Al Muthanna University
Collage of Engineering

Design of Reinforced Concrete Structures I

Shear of reinforced concrete beams-1

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Lecture (1)

<u>Shear of reinforced concrete beams</u>

Introduction

When a beam is loaded, bending moments and shear forces develop along the beam. To carry the loads safely, the beam must be designed for both types of forces. Flexural design is considered first to establish the dimensions of the beam section and the main reinforcement needed, as explained in the previous chapters. The beam is then designed for shear. If shear reinforcement is not provided, shear failure may occur. Shear failure is characterized by small deflections and lack of ductility, giving little or no warning before failure. The diagonal cracks are indication of shear behavior as shown in Figure (1). On the other hand, flexural failure is characterized by vertical cracks increased gradually, giving warning before total failure as shown in Figure (2). This is due to the ACI Code limitation on flexural reinforcement. The design for shear must ensure that shear failure does not occur before flexural failure.

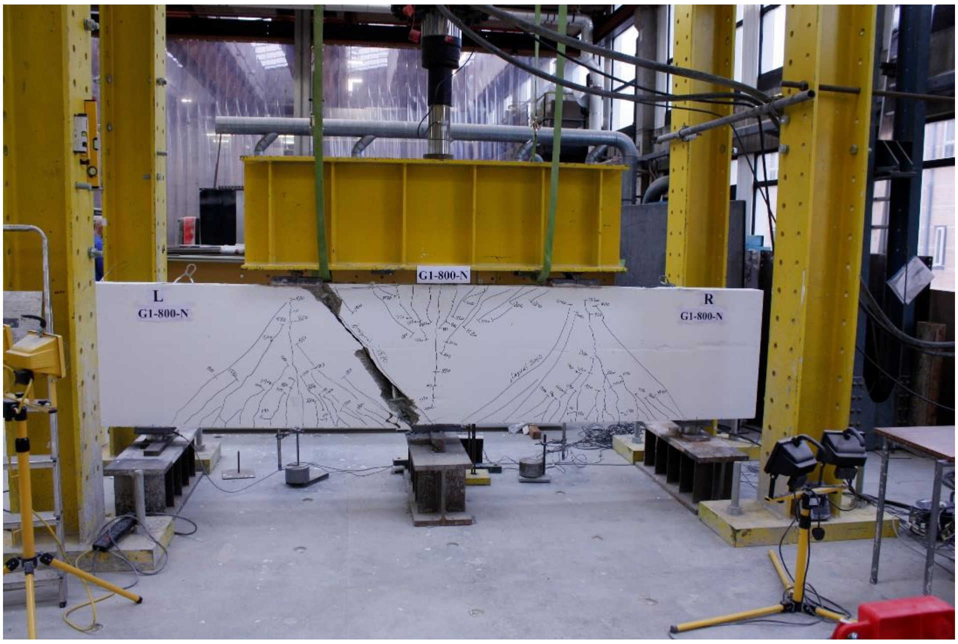

Figure (1) Shear Failure (diagonal crack)

Figure (2) flexural Failure (vertical crack)

Computation of Maximum Factored Shear Force (ACI 11.1.3)

Section 11.1.3 of ACI code describes three conditions that shall be satisfied in order to compute the maximum factored shear force Vu in accordance with 11.1.3.1 for non-prestressed members:

1. Support reaction (in direction of the applied shear force) introduces compression into the end regions of the member.
2. Loads are applied at or near the top of the member.
3. No concentrated load occurs between the face of the support and the location of the critical section, which is a distance (d) from the face of the support.

See Figure 3 (a), (b), and (c) for examples of support conditions where 11.1.3 would be applicable.

Conditions of (section 11.1.3) cannot be applied if:

(1) Members framing into a supporting member in tension, see Fig. 3 (d);

(2) Members loaded near the bottom, see Fig. 3 (e)

(3) Members subjected to an abrupt change in shear force between the face of the support and a distance (d) from the face of the support, see Fig. 3 (f).

In all of these cases, the critical section for shear must be taken at the face of the support.

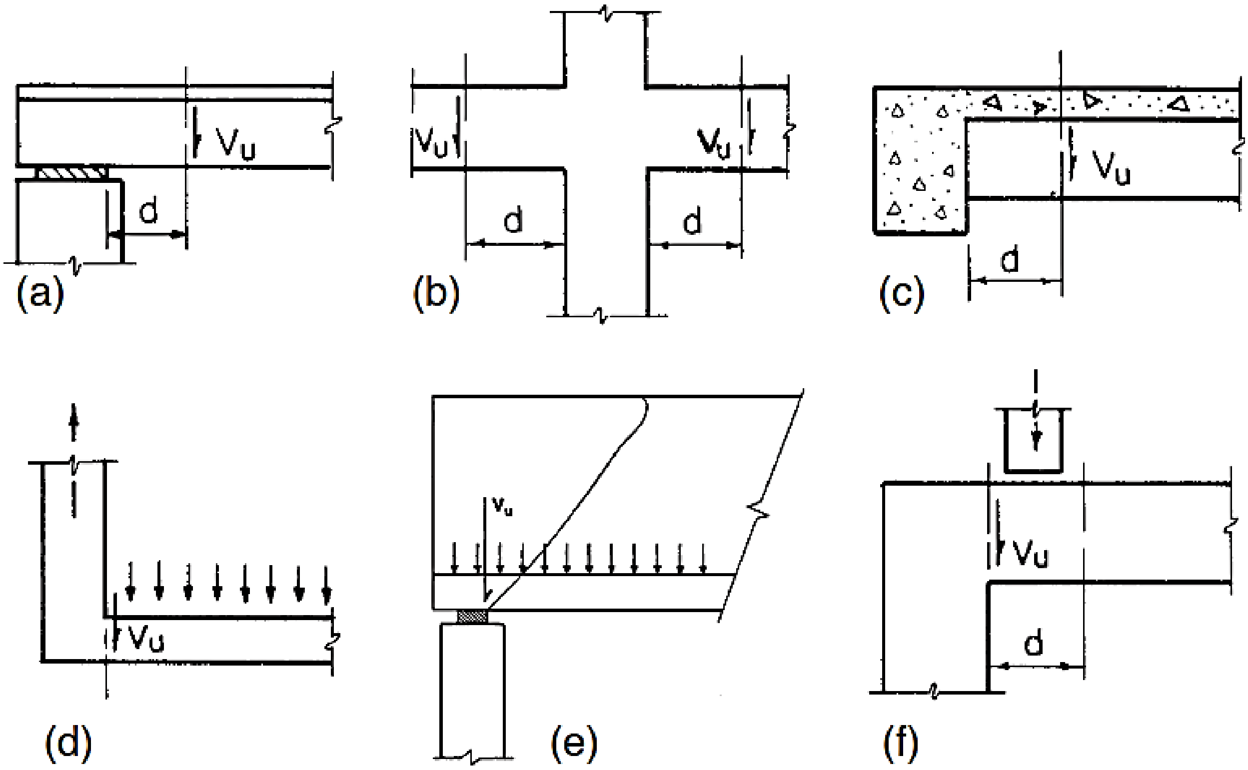

Figure (3): typical support conditions for locating factored shear force

Design to Resist Shear

$$V_u \leq \emptyset\, V_n$$

$$V_u = \emptyset\, [V_c + V_s]$$

Where:

$V_u = factored\ Shear\ force$

$\emptyset\, V_n = shear\ strength\ of\ member$

$\emptyset\ =\ strength\ redection\ factor = 0.75$

$V_c = shear\ strength\ provided\ by\ concrete$

$V_s = shear\ strength\ provided\ by\ steel\ reinforcement$

Shear Strength of Concrete (Section 11.1 of ACI)

The shear strength is based on an average shear stress on the full effective cross section ($b_w d$). In a member without shear reinforcement, shear is assumed to be carried by the concrete web.

In a member with shear reinforcement, a portion of the shear strength is assumed to be provided by the concrete and the remainder by the shear reinforcement.

The shear strength provided by the concrete, V_c, is considered to equal an average shear stress strength ($\frac{\lambda\sqrt{f_c'}}{6}$) times the effective cross-sectional area of the member, $b_w d$,

Where:

b_w is the width of a rectangular beam or width of the web of a T-beam or an I-beam.

$$V_c = \frac{\lambda\sqrt{f_c'}}{6} b_w d \quad \text{(ACI 11.3) (for max. value of } f_c' = 70\ MPa)$$

Where:

$\lambda = 1.0\ for\ normal\ weight\ concrete$

Because of a lack of test data and practical experience with concretes having compressive strengths greater than 70 MPa, the 1989 edition of the Code imposed a maximum value of 8.3 MPa on $\sqrt{f_c'}$ for use in the calculation of shear strength of concrete beams, joists, and slabs

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An alternative, the following shear force (from Section 11.2.1.2 of the code) may be used, which takes into account the effects of the longitudinal reinforcing and the moment and shear magnitudes. This value must be calculated separately for each point being considered in the beam.

Where:

$\lambda = 1.0\ for\ normal\ weight\ concrete$

$$V_c = \left(\lambda\sqrt{f_c'} + 120\rho_w \frac{V_u \, .d}{M_u}\right)\frac{b_w \, .d}{7} \le 0.3\lambda\sqrt{f_c'}\, b_w d \quad \text{(ACI 11.5)}$$

$$\rho_w = \frac{A_s}{b_w \, d}$$

$M_u V_u$ =Are the factored moment occurring simultaneously with, the factored shear at the section considered.

$$\frac{V_u . d}{M_u} \le 1$$

$\lambda = 1.0 \; for \; normal \; weight \; concrete$

Types of Shear Reinforcement (ACI 11.4.1)

Several types and arrangements of shear reinforcement are illustrated in Fig. 4 below. Spirals, circular ties, or hoops are explicitly recognized as types of shear reinforcement starting with the 1999 code.

Vertical stirrups are the most common type of shear reinforcement. Inclined stirrups and longitudinal bent bars are rarely used as they require special care during placement in the field. More details of shear reinforcement are shown in Figure 5.

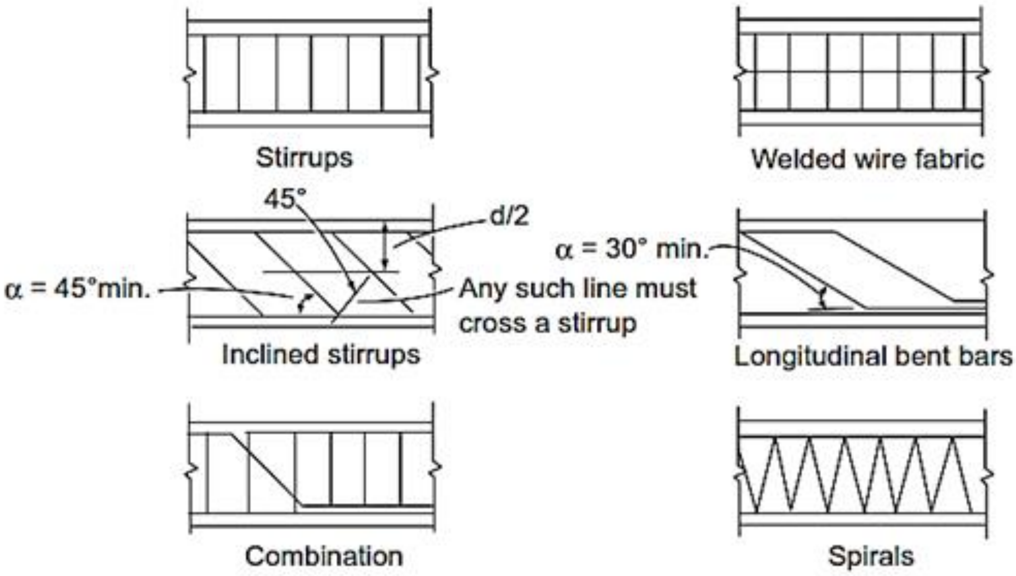


Figure 4 Types and Arrangements of Shear Reinforcement

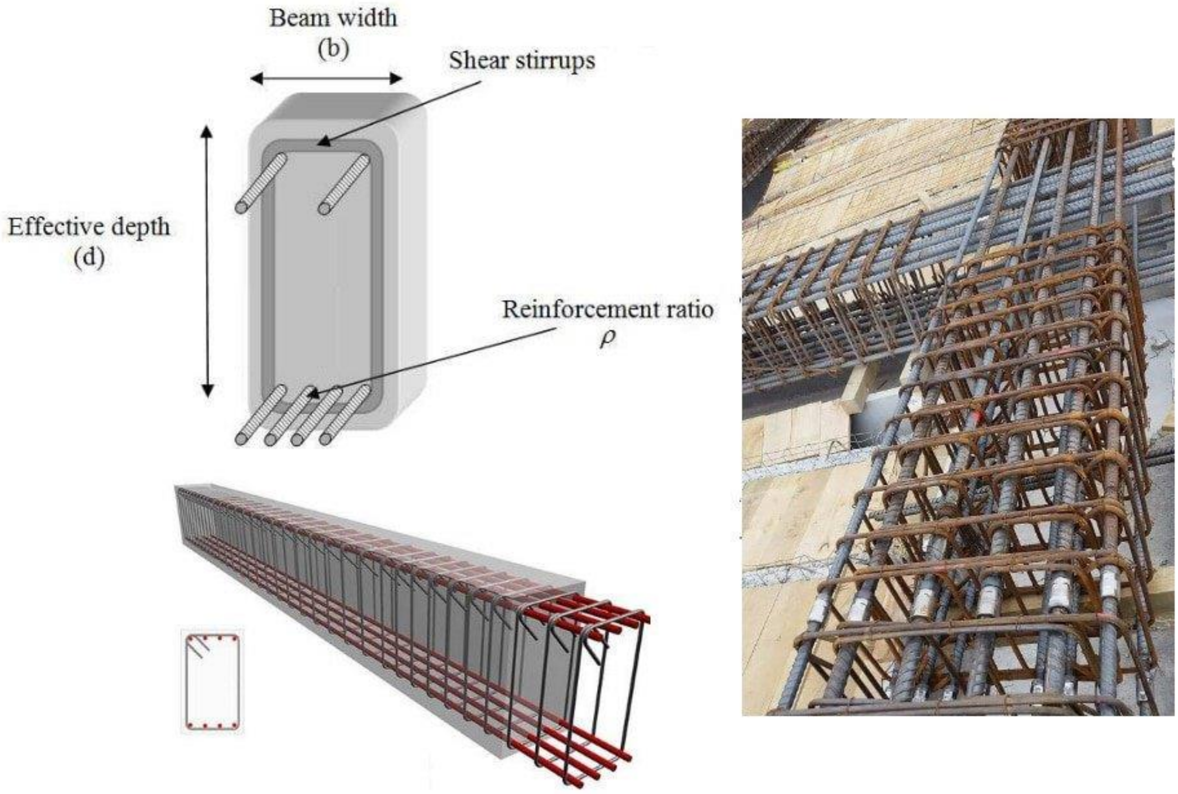


Figure 5: Datils of shear reinforcement

Web Reinforcement (Shear Reinforcement)

When the factored shear, Vu, is high, it shows that large cracks are going to occur unless some type of additional reinforcing is provided.

This reinforcing usually takes the form of stirrups that enclose the longitudinal reinforcing along the faces of the beam as shown in Figure 6.

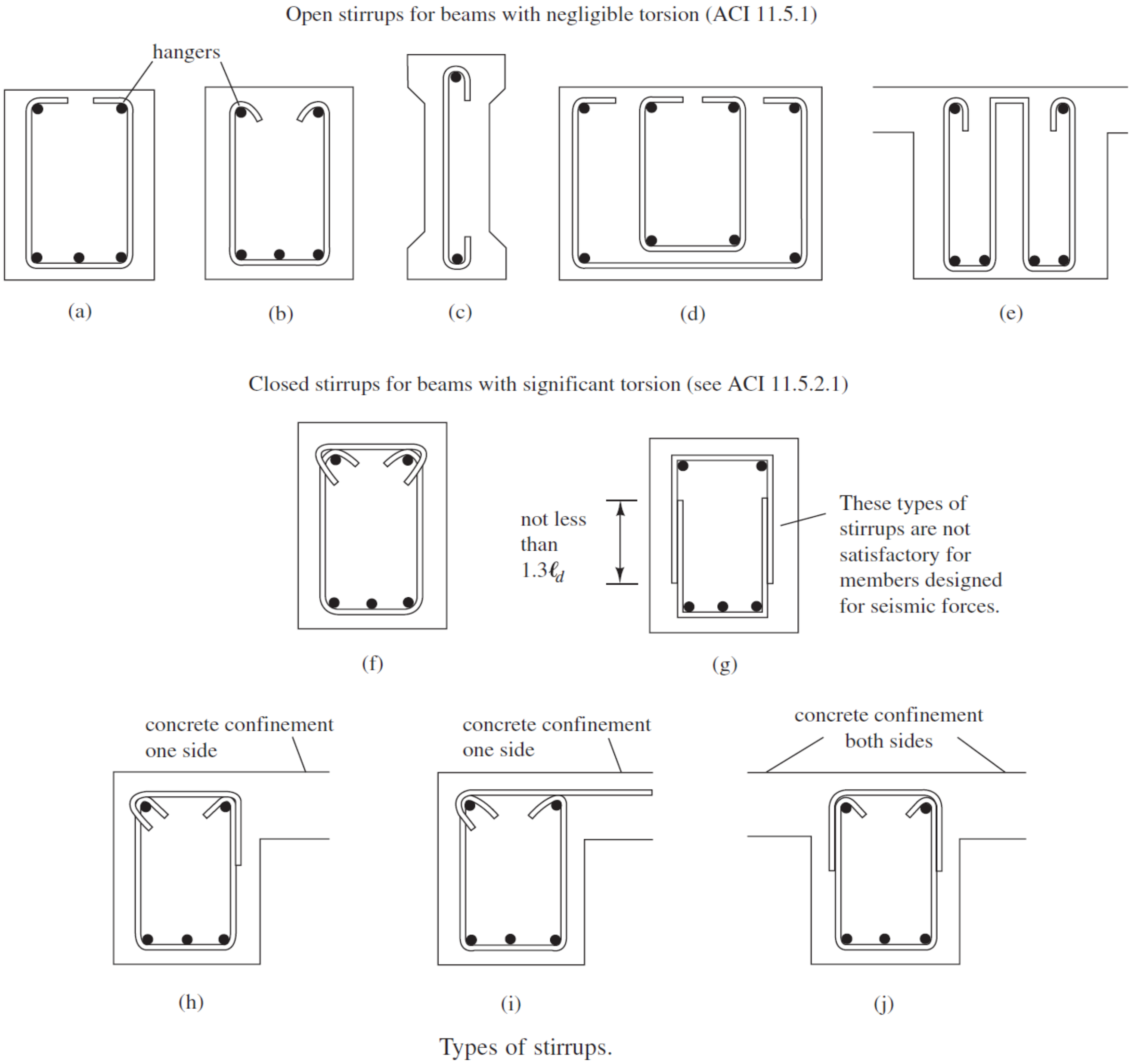


Figure 6: types of stirrups

Three Cases for Shear Reinforcement are available

- Case 1:

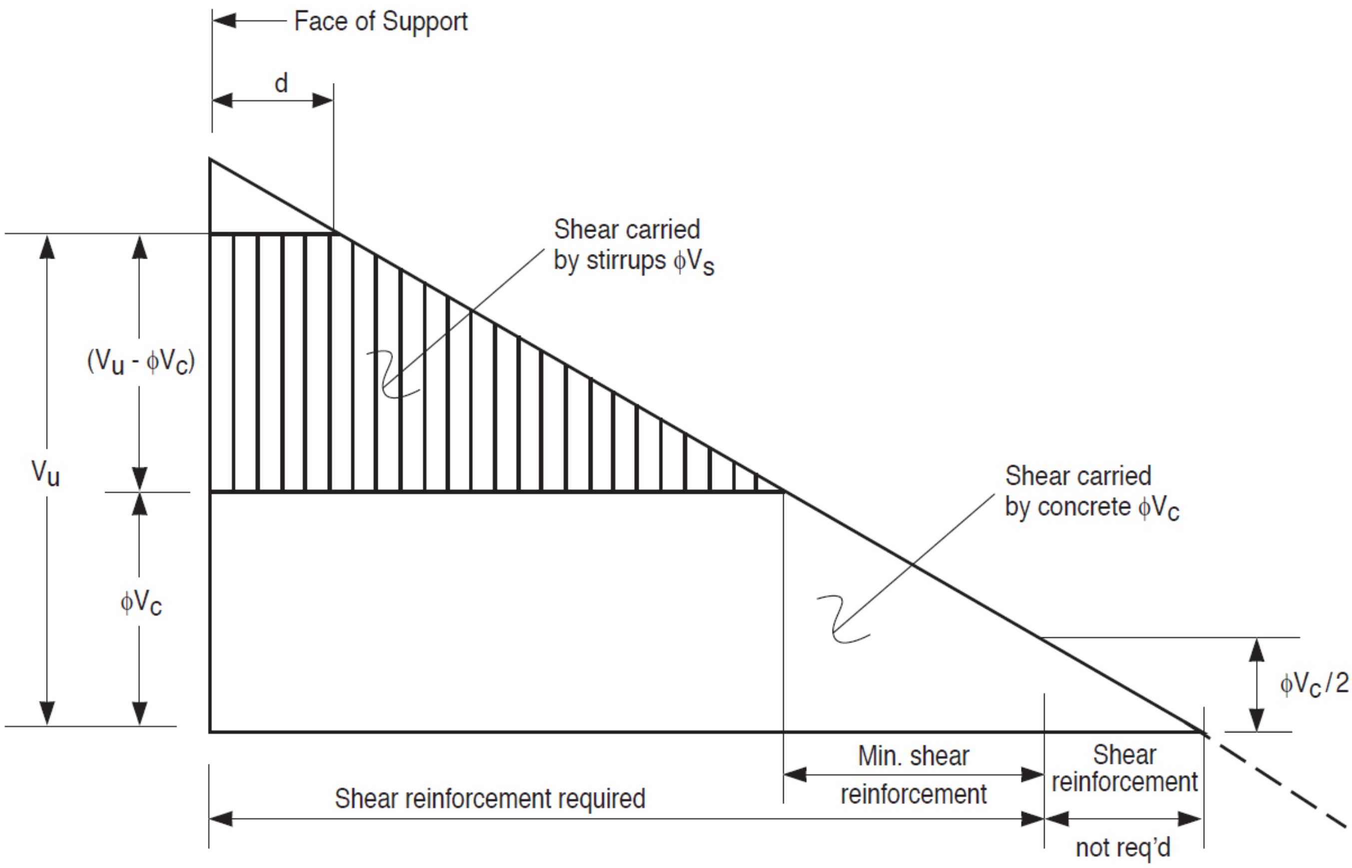
$$V_u \geq \emptyset\, V_c \textit{ shear reinforcement is required}$$

- Case 2:

$$0.5\, \emptyset\, V_c < V_u \leq \emptyset\, V_c \textit{ minimum shear reinforcement is required}$$

- Case 3:

$$V_u < 0.5\emptyset\, V_c \;\; \textit{No shear reinforcement is required}$$

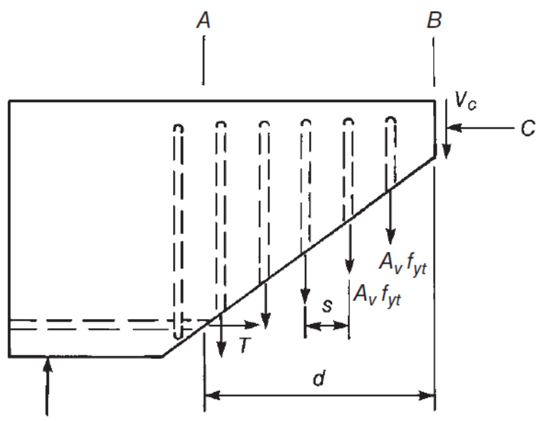


Case 1: $V_u \geq \emptyset\, V_c$

$$V_u = \emptyset\, [V_c + V_s]$$

$$V_s = \frac{V_u}{\emptyset} - V_c$$

a- Vertical stirrup



(a) Vertical stirrups.

$$\boxed{V_s = \frac{A_v.fy.d}{s} \leq \frac{2}{3}\sqrt{f_c'}\, b_w d \quad \text{(ACI 11.15)}}$$

$$\boxed{s = \frac{A_v.fy.d}{V_s}}$$

$$S \leq S_{max.}$$

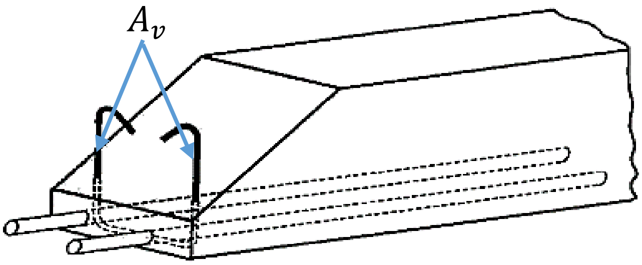
Where:

$\frac{2}{3}\sqrt{f_c'}b_w d$= maximum shear force resisted by shear reinforcement in section

If this condition is not satisfied, the dimensions may need to be increased (ACI 11.4.7.9)

A_v=cross section area of legs of stirrups (in the case of the U-shaped stirrup it is twice the area of one bar)

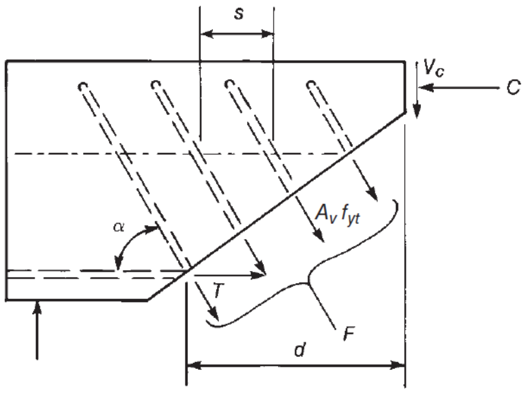
S=spacing of stirrup



b- Inclined stirrup

$$V_s = \frac{A_v\, fy\, d(sin\alpha + cos\alpha)}{s} \leq \frac{2}{3}\sqrt{f_c'}b_w d \quad \text{(ACI 11.16)}$$

$$s = \frac{A_v\, fy\, d(sin\alpha + cos\alpha)}{V_s}$$



(b) Inclined stirrups.

$$\boldsymbol{S \leq S_{max.}}$$

Where:

α=is the angle between inclined stirrup and longitudinal reinforcement.

Case 2:

$0.5\, \emptyset\, V_c < V_u \leq \emptyset\, V_c$ ***minimum shear reinforcement is required***

When the factored shear force Vu exceeds one-half the shear strength provided by concrete ($V_u > \emptyset \frac{V_c}{2}$), a minimum amount of shear reinforcement must be provided.

$$A_{v\ \ min.} = \frac{1}{16}\sqrt{f_c'}\frac{b_w\, s}{fy} \geq \frac{b_w\, s}{3\, fy} \quad \text{(ACI 11.13)}$$

Spacing Limits (maximum spacing) for Shear Reinforcement (ACI11.4.5)

$$\text{If } V_s \leq \frac{1}{3}\sqrt{f_c'}\,b_w d \quad \rightarrow S_{max.} = \min.of \begin{cases} d/2 \\ 600 \\ \dfrac{3\, A_v\, fy}{b_w} \\ \dfrac{16 A_v\, fy}{\sqrt{f_c'}\, b_w} \end{cases}$$

$$\text{If } V_s > \frac{1}{3}\sqrt{f_c'}\,b_w d \quad \rightarrow S_{max.} = \min.of \begin{cases} d/4 \\ 300 \\ \dfrac{3\, A_v\, fy}{b_w} \\ \dfrac{16 A_v\, fy}{\sqrt{f_c'}\, b_w} \end{cases}$$

Design Procedure for Shear Reinforcement

$$V_u @ d \leq \emptyset\, V_n$$

$$V_u @ d = \emptyset\, [V_c + V_s]$$

1. Determine maximum factored shear force (Vu) at a distance d from the face of the support.
2. Determine shear strength provided by the concrete V_c per Eq. (11.3) or Eq. (11.5)

$$V_c = \frac{\lambda \sqrt{f_c'}}{6} b_w d \quad \text{(ACI 11.3)}$$

$$V_c = \left(\lambda \sqrt{f_c'} + 120 \rho_w \frac{V_u d}{M_u}\right) \frac{b_w d}{7} \leq 0.3 \lambda \sqrt{f_c'}\, b_w d \quad \text{(ACI 11.5)}$$

$\lambda = 1.0\ for\ normal\ weight\ concrete$

$$\rho_w = \frac{A_s}{b_w d}$$

$M_u V_u$ =Are the factored moment occurring simultaneously with the factored shear at the section considered.

$$\frac{V_u . d}{M_u} \leq 1$$

3- Check V_u with $\emptyset\, V_c$ and $(0.5\ \emptyset\, V_c)$

Case 1:

$$V_u \geq \emptyset\, V_c\ shear\ reinforcement\ is\ required$$

4. if $V_u \geq \emptyset\, V_c$, then compute $V_s = \frac{V_u}{\emptyset} - V_c$ at the critical section.

$$\boxed{V_s \leq \frac{2}{3} \sqrt{f_c'} b_w d \quad \text{(ACI 11.15)}}$$

If $V_s > \frac{2}{3} \sqrt{f_c'} b_w d$, increase the size of the section or the concrete compressive strength.

5. Select diameter for the vertical stirrups (Av) and calculate (S)

$$\boxed{s = \frac{A_v . fy . d}{V_s}}$$

Vertical stirrups

$$s = \frac{A_v\, fy\, d(sin\alpha + cos\alpha)}{V_s}$$

Inclined stirrup

Where:

α= Is the angle between inclined stirrup and longitudinal reinforcement.

6- Check the spacing limit $S_{max.}$.

$$\text{If } \ V_s \leq \tfrac{1}{3}\sqrt{f_c'}\, b_w d \quad \rightarrow S_{max.} = \min.of \begin{cases} d/2 \\ 600 \\ \frac{3\, A_v\, fy}{b_w} \\ \frac{16 A_v\, fy}{\sqrt{f_c'}\, b_w} \end{cases}$$

$$\text{If } \ V_s > \tfrac{1}{3}\sqrt{f_c'}\, b_w d \quad \rightarrow S_{max.} = \min.of \begin{cases} d/4 \\ 300 \\ \frac{3\, A_v\, fy}{b_w} \\ \frac{16 A_v\, fy}{\sqrt{f_c'}\, b_w} \end{cases}$$

Case 2:

$0.5\, \emptyset\, V_c < V_u \leq \emptyset\, V_c$ *minimum shear reinforcement is required*

$$A_{v\ min.} = \frac{1}{16}\sqrt{f_c'}\frac{b_w\, s}{fy} \geq \frac{b_w\, s}{3\, fy} \quad \text{(ACI 11.13)}$$

Use $S_{max.}$ calculated from point (6)

Case 3:

$V_u < 0.5\emptyset\, V_c$ *No shear reinforcement is required*

Note

1- Where stirrups are required, it is usually more expedient to select a bar size and type (U-stirrups (2 legs)) and determine the required spacing.

2- Larger stirrup diameters at wider spacing are usually more cost effective than smaller stirrup sizes at closer spacing because the stirrups of closer spacing required high costs for fabrication and placement.

Design of Reinforced Concrete Structures I

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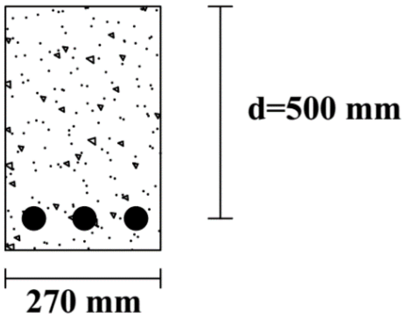
Shear of reinforced concrete beams-2

Dr Othman Hameed

Lecture (2)

Example-1: A rectangular reinforced concrete beam with b_w =270 mm, d= 500 mm, $f_c' = 21\ MPa$ and fy=276 MPa. Determine the required spacing of Ø 10 mm stirrups if:

1- Vu=35 kN
2- Vu=92 kN
3- Vu=236 kN
4- Vu=473 kN



Solution:

1- Vu=35 kN

$$V_c = \frac{\lambda\sqrt{f_c'}}{6} b_w d = \frac{\sqrt{21}}{6} * 270 * 500 * 10^{-3} = 103.1\ kN$$

$$\emptyset V_c = 0.75 * 103.1 = 77.33\ kN$$

$$0.5\ \emptyset V_c = 0.5 * 77.33 = 38.66\ kN$$

$$V_u = 35\ kN < \emptyset \frac{V_c}{2} = 38.66\ kN$$

No shear reinforcement required

2- Vu=92 kN

$$V_u = 92\ kN > \emptyset V_c = 77.33\ kN \ \rightarrow shear\ reinforcemenr\ required$$

$$V_u = \emptyset(V_c + V_s) \ \rightarrow V_s = \frac{V_u}{\emptyset} - V_c \quad \rightarrow V_s = \frac{92}{0.75} - 103.1 = 19.56\ kN$$

- $check\ the\ V_s\ limitation\ where\ V_s \le \frac{2}{3}\sqrt{f_c'}\ b_w d$

$$V_s = 19.56\ kN < \frac{2}{3}\sqrt{21} * 270 * 500 * 10^{-3} = 412.4kN \quad ok$$

$$S = \frac{(\frac{\pi}{4}(10)^2) * 2 * 276 * 500}{19.56 * 10^3} = 1108.2\, mm$$

- *Check the spacing limitation (S)*

$$V_s = 19.56\, kN < \frac{1}{3}\sqrt{21} * 270 * 500 * 10^{-3} = 206.2\, kN \quad ok$$

If $V_s \leq \frac{1}{3}\sqrt{f_c'} b_w d \quad \rightarrow S_{max.} = \min. of \begin{cases} {}^{d}/_{2} = 250mm \ \ control \\ 600 \\ \frac{3\, A_v\, fy}{b_w} = 481mm \\ \frac{16 A_v\, fy}{\sqrt{f_c'}\, b_w} = 560mm \end{cases}$

S>$S_{max.}$ Use S=250 mm   Ø10 mm@250 mm

3- Vu=236 kN

$$V_u = 236\, kN > \emptyset V_c = 77.33 kN \ \rightarrow shear\ reinforcemenr\ required$$

$$V_u = \emptyset(V_c + V_s) \ \rightarrow V_s = \frac{V_u}{\emptyset} - V_c \quad \rightarrow V_s = \frac{236}{0.75} - 103.1 = 211.56 kN$$

- *check the V_s limitation where $V_s \leq \frac{2}{3}\sqrt{f_c'}\, b_w d$*

$$V_s = 211.56\, kN < \frac{2}{3}\sqrt{21} * 270 * 500 * 10^{-3} = 412.4 kN \quad ok$$

$$S = \frac{(\frac{\pi}{4}(10)^2) * 2 * 276 * 500}{211.56 * 10^3} = 102.46\, mm$$

- *Check the spacing limitation (S)*

$$V_s = 211.56\, kN > \frac{1}{3}\sqrt{21} * 270 * 500 * 10^{-3} = 206.2\, kN \quad ok$$

If $V_s > \frac{1}{3}\sqrt{f_c'}b_w d \quad \rightarrow S_{max.} = \text{min. of} \begin{cases} d/4 = 125mm \ \ \textcolor{red}{control} \\ 300\ mm \\ \frac{3\,A_v\,fy}{b_w} = 481mm \\ \frac{16A_v\,fy}{\sqrt{f_c'}\,b_w} = 560mm \end{cases}$

S< Smax. Use S=100 mm Ø10 mm@100 mm

4- Vu=473 kN

$$V_u = 473\ kN > \emptyset V_c = 77.33kN \ \rightarrow shear\ reinforcemenr\ required$$

$$V_u = \emptyset(V_c + V_s) \ \rightarrow V_s = \frac{V_u}{\emptyset} - V_c \quad \rightarrow V_s = \frac{473}{0.75} - 103.1 = 527.56kN$$

- $check\ the\ V_s\ limitation\ where\ V_s \le \frac{2}{3}\sqrt{f_c'}\ b_w d$

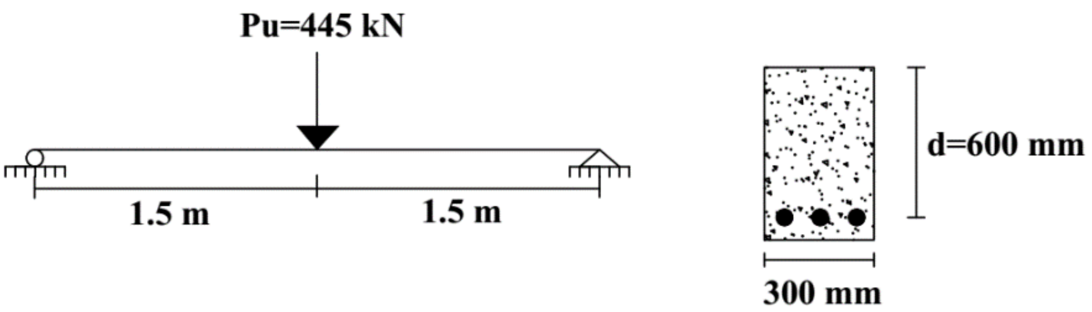
$$V_s = 527.56\ kN > \frac{2}{3}\sqrt{21} * 270 * 500 * 10^{-3} = 412.4kN \quad not\ ok$$

Increase section dimension

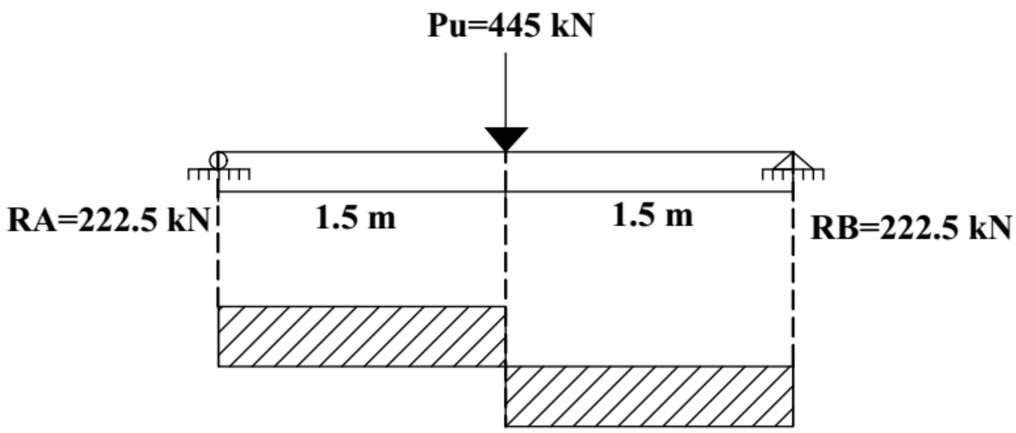
Example-2: For a rectangular reinforced concrete beam of ultimate load (factored load) and details shown in the below figure, neglect the beam weight and find the spacing (S) of Ø10 mm stirrups at a critical section if:

1- Stirrups are vertical.
2- Stirrups are inclined by 45°.

Use $\frac{f_c'}{fy} = \frac{28}{400} MPa$



Solution:



1-Stirrups are vertical.

1- Find an ultimate shear force @distance (d)

$V_u = 222.5\ kN$

2- Find the shear force provided by concrete.

$$V_c = \frac{\lambda\sqrt{f_c'}}{6} b_w d = \frac{\sqrt{28}}{6} * 300 * 600 * 10^{-3} = 159\ kN$$

$\emptyset V_c = 0.75 * 159 = 119.25\ kN$

$$V_u = 222.5\ kN > \emptyset V_c = 119.25\ kN \rightarrow shear\ reinforcement\ is\ required$$

3- Find shear force resisted by shear reinforcement

$$V_u = \emptyset(V_c + V_s) \rightarrow V_s = \frac{V_u}{\emptyset} - V_c \quad \rightarrow V_s = \frac{222.5}{0.75} - 159 = 138\ kN$$

4- $check\ the\ V_s\ limitation\ where\ V_s \leq \frac{2}{3}\sqrt{f_c'}\ b_w d$

$$V_s = 138\ kN < \frac{2}{3}\sqrt{28} * 300 * 600 * 10^{-3} = 635\ kN \quad ok$$

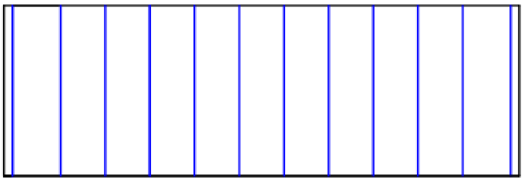
5- $S = \frac{A_v * fy * d}{Vs} = \frac{(\frac{\pi}{4}(10)^2)*2*400*600}{138*1000} = 273.2 \cong 270\ mm$

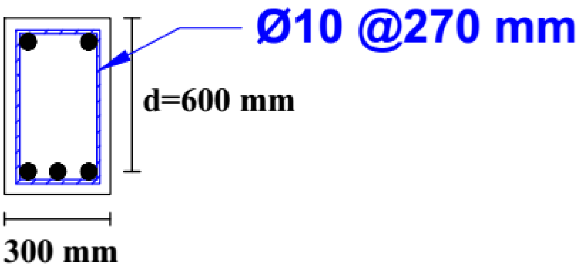
6- *Check the spacing limitation (S)*

$$V_s = 138\ kN < \frac{1}{3}\sqrt{28} * 300 * 600 * 10^{-3} = 318\ kN \quad ok$$

If $V_s \leq \frac{1}{3}\sqrt{f_c'}b_w d \quad \rightarrow S_{max.} = \min. of \begin{cases} d/2 = 300mm \ \textcolor{red}{control} \\ 600\ mm \\ \frac{3\ A_v\ fy}{b_w} = 628\ mm \\ \frac{16 A_v\ fy}{\sqrt{f_c'}\ b_w} = 633\ mm \end{cases}$

S< Smax Use Ø10mm @270mm





Note: add 50 mm as a cover for the stirrups

2-Stirrups are inclined by 45˚.

Same previous steps

$$V_s = 138\ kN < \frac{2}{3}\sqrt{28} * 300 * 600 * 10^{-3} = 635\ kN \quad ok$$

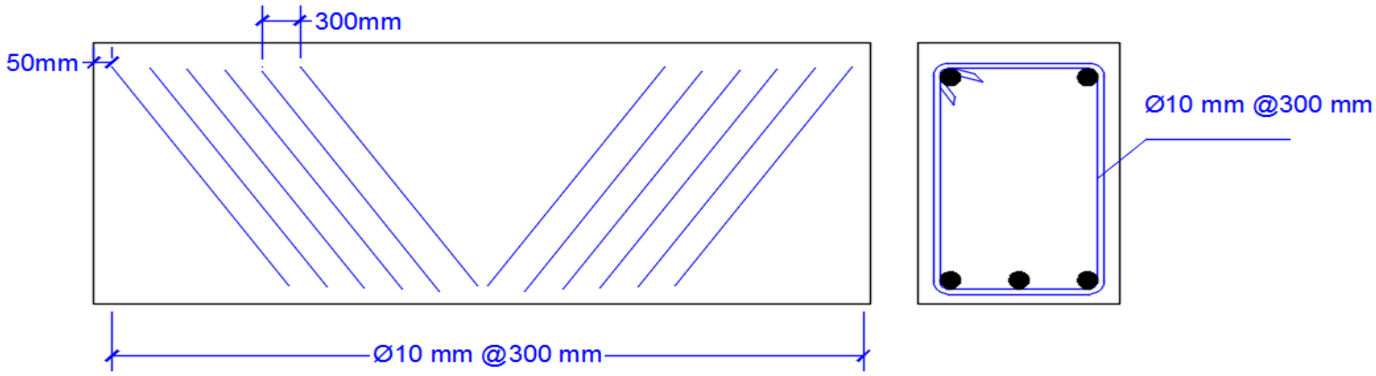
$$V_s = \frac{A_v\ fy\ d(sin\alpha + cos\alpha)}{s} \leq \frac{2}{3}\sqrt{f_c'}b_w d$$

$$S = \frac{(\frac{\pi}{4}(10)^2) * 2 * 400 * 600 * (sin45 + cos45)}{138 * 10^3} = 386\ mm$$

$$V_s = 138\ kN < \frac{1}{3}\sqrt{28} * 300 * 600 * 10^{-3} = 318\ kN \quad ok$$

If $V_s \leq \frac{1}{3}\sqrt{f_c'}b_w d \quad \rightarrow S_{max.} = \min. of \begin{cases} d/2 = 300mm \ \textcolor{red}{control} \\ 600\ mm \\ \frac{3\ A_v\ fy}{b_w} = 628\ mm \\ \frac{16 A_v\ fy}{\sqrt{f_c'}\ b_w} = 633\ mm \end{cases}$

S> Smax   Use Ø10mm @300mm



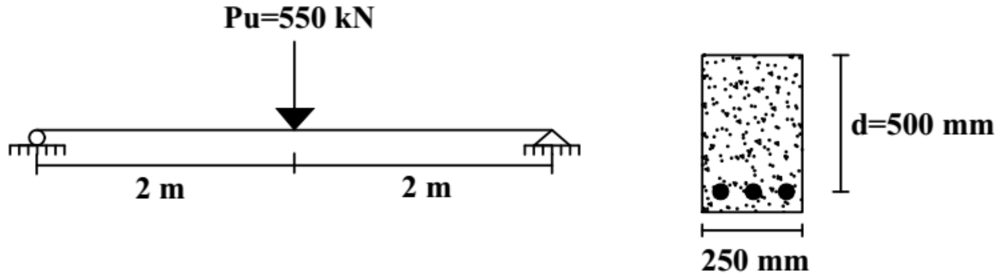
HW-1

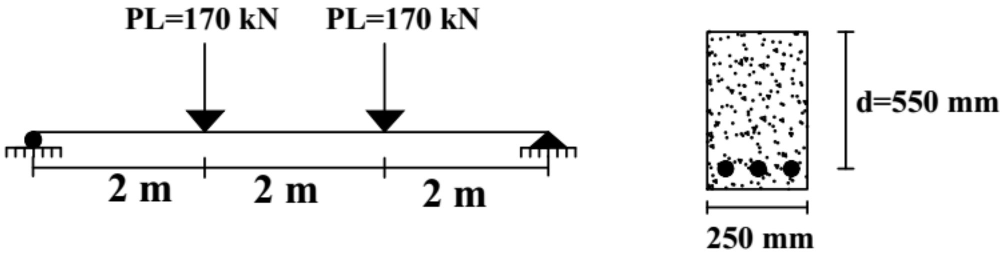
For a rectangular reinforced concrete beam of ultimate load (factored load) and details shown in the below figure, neglect the beam weight and find the spacing (S) of Ø10 mm stirrups at a critical section if:

1-Stirrups are vertical.

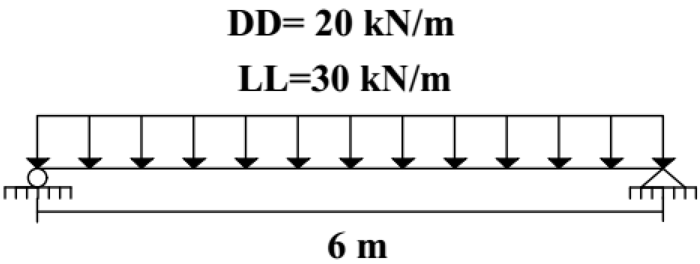
2-Stirrups are inclined by 45°.

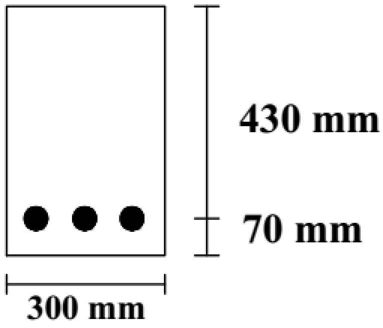
Use $\frac{f_c'}{fy} = \frac{30}{420} MPa$





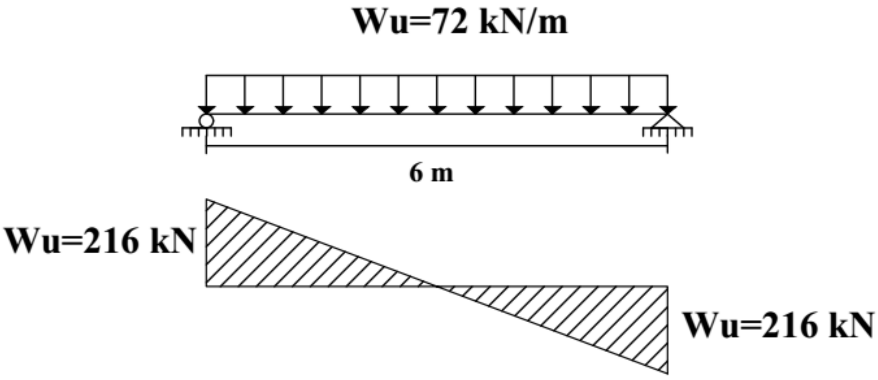
Example-3: Design the spacing of a 10 mm stirrup for the beam shown in figure to carry uniform service load of (D.L=20 kN/m and L.L=30 kN/m). Use $\frac{f_c'}{fy} = \frac{25}{420} MPa$ and $d = 430\, mm$.





Solution:

$W_u = 1.2 * 20 + 1.6 * 30 = 72$ kN/m



1- Find the shear force at the critical section

$$V_u = 216 - 72 * (0.43) = 185.04\, kN$$

2- Find the shear force provided by concrete.

$$V_c = \frac{\lambda \sqrt{f_c'}}{6} b_w d = \frac{\sqrt{25}}{6} * 300 * 430 * 10^{-3} = 107.5\, kN$$

$$\emptyset V_c = 0.75 * 142.9 = 80.625\, kN$$

$$V_u = 185.04\, kN > \emptyset V_c = 80.625\, kN$$

3- Find shear force resisted by shear reinforcement

$$V_u = \emptyset(V_c + V_s) \quad \rightarrow V_s = \frac{V_u}{\emptyset} - V_c \qquad \rightarrow V_s = \frac{185.04}{0.75} - 107.5 = 139.22\ kN$$

4- *check the* V_s *limitation where* $V_s \leq \frac{2}{3}\sqrt{f_c'}\, b_w d$

$$V_s = 139.22 < \frac{2}{3}\sqrt{25} * 300 * 430 * 10^{-3} = 430\ kN \quad ok$$

$$S = \frac{A_v * fy * d}{Vs} = \frac{\left(\frac{\pi}{4}(10)^2\right) * 2 * 420 * 430}{139.22 * 1000} = 203\ \cong 200\ mm$$

5- *Check the spacing limitation (S)*

$$V_s \leq \frac{1}{3}\sqrt{f_c'}\, b_w d$$

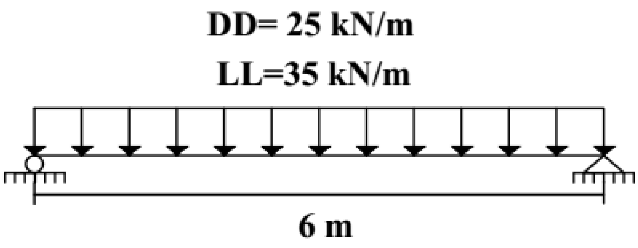
$$V_s = 139.22 < \frac{1}{3}\sqrt{25} * 300 * 430 * 10^{-3} = 215\ kN \quad ok$$

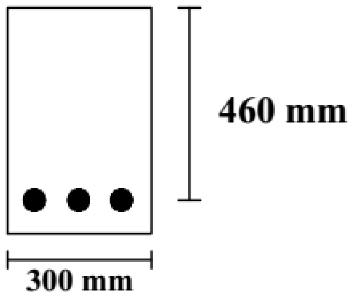
If $V_s \leq \frac{1}{3}\sqrt{f_c'} b_w d \qquad \rightarrow S_{max.} = \text{min.}\ of \begin{cases} \frac{d}{2} \\ 600\ mm \\ \frac{3\ A_v\ fy}{b_w} \\ \frac{16 A_v\ fy}{\sqrt{f_c'}\ b_w} = 665mm \end{cases}$

$$S_{max.} = \text{min.}\ of \begin{cases} \frac{430}{2} = 215\ mm\ \ control \\ 600\ mm \\ \frac{3\ \left(\frac{\pi}{4}(10)^2\right)\ * 2 * 420}{300} = 660mm \\ \frac{16\left(\frac{\pi}{4}(10)^2\right)\ * 2 * 420}{\sqrt{25} * 300} = 703mm \end{cases}$$

S < S max. Use Ø10mm @200 mm

HW-2: Design the spacing of a 10 mm stirrup for the beam shown in figure to carry uniform service load of (D.L=25 kN/m and L.L=35 kN/m) at the critical section. Use $\frac{f_c'}{fy} = \frac{28}{420} MPa$ and $d = 460\ mm$.





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Design of Reinforced Concrete Structures I

Shear of reinforced concrete beams-3

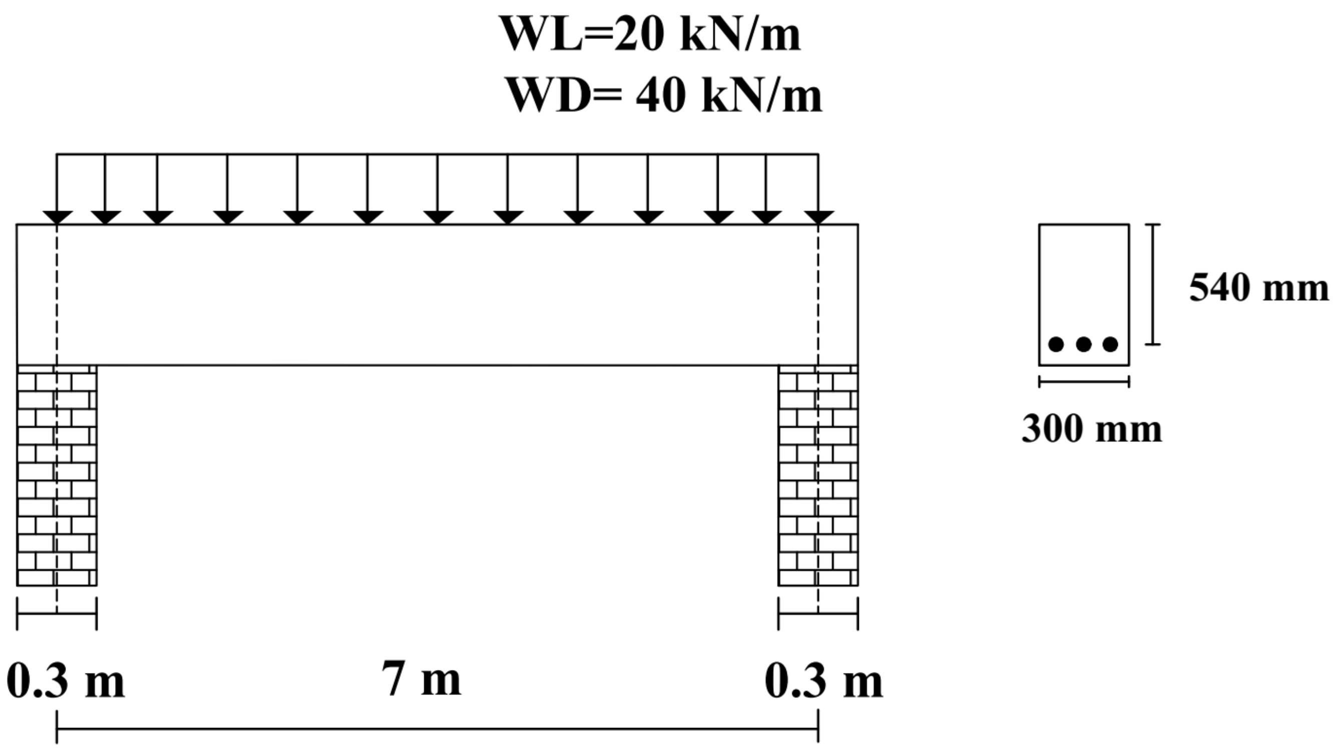
Dr Othman Hameed

Lecture (3)

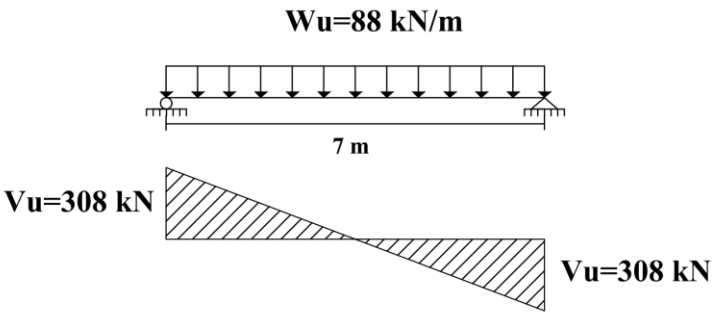
Example-4: Design the spacing of a 10 mm stirrup along the beam shown in the figure to carry uniform service load of (D.L=20 kN/m and L.L=40 kN/m). Use $\frac{f_c'}{fy} = \frac{25}{420} MPa$ and $d = 540\ mm$.

ملاحظة

عندما يُطلب تصميم القص على طول العتب along the beam ، يجب الانتباه الى توزيع حديد القص حسب الاحمال المسلطة. لان هذه الاحمال قد تتغير بزيادة او نقصان المسافة من المسند. ولكن عندما لايذكر ذلك او يطلب فقط تصميم حديد القص بالمناطق الحرجة فيمكن ايجاد القص عند المسافة d من المسند.



Solution:



$W_u = 1.2$ D.L+1.6 L.L= 88 kN/m

1- Find the shear force at the critical section

$$V_{u@d} = 308 - 88 * \left(0.54 + \frac{0.3}{2}\right) = 247.3\ kN$$

2- Find the shear force provided by concrete.

$$V_c = \frac{\lambda \sqrt{f_c'}}{6} b_w d = \frac{\sqrt{28}}{6} * 300 * 540 * 10^{-3} = 142.9\ kN$$

$$\emptyset V_c = 0.75 * 142.9 = 107.2\ kN$$

$$V_u = 247.3kN > \emptyset V_c = 142.9\ kN$$

3- Find shear force resisted by shear reinforcement

$$V_u = \emptyset(V_c + V_s) \ \ \rightarrow V_s = \frac{V_u}{\emptyset} - V_c \quad \rightarrow V_s = \frac{247.3}{0.75} - 142.9 = 186.83\ kN$$

4- $check\ the\ V_s\ limitation\ where\ V_s \le \frac{2}{3}\sqrt{f_c'}\ b_w d$

$$V_s = 186.83 < \frac{2}{3}\sqrt{28} * 300 * 540 * 10^{-3} = 571.48kN \quad ok$$

$$S = \frac{A_v * fy * d}{Vs} = \frac{\left(\frac{\pi}{4}(10)^2\right) * 2 * 420 * 540}{186.83 * 1000} = 190.68\ \cong 190mm$$

5- *Check the spacing limitation (S)*

$$V_s \le \frac{1}{3}\sqrt{f_c'}\ b_w d$$

$$V_s = 186.83 < \frac{1}{3}\sqrt{28} * 300 * 540 * 10^{-3} = 285.54\ kN \quad ok$$

If $V_s \le \frac{1}{3}\sqrt{f_c'} b_w d \quad \rightarrow S_{max.} = \min. of \begin{cases} d/2 = 270mm\ \ control \\ 600\ mm \\ \frac{3\ A_v\ fy}{b_w} = 660mm \\ \frac{16 A_v\ fy}{\sqrt{f_c'}\ b_w} = 665mm \end{cases}$

S < S max. Use Ø10mm @190mm

6- Classify the factored shear force

$$V_u = \frac{1}{2}\emptyset V_c = 0.5 * 107.2 = 53.6\ kN$$

From triangles similarity $\quad \frac{308}{3.5} = \frac{53.6}{x} \rightarrow x = 0.61\ m$

$$V_u = \emptyset V_c = 107.2\ kN$$

From triangles similarity $\quad \frac{308}{3.5} = \frac{107.2}{x} \rightarrow x = 1.22\ m$

Stirrups is required from 0 to 2280 mm (3500-1220=2280) (*Use Ø10mm @190mm) (Zone C)*

Minimum shear reinforcement is required from 2280 to 2890 mm (3500-610=2890) *(Zone B)*

No shear reinforcement is required from 2890 mm to 3500 mm (Zone A)

Minimum shear reinforcement for $\frac{1}{2}\emptyset V_c = 53.6\ kN < V_u < \emptyset V_c = 107.2\ kN$

$use\ \ S_{max.} = 270\ mm\ for\ zone\ B\ (from\ 2280\ mm\ to\ 2890\ mm)$

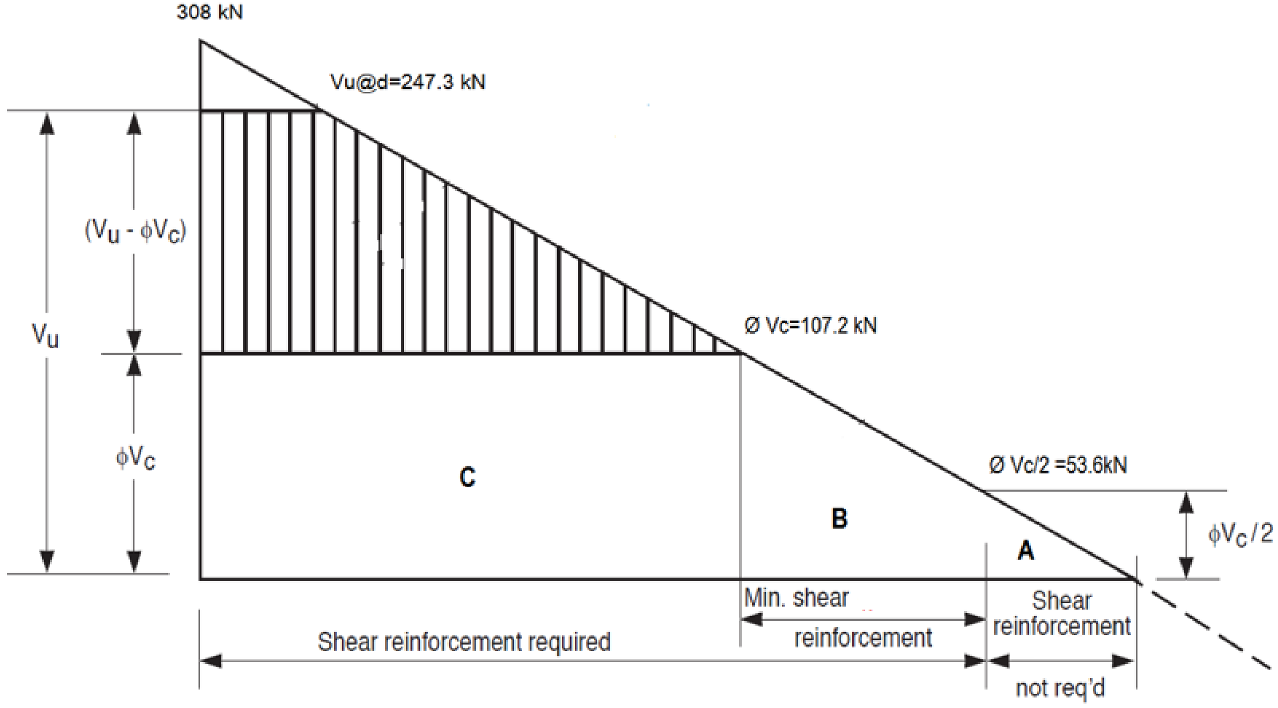
$$A_{v\ \ min.} = \frac{1}{16}\sqrt{f_c'}\frac{b_w\ s}{fy} \ge \frac{b_w\ s}{3\ fy}$$

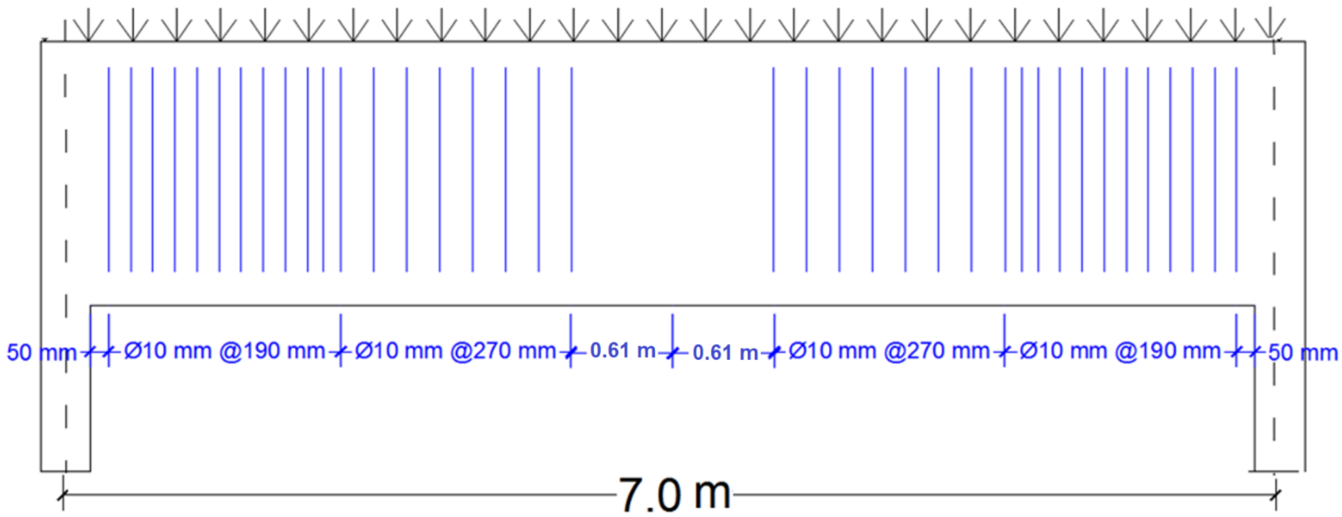
$$A_{v\ \ min.} = \frac{1}{16}\sqrt{28}\frac{300\ x\ 270}{420} = 63.78\ mm^2$$

$$\frac{b_w\ s}{3\ fy} = \frac{300\ x\ 270}{3\ x420} = 64.3\ mm^2$$

Use $A_{v\ min.} = 64.3\ mm^2$

Use Ø10mm @270mm (Zone B)

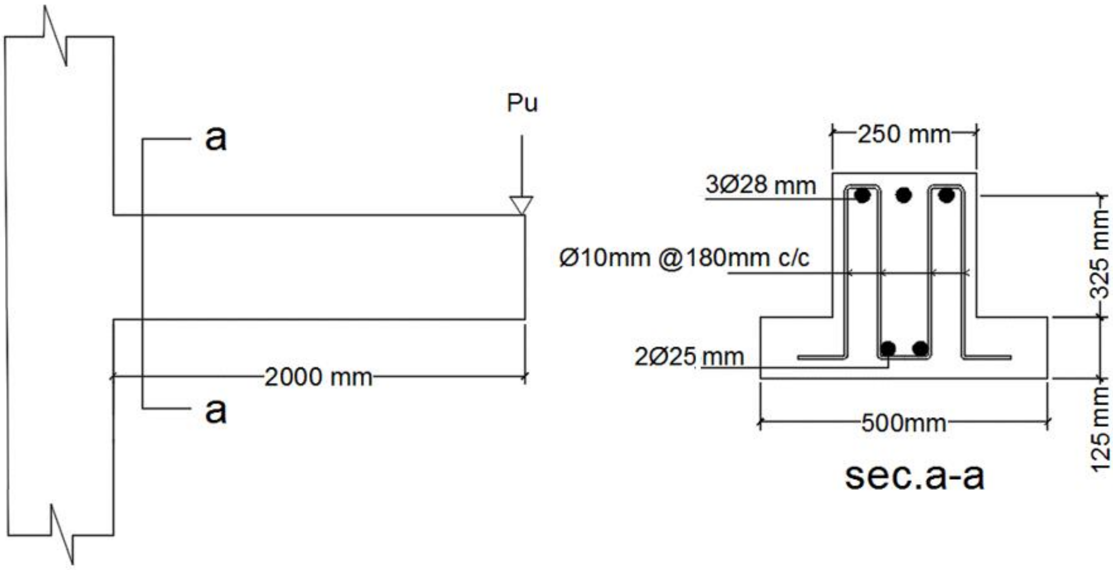

308 kN
Vu@d=247.3 kN
(Vu - ϕVc)
Vu
ϕVc
Ø Vc=107.2 kN
Ø Vc/2 =53.6kN
ϕVc/2
C
B
A
Min. shear reinforcement
Shear reinforcement not req'd
Shear reinforcement required


50 mm
Ø10 mm @190 mm
Ø10 mm @270 mm
0.61 m
0.61 m
Ø10 mm @270 mm
Ø10 mm @190 mm
50 mm
7.0 m

Example-5: According to the shear requirement, neglect the beam weight and find the maximum load Pu can be carried by the beam shown in Figure below by using:

1- Simple formula (ACI 11.3)
2- By using the effect of tension reinforcement (ACI 11.5)

Use $\frac{f_c'}{fy} = \frac{25}{400} MPa$



Solution:

1:

Vu@d =Pu

$V_u@\text{d} \leq \emptyset V_n$

$V_u@\text{d} = \emptyset(V_c + V_s)$

$$V_c = \frac{\lambda\sqrt{f_c'}}{6} b_w d = \frac{\sqrt{25}}{6} * 250 * 450 * 10^{-3} = 93.75\ kN$$

$$Vs = \frac{A_v * fy * d}{S} = \frac{(\frac{\pi}{4}(10)^2) * 4 * 400 * 450}{180 * 10^3}$$

$$= 314.16\ kN\ (@\ distance\ d\ from\ the\ face\ of\ the\ support)$$

$$V_u = 0.75(93.75 + 314.16) = 305.9\ kN \quad \rightarrow P_u = 305.9kN$$

2:

$$V_c = \left(\lambda\sqrt{f_c'} + 120\rho_w \frac{V_u*d}{M_u}\right)\frac{b_w*d}{7} \leq 0.3\lambda\sqrt{f_c'}\, b_w d \quad \text{(ACI 11.5)}$$

$$\rho_w = \frac{As}{b\ d} = \frac{1847.25}{250*450} = 0.0164$$

$$V_u@d = P_u\ , M_u@d = (2-0.45)P_u = 1.55P_u,\ \frac{V_u*d}{M_u} = \frac{0.45P_u}{1.55P_u} = 0.29 < 1.0\ ok$$

$$V_c = \left(1*\ \sqrt{25} + 120*0.0164*0.29\right)\frac{250*450}{7}*10^{-3} = 89.54\ kN$$

$$\leq 0.3\lambda\sqrt{f_c'}\, b_w d = 168.75\ kN\ ok$$

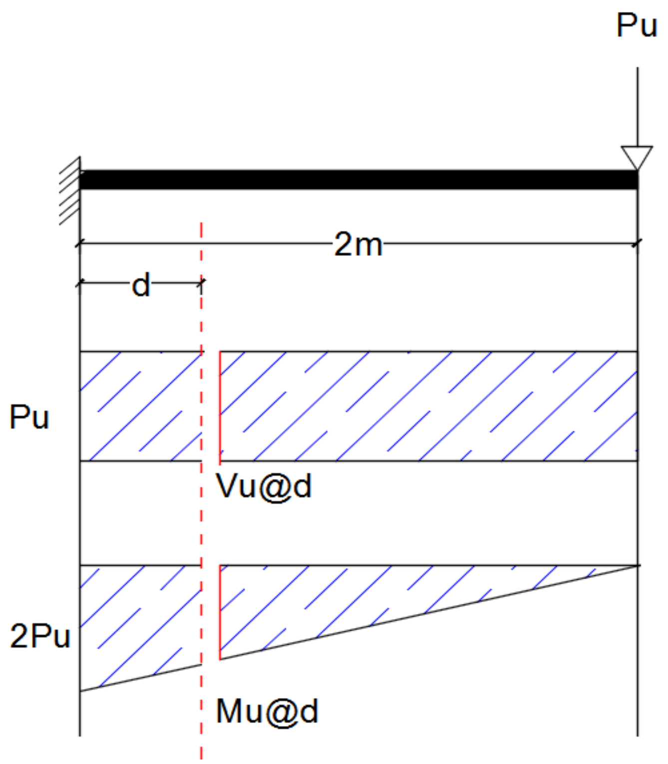
Vu@d =Pu

V_u@d $\leq \emptyset V_n$

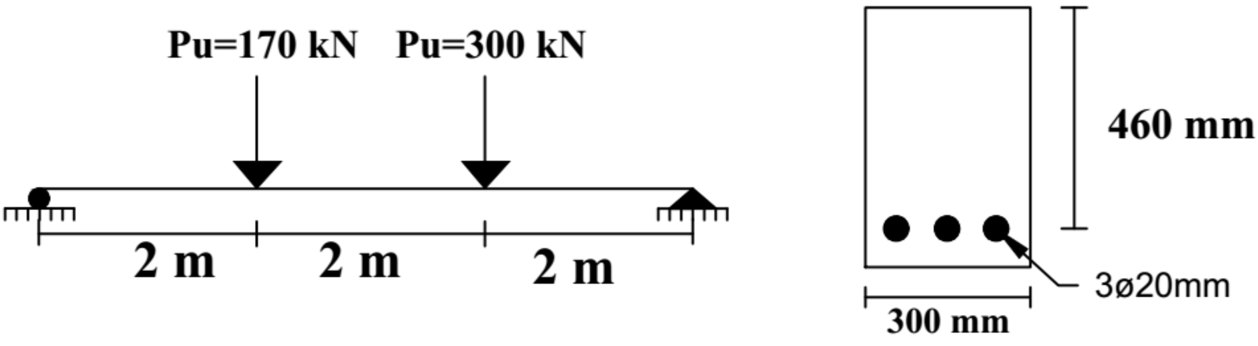
V_u@d $= \emptyset(V_c + V_s)$

$$V_u = 0.75(89.54 + 314.16) = 302.77kN$$

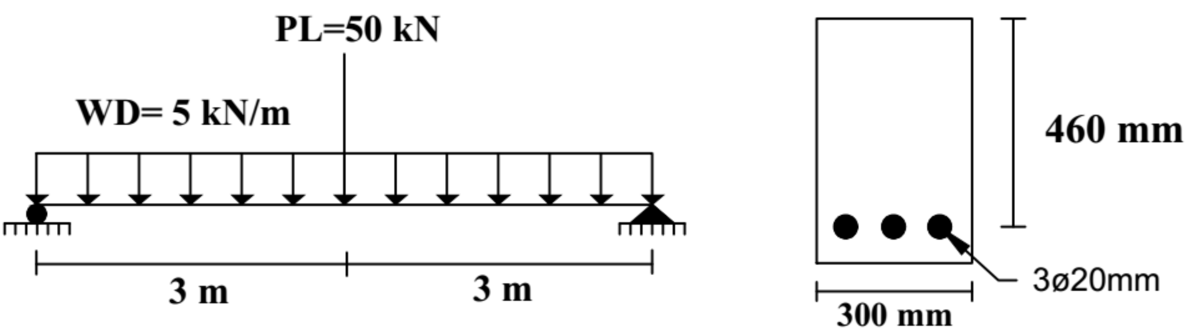
$$P_u = 302.77kN$$



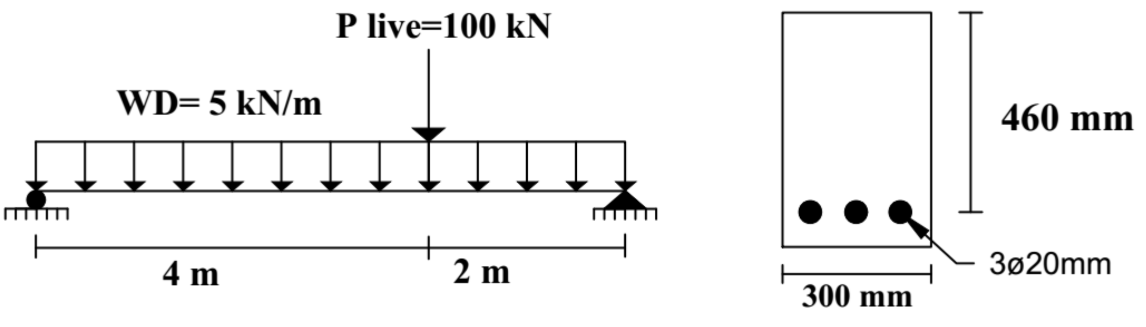
HW-3: Design the spacing of a 10 mm stirrup for the beams shown in the below figures. Use $\frac{f_c'}{fy} = \frac{28}{420} MPa$



HW-4: Design the spacing of a 10 mm stirrup for the beams shown in the below figures. Use $\frac{f_c'}{fy} = \frac{28}{420} MPa$



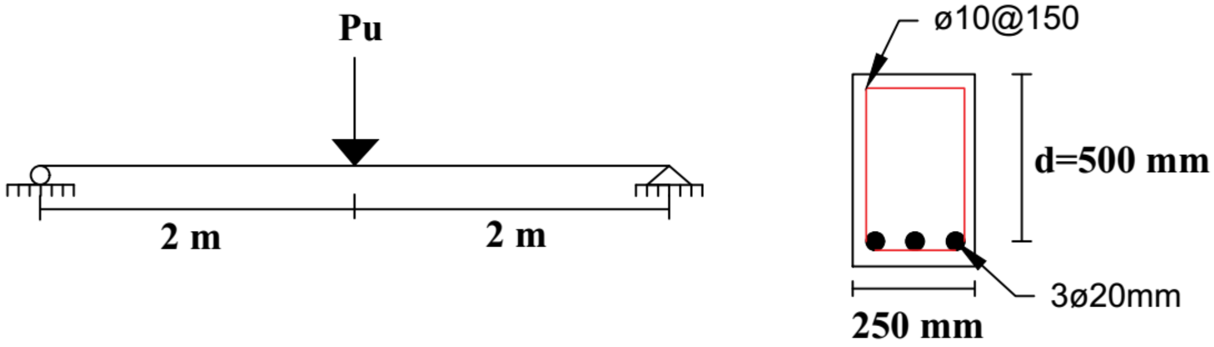
HW-5: Design the spacing of a 10 mm stirrup for the beams shown in the below figures. Use $\frac{f_c'}{fy} = \frac{25}{420} MPa$



HW-6: According to the shear requirement, neglect the beam weight and find the maximum load Pu can be carried by the beam shown in Figure below by using:

1- Simple formula (ACI 11.3)
2- By using the effect of tension reinforcement (ACI 11.5)

Use $\frac{f_c'}{fy} = \frac{25}{400} MPa$

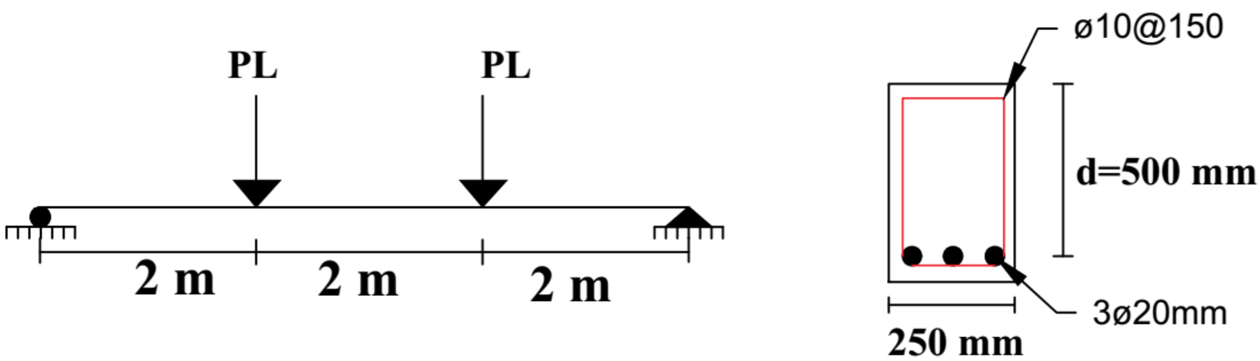


HW-7: According to the shear requirement, neglect the beam weight and find the maximum live load (PL) can be carried by the beam shown in Figure below by using:

1-Simple formula (ACI 11.3)

2-By using the effect of tension reinforcement (ACI 11.5)

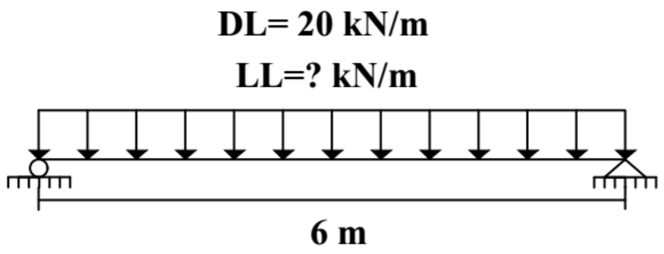
Use $\frac{f_c'}{fy} = \frac{25}{400} MPa$

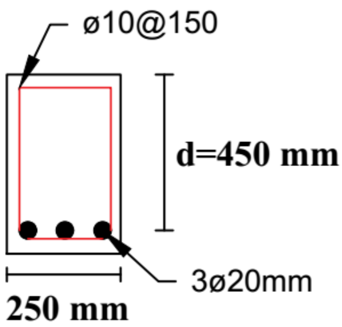


HW-8: According to the shear requirement, neglect the beam weight and find the maximum distributed live load (LL) can be carried by the beam shown in the figure below by using:

1- Simple formula (ACI 11.3)
2- By using the effect of tension reinforcement (ACI 11.5)

Use $\frac{f_c'}{fy} = \frac{25}{400} MPa$





Design of Reinforced Concrete Structures I

Al Muthanna University
Collage of Engineering

Serviceability limit states-Crack width

Dr Othman Hameed

Lecture (4)

Serviceability limit states

Introduction

In the beam lectures, limit-states design was discussed. The limit states were divided into two groups:

a. Those leading to collapse and,

b. Those, which disrupt the use of structures, but do not cause collapse.

These were referred to as ultimate limit states and serviceability limit states, respectively. Excessive crack widths, excessive deflections, and undesirable vibrations cause the major serviceability limit states for reinforced concrete structures. Cracks width and deflection will be discussed in this chapter.

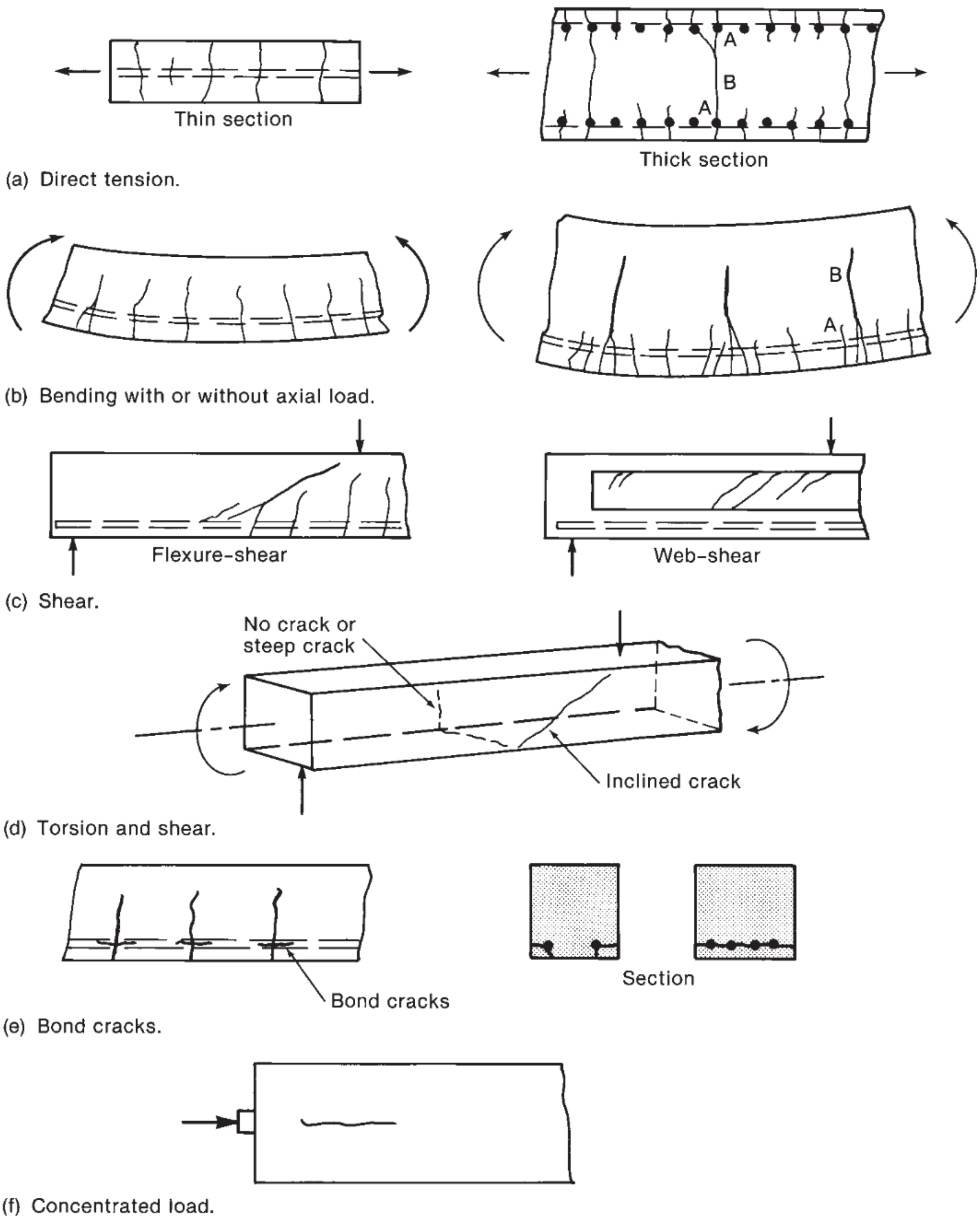
The crack width and deflection are measured at service loads (with no factor). The terms service loads and working loads refer to loads encountered in the everyday use of the structure. Service loads are generally taken to be the specified loads without load factors.

1. Cracking

2.1 Type of Cracking

Tensile stresses induced by loads, moments, shears, and torsion cause distinctive crack patterns, as shown in Figure below.

- Members loaded in direct tension through the entire cross section, with a crack spacing ranging from 0.75 to 2 times the minimum thickness of the member.
- In the case of a very thick tension member with reinforcement in each face, small surface cracks develop in the layer containing the reinforcement (Fig. a).
- Members subjected to bending moments develop flexural cracks, as shown in Fig.b. These vertical cracks extend almost to the zero-strain axis (neutral axis) of the member.
- Cracks due to shear have a characteristic inclined shape, as shown in Fig. c. Such cracks extend upward as high as the neutral axis and sometimes into the compression zone.
- Torsion cracks are similar. In pure torsion, they spiral around the beam (Fig.d).
- Bond stresses lead to splitting along the reinforcement, as shown in Fig.e.
- Concentrated loads will sometimes cause splitting cracks or “bursting cracks” of the type shown in Fig. f.



Tybes of cracks due to loading

1.1 Control of Cracking under Service

As a reinforced concrete beam deflects, the tension side of the beam cracks wherever the low tensile strength of the concrete is exceeded. The more the beam deflects, the greater the length and width of cracks. Although cracking cannot be prevented, it is possible by careful detailing of the steel to produce beams that develop a large number of narrow, closely spaced cracks in preference to a few wide cracks.

The maximum crack width the designer should permit depends on exposure conditions.

- If concrete is exposed to seawater or cycles of wetting and drying, the maximum width of any crack should not exceed (0.15 mm) or at the far limit (0.2 mm).
- For members protected against weather, crack widths up to (0.41 mm) are permitted by the ACI Code.
- ACI Committee 224, in a report on cracking, presented a set of approximately permissible maximum crack widths for reinforced concrete members subject to different exposure situations. These values are summarized in Table below.

Permissible Crack Widths

Members Subjected to	Permissible Crack Widths (in.)	(mm)
Dry air	0.016	0.41
Moist air, soil	0.012	0.30
Deicing chemicals	0.007	0.18
Seawater and seawater spray	0.006	0.15
Use in water-retaining structures	0.004	0.10

Experimental studies show that the width of cracks varies directly with the *magnitude of the steel stress* and demonstrate that a large number of small bars well distributed through the tension zone of the beam is more effective in reducing the width of cracks than a small number of larger-diameter bars used to supply the same area of steel.

The following equation was developed for estimating the maximum widths of cracks that will occur in the tension faces of flexural members.

$$w = 0.076\beta f_s \sqrt[3]{d_c A}$$

Where:

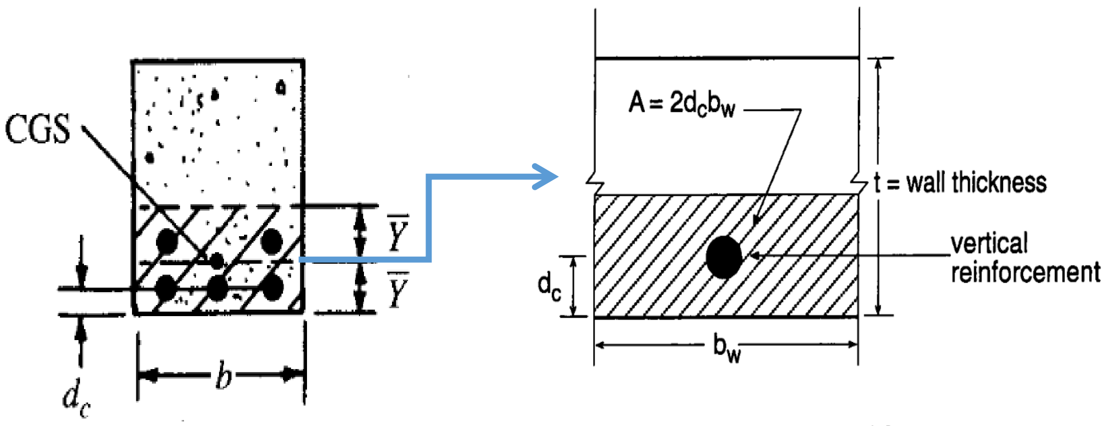
w= Maximum width of crack, thousandths of an inch (0.03 mm for SI units)

β = Can be taken as 1.2 for beams and 1.35 for slabs

f_s = Stress in steel due to service loads, (MPa)

d_c = Distance from tension surface to center of the row of reinforcing bars closest to outside surface

A = Effective tension area of concrete divided by the number of reinforcing bars



Effective Tension Area of Concrete

Use $\beta = 1.2$ then

$$w = 0.091 f_s \sqrt[3]{d_c A} * 10^{-3}$$

When we let w/0.091 = z

$$z = f_s \sqrt[3]{d_c A} * 10^{-3}$$

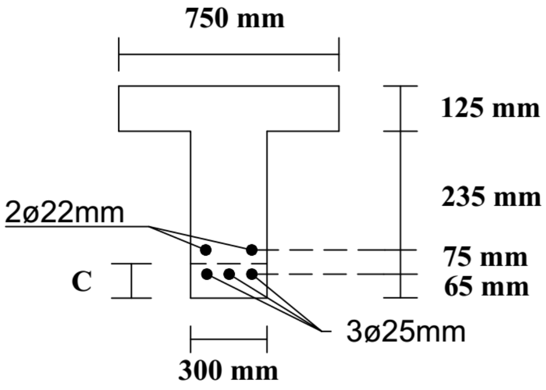
where f_s the stress in the steel (MPa), may be taken as 0.6*fy*.

ACI 95, section 10.6.4 specifies that z is not to exceed (25 MN/m) for exterior exposure or (30 MN/m) for interior exposure.

These values correspond to maximum crack widths of (0.33 mm) and (0.4 mm), respectively.

Example: -

Determine whether the reinforcement pattern in the figure below satisfies the requirement of ACI code for the crack width. The beam is under exterior exposure. f_y=420 MPa



Solution

Find the center of reinforcement

$$A_s = \frac{\pi}{4}22^2 \times 2 + \frac{\pi}{4}25^2 \times 3 = 759.88 + 1471.8 = 2231.68\, mm^2$$

$$C = \frac{1471.8 \times 65 + 759.88 \times (75+65)}{759.88 + 1471.8} = 90.53\, mm$$

$$f_s = 0.6 f_y = 0.6 \times 420 = 252\, MPa$$

$$d_c = 65\, mm$$

$$\text{No of bars} = \frac{\text{Total area of bars}}{\text{Area of largest one}} = \frac{2231.68}{\frac{\pi}{4}25^2} = 4.55$$

$$\text{A} = \frac{\text{Effective tension area of concrete}}{\text{number of reinforcing bars}} = \frac{300 \times (90.53 \times 2)}{4.55} = 11938\, mm^2$$

$$z = f_s \sqrt[3]{d_c A} * 10^{-3}$$

$$z = 252 \times \sqrt[3]{65 \times 11938} * 10^{-3} = 23.2\frac{MN}{m} < 25\frac{MN}{m}$$

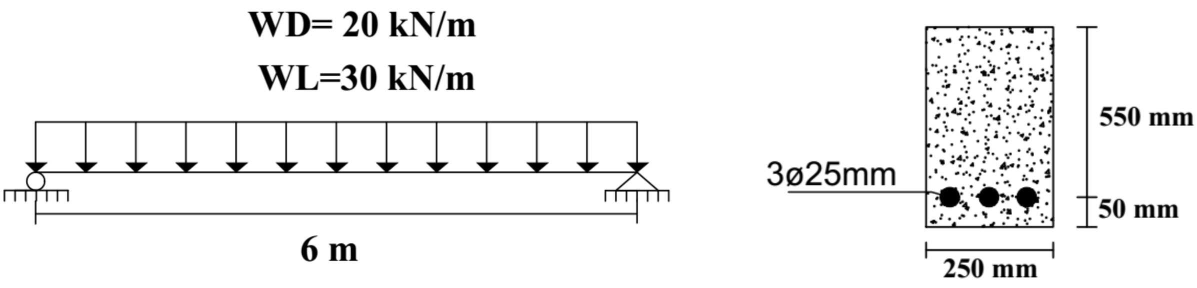
Crack width < 0.33 mm $\therefore ok$

HW-1

For the beam shown below. The beam carries a service dead load (W.D) of 20 kN/m and a service live load (W.L) of 30 kN/m. Use $\frac{f_c'}{fy} = \frac{25}{420} MPa$ and:

1- Find the spacing (S) of Ø10 mm stirrups at a critical section.

2- Determine whether the reinforcement pattern in the figure below satisfies the requirement of ACI code for the crack width. The beam is under an exterior exposure.



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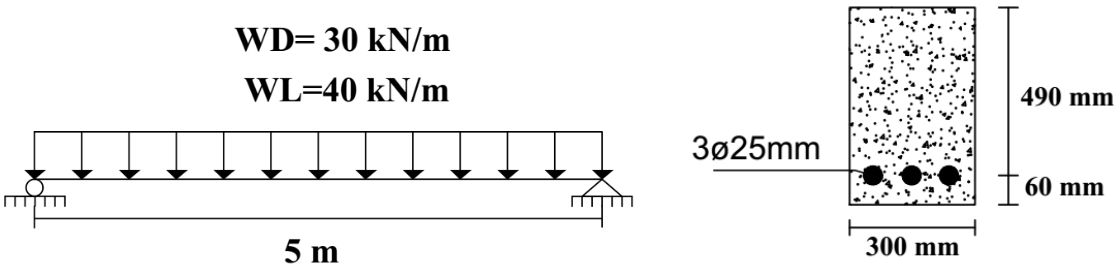
HW-2

For the beam shown below. The beam carries a service dead load (W.D) of 30 kN/m and a service live load (W.L) of 40 kN/m. Use $\frac{f_c'}{fy} = \frac{30}{400} MPa$ and:

1- Find the spacing (S) of Ø10 mm stirrups at a critical section.

2- Determine whether the reinforcement pattern in the figure below satisfies the requirement of ACI code for the crack width. The beam is under an exterior exposure.



Design of Reinforced Concrete Structures I

Al Muthanna University
Collage of Engineering

Serviceability limit states-Deflection-1

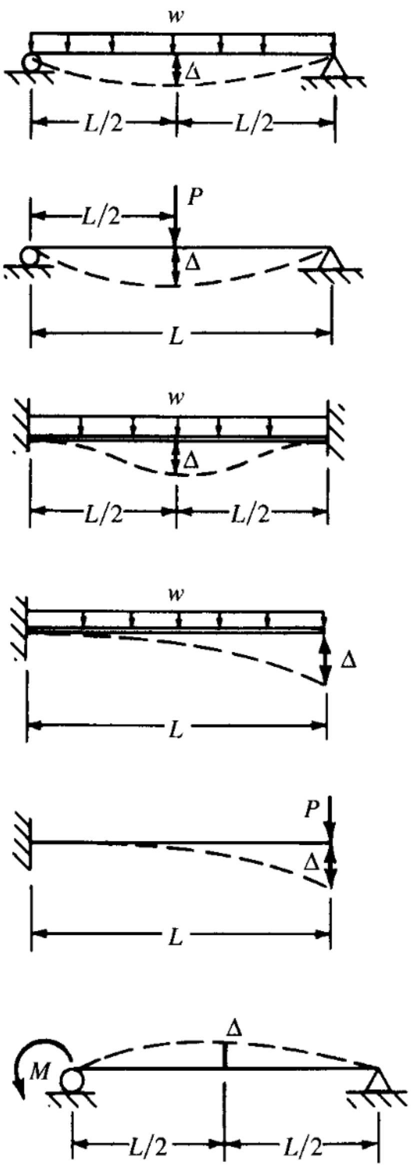
Dr Othman Hameed

Lecture 5

2. Deflection under Service Load

2.1 Computation of Immediate Deflections

Elastic equations shown in Figure below are used to compute the immediate deflections of a reinforced concrete beam. Other useful deflection equations are tabulated in engineering textbooks and design manuals, e.g., the AISC Steel Construction Manual.



(a) $\Delta = \frac{5wL^4}{384EI}$

(b) $\Delta = \frac{PL^3}{48EI}$

(c) $\Delta = \frac{wL^4}{384EI}$

(d) $\Delta = \frac{wL^4}{8EI}$

(e) $\Delta = \frac{PL^3}{3EI}$

(f) $\Delta = \frac{ML^2}{16EI}$

2.2 Effective Moment of Inertia

In a reinforced concrete beam, the effective cross section varies along the length of the member. In regions of low moment, where no cracks exist, the effective moment of inertia should logically be based on the gross transformed area of the cross section. At sections of high moment, where cracking is extensive, the effective moment of inertia is more properly based on the properties of the cracked transformed cross section. To account for the variation of the moment of inertia along the beam axis, ACI 24.2.3.5 requires the use in elastic-deflection equations of an effective moment of inertia 'I_e, which is computed below.

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 Ig + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] I_{cr} \leq Ig \quad \text{(ACI24.2.3.5.a)}$$

Where:

M_{cr} =Cracking moment= $\frac{f_r \times Ig}{y_t}$

f_r = modulus of rupture $= 0.62 \times \sqrt{f_c'}$

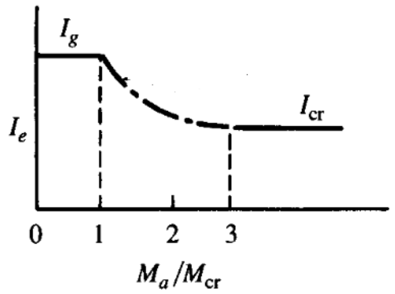
y_t = distance from centroid of gross section to extreme fiber in tension

Ma = maximum moment in member at stage for which deflection is being computed

Ig = moment of inertia of gross section neglecting area of tension steel

I_{cr}= moment of inertia of transformed cracked cross section

Equation above should be used when $1 \leq Ma/Mcr \leq 3$.
If $Ma/Mcr > 3$, the cracking will be extensive and I_e = I_{cr} with no significant error.
If Ma/Mcr < 1, no cracking is likely and I_e = Ig as shown below.



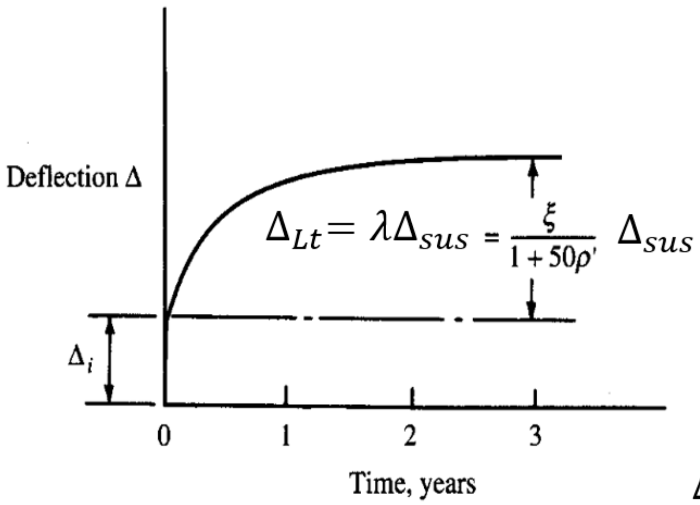
Variation of effective moment of inertia with maximum moment.

2.3 Long-term Deflections Due to Creep and Shrinkage

Deflections of concrete beams consist of two components,

1- An initial deflection Δ_i that occurs simultaneously with the application of load

2- A long-term or additional increment of deflection Δ_{Lt} (produced by creep and shrinkage) that takes place over time. ($\Delta_{Lt} = \lambda\Delta_{sus}$).



Variation of deflection with time. Δ_i = initial elastic deflection, Δ_{Lt} = long-term deflection.

Total deflection, $\Delta_t = \Delta_i + \Delta_{Lt}$

* The total deflection (Δ_t) is

$$\Delta_t = \Delta_i + \Delta_{Lt}$$

The increase with time of the long-term deflection is shown in Figure below.

To estimate the magnitude of the additional deflection Δ_a that occurs with time, the ACI (24.2.4.1.1) specifies that the instantaneous deflection Δ_i, produced by the sustained portion of the applied load is to be multiplied by the empirical factor λ, i.e., $\Delta_a = \lambda\Delta_i$ where

$$\lambda = \frac{\xi}{1 + 50\rho'}$$

$$\rho' = \frac{As'}{b_w d}$$

ξ= values of the time-dependent factor for sustained loads, ξ, shall be in accordance with ACI Table 24.2.4.1.3.

Table 24.2.4.1.3 –Time-dependent factor for sustained loads

Sustained load duration, months	Time-dependent factor ξ
3	1.0
6	1.2
12 **(one year)**	1.4
60 or more **(more than 5 years)**	2.0

2.4 Maximum Permissible Calculated Deflection

Table 24.2.2—Maximum permissible calculated deflections

Member	Condition		Deflection to be considered	Deflection limitation
Flat roofs	Not supporting or attached to nonstructural elements likely to be damaged by large deflections		Immediate deflection due to maximum of L_r, S, and R	$\ell/180$[1]
Floors			Immediate deflection due to L	$\ell/360$
Roof or floors	Supporting or attached to non-structural elements	Likely to be damaged by large deflections	That part of the total deflection occurring after attachment of nonstructural elements, which is the sum of the time-dependent deflection due to all sustained loads and the immediate deflection due to any additional live load[2]	$\ell/480$[3]
		Not likely to be damaged by large deflections		$\ell/240$[4]

[1]Limit not intended to safeguard against ponding. Ponding shall be checked by calculations of deflection, including added deflections due to ponded water, and considering time-dependent effects of sustained loads, camber, construction tolerances, and reliability of provisions for drainage.

[2]Time-dependent deflection shall be calculated in accordance with 24.2.4, but shall be permitted to be reduced by amount of deflection calculated to occur before attachment of nonstructural elements. This amount shall be calculated on basis of accepted engineering data relating to time-deflection characteristics of members similar to those being considered.

[3]Limit shall be permitted to be exceeded if measures are taken to prevent damage to supported or attached elements.

[4]Limit shall not exceed tolerance provided for nonstructural elements.

Notes

1- Immediate deflection $(\Delta_{\mathrm{i}})_{\mathrm{D+L}}$

2- Sustained deflection $\Delta_{sus} = D + \%L.L$

3- Long term deflection $(\Delta_{Lt} = \lambda\Delta_{sus})$.

4- Total deflection $\Delta_t = \Delta_i + \Delta_{Lt}$

5- Maximum Permissible Deflection

A- Not supporting or attached to nonstractural elements likely to be damaged by large deflection

A.1- Roof (L/180)
A.2- Floor (L/360)

B- Supporting or attached to nonstractural

B.1- Likely to be damaged by large deflection (L/480)
B.2- Not likely to be damaged by large deflection (L/240)

ملاحظة حول تدقيق الحدود المسموح بها:

لحساب الجزء الذي يؤثر على القواطع غير الانشائية نتبع التالي:

A. The structures **not** supporting nonstructural element likely to be damaged by large deflection.

عندما يكون المنشأ لا يتحمل او يتصل بأجزاء غير انشائية يمكن ان تتضرر بالهطول.

- يتم حساب الهطول الناتج عن الحمل الحي $(\Delta_i)_L$ ومقارنته مع الحدود المسموح بها (Δ_{max}).

$$\Delta_{max} = \begin{cases} \text{flat roof} = L/180 \\ \text{floors} \quad = L/360 \end{cases}$$

B. Supporting or attached to nonstructural element.

عندما يكون المنشأ يحمل او يتصل بأجزاء غير انشائية

- يتم حساب الهطول الناتج من (الحمل الحي + الهطول طويل الامد) ومقارنته مع الحدود المسموح بها(Δ_{max})

$$\Delta = (\Delta_i)_L + \Delta_{\text{long term}}$$

$$\Delta_{max} = \begin{cases} \text{damged by large deflection} = L/480 \\ \text{not damgeded} \qquad\qquad\quad = L/240 \end{cases}$$

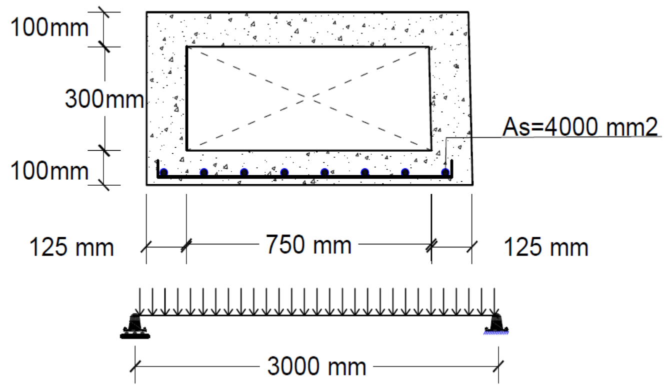
ملاحظة حول حساب الهطول الناتج عن الحمل الحي نتبع الخطوات التالية:

١. حساب الهطول الناتج عن الحمل الحي + الحمل الميت.
٢. حساب الهطول الناتج عن الحمل الميت.

$$(\Delta_i)_L = (\Delta_i)_{D+L} - (\Delta_i)_D$$

Ex-1

By using the working stress design method, find:



a- The maximum uniform distributed live load can be carry by the simply supported reinforced concrete beam of section and details shown in Figure. Use fs=165 MPa, f_c=12.5 MPa, effective depth= 330 mm, dead load = 30kN/m and n=8.

b- If the live load = 20 kN/m, and $f_c' = 28MPa$,comput the immediate deflection preduce by total load. For dead load, use the self weight of the beam only.

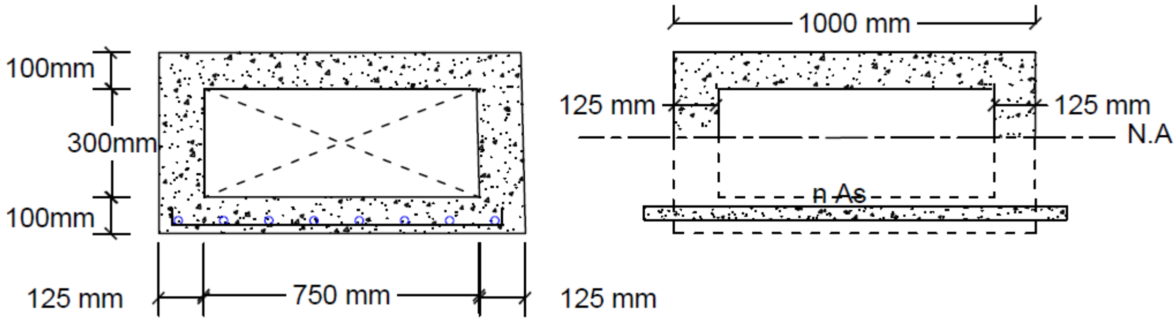
note:$\Delta_{max.} = \frac{5}{384}\frac{wl^4}{EI}$

Solution:

a-

1- Find N.A

$$750 * 100 * (y - 50) + 250 * y * \frac{y}{2} = n * As * (d - y) \rightarrow y = 117.58\ mm$$



2- Find moment of inertia about N.A

$$I_{cr} = \frac{b.h^3}{3} + n\,As(d - y)^2$$

$$I_{cr} = \frac{1000 * 117.6^3}{3} - \frac{750 * (117.6 - 100)^3}{3} + 8 * \ 4000 * (330 - 117.6)^2$$
$$= 1.98 * 10^9 mm^4$$

3- Find the moment resisted by section

$$f_c = \frac{M.y}{I} \rightarrow 12.5 = \frac{M * 10^6 * 117.6}{1.98 * 10^9} \rightarrow M = 210.9\ KN.m$$

$$f_s = n.\frac{M.(d - y)}{I} \rightarrow 165 = 8 * \frac{M * 10^6 * (330 - 117.6)}{1.98 * 10^9} \rightarrow M = 192.27KN.m\ \ control$$

4- Find W $M = \frac{W.l^2}{8} \rightarrow 192.27 = \frac{w*3^2}{8} \rightarrow W = 170.9\frac{KN}{m} = 30 + L.L \rightarrow L.L = 140.9 kN/m$

b- $\Delta = \frac{5}{384}\frac{wl^4}{EI}$

W= self weight +live load

Self weight =((1*0.5)-(0.75*0.3))*24=6.6 KN/m

$$Wt = (6.6 + 20) = 26.6\frac{kN}{m}\ ,\quad l = 3\,m\,, E = 4700\sqrt{28} = 24870MPa$$

$$M_a = \frac{w * l^2}{8} = 29.9\ kN.m$$

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 Ig + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] I_{cr} \le Ig$$

$$M_{cr} = \frac{f_r * y_t}{Ig}$$

$$f_r = 0.62 * \sqrt{28} = 3.28 \text{ MPa}$$

$$y_t = \frac{500}{2} = 250\ mm$$

$$Ig = \frac{1000 * 500^3}{12} - \frac{750 * 300^3}{12} = 8.729 * 10^9\ mm^4$$

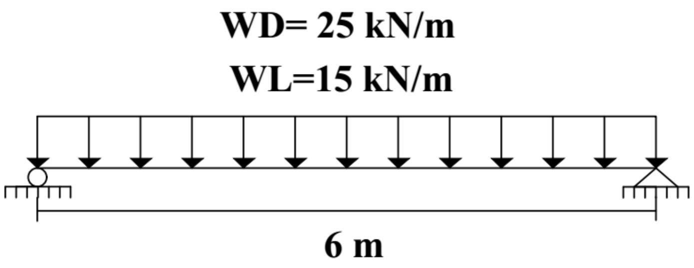
$$M_{cr} = \frac{3.28 * 8.729 * 10^9}{250} = 114.52\ kN.m$$

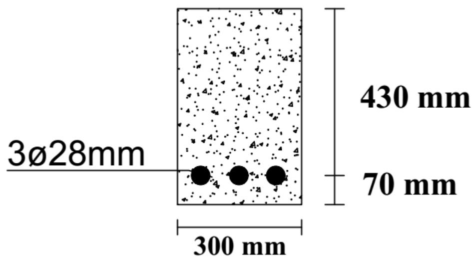
$$\frac{M_a}{M_{cr}} = \frac{29.9}{114.52} = 0.26 < 1.0 \qquad I_e = I_g$$

$$\Delta = \frac{5}{384}\frac{wl^4}{EI} = \frac{5}{384} * \frac{26.6 * 3000^4}{24870 * 8.729 * 10^9} = 0.129\ mm$$

Ex-2

For the beam and the cross section shown below, D.L=25 kN/m, L.L=15 kN/m, sustained live load=30% of live load. Use n=9 and f_c'=25 MPa.





Find

1. Immediate deflection due to dead load + live load.
2. Find the long-term deflection after 5 years if the sustained live load=30% of live load
3. Total deflection after 5 years.

Solution:

1- Immediate deflection due to (dead load + live load).

$$\Delta = \frac{5WL^4}{384EI}$$

$$L = 6\,m\,; \quad w = 25 + 15 = 40\,kN/m$$

$$E = 4700\sqrt{f'_c} = 23500\,N/mm^2$$

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] I_{cr} \le I_g$$

$$M_a = \frac{WL^2}{8} = \frac{40 \times 6^2}{8} = 180\,kN.m$$

$$M_{cr} = \frac{f_r\,I_g}{y_t} = \frac{0.62\sqrt{25} \times \frac{300 \times 500^3}{12}}{500/2} \times 10^{-6} = 38.75\,kN.m$$

$\frac{M_a}{M_{cr}} = \frac{180}{38.75} = 4.64 > 3 \qquad I_e = I_{cr}$

Find I_{cr}

$\frac{by^2}{2} = n \times A_s(d-y)$

$A_s = 3 \times \frac{\pi}{4}\, 28^2 = 1847.25\, mm^2$

$\frac{300 \times y^2}{2} = 9 \times 1847.25\,(430-y)$

$150\, y^2 = 9 \times 1847.25\,(430-y)$

$y^2 = 110.835\,(430-y)$

$y^2 + 110.835y - 4765.05 = 0$

$y = 169.8 = 170\, mm$

$I_{cr} = \frac{300 \times 170^3}{3} + 9 \times 18470.25\,(430-170)^2 = 1.615 \times 10^9\ mm^4$

$(\Delta_{\mathrm{i}})_{\mathrm{D+L}} = \frac{5 \times 40 \times 6000^4}{384 \times 23500 \times 1.615 \times 10^9} = 17.78\, mm$

----

2. The long-term deflection after 5 years if the sustained live load=30% of live load

$\Delta_{sus} = \frac{5WL^4}{284EI}$

$W_{sus} = D.L + 0.3\, L.L = 25 + 0.3 \times 15 = 29.5\ kN/m$

$M_{sus} = \frac{WL^2}{8} = \frac{29.5 \times 6^2}{8}$ =132.75 kN.m

$\frac{M_a}{M_{cr}} = \frac{132.75}{38.75} = 3.425 > 3$ use $\mathrm{I_e} = \mathrm{I_{cr}} = 1.615 \times 10^9\, mm^4$

$\Delta_{sus} = \frac{5 \times 29.5 \times 6000^4}{384 \times 23500 \times 1.615 \times 10^9} = 13.116\, mm$

$$\lambda = \frac{\xi}{1 + 50\rho'}$$

$$\lambda = \frac{2}{1 + 50(0)}$$

$$\Delta_{LT} = \lambda\, \Delta_{sus} = 2 \times 13.116 = 26.23\ mm$$

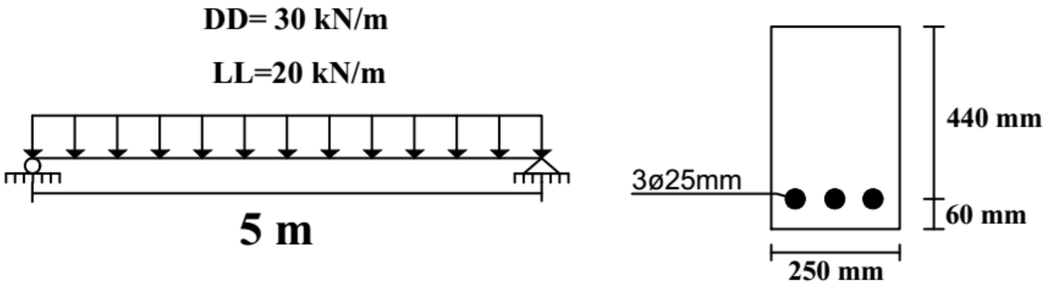
3. Total deflection after 5 years.

$\Delta_{total} = (\Delta_{\mathrm{i}})_{\mathrm{D+L}} + \Delta_{LT}$= 17.78 +26.23=44.01 mm

---

HW-1

For the beam and the cross section shown below, D.L=30 kN/m, L.L=20 kN/m, sustained live load=30% of live load. Use n=9 and f_c'=25 MPa.



Find

1- Immediate deflection due to dead load + live load.

2- Find the long-term deflection after 5 years if the sustained live load=30% of live load
3- Total deflection after 5 years.

Design of Reinforced Concrete Structures I

Al Muthanna University
Collage of Engineering

Serviceability limit states-Deflection-2

Dr Othman Hameed

Lecture 6

2. Deflection under Service Load

2.4 Maximum Permissible Calculated Deflection

Table 24.2.2—Maximum permissible calculated deflections

Member	Condition		Deflection to be considered	Deflection limitation
Flat roofs	Not supporting or attached to nonstructural elements likely to be damaged by large deflections		Immediate deflection due to maximum of L_r, S, and R	$\ell/180$[1]
Floors			Immediate deflection due to L	$\ell/360$
Roof or floors	Supporting or attached to non-structural elements	Likely to be damaged by large deflections	That part of the total deflection occurring after attachment of nonstructural elements, which is the sum of the time-dependent deflection due to all sustained loads and the immediate deflection due to any additional live load[2]	$\ell/480$[3]
		Not likely to be damaged by large deflections		$\ell/240$[4]

[1]Limit not intended to safeguard against ponding. Ponding shall be checked by calculations of deflection, including added deflections due to ponded water, and considering time-dependent effects of sustained loads, camber, construction tolerances, and reliability of provisions for drainage.

[2]Time-dependent deflection shall be calculated in accordance with 24.2.4, but shall be permitted to be reduced by amount of deflection calculated to occur before attachment of nonstructural elements. This amount shall be calculated on basis of accepted engineering data relating to time-deflection characteristics of members similar to those being considered.

[3]Limit shall be permitted to be exceeded if measures are taken to prevent damage to supported or attached elements.

[4]Limit shall not exceed tolerance provided for nonstructural elements.

Notes

1- Immediate deflection $(\Delta_{\mathrm{i}})_{\mathrm{D+L}}$

2- Sustained deflection $\Delta_{sus} = D + \%L.L$

3- Long term deflection $(\Delta_{Lt} = \lambda \Delta_{sus})$.

4- Total deflection $\Delta_t = \Delta_i + \Delta_{Lt}$

5- Maximum Permissible Deflection

A- Not supporting or attached to nonstractural elements likely to be damaged by large deflection

A.1- Roof (L/180)
A.2- Floor (L/360)

B- Supporting or attached to nonstractural

B.1- Likely to be damaged by large deflection (L/480)
B.2- Not likely to be damaged by large deflection (L/240)

<u>ملاحظة حول تدقيق الحدود المسموح بها:</u>

لحساب الجزء الذي يؤثر على القواطع غير الانشائية نتبع التالي:

A. The structures **<u>not</u>** supporting nonstructural element likely to be damaged by large deflection.

عندما يكون المنشأ لا يتحمل او يتصل بأجزاء غير انشائية يمكن ان تتضرر بالهطول.

- يتم حساب الهطول الناتج عن الحمل الحي $(\Delta_i)_L$ ومقارنته مع الحدود المسموح بها (Δ_{max}).

$$\Delta_{max} = \begin{cases} \text{flat roof} = L/180 \\ \text{floors} \quad = L/360 \end{cases}$$

B. Supporting or attached to nonstructural element.

عندما يكون المنشأ يحمل او يتصل بأجزاء غير انشائية

- يتم حساب الهطول الناتج من (الحمل الحي + الهطول طويل الامد) ومقارنته مع الحدود المسموح بها(Δ_{max})

$$\Delta = (\Delta_i)_L + \Delta_{\text{long term}}$$

$$\Delta_{max} = \begin{cases} \text{damged by large deflection} = L/480 \\ \text{not damgeded} \qquad\qquad\quad = L/240 \end{cases}$$

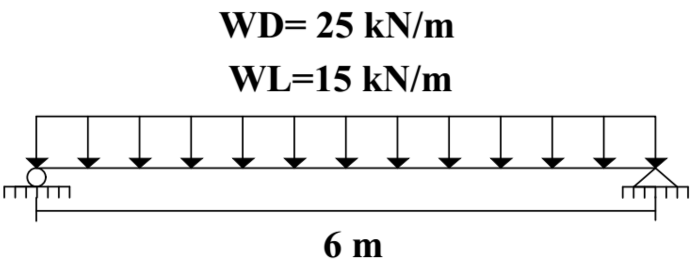
<u>ملاحظة حول حساب الهطول الناتج عن الحمل الحي نتبع الخطوات التالية:</u>

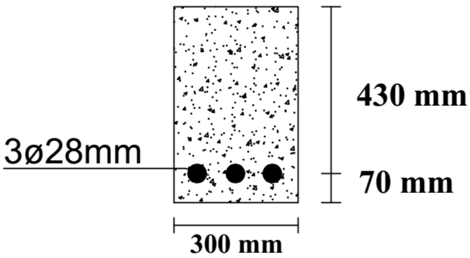
١. حساب الهطول الناتج عن الحمل الحي + الحمل الميت.
٢. حساب الهطول الناتج عن الحمل الميت.

$$(\Delta_i)_L = (\Delta_i)_{D+L} - (\Delta_i)_D$$

Ex-2

For the beam and the cross section shown below, D.L=25 kN/m, L.L=15 kN/m, sustained live load=30% of live load. Use n=9 and f_c'=25 MPa.





Find

1. Immediate deflection due to dead load + live load.
2. Find the long-term deflection after 5 years if the sustained live load=30% of live load
3. Total deflection after 5 years.
4. Check if the beam satisfies the deflection requirement if the beam is a part of a member constructed to support nonstructural element likely to be damaged by large deflection.
5. Check if the beam satisfies the deflection requirement if the beam is a part of a floor not supporting nonstructural element likely to be damaged by large deflection.

Solution:

1- Immediate deflection due to (dead load + live load).

$$\Delta = \frac{5WL^4}{384EI}$$

$$L = 6\,m\,; \quad w = 25 + 15 = 40\,kN/m$$

$$E = 4700\sqrt{f'_c} = 23500\,N/mm^2$$

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] I_{cr} \leq I_g$$

$$M_a = \frac{WL^2}{8} = \frac{40 \times 6^2}{8} = 180\,kN.m$$

$$M_{cr} = \frac{f_r\,I_g}{y_t} = \frac{0.62\sqrt{25} \times \frac{300 \times 500^3}{12}}{500/2} \times 10^{-6} = 38.75\,kN.m$$

$$\frac{M_a}{M_{cr}} = \frac{180}{38.75} = 4.64 > 3 \qquad I_e = I_{cr}$$

Find I_{cr}

$$\frac{by^2}{2} = n \times A_s(d - y)$$

$$A_s = 3 \times \frac{\pi}{4}\,28^2 = 1847.25\,mm^2$$

$$\frac{300 \times y^2}{2} = 9 \times 1847.25\,(430 - y)$$

$$150\,y^2 = 9 \times 1847.25\,(430 - y)$$

$$y^2 = 110.835\,(430 - y)$$

$$y^2 + 110.835y - 4765.05 = 0$$

$$y = 169.8 = 170\,mm$$

$$I_{cr} = \frac{300 \times 170^3}{3} + 9 \times 18470.25\,(430 - 170)^2 = 1.615 \times 10^9\ mm^4$$

$$(\Delta_\mathrm{i})_\mathrm{D+L} = \frac{5 \times 40 \times 6000^4}{384 \times 23500 \times 1.615 \times 10^9} = 17.78\,mm$$

---

2. The long-term deflection after 5 years if the sustained live load=30% of live load

$$\Delta_{sus} = \frac{5WL^4}{384EI}$$

$W_{sus} = D.L + 0.3\ L.L = 25 + 0.3\ \times 15 = 29.5\ kN/m$

$M_{sus} = \frac{WL^2}{8} = \frac{29.5 \times 6^2}{8}$ =132.75 kN.m

$\frac{M_a}{M_{cr}} = \frac{132.75}{38.75} = 3.425\ > 3\ \text{ use } \mathrm{I_e} = \mathrm{I_{cr}} = 1.615\ \times 10^9\ mm^4$

$\Delta_{sus} = \frac{5 \times 29.5 \times 6000^4}{384 \times 23500 \times 1.615 \times 10^9} = 13.116\ mm$

$$\lambda = \frac{\xi}{1 + 50\rho'}$$

$$\lambda = \frac{2}{1 + 50(0)}$$

$$\Delta_{LT} = \lambda\ \Delta_{sus} = 2 \times 13.116 = 26.23\ mm$$

3. Total deflection after 5 years.

$\Delta_{total} = (\Delta_\mathrm{i})_\mathrm{D+L} + \Delta_{LT}$ = 17.78 +26.23=44.01 mm

---

4. Check if the beam satisfies the deflection requirement if the beam is a part of a member constructed to support nonstructural element likely to be damaged by large deflection.

$\Delta = \Delta_{LT} + (\Delta_\mathrm{i})_\mathrm{L}$

$$\Delta_D = \frac{5WL^4}{384EI}$$

$w = 25\ kN/m\ ;\quad L = 6\ m\ ; E = 23500\ N/mm^2$

$$M_a = \frac{WL^2}{8} = 112.5\ kN.m$$

$$\frac{M_a}{M_{cr}} = \frac{112.5}{38.75} = 2.9 < 3$$

$$I_e = \left(\frac{38.75}{112.5}\right)^3 \times \frac{300 \ \times 500^3}{12} + \left[1 - \left(\frac{38.75_{cr}}{112.5}\right)^3\right] 1.615 \times 10^9$$

$I_e = 1.681 \times 10^9 mm^4 < I_g = 3.125 \times 10^9 \ mm^4$ ∴ ok

$$\Delta_D = \frac{5WL^4}{384EI} = \frac{5 \times 25 \times 6000^4}{384 \times 23500 \times 1.681 \times 10^9} = 10.68 \ mm$$

$\Delta_{\mathrm{L}} = \ \Delta_{D+L} - \ \Delta_D$

=17.78-10.68=7.1 mm

$\Delta = \ \Delta_{\mathrm{LT}} + \ \Delta_L = 26.23 + 7.1 = 33.33 \ mm$

$$\Delta_{\mathrm{max}} = \frac{L}{480} = \frac{6000}{480} = 12.5 \ mm$$

$33.33 \ mm > 12.5 \ mm \ \therefore not \ ok$

----

5. Check if the beam satisfies the deflection requirement if the beam is a part of a floor not supporting nonstructural element likely to be damaged by large deflection.

$\Delta_{\mathrm{L}} = \ \Delta_{D+L} - \ \Delta_D$

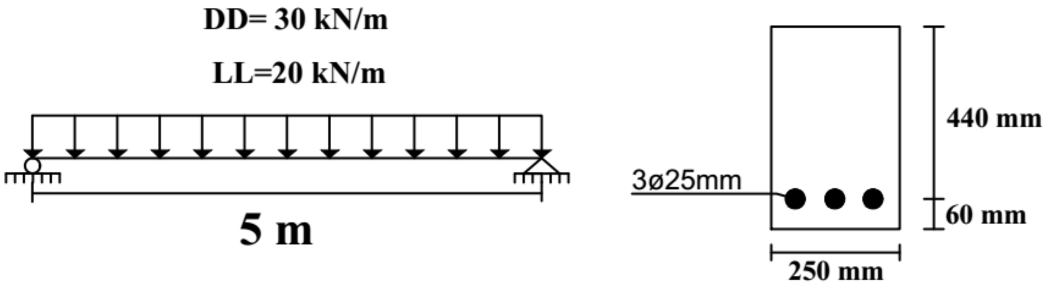
=17.78-10.68=7.1 mm

$$\Delta_{\mathrm{max}} = \frac{L}{360} = \frac{6000}{360} = 16.67 \ mm$$

$7.1 \ mm < 16.67 \ mm \ \therefore \ ok$

HW-2

For the beam and the cross section shown below, D.L=30 kN/m, L.L=20 kN/m, sustained live load=30% of live load. Use n=9 and f_c'=25 MPa.



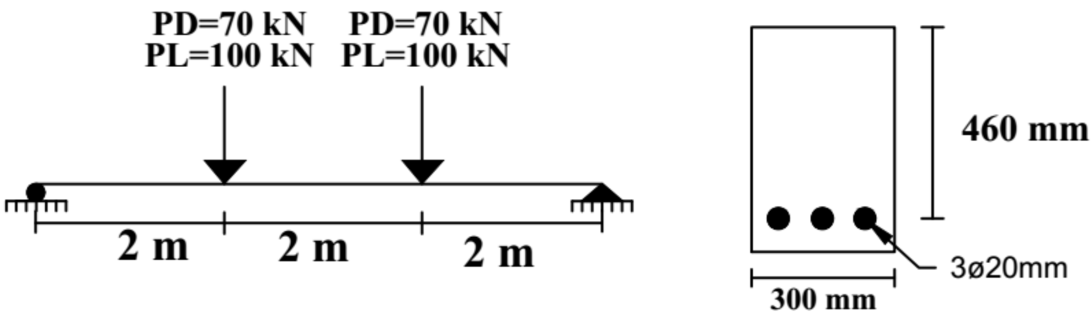
Find

1- Immediate deflection due to dead load + live load.

2- Find the long-term deflection after 5 years if the sustained live load=30% of live load
3- Total deflection after 5 years.
4- Check if the beam satisfies the deflection requirement if the beam is a part of a floor not supporting nonstructural element likely to be damaged by large deflection.
5- Check if the beam satisfies the deflection requirement if the beam is a part of a member constructed to support nonstructural element likely to be damaged by large deflection.

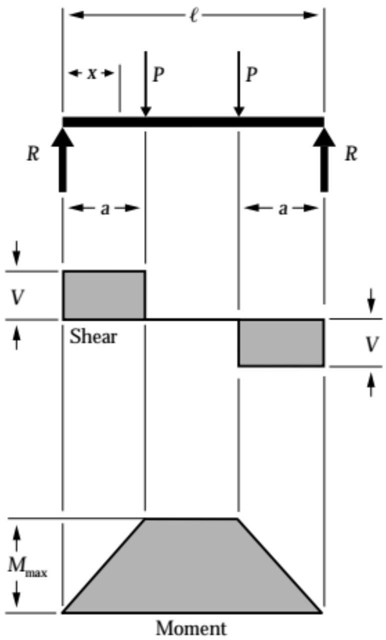
HW-3

For the beam and the cross section shown below, use n=9 and f_c'=25 MPa and find



1- Immediate deflection due to dead load + live load.

2- Find the long-term deflection after 5 years if the sustained live load=30% of live load
3- Total deflection after 5 years.



$R = V \dots \dots = P$

M_{max} (between loads) $\dots = Pa$

M_x (when $x < a$) $\dots = Px$

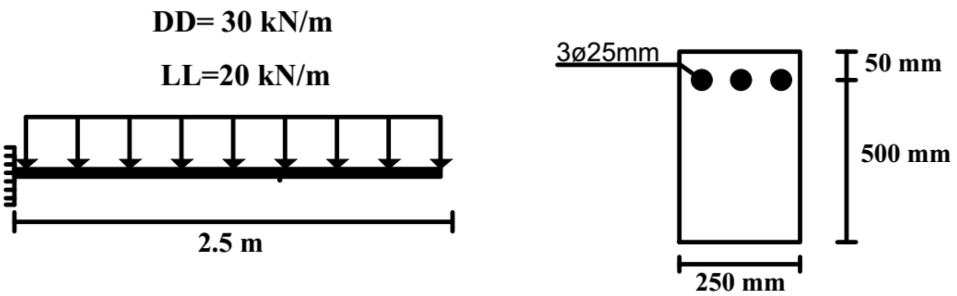
Δ_{max} (at center) $\dots = \dfrac{Pa}{24EI}(3\ell^2 - 4a^2)$

Δ_x (when $x < a$) $\dots = \dfrac{Px}{6EI}(3\ell a - 3a^2 - x^2)$

Δ_x $\left(\text{when } x > a \text{ and } < (\ell - a)\right) \dots = \dfrac{Pa}{6EI}(3\ell x - 3x^2 - a^2)$

HW-4

For the beam and the cross section shown below, use n=9 and f_c'=30 MPa and find



Find

1- Immediate deflection due to dead load + live load.

2- Find the long-term deflection after 5 years if the sustained live load=30% of live load

3- Total deflection after 5 years.

4- Check if the beam satisfies the deflection requirement if the beam is a part of a floor not supporting nonstructural element likely to be damaged by large deflection.

5- Check if the beam satisfies the deflection requirement if the beam is a part of a member constructed to support nonstructural element not likely to be damaged by large deflection.

Al Muthanna University
Collage of Engineering

Design of Reinforced Concrete Structures I

One way slabs-1

Dr Othman Hameed

Lecture (7)
One way slabs

1. Types of Slab

In reinforced concrete constructions, slabs are used to provide flat and useful surfaces. It may be supported by reinforced concrete beams (and is usually cast monolithically with such beams), by masonry or reinforced concrete walls, by structural steel members, or directly by columns.

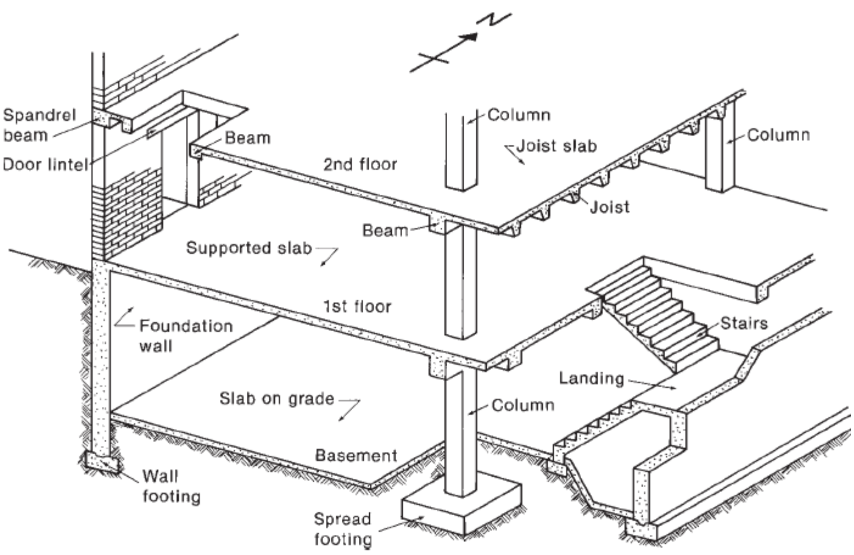
Slabs may be supported on two opposite sides only, as shown in Figure (a) in which case the structural action of the slab is essentially one-way, the loads being carried by the slab in the direction perpendicular to the supporting beams.

There may be beams on all four sides, as shown in Figure (b) so that two-way slab action is obtained. Intermediate beams, as shown in Figure (c), may be provided.

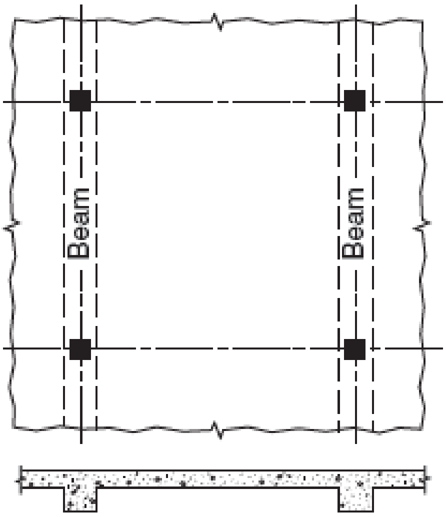
If the ratio of length to width of one slab panel is larger than about 2, most of the load is carried in *the short direction* to the supporting beams and one-way action is obtained in effect, even though supports are provided on all sides.

Concrete slabs in some cases may be carried directly by columns, as shown in Figures (d) and €, without the use of beams or girders. Such slabs are described as *flat plates* and are commonly used where spans are not large and loads not particularly heavy.

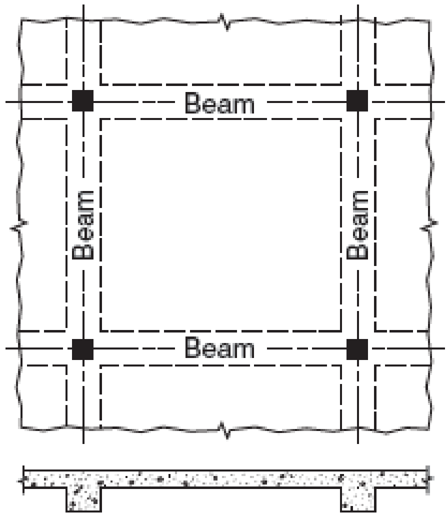
Closely related to the flat plate slab is the *two-way joist*, also known as a *grid* or *waffle slab*, shown in Fig. f.



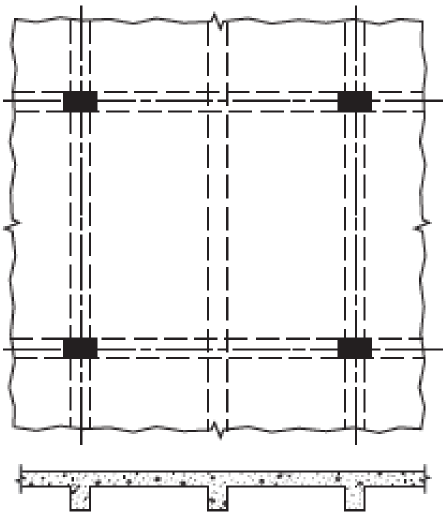
One way slabs

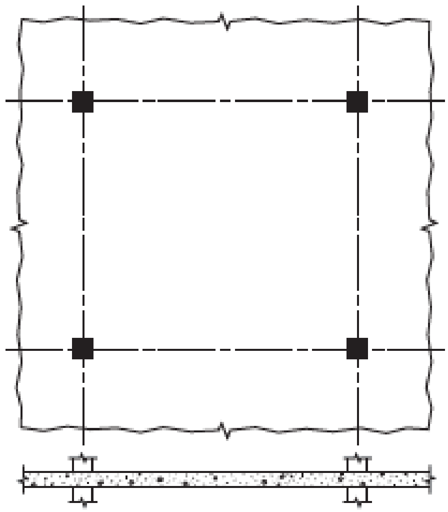


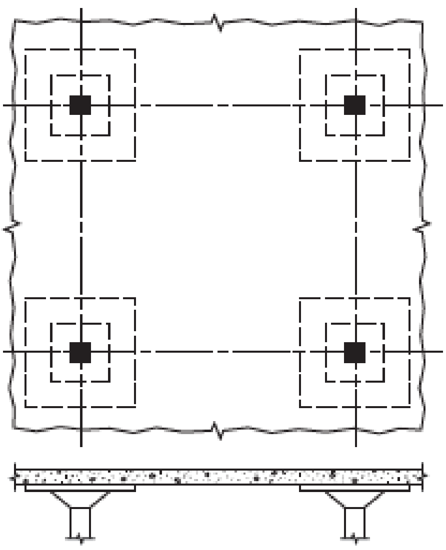
(*a*) One-way slab

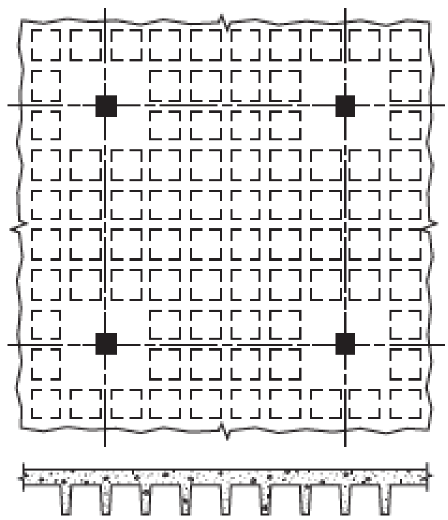


(*b*) Two-way slab


(*c*) One-way slab


(*d*) Flat plate


(*e*) Flat slab

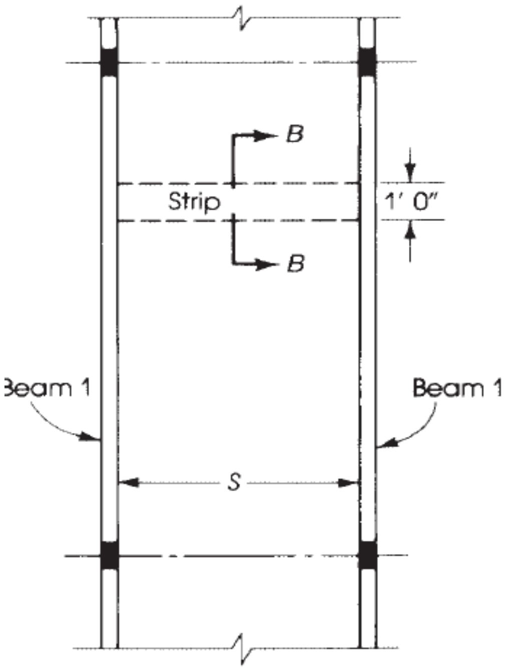

(*f*) Grid or waffle slab

2. One Way Slab

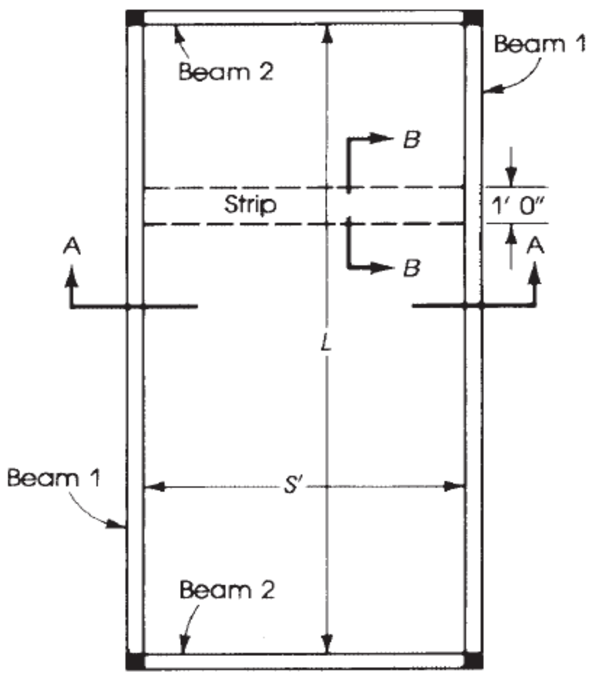
The slab is called one-way slab if:

1- If a slab is supported on two opposite sides only, it will bend or deflect in a direction perpendicular to the supported edges.
 The structural action is one way, and the loads are carried by the slab in the short direction. This type of slab is called a one-way slab.

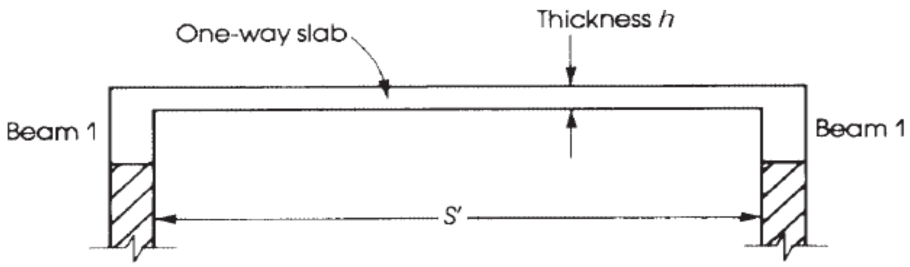
2- If the slab is supported on four sides and **the ratio of the long side to the short side is equal to or greater than 2**, then it is called a one-way solid slab.



(a)



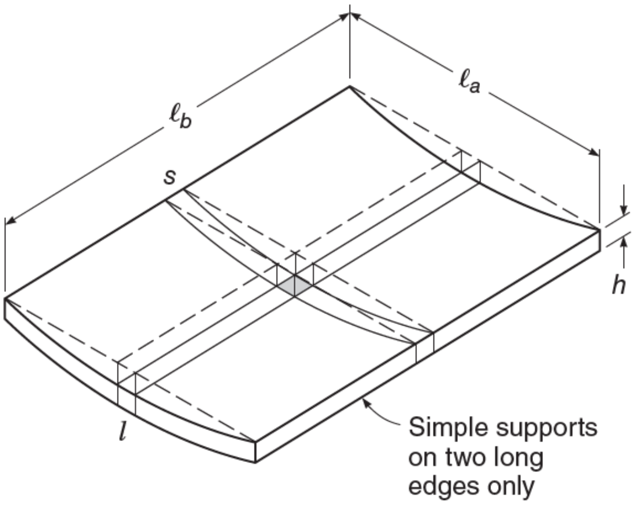
(b)



(c)

- **Why the ratio of $\frac{\textbf{\textit{long side of slab}}}{\textbf{\textit{short side of slab}}} \geq 2$ *is taken to be one way slab*?**

The slab shown in Figure has a simple support condition.



The deflection at mid span is equal in short and long direction, so

$\delta_a = \delta_b \quad at\ mid\ span$

$\delta_a = deflection\ in\ short\ direction$

$\delta_b = deflection\ in\ long\ direction$

$$\frac{5\,w_a l_a^4}{384EI} = \frac{5\,w_b l_b^4}{384EI}$$

$\frac{w_a}{w_b} = \left(\frac{l_b}{l_a}\right)^4 \qquad let\ \frac{l_b}{l_a} = 2 \qquad \rightarrow \frac{w_a}{w_b} = 16$

$w_a = 16\,w_b$

$w = w_a + w_b\ = 16\,w_b + w_b = 17\,w_b \qquad \rightarrow w_b = 0.06\,w \quad and\ w_a = 0.94\,w$

94% of the load applied on the slab will be distributed to the short direction, while only 6% of the load will go to the long direction. Therefore, the ratio of 2 was taken.

3. Design Requirement of One Way Solid Slab

3.1- Minimum Thickness of One Way Slab (ACI Table 9.5a)

The minimum thickness of one-way slabs using grade 420 steel can be defined according to the ACI Code, 9.5.2.1, Table 9.5a, for solid slabs and for beams or ribbed one-way slabs.

TABLE 9.5(a) — MINIMUM THICKNESS OF NONPRESTRESSED BEAMS OR ONE-WAY SLABS UNLESS DEFLECTIONS ARE CALCULATED

	Minimum thickness, ***h***			
	Simply supported	One end continuous	Both ends continuous	Cantilever
Member	Members not supporting or attached to partitions or other construction likely to be damaged by large deflections			
Solid one-way slabs	**ℓ/20**	**ℓ/24**	**ℓ/28**	**ℓ/10**
Beams or ribbed one-way slabs	**ℓ/16**	**ℓ/18.5**	**ℓ/21**	**ℓ/8**

Notes:
Values given shall be used directly for members with normalweight concrete and Grade 420 reinforcement. For other conditions, the values shall be modified as follows:
a) For lightweight concrete having equilibrium density, w_c, in the range of 1440 to 1840 kg/m^3, the values shall be multiplied by **(1.65 – 0.0003w_c)** but not less than 1.09.
b) For f_y other than 420 MPa, the values shall be multiplied by **(0.4 + f_y/700)**.

3.2- Shrinkage and Temperature reinforcement Ratio ($\rho_{shr.}$) (ACI 7.12.1.2)

Area of shrinkage and temperature reinforcement should be as follows, but not less than 0.0014:

$$\text{for } f_y = 280\ MPa\ to\ 350\ MPa \quad \rightarrow \rho_{shr.} = 0.002$$

$$\text{for } f_y = 420\ MPa \quad \rightarrow \rho_{shr.} = 0.0018$$

$$\text{for } f_y\ exceeding\ 420\ MPa \quad \rightarrow \rho_{shr.} = \frac{0.0018 * 420}{f_y} \geq 0.0014$$

$$As_{shr.} = \rho_{shr.} b\ h$$

3.3- Minimum Steel Reinforcement (ACI 10.5.4)

$$\rho_{min.} \geq \rho_{shr.}$$

$$As_{min\,.} = \rho_{min.} b\, h$$

3.4- Spacing of Steel Reinforcement (ACI 10.5.4)

a- For flexural reinforcement (ACI10.5.4) (main reinforcement)

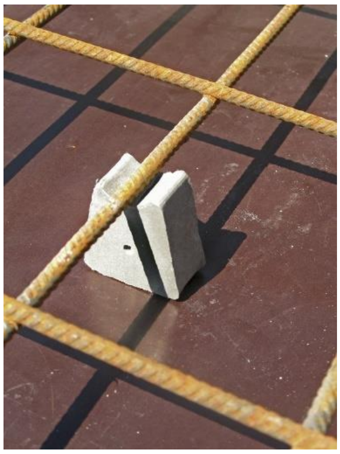
Maximum spacing of this reinforcement shall not exceed three times the thickness, nor 450 mm.

$$S_{max.} = min.of \begin{cases} three\ times\ slab\ thickness\ (3t) \\ 450\ mm \end{cases}$$

In practice $S_{max.} \leq 1.5\ thickness\ of\ slab$

b- For shrinkage reinforcement (ACI 7.12.2.2)

Shrinkage and temperature reinforcement shall be spaced not more than five times the slab thickness, nor farther apart than 450 mm



$$S_{max.} = min.of \begin{cases} five\ \ times\ slab\ thickness\ (5t) \\ 450\ mm \end{cases}$$

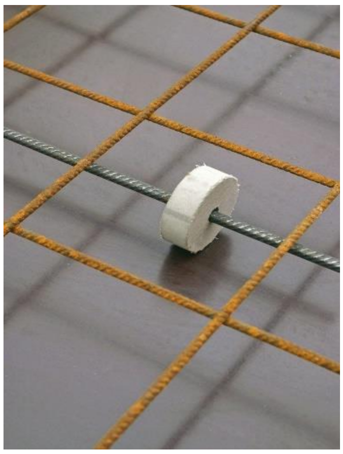
3.5- Minimum Cover (ACI 7.7.1)

- Concrete exposed to earth or weather
  For Ø <16 mm ---------- 40 mm

  For Ø >16 mm ----------50 mm

- Concrete not exposed to earth or weather
  For Ø <32 mm ---------- 20 mm

  Otherwise ----------------50 mm



3.6- Shear Capacity of One Way Slab.

$V_u \leq ØV_c$

$$V_c = \frac{1}{6}\sqrt{f_c'}\,b\ d$$

Otherwise increase slab thickness

4. Load Assigned to Slab

$w_u = 1.2\ D_L + 1.6\ L_L$

a- Dead load

- Self-weight of slab.
- Weight of finishing material.
- Weight of partition.

b- Live load

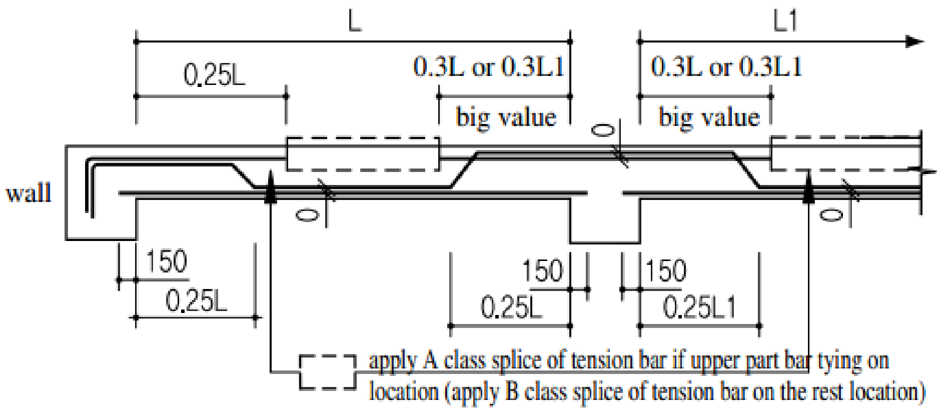
It depends on a function for which the slab has been constructed.

Minimum live Load values on slabs

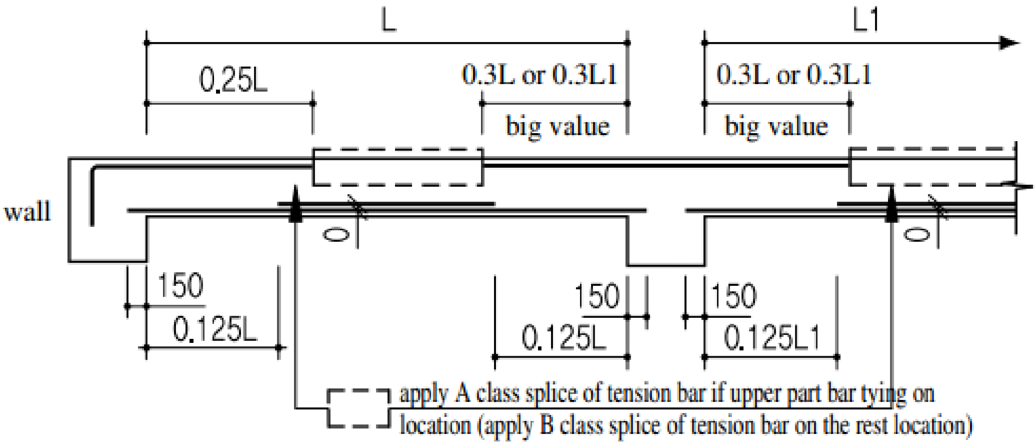
Type of Use	Uniform Live Load kN/m^2
Residential	2
Residential balconies	3
Computer use	5
Offices	2
Warehouses	
▪ Light storage	6
▪ Heavy Storage	12
Schools	
▪ Classrooms	2
Libraries	
▪ Rooms	3
▪ Stack rooms	6
Hospitals	2
Assembly Halls	
▪ Fixed seating	2.5
▪ Movable seating	5
Garages (cars)	2.5
Stores	
▪ Retail	4
▪ Wholesale	5
Exit facilities	5
Manufacturing	
▪ Light	4
▪ Heavy	6

5. Detail of Reinforcement for One Way Solid Slab.

A- Bent bar type



B- Cut bar type



6. Summary of One Way Solid Slab Design Procedure

a) Select a strip of 1 meter width in short direction.
b) Choose a slab thickness to satisfy deflection requirement.
c) Calculate the factored load.
d) Draw the shear and bending moment for each strip.
e) Check the adequacy of slab thickness in term of shear resistance.

$$V_u \leq \emptyset V_c \qquad\qquad V_c = \frac{1}{6}\sqrt{f_c'}\,b\,d$$

f) Design the flexural reinforcement. (use the equation of singly reinforcement)

$$Mu \leq \emptyset Mn \qquad\qquad Mu = \emptyset\,\rho\,bd^2 fy\left(1 - 0.59\frac{fy}{f_c'}\,\rho\right)$$

do not forget that

$$\emptyset = 0.9 \text{ if } \rho_t = 0.85\beta_1 \frac{f_c^{`}}{fy}\frac{0.003}{0.003+0.005} > \rho$$

g) Check the minimum steel reinforcement ratio (should be more than that of temperature and shrinkage)

Important notes

$\rho_{\,min.} \geq \rho_{\,shr.}$ [for $\rho_{\,min,}$ use $\rho_{\,shr.}$]
$A_s \geq A_{s\,min}$
If $A_s < A_{s\,min}$ then, use $A_s = A_{s\,min}$
$A_{s\ \ min} = \rho_{min.} * 1000 * h$

h) Check $\rho_{min.} < \rho < \rho_{max.}$

$$\rho_{max.} = 0.85\,\beta_1 \frac{f_c^{`}}{fy}\frac{0.003}{(0.003+0.004)}$$

i) Calculate $S_{max.}$ (for main reinforcement or flexural reinforcement)

$$S_{max.} = min.\text{ of } \begin{cases}\text{three times slab thickness } (3t) \\ 450\ mm\end{cases}$$

j) Compute the area of temperature and shrinkage reinforcement and find $S_{max.}$

$$S_{max.} = min.\,of \begin{cases}\text{five times slab thickness (5t)} \\ \text{450 mm}\end{cases}$$

k) Draw the detail of section and reinforcement.

Design of Reinforced Concrete Structures I

Al Muthanna University
Collage of Engineering

One way slabs-2

Dr Othman Hameed

Lecture (8)

One way slabs

One Way Solid Slab Design Procedure

a) Select a strip of 1 meter width in short direction.
b) Choose a slab thickness to satisfy deflection requirement.
c) Calculate the factored load.
d) Draw the shear and bending moment for each strip.
e) Check the adequacy of slab thickness in term of shear resistance.

$$V_u \leq \emptyset V_c \qquad V_c = \frac{1}{6}\sqrt{f_c'}\,b\,d$$

f) Design the flexural reinforcement. (use the equation of singly reinforcement)

$$Mu \leq \emptyset Mn \qquad Mu = \emptyset\,\rho\,bd^2 fy\left(1 - 0.59\frac{fy}{f_c'}\,\rho\right)$$

do not forget that

$$\emptyset = 0.9 \text{ if } \rho_t = 0.85\beta_1 \frac{f_c^`}{fy}\frac{0.003}{0.003+0.005} > \rho$$

g) Check the minimum steel reinforcement ratio (should be more than that of temperature and shrinkage)

Important notes

$\rho_{\ min.} \geq \rho_{\ shr.}$

If $\rho < \rho_{min.}$ then, use $\rho = \rho_{min.}$

$A_s = A_{s\ \ min} = \rho_{min.} * 1000 * h$

$A_s = n * A_{sb}$

$A_s = \frac{1000}{s} * A_{sb}$

h) Check $\rho_{min.} < \rho < \rho_{max.}$

$$\rho_{max.} = 0.85\,\beta_1 \frac{f_c^`}{fy}\frac{0.003}{(0.003+0.004)}$$

i) Calculate $S_{max.}$

$$S_{max.} = min.\text{of}\begin{cases}\text{three times slab thickness } (3t)\\ 450\ mm\end{cases}$$

j) Compute the area of temperature and shrinkage reinforcement and find $S_{max.}$

$$S_{max.} = min.of \begin{cases} \text{five times slab thickness (5t)} \\ 450 \text{ mm} \end{cases}$$

k) Draw the detail of section and reinforcement.

Example-1: A reinforced concrete slab is built integrally with a spandrel beam. The slab consists of two equal parts as shown in the figure. Design the slab to carry service live load of 4.8 kN/m^2.

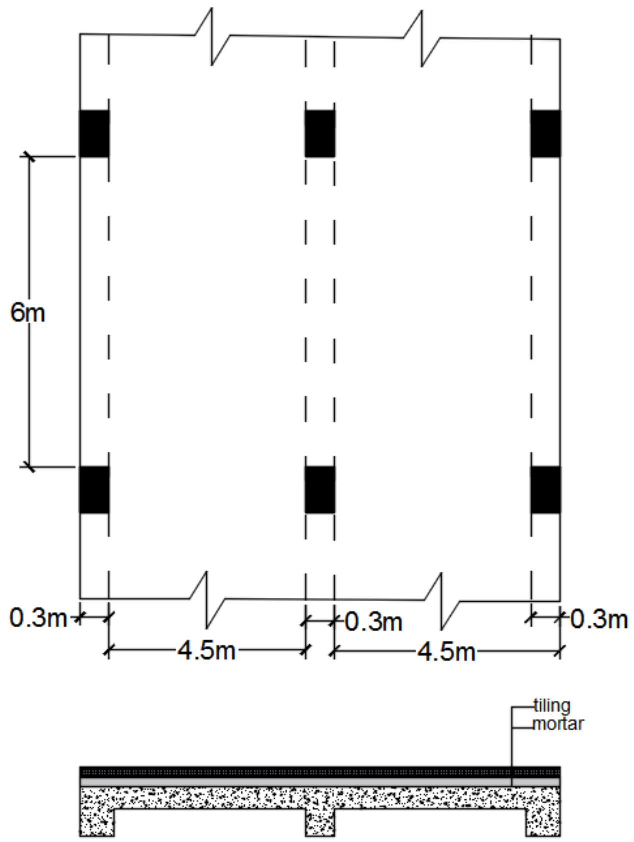
$f_c' = 28\ MPa\ and\ f_y = 420\ MPa$, use bar diameter of 12 mm for flexural reinforcement and $\emptyset\ 10\ mm$ for temperature and shrinkage.

The dead load is due to self weight plus weight of:

- Tiles 2 cm
- mortar 2 cm

(use density=25 kN/m^3 for concrete, mortar and tiles)

ملاحظة:- لايتم اضافة الاحمال الاضافية الى الحمل الميت الا اذا تم ذكرها في السؤال أما في الواقع العملي يجب أن تضاف.



Solution:

1- The slab is supported on two opposite beams, so the type of slab is one way solid slab.

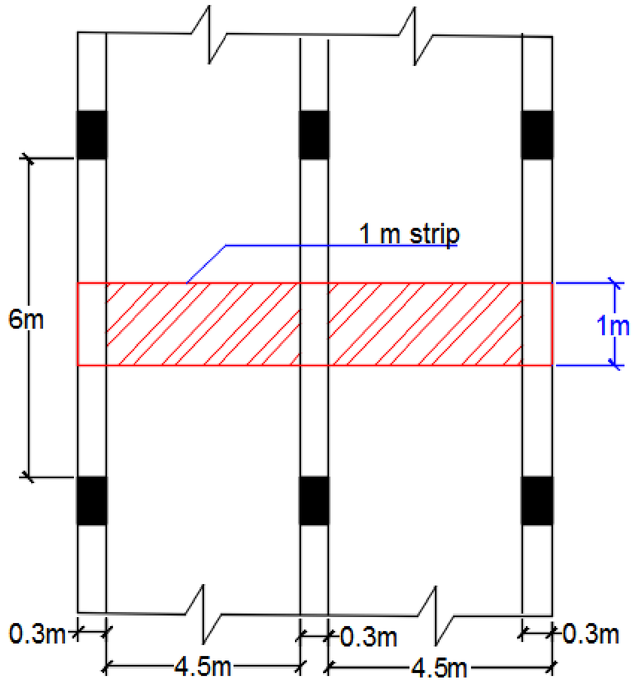
2- Take a strip of 1m width in short direction.

3- Choose the slab thickness according to ACI 9.5a

The slab is one end continuous

$$h = \frac{ln}{24} = \frac{4500}{24} = 187.5 \cong 190\ mm$$

$$d = h - cover - \frac{\emptyset}{2} = 190 - 20 - 6 = 164\ mm$$



4- Find the load assigned to slab.

a) Dead load

- Self weight of slab $= 0.19 * 25 = 4.75\ {kN}/{m^2}$
- Tiling & mortar $= 0.04 * 25 = 1\ {kN}/{m^2}$

- Total dead load $= 5.75\ kN/m^2$

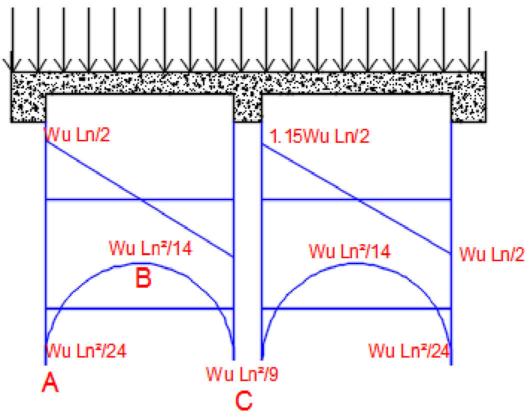
b) Live load

$$= 4.8\ {kN}/{m^2}$$

$$W_u = 1.2 * 5.75 + 1.6 * 4.8 = 14.58\ {kN}/{m^2}$$

$$\frac{W_u}{1m\ strip} = 14.58 * b(1m)$$

$$= 14.58\ {{kN}/{m}}/{1\ m}$$



5- Draw the shear and bending moment according to ultimate load.

6- Check the slab thickness according to shear requirement.

$$V_{u\ max.} = 1.15\frac{W_u\, l_n}{2} = 37.7\ {kN}/{1\ m}$$

$$V_{u\ @\ d} = 1.15\frac{W_u\ l_n}{2} - W_u.d$$

$$V_{u\ @\ d} = 37.7\ - 14.58 * 0.164 = 35.31\ {}^{kN}/_{1\ m}$$

$$\emptyset V_c = 0.75 * \frac{\sqrt{f_c'}}{6}\ b\ d$$

$$\emptyset V_c = 0.75 * \frac{\sqrt{28}}{6} * 1000 * 164 * 10^{-3} = 108.47\ {}^{kN}/_{1m}$$

$$V_{u\ @d.} = 35.31 < \emptyset V_c = 108.47$$

The slab thickness is enough

7- Design the flexural reinforcement

Section	Factor	Moment
A	$\frac{W_u l_n^2}{24}$	-12.3 $kN.m\ /\ m$
B	$\frac{W_u l_n^2}{14}$	+21.1 $kN.m/\ m$
C	$\frac{W_u l_n^2}{9}$	-32.8 $kN.m/\ m$

- Design section C

$$M_u = \ -38.8\ kN.m/1m$$
$$\emptyset Mn \geq Mu$$

$$Mu = \emptyset\ \rho\ bd^2 fy\ \left(1 - 0.59\frac{fy}{f_c'}\ \rho\right)$$

Assume $\emptyset = 0.9$ to be checked later

$$\rho = \frac{1 \pm \sqrt{1 - \frac{2.36 * M_u * 10^6}{\emptyset\ b\ d^2 f_c'}}}{1.18\left(\frac{fy}{f_c'}\right)}$$

$$\rho = \frac{1 \pm \sqrt{1 - \frac{2.36 * 32.8 * 10^6}{0.9 * \ 1000 * \ 164^2 * 28}}}{1.18\left(\frac{420}{28}\right)}$$

$$0.0388 = 0.9 * \ \rho * 1 * 0.164^2 * 420 \left(1 - 0.59 \frac{420}{28} \rho\right) \ \rightarrow \rho = 0.00332$$

$$\rho_t = 0.85 * 0.85 \frac{28}{420} \frac{0.003}{0.003 + 0.005} = 0.018 > \rho \ \therefore \emptyset = 0.9$$

$$\rho_{\ min.} \geq \rho_{\ shr.} \qquad for \ fy = 420 \ \rightarrow \rho_{\ shr.} = 0.0018$$

$\rho_{min.} < \rho < \rho_{max.} \quad ok$

$A_s = 0.00332 * 1000 * 164 = 544.48 \ mm^2 \ /1m$

$A_{s \ \ min} = 0.0018 * 1000 * 190 = 342 \ mm^2 \ /1m$

$A_s > \ A_{s \ \ min} \ \ ok$

$$n = \frac{As}{area \ of \ one \ bar} = \frac{544.48}{113.1} = 4.8 \ bar/1m$$

$$S = \frac{1000}{4.8} = 208 \ mm = 200 \ mm$$

$$S_{max.} = min. of \begin{cases} three \ times \ slab \ thickness \ (3t) = 3 * 190 = 570mm \\ 450 \ mm \end{cases}$$

$S = 200 \ mm < \quad S_{max.} = 450 \ mm$

Use Ø12 mm @200mm c/c

- <u>Design section A</u>

<u>$M_u = \ -12.3 \ kN.m/1m$</u>

$\emptyset Mn \geq Mu$

$$Mu = \emptyset \ \rho \ bd^2 fy \left(1 - 0.59 \frac{fy}{f_c'} \ \rho\right)$$

Assume $\emptyset = 0.9$ to be checked later

$$0.123 = 0.9 * \ \rho * 1 * 0.164^2 * 420 \left(1 - 0.59 \frac{420}{28} \rho\right) \ \rightarrow \rho = 0.00122$$

$$\rho_t = 0.85\,\beta_1 \frac{f_c^{`}}{fy} \frac{\varepsilon_c}{\varepsilon_c + 0.005}$$

$$\rho_t = 0.85 * 0.85 \frac{28}{420} \frac{0.003}{0.003 + 0.005} = 0.018 > \rho \;\; \therefore \emptyset = 0.9$$

$$\rho_{\ min.} \geq \rho_{\ shr.} \qquad for\ fy = 420 \ \rightarrow \rho_{\ shr.} = 0.0018$$

$$A_s = 0.00122 * 1000 * 164 = 200.8 \ mm^2 \ /1m$$
$$A_{s\ \ min} = 0.0018 * 1000 * 190 = 342\ mm^2\ /1m$$
$$A_s < A_{s\ \ min} \rightarrow \quad use \qquad A_{s\ \ min} = 342\ mm^2\ /1m$$
$$n = \frac{As}{area\ of\ one\ bar} = \frac{342}{113.1} = 3\ bar/1m$$
$$S = \frac{1000}{3} = 333.33\ mm = 300\ mm$$
$$S_{max.} = min.of \begin{cases} three\ times\ slab\ thickness\ (3t) = 3 * 190 = 570mm \\ 450\ mm \end{cases}$$

$$S = 300\ mm < \quad S_{max.} = 450\ mm$$

Use Ø12 mm @300mm c/c

- Design section B

$$\underline{M_u = \ +21.1\ kN.m/1m}$$

$$\emptyset Mn \geq Mu$$

$$Mu = \emptyset\,\rho\,bd^2 fy \left(1 - 0.59 \frac{fy}{f_c'}\,\rho\right)$$

Assume $\emptyset = 0.9$ to be checked later

$$0.21.1 = 0.9 * \ \rho * 1 * 0.164^2 * 420 \left(1 - 0.59 \frac{420}{28}\,\rho\right) \ \rightarrow \rho = 0.00221$$

$$\rho_t = 0.85\,\beta_1 \frac{f_c^{`}}{fy} \frac{\varepsilon_c}{\varepsilon_c + 0.005}$$

$$\rho_t = 0.85 * 0.85 \frac{28}{420} \frac{0.003}{0.003 + 0.005} = 0.018 > \rho \;\; \therefore \emptyset = 0.9$$

$$\rho_{\ min.} \geq \rho_{\ shr.} \qquad for\ fy = 420 \ \rightarrow \rho_{\ shr.} = 0.0018$$

$$\rho_{min.} < \rho < \rho_{max.} \quad ok$$

$$A_s = 0.00221 * 1000 * 164 = 346\ mm^2\ /1m$$

$$A_{s\ min} = 342\ mm^2\ /1m$$

$$n = \frac{As}{area\ of\ one\ bar} = \frac{346}{113.1} = 3\ bar/1m$$

$$S = \frac{1000}{3} = 333.33\ mm = 300\ mm$$

$$S_{max.} = min.of \begin{cases} \text{three times slab thickness } (3t) = 3 * 190 = 570mm \\ 450\ mm \end{cases}$$

$$S = 300\ mm < \quad S_{max.} = 450\ mm$$

Use Ø12 mm @300mm c/c

8- Temperature and shrinkage reinforcement.

$$A_{s\ shr.} = 0.0018 * 1000 * 190 = 342\ mm^2\ /1m$$

Use Ø10 mm as shrinkage reinforcement

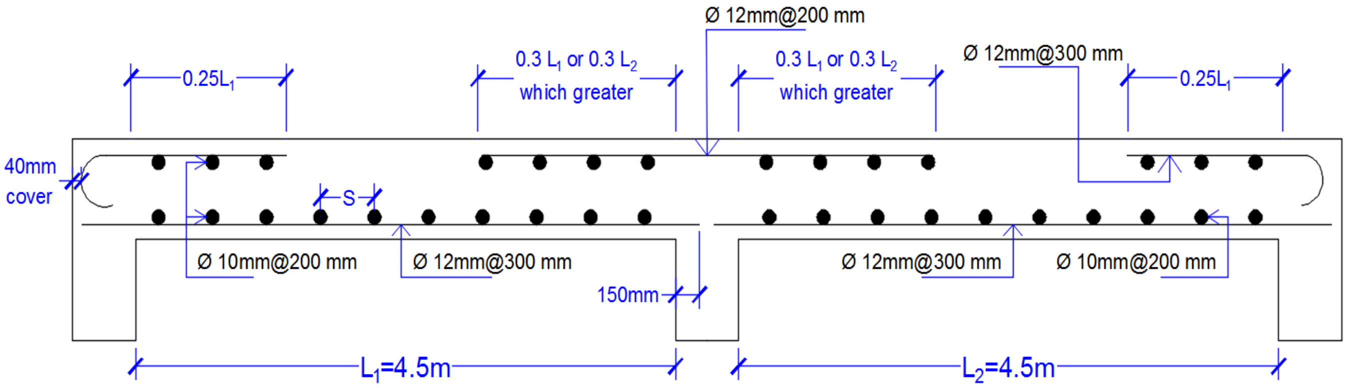
$$n = \frac{As}{area\ of\ one\ bar} = \frac{342}{78.5} = 4.35\ bar/1m$$

$$S = \frac{1000}{4.35} = 229\ mm = 200\ mm$$

$$S_{max.} = min.of \begin{cases} three\ times\ slab\ thickness\ (5t) = 5 * 190 = 950mm \\ 450\ mm \end{cases}$$

$S = 200\ mm < \quad S_{max.} = 450\ mm \rightarrow$ Use Ø10 mm @200mm c/c

9- Sketch detail of reinforcement

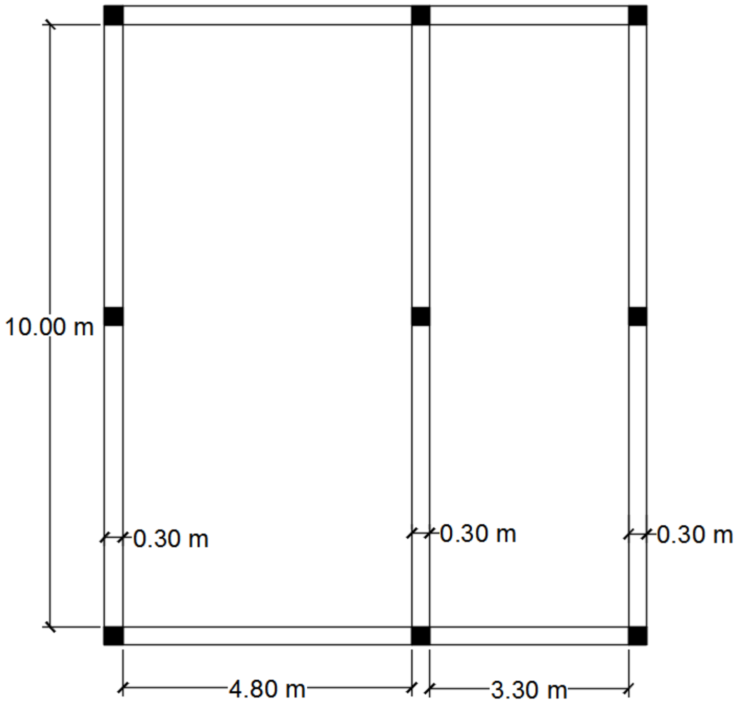


H.W: Design the slab of the plan shown below. The dead load is due to self-weight plus weight of:

- Tiles 3 cm (density 20 kN/m^3)
- sand 7 cm (density 20 kN/m^3)
- mortar 2 cm (density 20 kN/m^3)
- plaster 2 cm (density 20 kN/m^3)
- partition 2 kN/m^2

Live load =2.5 kN/m^2

$$f_c' = 25\ MPa\ and\ f_y = 420\ MPa$$



ملاحظة

الطول المستخدم لايجاد العزوم حسب طريقة (ACI التقريبية) هي الطول الصافي(l_n)

الطول المستخدم لايجاد سمك السقف حسب طريقة (ACI) هي الطول الصافي(l_n)

Design of Reinforced Concrete Structures I

Al Muthanna University
Collage of Engineering

One way slabs-3

Dr Othman Hameed

Lecture (9)

One way slabs

One Way Solid Slab Design Procedure

1- Find h and d

$$d = h - cover - \frac{\emptyset}{2}$$

2- Find ρ from

$$A_s = n * A_{sb}$$

$$n = \frac{1000}{S}$$

$$\rho = \frac{A_s}{b\ d}$$

3- Check $\rho_{min.} < \rho < \rho_{max.}$

$$\rho_{\ min.} = \rho_{\ shr.}$$

$$\rho_{max.} = 0.85\ \beta_1\ \frac{f_c^{`}}{fy}\ \frac{0.003}{(0.003 + 0.004)}$$

4- Find Mu from

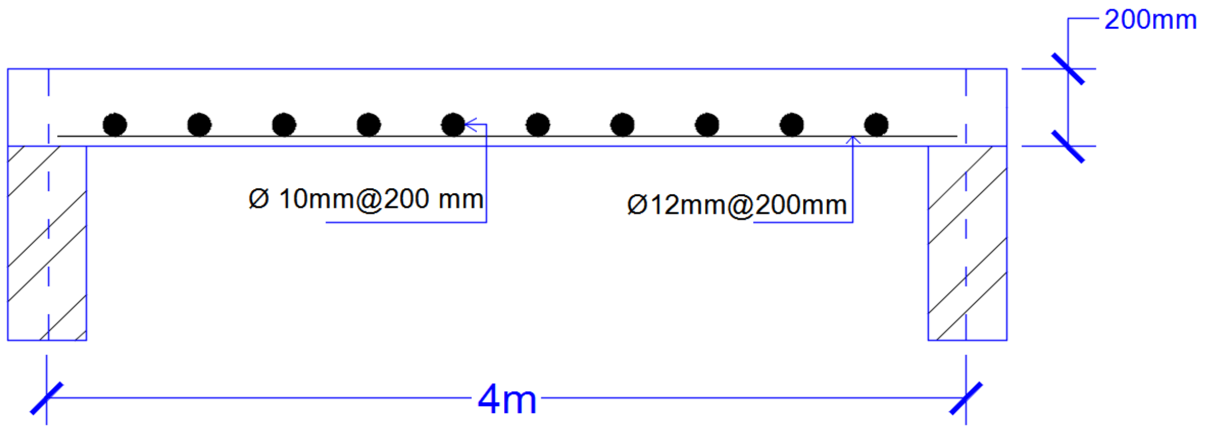
$\emptyset = 0.9$ if $\rho_t = 0.85\beta_1\ \frac{f_c^{`}}{fy}\ \frac{0.003}{0.003+0.005} > \rho$

$$Mu = \emptyset\ \rho\ bd^2 fy\left(1 - 0.59\frac{fy}{f_c'}\ \rho\right)$$

5- Find Wu or WL

Example-2: For the simply supported reinforced concrete one way solid slab of section and detail shown below, find the maximum live load can be carried by this slab.

$f_c' = 25\ MPa\ and\ fy = 420\ MPa$



Solution:

$$A_s = n * A_{sb}$$

$$A_s = \frac{1000}{S} * A_{sb}$$

$$n = \frac{1000}{S} = \frac{1000}{200} = 5\emptyset12/1m$$

$$As = 5 * 113.3 = 565.5\ mm^2$$

$$d = h - cover - \frac{\emptyset}{2} = 200 - 20 - 6 = 174\ mm$$

$$\rho = \frac{A_s}{b\ d} = \frac{565.5}{1000 * 174} = 0.00324$$

$$\rho_{\ min.} \geq \rho_{\ shr.} \qquad for\ f_y = 420\ \rightarrow \rho_{\ shr.} = 0.0018$$

$$\rho_{max.} = 0.85\ \beta_1 \frac{f_c^{`}}{fy} \frac{0.003}{(0.003 + 0.004)} = 0.0184$$

$$\rho_{min.} < \rho < \rho_{max.} \quad ok$$

$$\emptyset Mn \geq Mu$$

$$Mu = \emptyset\ \rho\ bd^2 fy \left(1 - 0.59 \frac{fy}{f_c'}\ \rho\right)$$

$$\rho_t = 0.85 * 0.85 \frac{25}{420} \frac{0.003}{0.003 + 0.005} = 0.0161 > \rho\ \therefore \emptyset = 0.9$$

$$Mu = 0.9 * 0.00324 * 1000 * 174^2 * 420 \left(1 - 0.59 \frac{420}{25}\ 0.00324\right) * 10^{-6}$$
$$= 35.88\ kN.m/1m$$

$$Mu = \frac{Wu\ l^2}{8} \rightarrow 35.88 = \frac{Wu\ 4^2}{8} \rightarrow Wu = 17.94\ \frac{kN}{m}/1m$$

$$V_{u\ @\ d} = \frac{W_u\ l}{2} - W_u.d$$

$$V_{u\ @\ d} = \frac{W_u * 4}{2} - 0.174 * W_u$$

$$V_{u\ @\ d} = 1.826\ W_u$$

$$\emptyset V_c = 0.75 * \frac{\sqrt{f_c'}}{6}\ b\ d$$

$$\emptyset V_c = 0.75 * \frac{\sqrt{25}}{6} * 1000 * 174 * 10^{-3} = 108.75\ kN/1m$$

$$\emptyset V_c \geq V_{u\ @\ d}$$

$$108.75 = 1.826\ W_u \rightarrow W_u = 59.56\ \frac{kN}{m}/1m$$

Use the minimum one ($Wu = 17.94\ \frac{kN}{m}/1m$)

$$D.L = 0.2 * 24 = 4.8\ kN/m^2$$

$$Wu = 1.2 * D.L + 1.6 * L.L$$

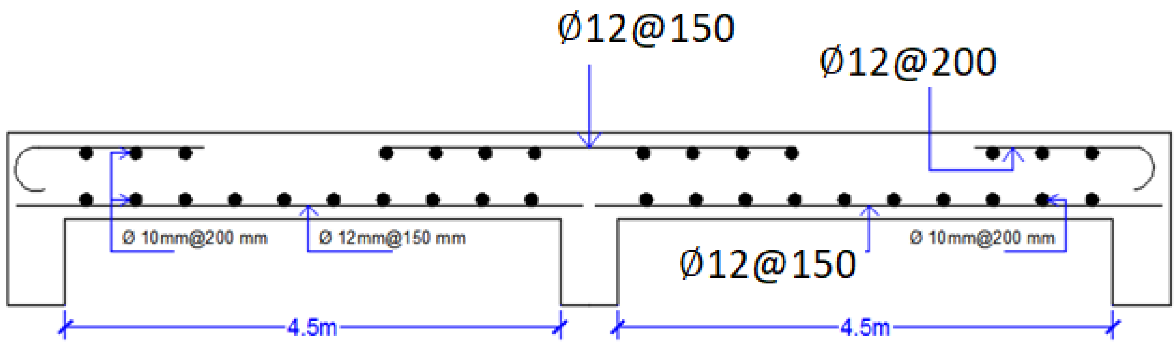
$$17.94 = 1.2 * 4.8 + 1.6 * L.L$$

$$L.L = 7.61\ kN/m^2$$

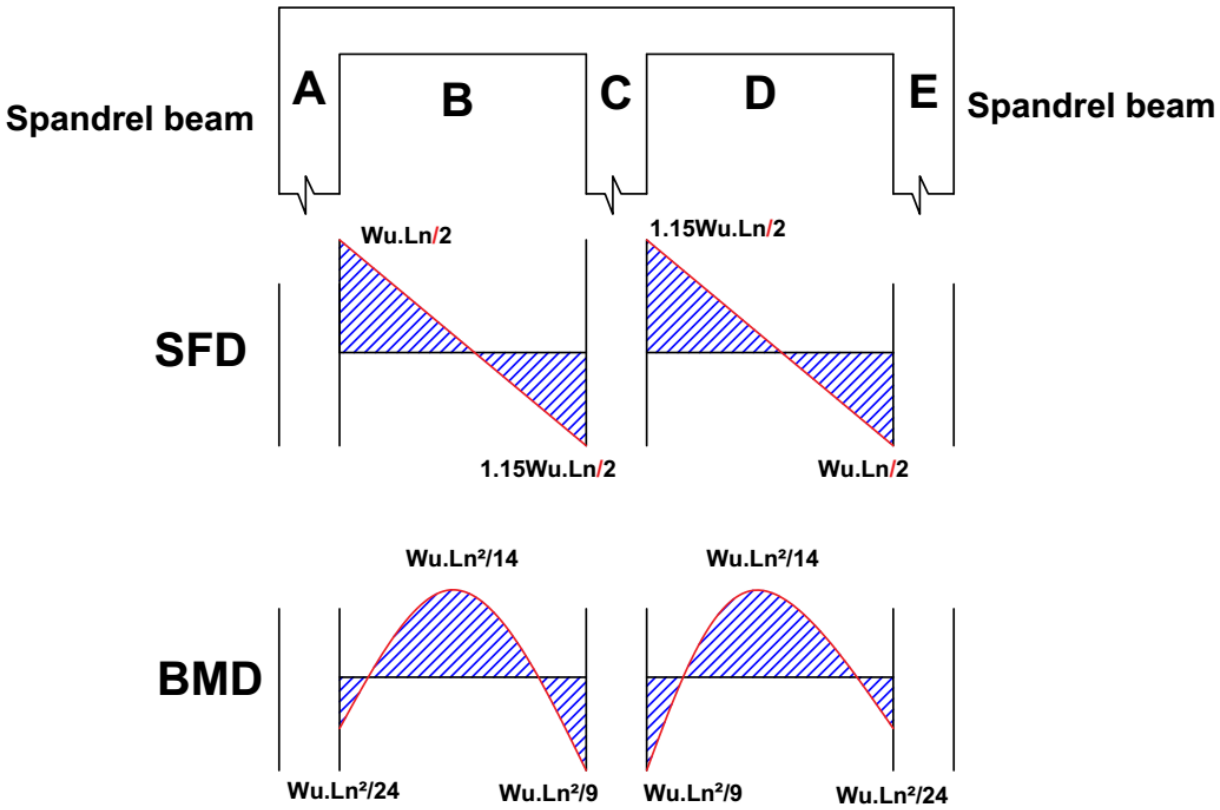
Example-3: A reinforced concrete slab is built integrally with a spandrel beam. The slab consists of two equal parts as shown in the figure. Use $f_c' = 28\ MPa\ and\ f_y = 420\ MPa$. Find the maximum live load can be carried by this slab. The dead load is due to the self-weight plus weight of:

- Tiles 2 cm
- mortar 2 cm

(Use density=24 kN/m^3 for concrete, mortar and tiles)



Solution



$$h = \frac{ln}{24} = \frac{4500}{24} = 187.5 \ \cong 190\ mm$$

$$d = h - cover - \frac{\emptyset}{2} = 190 - 20 - 6 = 164\ mm$$

Draw the shear and bending moment according to ultimate load.

Section	Factor
Moment (A)	$\frac{W_u l_n^2}{24}$
Moment (B)	$\frac{W_u l_n^2}{14}$
Moment (C)	$\frac{W_u l_n^2}{9}$
Shear	$\frac{1.15\ W_u l_n}{2}$

Moment @ (A)=$\frac{W_u l_n^2}{24}$

$$A_s = n * A_{sb}$$

$$A_s = \frac{1000}{S} * A_{sb}$$

$$n = \frac{1000}{S} = \frac{1000}{200} = 5$$

$$As = 5 * 113 = 565\ mm^2$$

$$\rho = \frac{A_s}{b\ d} = \frac{565}{1000 * 164} = 0.00344$$

$\rho_{\ min.} \geq \rho_{\ shr.}$ $\quad$ $for\ f_y = 420\ \rightarrow \rho_{\ shr.} = 0.0018$

$$\rho_{max.} = 0.85\ \beta_1\ \frac{f_c^`}{fy}\ \frac{0.003}{(0.003 + 0.004)} = 0.0206$$

$\rho_{min.} < \rho < \rho_{max.}$ $\quad ok$

$$\emptyset Mn \geq Mu$$

$$Mu = \emptyset\ \rho\ bd^2 fy\left(1 - 0.59\frac{fy}{f_c'}\ \rho\right)$$

$$\rho_t = 0.85 * 0.85\frac{28}{420}\ \frac{0.003}{0.003 + 0.005} = 0.0181 > \rho\ \ \therefore \emptyset = 0.9$$

$$Mu = 0.9 * 0.00344 * 1000 * 164^2 * 420\left(1 - 0.59\frac{420}{28}\ 0.00344\right) * 10^{-6}$$
$$= 34\ kN.m/1m$$

$$Mu = \frac{W_u l_n^2}{24} \rightarrow 34 = \frac{Wu * \ 4.5^2}{24} \rightarrow \ Wu = 40.3\ \ \frac{kN}{m}/1m$$

Moment @ (B)=$\frac{W_u l_n^2}{14}$

$$A_s = n * A_{sb}$$

$$A_s = \frac{1000}{S} * A_{sb}$$

$$n = \frac{1000}{S} = \frac{1000}{150} = 6.66$$

$$As = 6.66 * 113 = 752.6\ mm^2$$

$$\rho = \frac{A_s}{b\ d} = \frac{752.6}{1000 * 164} = 0.00459$$

$\rho_{\ min.} \geq \rho_{\ shr.}$ $\qquad$ $for\ f_y = 420\ \rightarrow \rho_{\ shr.} = 0.0018$

$$\rho_{max.} = 0.85\ \beta_1\ \frac{f_c^{`}}{fy}\ \frac{0.003}{(0.003 + 0.004)} = 0.0206$$

$\rho_{min.} < \rho < \rho_{max.}$ $\quad$ ok

$$\emptyset Mn \geq Mu$$

$$Mu = \emptyset\ \rho\ bd^2 fy\left(1 - 0.59\frac{fy}{f_c'}\ \rho\right)$$

$$\rho_t = 0.85 * 0.85\frac{28}{420}\ \frac{0.003}{0.003 + 0.005} = 0.0181 > \rho\ \ \therefore \emptyset = 0.9$$

$$Mu = 0.9 * 0.00459 * 1000 * 164^2 * 420 \left(1 - 0.59 \frac{420}{28}\, 0.00459\right) * 10^{-6}$$
$$= 44.77\ kN.m/1m$$

$$Mu = \frac{W_u l_n^2}{14} \rightarrow 44.77 = \frac{Wu * \ 4.5^2}{14} \rightarrow Wu = 30.95\ \frac{kN}{m}/1m$$

Moment @ (C)=$\frac{W_u l_n^2}{9}$

$$A_s = n * A_{sb}$$

$$A_s = \frac{1000}{S} * A_{sb}$$

$$n = \frac{1000}{S} = \frac{1000}{150} = 6.66$$

$$As = 6.66 * 113 = 752.6\, mm^2$$

$$\rho = \frac{A_s}{b\ d} = \frac{752.6}{1000 * 164} = 0.00459$$

$$\rho_{\ min.} \geq \rho_{\ shr.} \qquad for\ f_y = 420 \ \rightarrow \rho_{\ shr.} = 0.0018$$

$$\rho_{max.} = 0.85\, \beta_1 \frac{f_c^`}{fy} \frac{0.003}{(0.003 + 0.004)} = 0.0206$$

$$\rho_{min.} < \rho < \rho_{max.} \quad ok$$

$$\emptyset Mn \geq Mu$$

$$Mu = \emptyset\, \rho\, bd^2 fy \left(1 - 0.59 \frac{fy}{f_c'}\, \rho\right)$$

$$\rho_t = 0.85 * 0.85 \frac{28}{420} \frac{0.003}{0.003 + 0.005} = 0.0181 > \rho\ \therefore \emptyset = 0.9$$

$$Mu = 0.9 * 0.00459 * 1000 * 164^2 * 420 \left(1 - 0.59 \frac{420}{28}\, 0.00459\right) * 10^{-6}$$
$$= 44.77\ kN.m/1m$$

$$Mu = \frac{W_u l_n^2}{9} \rightarrow 44.77 = \frac{Wu * \ 4.5^2}{9} \rightarrow \boxed{Wu = 19.9 \ \frac{kN}{m}/1m}$$

Find the load from the shear

$$V_{u \ @ \ d} = \frac{1.15 \ W_u \ l}{2} - W_u . d$$

$$V_{u \ @ \ d} = \frac{1.15 W_u * 4.5}{2} - 0.164 * W_u$$

$$V_{u \ @ \ d} = 2.4235 \ W_u$$

$$\emptyset V_c = 0.75 * \frac{\sqrt{f_c'}}{6} \ b \ d$$

$$\emptyset V_c = 0.75 * \frac{\sqrt{28}}{6} * 1000 * \ 164 * 10^{-3} = 108.47 \ kN/1m$$

$$\emptyset V_c \geq V_{u \ @ \ d}$$

$$108.47 = 2.4235 \ W_u \ \rightarrow \ W_u = 44.77 \ \frac{kN}{m}/1m$$

Use the minimum value of Wu which is 19.9 $\frac{kN}{m}/1m$

$$Wu = 1.2Wd + 1.6Wl$$

Find the dead load

- Self weight of slab $= 0.19 * 24 = 4.56 \ {}^{kN}/{}_{m^2}$
- Tiling & mortar $= 0.04 * 24 = 0.96 \ {}^{kN}/{}_{m^2}$

- Total dead load $= 5.52 \ kN/m^2$

Find the live load

$$Wu = 1.2Wd + 1.6Wl$$

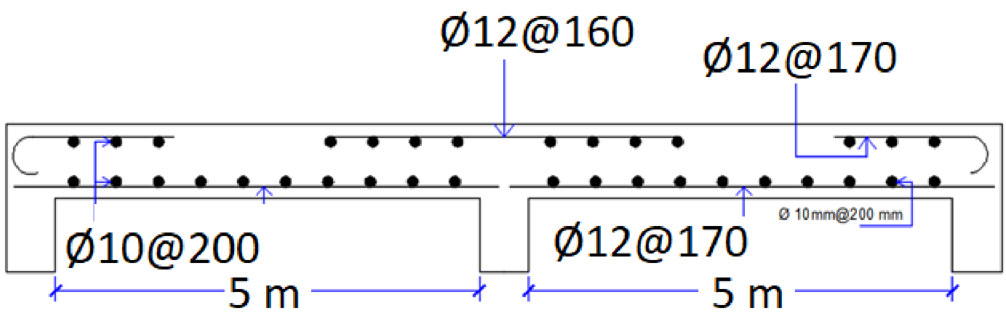
$$19.9 = 1.2 * 5.52 \ + 1.6Wl$$

$$Wl = 8.3 \ kN/m^2$$

H.W: A reinforced concrete slab is built integrally with a spandrel beam. The slab consists of two equal parts as shown in the figure. Use $f_c' = 25\ MPa\ and\ f_y = 420\ MPa$. Find the maximum live load can be carried by this slab. The dead load is due to

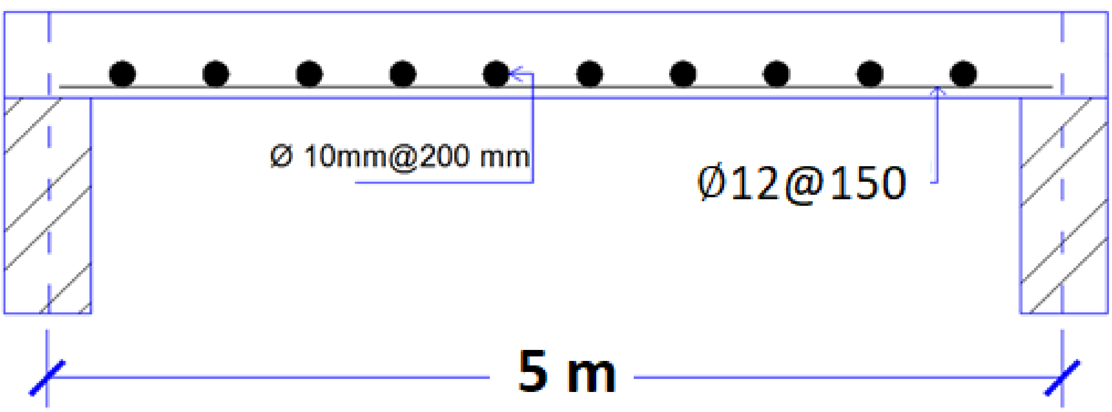
- The self-weight
- Tiles 2 cm
- mortar 2 cm

(Use density=24 kN/m^3 for concrete, mortar and tiles)



HW: For the simply supported reinforced concrete one way solid slab of section and detail shown below, find the maximum live load can be carried by this slab.

$f_c' = 27\ MPa\ and\ fy = 420\ MPa$



Design of Reinforced Concrete Structures I

Al Muthanna University
Collage of Engineering

One way slabs-4

Dr Othman Hameed

Lecture (10)

One way slabs

ملاحظة

عندما يتم اعطاء التسليح في السؤال و المطلوب هو (هل التسليح كافي), في هذه الحالة يمكن حل السؤال بطريقتين

١- استخراج العزم الداخلي (الذي يوفره التسليح الموجود) ومقارنته مع العزم الخارجي (المحسوب من الاحمال)

If $\emptyset Mn \geq Mu \rightarrow$ ok

$\emptyset Mn < Mu \rightarrow$ Not ok

٢- استخراج التسليح (حسب الحمل الخارجي) ومقارنته مع التسليح المعطى في السؤال,
- اذا كان التسليح المحسوب اقل من التسليح المعطى فهذا معناه أن التسليح المعطى في السؤال كافي ومناسب
- اذا كان التسليح المحسوب أكبر من التسليح المعطى في السؤال فهذا معناه أن التسليح المعطى في السؤال غير كافي

ملاحظة

لمعرفة قيمة التسليح الاكبر في السقوف, يجب حساب قيمة As

$\emptyset 12@200mm > \emptyset 12@300mm$

because

For $\emptyset 12@200mm$

$$n = \frac{1000}{200} = 5 \ \rightarrow As = 5 * \frac{\pi}{4} 12^2 = 565.5 \ mm^2$$

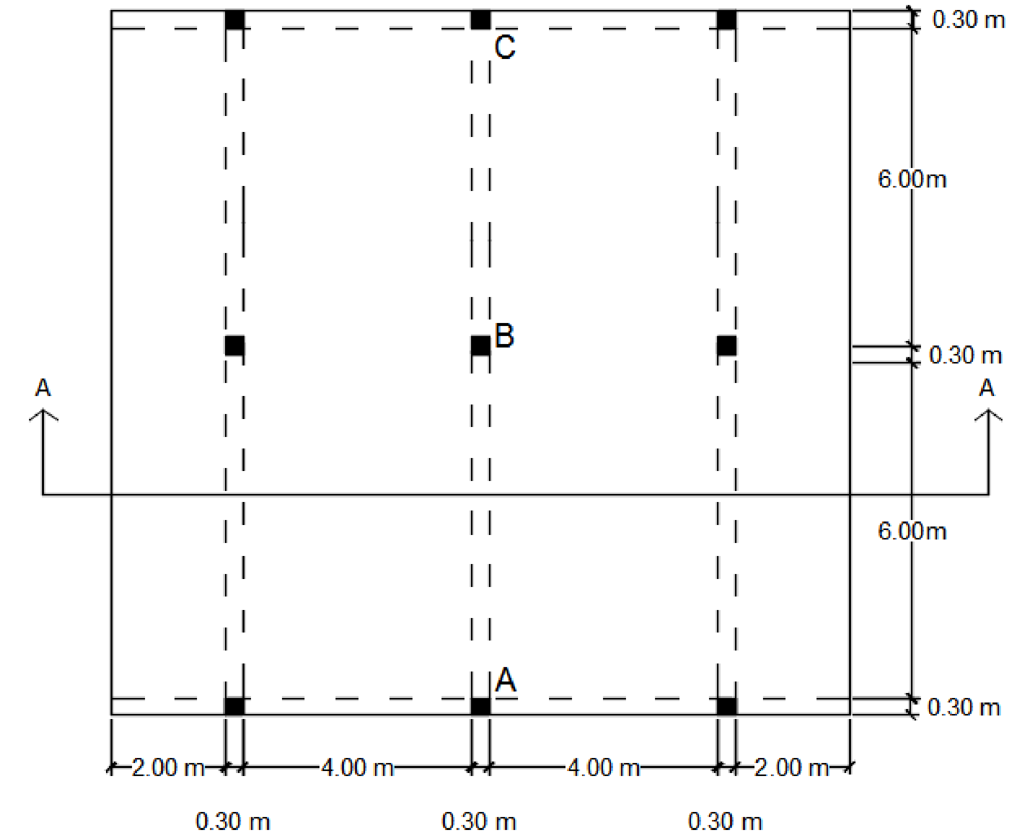
For $\emptyset 12@300mm$

$$n = \frac{1000}{300} = 3.33 \ \rightarrow As = 3.33 * \frac{\pi}{4} 12^2 = 376.4 \ mm^2$$

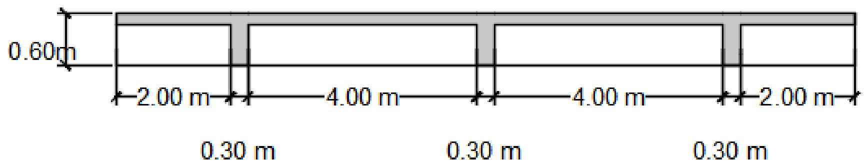
$\therefore \emptyset 12@200mm > \emptyset 12@300mm$

Example-4: The floor system shown in the Figure below supports a service live load of 4 kN/m^2 and a service dead load 3 kN/m^2 (not include slab weight). Use $f_c' = 28\ MPa\ and\ fy = 420\ MPa$ to answer the flowing.

1- Classify the floor system into one way or two way solid slab.
2- What is the minimun slab thickness that should be used to control deflection and shear requirement? (use one thickness for all slabs)
3- By using Ø 12 mm rebar, what is the required positive reinforcement?
4- If the top reinforcement at interior support is Ø 12 mm @200 mm, show if this reinforcemnt is adquate to support the applied load?



Floor Plane



Section A-A

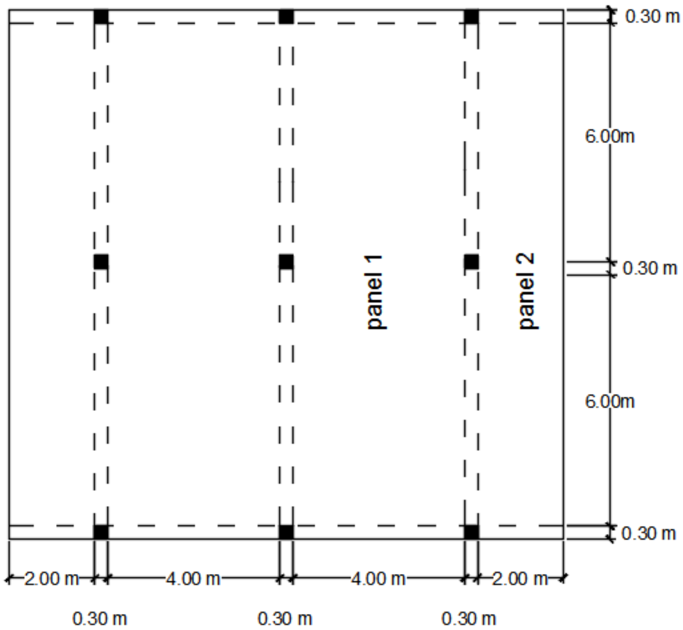
Solution:

1- For Panel 1

$$l/_s = {}^{12.3}/_4 = 3 > 2 \rightarrow one\ way\ slab$$

For Panel 2

$$l/_s = {}^{12.3}/_2 = 6.15 > 2 \rightarrow$$
$one\ way\ slab$



2- Minimum slab thickness

For panel 1 (both ends continues)

$$h = \frac{l}{28} = \frac{4000}{28} \cong 143\ mm$$

For panel 2 (cantilever slab)

$$h = \frac{l}{10} = \frac{2000}{10} = 200\ mm\ \ control$$

$$d = 200 - 20 - \frac{12}{2} = 168\ mm$$

- Check the slab thickness according to shear requirement.

Self weight $= 0.2 * 24 = 4.8\ kN/m^2$

$$W_u = 1.2 * (3 + 4.8) + 1.6 * (4) = 15.76\ kN/m^2 = 15.76\ \ kN/m/1m$$

$$V_{u,\ max} = 1.15 * \frac{W_u l_n}{2} = 1.15 * \frac{15.76 * 4}{2} = 36.25\ kN/1m$$

$$V_{u\ @\ d} = 1.15\frac{W_u\ l_n}{2} - W_u.d$$

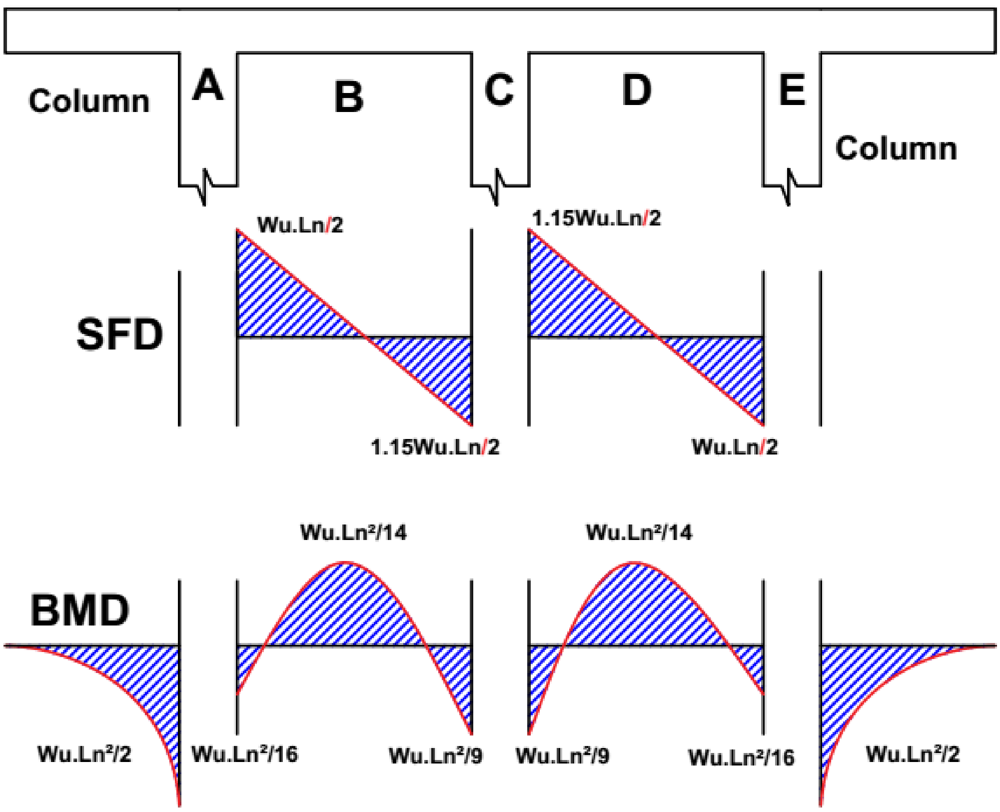
$$V_{u\ @\ d} = 36.25\ - 15.76 * 0.168 = 33.6\ {}^{kN}/_{1\ m}$$

$$\emptyset V_c = 0.75 * \frac{\sqrt{f_c'}}{6}\ b\ d$$

$$\emptyset V_c = 0.75 * \frac{\sqrt{28}}{6} * 1000 * 168 * 10^{-3} = 111.12\ {}^{kN}/_{1m}$$

$$V_{u\ max.} = 33.6\ kN/1m < \emptyset V_c = 111.12\ kN/1m$$

The slab thickness is enough



3- $M^+ = \frac{W_u l_n^2}{14} = \frac{15.76*4^2}{14} = 18.01\ kN.m/1m$

$$\emptyset Mn \geq Mu$$

$$Mu = \emptyset\ \rho\ bd^2 fy\left(1 - 0.59\frac{fy}{f_c'}\ \rho\right)$$

$$Assume\ \emptyset = 0.9$$

$$\rho = \frac{1 \pm \sqrt{1 - \frac{2.36 * 18.01 * 10^6}{0.9 *\ 1000 * 168^2 * 28}}}{1.18\left(\frac{420}{28}\right)}$$

$\rho = 0.00171$

$A_s = 0.00171 * 1000 * 168 = 287.28\, mm^2\ /1m$

for $fy = 420\ \rightarrow \rho_{\ shr.} = 0.0018$

$\rho_{\ min.} = \rho_{\ shr.}$

$$\rho_t = 0.85 * 0.85 \frac{28}{420} \frac{0.003}{0.003 + 0.005} = 0.018 > \rho\ \ \therefore ok\ \emptyset = 0.9$$

$A_{s\ \ min} = 0.0018 * 1000 * 200 = 360\, mm^2\ /1m$

$A_s < A_{s\, min}\ \ \rightarrow use\ A_s = A_{s\, min}$

$$n = \frac{As}{\text{area of one bar}} = \frac{360}{113.1} = 3.18\ bar/1m$$

$$S = \frac{1000}{3.18} = 314\ mm = 300\ mm$$

$$S_{max.} = min. of \begin{cases} three\ times\ slab\ thickness\ (3t) = 3 * 200 = 600mm \\ 450\ mm \end{cases}$$

$S = 300\ mm < \quad S_{max.} = 450\ mm$

Use Ø12 mm @300mm c/c

4- $M_u^- = \frac{W_u l_n^2}{9} = \frac{15.76*4^2}{9} = 26.24\ kN.m/1m$

$$n = \frac{1000}{S} = \frac{1000}{200} = 5\emptyset 12/1m$$

$$As_{provide} = \emptyset 12@200mm = \frac{5\emptyset 12}{1\ m} \rightarrow \rho = \frac{5 * 113.1}{1000 * 168} = 0.00336 > \rho_{min.}$$

$$\rho_t = 0.85 * 0.85 \frac{28}{400} \frac{0.003}{0.003 + 0.005} = 0.0189 > \rho\ \therefore \emptyset = 0.9$$

$$\emptyset Mn = \emptyset\ \rho\ bd^2 fy\ \left(1 - 0.59 \frac{fy}{f_c'}\ \rho\right)$$

$$\emptyset Mn = 0.9 * 0.00336 * 1000 * 168^2 * 420\left(1 - 0.59\frac{420}{28} * 0.00336\right) * 10^{-6}$$

$$= 34.78\ kN.m/1m$$

$\emptyset Mn = 34.78 > M_u^- = 26.24$ this reinforcemnt is adquate to support the applied load

---

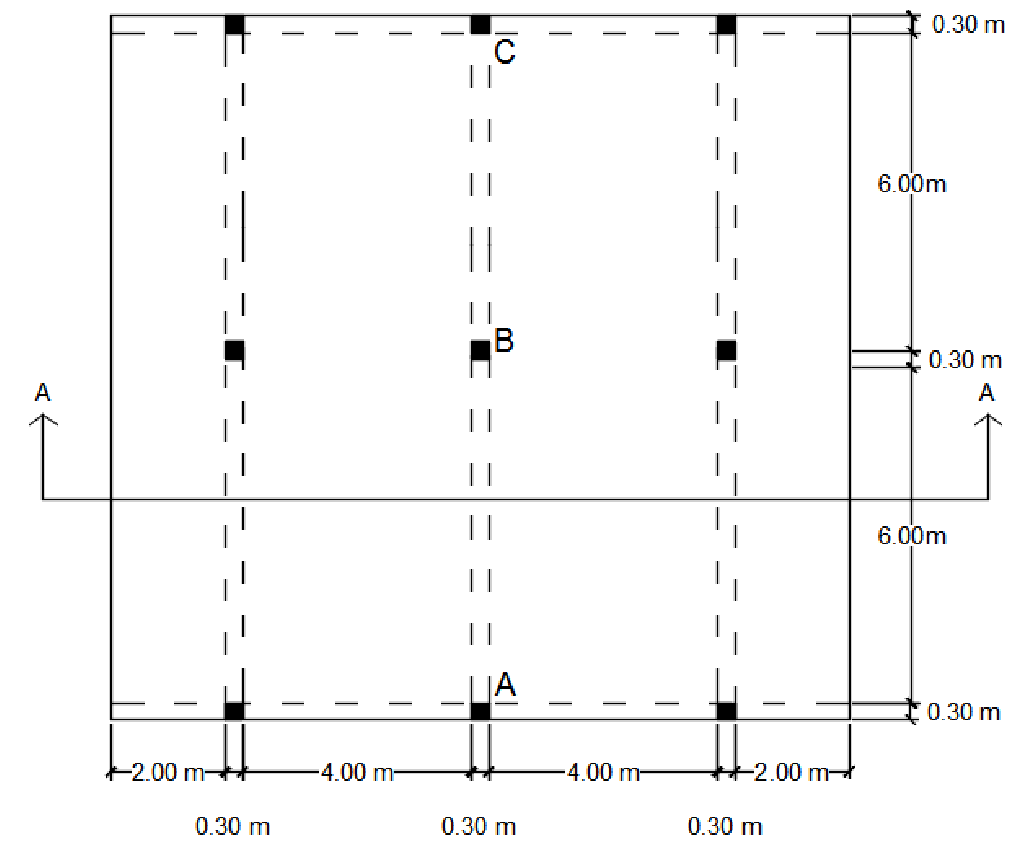
HW: The floor system shown in the Figure below supports a service live load of 5 kN/m^2 and a service dead load 3 kN/m^2 (not include slab weight). Use $f_c' = 23\ MPa\ and\ fy = 420\ MPa$ to answer the flowing.

1- Classify the floor system into one way or two way solid slab.

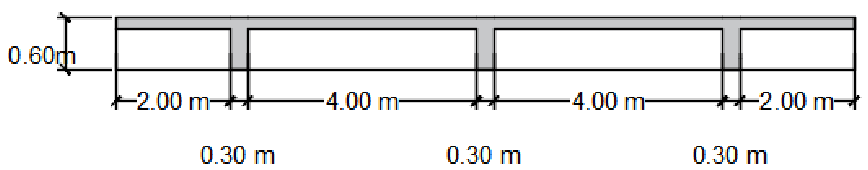
2- What is the minimun slab thickness that should be used to control deflection and shear requirement? (use one thickness for all slabs)

3- By using Ø 12 mm rebar, what is the required positive reinforcement?

4- If the top reinforcement at interior support is Ø 12 mm @150 mm, show if this reinforcemnt is adquate to support the applied load?



Floor Plane



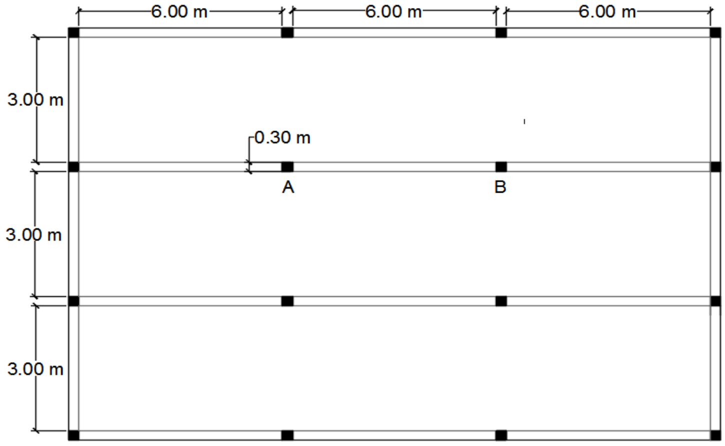
Section A-A

Problems

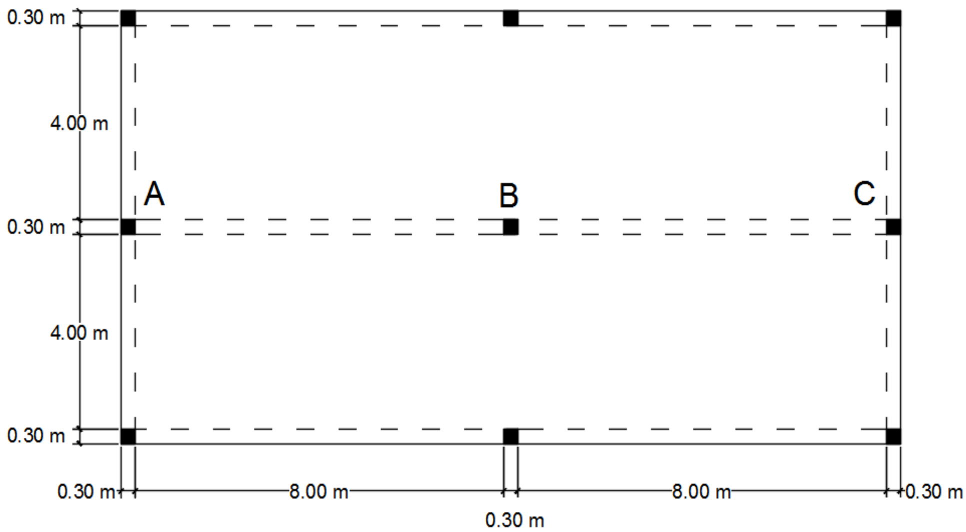
1- For the floor system shown in Figure below, support service live load 4 KN/m^2 and service dead load $5KN/m^2$. Answer the flowing.

a) Classify the floor system into one way or two way solid slab.
b) What is the minimun slab thickness that should be used to control deflection and shear requirement?
c) By using Ø 12 mm rebar, what is the required positive and nagative reinforcement?
d) Sketch the detail of reinforcement.

Use: $fy = 420\ Mpa,\ f_c' = 30\ Mpa$,



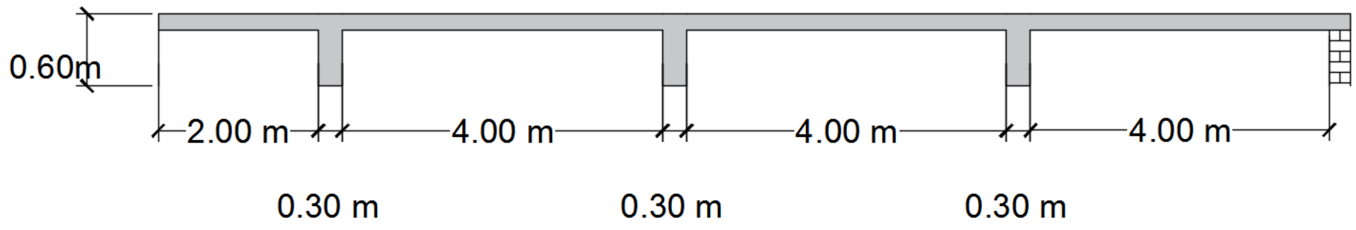
2- The floor of building shown below is design to carry its self-weight, service dead load of 6 kN/m^2 and service live load of 4 kN/m^2. The slab thickness is 170 mm reinforced by Ø10 mm @150 mm top and bottom.



1- Check the adequacy of the slab thickness.
2- Can the floor carry the applied load?

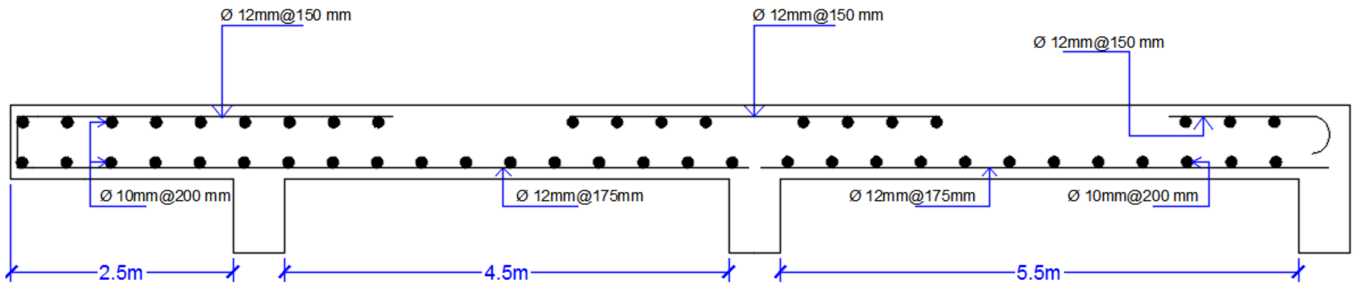
use: $fy = 400\ Mpa,\ f_c' = 30\ Mpa\ , b_w = 300mm$

3- Design the floor system shown below to support service live load=5 kN/m^2, service dead load =4 kN/m^2 $use\ \ fy = 420\ Mpa$ and $f_c' = 25\ Mpa$.



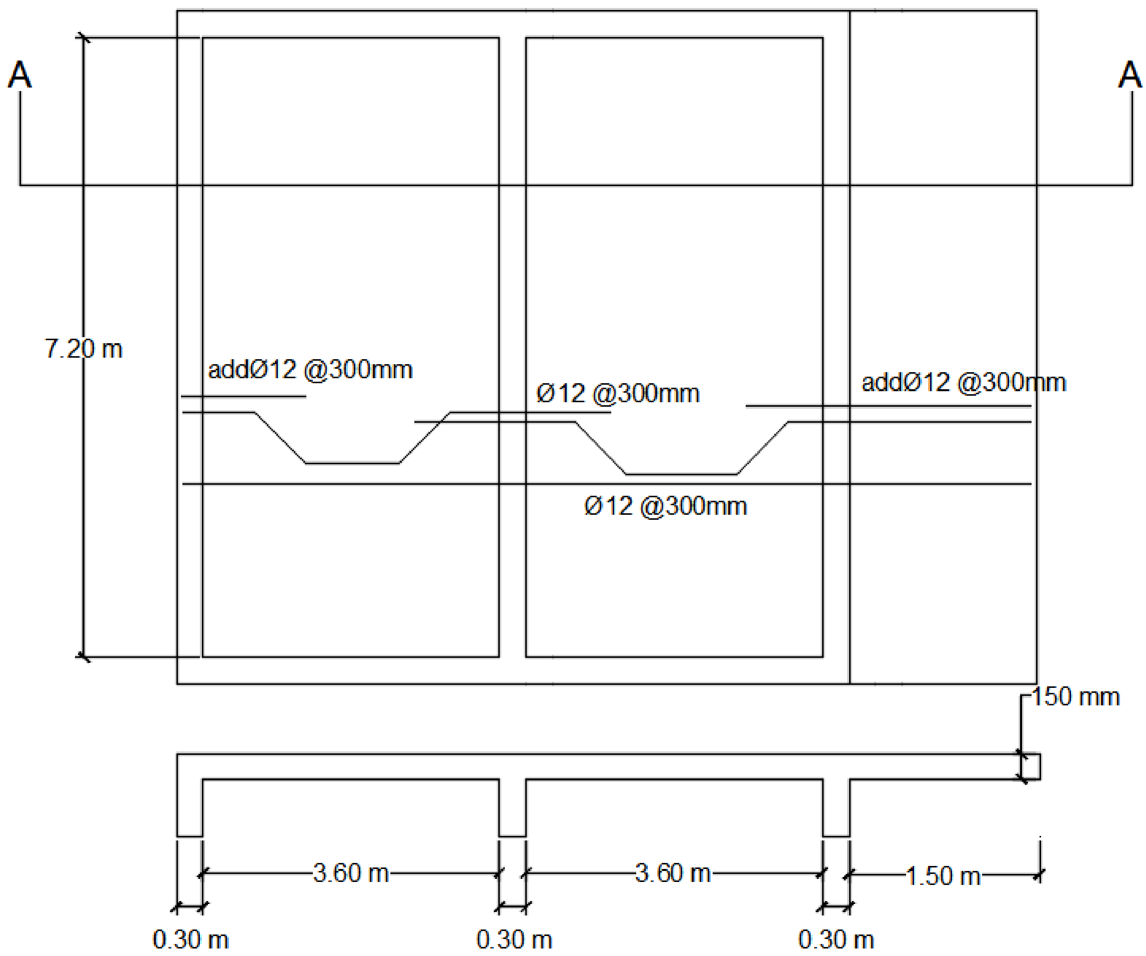
4- Find the maximum ultimate load can be carry by the floor system shown below.

$use\ \ fy = 400\ Mpa$ and $f_c' = 28\ Mpa$.



5-

1- If the service dead load = 4 kN/m2, find the maximum live load can be carried by the floor system shown below?
2- Based on the result of (1) check the adequacy of the slab thickness.



Section A-A

Design of Reinforced Concrete Structures I

Al Muthanna University
Collage of Engineering

Two way slabs-1

Dr Othman Hameed

Lecture (11)

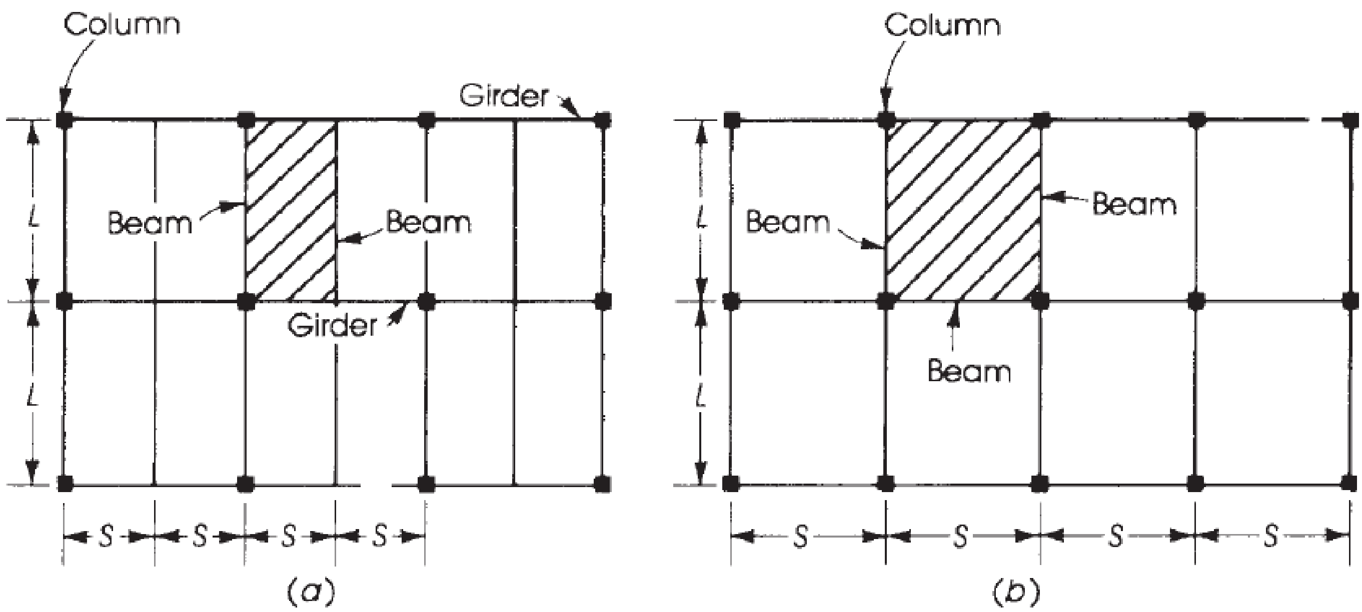
Two way slabs

Two way solid slabs

1. Introduction

When the slab is supported on all four sides and the length, L, is less than twice the width, S, the slab will deflect in two directions, and the loads on the slab are transferred to all four supports. This slab is referred to as a two-way slab.

The bending moments and deflections in such slabs are less than those in one-way slabs; thus, the same slab can carry more loads when supported on four sides. The load in this case is carried in two directions, and the bending moment in each direction is much less than the bending moment in the slab if the loads were carried in a one direction only. Typical slab–beam–girder arrangements of one-way and two-way slabs are shown in the Figure below.



(a) One-way slab, $L/S > 2$, and (b) two-way slab, $L/S \leq 2$.

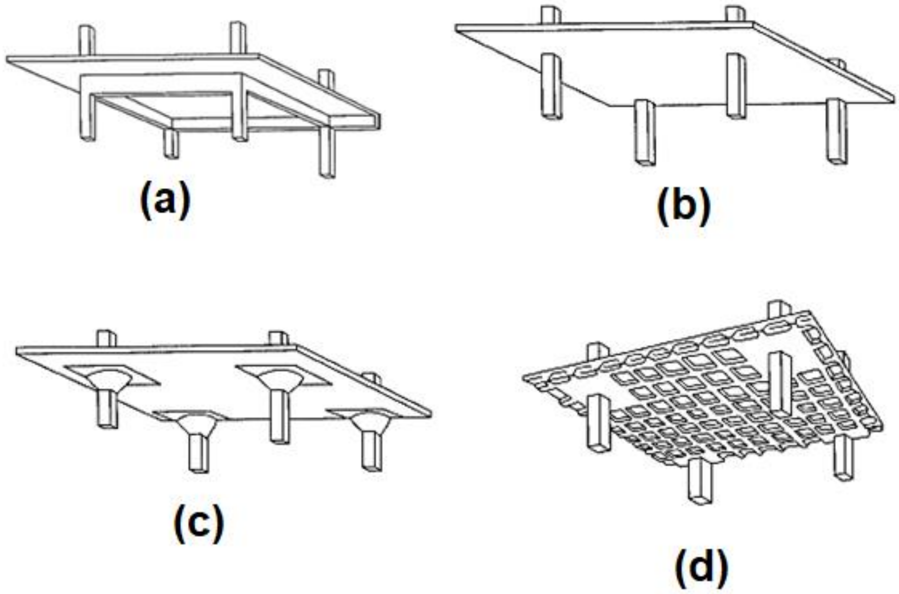
2. Type of Two Way Slab

1- Two-Way Slabs on Beams: This case occurs when the two-way slab is supported by beams on all four sides. The loads from the slab are transferred to all four supporting beams, which, in turn, transfer the loads to the columns, as shown in Figure (a) below.

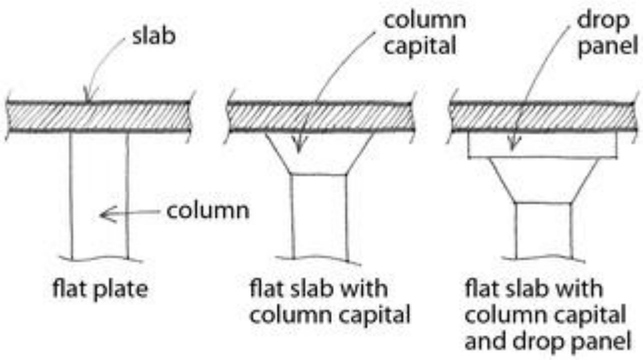
2- Flat-Plate: A flat-plate floor is a two-way slab system consisting of a uniform slab that rests directly on columns and does not have beams or column capitals as shown in Figure (c) below. In this case, the column tends to punch through the slab, producing diagonal tensile stresses. Therefore, a general increase in the slab thickness is required or special reinforcement is used.

3- Flat plate with column capital and/or panel drop: A flat slab is a two-way slab reinforced in two directions that usually does not have beams or girders. The loads are transferred to the drop panel and column capitals, as shown in Figure (b) below.

4- Two-Way Ribbed Slabs and the Waffle Slab System: This type of slab consists of a floor slab with a length-to-width ratio less than 2. The thickness of the slab is usually 5 to 10 cm and is supported by ribs (or joists) in two directions. The ribs are arranged in each direction at spacings of about, producing square or rectangular shapes. The ribs can also be arranged at or from the centerline of slabs, producing architectural shapes at the soffit of the slab, as shown in Figure (d) below.



Types of two way slab: (a) slab on beams, (b) flat plate, (c) flat slab (with column capital and/or drop panel, (d) waffle slab



Flat slab with column capital and drop panel

The ACI Code specifies two methods for the design of two-way slabs:

1. **The direct design method DDM (ACI Code, Section 13.6)**

Is an approximate procedure for the analysis and design of two-way slabs. It is limited to slab systems subjected to uniformly distributed loads and supported on **equally or nearly equally spaced columns**.

The method uses a set of coefficients to determine the design moments at critical sections. Two-way slab systems that do not meet the limitations of the ACI Code, Section 13.6.1, must be analyzed by more accurate procedures.

2. **The equivalent frame method EFM (ACI Code, Section 13.7)**

Is one in which a three-dimensional building is divided into a series of two-dimensional equivalent frames by cutting the building along lines midway between columns.

The resulting frames are considered separately in the longitudinal and transverse directions of the building and treated floor by floor.

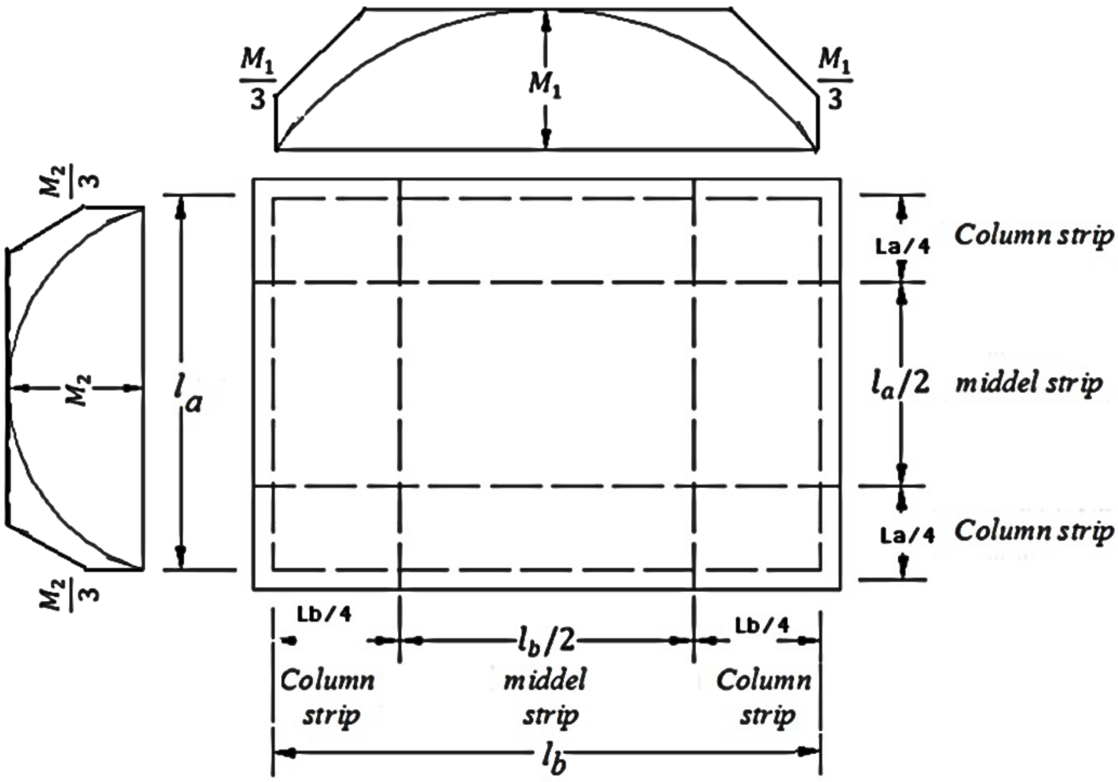
The systems that do not meet the requirements permitting analysis by the "direct design method" of the present code, has led many engineers to continue to use the design method of the 1963 ACI Code **(The coefficient method)**

The Coefficient Method (method three)

is a quick hand-method of calculating the moments in two-way slabs supported by edge beams. The Coefficient Method was first included in the 1963 edition of the ACI Code as a method to design two-way slabs supported on all four sides by walls, steel beams, or deep beams. The Coefficient Method is not included in current versions of the ACI Code 318, but it can still be used for two-way slab systems with edge beams.

Design Two Way Solid Slab by Coefficient Method (method 3)

The panel must be divided into middle strips and column strips in both the short and long directions. The width of the middle strip in each direction is equal to ½ the clear span length. The 2 edge strips are then ¼ the width of the clear span length.



Moment at column strip and middle strip are computed by:

$$M_a = C_a W_u L_a^2$$

$$M_b = C_b W_u L_b^2$$

Where:

$M_a = moment\ in\ short\ dirction$

$M_b = moment\ in\ long\ dirction$

$C_a \& \, C_b = tabulated\ moment\ coeffecients$

$L_a \& L_b = clear\ span\ in\ short\ and\ long\ direction, respectivally$

$W_u = uniform\ ultimate\ load\ kN/m^2$

$$width\ of\ middle\ strip = \frac{1}{2}\ panel$$

$$width\ of\ colume\ strip = \frac{1}{4}\ panel$$

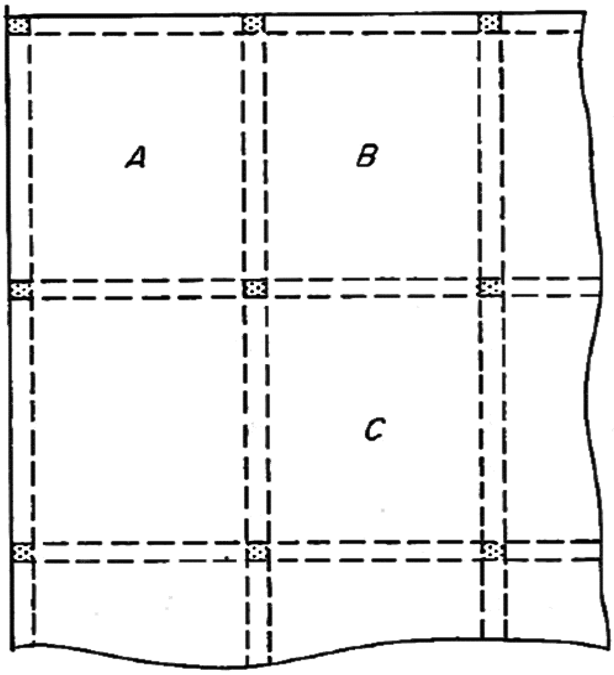
As expected in two-way slabs, the moments in both directions are larger in the **center portion** of the slab than the edges. Therefore, the middle strip must be designed for the maximum tabulated moment. In the edge strips, the strips must be designed for 1/3 of the maximum value of the calculated moment.

Table of coefficients

The ACI Coefficient Tables are designed to give you appropriate coefficients based on the edge conditions of the slab. To give you an idea of different edge conditions, see the floor plan below:

It is seen that some panels, such as ***A***, have two discontinuous exterior edges, while the other edges are continuous with their neighbors. Panel ***B*** has one edge discontinuous and three continuous edges, the interior panel ***C*** has all edges continuous, and so on.

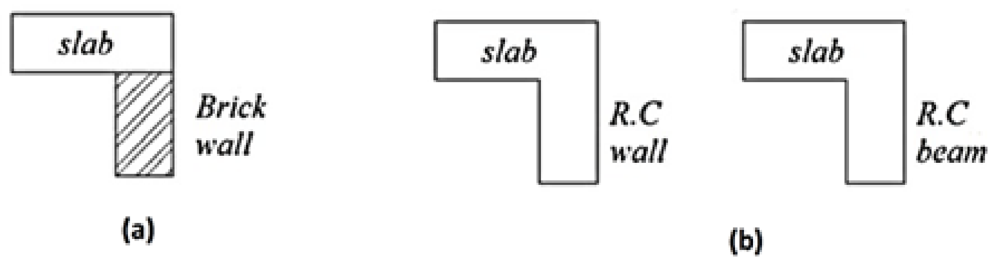
At a continuous edge in a slab, moments are negative, just as at interior supports of



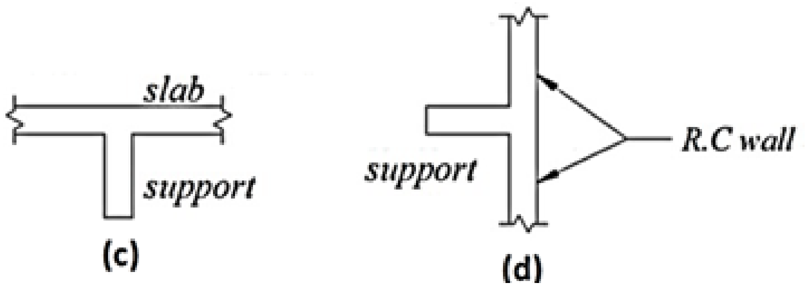
Plan of a typical two-way slab floor with beams on column lines.

continuous beams. Also, the magnitude of the positive moments depends on the conditions of continuity at all four edges.

Note



(a)discontinuous slab or (b)supporting have small torsional resistance



(c) slab is continuous a cross support (d) fixed

__Table 1__: gives the moment coefficients for ***Negative Moments at Continuous Edges.*** The coefficient you use depends on the ratio of l_a/l_b and the edge conditions of the panel in question. The maximum negative edge moment occurs when both panels adjacent to an edge are fully loaded. <u>Negative moments at discontinuous (free) edges are assumed to be 1/3 of the positive moment in the same direction.</u>

Table 1 - Coefficients for Negative Moments in Slabs

$$M_a^- = C_{a,neg} w_u l_a^2$$
$$M_b^- = C_{b,neg} w_u l_b^2$$

where w_u = total factored uniform load (DL + LL)

Ratio $m = \frac{l_a}{l_b}$		Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9
1.00	$C_{a,neg}$	-	0.045	0.076	0.050	0.075	0.071	0.071	0.033	0.061
	$C_{b,neg}$		0.045		0.050				0.061	0.033
0.95	$C_{a,neg}$	-	0.050	0.072	0.055	0.079	0.075	0.067	0.038	0.065
	$C_{b,neg}$		0.041		0.045				0.056	0.029
0.90	$C_{a,neg}$	-	0.055	0.070	0.060	0.080	0.079	0.062	0.043	0.068
	$C_{b,neg}$		0.037		0.040				0.052	0.025
0.85	$C_{a,neg}$	-	0.060	0.065	0.066	0.082	0.083	0.057	0.049	0.072
	$C_{b,neg}$		0.031		0.034				0.046	0.021
0.80	$C_{a,neg}$	-	0.065	0.061	0.071	0.083	0.086	0.051	0.055	0.075
	$C_{b,neg}$		0.027		0.029				0.041	0.017
0.75	$C_{a,neg}$	-	0.069	0.056	0.076	0.085	0.088	0.044	0.061	0.078
	$C_{b,neg}$		0.022		0.024				0.036	0.014
0.70	$C_{a,neg}$	-	0.074	0.050	0.081	0.086	0.091	0.038	0.068	0.081
	$C_{b,neg}$		0.017		0.019				0.029	0.011
0.65	$C_{a,neg}$	-	0.077	0.043	0.085	0.087	0.093	0.031	0.074	0.083
	$C_{b,neg}$		0.014		0.015				0.024	0.008
0.60	$C_{a,neg}$	-	0.081	0.035	0.089	0.088	0.095	0.024	0.080	0.085
	$C_{b,neg}$		0.010		0.011				0.018	0.006
0.55	$C_{a,neg}$	-	0.084	0.028	0.092	0.089	0.096	0.019	0.085	0.086
	$C_{b,neg}$		0.007		0.008				0.014	0.005
0.50	$C_{a,neg}$	-	0.086	0.022	0.094	0.090	0.097	0.014	0.089	0.088
	$C_{b,neg}$		0.006		0.006				0.010	0.003

<u>**Table 2:**</u> gives the moment coefficients for ***Positive Moment due to Dead Load***

Table 2 - Coefficients for Dead Load Positive Moments in Slabs

$$M_{a,DL}{}^{+} = C_{a,DL} w_{DL} l_a{}^2$$
$$M_{b,DL}{}^{+} = C_{b,DL} w_{DL} l_b{}^2$$

where w_{DL} = uniform factored Dead Load (DL)

Ratio $m = \frac{l_a}{l_b}$		Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9
1.00	$C_{a,DL}$	0.036	0.018	0.018	0.027	0.027	0.033	0.027	0.020	0.023
	$C_{b,DL}$	0.036	0.018	0.027	0.027	0.018	0.027	0.033	0.023	0.020
0.95	$C_{a,DL}$	0.040	0.020	0.021	0.030	0.028	0.036	0.031	0.022	0.024
	$C_{b,DL}$	0.033	0.016	0.025	0.024	0.015	0.024	0.031	0.021	0.017
0.90	$C_{a,DL}$	0.045	0.022	0.025	0.033	0.029	0.039	0.035	0.025	0.026
	$C_{b,DL}$	0.029	0.014	0.024	0.022	0.013	0.021	0.028	0.019	0.015
0.85	$C_{a,DL}$	0.050	0.024	0.029	0.036	0.031	0.042	0.040	0.029	0.028
	$C_{b,DL}$	0.026	0.012	0.022	0.019	0.011	0.017	0.025	0.017	0.013
0.80	$C_{a,DL}$	0.056	0.026	0.034	0.039	0.032	0.045	0.045	0.032	0.029
	$C_{b,DL}$	0.023	0.011	0.020	0.016	0.009	0.015	0.022	0.015	0.010
0.75	$C_{a,DL}$	0.061	0.028	0.040	0.043	0.033	0.048	0.051	0.036	0.031
	$C_{b,DL}$	0.019	0.009	0.018	0.013	0.007	0.012	0.020	0.013	0.007
0.70	$C_{a,DL}$	0.068	0.030	0.046	0.046	0.035	0.051	0.058	0.040	0.033
	$C_{b,DL}$	0.016	0.007	0.016	0.011	0.005	0.009	0.017	0.011	0.006
0.65	$C_{a,DL}$	0.074	0.032	0.054	0.050	0.036	0.054	0.065	0.044	0.034
	$C_{b,DL}$	0.013	0.006	0.014	0.009	0.004	0.007	0.014	0.009	0.005
0.60	$C_{a,DL}$	0.081	0.034	0.062	0.053	0.037	0.056	0.073	0.048	0.036
	$C_{b,DL}$	0.010	0.004	0.011	0.007	0.003	0.006	0.012	0.007	0.004
0.55	$C_{a,DL}$	0.088	0.035	0.071	0.056	0.038	0.058	0.081	0.052	0.037
	$C_{b,DL}$	0.008	0.003	0.009	0.005	0.002	0.004	0.009	0.005	0.003
0.50	$C_{a,DL}$	0.095	0.037	0.080	0.059	0.039	0.061	0.089	0.056	0.038
	$C_{b,DL}$	0.006	0.002	0.007	0.004	0.001	0.003	0.007	0.004	0.002

<u>Table 3</u>: gives the moment coefficient for ***Positive Moment due to Live Load***. This table is used in the same manner as Table 2. The reason for the separation of Dead and Live load positive moments is due to Live load placement to achieve maximum effect. For live load, the maximum positive moment in the panel occurs when the full live load is on the panel and not on any adjacent panel. This produces rotations at all continuous edges of the panel which require restraining moments. Dead load across all the panels creates rotations that cancel each other out (or closely enough).

Table 3 - Coefficients for Live Load Positive Moments in Slabs

$$M_{a,LL}{}^{+} = C_{a,LL} w_{LL} l_a{}^2$$
$$M_{b,LL}{}^{+} = C_{b,LL} w_{LL} l_b{}^2$$

where w_{LL} = uniform factored Live Load (LL)

Ratio $m = \frac{l_a}{l_b}$		Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9
1.00	$C_{a,LL}$	0.036	0.027	0.027	0.032	0.032	0.035	0.032	0.028	0.030
	$C_{b,LL}$	0.036	0.027	0.032	0.032	0.027	0.032	0.035	0.030	0.028
0.95	$C_{a,LL}$	0.040	0.030	0.031	0.035	0.034	0.038	0.036	0.031	0.032
	$C_{b,LL}$	0.033	0.025	0.029	0.029	0.024	0.029	0.032	0.027	0.025
0.90	$C_{a,LL}$	0.045	0.034	0.035	0.039	0.037	0.042	0.040	0.035	0.036
	$C_{b,LL}$	0.029	0.022	0.027	0.026	0.021	0.025	0.029	0.024	0.022
0.85	$C_{a,LL}$	0.050	0.037	0.040	0.043	0.041	0.046	0.045	0.040	0.039
	$C_{b,LL}$	0.026	0.019	0.024	0.023	0.019	0.022	0.026	0.022	0.020
0.80	$C_{a,LL}$	0.056	0.041	0.045	0.048	0.044	0.051	0.051	0.044	0.042
	$C_{b,LL}$	0.023	0.017	0.022	0.020	0.016	0.019	0.023	0.019	0.017
0.75	$C_{a,LL}$	0.061	0.045	0.051	0.052	0.047	0.055	0.056	0.049	0.046
	$C_{b,LL}$	0.019	0.014	0.019	0.016	0.013	0.016	0.020	0.016	0.013
0.70	$C_{a,LL}$	0.068	0.049	0.057	0.057	0.051	0.060	0.063	0.054	0.050
	$C_{b,LL}$	0.016	0.012	0.016	0.014	0.011	0.013	0.017	0.014	0.011
0.65	$C_{a,LL}$	0.074	0.053	0.064	0.062	0.055	0.064	0.070	0.059	0.054
	$C_{b,LL}$	0.013	0.010	0.014	0.011	0.009	0.010	0.014	0.011	0.009
0.60	$C_{a,LL}$	0.081	0.058	0.071	0.067	0.059	0.068	0.077	0.065	0.059
	$C_{b,LL}$	0.010	0.007	0.011	0.009	0.007	0.008	0.011	0.009	0.007
0.55	$C_{a,LL}$	0.088	0.062	0.080	0.072	0.063	0.073	0.085	0.070	0.063
	$C_{b,LL}$	0.008	0.006	0.009	0.007	0.005	0.006	0.009	0.007	0.006
0.50	$C_{a,LL}$	0.095	0.066	0.088	0.077	0.067	0.078	0.092	0.076	0.067
	$C_{b,LL}$	0.006	0.004	0.007	0.005	0.004	0.005	0.007	0.005	0.004

Table 4 provides the coefficients for determining shear in the slab and loads on edge beams.

Table 4 - Coefficients for Shear in Slabs

Ratio of load W in l_a and l_b directions for Shear in Slab and Load on Supports for Beams

Ratio $m = \frac{l_a}{l_b}$		Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9
1.00	W_a	0.50	0.50	0.17	0.50	0.83	0.71	0.29	0.33	0.67
	W_b	0.50	0.50	0.83	0.50	0.17	0.29	0.71	0.67	0.33
0.95	W_a	0.55	0.55	0.20	0.55	0.86	0.75	0.33	0.38	0.71
	W_b	0.45	0.45	0.80	0.45	0.14	0.25	0.67	0.62	0.29
0.90	W_a	0.60	0.60	0.23	0.60	0.88	0.79	0.38	0.43	0.75
	W_b	0.40	0.40	0.77	0.40	0.12	0.21	0.62	0.57	0.25
0.85	W_a	0.66	0.66	0.28	0.66	0.90	0.83	0.43	0.49	0.79
	W_b	0.34	0.34	0.72	0.34	0.10	0.17	0.57	0.51	0.21
0.80	W_a	0.71	0.71	0.33	0.71	0.92	0.86	0.49	0.55	0.83
	W_b	0.29	0.29	0.67	0.29	0.08	0.14	0.51	0.45	0.17
0.75	W_a	0.76	0.76	0.39	0.76	0.94	0.88	0.56	0.61	0.86
	W_b	0.24	0.24	0.61	0.24	0.06	0.12	0.44	0.39	0.14
0.70	W_a	0.81	0.81	0.45	0.81	0.95	0.91	0.62	0.68	0.89
	W_b	0.19	0.19	0.55	0.19	0.05	0.09	0.38	0.32	0.11
0.65	W_a	0.85	0.85	0.53	0.85	0.96	0.93	0.69	0.74	0.92
	W_b	0.15	0.15	0.47	0.15	0.04	0.07	0.31	0.26	0.08
0.60	W_a	0.89	0.89	0.61	0.89	0.97	0.95	0.76	0.80	0.94
	W_b	0.11	0.11	0.39	0.11	0.03	0.05	0.24	0.20	0.06
0.55	W_a	0.92	0.92	0.69	0.92	0.98	0.96	0.81	0.85	0.95
	W_b	0.08	0.08	0.31	0.08	0.02	0.04	0.19	0.15	0.05
0.50	W_a	0.94	0.94	0.76	0.94	0.99	0.97	0.86	0.89	0.97
	W_b	0.06	0.06	0.24	0.06	0.01	0.03	0.14	0.11	0.03

Al Muthanna University
Collage of Engineering

Design of Reinforced Concrete Structures I

Two way slabs-2

Dr Othman Hameed

Lecture (12)

Two way slabs

6 Slab Reinforcement According to ACI 13.3

The main reinforcement for the two-way edge-supported slab panel should be placed orthogonally (parallel and perpendicular) to the slab edges. The reinforcement in the short direction (***l**$_a$*) should be placed below the reinforcement in the long direction (***l**$_b$*). Negative reinforcement should be placed perpendicular to the supporting edge beams.

1- Area of reinforcement in each direction for two-way slab systems shall be determined from moments at critical sections but not less than minimum reinforcement.

2- Spacing of reinforcement at critical sections shall not exceed two times the slab thickness.

$$S_{max.} \leq 2t \qquad where: t = \ slab\ thickness$$

3- Positive moment reinforcement perpendicular to a discontinuous edge should extend to the edge of slab and have embedment, straight or hooked, at least 150 mm in spandrel beams, columns, or walls.

4- Negative moment reinforcement perpendicular to a discontinuous edge shall be bent, hooked, or otherwise anchored in spandrel beams, columns, or Walls.

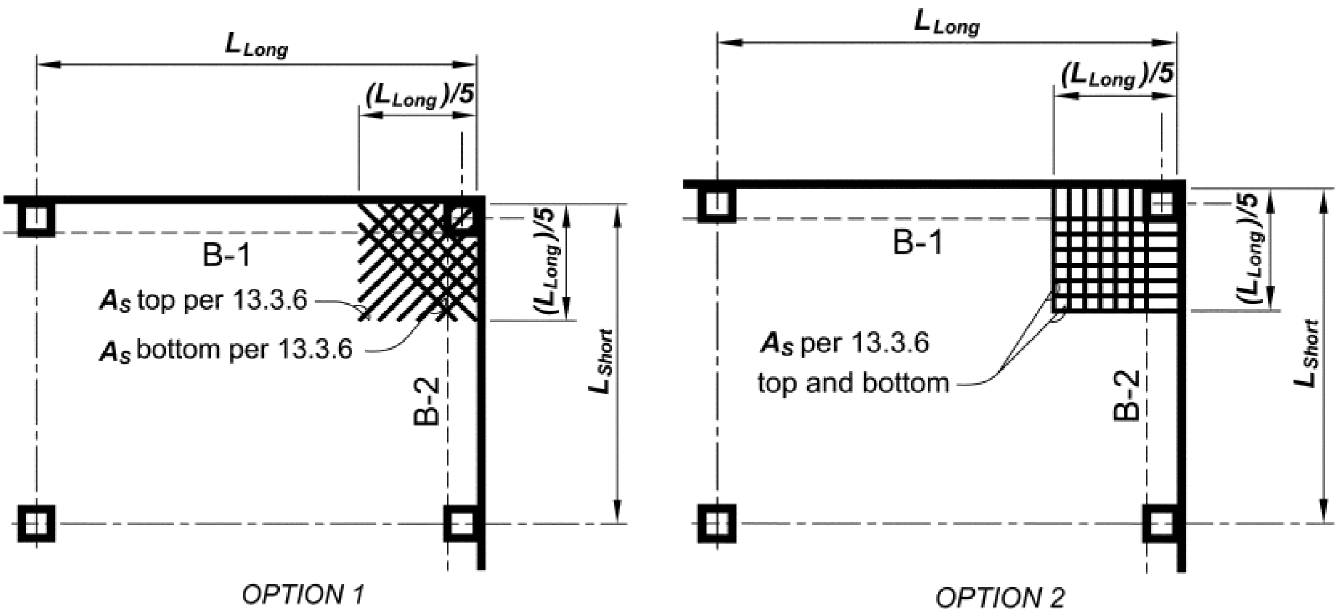
5- At exterior corners of slabs supported by edge walls or where one or more edge, top and bottom reinforcement shall be provided at exterior corners in accordance with 13.3.6.1 through 13.3.6.4.

13.3.6.1- Corner reinforcement in both top and bottom of the slab should be sufficient to resist a moment per unit width equal to the maximum positive moment per unit width in the slab panel.

13.3.6.2 - The moment shall be assumed to be about an axis perpendicular to the diagonal from the corner in the top of the slab and about an axis parallel to the diagonal from the corner in the bottom of the slab.

13.3.6.3- Corner reinforcement shall be provided for a distance in each direction from the corner equal to one-fifth the longer span.

13.3.6.4- Corner reinforcement shall be placed parallel to the diagonal in the top of the slab and perpendicular to the diagonal in the bottom of the slab.



7. Minimum Slab Thickness of Two Way Solid Slab

- Minimum slab thickness= $max.of \begin{cases} \frac{parameter\ of\ slab\ 2(l_a+l_b)}{180} \\ 90\ mm \end{cases}$

8. Shear Strength of Two Way Solid Slab

In a two way floor system, the slab must have adequate thickness to resist both bending moments and shear forces at the critical sections. To investigate the shear capacity of two way slabs, the following condition should be satisfied.

$$V_u@d \leq \emptyset \frac{1}{6}\sqrt{f_c'}b\ d$$

Procedure of solution

1- Check $\frac{L_{b\,(long)}}{L_{a\,(short)}} = \text{if} < 2$

2- Find the case of the slab and then find m= $\frac{L_{a\,(short)}}{L_{b\,(long)}}$

3- Find the slab thickness by applying

$$\text{Minimum slab thickness} = max.of \begin{cases} \dfrac{parameter\ of\ slab\ 2(l_a + l_b)}{180} \\ 90\ mm \end{cases}$$

4- Find effective depth in the short direction (d_s) and an effective depth in the long direction (d_l). Use table 4 to find the ratio of load that transfers to each direction

$$d_s = h - 20 - \frac{d_{b\,(short)}}{2}$$

$$d_l = h - 20 - d_{b\,(short)} - \frac{d_{b\,(long)}}{2}$$

5- Check shear at the critical section (short direction and use d_s)

$$V_u@d \ \le \emptyset \frac{1}{6}\sqrt{f_c'}b\ d$$

6- Find the moment transferred to the middle strip of the short direction (positive and negative moment) and the middle strip of the long direction (positive and negative moment).

For negative moment use table 1

For positive moment use table 2 and 3

7- Use the equation of singly reinforcement to find ρ (use d_s for shot direction and d_l for the long direction).

$$Mu \le \emptyset Mn \qquad\qquad Mu = \emptyset\ \rho\ bd^2 fy\ \left(1 - 0.59\frac{fy}{f_c'}\ \rho\right)$$

do not forget that

use $\emptyset = 0.9$ and b=1000 mm

8- Check the minimum steel reinforcement ratio

$$\text{for } f_y = 280\ MPa\ to\ 350\ MPa \quad \rightarrow \rho_{min.} = 0.002$$

$$\text{for } f_y = 400\ MPa\ and\ 420\ MPa \quad \rightarrow \rho_{min.} = 0.0018$$

$$\text{for } f_y\ exceeding\ 420\ MPa \quad \rightarrow \rho_{min.} = \frac{0.0018 * 420}{f_y} \geq 0.0014$$

$$As_{min.} = \rho_{min.} b\ h$$

If $\rho < \rho_{min.}$ then, use $\rho = \rho_{min.}$

$A_s = A_{s\ \ min} = \rho_{min.} * 1000 * h$

9- Check $\rho_{min.} < \rho < \rho_{max.}$

$$\rho_{max.} = 0.85\ \beta_1 \frac{f_c^`}{fy} \frac{0.003}{(0.003 + 0.004)}$$

10- Calculate S and $S_{max.}$

$$n = \frac{A_{s.}}{Area\ of\ one\ bar}$$

$$s = \frac{1000}{n}$$

Calculate $S_{max.}$

$$S_{max.} = min.\text{ of } \begin{cases} (2t) \\ 450\ mm \end{cases}$$

Then, chose the minimum one

11- Draw the detail of the section and reinforcement.

Design of Reinforced Concrete Structures I

Al Muthanna University
Collage of Engineering

Two way slabs-3

Dr Othman Hameed

Lecture (13)

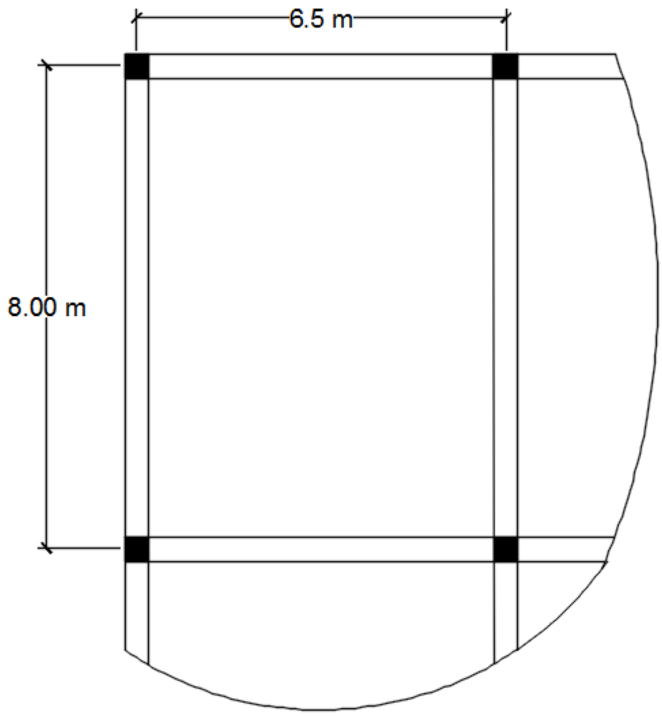
Two way solid slabs

Ex-1: For the reinforced concrete floor shown in the Figure below, design the slab to carry service live load 5 $\frac{kN}{m^2}$ and dead load due to:

Self-weight, 3 cm tiles, 2 cm mortar, 2 cm plaster and 2 $\frac{kN}{m^2}$ partition.

$f_c' = 20\ MPa, fy\ 400\ \ MPa\ and\ all\ beams\ have\ width\ 300\ mm$

Solution:



$$l_a = 6.5 - 0.3 = 6.2m$$

$$l_b = 8 - 0.3 = 7.7m$$

1- Find the type of the slab

$$\frac{l_b}{l_a} = \frac{7.7}{6.2} = 1.24 < 2 \rightarrow two\ way\ slab$$

2- Find slab thickness

$$h = \frac{2(l_a + l_b)}{180} = \frac{2 \times (6200 + 7700)}{180} = 154$$

$$\cong 160mm$$

3- Find the ultimate load

- Dead load

Material	$\gamma\ \frac{kN}{m^2}$	$W = \gamma \times h = \frac{kN}{m^2}$
Tiles	22	22X0.03=0.66
mortar	22	22X0.02=0.44
Self-weight of slab	25	25X0.16=4
plaster	22	22X0.02=0.44
partition		2
Total dead load		7.54

Factored dead load= 1.2*7.54=9.048 $\frac{kN}{m^2}$

- Live load

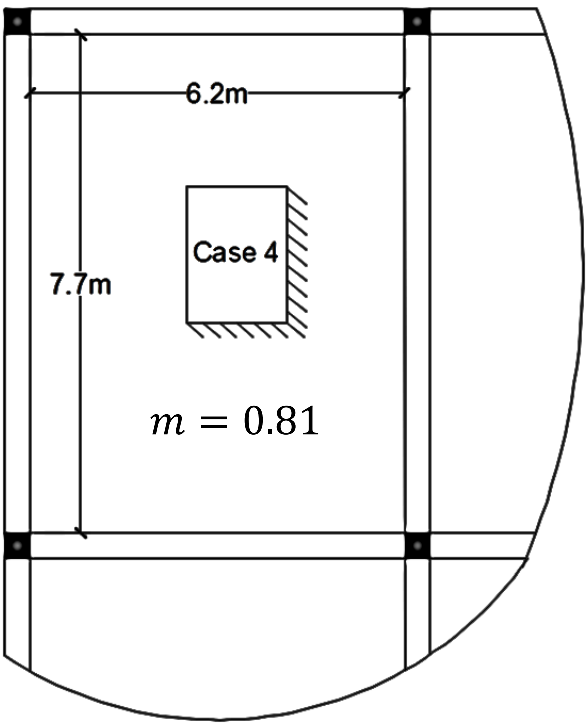
Factored live load= 1.6*5=8 $\frac{kN}{m^2}$

$$Wu = 9.048 + 8 = 17\ \frac{kN}{m^2}$$

4- Check the shear requirement

$m = \frac{l_a}{l_b} = 0.81$

To find the value of shear go to table 4



a- Short direction

$$W_a\left(\frac{l_a}{l_b} = 0.80\right) = 0.71$$

And

$$W_a\left(\frac{l_a}{l_b} = 0.85\right) = 0.66$$

$$W_a\left(\frac{l_a}{l_b} = 0.81\right) = 0.7$$

b- Long direction

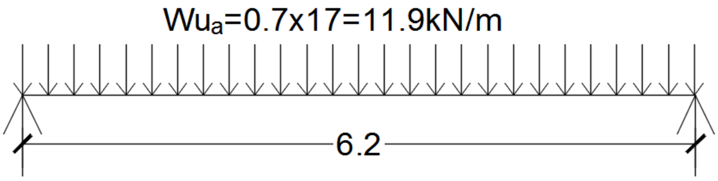
$$W_b\left(\frac{l_a}{l_b} = 0.80\right) = 0.29$$

And

$$W_b\left(\frac{l_a}{l_b} = 0.85\right) = 0.34$$

$$W_b\left(\frac{l_a}{l_b} = 0.81\right) = 0.3$$

The reactions of the slab are calculated from Table 4, which indicates that 70% of the load is transmitted in the short direction and 30% in the long direction.



$$\frac{w_{ua}\ l_a}{2} = 36.89 \qquad\qquad \frac{w_{ua}\ l_a}{2} = 36.89$$

$$Vu@d = 36.89 - 0.134 \times 11.9 = 35.3\ kN/1m$$

$$\emptyset V_c = 0.75 \times \frac{\sqrt{20}}{6} \times 1000 \times 134 \times 10^{-3} = 74.9kN/1m$$

$$\boxed{d = 160 - 20 - \frac{12}{2} = 134\ mm}$$

$\emptyset V_c > Vu@d\ \ ok$

the slab thickness is enough to resist the shear force

5- Find flexural moment

Short direction

1- Negative moment at continuous edge. Use table 1

Negative moment/short						
m	factor	length	load	Moment	ds	As
0.81	0.07	6.2	17	45.74	134	

$$M_a^- = C_a W l_a^2$$

$$C_a\left(\frac{l_a}{l_b} = 0.80\right) = 0.071 \;\&\; C_a\left(\frac{l_a}{l_b} = 0.85\right) = 0.066 \rightarrow C_a\left(\frac{l_a}{l_b} = 0.81\right) = 0.07$$

$$M_a^- = 0.07 \times 17 \times 6.2^2 = 45.74 \quad kN.m/m$$

2- Positive moment at mid span. Use table 2&3

Positive moment/ short										
m	Factor		length	load		Moment		Total moment	ds	As
	D	L		D	L	D	L			

$$M_{ad}^+ = C_{ad} W_d l_a^2 \text{...table 2}$$

$$C_{ad}\left(\frac{l_a}{l_b} = 0.80\right) = 0.039 \;\&\; C_{ad}\left(\frac{l_a}{l_b} = 0.85\right) = 0.036 \rightarrow C_{ad}\left(\frac{l_a}{l_b} = 0.81\right) = 0.038$$

$$M_{ad}^+ = 0.038 \times 9.048 \times 6.2^2 = 13.35 \; kN.m/m$$

$$M_{aL}^+ = C_{aL} W_L l_a^2 \text{...table 3}$$

$$C_{aL}\left(\frac{l_a}{l_b} = 0.80\right) = 0.048 \;\&\; C_{aL}\left(\frac{l_a}{l_b} = 0.85\right) = 0.043 \rightarrow C_{aL}\left(\frac{l_a}{l_b} = 0.81\right) = 0.047$$

$$M_{aL}^+ = 0.047 \times 8 \times 6.2^2 = 14.45 kN.m/m$$

$$M_a^+ = M_{ad}^+ + M_{aL}^+ = 13.35 + 14.45 = 27.8 \; kN.m/m$$

3- Negative moment at discontinuous edge= 1/3 positive moment

=1/3*27.8=9.27 kN.m/m

==

Long direction

1- Negative moment at continuous edge. Use table 1

Negative moment/Long direction						
m	factor	length	load	Moment	d_L	As

$$M_b^- = C_b W l_b^2$$

$$C_b\left(\frac{l_a}{l_b}=0.80\right)=0.029\ \&\ C_b\left(\frac{l_a}{l_b}=0.85\right)=0.034 \rightarrow C_b\left(\frac{l_a}{l_b}=0.81\right)=0.03$$

$$M_b^- = 0.03 \times 17 \times 7.7^2 = 30.23\ kN.m/m$$

2- Positive moment at mid span. Use table 2&3

Positive moment/Long direction										
m	Factor		length	load		Moment		Total moment	d_L	As
	D	L		D	L	D	L			

$M_{bd}^+ = C_{bd} W_d l_b^2$...table 2

$$C_{bd}\left(\frac{l_a}{l_b}=0.80\right)=0.016\ \&\ C_{bd}\left(\frac{l_a}{l_b}=0.85\right)=0.019 \rightarrow C_{bd}\left(\frac{l_a}{l_b}=0.81\right)=0.0166$$

$$M_{bd}^+ = 0.0166 \times 9.048 \times 7.7^2 = 8.91\ kN.m/m$$

$M_{bL}^+ = C_{bL} W_L l_b^2$...table 3

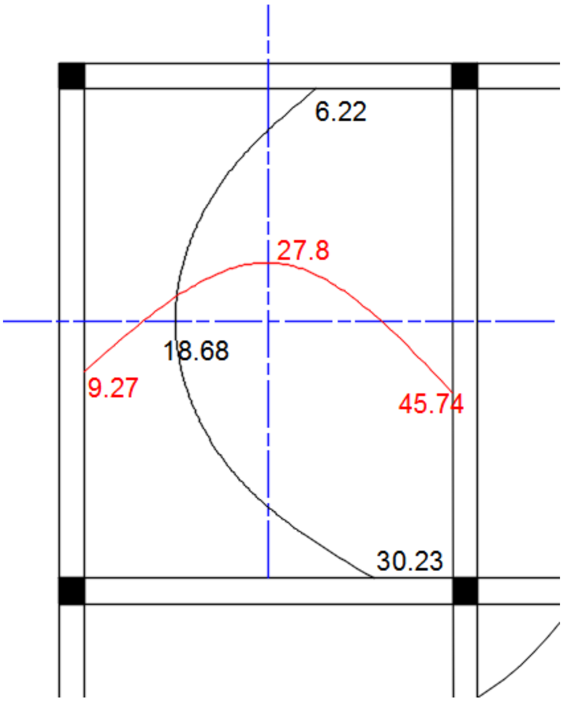
$$C_{bL}\left(\frac{l_a}{l_b}=0.80\right)=0.02\ \&\ C_{bL}\left(\frac{l_a}{l_b}=0.85\right)=0.023 \rightarrow C_{bL}\left(\frac{l_a}{l_b}=0.81\right)=0.0206$$

$$M_{bL}^+ = 0.0206 \times 8 \times 7.7^2 = 9.77\ kN.m/m$$

$$M_b^+ = M_{bd}^+ + M_{bL}^+ = 8.91 + 9.77 = 18.68\ kN.m/m$$

3- Negative moment at discontinuous edge= 1/3 positive moment

=1/3*18.68=6.22 kN.m/m



6- Find the Flexural reinforcement

Short direction

1- Negative moment at continues edge Mu=45.74 kN.m/m

$$d_a = 160 - 20 - \frac{12}{2} = 134\ mm$$

$$\rho_{min.} = 0.0018 \qquad \rightarrow As_{min.} = 0.0018 \times 1000 \times 160 = 288\ mm^2$$

$$\rho_{max.} = 0.85 \times \beta \times \frac{f_c'}{fy} \times \frac{3}{7} = 0.0154$$

$$\emptyset Mn \geq Mu$$

$$Mu = \emptyset\ \rho\ bd^2 fy\left(1 - 0.59\frac{fy}{f_c'}\ \rho\right)$$

$$\rho = \frac{1 \pm \sqrt{1 - \frac{2.36 * M_u * 10^6}{\emptyset\ b\ d^2 f_c'}}}{1.18\left(\frac{fy}{f_c'}\right)}$$

$$\rho = 0.00779$$

$$\rho_{min.} < \rho < \rho_{max.} \quad ok$$

$$As_{.} = 0.00779 \times 1000 \times 134 = 1043.86\ mm^2 > \ As_{min.} = 288\ mm^2$$

$$n = \frac{1044}{201.1} = 5.19\ bar\ in\ 1\ m \rightarrow s = \frac{1000}{5.19} = 192.6mm \approx 190mm$$

$$S_{max.} = min.of\ (2h = 320mm, 450mm)$$

S < $S_{max.}$ ok, Use Ø 16 @190 mm

2- Positive moment at mid span Mu=27.8 kN.m/m

$$\rho = 0.00454$$

$$\rho_{min.} < \rho < \rho_{max.} \quad ok$$

$$As_{.} = 0.00454 \times 1000 \times 134 = 608.36\ mm^2 > \ As_{min.} = 288\ mm^2$$

$$n = \frac{608.36}{113.1} = 5.38\ bar\ in\ 1\ m \rightarrow s = \frac{1000}{5.38} = 185.9\ mm \approx 185\ mm$$

$S_{max.} = min.of\ (2h = 320\ mm, 450\ mm)$ S < $S_{max.}$ ok, Use Ø 12 @185 mm

3- Negative moment at discontinuous edge

$$As_{.} = \frac{1}{3} \times 608.36 = 202.78\ mm^2 < \ As_{min.} = 288\ mm^2, use\ As_{min.}$$

$$S_{max.} = min.of\ (2h = 320\ mm, 450\ mm)$$

$$n = \frac{288}{113.1} = 2.54\ bar\ in\ 1\ m \rightarrow s = \frac{1000}{2.54} = 392\ mm > S_{max.} = 320\ mm$$

===

Long direction

1- Negative moment at continues edge Mu=30.23 kN.m/m

$$d_b = 160 - 20 - 12 - \frac{12}{2} = 122\ mm$$

$$\rho_{min.} = 0.0018 \qquad \rightarrow As_{min.} = 0.0018 \times 1000 \times 160 = 288\ mm^2$$

$$\rho_{max.} = 0.85 \times \beta \times \frac{f_c'}{fy} \times \frac{3}{7} = 0.0154$$

$$\emptyset Mn \geq Mu$$

$$Mu = \emptyset\ \rho\ bd^2 fy\left(1 - 0.59\frac{fy}{f_c'}\ \rho\right)$$

$$\rho = \frac{1 \pm \sqrt{1 - \frac{2.36 * M_u * 10^6}{\emptyset\ b\ d^2 f_c'}}}{1.18\left(\frac{fy}{f_c'}\right)}$$

$$\rightarrow \rho = 0.00608$$

$As_. = 0.00779 \times 1000 \times 122 = 741.76\ mm^2 > \ As_{min.} = 288\ mm^2\ ok$

$$n = \frac{741.76}{113.1} = 6.55\ bar\ in\ 1\ m \rightarrow s = \frac{1000}{6.55} = 150\ mm$$

$S_{max.} = min. of\ (2h = 320\ mm, 450\ mm)$

S < $S_{max.}$ ok, Use Ø 12 @150 mm

2- Positive moment at mid span Mu=16.68 kN.m/m

$$\rho = 0.00364$$

$$\rho_{min.} < \rho < \rho_{max.} \quad ok$$

$As_. = 0.00454 \times 1000 \times 122 = 444.36\ mm^2 > \ As_{min.} = 288mm^2$

$$n = \frac{444.36}{113.1} = 3.92\ bar\ in\ 1\ m \rightarrow s = \frac{1000}{3.92} = 255.1mm \approx 250mm$$

$S_{max.} = min. of\ (2h = 320\ mm, 450mm)$ S < $S_{max.}$ ok, Use Ø 12 @250 mm

3- Negative moment at discontinuous edge

$$As_. = \frac{1}{3} \times 444.36 = 148.12\ mm^2 < \ As_{min.} = 288\ mm^2, use\ As_{min.}$$

$S_{max.} = min. of\ (2h = 320\ mm, 450\ mm)$

$$n = \frac{288}{113.1} = 2.54\ bar\ in\ 1\ m \rightarrow s = \frac{1000}{2.54} = 392\ mm > S_{max.} = 320\ mm$$

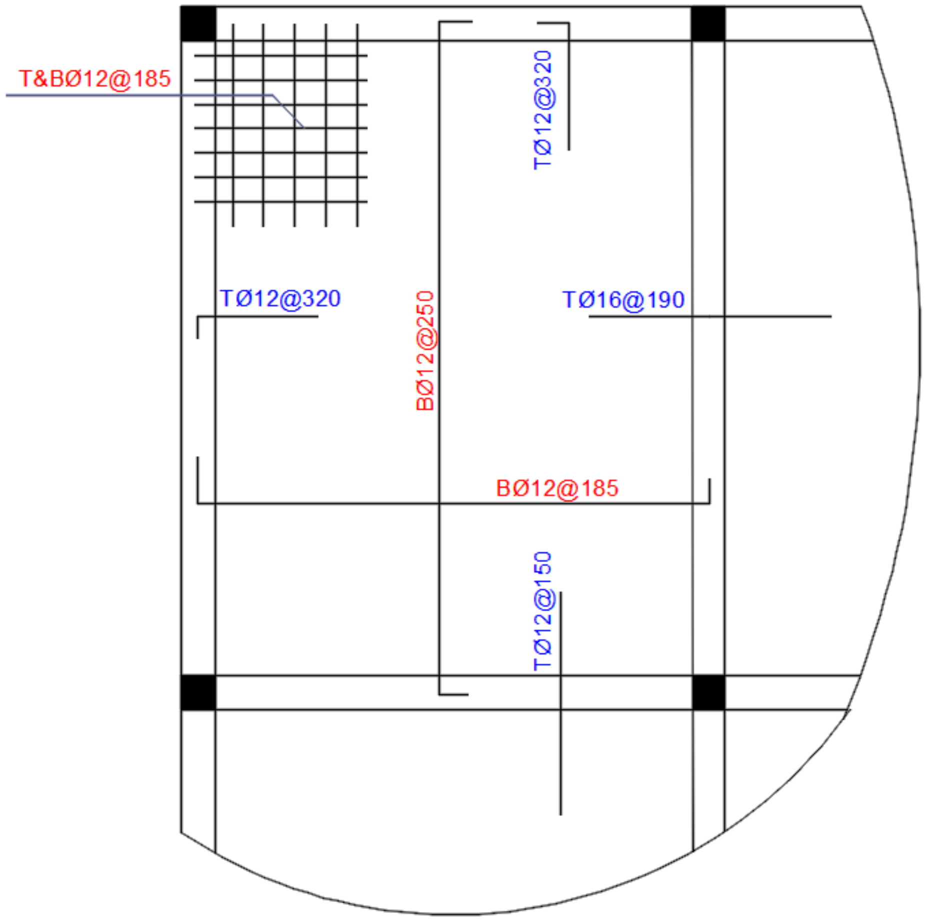
==

design the steel reinforcement for corner

The corner should be design according to max. bending positive moment of long and short direction.

$$\{\text{Mu short} = 27.8\ \text{kN.m/m}, \text{Mu long} = 18.68\ \text{kN.m/m}\}$$

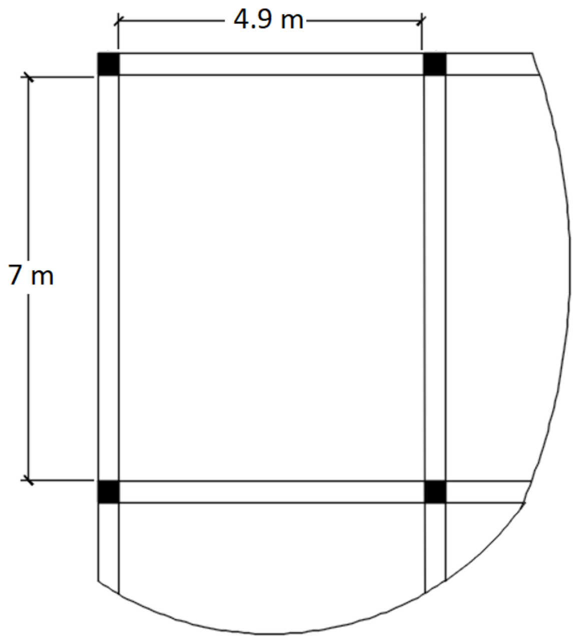
Use Ø 12 @185 mm



HW: For the reinforced concrete floor shown in the Figure below, design the slab to carry service live load 6 $\frac{kN}{m^2}$ and dead load due to:

Self-weight, 5 cm tiles, 2 cm mortar, 2 cm plaster and 3 $\frac{kN}{m^2}$ partition.

$f_c' = 25\ MPa, fy\ 420\ \ MPa$. For concrete use density=24 $\frac{kN}{m^3}$, for tiles, mortar and plaster used density=20 $\frac{kN}{m^3}$



Design of Reinforced Concrete Structures I

Al Muthanna University
Collage of Engineering

Two way slabs-4

Dr Othman Hameed

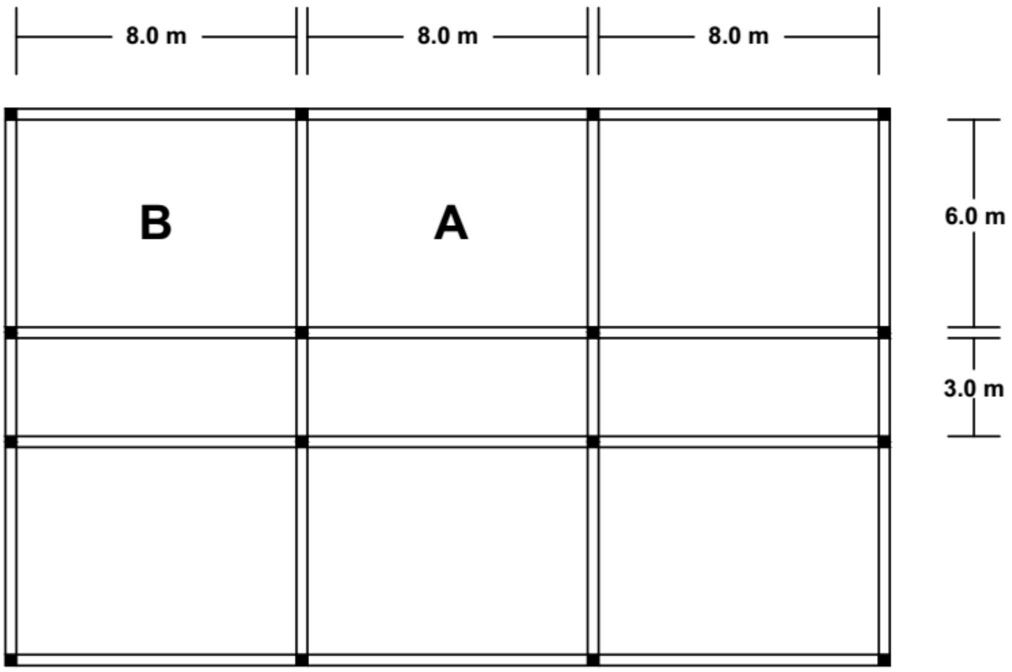
Lecture (14)

Two way solid slabs

Ex-2: For the reinforced concrete floor shown in the Figure below, design the slab (A) to carry service live load 5 $\frac{kN}{m^2}$ and dead load due to:

Self-weight, 3 cm tiles, 3 cm mortar, 2 cm plaster and 3 $\frac{kN}{m^2}$ partition.

$f_c' = 25\ MPa, fy = 420\ \ MPa$. For concrete use density 24 $\frac{kN}{m^3}$ and for tiles, mortar and plaster use density 22 $\frac{kN}{m^3}$.



Solusion

1- **Find the type of the slab**

$$\frac{l_b}{l_a} = \frac{8}{6} = 1.33\ < 2 \rightarrow\ two\ way\ slab$$

2- **Find slab thickness**

$$h = \frac{2(l_a + l_b)}{180} = \frac{2 \times (6000 + 8000)}{180} = 155.5 \cong 160mm$$

3- Find the ultimate load

- Dead load

Material	$\gamma\ \frac{kN}{m^3}$	$W = \gamma \times h = \frac{kN}{m^2}$
Tiles	22	22X0.03=0.66
mortar	22	22X0.03=0.66
Self-weight of slab	24	25X0.16=4
plaster	22	22X0.02=0.44
partition		3
Total dead load		8.6

Factored dead load= 1.2*8.6=10.32 $\frac{kN}{m^2}$

- Live load

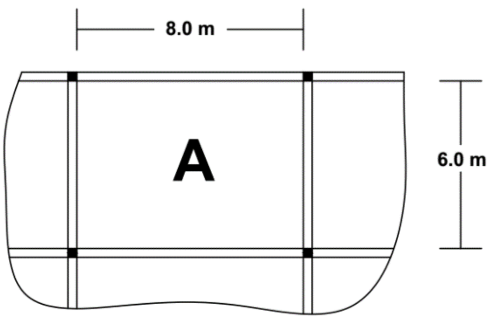
Factored live load= 1.6*5=8 $\frac{kN}{m^2}$

$$Wu = 10.32 + 8 = 18.32\ \frac{kN}{m^2}$$

4- Check the shear requirement

$m = \frac{l_a}{l_b} = \frac{6}{8} = 0.75$ (case 8)

To find the value of shear, go to table 4



a- Short direction $W_a\left(\frac{l_a}{l_b} = 0.75\right) = 0.61$

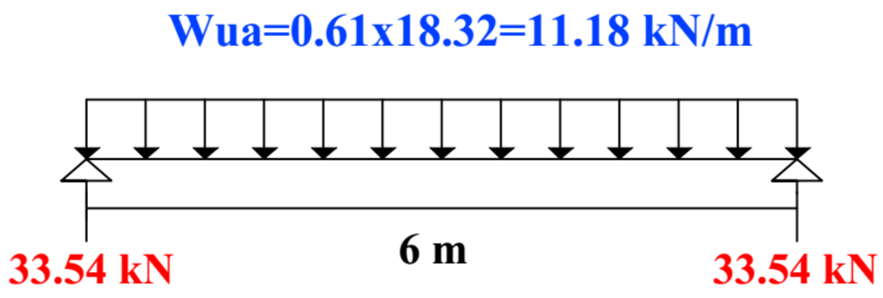
And

b- Long direction

$$W_b\left(\frac{l_a}{l_b} = 0.75\right) = 0.39$$

The reactions of the slab are calculated from Table 4, which indicates that 61% of the load is transmitted in the short direction and 39% in the long direction.

For short direction



$$\boxed{d_S = 160 - 20 - \frac{12}{2} = 134\,mm}$$

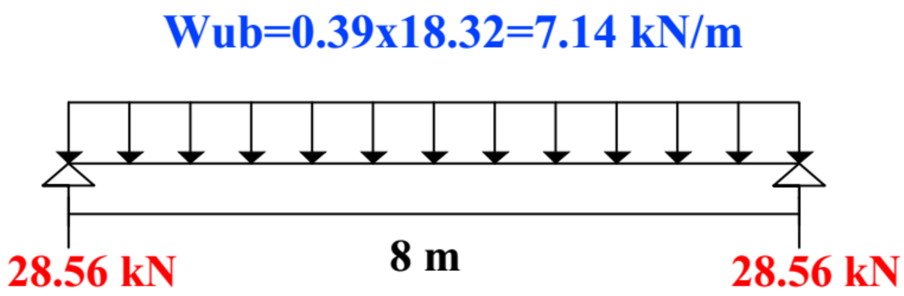
$$Vu@d = 33.54 - 0.134 \times 11.18 = 33.04\ kN/1m$$

$$\emptyset V_c = 0.75 \times \frac{\sqrt{25}}{6} \times 1000 \times 134 \times 10^{-3} = 83.75\ kN/1m$$

$$\emptyset V_c > Vu@d\ \ ok$$

the slab thickness is enough to resist the shear force

For long direction



$$\boxed{d_L = 160 - 20 - 12 - \frac{12}{2} = 122\,mm}$$

$$Vu@d = 28.56 - 0.122 \times 7.14 = 27.69\ kN/1m$$

$$\emptyset V_c = 0.75 \times \frac{\sqrt{25}}{6} \times 1000 \times 122 \times 10^{-3} = 76.25\ kN/1m$$

$$\emptyset V_c > Vu@d\ \ ok$$

$$the\ slab\ thickness\ is\ enough\ to\ resist\ the\ shear\ force$$

Find flexural moment

Short direction

1- Negative moment at continuous edge. Use table 1

Negative moment/short						
m	factor	length	load	Moment	ds	As
0.75	0.061	6	18.32	45.74	134	

$$M_a^- = C_a W l_a^2$$

$$C_a\left(\frac{l_a}{l_b} = 0.75\right) = 0.061$$

$$M_a^- = 0.061 \times 18.32 \times 6^2 = 40.23\ \ kN.m/m$$

2- Positive moment at mid span. Use table 2&3

$M_{ad}^+ = C_{ad} W_d l_a^2$...table 2

$$C_{ad}\left(\frac{l_a}{l_b} = 0.75\right) = 0.036$$

$$M_{ad}^+ = 0.036 \times 10.32\ \ \times 6^2 = 13.37\ kN.m/m$$

$M_{aL}^+ = C_{aL} W_L l_a^2$...table 3

$$C_{aL}\left(\frac{l_a}{l_b} = 0.75\right) = 0.049$$

$$M_{aL}^+ = 0.049 \times 8\ \ \times 6^2 = 14.11\ kN.m/m$$

$$M_a^+ = M_{ad}^+ + M_{aL}^+ = 13.37\ + 14.11 = 27.48\ kN.m/m$$

3- Negative moment at discontinuous edge= 1/3 positive moment

=1/3*27.48=9.16 kN.m/m

==

Long direction

1- Negative moment at continuous edge. Use table 1

Negative moment/Long direction						
m	factor	length	load	Moment	d_L	As
0.75	0.036	8	18.32		122	

$$M_b^- = C_b W l_b^2$$

$$C_b\left(\frac{l_a}{l_b} = 0.75\right) = 0.036$$

$$M_b^- = 0.036 \times 18.32 \times 8.0^2 = 42.2\ kN.m/m$$

2- Positive moment at mid span. Use table 2&3

$M_{bd}^+ = C_{bd} W_d l_b^2$...table 2

$$C_{bd}\left(\frac{l_a}{l_b} = 0.75\right) = 0.013$$

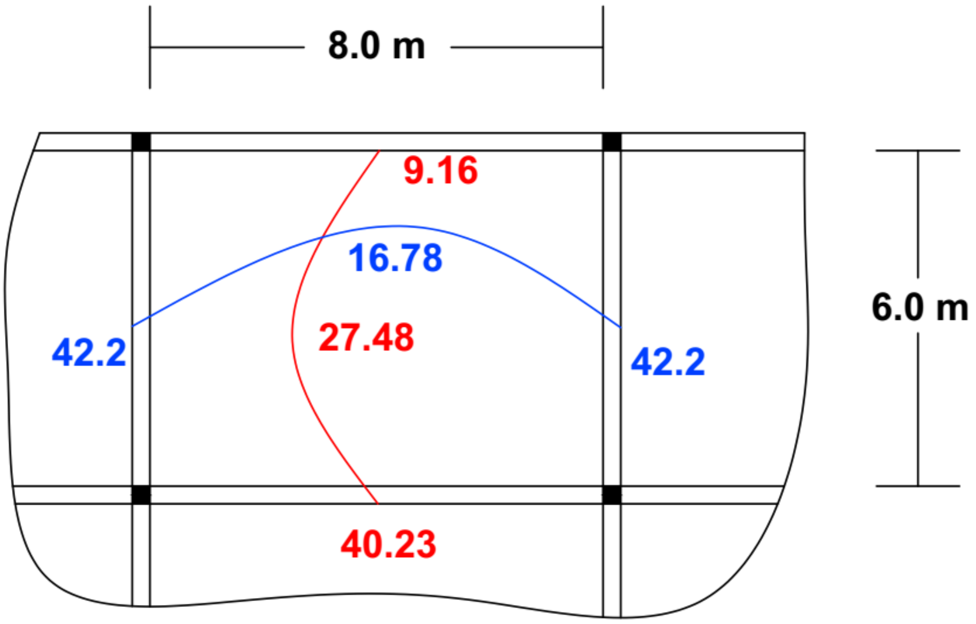
$$M_{bd}^+ = 0.013\ \times 10.32 \times 8.0^2 = 8.59\ kN.m/m$$

$M_{bL}^+ = C_{bL} W_L l_b^2$...table 3

$$C_{bL}\left(\frac{l_a}{l_b} = 0.75\right) = 0.016$$

$$M_{bL}^+ = 0.016 \times 8 \times 8.0^2 = 8.19\ kN.m/m$$

$$M_b^+ = M_{bd}^+ + M_{bL}^+ = 8.59 + 8.19 = 16.78\ kN.m/m$$



Find the Flexural reinforcement

Short direction

1- Negative moment at continues edge Mu=40.23 kN.m/m

$$d_a = 160 - 20 - \frac{12}{2} = 134\ mm$$

$$\rho_{min.} = 0.0018 \qquad \rightarrow As_{min.} = 0.0018 \times 1000 \times 160 = 288\ mm^2$$

$$\rho_{max.} = 0.85 \times \beta \times \frac{25}{420} \times \frac{3}{7} = 0.0184$$

$$\emptyset Mn \geq Mu$$

$$Mu = \emptyset\ \rho\ bd^2 fy \left(1 - 0.59 \frac{fy}{f_c'}\ \rho\right)$$

$$\rho = \frac{1 \pm \sqrt{1 - \frac{2.36 * M_u * 10^6}{\emptyset\ b\ d^2 f_c'}}}{1.18\left(\frac{fy}{f_c'}\right)}$$

$$\rho = 0.006324$$

$$As_{.} = 0.006324 \times 1000 \times 134 = 847.4\,mm^2 > \; As_{min.} = 288\,mm^2$$

$$n = \frac{847.4}{113.04} = 7.5\; bar\; in\; 1\; m \rightarrow s = \frac{1000}{7.5} = 133\; mm \approx 130mm$$

$$S_{max.} = min.of\;(2h = 320mm, 450mm)$$

S < $S_{max.}$ ok, Use Ø 12 @130 mm

Or Use Ø 16 @230 mm

2- Positive moment at mid span Mu=27.48 kN.m/m

$$\rho = \frac{1 \pm \sqrt{1 - \frac{2.36 * M_u * 10^6}{\emptyset\; b\; d^2 f_c'}}}{1.18\left(\frac{fy}{f_c'}\right)}$$

$$\rho = 0.004226$$

$$As_{.} = 0.004226 \times 1000 \times 134 = 566.2\,mm^2 > \; As_{min.} = 288\,mm^2$$

$$n = \frac{566.2}{113.1} = 5.0\; bar\; in\; 1\; m \rightarrow s = \frac{1000}{5.0} = 200\; mm$$

$S_{max.} = min.of\;(2h = 320\; mm, 450\; mm)$ S < $S_{max.}$ ok, Use Ø 12 @200 mm

3- Negative moment at discontinuous edge

$$As_{.} = \frac{1}{3} \times 566.2 = 188.73\,mm^2 < \; As_{min.} = 288\,mm^2, use\; As_{min.}$$

$$S_{max.} = min.of\;(2h = 320\; mm, 450\; mm)$$

$$n = \frac{288}{113.1} = 2.54\; bar\; in\; 1\; m \rightarrow s = \frac{1000}{2.54} = 392\; mm > S_{max.} = 320\; mm$$

Use Ø 12 @320 mm

===

Long direction

1- Negative moment at continues edge Mu=42.2 kN.m/m

$$d_b = 160 - 20 - 12 - \frac{12}{2} = 122\ mm$$

$$\rho_{min.} = 0.0018 \qquad \rightarrow As_{min.} = 0.0018 \times 1000 \times 160 = 288\ mm^2$$

$$\rho_{max.} = 0.85 \times \beta \times \frac{25}{420} \times \frac{3}{7} = 0.0184$$

$$\emptyset Mn \geq Mu$$

$$Mu = \emptyset\ \rho\ bd^2 fy \left(1 - 0.59 \frac{fy}{f_c'}\ \rho\right)$$

$$\rho = \frac{1 \pm \sqrt{1 - \frac{2.36 * M_u * 10^6}{\emptyset\ b\ d^2 f_c'}}}{1.18 \left(\frac{fy}{f_c'}\right)}$$

$$\rightarrow \rho = 0.008161$$

$$As_{.} = 0.008161 \times 1000 \times 122 = 995.6\ mm^2 > \ As_{min.} = 288\ mm^2\ ok$$

$$n = \frac{995.6}{113.1} = 8.81\ bar\ in\ 1\ m \rightarrow s = \frac{1000}{8.81} = 110\ mm$$

$$S_{max.} = min.of\ (2h = 320\ mm, 450\ mm)$$

S < $S_{max.}$ ok, Use Ø 12 @110 mm

Or, Use Ø 16 @200 mm

2- Positive moment at mid span Mu=16.78 kN.m/m

$$d_b = 160 - 20 - 12 - \frac{12}{2} = 122\ mm$$

$$\rho_{min.} = 0.0018 \qquad \rightarrow As_{min.} = 0.0018 \times 1000 \times 160 = 288\ mm^2$$

$$\rho_{max.} = 0.85 \times \beta \times \frac{25}{420} \times \frac{3}{7} = 0.0184$$

$$\emptyset Mn \geq Mu$$

$$Mu = \emptyset\, \rho\, bd^2 fy\left(1 - 0.59\frac{fy}{f_c'}\,\rho\right)$$

$$\rho = \frac{1 \pm \sqrt{1 - \dfrac{2.36 * M_u * 10^6}{\emptyset\, b\, d^2 f_c'}}}{1.18\left(\dfrac{fy}{f_c'}\right)}$$

$$\rightarrow \rho = 0.003076$$

$As_. = 0.003076 \times 1000 \times 122 = 375.3\, mm^2 > \; As_{min.} = 288\, mm^2$ *ok*

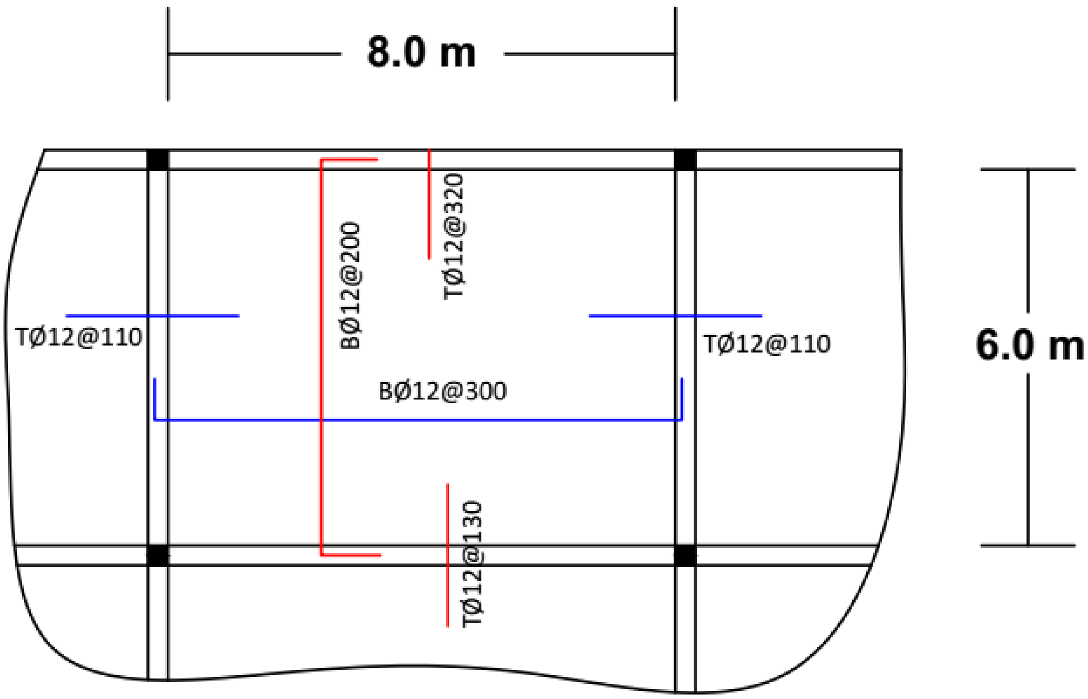
$$n = \frac{375.3}{113.1} = 8.81\, bar\ in\ 1\, m \rightarrow s = \frac{1000}{8.81} = 300\, mm$$

$S_{max.} = min.\,of\ (2h = 320\, mm, 450\, mm)$

S < $S_{max.}$ ok, Use Ø 12 @300 mm

===

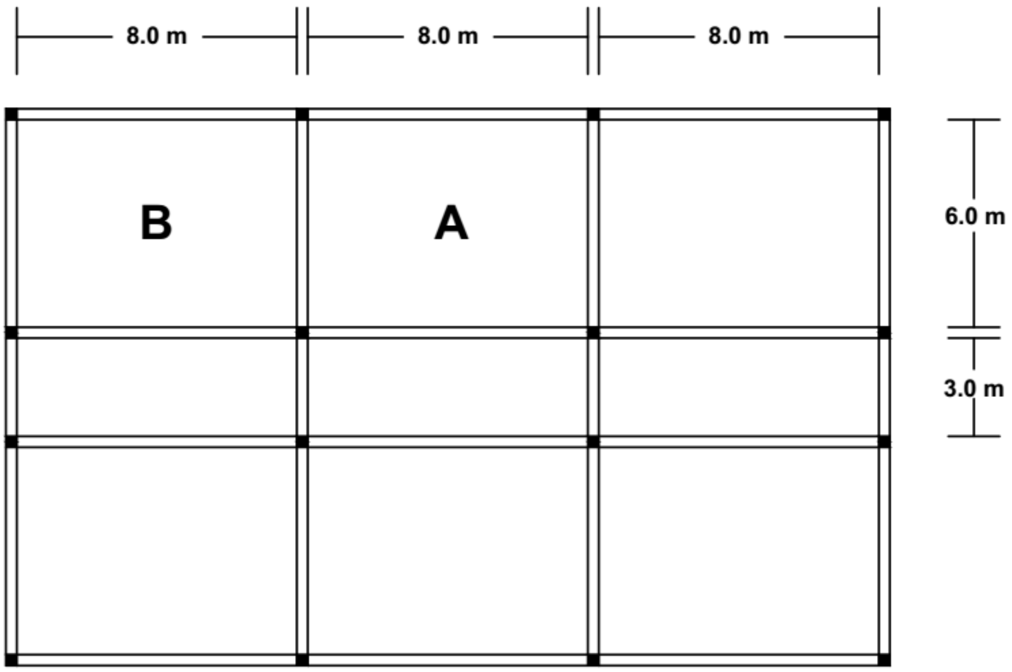
location	Short		Long	
	Moment kN.m/m	As	Moment kN.m/m	As
Positive	27.48	Ø 12 @200	16.78	Ø 12 @300
Negative continuous	40.23	Ø 12 @130	40.23	Ø 12 @110
Negative discontinuous	9.16	Ø 12 @320	-------------	-----------



HW-2: For the reinforced concrete floor shown in the Figure below, design the slab (B) to carry service live load 6 $\frac{kN}{m^2}$ and dead load due to:

Self-weight, 3 cm tiles, 3 cm mortar, 2 cm plaster and $4\frac{kN}{m^2}$ partition.

$f_c' = 22\ MPa, fy = 420\ \ MPa$. For concrete use density 24 $\frac{kN}{m^3}$ and for tiles, mortar and plaster use density 22 $\frac{kN}{m^3}$.



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Design of Reinforced Concrete Structures I

Two way slabs-5

Dr Othman Hameed

Lecture (15)

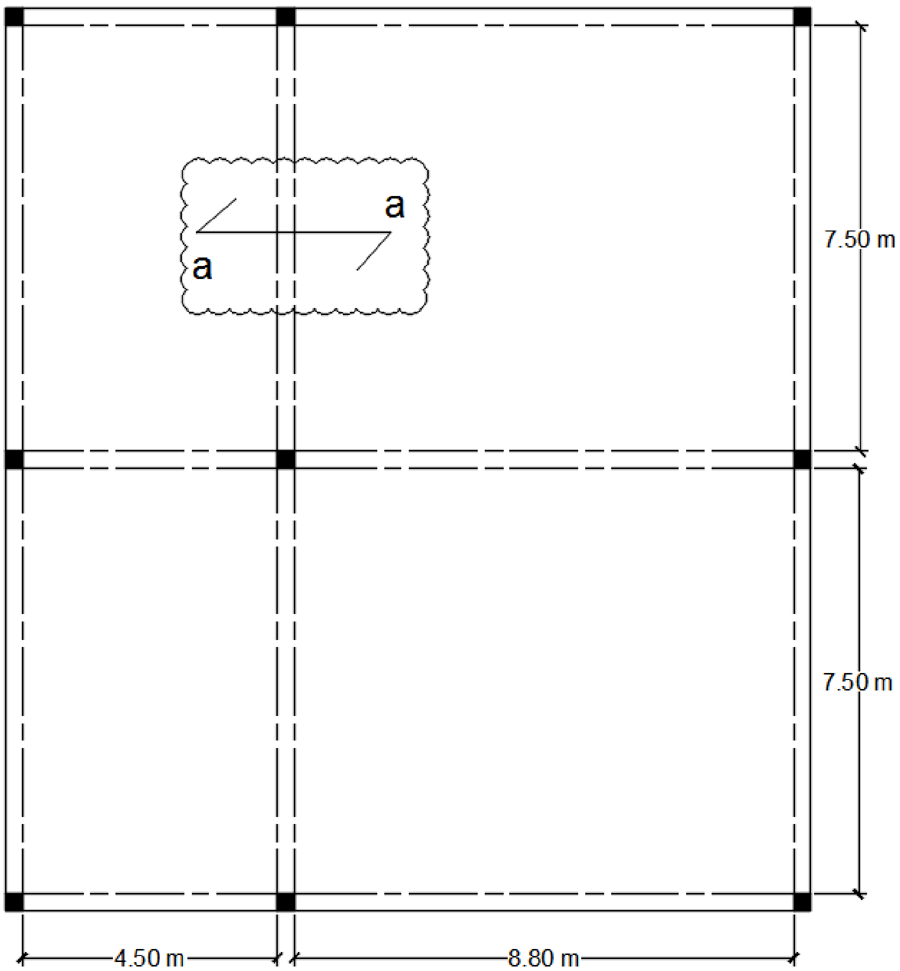
Two way solid slabs

Ex-3: For the floor system shown in the figure below use:

- live load 5 kN/m^2
- The service dead load consists of self-weight, 2 cm tiles, 2 cm mortar, and 1.5 cm plaster. (use concrete density 25 $\frac{kN}{m^3}$ while the density of tiles, mortar and plaster is 22 $\frac{kN}{m^3}$)
- $f_c' = 30\ MPa, f_y = 400\ \ MPa\ and\ all\ beam\ have\ width\ 300\ mm$

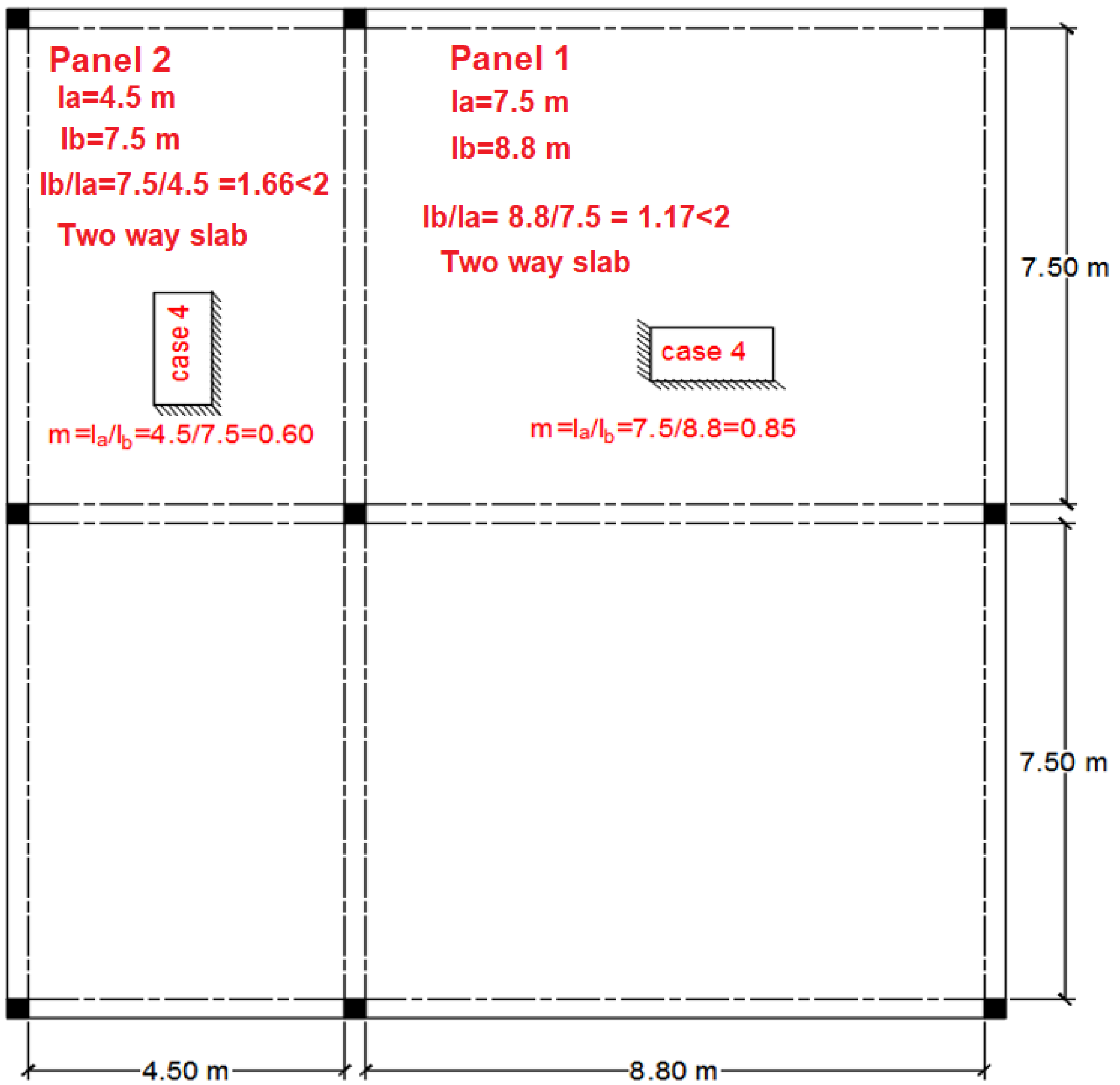
Answer the following:

1- Classify the floor system into one way or two way.
2- Find the slab thickness to satisfy deflection requirement. (use one thickness for all slabs)
3- Check the proposed slab thickness in previous step according to shear requirement.
4- Design the continuous edge (section a-a) (middle strip) shown in Figure.



Solution:

1)


Panel 2
la=4.5 m
lb=7.5 m
lb/la=7.5/4.5 =1.66<2
Two way slab
case 4
m =la/lb=4.5/7.5=0.60
Panel 1
la=7.5 m
lb=8.8 m
lb/la= 8.8/7.5 = 1.17<2
Two way slab
case 4
m =la/lb=7.5/8.8=0.85
7.50 m
7.50 m
4.50 m
8.80 m

2) Find the slab thickness

$$h = \frac{2(l_a + l_b)}{180} = \frac{2 \times (7500 + 8800)}{180} = 181 \cong 190mm$$

3) Check the proposed slab thickness according to shear requirement.

a- Find the ultimate load applied on slab

- Dead load

Material	$\gamma\ \frac{kN}{m^3}$	$W = \gamma \times h = \frac{kN}{m^2}$
Self-weight of slab	25	25*0.19=4.75
Tiles	22	22*0.02=0.44
Mortar	22	22*0.02=0.44
plaster	22	22*0.015=0.33
Total dead load		5.96

Factored dead load= 1.2*5.96=7.152 $\frac{kN}{m^3}$

b- Live load

Factored live load= 1.6*5=8 $\frac{kN}{m^2}$

$$Wu = 7.152 + 8 = 15.15\ \frac{kN}{m^2}$$

Go to table 4

Panel 1

Short direction

$$W_{u\,a} = 0.66 \times 15.15 = 10\ kN/m$$

Reaction= $\frac{W_{u\,a} \times l_a}{2} = \frac{10 \times 7.5}{2} = 37.5\ kN/m$

Long direction

$$W_{u\,b} = 0.34 \times 15.15 = 5.15\ kN/m$$

Reaction$= \frac{W_{u\,b} \times l_b}{2} = \frac{5.15 \times 8.8}{2} = 22.66\ kN/m$

Panel 2

Short direction

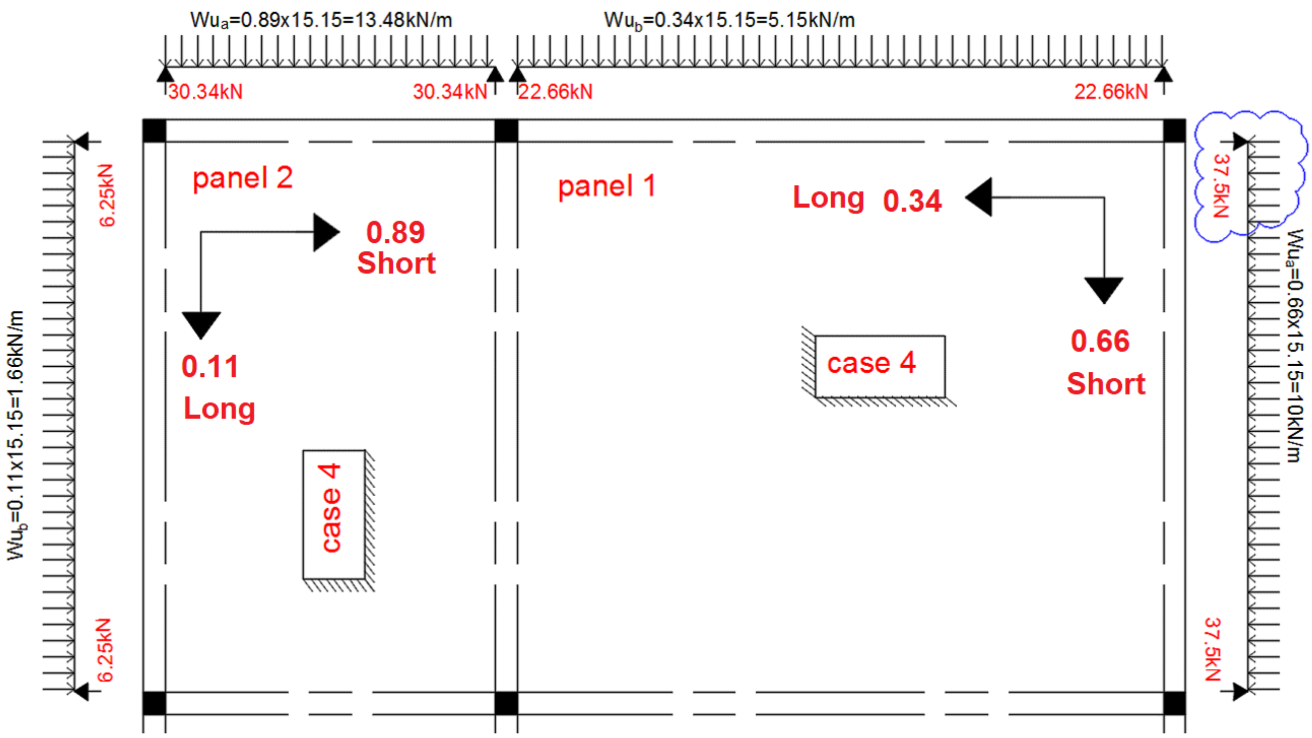
$$W_{u\,a} = 0.89 \times 15.15 = 13.48\ kN/m$$

Reaction$= \frac{W_{u\,a} \times l_a}{2} = \frac{13.48 \times 4.5}{2} = 30.33\ kN/m$

Long direction

$$W_{u\,b} = 0.11 \times 15.15 = 1.66\ kN/m$$

Reaction$= \frac{W_{u\,b} \times l_b}{2} = \frac{1.66 \times 7.5}{2} = 6.225\ kN/m$



$$d_a = 190 - 20 - \frac{12}{2} = 164\ mm$$

$$d_b = 190 - 20 - 12 - \frac{12}{2} = 152\ mm$$

$$V_u\ @\ d = 37.5 - 10 \times 0.164 = 35.86\ kN/1m$$

$$\emptyset V_c = 0.75 \times \frac{\sqrt{30}}{6} \times 1000 \times 164 \times 10^{-3} = 112.28\ kN/1m$$

$$\emptyset V_c > Vu@d\ \ ok$$

$$the\ slab\ thickness\ is\ enough\ to\ resist\ shear\ force$$

4) Design section a-a

Panle1

Long direction, la=8.8m, case4 and m=0.85

Negative moment at discontinuous edge

$$M_b^- = C_b W l_b^2$$

$$C_b\left(\frac{l_a}{l_b} = 0.85\right) = 0.034$$

$$M_b^- = 0.034 \times 15.15 \times 8.8^2 = 39.89\ kN.m/m$$

Panle2

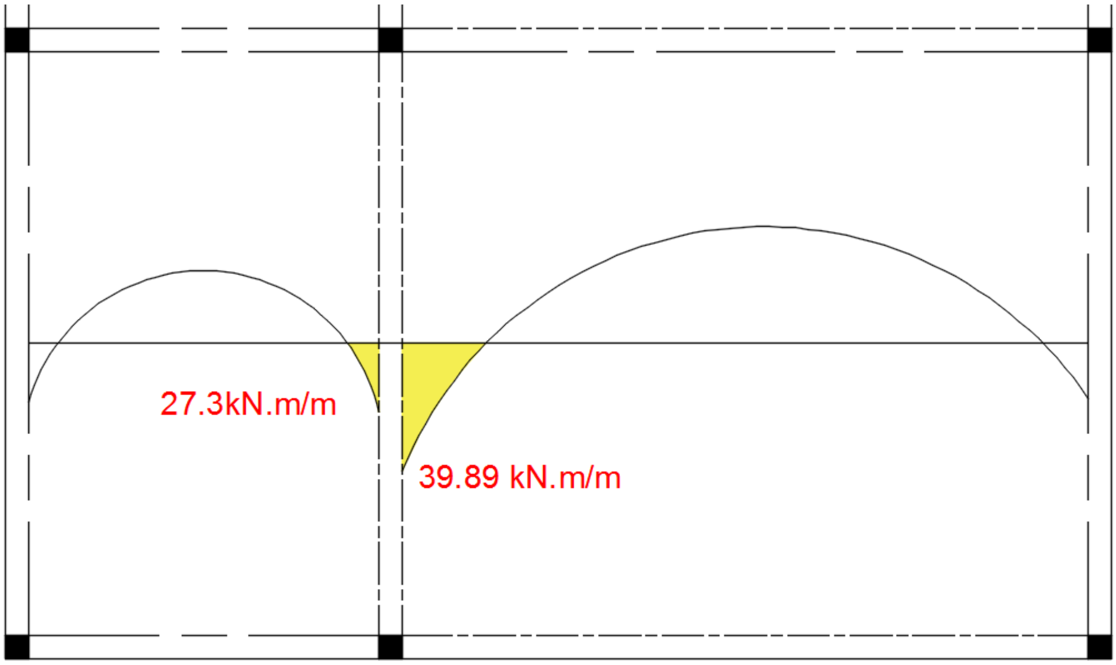
Short direction, la=4.5m, case4 and m=0.6

Negative moment at discontinuous edge

$$M_b^- = C_b W l_b^2$$

$$C_b\left(\frac{l_a}{l_b} = 0.6\right) = 0.089$$

$$M_b^- = 0.089 \times 15.15 \times 4.5^2 = 27.3\ kN.m/m$$



Note:

In continuous edge, there are two values for the negative moment, therefore the following condition should be considered:

$M_{u\,1} = the\ smallest\ value\ of\ moment$
$M_{u\,2} = the\ largest\ value\ of\ moment$

$$if\ \frac{M_{u\,1}}{M_{u\,2}} \geq 0.8\ \ design\ accoding\ to\ lagest\ value\ of\ moment$$

$$if\ \frac{M_{u\,1}}{M_{u\,2}} < 0.8\ \ the\ moment\ shold\ be\ redisterbution\ as\ shown\ below$$

$$M_u = M_{u\,1} + (\Delta M) \times \frac{\frac{1}{ls}}{\frac{1}{ls} + \frac{1}{lL}}$$

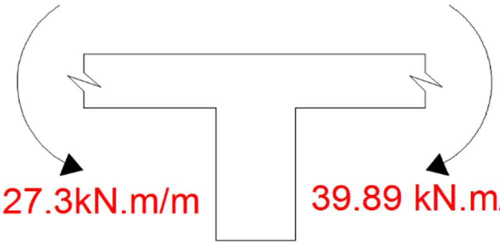
Where:

$M_u = design\ moment$

$\Delta M = M_{u\,2} - M_{u\,1}$

$ls = the\ length\ parallel\ to\ smallest\ moment$

$lL = the\ length\ parallel\ to\ largest\ moment$



$$\frac{27.3}{39.89} = 0.68 < 0.8$$

$$M_u = M_{u\,1} + (\Delta M) \times \frac{\frac{1}{ls}}{\frac{1}{ls} + \frac{1}{lL}} = 27.3 + (39.89 - 27.3) \times \frac{\frac{1}{4.5}}{\frac{1}{4.5} + \frac{1}{8.8}} = 35.6\ kN.m/m$$

Find the reinforcement

$$d_a = 190 - 20 - \frac{12}{2} = 164\ mm$$

$$d_b = 190 - 20 - 12 - \frac{12}{2} = 152\ mm$$

$$\rho_{min.} = 0.0018 \qquad \rightarrow As_{min.} = 0.0018 \times 1000 \times 190 = 342\ mm^2$$

$$\rho_{max.} = 0.85 \times \beta \times \frac{f_c'}{fy} \times \frac{3}{7} = 0.0154$$

Table 22.2.2.4.3—Values of β1 for equivalent rectangular Concrete stress distribution		
	$f_c'\ MPa$	β_1
a	$17 \le f_c' \le 28$	0.85
b	$28 < f_c' \le 55$	$0.85 - \frac{0.05\left(f_c^{`} - 28\right)}{7}$
c	$f_c' > 55$	0.65

$$\beta = 0.85 - \frac{0.05\left(f_c^{`} - 28\right)}{7} = 0.85 - \frac{0.05(30 - 28)}{7} = 0.836$$

$$\rho_{max.} = 0.85 \times 0.836 \times \frac{30}{400} \times \frac{3}{7} = 0.0228$$

$$\emptyset Mn \geq Mu$$

$$Mu = \emptyset\ \rho\ bd^2 fy \left(1 - 0.59\frac{fy}{f_c'}\ \rho\right)$$

$$\rho = \frac{1 \pm \sqrt{1 - \frac{2.36 * M_u * 10^6}{\emptyset\ b\ d^2 f_c'}}}{1.18\left(\frac{fy}{f_c'}\right)}$$

Use depth of the long direction ($d_a = 164\ mm$)

$$\rho = \frac{1 \pm \sqrt{1 - \frac{2.36 * 35.6 * 10^6}{0.9 *\ 1000 *\ 152^2 * 30}}}{1.18\left(\frac{400}{30}\right)}$$

$$\rho = 0.00443$$

$$As_{.} = 0.00443 \times 1000 \times 152 = 673.36\ mm^2 >\ As_{min.} = 342\ mm^2$$

$$n = \frac{673.36}{113.1} = 5.95\ bar\ in\ 1\ m \rightarrow s = \frac{1000}{5.95} = 168\ mm$$

$$S_{max.} = min.of\ (2h = 380\ mm, 450\ mm)$$

S < $S_{max.}$ ok, Use Ø 12 @160 mm

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Design of Reinforced Concrete Structures I

Load transferred to beams

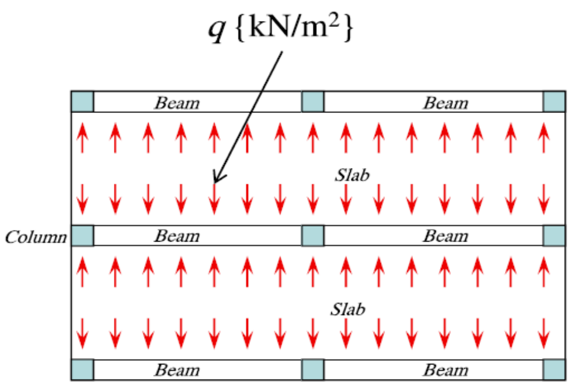
Dr Othman Hameed

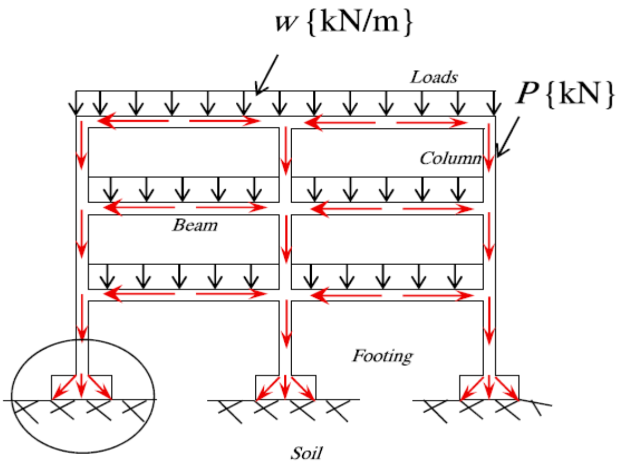
Lecture (16)

Load transferred to beams

Load Paths

Structural system transfer loads from the floors and roof to ground though load paths that need clearly identified in the design process.





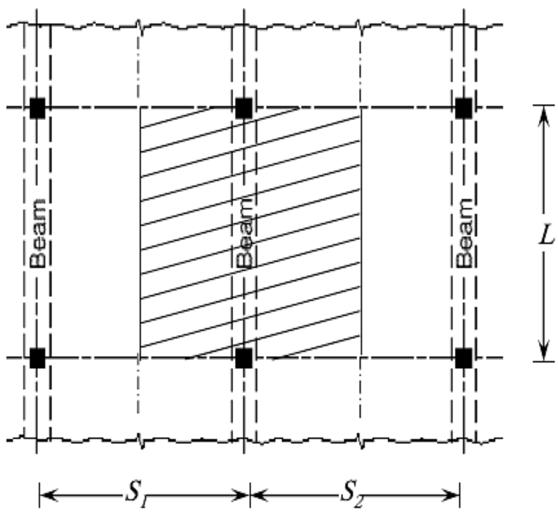
Identifying the correct path is important for determining the load carried by each structural member.

The distributed load over the floor has unit (force/ area) kN/m^2

In order to design the beam, the tributary load from the floor carried by beam and distributed over the span is determined. This load has unit (force/ distance) kN/m.

*** If L/S ≥ 2 the load will transfer to beam by one way path as shown in the below**

$$W\left(\frac{kN}{m}\right) = Load\ of\ slab\left(\frac{kN}{m^2}\right) \times \frac{S_1}{2}$$



*** If L/S < 2 the load will transfer on beam by two way which are bounded by 45-dgree lines drawn from corner of the panel as shown in figure below.**

Uniform distributed load transfer to short direction beam from each panel

$$W_{u\,S} = \frac{Wu\,.\,S}{3}$$

While the Uniform distributed load transfer to long direction beam from each panel

$$W_{u\,L} = \frac{Wu\,.\,S}{3} * \frac{3 - m^2}{2}$$

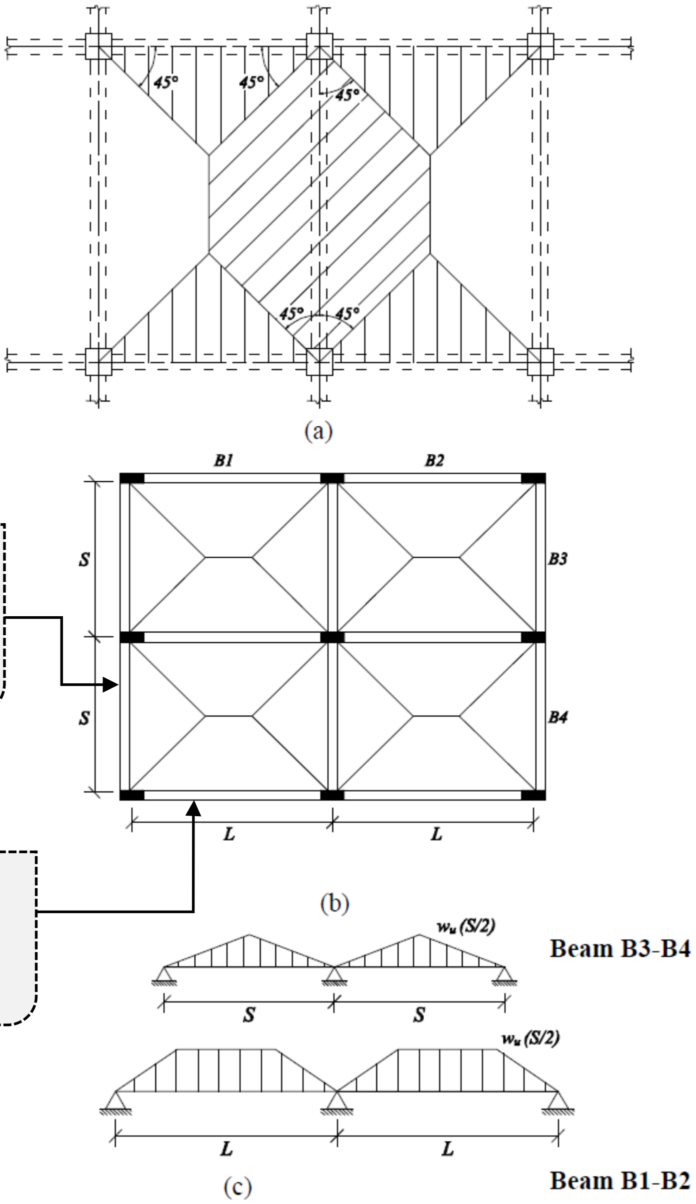


Figure (a) Tributary areas for slab loads on beams; (b) tributary areas for loads on continuous beams; (c) loads on beams *B3-B4* and *B1-B2*

Where:

W_u =uniform distributed load supported by slab (kN/m^2).

$W_{u\,L}$ = equivalent load transferred from slab to beam of long direction (kN/m).

$W_{u\,S}$ = equivalent load transferred from slab to beam of short direction (kN/m).

S = length of short direction.

L = length of long direction.

m = ratio of S/L

For Roof

The load transferred to beams of roof consists of:

1- Load transferred from slabs (one way or two ways)

2- Load transferred from walls (if any)

3- Load transferred from parapet (if any)

For Floor

The load transferred to beams of floor consists of:

1- Load transferred from slabs (one way or two ways)

2- Load transferred from partitions (if any)

Ex:1

The roof system shown in the figure below carries an ultimate load of 5 $\frac{kN}{m^2}$, find the following

1- The load transferred to the beam AB.

2- The load transferred to the beam CD

3- The load transferred to the beam EF

4- The load transferred to the beam DE

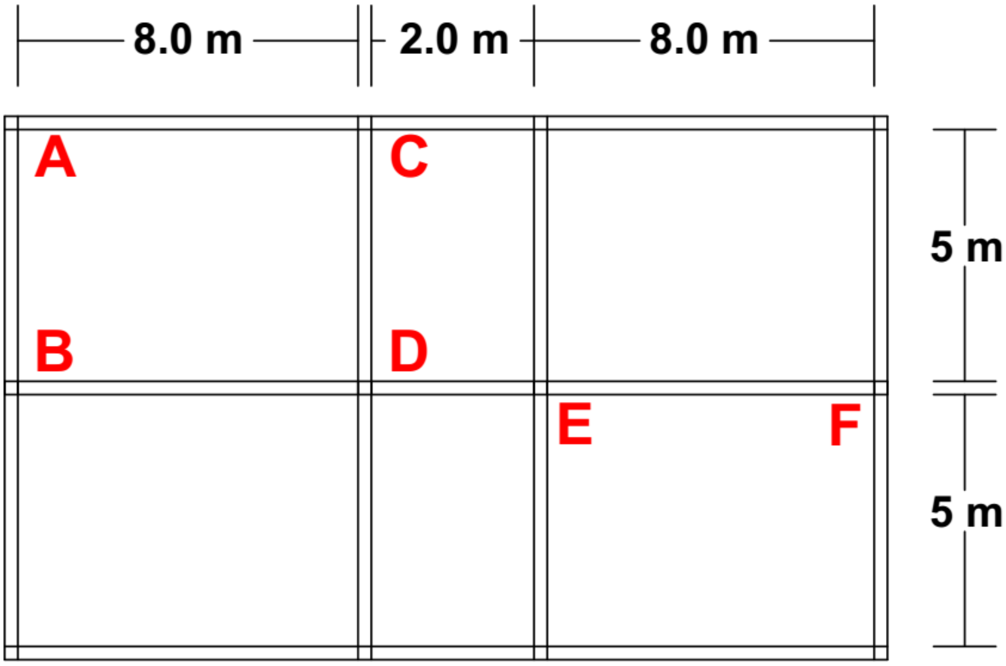
Notes

- The parapet dimension is 500 mm height and 200 mm thickness.

- A brick wall of 1.5 m height and 125 mm thickness is located at the edge beams of roof

- All beams of the first floor carry a brick wall of 3 m height and 250 mm thickness

- The densities of concrete and wall are 24 $\frac{kN}{m^3}$ and 18 $\frac{kN}{m^3}$, respectively.



Solution

1- The load transferred to the beam AB

- From slabs

$$W_{u\,s} = \frac{\mathrm{Wu}\,.\,S}{3} = \frac{5 \times 5}{3} = 8.33\ \frac{kN}{m}$$

- From parapets

$$1.2 \times 0.5 \times 0.2 \times 24 = 2.88\ \frac{kN}{m}$$

- From walls

$$1.2 \times 1.5 \times 0.125 \times 18 = 4.05\ \frac{kN}{m}$$

Total load transferred to beam AB$=8.33 + 2.88 + 4.05 = 15.26\ \frac{kN}{m}$

2- The load transferred to the beam CD

- From two way slab

$$\mathrm{W_{u\,S}} = \frac{\mathrm{Wu}\,.\ S}{3} = \frac{5 \times 5}{3} = 8.33\ \frac{kN}{m}$$

- From one way slab

$$W\left(\frac{kN}{m}\right) = Load\ of\ slab\left(\frac{kN}{m^2}\right) \times \frac{S}{2} = 5 \times \frac{2}{2} = 5\ \frac{kN}{m}$$

Total load transferred to beam CD=$8.33 + 5 = 13.33\ \frac{kN}{m}$

3- The load transferred to beam EF

$$\mathrm{W_{u\,L}} = 2 \times \frac{\mathrm{Wu}\,.\ S}{3} * \frac{3 - m^2}{2} = \frac{5\ \times\ 5}{3} * \frac{3 - \left(\frac{5}{8}\right)^2}{2} = 21.74\ \frac{kN}{m}$$

4- The load transferred to the beam DE

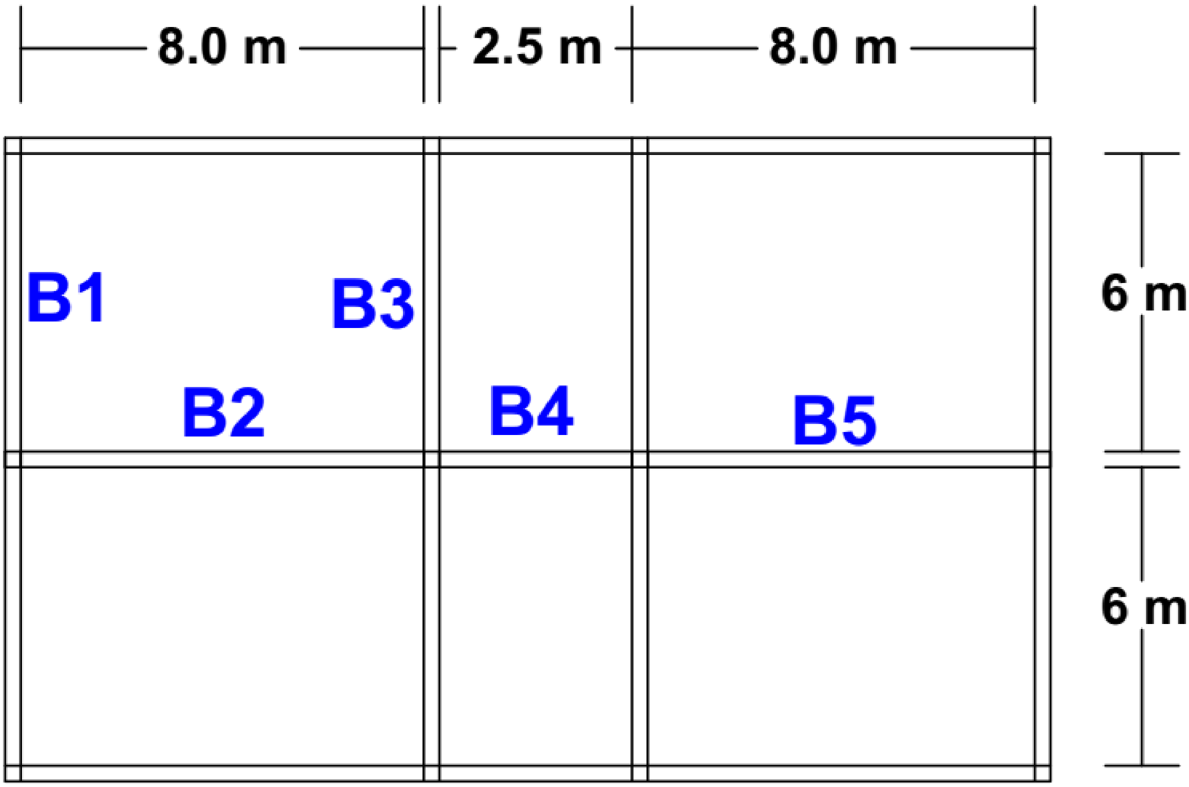
$$\mathrm{W_{u\,S}} = 0$$

Ex:2

The first floor system shown in the figure below carries a dead load of 4 $\frac{kN}{m^2}$ and a live load of 6 $\frac{kN}{m^2}$ find the load transferred to beams B1, B2, B3, B4, and b5

Notes

- The parapet dimension is 500 mm height and 200 mm thickness.
- A brick wall of 1.5 m height and 125 mm thickness is located at the edge beams of roof
- All beams of the first floor carry a brick wall of 3 m height and 250 mm thickness
- The densities of concrete and wall are 24 $\frac{kN}{m^3}$ and 20 $\frac{kN}{m^3}$, respectively.



Ex:3

The slab system shown in the figure below carries the following loads

- For the first floor, a dead load of 4 $\frac{kN}{m^2}$ and a live load of 6 $\frac{kN}{m^2}$

- For the roofs, a dead load of 4 $\frac{kN}{m^2}$ and a live load of 2 $\frac{kN}{m^2}$

1-Find the load transferred to beams B1, B2, and B3 of the first floor

2-Find the load transferred to beams B1, B2, and B3 of the roofs

3- Draw the frame 1-1

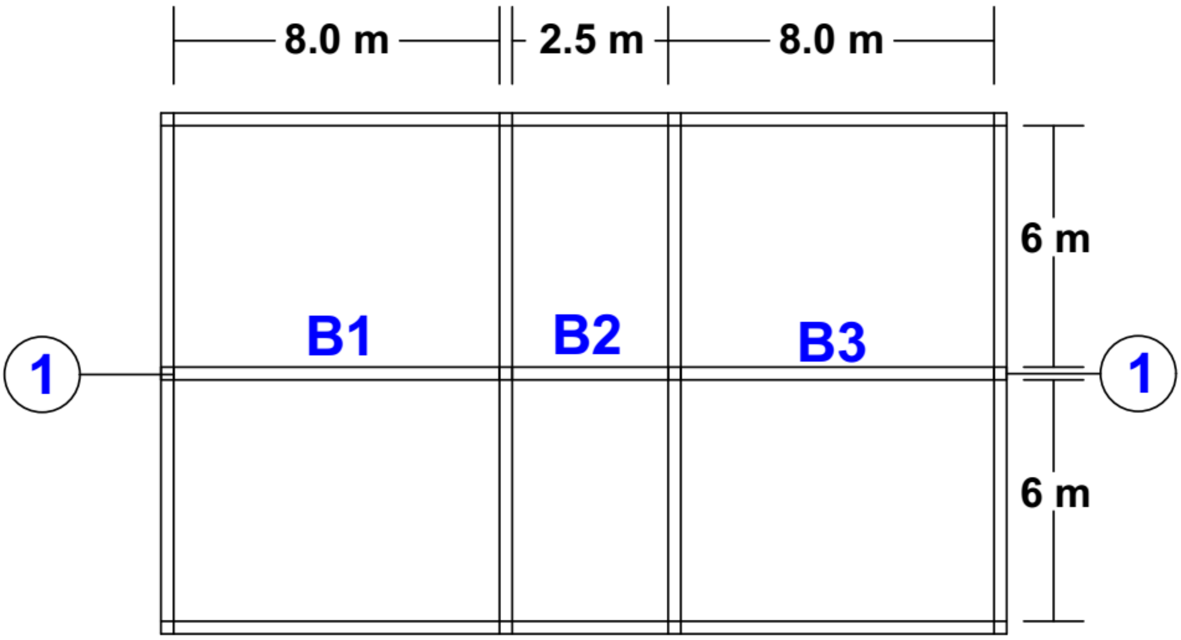
Notes

- The parapet dimension is 500 mm height and 200 mm thickness.

- A brick wall of 1.5 m height and 125 mm thickness is located at the edge beams of roof

- All beams of the first floor carry a brick wall of 3 m height and 250 mm thickness

- The densities of concrete and wall are 24 $\frac{kN}{m^3}$ and 20 $\frac{kN}{m^3}$, respectively.



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Design of Reinforced Concrete Structures I

Load transferred to beams-2

Dr Othman Hameed

Lecture (17)

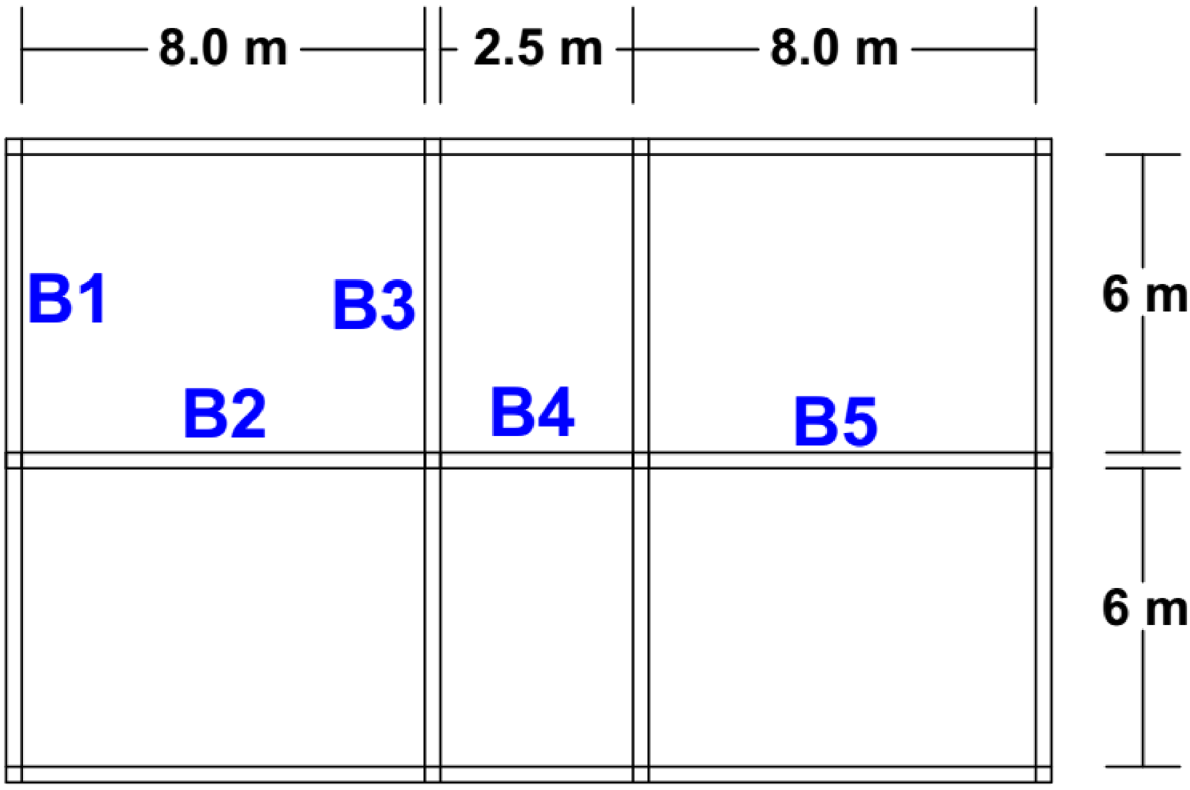
Load transferred to beams

Ex:2

The first floor system shown in the figure below carries a dead load of 4 $\frac{kN}{m^2}$ and a live load of 6 $\frac{kN}{m^2}$ find the load transferred to beams B1, B2, B3, B4, and b5

Notes

- The parapet dimension is 500 mm height and 200 mm thickness.
- A brick wall of 1.5 m height and 125 mm thickness is located at the edge beams of roof
- All beams of the first floor carry a brick wall of 3 m height and 250 mm thickness
- The densities of concrete and wall are 24 $\frac{kN}{m^3}$ and 20 $\frac{kN}{m^3}$, respectively.



Solution

$$W_u = 1.2W_D + 1.6W_L = 1.2 \times 4 + 1.6 \times 6 = 14.4\ \frac{kN}{m^2}$$

1- The load transferred to the beam B1

- From two way slab

$$\mathrm{W_{u\,S}} = \frac{\mathrm{Wu}\,.\,S}{3} = \frac{14.4 \times 6}{3} = 28.8\ \frac{kN}{m}$$

- From walls

$$1.2 \times 3 \times 0.25 \times 20 = 18\ \frac{kN}{m}$$

Total load transferred to beam B1=$28.8 + 18 = 46.8\ \frac{kN}{m}$

2- The load transferred to the beam B2

- From slabs

$$\mathrm{W_{u\,L}} = 2 \times \frac{\mathrm{Wu}\,.\,S}{3} * \frac{3 - m^2}{2} = 2 \times \frac{14.4 \ \times \ 6}{3} * \frac{3 - \left(\frac{6}{8}\right)^2}{2} = 70.2\ \frac{kN}{m}$$

- From walls

$$1.2 \times 3 \times 0.25 \times 20 = 18\ \frac{kN}{m}$$

Total load transferred to beam B2=$70.2\ + 18 = 88.2\ \frac{kN}{m}$

3- The load transferred to the beam B3

- From two way slabs

$$\mathrm{W_{u\,S}} = \frac{\mathrm{Wu}\,.\,S}{3} = \frac{14.4 \times 6}{3} = 28.8\ \frac{kN}{m}$$

- From one way slab

$$W\left(\frac{kN}{m}\right) = Load\ of\ slab\left(\frac{kN}{m^2}\right) \times \frac{S}{2} = 14.4 \times \frac{2.5}{2} = 18\ \frac{kN}{m}$$

- From walls

$$1.2 \times 3 \times 0.25 \times 20 = 18 \ \frac{kN}{m}$$

Total load transferred to beam B3=$28.8 + 18 + 18 = 64.8 \ \frac{kN}{m}$

4- The load transferred to the beam B4

$W_{u\,S} = 0$

- From walls

$$1.2 \times 3 \times 0.25 \times 20 = 18 \ \frac{kN}{m}$$

Total load transferred to beam B4=$0 + 18 = 18 \ \frac{kN}{m}$

5- The load transferred to the beam B5

- From slabs

$$W_{u\,L} = 2 \times \frac{Wu\,.\,S}{3} * \frac{3 - m^2}{2} = 2 \times \frac{14.4 \ \times \ 6}{3} * \frac{3 - \left(\frac{6}{8}\right)^2}{2} = 70.2 \ \frac{kN}{m}$$

- From walls

$$1.2 \times 3 \times 0.25 \times 20 = 18 \ \frac{kN}{m}$$

Total load transferred to beam B5=$70.2 \ + 18 = 88.2 \ \frac{kN}{m}$

Ex:3

The slab system shown in the figure below carries the following loads

- For the roofs, a dead load of 4 $\frac{kN}{m^2}$ and a live load of 2 $\frac{kN}{m^2}$

- For the first floor, a dead load of 4 $\frac{kN}{m^2}$ and a live load of 6 $\frac{kN}{m^2}$

1-Find the load transferred to beams B1, B2, and B3 of the roofs

2-Find the load transferred to beams B1, B2, and B3 of the first floor

3- Draw the frame 1-1

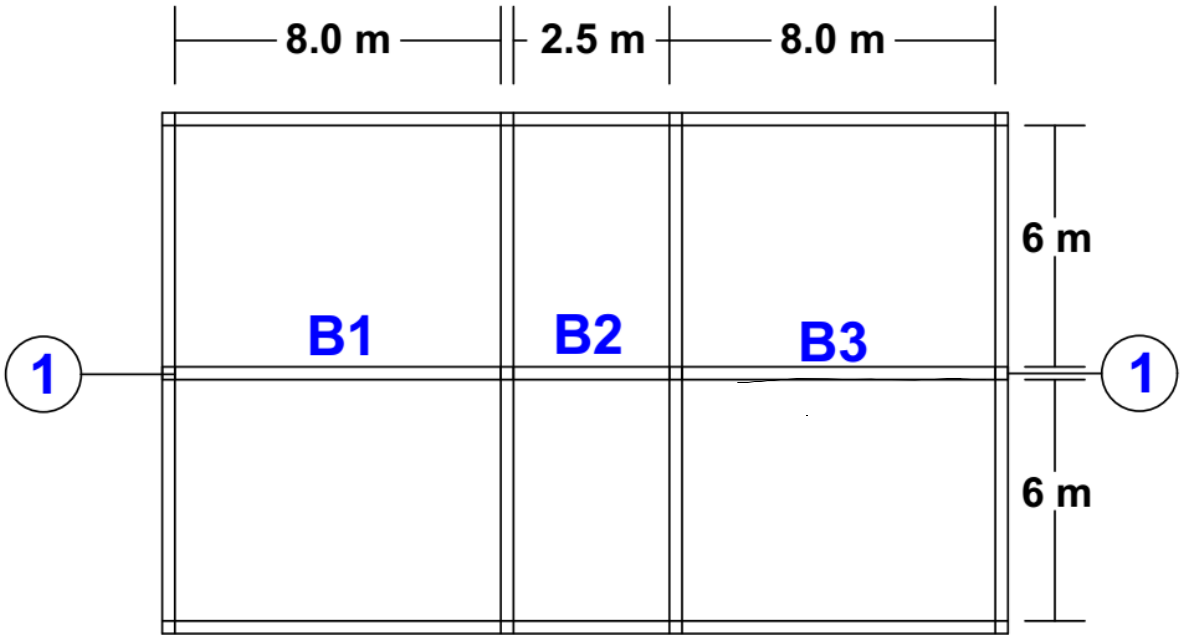
Notes

- The parapet dimension is 500 mm height and 200 mm thickness.

- A brick wall of 1.5 m height and 125 mm thickness is located at the edge beams of roof

- All beams of the first floor carry a brick wall of 3 m height and 250 mm thickness

- The densities of concrete and wall are 24 $\frac{kN}{m^3}$ and 20 $\frac{kN}{m^3}$, respectively.



1- load transferred to beams B1, B2, and B3 of the roofs

$$W_u = 1.2W_D + 1.6W_L = 1.2 \times 4 + 1.6 \times 2 = 8\ \frac{kN}{m^2}$$

- The load transferred to the beam B1

- From slabs

$$\mathrm{W_{u\,L}} = 2 \times \frac{\mathrm{Wu}\,.\,S}{3} * \frac{3 - m^2}{2} = 2 \times \frac{8 \times 6}{3} * \frac{3 - \left(\frac{6}{8}\right)^2}{2} = 39\ \frac{kN}{m}$$

- The load transferred to the beam B2=0

- The load transferred to the beam B3

- From slabs

$$\mathrm{W_{u\,L}} = 2 \times \frac{\mathrm{Wu}\,.\,S}{3} * \frac{3 - m^2}{2} = 2 \times \frac{8 \times 6}{3} * \frac{3 - \left(\frac{6}{8}\right)^2}{2} = 39\ \frac{kN}{m}$$

2- load transferred to beams B1, B2, and B3 of the first floor

$$W_u = 1.2W_D + 1.6W_L = 1.2 \times 4 + 1.6 \times 6 = 14.4\ \frac{kN}{m^2}$$

- The load transferred to beam B1

From slabs

$$\mathrm{W_{u\,L}} = 2 \times \frac{\mathrm{Wu}\,.\,S}{3} * \frac{3 - m^2}{2} = 2 \times \frac{14.4 \times 6}{3} * \frac{3 - \left(\frac{6}{8}\right)^2}{2} = 70.2\ \frac{kN}{m}$$

- From walls

$$1.2 \times 3 \times 0.25 \times 20 = 18\ \frac{kN}{m}$$

Total load transferred to beam B5=70.2 $+ 18 = 88.2\ \frac{kN}{m}$

- The load transferred to the beam B2

From walls

$$1.2 \times 3 \times 0.25 \times 20 = 18 \ \frac{kN}{m}$$

- The load transferred to the beam B3

From slabs

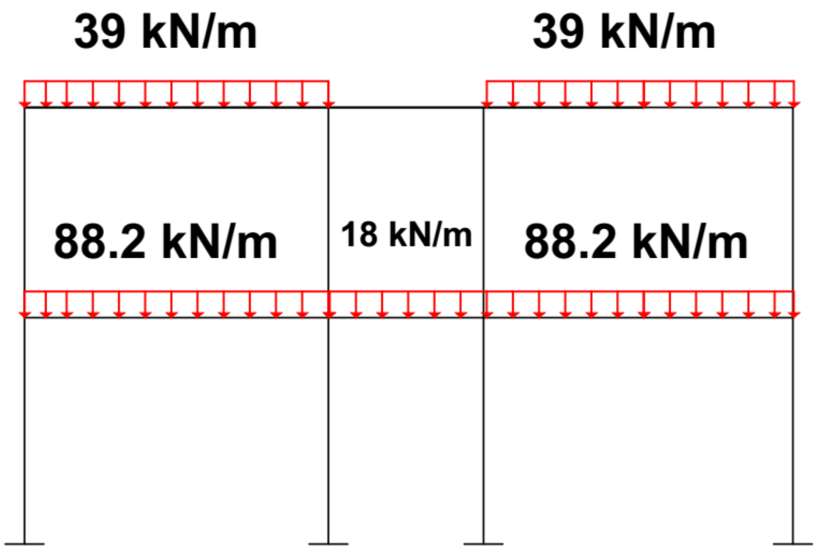
$$\mathrm{W_{u\,L}} = 2 \times \frac{\mathrm{Wu}\,.\,S}{3} * \frac{3 - m^2}{2} = 2 \times \frac{8 \ \times \ 6}{3} * \frac{3 - \left(\frac{6}{8}\right)^2}{2} = 70.2 \ \ \frac{kN}{m}$$

- From walls

$$1.2 \times 3 \times 0.25 \times 20 = 18 \ \frac{kN}{m}$$

Total load transferred to beam B5=70.2 $+ 18 = 88.2 \ \frac{kN}{m}$

3- Drawing of frame 1-1



Section 1-1