENON**

The process of compaction is accompanied by the expulsion of air only. In practice, soils of medium cohesion are compacted by means of rolling, while cohesionless soils are most effectively compacted by vibration. The degree of compaction of a soil is characterized by its dry density. The degree of compaction depends upon the moisture content, the amount of compactive effort or energy expended and the nature of the soil. The following

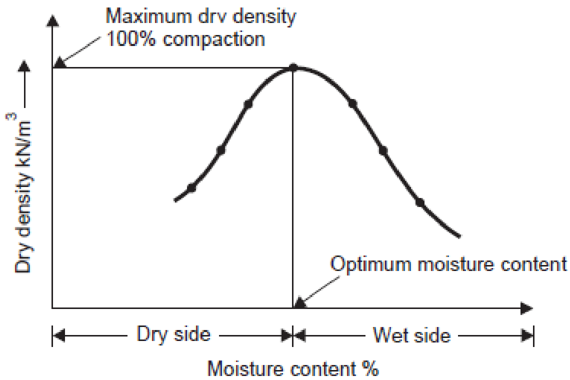
(*i*) Compaction increases the dry density of the soil, thus increasing its shear strength and bearing capacity through an increase in frictional characteristics;

(*ii*) Compaction decreases the tendency for settlement of soil; and,

(*iii*) Compaction brings about a low permeability of the soil.

- **Moisture Content—Dry Density Relationship.**

The relation between moisture content and dry density of a soil at a particular compaction energy or effort is shown in Fig. below.



The addition of water to a dry soil helps in bringing the solid particles together by coating them with thin films of water. At low water content, the soil is stiff and it is difficult to pack it together. As the water content is increased, water starts acting as a lubricant, the particles start coming closer due to increased workability and under a given amount of compactive effort, the soil-water-air mixture starts occupying less volume, thus effecting gradual increase in dry density.

The wet density and the moisture content are required in order to calculate the dry density as follows:

$$\gamma_d = \frac{\gamma}{(1+w)}, \text{ where}$$

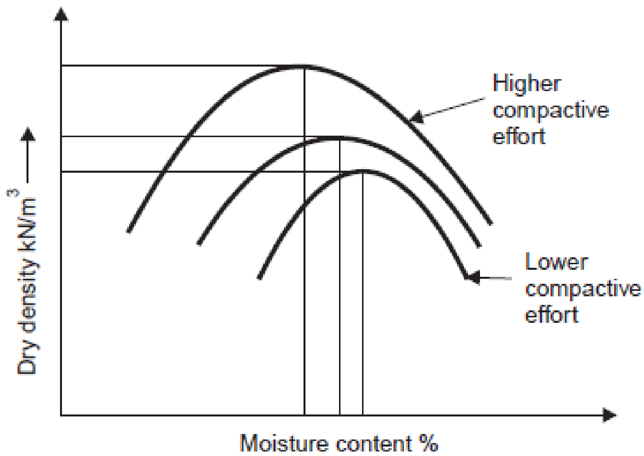
γ_d = dry density,

γ = wet (bulk) density,

w = water content, expressed as a fraction.

- **Effect of Compactive Effort**

Increase in compactive effort or the energy expended will result in an increase in the maximum dry density and a corresponding decrease in the optimum moisture content, as illustrated in Fig. below



j. 12.2 Effect of compactive effort on compaction characteristics

- **SATURATION (ZERO-AIR-VOIDS) LINE**

A line showing the relation between water content and dry density at a constant degree of saturation *S* may be established from the equation:

$$\gamma_d = \frac{G\gamma_w}{\left(1+\frac{wG}{S}\right)}$$

water content in %.

The lines thus obtained on a plot of γ_d versus *w* are called 95% saturation line, 90% saturation line and so on.

If one substitutes *S* = 100% and plots the corresponding line, one obtains the theoretical saturation line, relating dry density with water content for a soil containing no air voids. It is said to be 'theoretical' because it can never be reached in practice as it is impossible to expel the pore air completely by compaction. We then use

$$\gamma_d = \frac{G\gamma_w}{\left(1+\frac{wG}{100}\right)} \text{ for this situation.}$$

$$\gamma_d = \frac{G\gamma_w}{\left(1+\frac{wG}{100}\right)}$$ for this situation.

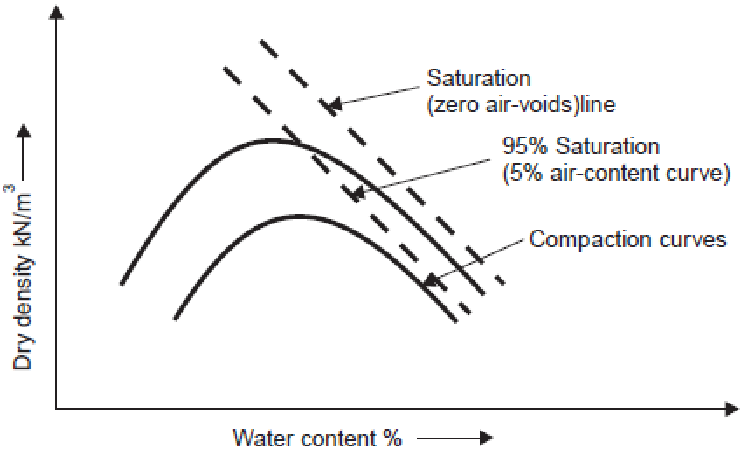


Fig. 12.3 Saturation lines superimposed on compaction curves

- **LABORATORY COMPACTION TESTS**

The various procedures used in the laboratory compaction tests involve application of impact loads, kneading, static loads, or vibration.

1- Standard Proctor Test (ASTM, D 698). Three layers with 2.5 Kg hammer weight and 305 mm height of drop with 25 blows.

2- Standard Modified Proctor Test (ASTM, D1557). Five layers with 5 Kg hammer weight and 457 mm height of drop with 25 blows.

- **IN-SITU OR FIELD COMPACTION**

The construction of a structural fill usually consists of two distinct operations—placing and spreading in layers and then compaction. The first part assumes greater significance in major jobs such as embankments and earth dams where the soil to be used as a construction material has to be excavated from a suitable borrow area and transported to the work site.

The phase of compaction may be properly accomplished by the use of appropriate equipment for compaction. The thickness of layers that can be properly compacted is known to be related to the type of soil and method or equipment of compaction.

Generally speaking, granular soils can be adequately compacted in thicker layers than fine-grained soils and clays; also, for a given soil type, heavy compaction equipment is capable of compacting thicker layers than light equipment.

Soil compaction or densification can be achieved by different means such as tamping action, kneading action, vibration, and impact. Compactors operating on the tamping, kneading and impact principle are effective in the case of cohesive soils, while those operating on the kneading, tamping and vibratory principle are effective in the case of cohesionless soils.

The primary types of compaction equipment are: ***(i) rollers, (ii) rammers and (iii) vibrators***. Of these, by far the most common are rollers.

Rollers are further classified as follows:

(*a*) Smooth-wheeled rollers,

(*b*) Pneumatic-tyred rollers,

(*c*) Sheepsfoot rollers, and

(*d*) Grid rollers.

Vibrators are classified as: **(a) Vibrating drum, (b) Vibrating pneumatic tyre (c) Vibrating plate, and (d) vibroflot.**

The methods available for the determination of *in-situ* unit weight are:

(*a*) Sand-replacement method, (*b*) Core-cutter method, (*c*) Volumenometer method, (*d*) Rubber balloon method, (*e*) Nuclear method, (*f*) Proctor plastic needle method.

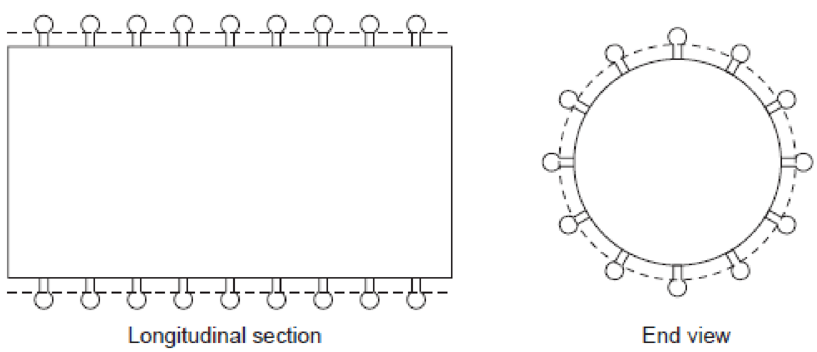
- **Types of In-situ Compaction Equipment**

Rollers

(a) Smooth-wheeled rollers: This type imparts static compression to the soil. There may be two or three large drums; if three drums are used, two large ones in the rear and one in the front is the common pattern. This type appears to be more suitable for compacting granular base courses and paving mixtures for highway and airfield work rather than for compacting earth fill. The common weight is ***80 kN*** to **100 kN** or 200 kN.

(b) Pneumatic-tyred rollers: This type compacts primarily by kneading action. The usual form is a box or container—mounted on two axles to which pneumatic-tyred wheels are fitted; the front axle will have one wheel less than the rear and the wheels are mounted in a staggered fashion so that the entire width between the extreme wheels is covered, This type is suitable for compacting most types of soil and has particular advantages with wet cohesive materials. The weight 120 kN (12 t) to 450 kN (45 t), although an exceptionally heavy capacity of 2000 kN (200 t)

c) *Sheepsfoot rollers:* This type of roller consists of a hollow steel drum provided with projecting studs or feet; the compaction is achieved by a combination of tamping and kneading. The drum can be filled with water or sand to provide and control the dead weight.



Initially, the projections sink into the loose soil and compact the soil near the lowest portion of the layer. This type of roller is found suitable for cohesive soils. It is unsuitable for granular soils as the studs tend to loosen these continuously.

The contact pressures of the feet may range from 700 kN/m^2 (7 kg/cm^2) to 4200 kN/m^2 (42 kg/cm^2) and weight per drum from 25 kN (2.5 t) to 130 kN (13 t).

Example 13. Compute the total, effective and pore pressure at a depth of 15 m below the bottom of a lake 6 m deep. The bottom of the lake consists of soft clay with a thickness of more than 15 m. The average water content of the clay is 40% and the specific gravity of soils may be assumed to be 2.65.

Water content $w_{sat} = 40\%$

Specific gravity of solids, $G = 2.65$

Void ratio, $e = w_{sat} \cdot G$

$$= 0.4 \times 2.65$$

$$= 1.06$$

$$\gamma_{sat} = \left(\frac{G + e}{1 + e}\right)\gamma_w$$

$$= \frac{(2.65 + 1.06)}{(1 + 1.06)} \times 9.81 \text{ kN/m}^3$$

$$= 17.67 \text{ kN/m}^3$$

Total stress at 15 m below the bottom of the lake:

$$\sigma = 6 \times 9.81 + 15 \times 17.67 = 323.9 \text{ kN/m}^2$$

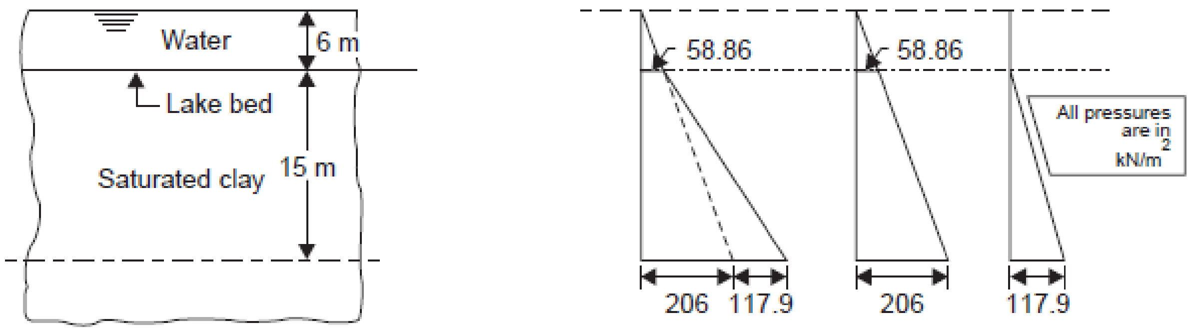
Neutral stress at 15 m below the bottom of the lake:

$$u = 21 \times 9.81 \text{ kN/m}^3 = \mathbf{206.0 \text{ kN/m}^2}$$

Effective stress at 15 m below the bottom of the lake:

$$\sigma' = 323.9 - 206.0 = \mathbf{117.9 \text{ kN/m}^2}$$

Also,
$$\bar{\sigma} = 15 \times \gamma' = 15 \times (\gamma_{sat} - \gamma_w)$$
$$= 15(17.67 - 9.81)$$
$$= 117.9 \text{ kN/m}^2$$



(a) Total pressure (b) Neutral pressure (c) Effective pressure

Example 14. A uniform soil deposit has a void ratio 0.6 and specific gravity of 2.65. The natural ground water is at 2.5 m below natural ground level. Due to capillary moisture, the average degree of saturation above ground water table is 50%. Determine the neutral pressure, total pressure and effective pressure at a depth of 6 m. Draw a neat sketch.

Void ratio, $e = 0.6$

Specific gravity $G = 2.65$

$$\gamma_{sat} = \left(\frac{G+e}{1+e}\right)\cdot\gamma_w = \frac{(2.65+0.60)}{(1+0.60)} \times 9.81 \text{ kN/m}^3$$
$$= 19.93 \text{ kN/m}^3$$

γ at 50% saturation

$$= \left(\frac{G+Se}{1+e}\right)\cdot\gamma_w = \frac{(2.65\times 0.5+0.60)}{(1+0.60)} \times 9.81 \text{ kN/m}^3 = 18.09 \text{ kN/m}^3.$$

Total pressure, σ at 6 m depth $= 2.5 \times 18.09 + 3.5 \times 19.93$

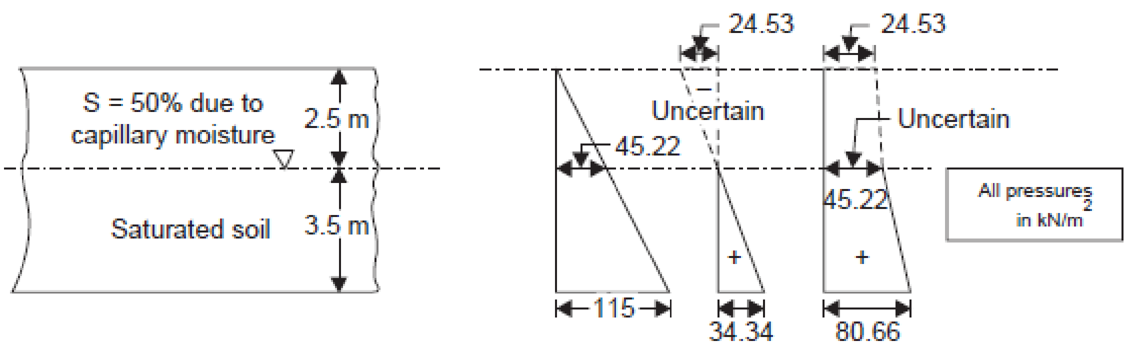
$= 115 \text{ kN/m}^2$

Neutral pressure, u at 6 m depth $= 3.5 \times 9.81 = \mathbf{34.34 \text{ kN/m}^2}$

Effective pressure, $\bar{\sigma}$ at 6 m depth $= (\sigma - u)$

$= 115.00 - 34.34 = \mathbf{80.66 \text{ kN/m}^2}$

The pressure diagrams are shown in Fig. 5.28.



(a) Total pressure (b) Neutral pressure
(c) Effective pressure

It may be pointed out that the pore pressure in the zone of partial capillary saturation is difficult to predict and hence the effective pressure in this zone is also uncertain. It may be a little more than what is given here.

Example 15: An earth embankment is compacted at a water content of 18% to a bulk density of 19.2 kN/m^3. If the specific gravity of the sand is 2.7, find the void ratio and the degree of saturation of the compacted embankment.

Water content, w = 18%, Bulk density, γ = 19.2 kN/m^3, Specific gravity, G = 2.7

Dry density, $\gamma_d = \dfrac{\gamma}{(1+w)} = \dfrac{19.2}{(1+0.18)} = 16.27$ kN/m^3

But $\gamma_d = \dfrac{G.\gamma_w}{(1+e)}$, where γ_w 9.81 kN/m^3

$\therefore$ $(1+e) = \dfrac{2.7 \times 9.81}{16.27} = 1.63$

Void ratio, $e = 0.63$

Also, $wG = S.e$

$\therefore$ The degree of saturation, $S = \dfrac{wG}{e} = \dfrac{0.18 \times 2.7}{0.63}$

$= 0.7714$

$\therefore$ The degree of saturation = **77.14%**

Example 17: A moist soil sample compacted into a mould of 1000 cm^3 capacity and weight 35 N, weighs 53.5 N with the mould. A representative sample of soil taken from it has an initial weight of 0.187 N and even dry weight of 0.1691 N. Determine (a) water content, (b) wet density, (c) dry density, (d) void ratio and (e) degree of saturation of sample. If the soil sample is so compressed as to have all air expelled, what will be the new volume and new dry density?

(a) Water content, $w = \dfrac{(0.1870 - 0.1691)}{0.1691} \times 100 = $ **10.58%**

Wet wt. of soil in the mould = (53.50 – 35.00) = 18.50 N

Volume of mould = 1000 cm^3

(*b*) Bulk density, $\gamma = \frac{18.5 \times (100)^3}{1000 \times 1000}$ **= 18.5 kN/m³**

(*c*) Dry density, $\gamma_d = \frac{\gamma}{(1+w)} = \frac{1850}{(1+0.1058)}$ **= 16.73 kN/m³**

$$\gamma_d = \frac{G\gamma_w}{(1+e)}$$

Assuming a value of 2.65 for grain specific gravity,

$$16.73 = \frac{2.65 \times 10}{(1+e)}, \text{since } \gamma_w \approx 10 \text{ kN/m}^3$$

$$(1+e) = \frac{2.65 \times 10}{16.73} = 1.584$$

(*d*) Void ratio, e = **0.584**

Also, $wG = S.e$

$$S = \frac{wG}{e} = \frac{0.1058 \times 2.65}{0.584} = 0.48$$

(*e*) Degree of saturation, S = **48%**

If air is fully expelled, the solid is fully saturated at that water content.

$\therefore \quad wG = e = 2.65 \times 0.1058 = 0.28$

New dry density $= \frac{2.65 \times 10}{(1+0.28)}$ **= 20.7 kN/m³**

New volume $= \frac{(1+0.28)}{(1+0.584)} \times 1000$ **= 808 cm³**

Example 18: The soil in a borrow pit has a void ratio of 0.90. A fill-in-place volume of 20,000 m³ is to be constructed with an in-place dry density of 18.84 kN/m³. If the owner of borrow area is to be compensated at 1.50 $ per cubic metre of excavation, determine the cost of compensation. If assuming grain specific gravity as 2.70 and taking γ_w as 9.81 kN/m³.

$$18.84 = \frac{2.70 \times 9.81}{(1+e_i)}$$

$$(1+e_i) = \frac{2.70 \times 9.81}{18.84} = 1.406$$

$e_i = 0.406$ (in-plane Void ratio)

Void-ratio of the soil in the borrow-pit,

$e_b = 0.90$

In-place volume of the fill, $V_i = 20{,}000$ m³

If the volume of the soil to be excavated from the borrow pit is V_b, then:

$$\frac{V_b}{V_i} = \frac{1+e_b}{1+e_i} = \frac{(1+0.90)}{(1+0.406)}$$

$$\therefore \qquad V_b = \frac{1.90}{1.406} \times 20{,}000 \text{ m}^3 = \mathbf{27.027} \text{ m}^3$$

a cost be paid to the owner of the borrow area = (1.50 × 27,027) = 40,540 $.

7- PERMEABILITY

7.1. Soil moisture

Water present in the void spaces of a soil mass is called ‘Soil water’. Specifically, the term ‘soil moisture’ is used to denote that part of the sub-surface water which occupies the voids in the soil above the ground water table.

7.1.1 Gravitational Water

‘Gravitational water’ is the water in excess of the moisture that can be retained by the soil. It translocate as a liquid and can be drained by the gravitational force. It is capable of transmitting hydraulic pressure.

Gravitational water can be subdivided into (a) free water (bulk water) and (b) Capillary water. Free water may be further distinguished as (i) Free surface water and (ii) Ground water.

(a) Free water (bulk water). It has the usual properties of liquid water. It moves at all times under the influence of gravity, or because of a difference in hydrostatic pressure head.

(i) Free surface water. Free surface water may be from precipitation, run-off جولة الاعادة, floodwater, melting snow and water from certain hydraulic operations. It is of interest when it comes into contact with a structure or when it influences the ground water in any manner. Rainfall and run-off are erosive agents which are capable of washing away soil and causing certain problems of strength and stability in the field of geotechnical engineering. The properties of free surface water correspond to those of ordinary water.

(ii) Ground water. Ground water is that water which fills up the voids in the soil up to the ground water table and translocate through them. It fills coherently and completely all voids. In such a case, the soil is said to be saturated. Ground water obeys the laws of hydraulics. The upper surface of the zone of full saturation of the soil, at which the ground water is subjected to atmospheric pressure, is called the ‘Ground water table’. The elevation of the ground water table at a given point is called the ‘Ground water level’.

(b) Capillary water. Water which is in a suspended condition, held by the forces of surface tension within the interstices and pores of capillary size in the soil, is called ‘capillary water’.

7.1.2 Held Water

‘Held water’ is that water which is held in soil pores or void spaces because of certain forces of attraction. It can be further classified as (a) Structural water and (b) Absorbed water. Sometimes, even ‘capillary water’ may be said to belong to this category of held water since the action of capillary forces will be required to come into play in this case.

7.2. Flow of Water through Soil (PERMEABILITY)

It is necessary for a Civil Engineer to study the principles of fluid flow and the flow of water through soil in order to solve problems involving

(*a*) The rate at which water flows through soil (for example, the determination of rate of leakage through an earth dam).

(*b*) Compression (for example, the determination of the rate of settlement of a foundation).

(*c*) Strength (for example, the evaluation of factors of safety of an embankment). The emphasis in this discussion is on the influence of the fluid on the soil through which it is flowing; in particular on the effective stress.

Soil, being a particulate material, has many void spaces between the grains because of the irregular shape of the individual particles; thus, soil deposits are porous media. In general, all voids in soils are connected to neighboring voids. Isolated voids are impossible in an assemblage of spheres, regardless of the type of packing; thus, it is hard to imagine isolated voids in coarse soils such as gravels, sands, and even silts. As clays consist of plate-shaped particles, a small percentage of isolated voids would seem possible.

Permeability is one of the most important of soil properties. The path of flow from one point to another is considered to be a straight one, on a macroscopic scale and the velocity of flow is considered uniform at an effective value.

The lower critical velocity v_c is governed by a dimensionless number, known as Reynold's number:

$$R = \frac{v \cdot D}{\upsilon}$$

$$R = \frac{v \cdot D \cdot \gamma_w}{\mu \cdot g}$$

where R = Reynold's number
v = Velocity of flow
D = Diameter of pipe/pore
υ = Kinematic viscosity of water
γ_w = Unit weight of water
μ = Viscosity of water, and
g = Acceleration due to gravity.
Reynolds found that v_c is governed by :

Darcy's Law

H. Darcy of France performed a classical experiment in 1856, using a set-up similar to that shown in Fig. (1), in order to study the properties of the flow of water through a sand filter bed. By measuring the value of the rate of flow or discharge, *q* for various values of the length of the sample, *L*, and pressure of water at top and bottom the sample, *h*1 and *h*2, Darcy found that *q* was proportional to (*h*1 – *h*2)/*L* or the hydraulic gradient, *i* :

q = k [(h1 – h2)/L] × A = k.i.A

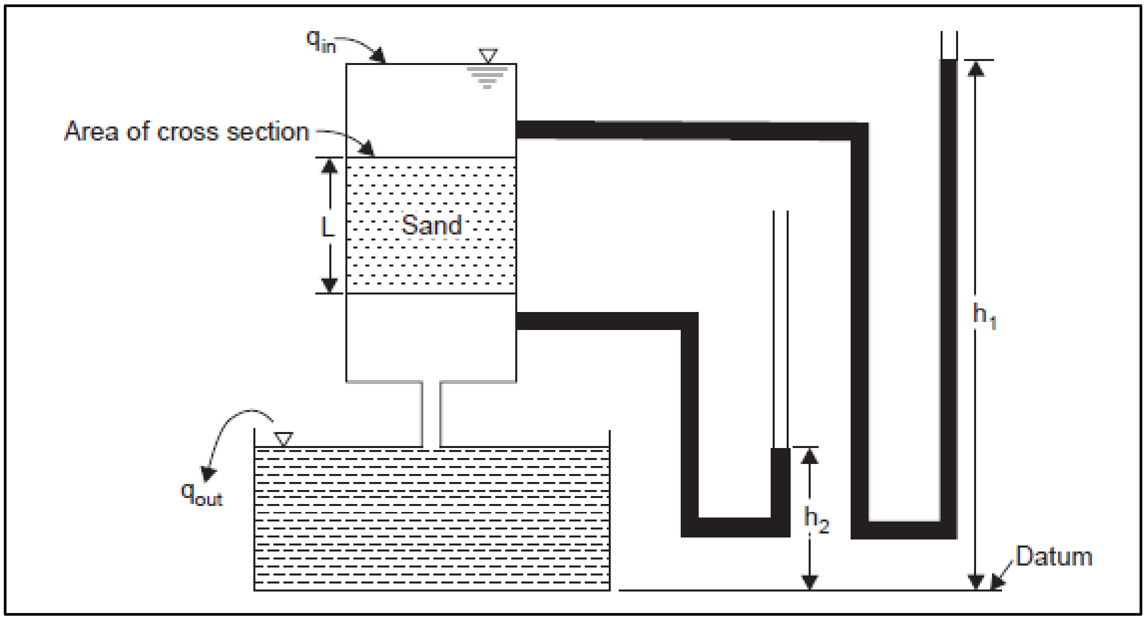


Fig. 1 Darcy's Experiment

where

q = the rate of flow or discharge

$\boldsymbol{k}$ = a constant, now known as Darcy's coefficient of permeability

$\boldsymbol{h_1}$ = the height above datum which the water rose in a standpipe inserted at the entrance of the sand bed,

$\boldsymbol{h_2}$ = the height above datum which the water rose in a stand pipe inserted at the exit end of the sand bed.

$\boldsymbol{L}$ = the length of the sample.

$\boldsymbol{A}$ = the area of cross-section of the sand bed normal to the general direction of flow.

$\boldsymbol{i} = (h1 - h2)/L$, the hydraulic gradient.

2.2 THE DETERMINATION OF PERMEABILITY

The permeability of a soil can be measured in either the laboratory or the field; laboratory methods are much easier than field methods. Field determinations of permeability are often required because permeability depends very much both on the microstructure—the arrangement of soil-grains—and on the macrostructure—such as stratification, and also because of the difficulty of getting representative soil samples. Laboratory methods permit the relationship of permeability to the void ratio to be studied and are thus usually run whether or not field determinations are made. The following are some of the methods used in the laboratory to determine permeability.

1. Constant head permeameter
2. Falling or variable head permeameter
3. Direct or indirect measurement during an Oedometer test
4. Horizontal capillarity test.

The following are the methods used in the field to determine permeability.

1. Pumping out of wells
2. Pumping into wells

In both these cases, the aquifer or the water-bearing stratum, can be ‘confined’ or ‘unconfined’.

Permeability may also be computed from the grain-size or specific surface of the soil, which constitutes an indirect approach.

The various methods will be studied in the following sub-sections.

2.2.1 Constant-Head Permeameter

A simple set-up of the constant-head permeameter is shown in Fig. 2.

The principle in this set-up is that the hydraulic head causing flow is maintained constant; the quantity of water flowing through a soil specimen of known cross-sectional area and length in a given time is measured. In highly impervious soils the quantity of water that can be collected will be small and, accurate measurements are difficult to make. Therefore, the constant head permeameter is mainly application cable to relatively pervious soils, although, theoretically speaking, it can be used for any type of soil.

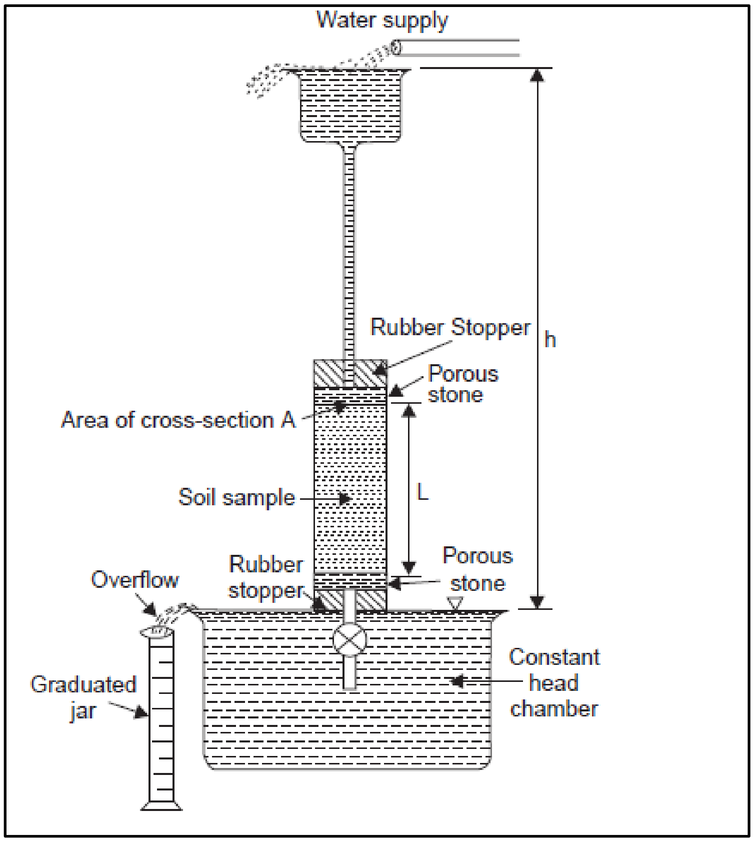


Fig. 2 Set-up of the constant-head permeameter

If the length of the specimen is large, the head lost over a chosen convenient length of the specimen may be obtained by inserting piezometers at the end of the specified length. If Q is the total quantity of water collected in the measuring jar after flowing through the soil in an elapsed time t, from Darcy's law,

$$q = Q/t = k.i.A$$
$$k = (Q/t).(1/iA) = (Q/t).(L/Ah) = QL/thA$$

where:

k = Darcy's coefficient of permeability

L and ***A*** = length and area of cross-section of soil specimen

h = hydraulic head causing flow.

The water should be collected only after a steady state of flow has been established.

2.2.2 Falling or Variable Head Permeameter

A simple set-up of the falling, or variable head permeameter is shown in Fig. 3.

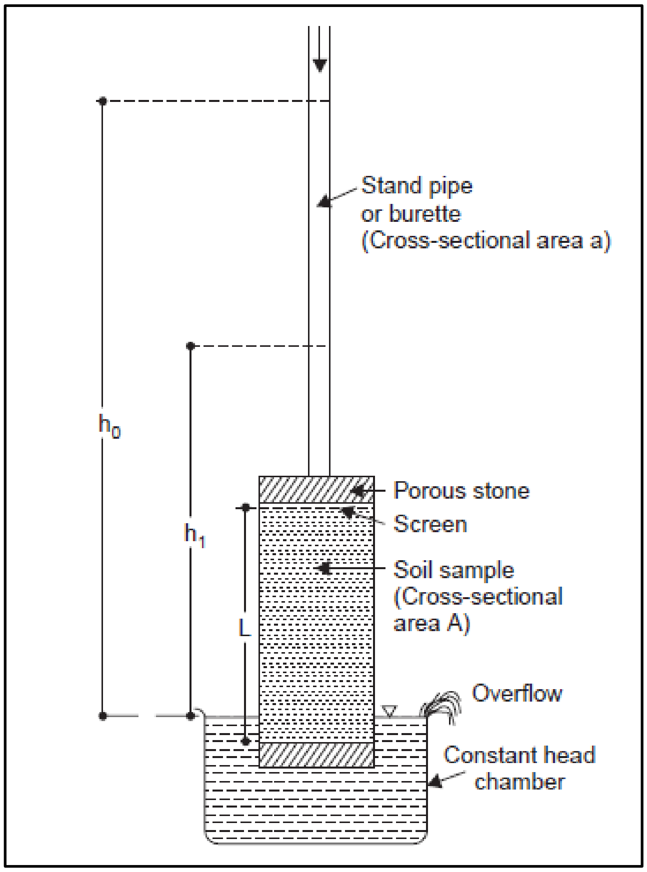


Fig. 3 falling, or variable, head permeameter

A better set-up in which the top of the standpipe is closed, with manometers and vacuum supply, may also be used to enhance the accuracy of the observations (Lambe and Whitman, 1969). The falling head permeameter is used for relatively less permeable soils where the discharge is small.

The water level in the stand-pipe falls continuously as water flows through the soil specimen. Observations should be taken after a steady state of flow has reached. If the head or height of water level in the standpipe above that in the constant head chamber falls from h_0 to h_1, corresponding to elapsed times t_0 and t_1, the coefficient of permeability, k, can be derived as follows :

Let – ***dh*** be the change in head in a small interval of time ***dt***. (***Negative sign indicates that the head decreases with increase in elapsed time***). From Darcy's law,

$$Q = (-a \cdot dh)/dt = k \cdot i \cdot a$$

$$-a dh/dt = K \cdot A \cdot h/L$$

$$\therefore \qquad (kh/L) \cdot A = -a \cdot \frac{dh}{dt}$$

or

$$(kA/aL) \cdot dt = -dh/h$$

Integrating both sides and applying the limits t_0 and t_1 for t, and h_0 and h_1 for h,

$$\frac{kA}{aL}\int_{t_0}^{t_1} dt = -\int_{h_0}^{h_1} \frac{dh}{h} = \int_{h_1}^{h_0} \frac{dh}{h}$$

$$\therefore \qquad (kA/aL)(t_1 - t_0) = \log_e (h_0/h_1) = 2.3 \log_{10} (h_0/h_1).$$

Transposing the terms,

$$k = \frac{2.303\, aL}{A\,(t_1 - t_0)} \cdot \log_{10} (h_0/h_1)$$

where

a = area of cross-section of standpipe

L and ***A*** = length and area of cross-section of the soil sample and the other quantities

$$k = \frac{2.303\, aL}{A(t_1 - t_0)} \cdot \log_{10} (h_0/h_1)$$

.2.23 Direct or Indirect Measurement During an Oeodometer Test

As discussed, the rate of consolidation of a soil depends directly on the permeability. The permeability can be computed from the measured rate of consolidation by using appropriate relationships. Since there are several quantities in addition to permeability that enter into the rate of consolidation-permeability relationship, this method is far from precise since these quantities cannot easily be determined with precision. Instead of the indirect approach, it would be better to run a constant-head permeability test on the soil sample in the oedometer or consolidation apparatus, at the end of a compression increment. This would yield precise results because of the directness of the approach.

2.2.4 Permeability from Horizontal Capillarity Test

The ‘Horizontal capillarity test’ or the ‘Capillarity-Permeability test’, used for determining the capillary head of a soil, can also be used to obtain the permeability of the soil. This is dealing with the phenomenon of ‘Capillarity’. The laboratory measurement of soil permeability, although basically straightforward, requires good technique to obtain reliable results.

2.2.5 Determination of Permeability—Field Approach

The average permeability of a soil deposit or stratum in the field may be somewhat different from the values obtained from tests on laboratory samples; the former may be determined by pumping tests in the field. But these are time-consuming and costlier.

A few terms must be understood in this connection. ‘***Aquifer***’ is a permeable formation which allows a significant quantity of water to move through it under field conditions. Aquifers may be ‘***Unconfined aquifers***’ or ‘***Confined aquifers***’. Unconfined aquifer is one in which the ground water table is the upper surface of the zone of saturation and it lies within the test stratum. It is also called ‘free’, ‘phreatic’ or ‘non-artesian’ aquifer. Confined aquifer is one in which ground water remains entrapped under pressure greater than atmospheric, by overlying relatively impermeable strata.

The following assumptions are relevant to the discussion that would follow:

1-The aquifer is homogeneous with uniform permeability and is of infinite area extent.

2- The flow is laminar and Darcy’s law is valid.

3- The flow is horizontal and uniform at all points in the vertical section.

4- The well penetrates the entire thickness of the aquifer.

5- Natural groundwater regime affecting the aquifer remains constant with time.

6- The velocity of flow is proportional to the tangent of the hydraulic gradient (Dupuit's assumption).

$V = i = \frac{h}{L}$

Unconfined Aquifer

A well penetrating an unconfined aquifer to its full depth is shown in Fig. 4.

Let $\boldsymbol{r_0}$ be radius of central well,

$\boldsymbol{r_1}$ and r_2 be the radial distances from the central well to two of the observation wells,

$\mathbf{z_1}$ and $\mathbf{z_2}$ be the corresponding heights of a drawdown curve above the impervious boundary,

$\boldsymbol{z_0}$ be the height of water level after pumping in the central well above the impervious boundary,

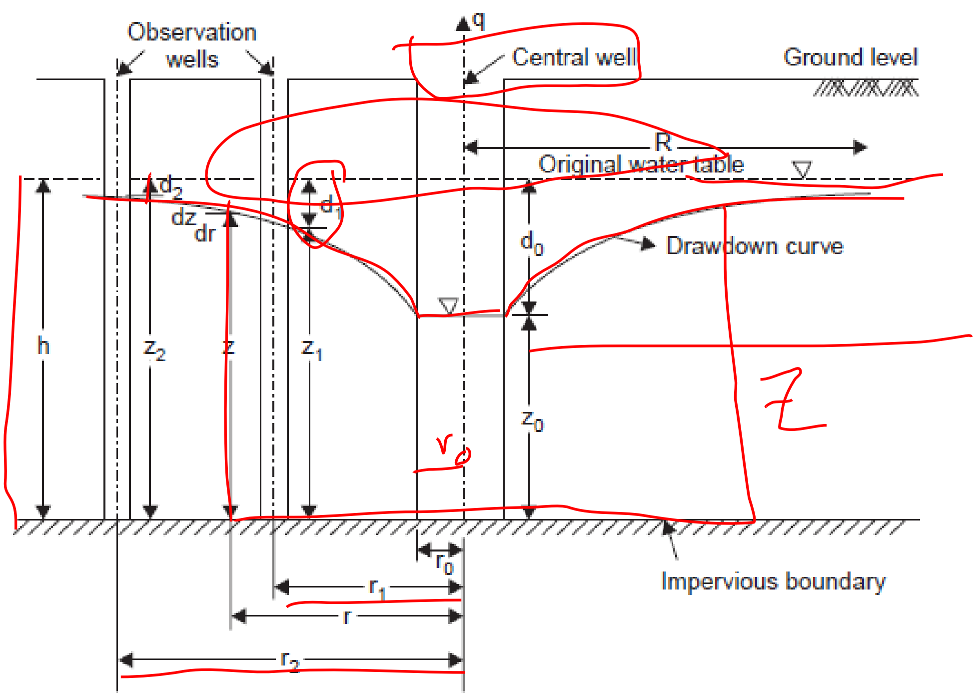


Fig. 4 Flow toward a well in an unconfined aquifer

$\boldsymbol{d_0}$, $\boldsymbol{d_1}$ and $\boldsymbol{d_2}$ be the depths of water level after pumping from the initial level of water table, or the drawdowns at the central well and the two observation wells respectively, ***h*** be the initial height of the water table above the impervious layer (h = $z_0 + d_0$, obviously) and,

R is the radius of influence or the radial distance from the central well of the point where the drawdown curve meets the original water table.

Let ***r*** and ***z*** be the radial distance and height above the impervious boundary at any point on the drawdown curve.

By Darcy's law, the discharge *q* is given by: $q = \frac{Q}{t}$ $\quad i = \frac{h}{L}$

q = k.A.dz/dr, $= k\,i\,A$

Since the hydraulic gradient, ***i***, is given by ***dz/dr*** by Dupuit's assumption.

Here, ***k*** is the coefficient of permeability.

But $$A = 2\pi rz.$$

$\therefore$ $$q = k \cdot 2\pi rz \cdot dz/dr$$

$$\int k.zdz = \int\left(\frac{q}{2\pi}\right) \cdot \frac{dr}{r}$$

Integrating between the limits r_1 and r_2 for *r* and z_1 and z_2 for *z*,

$$k\left\{\frac{z^2}{2}\right\}_{z_1}^{z_2} = (q/2\pi)\left\{\log_e r\right\}_{r_1}^{r_2}$$

$\therefore$ $$k.\frac{(z_2^2 - z_1^2)}{2} = (q/2\pi)\left\{\log_e \frac{r_2}{r_1}\right\}$$

$\therefore$ $$k = \frac{q}{\pi(z_2^2 - z_1^2)}\log_e\frac{r_2}{r_1} = \frac{q}{1.36(z_2^2 - z_1^2)}\log_{10}\frac{r_2}{r_1}$$

k can be evaluated if ***z*₁, *z*₂, *r*₁** and **r₂** are obtained from observations in the field. It can be noted that **z_1 = (h – d_1)** and **z_2 = (h – d_2)**.

If the extreme limits **z_0** and ***h*** at ***r_0*** and ***R*** are applied, the Equation reduces to

$$k = \frac{q}{1.36(h^2 - z_0^2)} \cdot \log_{10}\frac{R}{r_0}$$

Or

$$k = \frac{q}{1.36\, d_0(d_0 + 2z_0)} \cdot \log_{10} \frac{R}{r_0}$$

Confined Aquifer

A well penetrating a confined aquifer to its full depth is shown in Fig. 5.

By Darcy's law, the discharge *q* is given by :

q = k.A.dz/dr, as before.

But the cylindrical surface area of flow is given by $A = 2\pi rH$, in view of the confined

nature of the aquifer.

$\therefore q = k \cdot 2\pi rH \cdot dz/dr$

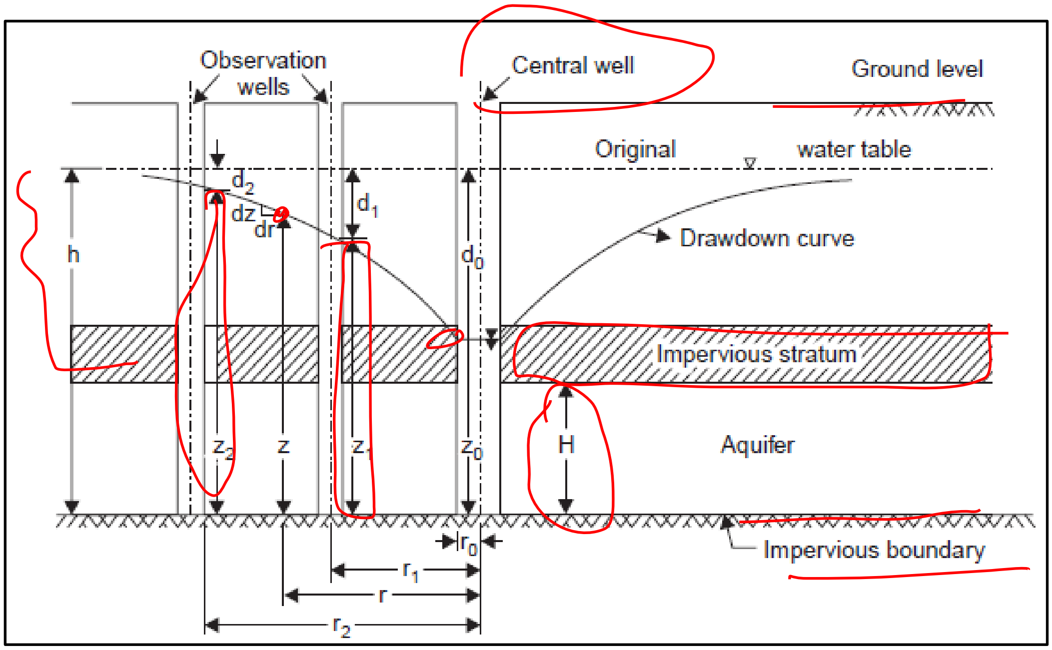


Fig. 5 Flow toward a well in a confined aquifer

$$\int k.dz = \int \frac{q}{2\pi H} \cdot \frac{dr}{r}.$$

$$k\Big[z\Big]_{z_1}^{z_2} = \frac{q}{2\pi H}\Big[\log_e r\Big]_{r_1}^{r_2}$$

$$k(z_2 - z_1) = \frac{q}{2\pi H} \cdot \log_e \frac{r_2}{r_1}$$

or $$k = \frac{q}{2\pi H\,(z_2 - z_1)} \cdot \log_e \frac{r_2}{r_1}$$

Since $z_1 = (h - d_1)$ and $z_2 = (h - d_2)$, $(z_2 - z) = (d_1 - d_2)$

Substituting, we have :

$$k = \frac{q}{2\pi H\,(d_1 - d_2)} \cdot \log_e \frac{r_2}{r_1} = \frac{q}{2.72\,H\,(d_1 - d_2)} \cdot \log_{10} \frac{r_2}{r_1}$$

Since the coefficient of transmissibility, T, by definition, is given by kH,

$$T = \frac{q}{2.72\,(d_1 - d_2)} \cdot \log_{10}(r_2/r_1)$$

2.3 Value of Permeability.

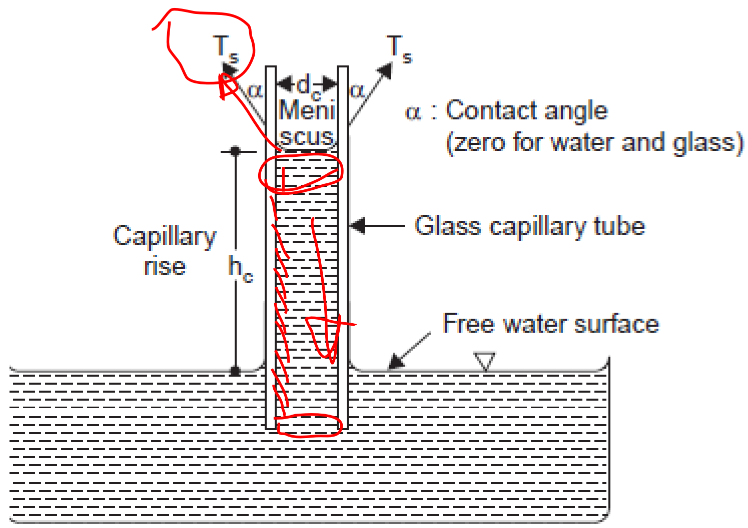
Soil description	*Coefficient of permeability mm/s*	*Degree of permeability (After Terzaghi and Peck, 1948)*
Coarse gravel	Greater than 1	High
Fine gravel—fine sand	1 to 10^{-2}	Medium
Silt-sand admixtures, loose silt, rock flour, and loess	10^{-2} to 10^{-4}	Low
Dense silt, clay-slit admixtures, non-homogeneous clays	10^{-4} to 10^{-6}	Very low
Homogeneous clays	Less than 10^{-6}	Almost impervious

3 Capillarity

The phenomenon in which water rises above the ground water table against the pull of gravity, but is in contact with the water table as its source, is referred to as 'Capillary rise' with reference to soils. The water associated with capillary rise is called 'capillary moisture'. The phenomenon by virtue of which a liquid rises in capillary tubes is, in general, called 'capillarity. All voids in soil located below the ground water table would be filled with water (except possibly for small pockets of entrapped air or gases). In addition, soil voids for a certain height above the water table will also be completely filled with water.

3.2 Capillary Rise in Soil

The rise of water in soils above the ground water table is analogous to the rise of water into capillary tubes placed in a source of water. However, the void spaces in a soil are irregular in shape and size, as they interconnect in all directions. Thus, the equations derived for regular shaped capillary tubes cannot be, strictly speaking, directly applicable to the capillary phenomenon associated with soil water. However, the features of capillary rise in tubes facilitate an understanding of factors affecting capillarity and help determine the order of a magnitude for a capillary rise in the various types of soils.



surface tension, is $\pi d_c \,.\, T_s$. But the weight of water column in the capillary tube is $\frac{\pi d_c^{\,2}}{4} \cdot h_c \cdot \gamma_w$,

where γ_w is the unit weight of water and h_c is the capillary rise.

$$\therefore \qquad \pi d_c \,.\, T_s = \frac{\pi}{4} d_c^{\,2} h_c . \gamma_w$$

$$\text{or} \qquad h_c = \frac{4T_s}{\gamma_w \,.\, d_c}$$

The value of T_s for water varies with temperature. At ordinary or room temperature, T_s is nearly 7.3 dynes/mm or 73×10^{-6} N/mm and γ_w may be taken as 9.81×10^{-6} N/mm^3.

$$h_c = \frac{4 \times 73 \times 10^{-6}}{9.81 \times 10^{-6}\, d_c} \approx \frac{30}{d_c}$$

This Equation indicates that even relatively large voids will be filled with capillary water if soil is close to the ground water table. As the height above the water table increases, only the smaller voids would be expected to be filled with capillary water.

The larger voids represent interference to an upward capillary flow and would not be filled. The soil just above the water table may become fully saturated with capillary water, but even this is questionable since it is dependent upon a number of factors.

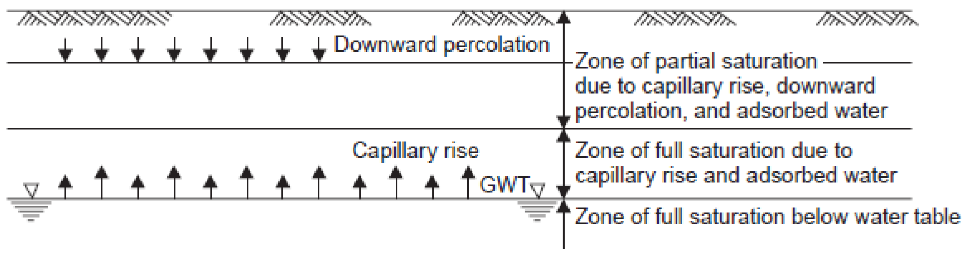


Fig. 5.15 Capillary fringe with zones of full and partial saturation

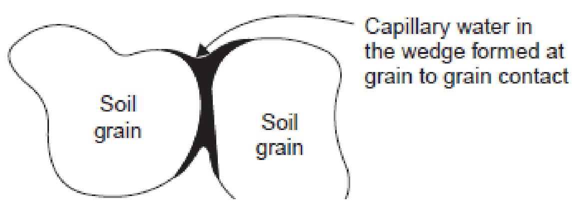


Table 2 for an idea of orders of magnitude.

Table 2 Typical ranges of capillary rise in soils (Mc Carthy, 1977)

Soil Designation	*Approximate Capillary height in mm*
Fine Gravel	20 – 100
Coarse sand	150
Fine sand	300 – 1000
Silt	1000 – 10000
Clay	10000 – 30000

Temperature plays an important role in the capillary rise in soil. At lower temperature capillary rise is more and vice versa. Capillary flow may also be induced from a warm zone towards a cold zone.

***Example* 9**. Determine the coefficient of permeability from the following data:

Length of sand sample = 25 cm

Area of cross section of the sample = 30 cm^2 A

Head of water = 40 cm h

Discharge = 200 ml in 110 s. Q

L = 25 cm

A = 30 cm^2

h = 40 cm (assumed constant)

Q = 200 ml. t = 110 s

$q = Q/t$ = 200/110 ml/s = 20/11 = 1.82 cm^3/s

$i = h/L$ = 40/25 = 8/5 = 1.60

$q = k \cdot i \cdot A$

$k = q/iA$ =20/(11 × 1.6 × 30) cm/s

= 0.03788 cm/s = **3.788 × 10^{-1} mm/s.**

Or $k = Q\times L/ (A \times t\times h)$ = (200 × 25)/(30× 110× 40)= 0.03788 cm/s = **3.788×10^{-1} mm/s.**

Example 10. The discharge of water collected from a constant head permeameter in a period of (15 minutes) is (500 ml). The internal diameter of the permeameter is (5 cm) and the measured difference in head between two gauging points (15 cm) vertically apart is (40 cm). Calculate the coefficient of permeability. If the dry weight of the (15 cm) long sample is (4.86 N) and the specific gravity of the solids is (2.65), calculate the seepage velocity.

Q = 500 ml; t = 15 × 60 = 900 s.

A = (π/4) × 5^2 = 6.25π cm^2 ; L = 15 cm ; h = 40 cm;

k = Q× L/ (A ×t× h) = (500 × 15)/(6.25× π ×900 ×40) = 0.106 mm/s

Superficial velocity v = Q/A×t =500/ (900 × 6.25×π) = 0.0283 cm/s = 0.283 mm/s

Dry weight of sample = 4.86 N

Volume of sample = A × L = 6.25 × π ×15 cm3 = 294.52 cm3

Dry density, ɤd= 4.86/294.52 = 16.5 kN/m3

$$\gamma_d = \frac{G\gamma_w}{(1+e)}$$

$$(1+e) = \frac{2.65 \times 10}{16.5} = 1.606, \quad \text{since} \quad \gamma_w \approx 10 \text{ kN/m}^3$$

$$e = 0.606$$

$$n = \frac{e}{(1+e)} = 0.3773 = 37.73\%$$

• ~

$$\therefore \quad \text{Seepage velocity, } v_s = v/n = \frac{0.283}{0.3773} = \mathbf{0.750 \text{ mm/s.}}$$

Example 11. A glass cylinder **5 cm** internal diameter and with a screen at the bottom was used as a falling head permeameter. The thickness of the sample was **10 cm.** With the water level in the tube at the start of the test as **50 cm** above the tail water, it dropped by **10 cm** in one minute, the tail water level remaining unchanged. Calculate the value of *k* for the sample of the soil. Comment on the nature of the soil. Falling head permeability test:

$h_1 = 50$ cm at $t_1 = 0$; $\quad$ $h_2 = 40$ cm at $t_2 = 60$s

so $t = t_2 - t_1 = 60$s

$A = (\pi/4) \times 5^2 = 6.25\,\pi$ cm^2 ; $\quad$ $L = 10$ cm

Since *a* is not given, let us assume *a* = *A*. (i.e. area of sample = area of stand pipe)

$$k = \frac{2.303\, aL}{A(t_1 - t_0)} \cdot \log_{10}(h_0/h_1)$$

$k = 2.303 \times (10/60) \log_{10} (50/40)$ cm/s

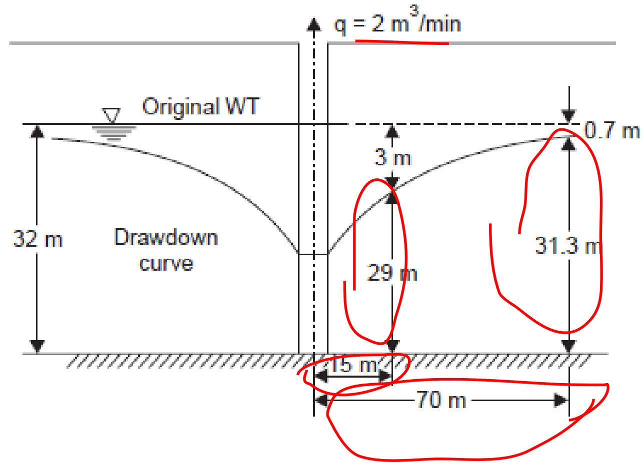
= 0.0372 cm/s

= **3.72 × 10–1 mm/s** $\quad$ The soil may be coarse sand or fine graved.

Example 12. An unconfined aquifer is known to be 32 m thick below the water table. A constant discharge of 2 cubic meters per minute is pumped out of the aquifer through a tube well till the water level in the tube well becomes steady. Two observation wells at distances of 15 m and 70 m from the tube well show falls of 3 m and 0.7 m respectively from their static water levels. Find the permeability of the aquifer.

Sol.

The conditions given are shown in Fig. below



We know,

$$k = \frac{q \log_e (r_2/r_1)}{\pi(z_2^2 - z_1^2)}$$

$$= \frac{2.303 \times 2 \log_{10} (70/15) \times 100}{60 \times \pi (31.3^2 - 29^2)} \text{ cm/s}$$

$$= \mathbf{1.18 \times 10^{-1}\ mm/s.}$$

the ratio $s = \frac{n_f}{n_d}$ is a characteristic of the flow net and is independent of the permeability k and

The value of s in this case is

$$s = \frac{n_f}{n_d} = 4/10 = 0.4$$

und, $q/L = k.H.s = 0.5 \times 1600 \times 0.4 \text{ mm}^3/\text{s/mm} = 320 \text{ mm}^3/\text{s/mm}$

Then, $q = (q/L) \times (400) = 320 \times 400 = 128000$

$= 1.28 \times 10^5 \text{ mm}^3/\text{s}.$

For example, at elevation 1000 mm,

The total head, $h = (8/10) \times H = (8/10) \times 1600 \text{ mm} = 1280 \text{ mm}$

Elevation head, $h_e = 1000 \text{ mm}$

$\therefore$ Pressure head, $h_p = (1280 - 1000) \text{ mm} = 280 \text{ mm}.$

The pore pressure at this elevation is $280 \times 9.81 \times 10^{-6}$ N/mm^2 or 2.75×10^{-3} N/mm^2. Similarly, the pressure heads at elevations 700 mm and 300 mm are 100 mm and – 140 mm, respectively, as shown at the left of the flow net.

- **FLOW NET FOR TWO-DIMENSIONAL FLOW**

During seepage analysis, a flow net can be drawn with as many flow lines as desired. The number of equipotential lines will be determined by the number of flow lines selected. Generally speaking, it is preferable to use the fewest flow lines that still permit reasonable depiction of the path along the boundaries and within the soil mass. For many problems, three or four flow channels (a channel being the space between adjacent flow lines) are sufficient. The first and second—flow under a sheet pile wall and flow under a concrete dam—are cases of confined flow since the boundary conditions are completely defined.

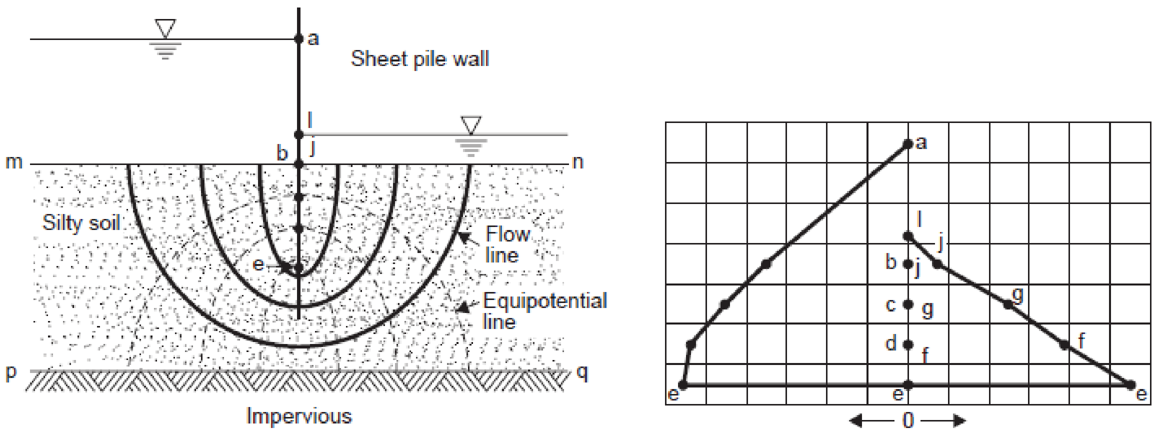
- **Flow under Sheet Pile Wall**

Figure below shows a sheet pile wall driven into a silty soil. The wall runs for a considerable length in a direction perpendicular to the paper; thus, the flow

underneath the sheet pile wall may be taken to be two-dimensional. The boundary conditions for the flow under the sheet pile wall are;

mb = upstream equipotential; **jn** = downstream equipotential; **bej** = flow line.

The flow net shown has been drawn within these boundaries. With the aid of flow net, we can compute the seepage under the wall, the pore pressure at any point and the hydraulic gradient at any point. A water pressure plot, such as that shown in Fig. 6.2 is useful in the structural design of the wall.



(a) Flow net (b) Water pressure on the wall

Fig. 9.2 Sheet pile wall

- **Flow under Concrete Dam**

Figures 9.3 to 9.6 show a concrete dam resting on an isotropic soil. The sections shown are actually those of the spillway portion. The upstream and tail water elevations are shown. The first one is with no cut-off walls, the second with cut-off wall at the heel as well as at the toe, the third with cut off-wall at the heel only and the fourth with cut-off wall at the toe only. The boundary flow lines and equipotential are known in each case and the flow nets are drawn as shown within these boundaries. The effect of the cut off walls is to reduce the under seepage, the uplift pressure on the underside of the dam and also the hydraulic gradient at the exit, called the ‘exit gradient’. A flow net can be understood to be a very powerful tool in developing a design and evaluating various schemes.

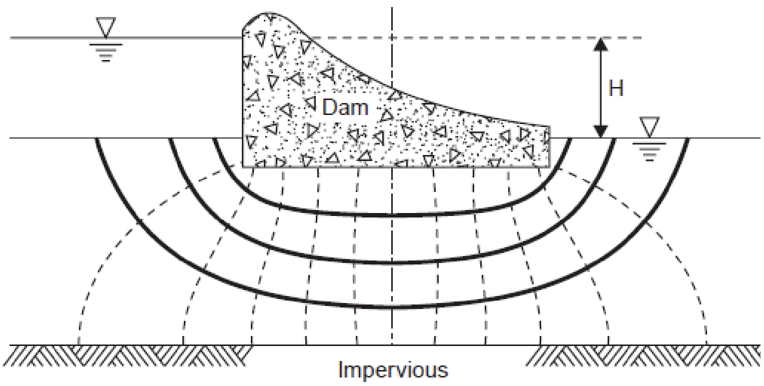


Fig. 9.3 Concrete dam with no cut-off walls

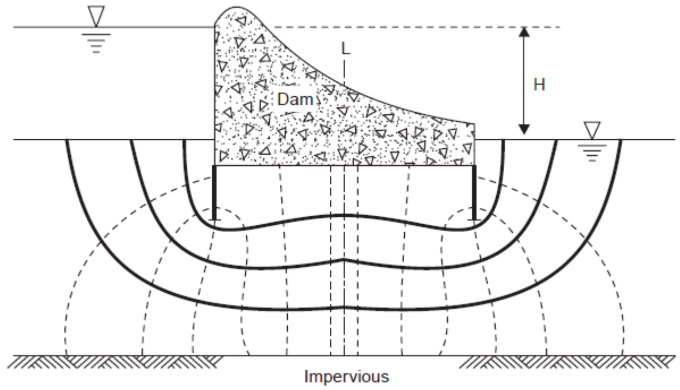


Fig. 9.4 Concrete dam with cut-offs at heel and at toe

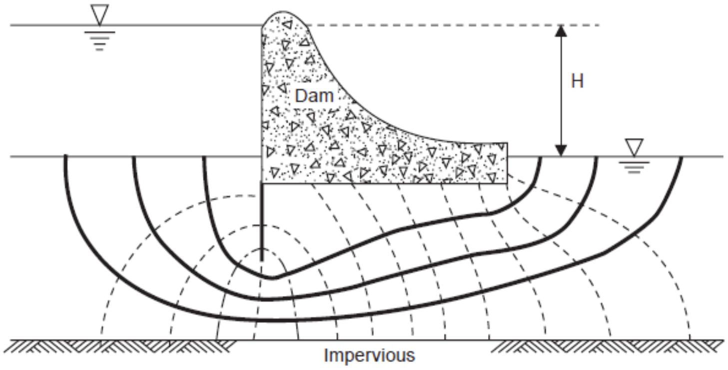


Fig. 9.5 Concrete dam with cut-off at heel

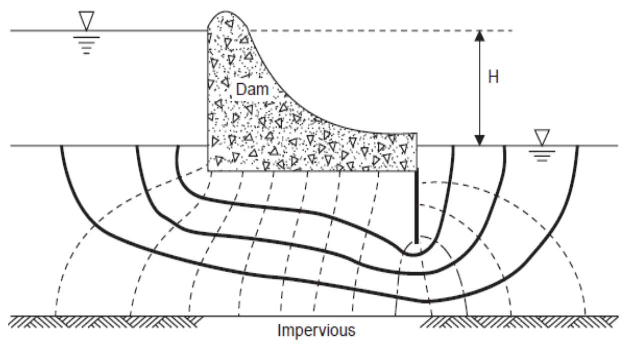


Fig. 9.6 Concrete dam with cut-off wall at toe

- **BASIC EQUATION FOR SEEPAGE**

The flow net was introduced in an intuitive manner in the preceding sections. The equation for seepage through soil which forms the theoretical basis for the flow net as well as other methods of solving flow problems will be derived in this section. The following assumptions are made:

1. Darcy's law is valid for flow through soil.

2. The hydraulic boundary conditions are known at entry and exit of the fluid (water) into the porous medium (soil).

3. Water is incompressible.

4. The porous medium is incompressible.

Let us consider an element of soil as shown in Fig. 9.7, through which laminar flow of water is occurring:

Let q be the discharge with components q_x, q_y and q_z in the X-,Y- and Z-directions respectively. $q = q_x + q_y + q_z$, obviously.

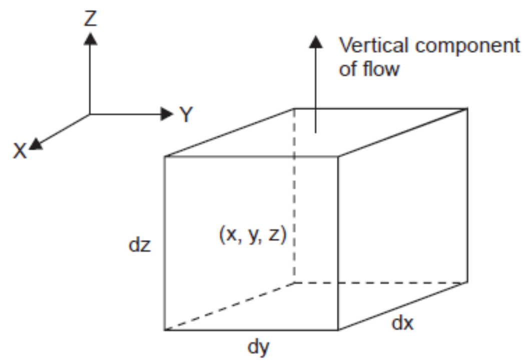


Fig. 9.7 Flow through an element of soil

By Darcy's law,

$q_z = k \cdot i \cdot A$,

where A is the area of the bottom face and q_z is the flow into the bottom face

$$= k_z \left(- \frac{\partial h}{\partial z} \right) dx \cdot dy,$$

where k_z is the permeability of the soil in the Z-direction at the point (x, y, z) and h is the total head. Flow out of the top of the element is given by:

$$q_z + \Delta q_z = \left(k_z + \frac{\partial k_z}{\partial z}, dz \right) \left(- \frac{\partial h}{\partial z} - \frac{\partial^2 h}{\partial z^2} \cdot dz \right) \cdot dx\, dy$$

Net flow into the element from vertical flow:

Δq_z = inflow – outflow

$$= k_z\left(-\frac{\partial h}{\partial z}\right)dxdy - \left(k_z + \frac{\partial k_z}{\partial z}\cdot dz\right)\left(\frac{\partial h}{-\partial z} - \frac{\partial^2 h}{\partial z^2}\cdot dz\right)dx\,dy$$

$$\Delta q_z = \left(k_z\cdot\frac{\partial^2 h}{\partial z^2} + \frac{\partial k_z \partial h}{\partial z^2} + \frac{\partial k_z}{\partial z}\cdot dz\,\frac{\partial^2 h}{\partial z^2}\right)dx\,dy\,dz$$

Assuming the permeability to be constant at all points in a given direction, (that is, the soil is homogeneous),

$$\frac{\partial k_z}{\partial z} = 0$$

$$\Delta q_z = \left(k_z\,\frac{\partial^2 h}{\partial z^2}\right)dx\,dy\,dz$$

Similarly, the net inflow in the *X*-direction is:

$$\Delta q_x = \left(k_x\cdot\frac{\partial^2 h}{\partial x^2}\right)dx\,dy\,dz$$

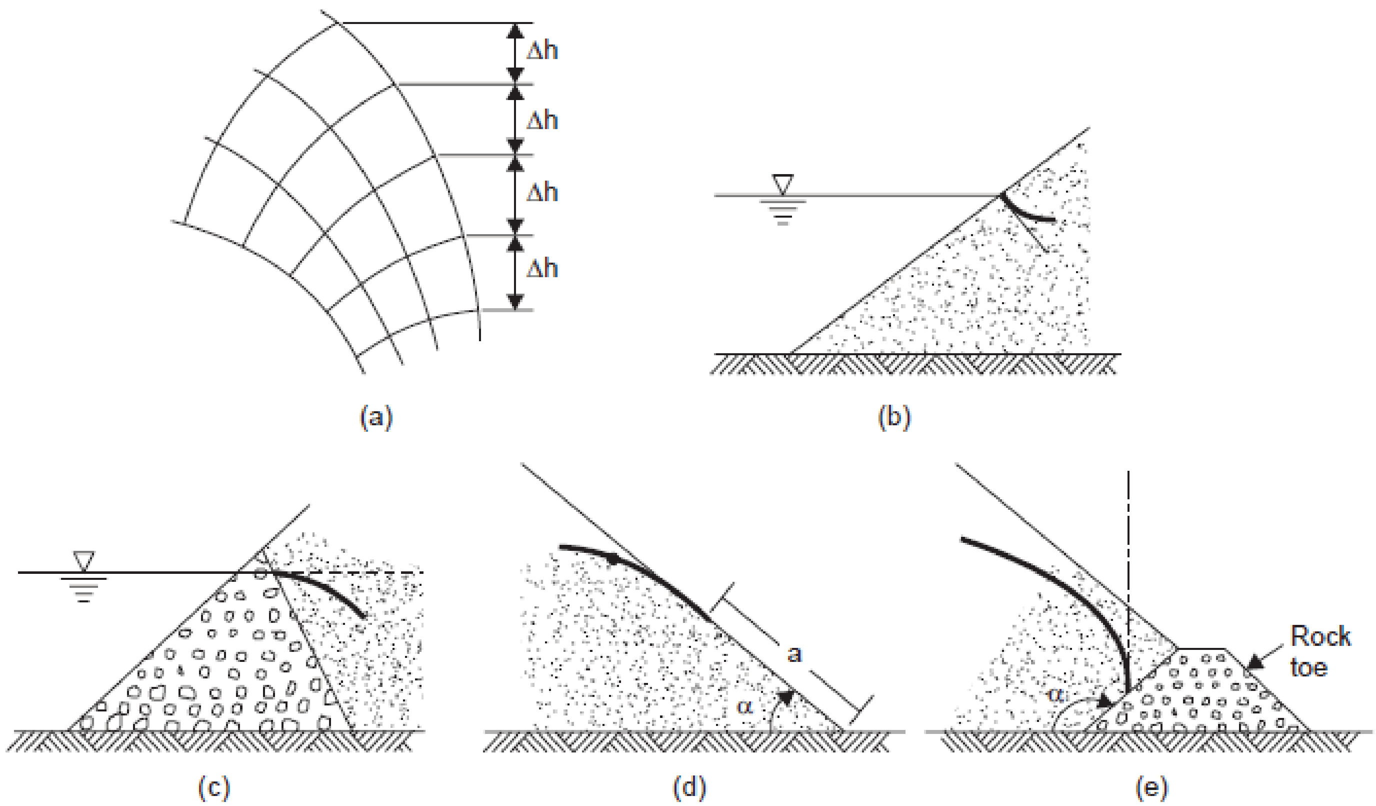
For two-dimensional flow, $q_y = 0$

$$\Delta q = \Delta q_x + \Delta q_z = \left(k_x\cdot\frac{\partial^2 h}{\partial x^2} + k_z\cdot\frac{\partial^2 h}{\partial z^2}\right)dx\,dy\,dz$$

- **TOP FLOW LINE IN AN EARTH DAM**

The flow net for steady seepage through an earth dam can be obtained by any one of the Methods available, including the graphical approach. However, since this is the case of an unconfined flow, the top flow line is not known and hence should be determined first. The top flow line is also known as the 'phreatic line', as the pressure is atmospheric on this line. Thus, the pressures in the dam section below the phreatic line are positive hydrostatic pressures.

The top flow line may be determined either by the graphical method or by the analytical method. Although the typical earth dam will not have a simple homogeneous section, such sections furnish a good illustration of the conditions that must be fulfilled by any top flow line. Furthermore, the location of a top flow line in a simple case can often be used for the first trial in the sketching of a flow net for a more complicated case. The top flow line must obey the conditions illustrated in Fig.

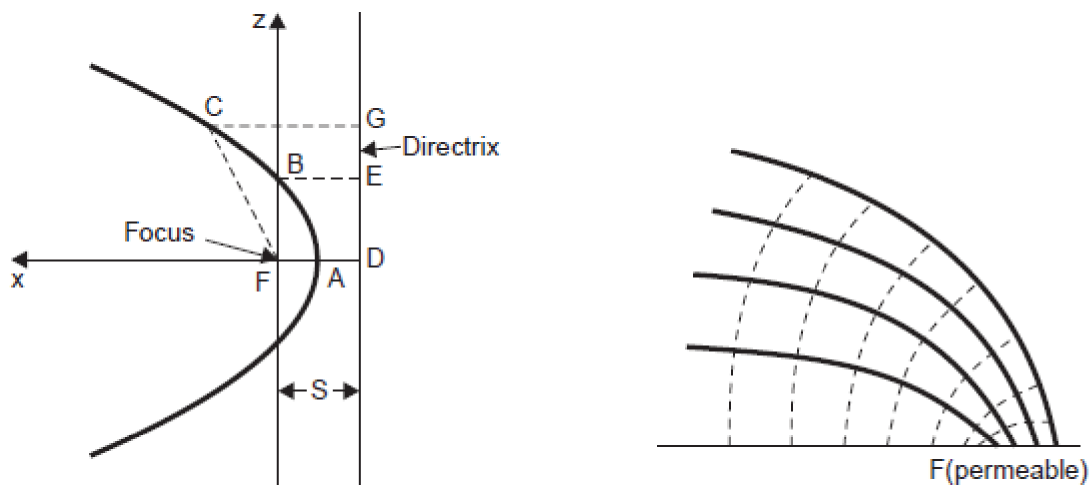


Since the top flow line is at atmospheric pressure, the only head that can exist along it is the elevation head. Therefore, there must be equal drops in elevation between the points at which successive equipotentials meet the top flow line, as in Fig (*a*).

At the starting point, the top flow line must be normal to the upstream slope, which is an equipotential line, as shown in Fig. (*b*). However, an exception occurs when the coarse material at the upstream face is so pervious that it does not offer appreciable resistance to flow, as shown in Fig. (*c*). Here, the upstream equipotential is the downstream boundary of the coarse material. The top flow line cannot be normal to this equipotential since it cannot rise without violating the condition illustrated in Fig. (*a*). Therefore, this line starts horizontally and zero initial gradient and zero velocity occur along it. This zero condition relieves the apparent inconsistency of deviation from a 90-degree intersection.

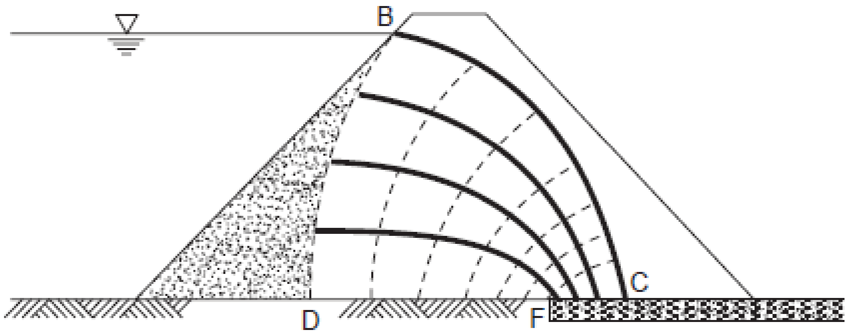
At the downstream end of the top flow line the particles of water tend to follow paths which conform as nearly as possible to the direction of gravity, as shown in Fig. (*d*); the top flow line here is tangential to the slope at the exit. This is also illustrated by the vertical exit condition into a rock-toe as shown in Fig (*e*).

- **Top Flow Line for an Earth Dam with a Horizontal Filter**

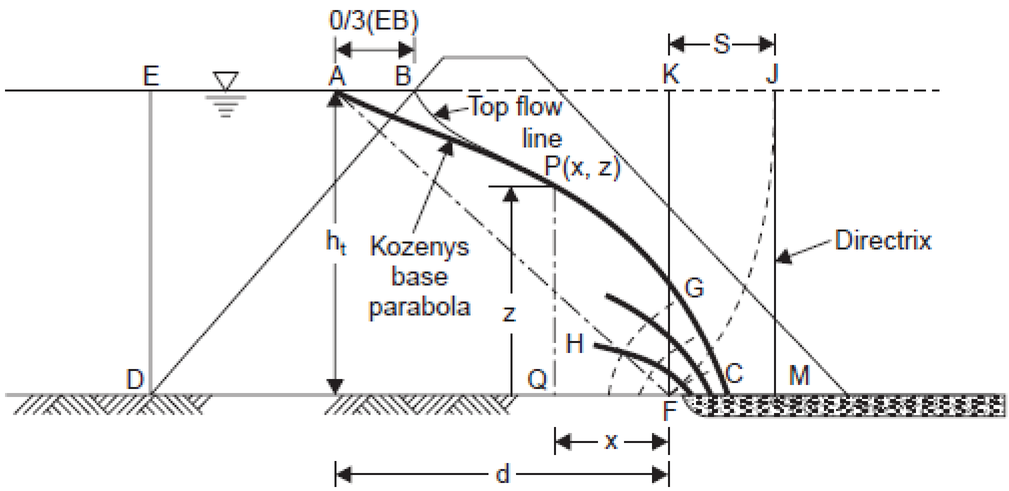


(a) Parabola

(b) Conjugate confocal parabolas (Kozeny, 1931)



(c) Earth dam with flow net consisting of confocal parabolas



BC and **DF** are flow lines, and **BD** and **FC** are equipotential. Casagrande (1937) suggests that **BA** is approximately equal to 0.3 times **BE** where **B** is the starting point of the Kozeny parabola at the upstream water level and **E** is on the upstream water level vertically above the heel **D** of the dam.

(*i*) Locate the point **A**, using **BA** = 0.3 (**BE**). A will be the starting point of the Kozeny

parabola.

(*ii*) With **A** as centre and **AF** as radius, draw an arc to cut the water surface (extended) in **J**. The vertical through **J** is the directrix. Let this meet the bottom surface of the dam in **M**.

(*iii*) The vertex **C** of the parabola is located midway between **F** and **M**.

(*iv*) For locating the intermediate points on the parabola the principle that it must be equidistant from the focus and the directrix will be used. For example, at any distance x from **F**, draw a vertical and measure **QM.** With **F** as center and **QM** as radius, draw an arc to cut the vertical through **Q** in **P**, which is the required point on the parabola.

(*v*) Join all such points to get the base parabola. The portion of the top flow line from **B** is sketched in such that it starts perpendicular to **BD**, which is the boundary equipotential and meets the remaining part of the parabola tangentially without any kink. The base parabola meets the filter perpendicularly at the vertex **C**.

The following analytical approach also may be used:

With the origin of co-ordinates at the focus, **PF** = **QM**

$$\sqrt{x^2 + z^2} = x + S$$

$$x = \frac{(z^2 - S^2)}{2S}$$

This is the equation to the parabola.

Analytically ***S*** may be got by substituting the coordinates of **A** (*d*, *ht*) in

$$S = \sqrt{d^2 + h_t^2} - d$$

For different values of *x*, *z* may be calculated and the parabola drawn. The corrections at the entry may then be incorporated.

The flow net **GCFH;** *nf* and *nd* are each equal to 3 for this net.

$$q = k\ .\ H\ .\ 3/3 = k\ .\ S$$

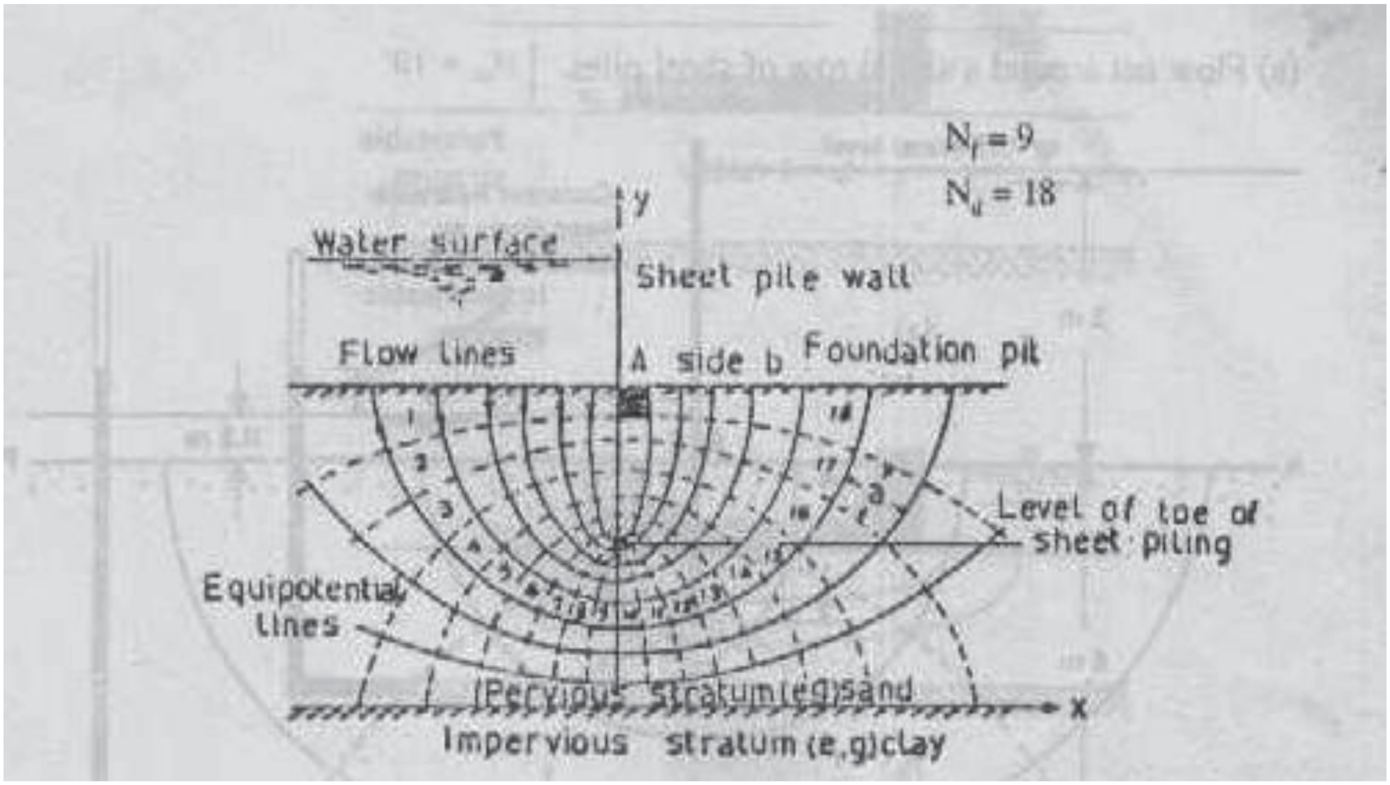
$$q = k\ .\ i\ .\ A$$

$$= k\ .\ \frac{d_z}{d_x}\ .\ z \text{ for unit length of the dam.}$$

But $z = (2x\,S + S^2)^{1/2}$ from

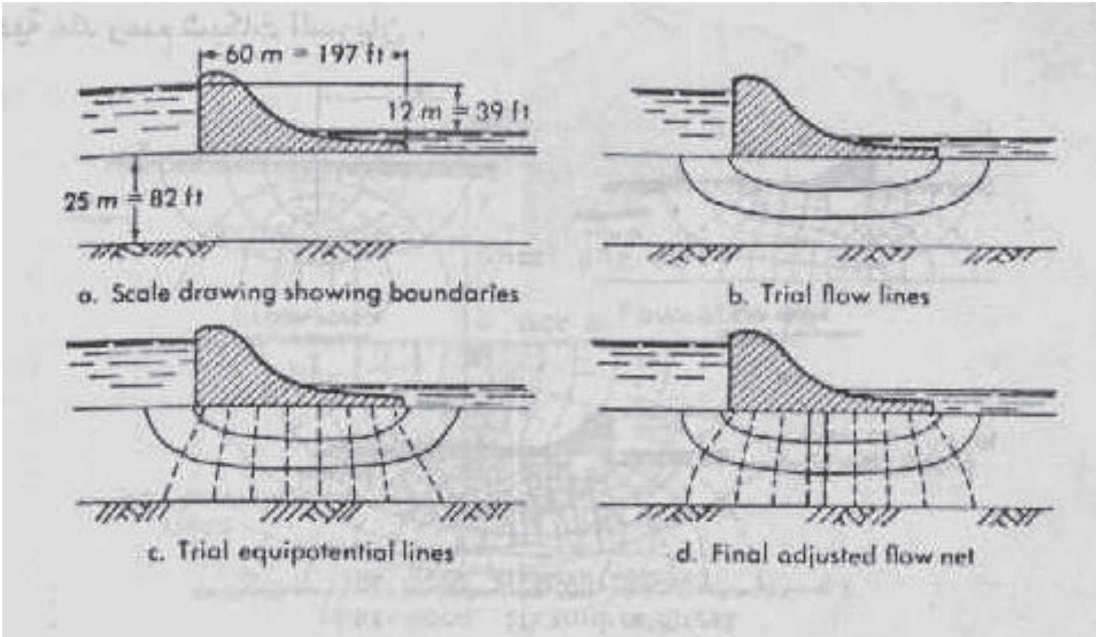
$$\frac{d_z}{d_x} = \frac{1}{2} \frac{2S}{(2xS + S^2)^{1/2}} = \frac{S}{(2xS + S^2)^{1/2}}$$

$$q = k \cdot \frac{S}{(2xS + S^2)^{1/2}} \cdot (2x\,S + S^2)^{1/2} = k.\ S,$$



Ex 19: compute the quantity of seepage under the dam in fig below if K= 1.5 x10^{-3} mm/sec. and level of water upstream is 18 m above the base of dam and downstream is 6 m above the base of the dam. The length of the dam is 250 m.

1- N_f= 3 and N_d=9.5
2- if K= 1.5 x10^{-3}, Δ h= 18-6 =12m
3- q per m =1.5 x10^{-3}x(3/9.5)x12 = 5.7x0^{-6} m^2/sec
4- q = 5.7x10^{-6} m^2/sec x 250 = 1.4 x 10^{-3} m^2/sec

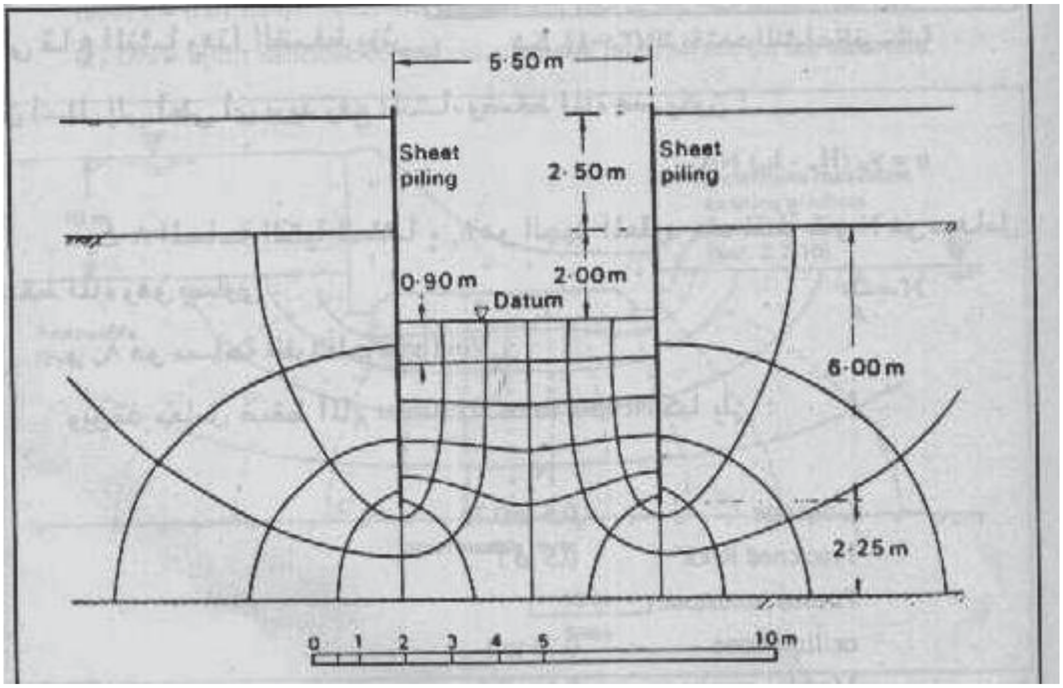


Ex 20: A river bed consist of a layer of sand 8-25m thick overlying impermeable rock, the depth of water is 2.5m. A long cofferdam 5.5m wide is formed by driving two lines of sheet piling to a depth of 6 m below the level of the river bed and excavation to a depth of 2.0 m below bed level is carried out within the cofferdam. The water level within the cofferdam is kept at excavation level by pumping. If the flow of water into cofferdam is 0.25 m^3/h per unit length, what is the coefficient of permeability of the sand? what is the hydraulic gradient immediately below the excavated surface.

Sol: the section and flow net appear in fig below in the flow net there are 6 flow channel and 10 equipotential drops. The total head loss is 4.5m. The coefficient of permeability is given by:

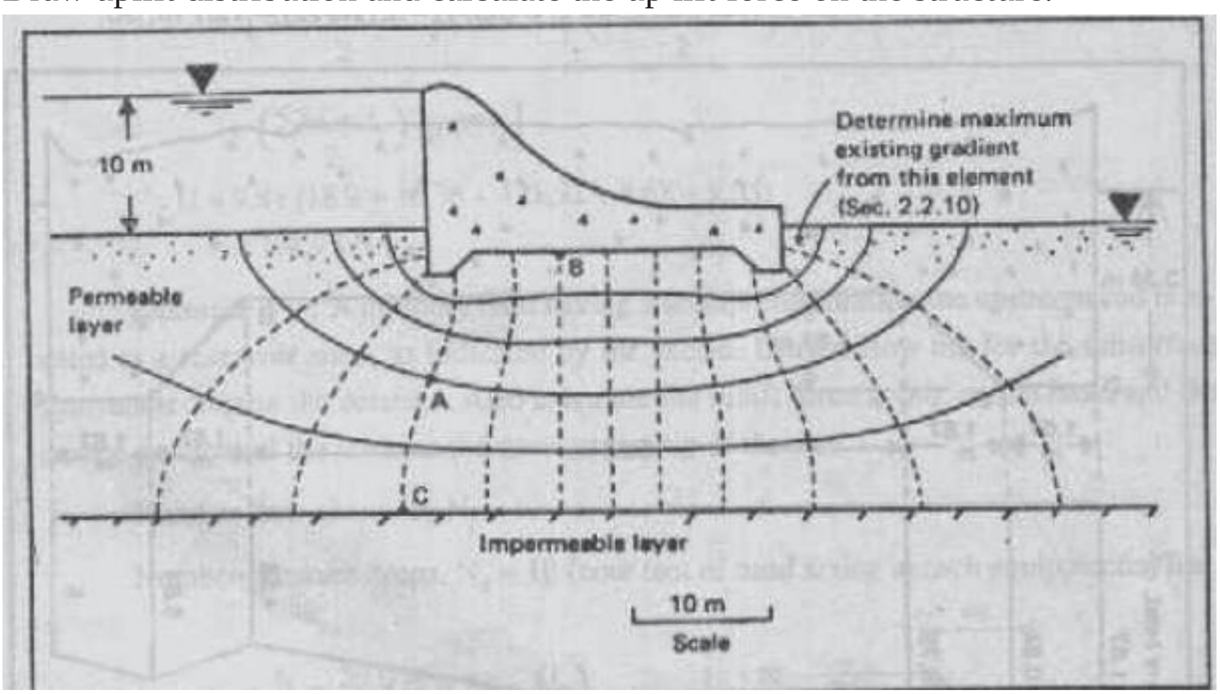
$$k = q \Big/ \left(h \frac{N_f}{N_d}\right)$$

$$= \frac{0.25}{4.50 \times 6/10 \times 60^2} = 2.6 \times 10^{-5} \text{ m/s}$$



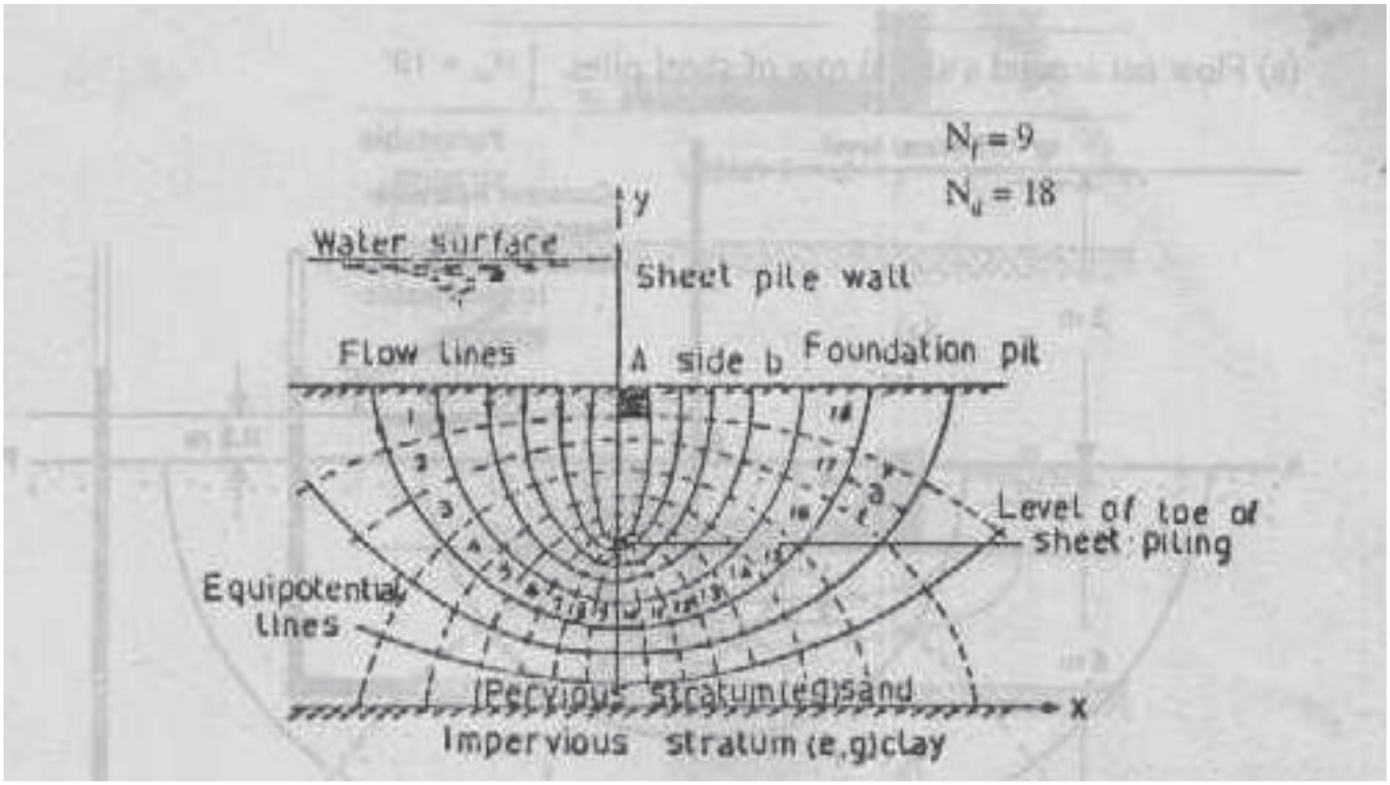
Ex 21: for the flow net shown below

a) How high would water rise if piezometer is place at A, B and C.
b) if K= 0.01 mm/s, determine the seepage loss of the dam in m^3/(day.m).
c) Draw uplift distribution and calculate the up lift force on the structure.



$$\frac{d_z}{d_x} = \frac{1}{2}\frac{2S}{(2xS+S^2)^{1/2}} = \frac{S}{(2xS+S^2)^{1/2}}$$

$$q = k \cdot \frac{S}{(2xS+S^2)^{1/2}} \cdot (2x\,S + S^2)^{1/2} = k.\,S,$$



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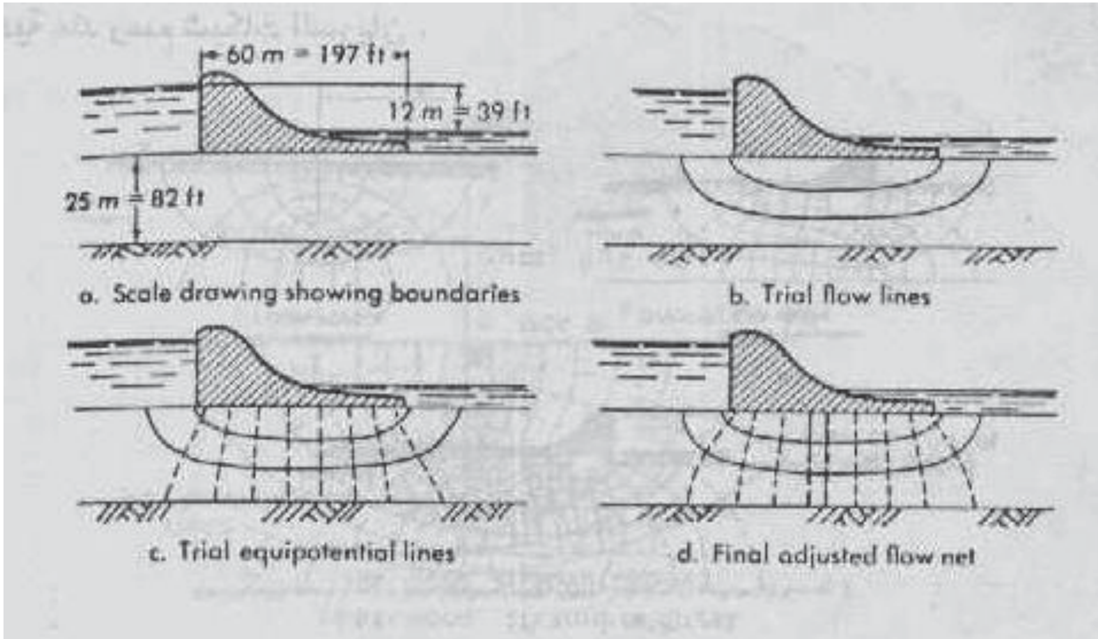
1- $N_f = 9$ and $N_d = 18$
2- if K= 1.5 x10^{-3}, Δ h= 18-6 =12m
3- q per m =1.5 x10^{-3}x(9/18)x12 = 9x10^{-3} m^2/sec
4- q = 9x10^{-3} m^2/sec x 250 = 2250 x 10^{-3} m^2/sec



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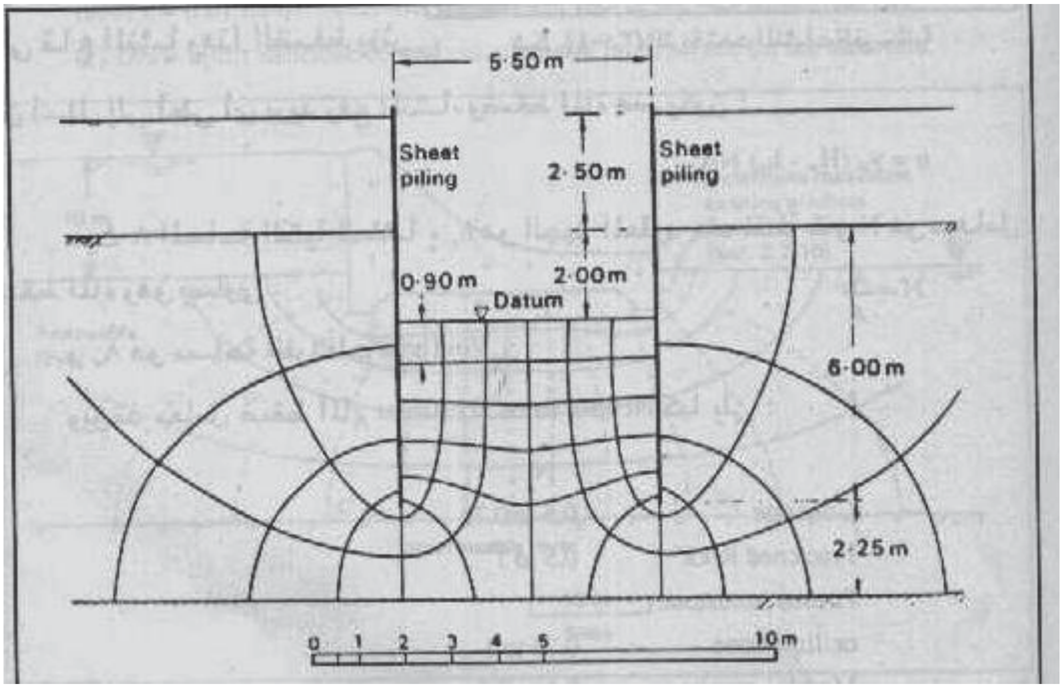

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a) How high would water rise if piezometer is place at A, B and C.
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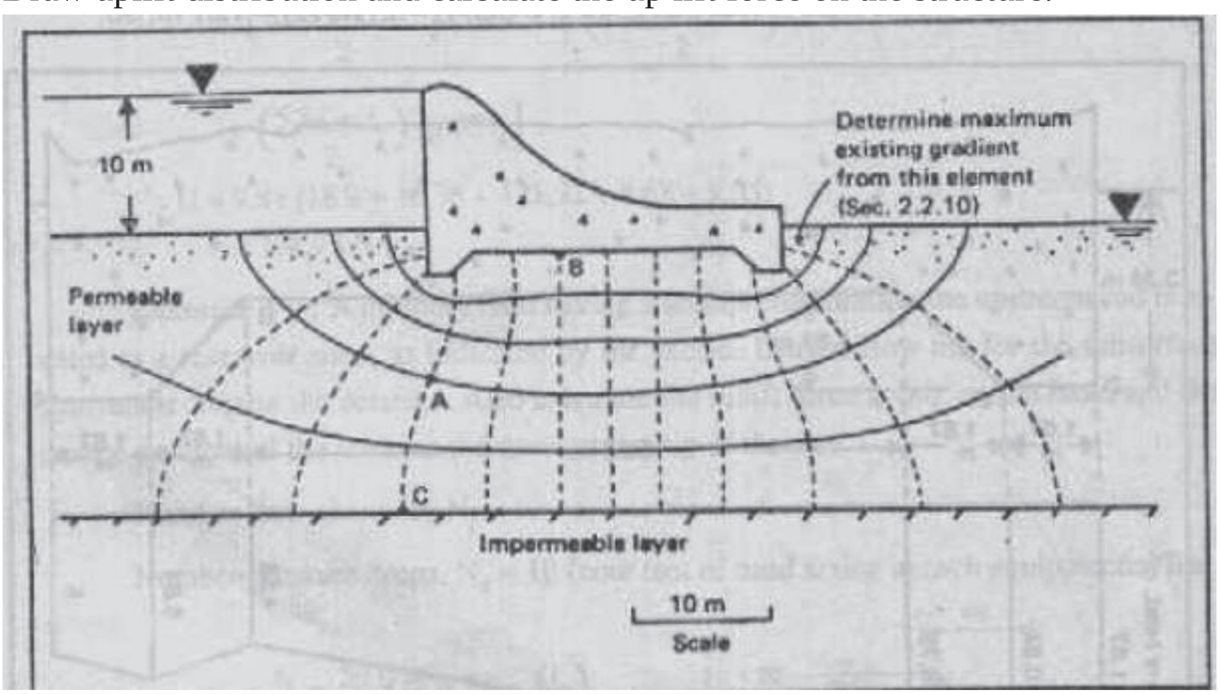


Sol. The maximum hydraulic head h is 10 m in the fig. N_d=12, Λh= h/ N_d=10/12=0.833

Part (a), (i) : To reach A, water has to go through three potential drops. So head lost is equal to 3 x 0.833 = 2.5m. Hence the elevation of the water level in the piezometer at A will be 10 - 2.5 = 7.5 m above the ground surface.

Part (a), (ii) : The water level in the piezometer above the ground level is 10 - 5 (0.833) = 5.84 m.

Part (a), (iii) : Points A and C are located on the same equipotential line. So water in a piezometer at C will rise to the same elevation as at A, i.e., 7.5 m above the ground surface.

Part (b) : The seepage loss is given by $q = kh (N_f/N_d)$. From Fig. 2.30, $N_f = 5$ and $N_d = 12$. Since .

$$k = 0.01 \text{ mm/s} = \left(\frac{0.01}{1000}\right)(60 \times 60 \times 24) = 0.864 \text{ m/day}$$

$$q = 0.864\,(10)\,(5/12) = 3.6 \text{ m}^3/(\text{day . m})$$

- **METHODS OF OBTAINING FLOW NETS**

The following methods are available for the determination of flow nets:

1. Graphical solution by sketching

2. Mathematical or analytical methods

3. Numerical analysis

4. Models

5. Analogy methods طر ق القياس او التماثل

All the methods are based on Laplace's equation.

6.8.1 Graphical Solution by Sketching

A flow net for a given cross-section is obtained by first transforming the cross-section (if the subsoil is anisotropic), and then sketching by trial and error, taking note of the boundary conditions.

(*a*) Every opportunity to study well-constructed flow nets should be utilised تستخدم to get the feel of the problem.

(*b*) Four to five flow channels are usually sufficient for the first attempt.

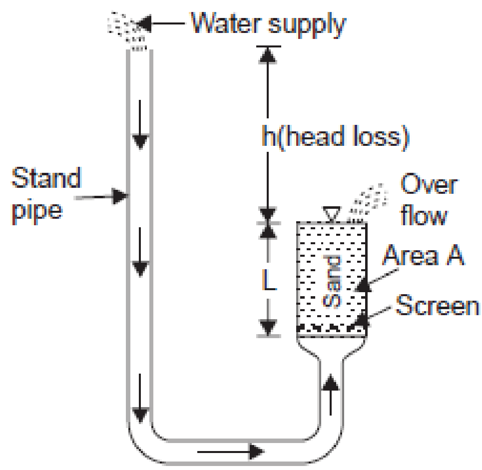
(*c*) The entire flow net should be sketched roughly بشكل تقريبي before details are adjusted تعدل.

(*d*) The fact that all transitions are smooth and are of elliptical or parabolic shape should be borne in mind.

(*e*) The boundary flow lines and boundary equipotentials should first be recognised and Sketched.

- **Quick sand**

Let us consider the upward flow of water through a soil sample as shown in Fig. below. Total upward water force on the soil mass at the bottom surface = $(h + L)\ \gamma_w \cdot A$. Total downward force at the bottom surface = Weight of the soil in the saturated condition



$$= \gamma_{sat} \,.\, L \,.\, A.$$

$$= \frac{(G+e)}{(1+e)} \,.\, \gamma_w \,.\, L \,.\, A.$$

$$(h+L)\,\gamma_w \,.\, A = \frac{(G+e)}{(1+e)} \,.\, \gamma_w\, L \,.\, A.$$

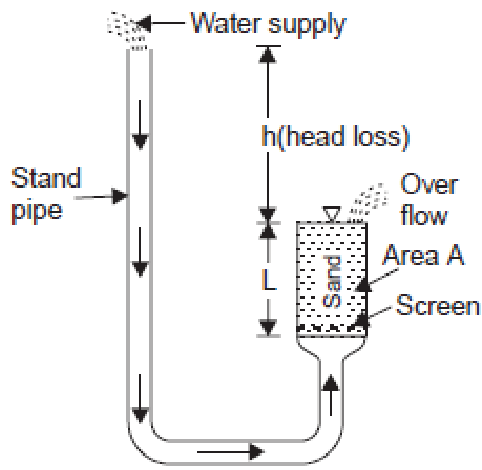
whence $i = h/L = (G - 1)/(1 + e)$

This means that an upward hydraulic gradient of magnitude $(G – 1)/(1 + e)$ will be just sufficient to start the phenomenon of ‘‘boiling’’ in sand. This gradient is commonly referred to as the ‘‘Critical hydraulic gradient’’, *ic*. Its value is approximately equal to unity.

SEEPAGE FORCES

The effective weight of the submerged mass is the submerged weight $\gamma'. LA$ or $\frac{(G-1)}{(1+e)}$ γ_w . LA. An upward force h . γ_w . A is dissipated, or transferred by viscous friction into an upward frictional drag *on the particles*. When quick condition is incipent, these forces are equal:

$$h\gamma_w \,.\, A = \frac{(G-1)}{(1+e)} \,.\, \gamma_w \,.\, L \,.\, A$$



$$= \gamma_{sat} \,.\, L \,.\, A.$$

$$= \frac{(G+e)}{(1+e)} \,.\, \gamma_w \,.\, L \,.\, A.$$

$$(h + L)\,\gamma_w \,.\, A = \frac{(G+e)}{(1+e)} \,.\, \gamma_w L \,.\, A.$$

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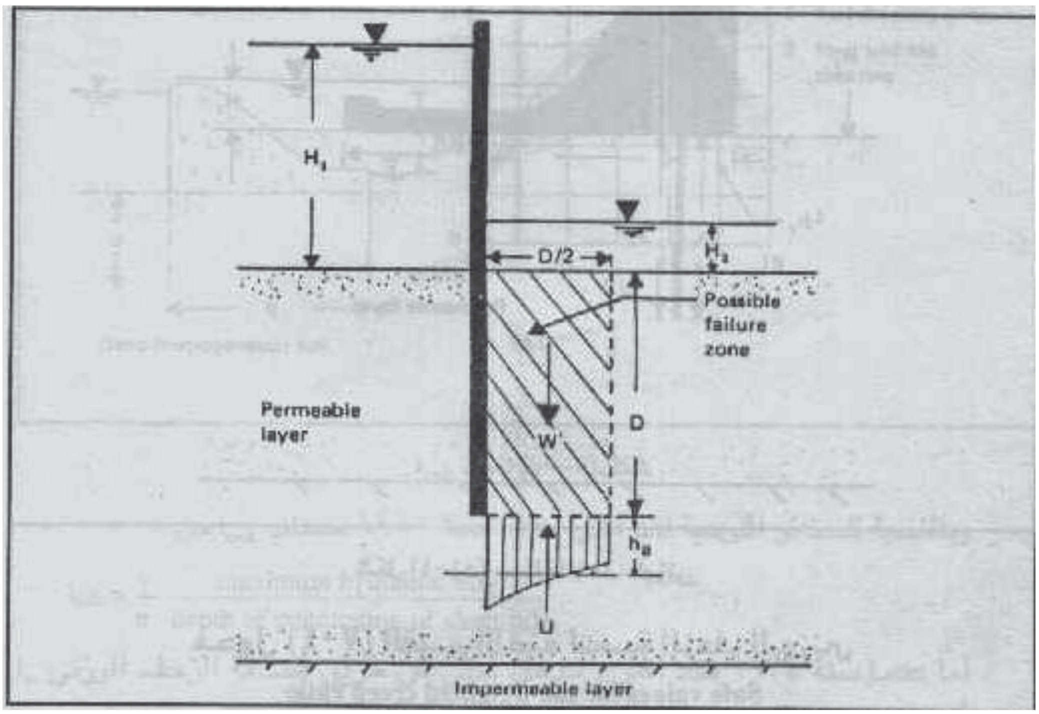
$$h\gamma_w \,.\, A = \frac{(G-1)}{(1+e)} \,.\, \gamma_w \,.\, L \,.\, A$$

The upward force of seepage is (h . γ_w A), as is in the left hand side of Eq. In uniform flow it is distributed uniformly throughout the volume of soil (L. A), and hence the seepage force j per unit volume is (h . γ_w A/LA), which equals (i . γ_w).

$$j = i \cdot \gamma_w$$



$$U = \frac{1}{2}\gamma_w D h_a$$

$$W' = \frac{1}{2}\gamma' D^2$$

$$F_S = \frac{W'}{U} = \frac{\frac{1}{2}\gamma' D^2}{\frac{1}{2}\gamma_w D h_a} = \frac{D\gamma'}{h_a \gamma_w}$$

Example 22: What is the critical gradient of a sand deposit of specific gravity = 2.65 and void ratio = 0.5 ?.

$G = 2.65$, $e = 0.50$

Critical hydraulic gradient, $i_c = (G - 1)(1 + e)$.

$$= \frac{(2.65 - 1)}{(1 + 0.50)} = \frac{1.65}{1.50} = \mathbf{1.1}.$$

Example 23: A 1.25 m layer of the soil (G = 2.65 and porosity = 35%) is subject to an upward seepage head of 1.85 m. What depth of coarse sand would be required above the soil to provide a factor of safety of 2.0 against piping assuming that the coarse sand has the same porosity and specific gravity as the soil and that there is negligible head loss in the sand.

$$G = 2.65;\ n = 35\% = 0.35;\ e = \frac{n}{(1-n)} = \frac{0.35}{0.65} = 7/13$$

Critical hydraulic gradient, $\quad i_c = \frac{(G-1)}{(1+e)} = \frac{(2.65-1)}{(1+7/13)} = \frac{1.65 \times 13}{20} = 1.0725$

With a factor of safety of 2.0 against piping,

Gradient, $\quad i = \frac{i_c}{2} = \frac{10725}{2} = 0.53625$

But $\quad i = h/L$

$\quad L = h/i = 1.850/0.53625 \text{ m} = 3.45 \text{ m}$

Available flow path = thickness of soil = 1.25 m.

$\therefore$ Depth of coarse sand required = **2.20 m.**

Example 24: A glass container with pervious bottom containing fine sand in loose state (void ratio = 0.8) is subjected to hydrostatic pressure from underneath until quick condition occurs in the sand. If the specific gravity of sand particles = 2.65, area of cross-section of sand sample = 10 cm^2 and height of sample = 10 cm, compute the head of water required to cause quicksand condition and also the seepage force acting from below.

$$e = 0.8,\ G = 2.65$$

$$i_c = \frac{(G-1)}{(1+e)} = \frac{(2.65-1)}{(1+0.8)} = \frac{1.65}{1.80} = 11/12 = 0.92$$

$$L = 10 \text{ cm. } h = L \cdot i_c = \frac{10 \times 11}{12} = 55/6 = \mathbf{9.17\ cm.}$$

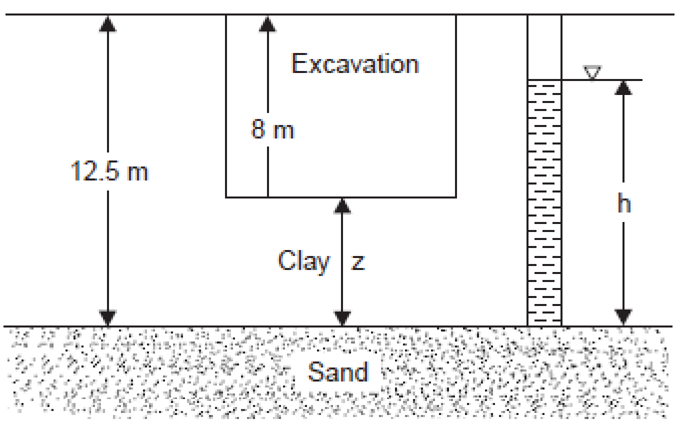
Seepage force per unit volume $= i \cdot \gamma_w$

$$= \frac{11}{12} \times 9.81 \text{ kN/m}^3$$

$$\text{Total seepage force} = \frac{11}{12} \times 9.81 \times \frac{10 \times 10}{100 \times 100 \times 100} \text{ kN}$$

$$\approx 0.0009 \text{ kN} = \mathbf{0.9\ N.}$$

Example 25: A large excavation was made in a stratum of stiff clay with a saturated unit weight of 18.64 kN/m^3. When the depth of excavation reached 8 m, the excavation failed as a mixture of sand and water rushed in. Subsequent borings indicated that the clay was underlain by a bed of sand with its top surface at a depth of 12.5 m. To what height would the water have risen above the stratum of sand into a drill hole before the excavation was started?



The effective stress at the top of sand stratum goes on getting reduced as the excavation proceeds due to relief of stress, the neutral pressure in sand remaining constant.

The excavation would fail when the effective stress reached zero value at the top of sand.

Effective stress at the top of sand stratum,

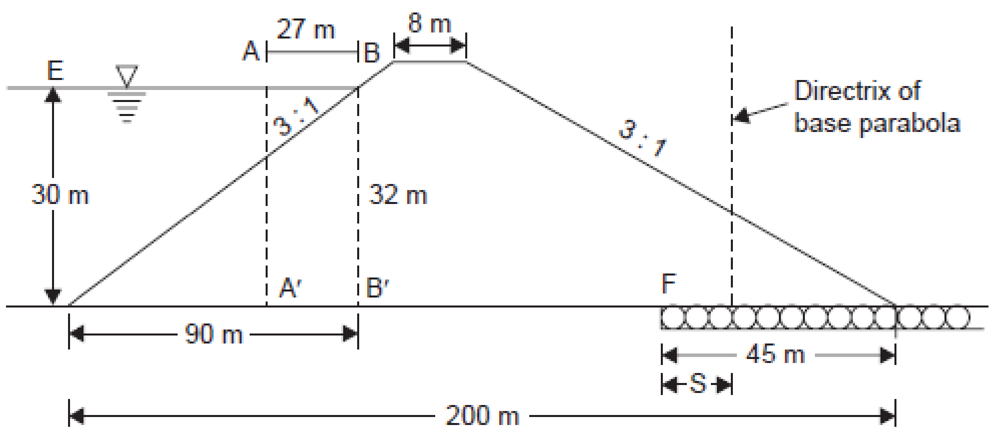
$$\bar{\sigma} = z \cdot \gamma_{sat} - h.\gamma_w$$

$$h\gamma_w = z \cdot \gamma_{sat}$$

$$h = \frac{z \cdot \gamma_{sat}}{\gamma_w} = \frac{(12.5 - 8) \times 18.64}{9.81} = 8.55 \text{ m}$$

Therefore, the water would have risen to a height of **8.55 m** above the stratum of sand into the drill hole before excavation under the influence of neutral pressure.

Example 26: An earth dam of homogeneous section with a horizontal filter is shown in Fig. If the coefficient of permeability of the soil is 3×10^{-2} mm/s, find the quantity of seepage per unit length of the dam.



AB = 0.3(**EB**) = 0.3 × 90 = 27 m

With respect of the focus, **F** (the end of the filter), as origin, the co-ordinates of **A**, the

starting point of the base parabola, are : x = **FA′** = 200 – 90 – 45 + 27 = 92 m

z = **A′A** = 30 m

The equation to the parabola is

$$\sqrt{x^2 + z^2} = x + S,$$

where S is the distance to the directrix from the focus, **F**.

$$\sqrt{92^2 + 30^2} = 92 + S$$

$$S = (\sqrt{92^2 + 30^2} - 92) \text{ m} = 4.77 \text{ m}$$

The quantity of seepage per metre unit length of the dam

$$\begin{aligned} q &= k \,.\, S \\ &= 3 \times 10^{-2} \times 10^{-3} \times 4.77 \text{ m}^3/\text{s} \\ &= 14.31 \times 10^{-5} \text{ m}^3/\text{S} \\ &= \mathbf{143.1 \text{ ml/s.}} \end{aligned}$$

9- SEEPAGE AND FLOW NETS

'Seepage' is defined as the flow of a fluid, usually water, through a soil under a hydraulic gradient. A hydraulic gradient is supposed to exist between two points if there exists a difference in the 'hydraulic head' at the two points. By hydraulic head is meant the sum of the position or datum head and pressure head of water. The discussion on flow nets and seepage relates to the practical aspect of controlling groundwater during and after construction of foundations below the groundwater table, earth dam and weirs on permeable foundations.

- **FLOW NET FOR ONE-DIMENSIONAL FLOW**

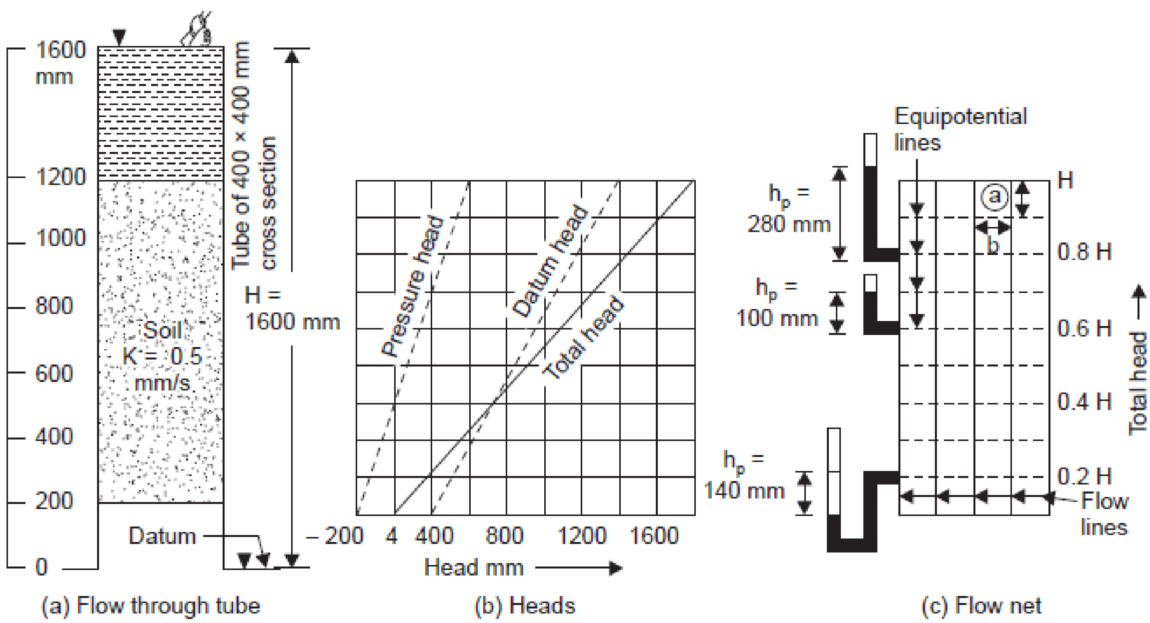


Figure (9.1) one-dimensional flow

Figure 9.1(*a*) shows a tube of square cross-section (400 mm × 400 mm) through which steady state vertical flow is occurring. The total head, elevation head and pressure are plotted in Fig. 9.1(*b*). The rate of seepage through the tube may be computed by Darcy's law: $q = k \, . \, i \, . \, A = 0.5 \times (1600/1000) \times 400 \times 400 = 1.28 \times 105 \text{ mm}^3/\text{s}$

If a dye صبغه is placed at the top of the soil and its movement through the soil is traced on a macroscopic scale, a vertical 'flow line', 'flow path', or 'stream line' would be obtained. The vertical edges of the tube are flow lines automatically; in addition to these, three more flow lines are shown at equal distances apart, for the sake of convenience. These five flow lines divide the vertical cross-section of the tube into four 'flow channels' of equal size Since there are four flow channels in, say, the *x*-direction, and the since the tube is a square one, there are also four flow channels in the *y*-direction, *i.e.*, perpendicular to the page. Thus there will be a total of 16 flow channels

Dashed lines indicate the lines along which the total head is a constant. These lines through points of equal total head are known as 'equipotential lines'.

Consider square *a* in the flow net–Fig. 9.1(*c*). The discharge q_a through this square is

$q_a = k \cdot i_a \cdot A_a$

The head lost in square *a* is given H/n_d , where H is the total head lost and n_d is the number of head drops in the flow net. i_a is then equal to $H/(n_d.\, l)$

Where l is the vertical dimension of square *a*.

$$\therefore \qquad q_a = k \,.\, \frac{H}{n_d \,.\, l} \,.\, b$$

Since a square net is chosen, $b = l$.

$$\therefore \qquad q_a = kH \,.\, \frac{l}{n_d} \,.$$

In order to obtain the flow per unit of length L perpendicular to the paper, q_a should be multiplied by the number of flow channels, say, n_f:

$$\therefore \qquad q/L = q_a \,.\, n_f = kH \,.\, \frac{n_f}{n_d}$$

$$\therefore \qquad q/L = k \,.\, H \,.\, \frac{n_f}{n_d} = kH.\, s$$

the ratio $s = \frac{n_f}{n_d}$ is a characteristic of the flow net and is independent of the permeability k and

The value of s in this case is

$$s = \frac{n_f}{n_d} = 4/10 = 0.4$$

ınd, $q/L = k.H.s = 0.5 \times 1600 \times 0.4 \text{ mm}^3\text{/s/mm} = 320 \text{ mm}^3\text{/s/mm}$

Then, $q = (q/L) \times (400) = 320 \times 400 = 128000$

$= 1.28 \times 10^5 \text{ mm}^3\text{/s}.$

For example, at elevation 1000 mm,

The total head, $h = (8/10) \times H = (8/10) \times 1600 \text{ mm} = 1280 \text{ mm}$

Elevation head, $h_e = 1000 \text{ mm}$

$\therefore$ Pressure head, $h_p = (1280 - 1000) \text{ mm} = 280 \text{ mm}.$

The pore pressure at this elevation is $280 \times 9.81 \times 10^{-6}$ N/mm^2 or 2.75×10^{-3} N/mm^2. Similarly, the pressure heads at elevations 700 mm and 300 mm are 100 mm and – 140 mm, respectively, as shown at the left of the flow net.

- **FLOW NET FOR TWO-DIMENSIONAL FLOW**

During seepage analysis, a flow net can be drawn with as many flow lines as desired. The number of equipotential lines will be determined by the number of flow lines selected. Generally speaking, it is preferable to use the fewest flow lines that still permit reasonable depiction of the path along the boundaries and within the soil mass. For many problems, three or four flow channels (a channel being the space between adjacent flow lines) are sufficient. The first and second—flow under a sheet pile wall and flow under a concrete dam—are cases of confined flow since the boundary conditions are completely defined.

- **Flow under Sheet Pile Wall**

Figure below shows a sheet pile wall driven into a silty soil. The wall runs for a considerable length in a direction perpendicular to the paper; thus, the flow

underneath the sheet pile wall may be taken to be two-dimensional. The boundary conditions for the flow under the sheet pile wall are;

mb = upstream equipotential; **jn** = downstream equipotential; **bej** = flow line.

The flow net shown has been drawn within these boundaries. With the aid of flow net, we can compute the seepage under the wall, the pore pressure at any point and the hydraulic gradient at any point. A water pressure plot, such as that shown in Fig. 9.2 is useful in the structural design of the wall.

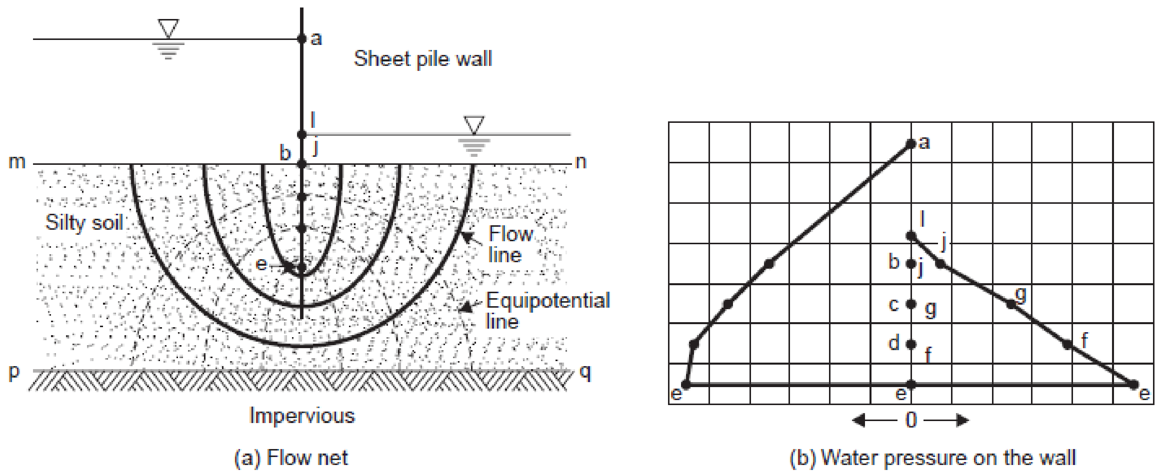


Fig. 9.2 Sheet pile wall

- **Flow under Concrete Dam**

Figures 9.3 to 9.6 show a concrete dam resting on an isotropic soil. The sections shown are actually those of the spillway portion. The upstream and tail water elevations are shown. The first one is with no cut-off walls, the second with cut-off wall at the heel as well as at the toe, the third with cut off-wall at the heel only and the fourth with cut-off wall at the toe only. The boundary flow lines and equipotential are known in each case and the flow nets are drawn as shown within these boundaries. The effect of the cut off walls is to reduce the under seepage, the uplift pressure on the underside of the dam and also the hydraulic gradient at the exit, called the 'exit gradient'. A flow net can be understood to be a very powerful tool in developing a design and evaluating various schemes.

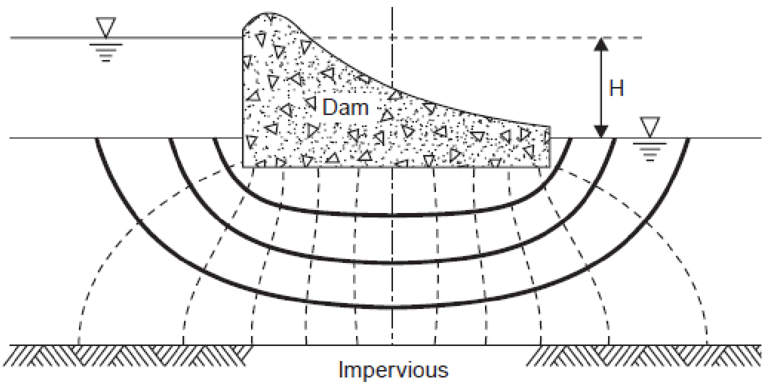


Fig. 9.3 Concrete dam with no cut-off walls

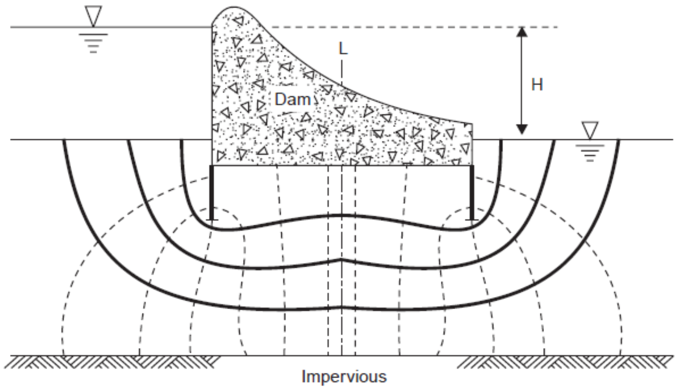


Fig. 9.4 Concrete dam with cut-offs at heel and at toe

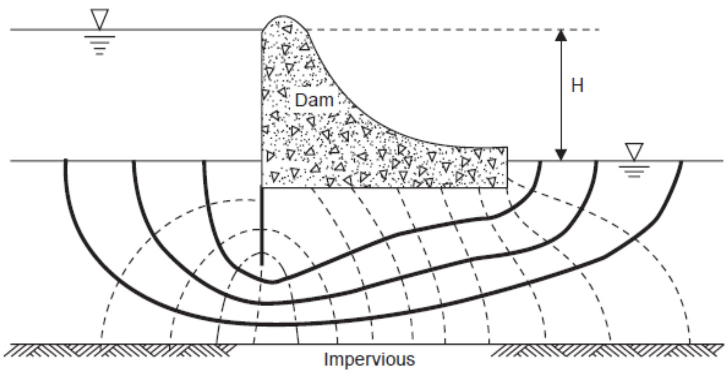


Fig. 9.5 Concrete dam with cut-off at heel

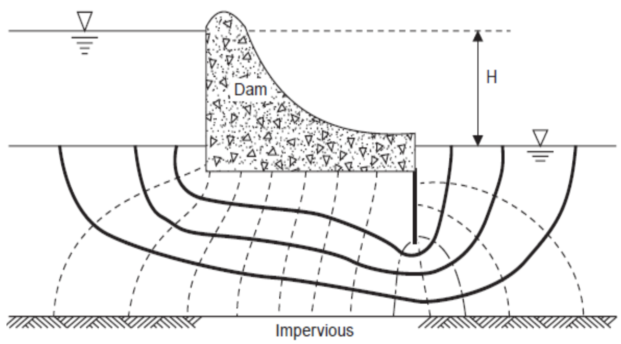


Fig. 9.6 Concrete dam with cut-off wall at toe

- **BASIC EQUATION FOR SEEPAGE**

The flow net was introduced in an intuitive manner in the preceding sections. The equation for seepage through soil which forms the theoretical basis for the flow net as well as other methods of solving flow problems will be derived in this section. The following assumptions are made:

1. Darcy's law is valid for flow through soil.

2. The hydraulic boundary conditions are known at entry and exit of the fluid (water) into the porous medium (soil).

3. Water is incompressible.

4. The porous medium is incompressible.

Let us consider an element of soil as shown in Fig. 9.7, through which laminar flow of water is occurring:

Let q be the discharge with components q_x, q_y and q_z in the X-,Y- and Z-directions respectively. $q = q_x + q_y + q_z$, obviously.

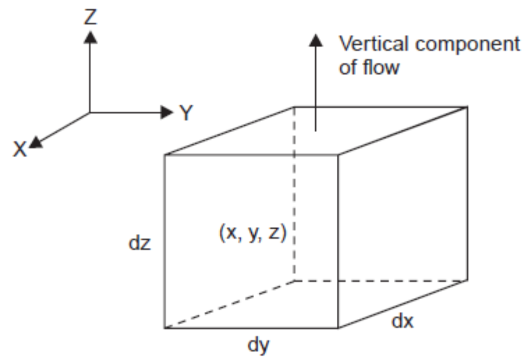


Fig. 9.7 Flow through an element of soil

By Darcy's law,

$q_z = k \cdot i \cdot A$,

where A is the area of the bottom face and q_z is the flow into the bottom face

$$= k_z\left(-\frac{\partial h}{\partial z}\right) dx \cdot dy,$$

where k_z is the permeability of the soil in the Z-direction at the point (x, y, z) and h is the total head. Flow out of the top of the element is given by:

$$q_z + \Delta q_z = \left(k_z + \frac{\partial k_z}{\partial z}, dz\right)\left(-\frac{\partial h}{\partial z} - \frac{\partial^2 h}{\partial z^2} \cdot dz\right) \cdot dx\, dy$$

Net flow into the element from vertical flow:

Δq_z = inflow – outflow

$$= k_z\left(-\frac{\partial h}{\partial z}\right) dxdy - \left(k_z + \frac{\partial k_z}{\partial z} \cdot dz\right)\left(\frac{\partial h}{-\partial z} - \frac{\partial^2 h}{\partial z^2} \cdot dz\right) dx\, dy$$

$$\Delta q_z = \left(k_z \cdot \frac{\partial^2 h}{\partial z^2} + \frac{\partial k_z \partial h}{\partial z^2} + \frac{\partial k_z}{\partial z} \cdot dz\, \frac{\partial^2 h}{\partial z^2}\right) dx\, dy\, dz$$

Assuming the permeability to be constant at all points in a given direction, (that is, the soil is homogeneous),

$$\frac{\partial k_z}{\partial z} = 0$$

$$\Delta q_z = \left(k_z \frac{\partial^2 h}{\partial z^2} \right) dx\, dy\, dz$$

Similarly, the net inflow in the *X*-direction is:

$$\Delta q_x = \left(k_x \cdot \frac{\partial^2 h}{\partial x^2} \right) dx\, dy\, dz$$

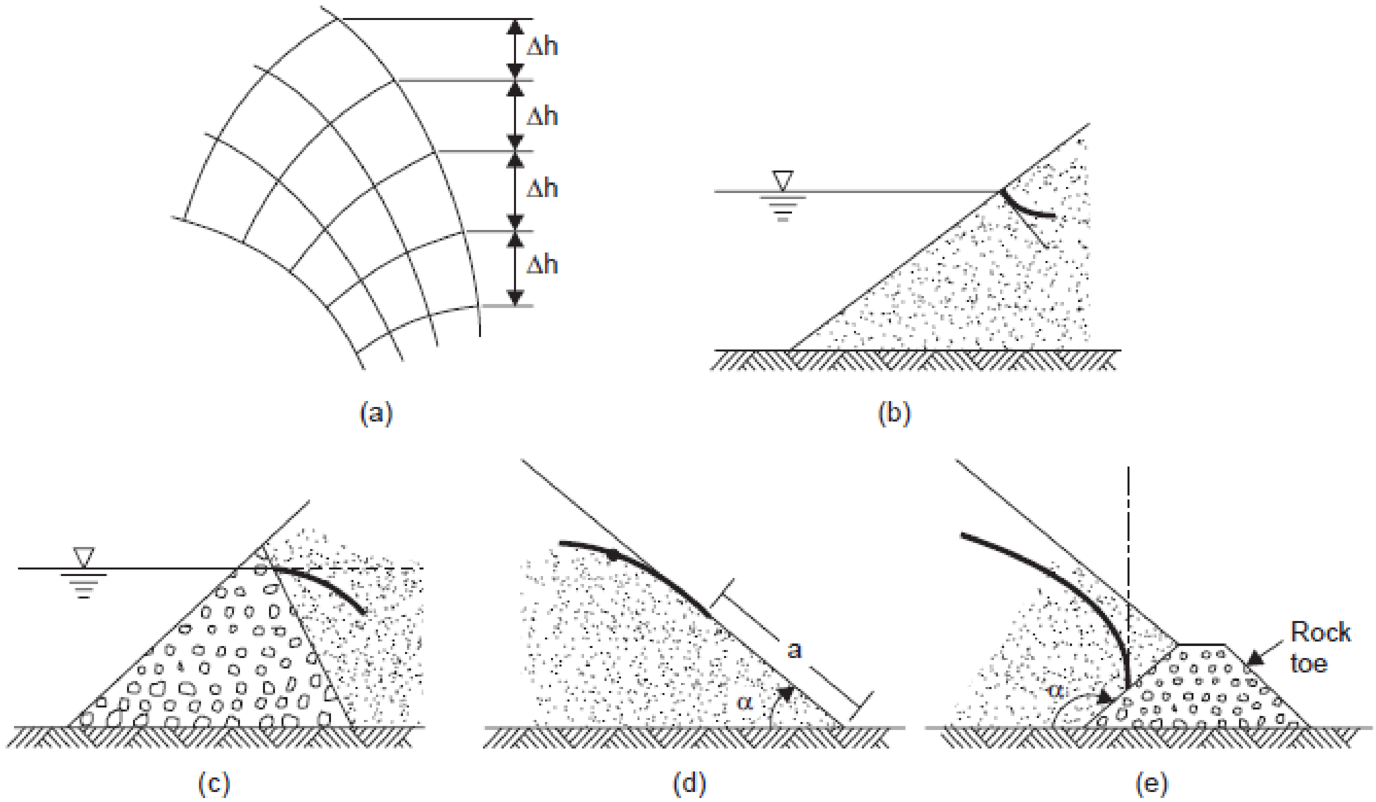
For two-dimensional flow, $q_y = 0$

$$\Delta q = \Delta q_x + \Delta q_z = \left(k_x \cdot \frac{\partial^2 h}{\partial x^2} + k_z \cdot \frac{\partial^2 h}{\partial z^2} \right) dx\, dy\, dz$$

- **TOP FLOW LINE IN AN EARTH DAM**

The flow net for steady seepage through an earth dam can be obtained by any one of the Methods available, including the graphical approach. However, since this is the case of an unconfined flow, the top flow line is not known and hence should be determined first. The top flow line is also known as the 'phreatic line', as the pressure is atmospheric on this line. Thus, the pressures in the dam section below the phreatic line are positive hydrostatic pressures.

The top flow line may be determined either by the graphical method or by the analytical method. Although the typical earth dam will not have a simple homogeneous section, such sections furnish a good illustration of the conditions that must be fulfilled by any top flow line. Furthermore, the location of a top flow line in a simple case can often be used for the first trial in the sketching of a flow net for a more complicated case. The top flow line must obey the conditions illustrated in Fig.

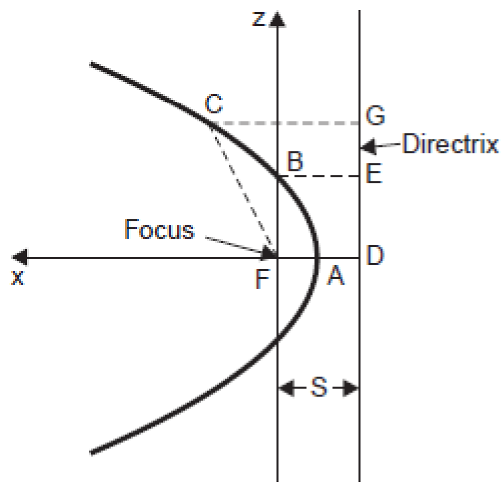


Since the top flow line is at atmospheric pressure, the only head that can exist along it is the elevation head. Therefore, there must be equal drops in elevation between the points at which successive equipotentials meet the top flow line, as in Fig (*a*).

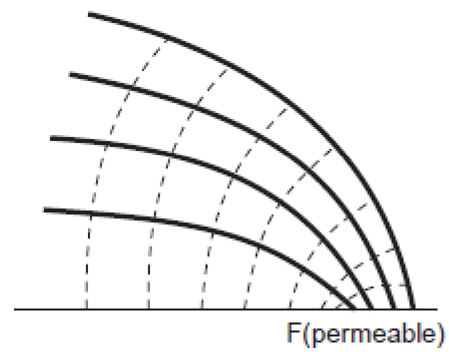
At the starting point, the top flow line must be normal to the upstream slope, which is an equipotential line, as shown in Fig. (*b*). However, an exception occurs when the coarse material at the upstream face is so pervious that it does not offer appreciable resistance to flow, as shown in Fig. (*c*). Here, the upstream equipotential is the downstream boundary of the coarse material. The top flow line cannot be normal to this equipotential since it cannot rise without violating the condition illustrated in Fig. (*a*). Therefore, this line starts horizontally and zero initial gradient and zero velocity occur along it. This zero condition relieves the apparent inconsistency of deviation from a 90-degree intersection.

At the downstream end of the top flow line the particles of water tend to follow paths which conform as nearly as possible to the direction of gravity, as shown in Fig. (*d*); the top flow line here is tangential to the slope at the exit. This is also illustrated by the vertical exit condition into a rock-toe as shown in Fig (*e*).

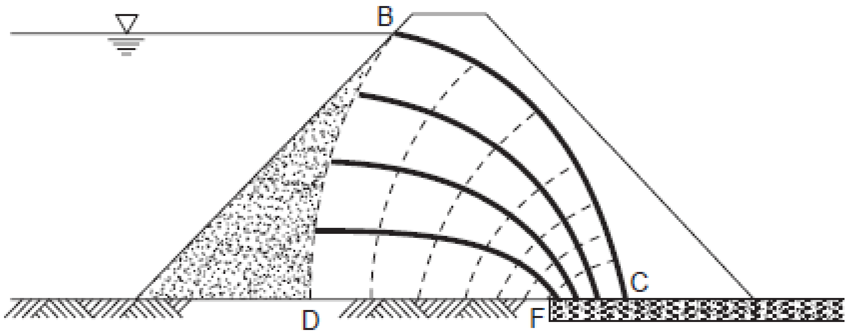
- **Top Flow Line for an Earth Dam with a Horizontal Filter**



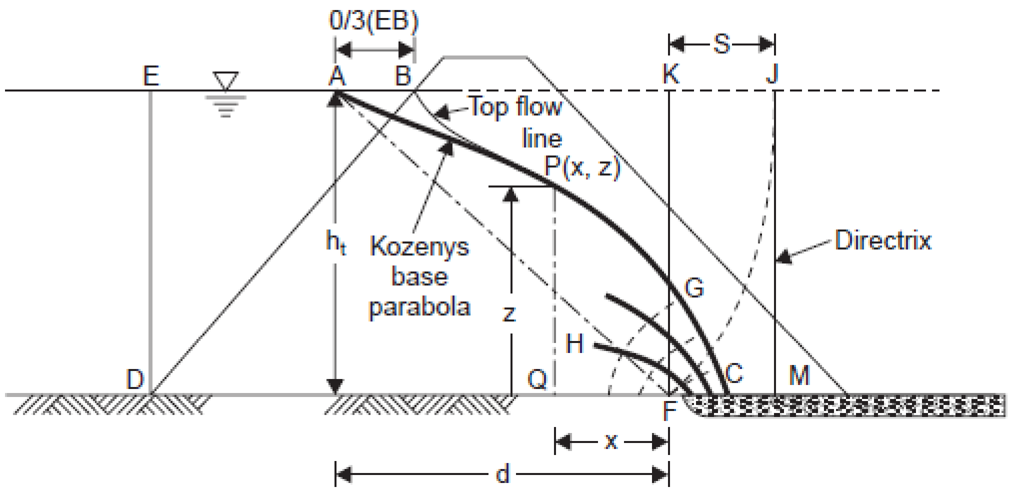
(a) Parabola



(b) Conjugate confocal parabolas (Kozeny, 1931)



(c) Earth dam with flow net consisting of confocal parabolas



BC and **DF** are flow lines, and **BD** and **FC** are equipotential. Casagrande (1937) suggests that **BA** is approximately equal to 0.3 times **BE** where **B** is the starting point of the Kozeny parabola at the upstream water level and **E** is on the upstream water level vertically above the heel **D** of the dam.

(*i*) Locate the point **A**, using **BA** = 0.3 (**BE**). A will be the starting point of the Kozeny

parabola.

(*ii*) With **A** as centre and **AF** as radius, draw an arc to cut the water surface (extended) in **J**. The vertical through **J** is the directrix. Let this meet the bottom surface of the dam in **M**.

(*iii*) The vertex **C** of the parabola is located midway between **F** and **M**.

(*iv*) For locating the intermediate points on the parabola the principle that it must be equidistant from the focus and the directrix will be used. For example, at any distance *x* from **F**, draw a vertical and measure **QM.** With **F** as center and **QM** as radius, draw an arc to cut the vertical through **Q** in **P**, which is the required point on the parabola.

(*v*) Join all such points to get the base parabola. The portion of the top flow line from **B** is sketched in such that it starts perpendicular to **BD**, which is the boundary equipotential and meets the remaining part of the parabola tangentially without any kink. The base parabola meets the filter perpendicularly at the vertex **C**.

The following analytical approach also may be used:

With the origin of co-ordinates at the focus, **PF** = **QM**

$$\sqrt{x^2 + z^2} = x + S$$

$$x = \frac{(z^2 - S^2)}{2S}$$

This is the equation to the parabola.

Analytically ***S*** may be got by substituting the coordinates of **A** (*d*, *ht*) in

$$S = \sqrt{d^2 + h_t^2} - d$$

For different values of *x*, *z* may be calculated and the parabola drawn. The corrections at the entry may then be incorporated.

The flow net **GCFH;** *nf* and *nd* are each equal to 3 for this net.

$$q = k \,.\, H \,.\, 3/3 = k \,.\, S$$

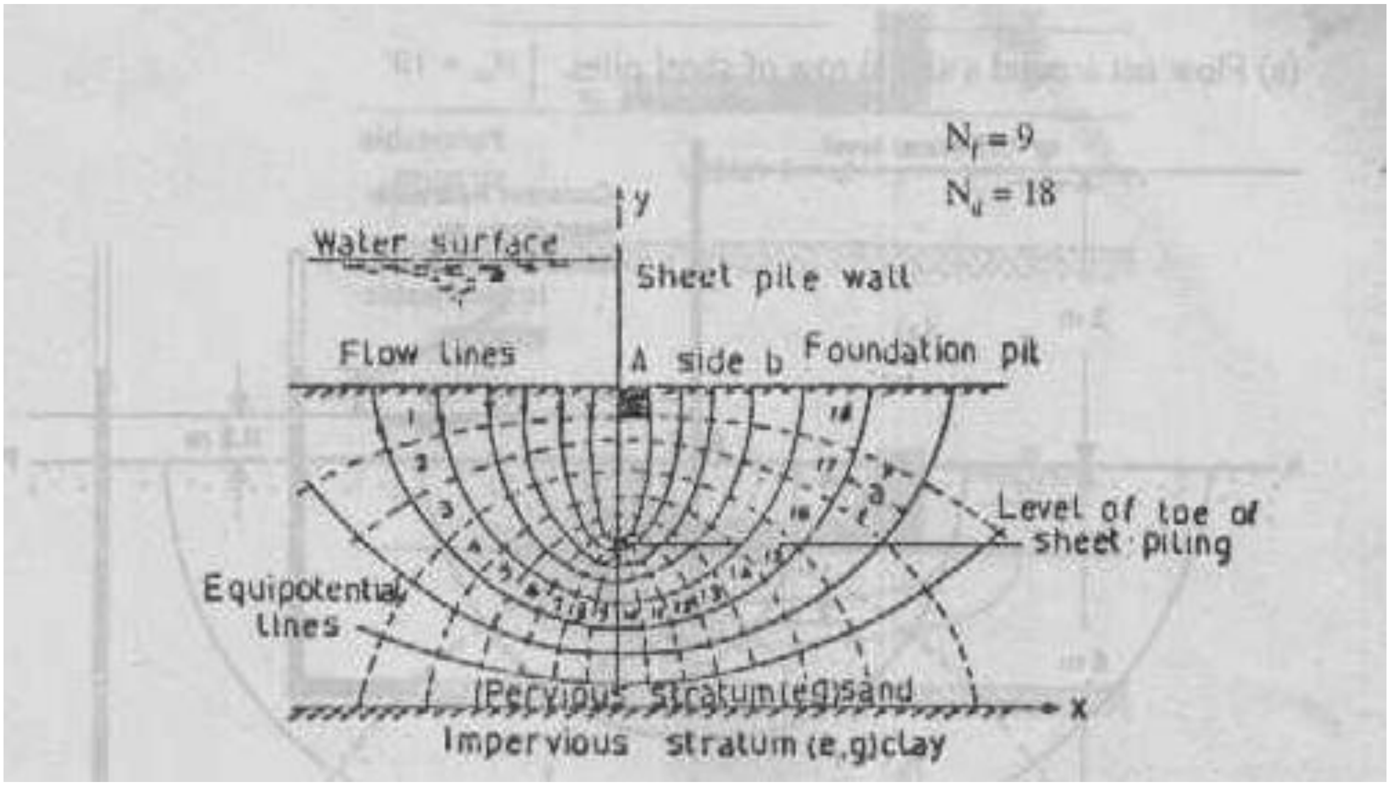
$$q = k \,.\, i \,.\, A$$

$$= k \,.\, \frac{d_z}{d_x} \,.\, z \text{ for unit length of the dam.}$$

But $z = (2x\,S + S^2)^{1/2}$ from

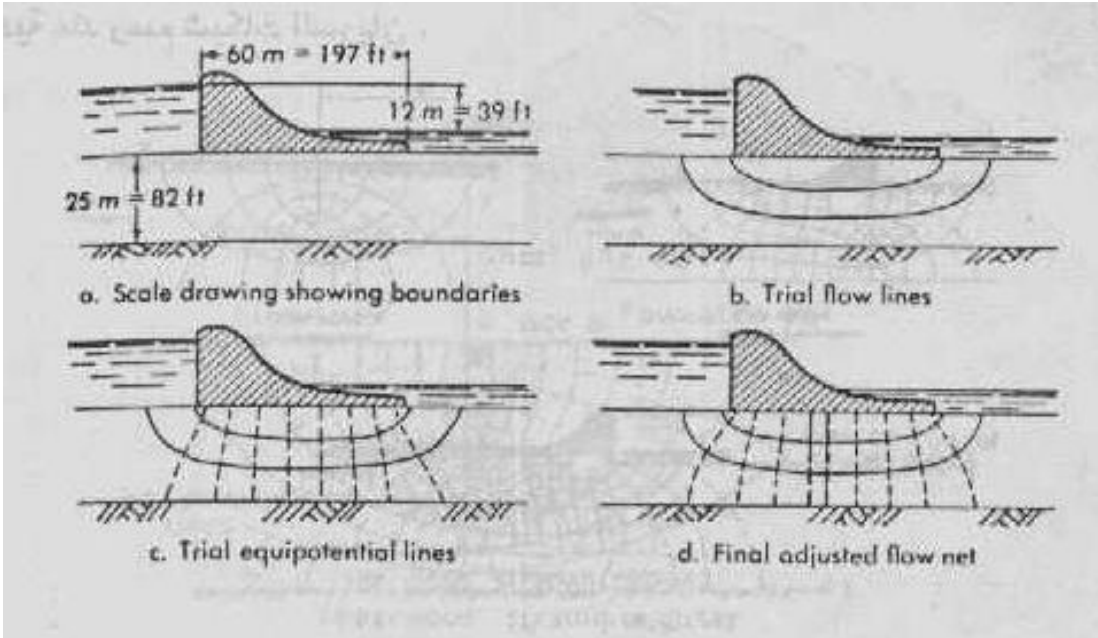
$$\frac{d_z}{d_x} = \frac{1}{2}\frac{2S}{(2xS + S^2)^{1/2}} = \frac{S}{(2xS + S^2)^{1/2}}$$

$$q = k \cdot \frac{S}{(2xS + S^2)^{1/2}} \cdot (2x\,S + S^2)^{1/2} = k.\,S,$$



Ex 19: compute the quantity of seepage under the dam in fig below if K= 1.5 x10^{-3} mm/sec. and level of water upstream is 18 m above the base of dam and downstream is 6 m above the base of the dam. The length of the dam is 250 m.

1- $N_f = 9$ and $N_d = 18$
2- if K= 1.5 x10^{-3}, Δ h= 18-6 =12m
3- q per m =1.5 x10^{-3}x(9/18)x12 = 9x10^{-3} m^2/sec
4- q = 9x10^{-3} m^2/sec x 250 = 2250 x 10^{-3} m^2/sec

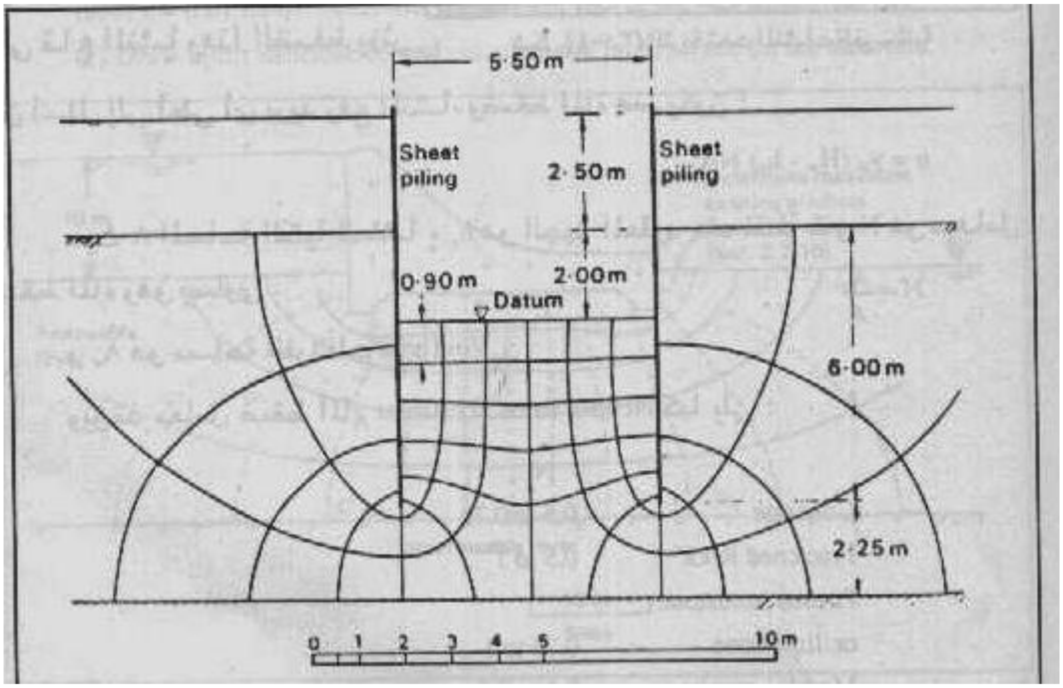


Ex 20: A river bed consist of a layer of sand 8-25m thick overlying impermeable rock, the depth of water is 2.5m. A long cofferdam 5.5m wide is formed by driving two lines of sheet piling to a depth of 6 m below the level of the river bed and excavation to a depth of 2.0 m below bed level is carried out within the cofferdam. The water level within the cofferdam is kept at excavation level by pumping. If the flow of water into cofferdam is 0.25 m^3/h per unit length, what is the coefficient of permeability of the sand? what is the hydraulic gradient immediately below the excavated surface.

Sol: the section and flow net appear in fig below in the flow net there are 6 flow channel and 10 equipotential drops. The total head loss is 4.5m. The coefficient of permeability is given by:

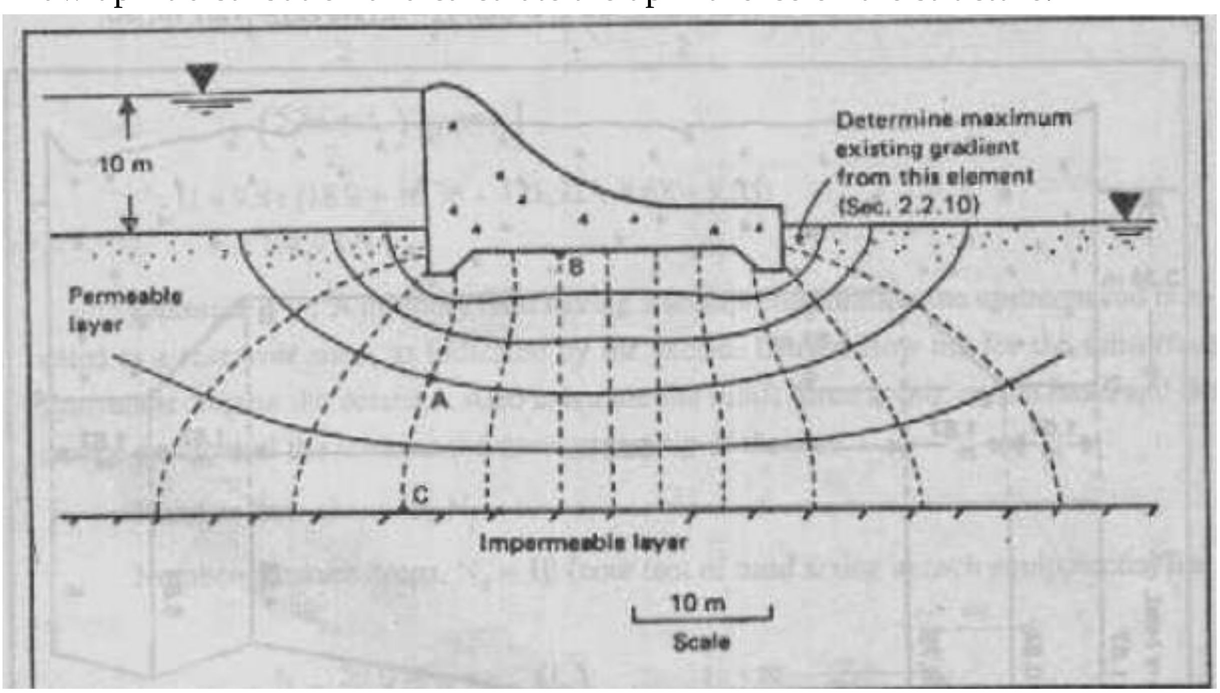
$$k = q \Big/ \left(h\frac{N_f}{N_d}\right)$$

$$= \frac{0.25}{4.50 \times 6/10 \times 60^2} = 2.6 \times 10^{-5} \text{ m/s}$$



Ex 21: for the flow net shown below

a) How high would water rise if piezometer is place at A, B and C.
b) if K= 0.01 mm/s, determine the seepage loss of the dam in m^3/(day.m).
c) Draw uplift distribution and calculate the up lift force on the structure.



Sol. The maximum hydraulic head h is 10 m in the fig. N_d=12, Λh= h/ N_d=10/12=0.833

Part (a), (i) : To reach A, water has to go through three potential drops. So head lost is equal to 3 x 0.833 = 2.5m. Hence the elevation of the water level in the piezometer at A will be 10 - 2.5 = 7.5 m above the ground surface.

Part (a), (ii) : The water level in the piezometer above the ground level is 10 - 5 (0.833) = 5.84 m.

Part (a), (iii) : Points A and C are located on the same equipotential line. So water in a piezometer at C will rise to the same elevation as at A, i.e., 7.5 m above the ground surface.

Part (b) : The seepage loss is given by $q = kh\,(N_f/N_d)$. From Fig. 2.30, $N_f = 5$ and $N_d = 12$. Since .

$$k = 0.01 \text{ mm/s} = \left(\frac{0.01}{1000}\right)(60 \times 60 \times 24) = 0.864 \text{ m/day}$$

$$q = 0.864\,(10)\,(5/12) = 3.6 \text{ m}^3/(\text{day . m})$$

- **METHODS OF OBTAINING FLOW NETS**

The following methods are available for the determination of flow nets:

1. Graphical solution by sketching

2. Mathematical or analytical methods

3. Numerical analysis

4. Models

5. Analogy methods طر ق القياس او التماثل

All the methods are based on Laplace's equation.

6.8.1 Graphical Solution by Sketching

A flow net for a given cross-section is obtained by first transforming the cross-section (if the subsoil is anisotropic), and then sketching by trial and error, taking note of the boundary conditions.

(*a*) Every opportunity to study well-constructed flow nets should be utilised تستخدم to get the feel of the problem.

(*b*) Four to five flow channels are usually sufficient for the first attempt.

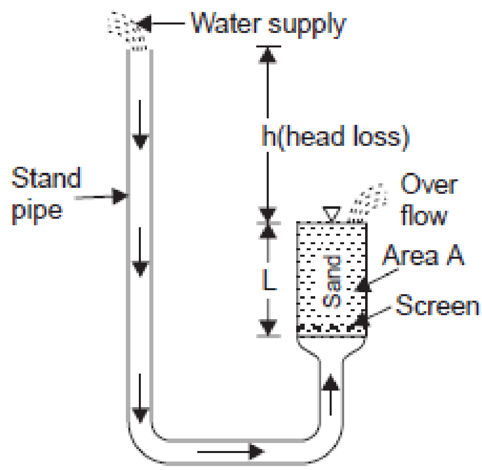
(*c*) The entire flow net should be sketched roughly بشكل تقريبي before details are adjusted تعدل.

(*d*) The fact that all transitions are smooth and are of elliptical or parabolic shape should be borne in mind.

(*e*) The boundary flow lines and boundary equipotentials should first be recognised and Sketched.

- **Quick sand**

Let us consider the upward flow of water through a soil sample as shown in Fig. below. Total upward water force on the soil mass at the bottom surface = $(h + L)\ \gamma_w$. A . Total downward force at the bottom surface = Weight of the soil in the saturated condition



$$= \gamma_{sat} \,.\, L \,.\, A.$$

$$= \frac{(G+e)}{(1+e)} \,.\, \gamma_w \,.\, L \,.\, A.$$

$$(h + L)\,\gamma_w \,.\, A = \frac{(G+e)}{(1+e)} \,.\, \gamma_w\, L \,.\, A.$$

whence $i = h/L = (G - 1)/(1 + e)$

This means that an upward hydraulic gradient of magnitude $(G - 1)/(1 + e)$ will be just sufficient to start the phenomenon of ''boiling'' in sand. This gradient is commonly referred to as the ''Critical hydraulic gradient'', *ic*. Its value is approximately equal to unity.

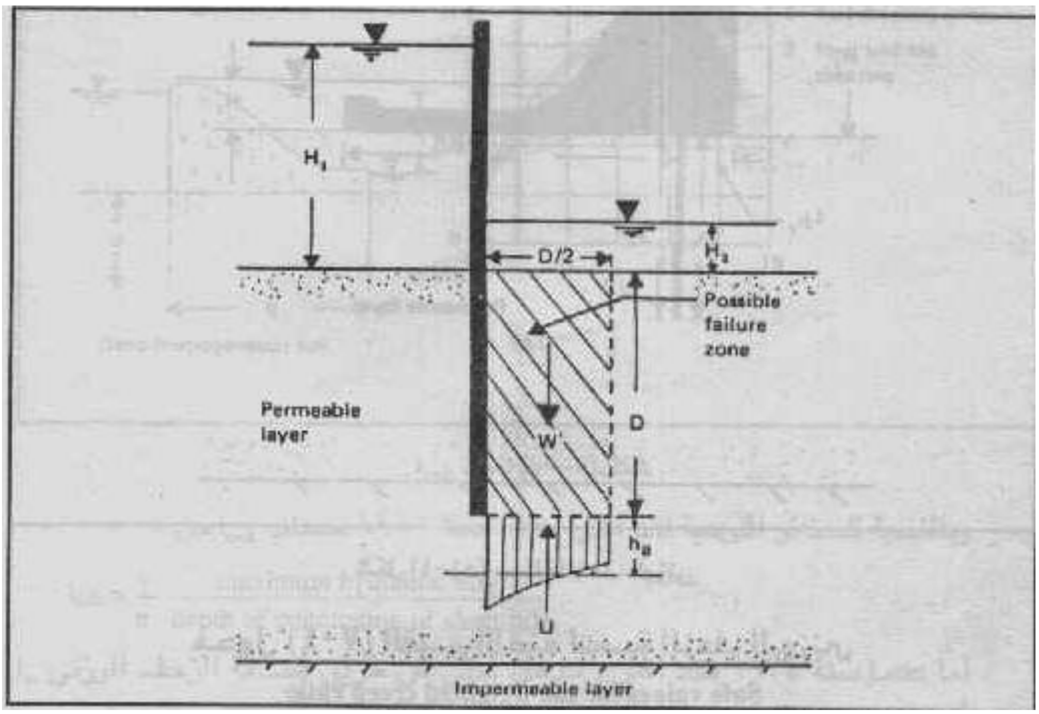
SEEPAGE FORCES

The effective weight of the submerged mass is the submerged weight γ'. LA or $\frac{(G-1)}{(1+e)}$ γ_w . LA. An upward force h . γ_w . A is dissipated, or transferred by viscous friction into an upward frictional drag *on the particles*. When quick condition is incipent, these forces are equal:

$$h\gamma_w \,.\, A = \frac{(G-1)}{(1+e)} \,.\, \gamma_w \,.\, L \,.\, A$$

The upward force of seepage is (h . γ_w A), as is in the left hand side of Eq. In uniform flow it is distributed uniformly throughout the volume of soil (L. A), and hence the seepage force j per unit volume is (h . γ_w A/LA), which equals (i . γ_w).

$$j = i \cdot \gamma_w$$



$$U = \frac{1}{2}\gamma_w\, D h_a$$

$$W' = \frac{1}{2}\gamma'\, D^2$$

$$F_S = \frac{W'}{U} = \frac{\frac{1}{2}\gamma' D^2}{\frac{1}{2}\gamma_w D h_a} = \frac{D\gamma'}{h_a\,\gamma_w}$$

Example 22: What is the critical gradient of a sand deposit of specific gravity = 2.65 and void ratio = 0.5 ?.

$G = 2.65$, $e = 0.50$

Critical hydraulic gradient, $i_c = (G - 1)(1 + e)$.

$$= \frac{(2.65 - 1)}{(1 + 0.50)} = \frac{1.65}{1.50} = \mathbf{1.1}.$$

Example 23: A 1.25 m layer of the soil (G = 2.65 and porosity = 35%) is subject to an upward seepage head of 1.85 m. What depth of coarse sand would be required above the soil to provide a factor of safety of 2.0 against piping assuming that the coarse sand has the same porosity and specific gravity as the soil and that there is negligible head loss in the sand.

$$G = 2.65;\ n = 35\% = 0.35;\ e = \frac{n}{(1-n)} = \frac{0.35}{0.65} = 7/13$$

Critical hydraulic gradient, $i_c = \frac{(G-1)}{(1+e)} = \frac{(2.65-1)}{(1+7/13)} = \frac{1.65 \times 13}{20} = 1.0725$

With a factor of safety of 2.0 against piping,

Gradient, $i = \frac{i_c}{2} = \frac{10725}{2} = 0.53625$

But $i = h/L$

$L = h/i = 1.850/0.53625 \text{ m} = 3.45 \text{ m}$

Available flow path = thickness of soil = 1.25 m.

$\therefore$ Depth of coarse sand required = **2.20 m.**

Example 24: A glass container with pervious bottom containing fine sand in loose state (void ratio = 0.8) is subjected to hydrostatic pressure from underneath until quick condition occurs in the sand. If the specific gravity of sand particles = 2.65, area of cross-section of sand sample = 10 cm^2 and height of sample = 10 cm, compute the head of water required to cause quicksand condition and also the seepage force acting from below.

$$e = 0.8,\ G = 2.65$$

$$i_c = \frac{(G-1)}{(1+e)} = \frac{(2.65-1)}{(1+0.8)} = \frac{1.65}{1.80} = 11/12 = 0.92$$

$$L = 10 \text{ cm. } h = L\ .\ i_c = \frac{10 \times 11}{12} = 55/6 = \mathbf{9.17\ cm.}$$

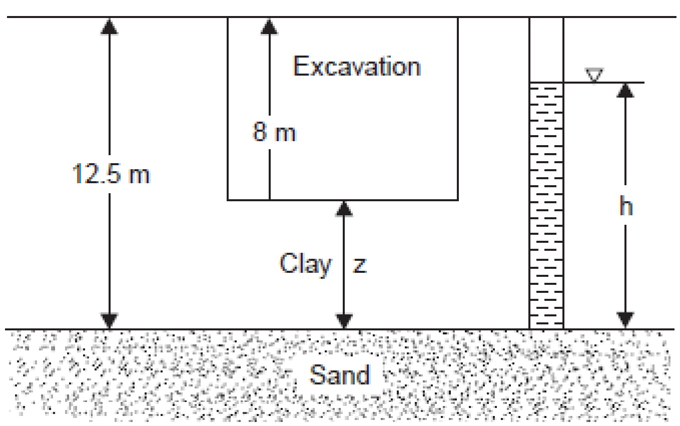
Seepage force per unit volume $= i\ .\ \gamma_w$

$$= \frac{11}{12} \times 9.81 \text{ kN/m}^3$$

$$\text{Total seepage force} = \frac{11}{12} \times 9.81 \times \frac{10 \times 10}{100 \times 100 \times 100} \text{ kN}$$

$$\approx 0.0009 \text{ kN} = \mathbf{0.9\ N.}$$

Example 25: A large excavation was made in a stratum of stiff clay with a saturated unit weight of 18.64 kN/m^3. When the depth of excavation reached 8 m, the excavation failed as a mixture of sand and water rushed in. Subsequent borings indicated that the clay was underlain by a bed of sand with its top surface at a depth of 12.5 m. To what height would the water have risen above the stratum of sand into a drill hole before the excavation was started?



The effective stress at the top of sand stratum goes on getting reduced as the excavation proceeds due to relief of stress, the neutral pressure in sand remaining constant.

The excavation would fail when the effective stress reached zero value at the top of sand.

Effective stress at the top of sand stratum,

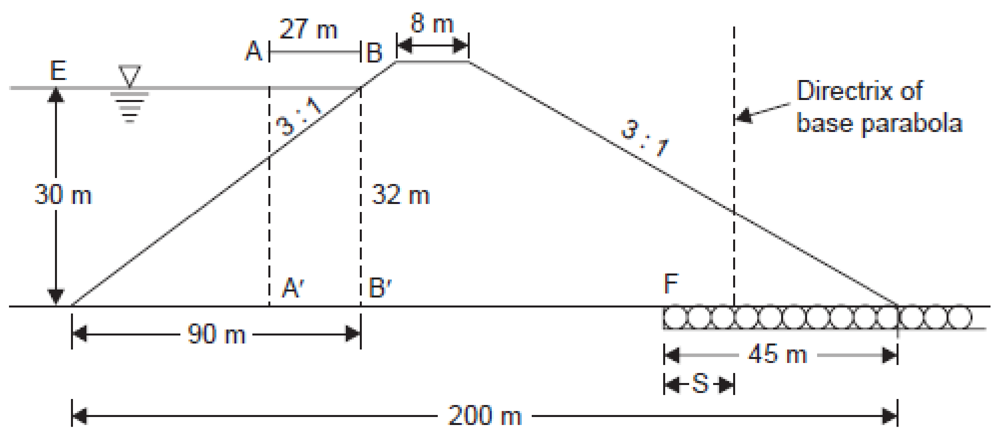
$$\bar{\sigma} = z \cdot \gamma_{sat} - h.\gamma_w$$

$$h\gamma_w = z \cdot \gamma_{sat}$$

$$h = \frac{z \cdot \gamma_{sat}}{\gamma_w} = \frac{(12.5 - 8) \times 18.64}{9.81} = 8.55 \text{ m}$$

Therefore, the water would have risen to a height of **8.55 m** above the stratum of sand into the drill hole before excavation under the influence of neutral pressure.

Example 26: An earth dam of homogeneous section with a horizontal filter is shown in Fig. If the coefficient of permeability of the soil is 3×10^{-2} mm/s, find the quantity of seepage per unit length of the dam.



AB = 0.3(**EB**) = 0.3 × 90 = 27 m

With respect of the focus, **F** (the end of the filter), as origin, the co-ordinates of **A**, the

starting point of the base parabola, are : x = **FA′** = 200 – 90 – 45 + 27 = 92 m

z = **A′A** = 30 m

The equation to the parabola is

$$\sqrt{x^2 + z^2} = x + S,$$

where *S* is the distance to the directrix from the focus, **F**.

$$\sqrt{92^2 + 30^2} = 92 + S$$

$$S = (\sqrt{92^2 + 30^2} - 92) \text{ m} = 4.77 \text{ m}$$

The quantity of seepage per metre unit length of the dam

$$\begin{aligned} q &= k \, . \, S \\ &= 3 \times 10^{-2} \times 10^{-3} \times 4.77 \text{ m}^3/\text{s} \\ &= 14.31 \times 10^{-5} \text{ m}^3/\text{S} \\ &= \mathbf{143.1 \ ml/s.} \end{aligned}$$

9- SEEPAGE AND FLOW NETS

'Seepage' is defined as the flow of a fluid, usually water, through a soil under a hydraulic gradient. A hydraulic gradient is supposed to exist between two points if there exists a difference in the 'hydraulic head' at the two points. By hydraulic head is meant the sum of the position or datum head and pressure head of water. The discussion on flow nets and seepage relates to the practical aspect of controlling groundwater during and after construction of foundations below the groundwater table, earth dam and weirs on permeable foundations.

- **FLOW NET FOR ONE-DIMENSIONAL FLOW**

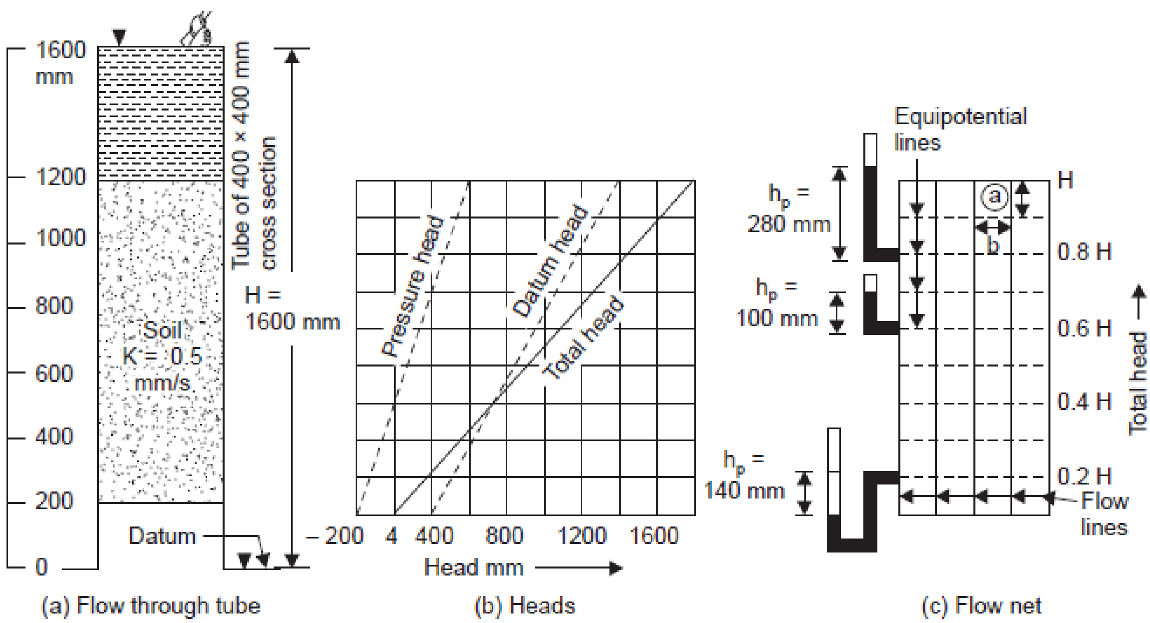


Figure (9.1) one-dimensional flow

Figure 9.1(*a*) shows a tube of square cross-section (400 mm × 400 mm) through which steady state vertical flow is occurring. The total head, elevation head and pressure are plotted in Fig. 9.1(*b*). The rate of seepage through the tube may be computed by Darcy's law: $q = k \cdot i \cdot A = 0.5 \times (1600/1000) \times 400 \times 400 = 1.28 \times 105\ mm^3/s$

If a dye صبغه is placed at the top of the soil and its movement through the soil is traced on a macroscopic scale, a vertical 'flow line', 'flow path', or 'stream line' would be obtained. The vertical edges of the tube are flow lines automatically; in addition to these, three more flow lines are shown at equal distances apart, for the sake of convenience. These five flow lines divide the vertical cross-section of the tube into four 'flow channels' of equal size Since there are four flow channels in, say, the *x*-direction, and the since the tube is a square one, there are also four flow channels in the *y*-direction, *i.e.*, perpendicular to the page. Thus there will be a total of 16 flow channels

Dashed lines indicate the lines along which the total head is a constant. These lines through points of equal total head are known as 'equipotential lines'.

Consider square *a* in the flow net–Fig. 9.1(*c*). The discharge q_a through this square is

$q_a = k \cdot i_a \cdot A_a$

The head lost in square *a* is given H/n_d , where H is the total head lost and n_d is the number of head drops in the flow net. i_a is then equal to $H/(n_d.\ l)$

Where l is the vertical dimension of square *a*.

$$\therefore \qquad q_a = k \ . \ \frac{H}{n_d\ .\ l}\ .\ b$$

Since a square net is chosen, $b = l$.

$$\therefore \qquad q_a = kH\ .\ \frac{l}{n_d}.$$

In order to obtain the flow per unit of length L perpendicular to the paper, q_a should be multiplied by the number of flow channels, say, n_f:

$$\therefore \qquad q/L = q_a\ .\ n_f = kH\ .\ \frac{n_f}{n_d}$$

$$\therefore \qquad q/L = k\ .\ H\ .\ \frac{n_f}{n_d} = kH.\ s$$

the ratio $s = \frac{n_f}{n_d}$ is a characteristic of the flow net and is independent of the permeability k and

The value of s in this case is

$$s = \frac{n_f}{n_d} = 4/10 = 0.4$$

ınd, $q/L = k.H.s = 0.5 \times 1600 \times 0.4 \text{ mm}^3\text{/s/mm} = 320 \text{ mm}^3\text{/s/mm}$

Then, $q = (q/L) \times (400) = 320 \times 400 = 128000$

$= 1.28 \times 10^5 \text{ mm}^3\text{/s}.$

For example, at elevation 1000 mm,

The total head, $h = (8/10) \times H = (8/10) \times 1600 \text{ mm} = 1280 \text{ mm}$

Elevation head, $h_e = 1000 \text{ mm}$

$\therefore$ Pressure head, $h_p = (1280 - 1000) \text{ mm} = 280 \text{ mm}.$

The pore pressure at this elevation is $280 \times 9.81 \times 10^{-6}$ N/mm^2 or 2.75×10^{-3} N/mm^2. Similarly, the pressure heads at elevations 700 mm and 300 mm are 100 mm and – 140 mm, respectively, as shown at the left of the flow net.

- **FLOW NET FOR TWO-DIMENSIONAL FLOW**

During seepage analysis, a flow net can be drawn with as many flow lines as desired. The number of equipotential lines will be determined by the number of flow lines selected. Generally speaking, it is preferable to use the fewest flow lines that still permit reasonable depiction of the path along the boundaries and within the soil mass. For many problems, three or four flow channels (a channel being the space between adjacent flow lines) are sufficient. The first and second—flow under a sheet pile wall and flow under a concrete dam—are cases of confined flow since the boundary conditions are completely defined.

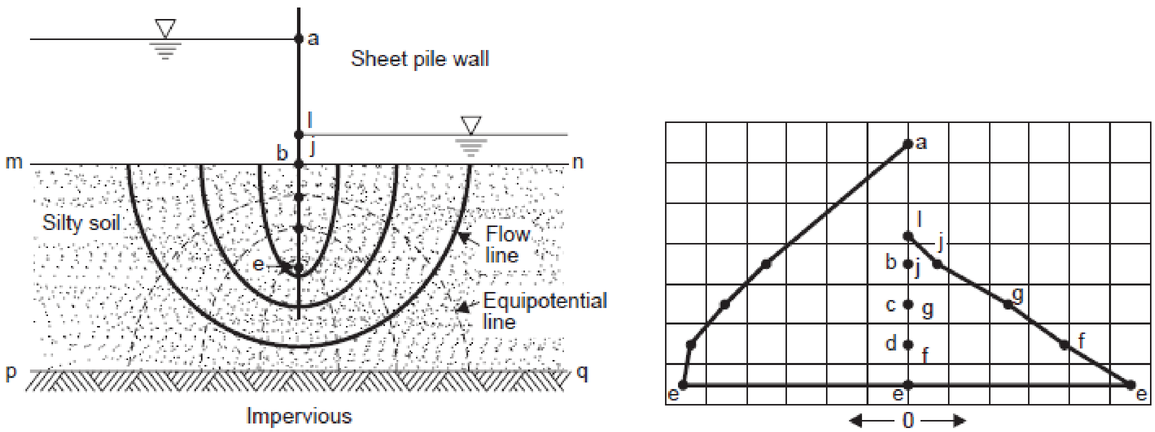
- **Flow under Sheet Pile Wall**

Figure below shows a sheet pile wall driven into a silty soil. The wall runs for a considerable length in a direction perpendicular to the paper; thus, the flow

underneath the sheet pile wall may be taken to be two-dimensional. The boundary conditions for the flow under the sheet pile wall are;

mb = upstream equipotential; **jn** = downstream equipotential; **bej =** flow line.

The flow net shown has been drawn within these boundaries. With the aid of flow net, we can compute the seepage under the wall, the pore pressure at any point and the hydraulic gradient at any point. A water pressure plot, such as that shown in Fig. 6.2 is useful in the structural design of the wall.



(a) Flow net (b) Water pressure on the wall

Fig. 9.2 Sheet pile wall

- **Flow under Concrete Dam**

Figures 9.3 to 9.6 show a concrete dam resting on an isotropic soil. The sections shown are actually those of the spillway portion. The upstream and tail water elevations are shown. The first one is with no cut-off walls, the second with cut-off wall at the heel as well as at the toe, the third with cut off-wall at the heel only and the fourth with cut-off wall at the toe only. The boundary flow lines and equipotential are known in each case and the flow nets are drawn as shown within these boundaries. The effect of the cut off walls is to reduce the under seepage, the uplift pressure on the underside of the dam and also the hydraulic gradient at the exit, called the 'exit gradient'. A flow net can be understood to be a very powerful tool in developing a design and evaluating various schemes.

❖ Characteristics of Grain-size Distribution Curves

D_{10} = the effective diameter is also termed the "Effective Size" of the soil.

"Coefficient of Uniformity" U:

$$U = \frac{D_{60}}{D_{10}}$$

where D_{60} = 60% finer size.

D_{10} = 10% finer size, or effective size.

Another parameter or index which represents the shape of the grain-size distribution curve is known as the "Coefficient of Curvature", ***(C_c)***, defined as:

$$C_c = \frac{(D_{30})^2}{D_{10} \cdot D_{60}}$$

Where

D_{30} = 30% finer size.

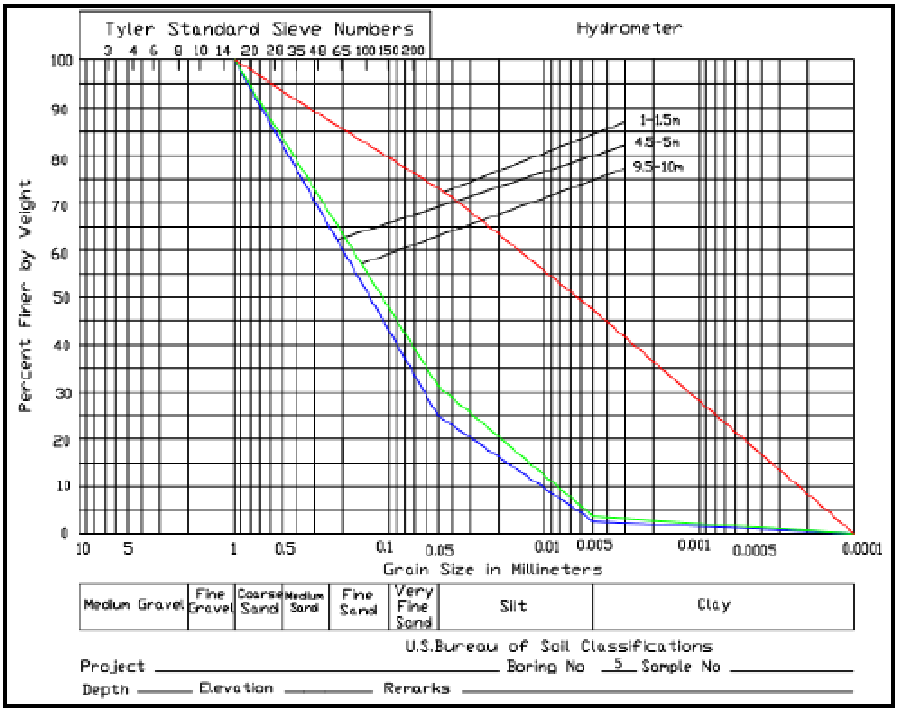
C_c should be 1 to 3 for a well-graded soil.

On the average,

for sands U = 10 to 20,

for silts U = 2 to 4, and

for clays U = 10 to 100



4. CONSISTENCY OF CLAY SOILS

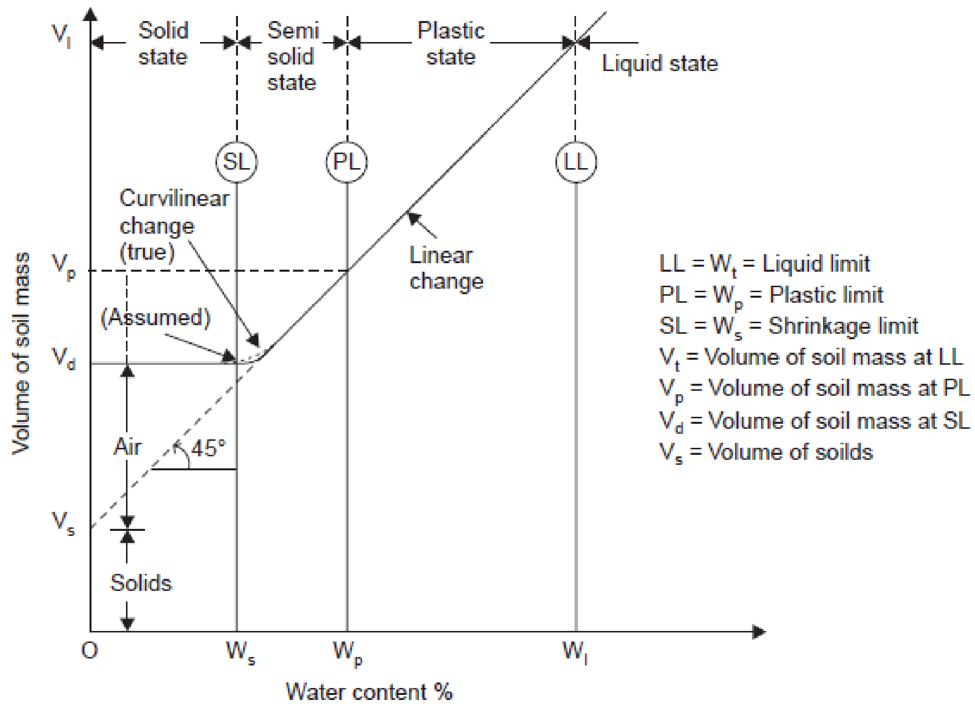
Consistency may also be looked upon as the degree of firmness of a soil and is often directly related to strength. This is applicable specifically to clay soils and is generally related to the water content.

Consistency is conventionally described as soft, medium stiff (or medium firm), stiff (or firm), or hard.

The water contents at which the soil passes from one of these states to the next have been arbitrarily designated as ‘consistency limits’ or ‘Atterberg limits’

1. حد التماسك (Cohesion limit) ذلك المحتوى المائي التي تلتصق عنده فقط فتات التربة سوية.
2. حد اللزوجة (Sticky limit) ذلك المحتوى المائي التي تلتصق عنده التربة بسطح معدني مثل سكينة المزج، ان هذا الحد مهم بالنسبة للمهندس الزراعي حيث انه يتعلق بالتربة التي تلتصق في حديدة او قرص المحراث عند حراثة التربة.
3. حد التقلص (Shrinkage limit) ذلك المحتوى المائي الذي دونه لايحدث اي نقصان او تقلص اضافي في حجم التربة.

4. حد اللدونة (Plastic limit) ذلك المحتوى المائي الذي دونه ستصبح التربة عديمة اللدونة ويدعى هذا الحد ايظا بحد المطاطية.
5. حد السيولة (liquid limit) ذلك المحتوى المائي الذي دونه تسلك التربة سلوكا لدنا وتكون التربة عند هذا المحتوى المائي على وشك ان تصبح مائعا لزجا.



Liquid Limit

'Liquid limit' (*LL* or w_L) is defined as the arbitrary limit of water content at which the soil is just about to pass from the plastic state into the liquid state. At this limit, the soil possesses a small value of shear strength, losing its ability to flow as a liquid. In other words, the liquid limit is the minimum moisture content at which the soil tends to flow as a liquid.

Plastic Limit

'Plastic limit' (*PL* or w_p) is the arbitrary limit of water content at which the soil tends to pass from the plastic state to the semi-solid state of consistency. Thus, this is the minimum water content at which the change in shape of the soil is accompanied by visible cracks, *i.e.*, when worked upon, the soil crumbles متفتت.

Shrinkage Limit

'Shrinkage limit' (*SL* or w_s) is the arbitrary limit of water content at which the soil tends to pass from the semi-solid to the solid state.

Plasticity Index

'Plasticity index' (*PI* or I_p) is the range of water content within which the soil exhibits plastic properties; that is, it is the difference between liquid and plastic limits.

PI (or I_p) = $(LL - PL) = (w_L - w_p)$

Table 3.4 Plasticity characteristics

Plasticity index	*Plasticity*
0	Non-plastic
1 to 5	Slight
5 to 10	Low
10 to 20	Medium
20 to 40	High
> 40	Very high

Table 3.5. Plasticity of the soil according to the liquid limit values

Low plasticity	w_L = < 35%
Intermediate plasticity	w_L = 35 - 50%
High plasticity	w_L = 50 - 70%
Very high plasticity	w_L = 70 - 90%
Extremely high plasticity	w_L = > 90%

Where: w_L is the liquid limit of soil.

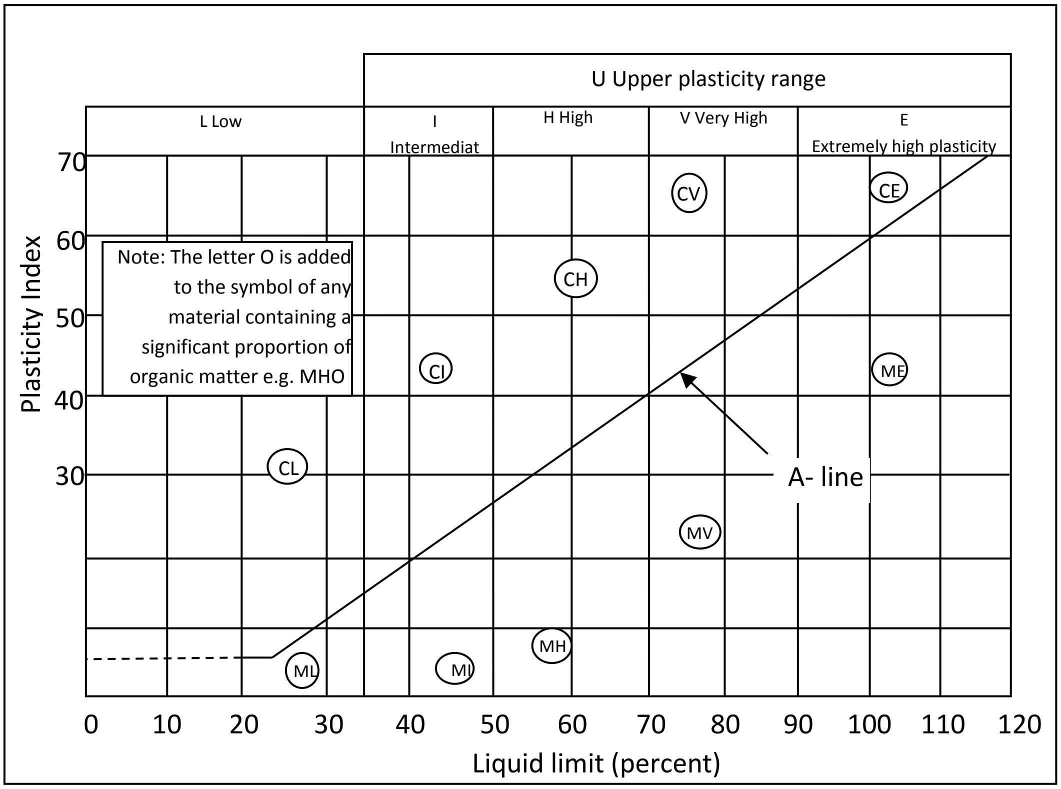


Figure. Plasticity chart for the classification of fine soils and the finer part of coarse soils (measurements made on material passing a 425 mm sieve, in accordance with BS 410).

Shrinkage Index

'Shrinkage index' (*SI* or *Is*) is defined as the difference between the plastic and shrinkage limits of a soil; in other words, it is the range of water content within which a soil is in a semisolid state of consistency.

SI (or I_s) = $(PL - SL) = (w_p - w_s)$

Liquidity Index

'Liquidity index (LI or I_L)' is the ratio of the difference between the natural water content and the plastic limit to the plasticity index:

$$LI(\text{or } I_L) = \frac{(w - PL)}{PI(\text{or } I_p)} = \frac{w - w_p}{I_p}$$

If $I_L = 0$, $w = PL$

$I_L = 1$, $w = LL$

$I_L > 1$, the soil is in liquid state.

$I_L < 0$, the soil is in semi-solid state and is stiff.

LI	*Consistency*
0.00 to 0.25	Stiff
0.25 to 0.50	Medium-soft
0.50 to 0.75	Soft
0.75 to 1.00	Very soft

5. ACTIVITY OF CLAYS

The presence of even small amounts of certain clay minerals can have significant effect on the properties of the soil.

'Activity (A)' is defined as the ratio of plasticity index to the percentage of clay-sizes:

$$A = \frac{I_p}{c}$$

where c is the percentage of clay sizes, i.e., of particles of size less than 0.002 mm.

Table 3.6 Activity classification

Activity	*Classification*
Less than 0.75	Inactive
0.75 to 1.25	Normal
Greater than 1.25	Active

Example 4: The liquid limit of a clay soil is **56%** and its plasticity index is **15%**. (*a*) In what state of consistency is this material at a water content of **45%**? (*b*) What is the plastic limit of the soil? (*c*) The void ratio of this soil at the minimum volume reached on shrinkage is 0.88. What is the shrinkage limit, if its grain specific gravity is 2.71?

Liquid limit, $W_L = 56\%$

Plasticity index, I_p = 15%

$Ip = w_L - w_p$,

∴ 15 = 56 – w_p , Whence the plastic limit, w_p = (56 – 15) = **41%**

∴ at a water content of 45%, the soil is in the *plastic state* of consistency.

Void ratio at minimum volume, e = 0.88

Grain specific gravity, G = **2.71**

Since at shrinkage limit, the volume is minimum and the soil is still saturated,

$e = w_s G$ or $w_s = e/G$ = 0.88/2.71 = 32.5%

∴ Shrinkage limit of the soil = 32.5%.

Example 5: A soil has a plastic limit of 25% and a plasticity index of 30. If the natural water content of the soil is 34%, what is the liquidity index and what is the consistency index ? How do you describe the consistency ?

Plastic limit, w_p = 25%, Plasticity index, I_p = 30

$Ip = w_L - w_p$

∴ Liquid limit, $w_L = I_p + w_p = 30 + 25 = 55\%$.

By Eq. 3.40,

Liquidity index, $I_L = \dfrac{(w - w_p)}{I_p}$

where w is the natural moisture content.

∴ Liquidity index, $I_L = \dfrac{(34 - 25)}{30} = \mathbf{0.30}$

By Eq. 3.39,

Consistency index, $I_c = \dfrac{(w_L - w)}{I_p}$

∴ Consistency index, $I_c = \dfrac{(55 - 34)}{30} = \mathbf{0.70}$

***Example 6*:** A clay sample has void ratio of 0.50 in the dry condition. The grain specific gravity has been determined as 2.72. What will be the shrinkage limit of this clay ?

The void ratio in the dry condition also will be the void ratio of the soil even at the shrinkage limit: but the soil has to be saturated at this limit. For a saturated soil,

$e = w\ \ G$ *i.e* $S=1$

or $w = e/G$

$\therefore w\ s = e/G$ = 0.50/2.72 = 18.4%, Hence the shrinkage limit for this soil is **18.4%**.

Ex the following data were obtained for fine grained soil

Liquid limit, W_L = 48%, Plastic Limit = 26%, clay content =25%, silt = 35%, sand = 10%, moisture content= 39%. Find the Activity and Liquidity Index.

$I._p$= 48-26 = 22%, Activity = I.P /clay content =22/25=0.88 Normal activity.

$$LI(\text{or } I_L) = \frac{(w - PL)}{PI(\text{or } I_p)} = \frac{w - w_p}{I_p}$$ = (39-26)/22=0.59 soft consistency.

6. *IDENTIFICATION AND CLASSIFICATION OF SOILS*

'Gravel', 'Sand', 'Silt' and 'Clay' are used to designate a soil and are based on the average grain-size or particle-size. Most natural soils are mixtures of two or more of these types, with or without organic matter for example, 'silty sand', 'sandy clay', etc. A soil consisting of approximately equal percentages of sand, silt, and clay is referred to as 'Loam'.

6.1 FIELD IDENTIFICATION OF SOILS.

Field identification of soils becomes easier if one understands how to distinguish gravel from sand, sand from silt, and silt from clay.

❖ ***Gravel from Sand***

4.75 mm < Gravel < 80mm.

0.075 mm< Sand < 4.75 mm.

❖ ***Sand from Silt***

It is possible to differentiate between the two by the 'Dispersion Test'. This test consists of pouring a spoonful of sample in a jar of water. If the material is sand, it will settle down in a minute or two, but, if it is silt, it may take 15 minutes to one hour.

In both these cases, nothing may be left in the suspension ultimately.

❖ ***Silt from Clay***

Few simple tests are performed.

Rolling Test. A thread is attempted to be made out of a moist soil sample with a diameter of about 3 mm. If the material is silt, it is not possible to make such a thread without disintegration and crumbling. If it is clay, such a thread can be made even to a length of about 30 cm and supported by its own weight when held at the ends. This is also called the 'Toughness test'.

Dispersion Test. A spoonful of soil is poured in a jar of water. If it is silt, the particles will settle in about 15 minutes to one hour. If it is clay, it will form a suspension which will remain as such for hours, and even for days, provided flocculation does not take place.

6.2 CLASSIFICATION SYSTEMS

The more common classification systems, some of which will be dealt with in greater detail in later sections, are enumerated below :

1. Preliminary Classification by Soil types or Descriptive Classification.

2. Geological Classification or Classification by Origin.

3. Classification by Structure.

4. Grain-size Classification or Textural Classification.

5. Unified Soil Classification System.

6. Indian Standard Soil Classification System.

Unified Soil Classification System

The Unified soil classification system was originally developed by A. Casagrande and adopted by the U.S.

1. Coarse-grained soils with up to 50% passing No. 200 ASTM Sieve.

2. Fine-grained soils with more than 50% passing No. 200 ASTM Sieve.

3. Organic soils.

The first two categories can be distinguished by their plasticity characteristics.

The third can be easily identified by its colour, odor and fibrous nature.

Each soil component is assigned a symbol as follows:

Gravels: *G*	Silt: *M*	Organic: *O*
Sand: *S*	Clay: *C*	Peat: *Pt*

Coarse-grained soils are further subdivided into well-graded (W) and poorly graded (P) varieties, depending upon the Uniformity coefficient, (C_u) and coefficient of Curvature (C_c):

Well-graded gravel, $C_u > 4$

Well-graded sand, $C_u > 6$

Well-graded soil, $C_c = 1$ to 3

Fine-grained soils are subdivided into those with low plasticity (***L***), with liquid limit less than **50%**, and those with high plasticity (***H***), with liquid limits more than **50%**.

The plasticity chart devised by Casagrande is used for identification of fine grained soils.

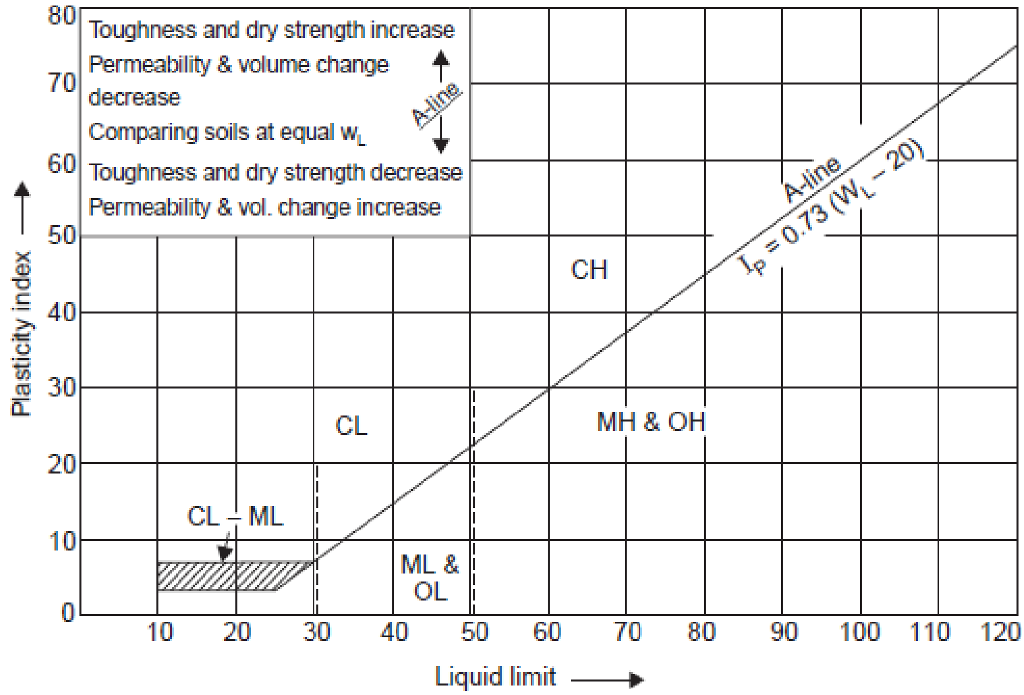


Fig. 4.3 Plasticity chart (unified soil classification)

W_L < 35% Low Plasticity.

35% < W_L < 50% Intermediate Plasticity.

W_L > 50 % High Plasticity.

Major Division	*Group Symbol*	*Typical Name*	*Classification Criteria*	*Field Identification Procedures (Excluding particles larger than 8 cm)*
Coarse grained soils	GW*	Well-graded gravels and gravel-sand mixtures, little or no fines.	$C_u = D_{60}/D_{10}$ Greater than 4	Wide range in grain-size and sub-stantial amounts of all intermediate particle sizes
More than 50% retain on No. 200 ASTM Sieve			$C_c = \dfrac{(D_{30})^2}{D_{10} \times D_{60}}$ Between 1 and 3	
50% or more of coarse fraction re-tained on No. 4 ASTM Sieve	GP	Poorly graded gravels and gravel-sand mixtures, little or no fines	Not meeting both Criteria for GW	Predominantly one size or a range of sizes with some in-termediate size missing
	GM	Silty gravels, gravel-sand-silt mixtures	Atterberg limits plot below *A*-line and plasticity index less than 4	Non-plastic fines (for identi-fication procedures see ML below)
Clean gravels Gravels with fines	GC	Clayey gravels, gravel-sand-clay mixtures	Atterberg limits plot above *A*-line and plasticity index greater than 7	Plastic fines (for identifica-tion procedures see CL be-low)
More than 50% of coarse fraction passes No. 4 ASTM sieve	SW	Well-graded sands and grav-elly sands, little or no fines	$C_u = D_{60}/D_{10}$ Greater than 6 $C_c = \dfrac{(D_{30})^2}{D_{10} \times D_{60}}$ Between 1 and 3	Wide range in grain-sizes and sub-stantial amounts of all intermediate particle sizes.
	SP	Poorly graded sands and gravelly sands, little or no fines	Not meeting both criteria for SW	Predominantly one size or a range of sizes with some in-termediate sizes missing.
Clean sands Sands with fines	SM	Silty sands, sand-silt mix-tures	Atterberg limits plot below *A*-line or plasticity index less than 4	Non-plastic fines (for identi-fication procedures see ML below)
	SC	Clayey sands, sand-clay mix-tures	Atterberg limits plot above *A*-line and plasticity index greater than 7	Plastic fines (for identifica-tion procedures CL below)

Major Division	Group Symbol	Typical Name	Classification Criteria	Identification procedures on Fraction smaller than No. 40 ASTM Sieve: Dry Strength	Dilatancy	Tough-ness
Fine grained soils 50% or more passes No. 200 ASTM Sieve						
Silt and clays	ML	Inorganic Silts, very fine sands, rock flour, silty or clayey fine sands		None to slight	Quick to slow	None
Liquid limit 50% or less	CL	Inorganic clays of low to me-dium plasticity, gravelly clays, sandy clays, silty clays, lean clays		Medium to high	None to very slow	Medium
	OL	Organic silts and organic silty clays of low plasicity	Check plasticity chart	Slight to medium	Slow	Slight
	MH	Inorganic silts, micaceous or diatomaceous fine sands or silts, elastic silts		Slight to medium	Slow to None	Slight to medium
Silt and clays	CH	Inorganic clays of high plas-ticity, fat clays		High to very high	None	High
Liquid limit greater than 50%	OH	Organic clays of medium to high plasticity		Medium to high	None to very slow	Slight to medium
Highly organic clays	Pt	Peat, muck and other highly organic soils	Fibrous organic matter, will char, burn, or glow	Readily identified by colour, odour, spongy feel, and frquently by fibrous texture.		

Note. "Boundary classification": soils possessing characteristics of two groups are designated by combinations of group symbols, for example, GW-GC, Well-graded, gravel-sand mixture with clay binder.

*Classification on the basis of percentage of fines

Less than 5% passing No. 200 ASTM Sieve.. GW, GP, SW, SP

More than 12% passing No. 200 ASTM Sieve.. GM, GC, SM, SC

5% to 12% passing No. 200 ASTM Sieve.. Border line classification requiring use of dual symbols.

Boundary Classification for Coarse-grained Soils

Coarse-grained soils with 5% to 12% fines (Passing No. 200 ASTM Sieve) are considered as border-line cases between clean and dirty gravels or sands as, for example, GW-GC, or SP-SM. Similarly, border-line cases might occur in dirty gravels and dirty sands, where *Ip* is between 4 and 7, as for example, GM-GC or SM-SC. It is, therefore, possible to have a border line case of a border line case. The rule for correct classification in such cases is to favour the non-plastic classification. For example, a gravel with 10% fines, a *Cu* of 20, a *Cc* of 2.0, and *Ip* of 6 would be classified GW—GM rather than GW—GC.

Boundary Classification for Fine-grained Soils

The fine-grained soils whose plot on the plasticity chart falls on, or practically on *A*-line, w_L= 35-lines, w_L = 50 and lines shall be assigned the proper boundary classification. Soils which plot above the *A*-line, or practically on it, and which have a plasticity index between 4 and 7 are classified ML—CL.

Classification Criteria for Fine-grained Soils(org. and inorganic)

The plasticity chart forms the basis for the classification of fine-grained soils, based on the laboratory tests. Organic silts and clays are distinguished from inorganic soils which have the same position on the plasticity chart, by odour and colour. In case of any doubt, the material may be oven-dried, remixed with water and retested for liquid limit. The plasticity of fine-grained organic soils is considerably reduced on oven-drying. Oven-drying also affects the liquid limit of inorganic soils, but only to a small extent. A reduction in liquid limit after oven drying to a value less than three-fourth of the liquid limit before oven-drying is positive identification of organic soils.

Procedure of Classification

The classification procedures for coarse-grained soils and for fine-grained soils, using this system, may be set out in the form of flow diagrams as shown in Figs. 4.5

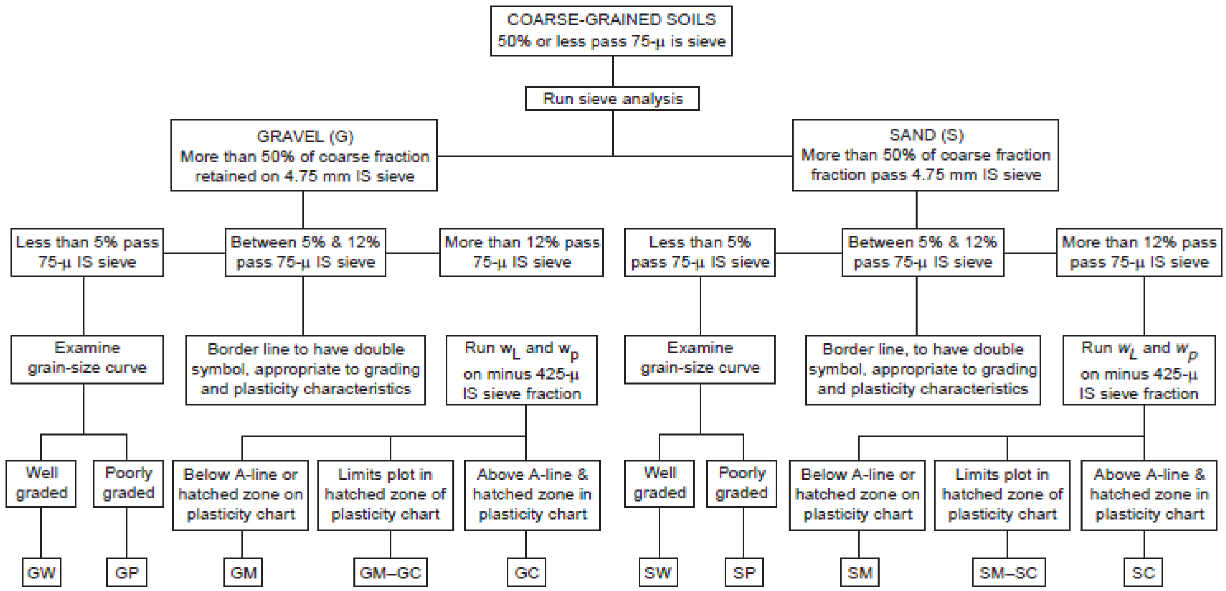


Fig. 4.5 Flow chart for classification of coarse-grained soils

Exampley7: Two soils S_1 and S_2 are tested in the laboratory for the consistency limits. The data available is as follows: Which soil is more plastic?

	Soil S_1	*Soil S_2*
Plastic limit, w_p	18%	20%
Liquid limit, w_L	38%	60%

Plasticity index, Ip for soil $S_1 = w_L - w_P = (38 - 18) = \mathbf{20}$

Ip for soil $S_2 = w_L - w_P = (60 - 20) = \mathbf{40}$

Obviously, Soil S_2 is the more plastic.

Example 8: A soil sample has a liquid limit of 20% and plastic limit of 12%. The following data are also available from sieve analysis:

Sieve size	*% passing*

2.032 mm	100
0.422 mm	85
0.075 mm	38

Classify the soil approximately according to Unified Classification.

- Since more than ***50%*** of the material is larger than ***75-μ*** size, the soil is a coarse-grained one.
- ***100%*** material passes ***2.032*** mm sieve (This will be the same as the per cent passing **4.75** mm sieve). it is classified as sand
- Since more than ***50%*** of coarse fraction is passing this sieve, since more than ***12%*** of the material passes the ***75-μ*** sieve, it must be ***SM*** or ***SC***.
- Now it can be seen that the plasticity index, Ip, is (20 – 12) = 8, which is > 7
- Also, if the values of w_L and I_P are plotted on the plasticity chart, the point falls above *A*-line. Hence the soil is to be classified as SC.

Example 1: One cubic meter of wet soil weighs 19.80 kN/m^3. If the specific gravity of soil particles is 2.70 and water content is 11%, find the void ratio, dry density and degree of saturation.

Bulk unit weight, $= 19.80$ kN/m^3

Water content, $w = 11\% = 0.11$

Dry unit weight, $\gamma_d = \dfrac{\gamma}{(1+w)} = \dfrac{19.80}{(1+0.11)}$ kN/m^3 $=$ **17.84 kN/m^3**

Specific gravity of soil particles $G = 2.70$

$$\gamma_d = \frac{G.\gamma_w}{1+e}$$

Unit weight of water, $\gamma_w = 9.81$ kN/m^3

$$\therefore \qquad 17.84 = \frac{2.70 \times 9.81}{(1+e)}$$

$$(1+e) = \frac{2.70 \times 9.81}{17.84} = 1.485$$

Void ratio, $e =$ **0.485**

Degree of Saturation, $S = wG/e$

$$\therefore \qquad S = \frac{0.11 \times 2.70}{0.485} = 0.6124$$

$\therefore$ Degree of Saturation $=$ **61.24%.**

Example 2: Determine the (*i*) Water content, (*ii*) Dry density, (*iii*) Bulk density, (*iv*) Void ratio and (*v*) Degree of saturation from the following data:

Sample size 3.81 cm dia. × 7.62 cm ht.

Specific gravity = 2.7

Wet weight, W = 1.668 N

Oven-dry weight, W_d = 1.400 N

Water content, $w = \dfrac{(1.668 - 1.400)}{1.40} \times 100\% =$ **19.14%**

Total volume of soil sample, $V = \dfrac{\pi}{4} \times (3.81)^2 \times 7.62$ cm^3

$= 86.87$ cm^3

Bulk unit weight, $\gamma = W/V = \dfrac{1.668}{86.87} = 0.0192$ N/cm^3

$=$ **18.84 kN/m^3**

Example 3: (i) A dry soil has a void ratio of ***0.65*** and its grain specific gravity is = ***2.80***. What is its unit weight?

(ii) Water is added to the sample so that its degree of saturation is ***60%*** without any change in void ratio. Determine the water content and unit weight.

(iii) The sample is next placed below water. Determine the true unit weight (not considering buoyancy) if the degree of saturation is ***95%*** and ***100%*** respectively.

(i) **Dry Soil**

Void ratio, $e = 0.65$

Grain specific gravity, $G = 2.80$

Unit weight, $$\gamma_d = \frac{G.\gamma_w}{(1+e)} = \frac{2.80 \times 9.8}{1.65} \text{ kN/m}^3 = \mathbf{16.65\ kN/m^3}.$$

(ii) **Partial Saturation of the Soil**

Degree of saturation, $S = 60\%$

Since the void ratio remained unchanged, $e = 0.65$

Water content, $$w = \frac{S.e}{G} = \frac{0.60 \times 0.65}{2.80} = 0.1393$$

$$= \mathbf{13.93\%}$$

$$\text{Unit weight} = \frac{(G + Se)}{(1+e)} . \gamma_w = \frac{(2.80 + 0.60 \times 0.65)}{1.65} 9.81 \text{ kN/m}^3$$

$$= \mathbf{18.97\ kN/m^3}.$$

(iii) **Sample below Water**

High degree of saturation $S = 95\%$

$$\text{Unit weight} = \frac{(G + Se)}{(1+e)} . \gamma_w = \frac{(2.80 + 0.95 \times 0.65)}{1.65} 9.81 \text{ kN/m}^3$$

$$= \mathbf{20.32\ kN/m^3}$$

3. DESCRIPTION AND CLASSIFICATION TESTS

1- Soil colour may vary widely, ranging from white through red to black; it mainly depends upon the mineral matter, quantity and nature of organic matter and the amount of colouring oxides of iron and manganese, besides the degree of oxidation.

2-**PARTICLE SHAPE**

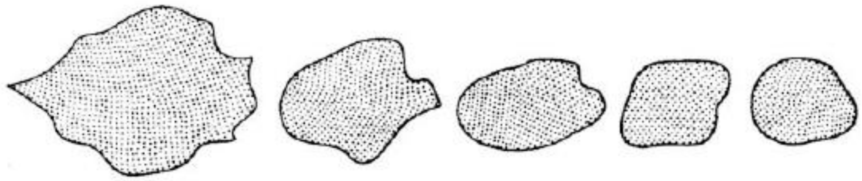


Fig. 3.1 Shapes of granular soil particles

3- WATER CONTENT

The water content of a soil in its natural state is termed its ‘Natural moisture content’, which characterizes its performance under the action of load and temperature.

4- *IN-SITU* UNIT WEIGHT

The *in-situ* unit weight refers to the unit weight of a soil in the undisturbed condition or of a compacted soil in-place.

5 -PARTICLE SIZE DISTRIBUTION (MECHANICAL ANALYSIS)

The particle-size distribution is found in two stages:

(*i*) Sieve analysis, for the coarse fraction.

(*ii*) Sedimentation analysis or wet analysis, for the fine fraction.

- ‘**Sieving**’ is the most direct method for determining particle sizes, but there are practical lower limits to sieve openings that can be used for soils. This lower limit is approximately at the smallest size attributed to sand particles (75μ or 0.075 mm).

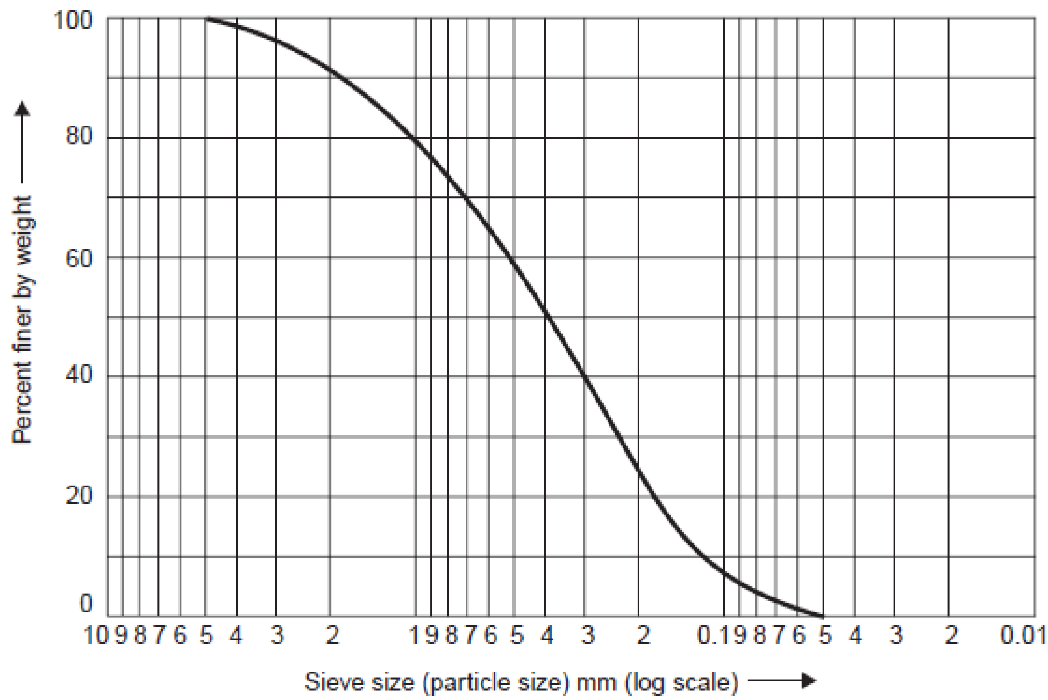
Certain sieve sizes have been standardized by certain Standard Organizations such as:

- ***British Standards Organization (B.S.)***
- ***American Society for Testing Materials (A.S.T.M.)***
- ***Indian Standards Institution (I.S.I.)***

The Indian standard nomenclature is as follows:

Gravel ...	80 mm to 4.75 mm
Sand ...	4.75 mm to 0.075 mm
Silt ...	0.075 mm to 0.002 mm
Clay ...	Less than 0.002 mm

Logarithmic scales for the particle diameter gives a very convenient representation of the sizes because a wide range of particle diameter can be shown in a single plot; also a different scale need not be chosen for representing the fine fraction with the same degree of precision as the coarse fraction.



❖ **Sedimentation Analysis (Wet Analysis)**

The soil particles less than (**0.075 mm**) size can be further analyzed for the distribution of the various grain-sizes of the order of silt and clay be ‘sedimentation analysis’ or ‘wet analysis’ By Stokes’ law, the terminal velocity of the spherical particle is given by Since, usually (***D)*** is to be expressed in **mm**, while (***v***) is to be expressed in (cm/sec), an (μ_τ)in (N-sec/m^2), Eq. 3.15 may be rewritten as follows:

$$v = \frac{1}{180} \frac{(\gamma_s - \gamma_\tau)}{\mu_\tau} . D^2$$

γ_s = unit weight of the material of falling sphere in g/cm^3.

γ_τ = unit weight of the liquid medium in g/cm^3.

μ_τ = viscosity of the liquid medium in N sec/cm^2.

D = diameter of the spherical particle in cm.

v = the terminal velocity, is obtained in cm/s.

Noting that $\gamma_s = G.\gamma_w$,

$$v = \frac{1}{180} \cdot \frac{\gamma_w (G - 1)}{\mu_w} . D^2$$

At 20°C, $\gamma_w = 0.9982 \text{ g/cm}^3 = 0.9982 \times 9.810 \text{ kN/m}^3$

$= 9.792 \text{ kN/m}^3$

$\mu_w = 0.001 \text{ N-sec/m}^2$

Soil mechanic

1- DISCRIBTION AND CLASSIFICATION

SOIL FORMATION

1.1- **RESIDUAL SOILS**

Soils which are formed by weathering of rocks may remain in position at the place of region. In that case these are "Residual Soils".

1.2- TRANSPORTED SOILS

These may get transported from the place of origin by various agencies such as wind, water, ice, gravity, etc. Transported soils may be further subdivided, depending upon the transporting agency and the place of deposition, as under:

Alluvial soils. Soils transported by rivers and streams: Sedimentary clays.

Aeoline soils. Soils transported by wind: loess.

Glacial soils. Soils transported by glaciers: Glacial till. الحرث الجليدي

Lacustrine soils. Soils deposited in lake beds: Lacustrine silts and lacustrine clays.

Marine soils. Soils deposited in sea beds: Marine silts and marine clays.

2- COMPOSITION OF SOIL

A soil mass as it exists in nature is a more or less random accumulation of soil particles, water and air-filled spaces as shown in Fig. 2.1 (*a*). For purposes of analysis it is convenient to represent this soil mass by a block diagram, called 'Phase-diagram', as shown in Fig. 2.1 (*b*). It may be noted that the separation of solids from voids can only be imagined.

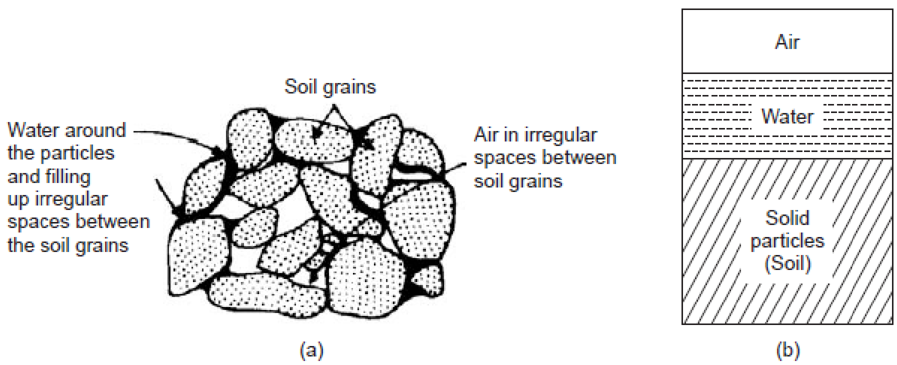


Fig. 2.1 (a) Actual soil mass, (b) Representation of soil mass by phase diagram

When the soil voids are completely filled with water, the gaseous phase being absent, it is said to be '**fully saturated**' or merely مجرد'saturated'. Fig. 3.2 (*a*)

When there is no water at all in the voids, the voids will be full of air, the liquid phase being absent; the soil is said to be **dry**. Fig. 2.2(*b*).

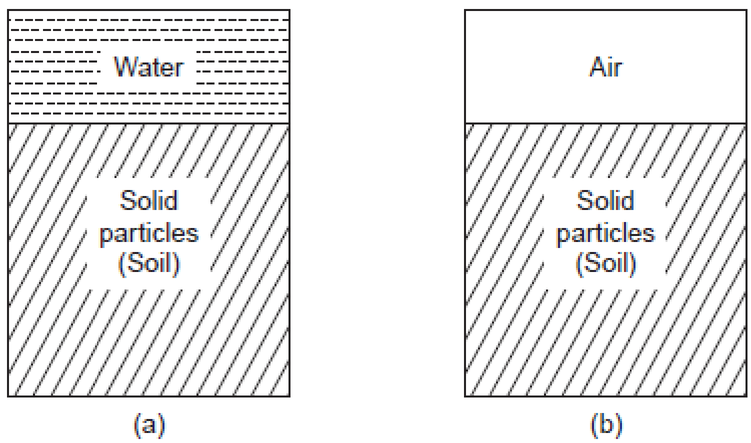
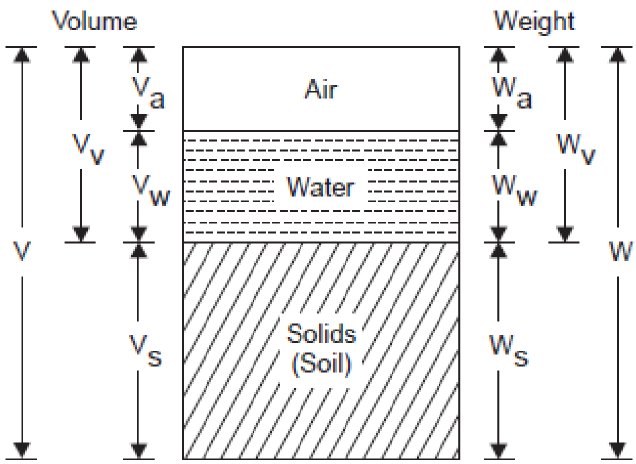


Fig. 2.2 (a) Saturated soil, (b) Dry soil represented as two-phase systems

2.1 Soil Phase



V_a = Volume of air

V_w = Volume of water

V_v = Volume of voids

V_s = Volume of solids

V = Total volume of soil mass

W_a = Weight of air (negligible or zero)

W_w = Weight of water

W_v = Weight of material occupying void space

W_s = Weight of solids

W = Total weight of solid mass

$W_v = W_w$

Fig. 2.3. Soil-phase diagram (volumes and weights of phases)

Porosity‘: is the ratio of the volume of voids to the total volume of the soil mass.

$$n = \frac{V_v}{V} \times 100$$

$$V_v = V_a + V_w \,;\, V = V_a + V_w + V_s$$

Void Ratio : the ratio of the volume of voids to the volume of solids in the soil mass

$$e = \frac{V_v}{V_s}$$

$$V_v = V_a + V_w$$

Degree of Saturation: the ratio of the volume of water in the voids to the volume of voids.

$$S = \frac{V_w}{V_v} \times 100$$

Here $V_v = V_a + V_w$

For a fully saturated soil mass, $V_w = V_v$.

Therefore, for a saturated soil mass $S = 100\%$.

For a dry soil mass, V_w is zero.

Therefore, for a perfectly dry soil sample S is zero.

Percent Air Voids: the ratio of the volume of air voids to the total volume of the soil mass

$$n_a = \frac{v_a}{V} \times 100$$

'***Water content' or 'Moisture content'***: the ratio of the weight of water to the weight of solids (dry weight) of the soil mass.

$$w = \frac{W_w}{W_s \text{ (or } W_d)} \times 100$$

$$= \frac{(W - W_d)}{W_d} \times 100$$

Saturated Unit Weight: the bulk unit weight of the soil mass in the saturated condition. This is denoted by the letter symbol (**$\gamma_{sat.}$**)

Submerged (Buoyant) Unit Weight :it is the submerged weight of soil solids $(W_s)_{sub}$ per unit of total volume, V of the soil. It is denoted by the letter symbol γ'.

$$\gamma' = \frac{(W_s)_{sub}}{V}$$

$$\gamma' = \gamma_{sat} - \gamma_w$$

Specific Gravity of Solids: the ratio of the unit weight of solids (absolute unit weight of soil) to the unit weight of water at the standard temperature (4°C).

$$G = \frac{\gamma_s}{\gamma_o}$$

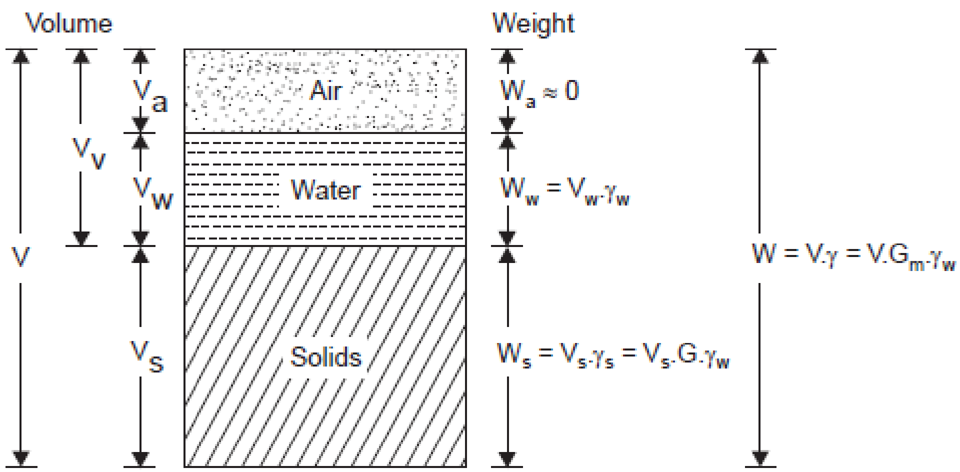


Fig. 3.4. Soil phase diagram showing additional equivalents on the weight side.

2.2 Relationships Involving Porosity, Void Ratio, Degree of Saturation, Water Content, Percent Air Voids and Air Content

Vs = $\frac{w_s}{\gamma_s} = \frac{w_s}{G\gamma_w}$

$$n = \frac{V_v}{V}, \text{ as a fraction}$$

$$= \frac{V - V_s}{V} = 1 - \frac{V_s}{V} = 1 - \frac{W_s}{G\gamma_w V}$$

$$n = 1 - \frac{W_d}{G\gamma_w V}$$

$$e = \frac{V_v}{V_s}$$

$$= \frac{(V - V_s)}{V_s} = \frac{V}{V_s} - 1 = \frac{VG\gamma_w}{W_s} - 1$$

$$e = \frac{V.G.\gamma_w}{W_d} - 1$$

$$n = \frac{V_v}{V} \qquad e = \frac{V_v}{V_s}$$

$$1/n = V/V_v = \frac{V_s + V_v}{V_v} = \frac{V_s}{V_v} + \frac{V_v}{V_v} = 1/e + 1 = \frac{(1+e)}{e}$$

$$n = \frac{e}{(1+e)}$$

$$\gamma_{sat} = \frac{W}{V}$$

$$\gamma_{sat} = \frac{\frac{W}{V_s}}{\frac{V}{V_s}} = \frac{\frac{W_s + W_w}{V_s}}{\frac{V_s + V_v}{V_s}} = \frac{\gamma_s + \frac{W_w}{V_s}}{1 + \frac{V_v}{V_s}} = \frac{\frac{\gamma_s . \gamma_w}{\gamma_w} + \frac{W_w . \gamma_w}{V_s . \gamma_w}}{1+e} = \frac{\gamma_w (\frac{\gamma_s}{\gamma_w} + \frac{V_w}{V_s})}{1+e}$$

$$= \frac{\gamma_w (G + \frac{V_w . V_v}{V_s . V_v})}{1+e} = \frac{\gamma_w (G + S.e)}{1+e}$$

$$S = \frac{v_w}{v_v} = \frac{v_w}{e.v_s} = \frac{v_w . G\gamma_w}{e.w_s} = \frac{v_w . G.w_w}{e.w_s.v_w} = \frac{\omega . G}{e}$$

$$\gamma_{sat} = \left(\frac{G+e}{1+e}\right).\gamma_w$$

$$\gamma_d = \frac{G.\gamma_w}{(1+e)} \qquad \gamma' = \frac{(G-1)}{(1+e)}.\gamma_w$$

Water content, $w = \frac{\text{weight of water}}{\text{weight of solids}} = \frac{S.e.\gamma_w}{G.\gamma_w} = S.e/G$

or $w.G = S.e$

$$\gamma = \frac{\text{total weight}}{\text{total volume}} = \frac{(G + S.e)}{(1+e)}.\gamma_w = \frac{G\gamma_w(1+w)}{(1+e)}$$

$$\gamma_d = \frac{\text{weight of solids}}{\text{total volume}} = \frac{G.\gamma_m}{(1+e)}$$

Example 1: One cubic meter of wet soil weighs 19.80 kN/m^3. If the specific gravity of soil particles is 2.70 and water content is 11%, find the void ratio, dry density and degree of saturation.