Bases for Comparison of Alternatives

A basis for Comparison is an index containing particular information about a series of receipts and disbursements representing an investment opportunity.

When expressed in terms of a common base, real differences become directly comparable and may be used for decision making.

Remember: when receipts are more than disbursement , the project is feasible

Common Bases for Comparison

1. Present –Worth Amount

The present worth amount is a net equivalent amount in the present that represents the difference between the equivalent disbursements and the equivalent receipts for an investment's cash flow for a selected interest rate.

So, the present worth of an investment alternative at interest rate I with a life of n years can be expressed as

$$PW(i) = Fo\frac{1}{(1+i)^0} + F1\frac{1}{(1+i)^1} + F2\frac{1}{(1+i)^2} + \ldots + Fn\frac{1}{(1+i)^n}$$

$$PW(i) = \sum_{t=0}^{n} \frac{Ft}{(1+i)t}$$

Present-Worth Amount

Features of PWA

1. Consider the time value of money according to the value of i
2. Concentrate the equivalent value of any cash in single index
3. The interest rate i has the biggest effect on decision making using PWA (Explanation to be followed)

Now, keep in your mind

1. *Disbursements are(-) sign*
2. *Receipts are (+) sign*
3. *Cash flow diagram are very helpful*
4. *If PWA is +, the investment is feasible*

Present-Worth Amount

Ex/ A cash flow of $1,000 was invested in a project. The receipts at the end of 1st, 2nd, 3rd, and 4th year were $400 each. Using the PWA, determine the feasibility of the project. The interest rate is 10%.

$$PW(10\%) = -\$1{,}000 + \frac{\$400}{(1+0.1)^1} + \frac{\$400}{(1+0.1)^2} + \frac{\$400}{(1+0.1)^3} + \frac{\$400}{(1+0.1)^4}$$

$$PW(10\%) = \$268$$

The project is feasible

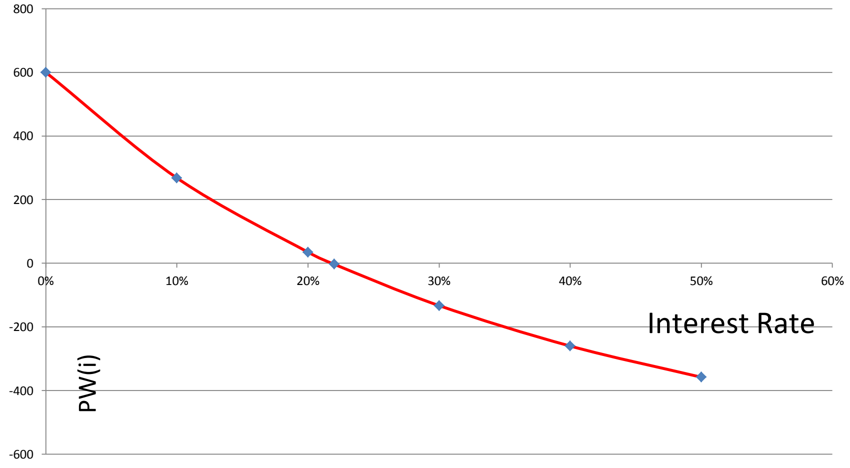
Present-Worth Amount

For the previous example, find the present worth amount as a function of interest rate and graph the results.

Solution/

i	$PW(i) = -\$1{,}000 + \$400\left\{\sum_{t=1}^{4}\left[\frac{1}{(1+i)^t}\right]\right\}$
0%	**$600**
10%	**$268**
20%	**35**
22%	**-3**
30%	**-133**
40%	**-260**
50%	**-358**

Present-Worth Amount



Common Bases for Comparison

2. Annual Equivalent Amount (AE)

- AE has characteristics similar to PW
- Cash flow is converted to a series of equal annual amounts
- First, convert all disbursements and receipts to PW and then multiply by the "Annual Payment Factor"
- So, the Annual Equivalent Amount for interest rate i and n years can be defined as:

$$AE(i) = \left[\sum_{t=0}^{n} \frac{Ft}{(1+i)^t} \right] \left[\frac{i(1+i)^n}{(1+i)^n - 1} \right]$$

Annual Equivalent Amount (AE)

Two investment opportunities are expected to produce the following receipts and disbursements

End of Year	Alt. 1	Alt. 2
0	-$1,000	-$1,200
1	$1,100	$1,100
2	$1,210	$1,210
3	$1,130	$1,330

Determine the ratio AE(Alt.1)/AE(Alt.2) if the interest rate is 10%

Annual Equivalent Amount (AE)

Alt. 1

$$AE = \left[-\$1{,}000 + \frac{\$1{,}100}{(1+0.1)^1} + \frac{\$1{,}210}{(1+0.1)^2} + \frac{\$1{,}130}{(1+0.1)^3} \right] \left[\frac{0.1(1+0.1)^3}{(1+0.1)^3 - 1} \right]$$

$$AE = \$743.5$$

Alt. 2

$$AE = \left[-\$1{,}200 + \frac{\$1{,}100}{1.1^1} + \frac{\$1.210}{1.1^2} + \frac{1330}{1.1^3} \right] \left[\frac{0.1(1.1)^3}{1.1^3 - 1} \right]$$

$$AE = \$723.48$$

AE(Alt.1)/AE(Alt.2) = 1.028

Common Bases for Comparison

3. Future Worth Amount (FW)

FW is a basis for comparison calculated at a future point in time for some interest rate.

The future-worth amount n years from the present is

$$FW = Fo(1+i)^n + F1(1+i)^{n-1} + \ldots\ldots + Fn-1(1+i)^1 + Fn(1+i)^0$$

$$FW = \sum_{t=0}^{n} Ft(1+i)^{n-t}$$

Future-Worth Amount (FW)

Another way to calculate the FW is to first determine the present worth amount of the cash flow and then convert to its future equivalent n years

$$FW(i) = PW(i)\left[(1+i)^n\right]$$

$FW(i) = PW(i)\left[(1+i)^n\right]$

Common Bases for Comparison

- ✓ The future worth, annual equivalent, and present worth are consistent base for comparison
- ✓ As long as I and n are fixed and alternatives A and B are being compared

$$\frac{PW(i)_A}{PW(i)_B} = \frac{AE(i)_A}{AE(i)_B} = \frac{FW(i)_A}{FW(i)_B}$$

Nominal and Effective Interest Rate

In Practice, cash flows or loan agreement may require that interest be paid more frequently, such as each half years, each quarter, or even each month.

- When actual "effective" rate of interest is 3% compounded each six months, the annual or "nominal" interest is quoted as "6% per year compounded semiannually"
- For an ***effective*** rate of 1.5% compounded at each three months, the ***nominal*** interest quoted as "6% per year compounded quarterly"

Nominal and Effective Interest Rate

Thus, the nominal rate of interest is expressed on an annual basis and is determined by multiplying the actual or effective interest rate per interest period times the number of interest period per year

r = nominal interest rate per year

i = effective interest per compounding period

c = number of compounding periods per year

The *nominal* interest rate is

$$r = c.i$$

and *effective* interest rate per compounding period is

$$i = r/c$$

Nominal and Effective Interest Rate

Consider a nominal interest rate of 12% compounded semiannually. The value of $1 at the end of one year when $1 is compounded at 6% for each half year

F = \$1 (1.06) (1.06) = \$1 $(1.06)^2$ = \$1.1236

The actual interest = \$1.1236 -\$1 = \$0.1236

Effective annual interest rate = \$0.1236/\$1 = 12.36%

If i_a= *effective annual interest*

$$i_a = \left(1+\frac{r}{c}\right) - 1$$

$$i_a = \left(1+\frac{0.12}{2}\right)^2 - 1 = 0.1236 = 12.36\%$$

Nominal and Effective Interest Rate

Effective annual interest rate for various compounding periods at a nominal rate of 18%

Compounding Frequency	No. of Periods per year	Effective interest rate per period	Effective Annual Interest Rate
Annually	1	18.0000%	18%
Semiannually	2	9.0000%	18.81%
Quarterly	4	4.5000%	19.2517%
Monthly	12	1.5000%	19.5618%
Weekly	52	0.3462%	19.6843%
Daily	365	0.0493%	19.7142%

Nominal and Effective Interest Rate

It is desired to find the compound amount of $1000 four years from now at nominal annual interest year of 18% compounded semiannually.

To find effective annual rate

$$i_a = (1+\frac{r}{c})^c - 1 = 0.1881$$
$$n = 4$$
$$F = \$1000(1+0.1881)^4 = \$1{,}992.60$$

Or effective interest rate for one-half year *i*=0.18/2=0.09

n=8 (six-month period)

F=$1000 $(1+0.09)^8$=$1,922.60

DEPRECIATION

Depreciation may be defined as the lessening in value of a physical assets with the passage of time.

- With the possible exception of land and collectible items, this phenomenon is a characteristic of all physical assets.

TYPES OF DEEPRECIATION

1. Physical depreciation
2. Functional Depreciation
3. Accidents

Physical Depreciation

Depreciation resulting in physical impairment of an asset is known as Physical Depreciation.

- ❑ Such tangible ways as the wearing of particles of metal from bearing and corrosion of the tubes in a heat exchanger
- ❑ This depreciation results in the lowering of the ability of a physical asset to render its intended service.

Causes

1. Deterioration due to action of the elements including corrosion of pipes, chemical decomposition, bacterial action and so on. It is independent of time
2. Wear and tear due to abrasion, shock, vibration, impact and the like

Functional Depreciation

Functional Depreciation results from a change in the demand for the service it can render.

- ❑ The demand for the service may be changed because it is more profitable to use more efficient unit, there is no longer work for the asset to do, or the work to be done exceeds the capacity of asset. Depreciation may be result of:

1. Discovery of another asset that is sufficient superior to make it uneconomical to continue
2. Inability to meet the demand placed upon it. Especially, when this demand not contemplated when the asset was acquired.

Importance of Depreciation

An asset is a unit of capital. Expenditures shall be accurately calculated in calculations of cost.

The depreciation is a cost and it is time and service dependant. It is complicated as we have more than one variables.

1. The accountant attempts to spread the loss in value over the life of the asset, so that the profit and loss statement is a more accurate reflection of the business.
2. To have continuous monetary measure of the value of an enterprise assets.

Value-Time Function

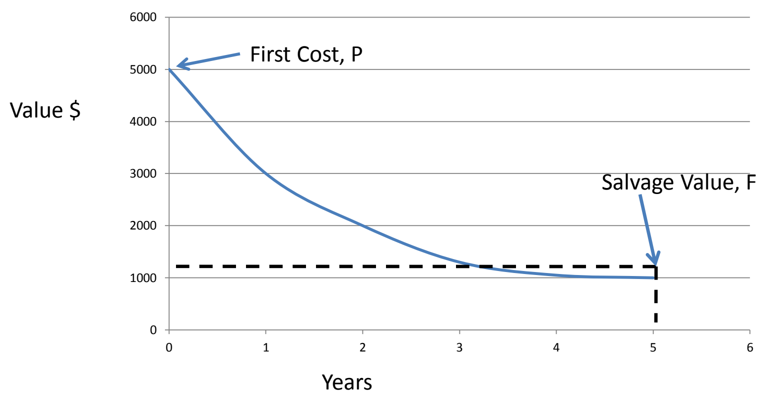
It is customary to assume that the value of an asset decreases yearly in accordance with one of the several mathematical functions.

Choice of a particular model to represent the lessening in the value of asset over time is a difficult task.

Accountants use the term of ***book value*** to represent the original value of an asset less its accumulated depreciation at any point in time.

The ***book value*** at the end of any year is equal to the book value at the beginning of the year less the depreciation expense charged during the year

Value-Time Function



Value-Time Function

P=first cost of the asset

F=Estimated salvage value

B_t= Book value at end of year t

D_t= Depreciation charge during year t

n= Estimated life of asset

$$Bt = B_{t-1} - D_t$$

Value-Time Function

The table represents the calculations of book value at the end of each year for an asset with a first cost of $12,000 , and estimated life of 5 years, and a salvage value of zero with assumed depreciation charges

End of year	Depreciation Charge During Year Dt	Book Value at End of year Bt
0	------	$12,000
1	$4,000	$8,000
2	$3,000	$5,000
3	$2,000	$3,000
4	$2,000	$1,000
5	$1,000	000000

Engineering Economy

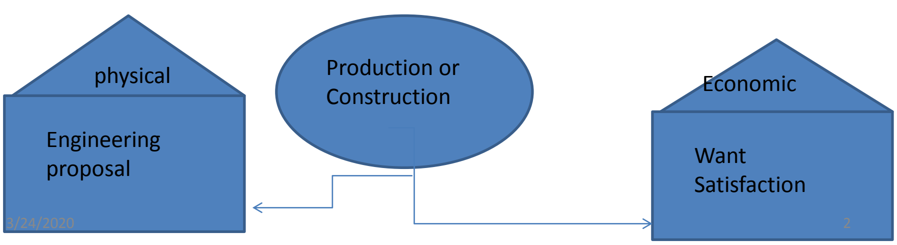
Definition: Engineering Economy involves formulating, estimating, and evaluating of economic outcomes when alternatives to accomplish a defined purpose are available.
Engineering Economy: is a collection of mathematical techniques that simplify the economic comparison
Engineering Economy: is an assistant decision tool by which the best economic solution can be selected.
YOU CAN FIND TENS OF DEFINITIONS.
My definition: The engineering economy is one of the engineer's tool to satisfy the requirements of a project.

Economic laws can be no more exact than the description of behavior of people (sometimes animals) acting singly and collectively

Physical and Economic Environments of Projects

Each project has two important interconnected environments *The Physical* and *the Economic:*

In physical part, engineers deal with physical laws (Newton, Venturi), formula and facts
In Economic part, engineers look for Satisfaction of stockholders, owners. These are people and they have various actions and desires.



Physical and Economical Efficiency

Success of projects is measured by ***Efficiency***

Efficiency (Physical) = output/input

When such physical units are involves, efficiency will always be less than the unity or less than 100%.

It is well known that physical efficiencies over 100% are not possible

Efficiency (Economic) = Worth/cost
Economic Efficiency can exceed 100% and must do so for economic undertakings to be successful

Efficiency

In the conversion of energy in a certain plant, assume that the plant efficiency is only 36%. Assume that the output in the form electrical energy have an economic worth of $14.65 per million and that input in the form of coal have an economic cost of $1.80 per million.

Find the economic efficiency and determine whether the project is successful or not?

Profit and Interest

There is a big confusion between Profit and Interest

Please Note:

Profit is the amount of money that paid to the ***lender*** as a rental of using his money

Interest: is the rental amount charged by financial institutions for the use of money . It is paid by the ***borrower***

Profit is a taxable money because it is an income
Interest is non a taxable money because it is a cost

Tom and Smith

Tom lent Smith $1 million to be returned after one year $1.1 million

Terminology

Tom is the Lender
Smith is the Borrower
$1 million is the Capital
Profit that Tom gained is $0.1 million
Tom should pay a tax for the $0.1 million because it is an income
Interest that Smith paid is $0.1 million
Smith can claim the 0.1 million as a cost (non taxable)

Concepts

So in order to have simple and right financial operation, there should be

- **Capital**
- **Determined time of return**
- **Agreement on the interest for the use of money**

Now Could you make a simple financial operation

Interest and Interest Rate

Interest (I): The term Interest is used to designate a rental amount charged by financial institutions for the use of money.

Physically, it is the increase added to the original borrowed or invested money

Interest (I) = Total accumulated – original invested money

Cont'd

Interest Rate(i) or the rate of capital growth is the rate of gain received from an investment. Usually, this rate of gain is stated on a per-year basis an it represents the percentage gain realized on the money committed to the undertaking.

Thus an 11% interest rate indicates that for every dollar of money used, an additional $0.11 must be returned as payment for the use of that money.

Interest Rate is determined by market forces involving supply and demand. It is mutual agreement between Lender and borrower and is known as *Market Rate*

Interest and Interest Rate

Ex./A furniture Company borrowed an amount of $ 10 million from Al Rafidain Bank. The borrowed money should be returned at the end of financial year as $10.6. Determine the interest and the interest rate?

Interest = Accumulated amount – original borrowed amount

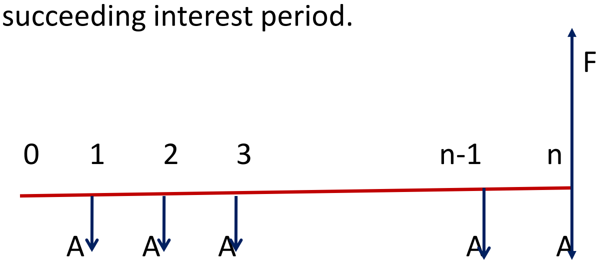
Interest Rate = Interest X 100 / original money

For discussion: if the money should be returned after 2 years, what are interest and interest rate

INTEREST FORMULAS

3. Equal- Payment-Series Compounded-Amount Factor

It is used to find the single future value that would accumulate from a series of equal payments occurring at the end of succeeding interest period.



Equal-Payment-Series Compounded-Amount Factor

What is the compounded amount of a series of five $100 payment made at the end of each year at 12% interest compounded annually?

End of year	Payment	Compounded Amount at end of 5^{th} year	Total Compounded amount
1	$100	$\$100\ (1.12)^4 = \$157$	
2	$100	$\$100\ (1.12)^3 = \$141$	
3	$100	$\$100\ (1.12)^2 = \$125$	
4	$100	$\$100\ (1.12)^1 = \$112$	
5	$100	$\$100\ (1.12)^0 = \$100$	$635

Equal-Payment-Series Compounded-Amount Factor

If A represents a series of n equal payment

$F = A(1) + A(1+i) + \ldots\ldots\ldots\ldots\ldots\ldots\ldots + A(1+i)^{n-1}$

Multiplying this equation by (1+i) results in

$F(1+i) = A(1+i) + A(1+i)^2 + \ldots\ldots\ldots\ldots + A(1+i)^n$

Subtracting the first equation from the second equation and solving for F gives

$$F= A [(1+i)^n -1]/i$$

The resulting factor $[(1+i)^n -1]/I$ is known as *Equal-Payment-Series Compound Amount factor*

Interest Formulas

4. Equal-Payment-Series Sinking-fund Factor

The equal-payment-series compound amount relationship can be solved for A as follows

$$A = F . i /[(1+i)^n -1]$$

The resulting factor $i /[(1+i)^n -1]$ is known as *the equal-payment-series sinking-fund factor.*

Ex/ It is desired to accumulate $635 by making a series of five equal annual payment at 12% interest compounded annually. What is the required amount of each payment?

Interest Formulas

Equal payment Series Capital-Recovery Factor

As
$$A = F\left[\frac{i}{(1+i)^n - 1}\right]$$
$$A = P\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right]$$

is Equal Payment Series Capital-Recovery factor
$$\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right]$$

Interest Formulas

A $1,000 invested at 15% interest compound annually will provide for eight equal year-end payment of

$$A = \$1000\left[\frac{0.15(1+0.15)^8}{(1+0.15)^8 - 1}\right]$$
$$A = \$1000(0.2229) = \$233$$

0.2229 is called Equal Payment Series Capital Recovery Facto

Interest Formulas

Equal Payment Series Present Worth

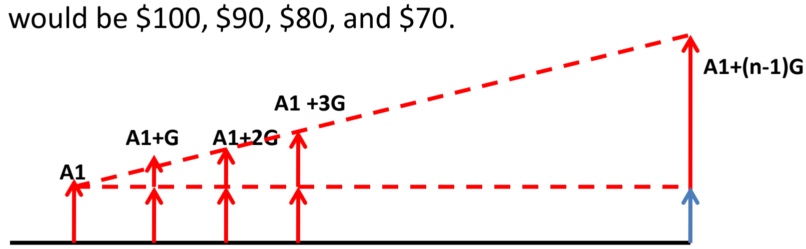
To find what single payment must be deposited now so that equal end of period can be made

$$P = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

Interest Formulas

5. Uniform Gradient-Series Factor

Payment may increase or decrease by a constant amount, for Example, a series of payment that would uniformly increasing $100, $125, $150, and $175 occurring at the first, second, third, and fourth year. Similarly, a uniformly decreasing series would be $100, $90, $80, and $70.



Uniform Gradient Series Factor

A1 = payment at the end of the first year

G = Annual change or Gradient

n = the number of years

A = Equivalent equal annual payment

One way of evaluating such a series is to apply interest formulas developed previously to each payment (time consuming method). Another approach is to reduce the uniformly increasing series of payment to an equivalent equal-payment series

Uniform Gradient Series Factor

For increasing Gradient

$$A = A1 + G\left[\frac{1}{i} - \frac{n}{(1+i)^n - 1}\right]$$

For Decreasing Gradient

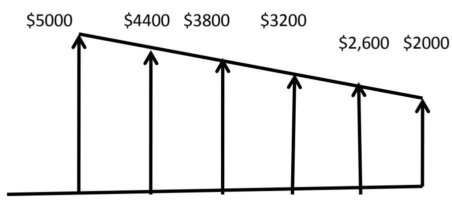
$$A = A1 - G\left[\frac{1}{i} - \frac{n}{(1+i)^n - 1}\right]$$

Ex/ Assume that an individual is planning to save $1000 from income during this year and can increase this amount by $200 for each of the following nine years. The rate of interest is 8% compounded annually. What equal annual series beginning at the end of year 1 and ending at year 10 would produce the same accumulation amount?

Uniform Gradient Series Factor

A = A1 + G [(1/i) – {n/(1+i)^n – 1}]
A = \$1000 + \$200 (3.8713) = \$1,774 per year

Find the equal-annual series equivalent to the decreasing gradient series as shown



A = A1 – G (2.2498) = \$5000 - \$600 (2.2498) = \$3,650

Interest Formulas

6. Geometric-Gradient Series Factor

Annual payments increase or decrease over time, not by a constant amount, but by a constant percentage.

If g is used to designate the percent ge change in the magnitude of the payment (can be positive or negative)

g\`= [(1+i)/(1+g)] – 1 Geometric Gradient Series factor

$$P = \frac{F1}{1+g}\left[\frac{(1+g`)^n - 1}{g`(1+g`)^n}\right]$$

where F1 is the first payment

Geometric Gradient Series Factor

Ex/ Suppose that receipts from a certain venture are estimated to increase by 7% per year from a first year base of $360,000. Determine the present worth of 10 years of such receipts at an interest rate of 15%

$$g` = \frac{1+0.15}{1+0.07} - 1 = 7.48\%$$

$$p = \$360 \frac{6.8704}{1.07} = \$2,311,536$$

Geometric Gradient Series

Ex/ Suppose that a shallow oil well is expected to produce 12,000 barrel of oil during its first year at $21 per barrel. If its yield is expected to decrease by 10% per year. What is the present worth of the anticipated gross revenue at an interest of 17% over the next seven years?

$$g` = \frac{1+0.17}{1-0.10} - 1 = 0.30$$

$$P = 21(12,000) \frac{2.9247}{1-0.10} = \$818,916$$

Interest Formula Relationship

FIND	GIVEN	FORMULA
F	P	
P	F	
F	A	**F = A [(1+i)^n -1]/i**
A	F	**A = F . i /[(1+i)^n -1]**
A	G	**A = A1 + G [(1/i) – {n/(1+i)^n – 1}]**
P	F1	$P = \frac{F1}{1+g}\left[\frac{(1+g`)^n - 1}{g`(1+g`)^n}\right]$

Bases for Comparison of Alternatives

INTERNAL RATE OF RETURN (IRR)

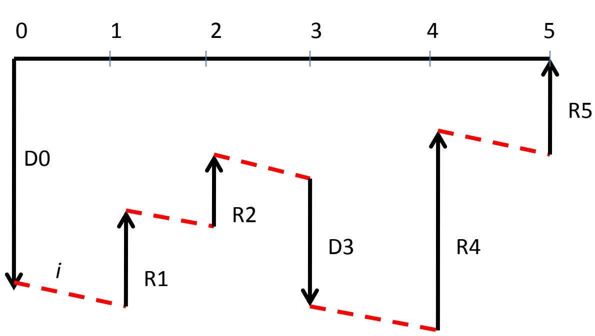
The Internal Rate of Return is a widely accepted index of profitability.

It is defined as the interest rate that causes the equivalent receipts of a cash flow to equal disbursements of that cash flow.

It is also defined as the interest rate that reduces the present worth of a series of receipts and disbursements to zero.

IRR

Suppose that investment (Disbursement) of D0 at year 0, Receipt of R1 at year 1, Receipt of R2 at year 2, Disbursement D3 at year 3, Receipt R4 at year 4, and Receipts of R5 at year 5.



IRR

In the IRR method, the unknown is i. In other words, we look for the value of i that makes the summation Present Worth of receipts and disbursement equal to zero.

Step 1. Convert all disbursements and receipts to Present Worth at time 0. (Remember Disbursements – and Receipts +)

Step 2. Make the summation equal to zero

Step 3. Find out i that makes the equation equal to zero

Step 4. Compare i to Minimum Attractive Rate of Return (MARR)

If (i) is greater or equal to MARR, the project is acceptable.

IRR

Accordingly

$$-D0 + R1\left(\frac{1}{(1+i)^1}\right) + R2\left(\frac{1}{(1+i)^2}\right) - D3\left(\frac{1}{(1+i)^3}\right) + R4\left(\frac{1}{(1+i)^4}\right) + R5\left(\frac{1}{(1+i)^5}\right) = 0$$

i can be found by trial and error

Compare i with MARR and decide if the project is acceptable or not?

IRR

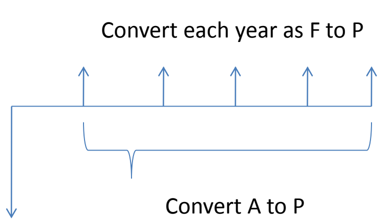
Ex/ An amount of $10,000 can be invested in a project with zero salvage value of five years useful life. The annual receipt is $4,838 and the annual cost of Operation and Maintenance (O&P) is $2,000 and the tax is $200 per year. If the MARR is 10%, determine if the project is desirable or not using the the method of Internal Rate of Return?

Sol/

The annual net = $4,838 - $2,000 -$200 =$2,638

Draw of Cash flow Diagram will be helpful

IRR



$$-10,000+\frac{2638}{(1+i)}+\frac{2638}{(1+i)^2}+\frac{2638}{(1+i)^3}+\frac{2638}{(1+i)^4}+\frac{2638}{(1+i)^5}=0$$

$$-10000+2638\left[\frac{(1+i)^5-1}{i(1+i)^5}\right]=0$$

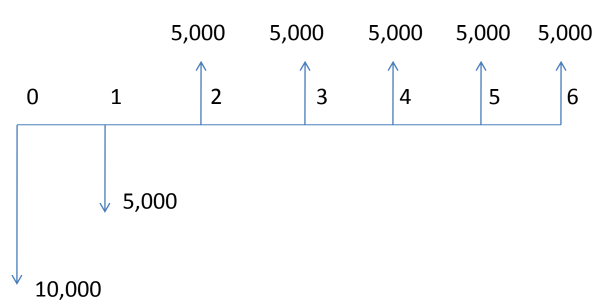
IRR

In both cases i = 10%

The project is desirable as i=MARR

IRR

- Determine if the project (whose net cash flow diagram below) is acceptable or not using IRR method. The MARR is 20%



SIMPLE INTEREST

Under simple interest, the interest to be paid upon repayment of a loan is proportional to the length of time the principal sum has been borrowed.

Let I represents the interest earned, P the principal, n the interest period, and i the interest rate:

$$I = Pni$$

SIMPLE INTEREST

Suppose that $1,000 is borrowed at a simple interest rate of 18% per annum. At the end of the year, the interest would be

$$I = \$1,000 \times 1 \times 0.18 = \$180$$

Where
$1,000 is P,
1 is interest period
0.18 is the interest rate

The principal plus interest would be **$1,180** and would be due at the end of the year

Simple Interest

✓Simple interest may be made for and period of time (period interest)

✓For a fraction of a year, it is common to consider the year as composed of 12 months of 30 days each, or 360 days.

Ex/ on a loan of $100 at a simple interest rate of 18% per annum, for the period Feb. 1 to April 20, the interest would be

$$I = \$100 \times (80/360) \times 0.18 = \$4$$

Simple Interest

Ex/ A contractor borrowed $1,000,000 from the central bank at a simple interest rate of 12% for 3 year. Calculate the amount owed at the end of each year?

End of year	Amount Borrowed	Interest *I*	Amount Owed at end of year
0	$1,000,000	------------	$1,000,000
1	------------	$120,000	$1,120,000
2	--------------	$120,000	$1,240,000
3	--------------	$120,000	$1,360,000

At the end of the 3 year, the money should paid an interest of
$I = \$1,000,000 \times 3 \times 0.12 = \$360,000$
Total amount to be paid by Contractor = $1,000,000 + $360,000 = $1,360,000

Simple Interest

Ex/ What is the principal (P) that had been borrowed 5 years ago at 7% simple interest rate and should be retained now as $5000. Find the total interest that was added to the principal during the five years?

Total Amount = Principal + Interest
S = P + I
$5000 = P + Pni
$5000 = P + (Px5x0.07)
$5000 = 1.35 P
P = $3703.7
Interest during 5 years = $3703.7 x 5 x 0.07 = $1296.3
OR $5000-$3703.7 = $1296.3

Cont'd

For the previous example, if additional $2,000 was borrowed at the end of the 3^{rd} year, calculate the amount owed at the end of 4^{th} and 5^{th} years?

End of year	Amount Borrowed	Interest	Amount Owed at the end of year
0	$3703.704	-----------	$3703.704
1	$3703.704	$259.259	$3962.963
2	$3703.704	$259.259	$4222.222
3	$3703.704 + $2,000	$259.259	$6481.481
4	$5703.704	$399.259	$6880.740
5	$5703.704	$399.259	$7280
	Total Interest	$1,576.295	

Pni + =

$6481.481 = $4,222.222 +$2,000 + $259.259

Compound Interest

The interest is charged on the total amount owed (principal plus interest). The interest owed in the previous year becomes part of the total amount for this year

At the end of the first year
Principal (P), interest (Pni) or (Pi) as n=1
Total amount owed = P + Pi = $P(1+i)^1$

At the end of Second Year
Principal, P(1+i)   Interest, P(1+i)i
Total Amount Owed = P(1+i) + P(1+i)i = $P(1+i)^2$

Cont'd

End of year	Amount Borrowed	Interest	Compound Amount Owed
0	P		P
1	P	Pi	P+Pi = $P(1+i)^1$
2	$P(1+i)^1$	P(1+i) i	$P(1+i)^1 + P(1+i)I = P(1+i)^2$
3	$P(1+i)^2$	$P(1+i)^2i$	$P(1+i)^2+P(1+i)^2i = P(1+i)^3$
4	$P(1+i)^3$	$P(1+i)^3i$	$P(1+i)^3+ P(1+i)^3i = P(1+i)^4$
n	$P(1+i)^{n-1}$	$P(1+i)^{n-1}i$	$P(1+i)^n$

So, total amount owed at the end of n interest period is $P(1 + \text{interest rate})^{\text{interest period}}$

The interest at n interest period is $P(1+\text{interest rate})^{\text{interest period } -1}i$

Compound Interest

A private business company borrowed $500,000 from a bank at a compound interest rate of 10% per annum. What is the total owed money that should be returned at the end of 7 years and what is the interest at the end of 4th year?

Solution:

$P = \$500,000 \quad n=7 \quad i=0.1$

Total Amount Owed (F) = $P(1+i)^n$

$= \$500,000\,(1+0.1)^7 = \$974,358.55$

Interest at the 4th year = $P(1+i)^{n-1}i = \$500,000\,(1.1)^3 \times 0.1$

$= \$66,550$

Cont'd

Provide Table of Calculations

End of year	Amount Borrowed	Interest	Compound Amount owed
0	$500,000		$500,000
1	$500,000	$50,000	$550,000
2	$550,000	$55,000	$605,000
3	$605,000	$60,500	$665,500
4	$665,500	$66,550	$732,050
5	$732,050	$73,205	$805,255
6	$805,255	80,525.5	$885,780.5
7	$885,780.5	$88,578.05	$974,358.55

UNITS-OF-PRODUCTION DEPRECIATION

- Value-time model (depreciation methods) may not be advisable.
- An Alternative is to assume that depreciation occurs on the basis of work performed regardless the duration of the asset's life.
- *A trencher might be depreciated on the basis of pipeline trench completed.*

UNITS-OF-PRODUCTION DEPRECIATION

Ex/ A trencher has a first cost of $11,000 and a salvage value of $600. If it is estimated that the trencher would dig 1,500,000 linear feet of pipeline trench over its life.

$$Depreciation / foot = \frac{\$11{,}000 - \$600}{1{,}500{,}000} = \$0.006933 / foot$$

Calculate the undepreciated capital at the end of the first year if 300,000 feet of pipeline trench were dug during the first year

Undepreciated Capital = $11,000 – (300,000x$0.006933)

= $8,920.10

DEPLETION

- ✓ Depletion differs in theory from Depreciation
- ✓ Depreciation results from use and passage of time
- ✓ Depletion results from intentional, Piecemeal Removal of certain types of assets.
- ✓ Depletion refers to an activity that tends to exhaust a supply
- ✓ Depletion literally means Emptying
- ✓ Depletion indicates a lessening in the value with passage of time
- ✓ Removal of coal from mine, timber from forest, stone from quarry, and oil from reservoir.

DEPLETION

- ➢ In depletion, there is no recovery of capital, while in depreciation the assets involved may be replaced with a like asset.
- ➢ The return of depletion can cover a) the profit earned on the venture and b) the owners' capital which was invested.

Ex/A reservoir containing an estimated 1,000,000 barrels of oil required an initial investment of $7,000,000 to develop. If 50,000 bbls of oil are produced from reservoir during the first year. Calculate the depletion charge at the end of the first year?

DEPLETION

The depletion rate = $7,000,000/1,000,000
= $7/bbl

Depletion Charge at the end of the first year
50,000 bbl x $7/bbl = $350,000

CAPITAL RECOVERY

Capital can be recovered as equal payments during the useful life of the asset. In this case the borrower shall include the interest rate in the calculation.

Do you remember? If an asset has a *first value of $5000* an estimated *salvage value of $1000*, and an *estimated life of 5 years . If the interest rate is 6%.*

Present worth of $5,000 = $5,000
Present worth of Salvage Value $1000 = $747 (P from F)
Total Present worth = (5,000 – 747) = $4,253

CAPITAL RECOVERY

By straight line method

End of year t	Depreciation Charge During year t	Book Value at end of year t	Sum of Depreciation and interest of undepreciated balance	Single payment Present worth
1	800	4200	800 +(5000x0.06) =1,100	$P = F/(1+i)^n$ = 1,038
2	800	3400	1,052	936
3	800	2600	1,004	843
4	800	1800	956	757
5	800	1000	908	679
Total present Value				**4,253**

CAPITAL RECOVERY

By Sum Years' Digits

End of year t	Depreciation Charge During year t	Book Value at end of year t	Sum of Depreciation and interest of undepreciated balance	Single payment Present worth
1	1,333	3667	1,333 +(5000x0.06) =1,633	$P = F/(1+i)^n$ = 1,541
2	1067	2600	1,287	1,145
3	800	1800	956	803
4	533	1267	641	508
5	267	1000	343	256
Total present Value				**4,253**

CAPITAL RECOVERY

We can find A from P

$$A = P\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right] = \$4{,}253\left[\frac{0.06(1.06)^5}{(1.06)^5 - 1}\right] = \$1{,}010$$

A can be directly determined from Capital Recovery (CR) formula

$$CR = (P - F)\left[\frac{i\ \ (1+i)^n}{(1+i)^n - 1}\right] + Fi$$

Where CR is “equal annual payment”=A

CAPITAL RECOVERY

$$CR = (5000 - 1000)\left[\frac{0.06(1.06)^5}{1.06^5 - 1}\right] + (1000x0.06) = \$1{,}010$$

Methods of Depreciation Calculations

1. **Straight Line Method of Depreciation**

Assume that the value of an asset decreases at a constant rate.

If an asset has a *first value of $5000* an estimated *salvage value of $1000*, and an *estimated life of 5 years*

The total depreciation is $4000

The depreciation per year is $4000/5 = $800

The depreciation rate is 1/5 = 20%

The annual depreciation and book value for each year is given in the following table

Straight Line Method of Depreciation

End of year t	Depreciation Charge During year t	Book Value at end of year t
0		\$5,000
1	\$800	\$4,200
2	\$800	\$3,400
3	\$800	\$2,600
4	\$800	\$1,800
5	\$800	\$1,000

Depreciation in any year Dt = (P-F)/n

The Book value at any year $B_t = P - t\left(\frac{P-F}{n}\right)$

Depreciation Rate per year = 1/n

Straight Line Method of Depreciation

End of year t	Depreciation Charge During year t	Book Value at end of year t
0	--------	p
1	(P-F)/n	$P-(\frac{P-F}{n})$
2	(P-F)/n	$P-2(\frac{P-F}{n})$
3	(P-F)/n	$P-3(\frac{P-F}{n})$
t	(P-F)/n	$P-t(\frac{P-F}{n})$
n	(P-F)/n	$P-n(\frac{P-F}{n})$

Straight Line Method of Depreciation

Ex/ An investor bought a factory for an amount of \$275,000. The cost of installation is \$75,000. If the estimated useful life is 30 years and the estimated salvage value is 10% of the purchase price, determine a)first cost (P), b) Salvage value (F), C) value of annual depreciation, and d)the book value at the end of 20 years?

a) First cost = \$275,000 + \$75,000 = \$350,000

b) Salvage value = 10% x \$275,000 = \$27,500

c) Dt= (P-F)/n = (\$350,000-\$27,500)/30 = \$10,750

d) $B_{20} = P - t(\frac{P-F}{n})$ = \$350,000 -20(\$350,000-\$27,500)/30

= \$135,000

Methods of Depreciation Calculations

2. Declining Balance Method of Depreciation

Assumes that an asset decreases in value faster early rather than in the latter portion of its service life. A fixed percentage is multiplied by the book value of the last year to get the depreciation of this year. Accordingly, the book value of the asset decreases through time, so does the size of the depreciation charge.

Declining Balance Method of Depreciation

For an asset of $5,000 first cost and an estimated life of 5 years and a depreciation rate of 30% per year

End of Year t	Depreciation Charge During Year t	Book value at end of Year t
0	------	$,5000
1	$5,000x30%=$1,500	$3,500
2	$3,500x30%=$1,050	$2,450
3	$2,450x30%=$735	$1,715
4	$1,715x30%=$515	$1,200
5	$1,200x30%=$360	$840

Declining Balance Method of Depreciation

If α=depreciation rate

Depreciation charge in any year t, $D_t = \alpha . B_{t-1}$

Book value at year t, $B_t = B_{t-1} - D_t$

$B_t = B_{t-1} - \alpha . B_{t-1} = (1 - \alpha) B_{t-1}$

In this method, the book value would never reach Zero, regardless the time span over which the asset was depreciated.

Declining Balance Method of Depreciation

End of Year t	Depreciation Charge During Year t	Book Value at End of year t
0	------	P
1	α x Bo = α(P)	(1-α)Bo = (1-α)P
2	α x B1 = α(1-α)P	$(1-\alpha)^2P$
3	$\alpha\ (1-\alpha)^2P$	$(1-\alpha)^3P$
t	$\alpha\ (1-\alpha)^{t-1}P$	$(1-\alpha)^tP$
n	$\alpha\ (1-\alpha)^{n-1}P$	$(1-\alpha)^nP$

$Dt = \alpha(1-\alpha)^{t-1}P$

$Bt = (1-\alpha)^tP$

Methods of Depreciation Calculations

3. Sum of the Years' Digits Method of Depreciation

Assume that the value of the asset decreases at a decreasing rate.

If an asset has an estimated life of 5 years, the sum of the years will be 1 + 2 + 3 + 4 + 5 = 15 years. If the first cost is $5,000 and the estimated salvage value is $1,000

- Depreciation during the first year = ($5,000-$1,000)x5/15
- Depreciation during the second year = ($5,000-$1,000)x4/15
- Depreciation during the third year = ($5,000-$1,000)x3/15

Sum of the Years' Digits Method of Depreciation

End of Year t	Depreciation Charge During Year t	Book Value at the End of year t
0	------	$5,000
1	$4,000 x5/15=$1,333	$3,667
2	$4000x4/15=$1,067	$2,600
3	$4000x3/15=$800	$1,800
4	$4,000x2/15=$533	$1,267
5	$4,000x1/15=$267	$1,000

Sum of years = 1+2+3+....+(n-1)+n = n(n+1)/2

Sum of the Years' Digits Method of Depreciation

Depreciation Charge at any year

$$Dt = \frac{n - t + 1}{n(n+1)/2}(P - F)$$

Book Value at the end of any year

$$Bt = P - \frac{(P - F)}{n(n+1)/2} \sum_{j=n-t+1}^{n} J$$

$$\sum_{j=n-t+1}^{n} J = \sum_{j=1}^{n} J - \sum_{j=1}^{n-t} J$$

Sum of the Years' Digits Method of Depreciation

An asset of $120 first price, 10 years service life and salvage value of $20. Using Sum years' digits method, calculate annual depreciation charge and the book value at the end of the sixth year?

$$Dt = \frac{n - t + 1}{n(n+1)/2}(P - F)$$

=($120-$20)x(10-6+1)x2/(10+1)x10

Dt = $9.09

$$Bt = P - \frac{(P - F)}{n(n+1)/2} \sum_{j=n-t+1}^{n} J$$

Sum of the Years' Digits Method of Depreciation

$$B_6 = \$120 - \frac{(\$120 - \$20)}{10(10+1)/2} \sum_{J=10-6+1}^{10} J$$

$$B_6 = \$120 - \frac{\$100}{55}(5+6+7+8+9+10)$$

$$B_6 = 38.18$$

EARNING POWER OF MONEY

- ✓ Funds borrowed are commonly exchanged for goods, services, or instruments of production
- ✓ If the investment is profitable, it means that the money earns more money
- ✓ This called *Earning Power of Money*

Show me what is the earning power of money

Mr. Digg and Earning Power of Money

Mr. Digg manually digs ditches for underground cable. He is paid \$0.40 per linear foot and average of 200 linear feet per day. Weather conditions limit this kind of work to 180 days per year. Mr. Digg buys a power ditcher for \$8,000 after borrowing the amount from bank at 14% interest. The machine will dig an average of 800 linear feet per day. By reducing the price to \$0.30 per linear foot he can get sufficient work to keep the machine busy when the weather will permit.

Estimated operating and maintenance cost for the machine is \$40 per working day. At the end of the year, the machine is worthless because it is worn out. Discuss the Earning Power of Money????????????????

Mr. Digg

Oh, God. What did
I do??

Annual Income (manual Digging) = \$0.40 x 200 x 180

= \$14,400 per year

With Machine:

Receipts = \$0.30 x 800 feet/day x 180 day/year = \$43,200

Disbursements

Repayment of Loan = \$8,000

Interest = \$8,000 x 0.14 = \$1,120

Operating & Maintenance = \$40 x 180 = \$7,200

Total Disbursements = \$8000 + \$1,120 + \$7,200 = \$16,320

Annual Income = \$43,200 - \$16,320 = \$26,880

What did Mr. Digg Do??

- ✓ Increase the annual income = \$26,880 - \$14,400 = \$12,480 is enjoyed by Mr. Digg
- ✓ Mr. Digg bought a machine (Market)
- ✓ Less price of Digging (\$0.30 instead of \$0.40) social effect
- ✓ Mr. Digg will pay more tax
- ✓ Mr. Digg employed operating and maintenance personnel
- ✓ Mr. Digg paid an interest to Bank
- ✓ More…………………………

This a typical example of Capitalism

TIME VALUE OF MONEY

A dollar in hand now is worth more than a dollar received n years from now

Why

1. Since money has an *earning power*, so that after n years the original dollars plus its interest will be larger amount than the $1 received at that time
2. Fact; money has a time value because the *Purchasing Power* of a dollar changes through time. During periods of *inflation* the amount of goods that can be bought for a particular amount of money decreases as the time of purchase occurs further out in the time

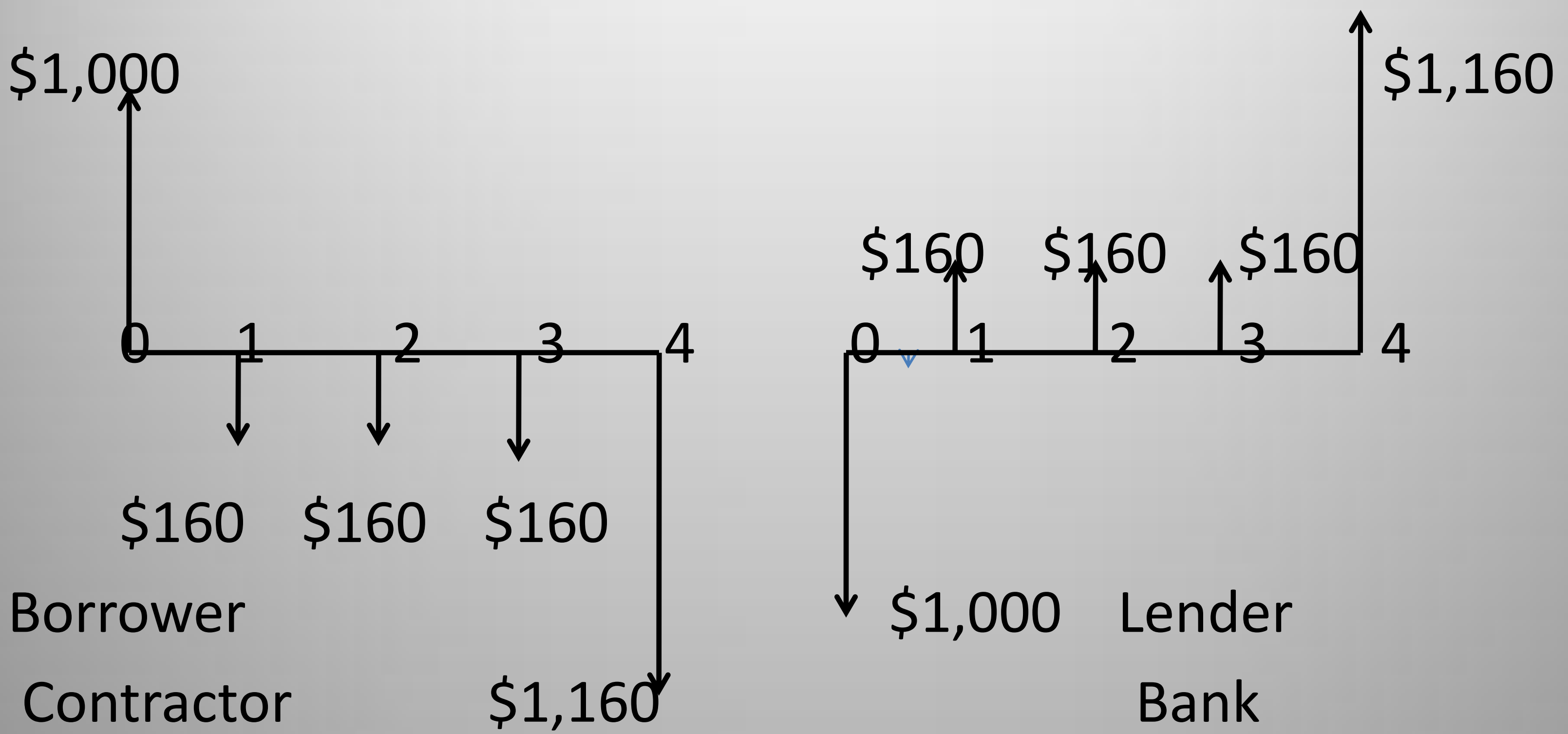
CASH FLOWS OVER TIME

Cash Flow Diagram provides all the information necessary for analyzing an investment proposal.

A cash flow diagram represents receipts received during a period of time by an upward arrow. The arrow's height may be proportional to the magnitude of receipts during the period. Similarly, disbursements during a periods are represented by a downward arrow.

Ex/ A contractor borrowed $1000 from a bank at an interest rate of 16% for 4 years. The contractor has to pay the interest at the end of each year. Draw cash flow diagram from the point of view of Contractor and the bank

Cash Flow Diagram



If receipts and disbursements are occurring simultaneously, a net cash flow can be calculated

Interest Formulas

1. ***Single-Payment Compound-Amount Factor***

If a present principal amount P is invested for n years (interest period) at compound interest rate I, The future amount F is

$$F = P\ (1+i)^n$$

The factor $(1+i)^n$ is known as *Single-payment Compound-Amount Factor*

Remember: No payment during the interest period

(Review Compound Interest)

Interest Formulas

2. Single-Payment Present-Worth Factor

The present principal amount P of a future amount F for n years (period interest) at compound interest of i

$P = F/(1+i)^n$

The factor $1/(1+i)^n$ is known as *Single-Payment Present –Worth Factor*

This is reverse of Single payment Compound Amount Factor

Cont'd

Ex/ How much must be invested now at 16% compounded annually so that \$1,811 can be received 4 years hence? Determine the single payment present worth factor?

$P = F/(1+i)^n = \$1,811/(1+0.16)^4 = \$1,000$

So, \$1,000 must be invested now

Single-Payment Present-Worth Factor $= 1/(1+i)^n = 1/(1.16)^4$

$= 0.5523$

Cont’d

- Remember

Single-Payment Compound-Amount Factor > 1

Single-Payment Present-Worth Factor < 1