Al Muthanna University
Collage of Engineering

Design of Reinforced Concrete Structures I

Chapter one
Introduction

Dr. Othman Hameed

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3. References

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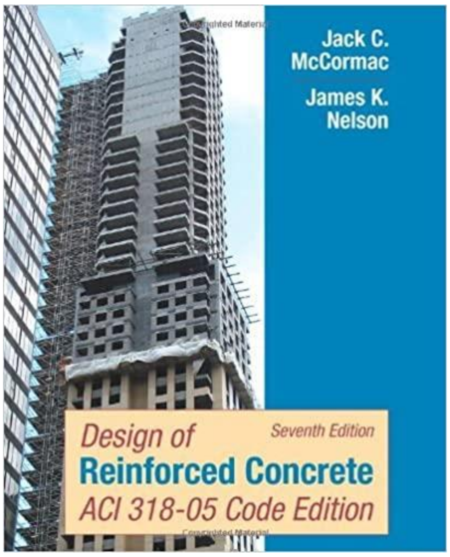
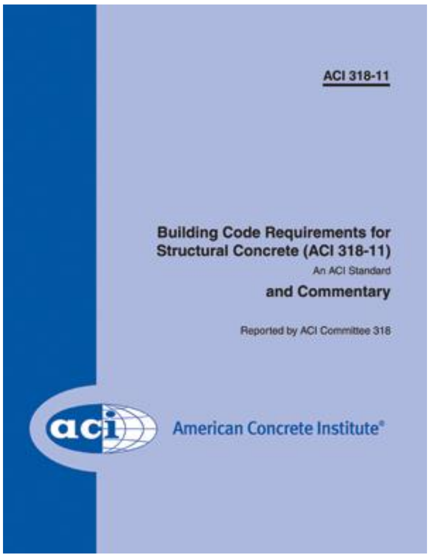
2- Building code requirement for structural concrete (ACI-318M-2011) and commentary





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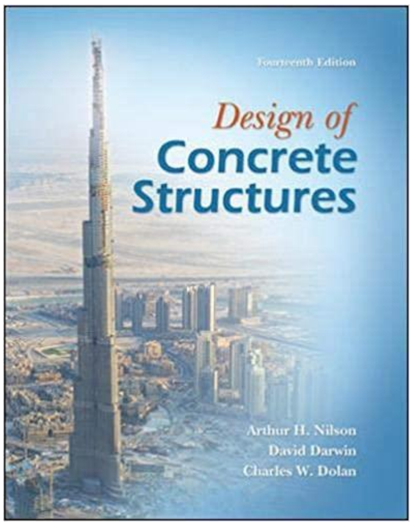

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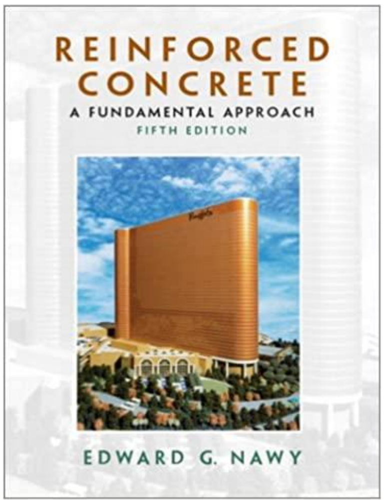

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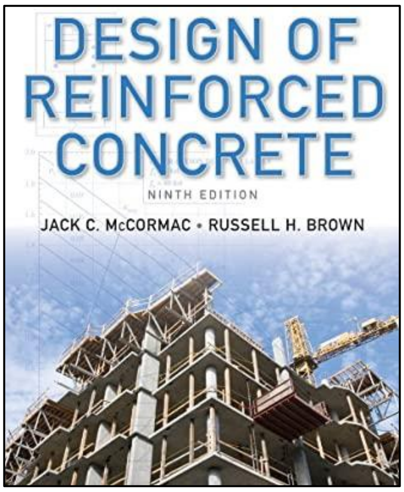

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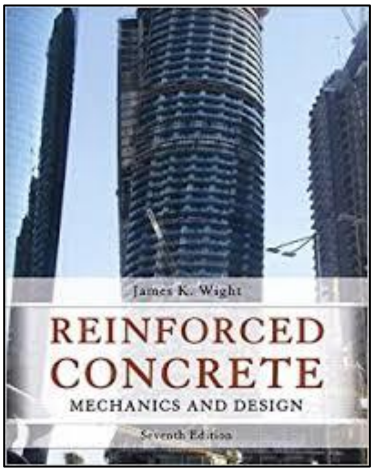


4. Overview

Concrete is a mixture of sand, gravel, crushed rock, or other aggregates held together with a paste of cement and water. Sometimes one or more admixtures are added to change certain characteristics of the concrete such as its workability, durability, and time of hardening.

In its unhardened, concrete can be placed in forms to produce a large variety of structural element. Although the hardened concrete by itself without reinforcement is strong in compression, it lacks tensile strength and therefore cracks easily.

Because unreinforced concrete is brittle, it cannot undergo large deformation under load and fails suddenly without warning. The additional of steel reinforcement to concrete reduces the negative effects of two principle inherent weaknesses, its susceptibility to cracking and its brittleness. Although steel is stiff, high strength material, it is also having several weakness that can be eliminated by encasing it in concrete.

- Concrete surrounding steel protect it from corrosion by moist air or salt water.
- At temperatures over 650c$^{\circ}$, the tensile strength of steel reduces rapidly. Since concrete is a good insulator, steel that is protected by concrete cover during several hours of exposure to intense heat.

Thus, steel and concrete form a synergistic relationship, each material improves the weakness of the other.

When the reinforcement is strongly bonded to concrete and ductile construction material is produced, this material called ***reinforced concrete***, is used extensively to construct foundations, structural frames, storage tanks, highways, dams, and innumerable other structural and building products.

The components of concrete structures can be broadly classified into:

a) Floor Slabs

Floor slabs are the main horizontal elements that transmit the moving live loads as well as the stationary dead loads to the vertical framing supports of a structure. They can be:

- ✓ Slabs on beams,
- ✓ Waffle slabs,
- ✓ Slabs without beams (Flat Plates) resting directly on columns,
- ✓ Composite slabs on joists.

They can be proportioned such that they act in one direction (one-way slabs) or proportioned so that they act in two perpendicular directions (two-way slabs).

b) Beams

Beams are the structural elements that transmit the tributary loads from floor slabs to vertical supporting columns. They are normally cast monolithically with the slabs and are structurally reinforced on one face, the lower tension side, or both the top and bottom faces. As they are cast monolithically with the slab, they form a T-beam section for interior beams or an L beam at the exterior support.

c) Columns

The vertical elements support the structural floor system. They are compression members subjected in most cases to both bending and axial load and are of major importance in the safety considerations of any structure.

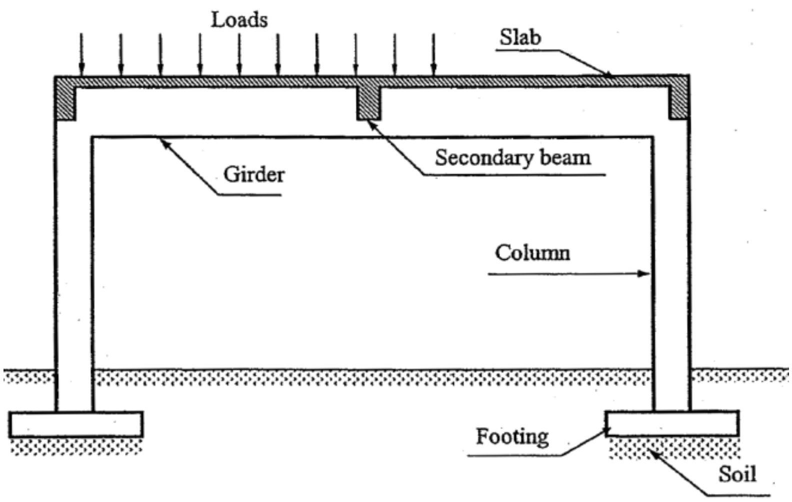
d) Walls

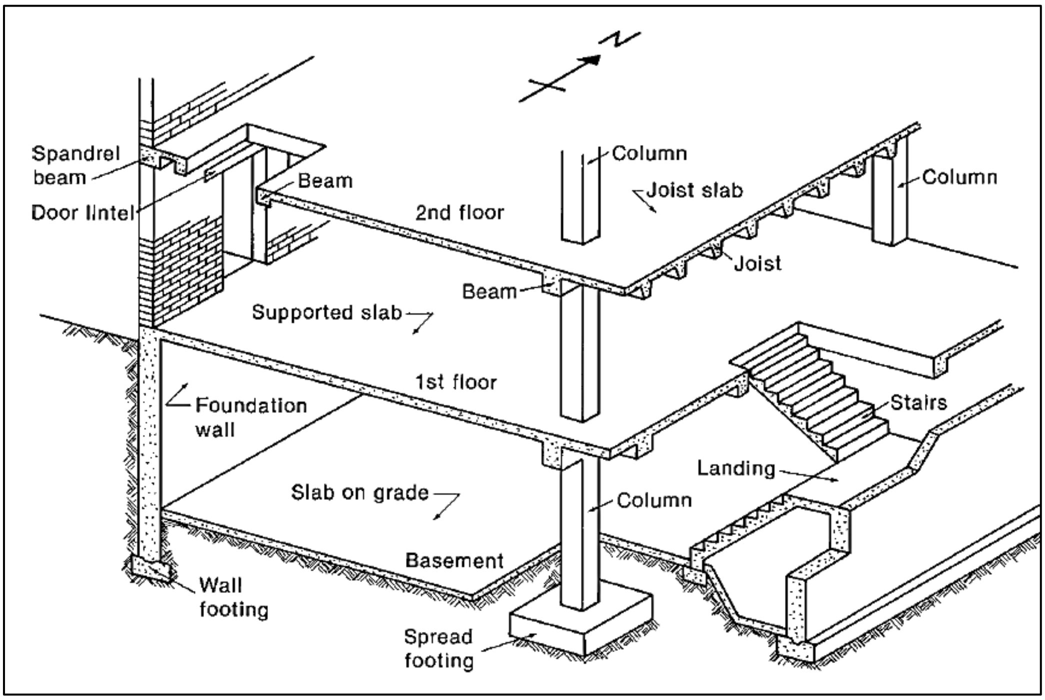
Walls are the vertical enclosures for building frames. They are not usually or necessarily made of concrete but of any material that esthetically fulfills the form and functional needs of the structural system. Additionally, structural concrete walls are often necessary as foundation walls, stairwell walls, and shear walls that resist horizontal wind loads and earthquake-induced loads.

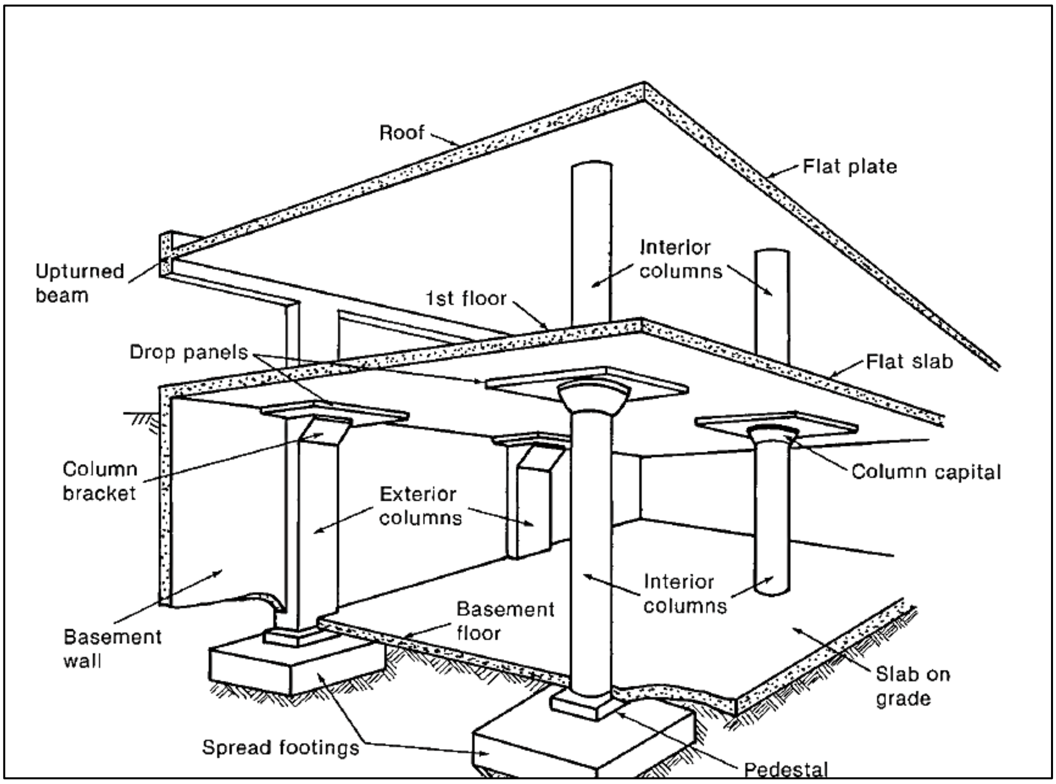
e) Foundations

Foundations are the structural concrete elements that transmit the weight of the superstructure to the supporting soil. They could be in many forms:

- ✓ Isolated footing - the simplest one. It can be viewed as an inverted slab transmitting a distributed load from the soil to the column.
- ✓ Combined footings supporting more than one column.
- ✓ Mat foundations, and rafts which are basically inverted slab and beam construction.
- ✓ Strip footing or wall footing supporting walls.
- ✓ Piles driven to rock.




Spandrel beam
Door lintel
Beam
Column
Joist slab
Column
2nd floor
Joist
Beam
Supported slab
1st floor
Foundation wall
Stairs
Landing
Slab on grade
Column
Basement
Wall footing
Spread footing


Roof
Flat plate
Upturned beam
Interior columns
1st floor
Drop panels
Flat slab
Column bracket
Column capital
Exterior columns
Interior columns
Basement floor
Basement wall
Slab on grade
Spread footings
Pedestal

5. Advantages of Reinforced Concrete as a Structural Material

1. It has considerable compressive strength per unit cost compared with most other materials.
2. Reinforced concrete has great resistance to the actions of fire and water and, in fact, is the best structural material available for situations where water is present.
3. Reinforced concrete structures are very rigid.
4. It is a low-maintenance material.
5. As compared with other materials, it has a very long service life.
6. It is usually the only economical material available for footings, floor slabs, basement walls, piers, and similar applications.
7. A special feature of concrete is its ability to be cast into an extraordinary variety of shapes from simple slabs, beams, and columns to great arches and shells.
8. A lower grade of skilled labor is required for erection as compared with other materials such as structural steel.

6. Disadvantages of Reinforced Concrete as a Structural Material

1. Concrete has a very low tensile strength, requiring the use of tensile reinforcing.
2. Forms are required to hold the concrete in place until it hardens.
3. The low strength per unit of weight of concrete leads to heavy members.
4. Similarly, the low strength per unit of volume of concrete means members will be relatively large, an important consideration for tall buildings and long-span structures. Two other characteristics that can cause problems are concrete's shrinkage and creep.

7. Compatibility of Concrete and Steel

Concrete and steel reinforcing work together beautifully in reinforced concrete structures because of

1- They will act together as a unit in resisting forces. The excellent bond obtained is the result of the chemical adhesion between the two materials,

2- Concrete and steel work well together because their coefficients of thermal expansion are quite close. For steel, the coefficient is 0.0000065 per unit length per degree Fahrenheit, while it varies for concrete from about 0.000004 to 0.000007 (average value: 0.0000055).

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Design of Reinforced Concrete Structures I

Chapter Two
Material Properties

Dr. Othman Hameed

Contents

1 Concrete

Plain concrete is made by mixing cement, fine aggregate, coarse aggregate, water, and frequently admixtures.

Structural concrete can be classified into:

- Lightweight concrete with a unit weight from about 1350 Kg/m^3 to 1850 Kg/m^3 produced from aggregates of expanded shale, clay, and slag.
- Normal-weight concrete with a unit weight from about 1800 Kg/m^3 to 2400 Kg/m^3 produced from the most commonly used aggregates— sand, gravel, crushed stone.
- Heavyweight concrete with a unit weight from about 3200 Kg/m^3 to 5600 Kg/m^3 produced from materials such as barite, limonite and magnetite. It is used for shielding against radiations in nuclear reactor containers and other structures.

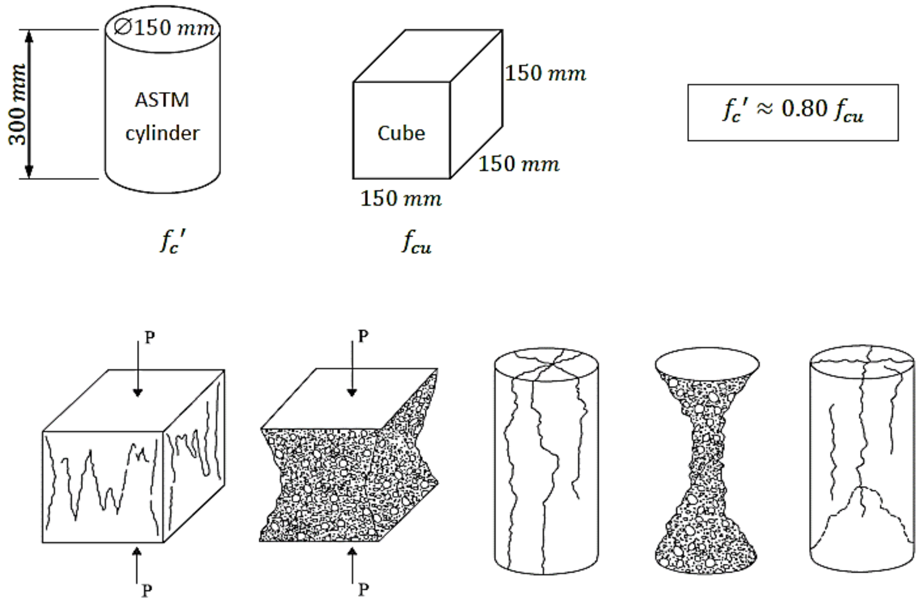
Expanded Shale

2 Mechanical Properties of Concrete

2.1 Compressive Strength of Concrete

The compressive strength of concrete is usually determined by loading (150mm) diameter by (300mm) high cylinders to failure in uniaxial compression. Cylinders are tested after they have hardened for 28 days. During the 28 days before testing, the cylinders are stored under water or placed in a constant-temperature room. Exposure to moisture speeds the gain in strength by increasing the hydration of the cement.

Additional details covering the preparation and testing of cylinders are covered by ASTM specifications. *The concrete strength depends on the size and shape of the test specimen and the manner of testing. For this reason, the cylinder (Ø150 mm by 300 mm high) strength is 80% of the 150 mm cube strength and 83% of the 200 mm cube strength.*



Cube and cylinder test specimens

2.2 Tensile Strength of Concrete

Experimental studies show that the tensile strength of concrete is highly variable and ranges from approximately 8 to 15 percent of the compressive strength f_c'. The large difference between the tensile and the compressive strengths of concrete is due in part to the many fine cracks that exist throughout the concrete.

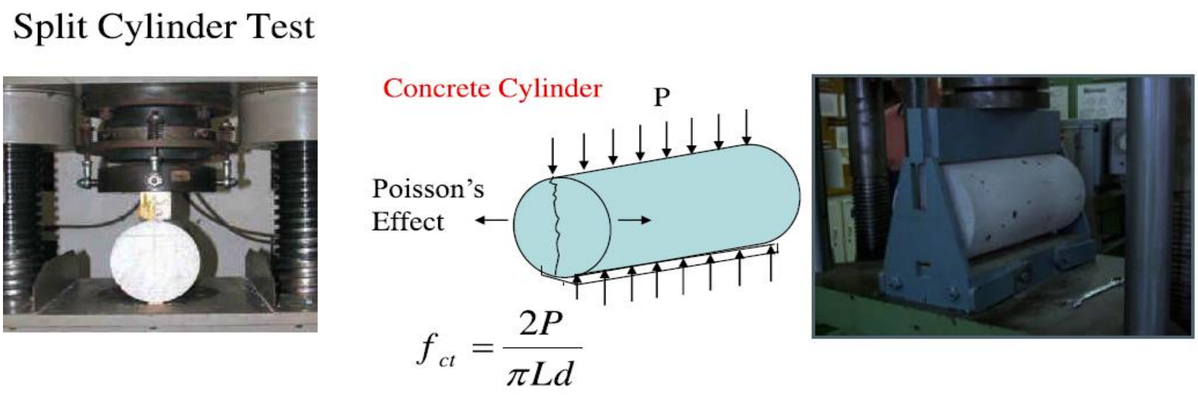
At moderate levels of stress, cracks do not influence the compressive strength of concrete significantly. Since compressive stress can push the sides of the crack together, both the cracked and uncracked areas are able to transmit compression stresses.

When concrete is stressed in tension, the distribution of stresses on the cross section changes. Since tension cannot be transferred across a crack, it is carried only on the uncracked area of the cross section. Because the effective area available to transmit tension is smaller than the gross area, therefore, most of design codes neglect the tensile strength in design.

Two indirect tests are used to measure the tensile strength of concrete.

A. The Split Cylinder Test (ASTM 496)

In this test, a standard compression test cylinder 15 cm in diameter and 30 cm in length is placed on its side and loaded in compression along a diameter until splitting occurs along the vertical diameter as shown in the Figure below.



Split cylinder test specimens

B. The Modulus of Rupture Test (ASTM C78)

In this test, a plain concrete beam 15 cm by 15 cm in cross section and 75 cm in length is loaded to failure in bending at the third points of a 60 cm span as shown in the Figure below.

The modulus of rupture is given by

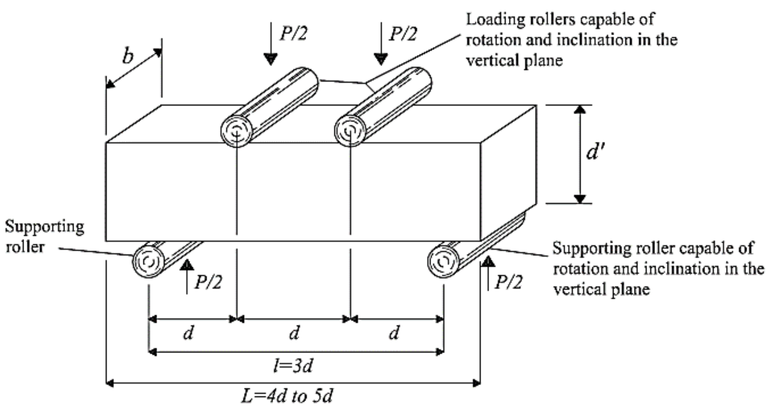
$$fr = \frac{\mathrm{MC}}{\mathrm{I}} = \frac{PL}{bh^2}$$

Where:

P = Applied load

b = width of specimen

h = depth of specimen.



Modulus of Rupture Test Specimens

The modulus of rupture of concrete ranges between 10 and 15% of the compressive strength. The ACI Code, Section 19.2.3 prescribes the value of the modulus of rupture as

$fr = 0.62\lambda \sqrt{f_c`}$ N/mm2 ....ACI 19.2.3

Where the modification factor λ for type of concrete is given as

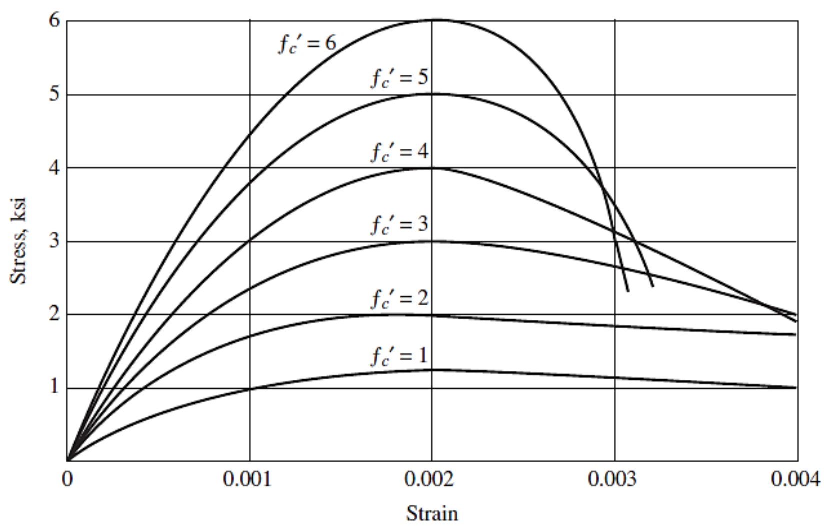
$\lambda =$ 1.0 for normal-weight concrete

0.85 for sand – lightweight concrete

0.75 For all – lightweight concrete

2.3. Stress-Strain Curve of concrete

Typical stress–strain curves for concretes of different strengths. All curves consist of an initial relatively straight elastic portion, reaching maximum stress at a strain of about 0.002; then rupture occurs at a strain of about 0.003.



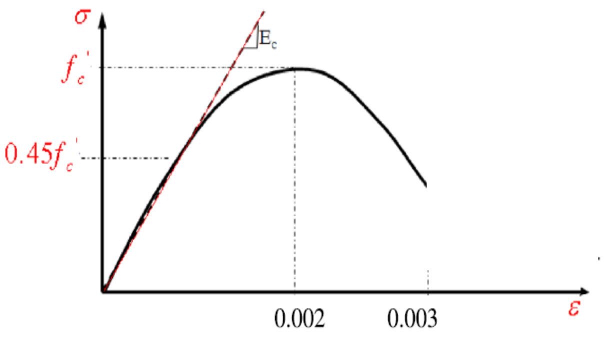
Typical concrete stress–strain curve, with short-term loading.

2.4 Static Modulus of Elasticity of Concrete

The modulus of elasticity, Ec, can be defined as the change of stress with respect to strain in the elastic range: $\mathrm{Ec} = \frac{\text{unit stress}}{\text{strain}}$

The modulus of elasticity is a measure of stiffness, or the resistance of the material to deformation.

Since the stress-strain diagram for concrete is nonlinear as evident in the above Figure, the slope of the curve is variable, making the determination of such modulus a tough task. The secant method is usually used to determine Ec, being the slope of the line drawn from a compressive stress of zero to a compressive stress of $0.45f_c'$.



Modulus of Elasticity of Concrete

The ACI Code, Section 19.2.2, gives a simple formula for calculating the modulus of elasticity of normal and lightweight concrete.

Ec = $0.043w^{1.5}\sqrt{f_c'}$ N/mm^2 ………..ACI 19.2.2.1.b

Where

w=unit weight of concrete 1400 to 2600 kg/m^3

f_c'= specified compressive strength of a standard concrete cylinder **in MPa**

w is approximately (2320 kg/m3); thus,

Ec = $4780\sqrt{f_c'}$ MPa

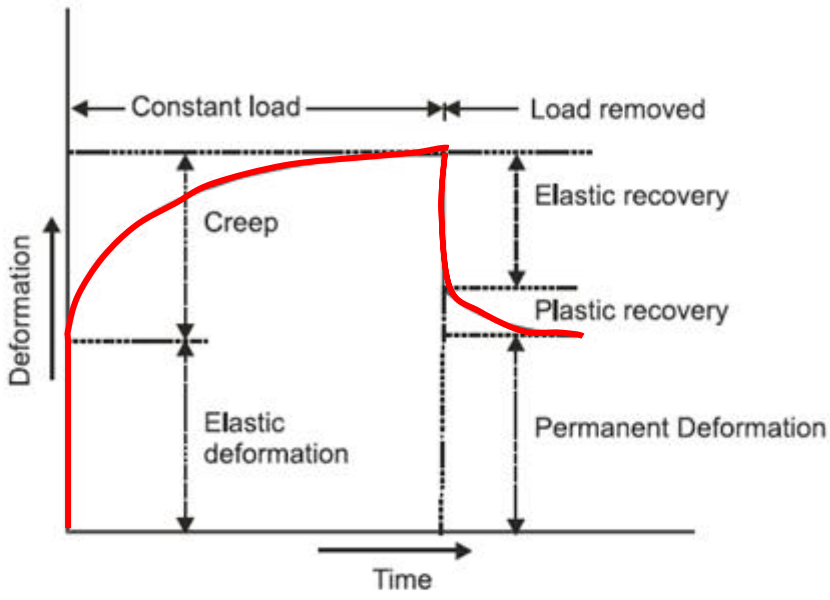
The ACI Code allows the use

Ec = $4700\sqrt{f'c}$ MPa

2.5 Creep

Creep is indicated when strain in a solid increases with time while the stress producing the strain is kept constant. Creep is a long-term deformation caused by the application of loads for long periods of time, usually years. The total deformation is divided into two parts;

The first is called instantaneous deformation occurring right after the application of loads, and the second which is time dependent is called creep. Long-term deformation increases at a slowing rate for a period of two to three years with maximum value recorded at a period of five years.



Creep strains due to loading at time and unloading at time

2.6 Shrinkage

Shrinkage of concrete is defined as the reduction in volume of concrete due to loss of moisture. If the concrete member is not restrained, no stresses will be produced. On the other hand, stresses will be developed in case of restraining the concrete member in any form. Once the allowable tensile stresses are exceeded, tension cracking will take place.

Shrinkage can be reduced through using a low water-cement ratio, good curing of concrete, nonporous aggregates, shrinkage reinforcement, and expansion joints.

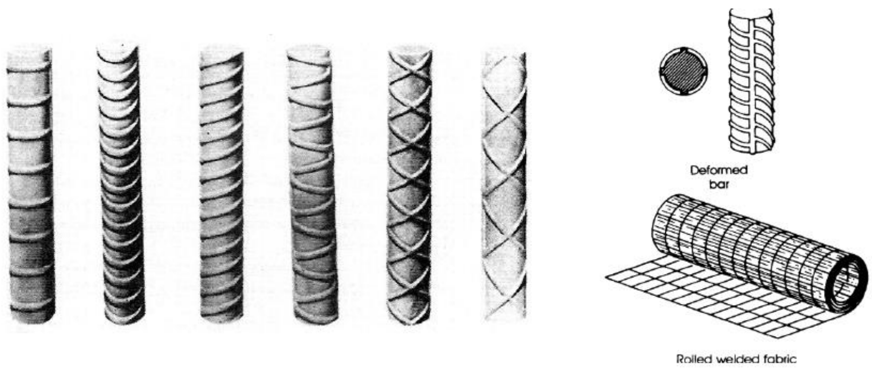
ACI 24.4.3 specifies that a minimum shrinkage and temperature reinforcement ratio of **0.0018** is to be used in one-way slabs perpendicular to the main reinforcement (for fy=400MPa).

3 Steel Reinforcement

Reinforcement, usually in the form of steel bars, is placed in the concrete member, mainly in the tension zone, to resist the tensile forces resulting from external load on the member. Reinforcement is also used to increase the member's compression resistance. Steel costs more than concrete, but it has a yield strength about 15 times the compressive strength of concrete.

Steel reinforcement may consist of:

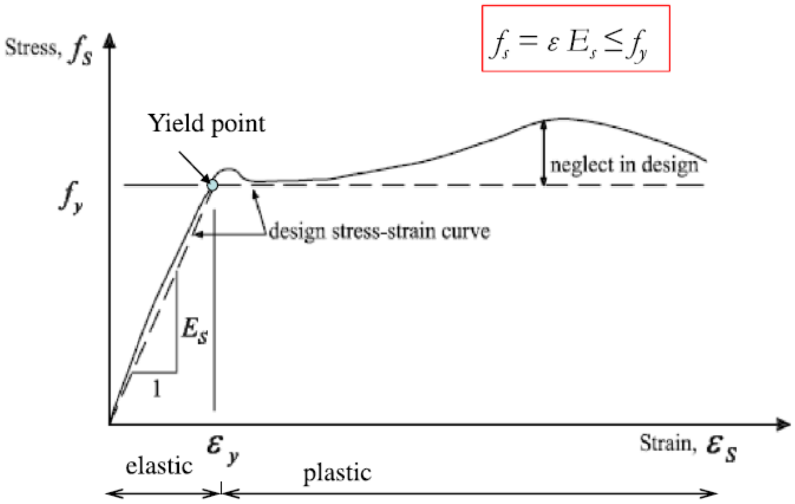
- ✓ Bars (deformed bars, as in picture below) – for usual construction.
- ✓ Welded wire fabric – is used in thins slabs, thin shells.
- ✓ Wire strands– are used for prestressed concrete.



The "Grade" of steel is the minimum specified yield stress (point) expressed in:

- ✓ For SI reinforcing bar Grades 300, 350, 420, and 520.
- ✓ For Inch-Pound reinforcing bar Grades 40, 50, 60, and 75.

The introduction of carbon and alloying additives in steel increases its strength but reduces its ductility. The proportion of carbon used in structural steels varies between 0.2 and 0.3%.



All steel grades have same modulus of elasticity E_s= 2x10^5 MPa
= 200 GPa

Two properties are of interest in the design of reinforced concrete structures;

The **first** is the modulus of elasticity, Es. It has been shown that the modulus of elasticity is constant for all types of steel. The ACI Code has adopted a value of **Es = (200000 MPa).** The modulus of elasticity is the slope of the stress–strain curve in the elastic range up to the proportional limit; Es =stress/strain.

Second is the yield strength, fy. Typical stress–strain curves for some steel bars are shown in Figure above. The yield strength or proof stress is considered the stress that leaves a residual strain of 0.2% on the release of load, or a total strain of 0.5 to 0.6% under load.

Bar sizes according to European Standard (EN 10080)

	W	Number of bars									
mm	N/m	1	2	3	4	5	6	7	8	9	10
6	**2.2**	28	57	85	113	141	170	198	226	254	283
8	**3.9**	50	101	151	201	251	302	352	402	452	503
10	**6.2**	79	157	236	314	393	471	550	628	707	785
12	**8.9**	113	226	339	452	565	679	792	905	1018	1131
14	**12.1**	154	308	462	616	770	924	1078	1232	1385	1539
16	**15.8**	201	402	603	804	1005	1206	1407	1608	1810	2011
18	**19.9**	254	509	763	1018	1272	1527	1781	2036	2290	2545
20	**24.7**	314	628	942	1257	1571	1885	2199	2513	2827	3142
22	**29.8**	380	760	1140	1521	1901	2281	2661	3041	3421	3801
24	**35.5**	452	905	1357	1810	2262	2714	3167	3619	4072	4524
25	**38.5**	491	982	1473	1963	2454	2945	3436	3927	4418	4909
26	**41.7**	531	1062	1593	2124	2655	3186	3717	4247	4778	5309
28	**45.4**	616	1232	1847	2463	3079	3695	4310	4926	5542	6158
30	**55.4**	707	1414	2121	2827	3534	4241	4948	5655	6362	7069
32	**63.1**	804	1608	2413	3217	4021	4825	5630	6434	7238	8042

Areas are in mm^2

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Design of Reinforced Concrete Structures I

Design Methods

Dr. Othman Hameed

Contents

1 Analysis and design of reinforced concrete structure

The analysis and design of a structural member may be regarded as the process of selecting the proper materials and determining the member dimensions.

In proportioning reinforced concrete structural members, main following items should be investigated:

- **Strength and serviceability**

Members are always designed with a capacity for load greater than required to support the anticipated service load. This extra capacity not only provides a factor of safety against failure by accidental overload or defective construction but also limits the level of stresses under service load to provide some control over deformation and cracking.

Deflection must be limited to ensure that floors will remain level within required tolerance and not vibrate. In addition, the crack width must be limited to preserve the architectural appearance of exposed surface and to protect reinforcement from attack by corrosion.

- **Stability**

The structure system design should prevent overturning, sliding or buckling under the action of the load.

There are two other considerations in design should be keep in mind: Economy and aesthetics.

Design involves finding the cross section dimensions and required reinforcement.

Analysis involves the determination of the capacity of a section of known dimensions, material properties, steel reinforcement and external load.

2 Design and Building Codes

A code is a set of technical specifications that control the design and construction of a certain type of structures.

Theoretical research, experiments, and past experience help in the process of setting these specifications. The purpose of such code is to set minimum requirements necessary for designing safe and sound structures. It also helps to provide protection for the public from dangers resulting from the use of inadequate design and construction techniques.

A structural code is originated and controlled by specialists who are concerned with the proper use of a specific material or who are involved with the safe design of a particular class of structures. Below are several important codes:

- The American Concrete Institute (ACI) Building Code 318-14, covering the design of reinforced concrete buildings.
- The American Association of State Highway and Transportation Officials (AASHTO), covering the design of highway bridges.
- The American Railroad Engineering Association (AREA), covering the design of railroad bridges.

ACI code contains provisions covering all aspect of reinforced concrete manufacture, design, and construction. It includes specifications on quality of materials, details on mixing and placing concrete, design assumptions for the analysis of continuous structures.

All design procedures used in these lectures are consistent with the specification of the ACI318-14.

3 Ductility versus Brittleness

A major objective of the ACI Code is to design concrete structures with adequate ductility since concrete is brittle without reinforcement.

The term ductility describes the ability of member to undergo large deformation without rupture as failure occurs. A structural-steel girder is an example of ductile member that can be bent and twisted through large angle without rupture. This capability prevents total structural collapse and provide protection to occupants of building. On the other hand, the term brittle describes member that fail suddenly, completely with little warning.

4 Load

Perhaps the most important and most difficult task faced by the structural designer is the accurate estimation of the loads that may be applied to a structure during its life.

After loads are estimated, the next problem is to decide the worst possible combinations of these loads that might occur at one time.

4.1 Type of Load

1. Dead loads

Dead loads are loads of constant magnitude that remain in one position. They include the weight of the structure under consideration for a reinforced concrete building, some dead loads are the frames, walls, floors, ceilings, Stairways, roofs, and plumbing.

2. Live Load

Live loads are loads that can change in magnitude and position. They include occupancy loads, warehouse materials, construction loads, overhead service cranes, equipment operating loads, and many others. In general, they are induced by gravity.

Some typical floor live loads that act on building structures are presented in Table.

Minimum live Load values on slabs

Type of Use	Uniform Live Load kN/m^2
Residential	2
Residential balconies	3
Computer use	5
Offices	2
Warehouses	
▪ Light storage	6
▪ Heavy Storage	12
Schools	
▪ Classrooms	2
Libraries	
▪ Rooms	3
▪ Stack rooms	6
Hospitals	2
Assembly Halls	
▪ Fixed seating	2.5
▪ Movable seating	5
Garages (cars)	2.5
Stores	
▪ Retail	4
▪ Wholesale	5
Exit facilities	5
Manufacturing	
▪ Light	4
▪ Heavy	6

3. Environmental Load

Environmental loads are loads caused by the environment in which the structure is located. For buildings, they are caused by rain, snow, wind, temperature change, and earthquake

✓ **Wind Load**

The wind load is a lateral load produced by wind pressure and gusts. It is a type of dynamic load that is considered static to simplify analysis. The magnitude of this force depends on the shape of the building, its height, the velocity of the wind and the type of terrain in which the building exists. Usually, this load is considered to act in combination with dead and live loads.

✓ **Earthquake load or Seismic load**

The earthquake load, which is also called seismic load, is a lateral load caused by ground motions resulting from earthquakes. The magnitude of such a load depends on the mass of the structure and the acceleration caused by the earthquake.

The provisions of the ACI Code provide enough ductility to allow concrete structures to stand earthquakes in low seismic risk regions. In moderate to high-risk regions, special arrangements and detailing are needed to guarantee ductility.

- **Load Factor ACI 5.3**

Load factors are numbers, almost larger than 1.0, which are used to increase the estimated loads applied to structures. They are used for loads applied to all types of members. The loads are increased to attempt to account for the uncertainties involved in estimating their magnitudes.

Section 5.3 of ACI Code presents load factors and combinations that are to be used for reinforced concrete design. The required strength U, or the load-carrying ability of a particular reinforced concrete member, must at least equal the largest value obtained by substituting into ACI equations in Table 5.3.1.

For the design of structural members, the factored design load is obtained by multiplying the dead load by a load factor and the specified live load by another load factor. Required strength ***U*** shall be at least equal to the effects of factored loads in ACI Table 5.3.1.

Table 5.3.1—Load combinations

Load combination	Equation	Primary load
$U = 1.4D$	(5.3.1a)	D
$U = 1.2D + 1.6L + 0.5(L_r \text{ or } S \text{ or } R)$	(5.3.1b)	L
$U = 1.2D + 1.6(L_r \text{ or } S \text{ or } R) + (1.0L \text{ or } 0.5W)$	(5.3.1c)	L_r or S or R
$U = 1.2D + 1.0W + 1.0L + 0.5(L_r \text{ or } S \text{ or } R)$	(5.3.1d)	W
$U = 1.2D + 1.0E + 1.0L + 0.2S$	(5.3.1e)	E
$U = 0.9D + 1.0W$	(5.3.1f)	W
$U = 0.9D + 1.0E$	(5.3.1g)	E

Where:

U= ultimate load,

D= dead load

L_r= roof live load

S = snow load

R = rain load

W = wind load

L= live load

E= seismic or earthquake load effects

5 Design Methods

Two methods of design for reinforced concrete have been dominant. The Working Stress method (WSDM) (elastic design) was the principal method used from the early 1900s until the early 1960s. Since the publication of the 1963 edition of the ACI Code, there has been a rapid transition to Ultimate Strength Design.

Ultimate Strength Design is identified in the code as the Strength Design Method. The 1956 ACI Code (ACI 318-56) was the first code edition which officially recognized and permitted the Ultimate Strength Design method and included it in an appendix. The 1963 ACI Code (ACI 318-63) dealt with both methods equally.

The 1971 ACI Code (ACI 318-71) was based fully on the strength approach for proportioning reinforced concrete members, except for a small section dedicated to what is called the Alternate Design Method. In the 1977 ACI Code (ACI 318-77) the Alternate Design Method was demoted to Appendix "B". It has been preserved in all editions of the code since 1977, including the 1999 edition mentioned in Appendix "A". In the 2002 code edition, the so called Alternate Design Method was taken out.

Working Stress Design Method (WSD)	Ultimate Strength Design Method (USD)
Section analysis and design under service loads	Section analysis and design under ultimate loads
Safety factor on the material strengths	Safety factors on the applied load or moment

6 Analysis methods

- **CLASSIC METHOD**

Slope deflection
Consistent deformation method
Moment distribution method

- **ADVANCED METHOD**

Stiffness method
Flexibility method

- **COMPUTER PROGRAM**

SAP (structure analysis program)
STAAD pro (structure analysis and design program)

- **ACI-COEFFICIENT METHOD**

Design of Reinforced Concrete Structures I

Behavior of Reinforced Concrete Beam under Load

Al Muthanna University
Collage of Engineering

Dr. Othman Hameed

Contents

1 Introduction

The addition of steel reinforcement that bonds strongly to concrete produces a relatively ductile material capable of transmitting tension and suitable for any structural elements, e.g., slabs, beam, columns. Reinforcement should be placed in the locations of anticipated tensile stresses and cracking areas. For example, the main reinforcement in a simple beam is placed at the bottom fibers where the tensile stresses develop as shown in Figure 4.1 (a). However, for a cantilever, the main reinforcement is at the top of the beam at the location of the maximum negative moment as shown in Figure 4.1 (b). Finally, Figure 4.1 (c) show a continuous beam, a part of the main reinforcement should be placed near the bottom fibers where the positive moments exist, and the other part is placed at the top fibers where the negative moments exist.

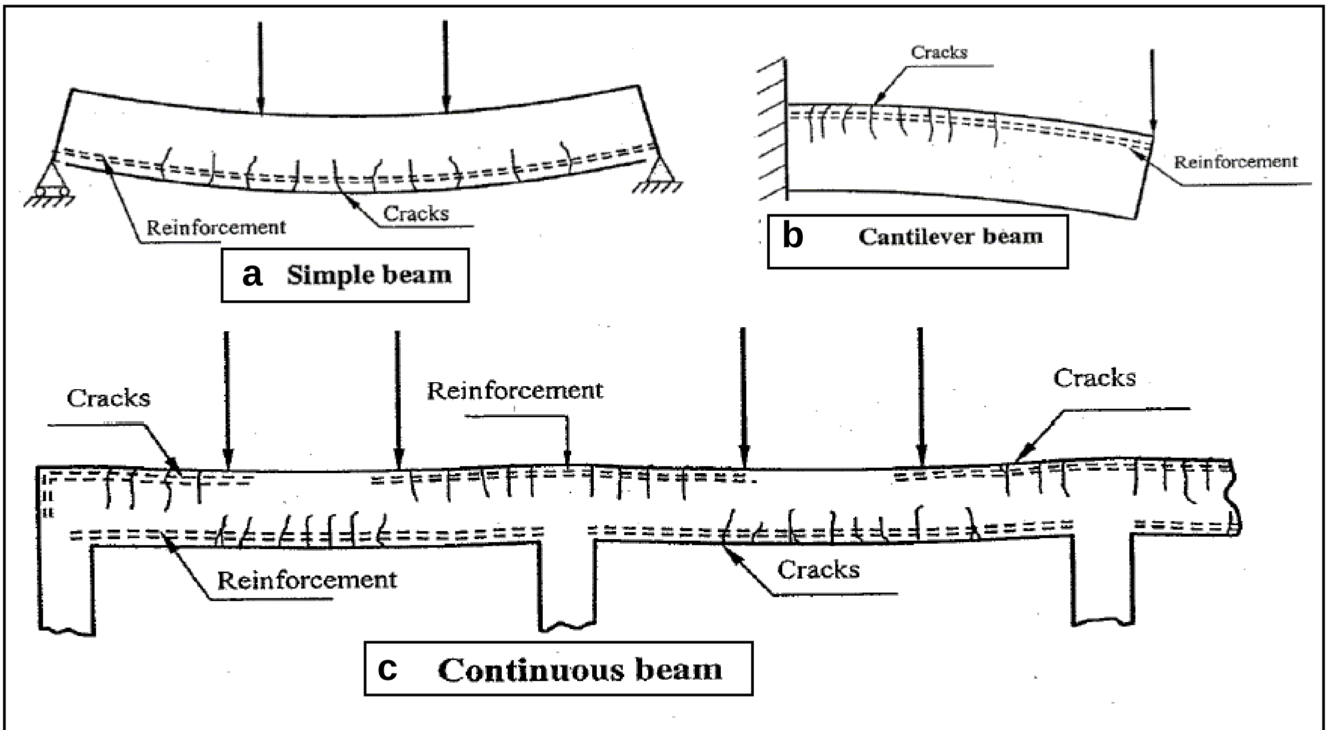
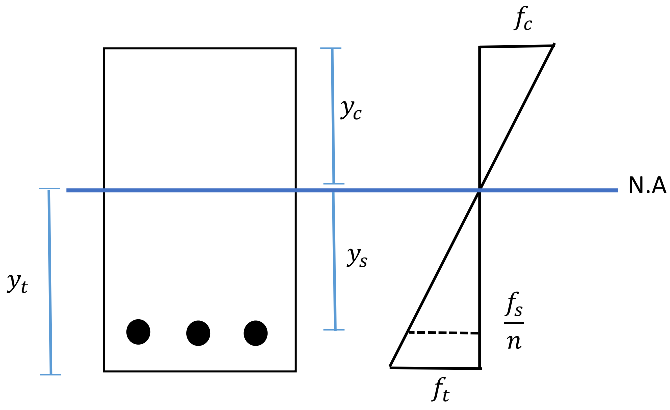


Figure 1:Reinforcement placement for different types of beams

2. Stresses in concrete and steel (review)



$$\sigma = \frac{M\,y}{I}$$

$$f_c = \frac{M\,y_c}{I}$$

$$f_t = \frac{M\,y_t}{I}$$

$$f_s = \frac{M\,y_s}{I}$$

$$f_r = \frac{M_r\,y_t}{I}$$

3 Behavior of Reinforced Concrete Beam under Load

In this section, it is assumed that a small transvers load is placed on the concrete beam with tensile reinforcing and that the load is gradually increased in magnitude until the beam fails. As this take place, we will find that the beam will go through three distinct stages before collapse occurs. These are:

1- The uncracked concrete stage.

2- The concrete cracked-elastic stresses stage.

3- The ultimate strength stage.

A relatively long beam is considered for this discussion so that shear will not have effect on its behavior.

3.1 Uncracked Concrete Stage $(f_t < f_r)$ or $(M < M_r)$ and $f_c < \frac{f_c^`}{2}$

At small load when the tensile stresses are less than the modulus of rupture $(f_t > f_r)$, the entire cross section of the beam resist bending. Both concrete and steel will resist the tension and concrete alone will resist the compression. Variation of stress and strain will be linear from the neutral axis to the outer fiber. Figure 2 shows the variation of stresses and strain for these small loads.

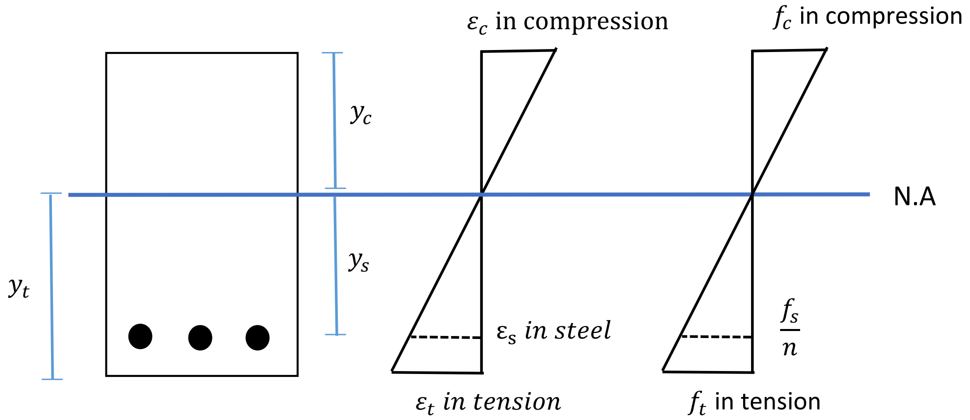


Figure 2: Uncracked Concrete Stage

$$f_r = \frac{M_r\, y_t}{I_g}$$

$$M_r = \frac{f_r\, I_g}{y_t}$$

$$f_r = 0.62\lambda\sqrt{f_c^`}$$

$$f_t = \frac{M\, y_t}{I}$$

Where

f_r: Modulus of rupture of concrete in MPa

λ : Modification factor reflecting the reduced mechanical properties of light weight concrete

$\lambda = 1$ for normal weight concrete

y_t: Distance from the neutral axis to the extreme fiber in tension

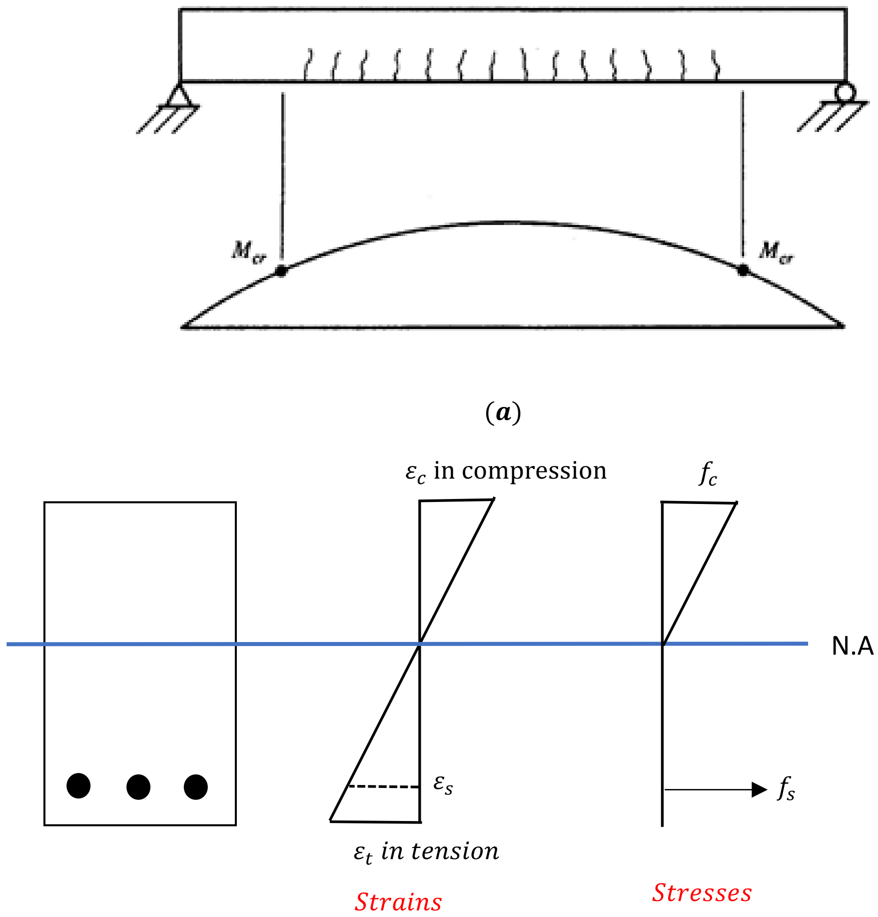
I_g: Gross moment of inertia (neglecting steel)

3.2 The concrete cracked-elastic stresses stage ($f_t \geq f_r$ and $f_c < \frac{f_c^`}{2}$)

As the load increased after the modulus of rupture of the concrete is exceeded ($f_t \geq f_r$), cracks begin to develop in the bottom of the beam. The moment at which these cracks begin to form (the tensile stresses at the bottom of the beam equal to the modulus of rupture ($f_t = f_r$) is referred to the cracking moment (M_{cr}). As the load further increased, these cracks quickly spread up to the neutral axis, and then neutral axis move upward. The cracks occur at those places along the beam where the actual moment is greater than cracking moment, as shown in Figure 3(a).

Because the concrete in cracked zone cannot resist the tensile stresses, the steel must do it. This stage will continuous as long as the compression stresses is less than one half of the concrete compressive strength f_c' and as long as the steel stresses is less than yield stresses.

In this stage, the compressive stresses vary linearly with the distance from the neutral axis as shown in Figure 3 (b).



(*a*)

(*b*)

Figure 3: Concrete cracked-elastic stresses stage

This stage will continue as long as

(a) $f_c < \frac{f_c^`}{2}$

Where

f_c is the compression stress

$f_c^`$ is the compression strength

(b) $f_s < f_y$

f_s is the tensile stress

f_y is the yield tensile strength

To compute the concrete and steel stresses of this stage, the transformed-area method is used

Transformed-area method

The steel bars are replaced with an equivalent area of fictitious concrete (n As), which is supposedly can resist tension.

Where

$$n = \frac{E_s}{E_c}$$

E_s is the modulus of elasticity of steel (200,000 MPa)

E_c is the modulus of elasticity of concrete ($4700\sqrt{f_c^`}$)

A_s is the area of steel bars

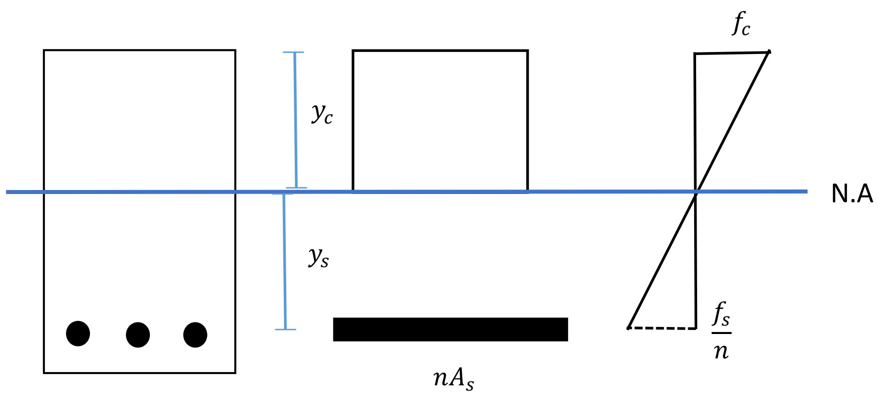


Figure 4: Transformed-area section

The main steps to find stresses are

a) Find the location of the neutral axis

b) Find moment of inertia of the transformed section (I)

c) Find the stresses as follows:

$$f_c = \frac{M\,y_c}{I}$$

$$f_s = \frac{M\,y_s}{I}\,n$$

d) if the permissible stresses are given and the resisting moment is required, the minimum moment of the above equations is taken

3.3 Beam Failure-Ultimate Strength Stage $\left(f_c \geq \frac{f_c^{`}}{2} \text{ and } f_s = f_y\right)$

As the load is increased further so that the compressive stresses are greater than (0.5 f_c'), the tensile cracks move upward, the concrete compression stresses begin to change from a straight line to nonlinear. For this initial discussion, it is assumed that the reinforcing bars have yielded. The stresses variation is much like that shown in Figure 5.

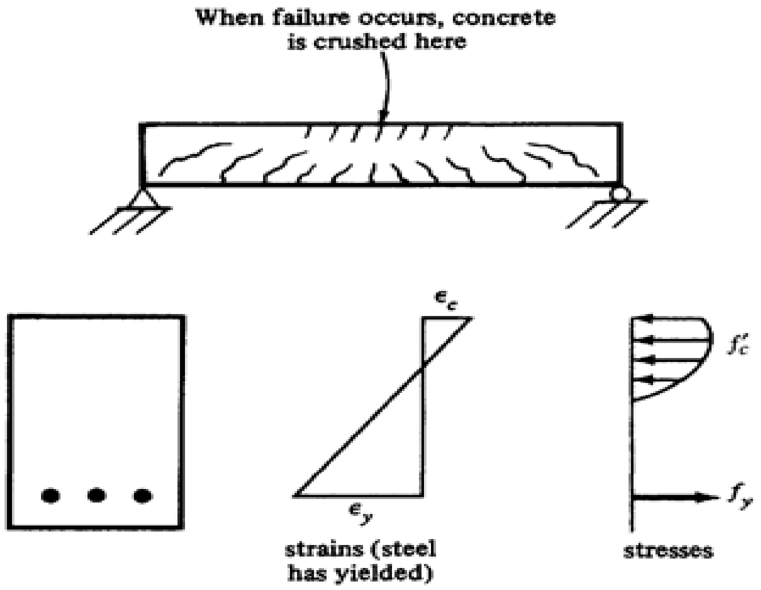


Figure 5: Ultimate Strength Stage

Summary

1 Uncracked Concrete section ($f_t < f_r$) and ($f_c < \frac{f_c^`}{2}$)

2 Cracked-elastic section stage ($f_t \geq f_r$ and $f_c < \frac{f_c^`}{2}$)

3 Cracked-inelastic section (Ultimate Strength Stage) ($f_c \geq \frac{f_c^`}{2}$ and $f_s = f_y$)

4 Failure

Design of Reinforced Concrete Structures I

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Working stress design method

Dr. Othman Hameed

Contents

1. Design Methods

Two design approaches for reinforced concrete members are available to the engineers. ***The first***, called working stress design method or elastic design method, is based on the prediction of stresses in member as they support the anticipated service loads. Service Load is the actual or maximum value of load that is expected to carry by the member.

In elastic design, the member is designed so that service load stresses do not exceed the allowable stresses.

working stress design method assumes that material behave elastically. Elastic design does not take into consideration the type of failure mode (ductile or brittle). Thus, the actual factor of safety against failure is, in fact, unknown.

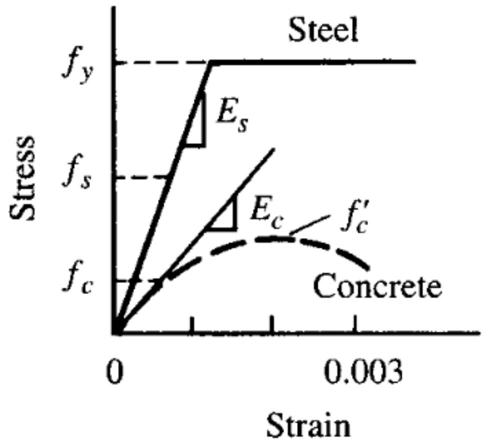
The second design approach, called ultimate strength design method or simply, strength design method, is based on predicting the load that produces failure rather than predicting stresses produced by service loads. In this approach the mode of failure is ductile rather than brittle manner.

Strength design method is more desirable approach as such method leads to a ductile failure that in turn results in local failure which is the foremost concern for reinforced concrete design. By controlling the ultimate strength of each member of a structure, the designer can control mode of failure of a total structural system. In this way, it is possible to design structures so that in the unlikely event of unanticipated overload, failure is confined to limited region instead of causing total collapse of entire system.

2. The Working-Stress Design Method

Before the introduction of the strength-design method in the ACI building code in 1956, the working stress design method was used in design. This method is based on the condition that:

1. The stresses caused by service loads without load factors are not to exceed the allowable stresses which are taken as a fraction of the ultimate stresses of the materials, f_c' for concrete and fy for steel as shown in Figure.



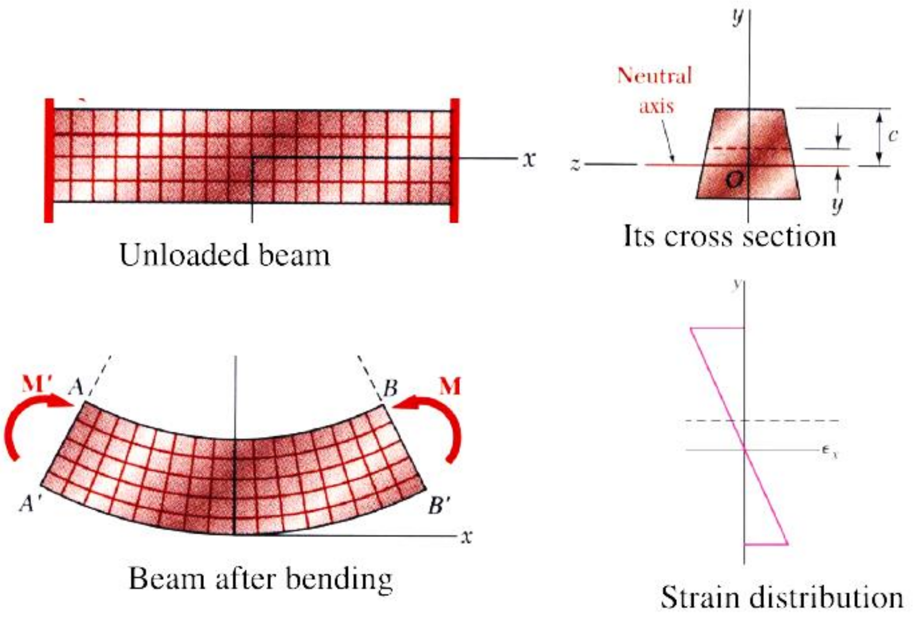
2. In this method, linear elastic relationship between stress and strain is assumed for both concrete and steel reinforcement.

The working stress-design method will generally result in designs that are more conservative than those based on the strength design method. Now only the design of sanitary structures holding fluids is based on the working-stress design method since keeping stresses low is a logical way to limit cracking and prevent leakage.

Although limited use is made of working-stress design today, the method is introduced here to develop an understanding of behavior when service loads are applied. Service loads cause the development of extensive flexural cracking in sections where the service moments exceed the cracking moment of the cross section. Once the cross section cracks, the steel alone must carry the tensile stress produced by moment.

2.1 Assumption

1. Plane sections remain plane after loading. This mean that the strain varies linearly over the depth of the section.



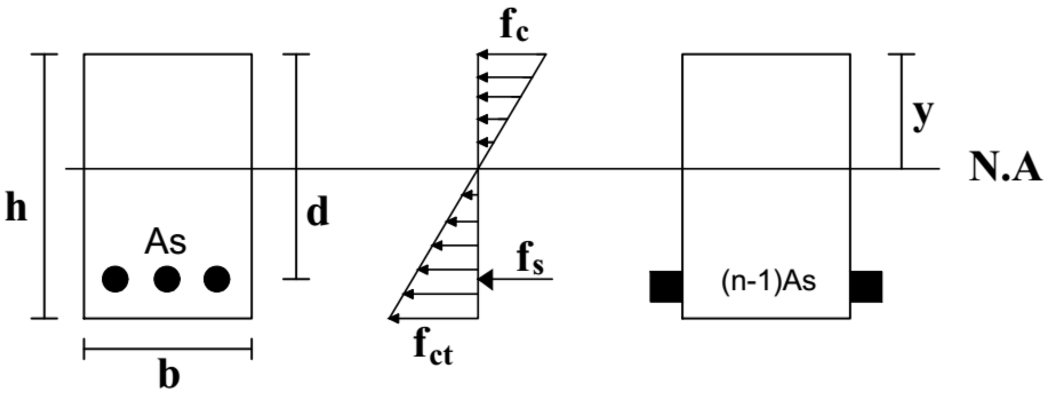
2. The strain in the reinforcement is equal to the strain in concrete at same level $\varepsilon_s = \varepsilon_c$ at same level . This means a good bond between the concert and steel
3. The concrete section at tension face is fully cracked so, the tensile strength of concrete is neglected in flexural strength.
4. The concrete is assumed to fail in compression when $\varepsilon_c = 0.003$
5. The allowable stresses are:

- $f_c = 0.45\, f_c'$
- $f_s = 140$ MPa for fy $(300 - 350$ MPa)
- $f_s = 170$ MPa for fy (400 MPa)
- $Es = 200000\ MPa$
- $Ec = 4700\sqrt{f_c'}$

3. Calculating I_{unc} and I_{cr} for rectangular section

3.1 singly reinforced section

a) Uncracked rectangular section with singly reinforcement



- Transfer steel area (As) to equivalent concrete area (nAs)
- Locate the natural axis from the extreme of compression fiber.

$$\frac{1}{2}\times b\times y^2=(n-1)As\times(d-y)+b\times\frac{(h-y)^2}{2}\quad\rightarrow \textbf{Find } \boldsymbol{y}$$

- Find the uncracked moment of inertia about N.A

$$I_{ucr}=\frac{b\times y^3}{3}+(n-1)As\times(d-y)^2+\ b\times\frac{(h-y)^3}{3}$$

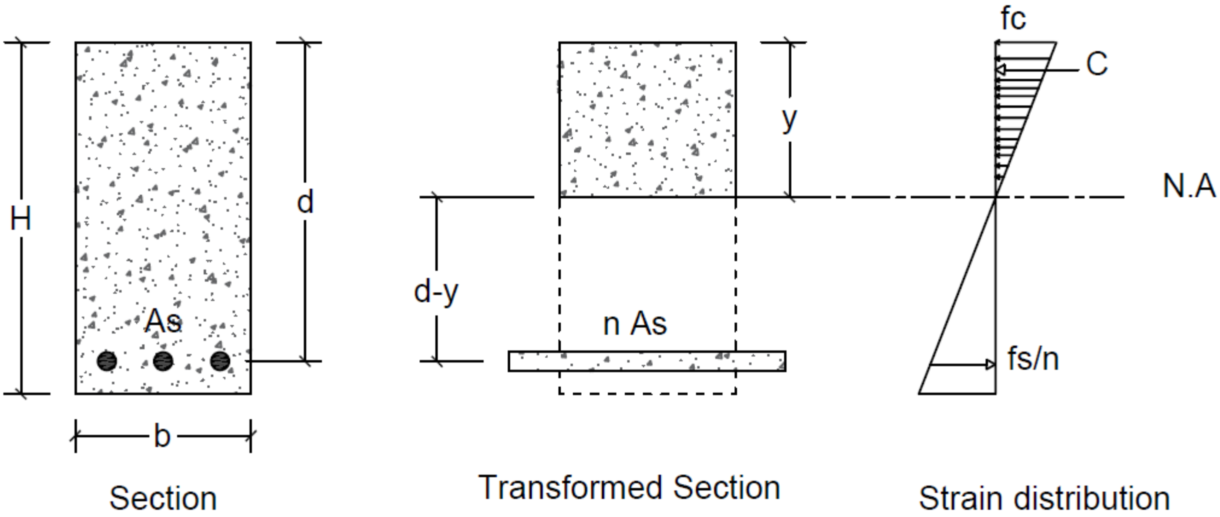
- Calculate the stresses in concrete & steel.

$$f_c=\frac{M\times y}{I_{ucr}}\ldots\ldots\ldots\ldots\ldots\ldots\ldots compression\ stresses$$

$$f_t=\frac{M\times(h-y)}{I_{ucr}}\ldots\ldots\ldots\ldots\ldots\ldots\ldots tension\ stresses$$

$$f_s=n\times\frac{M\times(d-y)}{I_{ucr}}\ldots\ldots\ldots\ldots tension\ \ stresses\ of\ steel\ bars$$

b) Cracked rectangular section with singly reinforcement



- Transfer steel area (As) to equivalent concrete area (nAs)
- Locate the natural axis from the extreme of compression fiber.

$$\frac{1}{2} \times b \times y^2 = n\ As \times (d - y) \qquad \rightarrow y$$

- Find the cracked moment of inertia about N.A

$$I_{cr} = \frac{b \times y^3}{3} + n\ As \times (d - y)^2$$

- Calculate the stresses in concrete & steel.

$$f_c = \frac{M \times y}{I_{cr}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ compression\ stresses$$

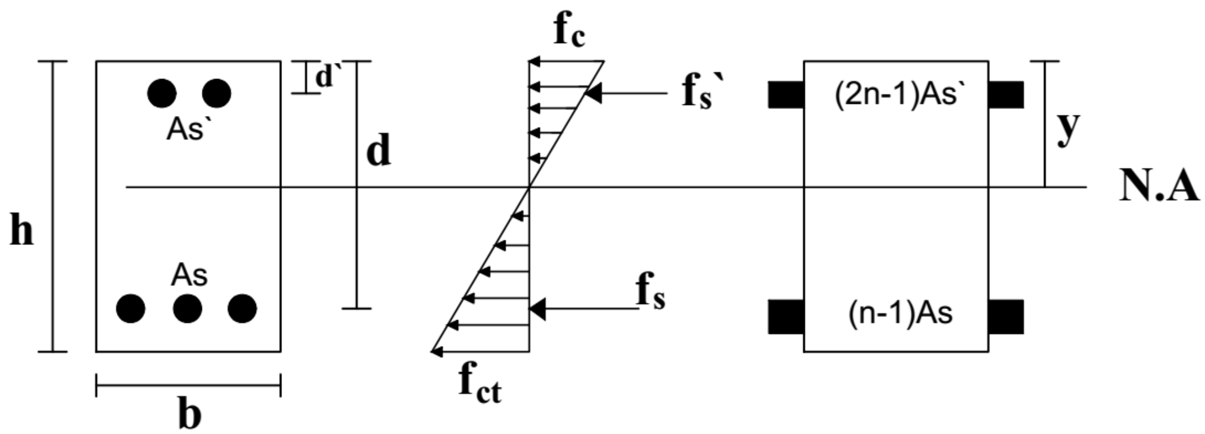
$$f_s = n \times \frac{M \times (d - y)}{I_{cr}} \ldots\ldots\ldots\ldots\ldots\ tension\ \ stresses$$

3.2 Doubly reinforced section

The compression zone of a reinforced concrete beam is occasionally reinforced with steel to raise the member's bending strength, to reduce long-term deflections produced by creep, or to increase ductility. Steel located in the compression zone, or compression steel, is denoted by A_s'.

If flexural stresses are to be evaluated, ACI Code A.5.5 specifies that the area of the compression steel be multiplied by **2n**. Doubling the modular ratio of the compression steel accounts for the increase in stress that occurs with time as the concrete in the compression zone creeps. The creep deformation of the concrete produces additional strain in the compression steel and gradually raises the level of stress to approximately twice that of the initial value.

a) Uncracked rectangular section with doubly reinforcement



- Locate the natural axis from the extreme of compression fiber.

$$\frac{1}{2} \times b \times y^2 + (2n-1)As \times \left(y - d`\right) = (n-1)As \times (d-y) + b \times \frac{(h-y)^2}{2} \quad \rightarrow \textbf{Find } \boldsymbol{y}$$

- Find the uncracked moment of inertia about N.A

$$I_{ucr} = \frac{b \times y^3}{3} + (2n-1)As` \times \left(y - d`\right)^2 + (n-1)As \times (d-y)^2 + b \times \frac{(h-y)^3}{3}$$

- Calculate the stresses in concrete & steel.

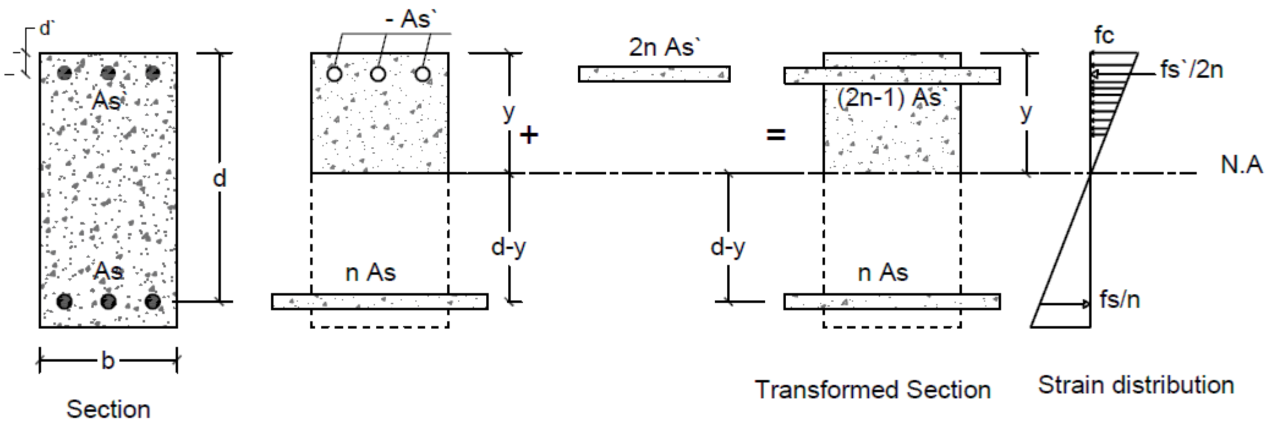
$$f_c = \frac{M \times y}{I_{ucr}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots compression\ stresses$$

$$f_t = \frac{M \times (h-y)}{I_{ucr}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots..tension\ stresses$$

$$f_s = n \times \frac{M \times (d-y)}{I_{ucr}} \ldots\ldots\ldots\ldots..tension\ \ stresses\ of\ steel\ bars\ in\ tension\ zone$$

$$f_s^{`} = 2n \times \frac{M \times (y-d^{`})}{I_{ucr}} \ldots\ldots\ldots\ldots..tension\ \ stresses\ in\ compression\ reinforcement$$

b) Cracked rectangular section with doubly reinforcement



- Transfer steel area (As) to equitant concrete area (nAs)
- Locate the natural axis from the extreme of compression fiber.

$$\frac{1}{2} \times b \times y^2 + (2n-1) \times As^{`} \times (y-d^{`}) = n\ As \times (d-y) \qquad \rightarrow y$$

- Find the cracked moment of inertia about N.A

$$I_{cr} = \frac{b \times y^3}{3} + (2n-1) \times As^{`} \times (y-d^{`})^2 + n\ As \times (d-y)^2$$

- Calculate the stresses in concrete & steel.

$$f_c = \frac{M \times y}{I_{cr}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..compression\ stresses$$

$$f_s = n \times \frac{M \times (d-y)}{I_{cr}} \ldots\ldots\ldots\ldots..tension\ \ stresses$$

$$f_s' = 2n \times \frac{M \times (y-d^{`})}{I_{cr}} \ldots\ldots\ldots..tension\ \ stresses\ in\ compression\ reinforcement$$

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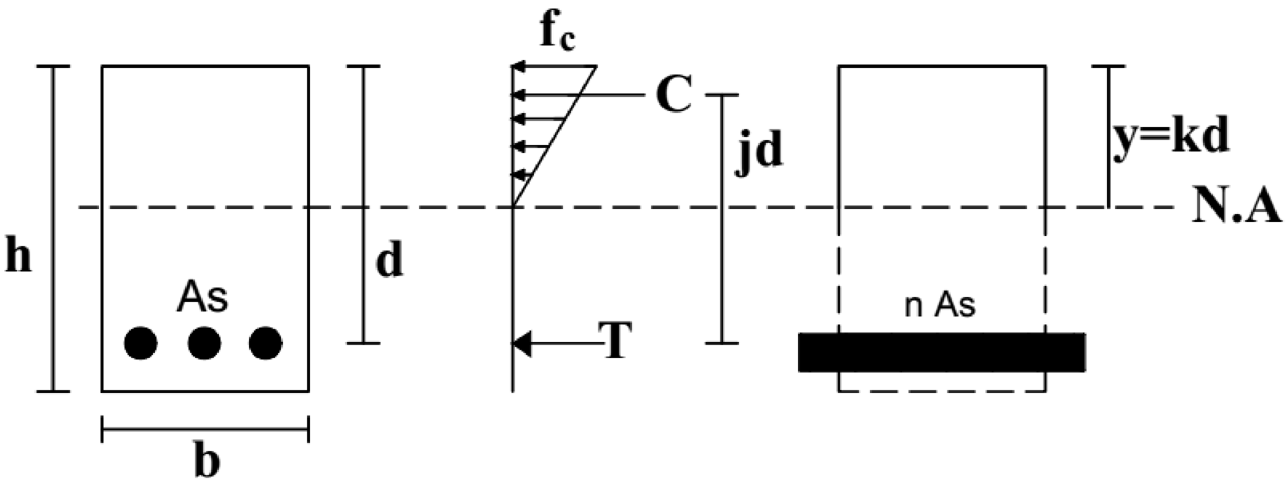
Design of Reinforced Concrete Structures I

Working stress design method-2

Dr. Othman Hameed

Other method to calculate the neutral axis of the cracked rectangular section

1) Cracked rectangular section with singly reinforcement



$$k = \sqrt{2n\rho + (n\rho)^2} - n\rho$$

$$y = kd$$

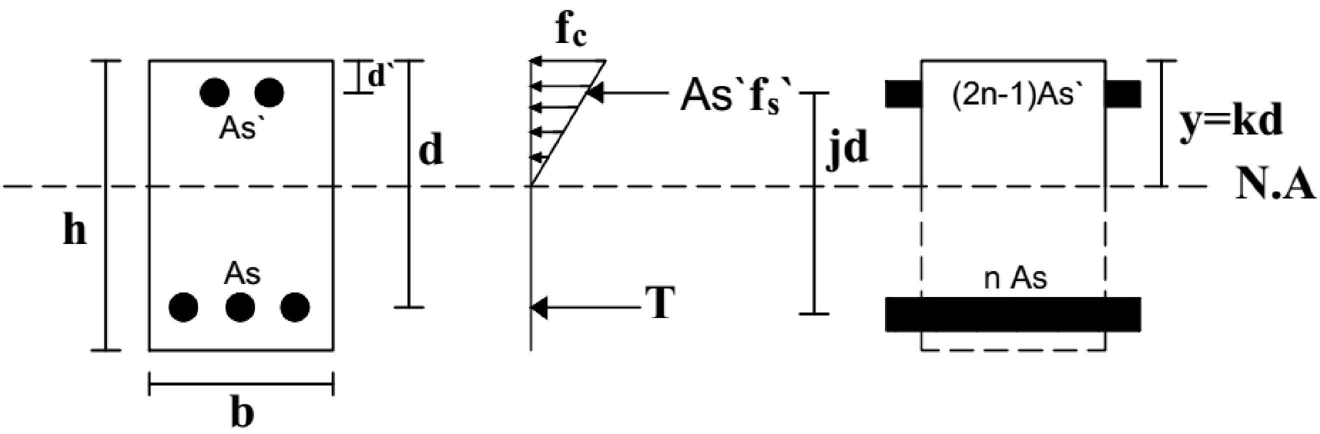
$$j = 1 - \frac{k}{3}$$

$$\rho = \frac{A_s}{bd}$$

$$M = Cjd = \frac{f_c\, kd}{2} bjd$$

$$M = Tjd = A_s\, f_s\, jd$$

2) Cracked rectangular section with doubly reinforcement



$$k = \sqrt{2(2n-1)\rho'\frac{d'}{d} + 2n\rho + n^2(2n\rho' + \rho - \frac{\rho'}{n})^2} - n(2\rho' + \rho - \frac{\rho'}{n})$$

$$y = kd$$

$$z = \frac{\frac{k^2 d}{6} + (2n-1)\rho' d'(1 - \frac{d'}{kd})}{\frac{k}{2} + (2n-1)\rho'(1 - \frac{d'}{kd})}$$

$$jd = d - z$$

$$\rho = \frac{A_s}{bd}$$

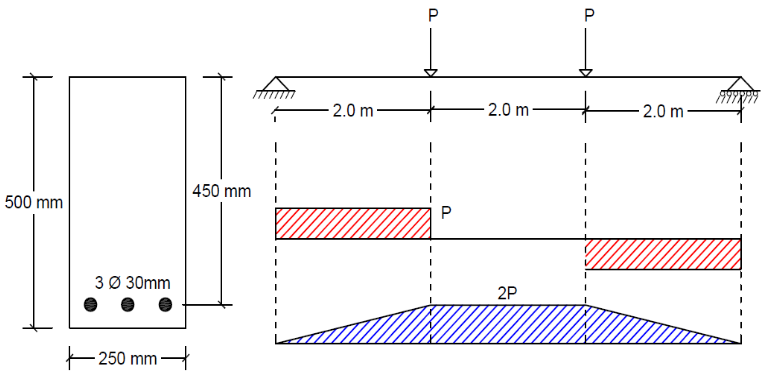
$$\rho' = \frac{A_s^{`}}{bd}$$

$$M = Cjd = (\frac{f_c\, kd}{2} b + A_s^{`} f_s^{`})jd$$

$$M = Tjd = A_s\, f_s\, jd$$

Ex1: For the beam of details shown in Figure, check the bending stress if:

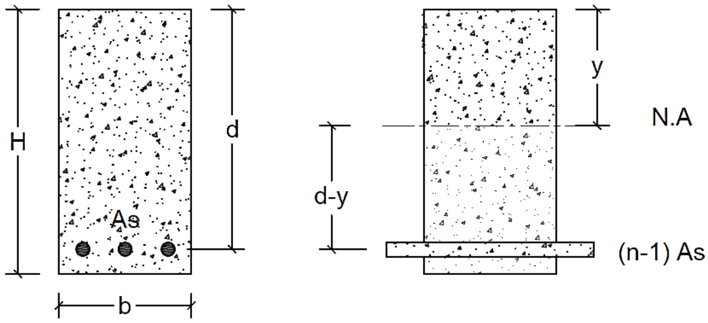
1- P= 17 kN
2- P= 32 kN
3- P= 90 kN, $\frac{f_c'}{fy} = \frac{30}{400}\,\mathrm{MPa}$



Solution:

Case 1, P= 17 kN, M= 2*17=34 kN.m

Assume the section is elastic uncracked



Section

Transformed Section

$$n = \frac{\mathrm{E_s}}{\mathrm{E_C}} = \frac{200000}{4700\sqrt{30}} = 7.8 \ \cong 8$$

$$\sigma = \frac{\mathrm{MC}}{\mathrm{I}}$$

$$A_s = 3 * \frac{\pi}{4} 30^2 = 2119.5\,mm^2$$

1- Find neutral axis

$$\sum \mathrm{M_{N.A}} = 0 \ \rightarrow 250\,.\frac{y^2}{2} = 250\,.\ (500-y)\ .\ \frac{(500-y)}{2} + (n-1).As.(d-y)$$

$$125\,y^2 = 125\ (500-y)^2 \ + 2119.5 * (8-1)\,(450-y)$$

$$y^2 = \ (500-y)^2 \ + 118.7(450-y)$$

$$y^2 = \ 250000 - 1000y + y^2 \ + 53415 - 118.7y$$

$$1118.7y = \ 303415$$

$$y = \ 271\ mm$$

2- Find the moment of inertia about N.A

$$\mathrm{I_{un}} = \frac{by^3}{3} + \frac{b\,(h-y)^3}{3} + (n-1).As\,.(d-y)^2$$

$$\mathrm{I_{un}} = \frac{250 * \ 271^3}{3} + \frac{250 * (500-271)^3}{3} + (8-1) * \ 2119.5 * \ (450-271)^2$$

$$= 3.13 \times 10^9 \ \ mm^4$$

3- Check the bending stresses

$$f_t = \frac{M(h-y)}{I_{un}} = \frac{34\times10^6\,(500-271)}{3.13\times10^9} = 2.49\text{ MPa} < f_r = 0.62\times\sqrt{30} = 3.39\text{ MPa}$$

so, the section is uncracked

$$f_c = \frac{M\,y}{I_{un}} = \frac{34\times10^6\times271}{3.13\times10^9} = 2.94\text{ MPa} < \frac{f_c'}{2} = 15\text{ MPa}, \text{……… elastic section}$$

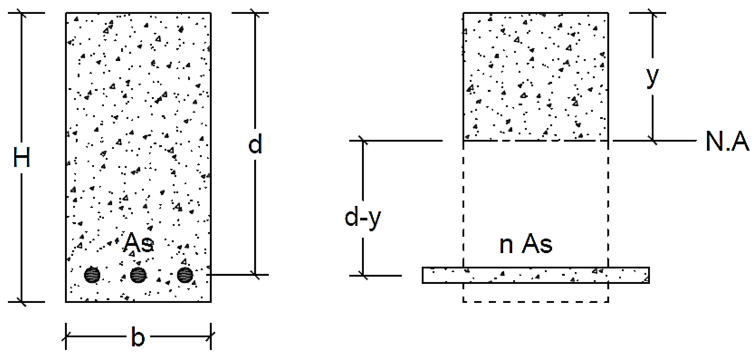
$$f_s = n.\frac{M(d-y)}{I_{un}} = 8\times\frac{34\times10^6\,(450-271)}{3.13\times10^9} = 15.55\text{ MPa}$$

Case 2, P=32 kN, M=2P= 64 kN.m

Assume the section is elastic uncracked

$$f_t = \frac{M(h-y)}{I_{un}} = \frac{64\times10^6\,(500-271)}{3.13\times10^9} = 4.68\text{ MPa} > f_r = 3.39\text{ MPa}, \text{the sec. is cracked}$$

1- Find new neutral axis for cracked sec.



Section

Transformed Section

$$\sum M_{N.A} = 0$$

$$250\times\frac{y^2}{2} = n.As\times(d-y) \Longrightarrow 125y^2 = 8\times2119.5(450-y)$$

$$y^2 = 135.65\,(450-y)$$

$$y^2 + 135.65\,y - 61042.5 = 0$$

$$y = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$y = \frac{-135.65\pm\sqrt{(135.65)^2+4\times1\times61042.5}}{2}$$

$$y = 188.4\,mm$$

y can also be calculated from

$$k = \sqrt{2n\rho + (n\rho)^2} - n\rho$$

$$y = kd$$

$$\rho = \frac{A_s}{bd}$$

$$\rho = \frac{2119.5}{250 * 450} = 0.01884$$

$$k = \sqrt{2 * 8 * 0.01884 + (8 * 0.01884)^2} - 8 * 0.01884 = 0.4186$$

$$y = 0.4186 * 450 = 188.4\,mm$$

2- Find the cracked moment of inertia about N.A

$$\mathrm{I_{cr}} = \frac{250\ \times 188.4^3}{3} + n.As\,.(450 - 188.4)^2 = 1.718 \times 10^9\,mm^4$$

3- Check the bending stresses

$$f_t = \frac{\mathrm{M(h-y)}}{\mathrm{I_{cr}}} = \frac{64 \times 10^6\ \times (500 - 188.4)}{1.718 \times 10^9} = 11.61\ \mathrm{MPa}\ > f_r = 3.39\ \mathrm{MPa} \ldots\ldots \text{cracked sec.}$$

$$f_c = \frac{\mathrm{M.y}}{\mathrm{I_{cr}}} = \frac{64 \times 10^6 \times 188.4}{1.718 \times 10^9} = 7.02\ \ \mathrm{MPa}\ < \frac{f_c'}{2} = 15\ \mathrm{MPa}\,, \ldots\ldots\ldots\ldots \text{elastic sec.}$$

$$f_s = n\,.\frac{\mathrm{M(d-\ y)}}{\mathrm{I_{cr}}} = 8\,.\frac{64 \times 10^6(450 - 188.4)}{1.718 \times 10^9} = 77.96\ \ \mathrm{MPa}$$

Case 3, P=90 KN, M= 2P= 180 kN.m

90 KN > 32 KN case 2 so, the sec. is cracked

$$f_t = \frac{\mathrm{M(h-y)}}{\mathrm{I_{cr}}} = \frac{180 \times 10^6\ \times (500 - 188.4)}{1.718 \times 10^9} = 32.65\ \mathrm{MPa}\ > f_r = 3.39\ \mathrm{MPa} \ldots\ldots \text{cracked sec.}$$

$$f_c = \frac{\mathrm{M.y}}{\mathrm{I_{cr}}} = \frac{180 \times 10^6 \times 188.4}{1.718 \times 10^9} = 19.74\ \ \mathrm{MPa} > \frac{f_c'}{2} = 15\ \mathrm{MPa}\,, \ldots\ldots\ldots \text{inelastic sec.}$$

If the section is inelastic, Hock's Law cannot be used

Design of Reinforced Concrete Structures I

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Working stress design method-3

Dr. Othman Hameed

Notes

- If the compression strength ($f_c^`$) is known and the stresses are required

1- Assume the section is uncracked

2- Calculate y_{unc} and I_{unc}

3- Calculate f_t

4- If $f_t < f_r$, calculate f_c and f_s

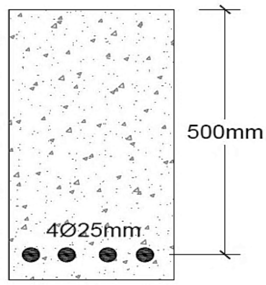
5- If $f_t > f_r$, calculate y_{cr} and I_{cr} and then determine f_t, f_c and f_s

- If the compression strength ($f_c^`$) is not known assume the section is cracked
- If the maximum moment is required, assume the section is cracked
- If the maximum load is required, assume the section is cracked
- If the maximum load, moment, or stresses before crack is required, assume the section is uncracked

---

Ex-2: A rectangular R.C beam of detail shown in Fig. subjected to bending moment of (150 kN.m), n=9, use the working stress method to find:

1- Maximum compression stresses in concrete.
2- Maximum tensile stresses in steel reinforcement.

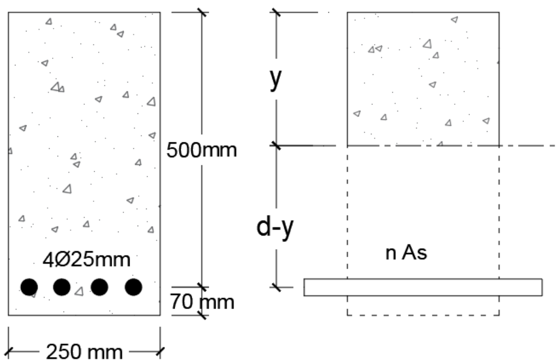


Section

Solution

As= 4 Ø25 mm= 1963.5 mm^2

1- Find location of N.A



Transformed Section

$$\frac{b.y^2}{2} = n.As.(d-y)$$

$$\frac{250 \times y^2}{2} = 9 \times 1963.5(500-y)$$

$$y^2 = 141.372(500-y)$$

$$y^2 + 141.372y - 70686 = 0$$

$$y = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$y = 204.4\ mm$$

y can also be calculated from

$$k = \sqrt{2n\rho + (n\rho)^2} - n\rho$$

$$y = kd$$

$$\rho = \frac{A_s}{bd}$$

$$\rho = \frac{1963.5}{250 * 500} = 0.015708$$

$$k = \sqrt{2 * 9 * 0.015708 + (9 * 0.015708)^2} - 9 * 0.015708 = 0.4088$$

$$y = 0.4088 * 500 = 204.4\,mm$$

2- Find the moment of inertia about N.A

$$I = \frac{by^3}{3} + n.As.(d - y)^2$$

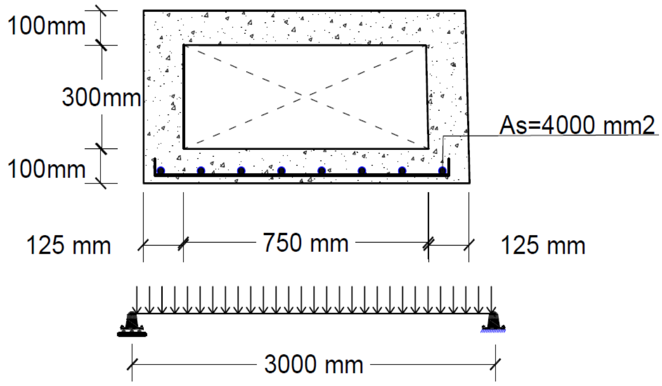
$$= \frac{250 \times 204.4^3}{3} + 9 \times 1963.5 \times (500 - 204.4)^2 = 2.25 \times 10^9 mm^4$$

3- Find the stresses in concrete and steel

$$f_c = \frac{M.y}{I} = \frac{150 \times 10^6 \times 204.4}{2.25 \times 10^9} = 13.62 \text{ MPa}$$

$$f_s = n\frac{M.(d - y)}{I} = 9 \times \frac{150 \times 10^6 \times (500 - 204.4)}{2.25 \times 10^9} = 177.36\text{MPa}$$

Ex-3: By using the working stress design method, find the maximum uniform distributed load that can be carried by the simply supported reinforced concrete beam of section and details shown in the Figure. Use fs=165 MPa, f_c=12.5 MPa, effective depth (d)= 430 mm and n=8.



Solution:

1- Find N.A

Let A= moment of area for flange (compression).
and B=moment of area for web (tension)

For any section except the rectangular, check the location of the neutral axis

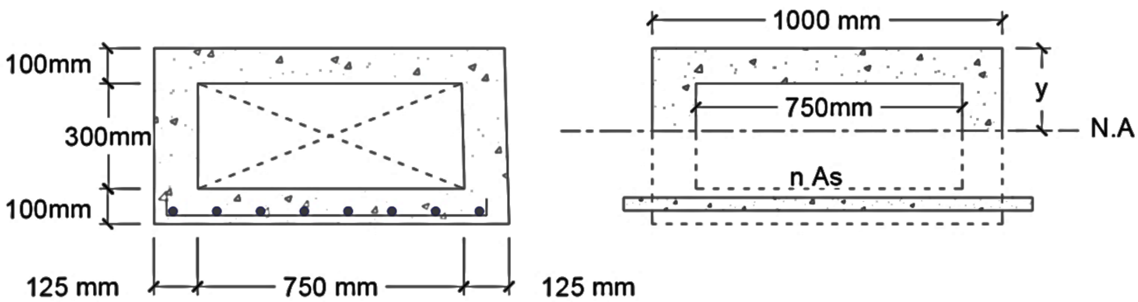
$If\ A < B\ so, the\ neutral\ axis\ locaed\ at\ the\ web$ (B)

$If\ A > B\ the\ neutral\ axis\ located\ at\ flange$ (A)

$$A = 1000 \times 100 \times 50 = 5{,}000{,}000\ mm^3$$

$$B = n\ As(d - 100) = 10{,}560{,}000\ mm^3$$

$B > A\ so, the\ neutral\ axis\ locaed\ at\ the\ web$



$$750 \times 100 \times (y - 50) + 2 \times 125 \times y \times \frac{y}{2} = n \times As \times (d - y)$$

$$75000(y - 50) + 125y^2 = 8 \times 4000 \times (430 - y)$$

$$600(y - 50) + y^2 = 256 \times (430 - y)$$

$$600y - 30000 + y^2 = 110080 - 256y$$

$$y^2 + 856y - 140080 = 0$$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$y = 140.56\ mm$$

2- Find moment of inertia about N.A

$$I_{cr} = \frac{b.h^3}{3} + n\,As(d-y)^2$$

$$I_{cr} = \frac{1000 \times 140.56^{\,3}}{3} - \frac{750 \times (140.56 \ - 100)^3}{3} + 8 \times \ 4000 \times (430 - 140.56\,)^2$$
$$= 3.59 \times 10^9 mm^4$$

3- Find the moment resisted by section

$$f_c = \frac{M.y}{I} \rightarrow 12.5 = \frac{M \times 10^6 \times 140.56}{3.59 \times 10^9} \rightarrow M = 319.26\,kN.m$$

$$f_s = n.\frac{M.(d-y)}{I} \rightarrow 165 = 8 \times \frac{M \times 10^6 \times (430 - 140.56)}{3.59 \times 10^9}$$

$$M = 255.82\,kN.m \ \ control$$

4- Find W

For simply supported beam under uniform distributed load, the moment capacity is

$$M = \frac{W.l^2}{8}$$

$$M = \frac{W.l^2}{8} \rightarrow 255.82 = \frac{w \times 3^2}{8} \rightarrow W = 227.4\,kN/m$$

Ex-4: For example 3, find the maximum uniform distributed live load that can be carried by the simply supported reinforced concrete beam of section and details shown in the above Figure. Use fs=165 MPa, f_c=12.5 MPa, effective depth (d)= 430 mm and n=8.

Solution

The procedure of solution is same of Ex-3

$$M = 255.82\,kN.m \ \ control$$

$$M = \frac{W.l^2}{8} \rightarrow 192.27 = \frac{w \times 3^2}{8} \rightarrow W = 227.4\,kN/m$$

W= Self weight +Live load

Self-weight = ((1*0.5) -(0.75*0.3)) *24=6.6 kN/m

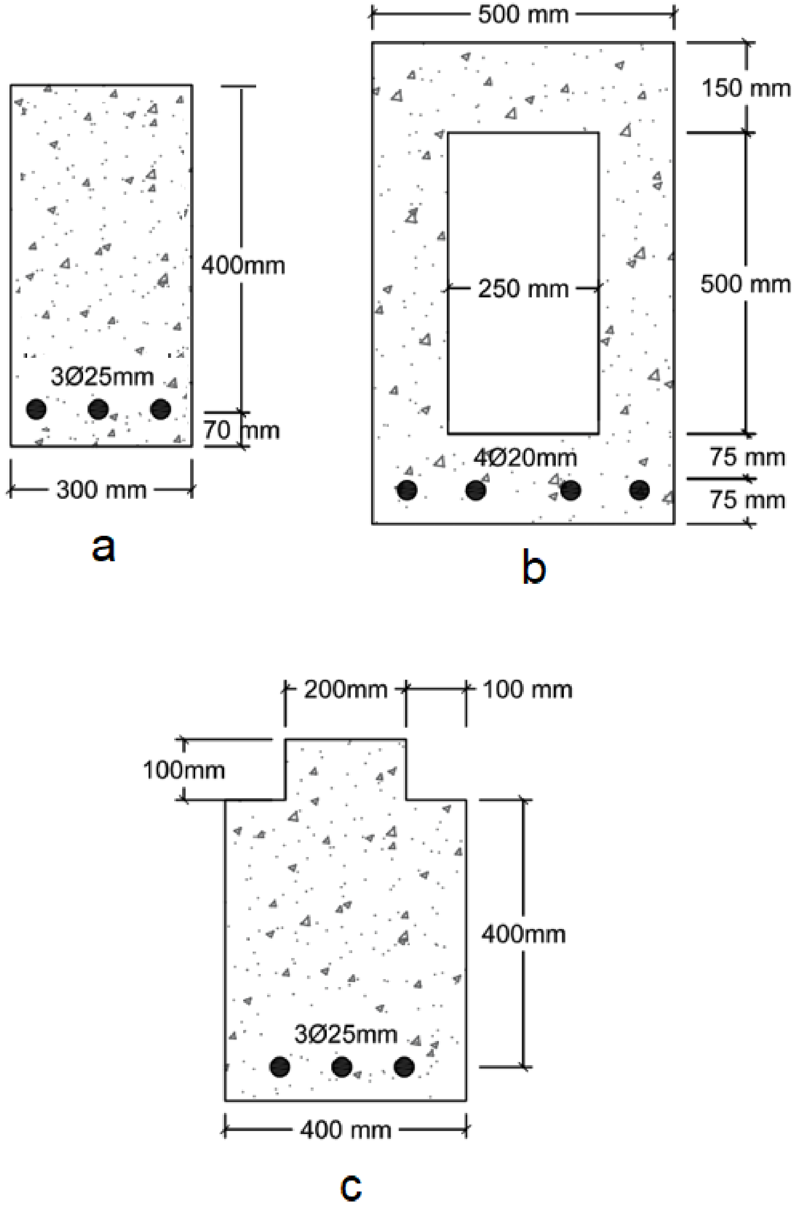
$W = W_d + W_l$

$227.4 = 6.6 + W_l$

$W_l = 220.8\,kN/m$

HW

By using the working stress design method find the maximum flexural moment can be carried by the cross sections of reinforced concrete beams of details shown in Figures a,b, and. Use fs=165 MPa, fc=12.5 MPa and n=8.



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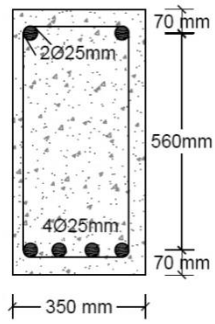
Design of Reinforced Concrete Structures I

Working stress design method-4

Dr. Othman Hameed

Ex-5: If the simply supported beam of the section shown below supports an external moment of 200 kN.m. Determine the maximum stress in the concrete and steel? Use n=10



Section

Solution:

As= 4 Ø25 mm= 1963.5 mm²

As`= 2 Ø25 mm= 981 mm²

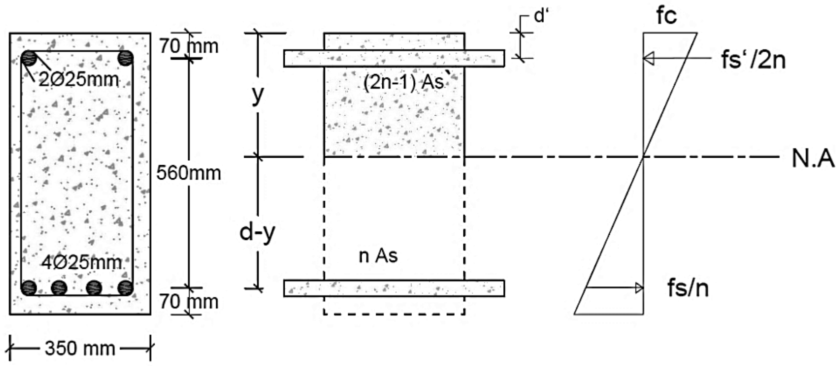
1- Find the neutral axis (N.A)

$$\frac{b.y^2}{2}+(2n-1).As`.(y-d`)=nAs(d-y)$$

$$\frac{350\times y^2}{2}+(2\times 10-1)\times 981\times(y-70)=10\times 1963.5\times(630-y)$$

$$175y^2+18639(y-70)=19635\times(630-y)$$

$$175y^2+38274y-13674780=0 \quad \rightarrow y=190.81\ mm$$



2- Find moment of inertia about N.A

$$I_{cr}=\frac{b.h^3}{3}+(2n-1).As`.(y-d`)^2+n.As.(d-y)^2$$

$$I_{cr}=\frac{350\times 190.81^3}{3}+(2\times 10-1)\times 981(190.81-70)^2+10\times\ 1963.5(630-190.81)^2$$

$$I_{cr}=8.1\times 10^8+2.72\times 10^8+3.787\times 10^9=4.87\times 10^9 mm^4$$

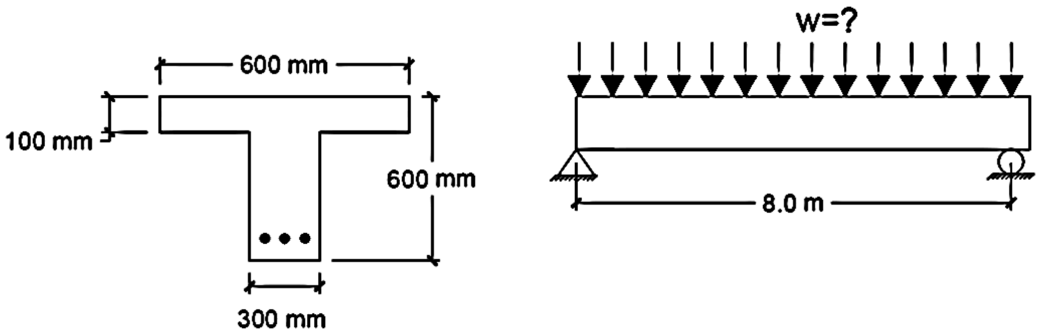
3- Calculate the stresses

$$f_c=\frac{M.y}{I}=\frac{200\times 10^6\times 190.81}{4.87\times 10^9}=7.83\ MPa$$

$$f_s = n.\frac{M.(d-y)}{I} = 10 \times \frac{200 \times 10^6 \times (630 - 190.81)}{4.87 \times 10^9} = 180.36\ MPa$$

$$f_s^{`} = 2n\frac{M.(y-d^{`})}{I} = 2 \times 10 \times \frac{200 \times 10^6 \times (190.81 - 70)}{4.87 \times 10^9} = 99.22\ MPa$$

<u>Ex-6:</u> For a T-shape reinforced concrete beam of detail shown in the figure below. Find the allowable uniform live load that can be carried over a simple span of 8.0 m.
Use $\mathrm{f_c'} = 30\ \mathrm{MPa}$, $\mathrm{fy} = 400\ \mathrm{MPa}$ and $\mathrm{n} = 8, \mathrm{d} = 538\ \mathrm{mm}$ and $A_s = 2413\ mm^2$.



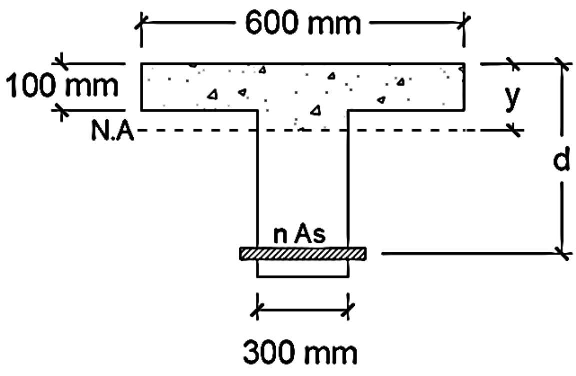
Solution:

1- Let A= moment area of flange (compression).
 and B=moment area of web (tension)

$A = 600 \times 100 \times 50 = 3{,}000{,}000\ mm^3$
$B = nA_s(d - h_f) = 8 \times 2413 \times (538 - 100) = 8{,}455{,}152\ mm^3$
$A < B\ so, the\ neutral\ axis\ locaed\ at\ the\ web$
$If\ A > B\ the\ neutral\ axis\ located\ at\ flange$

2- Find the neutral axis of the section.

$300 \times \frac{y^2}{2} + 2(150 \times 100)(y - 50) = 8 \times 2413 \times (538 - y)$
$150y^2 + 30000(y - 50) = 19304 \times (538 - y)$
$y^2 + 200(y - 50) = 128.7 \times (538 - y)$
$y^2 + 328.7y - 79240.6 = 0$
$y = 161.6\ mm$



3- Calculate the cracked moment of inertia about neutral axis.

$$I_{cr.} = \frac{300 \times 61.6^3}{3} + \left[\frac{600 \times 100^3}{12} + 600 \times 100 \times (161.6 - 50)^2\right]$$
$$+ 8 \times 2413 \times (538 - 161.6)^2 = 3.556 \times 10^9\ mm^4$$

4- Find the moment resisted by section

$$f_c = \frac{M.y}{I} \rightarrow 30 \times 0.45 = \frac{M \times 10^6 \times 161.6}{3.556 \times 10^9} \rightarrow M = 297.07\ kN.m$$

$$f_s = n.\frac{M.(d-y)}{I} \rightarrow 170 = 8 \times \frac{M \times 10^6 \times (538 - 161.6)}{3.556 \times 10^9}$$

$$M = 200.75\ kN.m \ \ control$$

5- Find W

$$M = \frac{W.l^2}{8} \rightarrow 200.75 = \frac{w \times 8^2}{8} \rightarrow W = 25.09\ kN/m$$

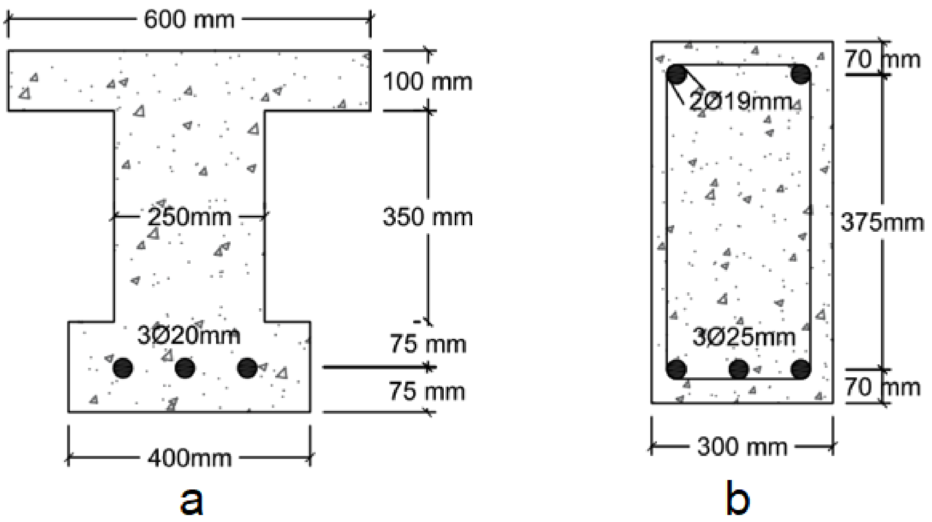
W= Self weight +Live load

$$Self\ weight = [(0.6 \times 0.1) + (0.5 \times 0.3)] \times 24 = 5.04\ kN/m$$

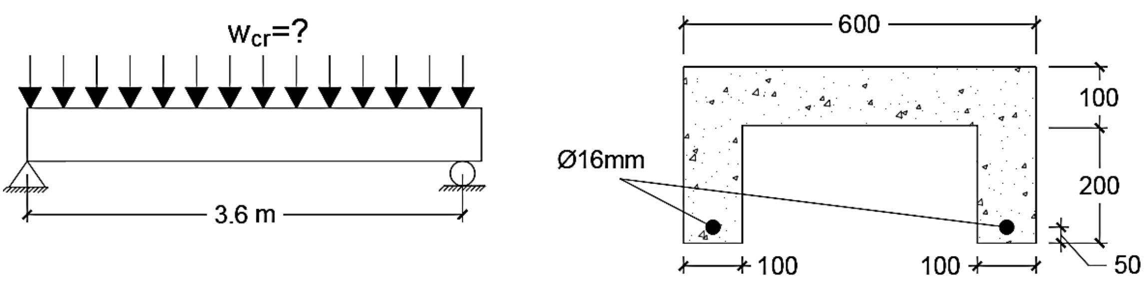
$$live\ load = W - self\ weight = 25.09 - 5.04 = 20.05\ kN/m$$

HW

1. By using the working stress design method, find the maximum flexural moment can be carried by the cross sections of reinforced concrete beams of details shown in Figures a and b. Use fs=165 MPa, fc=12.5 MPa and n=8.



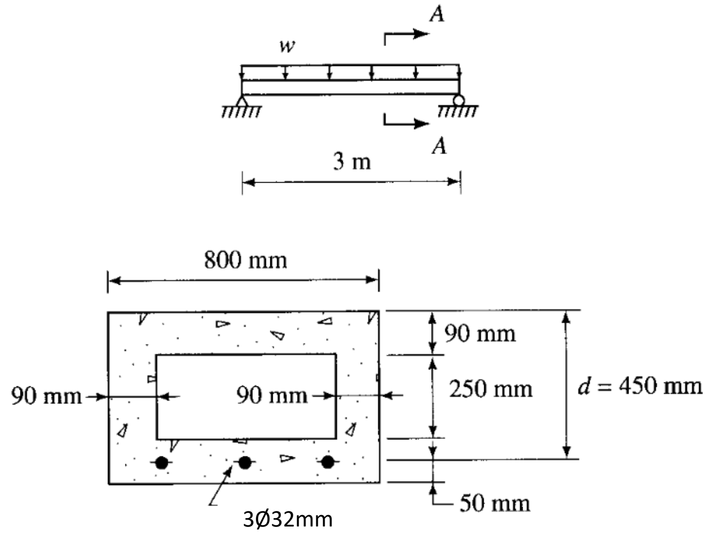
2. Determine the smallest value of uniform load that will cause a first crack for the reinforced concrete beam shown below. Use $f_c^{`} = 30\ MPa$



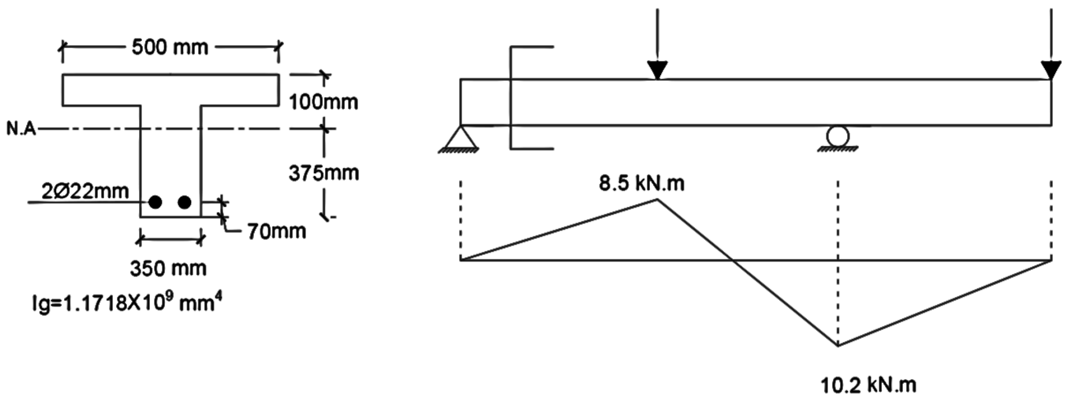
3. For the beam of details shown below, use $f_c^` = 40\ MPa$ and $fy = 400\ MPa$ to:

A- Compute the value of the uniform load that will produce <u>initial (first)</u> cracking of the reinforced concrete beam shown in the Figure below.

B- Compute the value of the maximum uniform load that can be carried by the reinforced concrete beam



4. The beam is constructed of reinforced concrete. Will it crack if the loads produce the moment curve shown in Figure below? If the beam cracks, indicate the location of the cracks. Use $f_c^` = 30\ MPa$ and $I_{unc} = 1.1718 \times 10^9\ mm^4$



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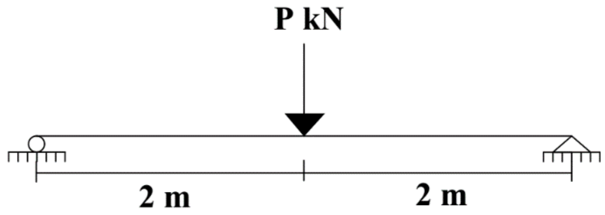
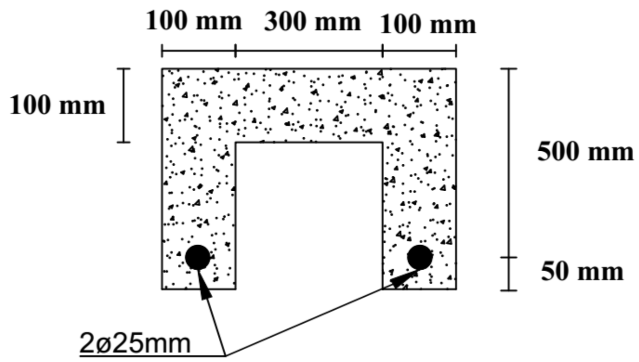
Design of Reinforced Concrete Structures I

Working stress design method-5

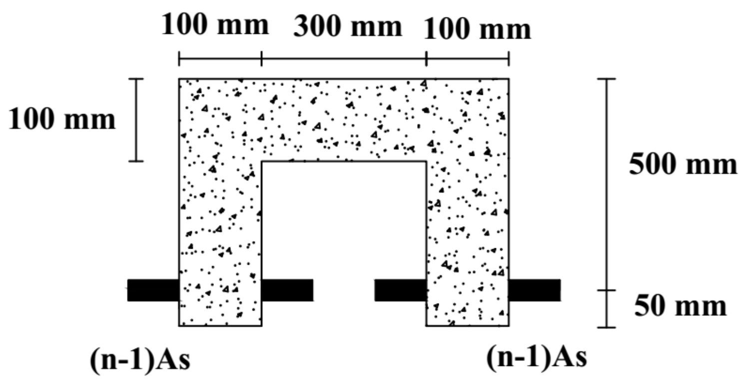
Dr. Othman Hameed

Ex-7: Determine the smallest value of the load (P) that will cause a first crack for the reinforced concrete beam shown below. Use $f_c^` = 30\ MPa$





Solution:



As= 1 Ø25 mm= 490.6 mm^2

$$n = \frac{E_s}{E_c} = 7.77$$

1- Let A= moment area of flange (compression).

and B=moment area of web (tension)

$$A = 500 \times 100 \times 50 = 2{,}500{,}000\ mm^3$$

$$B = 2 \times (n-1)A_s(500-100) + 450 \times 100 \times \frac{450}{2} =$$

$$B = 2 \times 6.77 \times 490.6 \times 400 + 10125000 = 12{,}782{,}089.6\ mm^3$$

$$B > A, \qquad the\ neutral\ axis\ located\ at\ the\ flange$$

2-Find the neutral axis (N.A) (uncracked section)

$$500 \times \frac{y^2}{2} - 300 \times \frac{(y-100)^2}{2} = 2 \times (n-1)\ As\ (500-y) + 2 \times 100 \times \frac{(550-y)^2}{2}$$

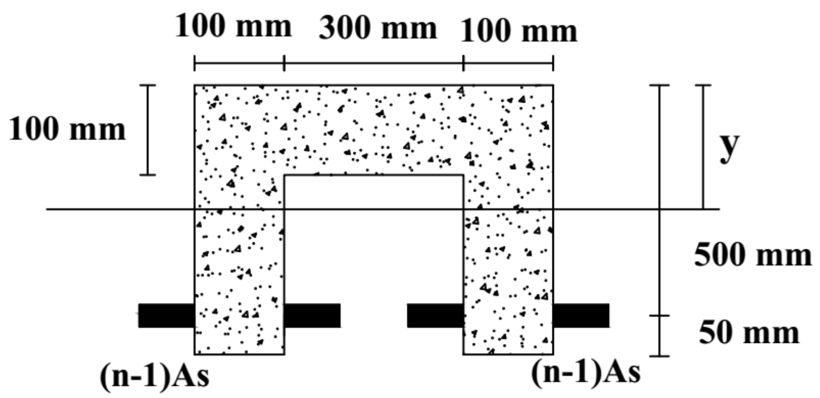
$$250y^2 - 150(y-100)^2 = 2 \times 6.77 \times 490.6(500-y) + 100 \times (550-y)^2$$

$$250y^2 - 150(y^2 - 200y + 10000) = 6642.7(500 - y) + 100 \times (302500 - 1100y + y^2)$$

$$100y^2 + 30000y - 1500000 = 3321350 - 6642.7y + 30250000 - 110000y + 100y^2$$

$$146642.7y = 35071350$$

$$y = 239.2\ mm$$



3-Find moment of inertia about N.A

$$I_{un} = \frac{500.y^3}{3} - \frac{300.(y-100)^3}{3} + 2 \times \frac{100.(550-y)^3}{3} + 2(n-1).As.(d-y)^2$$

$$I_{un} = \frac{500 \times 239.2^3}{3} - \frac{300.(239.2-100)^3}{3} + 2 \times \frac{100.(550-239.2)^3}{3} + 2 \times 6.77 \times 490.6\,(500-239.2)^2$$

$$I_{un} = 4.46 \times 10^9 mm^4$$

To calculate the load that cases the first crack, moment must be determined

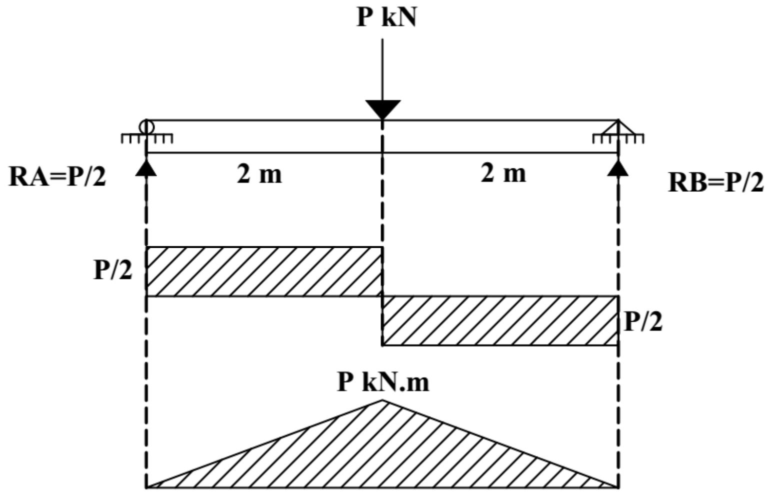
$$f_t = f_r$$

$$f_r = 0.62\sqrt{f_c^{`}} = 0.62\sqrt{30} = 3.4\ MPa$$

$$f_t = \frac{M.y_t}{I} = \frac{M \times 10^6 \times (550-y)}{4.53 \times 10^9}$$

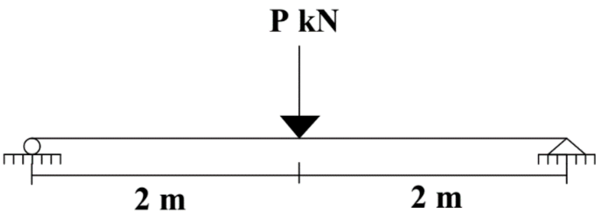
$$3.4 = \frac{M \times 10^6 \times (550-239.2)}{4.46 \times 10^9}$$

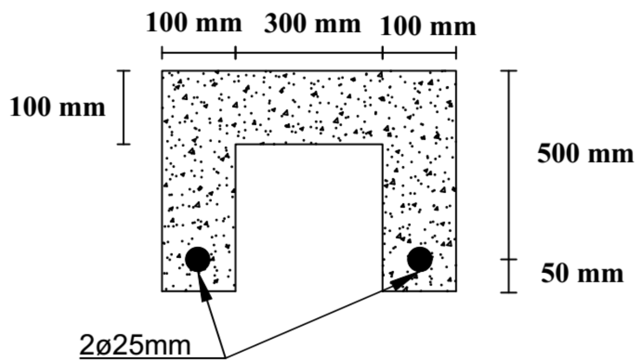
$$M = 48.8\ kN.M$$



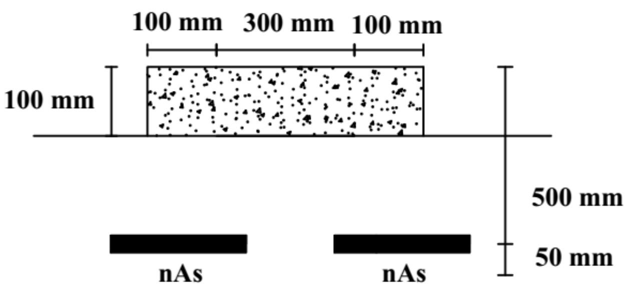
$$M = \frac{P \times L}{4} = P$$
$$P = 48.8\ kN$$

Ex-8: Determine the maximum value of the load (P) that can be carried by the beam of details shown below. Use $f_c^` = 30\ MPa$ and $fy = 400\ MPa$.





Solution:



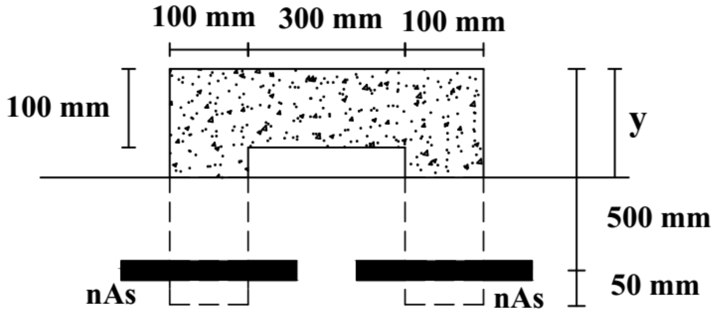
1-Let A= moment area of flange (compression).

and B=moment area of web (tension)

$A = 500 \times 100 \times 50 = 2{,}500{,}000\, mm^3$

$B = 2nA_s(500 - 100) = 2 \times 7.77 \times 490.6 \times 400 = 3049569.6\, mm^3$

2-Find the neutral axis (N.A)



3-Find moment of inertia about N.A (I_{un})

4- Find moment from stresses

$$f_c = \frac{M.y}{I} \rightarrow 30 \times 0.45 = \frac{M \times 10^6 \times y}{I_{un}}$$

$$f_s = n.\frac{M.(d-y)}{I} \rightarrow 170 = 7.77 \times \frac{M \times 10^6 \times (500 - y)}{I_{un}}$$

5- Find the load P

Design of Reinforced Concrete Structures I

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Analysis of singly reinforced section-1

Dr Othman Hameed

Lecture (10)

Ultimate Strength Design Method

After 1963, the ultimate-strength design method rapidly gained popularity because

- It is a more rational approach than does WSD,
- It uses a more realistic consideration of safety, and
- It provides more economical designs

Advantages of Ultimate Strength Design Method

1. The derivation of the strength design expressions takes into account the nonlinear shape of the stress–strain diagram.
2. A more realistic factor of safety is used in strength design.
3. A structure designed by the strength design method will have a more uniform safety factor against collapse throughout. The strength method takes considerable advantage of higher strength of steels, whereas working-stress design method did so only partly. The result is better economy for strength design.
4. The strength design method permits more flexible designs than did the working-stress method.

Assumptions

Reinforced concrete sections are heterogeneous (nonhomogeneous) materials, because they are made of two different materials, concrete and steel. Therefore, proportioning structural members by strength design approach is based on the following assumptions:

1. Strain in concrete is the same as in reinforcing bars at the same level, provided that the bond between the steel and concrete is adequate.
2. Strain in concrete is linearly proportional to the distance from the neutral axis.
3. The modulus of elasticity of all grades of steel is taken as Es = (200,000MPa N/mm2).
4. Plane cross sections continue to be plane after bending.
5. Tensile strength of concrete is neglected because

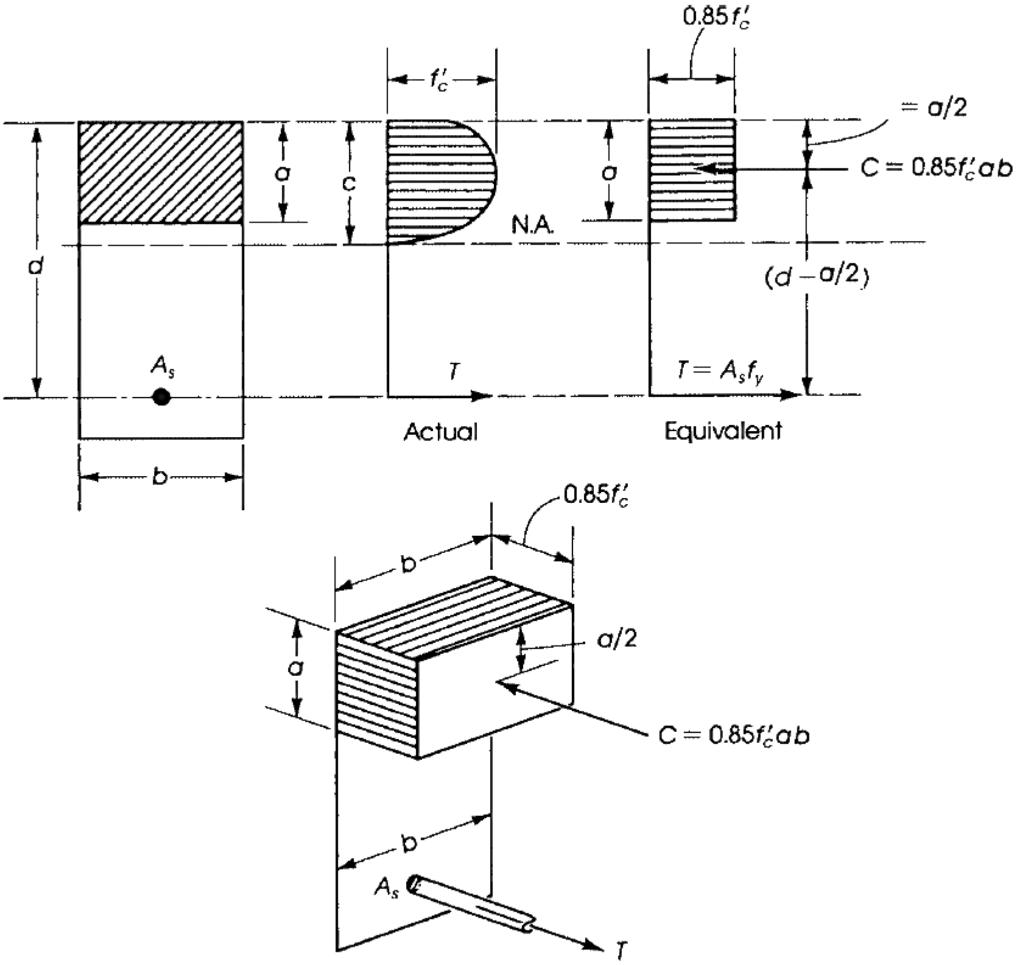
(a) Concrete's tensile strength is about 10% of its compressive strength,

(b) Cracked concrete is assumed to be not effective,

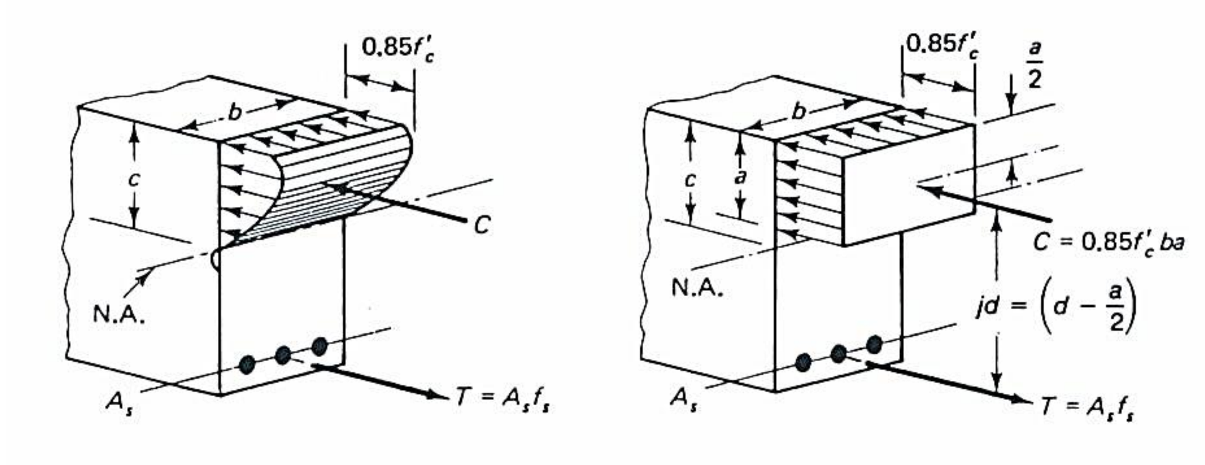
(c) Before cracking, the entire concrete section is effective in resisting the external moment.

6. At failure the maximum strain at the extreme compression fibers is assumed equal to 0.003 by the ACI Code provision.
7. For design strength, the shape of the compressive concrete stress distribution may be assumed to be rectangular, parabolic, or trapezoidal. In this text, a rectangular shape will be assumed (ACI Code, Section 22.2).

Analysis of Nominal Moment Strength for Singly Reinforced Beam Sections



The actual distribution of the compressive stress in a section has the form of rising parabola. It is time consuming to evaluate the volume of compressive stress block. An equivalent rectangular stress block can be used without loss of accuracy.



The flexural strength of the beam (Mn) can be calculated as shown below

Compression $\Longrightarrow$ $C = 0.85 f_c^{`} a\ b$

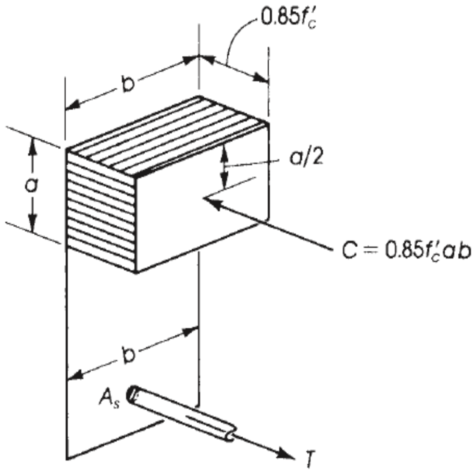
Tension $\Longrightarrow$ $T = As\,fy$

According ACI Code 22.2.2.4.1 through ACI Code 22.2.2.4.3, in the equivalent rectangular block an average stress of $0.85 f_c^{`}$ is used with a rectangle of depth $a = \beta_1 c$,

The values of β_1 shall be in accordance with Table 22.2.2.4.3.of ACI

Table 22.2.2.4.3—Values of β_1 for equivalent rectangular Concrete stress distribution		
	f_c' MPa	β_1
a	$17 \le f_c' \le 28$	0.85
b	$28 < f_c' \le 55$	$0.85 - \dfrac{0.05(f_c^{`} - 28)}{7}$
c	$f_c' > 55$	0.65

Based on these assumptions regarding the stress block, statics equations can easily be written for the sum of the horizontal forces and for the resisting moment produced by the internal couple. These expressions can then be solved separately for (a) and for the moment, Mn.



$$\sum Fx = 0 \quad \rightarrow \quad C = T$$

$$0.85\, f_c' ab = As\,fy \quad \rightarrow \quad a = \frac{As\,fy}{0.85\, f_c' b} = \frac{\rho\, fy\, d}{0.85\, f_c'} \quad \ldots (1)$$

Where: $\rho = \frac{As}{b\,d}$ = percentage of steel reinforcement to effective area

$$\sum M = 0$$

$$Mn = T\left(d - \frac{a}{2}\right) = As\,fy\,(d - \frac{a}{2})$$

<u>OR</u>

$$Mn = C\left(d - \frac{a}{2}\right) = 0.85\,a\,b\,(d - \frac{a}{2})$$

Because the reinforcing steel is limited to an amount such that it will yield well before the concrete reaches its ultimate strength, the value of the nominal moment, Mn, can be written as

$$Mn = T\left(d - \frac{a}{2}\right) = As\,fy\left(d - \frac{a}{2}\right) \ldots \ldots \ldots \ldots \ldots \ldots . (2)$$

Sub. (2) In (1)

$$Mn = \rho\, bd^2 fy \left(1 - 0.59 \frac{fy}{f_c'}\, \rho\right)$$

$$\phi Mn \geq Mu$$

Where:

ϕ= Strength reduction factor

Mn= nominal moment resisting capacity of the section

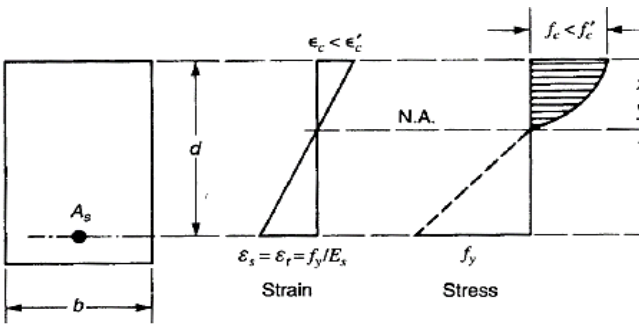
Mu= ultimate moment capacity applied on the section (from external load)

Types of Failure and Strain Limits

Three types of flexural failure of a structural member can be expected depending on the percentage of steel used in the section.

1- Tension Failure (ductile failure)

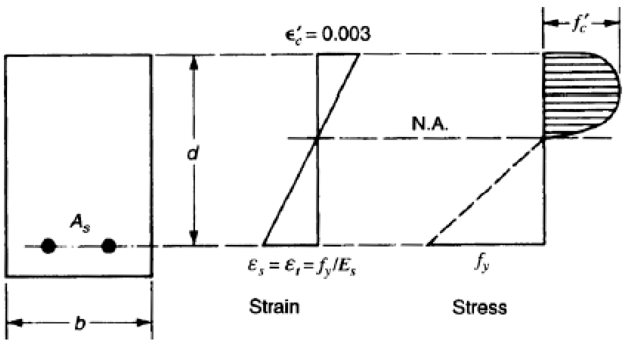
Steel may reach its yield strength before the concrete reaches its maximum strength. In this case, the failure is due to the yielding of steel reaching a high strain equal to or greater than 0.005. The section contains a relatively small amount of steel and the section is known (**under reinforcement section**) or ductile sec.



$$\left[\varepsilon_c < 0.003, \qquad \varepsilon_s > \varepsilon_y, \qquad \rho < \rho_b\right]$$

2- Balance Failure

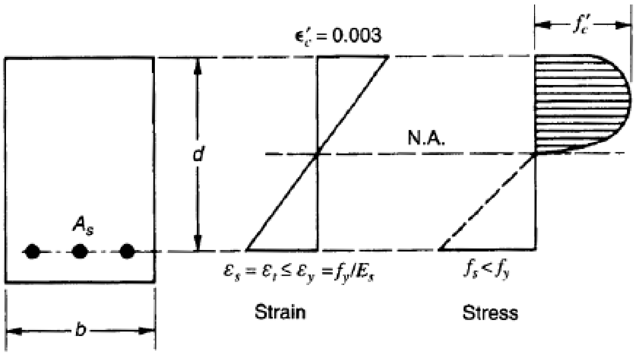
Steel may reach its yield strength at the same time as concrete reaches its ultimate strength so, the failure will be sudden. The section is known a balanced section.



$$\left[\varepsilon_c = 0.003, \qquad \varepsilon_s = \varepsilon_y, \qquad \rho = \rho_b\right]$$

3- Compression Failure (brittle failure)

Concrete may fail before the yield of steel, due to the presence of a high percentage of steel in the section.



In this case, the concrete strength and its maximum strain of 0.003 are reached, but the steel stress is less than the yield strength.

This section is called a compression-controlled or (**over reinforcement section**).

$$\left[\varepsilon_c = 0.003, \qquad \varepsilon_s < \varepsilon_y, \qquad \rho > \rho_b\right]$$

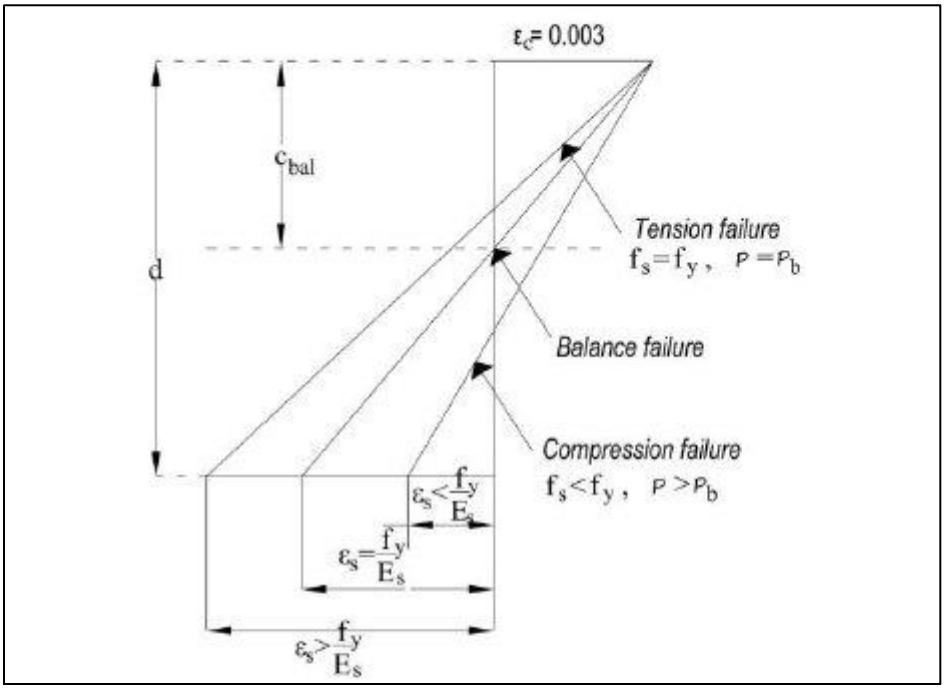
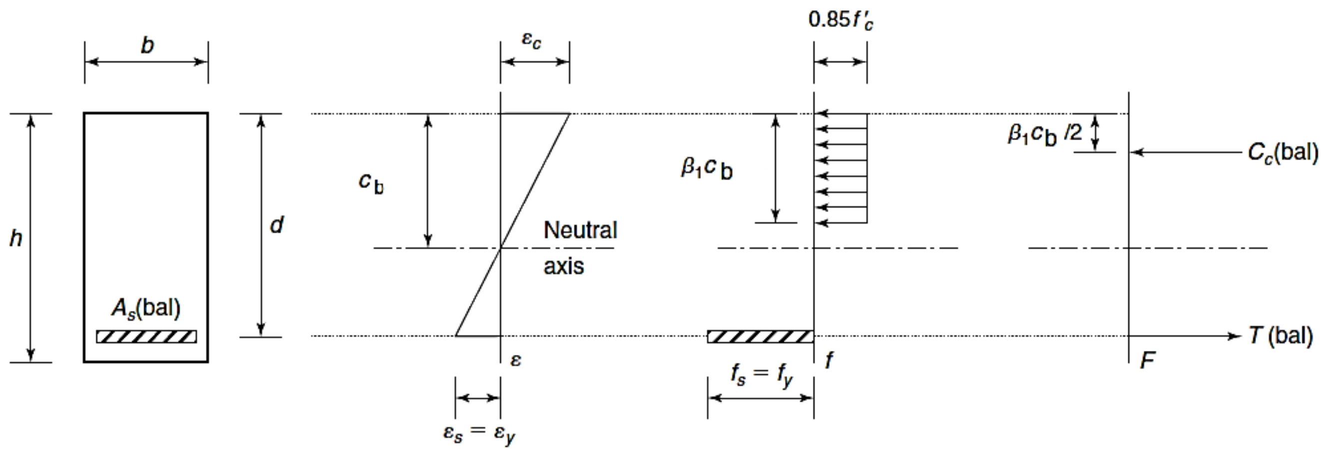


Figure shows types of failure

Which type of failure is most desirable?

The under-reinforcement beam is the most desirable beam because the failure mode is ductile, thus giving sufficient amount of warning before collapse and it's adopted by strength design method.

Balance Steel Reinforcement Ratio ρ_b



(a) Beam section. (b) Balanced strain distribution. (c) Stress distribution. (d) Internal forces.

From strain diagram

$$\frac{\varepsilon_{c(0.003)}}{c_b} = \frac{\varepsilon_y}{(d-c_b)} \quad \rightarrow 0.003d - 0.003c_b = \varepsilon_y\, c_b$$

$$c_b = \frac{0.003\ d}{(0.003+\varepsilon_y)} \times \frac{E_s}{Es} \quad = \boxed{\frac{600}{(600+fy)} \times d}$$

From stress diagram

$$\sum Fx = 0$$

$$0.85 f_c'\ a\ b = A_{sb}\ f_y$$

$$0.85 f_c'\ a\ b = \rho_b\ b\ d\ \ fy$$

$$\rho_b = \frac{0.85 f_c'}{fy\ d}\ a\ ; \qquad a = \beta_1 c_b$$

$$\boxed{\rho_b = 0.85\ \beta_1 \frac{f_c'}{fy}\ \frac{600}{(600+fy)}}$$

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Design of Reinforced Concrete Structures I

Analysis of singly reinforced section-2

Dr Othman Hameed

Lecture (11)

Design Requirement

1. Safety Provision

Structural members must always be proportioned to resist loads greater than the service or actual load in order to provide proper safety against failure. In the strength design method, the member is designed to resist factored loads, which are obtained by multiplying the service loads by load factors. Different factors are used for different loadings. Because dead loads can be estimated quite accurately, their load factors are smaller than those of live loads, which have a high degree of uncertainty

In addition to load factors, the ACI Code specifies another factor to allow an additional reserve in the capacity of the structural member. The nominal strength is generally calculated using an accepted analytical procedure based on statistics and equilibrium; however, in order to account for the degree of accuracy within which the nominal strength can be calculated, and for adverse variations in materials and dimensions, a strength reduction factor, Ø, should be used in the strength design method.

A) Load Factor ACI 5.3

For the design of structural members, the factored design load is obtained by multiplying the dead load by a load factor and the specified live load by another load factor.

Required strength ***U*** shall be at least equal to the effects of factored loads in ACI Table 5.3.1

Table 5.3.1—Load combinations

Load combination	Equation	Primary load
$U = 1.4D$	(5.3.1a)	D
$U = 1.2D + 1.6L + 0.5(L_r \text{ or } S \text{ or } R)$	(5.3.1b)	L
$U = 1.2D + 1.6(L_r \text{ or } S \text{ or } R) + (1.0L \text{ or } 0.5W)$	(5.3.1c)	L_r or S or R
$U = 1.2D + 1.0W + 1.0L + 0.5(L_r \text{ or } S \text{ or } R)$	(5.3.1d)	W
$U = 1.2D + 1.0E + 1.0L + 0.2S$	(5.3.1e)	E
$U = 0.9D + 1.0W$	(5.3.1f)	W
$U = 0.9D + 1.0E$	(5.3.1g)	E

Where:

U= ultimate load,

D= dead load or F = fluid load

L= live load

W= wind load, S= Snow load,

E=effect of horizontal and vertical earthquake-induced forces

B) Strength Reduction Factor (Ø) ACI 21.1

The nominal strength of a section, say Mn, for flexural members, calculated in accordance with the requirements of the ACI Code provisions must be multiplied by the strength reduction factor, ϕ, which is always less than 1. The purposes of strength reduction factors Ø are:

1. To account for the probability of under-strength members due to variations in material strengths and dimensions.
2. To account for inaccuracies in the design equations.
3. To reflect the available ductility and required reliability of the member under the load effects being considered.

Strength reduction factors Ø shall be in accordance with ACI Table 21.2.1

Table 21.2.1—Strength reduction factors ϕ

Action or structural element		ϕ	Exceptions
(a)	Moment, axial force, or combined moment and axial force	0.65 to 0.90 in accordance with 21.2.2	Near ends of pretensioned members where strands are not fully developed, ϕ shall be in accordance with 21.2.3.
(b)	Shear	0.75	Additional requirements are given in 21.2.4 for structures designed to resist earthquake effects.
(c)	Torsion	0.75	—
(d)	Bearing	0.65	—
(e)	Post-tensioned anchorage zones	0.85	—
(f)	Brackets and corbels	0.75	—
(g)	Struts, ties, nodal zones, and bearing areas designed in accordance with strut-and-tie method in Chapter 23	0.75	—
(h)	Components of connections of precast members controlled by yielding of steel elements in tension	0.90	—
(i)	Plain concrete elements	0.60	—
(j)	Anchors in concrete elements	0.45 to 0.75 in accordance with Chapter 17	—

Strength reduction factor for moment, axial force, or combined moment and axial force shall be in accordance with ACI Table 21.2.2.

Table 21.2.2—Strength reduction factor ϕ for moment, axial force, or combined moment and axial force

Net tensile stain ε_t	Classification	ϕ			
		Type of transverse reinforcement			
		Spirals conforming to 25.7.3		Other	
$\varepsilon_t \le \varepsilon_{ty}$	Compression-controlled	0.75	(a)	0.65	(b)
$\varepsilon_{ty} < \varepsilon_t < 0.005$	Transition[1]	$0.75 + 0.15\dfrac{(\varepsilon_t - \varepsilon_{ty})}{(0.005 - \varepsilon_{ty})}$	(c)	$0.65 + 0.25\dfrac{(\varepsilon_t - \varepsilon_{ty})}{(0.005 - \varepsilon_{ty})}$	(d)
$\varepsilon_t \ge 0.005$	Tension-controlled	0.90	(e)	0.90	(f)

[1]For sections classified as transition, it shall be permitted to use ϕ corresponding to compression-controlled sections.

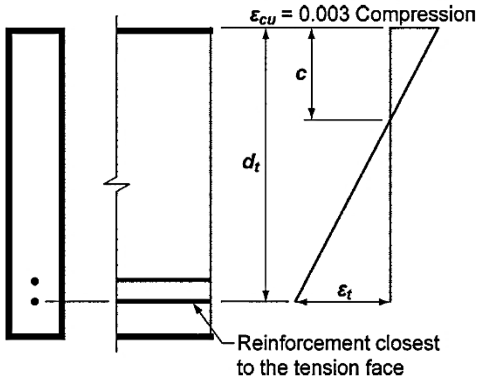
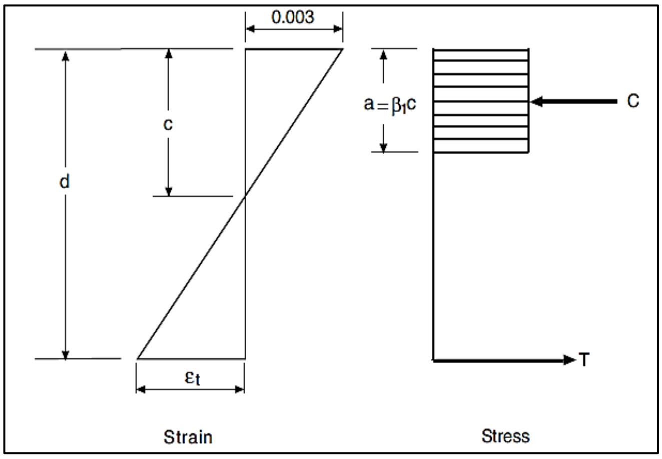


Fig. R21.2.2a—Strain distribution and net tensile strain in a nonprestressed member.



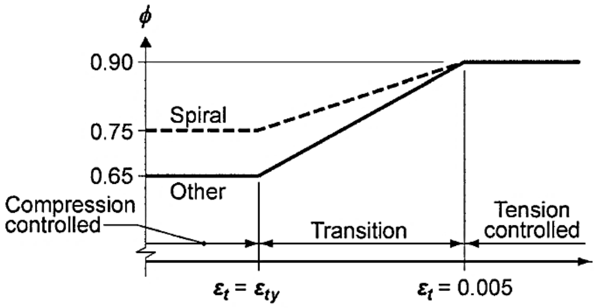


Fig. R21.2.2b—Variation of ϕ with net tensile strain in extreme tension reinforcement, ε_t.

ϕ= Strength reduction factor

1) $if\ \rho_t \ge \rho$, use $\phi = 0.9$ $\varepsilon_t \ge 0.005\ \ (tension\ control)$

$$\rho_t = 0.85\,\beta_1 \frac{f_c^{`}}{fy}\,\frac{\varepsilon_c}{\varepsilon_c + 0.005}$$

2) $if\ \rho_t < \ \rho, \emptyset\ must\ be\ calculated$

To calculate $\emptyset$*, the values of* ε_t *and* ε_{ty} *must be determined*

$$a = \frac{As\ fy}{0.85\ f_c'b} \ \rightarrow\ a = \beta_1\, c\ \rightarrow c = \frac{a}{\beta_1}$$

Find ε_t from

$$\frac{\varepsilon_t}{(d-c)} = \frac{\varepsilon_c = 0.003}{c}$$

A) If $\varepsilon_{ty} < \varepsilon_t < 0.005 \ \ (transition \ \ control)$

$$\phi = 0.65 + 0.25\frac{(\varepsilon_t - \varepsilon_{ty})}{(0.005 - \varepsilon_{ty})}$$

B) $If \ \varepsilon_t \leq \varepsilon_{ty} \ \ (compression \ \ control)$

$$\phi = 0.65$$

Where:

ε_t= Net tensile strain in extreme tension steel and can be calculated from strain diagram

ε_{ty} =Strain of steel bars and can be calculated from fy/Es

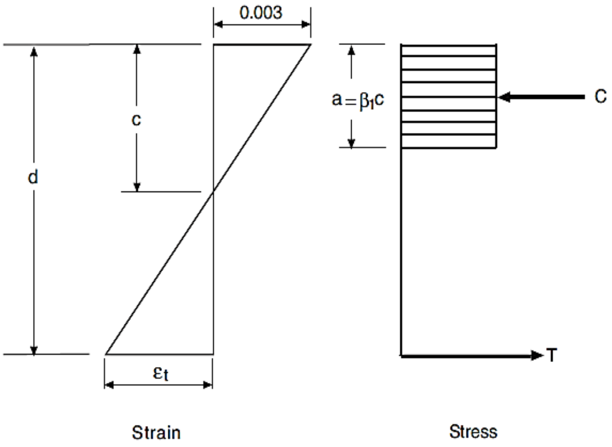
ρ_t = Maximum steel ratio at which the net tensile strain in steel exceed 0.005

For design strength, all sections must satisfy

$$\phi Mn \ \geq \ Mu$$

2. Maximum Steel Reinforcement Ratio $\rho_{max.}$

For the non-prestressed beam with ($P_u < 0.1\ Ag\ f_c^`$) ε_t shall not less than 0.004



From stress diagram

$$\sum Fx = 0$$

$$0.85 f_c^` \ a\ b = As_{max.}\ fy \qquad \div b\ d$$

$$\rho_{max.} = 0.85 \frac{f_c^`}{fy} \frac{a}{d} \ \dots\dots\dots\dots\dots (1)$$

$$a = \beta_1\ c \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (2)$$

From strain diagram

$$\frac{\varepsilon_{c(0.003)}}{c} = \frac{(\varepsilon_c + \varepsilon_t)}{d} \qquad \rightarrow c = \frac{\varepsilon_c}{(\varepsilon_c + \varepsilon_t)}\ d \ \dots\dots..(3)$$

Sub. 2 and 3 into 1

$$\rho_{max.} = 0.85\ \beta_1 \frac{f_c^`}{fy} \frac{\varepsilon_c}{(\varepsilon_c + \varepsilon_t)}$$

$$\varepsilon_c = 0.003$$

$$\varepsilon_t = 0.004$$

$$\rho_{max.} = 0.85\ \beta_1 \frac{f_c^`}{fy} \frac{0.003}{(0.003+0.004)}$$

3. Minimum Steel Reinforcement Ratio $\rho_{min.}$ ACI 9.6.1

Minimum area of flexural reinforcement, As_{min}, shall be provided at every section where tension reinforcement is required by analysis.

The steel reinforcement ratio shall be the grater of:

$$\rho_{min.} = max.of \begin{cases} \dfrac{\sqrt{f_c'}}{4\ fy} \\ \dfrac{1.4}{fy} \end{cases} \qquad \text{ACI9.6.1.2}$$

OR

$$As_{min.} = max.of \begin{cases} \dfrac{\sqrt{f_c'}}{4\,fy} bd \\ \dfrac{1.4}{fy} bd \end{cases}$$

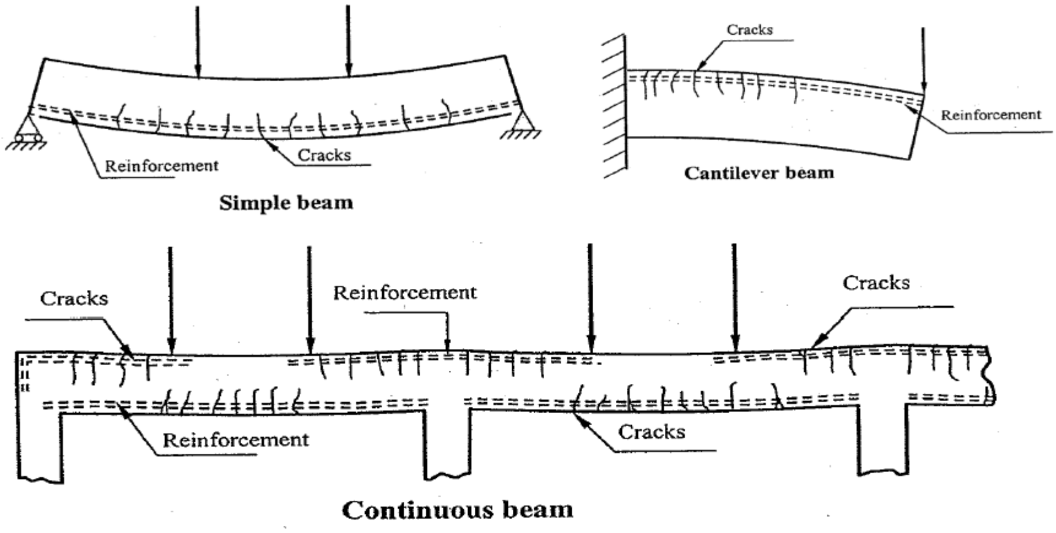
$$As_{min.} = \frac{\sqrt{f_c`}}{4\,fy}\; b\; d \geq \frac{1.4}{fy}\; b\; d$$

4. Location of Reinforcement

The reinforcement placed when the cracking occurred (tension region)

The tension stresses may be due to

- Flexural force
- Axial force
- Shrinkage



Reinforcement placement for different types of beams

5. Serviceability

The serviceability requirements ensure adequate performance at service load without excessive deflection and cracking.

Two method are given by ACI for controlling deflection

1- By calculating deflection and comparing with specification.
2- By using member thickness equal or greater than the value provided in ACI-2008 table 9.5.a

TABLE 9.5(a) — MINIMUM THICKNESS OF NONPRESTRESSED BEAMS OR ONE-WAY SLABS UNLESS DEFLECTIONS ARE CALCULATED

	Minimum thickness, ***h***			
	Simply supported	One end continuous	Both ends continuous	Cantilever
Member	Members not supporting or attached to partitions or other construction likely to be damaged by large deflections			
Solid one-way slabs	**ℓ/20**	**ℓ/24**	**ℓ/28**	**ℓ/10**
Beams or ribbed one-way slabs	**ℓ/16**	**ℓ/18.5**	**ℓ/21**	**ℓ/8**
Notes: Values given shall be used directly for members with normalweight concrete and Grade 60 reinforcement. For other conditions, the values shall be modified as follows: a) For lightweight concrete having equilibrium density, w_c, in the range of 90 to 115 lb/ft^3, the values shall be multiplied by **(1.65 – 0.005w_c)** but not less than 1.09. b) For f_y other than 60,000 psi, the values shall be multiplied by **(0.4 + f_y/100,000)**.				

Where (*l*) is the length of the member (center to center)

According to experience H= 10 cm for each 1 m of span length & b= 0.5 H

6. Detailing of Steel Reinforcement

A- Concrete cover

Concrete cover as protection of reinforcement against weather and other effects is measured from the concrete surface to the outermost surface of the steel to which the cover requirement applies.

According to ACI20.6.1.3.1, specify minimum clear cover in cast-in-place concrete should not be less than the value shown in table below.

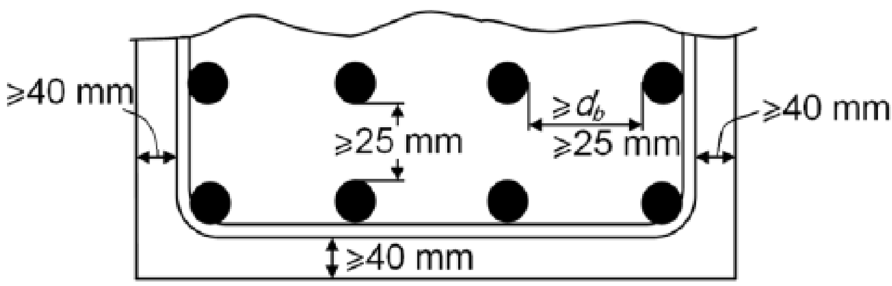
Table 20.6.1.3.1—Specified concrete cover for cast-in-place nonprestressed concrete members

Concrete exposure	Member	Reinforcement	Specified cover, mm
Cast against and permanently in contact with ground	All	All	75
Exposed to weather or in contact with ground	All	No. 19 through No. 57 bars	50
		No. 16 bar, MW200 or MD200 wire, and smaller	40
Not exposed to weather or in contact with ground	Slabs, joists, and walls	No. 43 and No. 57 bars	40
		No. 36 bar and smaller	20
	Beams, columns, pedestals, and tension ties	Primary reinforcement, stirrups, ties, spirals, and hoops	40

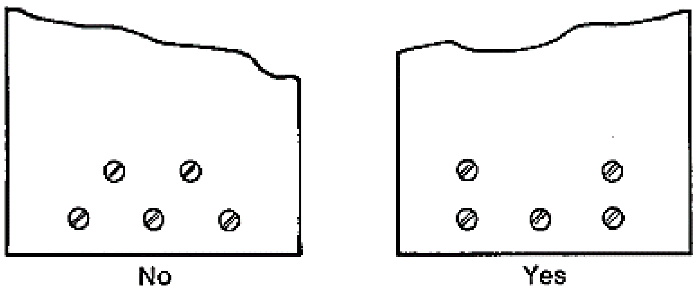
B- Spacing limits for reinforcement ACI 25.2

The minimum limits were originally established to permit concrete to flow easily into spaces between bars.

Based on ACI-25.2.1 For parallel non-prestressed reinforcement in a horizontal layer, clear spacing shall be at least the greatest of 25 mm, db, and (4/3)d_{agg}.



For parallel non-prestressed reinforcement placed in two or more horizontal layers, reinforcement in the upper layers shall be placed directly above reinforcement in the bottom layer with a clear spacing between layers of at least 25 mm.



Arrangement of bars In two layers (ACI Section 7.6.2).

Design of Reinforced Concrete Structures I

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Analysis of Singly Reinforced Sections-3

Dr Othman Hameed

Lecture (12)

Procedure of Analysis singly reinforced sections

1) $\phi Mn \geq Mu$

Mu: Is the ultimate factored moment (1.2 D.L+1.6 L.L)

2) $\rho = \frac{As}{b\,d}$

$\rho_{min.} < \rho < \rho_{max.}$

3) $\rho_{max.} = 0.85\,\beta_1\,\frac{f_c^`}{fy}\,\frac{0.003}{(0.003+0.004)}$

4) $\rho_{min.} = max.of\begin{cases}\frac{\sqrt{f_c'}}{4\,fy}\\ \frac{1.4}{fy}\end{cases}$

5) $\phi Mn = \phi\,\rho\,bd^2 fy\left(1 - 0.59\frac{fy}{f_c^{'}}\,\rho\right)$

β_1 can be calculated according to the table below

Table to calculate β_1		
	$f_c'\ MPa$	β_1
a	$17 \leq f_c' \leq 28$	0.85
b	$28 < f_c' \leq 55$	$0.85 - \frac{0.05(f_c^` - 28)}{7}$
c	$f_c' > 55$	0.65

To calculate ϕ

ϕ= Strength reduction factor

1) $if\ \rho_t \geq\ \rho\ ,$ use $\phi = 0.9$

$$\rho_t = 0.85\ \beta_1 \frac{f_c^{`}}{fy} \frac{\varepsilon_c}{\varepsilon_c + 0.005}$$

2) $if\ \rho_t <\ \rho, \emptyset\ must\ be\ calculated$

To calculate $\emptyset$, the values of ε_t and ε_{ty} must be determined

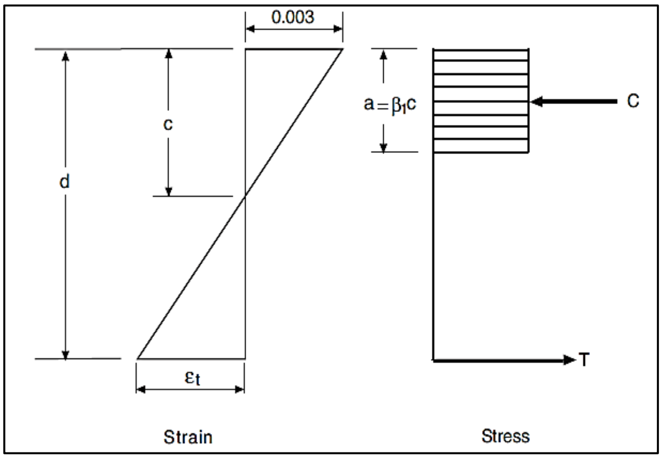
ε_t= Net tensile strain in extreme tension steel and can be calculated from strain diagram

ε_{ty} =Strain of steel bars and can be calculated from fy/Es

$$a = \frac{As\ fy}{0.85\ f_c' b} \ \rightarrow\ a = \beta_1\ c\ \rightarrow c = \frac{a}{\beta_1}$$

Find ε_t from

$$\frac{\varepsilon_t}{(d-c)} = \frac{\varepsilon_c = 0.003}{c}$$

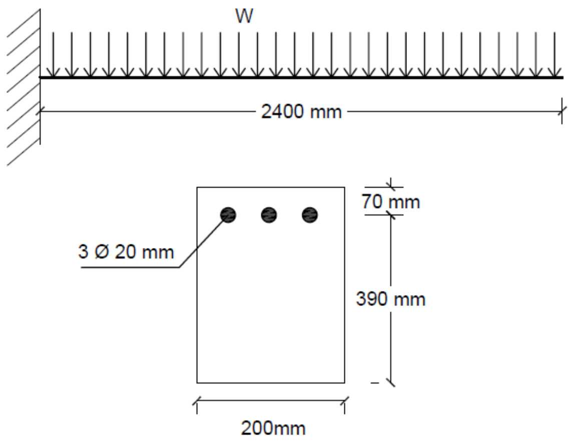


A) If $\varepsilon_{ty} < \varepsilon_t < 0.005\ \ (transition\ \ control)$

$$\phi = 0.65 + 0.25 \frac{(\varepsilon_t - \varepsilon_{ty})}{(0.005 - \varepsilon_{ty})}$$

B) $If\ \varepsilon_t \leq \varepsilon_{ty}\ \ (compression\ \ control)$

$$\phi = 0.65$$



Ex-1: The cantilever R.C beam of detail shown in the figure supports D.L=12 kN/m (include beam weight) and L.L= 10.5 kN/m.

Check the adequacy of the section. Use $\frac{f_c^{`}}{fy} = \frac{28}{420}$ MPa

Solution:

$$W_u = 1.2 \times D.L + 1.6 \times L.L = 31.2\ KN/m$$

$$M_u = \frac{W_u \times l^2}{2} = 89.85\ kN.m$$

$$\rho = \frac{As}{b\ d} = \frac{942.5}{200 \times 390} = 0.0121$$

$$\rho_{min.} = max. of \left\{ \frac{\sqrt{f_c^{`}}}{4\ fy} \quad \frac{1.4}{fy} \right\} = \{0.00315, \qquad 0.0033\}$$

$$\rho_{max.} = 0.85\ \beta_1 \frac{f_c^{`}}{fy} \frac{0.003}{(0.003 + 0.004)} = 0.0206 \qquad , f_c^{`} = 28MPa \rightarrow \beta_1 = 0.85$$

$$\rho_{min.} < \rho < \rho_{max.} \quad ok$$

$$\rho_t = 0.85\ \beta_1 \frac{f_c^{`}}{fy} \frac{\varepsilon_c}{\varepsilon_c + 0.005} = 0.018$$

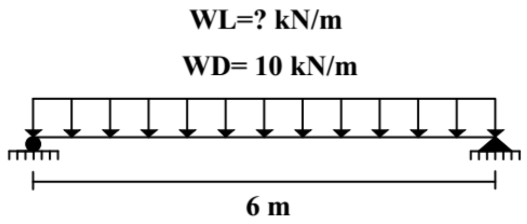
$$\rho_t > \rho \quad \rightarrow \emptyset = 0.9 \qquad (Tension\ section)$$

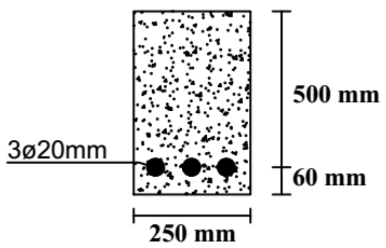
$$\emptyset Mn = \emptyset\ \rho\ bd^2 fy \left(1 - 0.59 \frac{fy}{f_c^{`}}\ \rho\right)$$

$$= 0.9 \times 0.0121 \times 200 \times 390^2 \times 420 \times \left(1 - 0.59 \times \frac{420}{28} \times 0.0121\right) = 124.13\ kN.m$$

$\emptyset Mn (124.13\ kN.m) > M_u\ (89.85\ KN.m)$ The section is adequate

Ex-2: The simply support R.C beam of detail shown in the figure below supports WD=10 kN/m (include beam weight). Find the maximum uniform live load that can be carried by the section. Use $\frac{f_c^`}{fy} = \frac{28}{420}$ MPa





Solution:

$\emptyset Mn \geq M_u$

$$\emptyset Mn = \emptyset\ \rho\ bd^2 fy \left(1 - 0.59 \frac{fy}{f_c^`}\ \rho\right)$$

$$\rho = \frac{As}{b\ d} = \frac{942.5}{250 \times 500} = 0.00754$$

$$\rho_{min.} = max.of \left\{\frac{\sqrt{f_c^`}}{4\ fy} \quad \frac{1.4}{fy}\right\} = \{0.00315, \qquad 0.0033\}$$

$$\rho_{max.} = 0.85\ \beta_1 \frac{f_c^`}{fy} \frac{0.003}{(0.003 + 0.004)} = 0.0206 \qquad , f_c^` = 28MPa\ \rightarrow \beta_1 = 0.85$$

$\rho_{min.} < \rho < \rho_{max.} \quad ok$

$$\rho_t = 0.85\ \beta_1 \frac{f_c^`}{fy} \frac{\varepsilon_c}{\varepsilon_c + 0.005} = 0.018$$

$\rho_t > \rho \quad \rightarrow \emptyset = 0.9 \qquad (Tension\ section)$

$$\emptyset Mn = \emptyset\ \rho\ bd^2 fy \left(1 - 0.59 \frac{fy}{f_c^`}\ \rho\right)$$

$$= 0.9 \times 0.00754 \times 250 \times 500^2 \times 420 \times \left(1 - 0.59 \times \frac{420}{28} \times 0.00754\right) = 166.25\ \mathrm{kN.m}$$

$$\emptyset Mn \geq M_u$$

$$166.25 = M_u$$

$$M_u = \frac{W_u \times l^2}{8} = 166.25\ \mathrm{kN.m}$$

$$W_u = 39.9\ \mathrm{kN/m}$$

$$\mathrm{W_u} = 1.2\ \times 10 + 1.6\ \times \mathrm{L.L} = 39.9\ \mathrm{kN/m}$$

$$\mathrm{L.L} = 15.56\ \mathrm{kN/m}$$

H.W.1: -

For the same Ex-1 above, if the L.L= 20 KN/m, can the beam carry this ultimate load?

H.W.2: -

For the same Ex-1 above, Find the maximum live load can be carried by this beam.

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Design of Reinforced Concrete Structures I

Design of Singly Reinforced Sections-1

Dr Othman Hameed

Lecture (13)

Design of singly reinforced beams

To design beams, the load is given and H, b, and As (ρ) need to be found

1) H and b are known and As is required

a) Find Mu

b) $d = h - 40 - 10 - \frac{d_b}{2}$ (if one layer)

c) Use the equation $Mu \le \phi Mn$ (assume $\phi = 0.9$ to be checked later)

$$Mu = \phi\, \rho\, bd^2 fy \left(1 - 0.59 \frac{fy}{f_c'}\, \rho\right)$$

Solve the above equation to calculate ρ

Use $\rho = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

d) Calculate $\rho_{max.} = 0.85\, \beta_1 \frac{f_c'}{fy} \frac{0.003}{(0.003+0.004)}$

e) Calculate $\rho_{min.} = max.of \begin{cases} \frac{\sqrt{f_c'}}{4\, fy} \\ \frac{1.4}{fy} \end{cases}$

f) check $\rho_{min.} \le \rho \le \rho_{max.}$

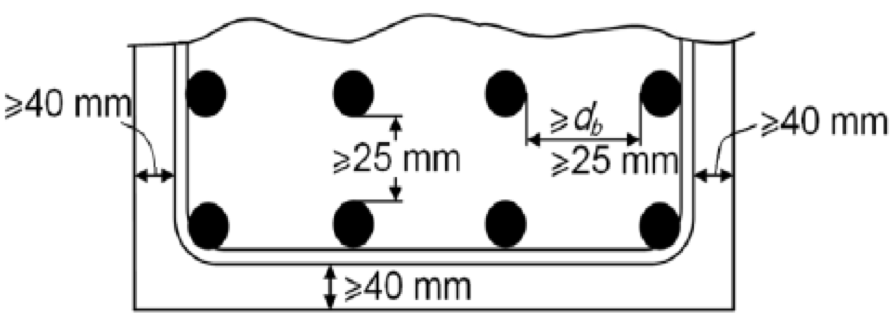
g) check $\rho_{t.} \ge \rho$ (to check the assumption of ϕ is ok)

$A_s = \rho\, b\, d$ $\qquad$ $n = \frac{A_s}{\frac{\pi}{4} {d_b}^2}$

h) Distribute the reinforcement and calculate S

$$s = \frac{b - 2\times 40 - 2\times 10 - n\times d_b}{(n-1)} > max \begin{cases} d_b \\ 25 \end{cases}$$

If not ok, use two layers



2) As (ρ) is available, H and b are unknown

a) Calculate $\rho_{max.} = 0.85\,\beta_1 \frac{f_c^{`}}{fy} \frac{0.003}{(0.003+0.004)}$

b) Calculate $\rho_{min.} = max.of \begin{cases} \frac{\sqrt{f_c'}}{4\,fy} \\ \frac{1.4}{fy} \end{cases}$

c) check $\rho_{min.} \leq \rho \leq \rho_{max.}$

d) Check $\rho_{t.} \geq \rho$

e) Find Mu

f) Use the equation $Mu \leq \phi Mn$

$$Mu = \phi\,\rho\,bd^2 fy\left(1 - 0.59\frac{fy}{f_c'}\,\rho\right)$$

Solve the above equation to find a relation between b and d

g) Assume b and find d (assume b=0.5d)

h) Calculate H

$$H = d + 40 + 10 + \frac{d_b}{2}\ \text{(if one layer)}$$

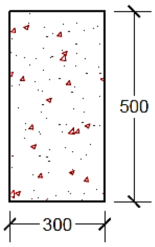
$$H = d + 40 + 10 + d_b + \frac{25}{2}\ \text{(if two layers)}$$

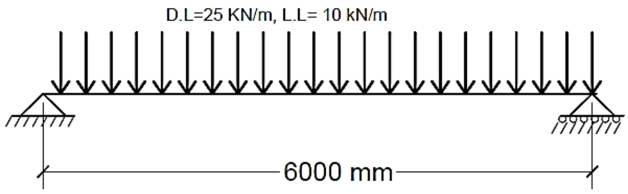
i) Check the calculated H with that of the below table

$H \geq h_{table}$

	Minimum thickness, ***h***			
	Simply supported	One end continuous	Both ends continuous	Cantilever
Beams or ribbed one-way slabs	***ℓ/16***	***ℓ/18.5***	***ℓ/21***	***ℓ/8***

Ex-1: Design the R.C beam of span and details shown in Figure to support the service load of (D.L=25kN/m **not includes beam weight**, L.L= 10kN/m). Use $\frac{f_c^`}{fy} = \frac{28}{420}$ MPa,





(use d_b=20 mm for design)

ملاحظة: عندما يتم ذكر عبارة not including beam weight هذا يعني يجب اضافة وزن العتب للحمل الميت.

Solution:

1- Find the factored load and moment

Beam weight=0.5*0.3*24= 3.6 kN/m

$$w_u = 1.2 \times (25 + 3.6) + 1.6 \times 10 = 50.32\ kN/m$$

$$\mathrm{M_u} = \frac{w_u \times l^2}{8} = \frac{50.32 \times 6^2}{8} = 226.44\ kN.m$$

$$d = h - 40 - 10 - \frac{d_b}{2}$$

$$d = 500 - 40 - 10 - \frac{20}{2} = 440\ mm$$

2- Find ρ and check it

$$\emptyset \mathrm{M_n} \geq \mathrm{M_u}$$

$$\mathrm{M_u} = \emptyset\ \rho\ bd^2 fy \left(1 - 0.59 \frac{fy}{f_c^`}\rho\right)$$

Assume $\phi = 0.9$ (to be checked later)

$$226.44 \times 10^6 = 0.9 \times \rho \times 300 \times 440^2 \times 420 \times \left(1 - 0.59 \frac{420}{28}\rho\right)$$

$$2.156 \times 10^{11}\rho^2 - 2.176 \times 10^{10}\rho + 226.44 \times 10^6 = 0, \qquad \rightarrow \rho = 0.0118$$

$$\rho_{max.} = 0.85\,\beta_1 \frac{f_c^`}{fy} \frac{0.003}{(0.003+0.004)} = 0.0206$$

$$f_c^` = 28MPa \;\rightarrow \beta_1 = 0.85$$

$$\rho_{min.} = max.of \left\{ \frac{\sqrt{f_c^`}}{4\,fy} \quad \frac{1.4}{fy} \right\} = \{0.0031, \qquad 0.0033\}$$

$$\rho_{min.} < \rho < \rho_{max.} \quad ok$$

$$\rho_t = 0.85\,\beta_1 \frac{f_c^`}{fy} \frac{\varepsilon_c}{\varepsilon_c + 0.005} = 0.018 \;>\; \rho = 0.0116 \qquad \rightarrow \emptyset = 0.9$$

3- Calculate area of steel

$$A_s = \rho\, b\, d = 0.0118 \times 300 \times 440 = 1557.6\; mm^2$$

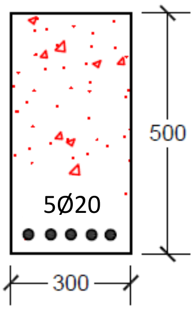
$$n = \frac{A_s}{\frac{\pi}{4} {d_b}^2}$$

$$n = \frac{15557.6}{\frac{\pi}{4} 20^2}$$

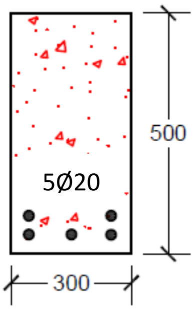
n=4.96, Use 5Ø20 mm

4- Draw the reinforced section

$$s = \frac{300 - 2\times 40 - 2\times 10 - 5\times 20}{(5-1)} = 25\; mm \;>\; d_b\; ok$$



OR



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Design of Reinforced Concrete Structures I

Design of Singly Reinforced Sections-2

Dr Othman Hameed

Lecture (14)

Design of singly reinforced beams

3) As (ρ) , h, d, and b are unknown

a) Assume a relation between b and d (use b=0.5d if not given in the question)
b) Assume a value for ρ (use $\rho = 0.75\ \rho_{max.}$ If not given in the question)
c) Use the equation $Mu \leq \phi Mn$

$$Mu = \phi\, \rho\, bd^2 fy \left(1 - 0.59 \frac{fy}{f'_c}\, \rho\right)$$

Solve the above equation to find b and d
d) Find H

$$H = d + 40 + 10 + \frac{d_b}{2} \text{ (if one layer)}$$

$$H = d + 40 + 10 + d_b + \frac{25}{2} \text{ (if two layers)}$$

e) Check the calculated H with that of the below table

$H \geq h_{table}$

	Minimum thickness, ***h***			
	Simply supported	One end continuous	Both ends continuous	Cantilever
Beams or ribbed one-way slabs	***ℓ/16***	***ℓ/18.5***	***ℓ/21***	***ℓ/8***

f) Use the equation $Mu \leq \phi Mn$ and calculated b and d to find <u>actual</u> ρ

$$Mu = \phi\, \rho\, bd^2 fy \left(1 - 0.59 \frac{fy}{f'_c}\, \rho\right)$$

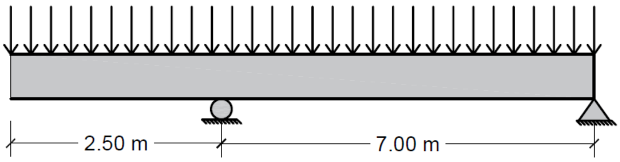
g) check $\rho_{min.} \leq \rho \leq \rho_{max.}$

h) check $\rho_{t.} \geq \rho$ (to check the assumption of ϕ is ok)

$A_s = \rho\, b\, d$ $n = \frac{A_s}{\frac{\pi}{4} {d_b}^2}$

i) Distribute the reinforcement and calculate S

$$s = \frac{b - 2\times40 - 2\times10 - n\times d_b}{(n-1)} > max \begin{cases} d_b \\ 25 \end{cases}$$ If not ok, use two layers



Ex-2: Design the rectangular overhang reinforced concrete beam shown in the figure to carry service loads of (D.L= 30 kN/m includes its own weight, L.L= 18 kN/m).

Assume $\rho = 0.75\ \rho_{max.}$ and $\mathrm{b} = 0.5\mathrm{d}$

Use $\frac{f_c^{`}}{fy} = \frac{25}{400}$ MPa,

(use d_b=25 mm for design)

Solution:

1- Find the factored load & draw S.F.D and B.M.D

Wu= 1.2*D.L+1.6* L.L=64.8 kN/m

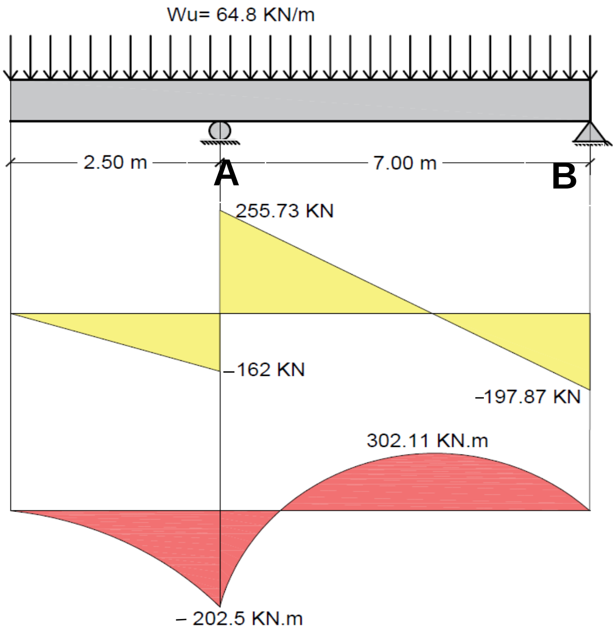
$\Sigma M@B = 0$

$$R_A \times 7 - 64.8 \times 9.5 \times \frac{9.5}{2} = 0$$

$$R_A = 417.73\ kN$$

$$R_A + R_B = 64.8 \times 9.5$$

$$R_B = 197.87$$



2- Design the dimension according to the maximum moment then check with ACI requirement.

Assume $\rho = 0.75\ \rho_{max.}$ & $\mathrm{b} = 0.5\mathrm{d}$

$$\rho_{max.} = 0.85\ \beta_1 \frac{f_c^{`}}{fy} \frac{0.003}{(0.003 + 0.004)} = 0.0193$$

$$\rho = 0.75 \times .0193 = 0.0145$$

$$\rho_t = 0.85\ \beta_1 \frac{f_c^{`}}{fy} \frac{0.003}{0.003 + 0.005} = 0.0169$$

$$\rho_t > \ \rho = 0.0145 \qquad \rightarrow \emptyset = 0.9$$

$\emptyset M_n \geq M_u$

$$M_u = \emptyset\ \rho\ bd^2 fy\left(1 - 0.59\frac{fy}{f_c^`}\ \rho\right)$$

(Assume b=0.5d)

$$302.11 \times 10^6 = 0.9 \times 0.0145 \times (0.5\mathrm{d}) \times d^2 \times 400\left(1 - 0.59\frac{400}{25} \times 0.0145\right)$$

$d = 512mm$

H= d + cover + stirrup + d_b/2 =512+40+10+12.5=575 mm

>$H_{min.}$ = (L/8 for cantilever =312 mm and L/18.5 one end con. =378 mm) ok

b= 0.5 d= 0.5 * 512= 256 mm use b= 250 mm

3- Design **positive** moment (302.11 kN.m)

$M_u \leq \emptyset M_n$

$$M_u = \emptyset\ \rho\ bd^2 fy\left(1 - 0.59\frac{fy}{f_c^`}\ \rho\right)$$

$$302.11 \times 10^6 = 0.9 \times \rho \times 250 \times 512^2 \times 400 \times \left(1 - 0.59\frac{400}{25}\rho\right)$$

$\rho^+ = 0.0149$

$$\rho_{min.} = max.of\left\{\frac{\sqrt{f_c^`}}{4\ fy},\ \frac{1.4}{fy}\right\} = \{0.003125, \quad 0.0035\}$$

$\rho_{min.} < \rho < \rho_{max.} \quad singly$

$$A_s^{\ +} = \rho\ b\ d = 0.0149 \times 250 \times 512 = 1970\ mm^2 \quad \rightarrow use\ 4\emptyset 25\ mm$$

4- Design **negative** reinforcement (202.5 *kN.m*)

$M_u \leq \emptyset M_n$

$$M_u = \emptyset\ \rho\ bd^2 fy\left(1 - 0.59\frac{fy}{f_c^`}\ \rho\right)$$

$$202.5 \times 10^6 = 0.9 \times \rho \times 250 \times 512^2 \times 400 \times \left(1 - 0.59\frac{400}{25}\rho\right)$$

$$\rho^- = 0.00941$$

$$\rho_{min.} < \rho < \rho_{max.} \quad singly$$

$$A_s^- = \rho\, b\, d = 0.0094 \times 250 \times 512 = 1204.5\ mm^2 \quad \rightarrow use\ 3\emptyset 25\ mm$$

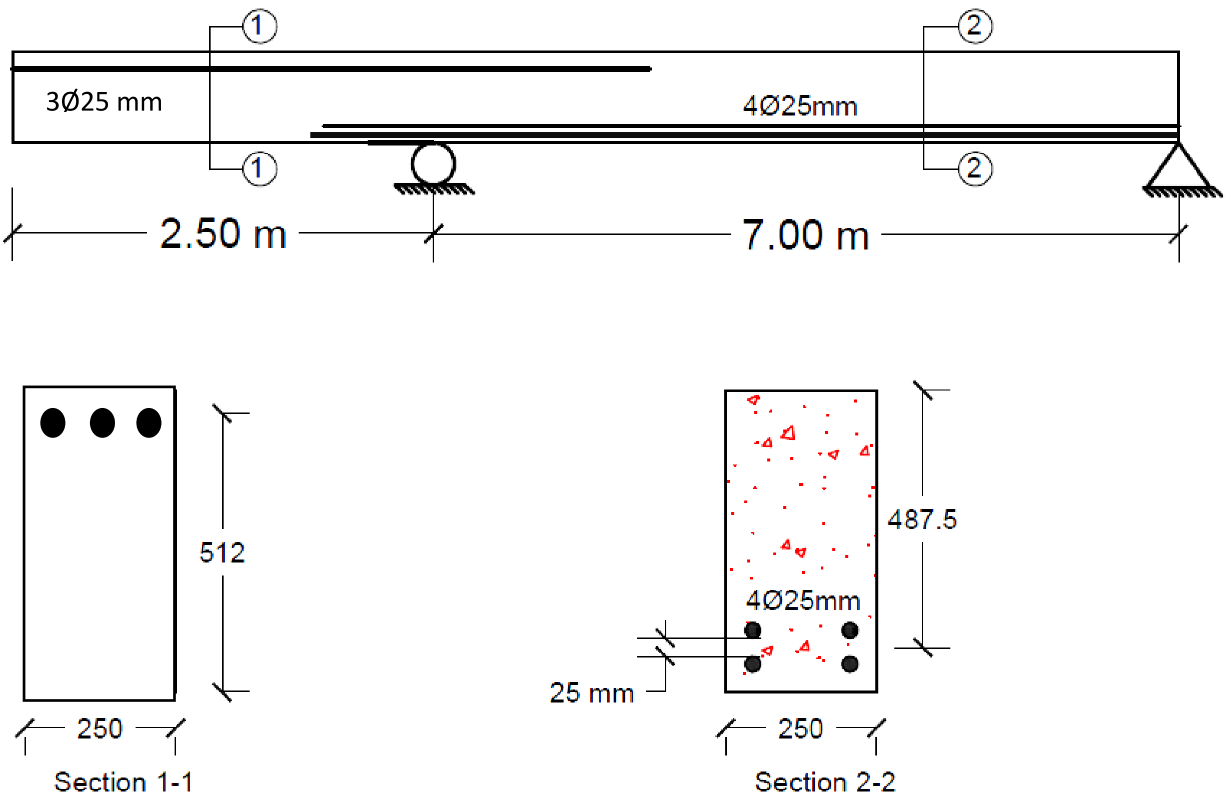
5- Draw detail of steel reinforcement

For positive moment

$$S = \frac{250 - 2 \times 40 - 2 \times 10 - 4 \times 25}{(4-1)} = 16\ mm < d_b(25mm) not\ ok, \qquad use\ two\ layer$$

For negative moment

$$S = \frac{250 - 2 \times 40 - 2 \times 10 - 3 \times 25}{(3-1)} = 37.5\ mm\ > 25\ and > \ d_b(25mm)\ ok$$

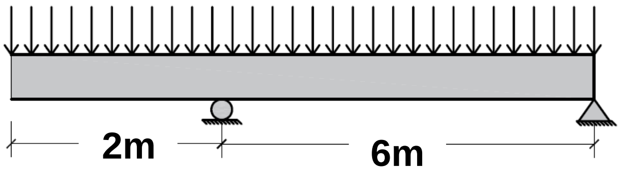


HW-1: Design the rectangular overhang reinforced concrete beam shown in the figure to carry service loads of (D.L= 20 kN/m includes its own weight, L.L= 25 kN/m).

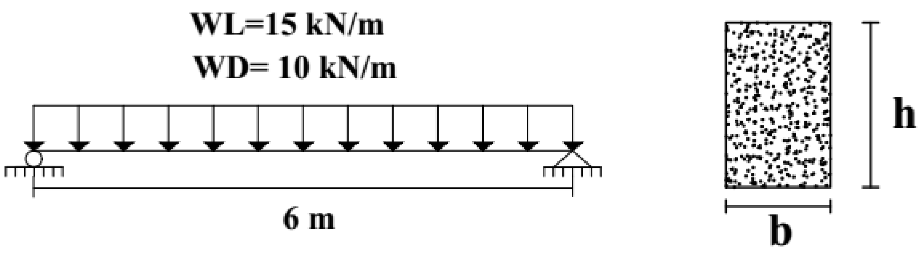
Assume $\rho = 0.6\ \rho_{max.}$ and $\mathrm{b} = 0.5\mathrm{d}$

Use $\frac{f\grave{c}}{fy} = \frac{25}{400}$ MPa,

(use d_b=25 mm for design)



HW-2: Design the rectangular reinforced concrete beam shown in the figure to carry service loads of (D.L= 10 kN/m includes its own weight, L.L= 15 kN/m).



Al Muthanna University
Collage of Engineering

Design of Reinforced Concrete Structures I

Analysis of Doubly Reinforced Sections-1

Dr Othman Hameed

Lecture (15)

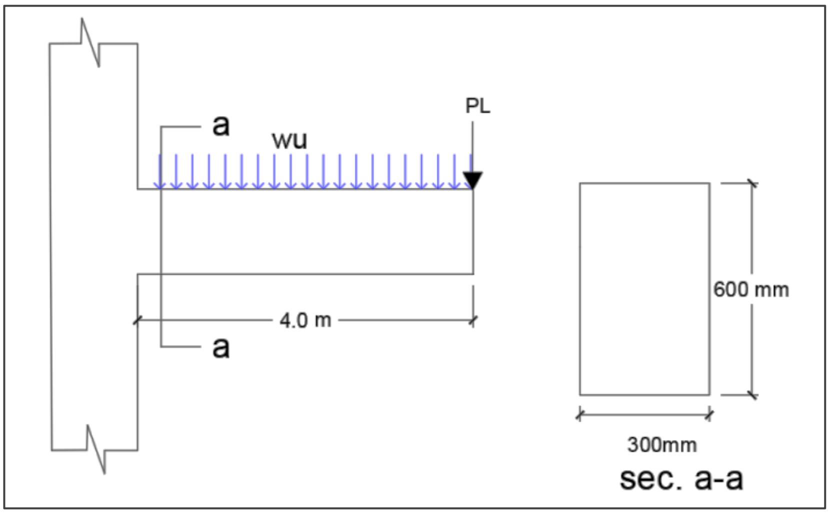
Doubly reinforcement

Analysis of doubly reinforced beams

$\rho > \rho_{max.}$

Example: For the beam of details shown below, design the beam to carry self-weight, live load WL=10 kN/m and concentrated live load PL=40 kN. Use $\frac{f_c'}{fy} = \frac{20}{300}$ MPa. Use d_b=25 mm.

Solution:



$Self\ weight = 0.6 \times 0.3 \times 24$

$= 4.32 KN/m$

$w_u = 1.2 \times 4.32 + 1.6 \times 10$

$= 21.2\ kN/m$

$P_u = 1.6 \times 40 = 64\ \text{KN}$

$$M_u = \frac{w_u \times l^2}{2} + P_u \times l$$

$= 425.6\ \text{kN.m}$

Assume Ø=0.9,

d=600-40-10-12.5=538mm

$\emptyset M_n \geq M_u$

$$M_u = \emptyset\ \rho\ bd^2 fy \left(1 - 0.59 \frac{fy}{f_c'}\ \rho\right)$$

$$0.4256\ (Mn.m) = 0.9 \times \rho \times 0.3 \times 0.538^2 \times 300 \times \left(1 - 0.59 \frac{300}{20} \times \rho\right)$$

$0.4256 = 23.44\ \rho(1 - 8.85\rho)$

$0.4256 = 23.44\ \rho - 207.44\ \rho^2$

$207.44\ \rho^2 - 23.44\ \rho + 0.4256 = 0$

$$\rho = \frac{-B \mp \sqrt{B^2 - 4AC}}{2A}$$

$\rho = 0.09\ or\ \rho = 0.0227 \rightarrow use\ \rho = 0.0227$

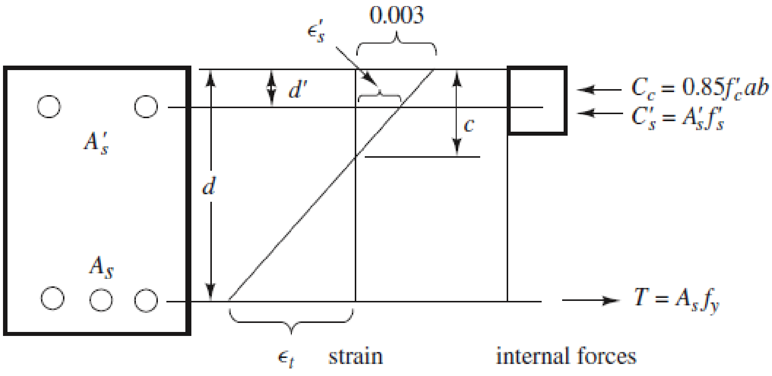
$$\rho_{max.} = 0.85\ \beta_1\ \frac{f_c'}{fy}\ \frac{0.003}{(0.003 + 0.004)} = 0.0206$$

$\rho = 0.0227 > \rho_{max.} = 0.0206 \qquad not\ ok$

To solve this problem

1- Increase the section dimension, or,
2- Use doubly reinforcement.

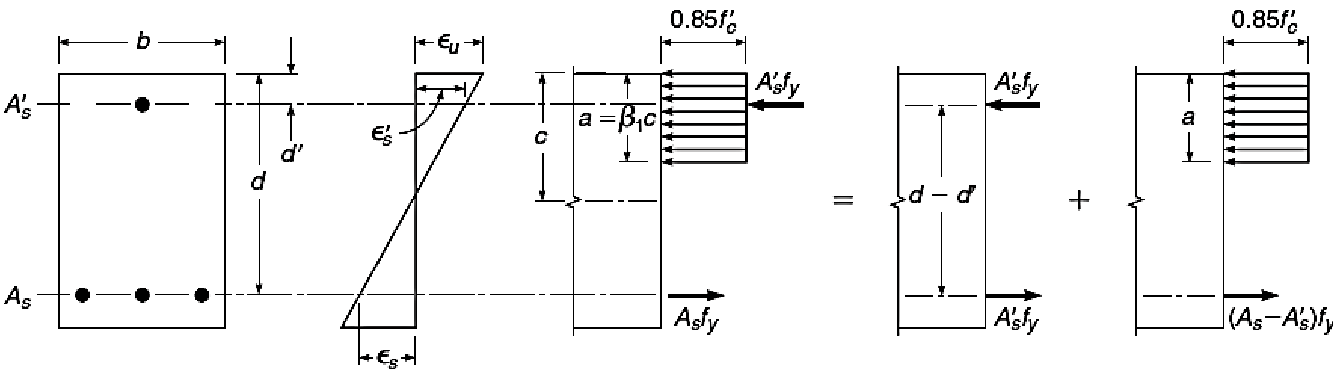
Design of Beam with Compression Steel (Doubly Reinforcement)



The doubly reinforcement is used because:

- Compression steel produces a marked improvement in behavior by raising the amount of compressive strain the concrete to sustain more before crushing.
- To reduce creep and increase ductility.
- To increase the moment capacity of section having limited dimensions
- To reduce the amount of long-term deflection

1- Balance Steel Ratio ρ_b



$$let\ \bar{\rho} = \frac{A_s}{b\ d}\ ,\qquad \rho' = \frac{A_s`}{b\ d}\qquad and\ \ \bar{\rho}_b = \frac{A_{sb}}{b\ d}$$

$$\sum \mathrm{Fx} = 0\,;\quad A_{sb} fy = A_s`\, fs' + 0.85 f_c'\ a\ b\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots.\ \ {\color{red}\div\ b\ d\ fy}$$

$$\bar{\rho}_b = \rho' \frac{fs'}{fy} + 0.85\ \frac{f_c'}{fy}\ \frac{a}{d}\ \ldots\ldots\ldots\ldots\ldots\ldots.{\color{red}(1)}$$

$$a = \beta_1 C$$

From strain diagram

$$\frac{C}{\varepsilon_u} = \frac{d}{\varepsilon_u + \varepsilon_y}\quad \rightarrow C = \frac{\varepsilon_u}{\varepsilon_u + \varepsilon_y}\ d\qquad where\ \ (\varepsilon_u = 0.003\ \ ,\qquad \varepsilon_y = \frac{fy}{Es})$$

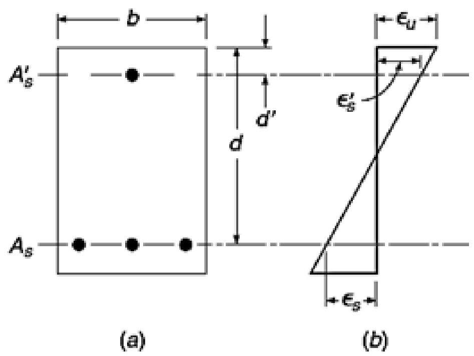
$$C = \frac{0.003}{0.003 + \frac{f_y}{200000}}\ d\ \ = \frac{600}{600 + fy}\ d\quad \ldots\ldots\ldots\ldots\ldots{\color{red}(2)\ sub.in\ (1)}$$

$$\bar{\rho}_b = \rho' \frac{fs'}{fy} + 0.85\ \beta_1\ \frac{f_c'}{fy}\ \frac{600}{600 + fy}$$

$$\bar{\rho}_b = \rho' \frac{fs'}{fy} + \rho_b\qquad\qquad {\color{red}for\ fs' < fy}$$

$$\bar{\rho}_b = \rho' + \rho_b\qquad\qquad {\color{red}for\ fs' =\ fy}$$

2- f_s' at balanced condition and ρ_{max}



$$\frac{\varepsilon_{cu}-\varepsilon s'}{d'}=\frac{\varepsilon_{cu}+\varepsilon_y}{d}$$

$$\varepsilon s'=\varepsilon_{cu}-\frac{d'}{d}(\varepsilon_{cu}+\varepsilon_y)$$

$$fs'=fy \quad \text{if} \quad \varepsilon s'\geq \varepsilon_y$$

$$fs'<fy \quad \text{if} \quad \varepsilon s'<\varepsilon_y$$

$$fs'=E_s\varepsilon s'=E_s\left[\varepsilon_{cu}-\frac{d'}{d}(\varepsilon_{cu}+\varepsilon_y)\right]$$

$$E_s=200{,}000\ MPa \qquad \varepsilon_{cu}=0.003$$

$$fs'=600-\frac{d'}{d}(600+fy) \qquad \rightarrow \ at\ balance\ condition$$

$$\frac{d'}{d}=\frac{600-fs'}{600+fy}$$

$$If\begin{cases}\frac{d'}{d}>0.33 & \rightarrow fs'<fy\\ \frac{d'}{d}\leq 0.33 & \rightarrow fs'=fy\end{cases}\Bigg\}\text{for } f_y=300\ MPa$$

$$If\begin{cases}\frac{d'}{d}>0.2 & \rightarrow fs'<fy\\ \frac{d'}{d}\leq 0.2 & \rightarrow fs'=fy\end{cases}\Bigg\}\text{for } f_y=400\ MPa$$

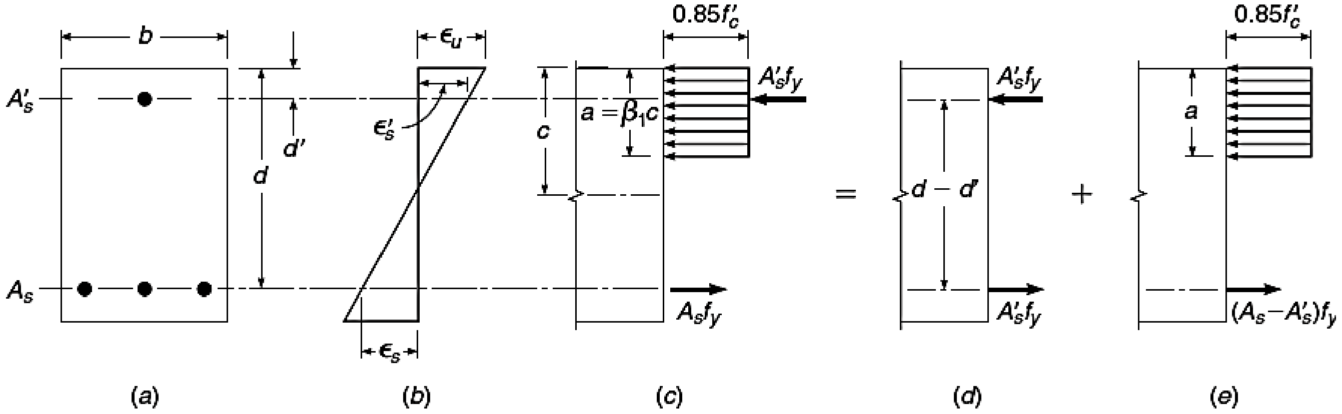
f_s' at ρ_{max}

$$\bar{\rho}_{max.}=0.85\,\beta_1\frac{f_c'}{fy}\frac{\varepsilon u}{\varepsilon u+0.004}+\rho'\frac{fs'}{fy}$$

Same above procedure, but use $\varepsilon_y=0.004$

$$fs'=600-1400\frac{d'}{d} \qquad \rightarrow \ at\ maximum\ steel\ ratio\ condition$$

3- Minimum steel ratio required to insure yielding of compression reinforcement $\bar{\rho}_{min.}$



$$\sum \mathrm{FX} = 0\,; \quad A_{s\,min} \;\; fy = A_s` fy + 0.85 f'_c \; a \; b \;\; \dots\dots\dots\dots\dots\dots\dots. \;\; \div\, b\; d\; fy$$

$$\bar{\rho}_{min} = \rho' + 0.85\, \frac{f'_c}{fy}\, \frac{a}{d} \;\; \dots\dots\dots\dots\dots\dots.\,(1)$$

$$a = \beta_1 C$$

From strain diagram

$$\frac{\varepsilon_u - \varepsilon s'}{d'} = \frac{\varepsilon_u}{c} \quad \rightarrow C = \frac{\varepsilon_u\, d'}{\varepsilon_u - \varepsilon_y} = \frac{600}{600 - fy}\; d' \qquad where \;\; (\varepsilon_u = 0.003 \;, \varepsilon_y = \frac{fy}{Es})$$

$$a = \beta_1 \frac{600}{600 - fy} \;\; d' \;\; \dots\dots\dots\dots\dots\dots\dots.\,(2) \;\; sub.\,in\;(1)$$

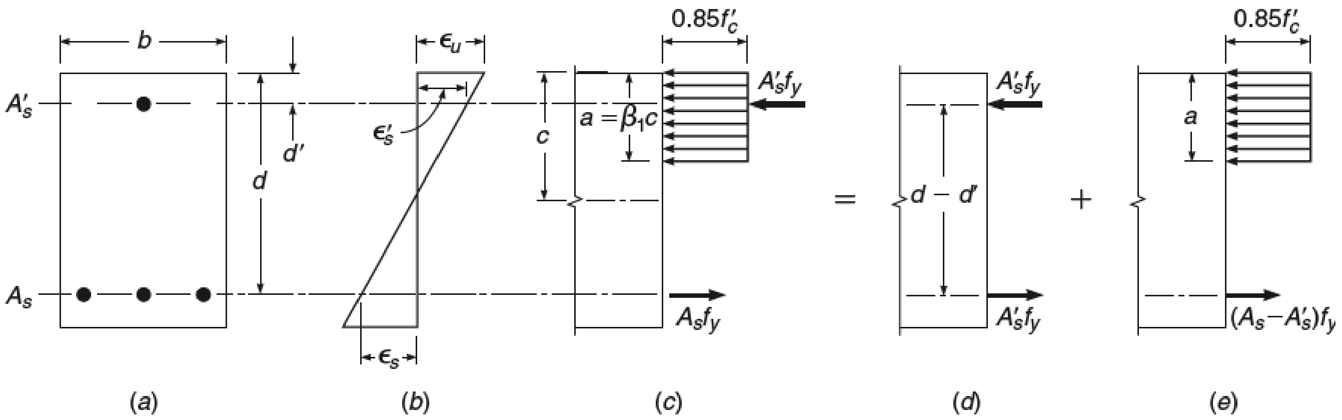
$$\bar{\rho}_{min} = \rho' + 0.85\; \beta_1\, \frac{f'_c}{fy}\, \frac{d'}{d}\, \frac{600}{600 - fy}$$

For Analysis

$$If \begin{cases} \bar{\rho} \geq \bar{\rho}_{min} & \rightarrow fs' = fy \;\; \dots\dots\dots\dots\dots\dots\dots for\; moment\; equation \\ \bar{\rho} < \bar{\rho}_{min} & \rightarrow fs' < fy \;\dots\dots\dots\dots\dots\dots\dots for\; moment\; equation \end{cases}$$

$$\bar{\rho} = \frac{A_s}{b\; d}$$

4- Bending moment capacity of doubly reinforced rectangular section



1- $fs' = fy$

$\emptyset M_n \geq M_u$

$$\emptyset M_n = \emptyset\left[0.85 f'_c\ a\ b\left(d - {}^{a}/_{2}\right) + As' fy(d - d')\right]$$

$$\sum \text{Fx} = 0\,; \quad A_s fy = As' fy + 0.85 f'_c\ a\ b \quad \rightarrow a = \frac{(As - As')fy}{0.85 f'_c b}$$

2- $fs' < fy$

$\emptyset M_n \geq M_u$

$$\emptyset M_n = \emptyset\left[0.85 f'_c\ a\ b\left(d - {}^{a}/_{2}\right) + As' fs'(d - d')\right]$$

$$\sum \text{FX} = 0\,; \quad A_s fy = As' fs' + 0.85 f'_c\ a\ b \quad \rightarrow a = \frac{(Asfy - As'fs')}{0.85 f'_c b} \quad \dots\dots\dots\dots\dots..(1)$$

$a = \beta_1 C$

From strain diagram

$$\frac{\varepsilon_u - \varepsilon s'}{d'} = \frac{\varepsilon_u}{c} \quad \rightarrow C = \frac{\varepsilon_u\, d'}{\varepsilon_u - \varepsilon s'} = \frac{600}{600 - fs'}\ d' \qquad where \left(\varepsilon_u = 0.003\ , \varepsilon s' = \frac{fs'}{Es}\right)$$

$$a = \beta_1 \frac{600}{600 - fs'}\ d' \quad \dots\dots\dots\dots\dots(2)$$

Use 1=2 and find a and fs'

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Design of Reinforced Concrete Structures I

Analysis of Doubly Reinforced Sections-2

Dr Othman Hameed

Lecture (16)

Doubly reinforcement

Analysis of doubly reinforced beams-2

Analysis Procedure for Doubly Reinforced Rectangular Section

$$\bar{\rho} = \rho = \frac{As}{b\,d} \quad and \qquad \rho' = \frac{As'}{b\,d}$$

1- Check $\begin{cases} \bar{\rho} \ \leq \ \rho_{max.} = 0.85\,\beta_1 \frac{f_c'}{fy}\,\frac{3}{7} & \rightarrow singly\,reinforcement \\ \bar{\rho} \ > \ \rho_{max.} = 0.85\,\beta_1 \frac{f_c'}{fy}\,\frac{3}{7} & \rightarrow doubly\;\;reinforcement \end{cases}$

2- Check $\bar{\rho} \ \leq \bar{\rho}_{max.} = 0.85\,\beta_1 \frac{f_c'}{fy}\,\frac{\varepsilon u}{\varepsilon u + 0.004} + \rho' \frac{fs'}{fy}$

Use $\varepsilon u = 0.003$

$$f_s' = 600 - 1400\frac{d'}{d} \leq fy \ldots\ldots\ldots\ldots\ldots\ldots..(\text{for } \bar{\rho}_{max.}\ equation\ only)$$

3- Check $\bar{\rho}_{min} \ = \rho' + 0.85\,\beta_1 \frac{f_c'}{fy}\,\frac{d'}{d}\,\frac{600}{600-fy}$

A- If $\bar{\rho} \ \geq \ \bar{\rho}_{min} \quad \ldots\ldots\ldots\ldots\ldots\ldots \qquad f_s' = fy$

$$\emptyset M_n = \emptyset\left[0.85 f_c'\,a\,b\left(d - {}^{a}/_{2}\right) + As' fy(d - d')\right]$$

$$a = \frac{(As - As')fy}{0.85 f_c' b}$$

B- If $\bar{\rho} \ < \ \bar{\rho}_{min} \quad \ldots\ldots\ldots\ldots\ldots\ldots \qquad f_s' < fy$

$$\emptyset M_n = \emptyset\left[0.85 f_c'\,a\,b\left(d - {}^{a}/_{2}\right) + As' fs'(d - d')\right]$$

$$\begin{cases} a = \dfrac{(Asfy - As'fs')}{0.85 f_c' b} \\ a = \beta_1 \dfrac{600}{600 - fs'} \quad d' \end{cases} \qquad solve\ and\ find\ (a)\ \&\ (fs')$$

Strength reduction factor

$$\rho'_t = 0.85\,\beta_1\,\frac{f_c'}{fy}\,\frac{\varepsilon_{cu}}{\varepsilon_{cu}+0.005} + \rho' = \rho_t + \rho' \qquad \text{(for } f_s' = fy\text{)}$$

$$\rho'_t = 0.85\,\beta_1\,\frac{f_c'}{fy}\,\frac{\varepsilon_{cu}}{\varepsilon_{cu}+0.005} + \rho'\,\frac{fs'}{fy} = \rho_t + \rho'\,\frac{fs'}{fy} \qquad \text{(for } f_s' < fy\text{)}$$

If $\rho'_t \geq \overline{\rho} \rightarrow \emptyset = 0.9$

If $\rho'_t < \overline{\rho} \rightarrow \emptyset\ must\ be\ calculated$

To calculate $\emptyset$, the values of ε_t and ε_{ty} must be determined

ε_{ty} = fy/Es

Find ε_t from

$$\frac{\varepsilon_t}{(d-c)} = \frac{0.003}{c}$$

$$c = \frac{a}{\beta_1}$$

$$a = \frac{(As - As')fy}{0.85 f_c' b}$$

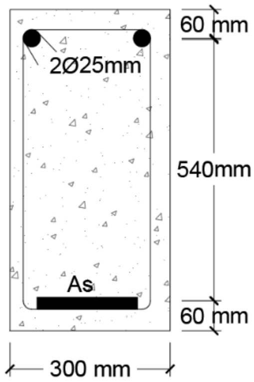
A) If $\varepsilon_{ty} < \varepsilon_t < 0.005$ $(transition\ \ control)$

$$\phi = 0.65 + 0.25\frac{(\varepsilon_t - \varepsilon_{ty})}{(0.005 - \varepsilon_{ty})}$$

B) $If\ \varepsilon_t \leq \varepsilon_{ty}$ $(compression\ \ control)$

$$\phi = 0.65$$

Ex-1: A rectangular beam of section and details shown in figure, what is the maximum moment can be carried by this beam if:



Section

1- $As = 2413\ mm^2$ (Bottom reinforcement) $As = 3\emptyset 32\ mm$

2- $As = 4826\ mm^2$(Bottom reinforcement) $As = 6\emptyset 32\ mm$

Use $\frac{f_c'}{fy} = \frac{28}{400}$ MPa .

Solution:

Case 1, $As = 3\emptyset 32\ mm = 2413\ mm^2$

$$\rho' = \frac{As'}{b\ d} = \frac{982}{300 \times 600} = 0.00545$$

$$\bar{\rho} = \frac{AS}{b\ d} = \frac{2413}{300 \times 600} = 0.0134$$

$$\rho_{max.} = 0.85\ \beta_1\ \frac{f_c'}{fy}\ \frac{0.003}{(0.003 + 0.004)} = 0.0216$$

$$\bar{\rho} < \rho_{max.} \qquad Singly\ Reinforcement$$

$$\rho_{min.} = max.of\ \left\{\frac{\sqrt{f_c'}}{4\ fy} \quad \text{and} \quad \frac{1.4}{fy}\right\} = \{0.0033, \mathbf{0.0035}\}$$

$$\rho_{min.} < \bar{\rho} < \rho_{max.} \qquad under\ Reinforcement$$

$$\rho_t = 0.85\ \beta_1 \frac{f_c'}{fy}\ \frac{\varepsilon_c}{\varepsilon_c + 0.005} = 0.0189 > \ \bar{\rho} = 0.0134 \quad \rightarrow \emptyset = 0.9$$

$Calculate\ \ \emptyset \mathrm{M_n}$

$$\emptyset M_n = \emptyset\ \rho\ bd^2 fy\ \left(1 - 0.59 \frac{fy}{f_c'}\ \rho\right)$$

$$\emptyset M_n = 0.9 \times 0.0134 \times 300 \times 600^2 \times 400 \times \left(1 - 0.59 \frac{400}{28}\ 0.0134\right) \times 10^{-6}$$

$$= 462.15\ kN.m$$

Case 2, $As = 6\emptyset 32\ mm = 4826\ mm^2$

$$\bar{\rho} = \frac{4826}{300 \times 600} = 0.0268 > \rho_{max.} = 0.0216 \quad \rightarrow \text{Doubly Reinforcement}$$

$$\bar{\rho}_{max.} = \rho_{max.} + \rho' \frac{fs'}{fy} \quad , \quad fs' = 600 - 1400\frac{d'}{d} = 460 > fy \quad \rightarrow \quad \text{use } fs' = fy$$

$$\bar{\rho}_{max.} = 0.0216 + 0.0055 = 0.0271$$

$$\bar{\rho} < \bar{\rho}_{max.} \qquad under\ Reinforcement$$

$$\bar{\rho}_{min} = \rho' + 0.85\ \beta_1 \frac{f'_c}{fy} \frac{d'}{d} \frac{600}{600-fy}$$

$$=0.0055 + 0.85 \times 0.85 \times \frac{28}{400} \times \frac{60}{600} \times \frac{600}{600-400} = 0.0206$$

$$\bar{\rho} = 0.0268 > \bar{\rho}_{min} = 0.0206 \qquad \rightarrow fs' = fy$$

$$\rho'_t = 0.85\ \beta_1 \frac{f'_c}{fy} \frac{\varepsilon_c}{\varepsilon_c + 0.005} + \rho' = 0.0189 + 0.00545 = 0.0244 < \bar{\rho} = 0.0268$$

$$\bar{\rho} > \rho'_t \rightarrow \emptyset\ must\ be\ calculated$$

To calculate $\emptyset$, the values of ε_t must be calculated and compered with ε_{ty} and 0.005

ε_{ty} = fy/Es = 400/200000 =0.002

$$a = \frac{(As - As')fy}{0.85 f'_c b} = 215.35\ mm$$

$$a = \beta_1 c \rightarrow c = \frac{a}{\beta_1} = 253.35mm\ (to\ be\ used\ to\ calculate\ \varepsilon_t)$$

$$\frac{\varepsilon_t}{(d-c)} = \frac{\varepsilon_c}{c} \rightarrow \varepsilon_t = 0.0041$$

$$\varepsilon_{ty} < \varepsilon_t < 0.005\ \ (transition\ \ control)$$

$$\phi = 0.65 + 0.25 \frac{(\varepsilon_t - \varepsilon_{ty})}{(0.005 - \varepsilon_{ty})}$$

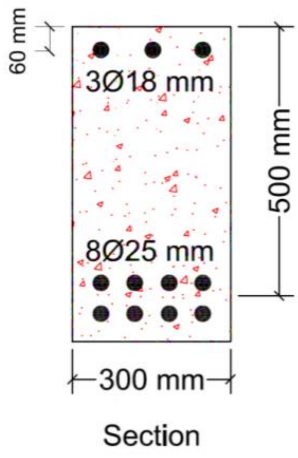
$$\emptyset = 0.65 + 0.25 \times \frac{0.0041 - 0.002}{0.005 - 0.002} = 0.825$$

$$\emptyset M_n = \emptyset\left[0.85 f'_c\ a\ b\ \left(d - {}^{a}/_{2}\right) + As' fy (d - d')\right]$$

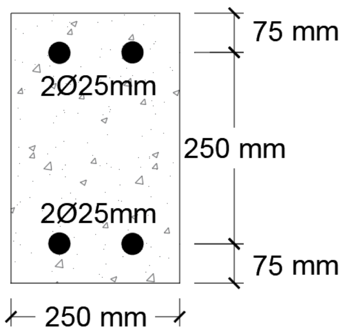
$$\emptyset M_n = 0.825\left[0.85 \times 28 \times 215.35 \times 300\ \left(600 - {}^{215.35}/_{2}\right) + 982 \times 400(600 - 60)\right] \times 10^{-6} = 799.5\ kN.m$$

Analysis

1- By ultimate strength design method, compute the maximum bending positive moment can be applied on the section shown in the figure. Let $fy = 400\ Mpa,\ f_c' = 30\ Mpa,$



2- Estimate the flexural design strength $\emptyset M_n$ of the cross section in the figure. Use $fy = 400\ Mpa\ and\ f_c' = 21\ Mpa.$



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Design of Reinforced Concrete Structures I

Design of Doubly Reinforced Sections

Dr Othman Hameed

Lecture (17)

Design of Doubly Reinforcement Rectangular Section

Procedure of design

1- Design the section as a singly reinforcement and find ρ (use the same procedure given for singly reinforcement $M_u = \phi\, \rho\, bd^2 fy \left(1 - 0.59 \frac{fy}{fc'}\, \rho\right)$)

2- If $\rho > \rho_{max.}$ $\rightarrow compression\ \ steel\ is\ required$ (doubly reinforcement)

3- Design A_{s1} from the maximum steel ratio (use $A_{s1} = \rho_{max.} bd$)

4- $M_u = \emptyset[M_{n1} + M_{n2}]$

5- Calculate $\emptyset M_{n1} = \phi\, \rho_{max}\, bd^2 fy \left(1 - 0.59 \frac{fy}{f_c'}\, \rho_{max}\right)$

To calculate ϕ, use $\varepsilon_t = 0.004$ and calculate ϕ from $\phi = 0.65 + 0.25 \frac{(\varepsilon_t - \varepsilon_{ty})}{(0.005 - \varepsilon_{ty})}$

6- $\emptyset M_{n2} = M_u - \emptyset M_{n1}$

7- $f_s' = \varepsilon_s' E = \left(\frac{C-d'}{C}\right) \times 600 \ \le fy$

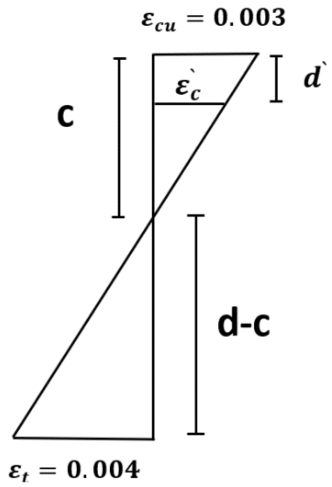
$$\frac{0.007}{d} = \frac{0.003}{c}$$

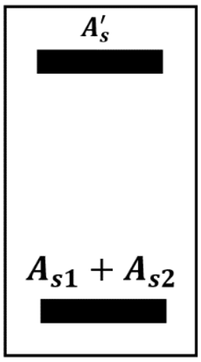
$$C = \frac{3}{7}\, d$$

8- $\emptyset M_{n2} = \emptyset As' f_s' (d - d')$ (find As')

9- $As_2 = As' \frac{f_s'}{fy}$

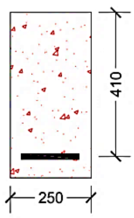
10- $As = As_1 + As_2 = \rho_{max.} bd + As_2$

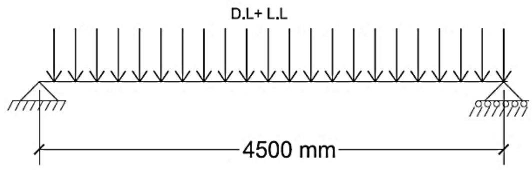




==

Ex-1: Design the beam of the section and detail shown in the below figure to carry service dead load =25kN/m (including beam weight) and service live load= 35 kN/m. Use:$f_c' = 20Mpa, fy = 400\ Mpa\ and\ d' = 60\ mm\ if\ its\ required$
(Use d_b=16 mm for compression and d_b=25 mm for tension reinforcement)





Find the factored moment and calculate the steel reinforcement ratio ρ as a singly reinforcement

$$w_u = 1.2 \times D.L + 1.6 \times L.L = 86\ kN/m$$

$$\mathrm{M_u} = \frac{w_u \times l^2}{8} = 217.7\ kN.m$$

$$\mathrm{M_u} = \emptyset\, \rho\, bd^2 fy \left(1 - 0.59 \frac{fy}{f_c'} \rho\right) \qquad let\ \emptyset = 0.9$$

$$0.2177 = 0.9\, \rho\, 0.25 \times 0.41^2 \times 400 \left(1 - 0.59 \frac{400}{20} \rho\right) \quad \rightarrow \rho = 0.0182$$

Check the steel reinforcement ratio limits

$$\rho_{max.} = 0.85\, \beta_1 \frac{f_c'}{fy} \frac{0.003}{(0.003 + 0.004)} = 0.0154 \quad \rightarrow \rho = 0.0182 > \rho_{max.} = 0.0154$$

$$\therefore doubly\ reinforcement\ and\ no\ need\ to\ check\ \rho_t\ (the\ assumption\ of \emptyset)$$

Calculate $\phi \mathrm{M_{n1}}\ from\ \ \rho = \rho_{max.}$

To calculate ϕ, use $\varepsilon_t = 0.004$ and calculate ϕ

$$\varepsilon_{ty} = \frac{fy}{200000} = 0.002$$

$$\phi = 0.65 + 0.25 \frac{(\varepsilon_t - \varepsilon_{ty})}{(0.005 - \varepsilon_{ty})} = 0.82$$

$$\emptyset\, \mathrm{M_{n1}} = \emptyset\, \rho_{max.}\, bd^2 fy \left(1 - 0.59 \frac{fy}{f_c'} \rho_{max.}\right) = 173.7\ kN.m$$

$$M_u = \emptyset[M_{n1} + M_{n2}] \rightarrow \quad \emptyset M_{n2} = M_u - \emptyset M_{n1}$$

Calculate $\emptyset M_{n2}$

$$\emptyset M_{n2} = 217.7 - 173.7 = 44\ kN.m$$

Calculate f_s'

$$f_s' = \left(\frac{C - d'}{C}\right) \times 600 \ \leq fy$$

$$C = \frac{3}{7}\ d = \frac{3}{7} \times 410 = 175.7\ mm$$

$$f_s' = \left(\frac{175.7 - 60}{175.7}\right) \times 600 = 395.1\ Mpa\ < fy = 400\ \ MPa$$

Calculate As'

$$\emptyset M_{n2} = \emptyset As' f_s'(d - d')\ ; \quad 44 \times 10^6 = 0.82 \times As' 395.1 \times (410 - 60)$$

$$As' = 388\ mm^2 \ \ (\text{compression reinforcement})$$

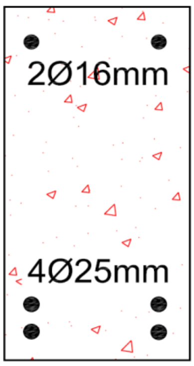
Use 2ø 16 mm

Tension reinforcement $=\rho_{max.} \times b \times d + As' \times \frac{f_s'}{fy}$

$$= 0.0154 \times 250 \times 410 + 388 \times \frac{395.1}{400} = 1962\ \ mm^2 \ \ \ ues\ 4\emptyset 25\ mm$$

$$s = \frac{250 - 2 \times 40 - 2 \times 10 - 4 \times 25}{4 - 1} = 16\ mm\ \ < d_b(25\ mm) not\ ok$$

$$\therefore use\ two\ layer$$



HomeWorks

Design

1- The R.C beam having span =8 m, H= 0.6 m and b=0.3 m subjected to a concentrated factored load at mid span of 300 kN, neglect the self-weight and design the beam for flexural.

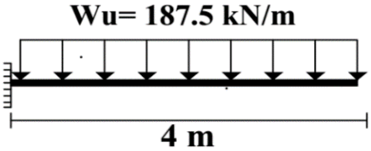
Let $fy = 400\ Mpa,\ f_c' = 30\ Mpa, d = 500\ mm\ and\ d' = 60\ mm$ (if its required)

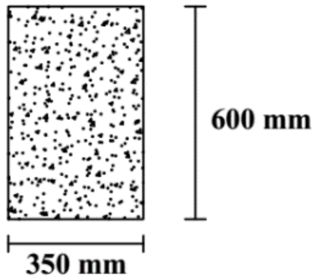
2- Design the necessary flexural steel reinforcement.

Let $fy = 400\ Mpa,\ f_c' = 35\ Mpa,$

$d = 520$ mm and $d' = 60$ mm (if its required)

Use db=25 mm





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Design of Reinforced Concrete Structures I

Simplified Method to calculate moment and shear-1

Dr Othman Hameed

Lecture (18)

Simplified Method of Analysis for Non-prestressed Continuous Beams and One-Way Slabs ACI 6.5

The approximate moments and shears give reasonable values for the stated conditions if the continuous beams and one-way slabs are part of a frame or continuous construction. Because the load patterns that produce critical values for moments in columns of frames differ from those for maximum negative moments in beams, column moments should be evaluated separately.

To apply this method, the conditions (a) through (e) need to be achieved:

(a) Members are prismatic

(b) Loads are uniformly distributed

(c) Live load ≤ 3 Dead load ($\frac{LL}{DL} \leq 3$)

(d) There are at least two spans

(e) The longer of two adjacent spans does not exceed the shorter by more than 20 percent. $\frac{long\ span - short\ span}{short\ span} \times 100\% \leq 20\ \%$

Moment (Mu) due to gravity loads shall be calculated in accordance with ACI-Table 6.5.2.

Table 6.5.2—Approximate moments for nonprestressed continuous beams and one-way slabs

Moment	Location	Condition	M_u
Positive	End span	Discontinuous end integral with support	$w_u\ell_n^2/14$
		Discontinuous end unrestrained	$w_u\ell_n^2/11$
	Interior spans	All	$w_u\ell_n^2/16$
Negative[1]	Interior face of exterior support	Member built integrally with supporting spandrel beam	$w_u\ell_n^2/24$
		Member built integrally with supporting column	$w_u\ell_n^2/16$
	Exterior face of first interior support	Two spans	$w_u\ell_n^2/9$
		More than two spans	$w_u\ell_n^2/10$
	Face of other supports	All	$w_u\ell_n^2/11$
	Face of all supports satisfying (a) or (b)	(a) slabs with spans not exceeding 3 m (b) beams where ratio of sum of column stiffnesses to beam stiffness exceeds 8 at each end of span	$w_u\ell_n^2/12$

[1]To calculate negative moments, ℓ_n shall be the average of the adjacent clear span lengths.

Shear (Vu) due to gravity loads shall be calculated in accordance with Table 6.5.4.

Table 6.5.4—Approximate shears for nonprestressed continuous beams and one-way slabs

Location	V_u
Exterior face of first interior support	$1.15w_u\ell_n/2$
Face of all other supports	$w_u\ell_n/2$

End Reactions

Reactions to a supporting beam, column or wall are obtained as the sum of shear forces acting on the sides of the support.

Integral and unrestrained supports are illustrated in Figure 1.

Exterior and interior supports are shown in Figure 2.

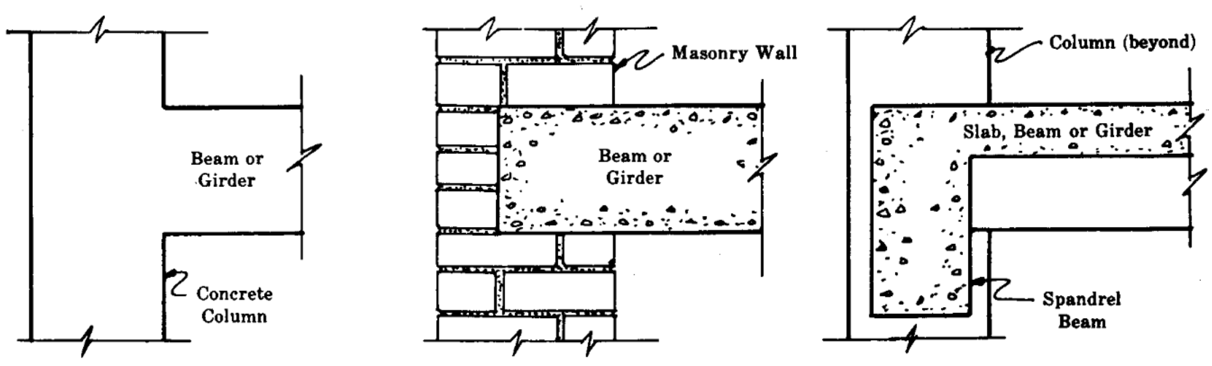


Figure 1

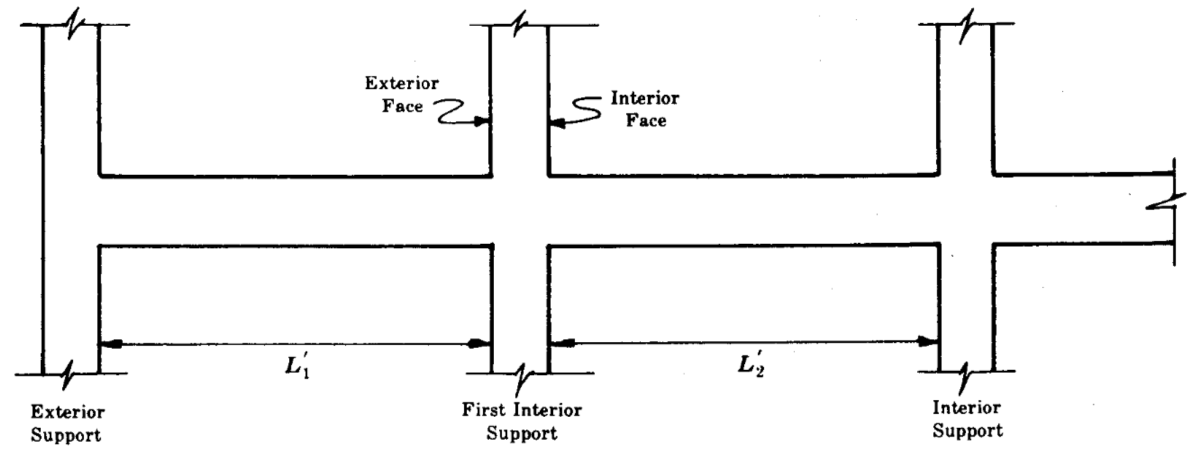
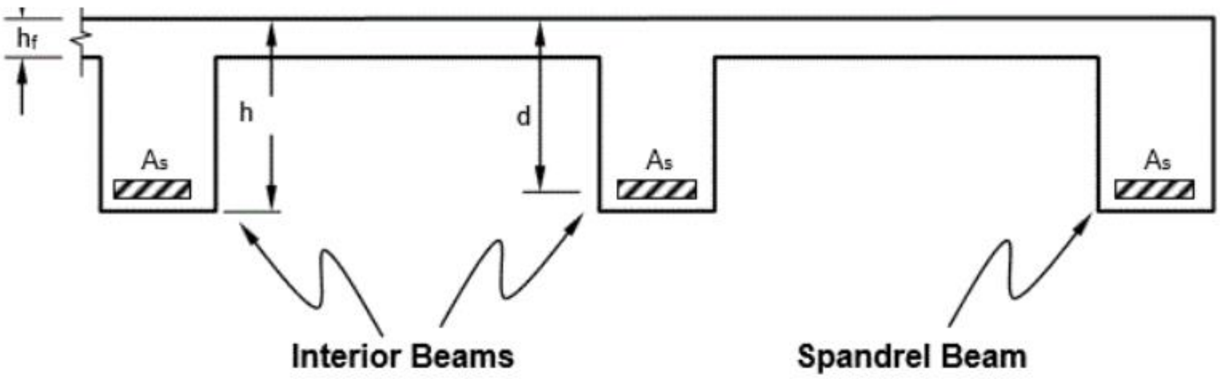
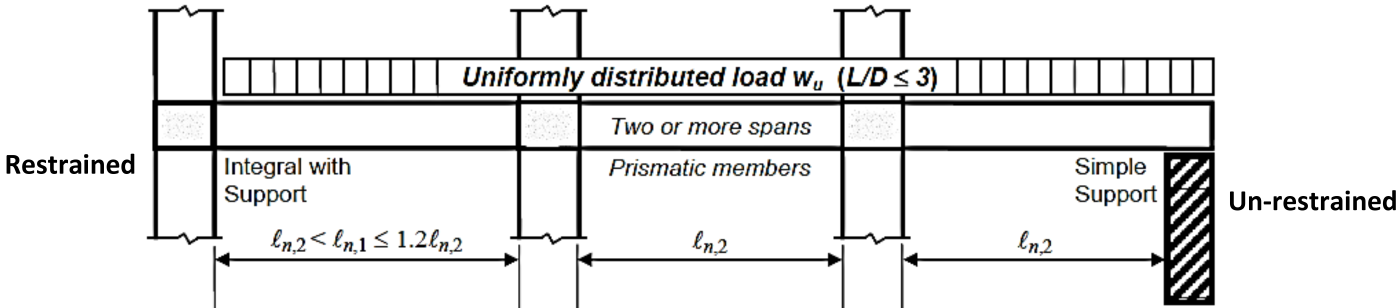


Figure 2



Example



	$\frac{w_u\ell_{n,1}^2}{14}$		$\frac{w_u\ell_{n,2}^2}{16}$		$\frac{w_u\ell_{n,2}^2}{11}$		**Positive Moment**
Spandrel Support	$\frac{w_u\ell_{n,1}^2}{24}$	$\frac{w_u\ell_{n,avg}^2}{10}$ *	$\frac{w_u\ell_{n,avg}^2}{11}$	$\frac{w_u\ell_{n,2}^2}{11}$	$\frac{w_u\ell_{n,2}^2}{10}$ *	0	**Negative Moment**
Column Support	$\frac{w_u\ell_{n,1}^2}{16}$						
Note A	$\frac{w_u\ell_{n,1}^2}{12}$	$\frac{w_u\ell_{n,avg}^2}{12}$	$\frac{w_u\ell_{n,avg}^2}{12}$	$\frac{w_u\ell_{n,2}^2}{12}$	$\frac{w_u\ell_{n,2}^2}{12}$ **	0	
	$\frac{w_u\ell_{n,1}}{2}$	$\frac{1.15w_u\ell_{n,1}}{2}$	$\frac{w_u\ell_{n,2}}{2}$	$\frac{w_u\ell_{n,2}}{2}$	$\frac{1.15w_u\ell_{n,2}}{2}$	$\frac{w_u\ell_{n,2}}{2}$	**Shear**

* $\frac{w_u\ell_n^2}{9}$ (2 spans)

$$\ell_{n,avg} = \frac{\ell_{n,1} + \ell_{n,2}}{2} \quad (8.3.3)$$

** $\frac{w_u\ell_{n,2}^2}{10}$ (for beams)

Note A: Applicable to slabs with spans $\le$ 10 ft and beams where the ratio of the sum of column stiffness to beam stiffness > 8 at each end of the span.

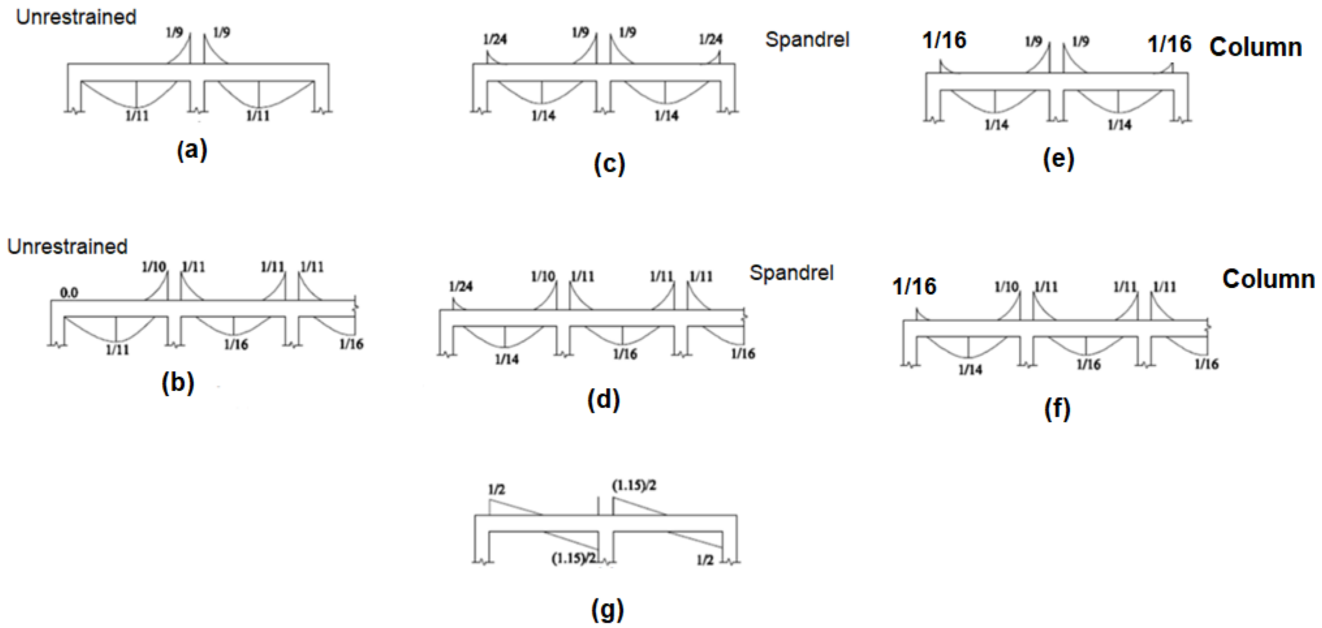


Figure: (a) two spans, exterior edge unrestrained; (b) more than two spans, exterior edge unrestrained; (c) two spans, support is spandrel beam; (d) more than two spans, support is spandrel beam; (e) two spans, support is column; (f) more than two spans, support is column; (g) shear force for two or more spans

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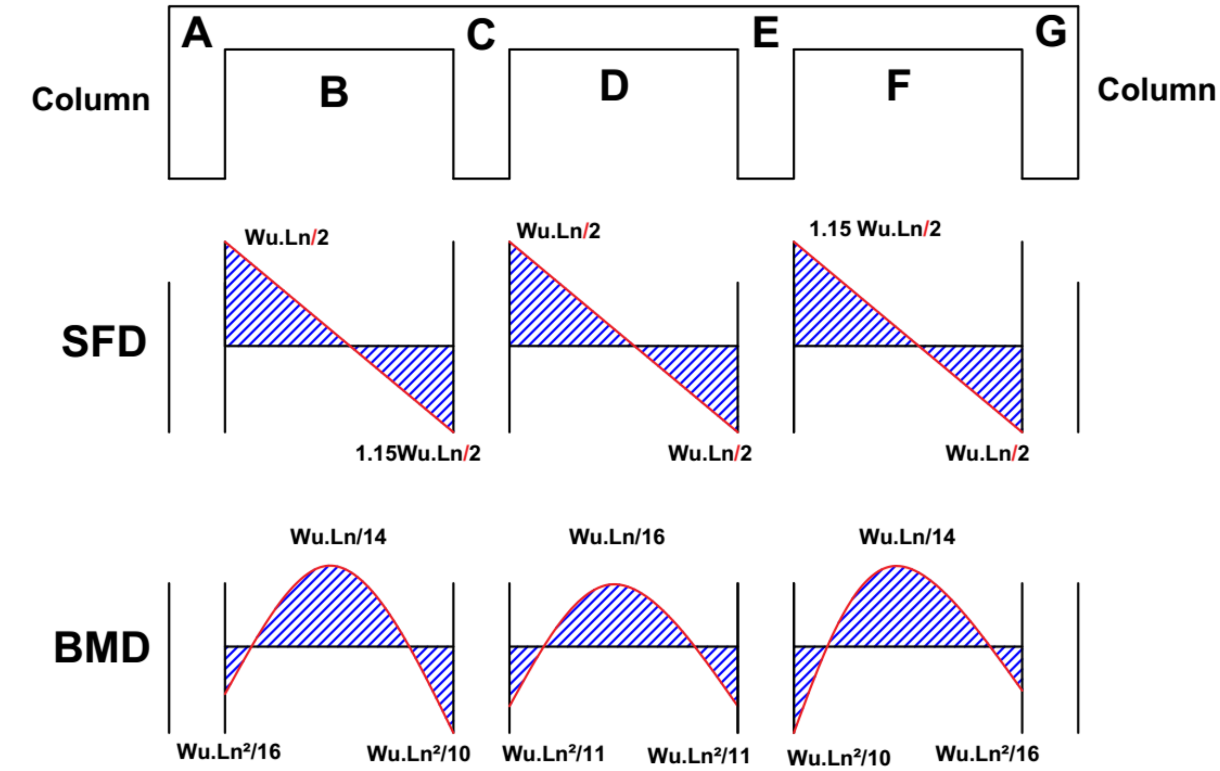
Design of Reinforced Concrete Structures I

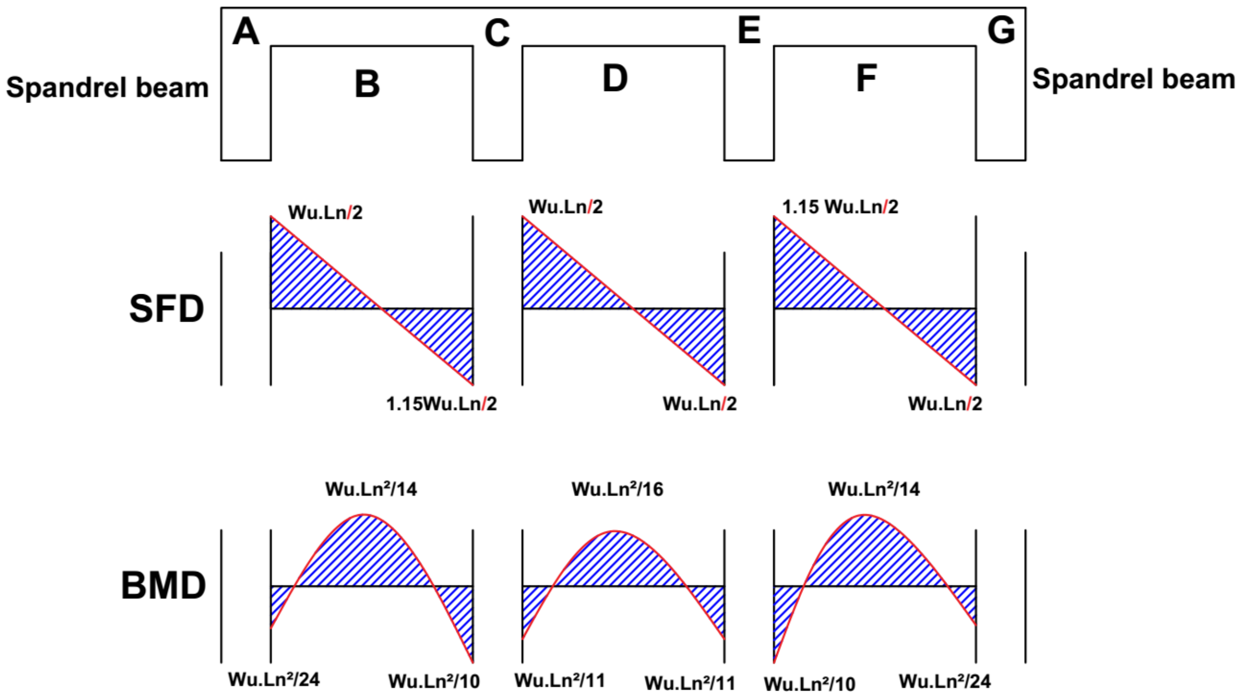
Simplified Method to calculate moment and shear-2

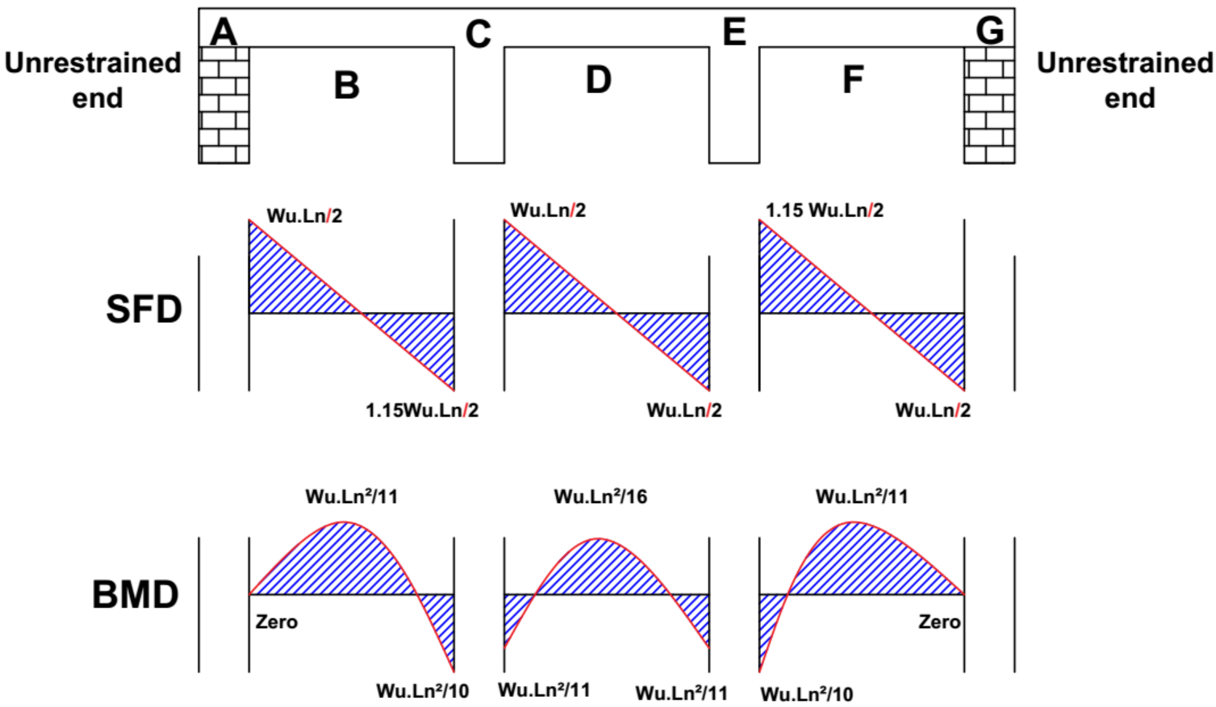
Dr Othman Hameed

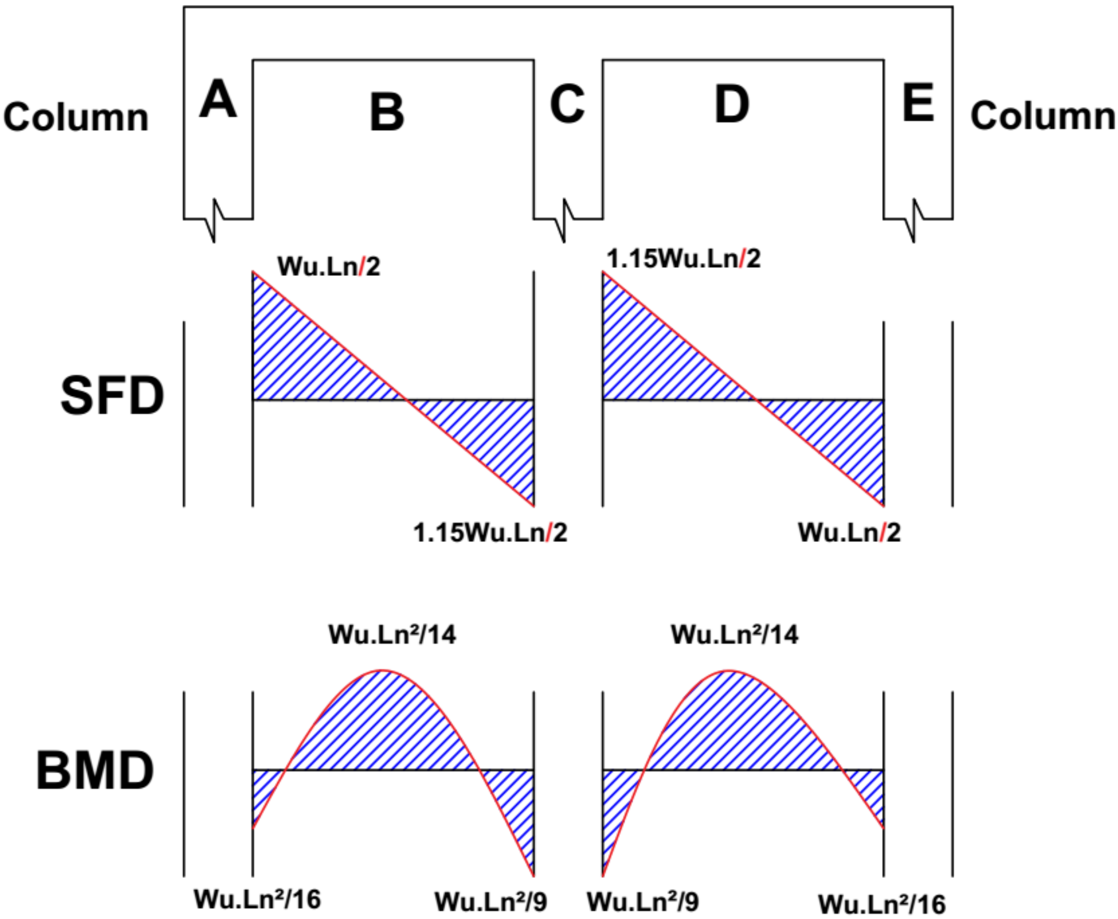
Lecture (19)

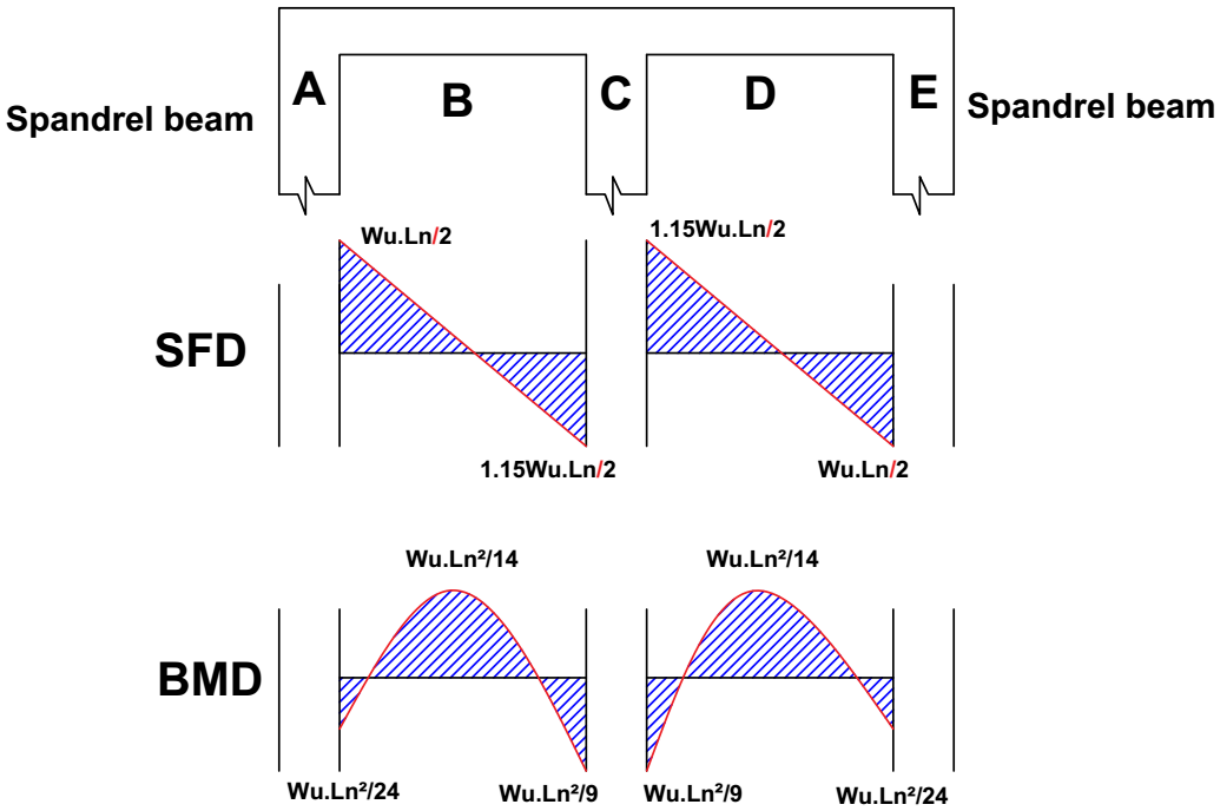
Simplified Method of Analysis for Non-prestressed Continuous Beams and One-Way Slabs ACI 6.5

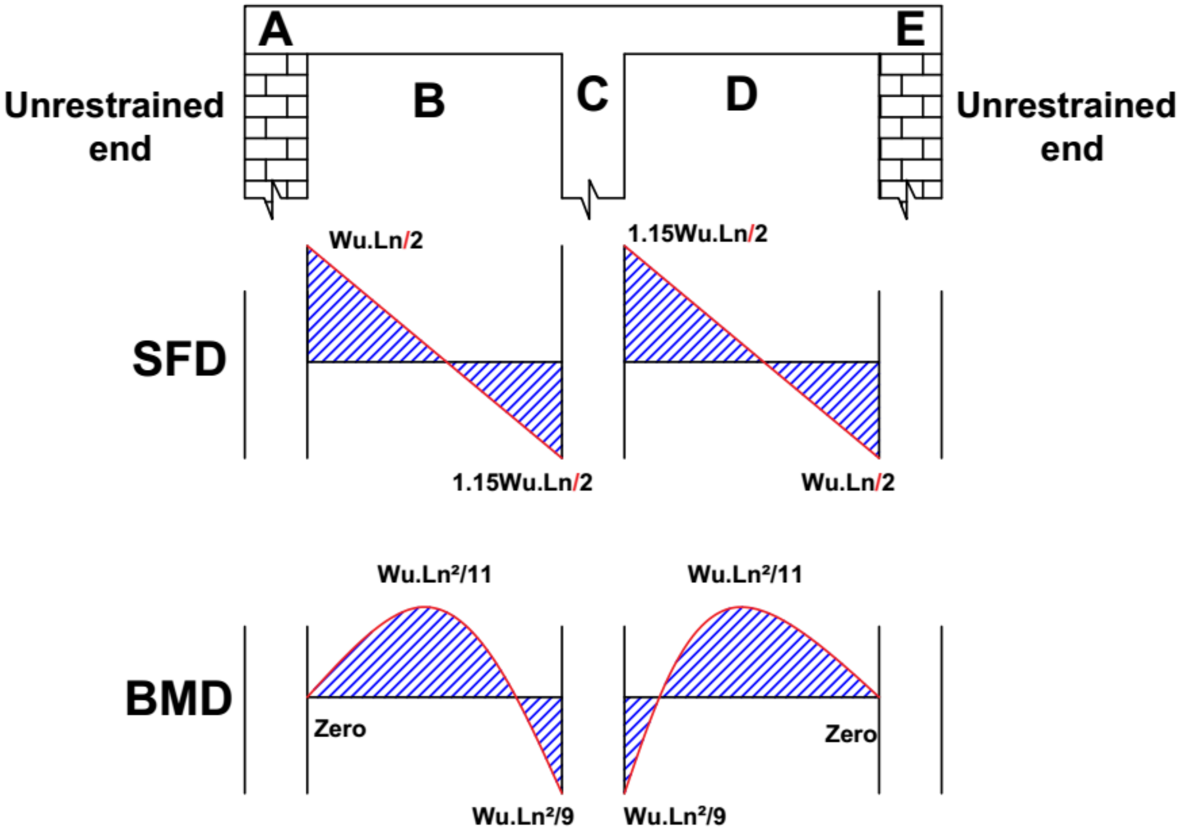




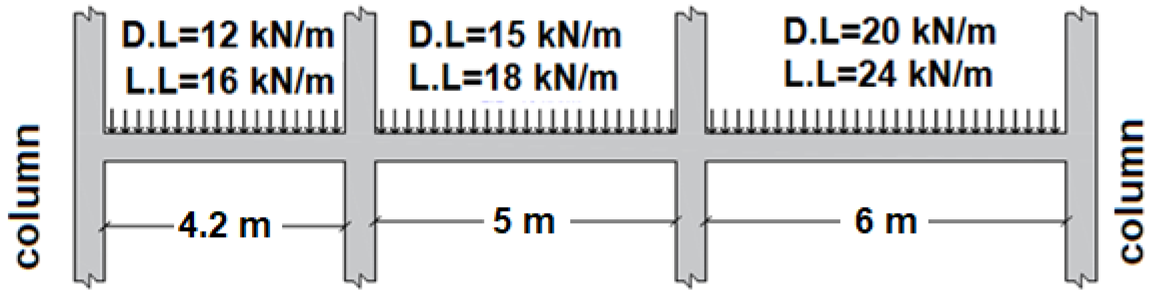

Unrestrained end
A
B
C
D
E
F
G
Unrestrained end
SFD
Wu.Ln/2
1.15Wu.Ln/2
Wu.Ln/2
Wu.Ln/2
1.15 Wu.Ln/2
Wu.Ln/2
BMD
Wu.Ln²/11
Zero
Wu.Ln²/10
Wu.Ln²/16
Wu.Ln²/11
Wu.Ln²/11
Wu.Ln²/11
Zero
Wu.Ln²/10


Column
A
B
C
D
E
Column
SFD
Wu.Ln/2
1.15Wu.Ln/2
1.15Wu.Ln/2
Wu.Ln/2
BMD
Wu.Ln²/14
Wu.Ln²/16
Wu.Ln²/9
Wu.Ln²/14
Wu.Ln²/9
Wu.Ln²/16


Spandrel beam
A
B
C
D
E
Spandrel beam
SFD
Wu.Ln/2
1.15Wu.Ln/2
1.15Wu.Ln/2
Wu.Ln/2
BMD
Wu.Ln²/14
Wu.Ln²/14
Wu.Ln²/24
Wu.Ln²/9
Wu.Ln²/9
Wu.Ln²/24


A
E
Unrestrained end
B
C
D
Unrestrained end
SFD
Wu.Ln/2
1.15Wu.Ln/2
1.15Wu.Ln/2
Wu.Ln/2
BMD
Wu.Ln²/11
Wu.Ln²/11
Zero
Zero
Wu.Ln²/9
Wu.Ln²/9

EX-1: A continuous beam shown in the below figure consists of three spans. The left and right ends are discontinuous and integral with the columns. Answer the following:

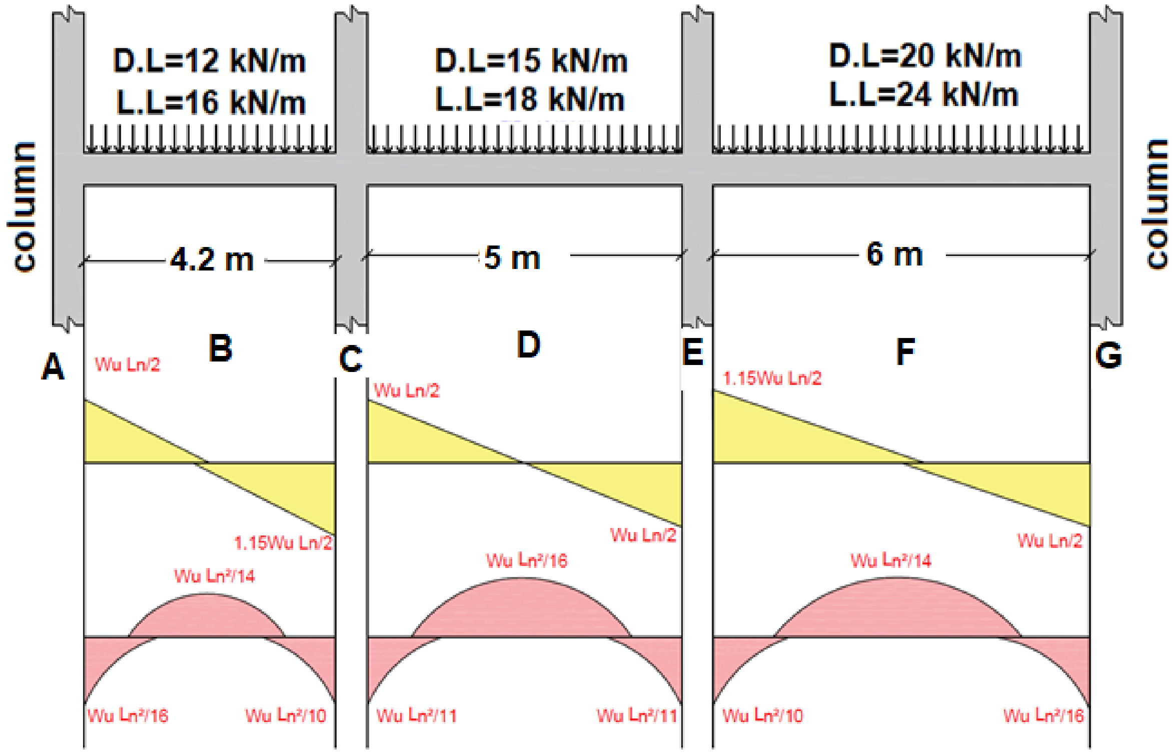


1- Draw the bending moment and shear force diagram according to ACI 8.3.3.

2- Find the Max. Positive and negative moment.

Solution:

$$w_{u1} = 1.2 \times 12 + 1.6 \times 16 = 40\,\mathrm{kN}/m$$
$$w_{u2} = 1.2 \times 15 + 1.6 \times 18 = 46.8\,\mathrm{kN}/m$$
$$w_{u3} = 1.2 \times 20 + 1.6 \times 24 = 62.4\,\mathrm{kN}/m$$



According to ACI-code ((coefficient method)) moment can be calculated as shown below:

$$M_{u\,A} = -\frac{w_u l_n^2}{16} = -\frac{40 \times 4.2^2}{16} = -44.1\ \text{kN.m}$$

$$M_{u\,B} = +\frac{w_u l_n^2}{14} = +\frac{40 \times 4.2^2}{14} = +50.4\ \text{kN.m}$$

$$M_{u\,C-L} = -\frac{w_u l_n^2}{10} = -\frac{40 \times 4.2^2}{10} = -70.56\ \text{kN.m}$$

$$M_{u\,C-R} = -\frac{w_u l_n^2}{11} = -\frac{46.8 \times 5^2}{11} = -106.36\ \text{kN.m}$$

Take the Max moment of both sides

Use $M_{u\,C} = -106.36\ \text{kN.m}$

$$M_{u\,D} = +\frac{w_u l_n^2}{16} = +\frac{46.8 \times 5^2}{16} = +73.13\ \text{kN.m}$$

$$M_{u\,E-L} = -\frac{w_u l_n^2}{11} = -\frac{46.8 \times 5^2}{11} = -106.36\ \text{kN.m}$$

$$M_{u\,E-R} = -\frac{w_u l_n^2}{10} = -\frac{62.4 \times 6^2}{10} = -223.2\ \text{kN.m}$$

Take the Max moment of both sides

$M_{u\,E} = -223.2\ \text{kN.m}$

$$M_{u\,F} = +\frac{w_u l_n^2}{14} = +\frac{62.4 \times (6)^2}{14} = +160.45\ \text{kN.m}$$

$$M_{u\,G} = -\frac{w_u l_n^2}{16} = -\frac{62.4 \times (6)^2}{16} = -140.4\ \text{KN.m}$$

2- Find the Max. Positive and negative moment.

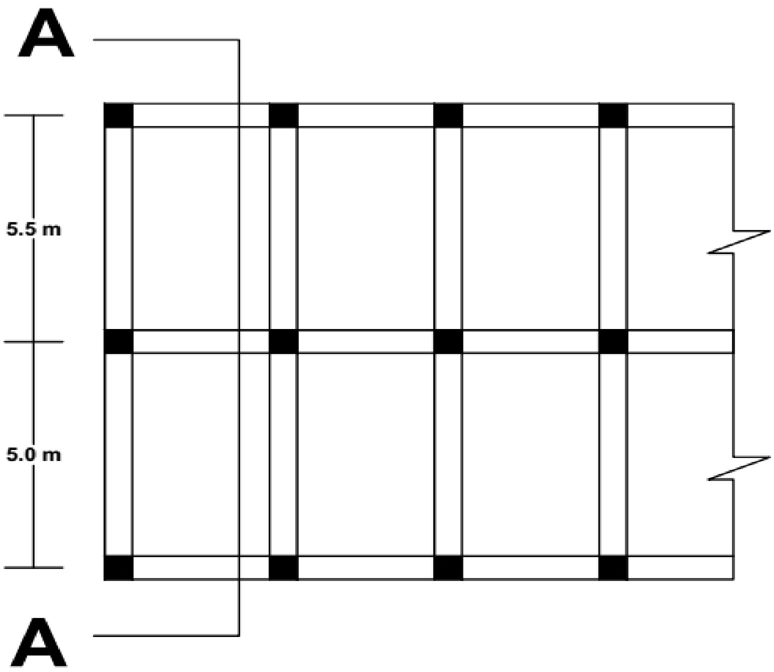
Max. positive moment $160.45\ \text{kN.m}$

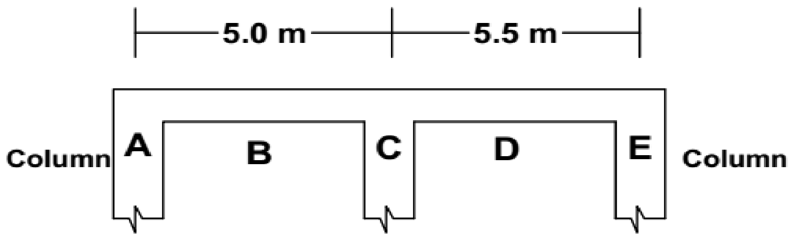
Max. negative moment $223.2\ \text{kN.m}$

EX-2: A continuous beam shown in the below figure consists of two spans and carries WD=10 kN/m and LL=15 kN/m. The left and right ends are discontinuous and integral with the columns. The dimensions of columns are 0.3X0.3 m. Answer the following:

1-Draw the shear force and bending moment diagram according to the approximate method of ACI code.

2-Find the Max. Positive and negative moment.



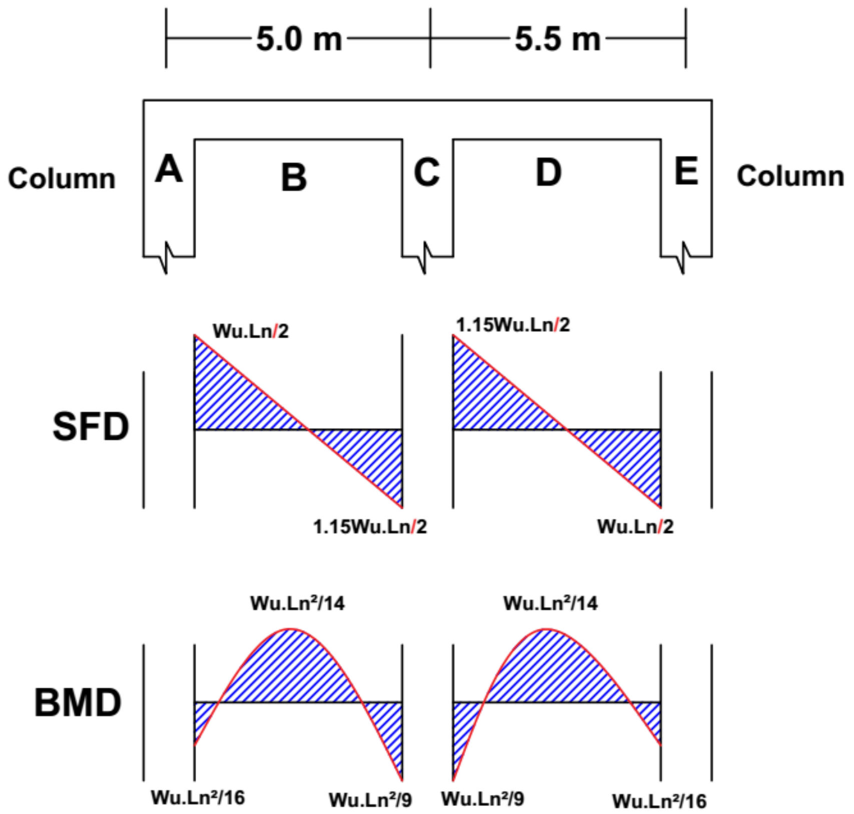


Section A-A

Solution

1-

$$w_u = 1.2 \times 10 + 1.6 \times 15 = 36\,\mathrm{kN}/m$$



2-

$$L_{n1} = 5 - 0.3 = 4.7\ m$$

$$L_{n2} = 5.5 - 0.3 = 5.2\ m$$

$$\mathrm{M_{u\,A}} = -\frac{w_u l_n^2}{16} = -\frac{36 \times 4.7^2}{16} = -49.7\ \ \mathrm{kN.m}$$

$$\mathrm{M_{u\,B}} = +\frac{w_u l_n^2}{14} = +\frac{36 \times 4.7^2}{14} = +56.8\ \mathrm{kN.m}$$

$$\mathrm{M_{u\,C-L}} = -\frac{w_u l_n^2}{9} = -\frac{36 \times 4.7^2}{9} = \mathbf{-88.36\ kN.m}$$

$$\mathrm{M_{u\,C-R}} = -\frac{w_u l_n^2}{9} = -\frac{36 \times 5.2^2}{9} = \mathbf{-108.16\ kN.m}$$

Take the Max moment of both sides

Use $\mathrm{M_{u\,C}} = \mathbf{-108.16\ kN.m}$

$$\mathrm{M_{u\,D}} = +\frac{w_u l_n^2}{14} = +\frac{36 \times 5.2^2}{14} = +69.53\ \mathrm{kN.m}$$

$$M_{u\,E} = -\frac{w_u l_n^2}{16} = -\frac{36 \times 5.2^2}{16} = -60.84\ \mathrm{kN.m}$$

Maximum positive moment=$\mathbf{+69.53\ kN.m}$

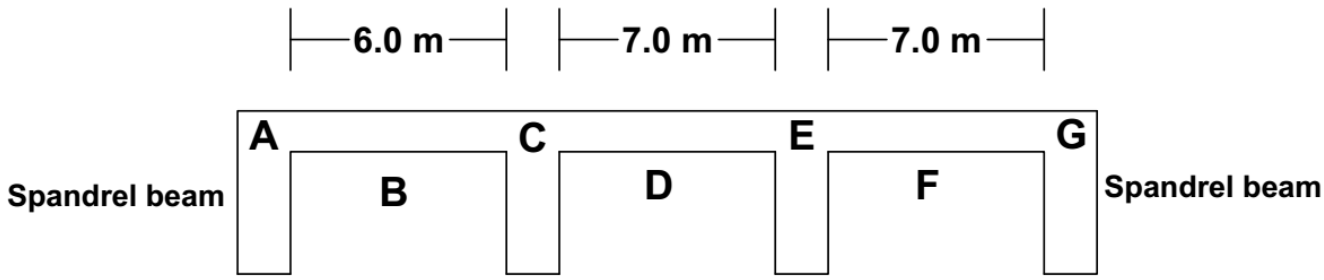
Maximum negative moment=$\mathbf{-108.16\ kN.m}$

HW: For the sections shown below, carry WD=12 kN/m and LL=20 kN/m. Answer the following:

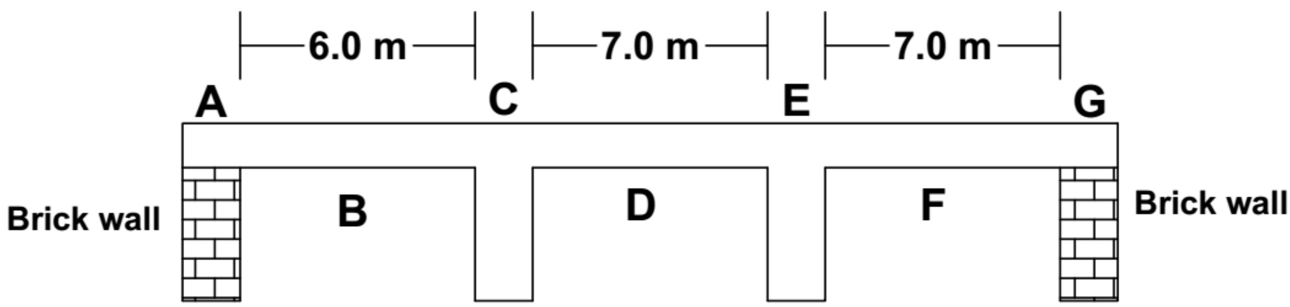
1-Draw the shear force and bending moment diagram according to the approximate method of ACI code.

2-Find the Max. Positive and negative moment.

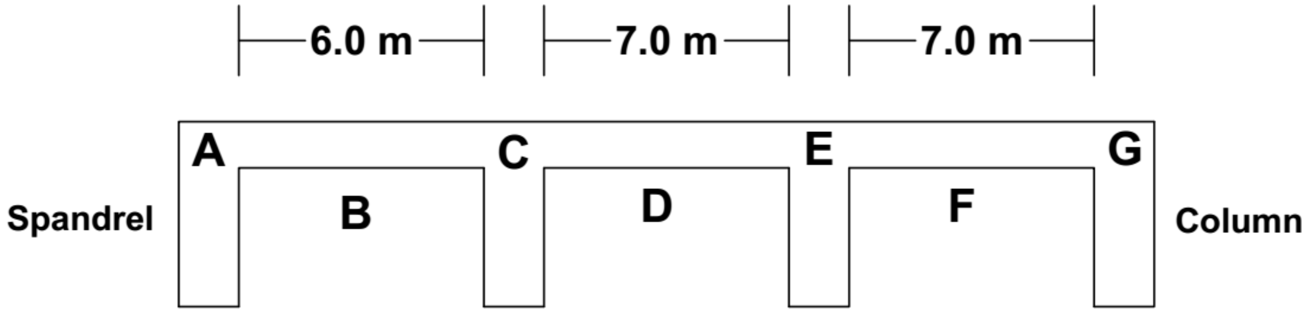
A)



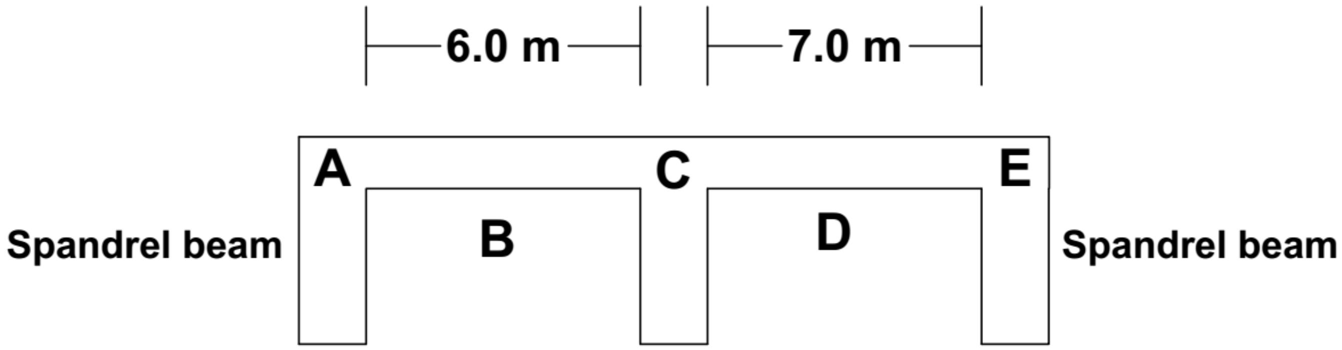
B)



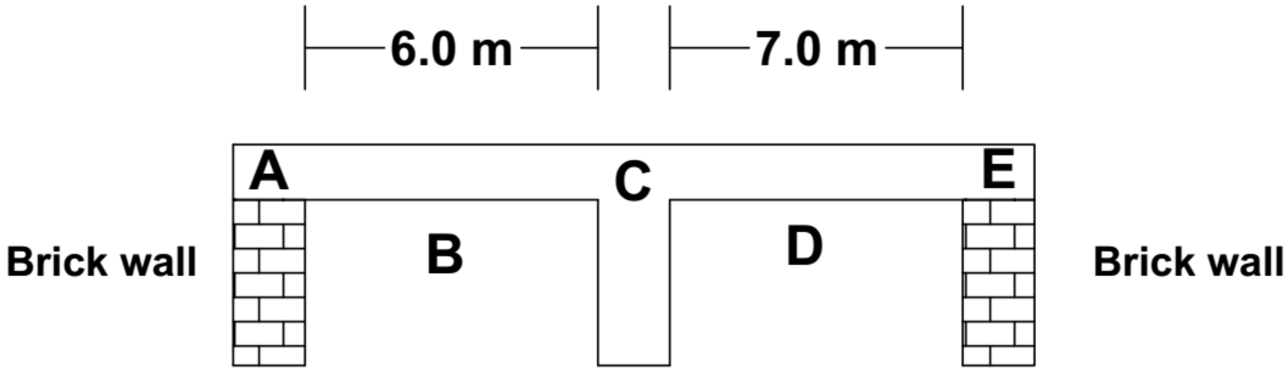
C)


6.0 m
7.0 m
7.0 m
A
C
E
G
Spandrel
B
D
F
Column

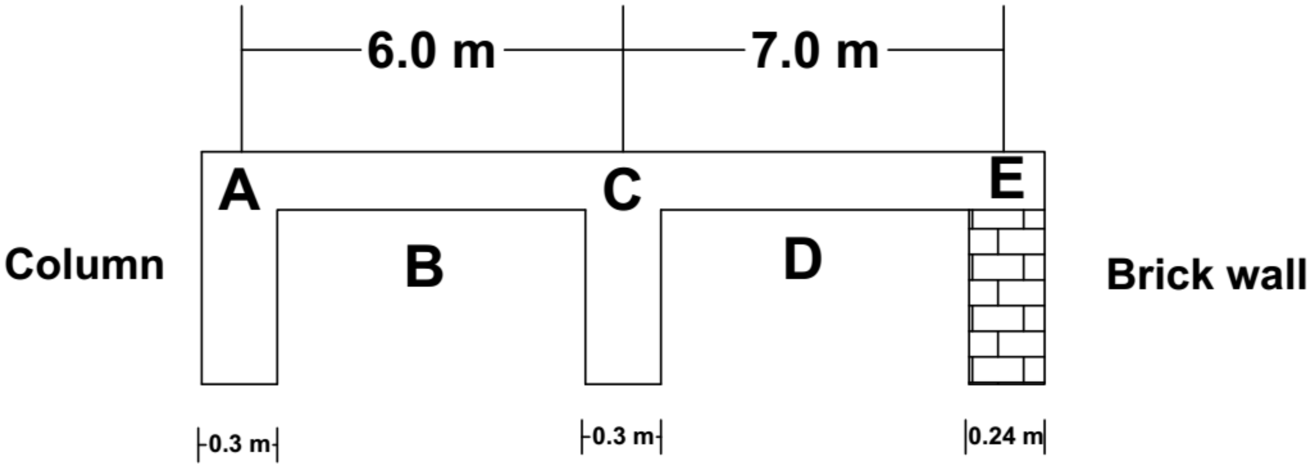
D)


6.0 m
7.0 m
A
C
E
Spandrel beam
B
D
Spandrel beam

E)


6.0 m
7.0 m
A
C
E
Brick wall
B
D
Brick wall

F) **important**



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Design of Reinforced Concrete Structures I

Analysis of T sections-1

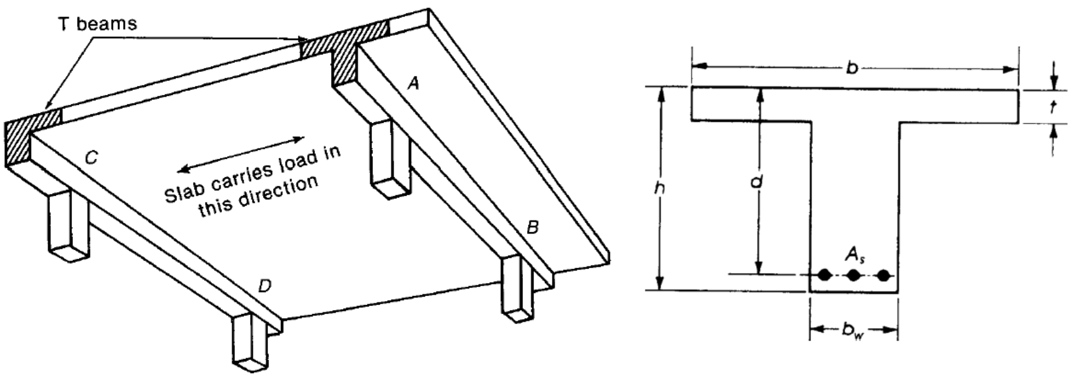
Dr Othman Hameed

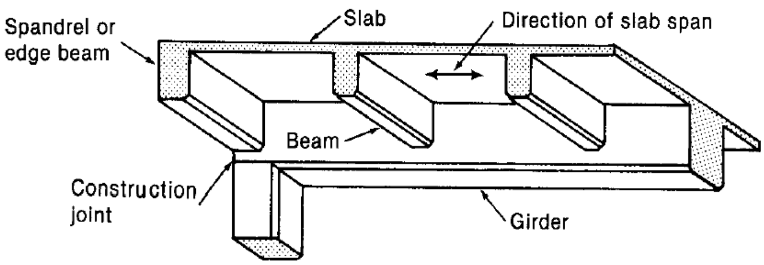
Lecture (20)

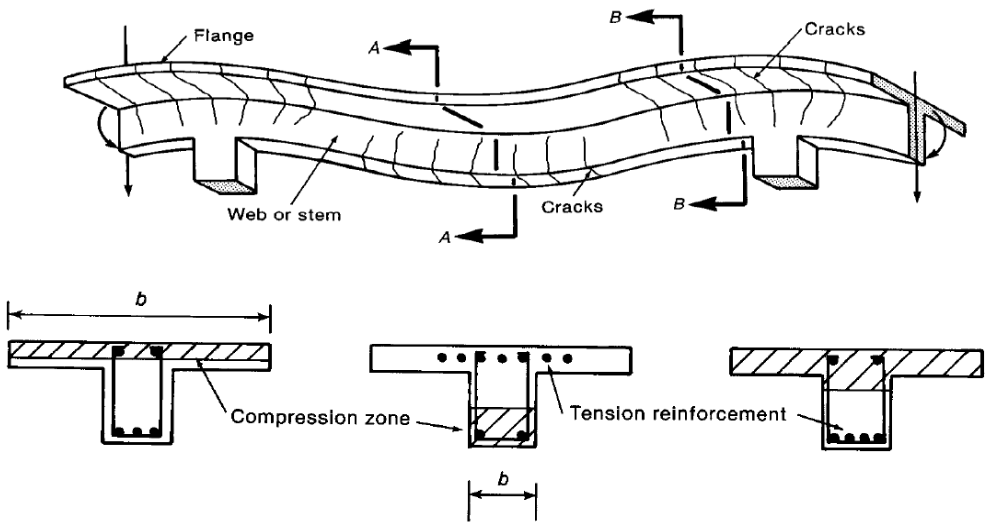
Analysis of T-sections

T- Beams

Reinforced concrete floor systems normally consist of slabs and beams that are placed monolithically. As a result, the two parts act together to resist loads. In effect, the beams have extra widths at their tops, called flanges, resulting T-shaped beams. The part of a T-beam below in the slab is referred to as the *web*. The beams may be L-shaped if the beam is at the end of a slab.







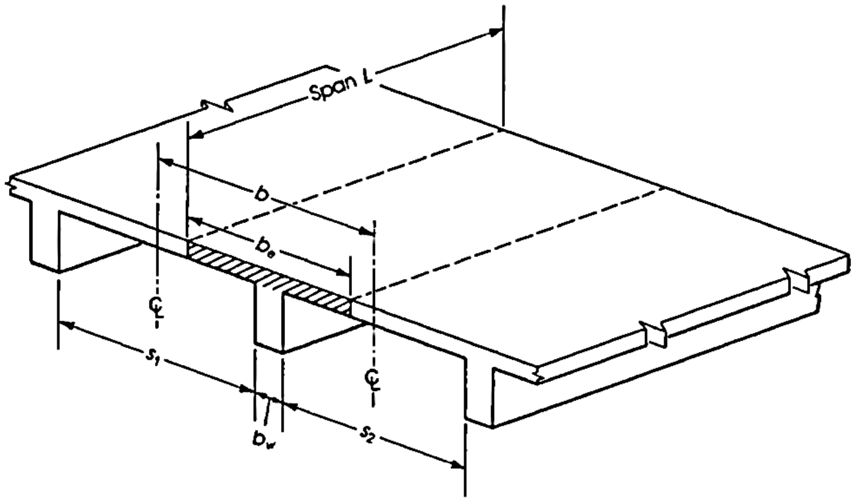
(b) Section A-A (rectangular compression zone).

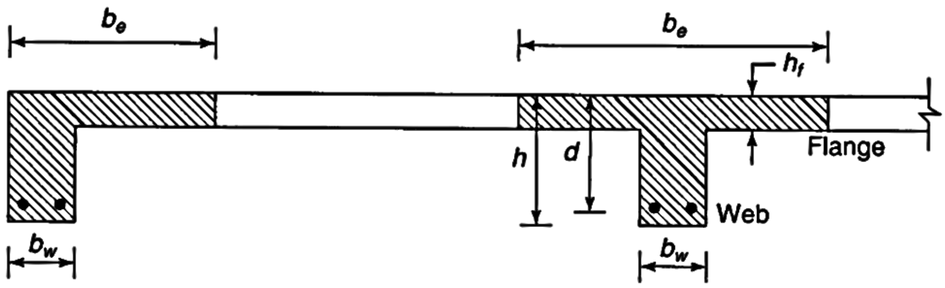
(c) Section B-B (negative moment).

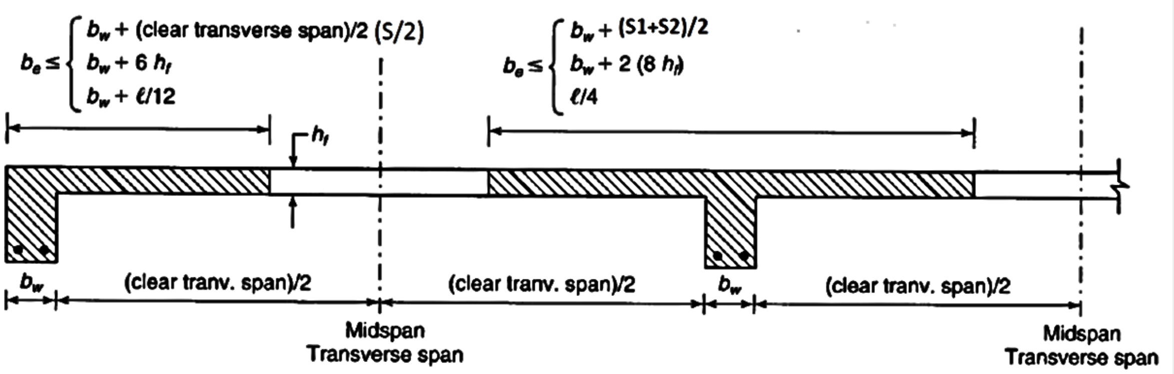
(d) Section A-A (T-shaped compression zone).

ACI code Provisions for Estimate (bf) ACI 6.3.2

The ACI Code definitions for the effective compression flange width for T- and inverted L-shapes in continuous floor systems as illustrated in figure below.





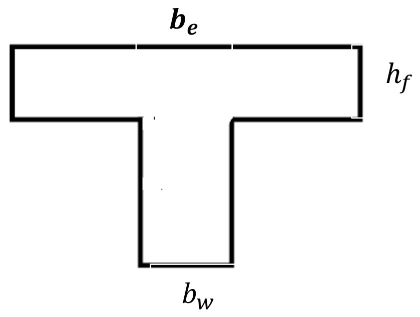


Isolated beams

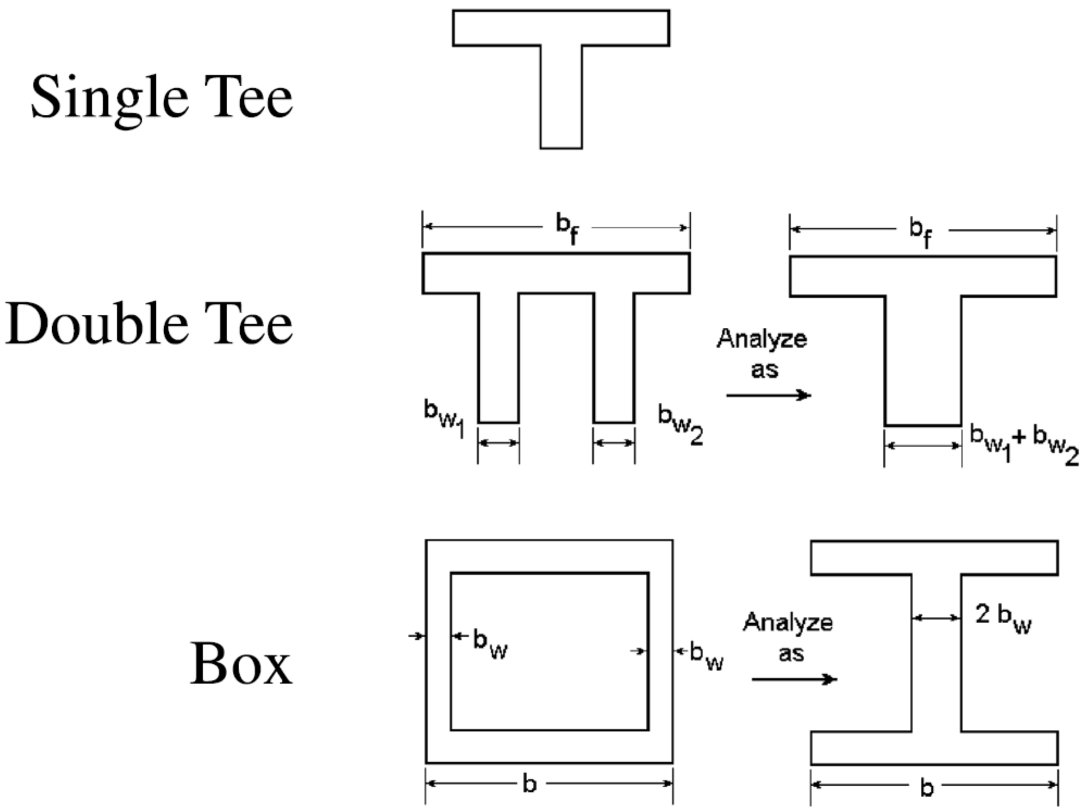
In the isolated beams, the T-shape is used to provide a flange for additional compression area. The flange thickness of isolated beams is calculated according (ACI 6.3.2.2)

$a)\ h_f \geq \frac{1}{2} b_w$

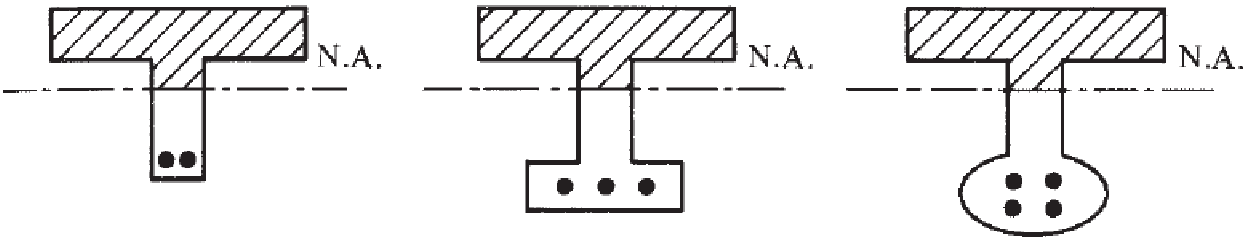
$b)\ be \leq\ 4b_w$



Various Possible Geometric of T-beam as shown below



A beam does not really have to look like a T beam to be one. This fact is shown by the beam cross sections shown in Figure below. For these cases the compression concrete is T shaped, and the shape or size of the concrete on the tension side, which is assumed to be cracked, has no effect on the theoretical resisting moments.

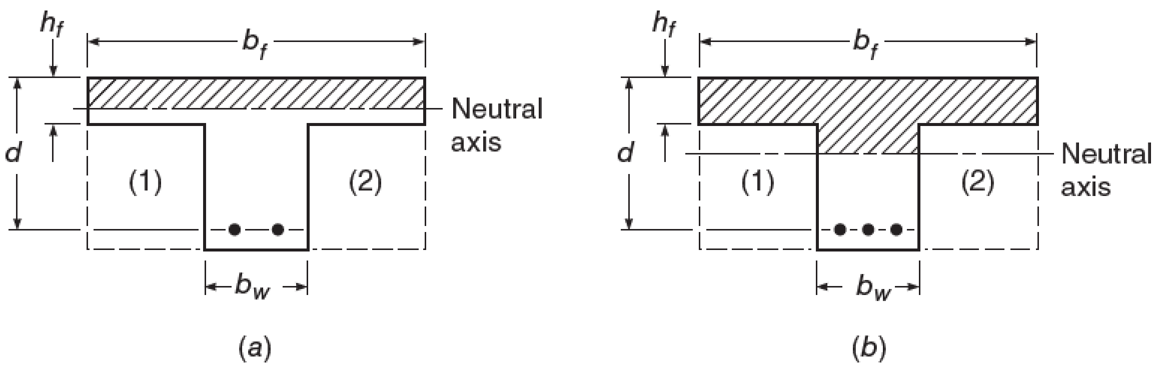


Analysis of T-section

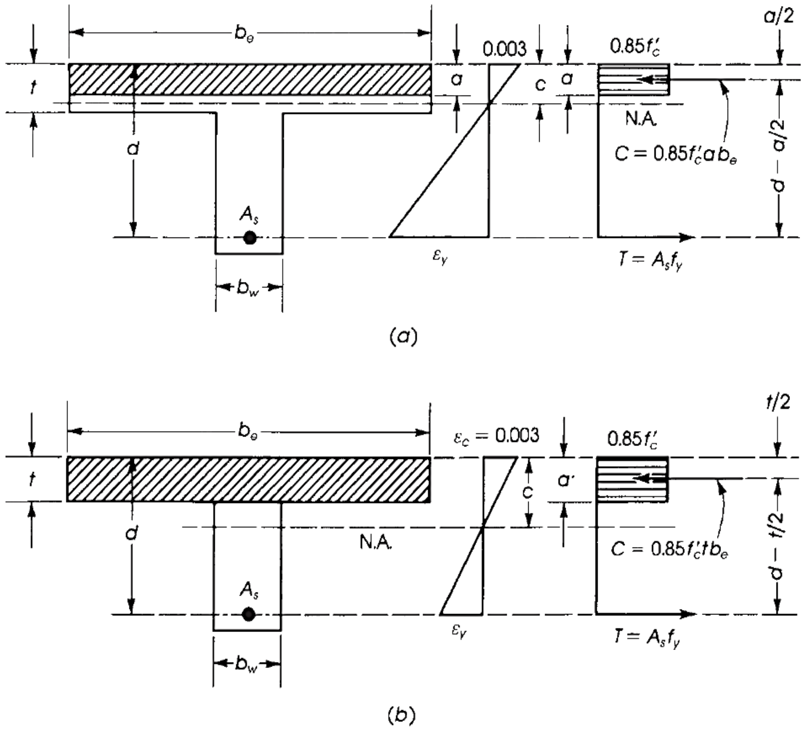
The neutral axis of a T beam may be either in the flange or in the web, depending upon the proportions of the cross section, the amount of tensile steel, and the strengths of the materials.

- If the depth to the neutral axis is $\leq$ the flange thickness(***h_f***), the beam can be analyzed as a rectangular beam of width equal to ***b_f***. As shown in figure a below.

- If the depth to the neutral axis is > the flange thickness(***h_f***), as in Figure (b) below, methods must be developed to account for the actual T-shaped compressive zone.



Case 1: $a \leq h_f$ $\quad$ $the\ section$ is $analysed\ as$ a $rectangular\ section.$



$$\sum Fx = 0 \qquad \rightarrow\ T = C$$

$$A_s\, fy = 0.85\ f_c'\, a\ b_e \qquad\qquad \rightarrow a = \frac{A_s\, fy}{0.85\ f_c'\ b_e} \leq h_f$$

$$\emptyset \mathrm{M_n} = \emptyset\ \rho\ b_e d^2 fy\ \left(1 - 0.59 \frac{fy}{f_c'}\ \rho\right)$$

Or

$$\emptyset \mathrm{M_n} = \emptyset\ A_s\ fy\ \left(d - \frac{a}{2}\right)$$

$$\rho = \frac{A_s}{b_e\ d}$$

$$\rho_{max.} = 0.85\ \beta_1 \frac{f_c'}{fy} \frac{0.003}{(0.003 + 0.004)}$$

$$\rho_{min.} = max.of \left\{ \frac{\sqrt{f_c'}}{4\ fy} \times \frac{b_w}{b_e} \quad \text{or} \quad \frac{1.4}{fy} \times \frac{b_w}{b_e} \right\}$$

Design of Reinforced Concrete Structures I

Al Muthanna University
Collage of Engineering

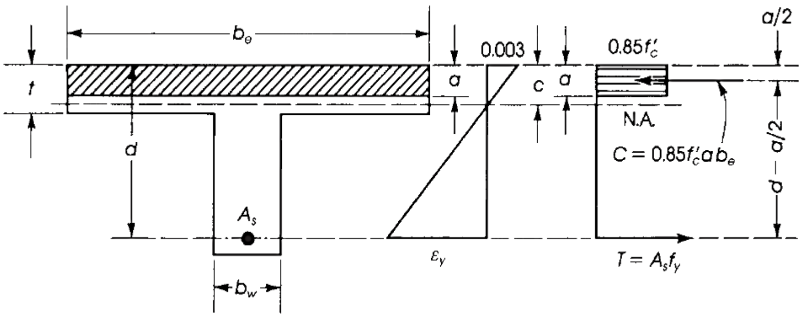
Analysis of T sections-2

Dr Othman Hameed

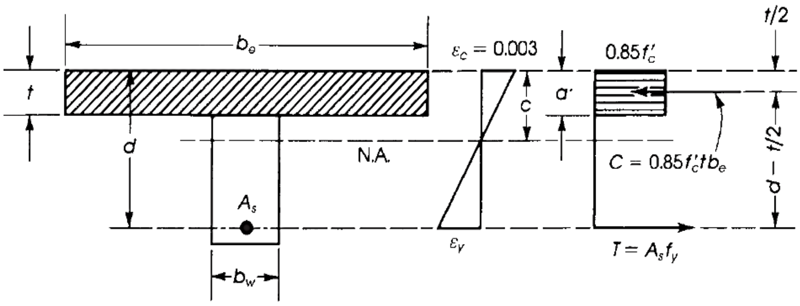
Lecture (21)

Analysis of T-sections

Case 1: $a \leq h_f$ $\quad$ *the section is analysed as rectangular section.*



(a)



(b)

$$\sum Fx = 0 \quad \rightarrow T = C$$

$$A_s\, fy = 0.85\, f_c'a\, b_e \qquad \rightarrow a = \frac{A_s\, fy}{0.85\, f_c'\, b_e} \leq h_f$$

$$\emptyset \mathrm{M_n} = \emptyset\, \rho\, b_e d^2 fy \left(1 - 0.59 \frac{fy}{f_c'}\, \rho\right)$$

Or

$$\emptyset \mathrm{M_n} = \emptyset\, A_s\, fy \left(d - \frac{a}{2}\right)$$

$$\rho = \frac{A_s}{b_e\, d}$$

$$\rho_{max.} = 0.85\, \beta_1 \frac{f_c'}{fy} \frac{0.003}{(0.003 + 0.004)}$$

$$\rho_{min.} = max. of \left\{ \frac{\sqrt{f_c'}}{4\, fy} \times \frac{b_w}{b_e} \quad \text{or} \quad \frac{1.4}{fy} \times \frac{b_w}{b_e} \right\}$$

$\rho_{min.} < \rho < \rho_{max.}$

Strength reduction factor for T-section ($a \leq h_f$)

$$\rho_t = 0.85\,\beta_1 \frac{f_c'}{fy}\frac{0.003}{(0.003+0.005)}$$

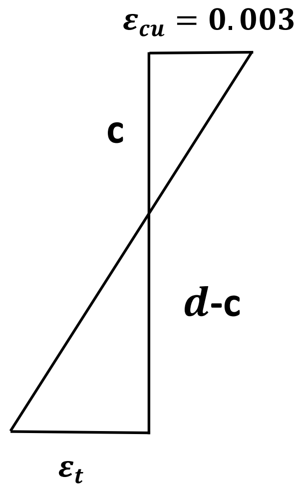
If $\rho_t \geq \rho \rightarrow \emptyset = 0.9$

If $\rho_t < \rho \rightarrow \emptyset\ must\ be\ calculated$

To calculate $\emptyset$, the values of ε_t and ε_{ty} must be determined

ε_{ty} = fy/Es

Find ε_t from



$$\frac{\varepsilon_t}{(d-c)} = \frac{0.003}{c}$$

$$c = \frac{a}{\beta_1}$$

$$a = \frac{A_s\,fy}{0.85\,f_c'\,b_e} \leq h_f$$

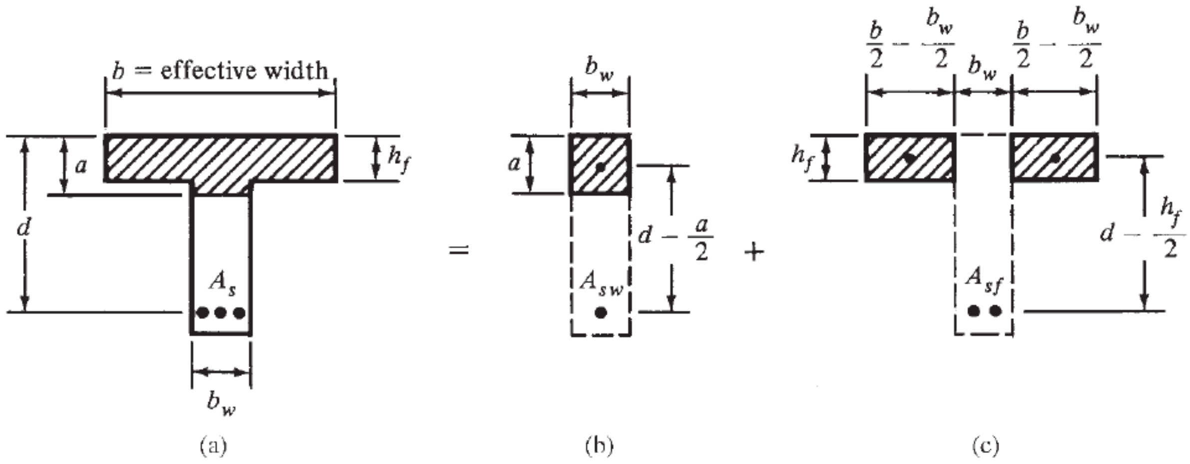
A) If $\varepsilon_{ty} < \varepsilon_t < 0.005$ $(transition\ \ control)$

$$\phi = 0.65 + 0.25\frac{(\varepsilon_t - \varepsilon_{ty})}{(0.005 - \varepsilon_{ty})}$$

B) $If\ \varepsilon_t \leq \varepsilon_{ty}$ $(compression\ \ control)$

$$\phi = 0.65$$

Case 2: $a > h_f$ *the section is analysed as* $T-$ *section.*



The analysis of T-section is similar to that of doubly reinforcement.

$$\emptyset M_n = \emptyset\,(M_{nw} + M_{nf}) = \emptyset[(A_s - A_{sf})fy \times (d - {}^{a}/_{2}) + A_{sf}\, fy\,(d - {}^{h_f}/_{2})]$$

$$\sum Fx = 0 \qquad \text{For flange case}$$

$$A_{sf}\, fy = 0.85 f_c'(b_e - b_w) \times h_f \quad \rightarrow \quad A_{sf} = \frac{0.85 f_c'(b_e - b_w) \times h_f}{fy}$$

$$\sum Fx = 0 \qquad \text{For web case}$$

$$A_{sw}\, fy = 0.85 f_c' b_w \times a$$

$$A_{sw} = A_s - A_{sf}$$

$$(A_s - A_{sf})\, fy = 0.85 f_c' b_w \times a \quad \rightarrow \quad a = \frac{(A_s - A_{sf})\, fy}{0.85 f_c' b_w}$$

- Balance Steel Ratio for T-beam

$$\rho_{total} = \frac{A_s}{b_w d}; \qquad \rho_f = \frac{A_{sf}}{b_w d}$$

$$\rho_{bw} = 0.85\,\frac{f_c'}{fy}\,\frac{600}{600+fy} + \rho_f \qquad = \rho_b + \rho_f$$

- Maximum Steel Ratio for T-beam

$$\rho_{max.} = 0.85\,\beta_1\,\frac{f_c'}{fy}\,\frac{0.003}{(0.003+0.004)} + \rho_f$$

- Minimum Steel Ratio for T-beam

$$\rho_{min.} = max.of \left\{\frac{\sqrt{f_c'}}{4\,fy} \quad \text{or} \quad \frac{1.4}{fy}\right\}$$

$$\rho_{min.} \le \rho_{total} \le \rho_{max.}$$

Strength reduction factor for T-section $(a > h_f)$

$$\rho_{wt} = 0.85\,\beta_1\,\frac{f_c'}{fy}\,\frac{\varepsilon_{cu}}{\varepsilon_{cu}+0.005} + \rho_f = \rho_t + \rho_f$$

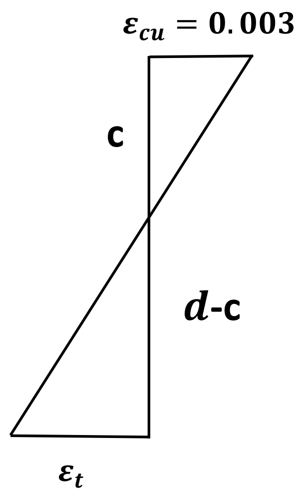
If $\rho_{wt} \geq \rho_{total} \rightarrow \emptyset = 0.9$

If $\rho_{wt} < \rho_{total} \rightarrow \emptyset\ must\ be\ calculated$

To calculate $\emptyset$*, the values of* ε_t *and* ε_{ty} *must be determined*

ε_{ty} = fy/Es

Find ε_t from



$$\frac{\varepsilon_t}{(d-c)} = \frac{0.003}{c}$$

$$c = \frac{a}{\beta_1}$$

$$a = \frac{(A_s - A_{sf})\,fy}{0.85 f_c' b_w}$$

C) If $\varepsilon_{ty} < \varepsilon_t < 0.005$ $(transition\ \ control)$

$$\phi = 0.65 + 0.25\,\frac{(\varepsilon_t - \varepsilon_{ty})}{(0.005 - \varepsilon_{ty})}$$

D) $If\ \varepsilon_t \leq \varepsilon_{ty}$ $(compression\ \ control)$

$$\phi = 0.65$$

Procedure of solution

1- Find be

For inner section (T-section)

$$b_e = \begin{cases} \frac{L}{4} \\ b_w + 16\,h_f \\ b_w + \frac{s_1+s_2}{2} \end{cases}$$

For edge section (L-section)

$$b_e = \begin{cases} \frac{L}{12} + b_w \\ b_w + 6\,h_f \\ b_w + \frac{s}{2} \end{cases}$$

2- Check if the section is T or rectangular.

$$a = \frac{A_s\,fy}{0.85\,f_c'\,b_e}$$

$if\ a \leq h_f$ the section is Rectangular (use the equation of case 1)

$if\ a > h_f$ the section is T (use the equation of case 2)

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Design of Reinforced Concrete Structures I

Analysis of T sections-3

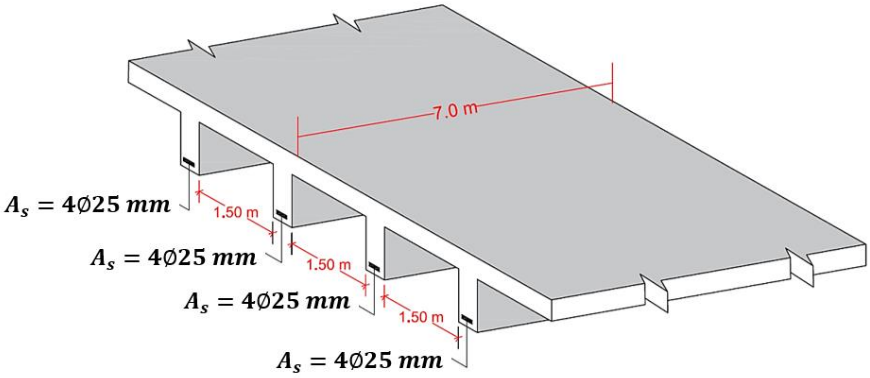
Dr Othman Hameed

Lecture (22)

Analysis of T-sections

Ex-1:

A series of reinforced concrete beams spaced at 1.5 m face to face have a simply supported span of 7.0 m. The beams support a reinforced concrete floor slab of 75mm thick. The effective depth (d)= 538 mm, web width =300mm, f_c'=28 Mp, fy = 420 Mpa. Calculate the bending moment capacity of interior beam. $A_s = 4\emptyset 25$ mm.



Solution:

1- Find the b_e=
$$\begin{cases} \frac{L}{4} = \frac{7000}{4} & = 1750mm \\ b_w + 16\,h_f = 300 + (16 \times 75) & = 1500mm \text{ control} \\ b_w + \frac{s_1+s_2}{2} = 300 + \frac{1500+1500}{2} & = 1800\ mm \end{cases}$$

2- Check if the section is T or rectangular.

$$a = \frac{A_s\,fy}{0.85\,f_c'\,b_e} = \frac{1964 \times 420}{0.85 \times 28 \times 1500} = 23.1\ mm < h_f\ (75mm) \rightarrow rectangular\ section$$

3- Find the moment capacity as previously discussed

$$\rho = \frac{As}{b\,d} = \frac{1964}{1500 \times 538} = 0.00243$$

$$\rho_{max.} = 0.85\,\beta_1\,\frac{f_c'}{fy}\,\frac{0.003}{(0.003+0.004)} = 0.0206$$

$$\rho_{min.} = max.of \left\{ \frac{\sqrt{f_c'}}{4\,fy} \times \frac{b_w}{b_e} \quad \frac{1.4}{fy} \times \frac{b_w}{b_e} \right\} = (0.00063,\ \ 0.00066)$$

$$\rho_{min.} < \rho < \rho_{max.}$$

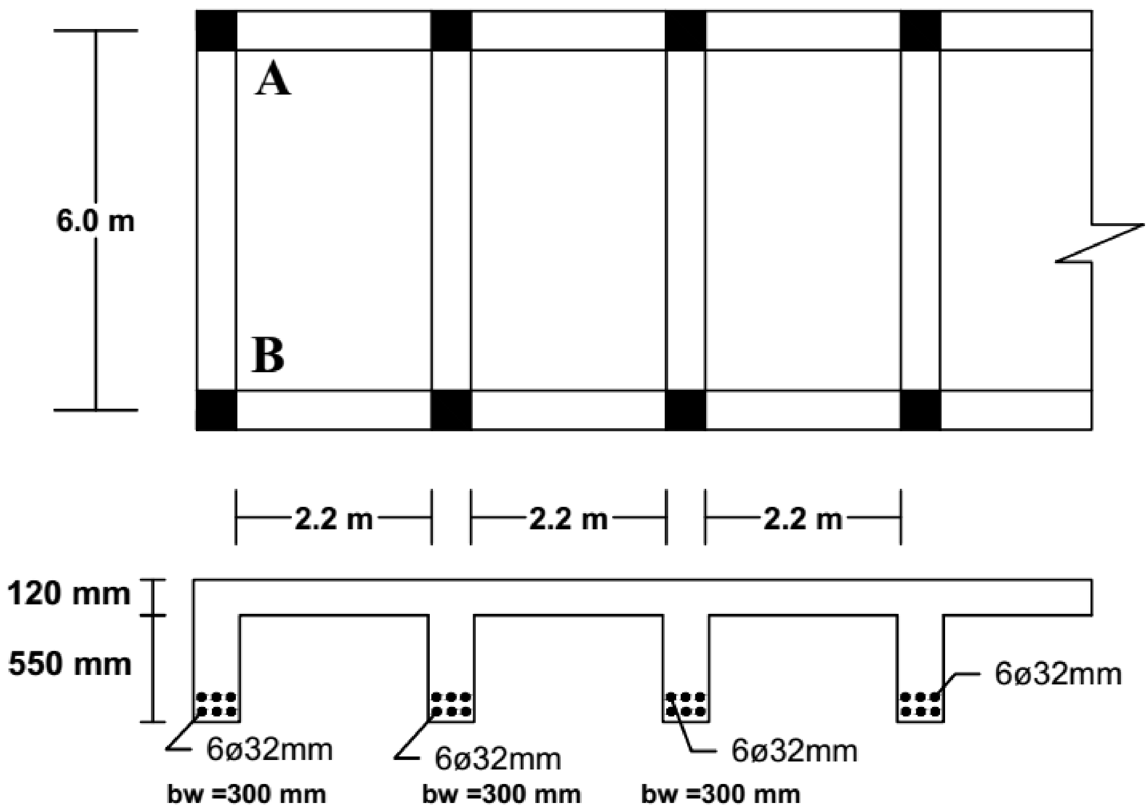
$$\rho_t = 0.85\,\beta_1\,\frac{f_c'}{fy}\,\frac{0.003}{(0.003+0.005)} = 0.018 > \rho = 0.00243 \quad \rightarrow \emptyset = 0.9$$

$$\emptyset M_n = \emptyset\, \rho\, b_e d^2 fy \left(1 - 0.59 \frac{fy}{f_c'}\, \rho\right)$$

$$M_u = 0.9 \times 0.00243 \times 1500 \times\ 538^2 \times 420 \left(1 - 0.59 \frac{420}{28}\ 0.00243\right) \times 10^{-6} = 390.2\ kN.m$$

Ex-2:

Determine the moment capacity of **exterior** beam (AB) of floor system shown in Figure below. The beam has a span of 6.0 m and $\frac{f_c'}{fy} = \frac{20}{400}$ Mpa.



Solution:

1- Find the b_e=
$$\begin{cases} \frac{L}{12} + b_w = \frac{6000}{12} + 300 & = 800mm \\ b_w + 6\,h_f = 300 + (6 \times 120) & = 1020mm \\ b_w + \frac{s}{2} = 300 + \frac{2200}{2} & = 1400\ mm \end{cases}$$

2- Check if the section is T or Rectangular

As=4825.5 mm²
d =670- 40- 10 -32-25/2 = 575.5 mm (2 layer)

$$a = \frac{A_s\, fy}{0.85\, f_c'\, b_e} = \frac{4825.5 \times 400}{0.85 \times 20 \times 800} = 141.9\ mm\ > h_f(120mm)\ \ \rightarrow T - section$$

3- Find A_{sf}

$$A_{sf} = \frac{0.85\ f_c'(b_e - b_w)h_f}{fy} = \frac{0.85 \times 20 \times (800 - 300) \times 120}{400} = 2550\ mm^2$$

4- Find (a)

$$a = \frac{(A_s - A_{sf})\ fy}{0.85 f_c' b_w} = \frac{(4825.5 - 2550)400}{0.85 \times 20 \times 300} = 178.47\ mm$$

5- Find strength redaction factor ø

$$\rho_{wt} = 0.85\ \beta_1 \frac{f_c'}{fy} \frac{\varepsilon_{cu}}{\varepsilon_{cu} + 0.005} + \rho_f$$

$$\rho_f = \frac{A_{sf}}{b_w d} = \frac{2550}{300*575.5} = 0.0148$$

$$\rho_{wt} = 0.85 * 0.85\ \frac{20}{400} \frac{0.003}{0.003 + 0.005} + 0.0148 = 0.01355 + 0.0148 = 0.0283$$

$$\rho_{total} = \frac{A_s}{b_w d} = \frac{4825.5}{300 * 575.5} = 0.0279$$

$$\rho_{total} \leq \rho_{wt} \rightarrow \emptyset = 0.9$$

6- Check $\rho_{max.}$ And $\rho_{min.}$

$$\rho_{max.} = 0.85\ \beta_1 \frac{f_c'}{fy} \frac{0.003}{(0.003 + 0.004)} + \rho_f = 0.01548 + 0.0148 = 0.0303$$

$$\rho_{min.} = max.of \left\{ \frac{\sqrt{f_c'}}{4\ fy} \quad \text{or} \quad \frac{1.4}{fy} \right\} = 0.00279 \text{ or } 0.0035$$

$$\rho_{min.} \leq \rho_{total} \leq \rho_{max.}$$

$$0.0035 \leq 0.0279 \leq 0.0303$$

7- Calculate the moment capacity

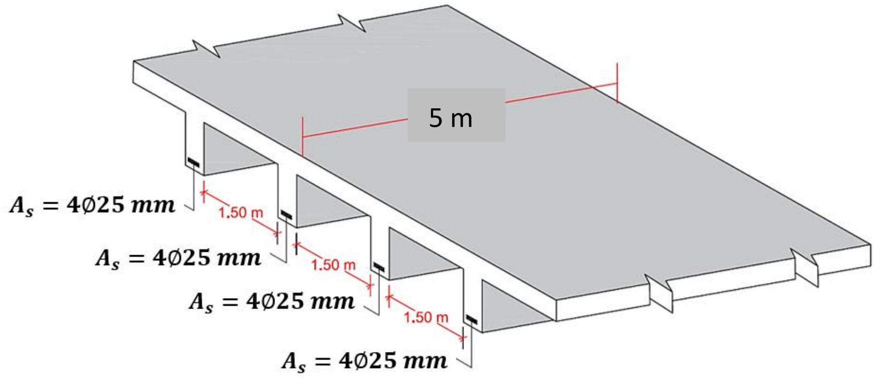
$$\emptyset M_n = \emptyset[(A_s - A_{sf})fy \times (d - {}^{a}/_{2}) + A_{sf}\ fy\ (d - {}^{h_f}/_{2})]$$

$$\emptyset M_n = 0.9[(4825.5 - 2550) \times 400 \times (575.5 - {}^{178.47}/_{2}) + 2550 \times 400\ (575.5 - {}^{120}/_{2})]$$

$$\times 10^{-6} = 871.5\ kN.m$$

HW-1:

A series of reinforced concrete beams spaced at 1.5 m face to face have a simply supported span of 5.0 m. The beams support a reinforced concrete floor slab of 100 mm thick. The effective depth (d)= 550 mm, web width =250mm, f_c'=30 Mp, fy = 420 Mpa. Calculate the bending moment capacity of interior beam. $A_s = 4\emptyset 25$ mm.

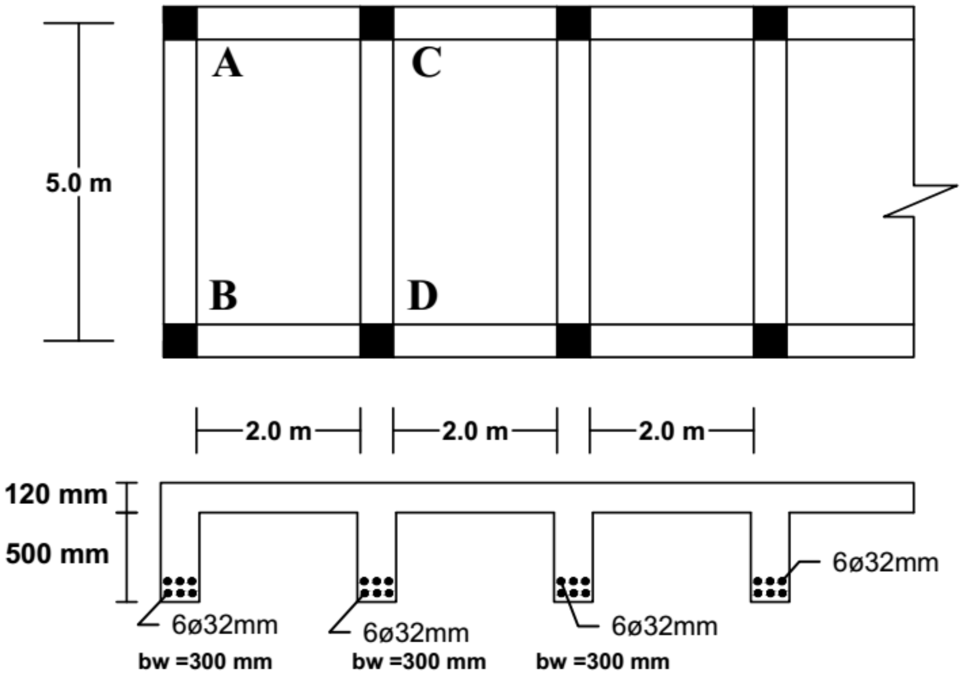


HW-2:

For the floor system shown in figure below. The beam has a span of 5.0 m and $\frac{f_c'}{fy} = \frac{25}{400}$ Mpa.

A- Determine the moment capacity of beam AB (**exterior** beam)

b- Determine the moment capacity of beam CD (**interior** beam)



Design of Reinforced Concrete Structures I

Al Muthanna University
Collage of Engineering

Design of T sections-1

Dr Othman Hameed

Lecture (23)

Equation to find ρ

$$\mathrm{M_u} = \emptyset\, \rho\, b\, d^2 fy \left(1 - 0.59 \frac{fy}{f_c'}\, \rho\right)$$

$$\left[\mathrm{M_u} = \emptyset\, \rho\, b\, d^2 fy \left(1 - 0.59 \frac{fy}{f_c'}\, \rho\right)\right] \div \emptyset\;\; b\, d^2 fy$$

$$\frac{\mathrm{M_u}}{\emptyset\;\; b\, d^2 fy} = \rho - 0.59\, \rho^2\, \frac{fy}{f_c'}$$

$$0.59\, \frac{fy}{f_c'} \rho^2 - \rho + \frac{\mathrm{M_u}}{\emptyset\;\; b\, d^2 fy} = 0$$

$$A = 0.59\, \frac{fy}{f_c'}$$

B= -1

$$C = \frac{\mathrm{M_u}}{\emptyset\;\; b\, d^2 fy}$$

$$\rho = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

ρ can be calculated as

$$\rho = \frac{1 \pm \sqrt{1 - \frac{2.36 * M_u * 10^6}{\emptyset\, b\, d^2 f_c'}}}{1.18\left(\frac{fy}{f_c'}\right)}$$

Design of T-Beams

1- Design of T Beams for Positive Moments

Design Procedure of T-Beams for Positive Moments

1- Establish (H) based on serviceability requirement and calculate (d).

2- Choose (bw). (ratio of d)

3- Find (be) according to ACI requirement.

4- Calculate (As) assume that a $\leq$ h_f with beam width (be) and ø =0.9 and then check.

$$M_u = \emptyset M_n$$

$$\emptyset M_n = \emptyset\, \rho\, b_e\, d^2 f_y \left(1 - 0.59 \frac{fy}{f'_c}\, \rho\right)$$

find $\rho \rightarrow As \rightarrow \quad a = \frac{A_s\, fy}{0.85\, f'_c\, b_e}$

5- If $a \leq h_f \quad \rightarrow$ the assumption is right and continue as a rectangular section

find ρ_{max} and ρ_{min}

6- If $a > h_f \quad \rightarrow$ the assumption is wrong and continue as a T − section

For T-section use the following procedure ($a > h_f$)

A- Find $A_{sf} = \frac{0.85 f'_c (b_e - b_w) \times h_f}{fy}$

B- Find $\emptyset M_{nf} = \emptyset\, A_{sf} fy \left(d - {}^{hf}/_{2}\right)$ and use $\emptyset = 0.9$

C- From $M_{u,total} = \emptyset(M_{nf} + M_{n\,w})$, find $\emptyset M_{n\,w} = M_{u,total} - \emptyset M_{nf}$

D-Use $\emptyset M_{nw} = \emptyset\, \rho_w\, b_w d^2 fy \left(1 - 0.59 \frac{fy}{f'_c}\, \rho_w\right)$ and find ρ_w

E- Find $A_{sw} = \rho_w\, b_w\, d$

F-$A_s = A_{sf} + A_{s\,w}$ and find $\rho_{total} = \frac{A_s}{b_w\, d}$

G- Check ρ_{total} with $\rho_{\,max.}$, $\rho_{\,min.}$ and $\rho_{\,t.}$

$$\rho_{\,max.} = 0.85\, \beta_1 \frac{f'_c}{fy} \frac{0.003}{(0.003 + 0.004)} + \rho_f \qquad , \qquad \rho_f = \frac{A_{sf}}{b_w d}$$

$$\rho_{\,min.} = max.of \left\{\frac{\sqrt{f'_c}}{4\, fy} \quad or \quad \frac{1.4}{fy}\right\}$$

$$\rho_{\,t} = 0.85\, \beta_1 \frac{f'_c}{fy} \frac{0.003}{(0.003 + 0.004)} + \rho_f$$

Ex-1:

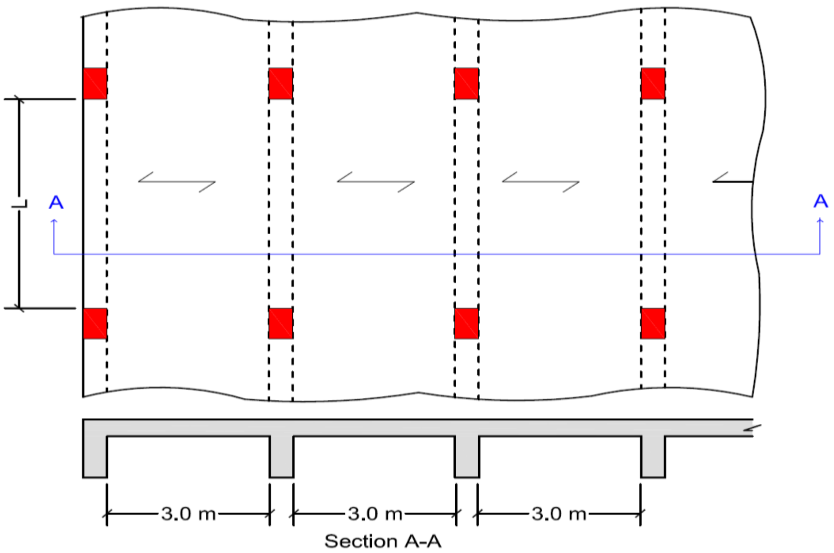
A floor system consists of 140 mm concrete slab supported by continuous beam with: Span $(L), b_w = 300\ mm, d = 550\ mm, f_c' = 28\ MPa\ and\ fy = 420\ MPa$

Determine the steel reinforcement required at mid span of interior beam to resist service dead load moment =320 kN.m and service live load moment =250 kN.m in the following case:

1- L= 8 m.

2- L =2 m.

Use db=25 mm



Solution:

Case 1: L= 8m

$M_u = 1.2M_D + 1.6M_L$

$M_u = 1.2 \times 320 + 1.6 \times 250 = 784\ kN.m$

1- Find the b_e= $\begin{cases} \frac{L}{4} = \frac{8000}{4} & = 2000\ mm \\ b_w + 16\,h_f = 300 + (16 \times 140) & = 2540\ mm \\ b_w + \frac{s_1+s_2}{2} = 300 + \frac{3000+3000}{2} & = 3300\ mm \end{cases}$

2- Calculate (As) assume that a = h_f with beam width (be) and ø =0.9 and then check.

$$M_u = \emptyset\, \rho\, b_e d^2 fy \left(1 - 0.59 \frac{fy}{f_c'} \rho\right)$$

$$0.784 = 0.9 \times \rho \times 2 \times 0.55^2 \times 420 \left(1 - 0.59 \frac{420}{28} \rho\right) \quad \rightarrow \rho = 0.00354$$

Or ρ can be calculated as below

$$\rho = \frac{1 \pm \sqrt{1 - \frac{2.36 * M_u * 10^6}{\emptyset\, b\, d^2 f_c'}}}{1.18\left(\frac{fy}{f_c'}\right)} \quad \rightarrow \quad \rho = \frac{1 \pm \sqrt{1 - \frac{2.36 * 784 * 10^6}{0.9 * 2000 * \ 550^2 * 420}}}{1.18\left(\frac{420}{28}\right)}$$

$\rho = 0.00354$ or $\rho = 0.1094$

Use $\rho = 0.00354$

$$As = \rho b_e d = 0.00354 \times 2000 \times 550 = 3894\ mm^2$$

3- Check the assumption

$$a = \frac{A_s\, fy}{0.85\, f_c'\, b_e} = \frac{3894 \times 420}{0.85 \times 28 \times 2000} = 34.35\ mm < h_f = 140\ mm \quad R. section\ \ ok$$

The assumption is right and continue as a rectangular section.

$$\rho_{max.} = 0.85\, \beta_1\, \frac{f_c'}{fy}\, \frac{0.003}{(0.003 + 0.004)} = 0.0206$$

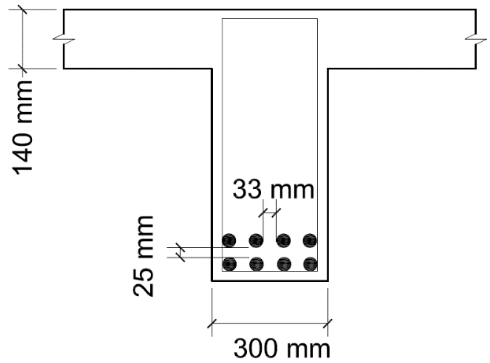
$$\rho_{min.} = max. of \left\{ \frac{\sqrt{f_c'}}{4\, fy} \times \frac{b_w}{b_e} \qquad \frac{1.4}{fy} \times \frac{b_w}{b_e} \right\} = (0.0005)$$

$$\rho_{min.} = (0.0005) < \rho = (0.00345) < \rho_{max.} = (0.0206)$$

$$\rho_t = 0.85\, \beta_1\, \frac{f_c'}{fy}\, \frac{0.003}{(0.003 + 0.005)} = 0.018 \quad > \rho \quad \rightarrow \emptyset = 0.9$$

$As = 3894\ mm^2$, use $8\emptyset 25\ mm$ two layers

$$s = \frac{300 - 2 \times 40 - 2 \times 10 - 4 \times 25}{4 - 1} = 33.3 > 25\ mm\ ok$$



Case 2: L=2 m

1- Find the b_e= $\begin{cases} \frac{L}{4} = \frac{2000}{4} & = 500mm \\ b_w + 16\,h_f = 300 + (16 \times 140) & = 2540mm \\ b_w + \frac{s_1+s_2}{2} = 300 + \frac{3000+3000}{2} & = 3300\ mm \end{cases}$

2- Calculate (As) assume that $a = h_f$ with beam width (be) and $ø$ =*0.9* and then check.

$$\mathrm{M_u} = \emptyset\,\rho\,b_e d^2 fy\left(1 - 0.59\frac{fy}{f_c'}\,\rho\right)$$

$$0.784 = 0.9 \times\ \rho \times 0.5 \times 0.55^2 \times 420\left(1 - 0.59\frac{420}{28}\,\rho\right) \quad \rightarrow \rho = 0.0159$$

Or ρ can be calculated as below

$$\rho = \frac{1 \pm \sqrt{1 - \frac{2.36 * M_u * 10^6}{\emptyset\,b\,d^2 f_c'}}}{1.18\left(\frac{fy}{f_c'}\right)} \quad \rightarrow \rho = 0.0159$$

$$As = \rho b_e d\ = 0.0159 \times 500 \times 550 = 4373\ mm^2$$

3- Check the assumption

$$a = \frac{A_s\,fy}{0.85\ f_c'\ b_e} = \frac{4373 \times 420}{0.85 \times 28 \times 500} = 154.3\ mm > h_f = 140\ mm \quad the\ section\ is\ T$$

The assumption is incorrect and continue as a T- section.

4- Find A_{sf} then find M_{uf}

$$A_{sf}\ fy = 0.85 f_c'(b_e - b_w) \times h_f \quad \rightarrow \quad A_{sf} = \frac{0.85 f_c'(b_e - b_w) \times h_f}{fy} = 1587\ mm^2$$

$$\emptyset M_{nf} = \emptyset\ A_{sf} fy\left(d - {}^{hf}/_{2}\right) = 288\ kN.m$$

$$M_{u,total} = \emptyset(M_{nf} + M_{n\,w}) \quad \rightarrow \emptyset M_{nw} = M_{u,total} - \emptyset M_{nf} = 784 - 288 = 496\ kN.m$$

$$\emptyset \mathrm{M_{nw}} = \emptyset\ \rho_w\ b_w d^2 fy\left(1 - 0.59\frac{fy}{f_c'}\,\rho\right)$$

$$496 \times 10^6 = 0.9\ \rho_w\ 300 \times 550^2 \times 420\left(1 - 0.59\frac{420}{28}\,\rho\right) \rightarrow \rho = 0.017$$

Or ρ can be calculated as below

$$\rho = \frac{1 \pm \sqrt{1 - \frac{2.36 * M_u * 10^6}{\emptyset\,b\,d^2 f_c'}}}{1.18\left(\frac{fy}{f_c'}\right)} \quad \rightarrow \rho = 0.017$$

$$A_{sw} = \rho_w b_w d = 0.017 \times 300 \times 550 = 2805\, mm^2$$

$$A_s = A_{sf} + A_{s\,w} = 1587 + 2805 = 4392\, mm^2$$

5- Check the limit of steel reinforcement

$$\rho_{total} = \frac{A_s}{b_w d} = \frac{4392}{300 \times 550} = 0.0266$$

$$\rho_{\,max.} = 0.85\, \beta_1 \frac{f_c'}{fy} \frac{0.003}{(0.003 + 0.004)} + \rho_f$$

$$\rho_{\,max.} = 0.85 \times 0.85 \frac{28}{420} \frac{0.003}{(0.003 + 0.004)} + \frac{1587}{300 \times 550} = 0.03$$

$$\rho_{\,min.} = max.of \left\{ \frac{\sqrt{f_c'}}{4\, fy} \quad or \quad \frac{1.4}{fy} \right\} = 0.0033$$

$$\rho_{\,t} = 0.85\, \beta_1 \frac{f_c'}{fy} \frac{0.003}{(0.003 + 0.004)} + \rho_f = 0.0276 > \rho_{total} = 0.0266 \quad \rightarrow \emptyset = 0.9$$

$$\rho_{min.} = (0.0033) < \rho_{total} = (0.0266) < \rho_{max.} = (0.03)$$

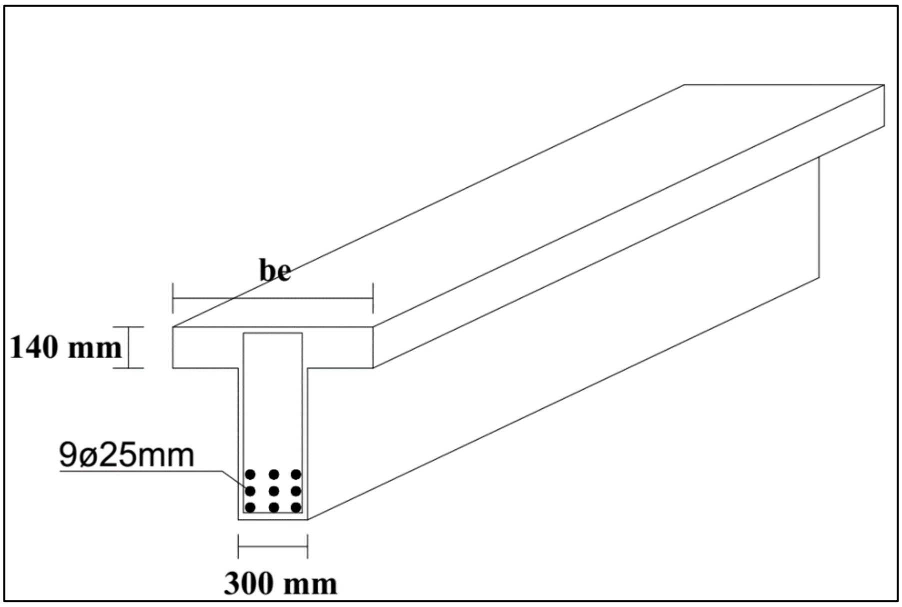
6- Sketch the section and show detail

$A_s = 4392\, mm^2$ try $d_b = 25\, mm$

$$n = \frac{A_s}{\frac{\pi}{4} {d_b}^2} \qquad \rightarrow n = \frac{4392}{\frac{\pi}{4} 25^2} = 8.9$$

Use 9Ø25 mm, three layers

$$S = \frac{300 - 2 \times 40 - 2 \times 10 - 3 \times 25}{(3 - 1)} = 62.5\, mm > 25\, and > d_b(25mm)\, ok$$



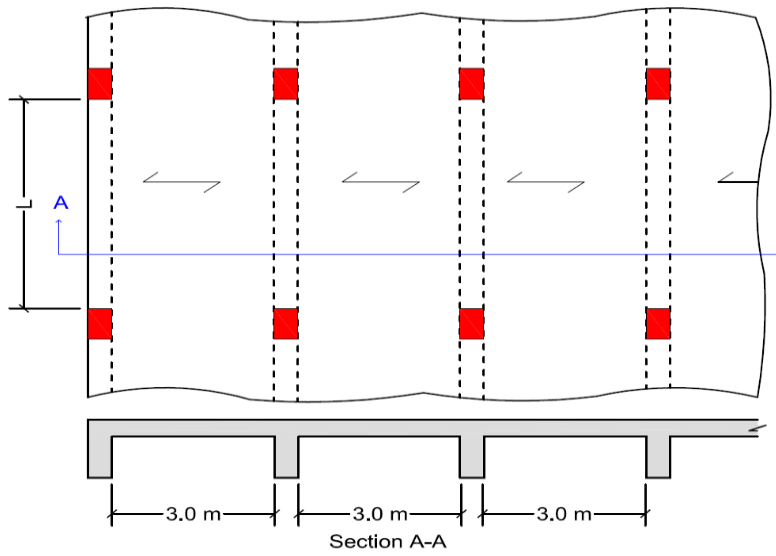
HW-1:

A floor system consists of 140 mm concrete slab supported by continuous beam with: Span $(L), b_w = 300\ mm, d = 500\ mm, f_c' = 28\ MPa\ and\ fy = 420\ MPa$. The service dead load moment =300 kN.m and service live load moment =270 kN.m in the following case:

1-Determine the steel reinforcement required at mid span of interior beam to resist service (Use db=25 mm).

A- L= 6 m.

B- L =2 m.



2-Determine the steel reinforcement required at mid span of exterior beam to resist service (Use db=25 mm).

A- L= 6 m.

B- L =2 m.

Design of Reinforced Concrete Structures I

Al Muthanna University
Collage of Engineering

Design of T sections-2

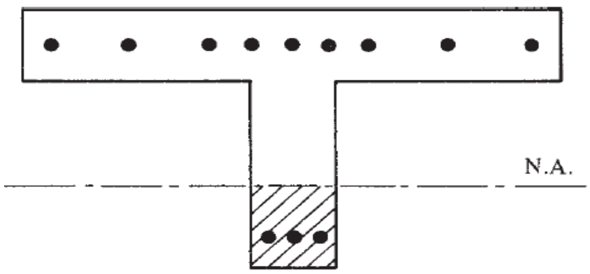
Dr Othman Hameed

Lecture (24)

Design of T-Beams

2- Design of T Beams for Negative Moments

When T beams are resisting negative moments, their flanges will be in tension and part of the web will be in compression, as shown in the Figure below. Obviously, for such situations, the rectangular beam design formulas will be used.

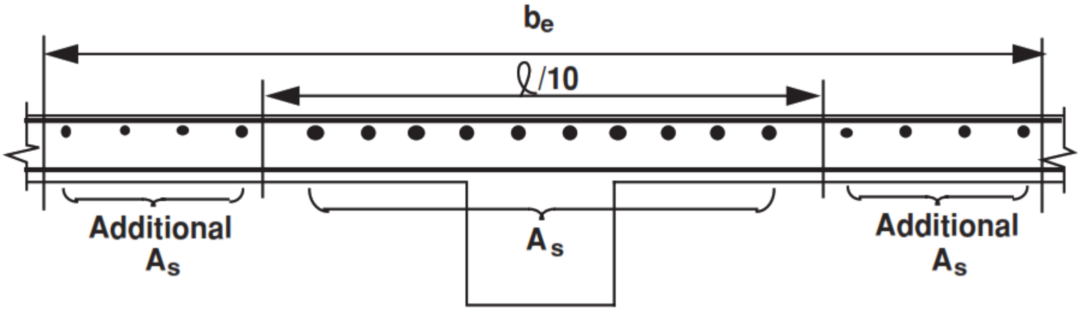


Section 10.5.1 of ACI (2005) stated that If flanges of T-beams are in tension, part of the flexural tension reinforcement shall be distributed over an effective flange, but not wider than ℓn/10.

If the effective flange width exceeds $\frac{L_n}{10}$., some additional longitudinal reinforcement must be added, as illustrated in Figure below. This additional longitudinal reinforcement must be provided in the outer portions of the flange. Section 10.5.1 does not specifically quantify the additional amount of reinforcement required. As a minimum, the amount for temperature and shrinkage reinforcement in should be provided.

$$\rho_{min.} = max.of \begin{Bmatrix} \frac{\sqrt{f'_c}}{4\,fy} & \frac{1.4}{fy} \end{Bmatrix}$$

For statically determinate members with their flanges in tension, (bw) in the above expression is to be replaced with either (2bw) or the width of the flange, whichever is smaller.



Negative Moment Reinforcement for Flanged Floor Beams

Notes

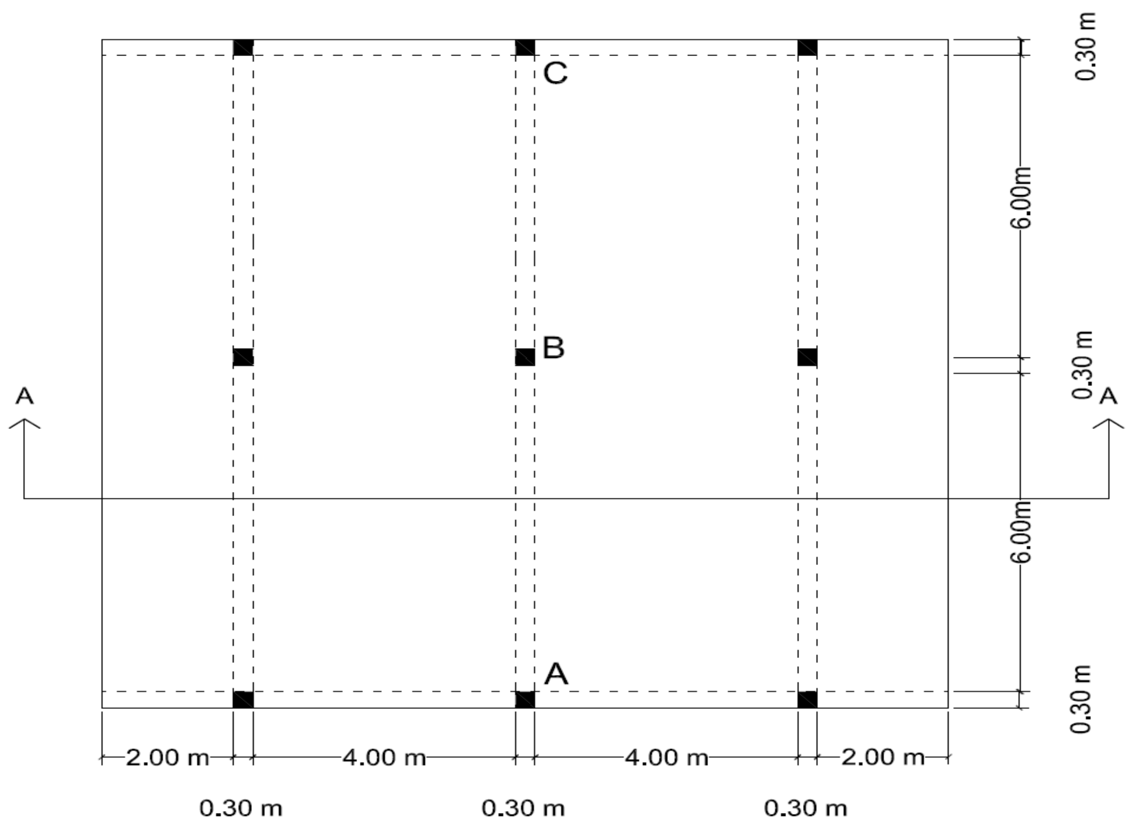
1- If the flange is under a negative moment, the T-section will be designed as a rectangular section (singly or doubly) with b_w width.

2- Flexural tension reinforcement shall be distributed over the minimum of be and $\frac{L_n}{10}$.

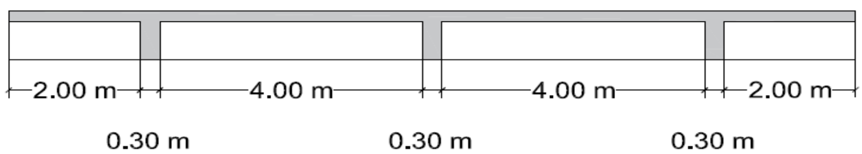
3- For $b_e > \frac{L_n}{10}$, the distance $b_e - \frac{L_n}{10}$ (if any) must be reinforced with $max.of \begin{Bmatrix} \frac{\sqrt{f_c'}}{4\,fy} & \frac{1.4}{fy} \end{Bmatrix}$, and ($b_w$) used to calculate As from in the expression is to be replaced with the minimum of (2bw) or the width of the flange (be)

Ex-1:

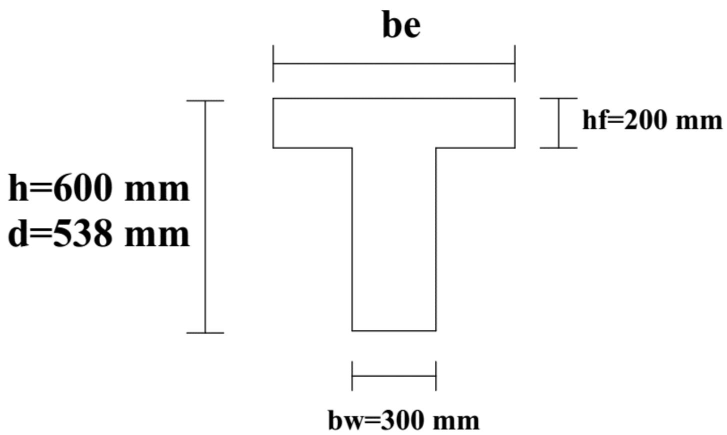
The floor system shown below having 200 mm slab thickness, The beam ABC supports uniform dead load of 13 kN/m (included beam weight) and uniform live load of 31 kN/m, use $f_c' = 28\ MPa$ and $fy = 400\ MPa$, h=600 mm, d=538 mm. Design the flexural reinforcement for interior beams A, B and C. Use db=16 mm.



Floor Plane



Section A-A



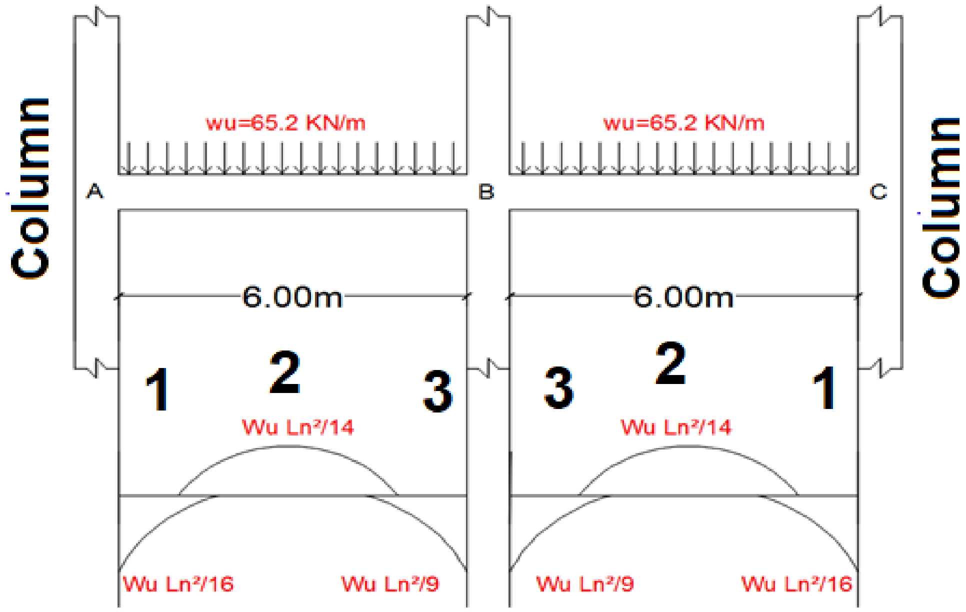
Solution

1- Find the load transformed from slab to beam

$$w_u = 1.2 \times WD + 1.6 \times WL$$

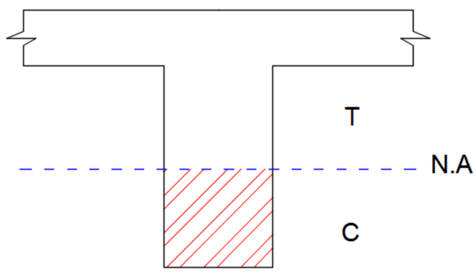
$$w_u = 1.2 \times 13 + 1.6 \times 31 = 65.2\ kN/m$$

2- Find the ultimate moment supported beam



Design the sections 1, 2 and 3

a. <u>Section 1-1</u>



$$M_u^- = \frac{w_u\, l^2}{16} = \frac{65.2 \times 6^2}{16} = 146.7\ kN.m$$

$$\mathrm{M_u} = \emptyset\, \rho\, b_w d^2 fy \left(1 - 0.59 \frac{fy}{f_c'}\, \rho\right)$$

$$\rho = \frac{1 \pm \sqrt{1 - \frac{2.36 * M_u * 10^6}{\emptyset\, b\, d^2 f_c'}}}{1.18\left(\frac{fy}{f_c'}\right)} \quad \rightarrow \rho = 0.004895$$

$$\rho_{max.} = 0.85\,\beta_1\,\frac{f_c'}{fy}\,\frac{0.003}{(0.003+0.004)} = 0.0217$$

$$\rho_{min.} = max.of\left\{\frac{\sqrt{f_c'}}{4\,fy}\ ,\ \frac{1.4}{fy}\right\} = (0.0035)$$

$$\rho_{min.} = (0.0035) < \rho = (0.004895) < \rho_{max.} = (0.0217)$$

$$\rho_t = 0.85\,\beta_1\,\frac{f_c'}{fy}\,\frac{0.003}{(0.003+0.005)} = 0.0189 \ > \rho \quad \rightarrow \emptyset = 0.9$$

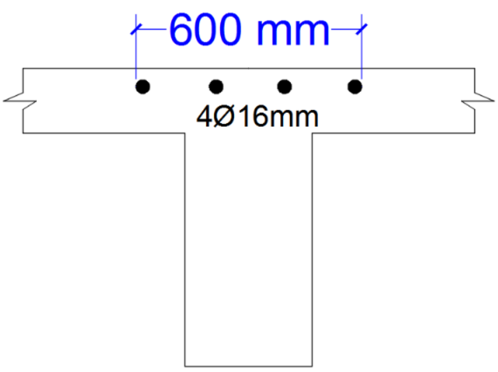
$As = 790\ mm^2$ $,use\ 4\emptyset16\ mm$

Section 10.5.1 of the ACI Code requires that part of the flexural steel in the top of the beam in the negative-moment region be distributed over the **effective width** of the flange or over a width equal to **one-tenth of the beam span**, whichever is smaller.

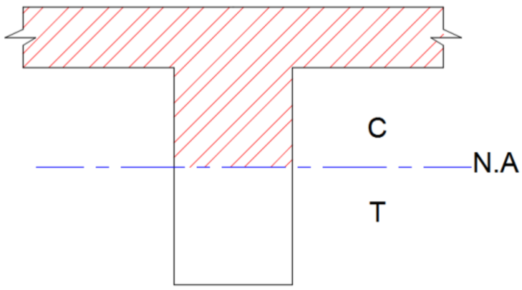
Find the b_e= $\begin{cases} \frac{L}{4} = \frac{6000}{4} & = 1500\ mm \\ b_w + 16\,h_f = 300 + (16\times200) & = 3500\ mm \\ b_w + \frac{s_1+s_2}{2} = 300 + \frac{4000+4000}{2} & = 4300\ mm \end{cases}$

The steel reinforcement will be distributed over the min. of (be=1500mm, $\frac{l}{10} = \frac{6000}{10} = 600\ mm$)

$$s = \frac{600-4\times16}{4-1} = 179 > 25\ mm\ ok$$



Section 2-2



$$M_u^+ = \frac{w_u\, l^2}{14} = \frac{65.2 \times 6^2}{14} = 167.65\ kN.m$$

1- Find the b_e

$$b_e = \begin{cases} \frac{L}{4} = \frac{6000}{4} & = 1500mm \\ b_w + 16\, h_f = 300 + (16 \times 200) & = 3500mm \\ b_w + \frac{s_1 + s_2}{2} = 300 + \frac{4000+4000}{2} & = 4300\ mm \end{cases}$$

2- Calculate (As) assume that $a = h_f$ with beam width (be) and ø =0.9 and then check.

$$M_u = \emptyset\, \rho\, b_e d^2 fy \left(1 - 0.59 \frac{fy}{f_c'}\, \rho\right)$$

$$M_u^+ = 167.65\ kN.m,\ b_e = 1500\ mm,\ d = 538\ mm,$$

$$\rho = \frac{1 \pm \sqrt{1 - \frac{2.36 * M_u * 10^6}{\emptyset\, b\, d^2 f_c'}}}{1.18\left(\frac{fy}{f_c'}\right)} \rightarrow \rho = 0.00108$$

$$As = \rho b_e d = 0.00108 \times 1500 \times 538 = 871.56\ mm^2$$

3- Check the assumption in

$$a = \frac{A_s\, fy}{0.85\, f_c'\, b_e} = \frac{871.56 \times 400}{0.85 \times 28 \times 1500} = 9.76\ mm < h_f = 200\ mm \qquad ok$$

The assumption is right and continuo as rectangular section.

$$\rho_{max.} = 0.85\, \beta_1 \frac{f_c'}{fy} \frac{0.003}{(0.003 + 0.004)} = 0.02017$$

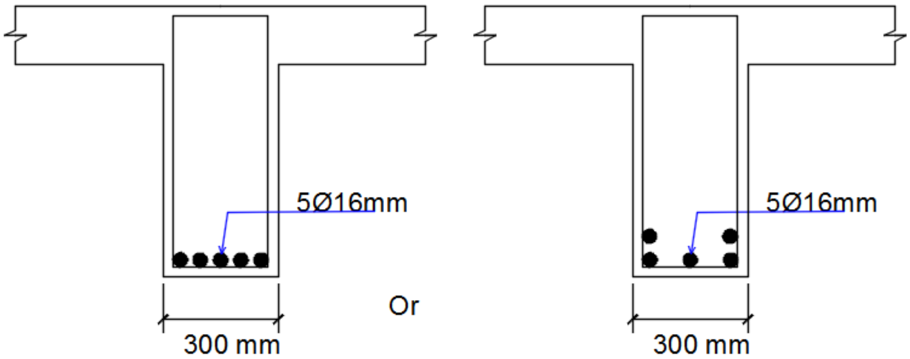
$$\rho_{min.} = max.of \left\{ \frac{\sqrt{f_c'}}{4\, fy} \times \frac{b_w}{b_e} \qquad \frac{1.4}{fy} \times \frac{b_w}{b_e} \right\} = (0.0007)$$

$$\rho_{min.} = (0.0007) < \rho = (0.00108) < \rho_{max.} = (0.0217)$$

$$\rho_t = 0.85\,\beta_1\,\frac{f_c'}{fy}\,\frac{0.003}{(0.003+0.005)} = 0.0189 \;\; > \rho \quad \rightarrow \emptyset = 0.9$$

$$As = 871.56\;mm^2 \;\;, use\;5\emptyset16\;mm$$

$$s = \frac{300 - 2\times40 - 2\times10 - 5\times16}{5-1} = 30 > 25\;mm\;ok$$



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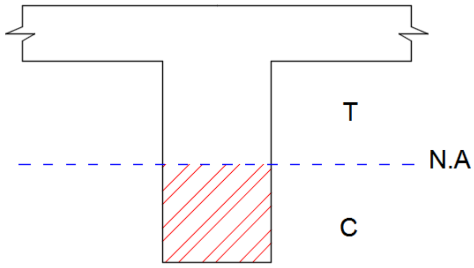
b. <u>Section 3-3</u>



$$M_u^{\,-} = \frac{w_u\,l^2}{9} = \frac{65.2\;\times 6^2}{9} = 260.8\;kN.m$$

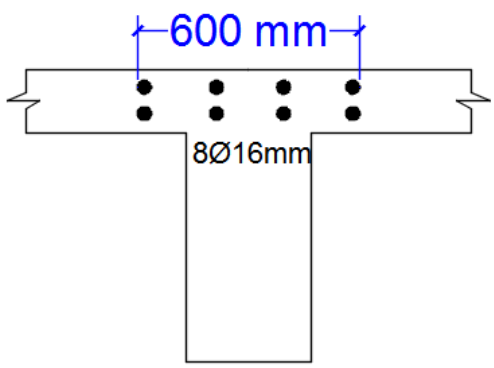
$$\mathrm{M_u} = \emptyset\,\rho\,b_w d^2 fy\left(1 - 0.59\frac{fy}{f_c'}\,\rho\right)$$

$$M_u^{\,+} = 260.8\;kN.m,\; b_e = 300\;mm,\; d = 538\;mm,$$

$$\rho = \frac{1 \pm \sqrt{1 - \dfrac{2.36 * M_u * 10^6}{\emptyset\;b\;d^2 f_c'}}}{1.18\left(\dfrac{fy}{f_c'}\right)} \qquad \rightarrow \rho = 0.00903$$

$$\rho_{max.} = 0.85\,\beta_1\,\frac{f_c'}{fy}\,\frac{0.003}{(0.003+0.004)} = 0.0217$$

$$\rho_{min.} = max.of\left\{\frac{\sqrt{f_c'}}{4\,fy} \qquad \frac{1.4}{fy}\right\} = (0.0035)$$



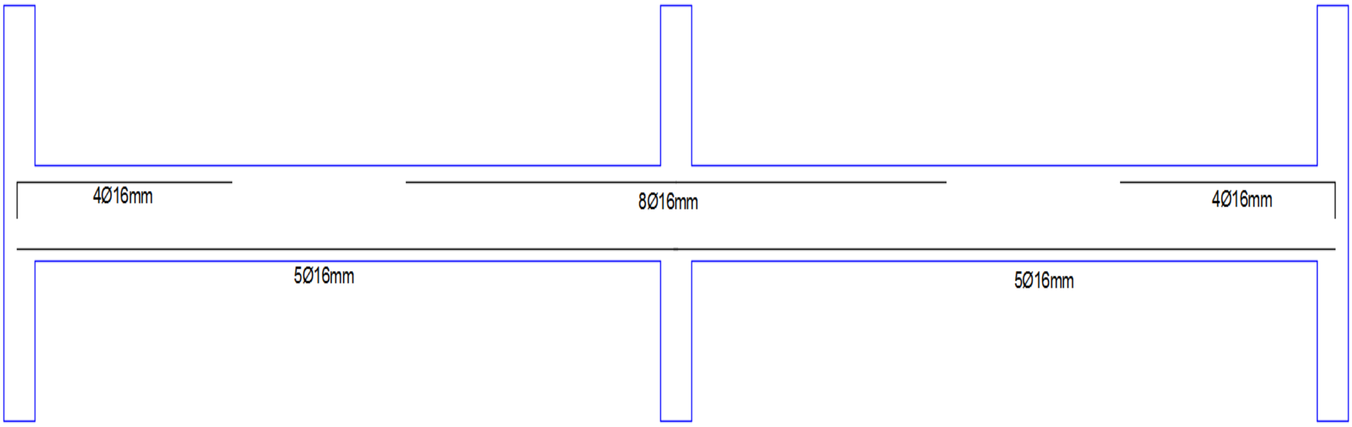
$$\rho_{min.} = (0.0035) < \rho = (0.00903) < \rho_{max.}$$
$$= (0.0217)$$

$$\rho_t = 0.85\,\beta_1 \frac{f_c'}{fy} \frac{0.003}{(0.003+0.005)} = 0.0189 \ > \rho \quad \rightarrow \emptyset = 0.9$$

$As = 1457.44\, mm^2$ $, use\ 8\emptyset 16\ mm\ in\ one\ or\ two\ layers$

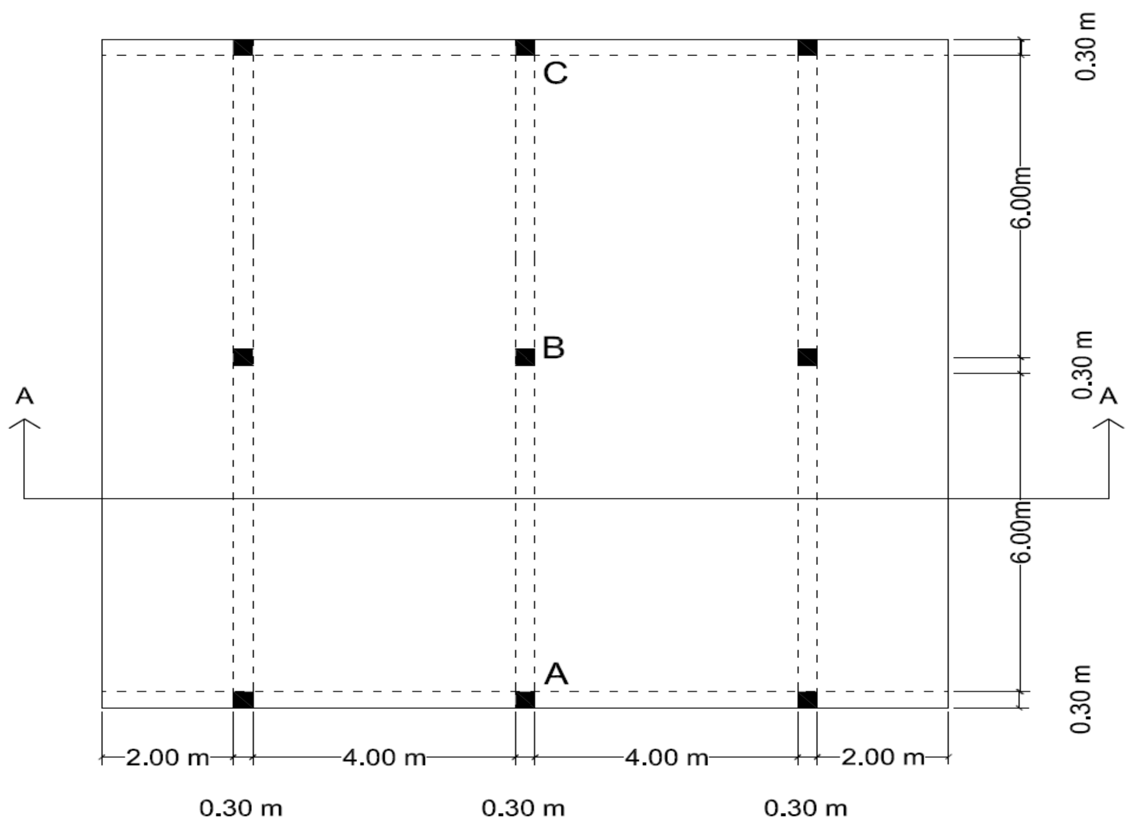
If two layers S will be as below

$$s = \frac{600 - 4 \times 16}{4-1} = 178 > 25\ mm\ ok$$

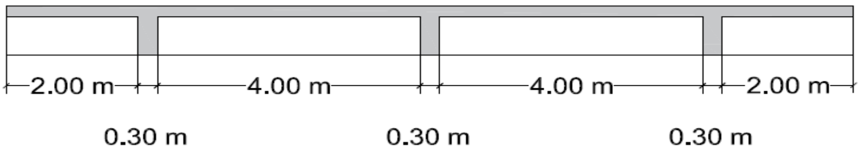


HW.1

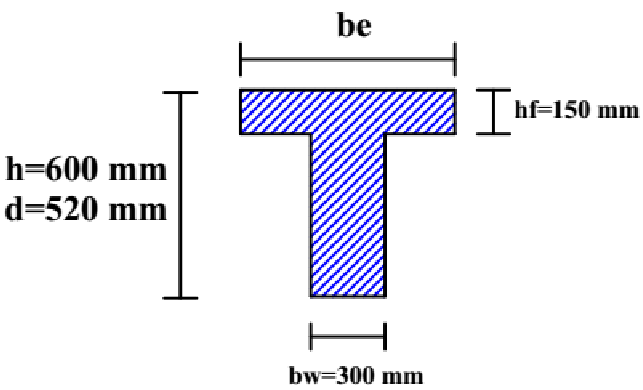
The floor system shown below having 150 mm slab thickness, The beam ABC supports uniform dead load of 20 kN/m (included beam weight) and uniform live load of 40 kN/m, use $f_c' = 28\ MPa$ and $fy = 420\ MPa$, h=600 mm, d=520 mm. Design the flexural reinforcement for interior beams A, B and C. Use db=20 mm.



Floor Plane



Section A-A

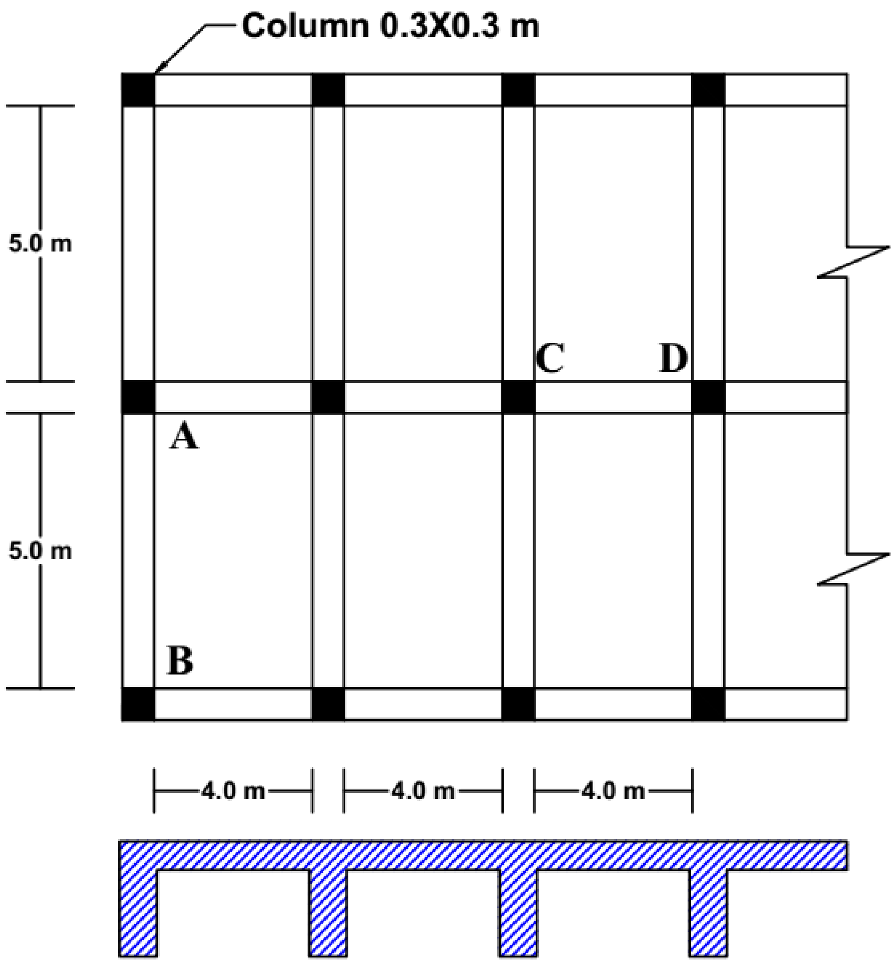


HW.2

The floor system shown below having 150 mm slab thickness, The beams AB and CD support a uniform dead load of 15 kN/m (included beam weight) and uniform live load of 40 kN/m, use $f_c' = 28\ MPa$ and $fy = 420\ MPa$, h=600 mm, d=500 mm.

1- Design the flexural reinforcement for exterior beam A B. Use db=20 mm.

2- Design the flexural reinforcement for interior beam CD. Use db=20 mm.



Al Muthanna University
Collage of Engineering

Design of Reinforced Concrete Structures I

Design of T sections-3

Dr Othman Hameed

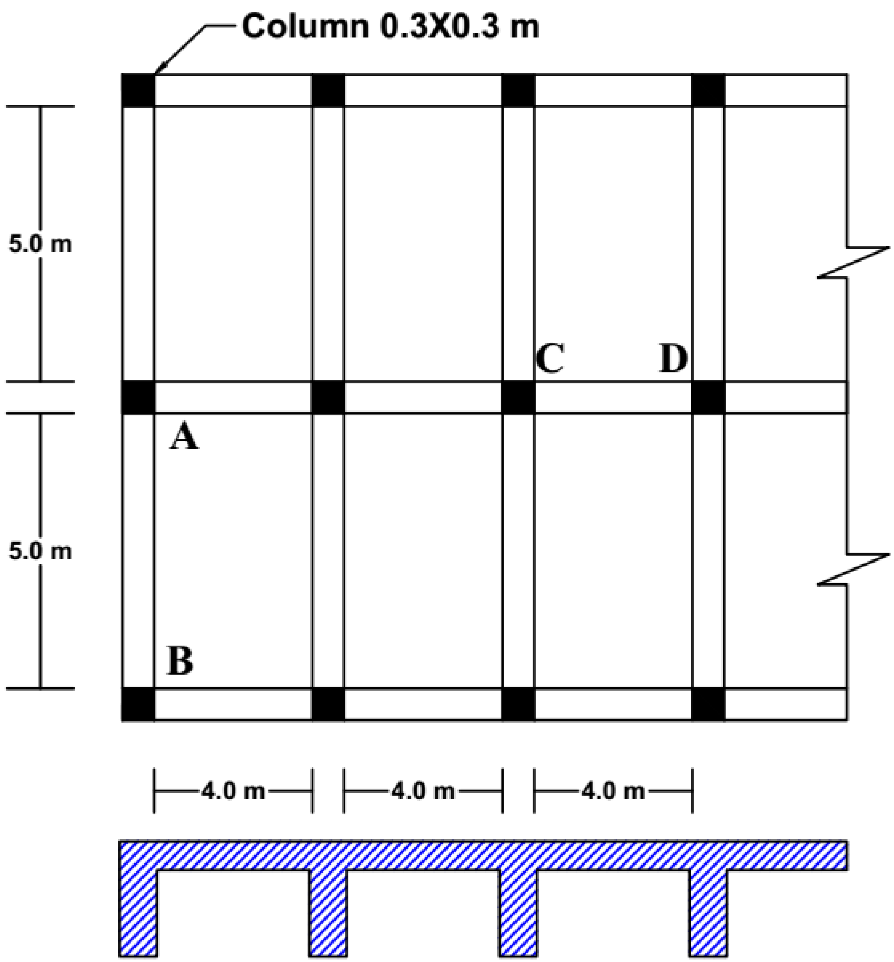
Lecture (25)

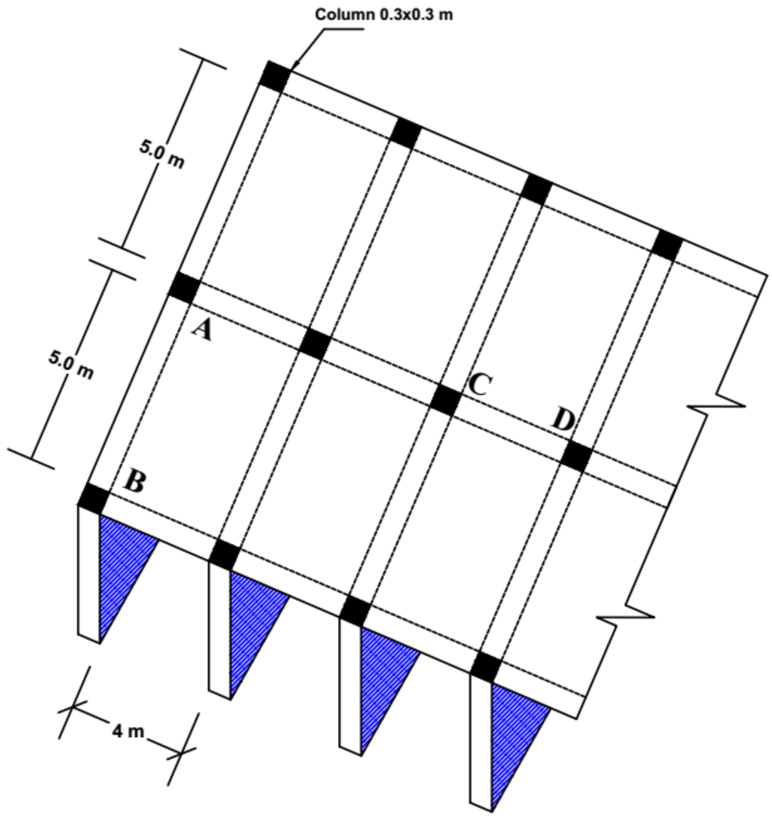
Design of T-Beams

HW.2

The floor system shown below having 150 mm slab thickness, the beams AB and CD support a uniform dead load of 15 kN/m (included beam weight) and uniform live load of 40 kN/m, use $f_c' = 28\ MPa$ and $fy = 420\ MPa$, h=600 mm, d=500 mm.

1- Design the flexural reinforcement for exterior beam AB. Use db=20 mm.
2- Design the flexural reinforcement for interior beam CD. Use db=20 mm.





Solution

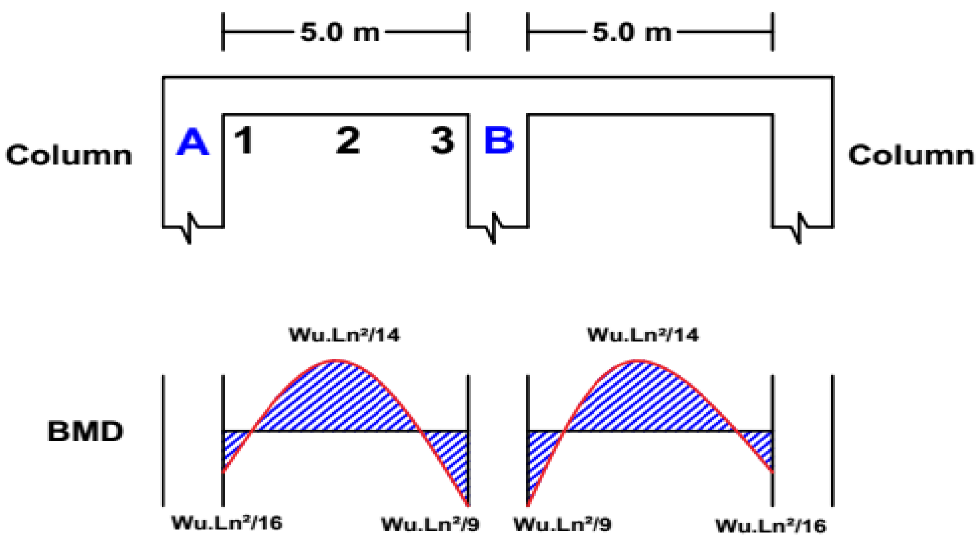
1- Design the flexural reinforcement for exterior beam AB. Use db=20 mm.

- Find the factored load

$$w_u = 1.2 \times WD + 1.6 \times WL$$

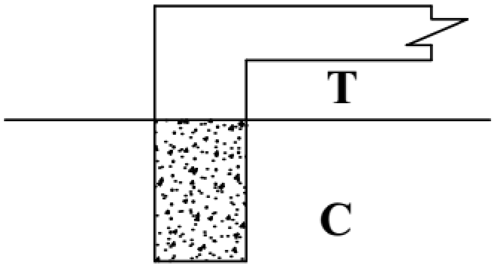
$$w_u = 1.2 \times 15 + 1.6 \times 40 = 82 \ kN/m$$

- Find the ultimate moment supported beam



Design the beam AB

Section 1-1



$$M_u^- = \frac{w_u l^2}{16} = \frac{82 \times 5^2}{16} = 128.125\ kN.m$$

$$\mathrm{M_u} = \emptyset\ \rho\ b_w d^2 fy \left(1 - 0.59 \frac{fy}{f_c'}\ \rho\right)$$

$$b_w = 300\ mm, d = 500\ mm, assume\ \emptyset = 0.9$$

$$f_c' = 28\ MPa \text{ and } fy = 420\ MPa$$

$$\rho = \frac{1 \pm \sqrt{1 - \frac{2.36 * M_u * 10^6}{\emptyset\ b\ d^2 f_c'}}}{1.18\left(\frac{fy}{f_c'}\right)} \quad \rightarrow \rho = 0.0047$$

$$\rho_{max.} = 0.85\ \beta_1\ \frac{f_c'}{fy}\ \frac{0.003}{(0.003 + 0.004)} = 0.0206$$

$$\rho_{min.} = max. of \left\{\frac{\sqrt{f_c'}}{4\ fy}\ \ ,\ \ \frac{1.4}{fy}\right\} = (0.0033)$$

$\rho_{min.} = (0.0033) < \rho = (0.0047) < \rho_{max.} = (0.0206)$, (Singly reinforced section)

$$\rho_t = 0.85\ \beta_1\ \frac{f_c'}{fy}\ \frac{0.003}{(0.003 + 0.005)} = 0.0181 \ \ > \rho \quad \rightarrow \emptyset = 0.9$$

$$As = 707.44\ mm^2\ \ , use\ 3\emptyset 20\ mm$$

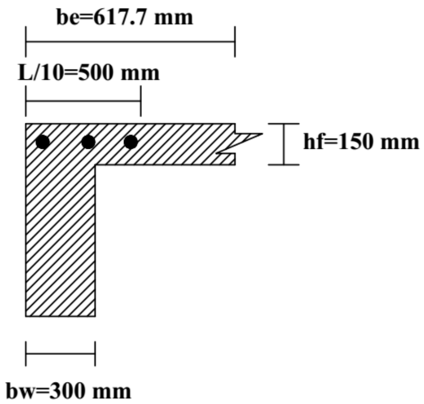
Section 10.5.1 of the ACI Code requires that part of the flexural steel in the top of the beam in the negative-moment region be distributed over the **effective width** of the flange or over a width equal to **one-tenth of the beam span**, whichever is smaller.

For edge section (L-section)

$$\mathrm{b_e} = \begin{cases} \frac{L}{12} + b_w = \frac{5000}{12} + 300 & = 716.7\ mm \\ b_w + 6\ h_f = 300 + 6\ x150 = & 1200\ mm \\ b_w + \frac{s}{2} = 300 + \frac{4000}{2} & = 2300\ mm \end{cases}$$

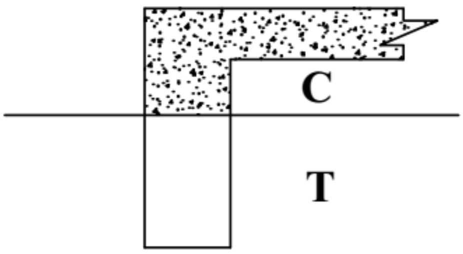
The steel reinforcement will be distributed over the min. of (be=716.7 mm, $\frac{l}{10} = \frac{5000}{10} = 500\ mm$)

$$s = \frac{500-40-10-3\times20}{3-1} = 195 > 25\ mm\ ok$$



Section 2-2

$$M_u{}^{+} = \frac{w_u\, l^2}{14} = \frac{82\times5^2}{14} = 146.43\ kN.m$$



Find the b_e

For edge section (L-section)

$$b_e = \begin{cases} \frac{L}{12} + b_w = \frac{5000}{12} + 300 = 716.7\ mm \\ b_w + 6\ h_f = 300 + 6\ x150 = 1200\ mm \\ b_w + \frac{s}{2} = 300 + \frac{4000}{2} = 2300\ mm \end{cases}$$

Calculate (As) assume that a $= h_f$ with beam width (be) and ø =0.9 and then check.

$$\mathrm{M_u} = \emptyset\ \rho\ b_e d^2 fy\left(1 - 0.59\frac{fy}{f_c'}\ \rho\right)$$

$$M_u{}^{+} = 146.43\ \ kN.m,\ b_e = 716.7\ mm,\ d = 500\ mm,$$

$$\rho = \frac{1 \pm \sqrt{1 - \frac{2.36 * M_u * 10^6}{\emptyset\ b\ d^2 f_c'}}}{1.18\left(\frac{fy}{f_c'}\right)} \quad \rightarrow \rho = 0.0022$$

$$As = \rho b_e d = 0.0022 \times 716.7 \times 500 = 788.37\ mm^2$$

- Check the assumption in

$$a = \frac{A_s\, fy}{0.85\ f_c'\ b_e} = \frac{788.37 \times 420}{0.85 \times 28 \times 716.7} = 19.4\ mm < h_f = 150\ mm \quad ok$$

The assumption is right and continuo as rectangular section.

$$\rho_{max.} = 0.85\ \beta_1\ \frac{f_c'}{fy}\ \frac{0.003}{(0.003 + 0.004)} = 0.0206$$

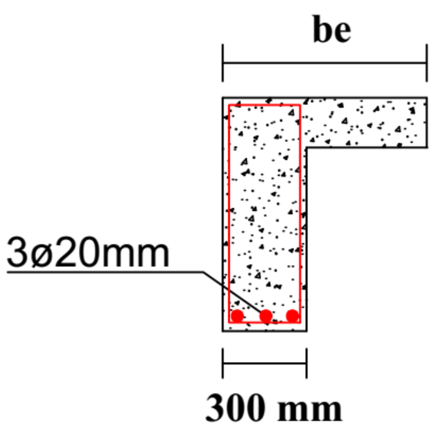
$$\rho_{min.} = max. of \begin{Bmatrix} \frac{\sqrt{f_c'}}{4\ fy} \times \frac{b_w}{b_e} & \frac{1.4}{fy} \times \frac{b_w}{b_e} \end{Bmatrix} = (0.00132\ or\ 0.00139)$$

$$\rho_{min.} = (0.00139) < \rho = (0.0022) < \rho_{max.} = (0.0206)$$

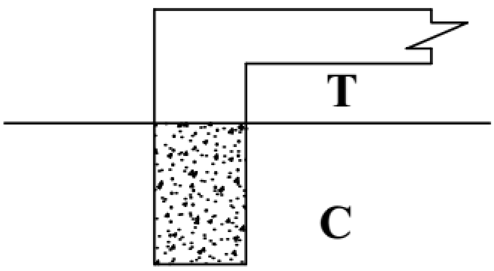
$$\rho_t = 0.85\ \beta_1\ \frac{f_c'}{fy}\ \frac{0.003}{(0.003 + 0.005)} = 0.01806 \ \ > \rho \quad \rightarrow \emptyset = 0.9$$

$$As = 788.37\ mm^2 \ \ , use\ 3\emptyset 20\ mm$$

$$s = \frac{300 - 2 \times 40 - 2 \times 10 - 3 \times 20}{3 - 1} = 70 > 25\ mm\ ok$$



Section 3-3



$$M_u^- = \frac{w_u\, l^2}{9} = \frac{82\ \times 5^2}{9} = 227.78\ kN.m$$

$$\mathrm{M_u} = \emptyset\ \rho\ b_w d^2 fy\ \left(1 - 0.59 \frac{fy}{f_c'}\ \rho\right)$$

$$M_u^+ = 227.78\ kN.m,\ b_w = 300\ mm,\ d = 500\ mm,$$

$$\rho = \frac{1 \pm \sqrt{1 - \frac{2.36 * M_u * 10^6}{\emptyset\, b\, d^2 f_c'}}}{1.18\left(\frac{fy}{f_c'}\right)} \qquad \rightarrow \rho = 0.0035$$

$$\rho_{max.} = 0.85\, \beta_1 \frac{f_c'}{fy} \frac{0.003}{(0.003 + 0.004)} = 0.0206$$

$$\rho_{min.} = max.of \begin{Bmatrix} \frac{\sqrt{f_c'}}{4\, fy} & \frac{1.4}{fy} \end{Bmatrix} = (0.0033)$$

$\rho_{min.} = (0.0033) < \rho = (0.0035) < \rho_{max.} = (0.0206)$ (Singly reinforced section)

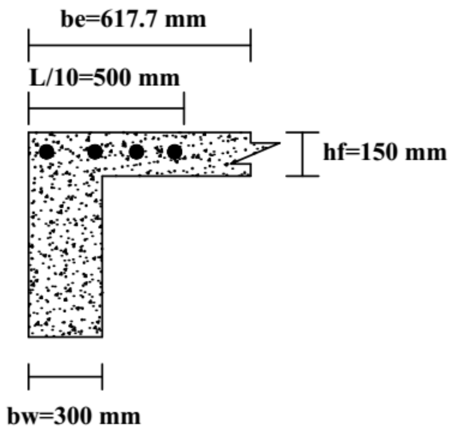
$$\rho_t = 0.85\, \beta_1 \frac{f_c'}{fy} \frac{0.003}{(0.003 + 0.005)} = 0.0181 \ > \rho \quad \rightarrow \emptyset = 0.9$$

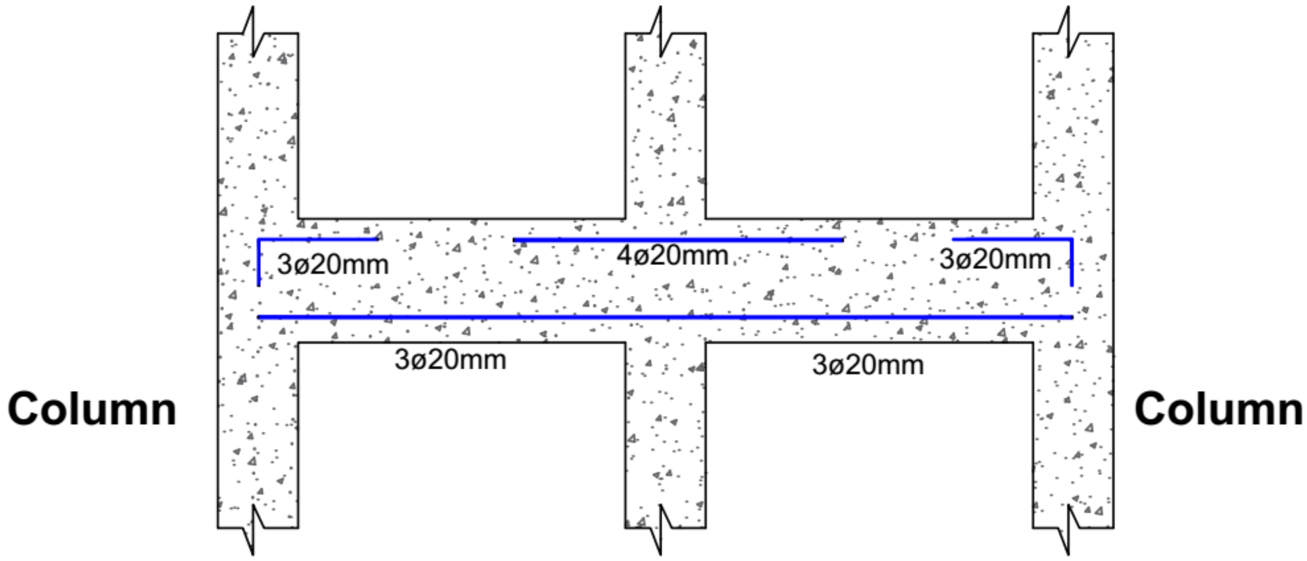
$As = 1243.35\ mm^2$, $use\ 4\emptyset 20\ mm\ in\ one\ \ layer$

The steel reinforcement will be distributed over the min. of (be=716.7 mm, $\frac{l}{10} = \frac{5000}{10} = 500\ mm$)

S will be as below

$$s = \frac{500 - 40 - 10 - 4 \times 20}{4 - 1} = 123 > 25\ mm\ ok$$





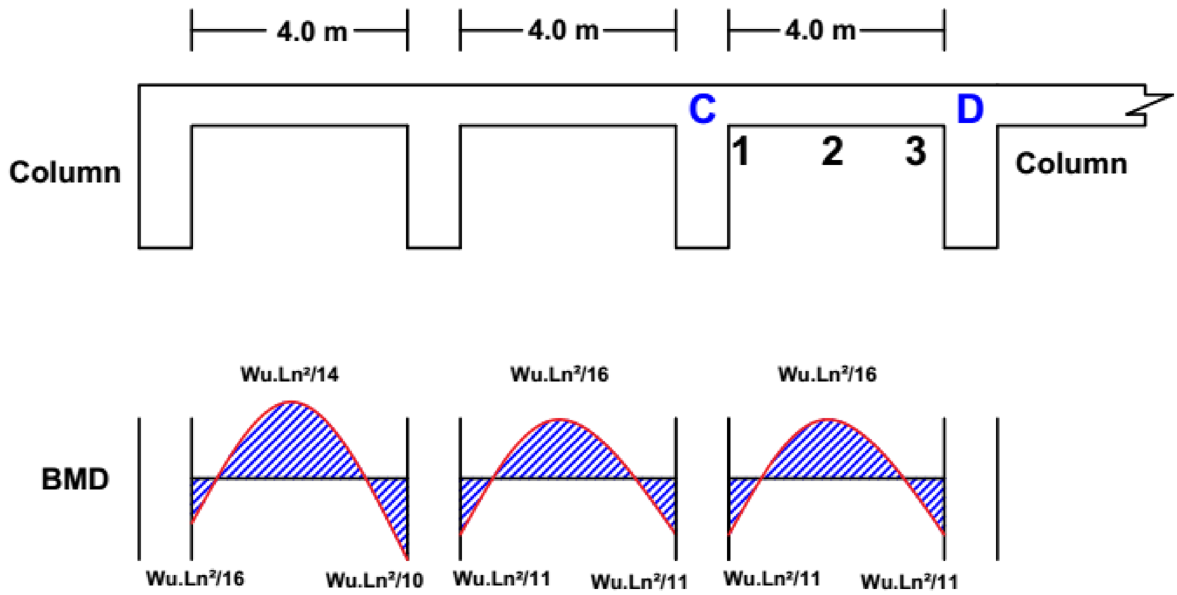
2- Design the flexural reinforcement for interior beam CD. Use db=20 mm.

- Find the factored load

$$w_u = 1.2 \times WD + 1.6 \times WL$$

$$w_u = 1.2 \times 15 + 1.6 \times 40 = 82\ kN/m$$

- Find the ultimate moment supported beam



Design the beam CD

Section 1-1 and 3-3

$$M_u^{-} = \frac{w_u\, l^2}{11} = \frac{82\ \times 4^2}{11} = 119.27\, kN.m$$

Section 2-2

$$M_u^{-} = \frac{w_u\, l^2}{16} = \frac{82\ \times 4^2}{16} = 82\, kN.m$$

Complete the solution----------