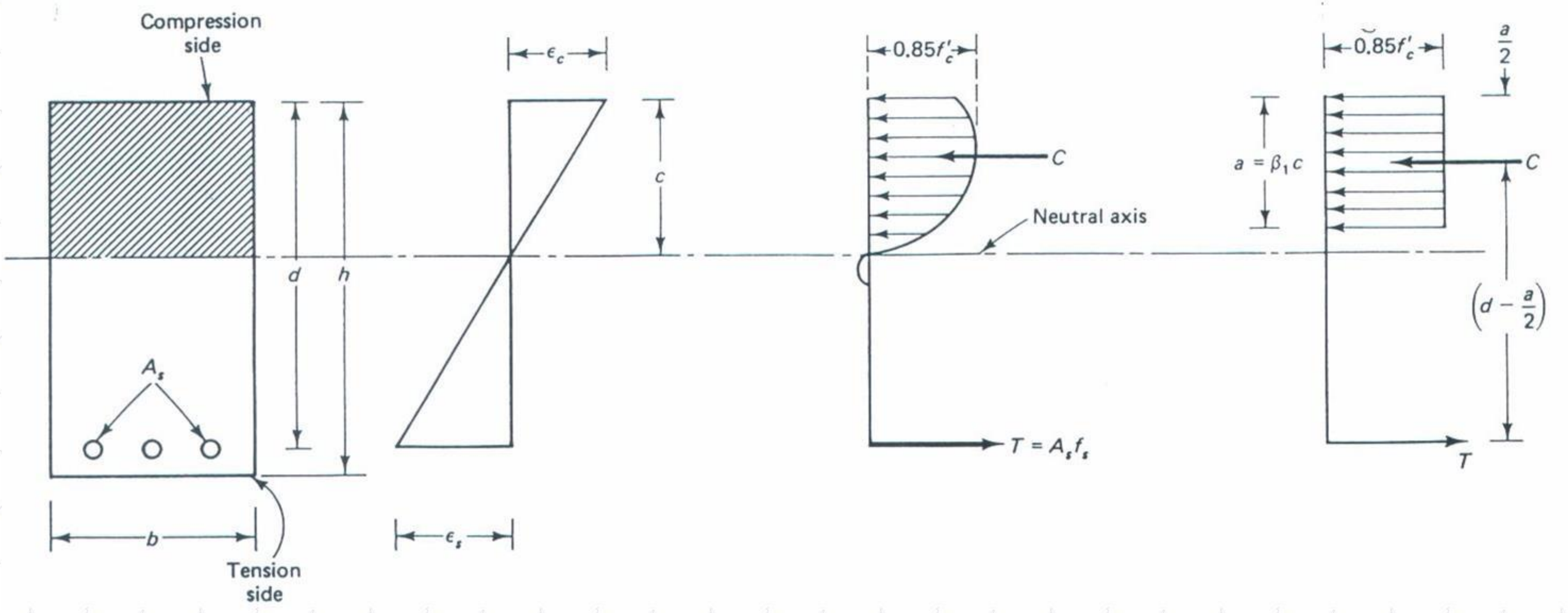
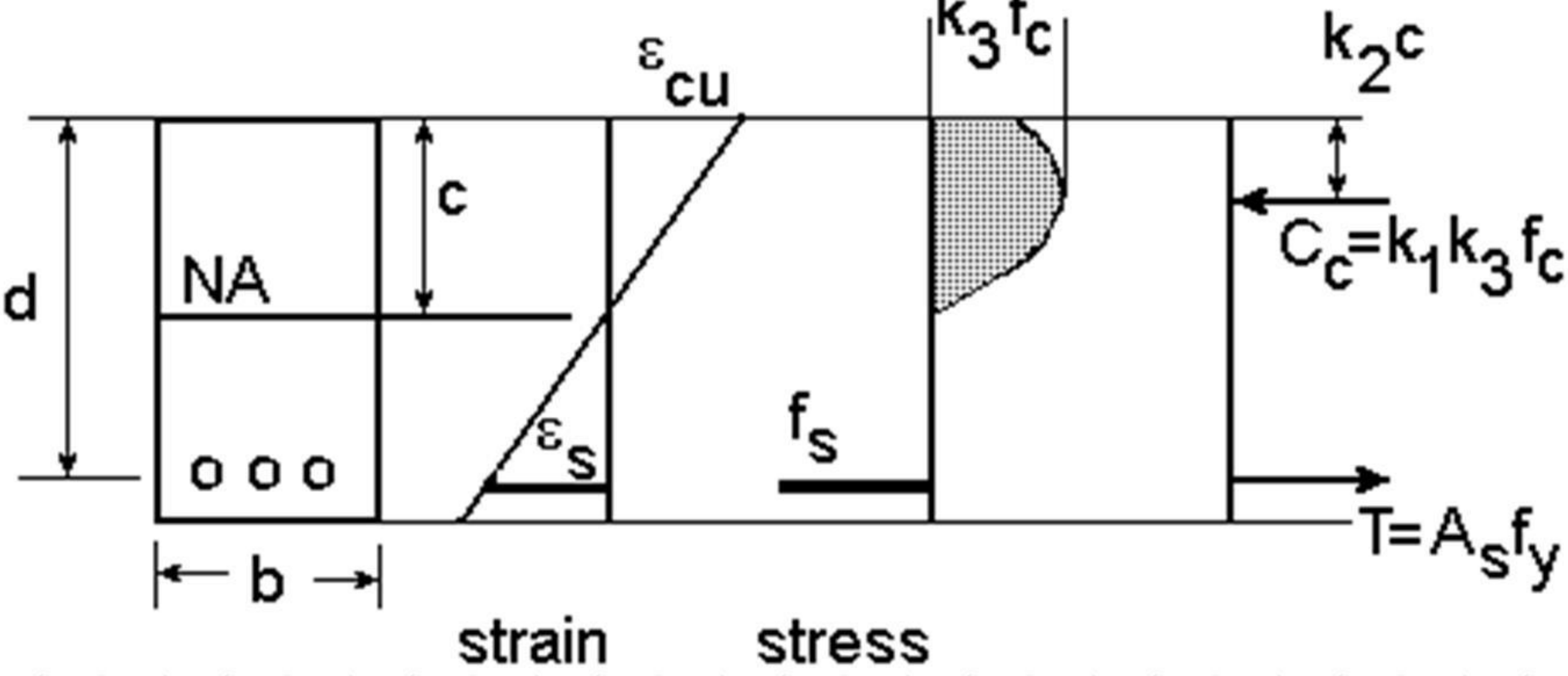
Flexural Stress

The concrete may exceed the ε_c at the outside edge of the compressive zone.



Flexural Stress

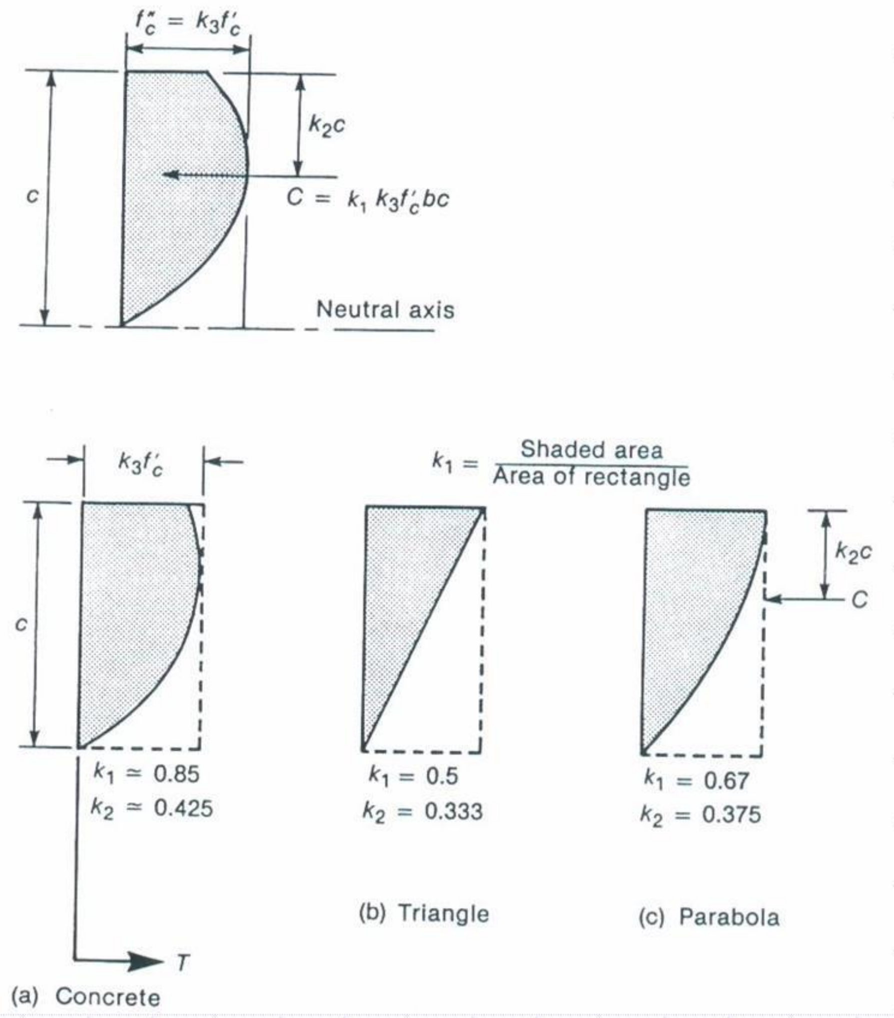
The compressive force is modeled as $C_c = k_1 k_3 f'_c\ b*c$ at the location $x = k_2*c$



Flexural Stress

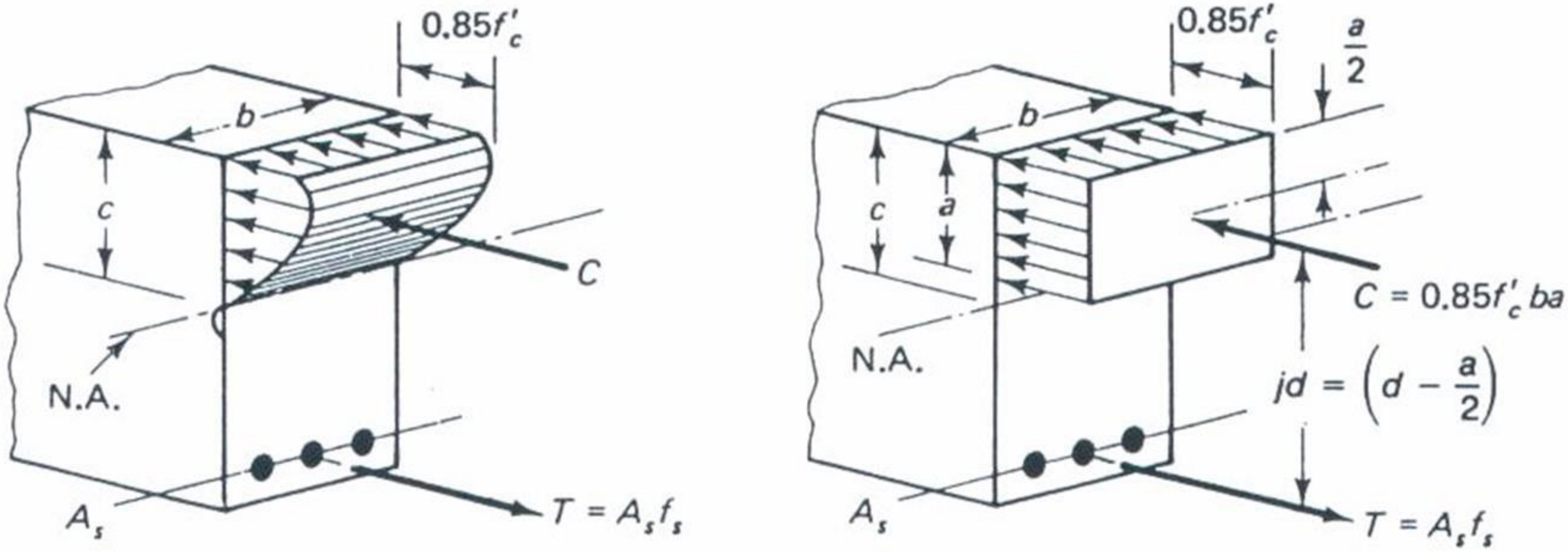
The compressive coefficients of the stress block at given for the following shapes.

k_3 is ratio of maximum stress at f_c in the compressive zone of a beam to the cylinder strength, f_c' (0.85 is a typical value for common concrete) (ACI 10.2.7)



Flexural Stress

The compressive zone is modeled with an equivalent stress block.



Flexural Stress

The equivalent rectangular concrete stress distribution has what is known as a β_1 coefficient is proportion of average stress distribution covers. (ACI 10.2.7.3)

$$\beta_1 = 0.85 \text{ for } f_c \leq 4000 \text{ psi}$$

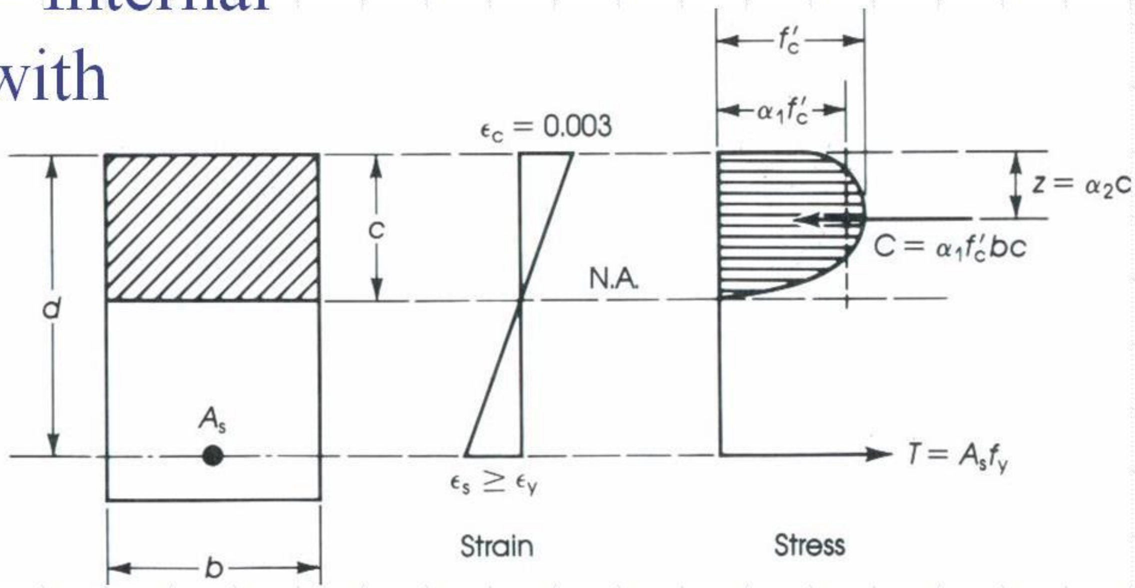
$$\beta_1 = 0.85 - 0.05 * \left[\frac{f_c - 4000}{1000} \right] \geq 0.65$$

Flexural Stress

Requirements for analysis of reinforced concrete beams

[1] Stress-Strain Compatibility – Stress at a point in member must correspond to strain at a point.

[2] Equilibrium – Internal forces balances with external forces



Flexural Stress

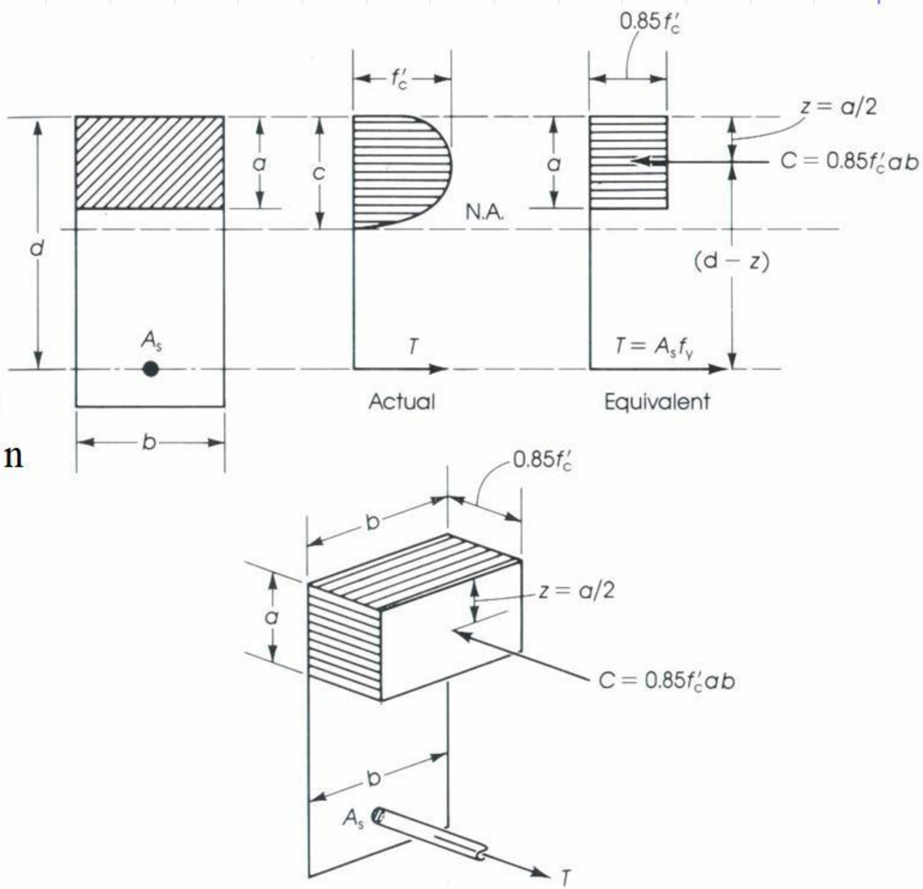
Example of rectangular reinforced concrete beam.

(1) Setup equilibrium.

$$\sum F_x = 0 \quad \Rightarrow \quad \mathrm{T} = \mathrm{C}$$

$$A_s f_s = 0.85 f_c ab$$

$$\sum M = 0 \quad \Rightarrow \quad \mathrm{T}\left(d - \frac{a}{2}\right) = \mathrm{M_n}$$



Flexural Stress

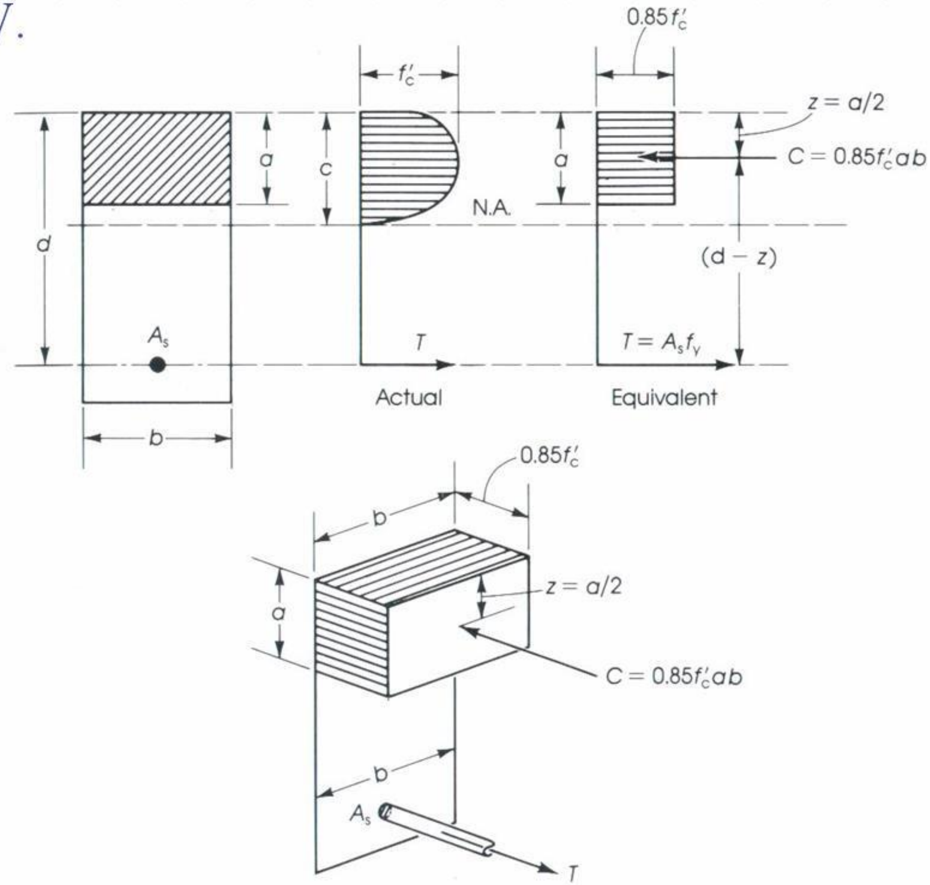
Example of rectangular reinforced concrete beam.

(2) Find flexural capacity.

$$T = A_s f_s$$

$$C = 0.85 f_c a b$$

$$a = \frac{A_s f_s}{0.85 f_c b}$$



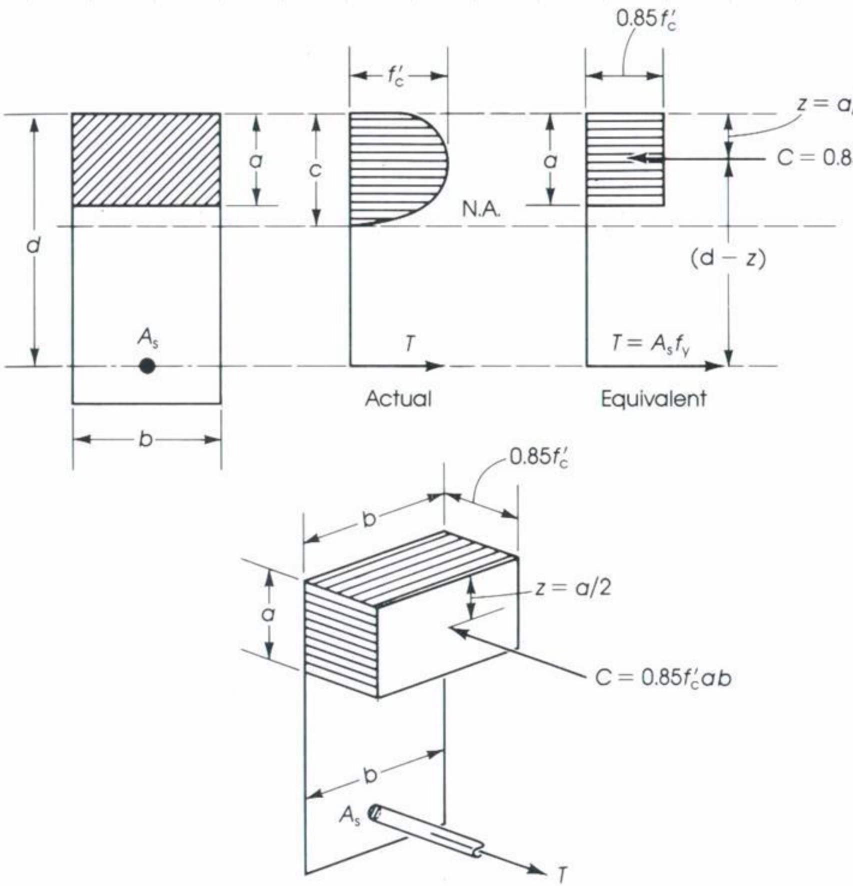
Flexural Stress

Example of rectangular reinforced concrete beam.

(2) Find flexural capacity.

$$M_n = T\,(\text{moment arm})$$

$$= A_s f_s \left(d - \frac{a}{2} \right)$$



Flexural Stress

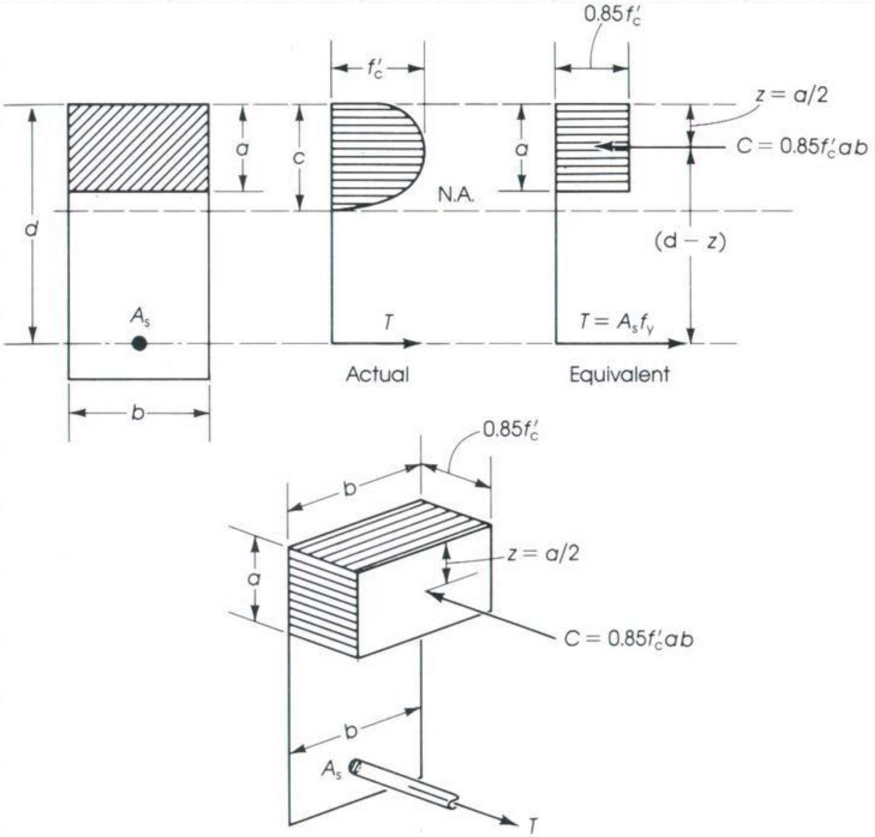
Example of rectangular reinforced concrete beam.

(3) Need to confirm $\varepsilon_s > \varepsilon_y$

$$\varepsilon_y = \frac{\sigma_y}{E_s}$$

$$c = \frac{a}{\beta_1}$$

$$\varepsilon_s = \frac{(d-c)}{c}\varepsilon_c > \varepsilon_y$$



Flexural Stress – Rectangular Example

Example of rectangular reinforced concrete beam.

Given a rectangular beam

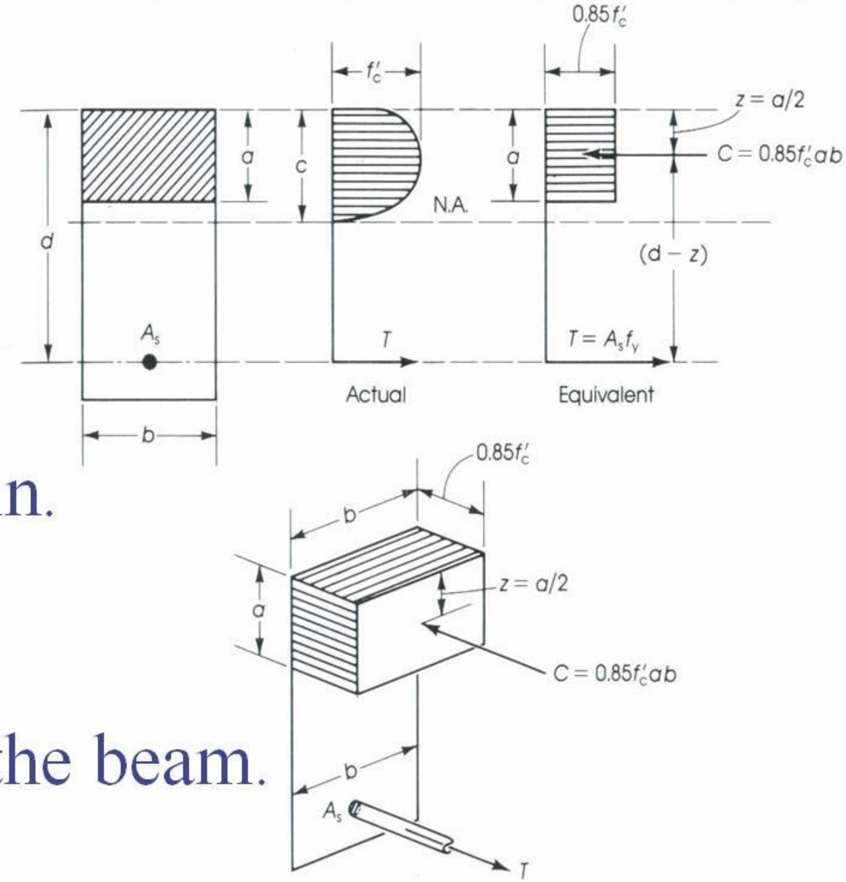
$f_c = 4000$ psi

$f_y = 60$ ksi (4 #7 bars)

b = 12 in. d = 15.5 in. h= 18 in.

Find the neutral axis.

Find the moment capacity of the beam.



Flexural Stress – Rectangular Example

Determine the area of steel, #7 bar has 0.6 in^2.

$$A_s = 4\left(0.6 \text{ in}^2\right) = 2.4 \text{ in}^2$$

The β value is $\beta_1 = 0.85$ because the concrete has a f_c =4000 psi.

$$\beta_1 \quad = \quad 0.85 \text{ for } f_c \leq \ 4000 \text{ psi}$$

Flexural Stress – Rectangular Example

From equilibrium (assume the steel has yielded)

$$C = T$$

$$0.85 f_c b a = f_y A_s$$

$$a = \frac{f_y A_s}{0.85 f_c b} = \frac{(60 \text{ ksi})(2.4 \text{ in}^2)}{0.85(4 \text{ ksi})(12 \text{ in})} = 3.53 \text{ in.}$$

The neutral axis is

$$c = \frac{a}{\beta_1} = \frac{3.53 \text{ in.}}{0.85} = 4.152 \text{ in.}$$

Flexural Stress – Rectangular Example

Check to see whether or not the steel has yielded.

$$\varepsilon_{\mathrm{y}} = \frac{f_{\mathrm{y}}}{\mathrm{E_s}} = \frac{60\ \text{ksi}}{29000\ \text{ksi}} = 0.00207$$

Check the strain in the steel

$$\varepsilon_{\mathrm{s}} = \left(\frac{d-c}{c}\right)(0.003)$$

Steel yielded!

$$= \left(\frac{15.5\ \text{in.} - 4.152\ \text{in.}}{4.152\ \text{in.}}\right)(0.003) = 0.0082 > 0.000207$$

Flexural Stress – Rectangular Example

Compute moment capacity of the beam.

$$M_n = A_s f_y \left(d - \frac{a}{2} \right)$$

$$= \left(2.4 \text{ in}^2 \right) \left(60 \text{ ksi} \right) \left(15.5 \text{ in.} - \frac{3.53 \text{ in.}}{2} \right)$$

$$= 1979 \text{ k-in.} \Rightarrow 164.8 \text{ k-ft.}$$