Foundation Engineering (2), 4th stage, Civil Engineering Dept., College of Engineering, Al-Muthanna University, 2020-2021

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Deep Foundation

- Capacity of Pile (deep) Foundation Adopting Pile-driving (Dynamic) Formulae

Pile-Driving Formulas: Dynamic equations are widely used in the field to determine whether a pile has reached a satisfactory bearing value at the predetermined depth

Engineering News (EN) Record formula:

- **The earliest formula**
- **derived from the work—energy theory:**

Energy imparted by the hammer per blow =(pile resistance)(penetration per hammer blow)

$$Q_u = \frac{W_R h}{S + C}$$

Where:

W_R = *weight of the ram*

h = *height of fall of the ram*

S = *penetration of pile per hammer blow*

C = a constant

For drop hammers:

C = 25.4 mm if S and h are in mm

1 in. if S and h are in inches

For steam hammers:

C = 2.54 mm if S and h are in mm

0.1 in. if S and h are in inches

Factor of safety (FS) = 6

For single and double-acting hammers, the term $W_R h$ can be replaced by EH_E, where E is the efficiency of the hammer and H_E is the rated energy of the hammer. Thus:

$$Q_u = \frac{EH_E}{S + C}$$

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Pile-Driving Formulas

Name	Formula
Modified EN formula	$$Q_u = \frac{EW_R h}{S + C} \frac{W_R + n^2 W_p}{W_R + W_p}$$

where E = efficiency of hammer
C = 2.54 mm if the units of S and h are in mm
C = 0.1 in. if the units of S and h are in in.
W_p = weight of the pile
n = coefficient of restitution between the ram and the pile cap

Typical values for E	
Single- and double-acting hammers	0.7–0.85
Diesel hammers	0.8–0.9
Drop hammers	0.7–0.9

Typical values for n	
Cast-iron hammer and concrete piles (without cap)	0.4–0.5
Wood cushion on steel piles	0.3–0.4
Wooden piles	0.25–0.3

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Danish formula (Olson and Flaate, 1967)

$$Q_u = \frac{EH_E}{S + \sqrt{\dfrac{EH_E L}{2A_p E_p}}}$$

where
E = efficiency of hammer
H_E = rated hammer energy
E_p = modulus of elasticity of the pile material
L = length of the pile
A_p = cross-sectional area of the pile

Janbu's formula (Janbu, 1953)

$$Q_u = \frac{EH_E}{K'_u S}$$

where

$$K'_u = C_d\left(1 + \sqrt{1 + \frac{\lambda'}{C_d}}\right)$$

$$C_d = 0.75 + 0.14\left(\frac{W_p}{W_R}\right)$$

$$\lambda' = \left(\frac{EH_E L}{A_p E_p S^2}\right)$$

For illustration of determination of the maximum load applied on a pile, the modified EN formula presented here:

$$Q_u = \frac{E W_R h}{S + C} \frac{W_R + n^2 W_p}{W_R + W_p}$$

S is the average penetration per hammer blow, which can also be expressed as:

$$S = \frac{1}{N}$$

where:
S in inches
N = number of hammer blows per 1 inch of penetration

$$Q_u = \frac{E W_R h}{(1/N) + 0.1} \frac{W_R + n^2 W_p}{W_R + W_p}$$

Suppose that a prestressed concrete pile 80 ft in length has to be driven by a hammer. The pile sides measure 10 in.

$A_p = 100\ in^2$

Weight of Concrete $= A_p L \gamma_c = (100\ in^2/144\ in^2/ft^2)\ (80\ ft)\ (150\ lb/ft^3) = 8.33$ kip

Weight of pile cap = 0.67 kip

So, total weight (w_p) = 9 kip

For the hammer, let

Rated energy = 19.2 kip-ft = $H_E = W_R h$

Weight of ram = 5 kip

Assume that the hammer efficiency is 0.85 and that n = 0.35. Substituting these values into Eq. yields :

$$Q_u = \left[\frac{(0.85)(19.2 \times 12)}{\frac{1}{N} + 0.1}\right]\left[\frac{5 + (0.35)^2(9)}{5 + 9}\right] = \frac{85.37}{\frac{1}{N} + 0.1}\ \text{kip}$$

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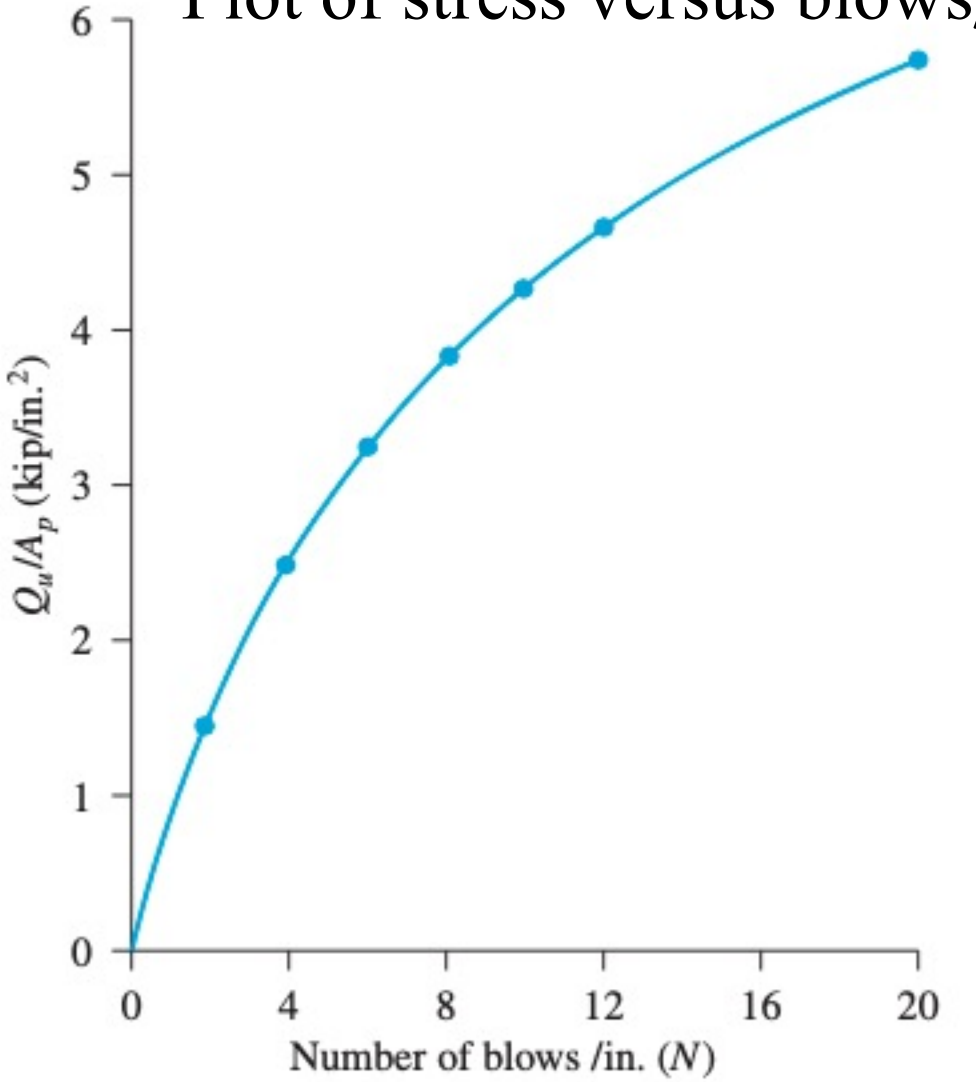
Now the following table can be prepared:

Q_u/A_p = driving stress

N	Q_u (kip)	A_p (in²)	Q_u/A_p (kip/in²)
0	0	100	0
2	142.3	100	1.42
4	243.9	100	2.44
6	320.1	100	3.20
8	379.4	100	3.79
10	426.9	100	4.27
12	465.7	100	4.66
20	569.1	100	5.69

Plot of stress versus blows/in.



Example

A precast concrete pile 12 in. × 12 in. in cross section is driven by a hammer. Given

Maximum rated hammer energy = 30 kip-ft
Hammer efficiency = 0.8
Weight of ram = 7.5 kip
Pile length = 80 ft
Coefficient of restitution = 0.4
Weight of pile cap = 550 lb
$E_p = 3 \times 10^6$ lb/in^2
Number of blows for last 1 in. of penetration = 8

Estimate the allowable pile capacity by the

a. Modified EN formula (use FS = 6)
b. Danish formula (use FS = 4)

Solution

Part a

$$Q_u = \frac{EW_Rh}{S + C}\frac{W_R + n^2W_p}{W_R + W_p}$$

$$\text{Weight of pile + cap} = \left(\frac{12}{12} \times \frac{12}{12} \times 80\right)(150\ \text{lb/ft}^3) + 550$$

$$= 12{,}550\ \text{lb} = 12.55\ \text{kip}$$

Given: $W_Rh = 30$ kip-ft.

$$Q_u = \frac{(0.8)(30 \times 12\ \text{kip-in.})}{\frac{1}{8} + 0.1} \times \frac{7.5 + (0.4)^2(12.55)}{7.5 + 12.55} = 607\ \text{kip}$$

$$Q_{all} = \frac{Q_u}{\text{FS}} = \frac{607}{6} \approx \mathbf{101\ kip}$$

Part b

$$Q_u = \frac{EH_E}{S + \sqrt{\dfrac{EH_EL}{2A_pE_p}}}$$

Use $E_p = 3 \times 10^6$ lb/in^2.

$$\sqrt{\frac{EH_EL}{2A_pE_p}} = \sqrt{\frac{(0.8)(30 \times 12)(80 \times 12)}{2(12 \times 12)\left(\dfrac{3 \times 10^6}{1000}\ \text{kip/in}^2\right)}} = 0.566\ \text{in.}$$

$$Q_u = \frac{(0.8)(30 \times 12)}{\frac{1}{8} + 0.566} \approx 417\ \text{kip}$$

$$Q_{all} = \frac{417}{4} \approx \mathbf{104\ kip}$$

Any Questions….????

Design of Deep (Pile) Foundations (b)

Static Approach

➢ Capacity of Pile (Ultimate and Allowable)

❖ End Bearing Capacity

✓ For cohesionless soil (sandy soil)

✓ For cohesive soil (clay soil)

✓ (C-Φ) soil

❖ Friction Capacity (skin friction capacity)

✓ For cohesionless soil (sandy soil)

✓ For cohesive soil (clay soil)

✓ (C-Φ) soil

Methods of determining 'Load carrying capacity'

The load carrying capacity of a pile can be determined by the following methods:

1) Dynamic formulae
2) Static formulae
3) Pile load tests, and
4) Penetration test

End Bearing Capacity (Point Load Capacity) of Pile,
a) Cohesionless (sand)soil

Meyerhof method to calculate the value of Q_P for sand and clay.

Calculation of Q_P for sand:

$Q_P = A_P \times q_P \leq Q_L$

A_P =Cross-sectional area (base area at the end of pile)

of the **end point** of the pile (**bearing area** between pile and soil).

$q_P = q' \times N_{q*}$

q' = **Effective vertical stress** at the level of the end of the pile.

$Nq*$ =Load capacity Factor(depends only on ϕ−value)

$Nq*$ is calculated from Meyerhof values bearing capacity,

Q_L = Limiting value for point resistance.

$Q_L = 0.5 \times A_P \times p_a \times N_{q*} \times \tan \phi$

p_a = atmospheric pressure = (100 kN/m^2 or 2000 lb/ft^2)

So, for sandy soil the value of Q_P is:

$Q_P = A_P \times q' \times N_{q*} \leq 0.5 \times A_P \times p_a \times N_{q*} \times \tan \phi$

Table 11.5 Interpolated Values of N_q^* Based on Meyerhof's Theory

Soil friction angle, ϕ (deg)	N_q^*
20	12.4
21	13.8
22	15.5
23	17.9
24	21.4
25	26.0
26	29.5
27	34.0
28	39.7
29	46.5
30	56.7
31	68.2
32	81.0
33	96.0
34	115.0
35	143.0
36	168.0
37	194.0
38	231.0
39	276.0
40	346.0
41	420.0
42	525.0
43	650.0
44	780.0
45	930.0

End Bearing Capacity (Point Load Capacity) of Pile,
b) Cohesive (clay)soil

Calculation of Q_P for Clay:

$Q_P = A_P \times c_u \times N_{c*}$

c_u = Cohesion for the soil supported the pile at its end.

N_{c*} = Bearing capacity factor for clay = 9 (when $\phi = 0.0$), $Q_P = 9 \times A_P \times c_u$

End Bearing Capacity (Point Load Capacity) of Pile,
c) C-ϕ

Calculation of QP for C – ϕ Soile:

If the supporting the pile from its end is C – **ϕ** soil:

$Q_P = (A_P \times q' \times N_{q*} \leq 0.5 \times A_P \times p_a \times N_{q*} \times \tan_\phi) + A_P \times c_u \times N_{c*}$

But here, the value of $\phi \neq 0.0 \rightarrow Nc* \neq 9$ (you will given it according ϕ)

But, if you are not given the value of Nc∗ at the existing value of ϕ → Assume Nc∗ = 9 and complete the solution.

Important Note:

The soil profile may consists of several sand layers, the value of friction angle (ϕ) which used to calculate Q_P as shown in the above equation is the friction angle for the soil that **supporting the pile end** (for last soil layer).

Frictional Resistance Capacity (Skin Friction) of Pile ,
a) For cohesionless soil

Calculation of Frictional Resistance(Q_s) Calculation of Q_s for sand:

The general formula for calculating Qs is:

$Q_s = P \times \sum f_i \times L_i$

P = pile perimeter = $\pi \times D$ (if the pile is circular, D = Pile diameter) = $4 \times D$ (if the pile is square, D = square dimension)

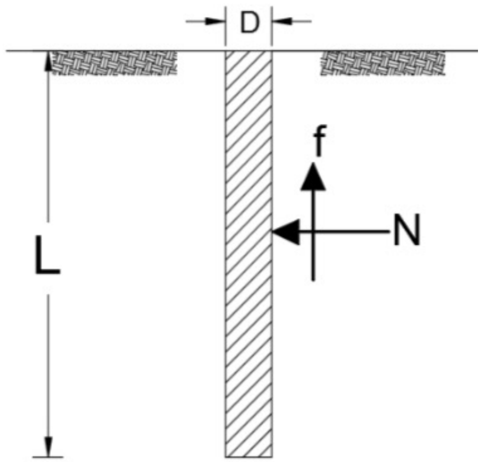
f_i = unit friction resistance at any depth

L_i = depth of each soil layer

Now, how we calculate the value of f_i (for each soil layer):

$f = \mu_s \times N$

Here the value of (f) is vertical, so N must be perpendicular to f (i.e. N must be horizontal) as shown in the following figure:



Frictional Resistance Capacity (Skin Friction) of Pile ,
a) For cohesionless soil, Cont'd

μ_s = friction coefficient between soil and pile = tan δ
δ = soil − pile friction angle = 0.8 $\varphi \rightarrow \mu_s = \tan(0.8\ \varphi)$ (for each layer) N = **Horizontal stress** from the soil to the pile
$\rightarrow N=\sigma'_v \times K$ (for each soil layer)
σ'_v = vertical effective stress for each layer
But, to calculate σ'_v for each soil layer, to be representative, we take the average value for σ'_v for each layer.
K = Effective earth pressure coefficient
$K=1-\sin\varphi$ or $K= 0.5+0.008D_r$, D_r = relative density (%)
If you are not given the relative density for each layer, use $K = 1 - \sin\varphi$ or you may given another formula to calculate K.

Frictional Resistance Capacity (Skin Friction) of Pile ,
a) For cohesionless soil, Cont'd

Now, N = $\sigma'_{v,av} \times K$ (for each soil layer)

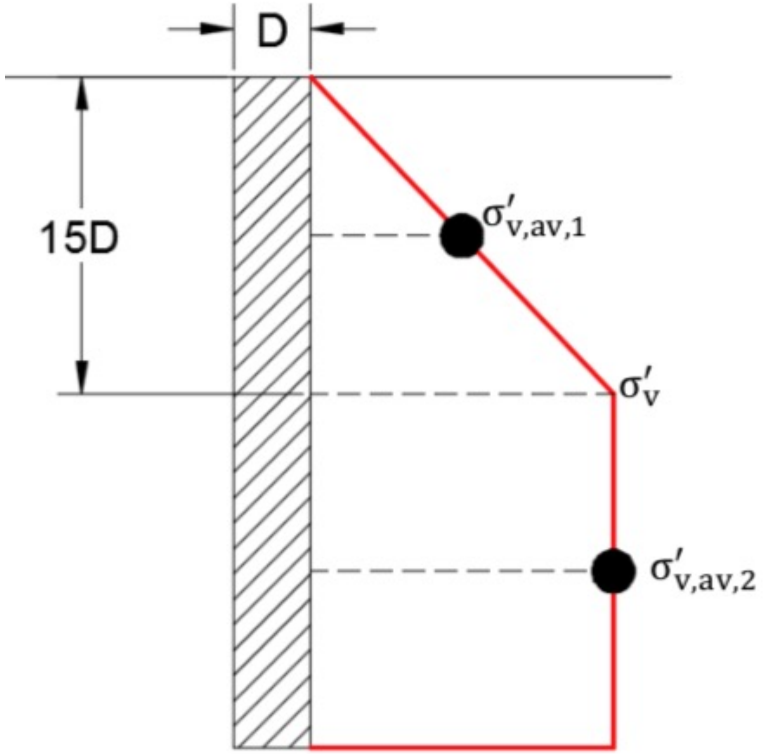
$\rightarrow\rightarrow$ $f = \tan(0.8\,\varphi) \times \sigma'_{v,av} \times K$ (for each soil layer)

Now, how we calculate the value of $\sigma'_{v,av}$ for each soil layer:

We draw the vertical effective stress along the pile, but the stress will linearly increase to a depth of (15D), after this depth the stress will be constant and will not increase. (this is true only if we deal with sandy soil). If there is one soil layer before reaching 15D:

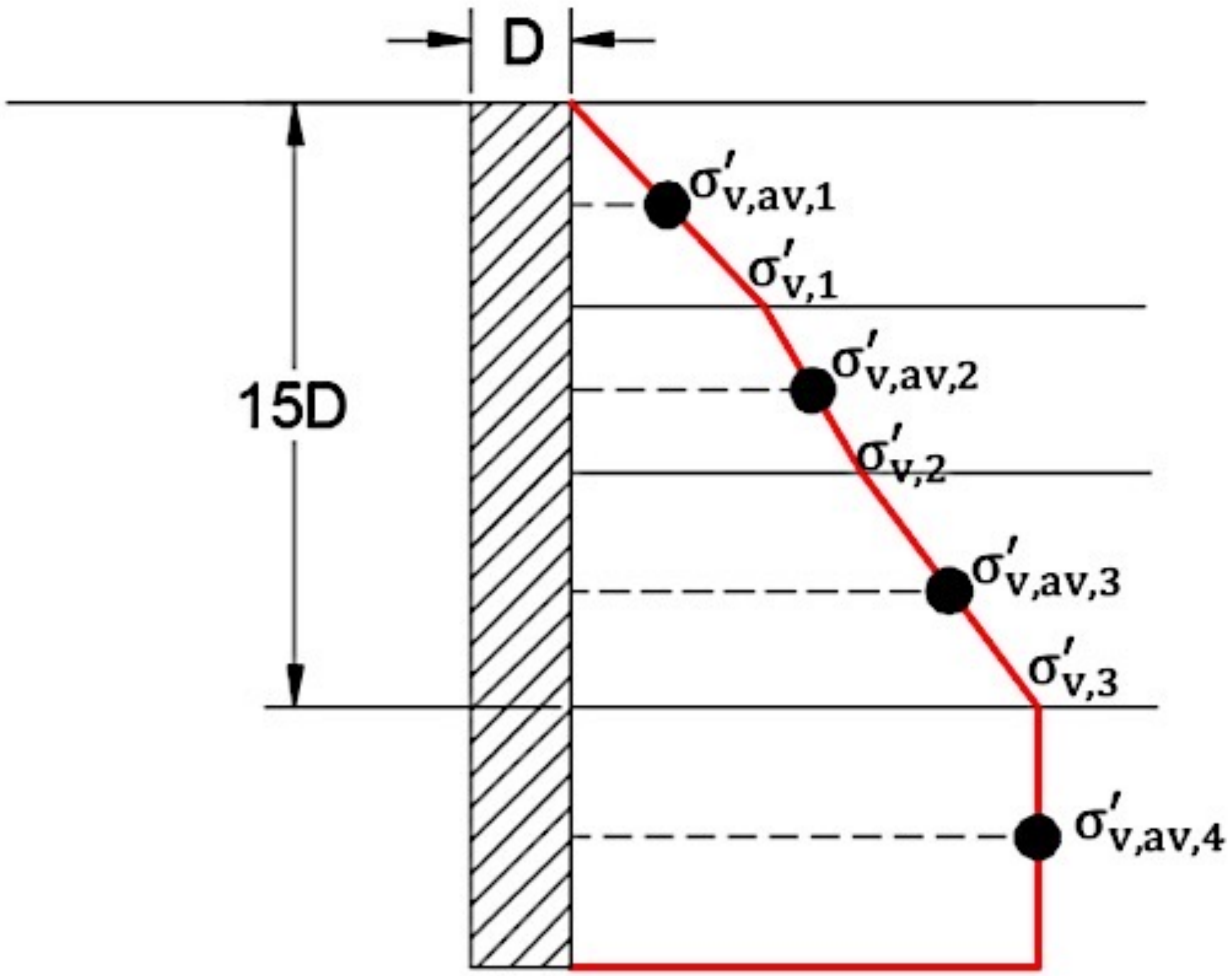
$$\sigma'_{v,av,1} = \frac{0 + \sigma'_v}{2} = 0.5\sigma'_v$$

$$\sigma'_{v,av,2} = \frac{\sigma'_v + \sigma'_v}{2} = \sigma'_v$$



Frictional Resistance Capacity (Skin Friction) of Pile ,
a) For cohesionless soil, Cont'd

If there are more than one soil layer before reaching 15D:



Frictional Resistance Capacity (Skin Friction) of Pile , a) For cohesionless soil, Cont'd

$$\sigma'_{v,av,1} = \frac{0 + \sigma'_{v,1}}{2} = 0.5\sigma'_{v,1}$$

$$\sigma'_{v,av,2} = \frac{\sigma'_{v,1} + \sigma'_{v,2}}{2}$$

$$\sigma'_{v,av,3} = \frac{\sigma'_{v,2} + \sigma'_{v,3}}{2}$$

$$\sigma'_{v,av,4} = \frac{\sigma'_{v,3} + \sigma'_{v,3}}{2} = \sigma'_{v,3}$$

Finally we can calculate the value of Q_s as following:

$$Q_s = P \times \sum \tan(0.8\phi_i) \times \sigma'_{v,av,i} \times K_i \times L_i$$

$i =$ each soil layer

Note:

We take soil layer every change in soil properties or every change in slope of vertical stress.

Frictional Resistance Capacity (Skin Friction) of Pile ,
b) For cohesive soil (Clay soil)

Calculation of Q_s for clay:

There are three methods used to calculate Qs in clay:

1. **λ Method**

$$Q_s = P \times \sum f_i \times L_i$$

But here we take the entire length of the pile:

$$Qs = P \times L \times \sum fi$$

$$\sum f_i = f_{av} = \lambda \times (\sigma'_{v.av} + 2c_{u,av})$$

$\sigma'_{v,av}$ = mean effective vertical stress for the entire embedment length

$c_{u,av}$ = mean undrained shear strength for the entire embedment length

λ = function of pile length (L)

Table 11.9 Variation of λ with pile embedment length, *L*

Embedment length, *L* (m)	λ
0	0.5
5	0.336
10	0.245
15	0.200
20	0.173
25	0.150
30	0.136
35	0.132
40	0.127
50	0.118
60	0.113
70	0.110
80	0.110
90	0.110

Frictional Resistance Capacity (Skin Friction) of Pile ,
b) For cohesive soil (Clay soil)

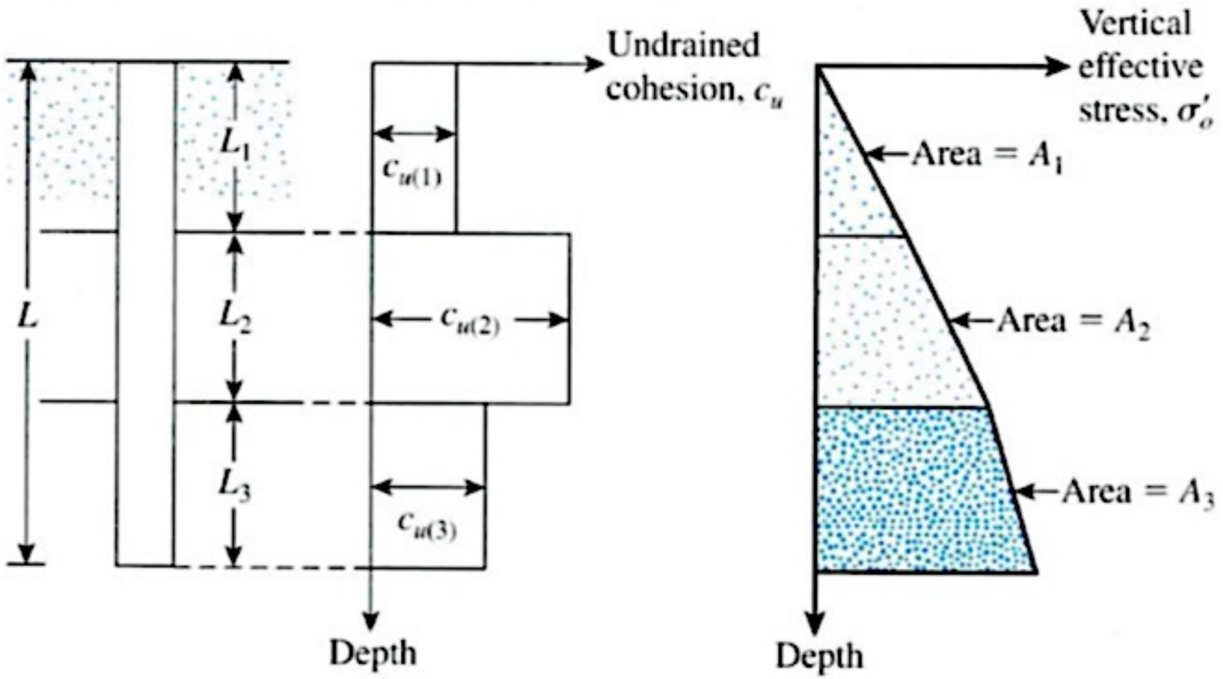
Calculation of $\sigma'_{v,av}$ and $c_{u,av}$

We prepare the following graph (assuming three soil layers).

Note that the soil is clay, and the stress is not constant after 15D, the stress is constant (after 15D) in sand only.

$$c_{u,av} = \frac{L_1 \times c_{u,1} + L_2 \times c_{u,2} + L_3 \times c_{u,3}}{L}$$

$$\sigma'_{v,av} = \frac{A_1 + A_2 + A_3}{L}$$



Frictional Resistance Capacity (Skin Friction) of Pile,
b) For cohesive soil (Clay soil), Cont'd

2. α Method

$$Q_s = P \times \sum f_i \times L_i$$

$$f_i = \alpha_i \times c_{u,i}$$

$$\alpha_i = \text{function of } \left(\frac{c_{u,i}}{p_{atm}}\right) \text{ (calculated from } \textbf{Table 11.10}$$

$$Q_s = P \times \sum \alpha_i \times c_{u,i} \times L_i$$

Table 11.10 Variation of α (interpolated values based on Terzaghi, Peck and Mesri, 1996)

$\frac{c_u}{p_a}$	α
≤ 0.1	1.00
0.2	0.92
0.3	0.82
0.4	0.74
0.6	0.62
0.8	0.54
1.0	0.48
1.2	0.42
1.4	0.40
1.6	0.38
1.8	0.36
2.0	0.35
2.4	0.34
2.8	0.34

Note: p_a = atmospheric pressure ≈ 100 kN/m² or 2000 lb/ft²

Frictional Resistance Capacity (Skin Friction) of Pile ,
b) For cohesive soil (Clay soil), Cont'd

3. β Method

$$Q_s = P \times \sum f_i \times L_i$$

$$f_i = \beta_i \times \sigma'_{v,av,i}$$

$\sigma'_{v,av,i}$ = average vertical effective stress for each clay layer

$$\beta_i = K_i \times \tan\phi_{R,i}$$

ϕ_R = drained friction angle of remolded clay (given for each layer)

K_i = earth pressure coefficient for each clay layer

$K = 1 - \sin\phi_R$ (for normally consolidated clay)

$K = (1 - \sin\phi_R) \times \sqrt{OCR}$ (for overconsolidated clay)

Frictional Resistance Capacity (Skin Friction) of Pile ,
c) For *(C-ϕ)* soil

For the soil ($C - \varphi$):

- ✓ Calculate Qs for sand alone and for clay alone and then sum the two values to get the total Qs

Allowable capacity of pile

From all above methods, the ultimate load that the pile could carry has been determined:

$Q_u = Q_p + Q_s$

The allowable load of pile can be determined using factor of safety (FS), in general, the value is between (2.5 to 4) and may taken to be (3) by the formula:

$$Q_{all} = \frac{Q_u}{FS} \quad (FS \geq 3)$$

The End

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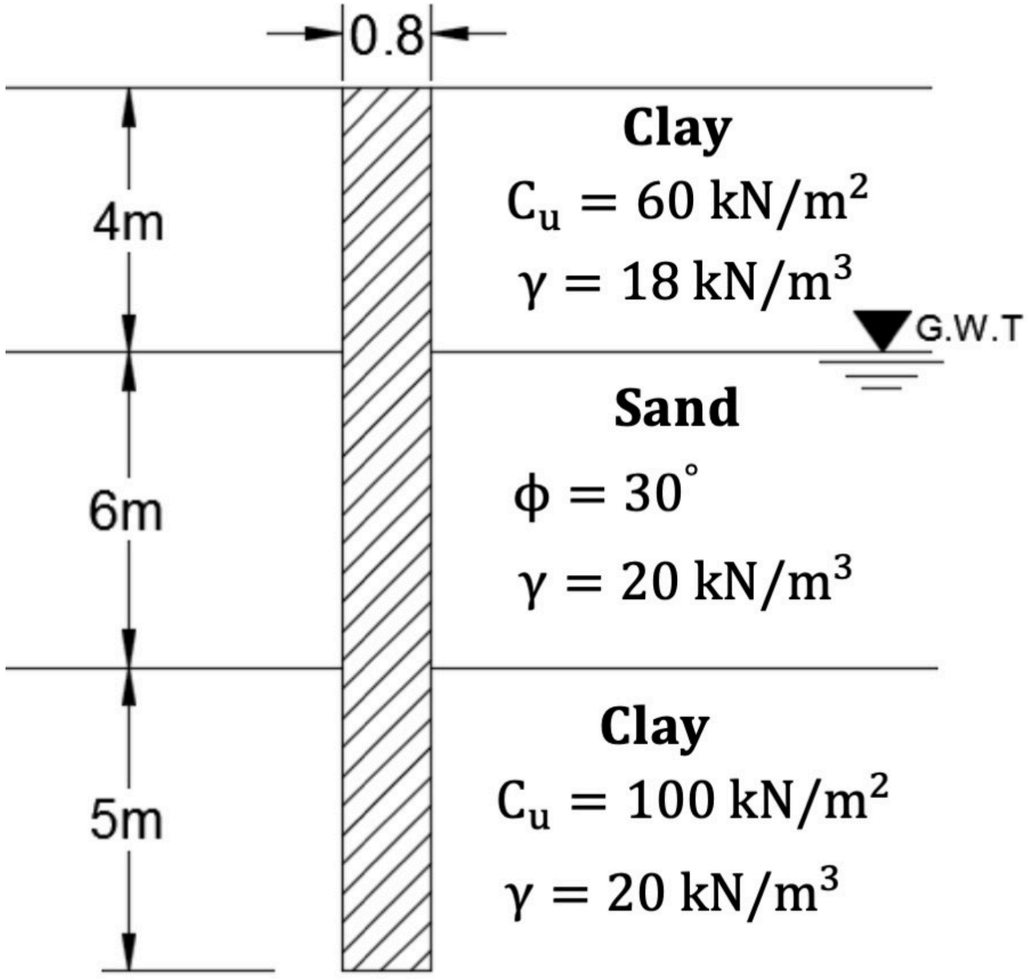
Design of Deep (Pile) Foundations (C)

Examples:

- End Bearing Capacity (Point Load Capacity)
- Frictional (Skin Friction) Resistance Capacity
- Ultimate and Allowable Bearing Capacity Load of Pile

Example (1)

Determine the ultimate load capacity of the 800 mm diameter concrete bored pile given in the figure below.



Example (1), Cont'd

Solution

Calculation of Q_P:

Note that the soil supporting the pile at its end is clay, so:

$Q_P = A_P \times c_u \times N_c^*$ $\quad N_c^* = 9$ (pure clay $\phi = 0.0$)

$A_P = \frac{\pi}{4} \times 0.8^2 = 0.502\ m^2$

$c_u = 100\ kN/m^2$ (for the soil supporting the pile at its end)

$Q_P = 0.502 \times 100 \times 9 = 452.4\ KN$

$Q_L = 0.0$ since $\tan\phi = 0.0 \rightarrow \boxed{Q_P = 452.4\ KN}$

Calculation of Q_s:

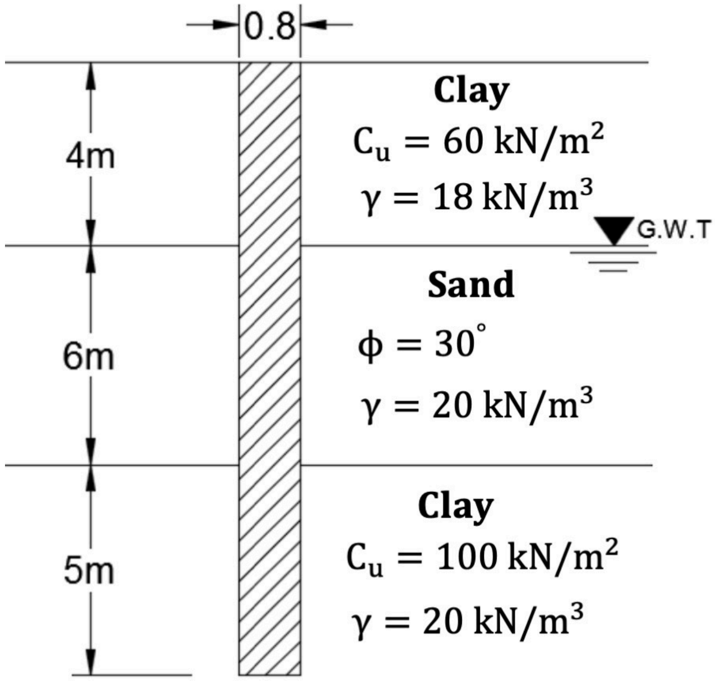
Since there are one sand layer and two clay layer, we solve firstly for sand and then for clay:

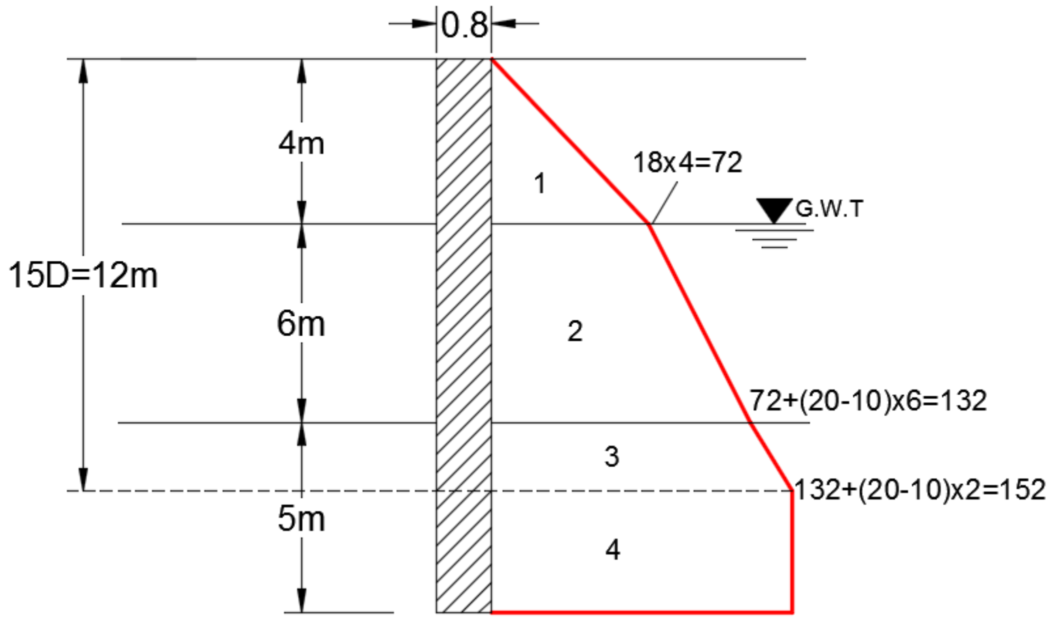
For sand:

The stress will increase till reaching 15D

$15D = 15 \times 0.8 = 12m$

Now we draw the vertical effective stress with depth:





Example (1), Cont'd

$$Q_s = P \times \sum (\tan(0.8\phi_i) \times \sigma'_{v,av,i} \times K_i) \times L_i$$

Note that the value of ϕ for layers 1, 3, and 4 is zero (clay), so we calculate Q_s only for the layer 2 (sand layer).

$$P = \pi \times D = \pi \times 0.8 = 2.51\ \text{m}$$

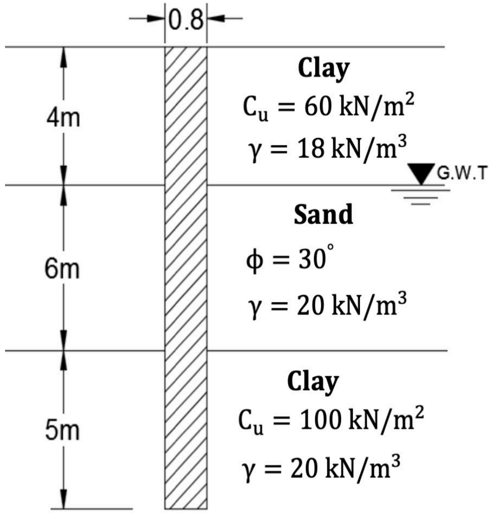
$$\sigma'_{v,av,2} = \frac{72 + 132}{2} = 102\ \ \text{kN/m}^2$$

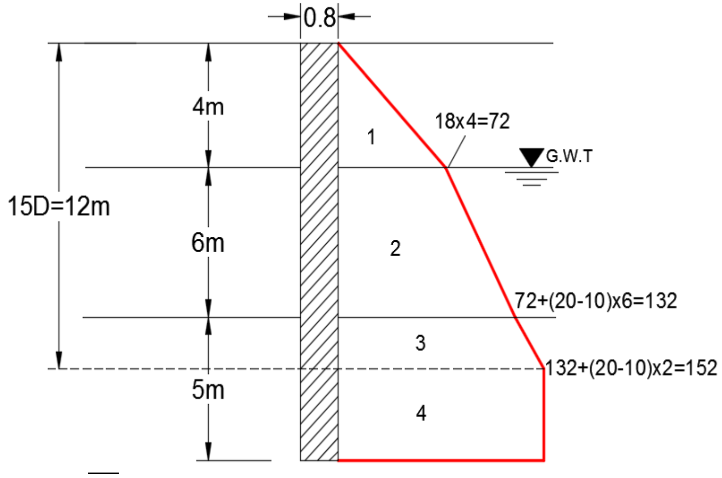
$$L_2 = 6\text{m}$$

$$\phi_2 = 30^\circ$$

$$K_2 = 1 - \sin\phi_2 = 1 - \sin 30 = 0.5$$

$$\rightarrow \boxed{Q_{s,sand} = 2.51 \times (\tan(0.8 \times 30) \times 102 \times 0.5) \times 6 = 342\ \text{kN}}$$





Example (1), Cont'd

For Clay:

If we want to use λ Method:

$$Q_s = P \times L \times f_{av}$$

$$f_{av} = \lambda \times \left(\sigma'_{v,av} + 2\, c_{u,av}\right)$$

$$P = 2.51\text{m} \quad L = 4 + 6 + 5 = 15\text{m}$$

$$\lambda = 0.2 \text{ (at } L = 15\text{m from Table 11.9)}$$

$$c_{u,av} = \frac{L_1 \times c_{u,1} + L_2 \times c_{u,2} + L_3 \times c_{u,3}}{L}$$

$$c_{u,av} = \frac{4 \times 60 + 6 \times 0 + 5 \times 100}{15} = 49.33\ \text{kN/m}^2$$

Table 11.9 Variation of λ with pile embedment length, *L*

Embedment length, *L* (m)	λ
0	0.5
5	0.336
10	0.245
15	0.200
20	0.173
25	0.150
30	0.136
35	0.132
40	0.127
50	0.118
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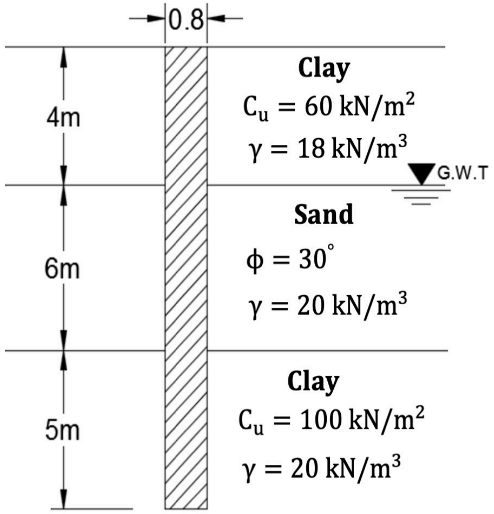
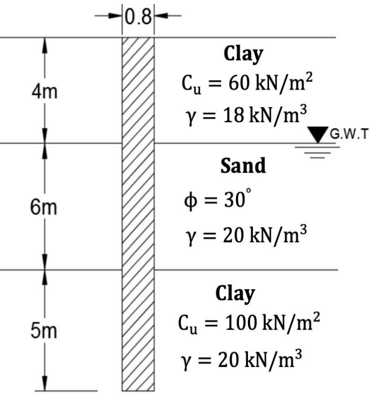
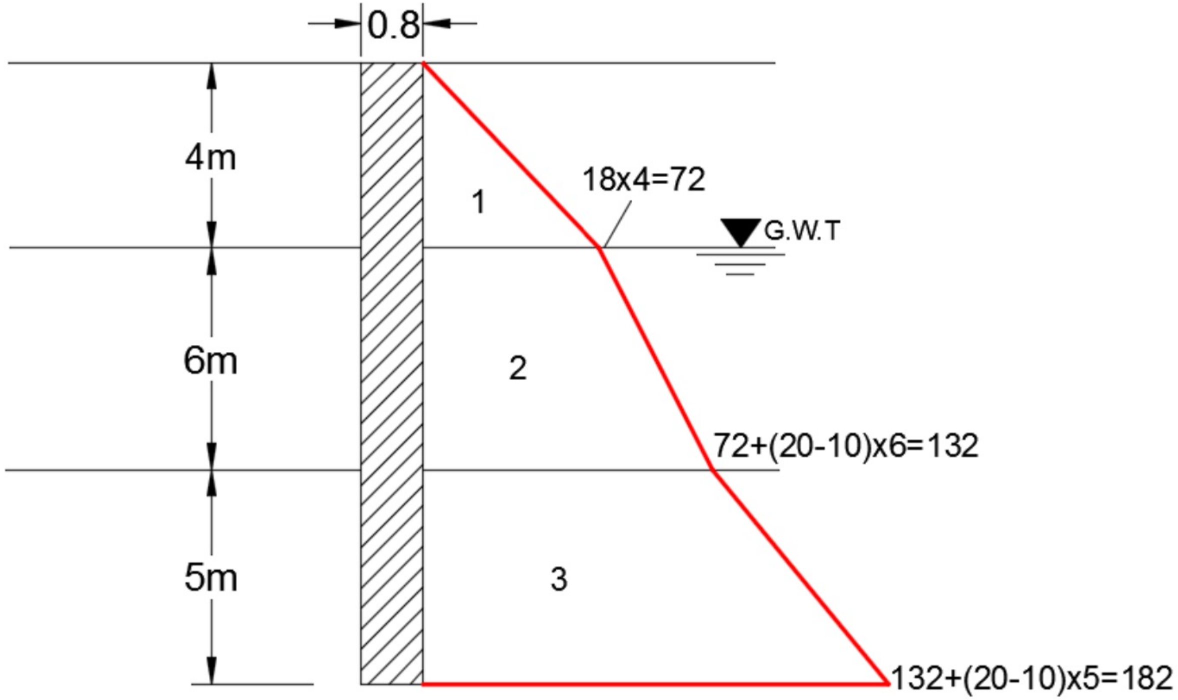


Example (1), Cont'd

$$\sigma'_{v,av} = \frac{A_1 + A_2 + A_3 + \cdots A_n}{L}$$

We draw the vertical effective pressure with depth:





$$A_1 = \frac{1}{2} \times 72 \times 4 = 144$$

$$A_2 = \frac{1}{2} \times (72 + 132) \times 6 = 612$$

$$A_3 = \frac{1}{2} \times (132 + 182) \times 5 = 785$$

$$\sigma'_{v,av} = \frac{144 + 612 + 785}{15} = 102.73\ kN/m^2$$

$$f_{av} = 0.2 \times (102.73 + 2 \times 49.33) = 40.28$$

$$\rightarrow \boxed{Q_{s,clay} = 2.51 \times 15 \times 40.28 = 1516.54\ kN}$$

Example (1), Cont'd

If we want to use α Method:

$$Q_s = P \times \sum \alpha_i \times c_{u,i} \times L_i$$

For layer (1)

$$c_{u,1} = 60 \rightarrow \frac{c_{u,1}}{p_{atm}} = \frac{60}{100} = 0.6 \rightarrow \alpha_1 = 0.62$$

For layer (2)

$$c_{u,2} = 0.0 \rightarrow \frac{c_{u,2}}{p_{atm}} = \frac{0}{100} = 0 \rightarrow \alpha_2 = 0$$

For layer (3)

$$c_{u,3} = 100 \rightarrow \frac{c_{u,3}}{p_{atm}} = \frac{100}{100} = 1 \rightarrow \alpha_3 = 0.48$$

$$Q_{s,clay} = 2.51 \times [(0.62 \times 60 \times 4) + 0 + (0.48 \times 100 \times 5)] = 975.88\ \text{kN}$$

$$\boxed{\begin{aligned} Q_{s,total} &= 342 + 1516.54 = 1858.54\ \text{kN (when using } \lambda - \text{method)} \\ Q_{s,total} &= 342 + 975.88 = 1317.88\ \text{kN (when using } \alpha - \text{method)} \end{aligned}}$$

$$\boxed{\begin{aligned} Q_u &= Q_p + Q_{s,total} \\ Q_u &= 452.4 + 1858.54 = 2310.94\ \text{kN (when using } \lambda - \text{method)} \checkmark. \\ Q_u &= 452.4 + 1317.88 = 1770.28\ \text{kN (when using } \alpha - \text{method)} \checkmark. \end{aligned}}$$

Example (2)

A pile is driven through a soft cohesive deposit overlying a stiff clay, the average undrained shear strength in the soft clay is 45 kPa. and in the lower deposit the average undrained shear strength is 160 kPa. The water table is 5 m below the ground and the stiff clay is at 8 m depth. The unit weights are 17.5 kN/m^3 and 19 kN/m^3 for the soft and the stiff clay respectively. **Estimate the length** of 500 mm diameter pile to carry a load of 500 kN with a safety factor of 4.

Using **(a):** α – method **(b):** λ – method

Solution

There is no given graph in this problem, so, firstly it requires to draw a graph for understanding the problem.

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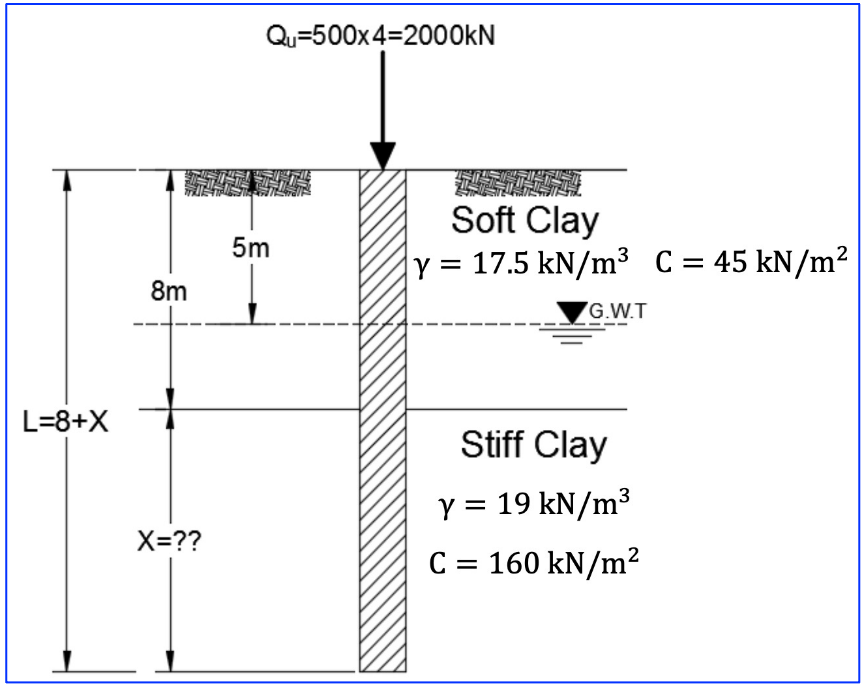
Example (2), Cont'd



$Q_{all} = 500\ \text{kN}$, $FS = 4 \rightarrow Q_u = 500 \times 4 = 2000\ \text{KN}$

$Q_u = Q_P + Q_s$

Calculation of Q_P:

Note that the soil supporting the pile from its end is clay, so:

$Q_P = A_P \times c_u \times N_c^*$ $\quad N_c^* = 9$ (pure clay $\phi = 0.0$)

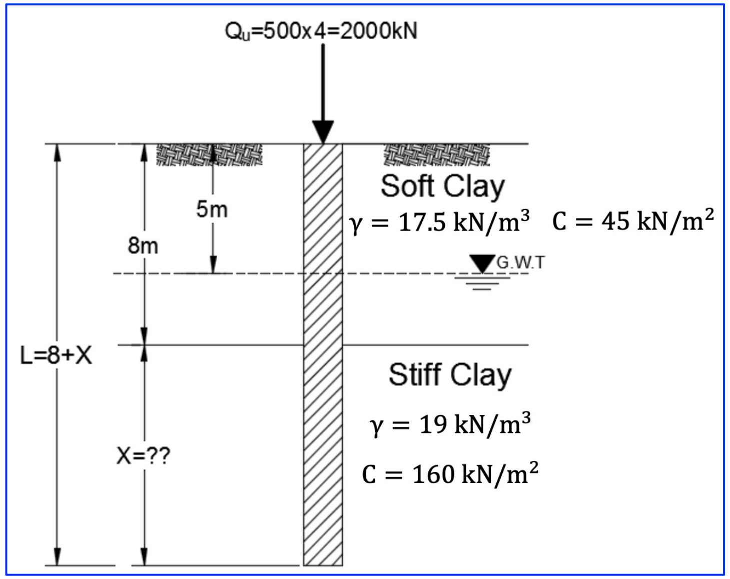
$A_P = \dfrac{\pi}{4} \times 0.5^2 = 0.196\ \text{m}^2$

$c_u = 160\ \text{kN/m}^2$ (for the soil supporting the pile at its end)

$Q_P = 0.196 \times 160 \times 9 = 282.24\ \text{KN}$

$Q_L = 0.0$ since $\tan\phi = 0.0 \rightarrow Q_P = 282.24\ \text{KN}$

Example (2), Cont'd



Calculation of Q_s:

Note that all layers are clay.

(a). α − method

$$Q_s = P \times \sum \alpha_i \times c_{u,i} \times L_i$$

$$P = \pi \times D = \pi \times 0.5 = 1.57m$$

For layer (1)

$$c_{u,1} = 45 \rightarrow \frac{c_{u,1}}{p_{atm}} = \frac{45}{100} = 0.45 \rightarrow \alpha_1 = 0.71 \text{ (by interpolation from table)}$$

For layer (2)

$$c_{u,2} = 160 \rightarrow \frac{c_{u,2}}{p_{atm}} = \frac{160}{100} = 1.6 \rightarrow \alpha_2 = 0.38$$

$$Q_s = 1.57 \times [(0.71 \times 45 \times 8) + (0.38 \times 160 \times X)] = 401.3 + 95.45X \text{ Kn}$$

$$Q_u = Q_P + Q_s = 282.24 + 401.3 + 95.45X = 683.54 + 95.45X$$

But $Q_u = 2000 \rightarrow 2000 = 683.54 + 95.45X \rightarrow X = 13.8m$

$\rightarrow\rightarrow \boxed{L = 8 + 13.8 = 21.8 \cong 22m ✓}$.

Table 11.10 Variation of α (interpolated values based on Terzaghi, Peck and Mesri, 1996)

$\frac{c_u}{p_a}$	α
≤ 0.1	1.00
0.2	0.92
0.3	0.82
0.4	0.74
0.6	0.62
0.8	0.54
1.0	0.48
1.2	0.42
1.4	0.40
1.6	0.38
1.8	0.36
2.0	0.35
2.4	0.34
2.8	0.34

Note: p_a = atmospheric pressure $\approx 100\ kN/m^2$ or $2000\ lb/ft^2$

Example (2), Cont'd

(b). λ – method

$Q_s = P \times L \times f_{av}$

$f_{av} = \lambda \times \left(\sigma'_{v,av} + 2\,c_{u,av}\right)$

$P = 1.57\text{m} \quad L = 8 + X$

We want to calculate λ from the table, but λ is a function of pile length which is required, so in this types of problems when the solution is required according λ – method you are strongly recommended to assume a reasonable value of L.

$\boxed{\text{Assume } X = 7\text{ m} \rightarrow L = 8 + 7 = 15}$

$\lambda = 0.2$ (at $L = 15\text{m}$ from Table 11.9)

$$c_{u,av} = \frac{L_1 \times c_{u,1} + L_2 \times c_{u,2} + L_3 \times c_{u,3}}{L}$$

$$c_{u,av} = \frac{8 \times 45 + 7 \times 160}{15} = 98.67\ \text{kN/m}^2$$

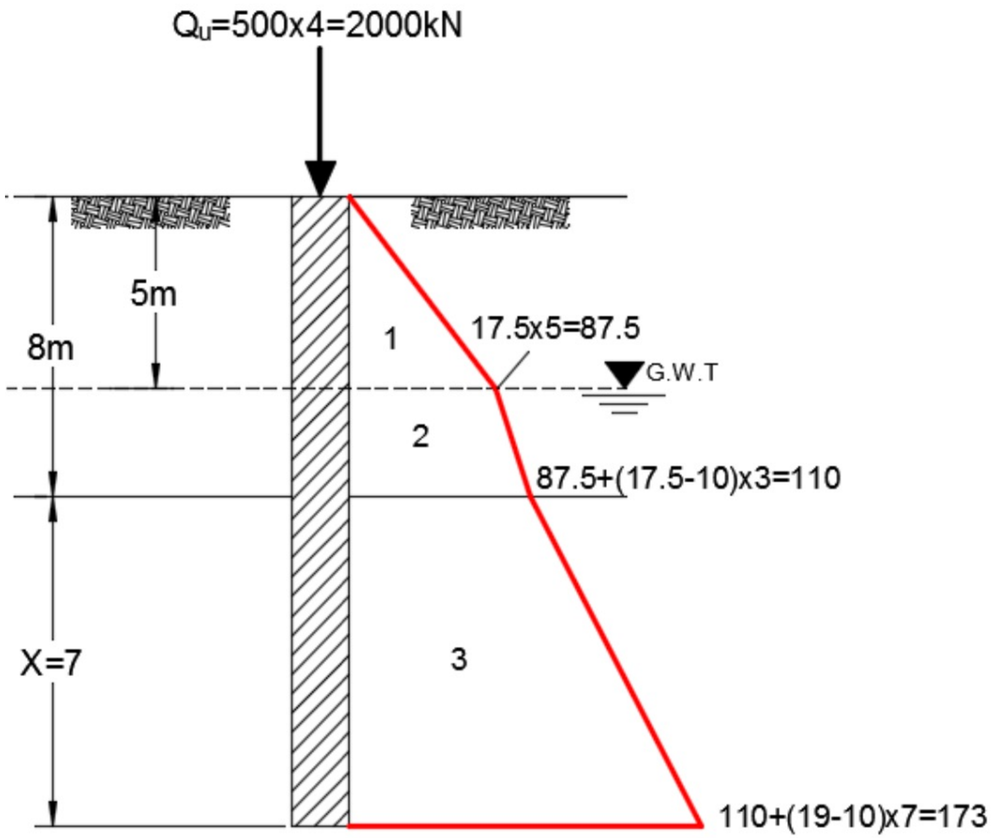
Table 11.9 Variation of λ with pile embedment length, L

Embedment length, L (m)	λ
0	0.5
5	0.336
10	0.245
15	0.200
20	0.173
25	0.150
30	0.136
35	0.132
40	0.127
50	0.118
60	0.113
70	0.110
80	0.110
90	0.110

Example (2), Cont'd

The vertical effective pressure with depth should be draft for determining average value of it : For the upper layer (Soft clay) assume the saturated unit weight is the same as the natural unit weight (17.5 kN/m^3) because no enough information about it.

First Trial with X= 7 m



$$\sigma'_{v,av} = \frac{A_1 + A_2 + A_3 + \cdots A_n}{L}$$

$$A_1 = \frac{1}{2} \times 87.5 \times 5 = 218.75$$

$$A_2 = \frac{1}{2} \times (87.5 + 110) \times 3 = 296.25$$

$$A_3 = \frac{1}{2} \times (110 + 173) \times 7 = 990.5$$

$$\sigma'_{v,av} = \frac{218.75 + 296.25 + 990.5}{15} = 100.36\ \mathrm{kN/m^2}$$

$$f_{av} = 0.2 \times (100.36 + 2 \ \times 98.67\,) = 59.54$$

$$\rightarrow Q_s = 1.57 \times 15 \times 59.54 = 1402.167\ \mathrm{kN}$$

$Q_u = Q_p + Q_s = 282.24 + 1402.167 = 1684.4 < 2000 \rightarrow\rightarrow\rightarrow$ We need to increase L to be closed from 2000

Example (2), Cont'd

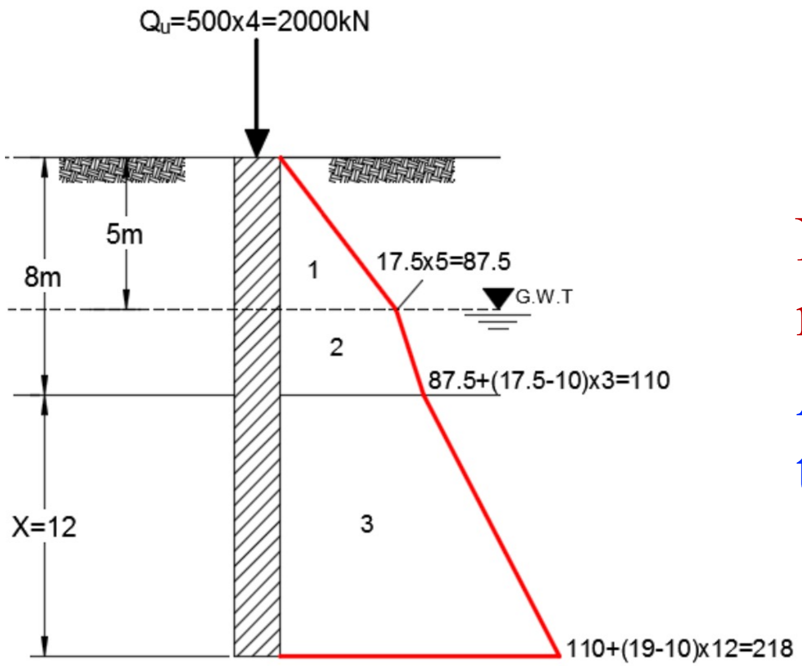
2nd Trial with X= 12 m

$$\text{Try } X = 12\text{ m} \rightarrow L = 8 + 12 = 20$$
$$\lambda = 0.173 \text{ (at L = 20m from Table 11.9)}$$

$$c_{u,av} = \frac{L_1 \times c_{u,1} + L_2 \times c_{u,2} + L_3 \times c_{u,3}}{L}$$

$$c_{u,av} = \frac{8 \times 45 + 12 \times 160}{20} = 114 \text{ kN/m}^2$$

$$\sigma'_{v,av} = \frac{A_1 + A_2 + A_3 + \cdots A_n}{L}$$



$$A_1 = \frac{1}{2} \times 87.5 \times 5 = 218.75$$
$$A_2 = \frac{1}{2} \times (87.5 + 110) \times 3 = 296.25$$
$$A_3 = \frac{1}{2} \times (110 + 218) \times 12 = 1968$$

Table 11.9 Variation of λ with pile embedment length, L

Embedment length, L (m)	λ
0	0.5
5	0.336
10	0.245
15	0.200
20	0.173

$$\sigma'_{v,av} = \frac{218.75 + 296.25 + 1968}{20} = 124.15 \text{ kN/m}^2$$
$$f_{av} = 0.173 \times (124.15 + 2 \times 114) = 60.92$$
$$\rightarrow Q_s = 1.57 \times 20 \times 60.92 = 1912.94 \text{ kN}$$
$$Q_u = Q_p + Q_s = 282.24 + 1912.94 = 2195.12 > 2000$$

Note that at X= 12 m the value of Qu is closed to 2000 but its need to slightly decrease, so we can say $X \cong 10\text{m} \rightarrow L \cong 18\text{m}$✓.

As you see, the solution is by trial and error, so when you assume the value for L, be logic and be realistic to save time.

Example (3)

A concrete pile is 20 m length and 360 mm x 360 mm in cross section. The pile is fully embedded in **sand** which unit weight is 16.8 kN/m^3 and $\phi = 30°$. You are given also $Nq* = 56.7$ for this angle of internal friction.

Calculate:

a) The ultimate load (Q_p), using Meyerhof's method.

b) Determine the frictional resistance (Q_s), if k = 1.3 and $\delta = 0.8\phi$.

c) Estimate the allowable load carrying capacity of the pile (Use FS = 4).

Example (3), Cont'd

Solution

a) $\mathbf{Q_P = ??}$

$$Q_P = A_P \times q' \times N_q^* \leq Q_L$$

$$Q_P = A_P \times q' \times N_q^*$$

$$A_P = 0.36 \times 0.36 = 0.1296\ m^2$$

$$q' = 16.8 \times 20 = 336\ kN/m^2$$

$$N_q^* = 56.7\ (\text{given})$$

$$\rightarrow Q_P = 0.1296 \times 336 \times 56.7 = 2469\ KN$$

Now we check for Upper limit:

$$Q_L = 0.5 \times A_P \times p_a \times N_q^* \times \tan\phi$$

$$p_a = \text{atmospheric pressure} = 100\ kPa\ ,\quad \phi = 30^{\circ}(\text{given})$$

$$Q_L = 0.5 \times 0.1296 \times 100 \times 56.7 \times \tan 30 = 212.13\ kN$$

$$Q_L < Q_P \rightarrow Q_P = Q_L = 212.13\ kN\ ✓.$$

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Example (3), Cont'd

b) $\mathbf{Q_s}$ **=??**

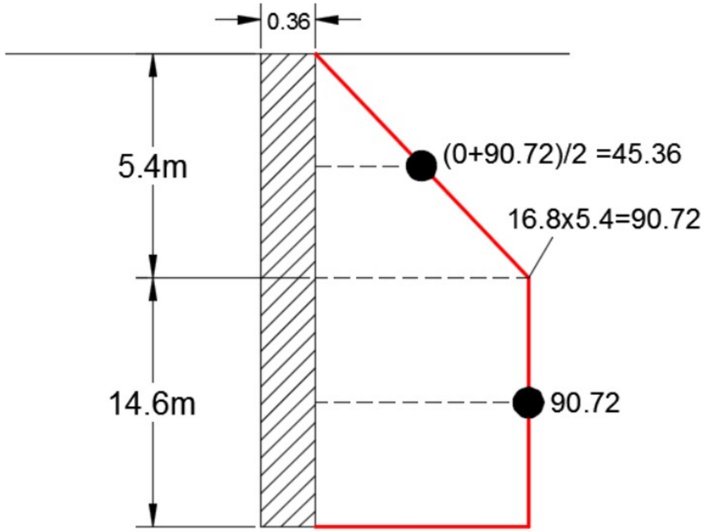
The soil is pure sand, so:

$$Q_s = P \times \sum \tan(0.8\phi_i) \times \sigma'_{v,av,i} \times K_i \times L_i$$

But, since the soil is sand the stress will varies at depth of 15D then will be constant on the remained pile length.

$15D = 15 \times 0.36 = 5.4m$

The shown figure is the pressure distribution with depth:



$P = 4 \times 0.36 = 1.44$ (Square cross section)

$K_1 = K_2 = 1.3$ (given)

$0.8\phi_i = 0.8 \times 30 = 24^{\circ}$

$Q_s = 1.44 \times 1.3 \times \tan(24) \left[\sigma'_{v,av,i} \times L_i\right]$

$Q_s = 0.8334[45.36 \times 5.4 + 90.72 \times 14.6]$

$\rightarrow Q_s = 1308 \text{ kN}$ ✓.

c) $\mathbf{Q_{all}}$ **=??**

$$Q_{all} = \frac{Q_u}{FS}$$

$Q_u = Q_P + Q_s = 212.13 + 1308 = 1520.13 \text{ kN}$, $FS = 4$ (given)

$$\rightarrow Q_{all} = \frac{1520.13}{4} = 380 \text{ kN} ✓.$$

Thank You

Foundation Engineering (2)

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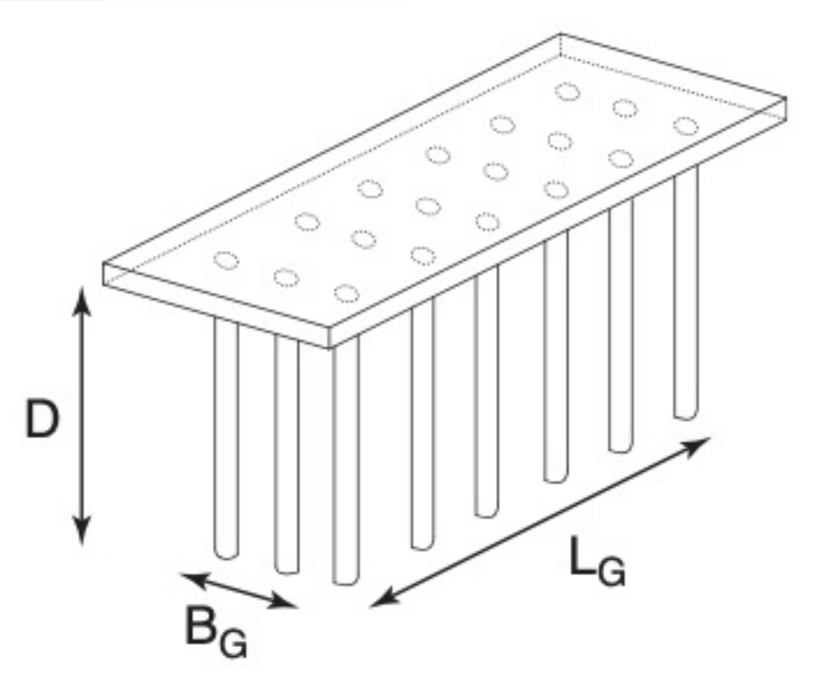
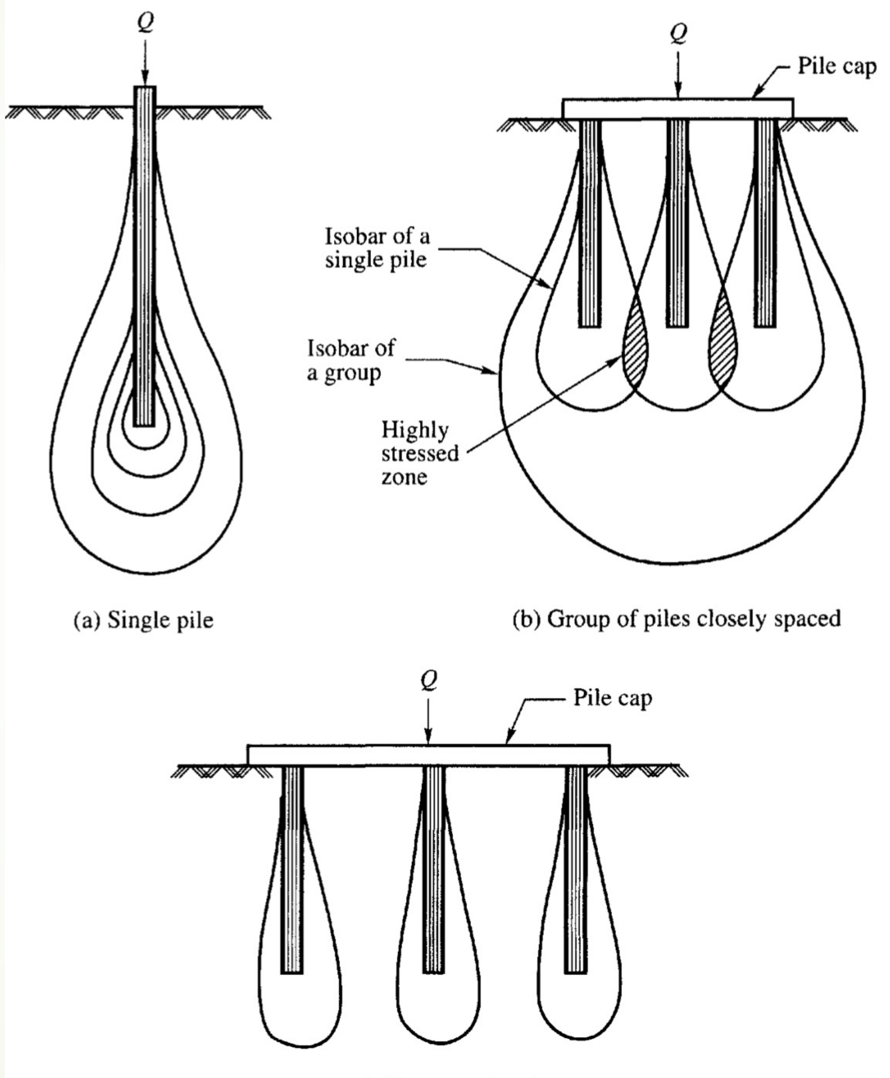
Design of Deep (Pile) Foundations (d): Piles Group

- Types and review
- Pattern of distribution of piles in group
- Capacity
- Settlement
- Negative skin friction

Number and spacing of piles in group

Piles are often installed in groups as in the Figure to carry higher loads under columns of buildings, bridges, dams, and other structures. As shown in figures a, b, and c, the stress isobars (bulbs) for single pile and for piles in groups.





(a) Single pile

(b) Group of piles closely spaced

(c) Group of piles with piles far apart

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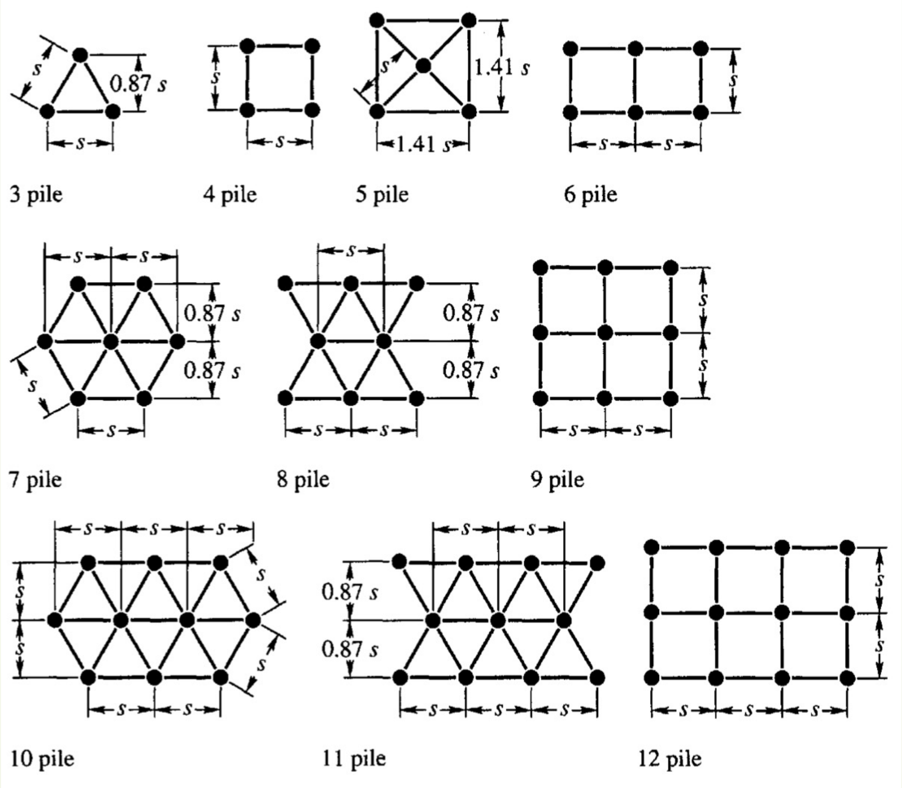
Spacing between piles depends on many factors:

- Overlapping stresses from the adjacent piles.
- Cost
- Efficiency of the group

Generally, the arrangement of the piles in group has various patterns depending on the types of pile and installation methods . The figure below may be considered as a guide for that.



4

The minimum allowable spacing of piles is usually stipulated in building codes.

- ✓ The spacings (*S*) for straight uniform diameter piles may vary from **2 to 6** times the diameter of the shaft.
- ✓ For friction piles, the minimum spacing recommended is **3d** where *d* is the diameter of the pile.
- ✓ For end bearing piles passing through relatively compressible strata, the spacing of piles shall not be less than **2.5d**.
- ✓ For end bearing piles passing through compressible strata and resting in stiff clay, the spacing may be increased to **3.5d**.
- ✓ For compaction piles, the spacing may be **Id**.

PILE GROUP EFFICIENCY

The spacing of piles is usually predetermined by practical and economical considerations. The design of a pile foundation subjected to vertical loads consists of

1. The determination of the ultimate load bearing capacity of the group Q_{gu} .
2. Determination of the settlement of the group, S_g , under an allowable load Q_{ga} .

The ultimate load of the group is generally different from the sum of the ultimate loads of individual piles Qu.

The factor of efficiency: $E_g = \dfrac{Q_{gu}}{\Sigma Q_u}$

Pile Group Efficiency Equation

There are many pile group equations. These equations are to be used very cautiously, and may in many cases be no better than a good guess. The Converse-Labarre Formula is one of the most widely used group-efficiency equations which is expressed as

$$E_g = 1 - \frac{\theta(n-1)m + (m-1)n}{90\,mn}$$

where
m = number of columns of piles in a group,
n = number of rows,
$\theta = \tan^{-1}(d/s)$ in degrees,
d = diameter of pile,
s = spacing of piles center to center.

Bearing capacity of pile groups (in compression) embedded in granular soil (Sand and gravel soils)

1) Driven piles:

$$Q_{ug} = n(e_s Q_{us} + e_p Q_{up})$$

where Q_{ug} is the ultimate capacity of the pile group, n is the number of piles in the group, e_s and e_p are the efficiencies of the pile group in friction and in point resistance respectively, and Q_{us} and Q_{up} are the ultimate capacity of one pile in friction and in point resistance respectively.

The pile group load test just described suggests e_s and e_p values of 0.67 and 1.8 respectively.

It is suggested that the efficiency of driven piles in loose sand be taken as 1.

Note: The above equation is not applicable when the tip (end) of pile rest at compressible stratum, so the bearing load of pile group will govern by the shear strength of the soil at the end rather than by the (efficiency) of the group within sand and gravel.

Bearing capacity of pile groups (in compression) embedded in cohesive soil (Clay soils)

1) Driven piles:

The effect of driving piles into cohesive soils (clays and silts) is very different from that of cohesionless soils. When piles are driven into clay soils, particularly when the soil is soft and sensitive, there will be considerable remolding of the soil. Besides there will be heaving of the soil between the piles since compaction during driving cannot be achieved in soils of such low permeability. There is every possibility of lifting of the pile during this process of heaving of the soil.

In case driven piles are to be used, the following steps should be favored:

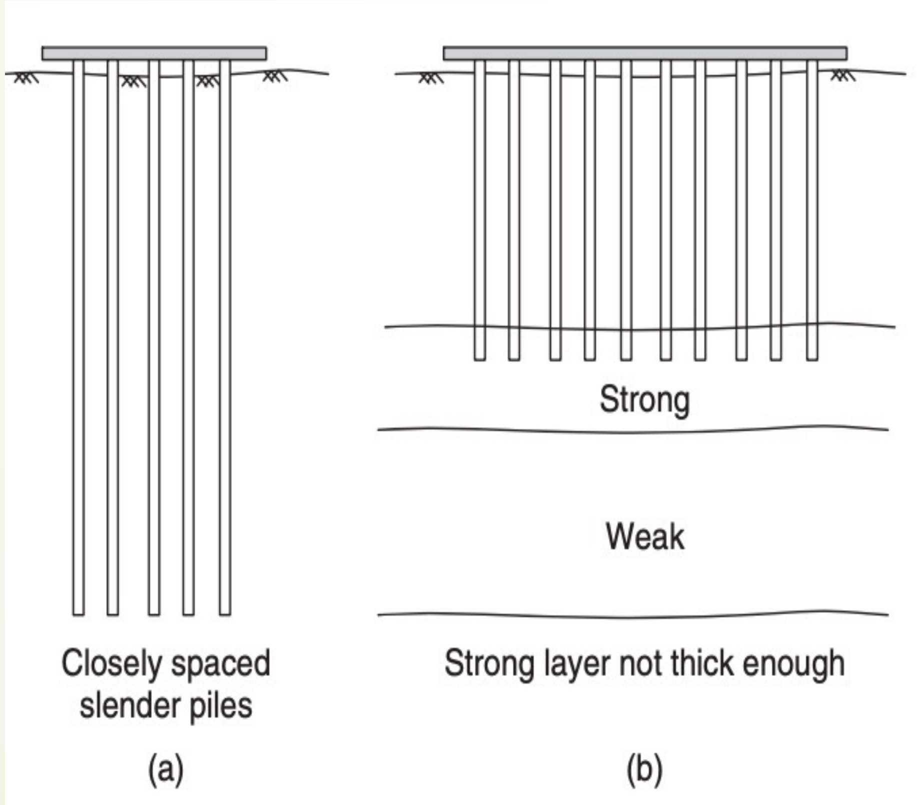
- ✓ Piles should be spaced at greater distances apart.
- ✓ Piles should be driven from the center of the group towards the edges, and,
- ✓ The rate of driving of each pile should be adjusted as to minimize the development of pore water pressure.

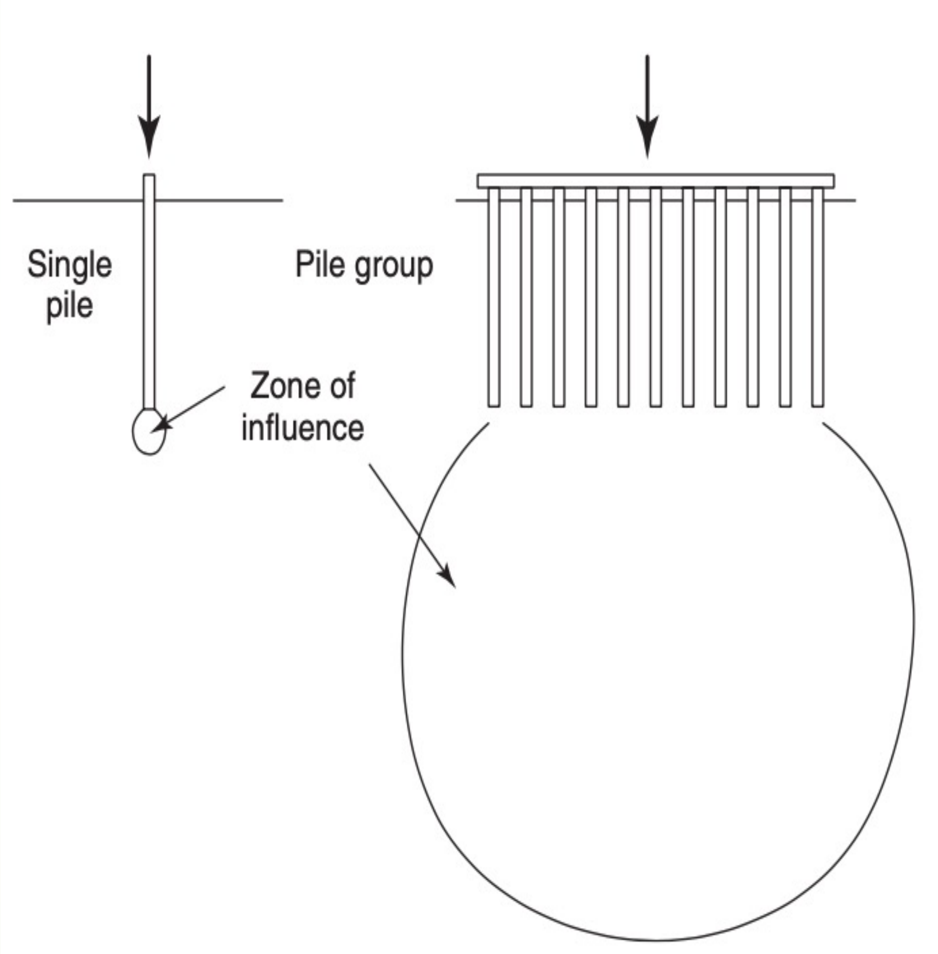
2) Bored piles:

Bored piles are, therefore, preferred to driven piles in cohesive soils.

Efficiency factor of pile groups in cohesive soil:

The efficiency ratio is less than unity at closer spacings and may reach unity at a spacing of about 8 diameters.





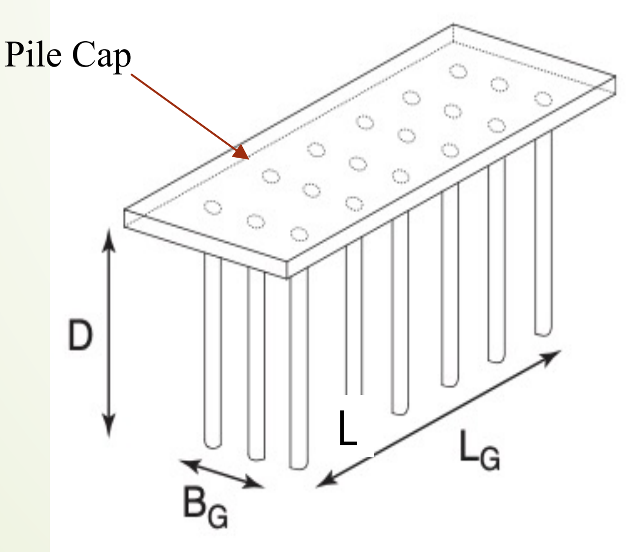
Experimental results have indicated that when a pile group installed in cohesive soils is loaded, it may fail by any one of the following ways:

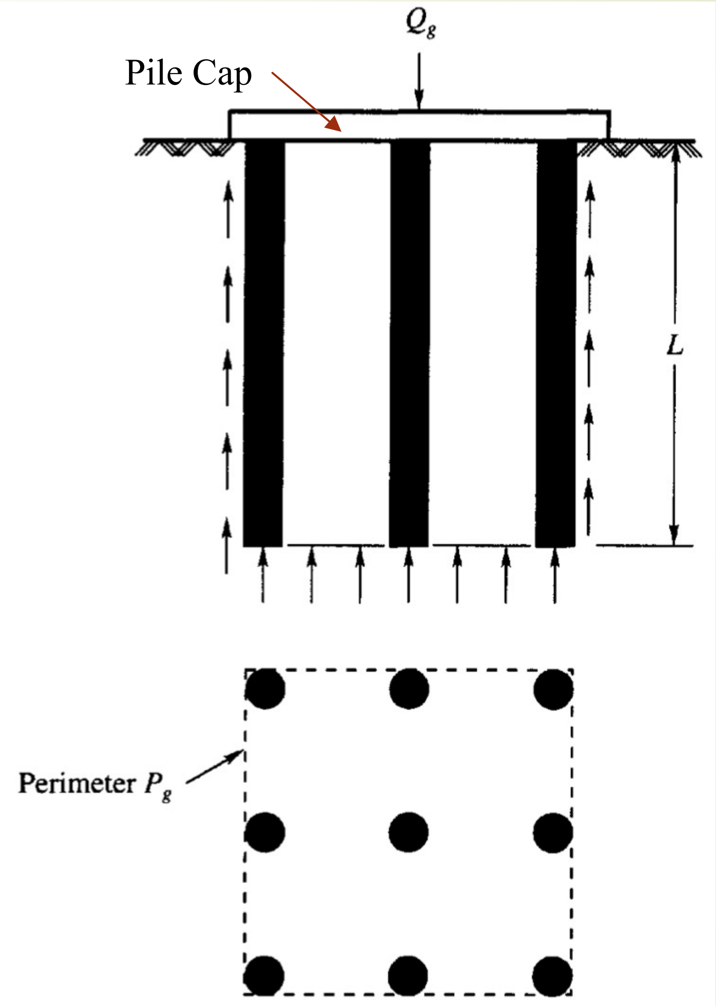
- May fail as a block (called block failure). It occurs when pile spacing (s) = (2-3) diameter of pile.
- Individual piles in the group may fail, when spacings between piles are wider.

1) Block model of failure and so, the capacity of pile groups may be written as:

$$Q_{ug} = cN_c\, A_g + P_g L\bar{c} \qquad (1)$$

where
- c = cohesive strength of clay beneath the pile group,
- $\bar{c}$ = average cohesive strength of clay around the group,
- L = length of pile,
- P_g = perimeter of pile group,
- A_g = sectional area of group, $A_g = B_g \times L_g$
- N_c = bearing capacity factor which may be assumed as 9 for deep foundations.





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The bearing capacity of a pile group on the basis of individual pile failure may be written as

$$Q_{gu} = nQ_u \tag{2}$$

where n = number of piles in the group,

Q_u = bearing capacity of an individual pile.

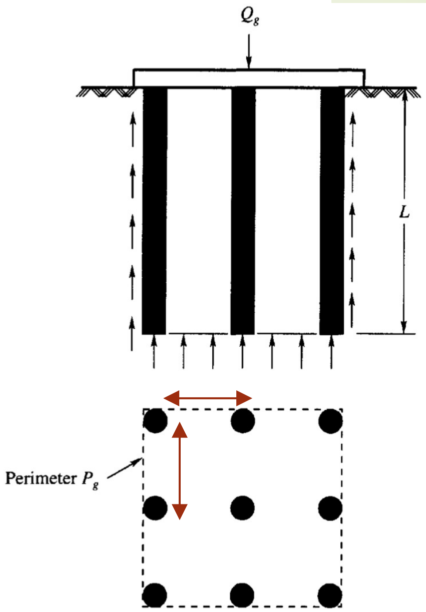
The bearing capacity of a pile group is normally taken as the smaller of the two given by Eqs. (1) and (2)

Example 1

A group of 9 piles with 3 piles in a row was driven into a soft clay extending from ground level to a great depth. The diameter and the length of the piles were 30 cm and 10 m respectively. The unconfined compressive strength of the clay is 70 kPa. If the piles were placed 90 cm center to center, compute the allowable load on the pile group on the basis of a shear failure criterion for a factor of safety of 2.5.

Solution

The allowable load on the group is to be calculated for two conditions: (a) block failure and (b) individual pile failure. The least of the two gives the allowable load on the group.



(a) Block failure As shown in the Figure and using eq. (1)

$$Q_{gu} = cN_cA_g + P_gL\bar{c} \quad \text{where} \quad N_c = 9, c = \bar{c} = 70/2 = 35 \text{ kN/m}^2$$

$$A_g = 2.1 \times 2.1 = 4.4 \text{ m}^2, \quad P_g = 4 \times 2.1 = 8.4 \text{ m}, \quad L = 10 \text{ m}$$

$$Q_{gu} = 35 \times 9 \times 4.4 + 8.4 \times 10 \times 35 = 4326 \text{ kN}, \quad Q_a = \frac{4326}{2.5} = 1730 \text{ kN}$$

(b) Individual pile failure

$$Q_u = Q_b + Q_f = q_b A_b + \alpha \bar{c} A_s. \text{ Assume } \alpha = 1.$$

Now, $q_b = cN_c = 35 \times 9 = 315 \text{ kN/m}^2$, $A_b = 0.07 \text{ m}^2$,

$$A_s = 3.14 \times 0.3 \times 10 = 9.42 \text{ m}^2$$

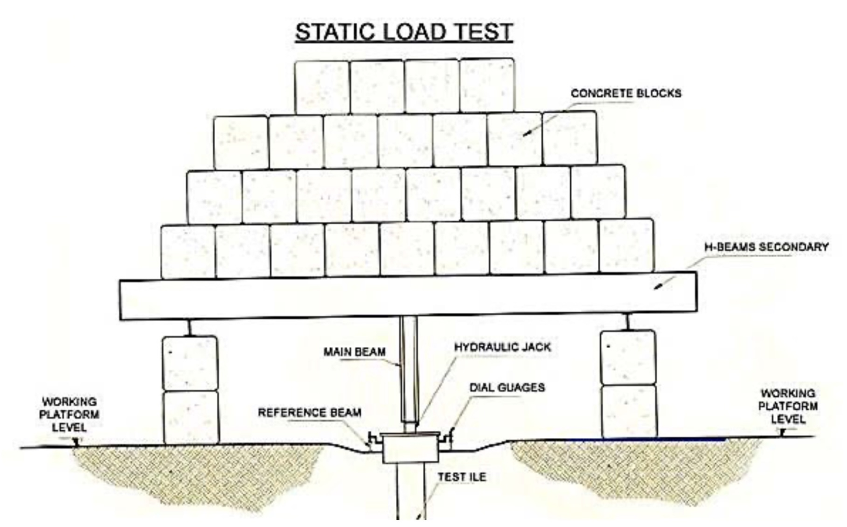
Substituting, $Q_u = 315 \times 0.07 + 1 \times 35 \times 9.42 = 352 \text{ kN}$

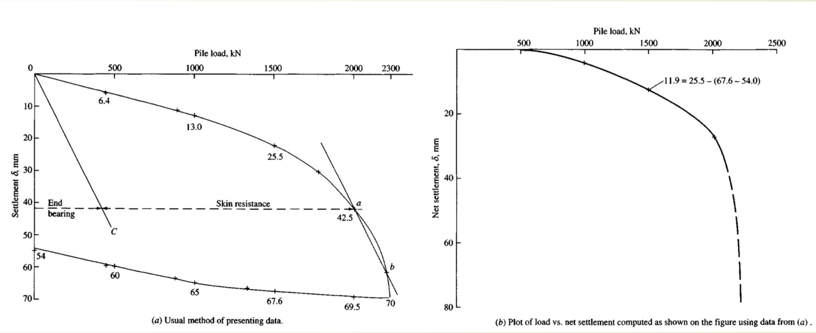
$$Q_{gu} = nQ_u = 9 \times 352 = 3168 \text{ kN}, \quad Q_a = \frac{3168}{2.5} = 1267 \text{ kN}$$

The allowable load is 1267 kN.

PILE-LOAD TESTS

The most reliable method to determine the load capacity of a pile is to load-test it. This consists in driving the pile to the design depth and applying a series of loads by some means. The usual procedure is to drive several of the piles in a group and use two or more of the adjacent piles for reactions to apply the load. A rigid beam spans across the test pile and is securely attached to the reaction piles. A large-capacity jack is placed between the reaction beam and the top of the test pile to produce the test load increments. For more details on procedure of this test, please, refer to ASTM D-1143

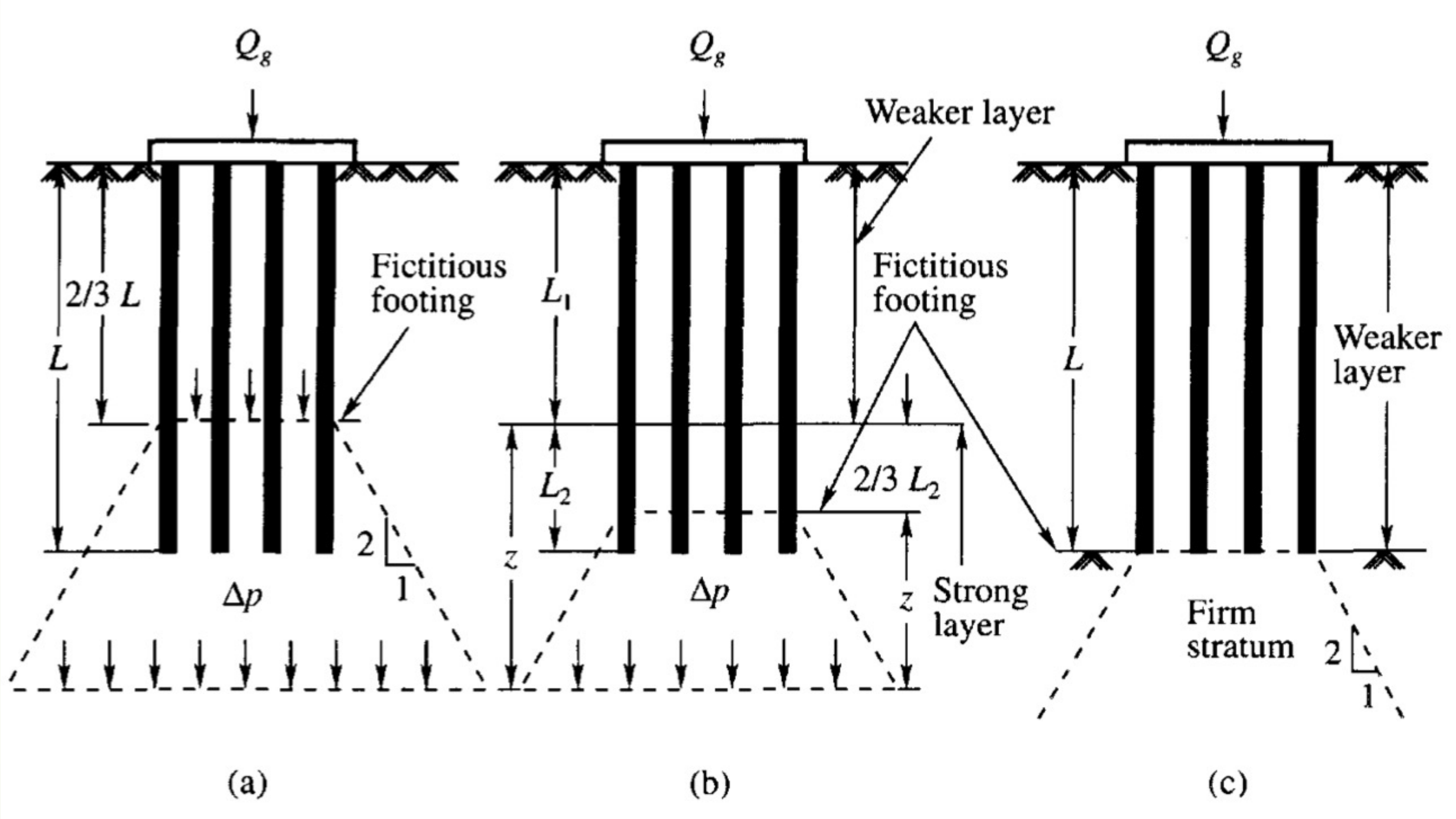




A typical results of Pile Test from (Bowels)

SETTLEMENT OF PILE GROUPS IN COHESIVE SOILS

The total settlements of pile groups may be calculated by making use of consolidation settlement equations. The problem involves evaluating the increase in stress Δp beneath a pile group when the group is subjected to a vertical load Q_g . The computation of stresses depends on the type of soil through which the pile passes. The methods of computing the stresses are explained in the Figure below:



- The soil in the first group given in the Figure (a) is homogeneous clay. The load Q_g is assumed to act on a fictitious footing at a depth (2/3)L from the surface and distributed over the sectional area of the group. The load on the pile group acting at this level is assumed to spread out at a 2 V : 1 H slope. The stress Δp at any depth z below the fictitious footing may be found as explained in soil mechanics.
- In the second group given in (b) of the Figure, the pile passes through a very weak layer of depth L_1 and the lower portion of length L_2 is embedded in a strong layer. In this case, the load Q_g is as summed to act at a depth equals to (2/3) L_2 below the surface of the strong layer and spreads at a 2 V : 1 H slope as before.
- In the third case shown in (c) of the Figure, the piles are point bearing piles. The load in this case is assumed to act at the level of the firm stratum and spreads out at a 2 V : 1 H slope as before.

Thank You

Foundation Engineering (2)

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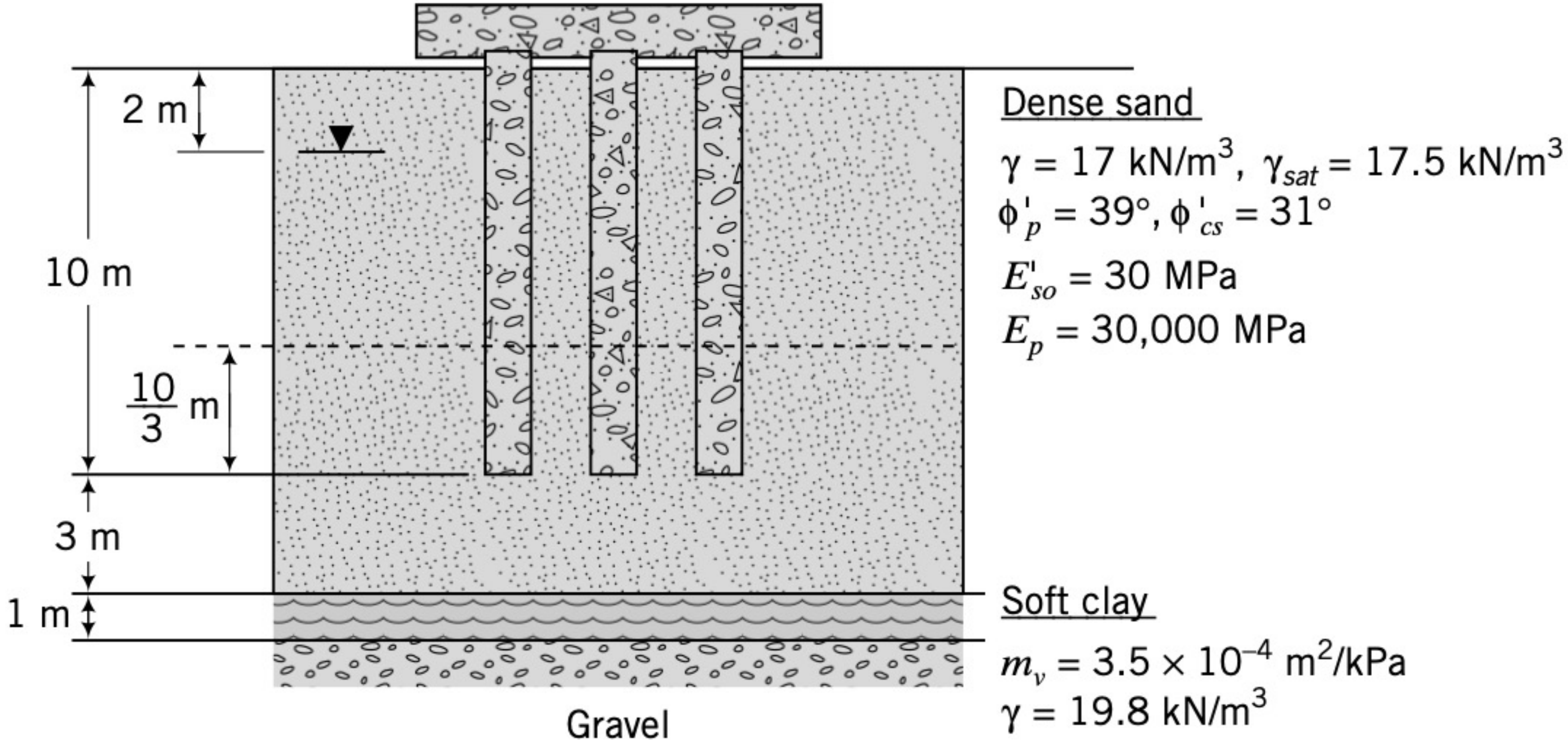
Design of Deep (Pile) Foundations (e)

- Examples
- Negative Skin Friction

Example: A 3x3 concrete pile group with a pile spacing of 1 m and pile diameter of 0.4 m supports a load of 2.5 MN (Figure).
(a) Determine the factor of safety for the pile group. (b) Calculate the total settlement of the pile group. The piles were driven.

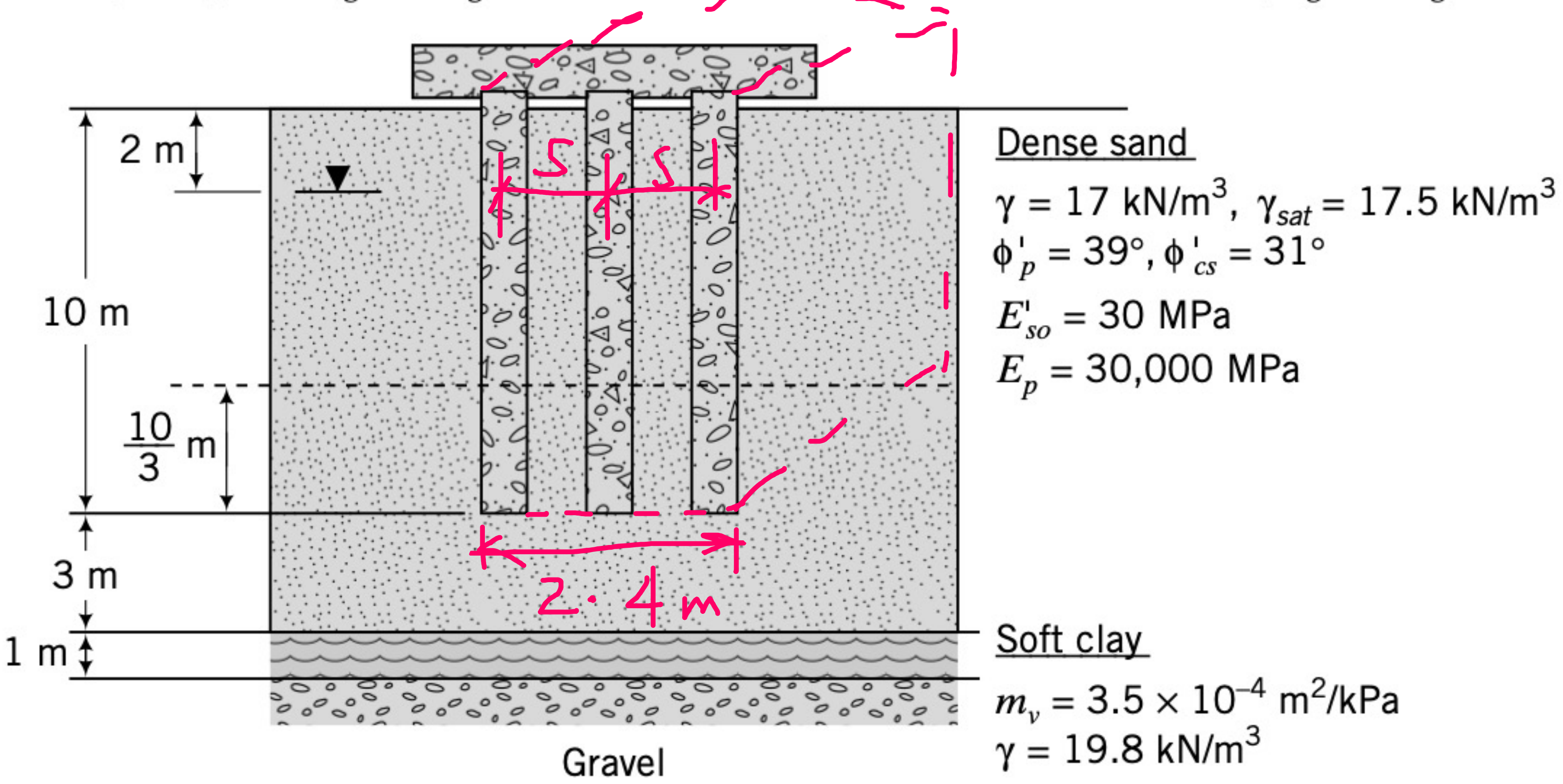


$$D = 0.4\ \text{m}, \quad \frac{L}{D} = \frac{10}{0.4} = 25; \quad n = 9\ \text{piles}, \quad s = 1\ \text{m}$$

Single pile: Perimeter $= \pi D = \pi \times 0.4 = 1.26$ m; $A_b = \frac{\pi D^2}{4} = \frac{\pi \times 0.4^2}{4} = 0.126\ \text{m}^2$

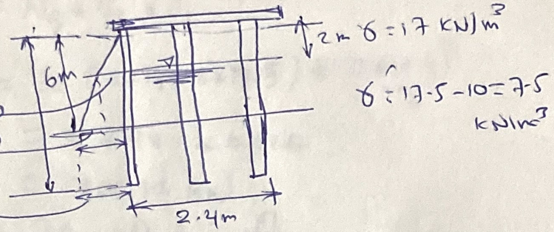
Group: $B_g = L_g = 2s + D = 2 \times 1 + 0.4 = 2.4$ m

Area of group $= L_g \times B_g = 2.4^2 = 5.76\ \text{m}^2$; Perimeter $= 2(L_g + B_g) = 2(2.4 + 2.4) = 9.6$ m



$For\ \phi = 31^o, Nq = 30$

Q_s ?



$\sigma_1 = 2*17 = 34$

$\sigma_2 = 4*7.5 = 30$

Total $= 3+34 = 68$

$\sigma = 68$

$\beta = 0.8\phi' = 0.8*31 = 24.8^\circ \Rightarrow \tan(24.8) = 0.462$

$h = 15D = 6m$,

$\sigma_{v,Avg} = \frac{0+34}{2} = 17\ kN/m^2 \rightarrow 2m$

$\sigma_{2,Avg} = \frac{34+68}{2} = 51\ kN/m^2 \Rightarrow (2-6m)$

$\sigma_{3,Avg} = \frac{68+68}{2} = 68 \Rightarrow (6-10m)$

$K = 0.8$

$Q_s = P\sum f_i = P\sum \sigma_{v_{avg,i}} * K_i * l_i * \beta$

$= \pi*0.4(17*0.8*2*0.462 + 51*0.8*4*0.462 + 68*0.8*4*0.462)$

$= \pi*0.4*0.462*0.8(34+204+272)$

$= 236.8\ kN$

$Q_p = N_q * \sigma'_{z_b} * A_b$

$= 30(2*17 + 8*7.5) * \pi*\frac{(0.4)^2}{4}$

$= 30*94*0.126$

$= 355.32\ kN$

Ultimate load capacity for pile

$= Q_s + Q_p = 236.8 + 355.32$

$= 592.12\ kN$

$Q_{ult_g} = 592.12*9 = 5329.1\ kN$

for Mode of group failure

$P_g = 2(L_g + B_g) = 9.6\ m$

~~Side surface Area = 10 * 9.6 = 96 m²~~

So for group, skin friction capacity

$= 236.8 * \frac{9.6m}{\pi*0.4} = 236.8*\frac{9.6}{1.26} = 1804.2\ kN$

end bearing for group $= 355.3 * \frac{5.76}{0.126} = 16242.3\ kN$

Total capacity $= 18046.5\ kN$

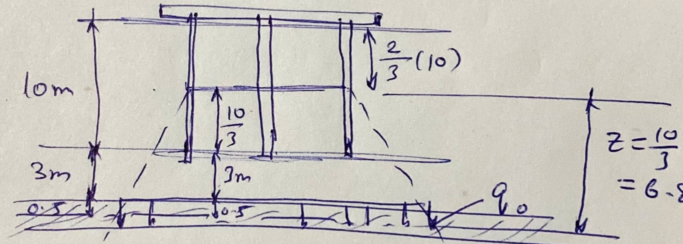
The minimum one is the govern :-

Q_u of the group $= 5329.1\ kN$

Q_{all} (Load from structure) $= 2500\ kN$ (given)

$F_s = \frac{Q_u}{Q_{all}} = \frac{5239.1}{2500} = 2.1 \simeq 2$

For Consolidation Settlement :-



$z = \frac{10}{3} + 3 + 0.5 = 6.83\ m$

$q_o = \frac{Q_{avg}}{(B_g+z)^2} = \frac{2500\ kN}{(2.4+6.83)} = 29.3\ kN/m^2$

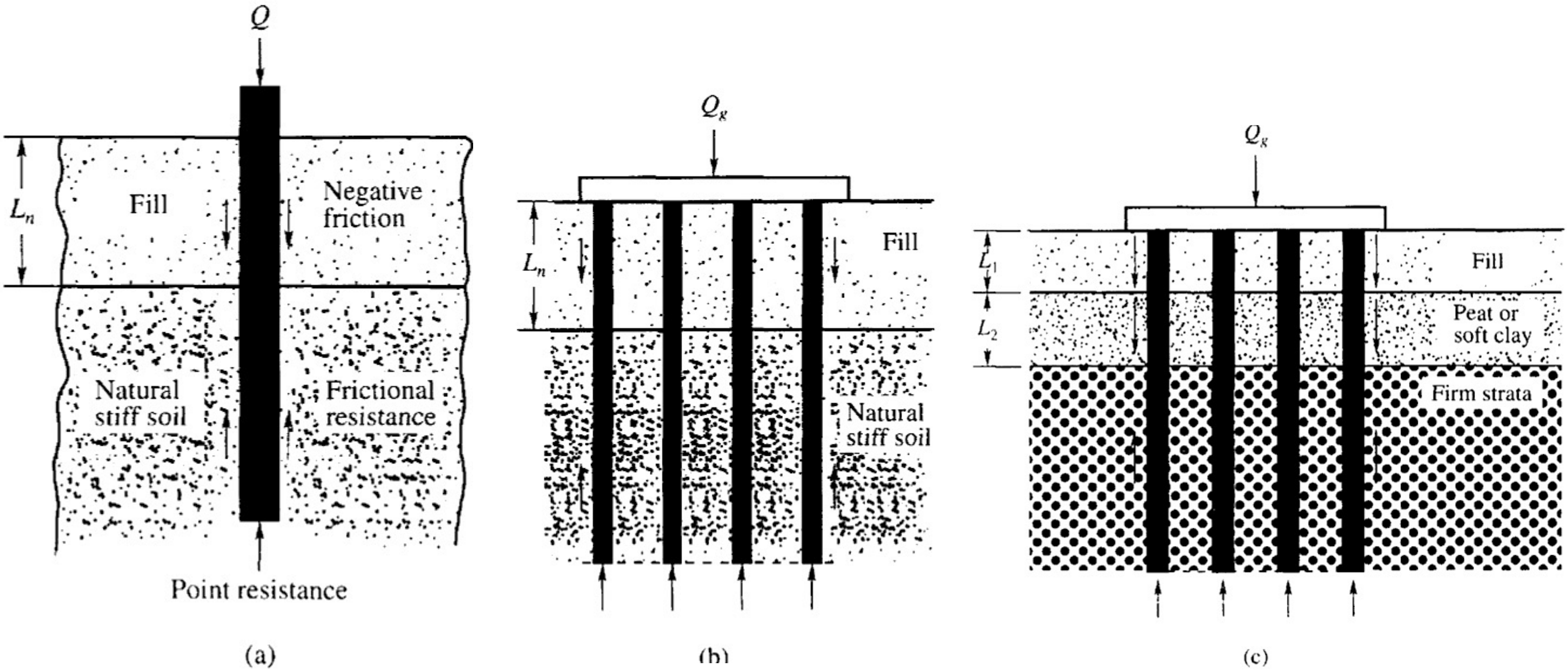
$S_c = m_v * H_o * q_o = 3.5*10^{-4}*1*29.3$

$= 102.7*10^{-4}\ m \simeq 10mm$

NEGATIVE FRICTION

Figure (a) shows a single pile and (b) a group of piles passing through a recently constructed cohesive soil fill. The soil below the fill had completely consolidated under its overburden pressure. When the fill starts consolidating under its own overburden pressure, it develops a drag on the surface of the pile. This drag on the surface of the pile is called '*negative friction*'. Negative friction may develop if the fill material is loose cohesionless soil. Negative friction can also occur when fill is placed over peat or a soft clay stratum as shown in Fig. (c). The superimposed loading on such compressible stratum causes heavy settlement of the fill with consequent drag on piles.
Negative friction may develop by lowering the ground water which increases the effective stress causing consolidation of the soil with resultant settlement and friction forces being developed on the pile.
Negative friction must be allowed when considering the factor of safety on the ultimate carrying capacity of a pile. The factor of safety, Fs, where negative friction is likely to occur may be written as:

$$F_s = \frac{\text{Ultimate carrying capacity of a single pile or group of piles}}{\text{Working load + Negative skin friction load}}$$



Negative Friction on Piles

Professor Dr. Hussein M. Ashour Al.Khuzaie; hma@mu.edu.iq

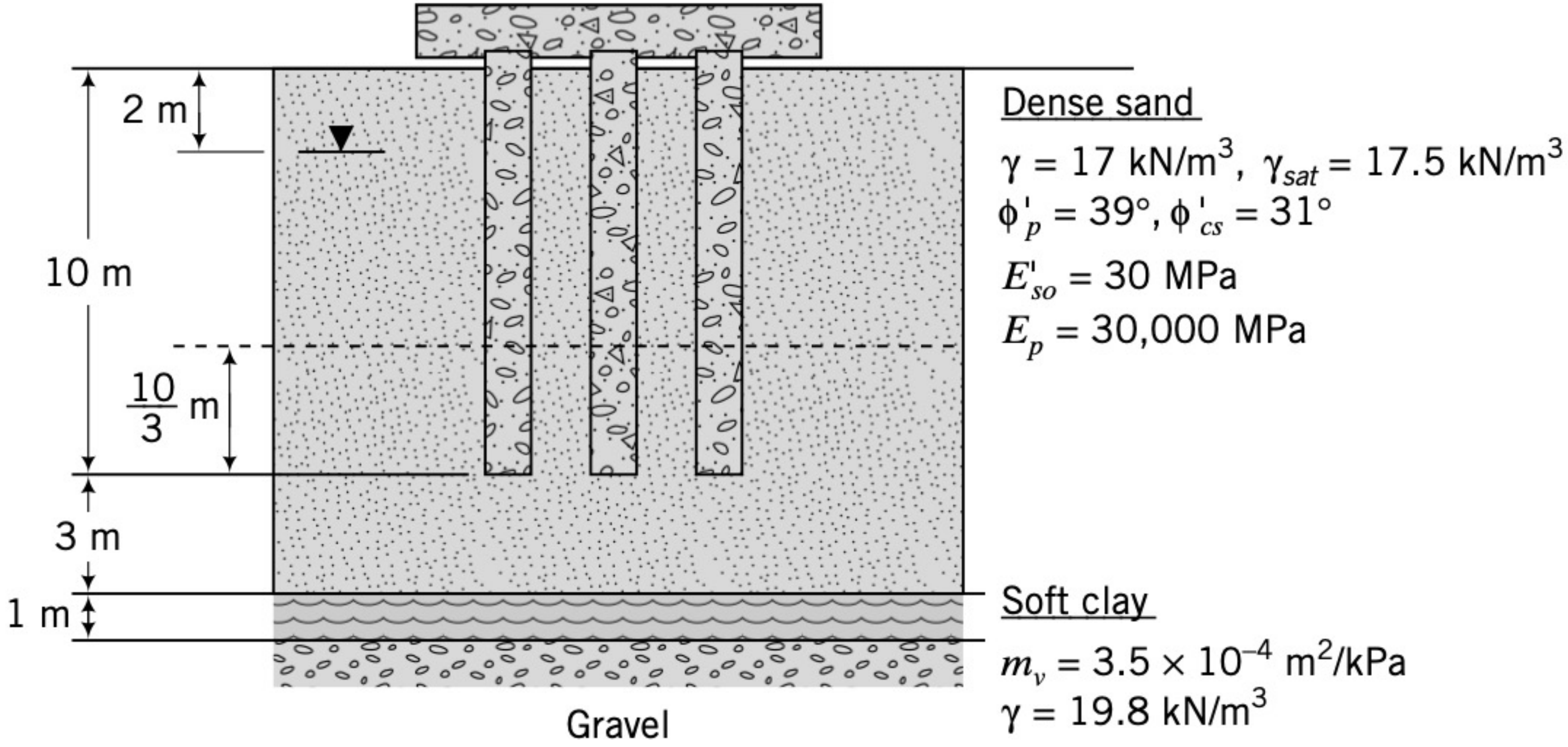
Questions?

Foundation Engineering (2)
Civil Dept., College of Engineering, Al-Muthanna University
Professor Dr. Hussein M. Al.Khuzaie; hma@mu.edu.iq

Design of Deep (Pile) Foundations (e)

- Examples
- Negative Skin Friction

Example: A 3x3 concrete pile group with a pile spacing of 1 m and pile diameter of 0.4 m supports a load of 2.5 MN (Figure).
(a) Determine the factor of safety for the pile group. (b) Calculate the total settlement of the pile group. The piles were driven.



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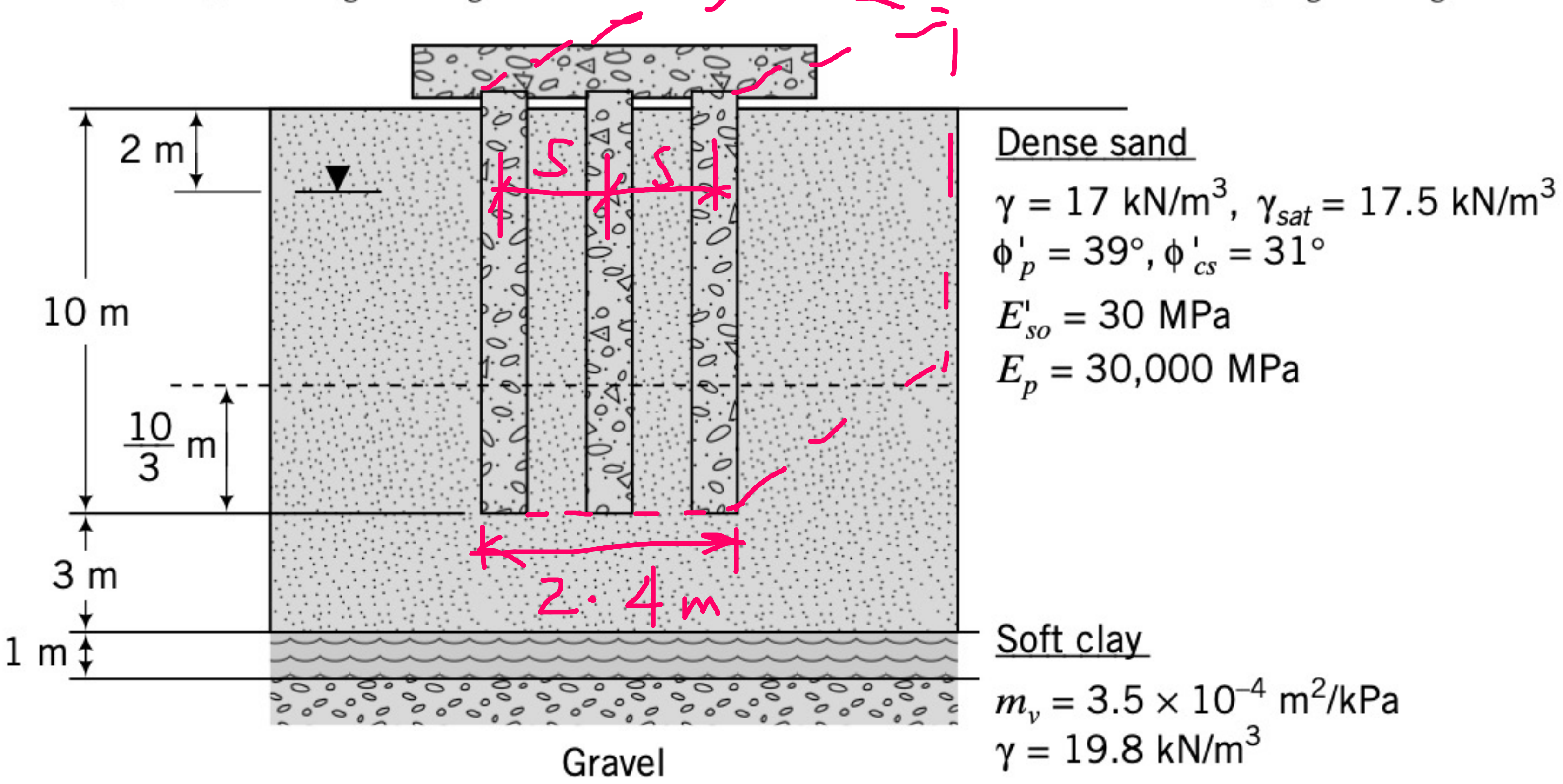

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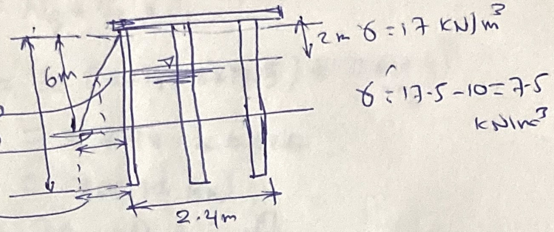

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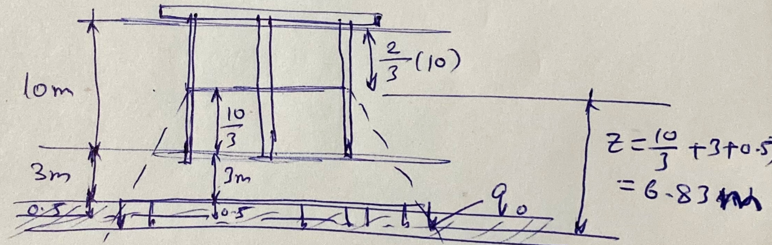

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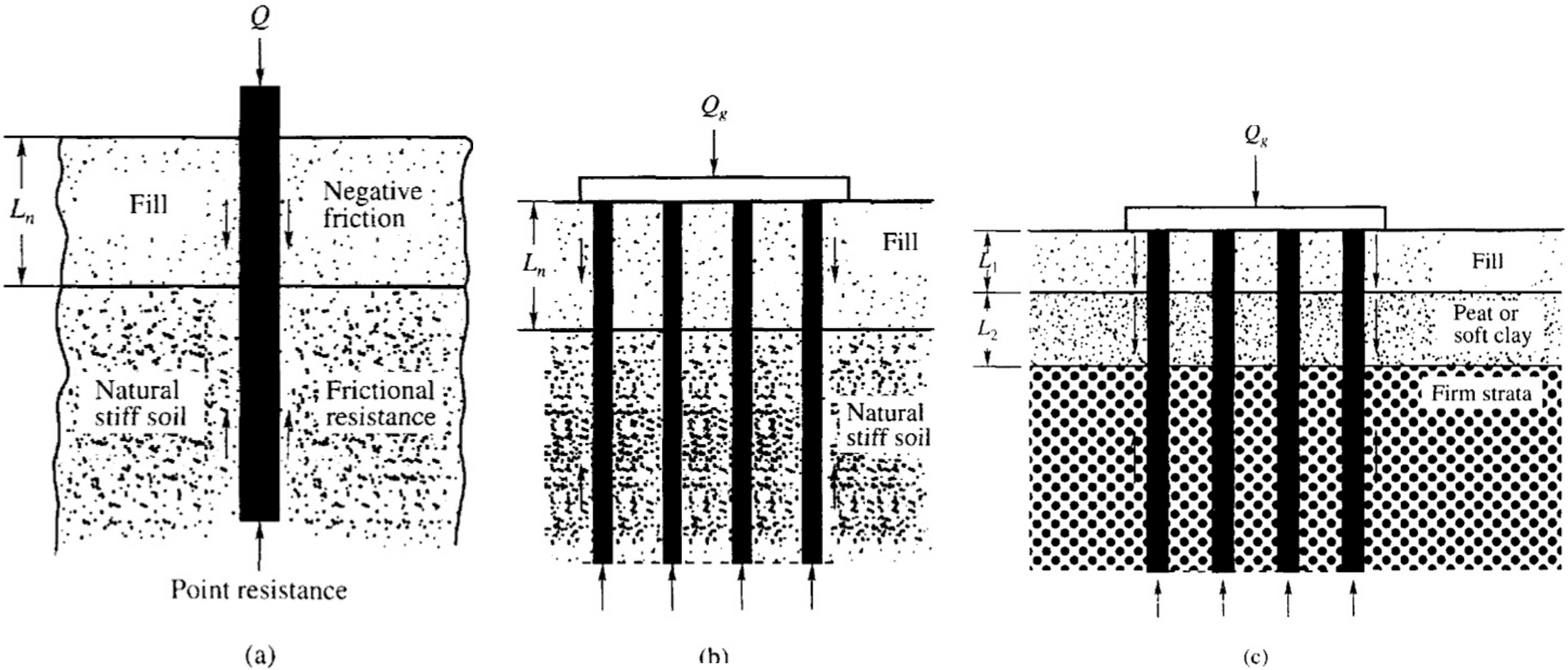

Negative Friction on Piles

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Questions?

Foundation Engineering (2)

Al-Muthanna University, College of Engineering, Civil Engineering Department
Professor Dr. Hussein Mandeel Ashour Al.Khuzaie (*hma@mu.edu.iq*)

Design of Shallow Foundations:

- Types
- Factors to be considered for depth of foundation
- Spread footing design and stress distribution

Introduction

A foundation is defined as that part of the structure that supports the weight of the structure and transmits the load to underlying soil or rock. Foundation engineering applies the knowledge of soil mechanics, rock mechanics, geology, and structural engineering to the design and construction of foundations for buildings and other structures.

Foundations : built for above-ground structures include shallow and deep foundations.

Retaining structures: include earth-filled dams and retaining walls. Earthworks include embankments, tunnels, dikes, levees, channels, reservoirs, deposition of hazardous waste and sanitary landfills.

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A proper design requires the following:

1. Determining the building purpose, probable service-life loading, type of framing, soil profile, construction methods, and construction costs

2. Determining the client/owner's needs

3. Making the design, but ensuring that it does not excessively degrade the environment, and provides a margin of safety that produces a tolerable risk level to all parties: the public, the owner, and the engineer

ADDITIONAL CONSIDERATIONS

The Following additional considerations that may have to be taken into account at specific sites.

1. Depth must be adequate to avoid lateral squeezing of material from beneath the foundation for footings and mats. Similarly, excavation for the foundation must take into account that this can happen to existing building footings on adjacent sites and requires that suitable precautions be taken. The number of settlement cracks that are found by owners of existing buildings when excavations for adjacent structures begin is truly amazing.
2. Depth of foundation must be below the zone of seasonal volume changes caused by freezing, thawing, and plant growth. Most local building codes will contain minimum depth requirements.

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3. The foundation scheme may have to consider expansive soil conditions. Here the building tends to capture upward-migrating soil water vapor, which condenses and saturates the soil in the interior zone, even as normal perimeter evaporation takes place. The soil in a distressingly large number of geographic areas tends to swell in the presence of substantial moisture and carry the foundation up with it.

4. In addition to compressive strength considerations, the foundation system must be safe against overturning, sliding, and any uplift (flotation).

5. System must be protected against corrosion or deterioration due to harmful materials present in the soil. Safety is a particular concern in reclaiming sanitary landfills but has application for marine and other situations where chemical agents that are present can corrode metal pilings, destroy wood sheeting/piling, cause adverse reactions with Portland cement in concrete footings or piles, and so forth.

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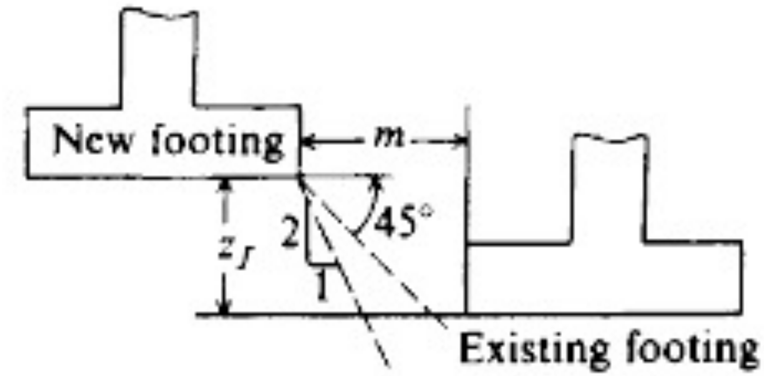
6. Foundation system should be adequate to sustain some later changes in site or construction geometry and be easily modified should changes in the superstructure and loading become necessary.
7. The foundation should be buildable with available construction personnel. For one-of-a-kind projects there may be no previous experience. In this case, it is necessary that all concerned parties carefully work together to achieve the desired result.
8. The foundation and site development must meet local environmental standards, including determining if the building is or has the potential for being contaminated with hazardous materials from ground contact (for example, radon or methane gas). Adequate air circulation and ventilation within the building are the responsibility of the mechanical engineering group of the design team.

•FOOTING DEPTH AND SPACING

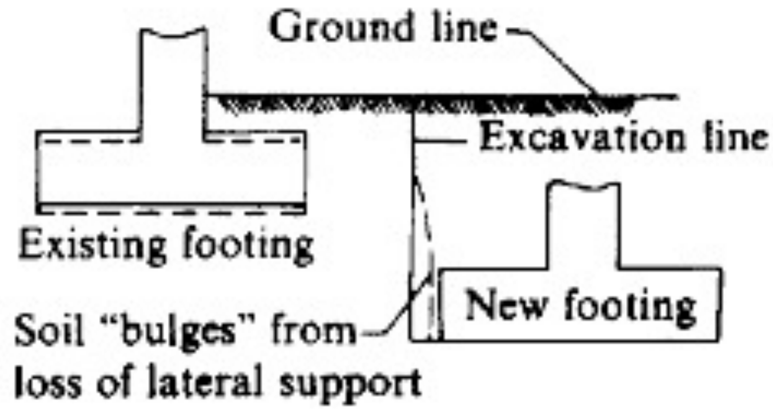
Footings should be carried below:

1. The frost line
2. Zones of high volume change due to moisture fluctuations
3. Topsoil or organic material
4. Peat and muck
5. Unconsolidated material such as abandoned (or closed) garbage dumps and similar filled-in areas.

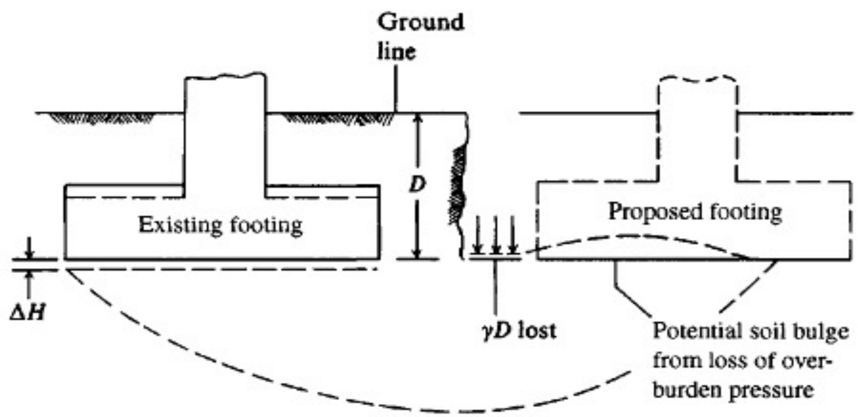
•SPACING



(a) An approximation for the spacing of footings
Make *m > zf*.



(b) Possible settlement of "existing" footing because of loss of lateral support of soil wedge beneath existing footing.



Potential settlement or instability from loss of overburden pressure.

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SELECTION OF FOUNDATION TYPE

Table below tabulates the use and application of the several general foundation types:

Foundation types and typical usage

Foundation type	Use	Applicable soil conditions
	Shallow foundations (generally $D/B \leq 1$)	
Spread footings, wall footings	Individual columns, walls	Any conditions where bearing capacity is adequate for applied load. May use on a single stratum; firm layer over soft layer or soft layer over firm layer. Check settlements from any source.
Combined footings	Two to four columns on footing and/or space is limited	Same as for spread footings above.
Mat foundations	Several rows of parallel columns; heavy column loads; use to reduce differential settlements	Soil bearing capacity is generally less than for spread footings, and over half the plan area would be covered by spread footings. Check settlements from any source.

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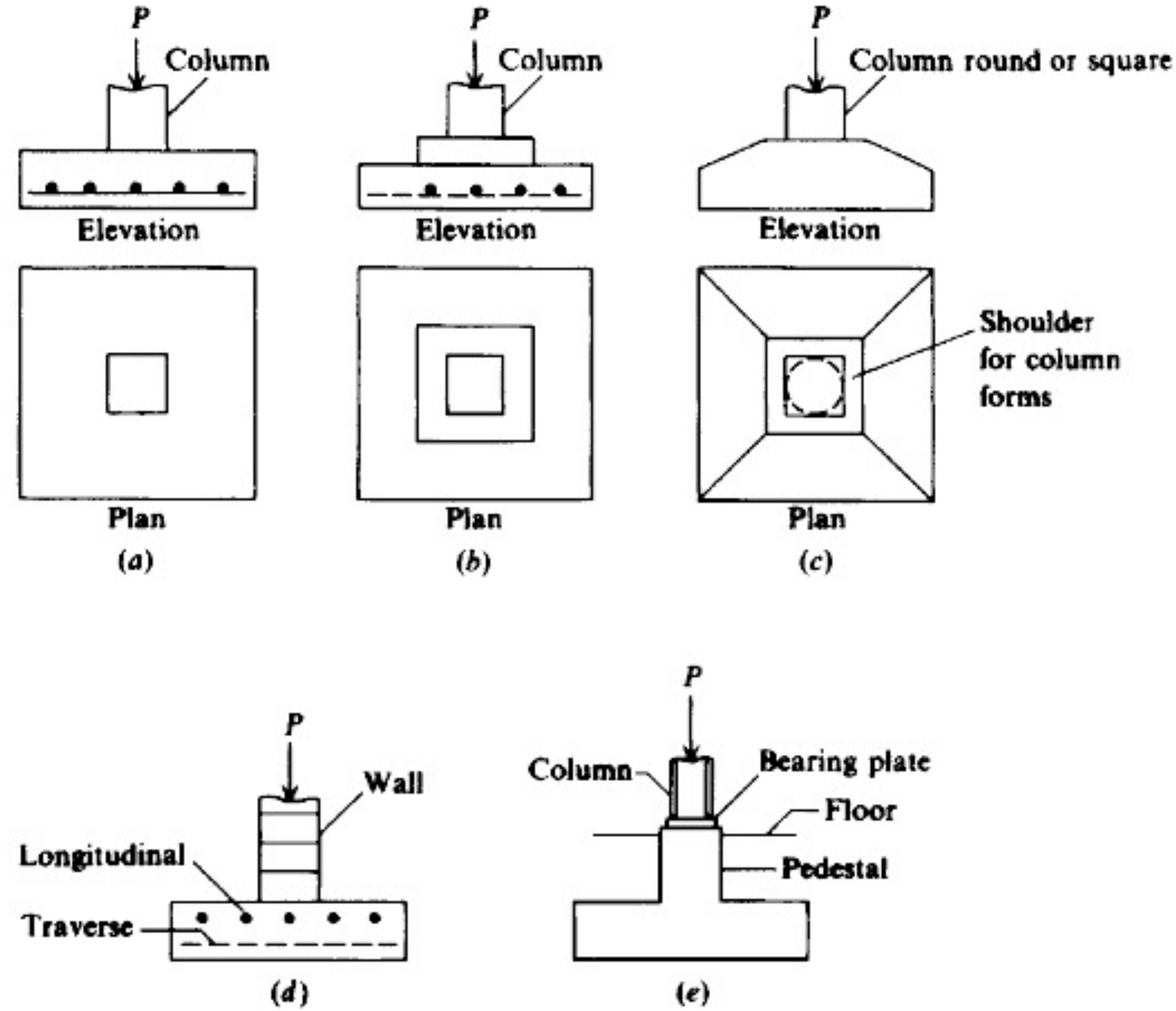
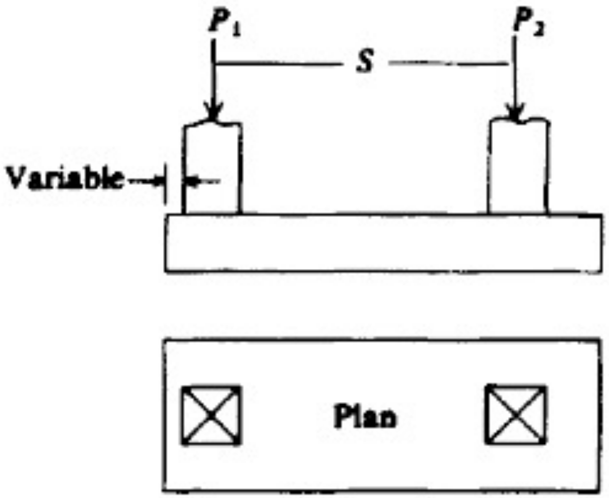
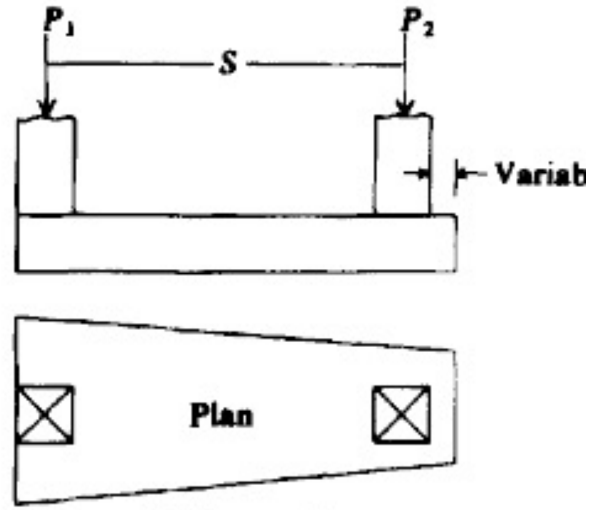
SHALLOW FOUNDATION TYPES



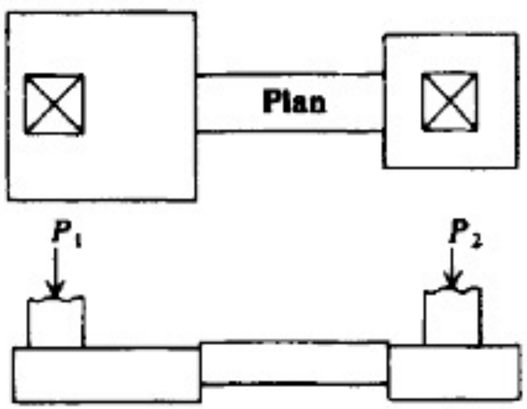
Typical footings, *(a) Single or spread footings; (b) stepped footing; (c) sloped footing; (d) wall* footing; *(e) footing with pedestal.*



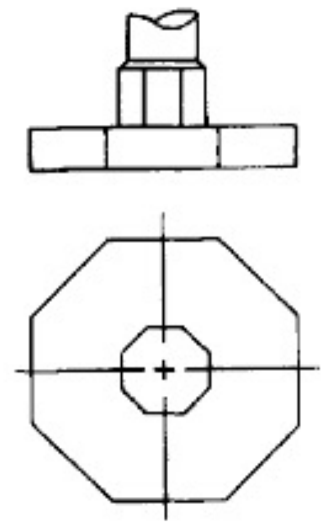
(a) Rectangular



(b) Trapezoid



(c) Strap


(d) Industrial tower

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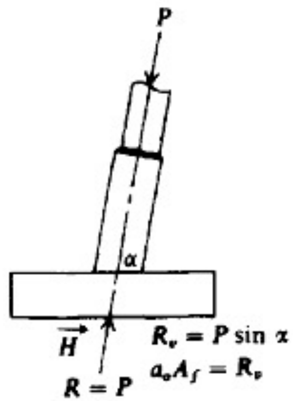
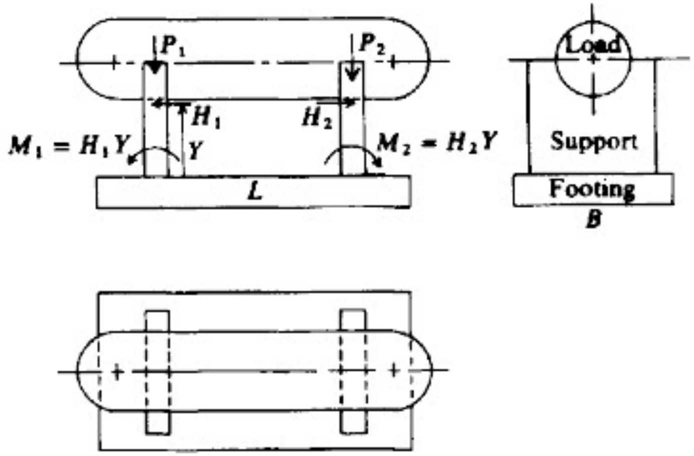
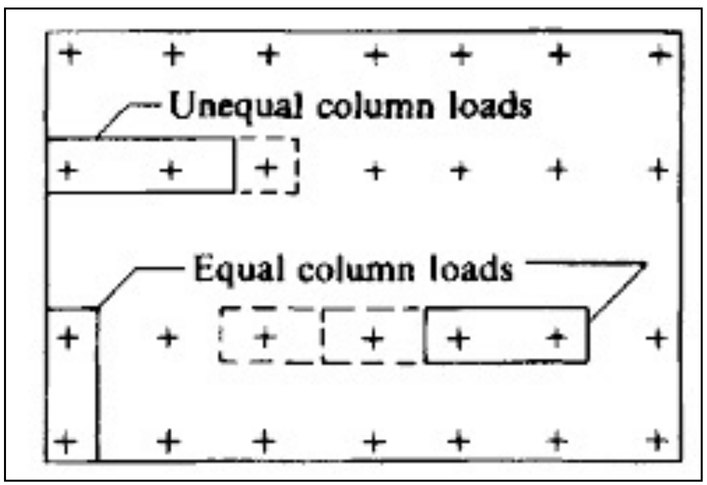


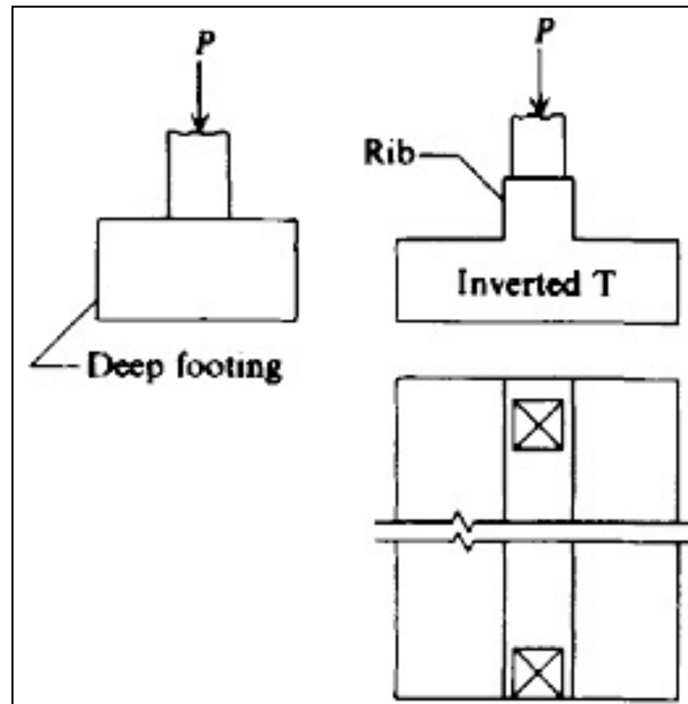
(e) Eccentric



(f) Industrial



Raft (Mat) Foundation

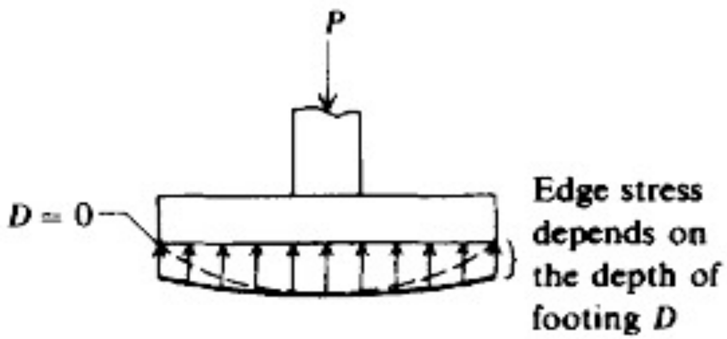


Other types of foundation

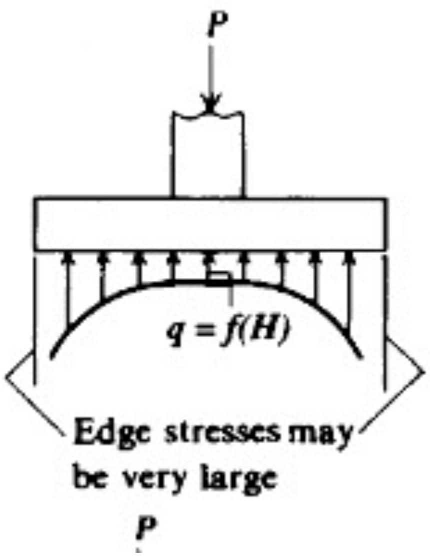
SPREAD FOOTING DESIGN

- A footing carrying a single column is called a *spread footing, since its function is to "spread"* the column load laterally to the soil so that the stress intensity is reduced to a value that the soil can safely carry. These members are sometimes called *single* or *isolated* footings.
- Footings are designed to resist the ***full dead load*** delivered by the column. The live load contribution may be either the full amount for one- or two-story buildings or a reduced value as allowed by the local building code for multistory structures.
- The **safety factor** ranges from **2 to 5** for **cohesionless materials** depending on density, effects of failure, and consultant caution, while for **cohesive soil** it can be taken between **3-6** , in this case ,the max. value for the factor can be furnished when the consolidation settlement might occur over a long period of time.

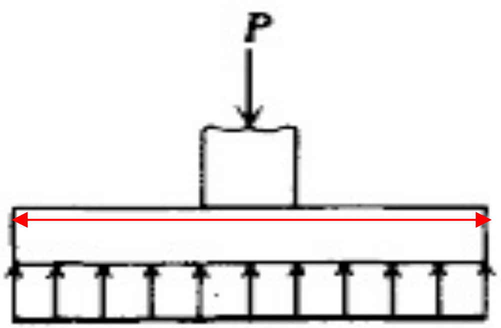
Probable pressure distribution beneath a rigid footing:



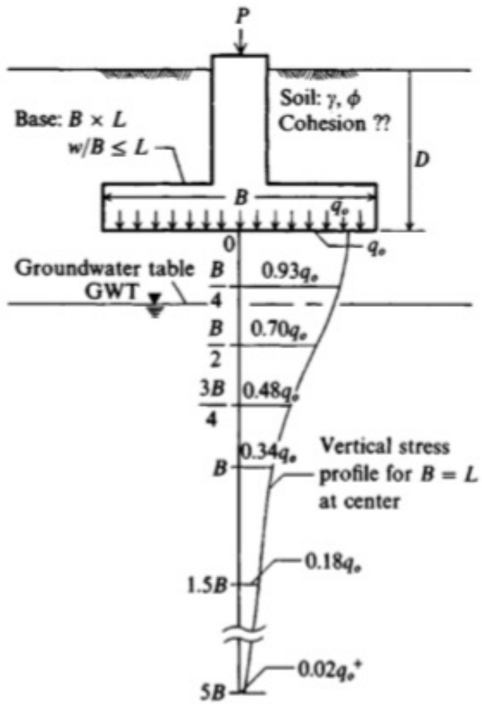
a) On a cohesionless soil;



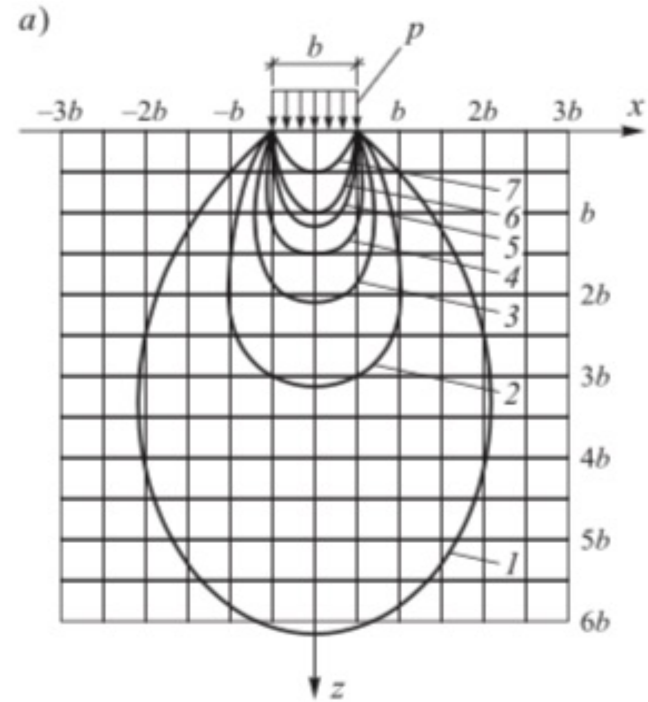
b) generally for cohesive soil;



Usually, assumed Linear Distribution.



(a) Spread foundation. Base contact pressure



a) 1 – 0,1p; 2 – 0,2 p; 3 – 0,3p; 4 – 0,4p; 5 – 0,5p; 6 – 0,6p; 7 – 0,8p;

Stress bulb

Stress of super structure with foundation mass weight distribution under the base of foundation.

Best wishes towards health and success

Foundation Engineering (2)

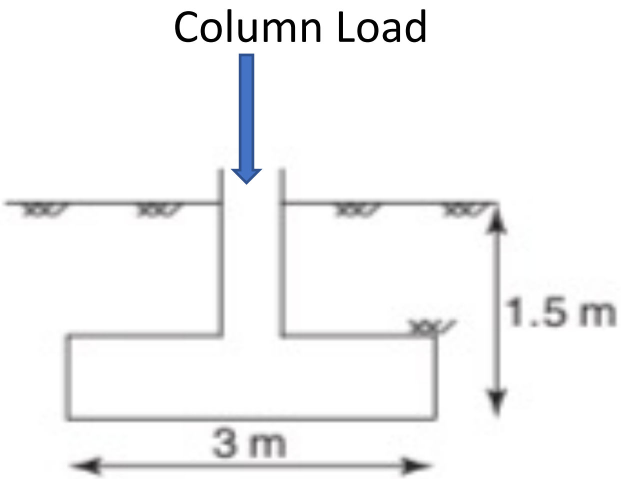
Al-Muthanna University, College of Engineering, Civil Engineering Department
Professor Dr. Hussein Mandeel Ashour Al.Khuzaie (hma@mu.edu.iq)

Design of Shallow Foundations: Geotechnical Design of Spread and Strip (Wall) footings:

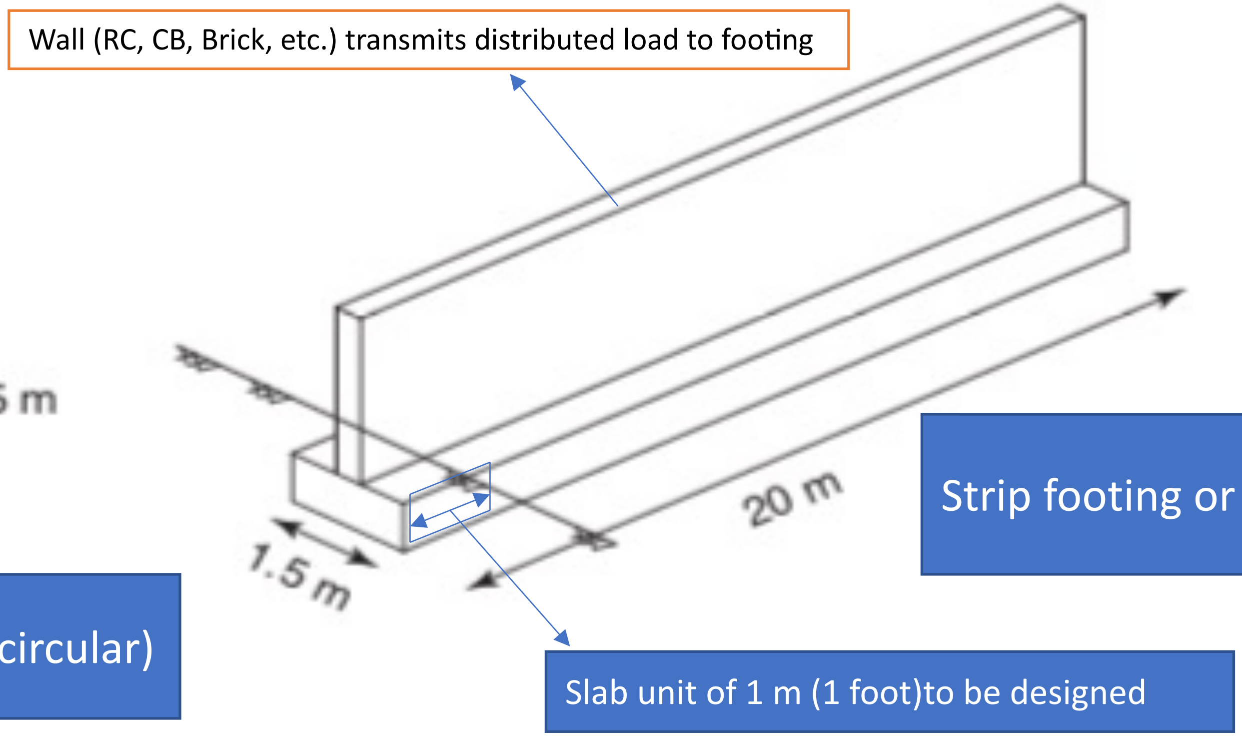
- Criteria and factors
- Base area determination (Geotechnical Design)
- General notes and remarks

General view of spread footing:

- ✓ Under column load (in general, concentrated)
- ✓ Under wall distributed load



Spread footing (square, rectangular, or circular)



Strip footing or wall footing

Geotechnical Design (Base area determination), or Proportioning

Building Code Requirements for Structural Concrete (ACI 318-11):

15.2.2 — Base area of footing (for shallow foundation) or number and arrangement of piles (for deep foundation)

shall be determined from unfactored forces and moments

transmitted by footing to soil or piles and *permissible soil pressure* or permissible pile capacity *determined through principles of soil mechanics (as studied in previous lectures).*

Steps of Geotechnical Design (Proportioning) of Footings:

1) Determine allowable bearing capacity (permissible stress of soil), using the methods had been explained in the previous lectures, with appropriate factor of safety.
2) Determine the load (unfactored live and dead loads, or any type of loadings) of super structure (including weight of foundation), using the structural methods of analysis which had been studied by theory of structures (manually) or by software.
3) Find the area of the foundation (base area), or as it is called proportion the foundation dimensions of plan area (foot print of footing).

So, that the stress at the level of base of footing (*p*) is small than the applied load.

Examples on Geotechnical Proportioning of spread (strip footing)

1) A column load of 2000 kN (Q) is to be supported by a square spread footing on a very stiff clay. Recommend (proportion) the size (Plan dimensions) of the footing after addressing the issue of bearing capacity, i.e. to be safe against shear failure. Soil properties: Su = 100 kPa, γ = 18 kN/m^3. If you need additional properties, assume reasonable values.

Solution:

Suppose: the depth of foundation D_f = 0.5 m, and

the factor of safety (FOS) =3

The soil is very stiff soil and for undrained condition, use Skempton's equation, so, $q_u = CN_c + D_f\,\gamma$, where, $C=S_u$=100 kPa, so,

$q_u = 100 * 6.3 + 0.5 * 18 = 639$ kPa

$q_{all} = q_u/FOS = 639/3 = 213$ kPa

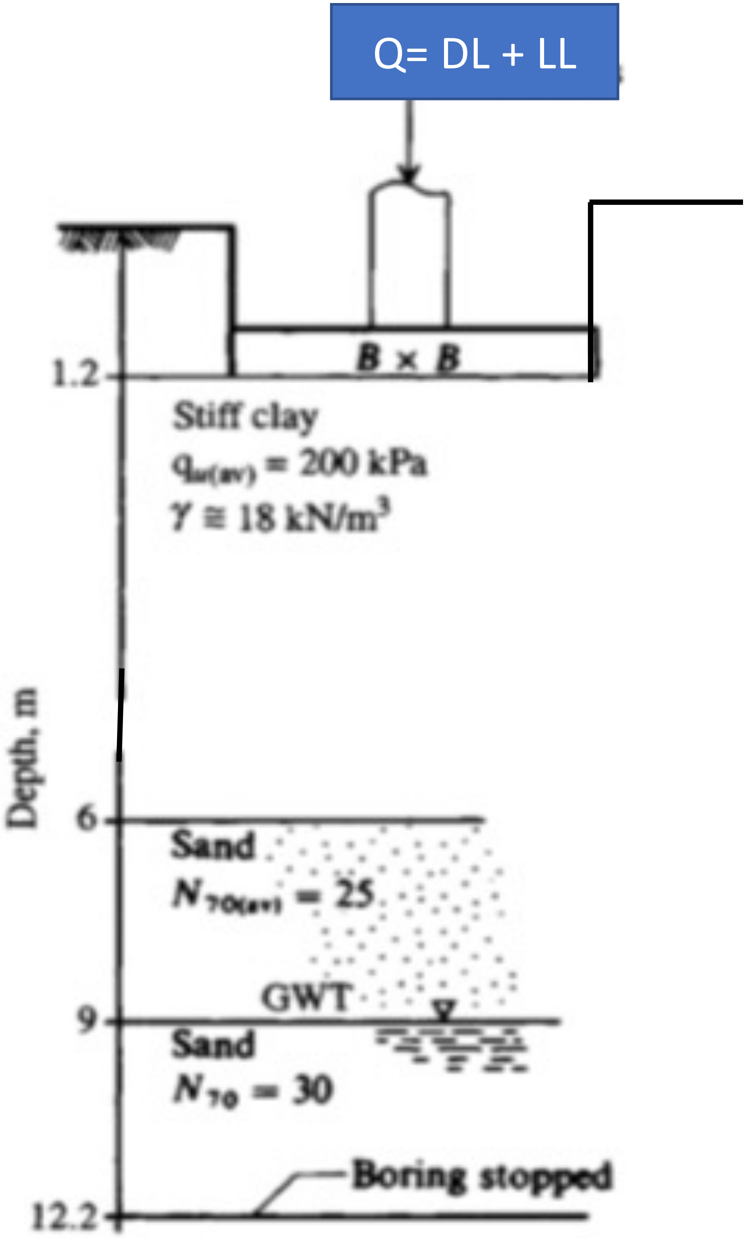
Applied pressure by column load (p) (unfactored load)=Q/A

$A = B^2 \succ p = 2000/B^2 \blacktriangleright p \leqq q_{all} \rightarrow 2000/B^2 = 213$,

So, $B = \sqrt{2000/213} = 3.06$ m use 3.5 m

Examples on Geotechnical Proportioning of spread (strip footing), cont'd

2) Design a spread footing for the average soil conditions and footing load given in the Fig. In this case the designer preferred to select the allowable soil pressure from a soil profile provided by the geotechnical engineer. Here, the $q_{un\,(av)}$ is the unconfined compression strength. The applied load is in form of dead load (DL) and live load (LL). DL = 350 kN and LL = 450 kN.



Solution:

Step 1. From the soil profile find qa. To start, we readily obtain $qa = qu$ from the average q_u . (FOS = 3. Estimate $\gamma_{clay} \approx 18.00$ kN/m^3 . So, we can include the qNq term (and $Nq = 1.0$):

q_a = 200 kPa + 1.2(18)(I) ≈ 220 kPa (Use 200 kPa).

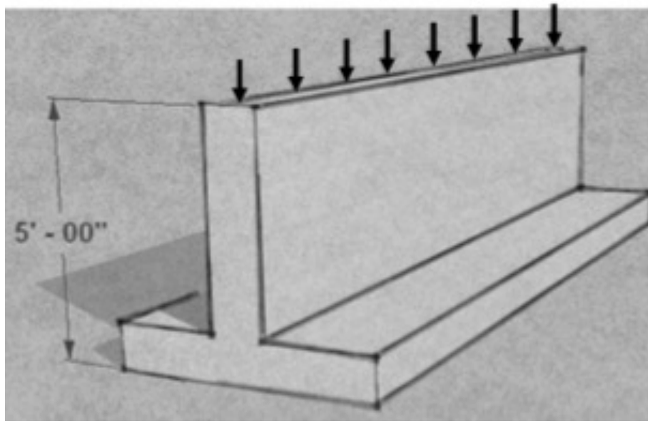
Step 2. Find tentative base dimensions B using a square footing, or

$Q = 350 + 450 = 800$ kN and

$B^2\, q_a = Q$

$$B = \sqrt{\frac{800}{200}} = 2\ m.$$

3) A 12 in thick concrete wall carries a service dead load of 10 kips/ft and a service live load of 12.5 kips/ft. the allowable soil pressure, q_a, is 5000 psf (Pound per Square Foot) at the level of the base of the footing, which is5 ft below the final ground surface. Proportion the footing of this wall.



Solution

Calculate bearing area, A_{req}

A_{req} = service load / q_a

Service load = DL + LL= 10 + 12.5 = 22.5 kips/ft

A_{req} = 22.5/5 = 4.5 ft^2 per foot of length of the wall

Trying a footing 4 ft 12 in wide .

General notes and remarks :

1- The lecture in PPT and video will be uploaded on both Classroom and Moodle (e-learning : mu . edu . iq) .

2- Assignments and quizzes will be announced on the classroom .

3- Any comment or request from you, please, send it by classroom .

With best wishes towards success

Foundation Engineering (2)

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Design of Shallow Foundation (3): Combined Footings

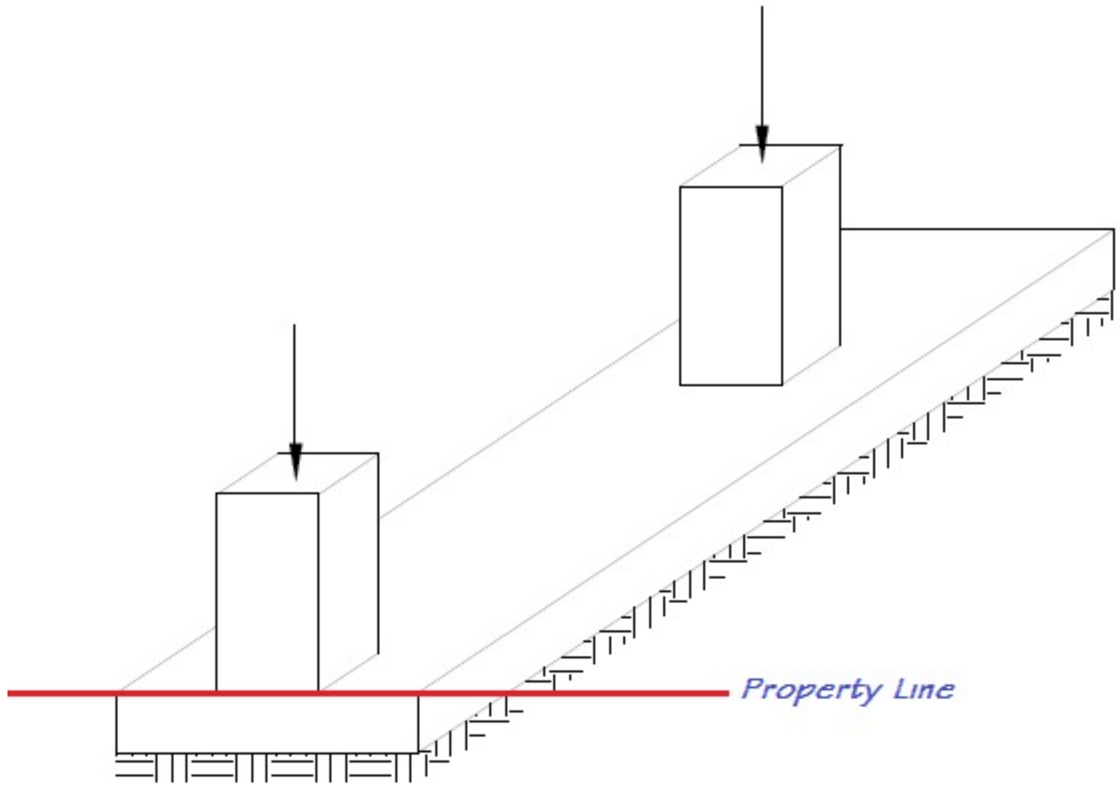
- ***Rectangular Footings.***
- ***Trapezoidal Footings.***
- ***Cantilever or Strap Footings***

A *combined footing, how and why?*

- **is usually used to support two to four columns of unequal loads. In such a case, the resultant of the applied loads would not coincide with the centroid of the footing, and the consequent the soil pressure would not be uniform.**
- **Another case where a combined footing is an efficient foundation solution is when there are two interior columns which are so close to each other that the two or more isolated footings stress zones in the soil areas would overlap.**
- **The area of the combined footing may be proportioned for a uniform settlement by making its centroid coincide with the resultant of the column loads supported by the footing.**
- **There are many instances when the load to be carried by a column and the soil bearing capacity are such that the standard spread footing design will require an extension of the column foundation beyond the property line. In such a case, two or more columns can be supported on a single rectangular foundation. If the net allowable soil pressure is known, the size of the foundation $B \times L$ can be determined.**

This photo shows an example of combined footings used in a heavy industrial plant, where the machinery loads place very large loads upon relatively confined space.
The use of combined footings helps spread out the loads out to the adjacent footings in order to minimize stresses in the footings and reduce the differential settlement between them.


09/06/2005



A third case of a useful application of a combined footing is if one (or several) columns are placed right at the property line. The footings for those columns can not be centered around the columns. The consequent eccentric load would generate a large moment in the footing. By tying the exterior footing to an interior footing through a continuous footing, the moment can be substantially reduced, and a more efficient design is attained.

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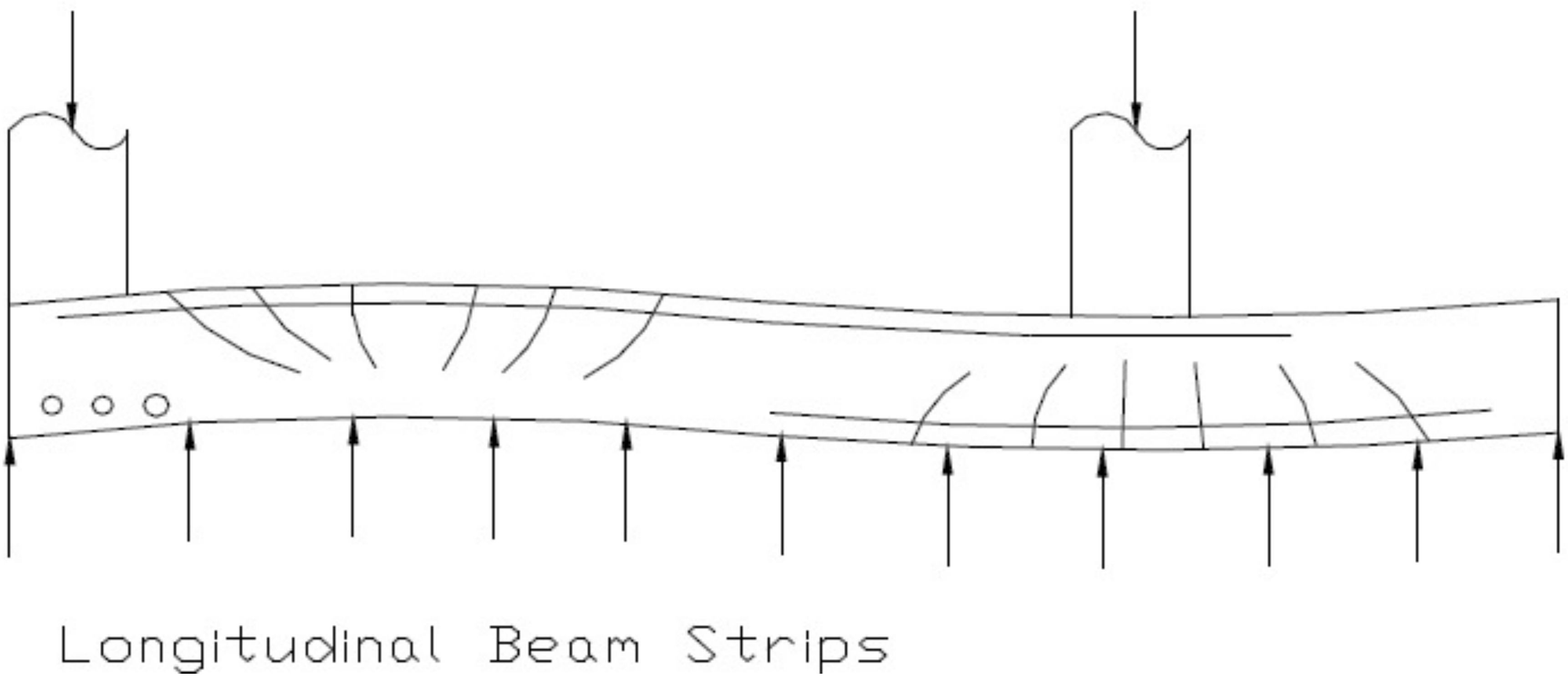
A combined footing will deform as shown in the sketch below. The eccentric loading condition upon the left end, due to the restrictions of a property line, will generate tensile stresses on the top of the footing. These stresses mean that a combined footing will require flexural reinforcement both at the top and the bottom of the footing.



Design Steps for geotechnical part of combined footing

1) Locate the point of application of the column loads on the footing.
2) Proportion the footing such that the resultant of loads passes through the center of footing.
3) Compute the area of footing such that the allowable soil pressure is not exceeded.

Rectangular Combined Footings.

Step #1. The required design area A of a footing can be found from,

$$A = \frac{Q_1 + Q_2}{q_{all\ net}} \qquad (1)$$

where Q_1, Q_2 are the loads in columns #1 and #2, and $q_{all\ (net)}$ is the net allowable soil bearing capacity.

Step #2. Determine the location of the resultant of the column loads.

$$x = \frac{Q_2 L_3}{Q_1 + Q_2} \qquad (2)$$

Step #3. For a uniform distribution of soil pressure under the footing, the resultant of the column loads should pass through the centroid of the foundation. Thus,

$$L = 2(L_2 + x) \qquad (3)$$

where L = length of the foundation

Step #4. Once the length L is determined from above, the value of L_1 can be obtained from,

$$L_1 = L - L_2 - L_3 \qquad (4)$$

The magnitude of L_2 will be known and depends on the location of the property line.

The width B of the foundation then is then found from,

$$B = \frac{A}{L} \qquad (5)$$

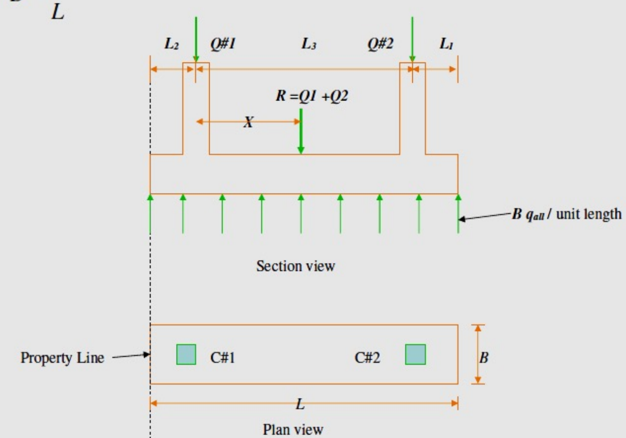


Figure 1

Trapezoidal Combined Footing.

This type of combined footing, shown in Figure 2, is sometimes used as an isolated spread foundation for a column that is required to carry a large load in a tight space. The size of the trapezoidal footing that will generate a uniform pressure on the soil can be found through the following procedure.

Step #1. If the net allowable soil pressure is known, determine the area of the footing,

$$A = \frac{Q_1 + Q_2}{q_{all\ net}}$$

From Figure 2, $A = [\,(B_1 + B_2)/2\,]\,L$ (6)

Step #2. Determine the location of the resultant for the column loads,

$$x = \frac{Q_2 L_3}{Q_1 + Q_2}$$

From the properties of a trapezoid, $$x + L_2 = [\frac{B_1 + 2B_2}{B_1 + B_2}]\frac{L}{3} \qquad (7)$$

With known values of A, L, x, and L_2, solve equations (6) and (7) to obtain B_1 and B_2. Note that for a trapezoid $(L/3) < (x+L_2) < (L/2)$.

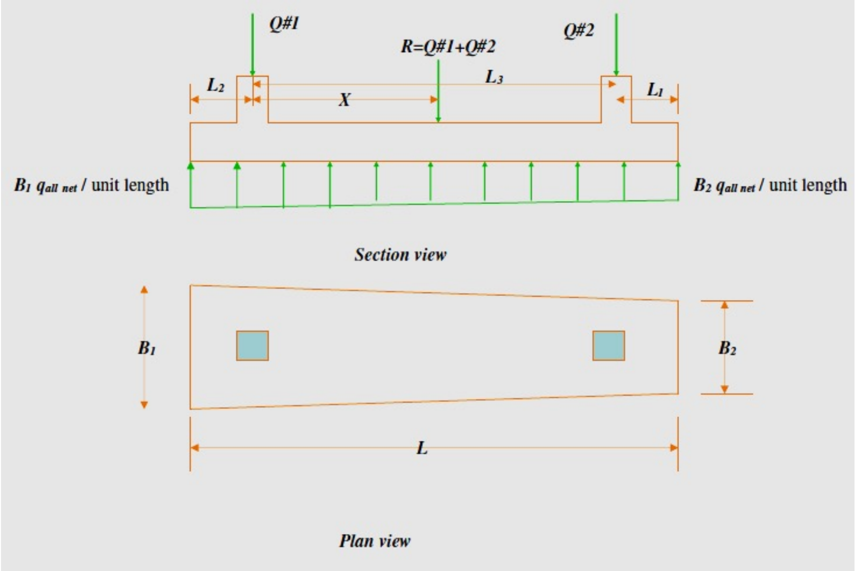


Figure 2

Cantilever or Strap Footings.

- **A *strap footing* is used to connect an eccentrically loaded column footing to an interior column.**
- **The strap is used to transmit the moment caused from an eccentricity to the interior column footing so that a uniform soil pressure is generated beneath both footings.**
- **The strap footing may be used instead of a rectangular or trapezoidal combined footing if the distance between columns is large and / or the allowable soil pressure is relatively large so that the additional footing area is not needed.**

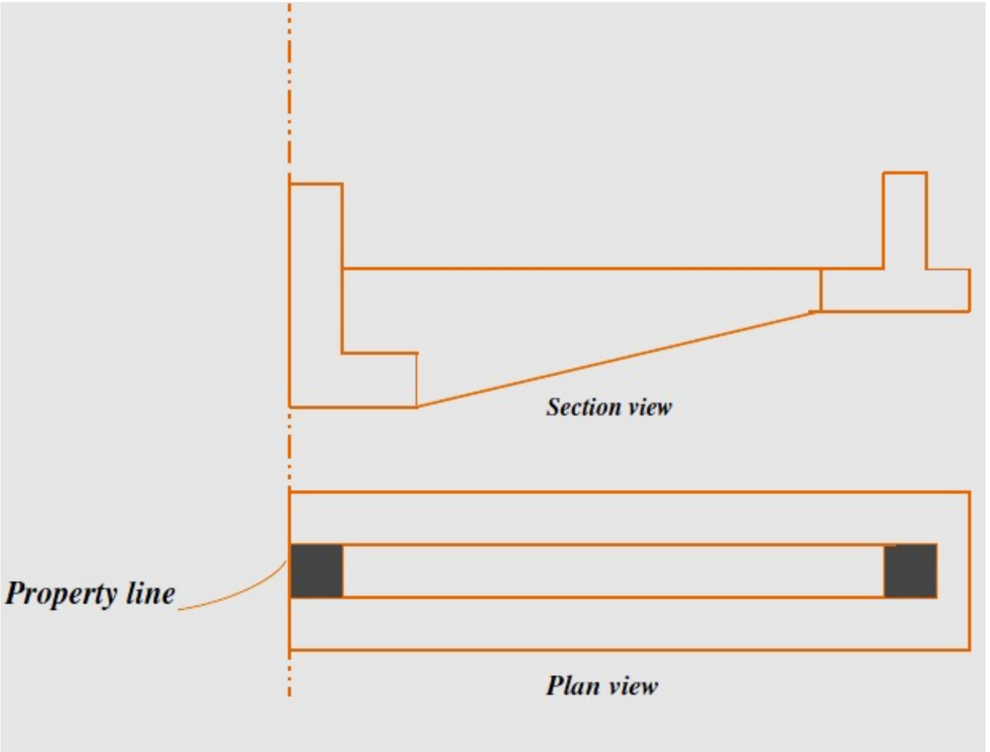


Figure 3

Thankyou

Foundation Engineering (2)

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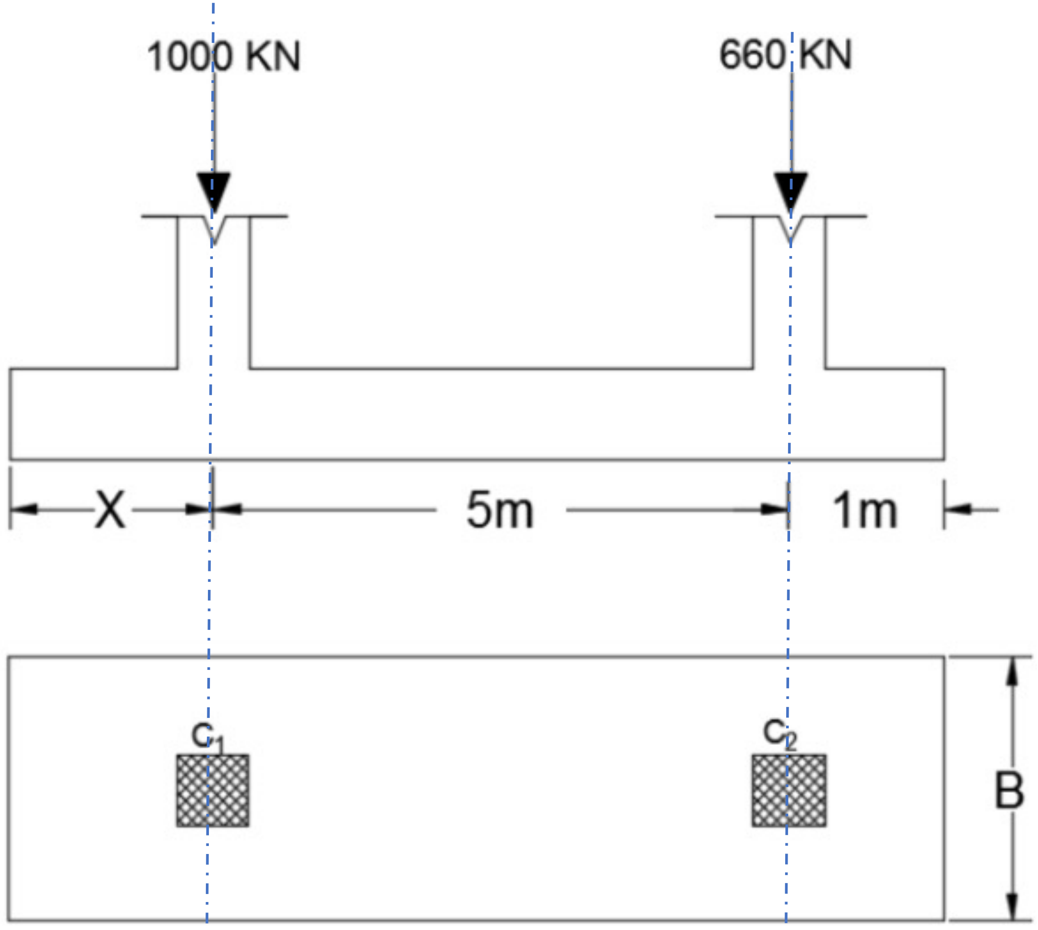
- Design of Shallow Foundation (4): Combined Footing (b):
- Examples on:
- Rectangular,
- Trapezoidal
- Strap

Example (1): Rectangular combined foundation

For the combined footing shown below:

- Find distance X so that the contact pressure is uniform.
- If $q_{all,net}$=140 kN/m^2, find B.
- Draw Shear Force (S.F.) and Bending Moment (B.M) diagrams.



Example (1), cont'd, Solution

To keep uniform contact pressure under the base, the resultant force R must be at the center of the foundation.

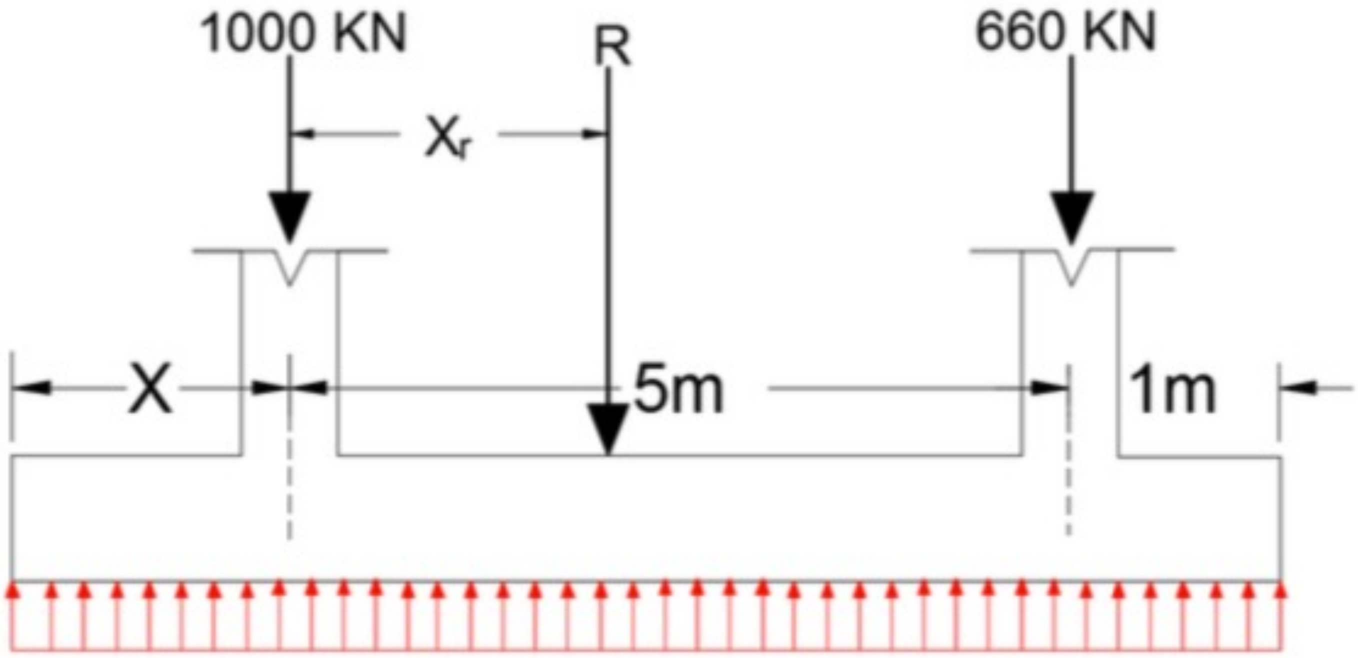
$Q1 = 1000$ KN

$Q2 = 660$ kN

R=Q1 +Q2 =1000+660

$R = 1660$ kN

The weight of the foundation and the soil is not given, so we neglect it.



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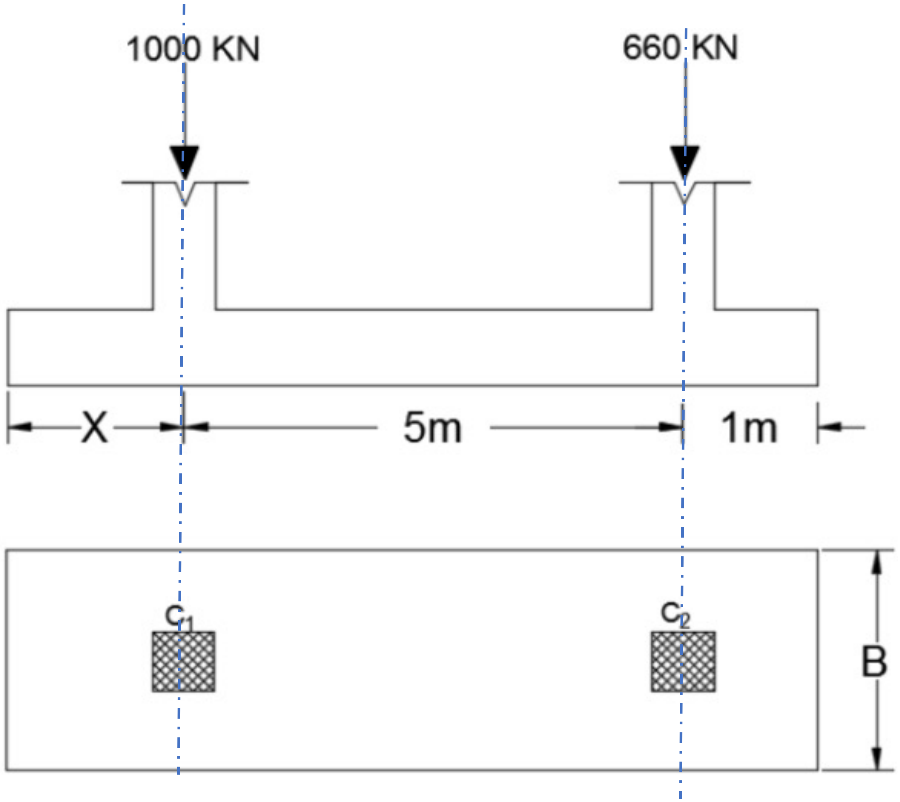
Example (1), cont'd, Solution

$$\sum M_{c_1} = 0.0 \rightarrow 660 \times 5 = 1660 \times X_r \rightarrow X_r = 1.98 \text{ m}$$



To keep uniform pressure: $X_r + X = \frac{L}{2}$

$$\frac{L}{2} = (5 - 1.98) + 1 = 4.02 \text{ m}$$

$$\frac{L}{2} = 4.02\text{m} \rightarrow L = 4.02 \times 2 = 8.04 \text{ m}$$

$$X_r + X = \frac{L}{2} \rightarrow 1.98 + X = 4.02 \rightarrow X = 2.04 \text{ m} \checkmark.$$

Calculation of B:

$$A_{req} = \frac{Q_{service}}{q_{all,net}} = B \times L \rightarrow \frac{1660}{140} = B \times 8.04 \rightarrow B = 1.47 \text{ m} \checkmark$$

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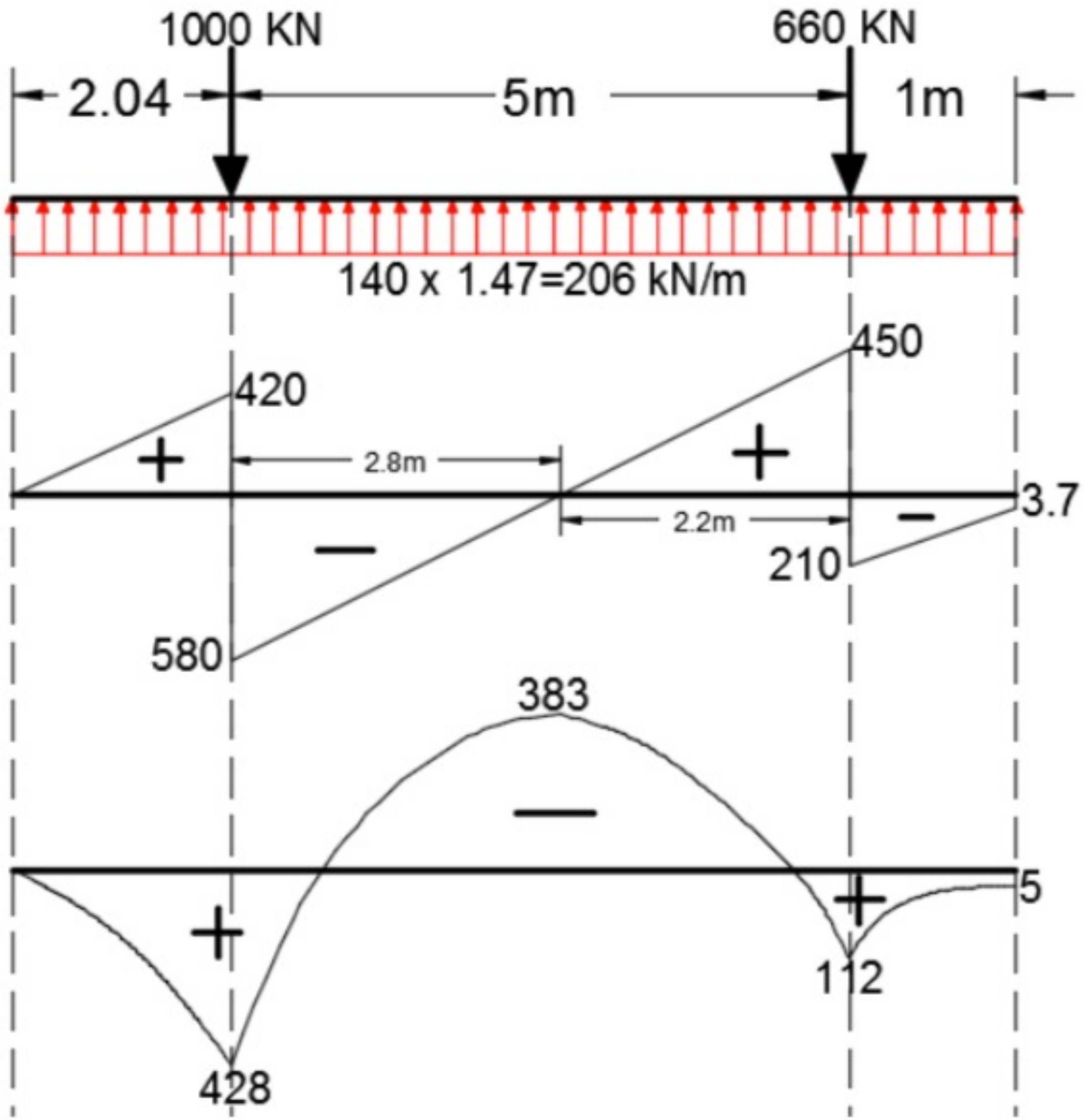
Example (1), cont'd, Solution

Drawing SFD and BMD:

To draw SFD and BMD we use factored loads (if givens), but in this problem we given the service loads directly, so we use service loads.

The free body diagram for the footing, SFD and BMD are shown in the figure.

Note: The shear and the moment should be zero at the 2nd end of the footing, these small figures are residue due to approximation.

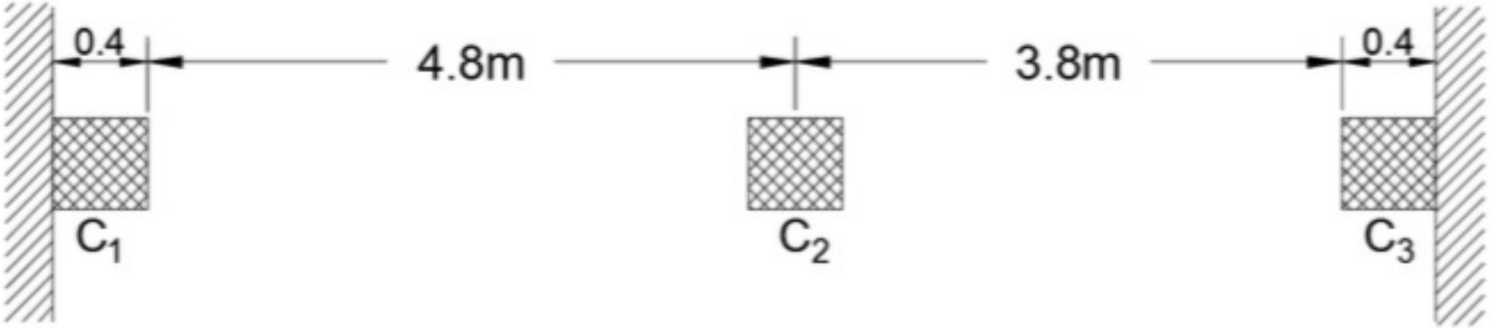


Note that the moment and shear for the footing is the opposite for beams such that the positive moments is at the supports and the minimum moments at the middle of spans, so when reinforced the footing, the bottom reinforcement mustn't cutoff at supports but we can cut it at the middle of the span, also the top reinforcement mustn't cutoff at the middle of the span and we can cut it at the supports (Exactly).

Example (2), Trapezoidal combined foundation

A- For the **set of columns** shown below**, it is required to determine the live load for the column C_1 to make the soil reaction uniform under the base of rectangular combined footing**, knowing that dead load is $(1\frac{1}{2})$ of live load for this column.

Column	DL (tons)	LL (tons)
C_1	DL_1	LL_1
C_2	80	46
C_3	55	35



B. Design a combined footing for the same figure above if C_1 has a dead load of 90 tons and live load of 54 tons, knowing that the extension is not permitted and the soil reaction is being uniform. ($q_{all,net}$=20.8 t/m^2).

Example (2), Trapezoidal combined foundation, cont'd

A.

For rectangular footing, to keep uniform pressure, the resultant force R must be in the center of the foundation.

$Q_1 = DL + LL$

$DL = 1.5\,LL \rightarrow Q_1 = 1.5\,LL + LL = 2.5\,LL$

$Q_2 = 80 + 46 = 126$ ton

$Q_3 = 55 + 35 = 90$ ton

$R = Q_1 + Q_2 + Q_3 = Q_1 + 126 + 90$

$R = Q_1 + 216$

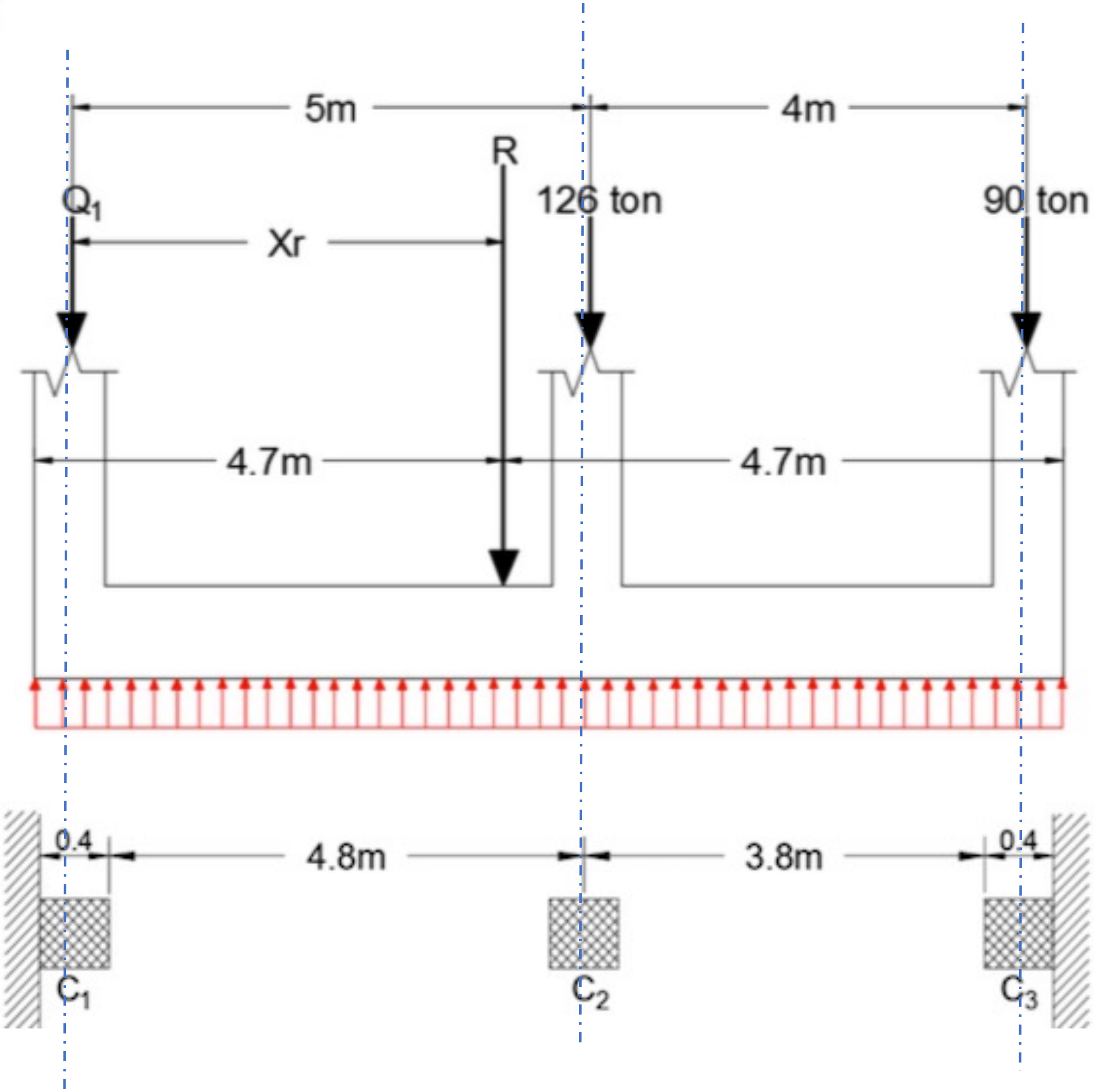
The weight of the foundation and the soil is not given, so we neglect it.

$L = 0.4 + 4.8 + 3.8 + 0.4 = 9.4\text{m} \rightarrow 0.5L = 4.7\text{m}$

$X_r = 4.7 - 0.2 = 4.5\text{m}$

$$\sum M_{c_1} = 0.0 \rightarrow 126 \times 5 + 90 \times 9 = (Q_1 + 216) \times 4.5 \rightarrow Q_1 = 104 \text{ ton}$$

$Q_1 = 104 = 2.5\,LL \rightarrow LL = 4.6$ ton ✓.



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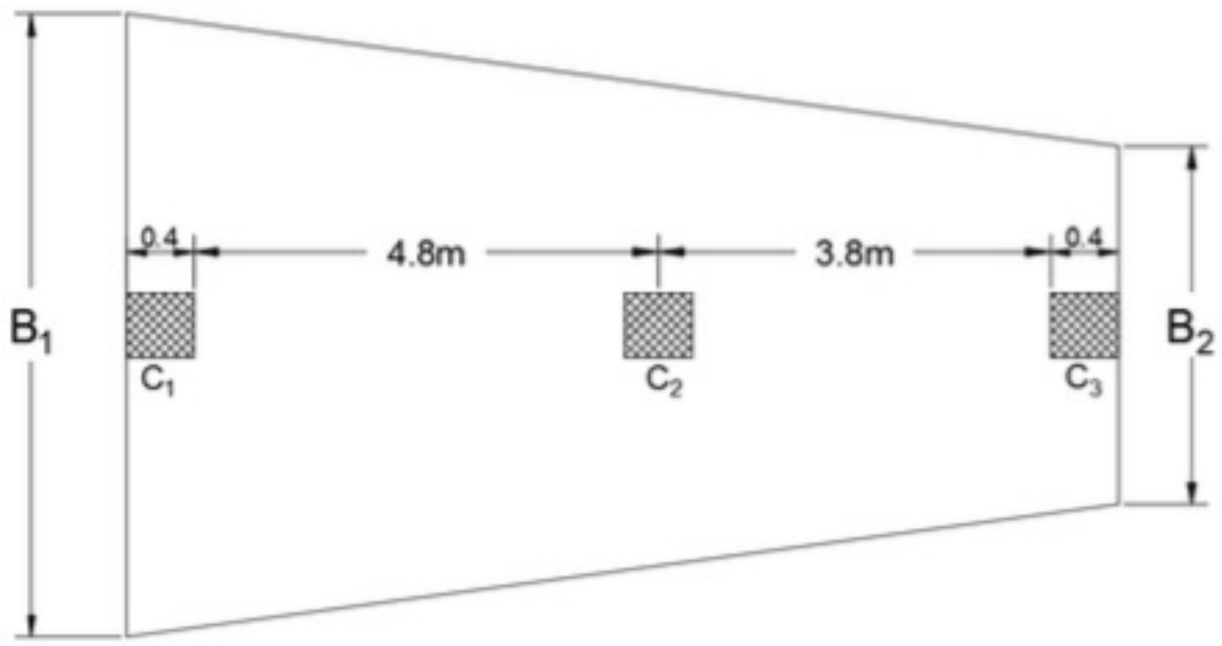
Example (2), Trapezoidal combined foundation,cont'd

B.

IfQ1 =90+54=144ton,

Design the footing to keep uniform contact pressure .

Note that if we want to use rectangular footing, the pressure will be uniform only when $Q1 = 104\ tons$, otherwise if we want to use rectangular footing the pressure will not be uniform, so to maintain uniform pressure under the given loading, it is required to design a **trapezoidal combined footing** (The largest width at largest load and smallest width at smallest column load) as shown:



Example (2), Trapezoidal combined foundation, cont'd

$$Q_1 = 144 \text{ ton}$$
$$Q_2 = 126 \text{ ton}$$
$$Q_3 = 90 \text{ ton}$$
$$R = Q_1 + Q_2 + Q_3 = 144 + 126 + 90 = 360 \text{ ton}$$
$$L = 0.4 + 4.8 + 3.8 + 0.4 = 9.4\text{m}$$

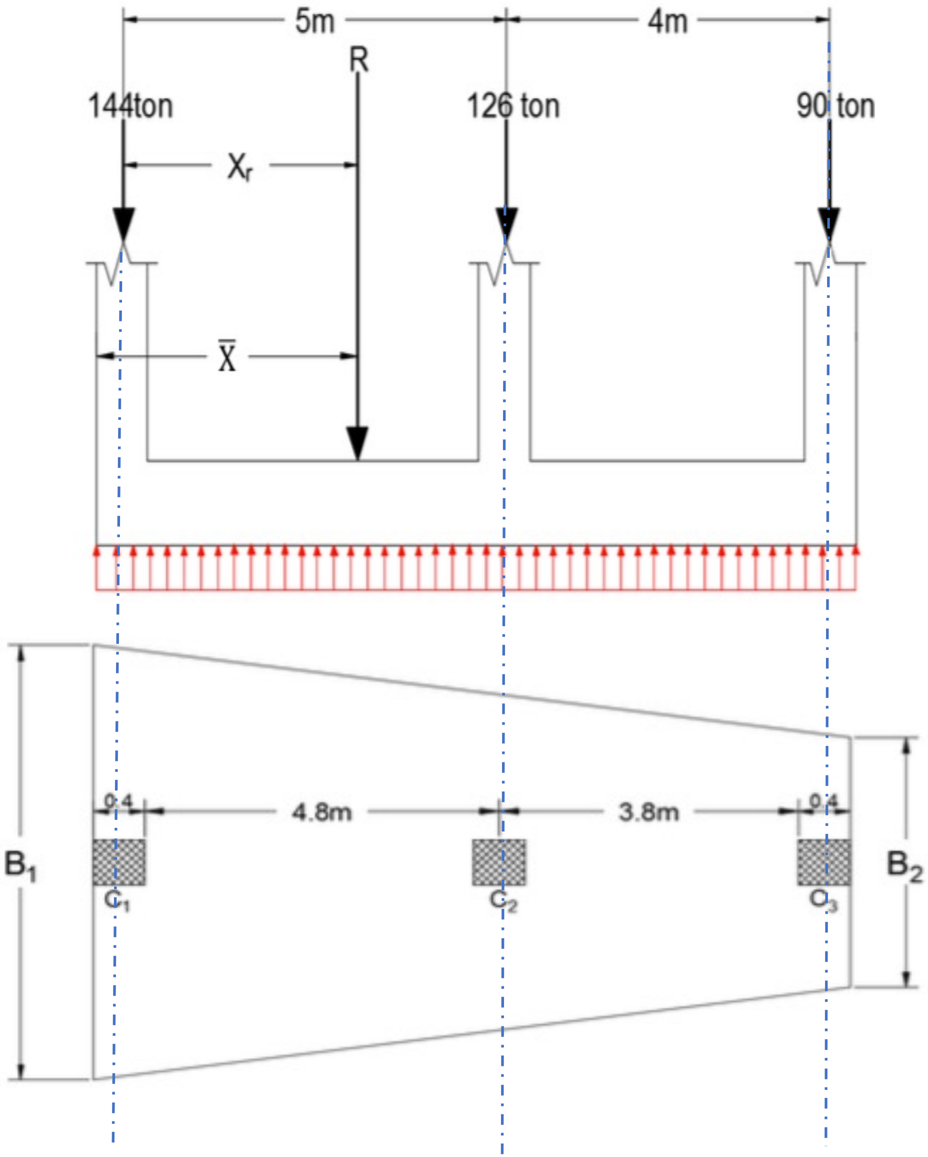
$$A_{req} = \frac{Q_{service}}{q_{all,net}} = \frac{L}{2}(B_1 + B_2) \rightarrow \frac{360}{20.8} = \frac{9.4}{2}(B_1 + B_2)$$
$$\rightarrow (B_1 + B_2) = 3.68 \rightarrow B_1 = 3.68 - B_2 \rightarrow\rightarrow \text{Eq. (1)}$$

$$\sum M_{c_1} = 0.0 \rightarrow 126 \times 5 + 90 \times 9 = 360 \times X_r \rightarrow X_r = 4 \text{ m}$$

$$X_r + 0.2 = \overline{X} \rightarrow 4.2 = \frac{L}{3}\left(\frac{B_1 + 2B_2}{B_1 + B_2}\right) \rightarrow 1.34 = \left(\frac{B_1 + 2B_2}{B_1 + B_2}\right) \rightarrow\rightarrow \text{Eq. (2)}$$

Substitute from Eq.(1) in Eq.(2):

$$1.34 = \left(\frac{3.68 - B_2 + 2B_2}{3.68 - B_2 + B_2}\right) \rightarrow B_2 = 1.25 \text{ m} \checkmark.$$
$$B_1 = 3.68 - 1.25 = 2.43 \text{ m} \checkmark.$$



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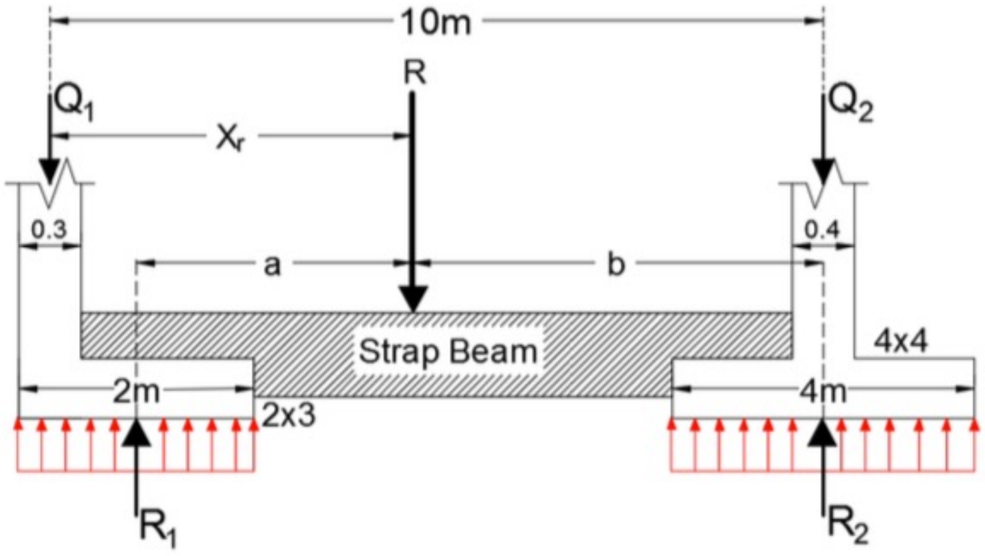
Example (2), Strap (Cantilever) foundation

For the strap footing shown below, if qall,net =250 kN/m², determine Q_1 and Q_2



Solution

$R_1 = A_1 \times q_{all,net} = (2 \times 3) \times 250 = 1500$ KN

$R_2 = A_2 \times q_{all,net} = (4 \times 4) \times 250 = 4000$ KN

$R = R_1 + R_2 = 1500 + 4000 = 5500$ KN $= Q_1 + Q_2$

$a + b = 10 + 0.15 - 1 = 9.15$m

$\sum M_{R_2} = 0.0$(after use of strap) $\rightarrow 1500 \times 9.15 = 5500 \times b \rightarrow b = 2.5$ m

$\rightarrow a = 9.15 - 2.5 = 6.65$ m $\rightarrow X_r = 6.65 + 1 - 0.15 = 7.5$m

$\sum M_{@\,Q_1} = 0.0 \rightarrow 10Q_2 = 5500 \times 7.5 \rightarrow Q_2 = 4125$ kN ✓.

$\rightarrow Q_1 = R - Q_2 = 5500 - 4125 = 1375$ kN ✓.

Best Wishes towards Success

Foundation Engineering (2)

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Design of Shallow Foundation (4): Mat foundation (b)

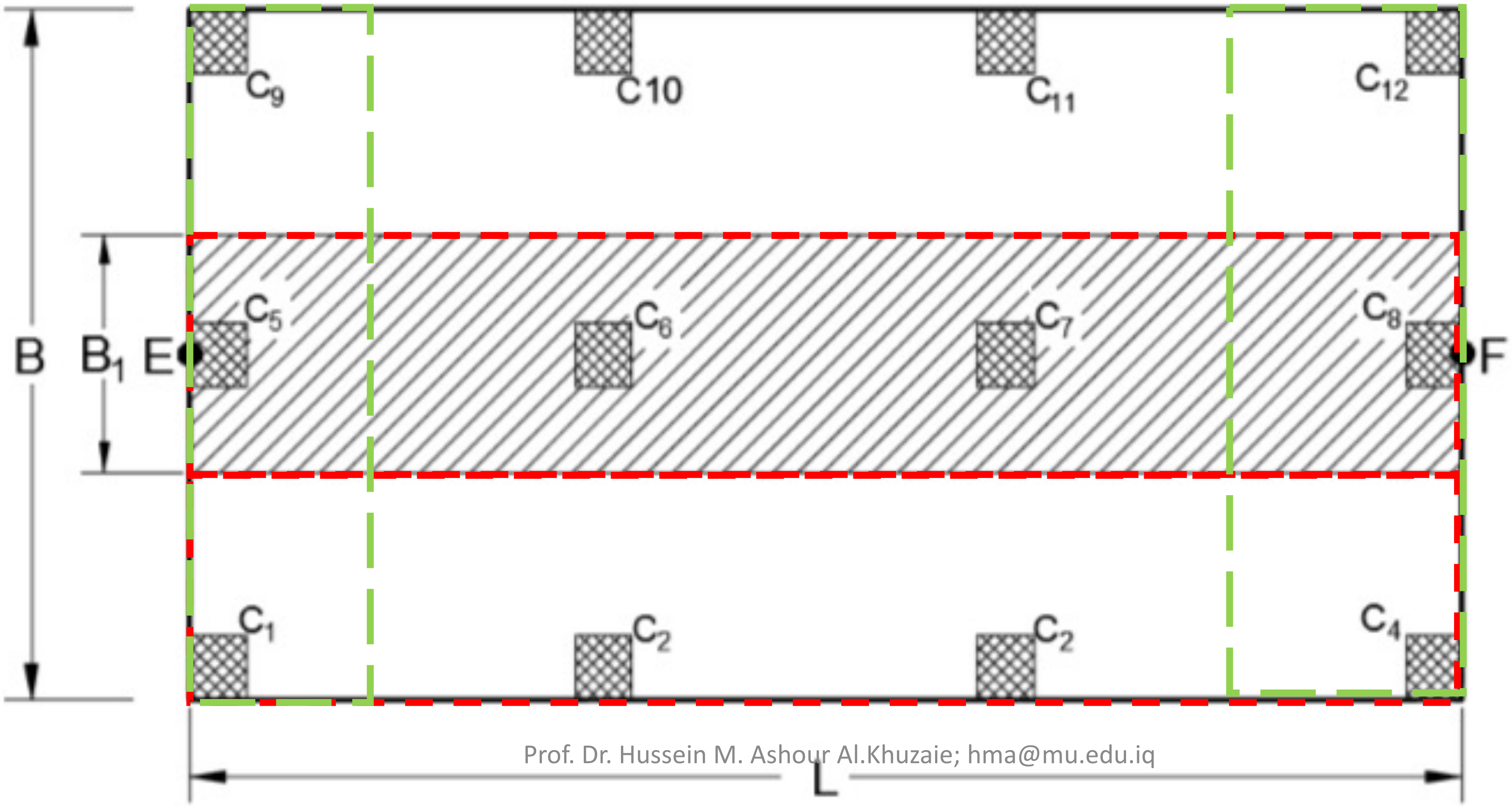
- Structural Design of Mat Footing (Conventional rigid design method)
- Example

Structural Design of Mat Foundation (b) (Ultimate Loads)

In structural design:

▶ Draw Shear Force Diagram (SFD) and Bending Moment Diagram (BMD):

Subdivide mat foundations into a strips in both directions, each strip must contains a line of columns.



Structural Design of Mat Foundation (b) (Ultimate Loads); Cont'd

For the previous mat, let we take a strip of width B_1 for the columns 5,6,7 and 8 as shown in figure below:

1. Locate the points E and F at the middle of strip edges.

2. Calculate the factored resultant force (R_u):

$$R_u = \sum Q_{ui}$$

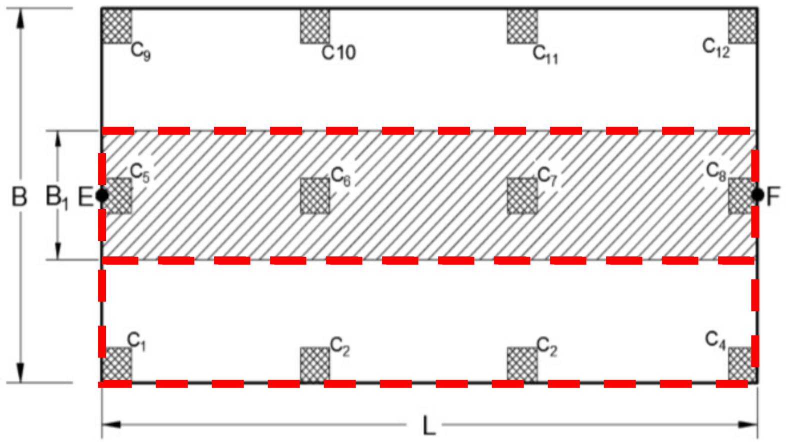
3. The eccentricities in X and Y directions remains unchanged because the location of the resultant force will not change since we factored all columns by the same factor.

4. Calculate the factored moment in X and Y directions:

$$M_{u,x} = e_y \times \sum Q_{ui} \qquad M_{u,y} = e_x \times \sum Q_{ui}$$

5. Calculate the stresses at points E and F (using factored loads and moments):

$$q = \frac{\sum Q_{ui}}{A} \pm \frac{M_{u,y}}{I_y} X \pm \frac{M_{u,x}}{I_x} Y$$



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Structural Design of Mat Foundation (b) (Ultimate Loads); Cont'd

6. Since the stress at points E and F is not equal, the pressure under the strip in not uniform, so we find the average stress under the strip:

$$q_{u,avg} = \frac{q_E + q_F}{2}$$

7. Now, we check the stability of the strip:

$$\left(\sum Q_{ui}\right)_{strip} = q_{u,avg} \times A_{strip}$$

If this check is ok, draw the SFD and BMD and then design the strip.

If not $\Rightarrow$ Go to step 8.

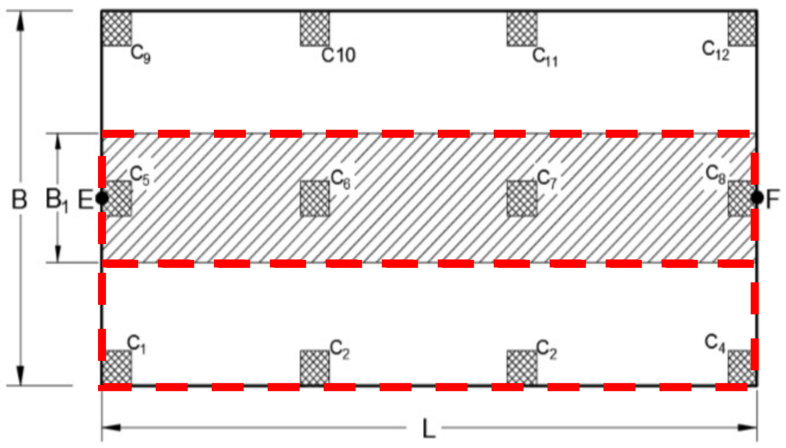
8. We have to modified the loads to make the strip stable by the following steps:

- Calculate the average load on the strip:

$$\text{Average Load} = \frac{(\sum Q_{ui})_{strip} + q_{u,avg} \times A_{strip}}{2}$$

- Calculate the modified columns loads:

$$(Q_{ui})_{mod} = Q_{ui} \times \frac{\text{Average Load}}{(\sum Q_{ui})_{strip}}$$

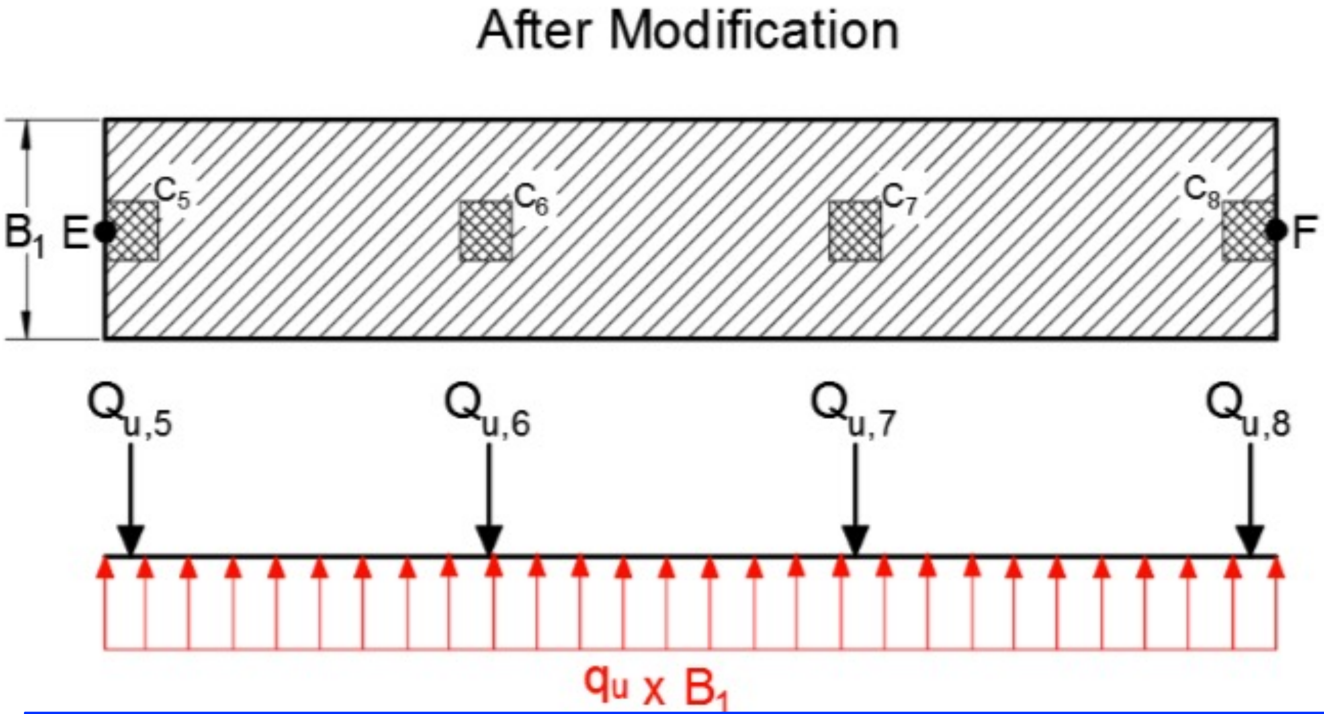


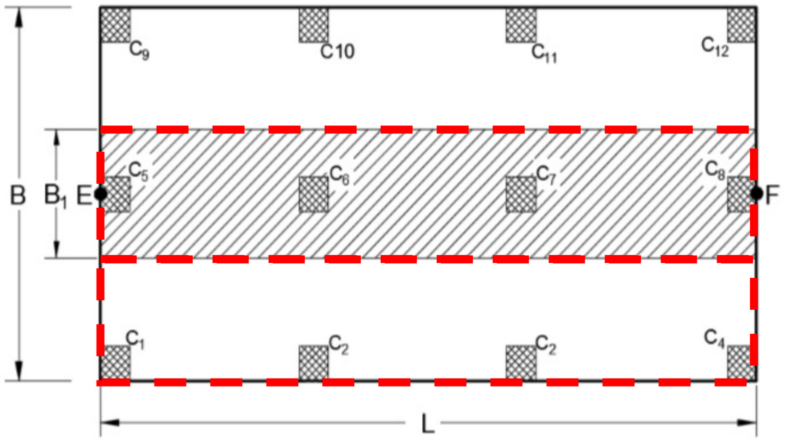
Structural Design of Mat Foundation (Ultimate Loads); Cont'd

- Calculate the modified soil pressure:

$$(q_{u,avg})_{mod} = q_{u,avg} \times \frac{\text{Average Load}}{q_{u,avg} \times A_{strip}}$$

$$\text{Now, } \left(\sum Q_{ui}\right)_{strip} = q_{u,avg} \times A_{strip} \rightarrow \text{Draw SFD and BMD}$$





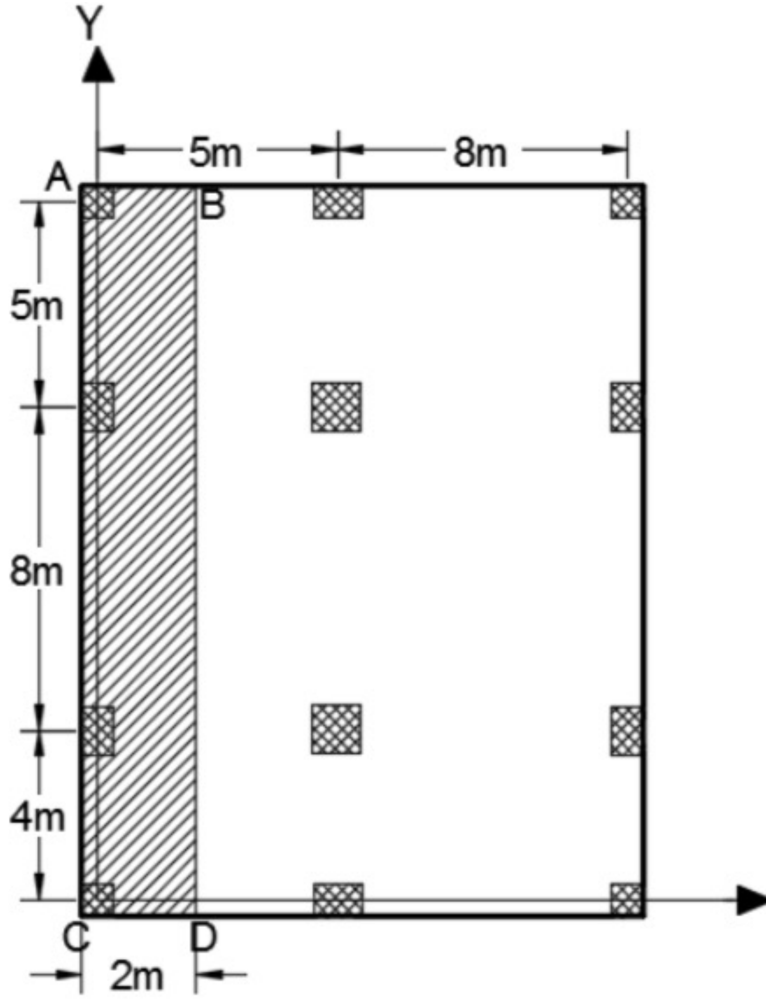
Note:
The same procedure can be used for the service load

Example:

For the shown mat foundation, If $q_{all,net}$=150 kN/m^2.

1. ~~Check the adequacy of the foundation dimensions. (it was presented in part –a-~~

2. **Draw SFD and BMD for the strip ABDC which is 2m width.**



	Interior Columns	Edge Columns	Corner Columns
Columns Dimensions	60cm x 60cm	60 cm x 40 cm	40 cm x 40 cm
Service Loads	1800 kN	1200 kN	600 kN
Factored Loads	2700 kN	1800 kN	900 kN

Structural Design of Mat Foundation (b) (Ultimate Loads); Cont'd

Solution of 2nd part: Drawing of SFD and BMD

Now we want to Draw SFD and BMD for strip ABCD (using factored loads)

Locate point E at the middle of the upper edge of strip (between A and B) and point F at the middle of the lower edge of strip (between C and D).

$$R_u = \sum Q_{ui} = 2 \times 2700 + 6 \times 1800 + 4 \times 900 = 19800 \text{ kN}$$

The eccentricities will not change since we factored all loads by the same factor

$$M_{u,x} = e_y \times \sum Q_{ui} = 0.3 \times 19800 = 5940 \text{ kN.m}$$

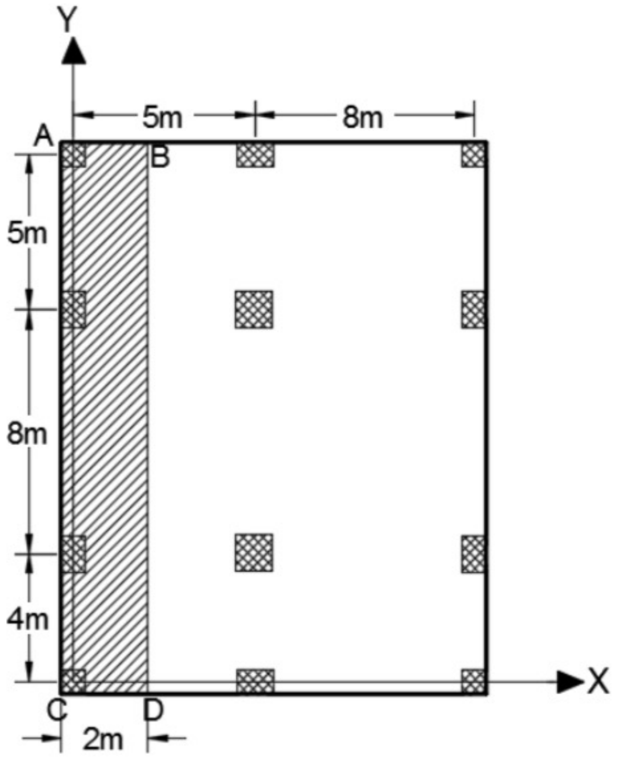
$$M_{u,y} = e_x \times \sum Q_{ui} = 0.68 \times 19800 = 13464 \text{ kN.m}$$

$$q_u = \frac{\sum Q_{ui}}{A} \pm \frac{M_{u,y}}{I_y} X \pm \frac{M_{u,x}}{I_x} Y \rightarrow q_u = \frac{19800}{13.4 \times 17.4} \pm \frac{13464}{3488.85} X \pm \frac{5940}{5882.6} Y$$

$$\rightarrow q_u = 84.92 \pm 3.86\, X \pm 1.01\, Y$$

If compression, use (+) sign

If Tension, use (–) sign.



Structural Design of Mat Foundation (b) (Ultimate Loads); Cont'd

If compression, use (+) sign

If Tension, use (–) sign.

X = horizontal distance from centroid to the points E and F at strip

$$= \frac{13.4}{2} - \frac{2}{2} = 5.7 \text{ m}$$

Y = vertical distance from centroid to points E and F at strip

$$= \frac{17.4}{2} = 8.7 \text{ m}$$

The shown figure explains the moments signs (compression or tension) and the strip ABCD with points E and F:

At point E:

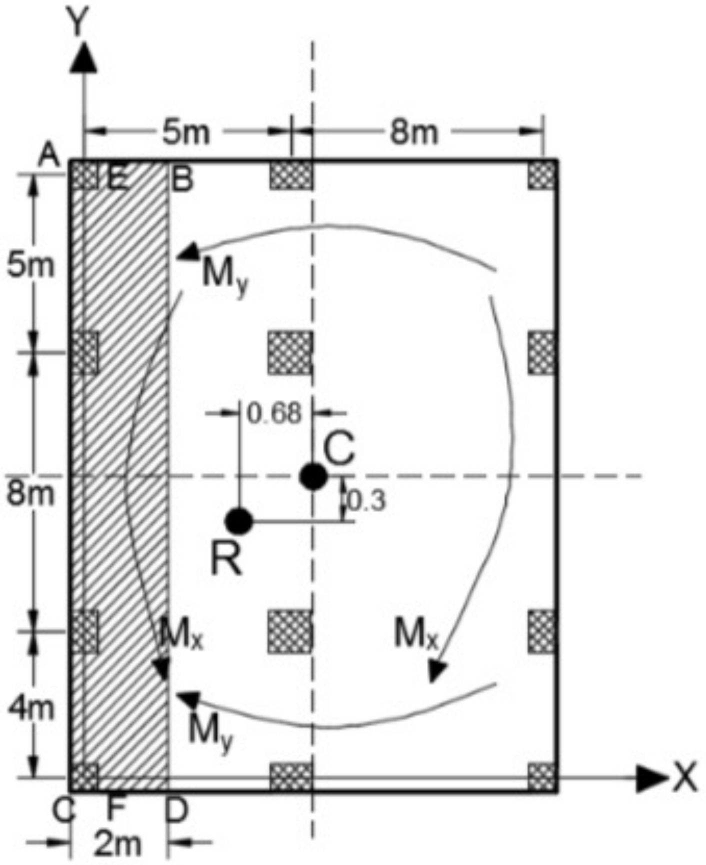
M_y = compression (+)

M_x = tension (–)

At point F:

M_y = compression (+)

M_x = compression (+)



Structural Design of Mat Foundation (b) (Ultimate Loads); Cont'd

$$q_{u,E} = 84.92 + 3.86 \times 5.7 - 1.01 \times 8.7 = 98.13\ \text{kN/m}^2$$

$$q_{u,F} = 84.92 + 3.86 \times 5.7 + 1.01 \times 8.7 = 115.7\ \text{kN/m}^2$$

$$q_{u,avg} = \frac{q_{u,E} + q_{u,F}}{2} = \frac{98.13 + 115.7}{2} = 106.9\ \text{kN/m}^2$$

Now, we check the stability of the strip:

$$\left(\sum Q_{ui}\right)_{strip} = 2 \times 1800 + 2 \times 900 = 5400\ \text{kN}$$

$$q_{u,avg} \times A_{strip} = 106.9 \times (17.4 \times 2) = 3720$$

$$\left(\sum Q_{ui}\right)_{strip} \neq q_{u,avg} \times A_{strip} \rightarrow \text{Not stable} \rightarrow \text{Loads must modified}$$

$$\text{Average Load} = \frac{(\sum Q_{ui})_{strip} + q_{u,avg} \times A_{strip}}{2} = \frac{5400 + 3720}{2} = 4560\ \text{kN}$$

$$(Q_{ui})_{mod} = Q_{ui} \times \frac{\text{Average Load}}{(\sum Q_{ui})_{strip}} = Q_{ui} \times \frac{4560}{5400} = 0.84 Q_{ui}$$

Structural Design of Mat Foundation (b) (Ultimate Loads); Cont'd

$$(Q_{ui})_{mod} = Q_{ui} \times \frac{\text{Average Load}}{(\sum Q_{ui})_{strip}} = Q_{ui} \times \frac{4560}{5400} = 0.84Q_{ui}$$

$$(Q_{u,\text{corner columns}})_{mod} = 0.84 \times 900 = 756\text{ Kn}$$

$$(Q_{u,\text{edge columns}})_{mod} = 0.84 \times 1800 = 1512\text{ kN}$$

$$(q_{u,avg})_{mod} = q_{u,avg} \times \frac{\text{Average Load}}{q_{u,avg} \times A_{strip}} = 106.9 \times \frac{4560}{3720} = 131\text{kN/m}^2$$

Now, we check the stability of the strip (after modification):

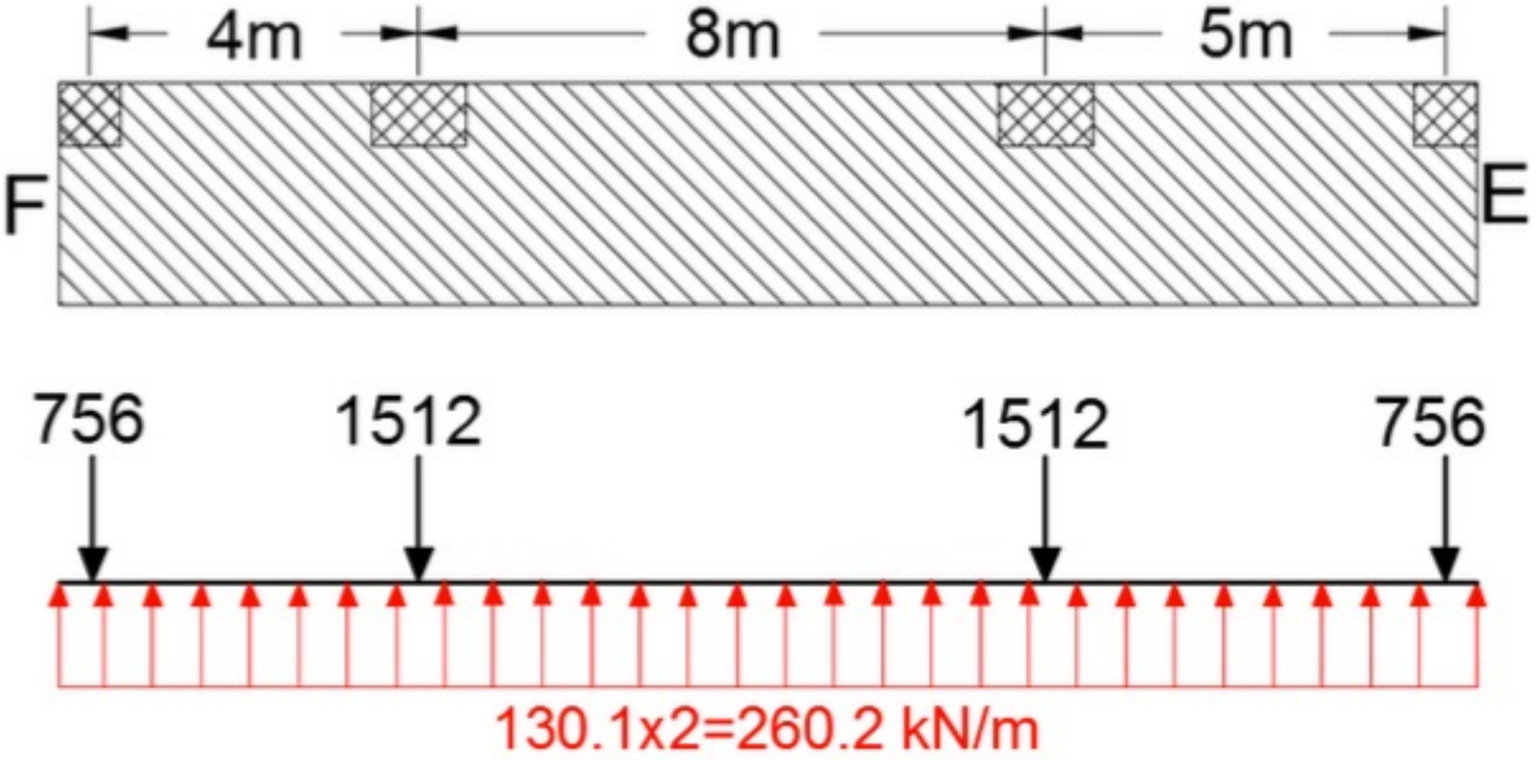
$$\left(\sum Q_{ui}\right)_{strip} = 2 \times 1512 + 2 \times 756 = 4536\text{ kN}$$

$$q_{u,avg} \times A_{strip} = 131 \times (17.4 \times 2) = 4558.8\text{ kN}$$

$$\left(\sum Q_{ui}\right)_{strip} \cong q_{u,avg} \times A_{strip} \rightarrow \text{can be considered stable}$$

Structural Design of Mat Foundation (b) (Ultimate Loads); Cont'd

The final loads after modification are as shown in the figure.



Now you can draw SFD and BMD but you will observe that the SFD and BMD will not enclosed to zero (at the end) and this because the vertical loads not exactly the same $4536 \cong 4558.8$.

I try to say " don't confused if SFD and BMD" are not enclosed to zero (i.e. trust yourself

The end of this lecture

Foundation Engineering (2)

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Design of Shallow Foundation (4): Mat (Raft) Foundation (a):

- General
- Compensated raft footing
- Geotechnical design and conventional approximate rigid method

Mat Foundations (General)

Definition: Mat foundation, which is sometimes referred to as a raft foundation, is a combined footing that may cover the entire area under a structure supporting several columns and walls.

Why? Mat foundations:

- ✓ Soils that have low load-bearing capacities, but that will have to support high column or wall loads.
- ✓ Spread and/or combined footings would have to cover ≥ 50 % of the building area.
- ✓ Mats may be supported by piles, which help reduce the settlement of a structure built over highly compressible soil. Where the water table is high, mats are often placed over piles to control buoyancy.

Types:

1. Flat plate (Figure a). The mat is of uniform thickness.
2. Flat plate thickened under columns (Figure b).
3. Beams and slab (Figure c).The beams run both ways, and the columns are located at the intersection of the beams.
4. Flat plates with pedestals (Figure d).
5. Slab with basement walls as a part of the mat (Figure e).The walls act as stiffeners for the mat.

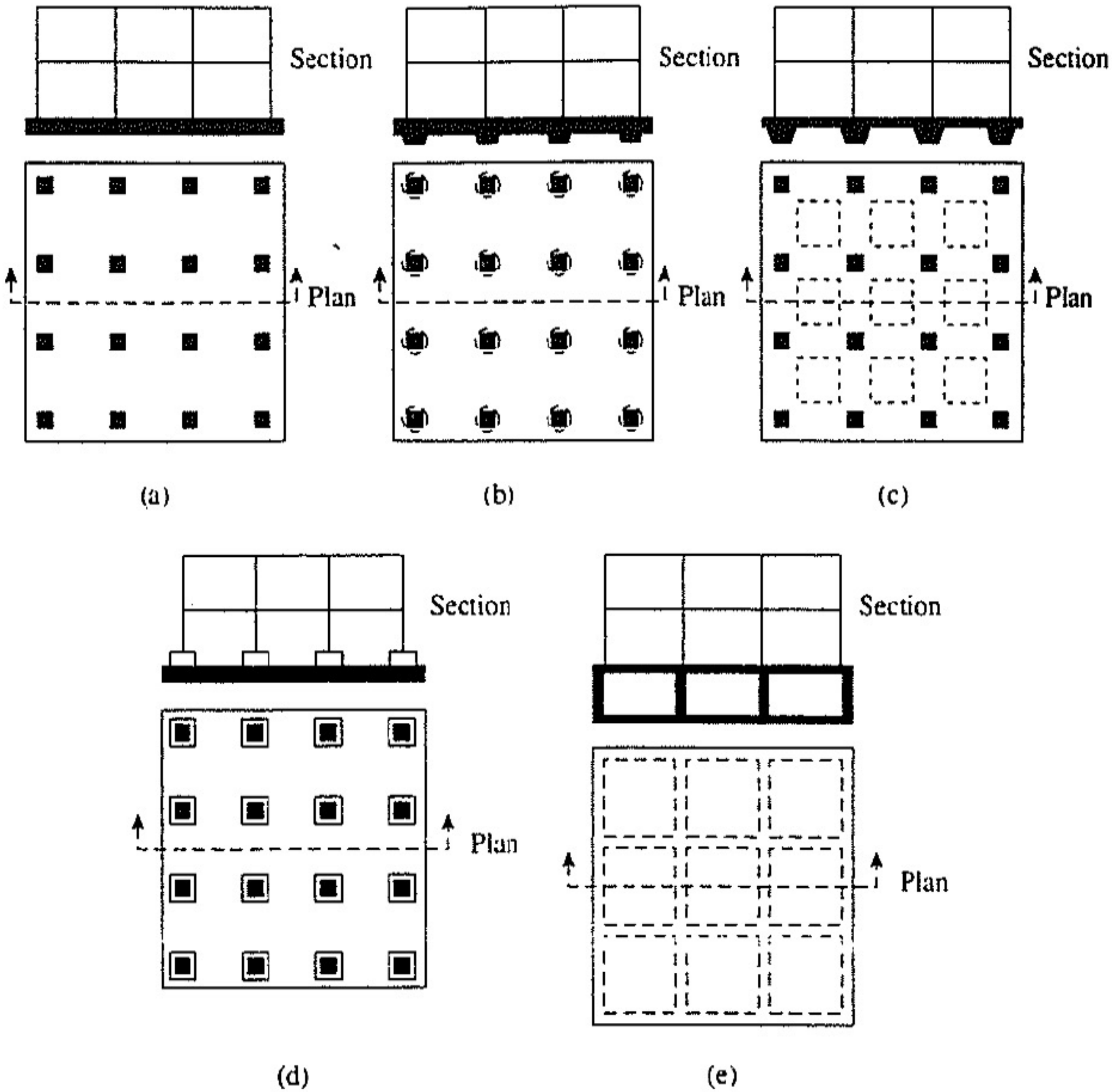


Figure 6.4 Common types of mat foundation

Bearing capacity of the Foundation

- <u>Bearing Capacity Analysis</u> follows the same approach as for spread footings

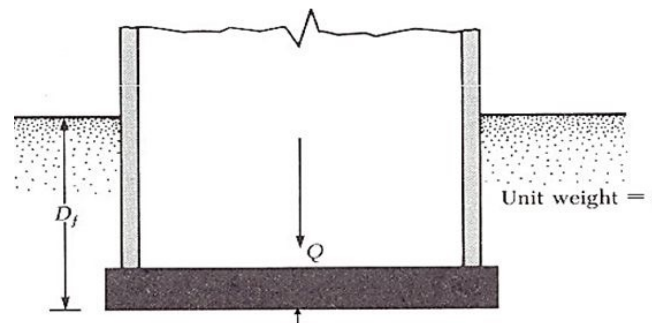
$$q_{ult} = c'N_c s_c d_c + \sigma'_{zD} N_q s_q d_q + 0.5\gamma' B N_\gamma s_\gamma d_\gamma$$

Foundation type	Expected maximum settlement, mm	Expected differential settlement, mm
Spread	25	20
Mat	50	20

Compensated raft footing

The net pressure increase in the soil under a mat foundation can be reduced by increasing the depth D_f of the mat. This approach is generally referred to as the *compensated foundation design* and is extremely useful when structures are to be built on very soft clays. In this design, a deeper basement is made below the *higher* portion of the superstructure, so that the net pressure increase in soil at any depth is relatively uniform.

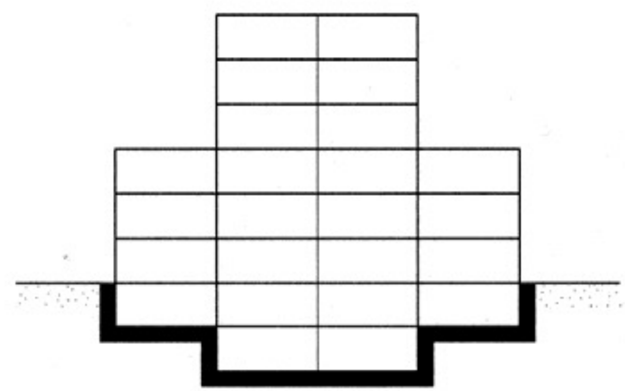
$$q = (Q/A) - \gamma D_f \leq q_{na}$$



For no increase of the net soil pressure; fully compensated raft **q**

$$0 = (Q/A) - \gamma D_f$$

$$D_f = Q / A\gamma$$



For partial compensation,

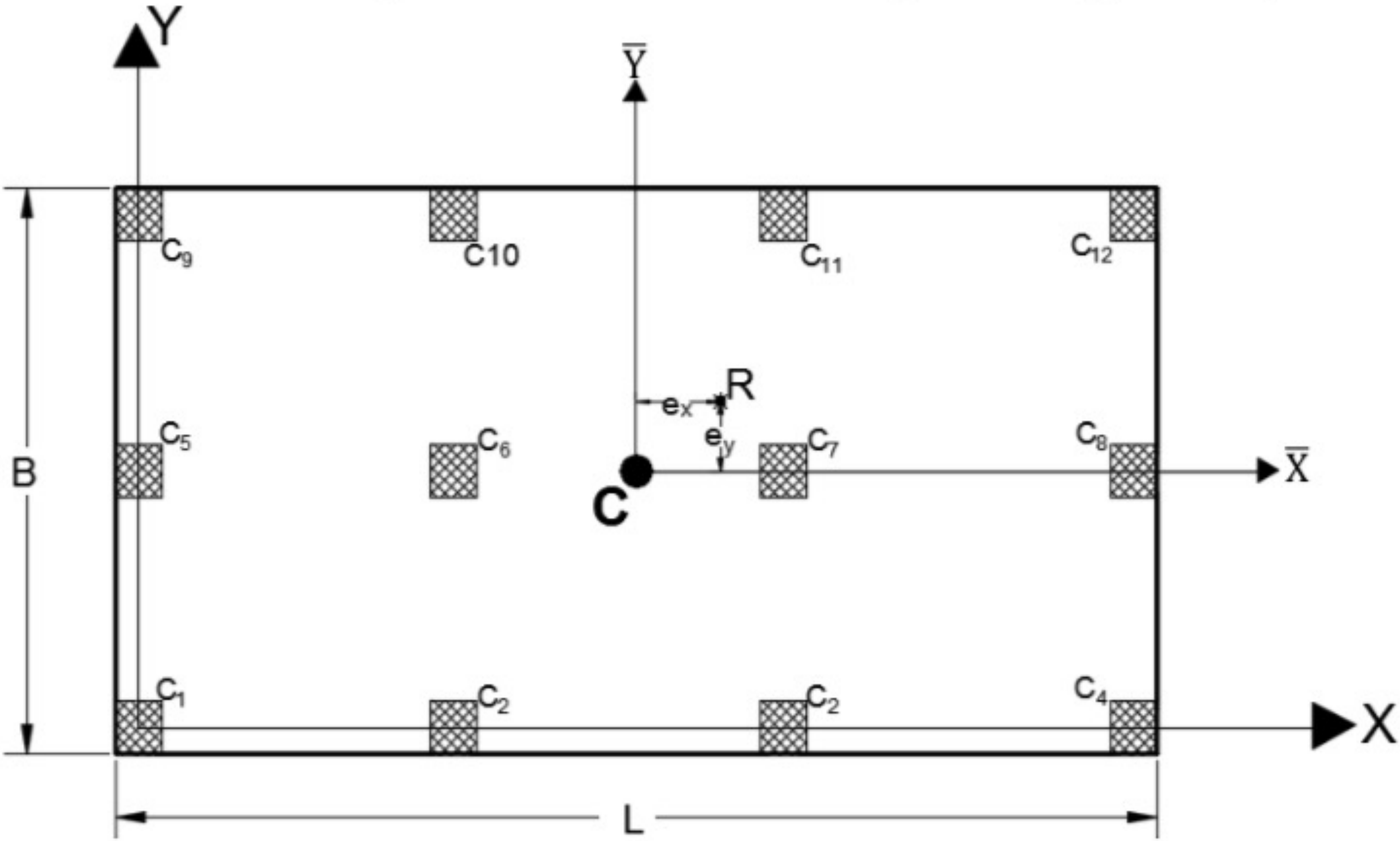
$$D_f < Q / A\gamma$$

Design of Mat Foundations

- Approximate Method
- Flexible Method
- Finite Difference Method
- Finite Element Method

In this lecture, only the basic concepts of the Conventional Rigid Design Method (CRCM) (Approximate method) will be presented.

Geometric Design of Mat Foundation (Working Loads)



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Procedures:

1. Determine the horizontal and vertical axes (usually at the center line of the horizontal and vertical edge columns) as shown.

2. Calculate the centroid of the mat [point C $(\overline{X}, \overline{Y})$]with respect to X and Y axes:

$$\overline{X} = \frac{\sum X_i \times A_i}{\sum A_i} \qquad \overline{Y} = \frac{\sum Y_i \times A_i}{\sum A_i}$$

A_i = shapes areas.

X_i = distance between y − axis and the center of the shape.

Y_i = distance between y − axis and the center of the shape.

If the mat is rectangular:

$$\overline{X} = \frac{L}{2} - \frac{w_{\text{vertical edge columns}}}{2}$$

$$\overline{Y} = \frac{B}{2} - \frac{w_{\text{horizontal edge columns}}}{2}$$

3. Calculate the resultant force R:

$$R = \sum Q_i$$

4. Calculate the location of resultant force R (X_R, Y_R) with respect to X and Y axes:

To find X_R take summation moments about Y-axis:

$$X_R = \frac{\sum Q_i \times X_{ri}}{\sum Q_i}$$

To find Y_R take summation moments about X-axis:

$$Y_R = \frac{\sum Q_i \times Y_{ri}}{\sum Q_i}$$

Q_i = load on column

X_{ri} = distance between columns center and Y − axis

Y_{ri} = distance between columns center and X − axis

5. Calculate the eccentricities:

$$e_x = |X_R - \overline{X}| \qquad e_y = |Y_R - \overline{Y}|$$

6. Calculate moments in X and Y directions:

$$M_x = e_y \times \sum Q_i \qquad M_y = e_x \times \sum Q_i$$

7. Calculate the stress under each corner of the mat:

$$q = \frac{\sum Q_i}{A} \pm \frac{M_y}{I_y} X \pm \frac{M_x}{I_x} Y$$

How we can know the sign (+ or −):

If compression (+) If tension (−)

Compression if the arrow of moment is at the required point

Tension if the arrow of moment is far away from the required point

X and Y are distances from the **centroid to the required point**

X and Y (take them always positive)

$I_x = I_{\overline{X}}$ = moment of inertia about centoid of mat (in x − direction)

$I_y = I_{\overline{Y}}$ = moment of inertia about centoid of mat (in y − direction)

$$I_x = \frac{B^3 L}{12} \qquad I_Y = \frac{L^3 B}{12}$$ (in case of rectangular foundation

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8. Check the adequacy of the dimensions of mat foundation:

Calculate q_{max} (maximum stress among all corners of the mat)

Calculate q_{min} (minimum stress among all corners of the mat)

$q_{max} \leq q_{all,net}$

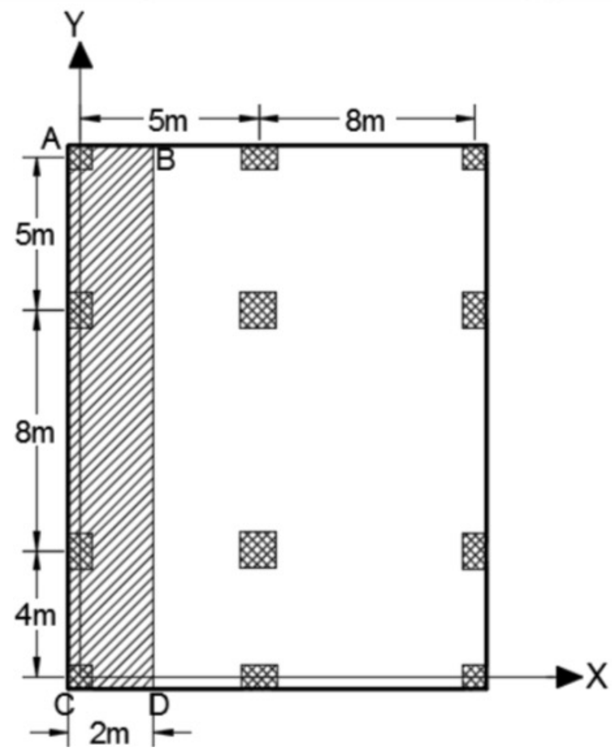
$q_{min} \geq 0.0$

If one of the two conditions doesn't satisfied, increase the dimensions of the footing.

Example (1):

For the shown mat foundation, If $q_{all,net}$=150 kN/m^2.
Check the adequacy of the foundation dimensions.

	Interior Columns	Edge Columns	Corner Columns
Columns Dimensions	60cm x 60cm	60 cm x 40 cm	40 cm x 40 cm
Service Loads	1800 kN	1200 kN	600 kN
Factored Loads	2700 kN	1800 kN	900 kN



Solution

Firstly we take the horizontal and vertical axes as shown in figure above.

Calculate the centroid of the footing with respect to these axes:

$$B = 5 + 8 + 2 \times 0.2 = 13.4\text{m} \ \ (\text{horizontal dimension})$$

$$L = 5 + 8 + 4 + 2 \times 0.2 = 17.4\text{m} \ (\text{certical dimension})$$

$$\overline{X} = \frac{13.4}{2} - \frac{0.4}{2} = 6.5\text{m} \ (\text{from } y - \text{axis})$$

$$\overline{Y} = \frac{17.4}{2} - \frac{0.4}{2} = 8.5 \text{ m} \ (\text{from } x - \text{axis})$$

Calculate the resultant force R:

$$R = \sum Q_i \, (\text{service loads}) = 2 \times 1800 + 6 \times 1200 + 4 \times 600 = 13200 \text{ kN}$$

Calculate the location of resultant force R (X_R, Y_R) with respect to X and Y axes:

$$X_R = \frac{\sum Q_i \times X_{ri}}{\sum Q_i}$$

$$= \frac{2 \times 1800 \times 5 + 2 \times 1200 \times 5 + 2 \times 1200 \times 13 + 2 \times 600 \times 13}{13200}$$

$$X_R = 5.82 \text{ m (from y} - \text{axis)}$$

Note that the moments of the first vertical line of columns will equal zero because y-axis is at the centerline of these columns.

$$Y_R = \frac{\sum Q_i \times Y_{ri}}{\sum Q_i}$$

$$= \frac{2 \times 1200 \times 4 + 1800 \times 4 + 2 \times 1200 \times 12 + 1800 \times 12 + 2 \times 600 \times 17 + 1200 \times 17}{13200}$$

$$Y_R = 8.2 \text{ m (from x} - \text{axis)}$$

Note that the moments of the first horizontal line of columns will equal zero because x-axis is at the centerline of these columns.

Calculate the eccentricities:

$$e_x = |X_R - \overline{X}| = |5.82 - 6.5| = 0.68$$

$$e_y = |Y_R - \overline{Y}| = |8.2 - 8.5| = 0.3\text{m}$$

Calculate moments in X and Y directions:

$$M_x = e_y \times \sum Q_i = 0.3 \times 13200 = 3960 \text{ kN.m}$$

$$M_y = e_x \times \sum Q_i = 0.68 \times 13200 = 8976 \text{ kN.m}$$

Calculate moment of inertia in X and Y directions:

$$I_x = \frac{B\,L^3}{12} = \frac{13.4 \times 17.4^3}{12} = 5882.6 \text{ m}^4$$

$$I_y = \frac{L\,B^3}{12} = \frac{17.4 \times 13.4^3}{12} = 3488.85 \text{ m}^4$$

Not that the value which **perpendicular** to the required axis will be tripled because it's the value that resist the moment.

Calculate the stresses:

$$q = \frac{\sum Q_i}{A} \pm \frac{M_y}{I_y} X \pm \frac{M_x}{I_x} Y \rightarrow q = \frac{13200}{13.4 \times 17.4} \pm \frac{8976}{3488.85} X \pm \frac{3960}{5882.6} Y$$

$$q = 56.61 \pm 2.57\,X \pm 0.67\,Y$$

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If compression, use (+) sign

If Tension, use (–) sign.

But we want to calculate q_{max} and q_{min}

$$q_{max} = 56.61 + 2.57\,X + 0.67\,Y$$

$$q_{min} = 56.61 - 2.57\,X - 0.67\,Y$$

X = maximum horizontal distance from centroid to the edge of mat

$$= \frac{13.4}{2} = 6.7 \text{ m}$$

Y = maximum vertical distance from centroid to the edge of mat

$$= \frac{17.4}{2} = 8.7 \text{ m}$$

$$\rightarrow q_{max} = 56.61 + 2.57 \times 6.7 + 0.67 \times 8.7 = 79.66 \text{ kN/m}^2$$

$$\rightarrow q_{min} = 56.61 - 2.57 \times 6.7 - 0.67 \times 8.7 = 33.56 \text{ kN/m}^2$$

$$q_{max} = 79.66 < q_{all,net} = 150 \rightarrow \text{Ok}$$

$$q_{min} = 33.56 > 0.0 \rightarrow \text{Ok}$$

The foundation Dimensions are adequate ✓.

End of this part

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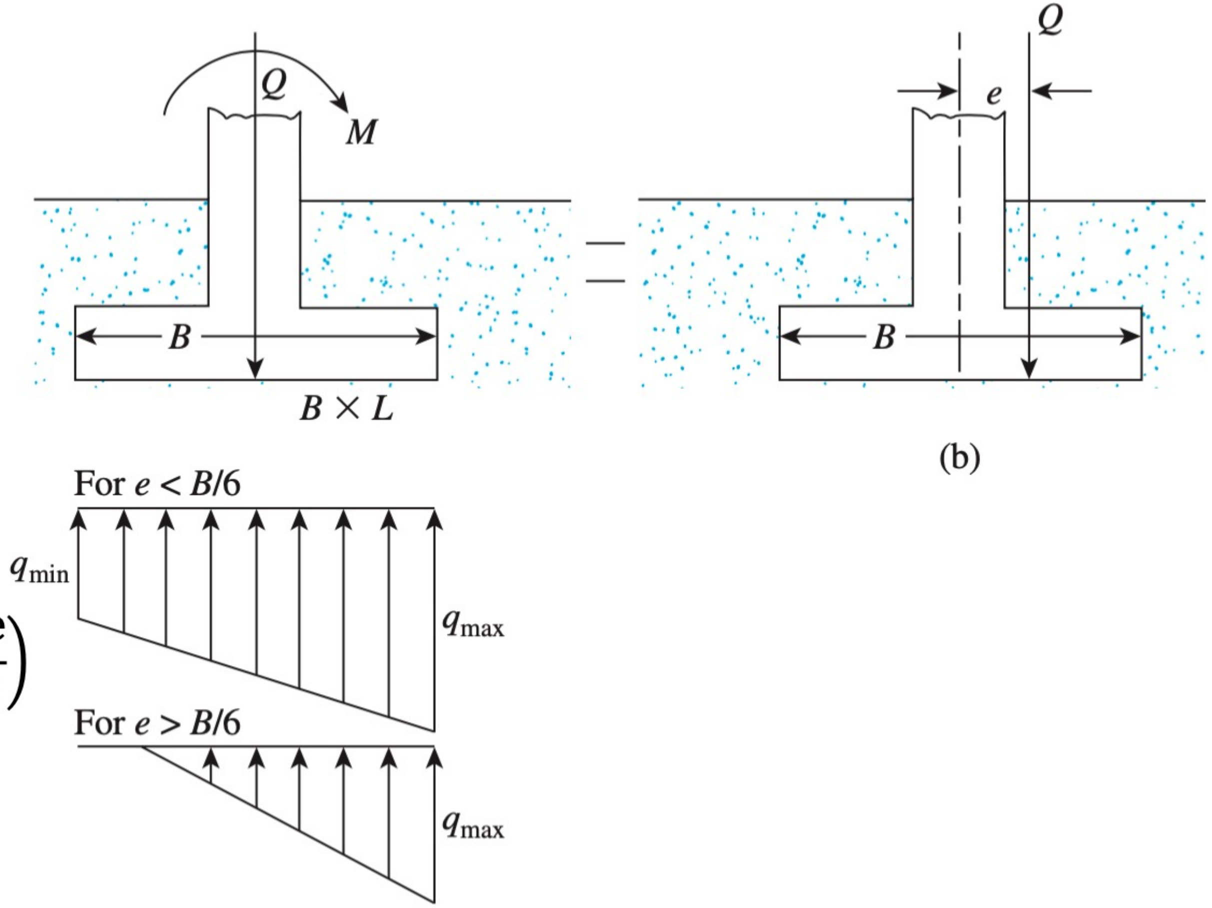
Miscellaneous Topics (1)

- **Bearing Capacity for:**
  - Eccentric Loadings:
    - ✓ One way eccentricity
    - ✓ Two way eccentricity
  - Layered Soils
  - Foundation on slope
  - Uplift Loadings

Eccentrically Loaded Foundations

In several instances, as it was shown with the base of a retaining wall, foundations are subjected to moments in addition to the vertical load, as shown in the Figure. In such cases, the distribution of pressure by the foundation on the soil is not uniform. The stress will be due to compression exerted by the concentrated load (Q) and the stress due to moment so that the stress equation will be in the form: $q = \frac{Q}{A} \pm \frac{MC}{I}$.

Here, M = Q*e; c = B/2 or c = L/2, depending On the direction of moment and on the axis that affects on.; I is the moment of inertia And also it may be about B axis or about L axis. For rectangular, $I_{x(b)} = B^3L/12$ or $I_{y\,(L)} = L^3B/12$



$$q = \frac{Q}{B \times L} \pm \frac{Q \times e \times B}{\frac{2\,B^3 \times L}{12}} \rightarrow q = \frac{Q}{B \times L} \pm \frac{6eQ}{B^2L} \rightarrow q = \frac{Q}{B \times L}\left(1 \pm \frac{6e}{B}\right)$$

$$\boxed{q = \frac{Q}{B \times L}\left(1 \pm \frac{6e}{B}\right) \text{ General Equation}}$$

The nominal distribution of pressure is:

$$q_{max} = \frac{Q}{BL} + \frac{6M}{B^2L}$$

and

$$q_{min} = \frac{Q}{BL} - \frac{6M}{B^2L}$$

where

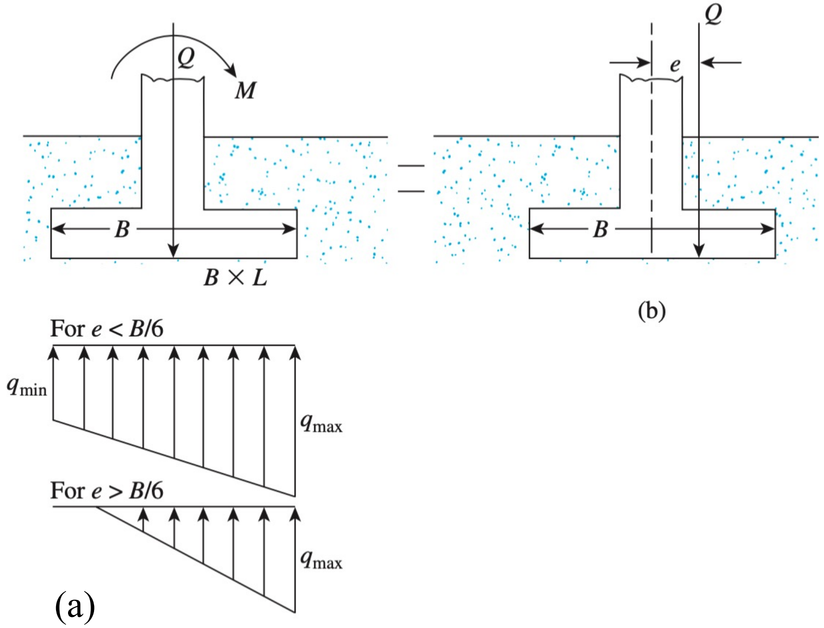
Q = total vertical load

M = moment on the foundation

In the Figure below shows a force system applied on the foundation. The distance is the eccentricity and it equals:

$$e = \frac{M}{Q}$$

So that Fig. (b) is the equivalent loading system of that loading due to Moment and Q.



(a)

(b)

Substituting Equation of (e) into Eqs. of q_{max} and q_{min} gives

$$q_{\max} = \frac{Q}{BL}\left(1 + \frac{6e}{B}\right)$$

$$q_{\min} = \frac{Q}{BL}\left(1 - \frac{6e}{B}\right)$$

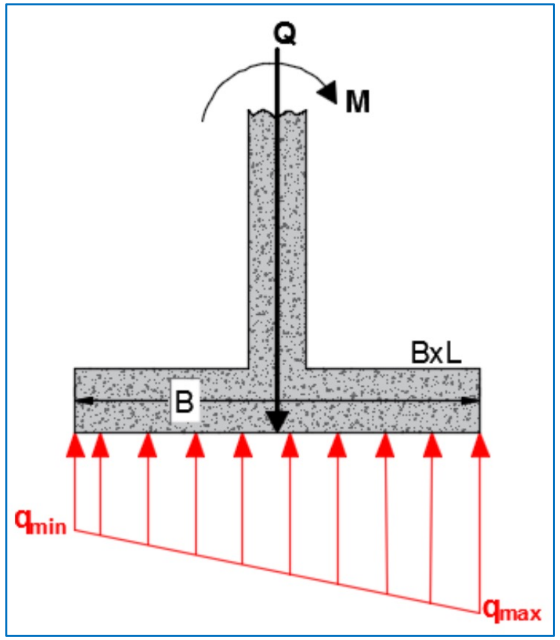
Note (See the next slide)

1) These two equations are valid when the eccentricity $e \leq B/6$.
2) q_{min} is zero for $e = B/6$.
3) q_{min} will be negative when $e > B/6$, which means that tension will develop. Because soil cannot take any tension, there will then be a separation between the foundation and the soil underlying it. .

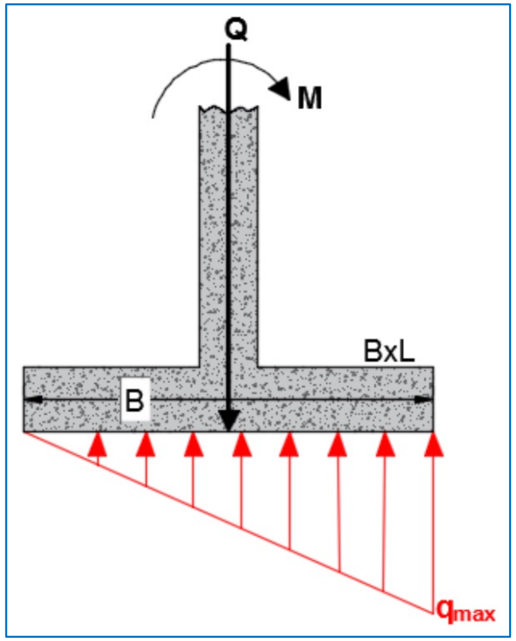
The factor of safety for such type of loading against bearing capacity failure can be evaluated as:

$$\text{FS} = \frac{Q_u}{Q}$$

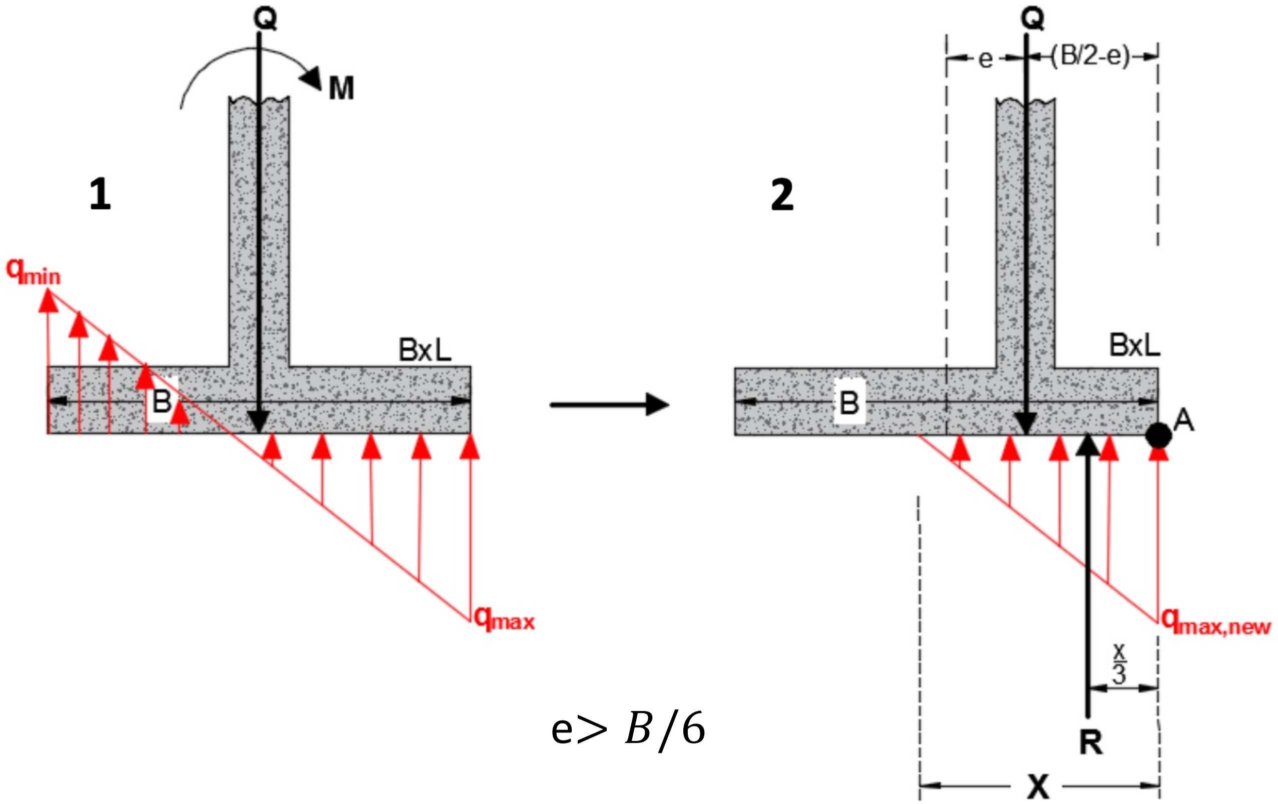
where Q_u = ultimate load-carrying capacity.



e< $B/6$



e = $B/6$



e> $B/6$

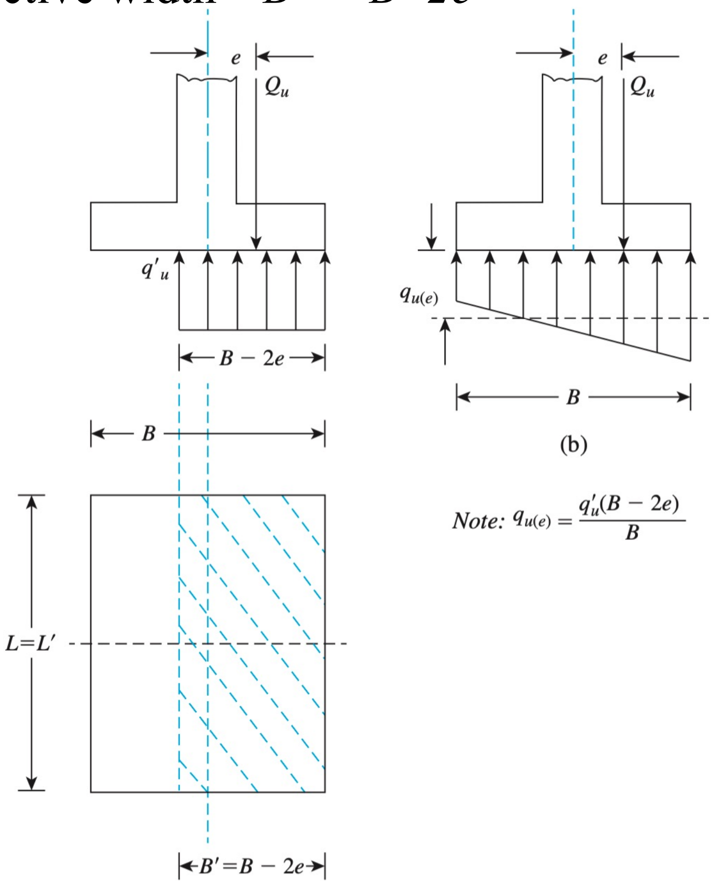
When e $\leq \frac{B}{6}$, this is accepted

When e $> \frac{B}{6}$, this is unaccepted

Ultimate Bearing Capacity under Eccentric Loading:
One-Way Eccentricity : Effective Area Method

step-by-step procedure for determining the ultimate load that the soil can support and the factor of safety against bearing capacity failure:

1) Determine the effective dimensions of the foundation as shown in the Figure : B' = effective width = B' = $B - 2e$
L' = effective length = L .



Note that if the eccentricity were in the direction of the length of the foundation, the value of L' would be equal to (L - 2e). The value of B' would equal B. The smaller of The two dimensions (i.e., L' and B') is the effective width of the foundation.

2) Use the same equation for determination of ultimate bearing capacity by Meyerhof, But, only substitute B by B' that was found by step (1) in above.

$$q'_u = c'N_cF_{cs}F_{cd}F_{ci} + qN_qF_{qs}F_{qd}F_{qi} + \frac{1}{2}\gamma B'N_\gamma F_{\gamma s}F_{\gamma d}F_{\gamma i}$$

3) The total ultimate load that the foundation can sustain is

$$Q_u = q'_u \overbrace{(B')(L')}^{A'}$$

where A' = effective area.

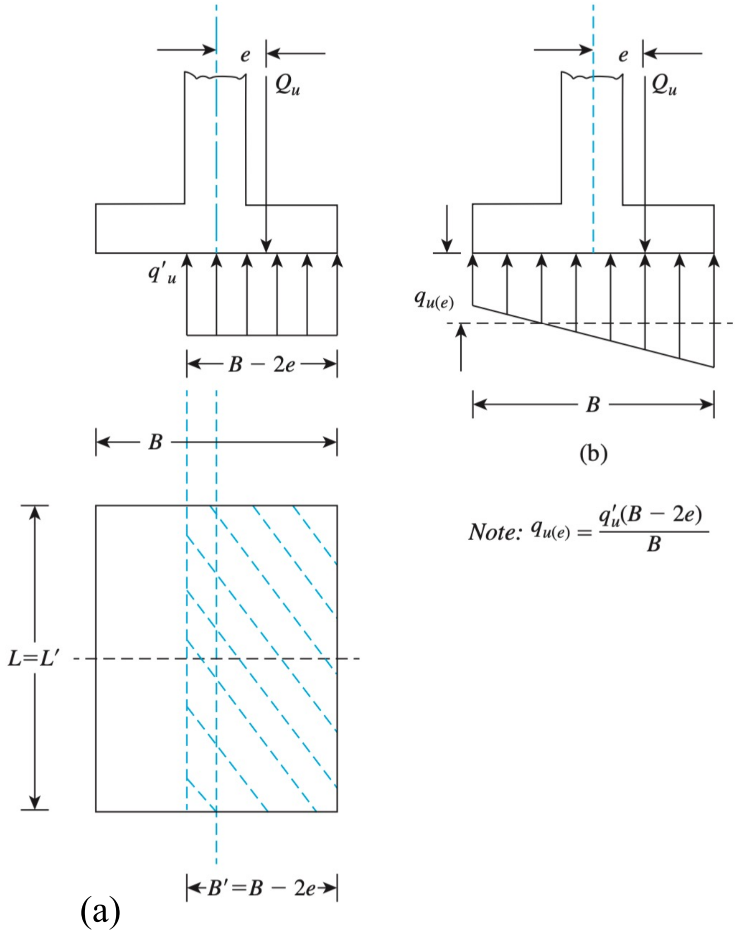
4) The factor of safety against bearing capacity failure is: $\mathrm{FS} = \dfrac{Q_u}{Q}$

Important Note: It is important to note that q'_u is the ultimate bearing capacity of a foundation of width $B' = (B-2e)$ with a centric load as in the Figure (a). However, the actual distribution of soil reaction at ultimate load will be of the type shown in Figure (b).

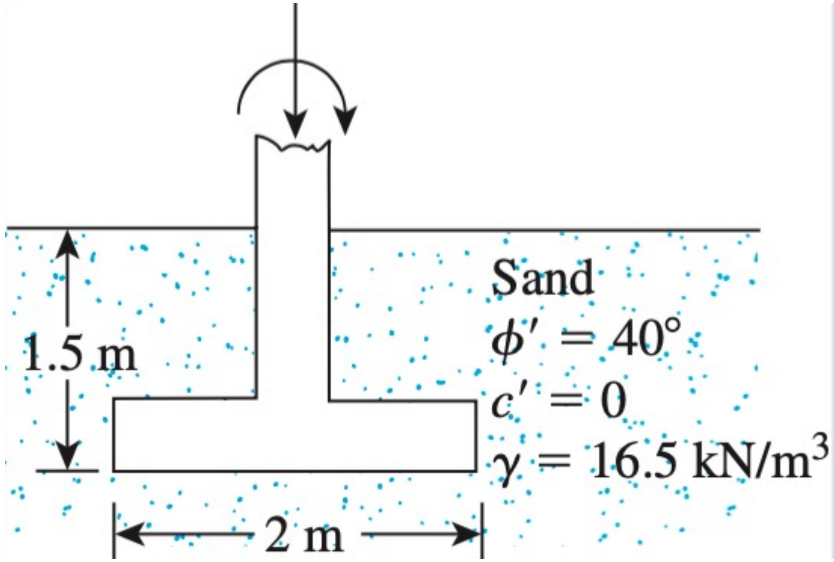
In Figure (b), $q_{u(e)}$ is the average load per unit area of the foundation

Thus:

$$q_{u(e)} = \frac{q'_u(B - 2e)}{B}$$



Example: A continuous (strip) foundation is shown in the Figure. If the load eccentricity is 0.2 m, determine the ultimate load, Q_u, per unit length of the foundation. Use Meyerhof's effective area method.



Solution

For $c' = 0$, Eq. gives:

$$q'_u = qN_qF_{qs}F_{qd}F_{qi} + \frac{1}{2}\gamma' B' N_\gamma F_{\gamma s}F_{\gamma d}F_{\gamma i}$$

where $q = (16.5)(1.5) = 24.75$ kN/m²

For $\phi' = 40°$, $N_q = 64.2$ and $N_\gamma = 109.41$. Also,

$$B' = 2 - (2)(0.2) = 1.6 \text{ m}$$

Because the foundation in question is a continuous foundation, B'/L' is zero. Hence, $F_{qs} = 1$, $F_{\gamma s} = 1$.

$$F_{qi} = F_{\gamma i} = 1$$

$$F_{qd} = 1 + 2\tan\phi'(1 - \sin\phi')^2\frac{D_f}{B} = 1 + 0.214\left(\frac{1.5}{2}\right) = 1.16$$

$$F_{\gamma d} = 1$$

and

$$q'_u = (24.75)(64.2)(1)(1.16)(1) + \left(\frac{1}{2}\right)(16.5)(1.6)(109.41)(1)(1)(1) = 3287.39 \text{ kN/m}^2$$

Consequently,

$$Q_u = (B')(1)(q'_u) = (1.6)(1)(3287.39) \approx \mathbf{5260\ kN}$$

Ultimate Bearing Capacity under Eccentric Loading:
Two-Way Eccentricity :

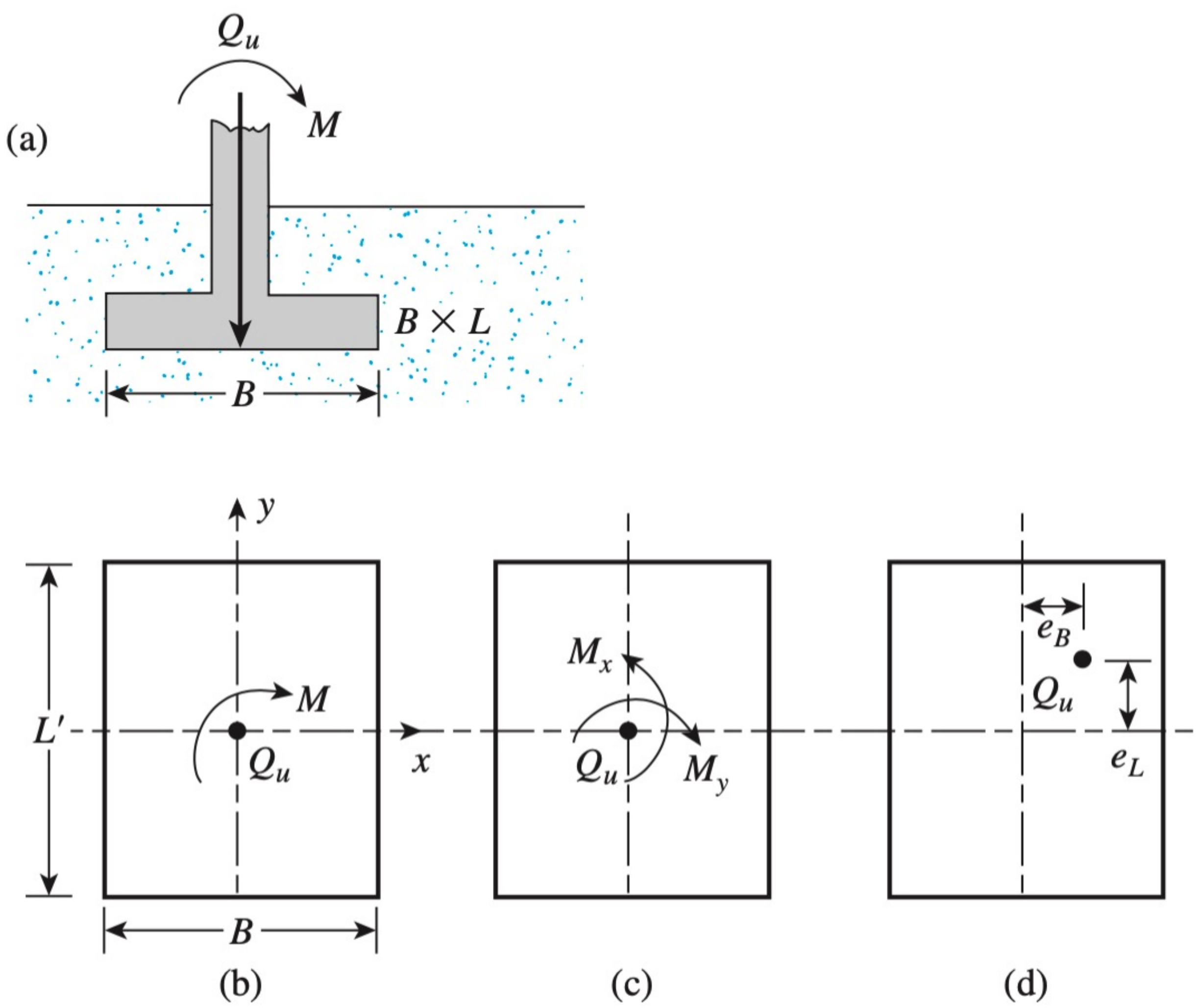


Figure 4.24 Analysis of foundation with two-way eccentricity

M about the x- and y-axes can be determined as M_x and M_y, respectively. (See Figure 4.24c.) This condition is equivalent to a load Q_u placed eccentrically on the foundation with $x = e_B$ and $y = e_L$ (Figure 4.24d). Note that

$$e_B = \frac{M_y}{Q_u} \tag{4.65}$$

and

$$e_L = \frac{M_x}{Q_u} \tag{4.66}$$

If Q_u is needed, it can be obtained from Eq. (4.52); that is,

$$Q_u = q'_u A'$$

where, from Eq. (4.51),

$$q'_u = c'N_c F_{cs} F_{cd} F_{ci} + qN_q F_{qs} F_{qd} F_{qi} + \tfrac{1}{2}\gamma B' N_\gamma F_{\gamma s} F_{\gamma d} F_{\gamma i}$$

and

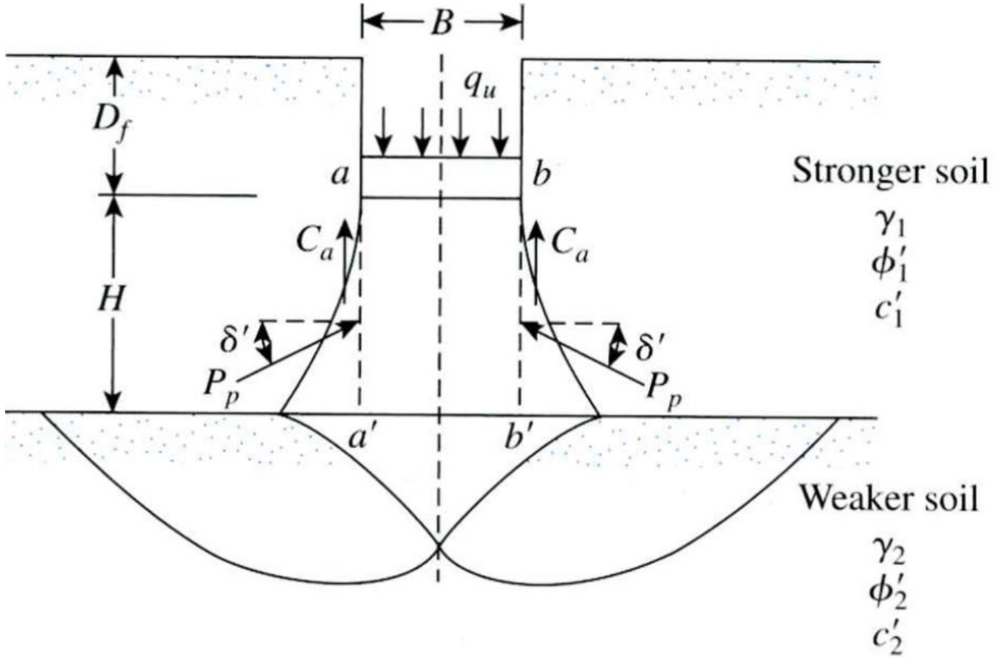
$$A' = \text{effective area} = B'L'$$

As before, to evaluate F_{cs}, F_{qs}, and $F_{\gamma s}$ (Table 4.3), we use the effective length L' and effective width B' instead of L and B, respectively. To calculate F_{cd}, F_{qd}, and $F_{\gamma d}$, we do not replace B with B'. In determining the effective area A', effective width B', and effective length L', five possible cases may arise (Highter and Anders, 1985).

Bearing Capacity of Layered Soils: Stronger soil Underlain by Weaker Soil

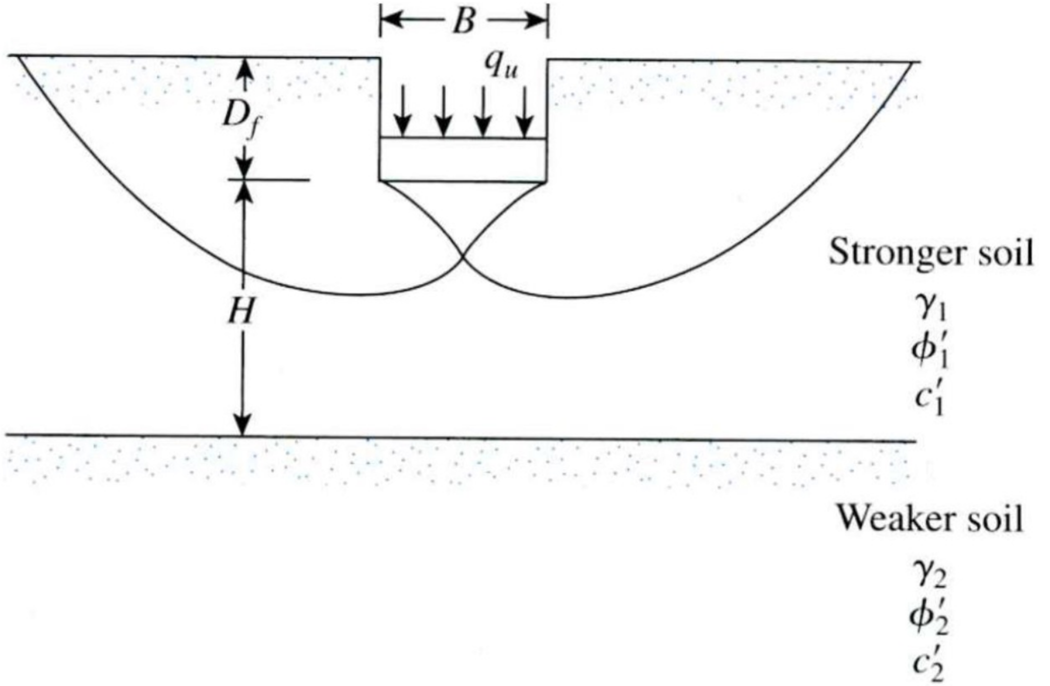
Soil Properties			
Layer	Unit weight	Friction angle	Cohesion
Top	γ_1	ϕ_1	c_1
Bottom	γ_2	ϕ_2	c_2

If the depth H is relatively small compared with the foundation width B



$$q_u = q_b + \frac{2c_a \times H}{B} + \gamma_1 H^2 \left(1 + \frac{2D_f}{H}\right) \times \frac{K_s \tan\phi_1}{B} - \gamma_1 \times H$$

If the depth ,H, is relatively large (thickness of top layer is large)



$$q_u = q_t = c_1 N_{c(1)} + q\, N_{q(1)} + 0.5B\gamma_1 N_{\gamma(1)}$$

$$q_u = q_b + \left(1 + \frac{B}{L}\right) \times \frac{2c_a \times H}{B}$$

$$+\gamma_1 H^2 \times \left(1 + \frac{B}{L}\right)\left(1 + \frac{2D_f}{H}\right) \times \frac{K_s \tan\phi_1}{B} - \gamma_1 \times H \leq q_t$$

$$q_t = c_1 N_{c(1)} F_{cs(1)} + q\, N_{q(1)} F_{qs(1)} + 0.5B\gamma_1 N_{\gamma(1)} F_{\gamma s(1)}$$

q = effective stress at the top of layer(1) = $\gamma_1 \times D_f$

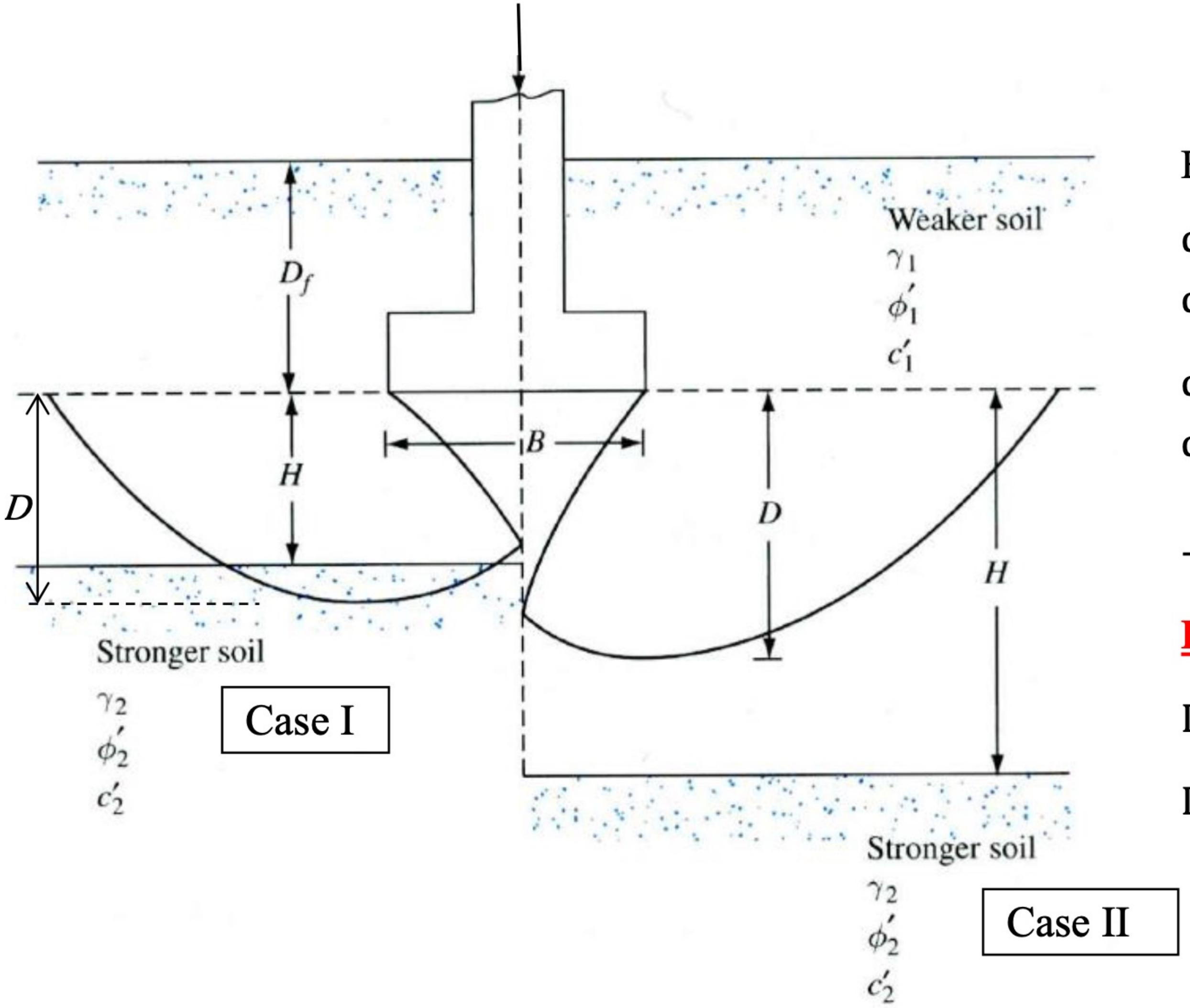
$$q_b = c_2 N_{c(2)} F_{cs(2)} + q\, N_{q(2)} F_{qs(2)} + 0.5B\gamma_2 N_{\gamma(2)} F_{\gamma s(2)}$$

q = effective stress at the top of layer(2) = $\gamma_1 \times (D_f + H)$

All depth factors will equal (1) because their considered in punching term.

Assume no inclination so, all inclination factors equal (1).

Bearing Capacity of Layered Soils: Weaker soil Underlain by Stronger Soil



Because failure surface is fully located on top (weaker soil).

$q_{t,weak} = c_1 N_{c(1)} F_{cs(1)} + q\, N_{q(1)} F_{qs(1)} + 0.5 B \gamma_1 N_{\gamma(1)} F_{\gamma s(1)}$

$q =$ effective stress at the top of layer(1) $= \gamma_1 \times D_f$

$q_{b,strong} = c_2 N_{c(2)} F_{cs(2)} + q\, N_{q(2)} F_{qs(2)} + 0.5 B \gamma_1 N_{\gamma(2)} F_{\gamma s(2)}$

$q =$ effective stress at the top of layer(1) by assuming the foundation is located directly above stronger soil layer at depth of D_f

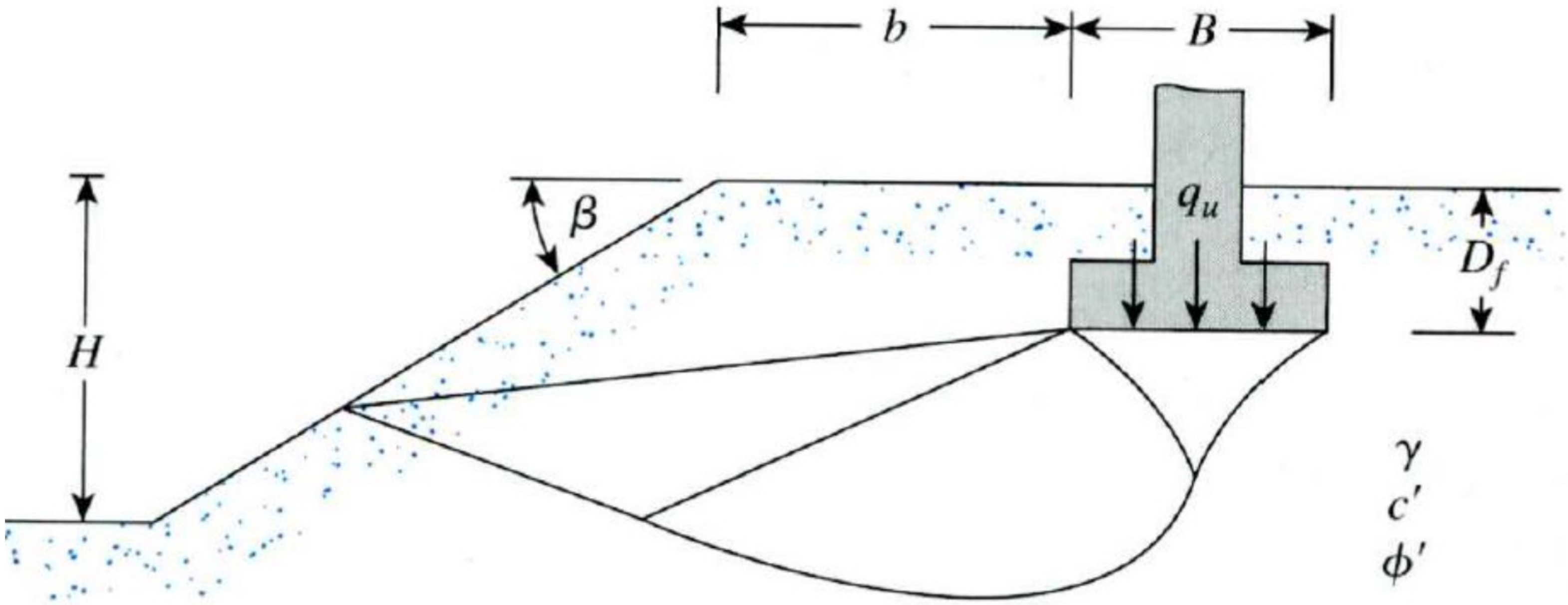
$\rightarrow q = \gamma_2 \times D_f$

Important Note:

$D = B$ (for **loose sand** and **clay**)

$D = 2B$ (for **dense sand**)

Bearing Capacity of Foundations on Top of a Slope



H = height of slope , β = angle between the slope and horizontal
b = distance from the edge of the foundation to the top of the slope

The ultimate bearing capacity for **continuous or strip** footing can be calculated by the following theoretical relation:

$$q_u = cN_{cq} + 0.5B\gamma N_{\gamma q}$$

For purely granular soil (c = 0.0):

$$q_u = 0.5B\gamma N_{\gamma q}$$

For purely cohesive soil (ϕ = 0.0):

$$q_u = cN_{cq}$$

Uplift Capacity of Foundations

The uplift capacity of the foundation should be determined from the cases when the superstructure exerted uplift pressure on the foundation, such as for towers of transmition lines for electricity lines or others facilities. For details you can refer to any text books contains this subject.

These miscellaneous subjects for useful information

Foundation Engineering (2)

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Design of Deep (Pile) Foundation (a)

- General (definition, why?)
- Types

Pile Foundations: Definition and why?

A typical deep foundation consists of a cluster of piles installed down to a certain depth in order to transfer the load to a more competent bearing layer or to distribute the load over a larger depth.

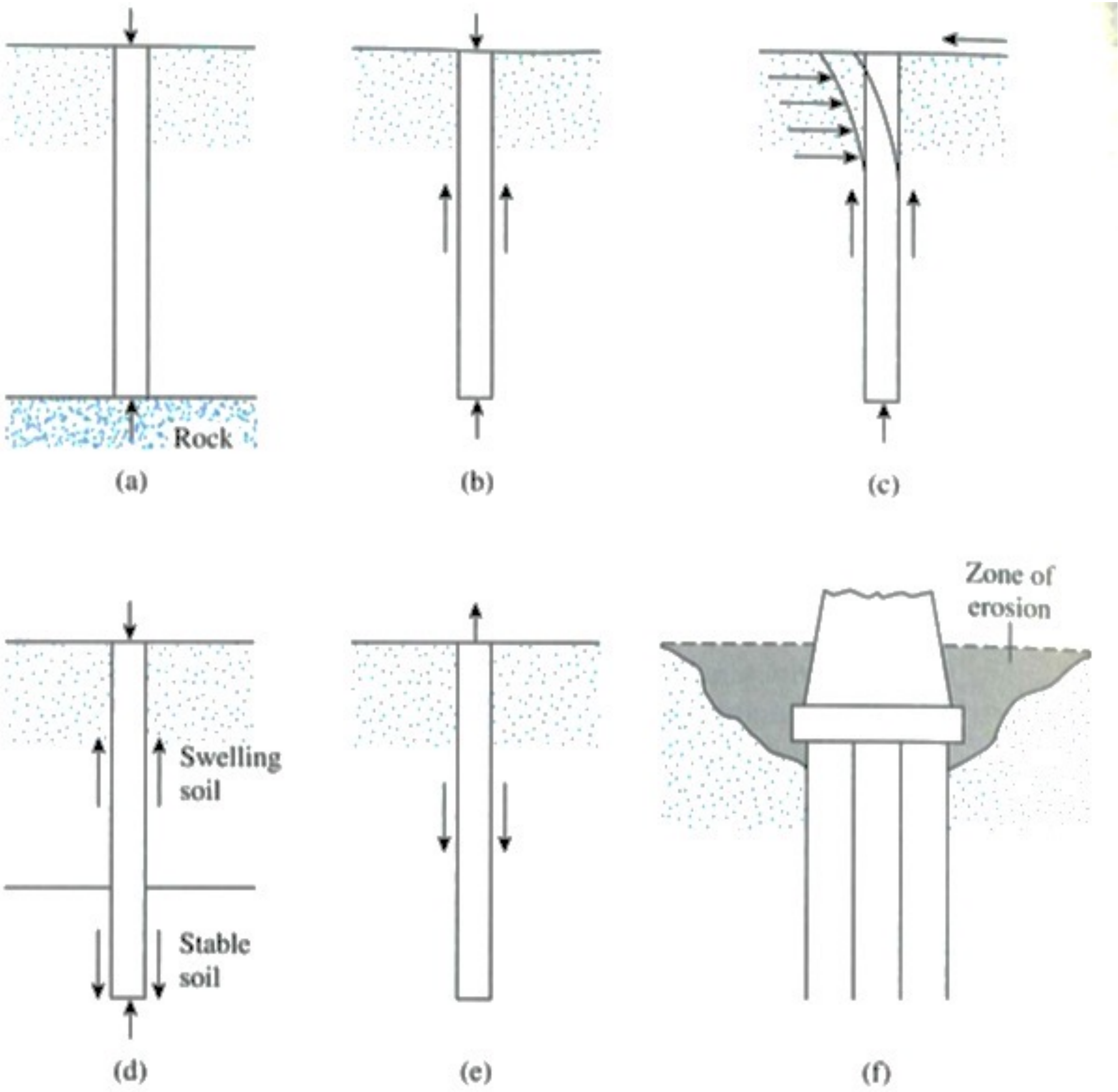
Deep foundations are required when the subsurface conditions immediately below the structure or ground surface are not suitable for the support of the structure. Thus, the foundation needs to be deepened or extended to a lower soil or rock layer, usually at a higher cost than that of shallow foundation :

Pile Foundations: Definition and why?, cont'd

Subsurface and/or loading conditions that lead to the use of deep foundations can include:

- ✓ Inadequate strength or compressibility of the soil immediately below the structure or ground surface to support the loading conditions using a shallow foundation.
- ✓ The soil immediately below the structure or current ground surface may be eroded by wind or water scouring during the life of the structure. These conditions can include structures to be built on sand dunes in a desert or river crossing bridges .
- ✓ Excessive movement of the soil immediately below the structure or ground surface could occur due to changes in moisture content (i.e., expansive or collapsible soil), or liquefiable under seismic (earthquake) loading conditions.
- ✓ For structures that impose large tensile force, lateral force, bending moment, or combinations of the above on the foundation, the deep foundation is likely to be more cost effective than a shallow foundation. Structures such as transmission towers, light poles, offshore oil rigs, wind turbines, or super high-rise buildings can involve such loading conditions.

Pile Foundations: Definition and why?, cont'd



(a): The bed rock so longer deep.
(b): The load resistance is du to friction.
(c): When, the structure subjected to horizontal forces (wind and/or earthquake.
(d): Below the surface, the soil is swelling or collapsible (problematic soil).
(e): For resisting pull out forces, as for transmition towers.
(f): Piers of bridge to be out of erosion effect.

Pile types

- **According to materials**

Table: Summary of various types of piles

Pile type	*Timber pile*	*Steel H-pile*	*Steel pipe pile*	*Pre-cast pre-stressed Pile*	*Cast-in-place pile cased*	*Cast-in-place pile uncased*
Typical length, m	6–23	6–30	9–36	9–15 (pre-cast) 15–36 (pre-stressed)	3–36	3–100
Typical design axial capacity, kN	156–670	450–1800	900–2200	450–1100	450–1300	450–45,000
Advantages	• Low cost • Renewable resource • Easy to handle/drive • Natural taper, higher shaft friction in granular soil than uniform piles	• Small displacement • High-load capacity • Easy to splice	• High-load capacity • Small displacement when open ended • Easy to splice	• High-load capacity • Corrosion resistance obtainable	• Can be driven with or without mandrel • Can be inspected after casing driving • Tapered section provides higher shaft friction in granular soil than uniform piles	• Significantly less noise and vibration than driven piles • Very large load can be carried by a single pile • Applicable to a wide variety of soil conditions • Changes in pile geometry possible during the progress of construction if ground conditions so dictate.
Disadvantages	• Low-axial capacity • Difficult to splice	• Vulnerable to corrosion • Not efficient as friction pile	• Open ended not efficient as friction pile	• Vulnerable to handling/driving damage	• Thin shells (mandrel driven) vulnerable to damage during driving • Displacement can be high	• Critical to construction procedure and workmanship

Source: NAVFAC, 1986. *Foundations and Earth Structures, Design Manual DM 7.02.* Department of the Navy, Naval Facilities Engineering Command, Alexandria, VA, 279p.

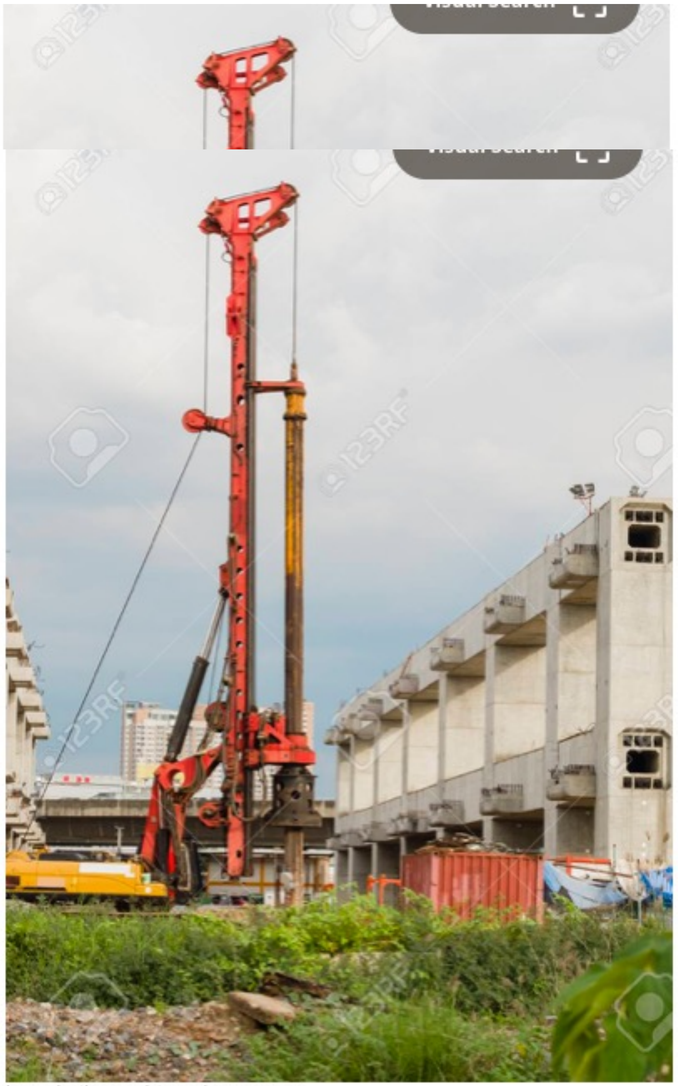
Pile types, Cont'd

- **According to method of placement:**

➢ **Driven piles are: precast prestressed concrete, steel (H and pipe).**

Pile Drivers

- Drop hammer
  - Hammer drops on piling
- Mechanical
  - Piston actuated by steam or compressed air
- Vibratory hammer



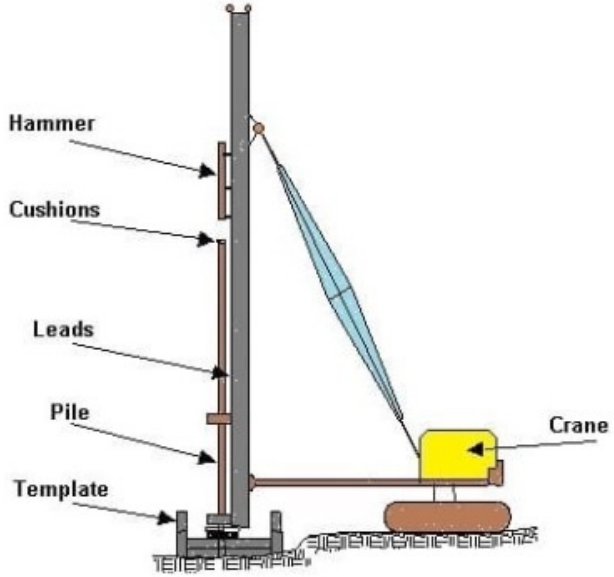
bored pile machine, dry process

Steel pipes

Impact driving of steel pipes used for the installation of foundation piles.

Precast impact driven piles

Impact driving of precast concrete piles belongs to the classic pile driving applications.



Some photos for boring piles and machines for driving piles

Pile types, Cont'd

- **According to method of placement:**

  - **Cast in place piles(bored piles)**

There are three main procedures for placing a bored pile:

1. Dry method (Figure 1)
2. Casing method (Figure 2)
3. Wet method (Figure 3)

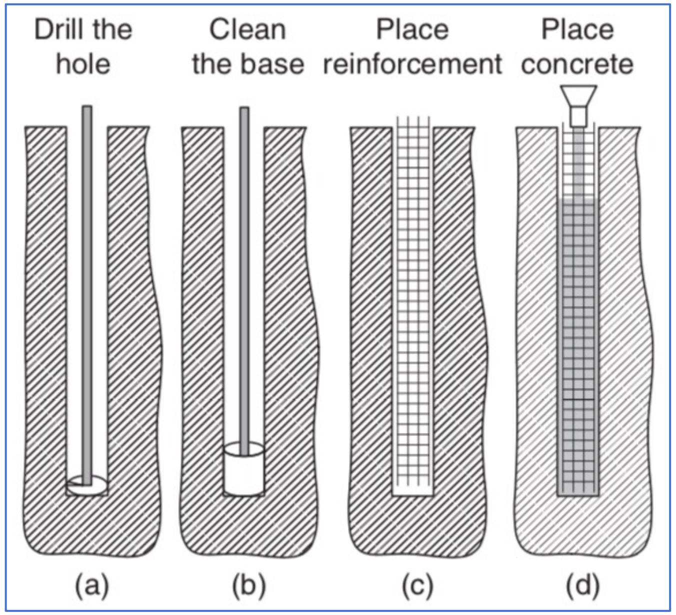


Fig. -1-: Dry method

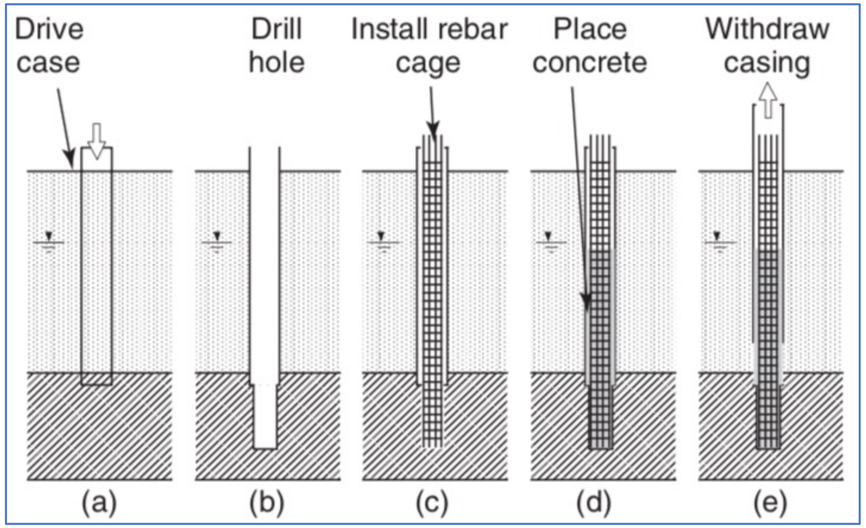


Fig. -2-: Casing method

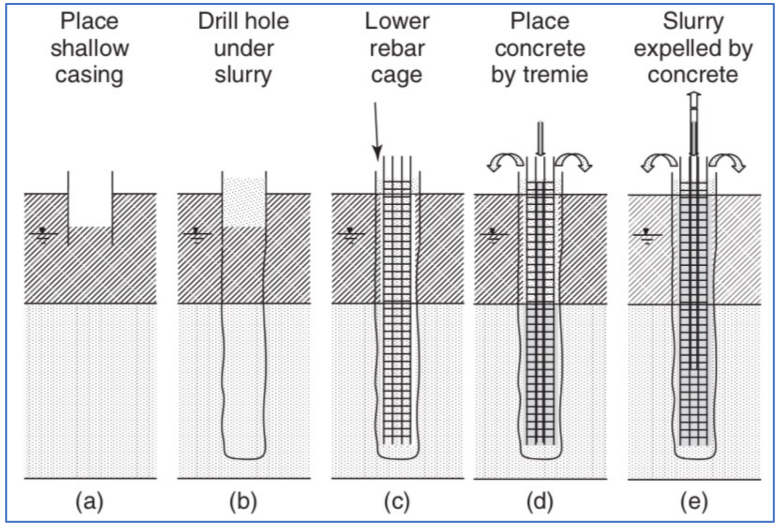
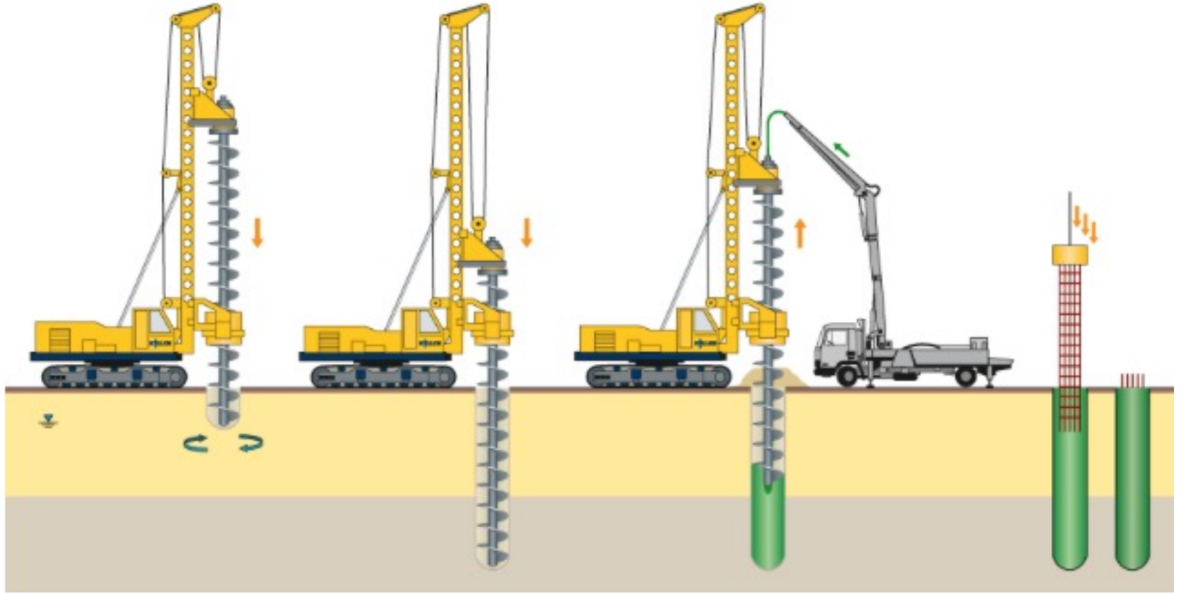


Fig. -3-: Wet method

Stages of boring and installation of cages of reinforcement and pouring piles



Video on Bored pile in process

This video can be opened by the following link by youtube. In this video you can watch the drilling for installing the reinforcement cage and then casting concrete in insitu.
https://www.youtube.com/watch?v=w0sHutDSQ8c

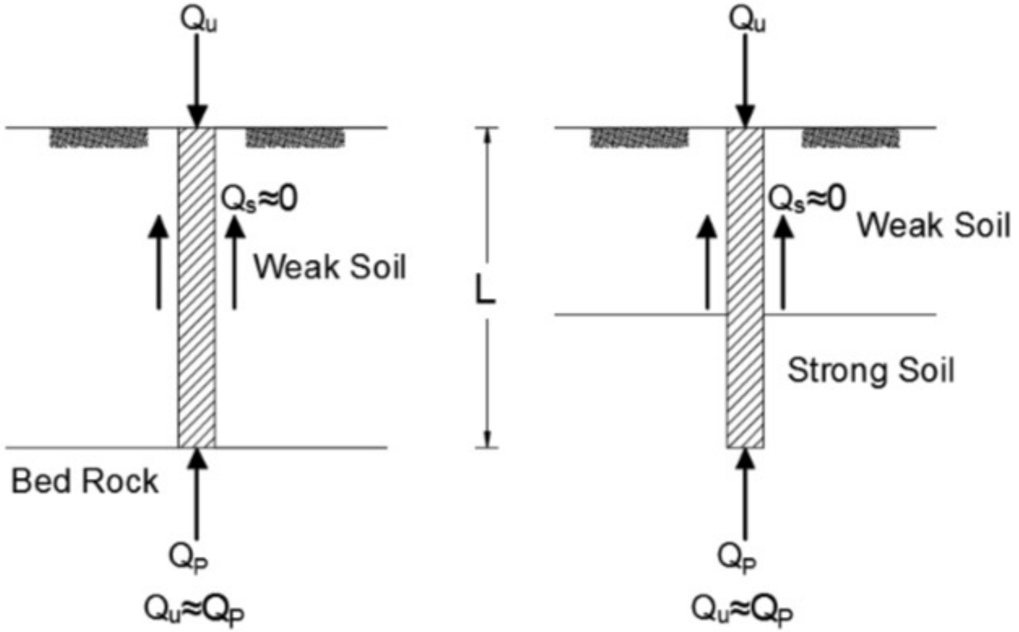
Pile types, Cont'd

- **method of load transfer:**

➢ **Point Bearing Piles:**

If the soil supporting the structure is weak soil, pile foundation will used to transmit the load to the strong soil layer or to the bed rock (if encountered), here the pile will resist the entire load depending on its end point load Q_P and the value of Q_s (frictional resistance) is very small in this case, so:

$Q_u = Q_P + Q_s$ $Q_s \approx 0.0 \rightarrow Q_u = Q_P$ (Point Bearing Piles).



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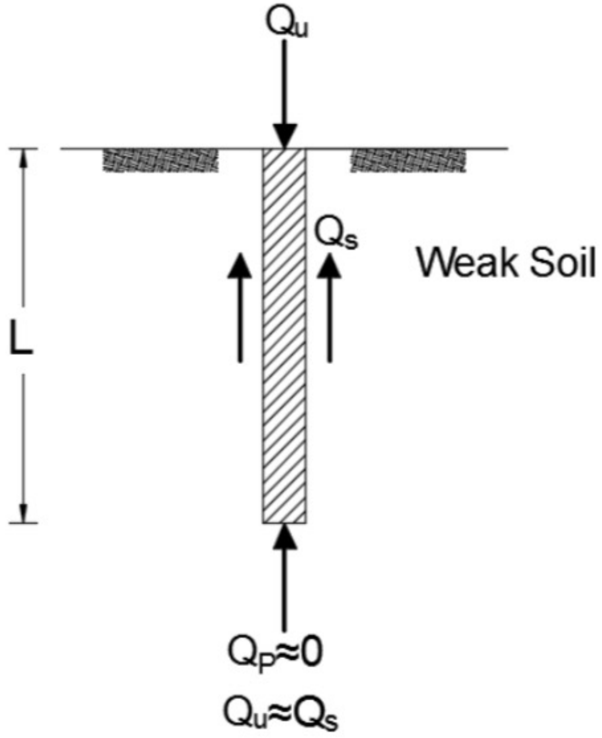
Pile types, Cont'd

- **method of load transfer; Cont'd:**

➢ Friction Piles:

When no strong layer or rock is present at reasonable depth at a site, point bearing piles becomes very long (to reach strong layer) and uneconomical. In these type of soil profiles, piles are driven through the softer (weaker) soil to specified depth, and here the point bearing load (Q_P) is very small and can be considered zero, however the load on the pile will resisted mainly by the frictional resistance between soil and pile (skin frictional force(Q_s) so:

$Q_u = Q_P + Q_s$, $Q_P \approx 0.0 \rightarrow Q_u = Q_s$ (Friction Piles)



End of this part of Pile footing theme

Foundation Engineering (2)

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Deep (Pile) Foundation

- Types of piles
- Installation
- load test of piles

CLASSIFICATION OF PILES:

- With respect to (W.R.T):

  1. Mode of construction
  2. Material of construction
  3. Method of loading
  4. Function of pile
  5. Shape
  6. Size

CLASSIFICATION W.R.T MODE OF CONSTRUCTION:

1. Pre-cast (Driven Piles)
2. Cast in-situ Piles (Bored Piles)

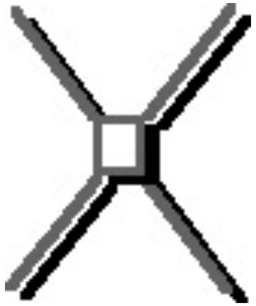
- Under sized Bore.(It is feasible because of less noise , under sized hole is dug and full size pile is driven.
- By driving the piles, the soil is displaced so type is

a) High volume displaced piles (vol. almost equal to vol.of pile).
b) No volume displaced piles.
c) Low volume displaced piles.

CLASSIFICATION W.R.T MATERIAL OF CONSTRUCTION:

1) Timber piles: (Trunk of a Wooden tree, the oldest pile)
2) Concrete pile
3) Steel pile
4) Composite pile: (Certain portion by one material and certain portion by other material)

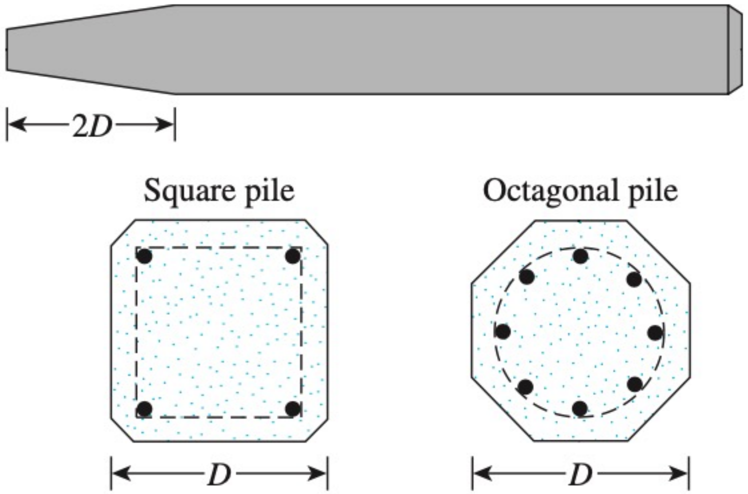
Steel piles cross-sections



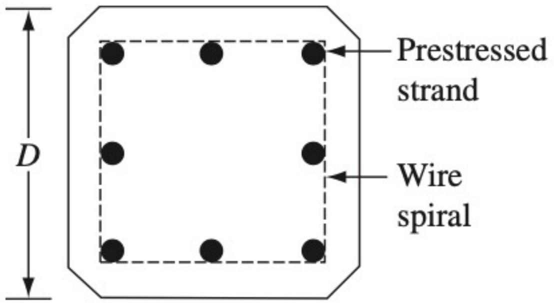
a) X- cross-section

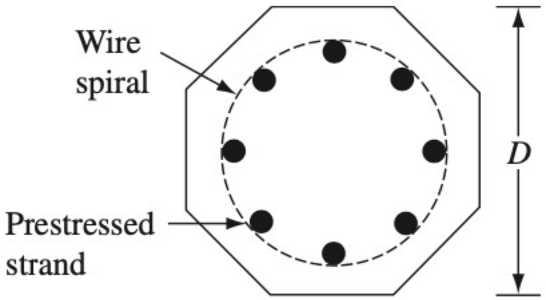
b) H - cross-section

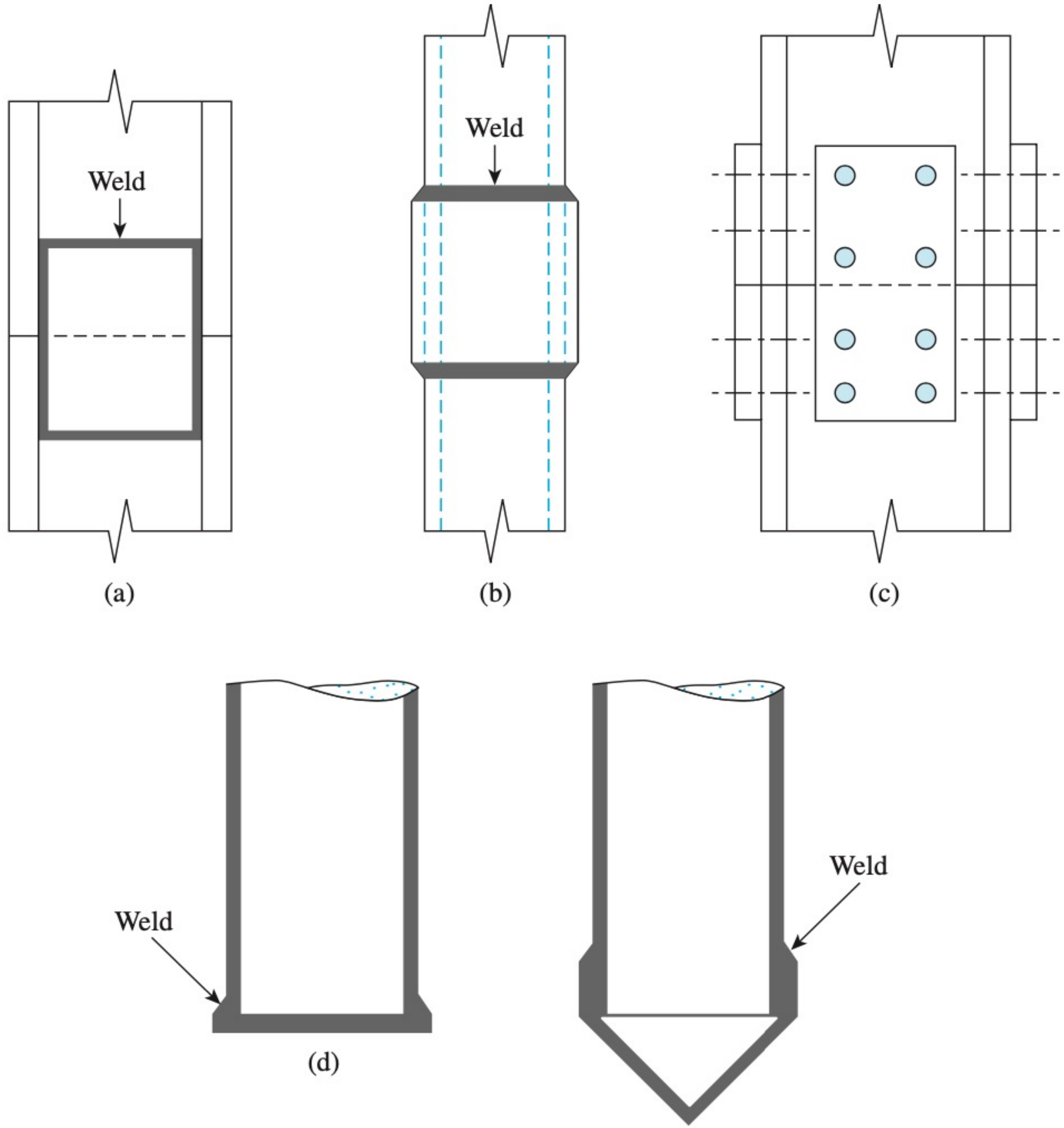
c) steel pipe

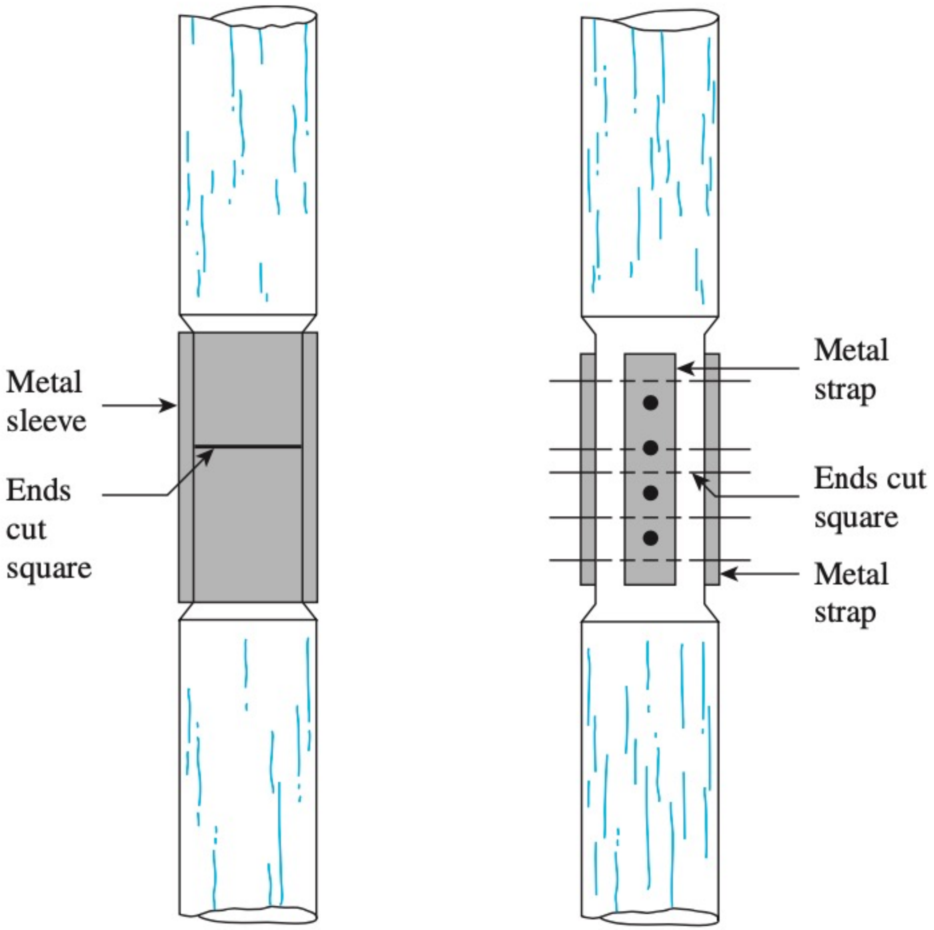


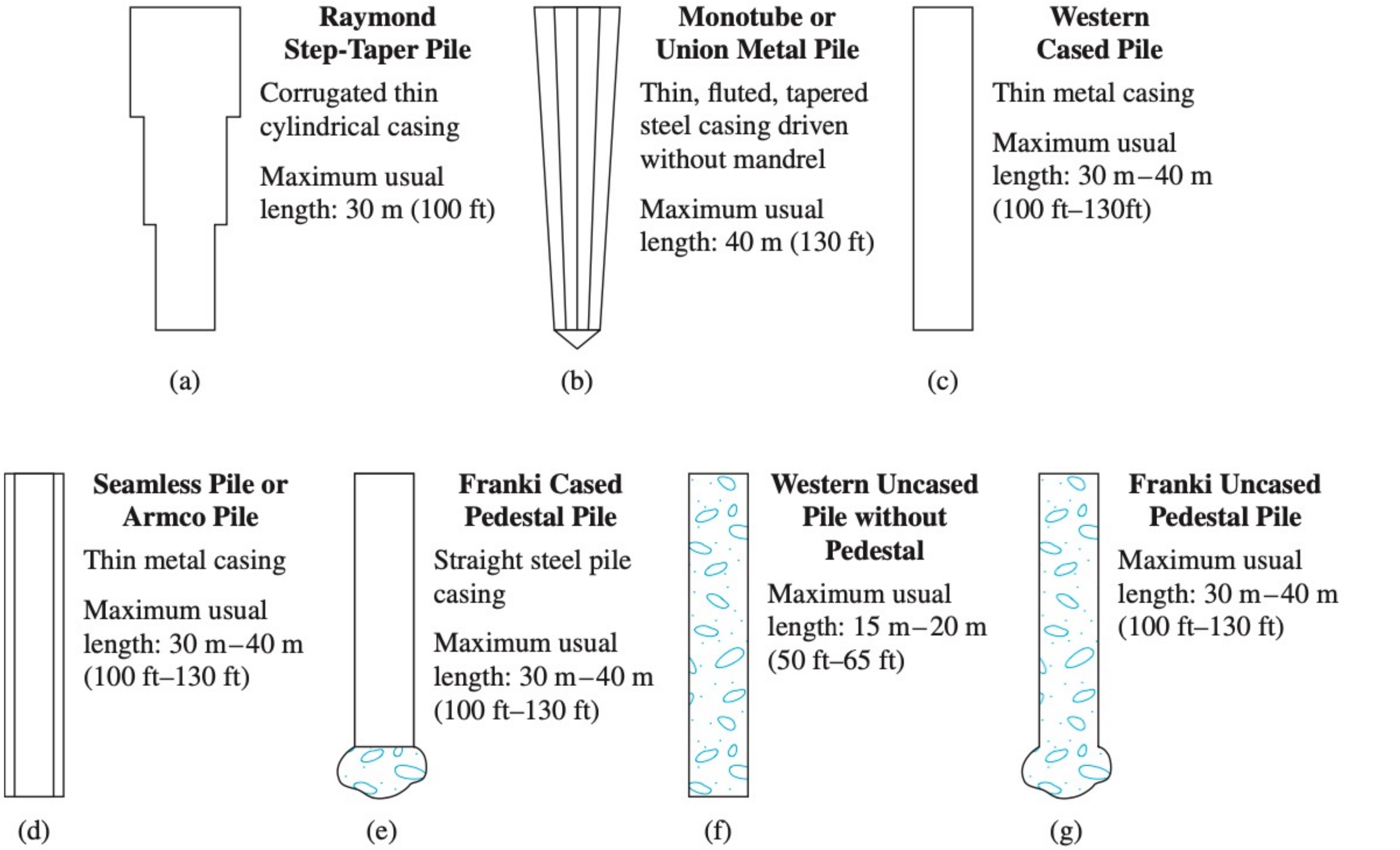
Precast piles with ordinary reinforcement






Weld
Weld
(a)
(b)
(c)
Weld
Weld
(d)


Metal
sleeve
Ends
cut
square
Metal
strap
Ends cut
square
Metal
strap



Cast-in-place concrete piles

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CLASSIFICATION W.R.T METHOD OF LOADING:

Some times skin friction is predominant and sometimes the End bearing so

1) Frictional Pile

If major part is taken by the shaft of pile. When very Weak soils of large depths are available.

2) End Bearing Pile

When a soil layer of reasonable strength is available at a reasonable depth.

3. Combination of Two. (Friction and End bearing piles)

CLASSIFICATION W.R.T FUNCTION OF PILE:

1) Compression pile: (To resist the comp. load)
2) Tension pile or Anchor pile
3) Compaction pile: (granular soil i.e. very loose sand can be compacted by driving the piles at one place, then are pulled out and driven at the next place, in this way sand is densified).
4) Fender piles: (Used near sea-part to protect the Harbour, just to absorb the impact of floating objects)
5) Batter piles: (Provided at an inclination their stability is more against overturning).
6) Sheet piles.(To reduce seepage or to provide lateral stability).


LEADS RAISE
Up to 15' above grade
FORWARD REACH
SIDE BATTER ANGLES
Up to 15°
BATTER ANGLES

CLASSIFICATION W.R.T SHAPE:

1. Round Piles
2. Square Piles
3. Octagonal Piles
4. I-Shaped Piles
5. Straight Piles
6. Tapered Piles
7. Bell-Bottom Piles
8. Screw Piles

CLASSIFICATION W.R.T SIZE:

1. Large Dia Pile: (> 24” (600 mm))
2. Small Dia Pile:(> 6” to 24” (150 – 600 mm)
3. Micro Dia Pile:(= 4” (100 mm) to 6” (150 mm)

(These are used for specific projects i.e. for Repair).

1. Root Pile(Rectangular) Used for special projects i,e for under pressing, Repair).

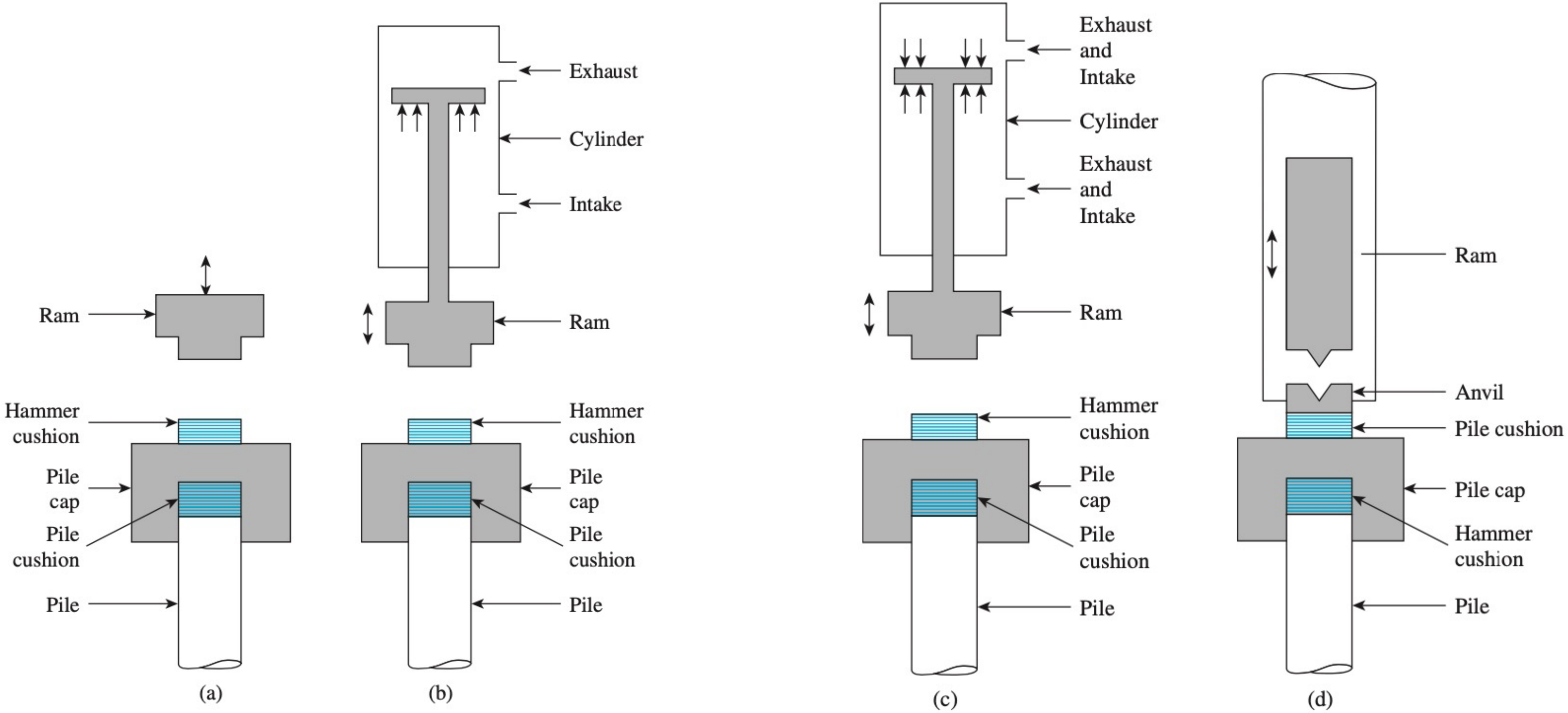
   If > 24” (600 mm) then These are called as pier.

<u>**Means of driving piles:**</u>

- hammers
- vibratory drivers
- jetting
- partial auguring

<u>**Types of hammer**</u>

(a) the drop hammer,
(b) the single-acting air or steam hammer,
(c) the double-acting and differential air or steam hammer,
(d) the diesel hammer.



Pile-driving equipment: (a) drop hammer; (b) single-acting air or steam hammer; (c) double-acting and differential air or steam hammer; (d) diesel hammer

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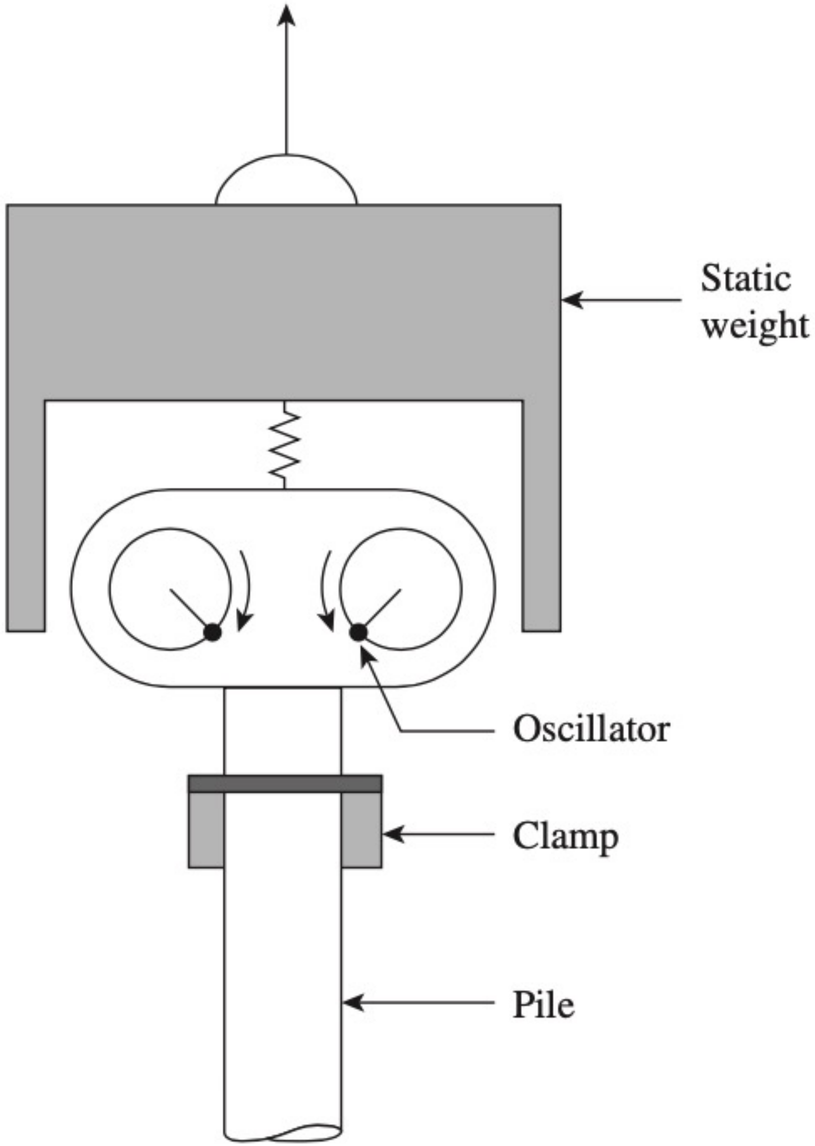


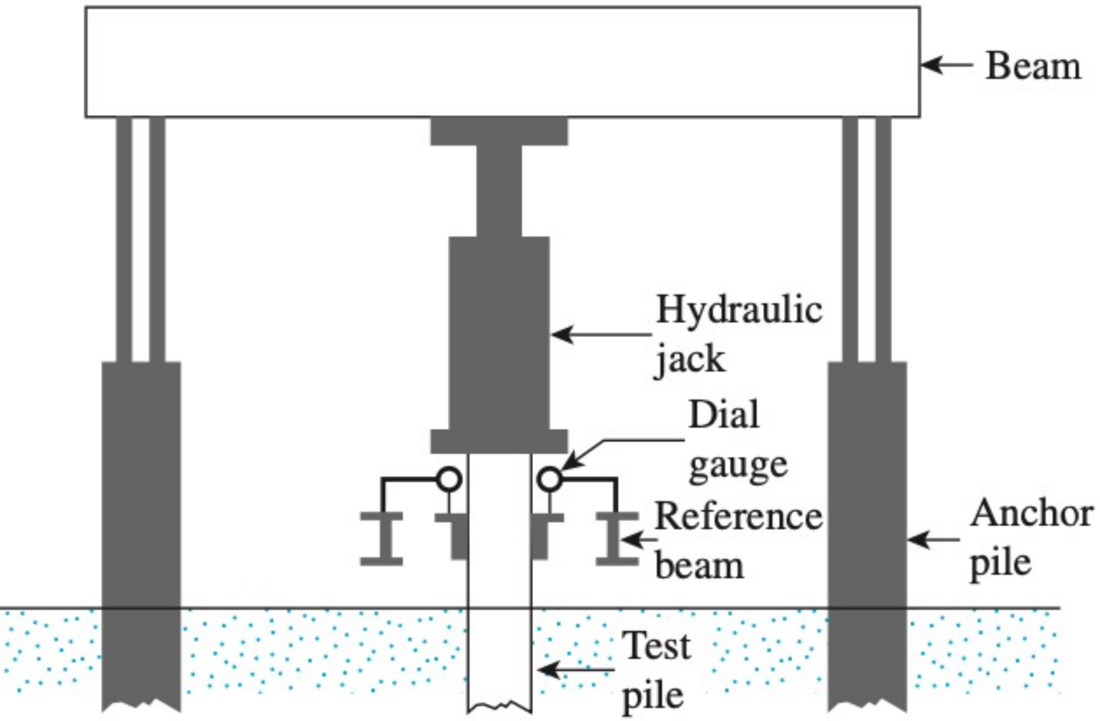
(e)

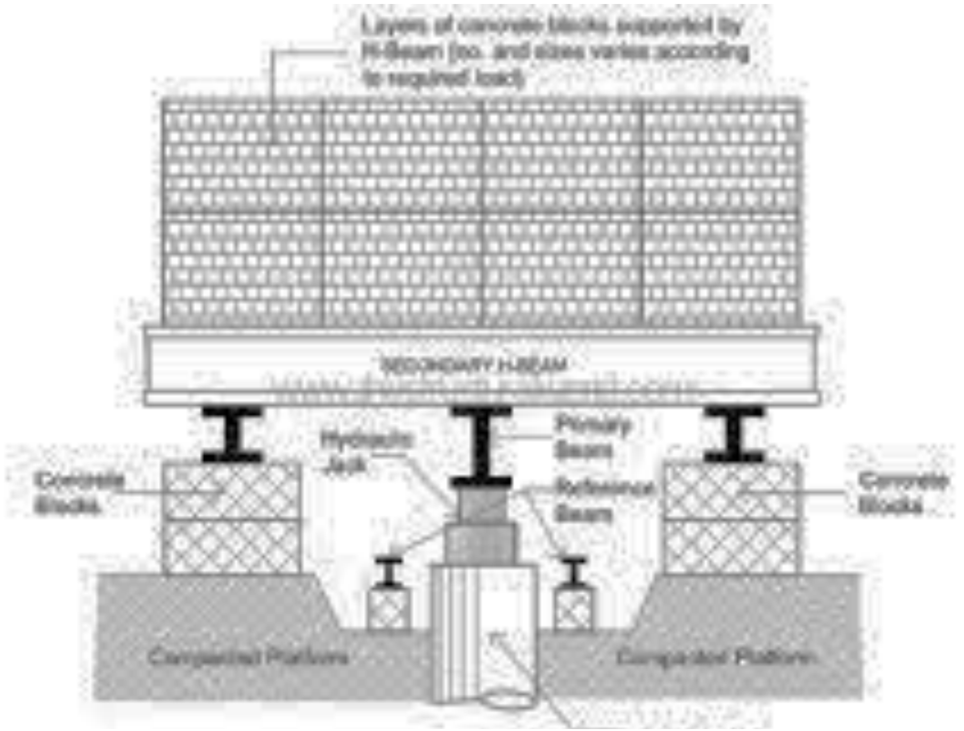
(f)

(*continued*) Pile-driving equipment: (e) vibratory pile driver; (f) photograph of a vibratory pile driver

Pile Load Tests

- ✓ A specific number of load tests must be conducted on piles.
- ✓ The primary reason is the unreliability of prediction methods.
- ✓ The vertical and lateral load- bearing capacity of a pile can be tested in the field.
- ✓ A schematic diagram of the pile load arrangement for testing *axial compression* in the field shown below.
- ✓ The load is applied to the pile by a hydraulic jack.
- ✓ Step loads are applied to the pile, and sufficient time is allowed to elapse after each load so that a small amount of settlement occurs.
- ✓ The settlement of the pile is measured by dial gauges.
- ✓ The amount of load to be applied for each step will vary, depending on local building codes.
- ✓ Most building codes require that each step load be about one-fourth of the proposed working load. The load test should be carried out to at least a total load of two times the proposed working load. After the desired pile load is reached, the pile is gradually unloaded.
- ✓ A load–settlement diagram obtained from field loading and unloading.





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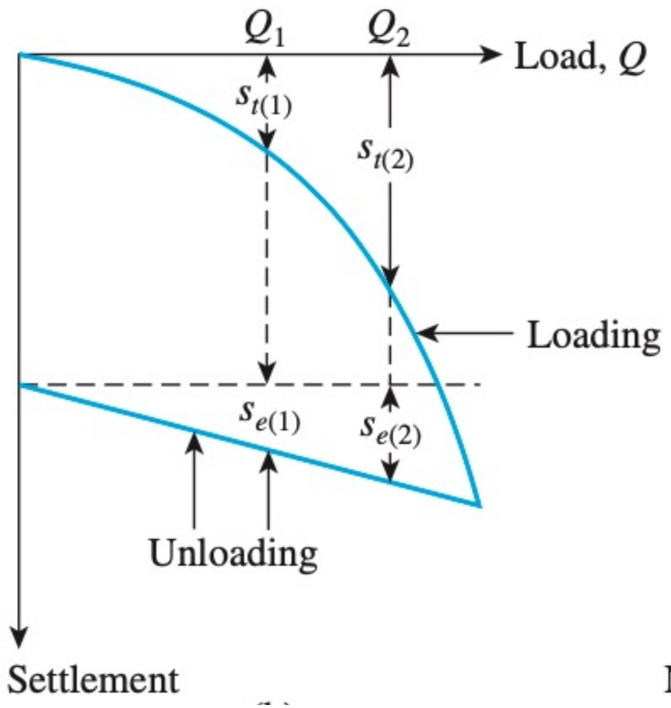

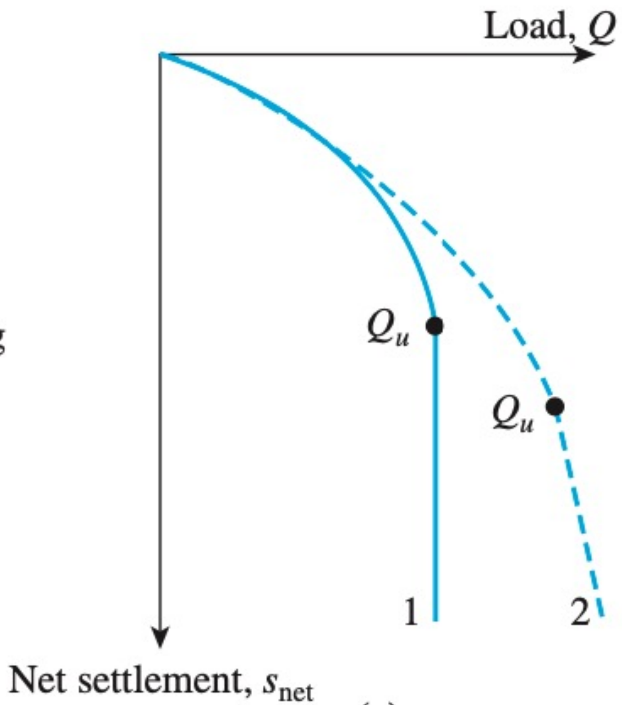

A: Load against total settlement. B: Load against net settlement

When $Q = Q_1$,

$$\text{Net settlement, } s_{net(1)} = s_{t(1)} - s_{e(1)}$$

When $Q = Q_2$,

$$\text{Net settlement, } s_{net(2)} = s_{t(2)} - s_{e(2)}$$

where

s_{net} = net settlement
s_e = elastic settlement of the pile itself
s_t = total settlement

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Any Questions?

Foundation Engineering (2)

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Retaining Walls (1):

- Types
- Overview of lateral earth pressure
- Stability against:
  - ✓ Overturning
  - ✓ Sliding
  - ✓ Base shear failure

General and Review

Structures that are built to retain vertical or nearly vertical earth banks or any other material are called *retaining walls*.

Retaining walls may be constructed of masonry or sheet piles. Some of the purposes for which retaining walls are used are shown in the Figures (Figure (1): (a to f)).

Retaining walls may retain water also. The earth retained may be natural soil or fill. Figure (2) shows main types of principal Retaining Walls (R.W.).

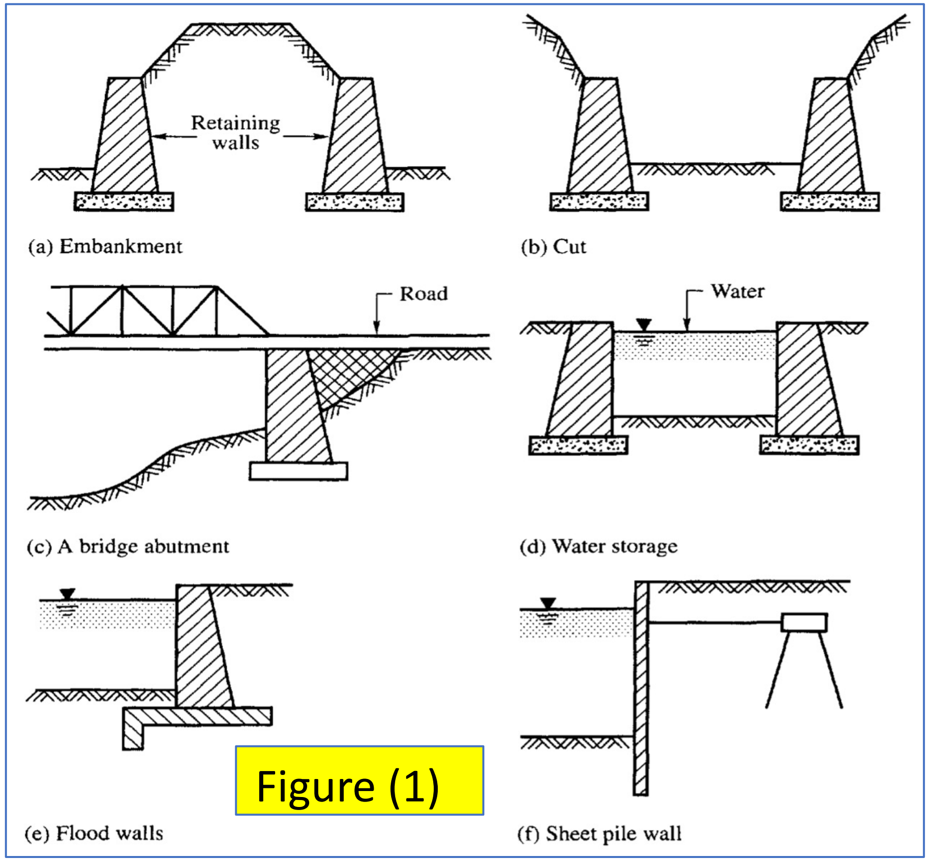


Figure (1)

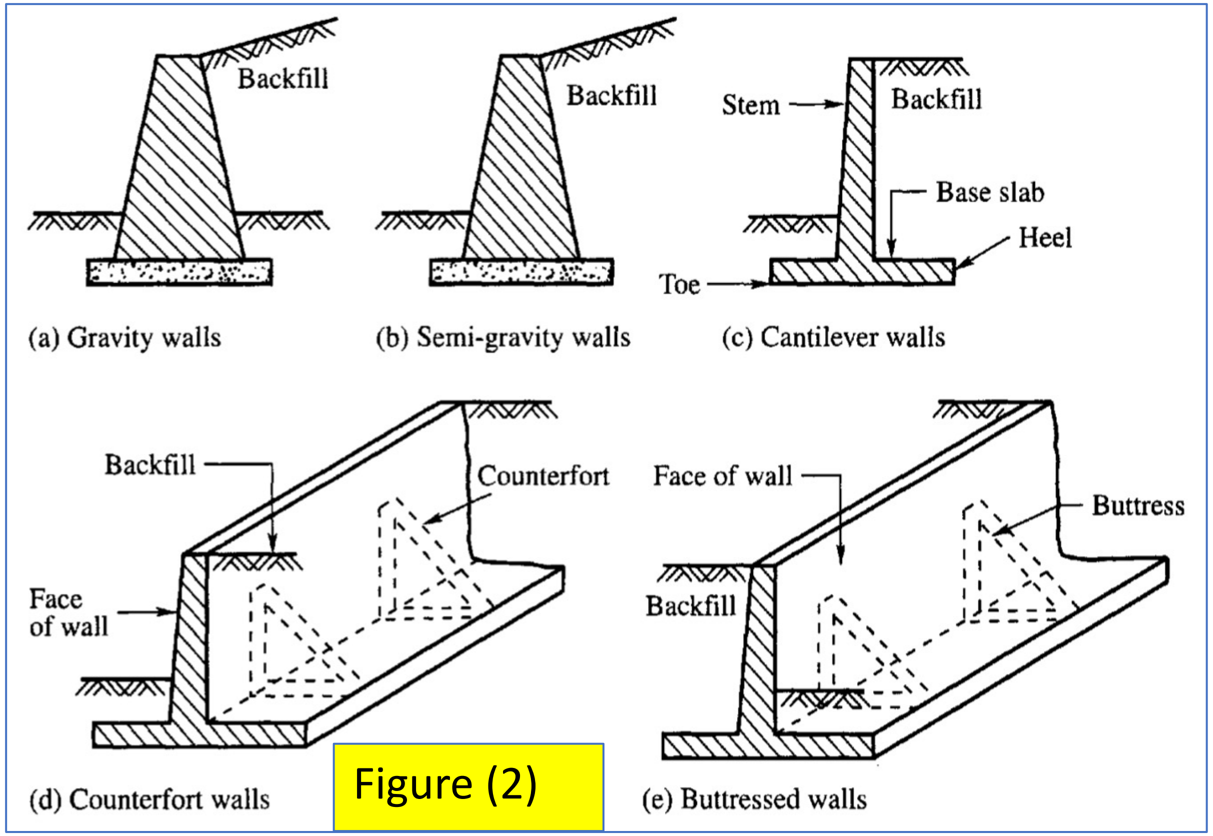
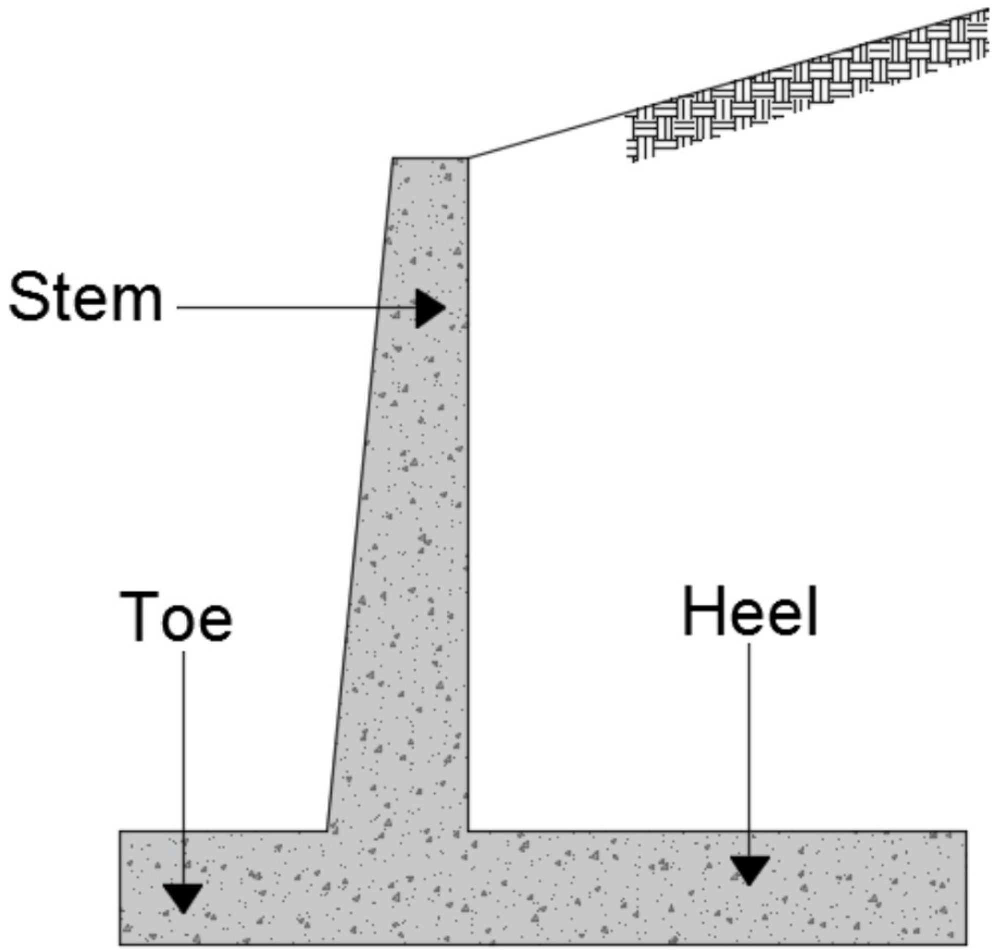


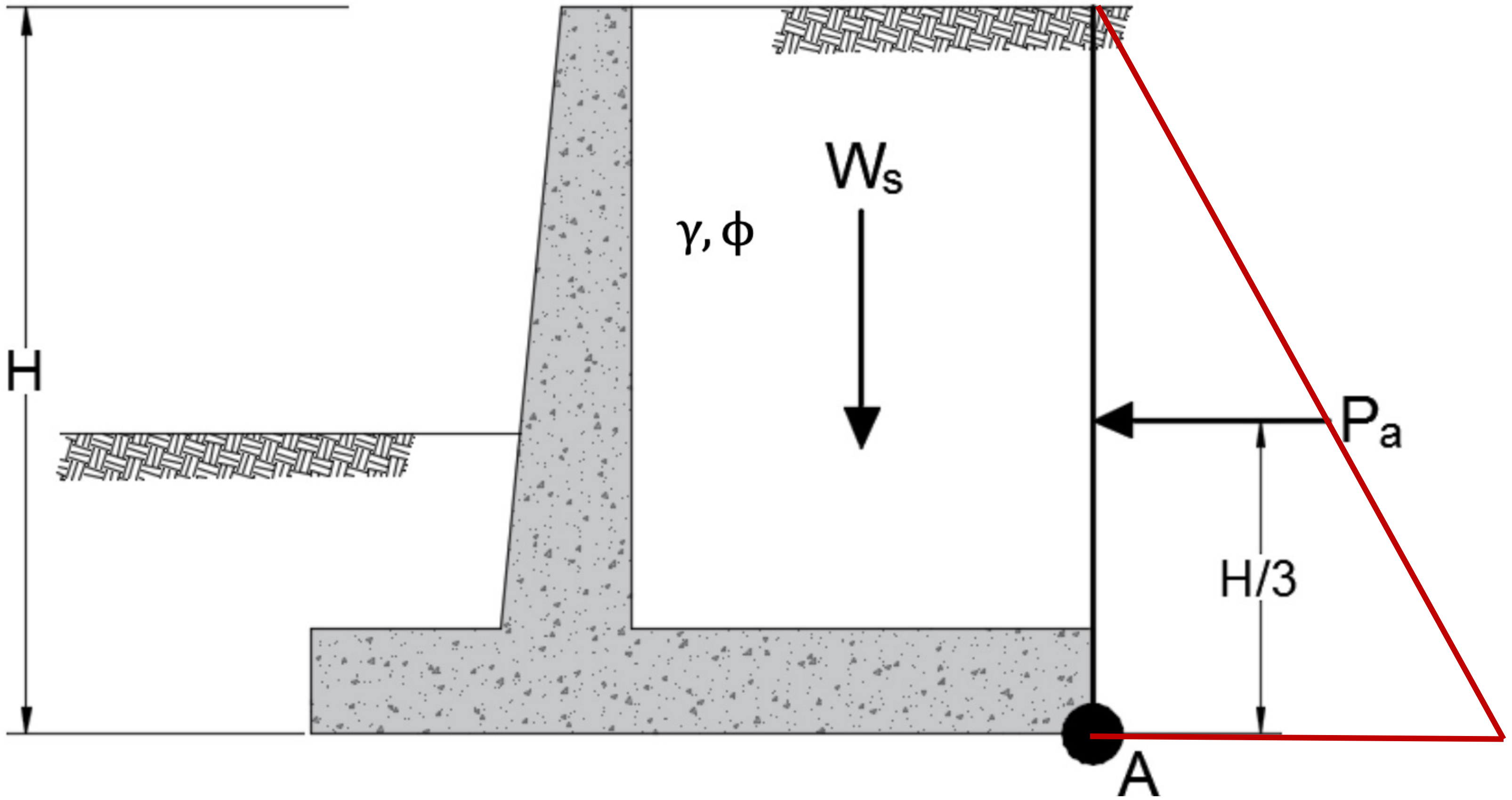
Figure (2)

Each retaining wall divided into three parts; stem, heel, and toe as shown for the following cantilever footing (as example):



Rankine Theory:

1. The wall is vertical and backfill is horizontal:

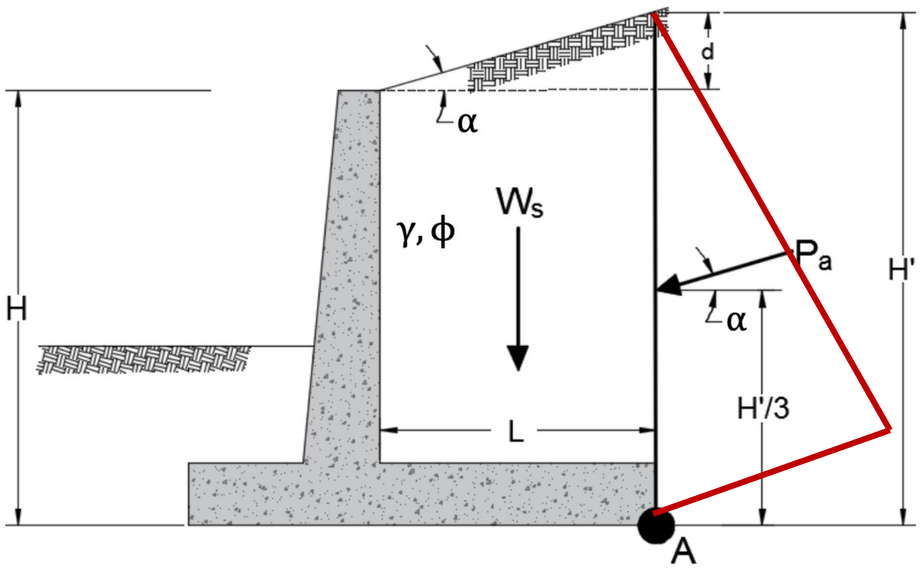


Here the active force P_a is horizontal and can be calculated as following:

$$P_a = \frac{1}{2}\gamma H^2 K_a \quad , \quad K_a = \frac{1 - \sin\phi'}{1 + \sin\phi'} = \tan^2\left(45 - \frac{\phi'}{2}\right)$$

Rankine Theory:

2. The wall is vertical and the backfill is inclined with horizontal by angle (α):



Here the active force Pa is inclined with angle (α) and can be calculated as following:

$$P_a = \frac{1}{2}\gamma H'^2 K_a$$

K_A = *coefficient of earth pressure for the active state* = $\tan^2(45° - \phi/2)$

Why H′? → Because the pressure is applied on the vertical line (according active theory) not on the wall, so we need the height of this vertical line H′

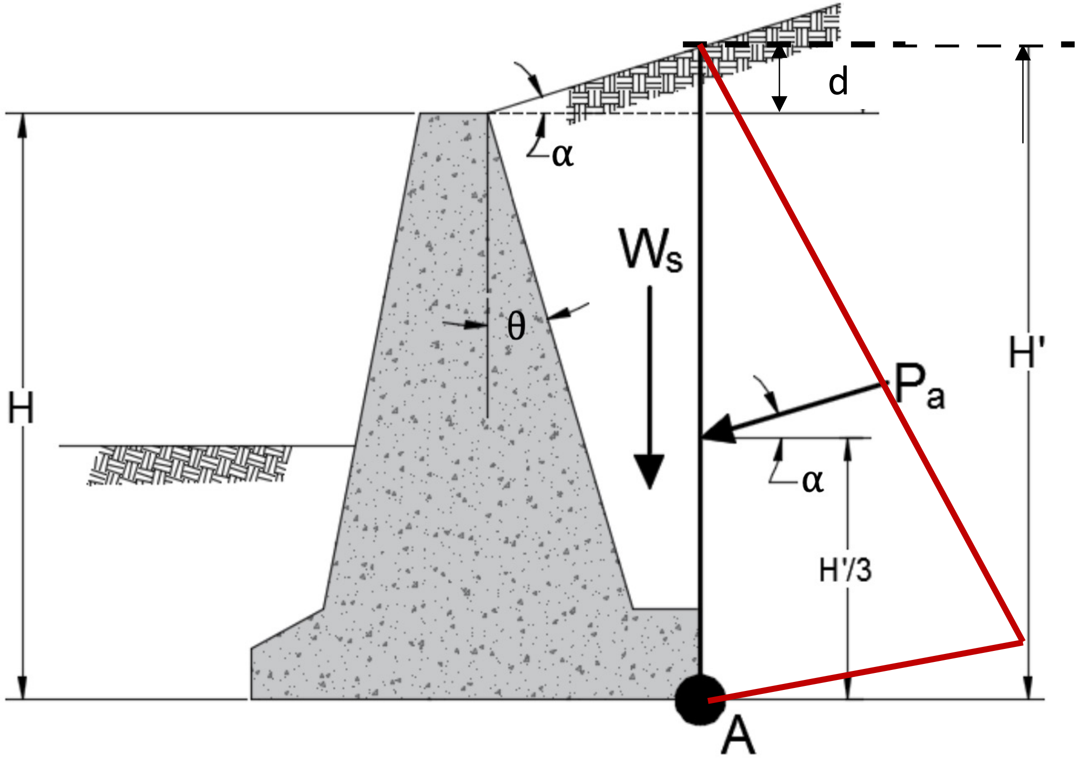
$H' = H + d \rightarrow\rightarrow d = L \tan\alpha$

Now the calculated value of Pa is inclined with an angle (α), so its analyzed in horizontal and vertical axes and then we use the horizontal and vertical components in design as will explained later.

$P_{a,h} = P_a \cos(\alpha)$, $P_{a,v} = P_a \sin(\alpha)$

Rankine Theory:

3. The wall is inclined by angle (θ)with vertical and the backfill is inclined with horizontal by angle (α):



Note that the force P_a is inclined with angle (α) and not depend on the inclination of the wall because the force applied on the vertical line and can be calculated as following:

$P_a = 1/2\ \gamma\ H'^2\ K_a$

What about K_a ?

K_a is depend on the inclination of the wall and inclination of the backfill because it's related to the soil itself and the angle of contact surface with this soil, so K_a can be calculated from the following equation or by using the table:

$$K_a = \frac{\cos(\alpha - \theta)\sqrt{1 + \sin^2\phi - 2\sin\phi\cos\psi_a}}{\cos^2\theta\left(\cos\alpha + \sqrt{\sin^2\phi - \sin^2\alpha}\right)}$$

$$\psi_a = \sin^{-1}\left(\frac{\sin\alpha}{\sin\phi}\right) - \alpha + 2\theta$$

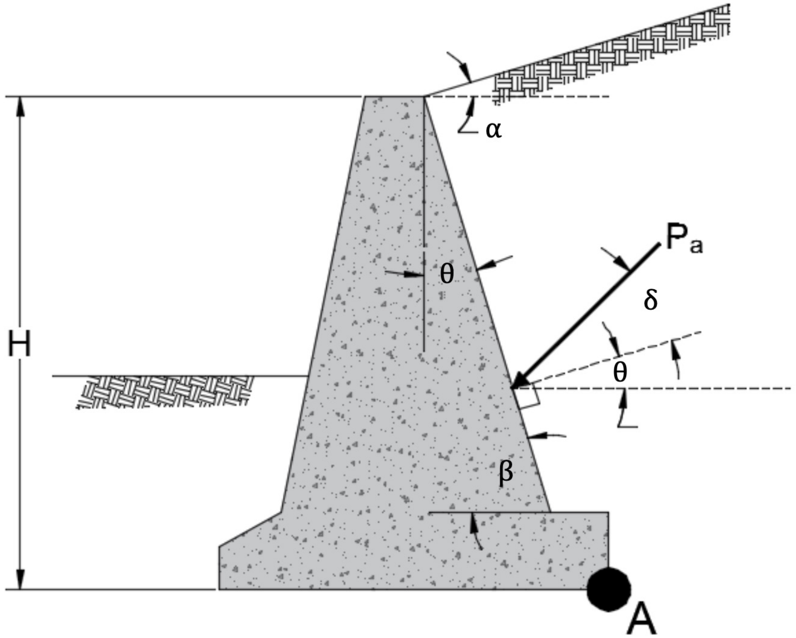
$$P_{a,h} = P_a\cos(\alpha) \quad , \quad P_{a,v} = P_a\sin(\alpha)$$

Table 13.2 Values of $K_{a(R)}$ [Eq. (13.29)]

↓ α (deg)	ϕ' (deg) → 28	30	32	34	36	38	40
0	0.361	0.333	0.307	0.283	0.260	0.238	0.217
5	0.366	0.337	0.311	0.286	0.262	0.240	0.219
10	0.380	0.350	0.321	0.294	0.270	0.246	0.225
15	0.409	0.373	0.341	0.311	0.283	0.258	0.235
20	0.461	0.414	0.374	0.338	0.306	0.277	0.250
25	0.573	0.494	0.434	0.385	0.343	0.307	0.275

Coulomb's Theory:

The force P_a is applied directly on the wall, so whole soil retained by the wall will be considered in P_a and thereby the weight of soil will not apply on the heel of the wall.



$P_a = 1/2\ \gamma\ H^2\ K_a$

Why H?, Here the force Pa is applied directly on the wall, so the lateral pressure of the soil is applied on the wall from start to end, so we only take the height of the wall (in coulomb theory).

Ka is calculated from **Tables** according the following angles: ϕ, α, β and δ

As shown, the force Pa is inclined with angle (δ + θ) with horizontal, so: $P_{a,h} = P_a \cos(\delta + \theta)$, $P_{a,v} = P_a \sin(\delta + \theta)$

What about Passive Force

You can always calculate passive force from Rankine theory even if its require to solve the problem based on coulomb's theory, because we concerned about Rankine and coulomb's theories in active lateral pressure. So, K_p can be determined by the following equation for no slope of back fill soil and the wall is vertical:

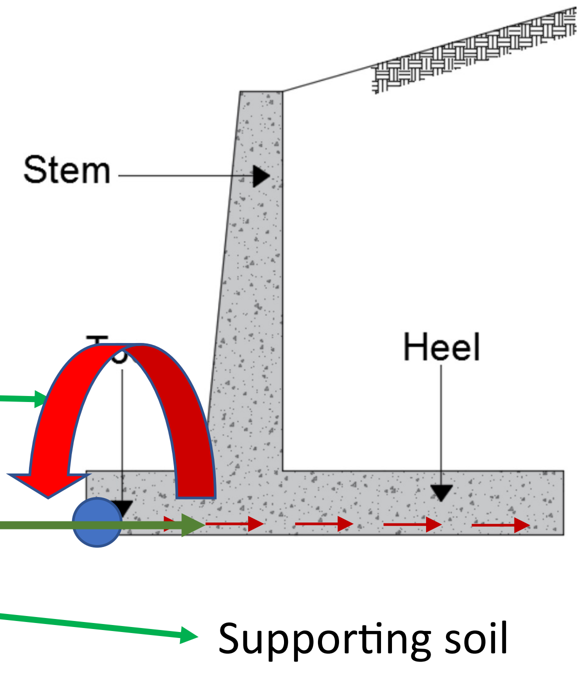
$$K_p = \frac{1+\sin\phi}{1-\sin\phi} = \tan^2\left(45^\circ + \frac{\phi}{2}\right)$$

Important Note:

Coulomb's theory can't be used in the following cases:

1. If the soil retained by the wall is (C – ϕ) soil, because Coulomb's theory deals only with granular soil (pure sand).
2. If wall friction angle between retained soil and the wall is equal zero.
3. If we asked to solve the problem using Rankine theory.

Stability of Retaining Wall



A retaining wall may be fail in any of the following modes of failure:

1. It may overturn about its toe.
2. It may slide along its base.
3. It may fail due to the loss of bearing capacity of the soil supporting the base.
4. It may undergo deep-seated shear failure.
5. It may go through excessive settlement.

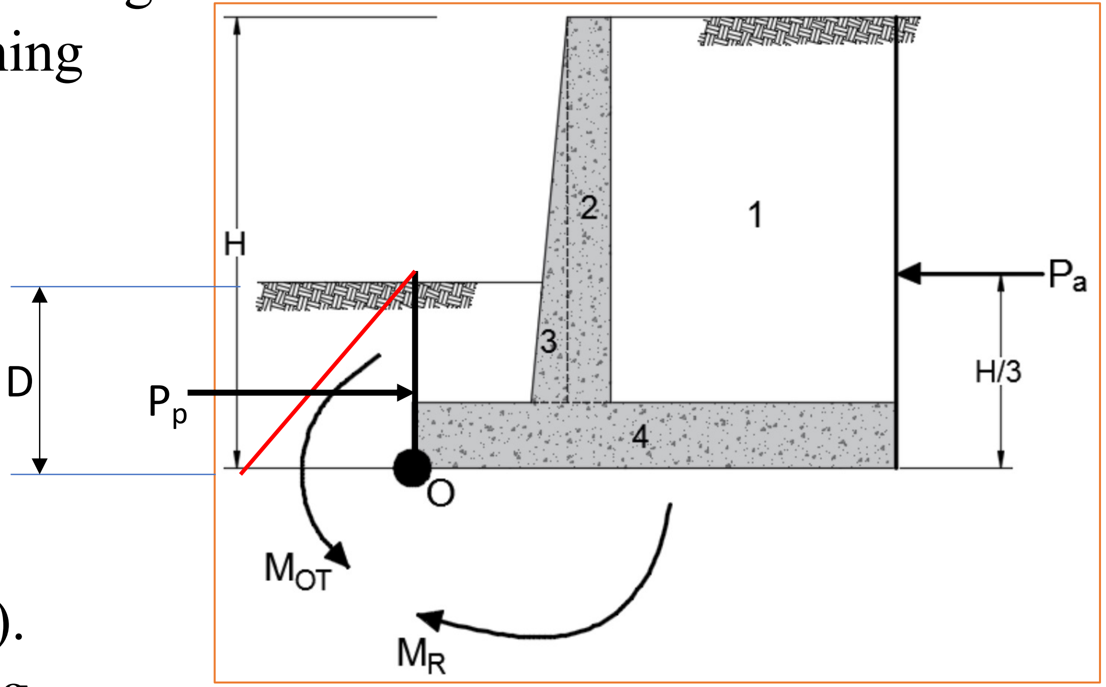
Stability of Retaining Wall, Cont'd

Remark: Rankine theory used to discusses the stability of these types of failures. Coulomb's theory will be the same with only difference mentioned above (active force applied directly on the wall).

1)Stability for Overturning:

The **horizontal** component of active force will causes overturning on retaining wall about point O by moment called "overturning moment"

$M_{OT} = P_{a,h} \times H/3$



The resisting moment (M_R) will be due to:

1- Vertical component of active force $P_{a,v}$ (if exist).
2- Weight of all soil above the heel of the retaining wall.
3- Weight of each element of retaining wall.
4- Passive force (we neglect it in this check for more safety).

Now, to calculate the moment from these all forces (resisting moment) we prepare the following table:

Force = Volume × unit weight but, we take a strip of 1 m length

→ Force = Area × unit weight

1)Stability for Overturning, Cont'd:

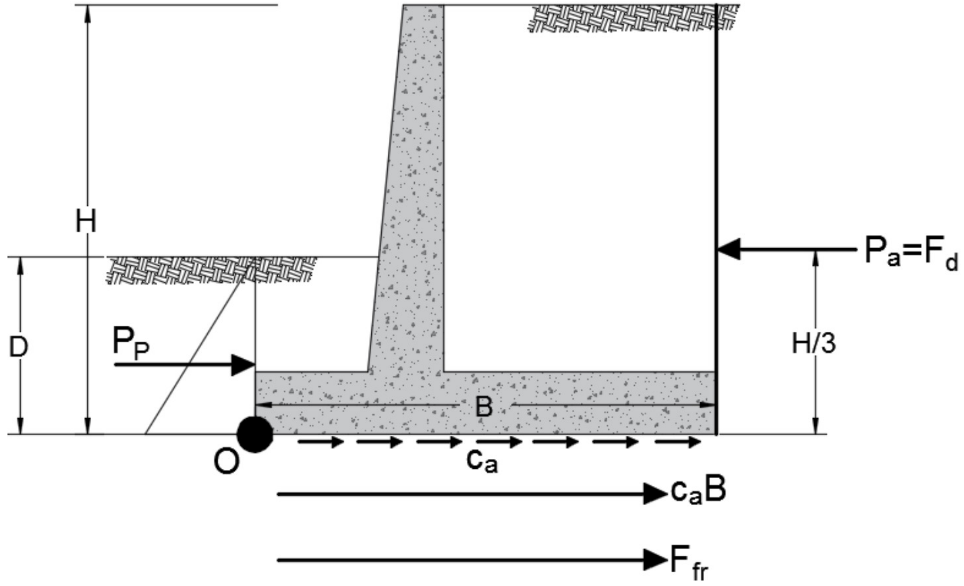
Section	Area	Weight/unit length of the wall	Moment arm measured from O	Moment about O
1	A_1	$W_1 = A_2 \times \gamma_1$	X_1	M_1
2	A_2	$W_2 = A_2 \times \gamma_c$	X_2	M_2
3	A_3	$W_3 = A_2 \times \gamma_c$	X_3	M_3
4	A_4	$W_4 = A_2 \times \gamma_c$	X_4	M_4
		$P_{a,v}$(if exist).	B	M_V
$\sum$		$\sum V$		$\sum M = M_R$

γ_1 = unit weight of the soil above the heel of RW

$$\boxed{FS_{OT} = \frac{M_R}{M_{OT}} \geq 2}$$

Remark: If you asked to consider passive force→ consider it in the resisting moment and the factor of safety remains 2. (So we neglect it here for safety).

Stability for Sliding along the Base



The horizontal component of active force may causes movement of the wall in horizontal direction (i.e. causes sliding for the wall), this force is called driving force $F_d = P_{a,h}$.

This driving force will be resisted by the following forces:

1. Adhesion between the soil (under the base) and the base of retaining wall: c_a = adhesion along the base of Retaining Wall (RW), (KN/m)

$C_a = c_a \times B$ = adhesion force under the base of RW (KN).

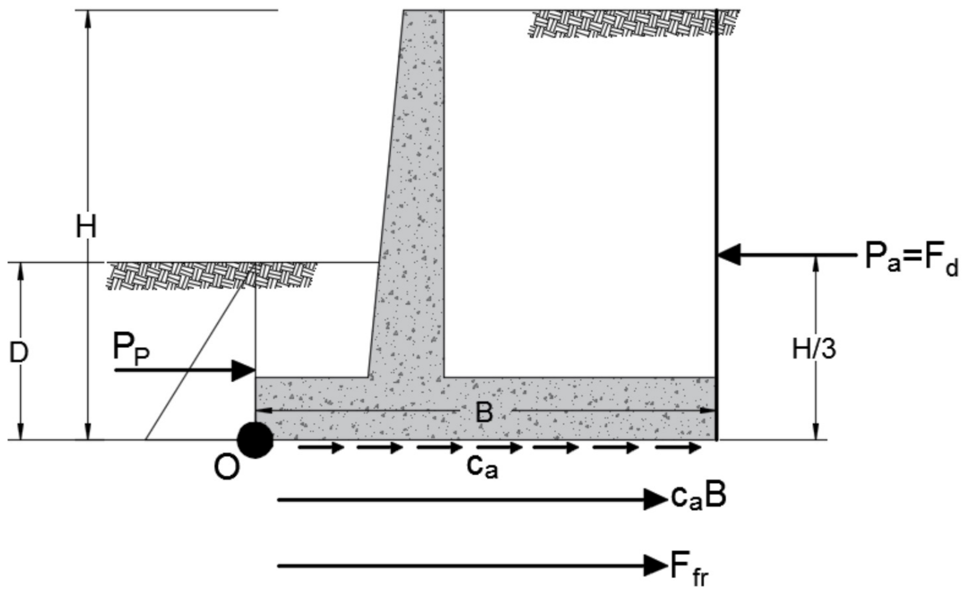
c_a can be calculated from the following relation:

$c_a = K_2c_2$, where, c_2 = cohesion of soil under the base

So adhesion force is:

$C_a = K_2c_2B$

Stability for Sliding along the Base, Cont'd



<u>2. Friction force</u> due to the friction between the soil and the base of RW: Always friction force is calculated from the following relation:

$F_{fr} = \mu_s N$

Here N is the sum of vertical forces calculated in the table of the first check (overturning)

$\rightarrow N = \sum V$ (including the vertical component of active force)

μ_s = coefficient of friction (related to the friction between soil and base) $\mu_s = \tan(\delta_2)$ $\delta_2 = K_1\varphi_2 \rightarrow\rightarrow \mu_s = \tan(K_1\varphi_2)$

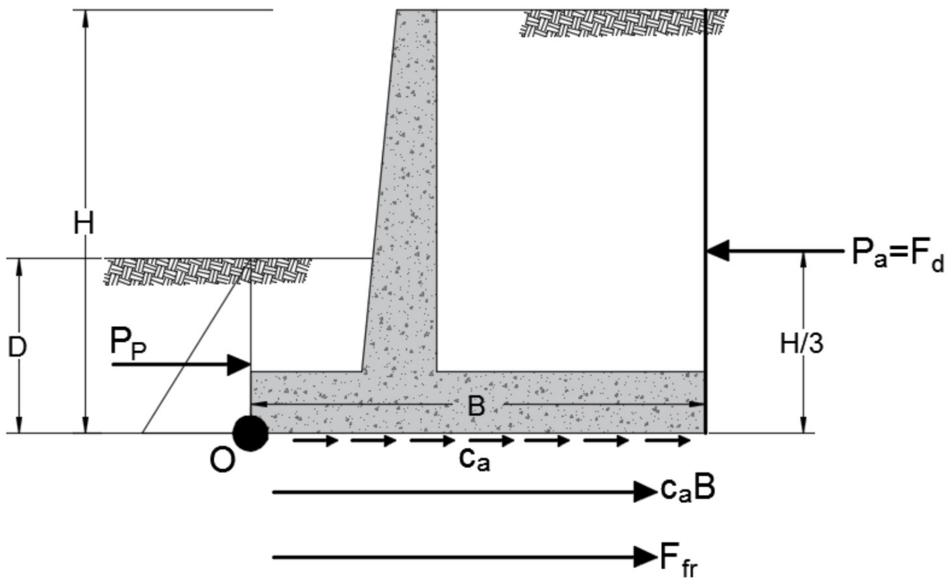
φ_2 = friction angle of the soil under the base.

$\rightarrow F_{fr} = \sum V \times \tan(K_1\varphi_2)$

Note:

$K_1 = K_2 = (1/2 \rightarrow 2/3)$ if you are not given them $\rightarrow$ take $K_1 = K_2 = 2/3$

Stability for Sliding along the Base, Cont'd



3. Passive force P_P. (Calculated using Rankine theory).
So the total resisting force F_R can be calculated as following:

$$F_R = \sum V \times \tan(K_1\varphi_2) + Kc_2B + P_P$$

Factor of safety against sliding:

$FS_S = F_R/F_d \geq 2$ (if we consider P_P in F_R)

$FS_S = F_R/F_d \geq 1.5$ (if we don't consider P_P in F_R)

Check Stability for Bearing Capacity Failure

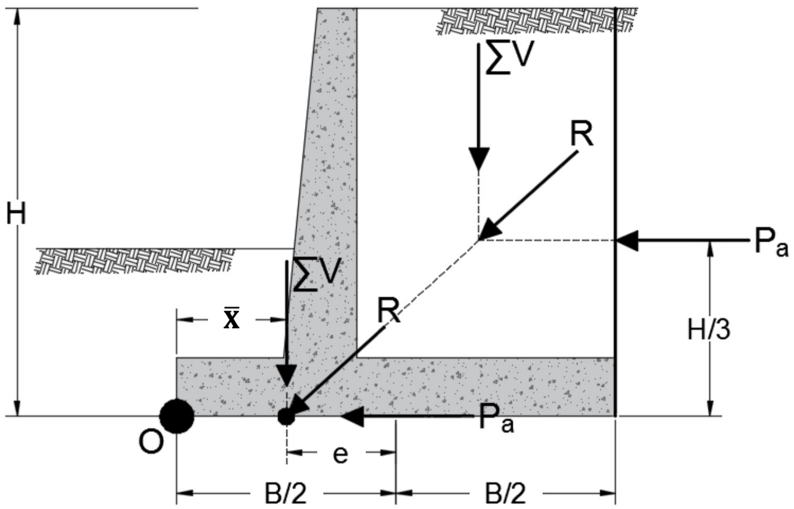
The resultant force (R) is not applied on the center of the base of retaining wall, so there is an eccentricity between the location of resultant force and the center of the base, this eccentricity **may be calculated as following:**

From the figure above, take summation moments about point O:

$M_O = \sum V \times \overline{X}$

From the first check (overturning) we calculate the overturning moment and resisting moment about point O,

so the difference between these two moments gives the net moment at O.



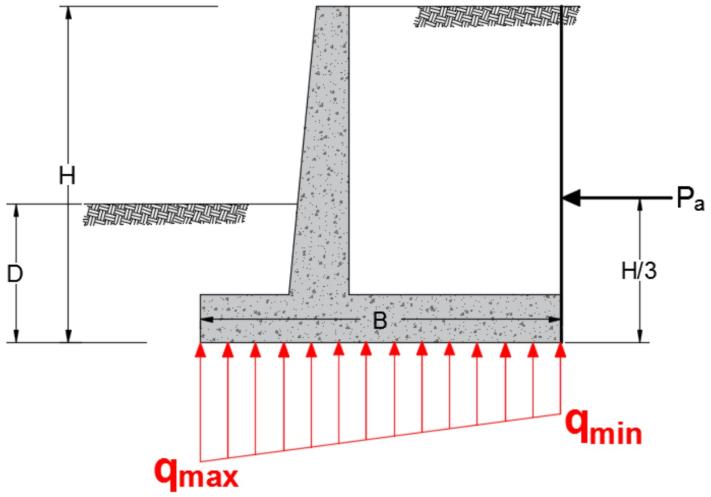
$$M_O = M_R - M_{OT}$$

$$\rightarrow M_R - M_{OT} = \sum V \times \overline{X} \rightarrow\rightarrow \overline{X} = \frac{M_R - M_{OT}}{\sum V}$$

$$e = \frac{B}{2} - \overline{X} = \checkmark \text{ (see the above figure).}$$

Note: Since there exist eccentricity, the pressure under the base of retaining wall is not uniform (there exist maximum and minimum values for pressure).

We calculate q_{max} and q_{min} as below.
Eccentricity in B-direction and retaining wall (RW) can be considered strip footing.



If $e < \frac{B}{6}$

$$q_{max} = \frac{\sum V}{B \times 1}\left(1 + \frac{6e}{B}\right)$$

$$q_{min} = \frac{\sum V}{B \times 1}\left(1 - \frac{6e}{B}\right)$$

If $e > \frac{B}{6}$

$$q_{max,new} = \frac{4\sum V}{3 \times 1 \times (B - 2e)}$$

Now, we must check for qmax:

$q_{max} \leq q_{all} \rightarrow q_{max} = q_{all}$ (at critical case)

$FS_{B.C} = \frac{q_u}{q_{max}} \geq 3$

Calculation of q_u:

q_u is calculated using Meyerhof equation as following:

$$q_u = cN_cF_{cs}F_{cd}F_{ci} + qN_qF_{qs}F_{qd}F_{qi} + 0.5B\gamma N_\gamma F_{\gamma s}F_{\gamma d}F_{\gamma i}$$

Where

$c =$ Cohesion of soil under the base

$q =$ Effective stress at the level of the base of retaining wall.

$q = \gamma_2 \times D_f$

D_f here is the depth of soil above the toe $= D$ (above figure)

$\rightarrow q = \gamma_2 \times D$

$\gamma =$ unit weight of the soil under the base of the RW.

Important Note:

May be a water table under the base or at the base or above the base (three cases discussed previously) is the same here, so be careful don't forget

$B = B' = B - 2e$

$N_c, N_q, N_\gamma =$ Myerhof bearing capacity factors according the friction angle for the soil under the base

$F_{cs} = F_{qs} = F_{\gamma s} = 1$ (since RW is considered a strip footing)

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Depth factors: (We use B not B')

Here since the depth D is restively small to width of the base B, in most **cases** $\frac{\mathbf{D}}{\mathbf{B}} \leq \mathbf{1} \rightarrow\rightarrow$

1. **For $\phi = 0.0$**

$$F_{cd} = 1 + 0.4\left(\frac{D_f}{B}\right)$$

$$F_{qd} = 1$$

$$F_{\gamma d} = 1$$

2. **For $\phi > 0.0$**

$$F_{cd} = F_{qd} - \frac{1 - F_{qd}}{N_c \tan\phi}$$

$$F_{qd} = 1 + 2\tan\phi\,(1 - \sin\phi)^2\left(\frac{D_f}{B}\right)$$

$$F_{\gamma d} = 1$$

Inclination Factors:

Note that the resultant force applied on the base of the foundation is not vertical, but it is inclined with angle $\beta = \Psi$ (with vertical), this angle can be calculated as following:

$$\beta = \Psi = \tan^{-1}\left(\frac{P_{a,h}}{\sum V}\right)$$

$$F_{ci} = F_{qi} = \left(1 - \frac{\beta^{\circ}}{90}\right)^2$$

$$F_{\gamma i} = \left(1 - \frac{\beta^{\circ}}{\phi^{\circ}}\right)$$

Next lecture will be Tutorial on this Theme

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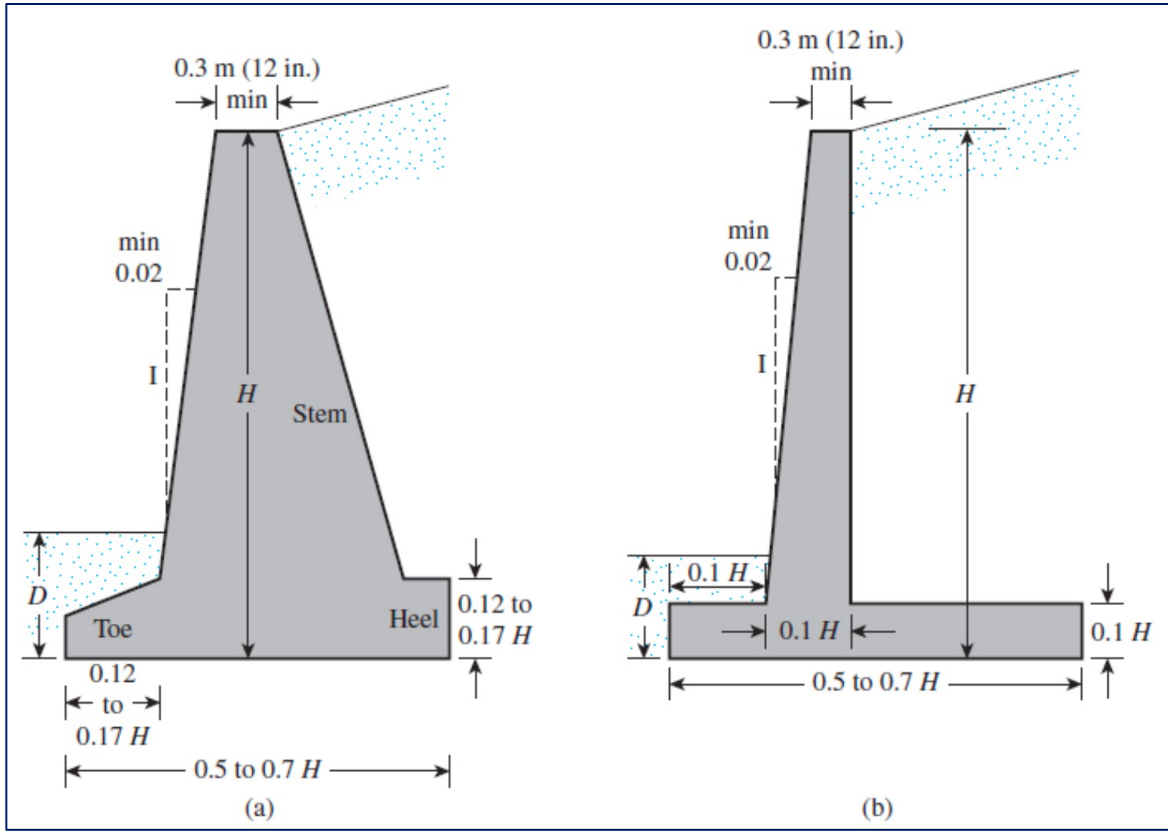
Retaining Walls (RW), (2)

- Proportioning of RW for Stability
- Construction Joints and Drainage from Backfill with Example
- Examples on Stability

Proportioning of RW (Gravity and Cantilever Walls)

1) **Proportioning** means assuming some of RW dimensions as trial to check the walls for stability. If the stability checks yield undesirable results, the sections can be changed and rechecked. The figure shows the general proportions of various retaining wall components that can be used for initial checks.
2) The **top of the stem** of any retaining wall should **not be less than about 0.3 m (≈12 in)** for proper placement of concrete.
3) The **depth, D**, to the bottom of the base slab should be a minimum of **0.6 m (≈2 ft).** However, the bottom of the base slab should be positioned below the seasonal frost line.
4) For **counterfort retaining walls**, the general proportion of the **stem and the base slab** is the **same as for cantilever walls**.
5) The **counterfort slabs** may be about **0.3 m (≈12 in) thick** and **spaced at center-to-center distances of 0.3H to 0.7H.**



(a): Gravity wall, (b): Cantilever wall

Construction Joints and Drainage from Backfill

Construction Joints

A retaining wall may be constructed with one or more of the following joints:

1. *Construction joints* (see Figure 8.14a) are vertical and horizontal joints that are placed between two successive pours of concrete. To increase the shear at the joints, keys may be used. If keys are not used, the surface of the first pour is cleaned and roughened before the next pour of concrete.
2. *Contraction joints* (Figure 8.14b) are vertical joints (grooves) placed in the face of a wall (from the top of the base slab to the top of the wall) that allow the concrete to shrink without noticeable harm. The grooves may be about 6 to 8 mm (≈0.25 to 0.3 in.) wide and 12 to 16 mm (≈0.5 to 0.6 in.) deep.
3. *Expansion joints* (Figure 8.14c) allow for the expansion of concrete caused by temperature changes; vertical expansion joints from the base to the top of the wall may also be used. These joints may be filled with flexible joint fillers. In most cases, horizontal reinforcing steel bars running across the stem are continuous through all joints. The steel is greased to allow the concrete to expand.

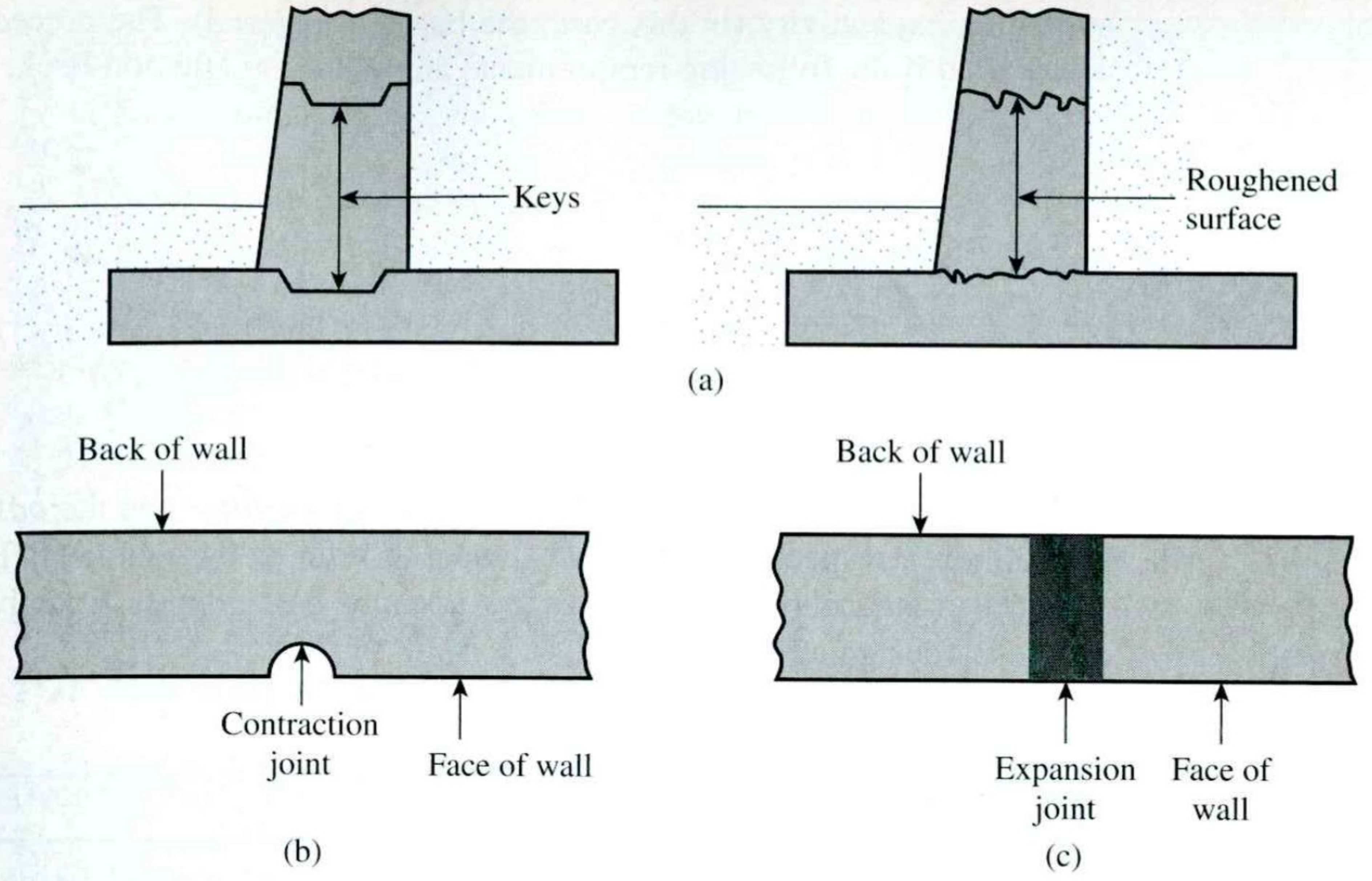


Figure 8.14 (a) Construction joints; (b) contraction joint; (c) expansion joint

Drainage from the Backfill

As the result of rainfall or other wet conditions, the backfill material for a retaining wall may become saturated, thereby increasing the pressure on the wall and perhaps creating an unstable condition. For this reason, adequate drainage must be provided by means of *weep holes* or *perforated drainage pipes*. (See Figure 8.15.)

When provided, weep holes should have a minimum diameter of about 0.1 m (4 in.) and be adequately spaced. Note that there is always a possibility that backfill material may be washed into weep holes or drainage pipes and ultimately clog them. Thus, a filter material needs to be placed behind the weep holes or around the drainage pipes, as the case may be; geotextiles now serve that purpose.

Two main factors influence the choice of filter material: The grain-size distribution of the materials should be such that (a) the soil to be protected is not washed into the filter and (b) excessive hydrostatic pressure head is not created in the soil with a lower hydraulic conductivity (in this case, the backfill material). The preceding conditions can be satisfied if the following requirements are met (Terzaghi and Peck, 1967):

$$\frac{D_{15(F)}}{D_{85(B)}} < 5 \qquad [\text{to satisfy condition (a)}] \tag{8.25}$$

$$\frac{D_{15(F)}}{D_{15(B)}} > 4 \qquad [\text{to satisfy condition (b)}] \tag{8.26}$$

In these relations, the subscripts F and B refer to the *filter* and the *base* material (i.e., the backfill soil), respectively. Also, D_{15} and D_{85} refer to the diameters through which 15% and 85% of the soil (filter or base, as the case may be) will pass. Example 8.3 gives the procedure for designing a filter.

Example on Filter

Figure 8.16 shows the grain-size distribution of a backfill material. Using the conditions outlined in Section 8.8, determine the range of the grain-size distribution for the filter material.

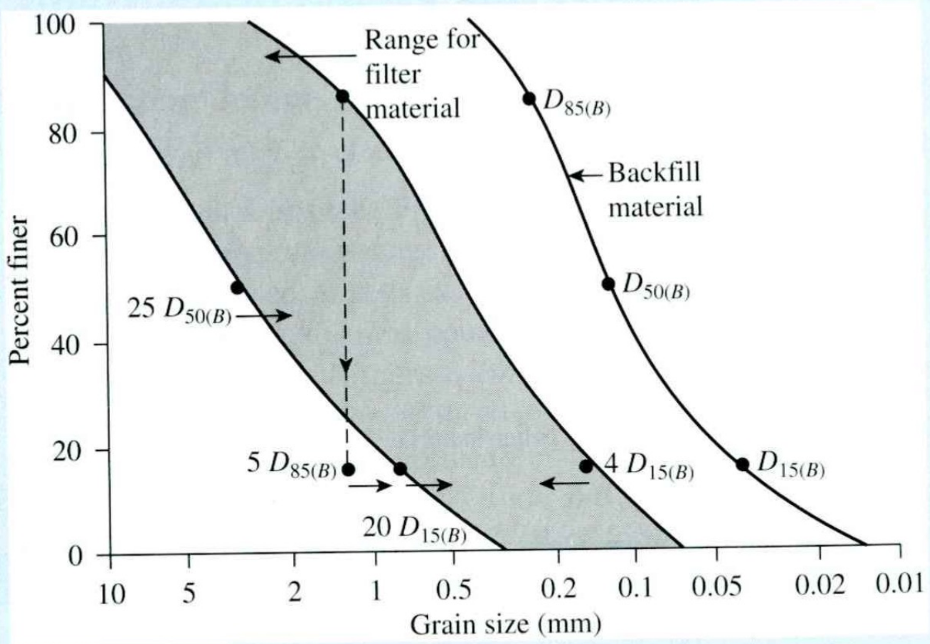


Figure 8.16 Determination of grain-size distribution of filter material

Solution

From the grain-size distribution curve given in the figure, the following values can be determined:

$$D_{15(B)} = 0.04 \text{ mm}$$

$$D_{85(B)} = 0.25 \text{ mm}$$

$$D_{50(B)} = 0.13 \text{ mm}$$

Conditions of Filter

1. $D_{15(F)}$ should be less than $5D_{85(B)}$; that is, $5 \times 0.25 = 1.25$ mm.
2. $D_{15(F)}$ should be greater than $4D_{15(B)}$; that is, $4 \times 0.04 = 0.16$ mm.
3. $D_{50(F)}$ should be less than $25D_{50(B)}$; that is, $25 \times 0.13 = 3.25$ mm.
4. $D_{15(F)}$ should be less than $20D_{15(B)}$; that is, $20 \times 0.04 = 0.8$ mm.

These limiting points are plotted in Figure 8.16. Through them, two curves can be drawn that are similar in nature to the grain-size distribution curve of the backfill material. These curves define the range of the filter material to be used.

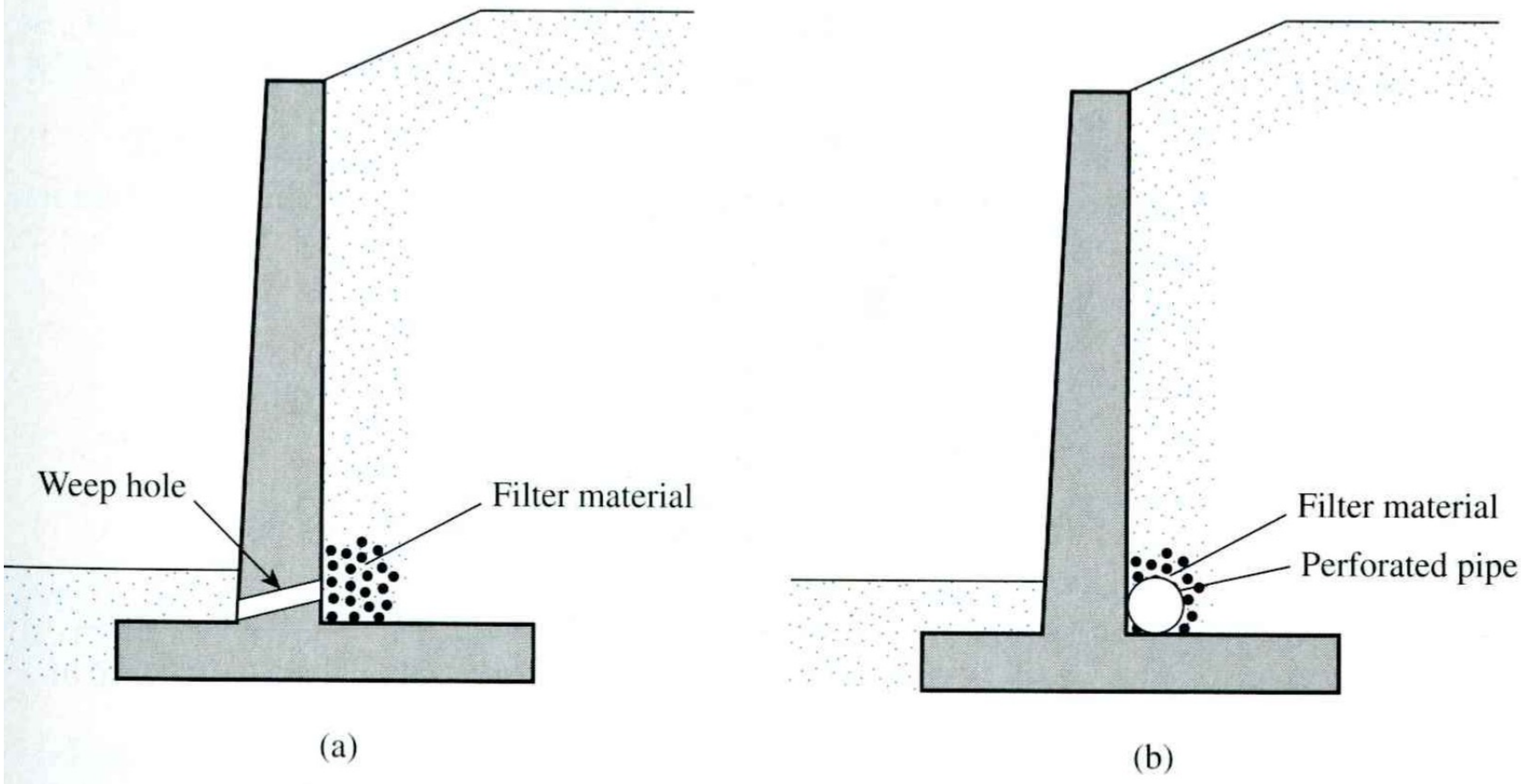


Figure 8.15 Drainage provisions for the backfill of a retaining wall: (a) by weep holes; (b) by a perforated drainage pipe

Examples

1- The cross section of a cantilever retaining wall is shown in Figure 8.12. Calculate t
factors of safety with respect to overturning, sliding, and bearing capacity.

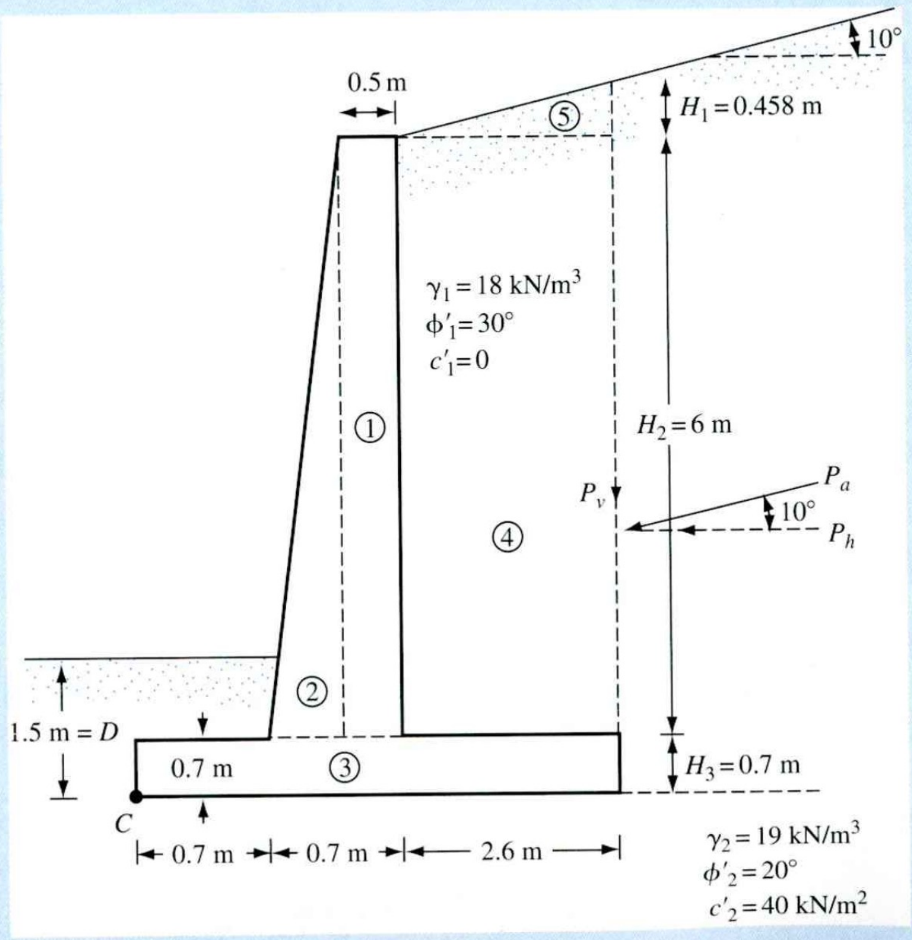


Figure 8.12 Calculation of stability of a retaining wall

$$K_a = \cos\alpha \frac{\cos\alpha - \sqrt{\cos^2\alpha - \cos^2\phi'}}{\cos\alpha + \sqrt{\cos^2\alpha - \cos^2\phi'}} \quad (7.19)$$

Table 7.1 Values of K_a [Eq. (7.19)]

α (deg) ↓	ϕ' (deg) → 28	29	30	31	32	33	34	35	36	37	38	39	40
0	0.3610	0.3470	0.3333	0.3201	0.3073	0.2948	0.2827	0.2710	0.2596	0.2486	0.2379	0.2275	0.2174
1	0.3612	0.3471	0.3335	0.3202	0.3074	0.2949	0.2828	0.2711	0.2597	0.2487	0.2380	0.2276	0.2175
2	0.3618	0.3476	0.3339	0.3207	0.3078	0.2953	0.2832	0.2714	0.2600	0.2489	0.2382	0.2278	0.2177
3	0.3627	0.3485	0.3347	0.3214	0.3084	0.2959	0.2837	0.2719	0.2605	0.2494	0.2386	0.2282	0.2181
4	0.3639	0.3496	0.3358	0.3224	0.3094	0.2967	0.2845	0.2726	0.2611	0.2500	0.2392	0.2287	0.2186
5	0.3656	0.3512	0.3372	0.3237	0.3105	0.2978	0.2855	0.2736	0.2620	0.2508	0.2399	0.2294	0.2192
6	0.3676	0.3531	0.3389	0.3253	0.3120	0.2992	0.2868	0.2747	0.2631	0.2518	0.2409	0.2303	0.2200
7	0.3701	0.3553	0.3410	0.3272	0.3138	0.3008	0.2883	0.2761	0.2644	0.2530	0.2420	0.2313	0.2209
8	0.3730	0.3580	0.3435	0.3294	0.3159	0.3027	0.2900	0.2778	0.2659	0.2544	0.2432	0.2325	0.2220
9	0.3764	0.3611	0.3463	0.3320	0.3182	0.3049	0.2921	0.2796	0.2676	0.2560	0.2447	0.2338	0.2233
10	0.3802	0.3646	0.3495	0.3350	0.3210	0.3074	0.2944	0.2818	0.2696	0.2578	0.2464	0.2354	0.2247
11	0.3846	0.3686	0.3532	0.3383	0.3241	0.3103	0.2970	0.2841	0.2718	0.2598	0.2482	0.2371	0.2263
12	0.3896	0.3731	0.3573	0.3421	0.3275	0.3134	0.2999	0.2868	0.2742	0.2621	0.2503	0.2390	0.2281
13	0.3952	0.3782	0.3620	0.3464	0.3314	0.3170	0.3031	0.2898	0.2770	0.2646	0.2527	0.2412	0.2301
14	0.4015	0.3839	0.3671	0.3511	0.3357	0.3209	0.3068	0.2931	0.2800	0.2674	0.2552	0.2435	0.2322
15	0.4086	0.3903	0.3729	0.3564	0.3405	0.3253	0.3108	0.2968	0.2834	0.2705	0.2581	0.2461	0.2346
16	0.4165	0.3975	0.3794	0.3622	0.3458	0.3302	0.3152	0.3008	0.2871	0.2739	0.2612	0.2490	0.2373
17	0.4255	0.4056	0.3867	0.3688	0.3518	0.3356	0.3201	0.3053	0.2911	0.2776	0.2646	0.2521	0.2401
18	0.4357	0.4146	0.3948	0.3761	0.3584	0.3415	0.3255	0.3102	0.2956	0.2817	0.2683	0.2555	0.2433
19	0.4473	0.4249	0.4039	0.3842	0.3657	0.3481	0.3315	0.3156	0.3006	0.2862	0.2724	0.2593	0.2467

Solution

From the figure,

$$H' = H_1 + H_2 + H_3 = 2.6 \tan 10° + 6 + 0.7$$
$$= 0.458 + 6 + 0.7 = 7.158 \text{ m}$$

The Rankine active force per unit length of wall $= P_p = \frac{1}{2}\gamma_1 H'^2 K_a$. For $\phi'_1 = 30°$ and $\alpha = 10°$, K_a is equal to 0.3532. (See Table 7.1.) Thus, a

$$P_a = \tfrac{1}{2}(18)(7.158)^2(0.3532) = 162.9 \text{ kN/m}$$

$$P_v = P_a \sin 10° = 162.9\,(\sin 10°) = 28.29 \text{ kN/m}$$

and

$$P_h = P_a \cos 10° = 162.9\,(\cos 10°) = 160.43 \text{ kN/m}$$

Factor of Safety against Overturning

The following table can now be prepared for determining the resisting moment:

Section no.[a]	Area (m^2)	Weight/unit length (kN/m)	Moment arm from point *C* (m)	Moment (kN-m/m)
1	$6 \times 0.5 = 3$	70.74	1.15	81.35
2	$\frac{1}{2}(0.2)6 = 0.6$	14.15	0.833	11.79
3	$4 \times 0.7 = 2.8$	66.02	2.0	132.04
4	$6 \times 2.6 = 15.6$	280.80	2.7	758.16
5	$\frac{1}{2}(2.6)(0.458) = 0.595$	10.71	3.13	33.52
		$P_v = 28.29$	4.0	113.16
		$\Sigma V = 470.71$		$1130.02 = \Sigma M_R$

[a]For section numbers, refer to Figure 8.12

$\gamma_{concrete} = 23.58\ \text{kN/m}^3$

The overturning moment

$$M_o = P_h\left(\frac{H'}{3}\right) = 160.43\left(\frac{7.158}{3}\right) = 382.79\ \text{kN-m/m}$$

and

$$\boxed{FS_{(overturning)} = \frac{\Sigma M_R}{M_o} = \frac{1130.02}{382.79} = \mathbf{2.95 > 2,\ OK}}$$

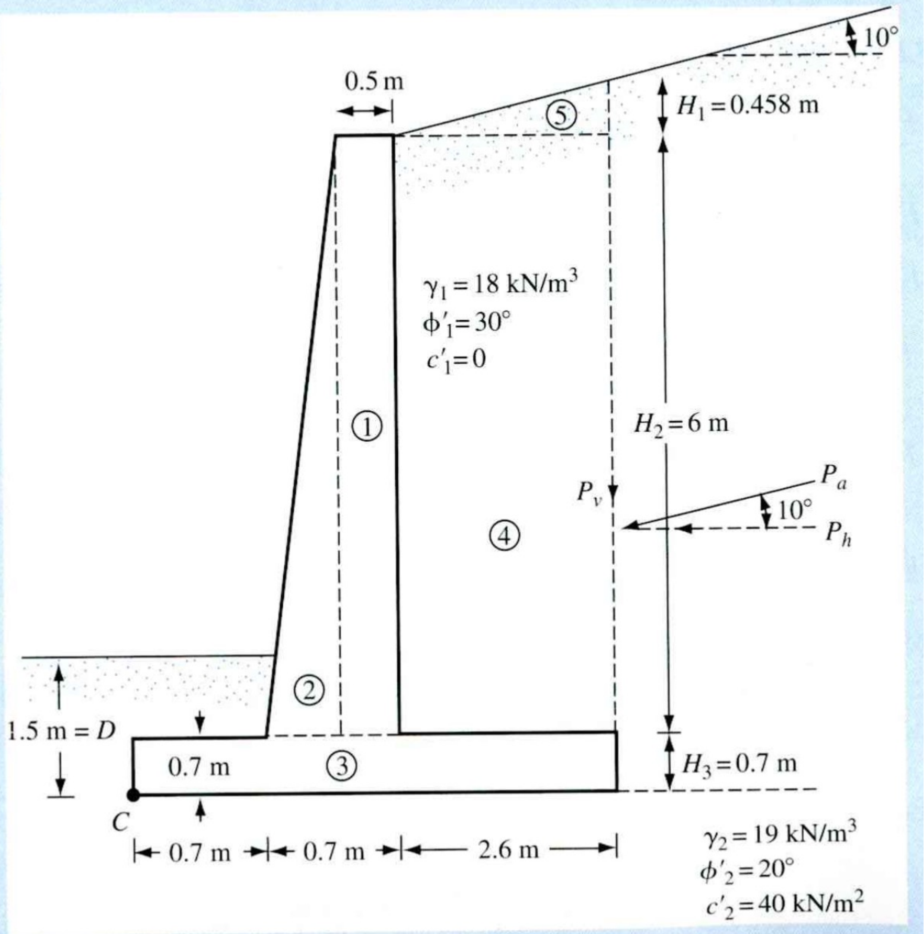


Figure 8.12 Calculation of stability of a retaining wall

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Factor of Safety against Sliding

From Eq. (8.11),

$$FS_{(sliding)} = \frac{(\Sigma V)\tan(k_1\phi_2') + Bk_2c_2' + P_p}{P_a\cos\alpha}$$

Let $k_1 = k_2 = \frac{2}{3}$. Also,

$$P_p = \tfrac{1}{2}K_p\gamma_2 D^2 + 2c_2'\sqrt{K_p}D$$

$$K_p = \tan^2\left(45 + \frac{\phi_2'}{2}\right) = \tan^2(45 + 10) = 2.04$$

and

$$D = 1.5 \text{ m}$$

So

$$P_p = \tfrac{1}{2}(2.04)(19)(1.5)^2 + 2(40)(\sqrt{2.04})(1.5)$$
$$= 43.61 + 171.39 = 215 \text{ kN/m}$$

Hence,

$$FS_{(sliding)} = \frac{(470.71)\tan\left(\frac{2\times 20}{3}\right) + (4)\left(\frac{2}{3}\right)(40) + 215}{160.43}$$

$$= \frac{111.56 + 106.67 + 215}{160.43} = \mathbf{2.7 > 1.5, OK}$$

Note: For some designs, the depth D in a passive pressure calculation may be taken to be *equal to the thickness of the base slab*.

$$FS_{(sliding)} = \frac{(\Sigma V)\tan(k_1\phi_2') + Bk_2c_2' + P_p}{P_a\cos\alpha} \tag{8.11}$$

Factor of Safety against Bearing Capacity Failure

$$M_{\text{net}} = \Sigma M_R - \Sigma M_o \qquad (8.16)$$

$$\overline{X} = \frac{M_{\text{net}}}{\Sigma V} \qquad (8.17)$$

$$e = \frac{B}{2} - \overline{X} \qquad (8.18)$$

Combining Eqs. (8.16), (8.17), and (8.18) yields

$$e = \frac{B}{2} - \frac{\Sigma M_R - \Sigma M_o}{\Sigma V} = \frac{4}{2} - \frac{1130.02 - 382.79}{470.71}$$

$$= 0.411 \text{ m} < \frac{B}{6} = \frac{4}{6} = 0.666 \text{ m}$$

$$q_{\max} = q_{\text{toe}} = \frac{\Sigma V}{B}\left(1 + \frac{6e}{B}\right) \qquad (8.20)$$

$$q_{\min} = q_{\text{heel}} = \frac{\Sigma V}{B}\left(1 - \frac{6e}{B}\right) \qquad (8.21)$$

Again, from Eqs. (8.20) and (8.21)

$$q_{\substack{\text{toe}\\\text{heel}}} = \frac{\Sigma V}{B}\left(1 \pm \frac{6e}{B}\right) = \frac{470.71}{4}\left(1 \pm \frac{6 \times 0.411}{4}\right) = 190.2 \text{ kN/m}^2 \text{ (toe)}$$

$$= 45.13 \text{ kN/m}^2 \text{ (heel)}$$

The ultimate bearing capacity of the soil can be determined from Eq. (8.22)

$$q_u = c_2' N_c F_{cd} F_{ci} + q N_q F_{qd} F_{qi} + \frac{1}{2}\gamma_2 B' N_\gamma F_{\gamma d} F_{\gamma i}$$

For $\phi_2' = 20°$ (see Table 3.3), $N_c = 14.83$, $N_q = 6.4$, and $N_\gamma = 5.39$. Also,

$$q = \gamma_2 D = (19)(1.5) = 28.5 \text{ kN/m}^2$$

$$B' = B - 2e = 4 - 2(0.411) = 3.178 \text{ m}$$

$$F_{cd} = F_{qd} - \frac{1 - F_{qd}}{N_c \tan\phi_2'} = 1.148 - \frac{1 - 1.148}{(14.83)(\tan 20)} = 1.175$$

$$F_{qd} = 1 + 2\tan\phi_2'(1 - \sin\phi_2')^2\left(\frac{D}{B'}\right) = 1 + 0.315\left(\frac{1.5}{3.178}\right) = 1.148$$

$$F_{\gamma d} = 1$$

$$F_{ci} = F_{qi} = \left(1 - \frac{\psi°}{90°}\right)^2$$

and

$$\psi = \tan^{-1}\left(\frac{P_a \cos\alpha}{\Sigma V}\right) = \tan^{-1}\left(\frac{160.43}{470.71}\right) = 18.82°$$

Table 3.3 Bearing Capacity Factors

ϕ'	N_c	N_q	N_γ	ϕ'	N_c	N_q	N_γ
0	5.14	1.00	0.00	26	22.25	11.85	12.54
1	5.38	1.09	0.07	27	23.94	13.20	14.47
2	5.63	1.20	0.15	28	25.80	14.72	16.72
3	5.90	1.31	0.24	29	27.86	16.44	19.34
4	6.19	1.43	0.34	30	30.14	18.40	22.40
5	6.49	1.57	0.45	31	32.67	20.63	25.99
6	6.81	1.72	0.57	32	35.49	23.18	30.22
7	7.16	1.88	0.71	33	38.64	26.09	35.19
8	7.53	2.06	0.86	34	42.16	29.44	41.06
9	7.92	2.25	1.03	35	46.12	33.30	48.03
10	8.35	2.47	1.22	36	50.59	37.75	56.31
11	8.80	2.71	1.44	37	55.63	42.92	66.19
12	9.28	2.97	1.69	38	61.35	48.93	78.03
13	9.81	3.26	1.97	39	67.87	55.96	92.25
14	10.37	3.59	2.29	40	75.31	64.20	109.41
15	10.98	3.94	2.65	41	83.86	73.90	130.22
16	11.63	4.34	3.06	42	93.71	85.38	155.55
17	12.34	4.77	3.53	43	105.11	99.02	186.54
18	13.10	5.26	4.07	44	118.37	115.31	224.64
19	13.93	5.80	4.68	45	133.88	134.88	271.76
20	14.83	6.40	5.39	46	152.10	158.51	330.35
21	15.82	7.07	6.20	47	173.64	187.21	403.67
22	16.88	7.82	7.13	48	199.26	222.31	496.01
23	18.05	8.66	8.20	49	229.93	265.51	613.16
24	19.32	9.60	9.44	50	266.89	319.07	762.89
25	20.72	10.66	10.88				

So

$$F_{ci} = F_{qi} = \left(1 - \frac{18.82}{90}\right)^2 = 0.626$$

and

$$F_{\gamma i} = \left(1 - \frac{\psi}{\phi_2'}\right)^2 = \left(1 - \frac{18.82}{20}\right)^2 \approx 0$$

Hence,

$$\begin{aligned} q_u &= (40)(14.83)(1.175)(0.626) + (28.5)(6.4)(1.148)(0.626) \\ &\quad + \tfrac{1}{2}(19)(5.93)(3.178)(1)(0) \\ &= 436.33 + 131.08 + 0 = 567.41 \text{ kN/m}^2 \end{aligned}$$

and

$$\boxed{\mathrm{FS}_{\text{(bearing capacity)}} = \frac{q_u}{q_{\text{toe}}} = \frac{567.41}{190.2} = \mathbf{2.98}}$$

Note: $\mathrm{FS}_{\text{(bearing capacity)}}$ is less than 3. Some repropertioning will be needed.

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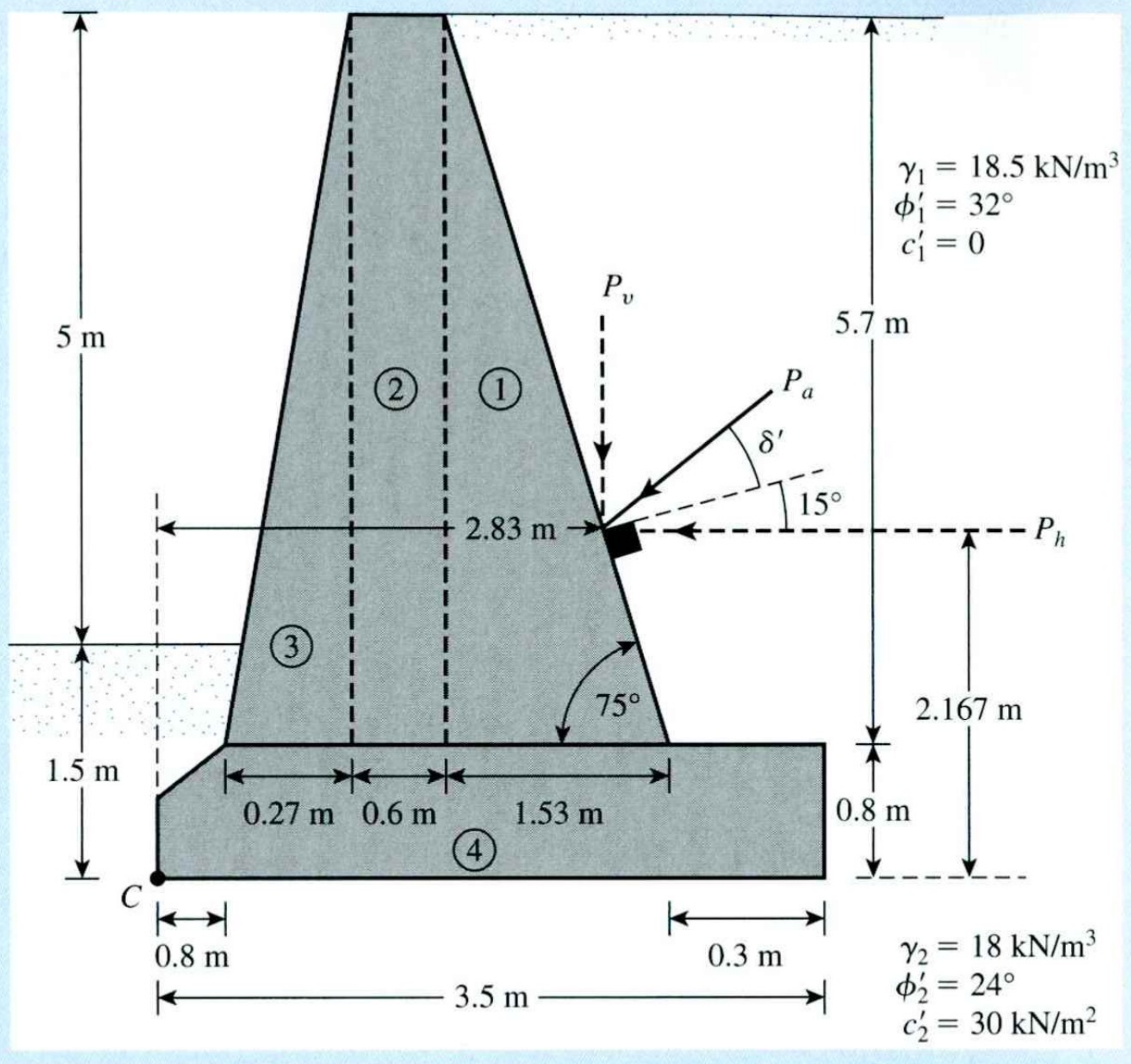
2- A gravity retaining wall is shown in Figure . Use $\delta' = 2/3\phi'_1$ and Coulomb's active

a. The factor of safety against overturning

b. The factor of safety against sliding

c. The pressure on the soil at the toe and heel



$$K_a = \text{Coulomb's active earth pressure coefficient}$$

$$= \frac{\sin^2(\beta+\phi')}{\sin^2\beta\,\sin(\beta-\delta')\left[1+\sqrt{\dfrac{\sin(\phi'+\delta')\sin(\phi'-\alpha)}{\sin(\beta-\delta')\sin(\alpha+\beta)}}\right]^2} \tag{7.26}$$

Table 7.4 Values of K_a [from Eq. (7.26)] for $\delta' = \frac{2}{3}\phi'$

		β (deg)					
α (deg)	φ′ (deg)	90	85	80	75	70	65
0	28	0.3213	0.3588	0.4007	0.4481	0.5026	0.5662
	29	0.3091	0.3467	0.3886	0.4362	0.4908	0.5547
	30	0.2973	0.3349	0.3769	0.4245	0.4794	0.5435
	31	0.2860	0.3235	0.3655	0.4133	0.4682	0.5326
	32	0.2750	0.3125	0.3545	0.4023	0.4574	0.5220
	33	0.2645	0.3019	0.3439	0.3917	0.4469	0.5117
	34	0.2543	0.2916	0.3335	0.3813	0.4367	0.5017
	35	0.2444	0.2816	0.3235	0.3713	0.4267	0.4919
	36	0.2349	0.2719	0.3137	0.3615	0.4170	0.4824
	37	0.2257	0.2626	0.3042	0.3520	0.4075	0.4732
	38	0.2168	0.2535	0.2950	0.3427	0.3983	0.4641
	39	0.2082	0.2447	0.2861	0.3337	0.3894	0.4553
	40	0.1998	0.2361	0.2774	0.3249	0.3806	0.4468
	41	0.1918	0.2278	0.2689	0.3164	0.3721	0.4384
	42	0.1840	0.2197	0.2606	0.3080	0.3637	0.4302
5	28	0.3431	0.3845	0.4311	0.4843	0.5461	0.6190
	29	0.3295	0.3709	0.4175	0.4707	0.5325	0.6056
	30	0.3165	0.3578	0.4043	0.4575	0.5194	0.5926
	31	0.3039	0.3451	0.3916	0.4447	0.5067	0.5800
	32	0.2919	0.3329	0.3792	0.4324	0.4943	0.5677
	33	0.2803	0.3211	0.3673	0.4204	0.4823	0.5558
	34	0.2691	0.3097	0.3558	0.4088	0.4707	0.5443
	35	0.2583	0.2987	0.3446	0.3975	0.4594	0.5330
	36	0.2479	0.2881	0.3338	0.3866	0.4484	0.5221
	37	0.2379	0.2778	0.3233	0.3759	0.4377	0.5115
	38	0.2282	0.2679	0.3131	0.3656	0.4273	0.5012
	39	0.2188	0.2582	0.3033	0.3556	0.4172	0.4911
	40	0.2098	0.2489	0.2937	0.3458	0.4074	0.4813
	41	0.2011	0.2398	0.2844	0.3363	0.3978	0.4718
	42	0.1927	0.2311	0.2753	0.3271	0.3884	0.4625
10	28	0.3702	0.4164	0.4686	0.5287	0.5992	0.6834
	29	0.3548	0.4007	0.4528	0.5128	0.5831	0.6672
	30	0.3400	0.3857	0.4376	0.4974	0.5676	0.6516
	31	0.3259	0.3713	0.4230	0.4826	0.5526	0.6365
	32	0.3123	0.3575	0.4089	0.4683	0.5382	0.6219
	33	0.2993	0.3442	0.3953	0.4545	0.5242	0.6078
	34	0.2868	0.3314	0.3822	0.4412	0.5107	0.5942
	35	0.2748	0.3190	0.3696	0.4283	0.4976	0.5810
	36	0.2633	0.3072	0.3574	0.4158	0.4849	0.5682
	37	0.2522	0.2957	0.3456	0.4037	0.4726	0.5558
	38	0.2415	0.2846	0.3342	0.3920	0.4607	0.5437
	39	0.2313	0.2740	0.3231	0.3807	0.4491	0.5321
	40	0.2214	0.2636	0.3125	0.3697	0.4379	0.5207
	41	0.2119	0.2537	0.3021	0.3590	0.4270	0.5097
	42	0.2027	0.2441	0.2921	0.3487	0.4164	0.4990
15	28	0.4065	0.4585	0.5179	0.5868	0.6685	0.7670

(continued)

Solution

The height

$$H' = 5 + 1.5 = 6.5 \text{ m}$$

Coulomb's active force is

$$P_a = \tfrac{1}{2}\gamma_1 H'^2 K_a$$

With $\alpha = 0°$, $\beta = 75°$, $\delta' = 2/3\phi'_1$, and $\phi'_1 = 32°$, $K_a = 0.4023$. (See Table 7.4.) So,

$$P_a = \tfrac{1}{2}(18.5)(6.5)^2(0.4023) = 157.22 \text{ kN/m}$$

$$P_h = P_a \cos(15 + \tfrac{2}{3}\phi'_1) = 157.22 \cos 36.33 = 126.65 \text{ kN/m}$$

and

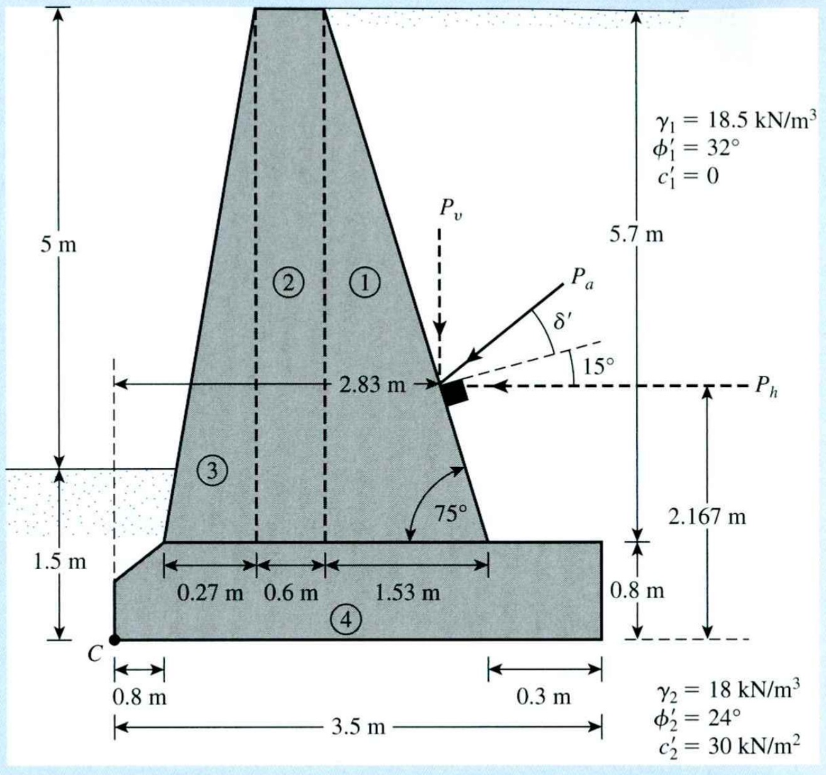
$$P_v = P_a \sin(15 + \tfrac{2}{3}\phi'_1) = 157.22 \sin 36.33 = 93.14 \text{ kN/m}$$

Part a: Factor of Safety against Overturning

From Figure 8.13, one can prepare the following table:

Area no.	Area (m^2)	Weight* (kN/m)	Moment arm from C (m)	Moment (kN-m/m)
1	$\frac{1}{2}(5.7)(1.53) = 4.36$	102.81	2.18	224.13
2	$(0.6)(5.7) = 3.42$	80.64	1.37	110.48
3	$\frac{1}{2}(0.27)(5.7) = 0.77$	18.16	0.98	17.80
4	$\approx (3.5)(0.8) = 2.8$	66.02	1.75	115.54
		$P_v = 93.14$	2.83	263.59
		$\Sigma V = 360.77$ kN/m		$\Sigma M_R = 731.54$ kN-m/m

*$\gamma_{concrete} = 23.58$ kN/m^3



Note that the weight of the soil above the back face of the wall is not taken into account in the preceding table. We have

$$\text{Overturning moment} = M_o = P_h\left(\frac{H'}{3}\right) = 126.65(2.167) = 274.45 \text{ kN-m/m}$$

Hence,

$$FS_{(overturning)} = \frac{\Sigma M_R}{\Sigma M_o} = \frac{731.54}{274.45} = \mathbf{2.67 > 2, OK}$$

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Part b: Factor of Safety against Sliding

We have

$$FS_{(\text{sliding})} = \frac{(\Sigma V)\tan\left(\frac{2}{3}\phi_2'\right) + \frac{2}{3}c_2'B + P_p}{P_h}$$

$$P_p = \tfrac{1}{2}K_p\gamma_2 D^2 + 2c_2'\sqrt{K_p}D$$

and

$$K_p = \tan^2\left(45 + \frac{24}{2}\right) = 2.37$$

Hence,

$$P_p = \tfrac{1}{2}(2.37)\,(18)\,(1.5)^2 + 2(30)\,(1.54)\,(1.5) = 186.59\ \text{kN/m}$$

So

$$FS_{(\text{sliding})} = \frac{360.77\tan\left(\frac{2}{3}\times 24\right) + \frac{2}{3}(30)\,(3.5) + 186.59}{126.65}$$

$$= \frac{103.45 + 70 + 186.59}{126.65} = \mathbf{2.84}$$

If P_p is ignored, the factor of safety is **1.37**.

$$M_{net} = \Sigma M_R - \Sigma M_o \quad (8.16)$$

$$\overline{X} = \frac{M_{net}}{\Sigma V} \quad (8.17)$$

$$e = \frac{B}{2} - \overline{X} \quad (8.18)$$

Part c: Pressure on Soil at Toe and Heel

From Eqs. (8.16), (8.17), and (8.18),

$$e = \frac{B}{2} - \frac{\Sigma M_R - \Sigma M_o}{\Sigma V} = \frac{3.5}{2} - \frac{731.54 - 274.45}{360.77} = 0.483 < \frac{B}{6} = 0.583$$

$$q_{toe} = \frac{\Sigma V}{B}\left[1 + \frac{6e}{B}\right] = \frac{360.77}{3.5}\left[1 + \frac{(6)(0.483)}{3.5}\right] = \mathbf{188.43\ kN/m^2}$$

$$q_{max} = q_{toe} = \frac{\Sigma V}{B}\left(1 + \frac{6e}{B}\right) \quad (8.20)$$

$$q_{min} = q_{heel} = \frac{\Sigma V}{B}\left(1 - \frac{6e}{B}\right) \quad (8.21)$$

and

$$q_{heel} = \frac{V}{B}\left[1 - \frac{6e}{B}\right] = \frac{360.77}{3.5}\left[1 - \frac{(6)(0.483)}{3.5}\right] = \mathbf{17.73\ kN/m^2}$$

Meet You Again

Foundation Engineering)2)

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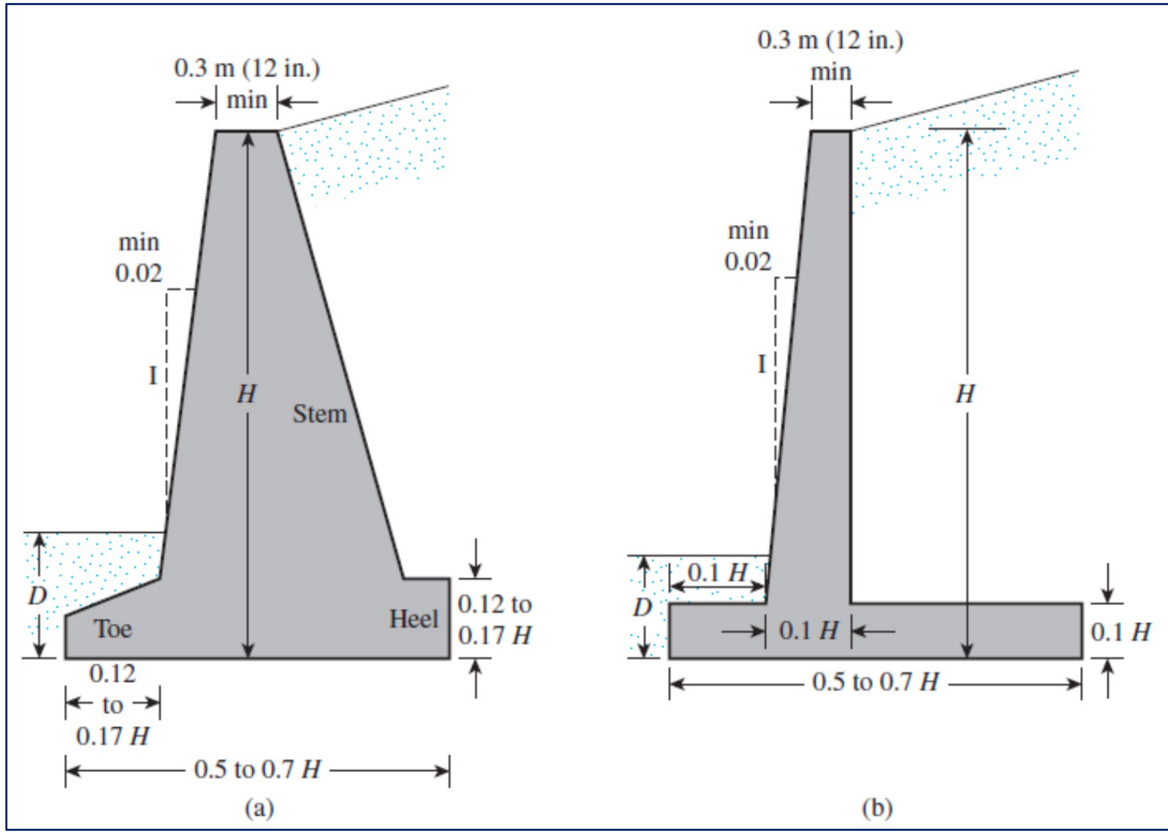
Retaining Walls (RW), (2)

- Proportioning of RW for Stability
- Construction Joints and Drainage from Backfill with Example
- Examples on Stability

Proportioning of RW (Gravity and Cantilever Walls)

1) **Proportioning** means assuming some of RW dimensions as trial to check the walls for stability. If the stability checks yield undesirable results, the sections can be changed and rechecked. The figure shows the general proportions of various retaining wall components that can be used for initial checks.
2) The **top of the stem** of any retaining wall should **not be less than about 0.3 m (≈12 in)** for proper placement of concrete.
3) The **depth, D**, to the bottom of the base slab should be a minimum of **0.6 m (≈2 ft).** However, the bottom of the base slab should be positioned below the seasonal frost line.
4) For **counterfort retaining walls**, the general proportion of the **stem and the base slab** is the **same as for cantilever walls**.
5) The **counterfort slabs** may be about **0.3 m (≈12 in) thick** and **spaced at center-to-center distances of 0.3H to 0.7H.**



(a): Gravity wall, (b): Cantilever wall

Construction Joints and Drainage from Backfill

Construction Joints

A retaining wall may be constructed with one or more of the following joints:

1. *Construction joints* (see Figure 8.14a) are vertical and horizontal joints that are placed between two successive pours of concrete. To increase the shear at the joints, keys may be used. If keys are not used, the surface of the first pour is cleaned and roughened before the next pour of concrete.
2. *Contraction joints* (Figure 8.14b) are vertical joints (grooves) placed in the face of a wall (from the top of the base slab to the top of the wall) that allow the concrete to shrink without noticeable harm. The grooves may be about 6 to 8 mm (≈0.25 to 0.3 in.) wide and 12 to 16 mm (≈0.5 to 0.6 in.) deep.
3. *Expansion joints* (Figure 8.14c) allow for the expansion of concrete caused by temperature changes; vertical expansion joints from the base to the top of the wall may also be used. These joints may be filled with flexible joint fillers. In most cases, horizontal reinforcing steel bars running across the stem are continuous through all joints. The steel is greased to allow the concrete to expand.

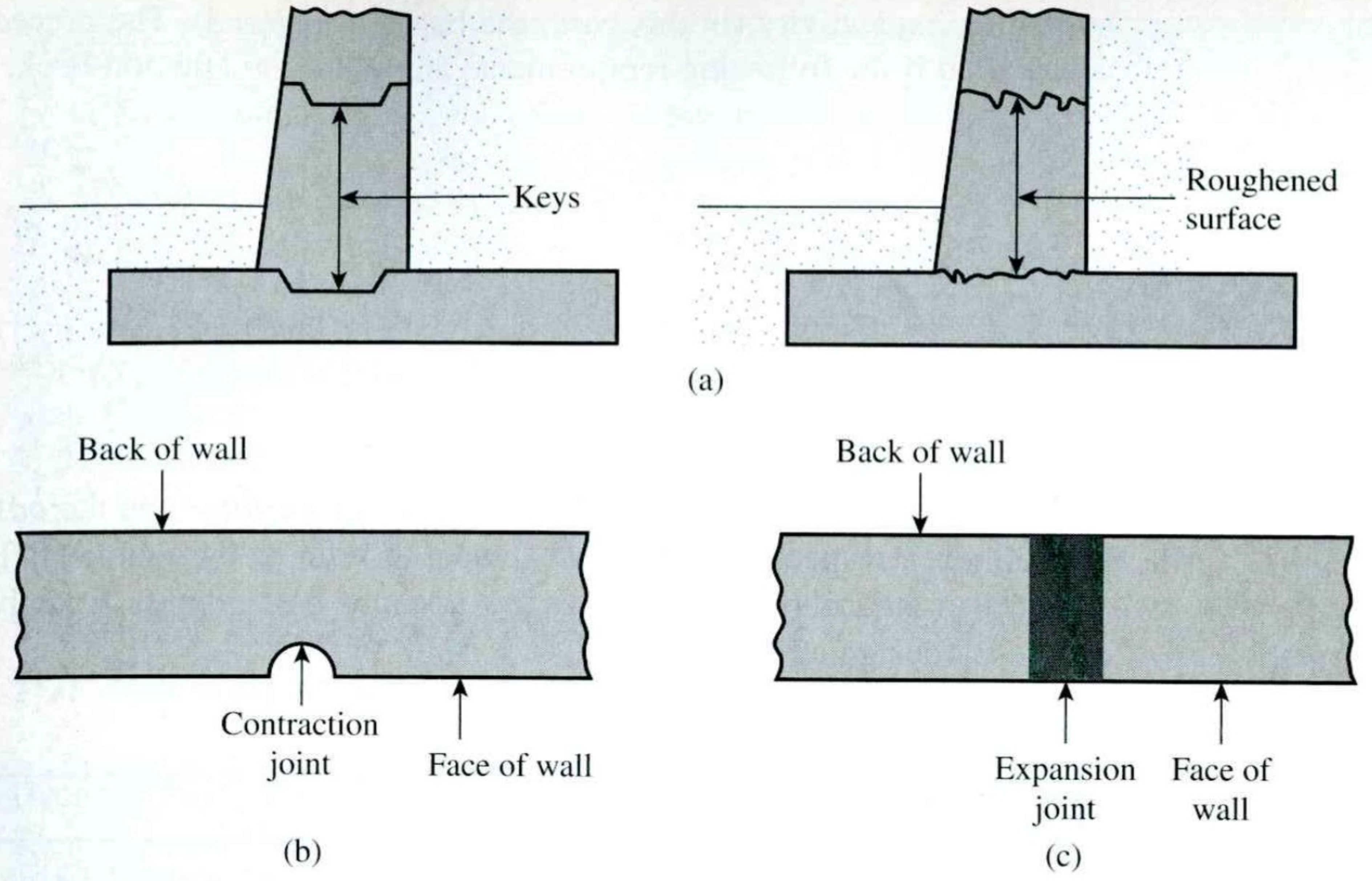


Figure 8.14 (a) Construction joints; (b) contraction joint; (c) expansion joint

Drainage from the Backfill

As the result of rainfall or other wet conditions, the backfill material for a retaining wall may become saturated, thereby increasing the pressure on the wall and perhaps creating an unstable condition. For this reason, adequate drainage must be provided by means of *weep holes* or *perforated drainage pipes*. (See Figure 8.15.)

When provided, weep holes should have a minimum diameter of about 0.1 m (4 in.) and be adequately spaced. Note that there is always a possibility that backfill material may be washed into weep holes or drainage pipes and ultimately clog them. Thus, a filter material needs to be placed behind the weep holes or around the drainage pipes, as the case may be; geotextiles now serve that purpose.

Two main factors influence the choice of filter material: The grain-size distribution of the materials should be such that (a) the soil to be protected is not washed into the filter and (b) excessive hydrostatic pressure head is not created in the soil with a lower hydraulic conductivity (in this case, the backfill material). The preceding conditions can be satisfied if the following requirements are met (Terzaghi and Peck, 1967):

$$\frac{D_{15(F)}}{D_{85(B)}} < 5 \qquad [\text{to satisfy condition (a)}] \tag{8.25}$$

$$\frac{D_{15(F)}}{D_{15(B)}} > 4 \qquad [\text{to satisfy condition (b)}] \tag{8.26}$$

In these relations, the subscripts F and B refer to the *filter* and the *base* material (i.e., the backfill soil), respectively. Also, D_{15} and D_{85} refer to the diameters through which 15% and 85% of the soil (filter or base, as the case may be) will pass. Example 8.3 gives the procedure for designing a filter.

Example on Filter

Figure 8.16 shows the grain-size distribution of a backfill material. Using the conditions outlined in Section 8.8, determine the range of the grain-size distribution for the filter material.

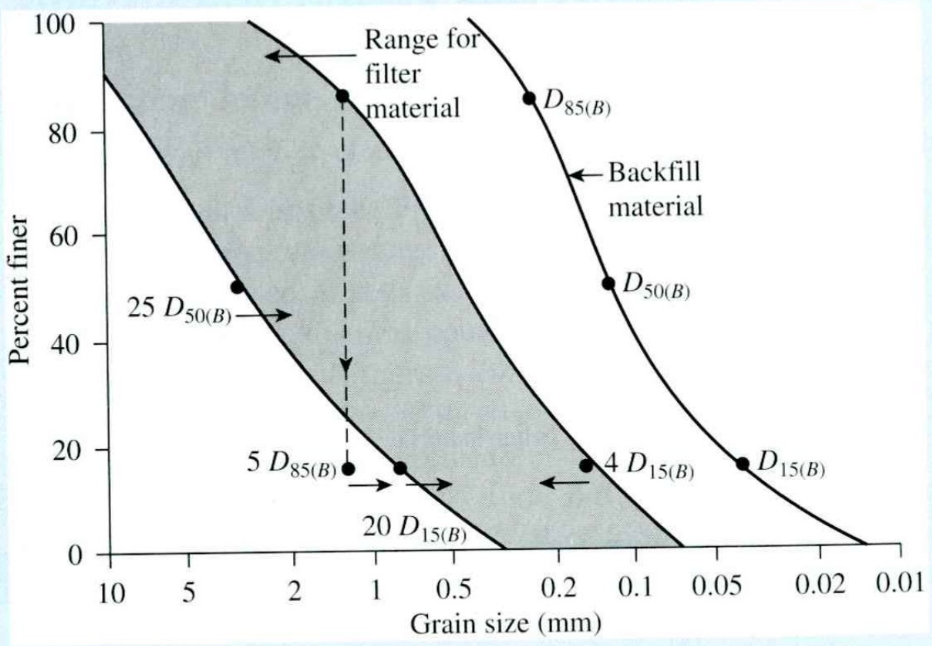


Figure 8.16 Determination of grain-size distribution of filter material

Solution

From the grain-size distribution curve given in the figure, the following values can be determined:

$$D_{15(B)} = 0.04 \text{ mm}$$

$$D_{85(B)} = 0.25 \text{ mm}$$

$$D_{50(B)} = 0.13 \text{ mm}$$

Conditions of Filter

1. $D_{15(F)}$ should be less than $5D_{85(B)}$; that is, $5 \times 0.25 = 1.25$ mm.
2. $D_{15(F)}$ should be greater than $4D_{15(B)}$; that is, $4 \times 0.04 = 0.16$ mm.
3. $D_{50(F)}$ should be less than $25D_{50(B)}$; that is, $25 \times 0.13 = 3.25$ mm.
4. $D_{15(F)}$ should be less than $20D_{15(B)}$; that is, $20 \times 0.04 = 0.8$ mm.

These limiting points are plotted in Figure 8.16. Through them, two curves can be drawn that are similar in nature to the grain-size distribution curve of the backfill material. These curves define the range of the filter material to be used. ■

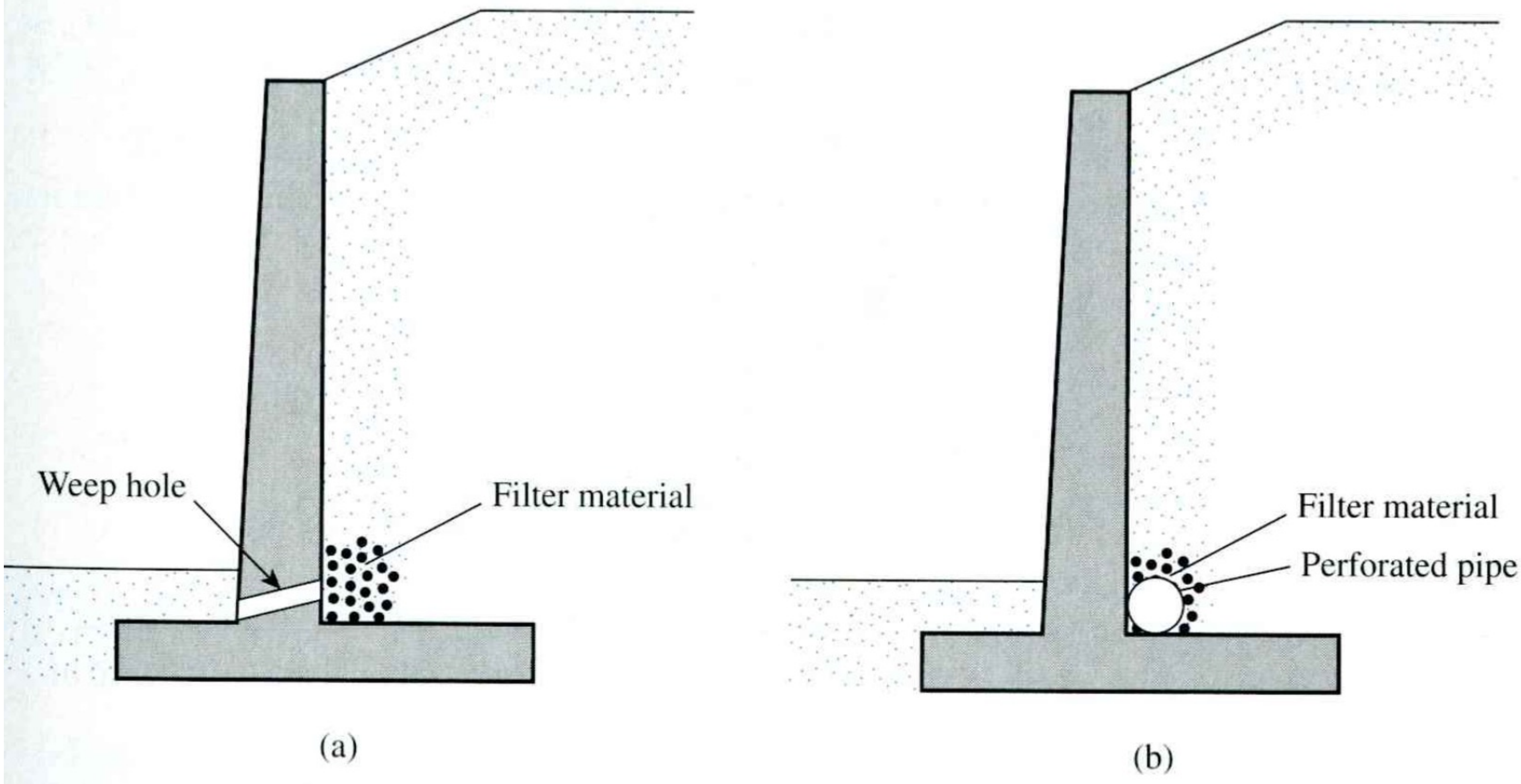


Figure 8.15 Drainage provisions for the backfill of a retaining wall: (a) by weep holes; (b) by a perforated drainage pipe

Examples

1- The cross section of a cantilever retaining wall is shown in Figure 8.12. Calculate the factors of safety with respect to overturning, sliding, and bearing capacity.

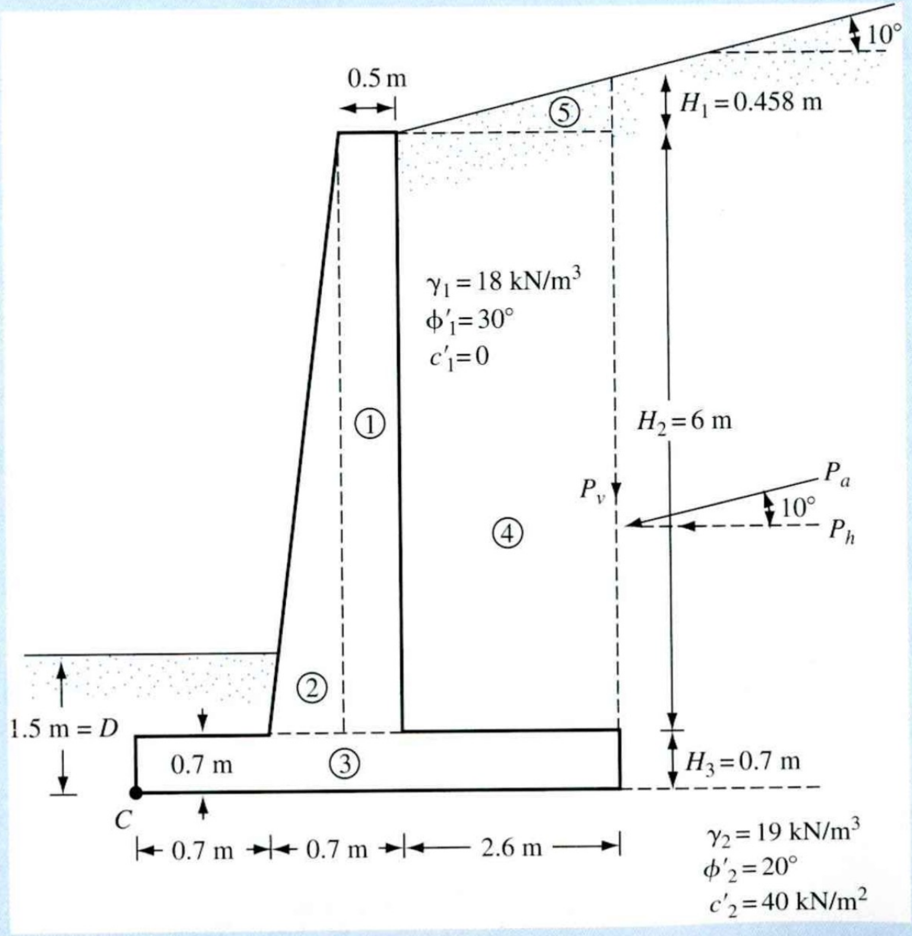


Figure 8.12 Calculation of stability of a retaining wall

$$K_a = \cos\alpha \frac{\cos\alpha - \sqrt{\cos^2\alpha - \cos^2\phi'}}{\cos\alpha + \sqrt{\cos^2\alpha - \cos^2\phi'}} \quad (7.19)$$

Table 7.1 Values of K_a [Eq. (7.19)]

α (deg) ↓	ϕ' (deg) → 28	29	30	31	32	33	34	35	36	37	38	39	40
0	0.3610	0.3470	0.3333	0.3201	0.3073	0.2948	0.2827	0.2710	0.2596	0.2486	0.2379	0.2275	0.2174
1	0.3612	0.3471	0.3335	0.3202	0.3074	0.2949	0.2828	0.2711	0.2597	0.2487	0.2380	0.2276	0.2175
2	0.3618	0.3476	0.3339	0.3207	0.3078	0.2953	0.2832	0.2714	0.2600	0.2489	0.2382	0.2278	0.2177
3	0.3627	0.3485	0.3347	0.3214	0.3084	0.2959	0.2837	0.2719	0.2605	0.2494	0.2386	0.2282	0.2181
4	0.3639	0.3496	0.3358	0.3224	0.3094	0.2967	0.2845	0.2726	0.2611	0.2500	0.2392	0.2287	0.2186
5	0.3656	0.3512	0.3372	0.3237	0.3105	0.2978	0.2855	0.2736	0.2620	0.2508	0.2399	0.2294	0.2192
6	0.3676	0.3531	0.3389	0.3253	0.3120	0.2992	0.2868	0.2747	0.2631	0.2518	0.2409	0.2303	0.2200
7	0.3701	0.3553	0.3410	0.3272	0.3138	0.3008	0.2883	0.2761	0.2644	0.2530	0.2420	0.2313	0.2209
8	0.3730	0.3580	0.3435	0.3294	0.3159	0.3027	0.2900	0.2778	0.2659	0.2544	0.2432	0.2325	0.2220
9	0.3764	0.3611	0.3463	0.3320	0.3182	0.3049	0.2921	0.2796	0.2676	0.2560	0.2447	0.2338	0.2233
10	0.3802	0.3646	0.3495	0.3350	0.3210	0.3074	0.2944	0.2818	0.2696	0.2578	0.2464	0.2354	0.2247
11	0.3846	0.3686	0.3532	0.3383	0.3241	0.3103	0.2970	0.2841	0.2718	0.2598	0.2482	0.2371	0.2263
12	0.3896	0.3731	0.3573	0.3421	0.3275	0.3134	0.2999	0.2868	0.2742	0.2621	0.2503	0.2390	0.2281
13	0.3952	0.3782	0.3620	0.3464	0.3314	0.3170	0.3031	0.2898	0.2770	0.2646	0.2527	0.2412	0.2301
14	0.4015	0.3839	0.3671	0.3511	0.3357	0.3209	0.3068	0.2931	0.2800	0.2674	0.2552	0.2435	0.2322
15	0.4086	0.3903	0.3729	0.3564	0.3405	0.3253	0.3108	0.2968	0.2834	0.2705	0.2581	0.2461	0.2346
16	0.4165	0.3975	0.3794	0.3622	0.3458	0.3302	0.3152	0.3008	0.2871	0.2739	0.2612	0.2490	0.2373
17	0.4255	0.4056	0.3867	0.3688	0.3518	0.3356	0.3201	0.3053	0.2911	0.2776	0.2646	0.2521	0.2401
18	0.4357	0.4146	0.3948	0.3761	0.3584	0.3415	0.3255	0.3102	0.2956	0.2817	0.2683	0.2555	0.2433
19	0.4473	0.4249	0.4039	0.3842	0.3657	0.3481	0.3315	0.3156	0.3006	0.2862	0.2724	0.2593	0.2467
20	0.4605	0.4365	0.4142	0.3934	0.3739	0.3555	0.3381	0.3216	0.3060	0.2911	0.2769	0.2634	0.2504
21	0.4758	0.4498	0.4259	0.4037	0.3830	0.3637	0.3455	0.3283	0.3120	0.2965	0.2818	0.2678	0.2545
22	0.4936	0.4651	0.4392	0.4154	0.3934	0.3729	0.3537	0.3356	0.3186	0.3025	0.2872	0.2727	0.2590
23	0.5147	0.4829	0.4545	0.4287	0.4050	0.3832	0.3628	0.3438	0.3259	0.3091	0.2932	0.2781	0.2638
24	0.5404	0.5041	0.4724	0.4440	0.4183	0.3948	0.3731	0.3529	0.3341	0.3164	0.2997	0.2840	0.2692
25	0.5727	0.5299	0.4936	0.4619	0.4336	0.4081	0.3847	0.3631	0.3431	0.3245	0.3070	0.2905	0.2750

Solution

From the figure,

$$H' = H_1 + H_2 + H_3 = 2.6 \tan 10° + 6 + 0.7$$
$$= 0.458 + 6 + 0.7 = 7.158 \text{ m}$$

The Rankine active force per unit length of wall $= P_p = \frac{1}{2}\gamma_1 H'^2 K_a$. For $\phi'_1 = 30°$ and $\alpha = 10°$, K_a is equal to 0.3532. (See Table 7.1.) Thus, a

$$P_a = \tfrac{1}{2}(18)(7.158)^2(0.3532) = 162.9 \text{ kN/m}$$

$$P_v = P_a \sin 10° = 162.9 (\sin 10°) = 28.29 \text{ kN/m}$$

and

$$P_h = P_a \cos 10° = 162.9 (\cos 10°) = 160.43 \text{ kN/m}$$

Factor of Safety against Overturning

The following table can now be prepared for determining the resisting moment:

Section no.[a]	Area (m²)	Weight/unit length (kN/m)	Moment arm from point *C* (m)	Moment (kN-m/m)
1	$6 \times 0.5 = 3$	70.74	1.15	81.35
2	$\frac{1}{2}(0.2)6 = 0.6$	14.15	0.833	11.79
3	$4 \times 0.7 = 2.8$	66.02	2.0	132.04
4	$6 \times 2.6 = 15.6$	280.80	2.7	758.16
5	$\frac{1}{2}(2.6)(0.458) = 0.595$	10.71	3.13	33.52
		$P_v = 28.29$	4.0	113.16
		$\Sigma V = 470.71$		$1130.02 = \Sigma M_R$

[a]For section numbers, refer to Figure 8.12

$\gamma_{concrete} = 23.58 \text{ kN/m}^3$

The overturning moment

$$M_o = P_h\left(\frac{H'}{3}\right) = 160.43\left(\frac{7.158}{3}\right) = 382.79 \text{ kN-m/m}$$

and

$$\text{FS}_{(\text{overturning})} = \frac{\Sigma M_R}{M_o} = \frac{1130.02}{382.79} = \mathbf{2.95 > 2, OK}$$

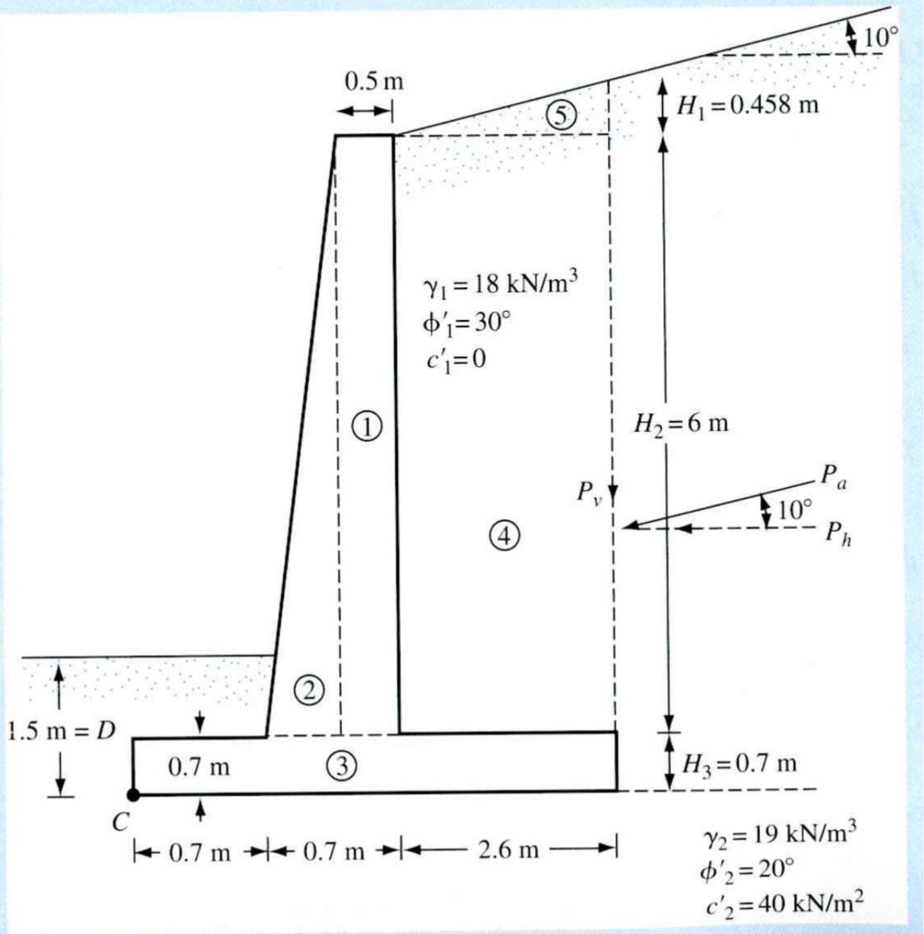


Figure 8.12 Calculation of stability of a retaining wall

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Factor of Safety against Sliding

From Eq. (8.11),

$$FS_{(sliding)} = \frac{(\Sigma V)\tan(k_1\phi'_2) + Bk_2c'_2 + P_p}{P_a\cos\alpha}$$

Let $k_1 = k_2 = \frac{2}{3}$. Also,

$$P_p = \frac{1}{2}K_p\gamma_2 D^2 + 2c'_2\sqrt{K_p}D$$

$$K_p = \tan^2\left(45 + \frac{\phi'_2}{2}\right) = \tan^2(45 + 10) = 2.04$$

and

$$D = 1.5 \text{ m}$$

So

$$P_p = \frac{1}{2}(2.04)(19)(1.5)^2 + 2(40)(\sqrt{2.04})(1.5)$$
$$= 43.61 + 171.39 = 215 \text{ kN/m}$$

Hence,

$$FS_{(sliding)} = \frac{(470.71)\tan\left(\frac{2\times 20}{3}\right) + (4)\left(\frac{2}{3}\right)(40) + 215}{160.43}$$

$$= \frac{111.56 + 106.67 + 215}{160.43} = \mathbf{2.7 > 1.5, OK}$$

Note: For some designs, the depth D in a passive pressure calculation may be taken to be *equal to the thickness of the base slab*.

$$FS_{(sliding)} = \frac{(\Sigma V)\tan(k_1\phi'_2) + Bk_2c'_2 + P_p}{P_a\cos\alpha} \quad (8.11)$$

Factor of Safety against Bearing Capacity Failure

$$M_{net} = \Sigma M_R - \Sigma M_o \tag{8.16}$$

$$\overline{X} = \frac{M_{net}}{\Sigma V} \tag{8.17}$$

$$e = \frac{B}{2} - \overline{X} \tag{8.18}$$

Combining Eqs. (8.16), (8.17), and (8.18) yields

$$e = \frac{B}{2} - \frac{\Sigma M_R - \Sigma M_o}{\Sigma V} = \frac{4}{2} - \frac{1130.02 - 382.79}{470.71}$$

$$= 0.411 \text{ m} < \frac{B}{6} = \frac{4}{6} = 0.666 \text{ m}$$

$$q_{max} = q_{toe} = \frac{\Sigma V}{B}\left(1 + \frac{6e}{B}\right) \tag{8.20}$$

$$q_{min} = q_{heel} = \frac{\Sigma V}{B}\left(1 - \frac{6e}{B}\right) \tag{8.21}$$

Again, from Eqs. (8.20) and (8.21)

$$q_{\substack{toe\\heel}} = \frac{\Sigma V}{B}\left(1 \pm \frac{6e}{B}\right) = \frac{470.71}{4}\left(1 \pm \frac{6 \times 0.411}{4}\right) = 190.2 \text{ kN/m}^2 \text{ (toe)}$$

$$= 45.13 \text{ kN/m}^2 \text{ (heel)}$$

The ultimate bearing capacity of the soil can be determined from Eq. (8.22)

$$q_u = c_2' N_c F_{cd} F_{ci} + q N_q F_{qd} F_{qi} + \frac{1}{2} \gamma_2 B' N_\gamma F_{\gamma d} F_{\gamma i}$$

For $\phi_2' = 20°$ (see Table 3.3), $N_c = 14.83$, $N_q = 6.4$, and $N_\gamma = 5.39$. Also,

$$q = \gamma_2 D = (19)(1.5) = 28.5 \text{ kN/m}^2$$

$$B' = B - 2e = 4 - 2(0.411) = 3.178 \text{ m}$$

$$F_{cd} = F_{qd} - \frac{1 - F_{qd}}{N_c \tan\phi_2'} = 1.148 - \frac{1 - 1.148}{(14.83)(\tan 20)} = 1.175$$

$$F_{qd} = 1 + 2\tan\phi_2'(1 - \sin\phi_2')^2\left(\frac{D}{B'}\right) = 1 + 0.315\left(\frac{1.5}{3.178}\right) = 1.148$$

$$F_{\gamma d} = 1$$

$$F_{ci} = F_{qi} = \left(1 - \frac{\psi°}{90°}\right)^2$$

and

$$\psi = \tan^{-1}\left(\frac{P_a \cos\alpha}{\Sigma V}\right) = \tan^{-1}\left(\frac{160.43}{470.71}\right) = 18.82°$$

Table 3.3 Bearing Capacity Factors

ϕ'	N_c	N_q	N_γ	ϕ'	N_c	N_q	N_γ
0	5.14	1.00	0.00	26	22.25	11.85	12.54
1	5.38	1.09	0.07	27	23.94	13.20	14.47
2	5.63	1.20	0.15	28	25.80	14.72	16.72
3	5.90	1.31	0.24	29	27.86	16.44	19.34
4	6.19	1.43	0.34	30	30.14	18.40	22.40
5	6.49	1.57	0.45	31	32.67	20.63	25.99
6	6.81	1.72	0.57	32	35.49	23.18	30.22
7	7.16	1.88	0.71	33	38.64	26.09	35.19
8	7.53	2.06	0.86	34	42.16	29.44	41.06
9	7.92	2.25	1.03	35	46.12	33.30	48.03
10	8.35	2.47	1.22	36	50.59	37.75	56.31
11	8.80	2.71	1.44	37	55.63	42.92	66.19
12	9.28	2.97	1.69	38	61.35	48.93	78.03
13	9.81	3.26	1.97	39	67.87	55.96	92.25
14	10.37	3.59	2.29	40	75.31	64.20	109.41
15	10.98	3.94	2.65	41	83.86	73.90	130.22
16	11.63	4.34	3.06	42	93.71	85.38	155.55
17	12.34	4.77	3.53	43	105.11	99.02	186.54
18	13.10	5.26	4.07	44	118.37	115.31	224.64
19	13.93	5.80	4.68	45	133.88	134.88	271.76
20	14.83	6.40	5.39	46	152.10	158.51	330.35
21	15.82	7.07	6.20	47	173.64	187.21	403.67
22	16.88	7.82	7.13	48	199.26	222.31	496.01
23	18.05	8.66	8.20	49	229.93	265.51	613.16
24	19.32	9.60	9.44	50	266.89	319.07	762.89
25	20.72	10.66	10.88				

So

$$F_{ci} = F_{qi} = \left(1 - \frac{18.82}{90}\right)^2 = 0.626$$

and

$$F_{\gamma i} = \left(1 - \frac{\psi}{\phi'_2}\right)^2 = \left(1 - \frac{18.82}{20}\right)^2 \approx 0$$

Hence,

$$\begin{aligned} q_u &= (40)(14.83)(1.175)(0.626) + (28.5)(6.4)(1.148)(0.626) \\ &\quad + \tfrac{1}{2}(19)(5.93)(3.178)(1)(0) \\ &= 436.33 + 131.08 + 0 = 567.41 \text{ kN/m}^2 \end{aligned}$$

and

$$\boxed{\text{FS}_{\text{(bearing capacity)}} = \frac{q_u}{q_{\text{toe}}} = \frac{567.41}{190.2} = \mathbf{2.98}}$$

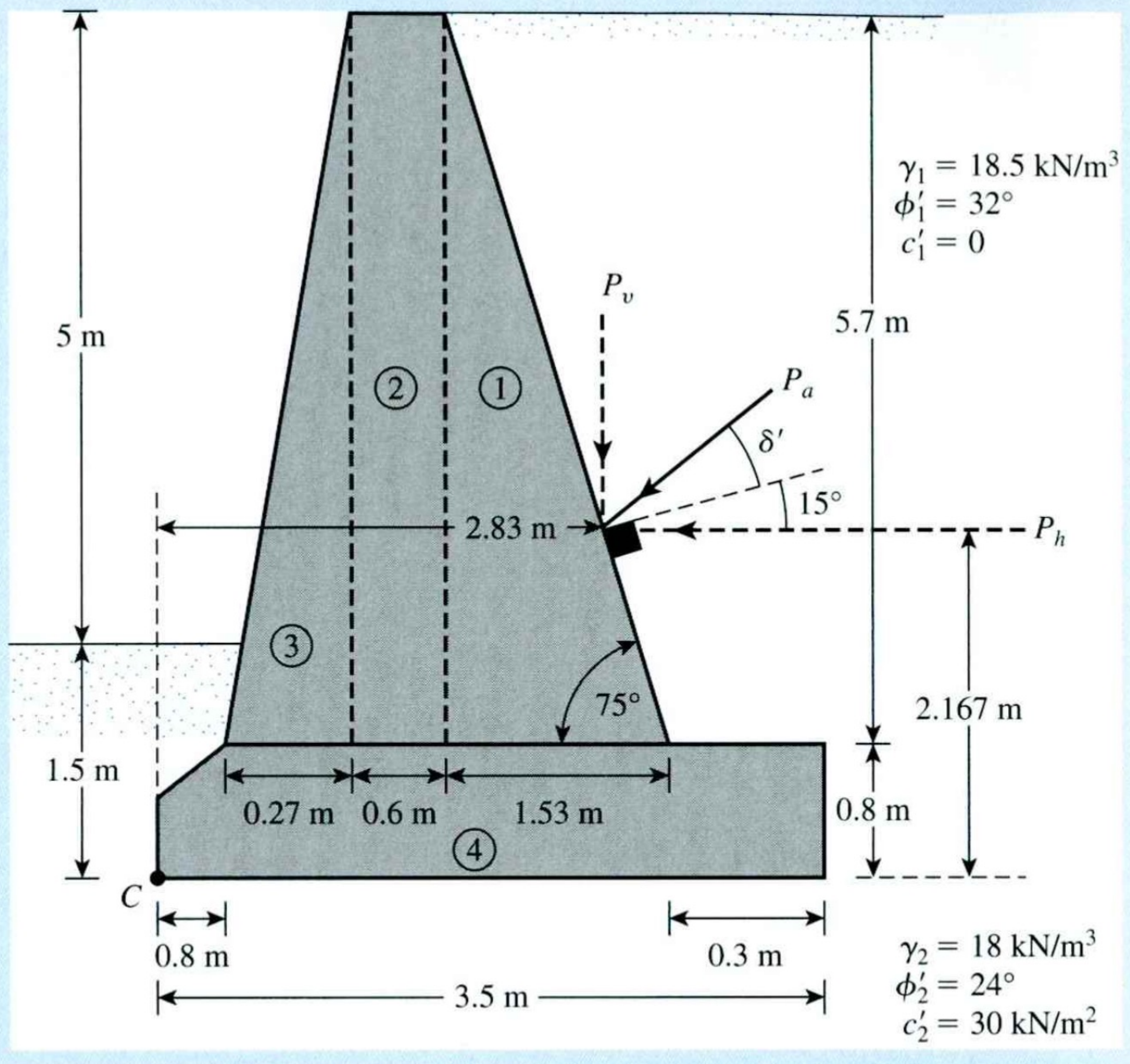
Note: $\text{FS}_{\text{(bearing capacity)}}$ is less than 3. Some repropertioning will be needed.

2- A gravity retaining wall is shown in Figure . Use $\delta' = 2/3\phi'_1$ and Coulomb's active

a. The factor of safety against overturning

b. The factor of safety against sliding

c. The pressure on the soil at the toe and heel



$$K_a = \text{Coulomb's active earth pressure coefficient}$$

$$= \frac{\sin^2(\beta + \phi')}{\sin^2\beta \sin(\beta - \delta')\left[1 + \sqrt{\frac{\sin(\phi' + \delta')\sin(\phi' - \alpha)}{\sin(\beta - \delta')\sin(\alpha + \beta)}}\right]^2} \tag{7.26}$$

Solution

The height

$$H' = 5 + 1.5 = 6.5 \text{ m}$$

Coulomb's active force is

$$P_a = \tfrac{1}{2}\gamma_1 H'^2 K_a$$

With $\alpha = 0°$, $\beta = 75°$, $\delta' = 2/3\phi'_1$, and $\phi'_1 = 32°$, $K_a = 0.4023$. (See Table 7.4.) So,

$$P_a = \tfrac{1}{2}(18.5)(6.5)^2(0.4023) = 157.22 \text{ kN/m}$$

$$P_h = P_a \cos(15 + \tfrac{2}{3}\phi'_1) = 157.22 \cos 36.33 = 126.65 \text{ kN/m}$$

and

$$P_v = P_a \sin(15 + \tfrac{2}{3}\phi'_1) = 157.22 \sin 36.33 = 93.14 \text{ kN/m}$$

Table 7.4 Values of K_a [from Eq. (7.26)] for $\delta' = \frac{2}{3}\phi'$

		β (deg)					
α (deg)	ϕ' (deg)	90	85	80	75	70	65
0	28	0.3213	0.3588	0.4007	0.4481	0.5026	0.5662
	29	0.3091	0.3467	0.3886	0.4362	0.4908	0.5547
	30	0.2973	0.3349	0.3769	0.4245	0.4794	0.5435
	31	0.2860	0.3235	0.3655	0.4133	0.4682	0.5326
	32	0.2750	0.3125	0.3545	0.4023	0.4574	0.5220
	33	0.2645	0.3019	0.3439	0.3917	0.4469	0.5117
	34	0.2543	0.2916	0.3335	0.3813	0.4367	0.5017
	35	0.2444	0.2816	0.3235	0.3713	0.4267	0.4919
	36	0.2349	0.2719	0.3137	0.3615	0.4170	0.4824
	37	0.2257	0.2626	0.3042	0.3520	0.4075	0.4732
	38	0.2168	0.2535	0.2950	0.3427	0.3983	0.4641
	39	0.2082	0.2447	0.2861	0.3337	0.3894	0.4553
	40	0.1998	0.2361	0.2774	0.3249	0.3806	0.4468
	41	0.1918	0.2278	0.2689	0.3164	0.3721	0.4384
	42	0.1840	0.2197	0.2606	0.3080	0.3637	0.4302
5	28	0.3431	0.3845	0.4311	0.4843	0.5461	0.6190
	29	0.3295	0.3709	0.4175	0.4707	0.5325	0.6056
	30	0.3165	0.3578	0.4043	0.4575	0.5194	0.5926
	31	0.3039	0.3451	0.3916	0.4447	0.5067	0.5800
	32	0.2919	0.3329	0.3792	0.4324	0.4943	0.5677
	33	0.2803	0.3211	0.3673	0.4204	0.4823	0.5558
	34	0.2691	0.3097	0.3558	0.4088	0.4707	0.5443
	35	0.2583	0.2987	0.3446	0.3975	0.4594	0.5330
	36	0.2479	0.2881	0.3338	0.3866	0.4484	0.5221
	37	0.2379	0.2778	0.3233	0.3759	0.4377	0.5115
	38	0.2282	0.2679	0.3131	0.3656	0.4273	0.5012
	39	0.2188	0.2582	0.3033	0.3556	0.4172	0.4911
	40	0.2098	0.2489	0.2937	0.3458	0.4074	0.4813
	41	0.2011	0.2398	0.2844	0.3363	0.3978	0.4718
	42	0.1927	0.2311	0.2753	0.3271	0.3884	0.4625
10	28	0.3702	0.4164	0.4686	0.5287	0.5992	0.6834
	29	0.3548	0.4007	0.4528	0.5128	0.5831	0.6672
	30	0.3400	0.3857	0.4376	0.4974	0.5676	0.6516
	31	0.3259	0.3713	0.4230	0.4826	0.5526	0.6365
	32	0.3123	0.3575	0.4089	0.4683	0.5382	0.6219
	33	0.2993	0.3442	0.3953	0.4545	0.5242	0.6078
	34	0.2868	0.3314	0.3822	0.4412	0.5107	0.5942
	35	0.2748	0.3190	0.3696	0.4283	0.4976	0.5810
	36	0.2633	0.3072	0.3574	0.4158	0.4849	0.5682
	37	0.2522	0.2957	0.3456	0.4037	0.4726	0.5558
	38	0.2415	0.2846	0.3342	0.3920	0.4607	0.5437
	39	0.2313	0.2740	0.3231	0.3807	0.4491	0.5321
	40	0.2214	0.2636	0.3125	0.3697	0.4379	0.5207
	41	0.2119	0.2537	0.3021	0.3590	0.4270	0.5097
	42	0.2027	0.2441	0.2921	0.3487	0.4164	0.4990
15	28	0.4065	0.4585	0.5179	0.5868	0.6685	0.7670

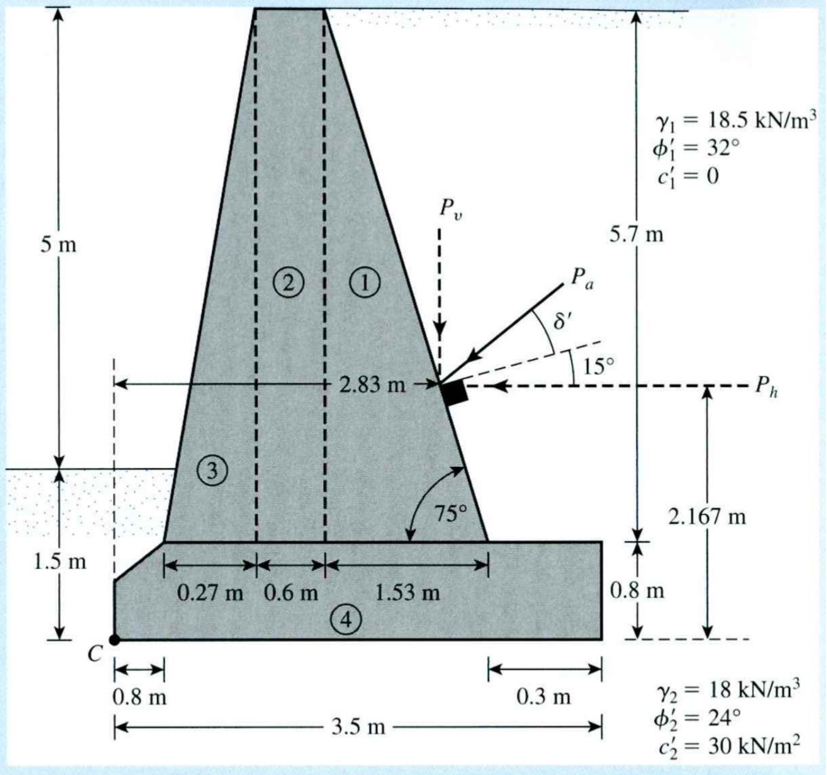
(continued)

Part a: Factor of Safety against Overturning

From Figure 8.13, one can prepare the following table:

Area no.	Area (m^2)	Weight* (kN/m)	Moment arm from *C* (m)	Moment (kN-m/m)
1	$\frac{1}{2}(5.7)(1.53) = 4.36$	102.81	2.18	224.13
2	$(0.6)(5.7) = 3.42$	80.64	1.37	110.48
3	$\frac{1}{2}(0.27)(5.7) = 0.77$	18.16	0.98	17.80
4	$\approx (3.5)(0.8) = 2.8$	66.02	1.75	115.54
		$P_v = 93.14$	2.83	263.59
		$\Sigma V = 360.77$ kN/m		$\Sigma M_R = 731.54$ kN-m/m

*$\gamma_{concrete} = 23.58$ kN/m^3



Note that the weight of the soil above the back face of the wall is not taken into account in the preceding table. We have

$$\text{Overturning moment} = M_o = P_h\left(\frac{H'}{3}\right) = 126.65(2.167) = 274.45 \text{ kN-m/m}$$

Hence,

$$\text{FS}_{(\text{overturning})} = \frac{\Sigma M_R}{\Sigma M_o} = \frac{731.54}{274.45} = \mathbf{2.67 > 2, OK}$$

Part b: Factor of Safety against Sliding

We have

$$FS_{(\text{sliding})} = \frac{(\Sigma V)\tan\left(\frac{2}{3}\phi'_2\right) + \frac{2}{3}c'_2 B + P_p}{P_h}$$

$$P_p = \tfrac{1}{2}K_p\gamma_2 D^2 + 2c'_2\sqrt{K_p}D$$

and

$$K_p = \tan^2\left(45 + \frac{24}{2}\right) = 2.37$$

Hence,

$$P_p = \tfrac{1}{2}(2.37)(18)(1.5)^2 + 2(30)(1.54)(1.5) = 186.59 \text{ kN/m}$$

So

$$FS_{(\text{sliding})} = \frac{360.77\tan\left(\frac{2}{3}\times 24\right) + \frac{2}{3}(30)(3.5) + 186.59}{126.65}$$

$$= \frac{103.45 + 70 + 186.59}{126.65} = \mathbf{2.84}$$

If P_p is ignored, the factor of safety is **1.37**.

$$M_{net} = \Sigma M_R - \Sigma M_o \quad (8.16)$$

$$\overline{X} = \frac{M_{net}}{\Sigma V} \quad (8.17)$$

$$e = \frac{B}{2} - \overline{X} \quad (8.18)$$

Part c: Pressure on Soil at Toe and Heel

From Eqs. (8.16), (8.17), and (8.18),

$$e = \frac{B}{2} - \frac{\Sigma M_R - \Sigma M_o}{\Sigma V} = \frac{3.5}{2} - \frac{731.54 - 274.45}{360.77} = 0.483 < \frac{B}{6} = 0.583$$

$$q_{toe} = \frac{\Sigma V}{B}\left[1 + \frac{6e}{B}\right] = \frac{360.77}{3.5}\left[1 + \frac{(6)(0.483)}{3.5}\right] = \mathbf{188.43\ kN/m^2}$$

$$q_{max} = q_{toe} = \frac{\Sigma V}{B}\left(1 + \frac{6e}{B}\right) \quad (8.20)$$

$$q_{min} = q_{heel} = \frac{\Sigma V}{B}\left(1 - \frac{6e}{B}\right) \quad (8.21)$$

and

$$q_{heel} = \frac{V}{B}\left[1 - \frac{6e}{B}\right] = \frac{360.77}{3.5}\left[1 - \frac{(6)(0.483)}{3.5}\right] = \mathbf{17.73\ kN/m^2}$$

Meet You Again

Foundation Engineering (2)

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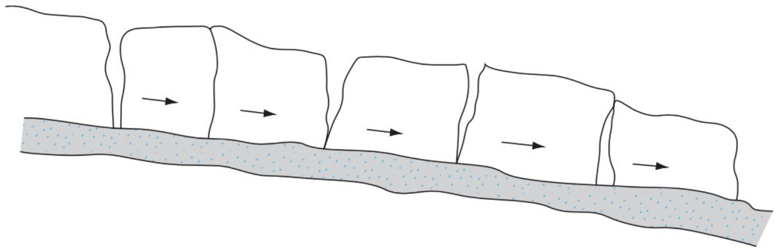
Slope Stability:

- General and types of slope failures
- Factor of safety
- Methods of analysis of slope stability

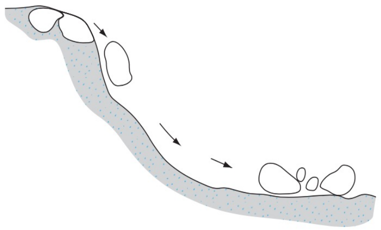
Modes of slope failure

An exposed ground surface that stands at an angle with the horizontal is called an unrestrained slope. The slope can be natural or man-made. It can fail in various modes. They are

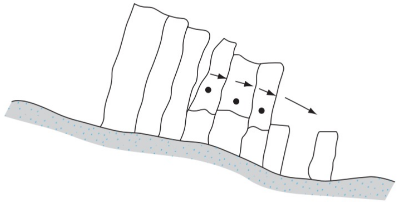
- ✓ **Fall..**
- ✓ **Topple.**
- ✓ **Slide.**
- ✓ **Spread.**
- ✓ **Flow.**



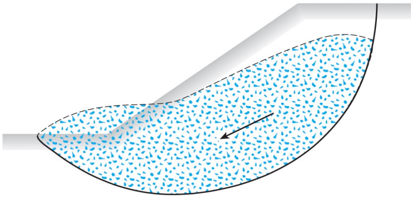
4:Slope failure by lateral "spreading"



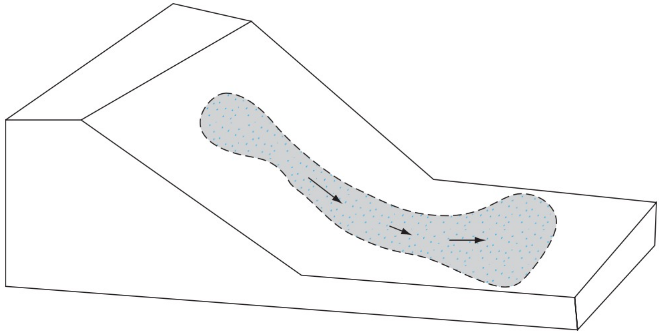
1: "Fall" type of landslide



2: Slope failure by "toppling"



3:Slope failure by "sliding"



5: Slope failure by "flowing"

This lecture primarily relates to the quantitative analysis that fall under the category ***of slide (type 3).***

Factor of Safety

The main task of slope stability analysis is to determine the factor of safety.

The factor of safety is defined as:

$$F_s = \frac{\tau_f}{\tau_d}$$

Where,

F_s factor of safety with respect to strength

τ_f : average shear strength of the soil

τ_d : average shear stress developed along the potential failure surface

$$\tau_f = c' + \sigma' \tan \phi'$$

where c' = cohesion

ϕ' = angle of friction

σ' = normal stress on the potential failure surface

$$\tau_d = c'_d + \sigma' \tan \phi'_d$$

where C'_d and $\emptyset'_d$ are, respectively, the cohesion and the angle of friction that develop along the potential failure surface

Factor of Safety, Cont'd

- ✓ The factor of safety with respect to cohesion, $F_{c'}$:

$$F_{c'} = \frac{c'}{c'_d}$$

- ✓ The factor of safety with respect to friction, $F_{\phi'}$:

$$F_{\phi'} = \frac{\tan \phi'}{\tan \phi'_d}$$

When we compare the above Equations and when $F_{c'}$ becomes equal to $F_{\phi'}$ it gives the factor of safety with respect to strength, or, if

$$\frac{c'}{c'_d} = \frac{\tan \phi'}{\tan \phi'_d}$$

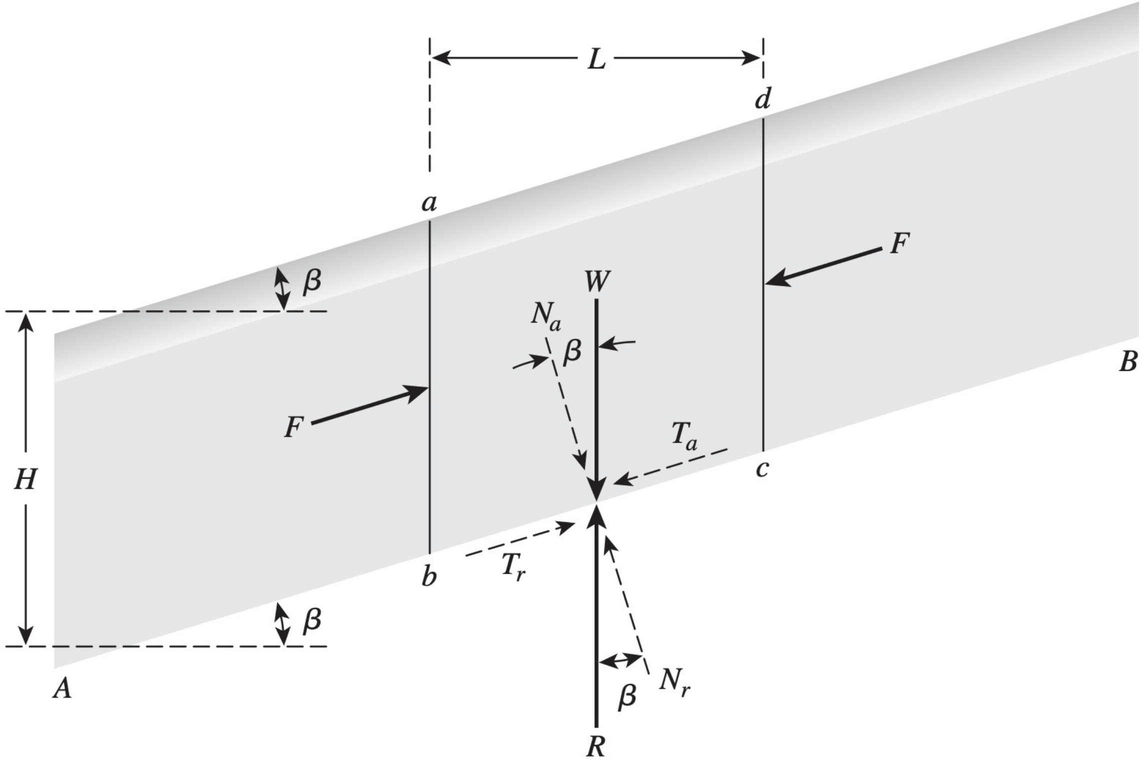
So, $$F_s = F_{c'} = F_{\phi'}$$

- When F_s is equal to 1, the slope is in a state of impending failure.
- A value of **1.5** for the **factor of safety** with respect to strength is **acceptable** for the design of a stable slope.

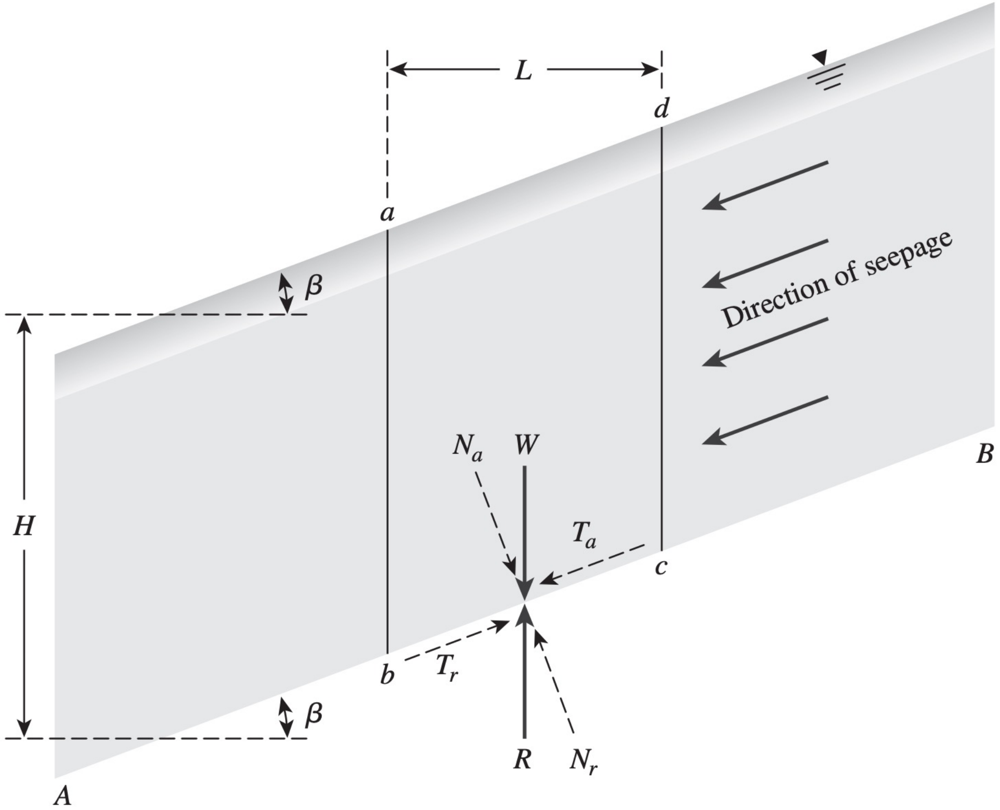
Stability Slopes

1) Infinite slopes:

a) Analysis of infinite slope (without seepage)



b) Analysis of infinite slope (with seepage)

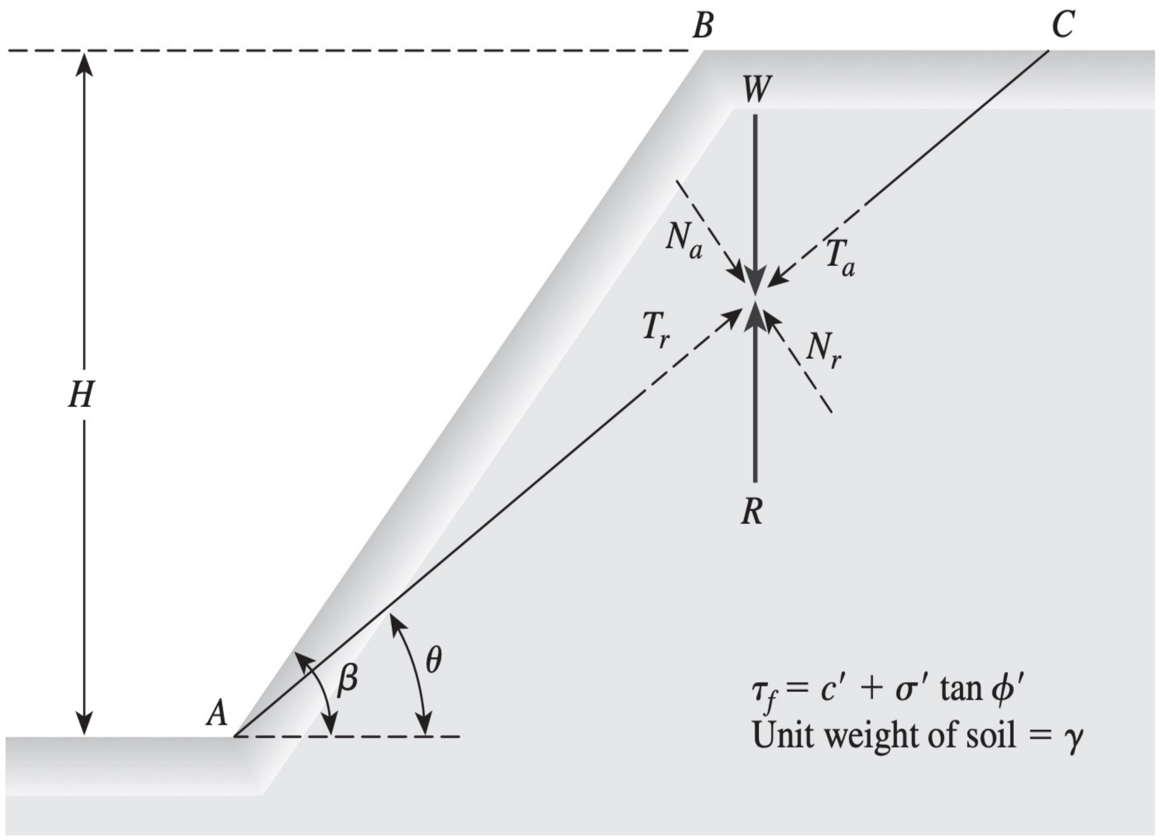


Stability Slopes

2) Finite slopes

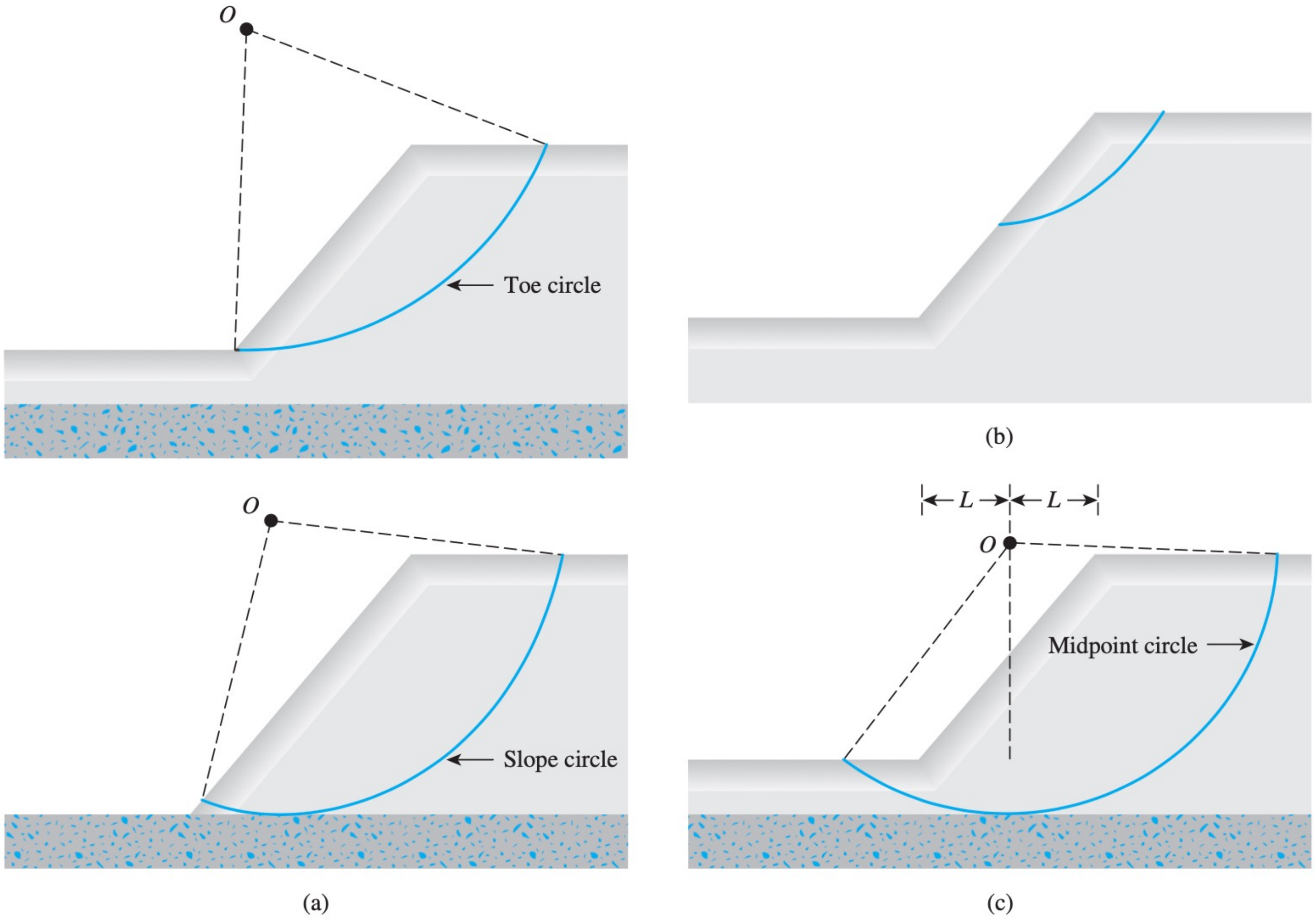
Analysis of Finite Slopes with Plane Failure Surfaces

- Finite slope analysis— Cullman's method



Stability Slopes, Cont'd

- **Analysis of Finite Slopes with Circular Failure Surfaces**



Modes of failure of finite slope: (a) slope failure; (b) shallow slope failure; (c) base failure

Stability Slopes, Cont’d

- **Analysis of Finite Slopes with Circular Failure Surfaces**

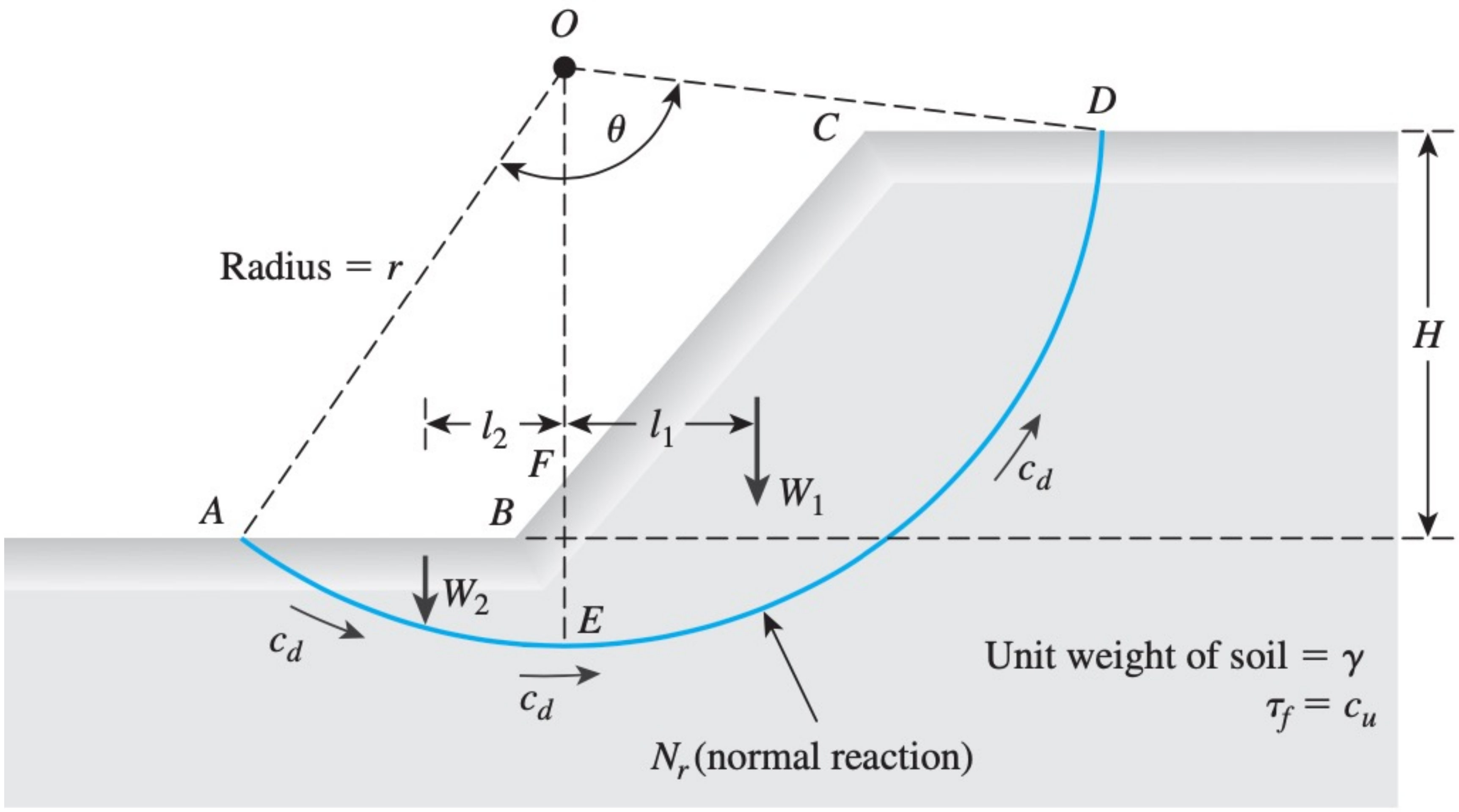
Types of Stability Analysis Procedures

- ***Mass procedure***: In this case, the mass of the soil above the surface of sliding is taken as a unit. This procedure is useful when the soil that forms the slope is assumed to be homogeneous, although this is not the case in most natural slopes.

- ***Method of slices***: In this procedure, the soil above the surface of sliding is divided into a number of vertical parallel slices. The stability of each slice is calculated separately. This is a versatile technique in which the nonhomogeneity of the soils and pore water pressure can be taken into consideration. It also accounts for the variation of the normal stress along the potential failure surface.

Stability Slopes, Cont'd

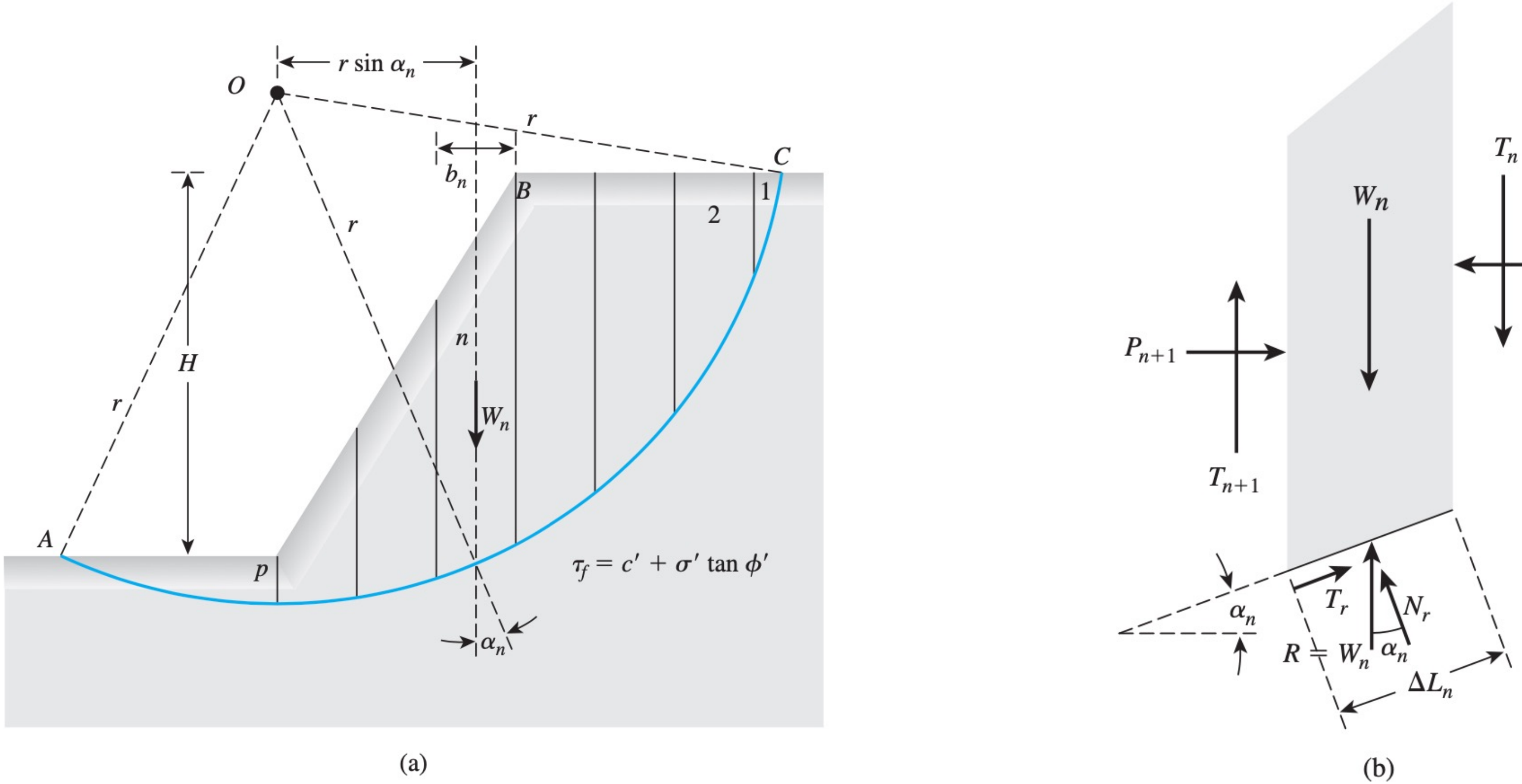
- **Analysis of Finite Slopes with Circular Failure Surfaces, Mass procedure**



Stability analysis of slope in homogeneous saturated clay soil

Stability Slopes, Cont'd

- **Analysis of Finite Slopes with Circular Failure Surfaces, Slices procedure**
  - ✓ **Ordinary Method of Slices**



Stability analysis by ordinary method of slices: (a) trial failure surface; (b) forces acting on *n*th slice

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Stability Slopes, Cont'd

✓ Ordinary Method of Slices, Cont'd

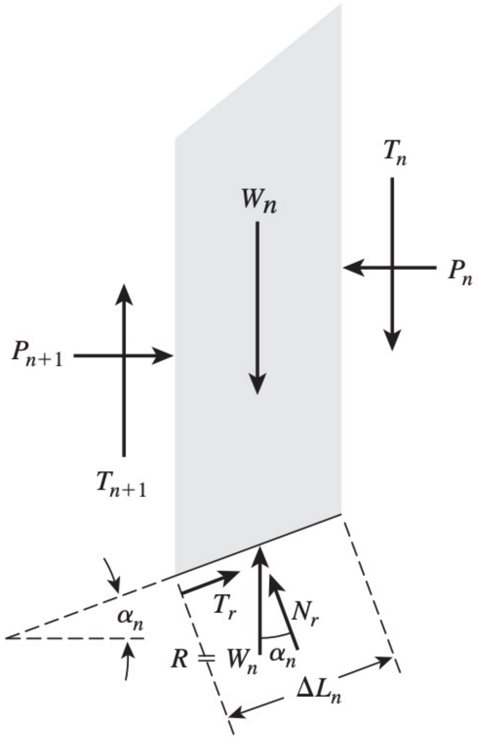
For equilibrium consideration,

$$N_r = W_n \cos \alpha_n$$

The resisting shear force can be expressed as

$$T_r = \tau_d(\Delta L_n) = \frac{\tau_f(\Delta L_n)}{F_s} = \frac{1}{F_s}[c' + \sigma' \tan \phi']\Delta L_n$$

The normal stress, σ', in the above Eq. is equal to: $\dfrac{N_r}{\Delta L_n} = \dfrac{W_n \cos \alpha_n}{\Delta L_n}$

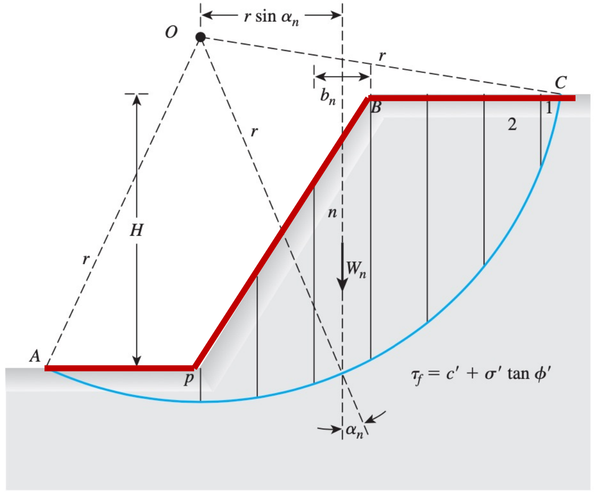


For equilibrium of the trial wedge *ABC*, the moment of the driving force about *O* equals the moment of the resisting force about *O*, or:

$$\sum_{n=1}^{n=p} W_n r \sin \alpha_n = \sum_{n=1}^{n=p} \frac{1}{F_s}\left(c' + \frac{W_n \cos \alpha_n}{\Delta L_n} \tan \phi'\right)(\Delta L_n)(r)$$

or

$$F_s = \frac{\displaystyle\sum_{n=1}^{n=p} (c'\Delta L_n + W_n \cos \alpha_n \tan \phi')}{\displaystyle\sum_{n=1}^{n=p} W_n \sin \alpha_n}$$

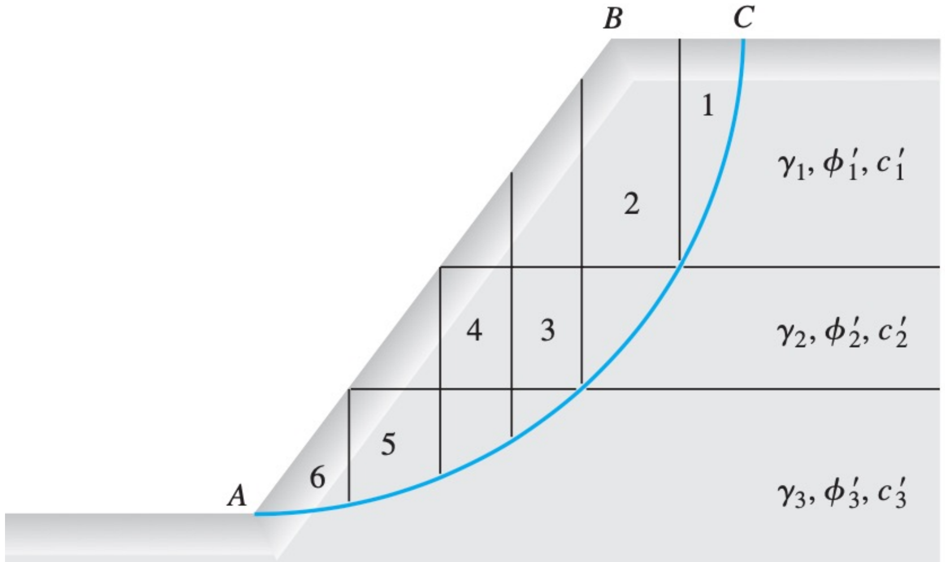


Stability Slopes, Cont'd

✓ Ordinary Method of Slices, Cont'd

Notes:

1) ΔL_n in the above Eq. is approximately equal to $(b_n)/(\cos \alpha_n)$, where b_n = the width of the nth slice.
2) Note that the value of α_n may be either positive or negative. The value of α_n is positive when the slope of the arc is in the same quadrant as the ground slope. To find the minimum factor of safety; that is, the factor of safety for the critical circle—one must make several trials by changing the center of the trial circle. This method generally is referred to as the *ordinary method of slices*.
3) The method of slices can be extended to slopes with layered soil, as shown in the Figure. The general procedure of stability analysis is the same. However, some minor points should be kept in mind. When the previous Eq. is used for the factor of safety calculation, the values of ϕ' and c' will not be the same for all slices.



Stability analysis, by ordinary method of slices, for slope in layered soils

Stability Slopes, Cont'd

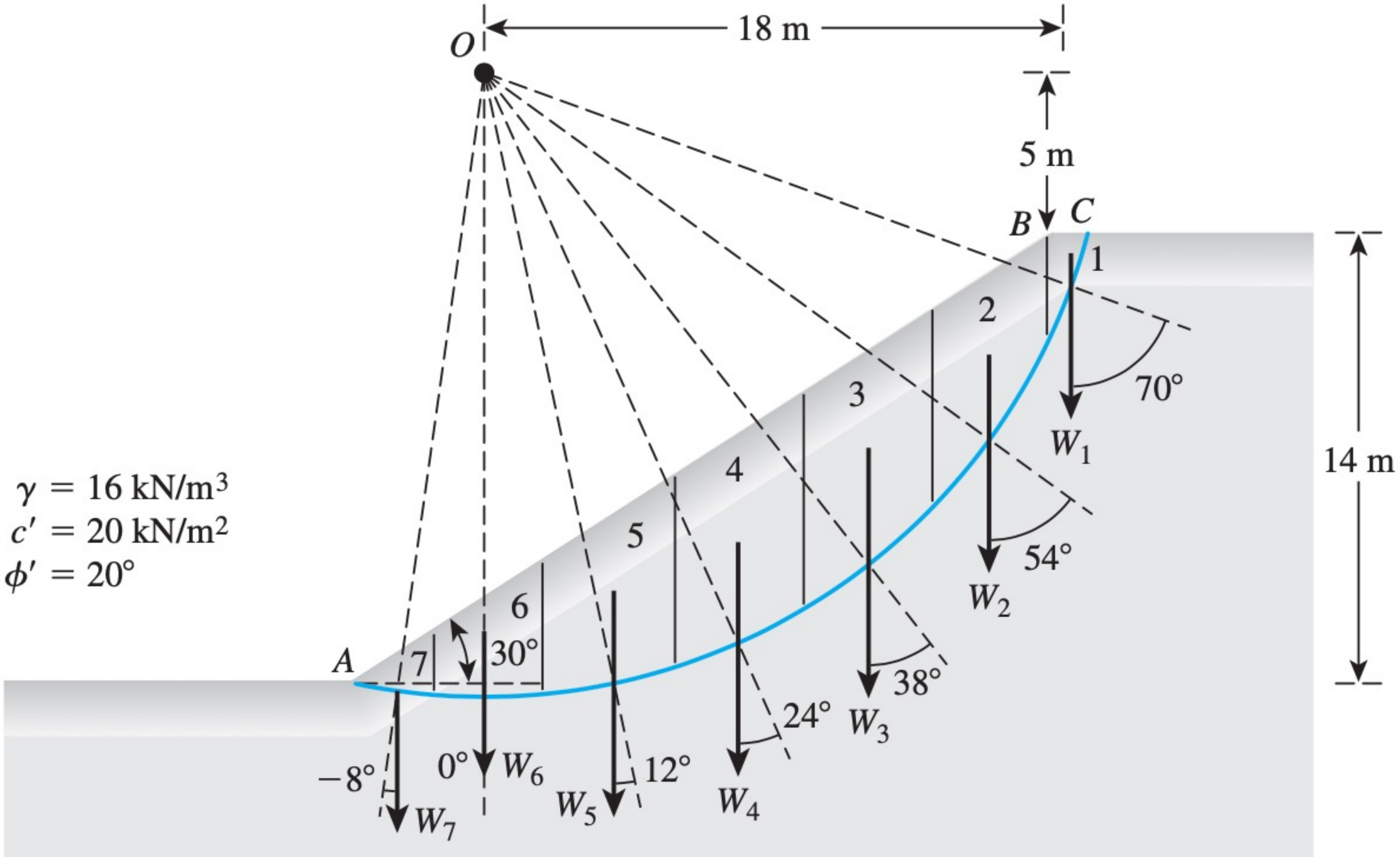
✓ Ordinary Method of Slices, Cont'd

If total shear strength parameters (that is, $\tau_f = c + tan\ \phi$) were used, Eq. of the factor of safety would be as in the form:

$$F_s = \frac{\sum_{n=1}^{n=p} (c \Delta L_n + W_n \cos \alpha_n \tan \phi)}{\sum_{n=1}^{n=p} W_n \sin \alpha_n}$$

Example

For the slope shown in Figure shown below, find the factor of safety against sliding for the trial slip surface *AC*. Use the ordinary method of slices.



Stability analysis of a slope by ordinary method of slices

Solution:

The sliding wedge is divided into seven slices, as shown in the Fig., then following table presents the results.

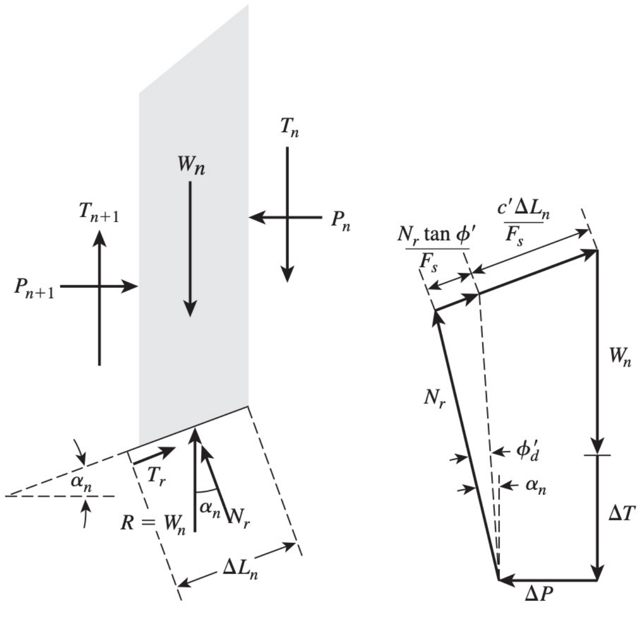
Slice no. (1)	W (kN/m) (2)	α_n (deg) (3)	$\sin \alpha_n$ (4)	$\cos \alpha_n$ (5)	ΔL_n (m) (6)	$W_n \sin \alpha_n$ (kN/m) (7)	$W_n \cos \alpha_n$ (kN/m) (8)
1	22.4	70	0.94	0.342	2.924	21.1	7.66
2	294.4	54	0.81	0.588	6.803	238.5	173.1
3	435.2	38	0.616	0.788	5.076	268.1	342.94
4	435.2	24	0.407	0.914	4.376	177.1	397.8
5	390.4	12	0.208	0.978	4.09	81.2	381.8
6	268.8	0	0	1	4	0	268.8
7	66.58	−8	−0.139	0.990	3.232	−9.25	65.9
					Σ Col. 6 = 30.501 m	Σ Col. 7 = 776.75 kN/m	Σ Col. 8 = 1638 kN/m

$$F_s = \frac{(\Sigma \text{ Col. } 6)(c') + (\Sigma \text{ Col. } 8)\tan \phi'}{\Sigma \text{ Col. } 7}$$

$$= \frac{(30.501)(20) + (1638)(\tan 20)}{776.75} = \mathbf{1.55}$$

$$F_s = \frac{\sum_{n=1}^{n=p} (c'\Delta L_n + W_n \cos \alpha_n \tan \phi')}{\sum_{n=1}^{n=p} W_n \sin \alpha_n}$$

Bishop's Simplified Method of Slices

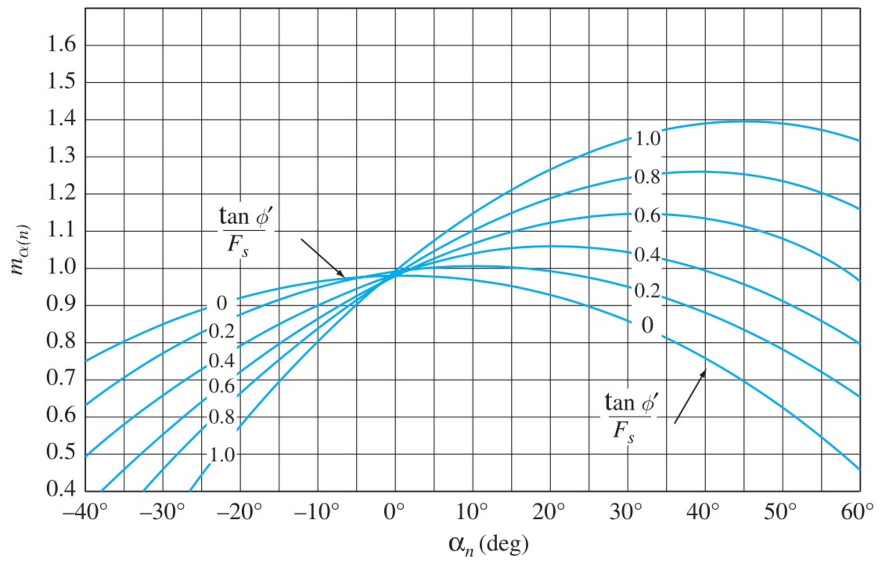


Bishop's simplified method of slices: (a) forces acting on the *n*th slice;
(b) force polygon for equilibrium

$$F_s = \frac{\sum_{n=1}^{n=p} (c'b_n + W_n \tan\phi' + \Delta T \tan\phi') \frac{1}{m_{\alpha(n)}}}{\sum_{n=1}^{n=p} W_n \sin\alpha_n}$$

Where:

$$m_{\alpha(n)} = \cos\alpha_n + \frac{\tan\phi' \sin\alpha_n}{F_s}$$



Variation of $m_{\alpha(n)}$ with α_n and $\tan\phi'/F_s$

For simplicity, if we let $\Delta T = 0$, the Eq. becomes

$$F_s = \frac{\sum_{n=1}^{n=p} (c'b_n + W_n \tan\phi') \frac{1}{m_{\alpha(n)}}}{\sum_{n=1}^{n=p} W_n \sin\alpha_n}$$

The End

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Slope Stability:

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- Factor of safety
- Methods of analysis of slope stability

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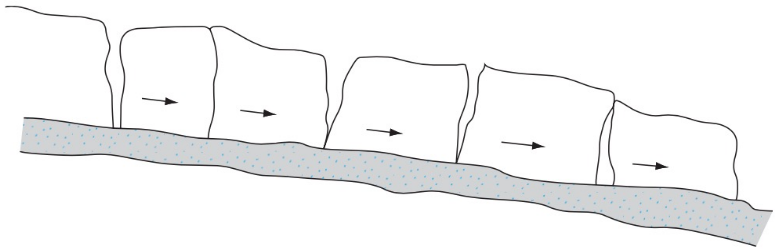
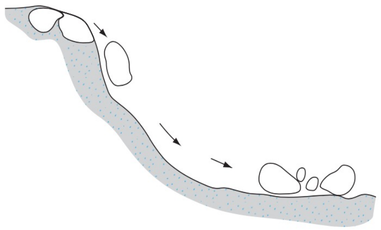
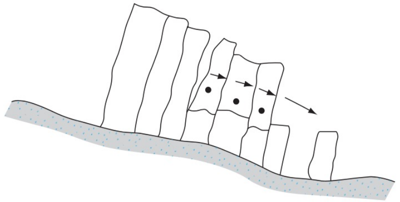


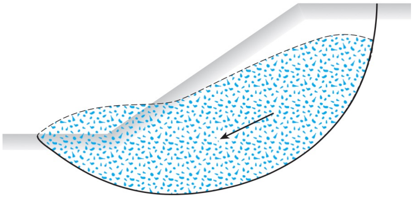
4:Slope failure by lateral "spreading"



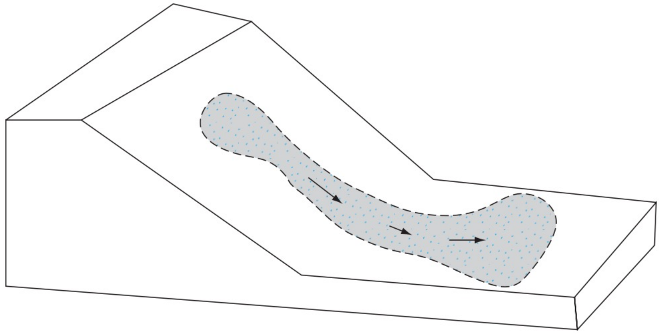
1: "Fall" type of landslide



2: Slope failure by "toppling"



3:Slope failure by "sliding"



5: Slope failure by "flowing"

This lecture primarily relates to the quantitative analysis that fall under the category of *slide (type 3).*

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The main task of slope stability analysis is to determine the factor of safety.
The factor of safety is defined as:

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Where,

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Factor of Safety, Cont'd

- ✓ The factor of safety with respect to cohesion, $F_{c'}$:

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Stability Slopes

1) Infinite slopes:

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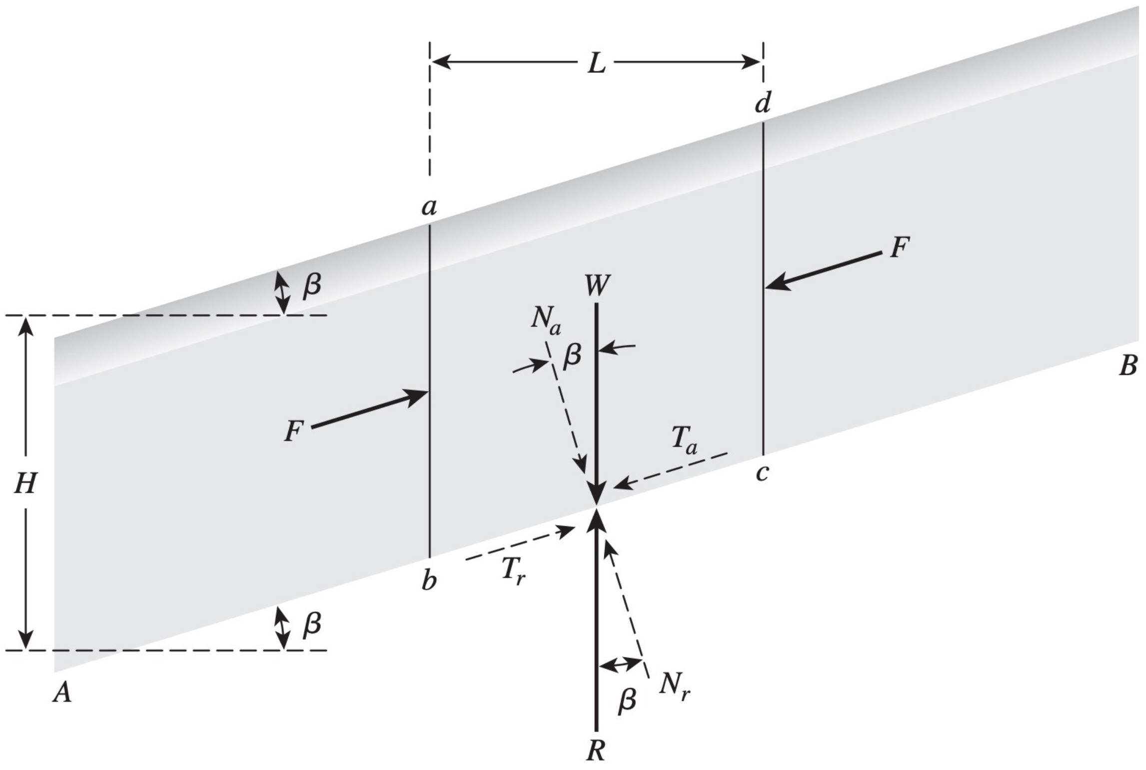
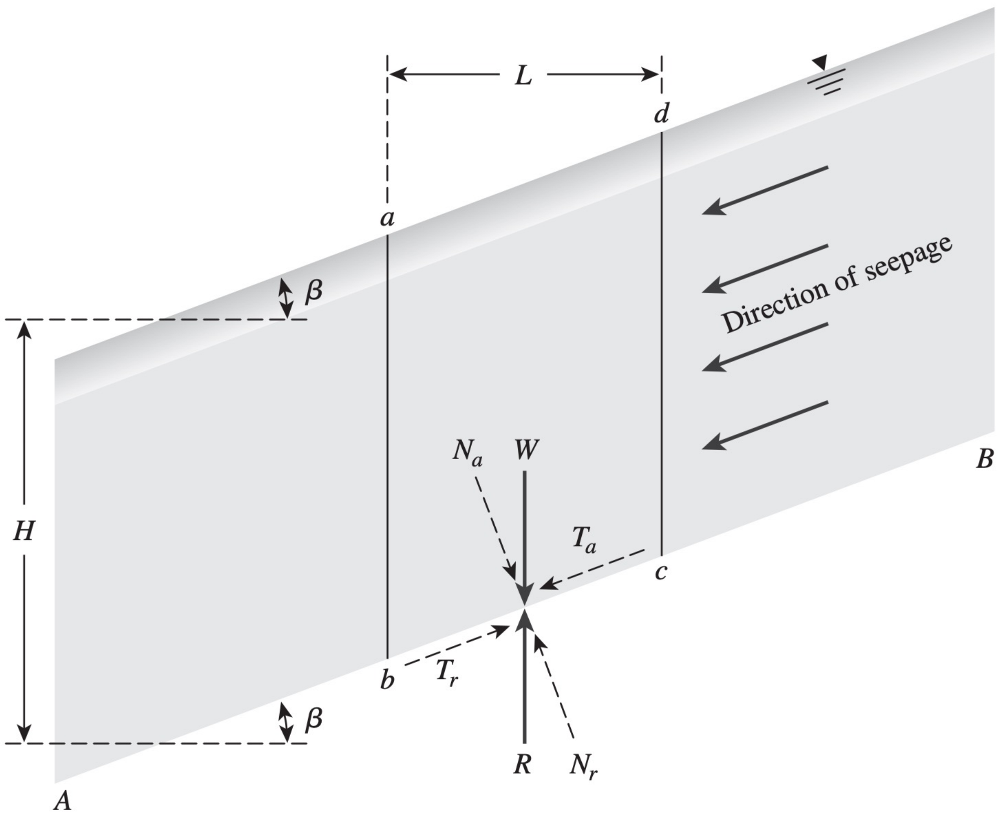


b) Analysis of infinite slope (with seepage)

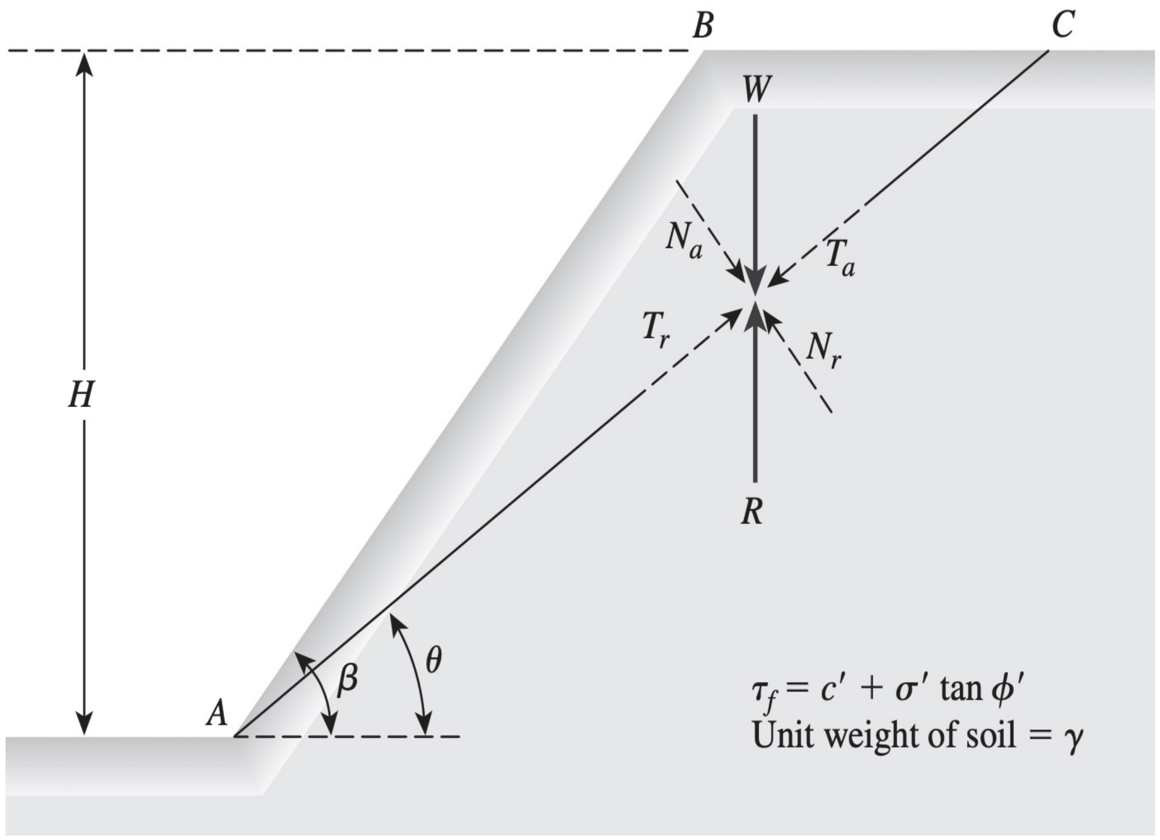


Stability Slopes

2) Finite slopes

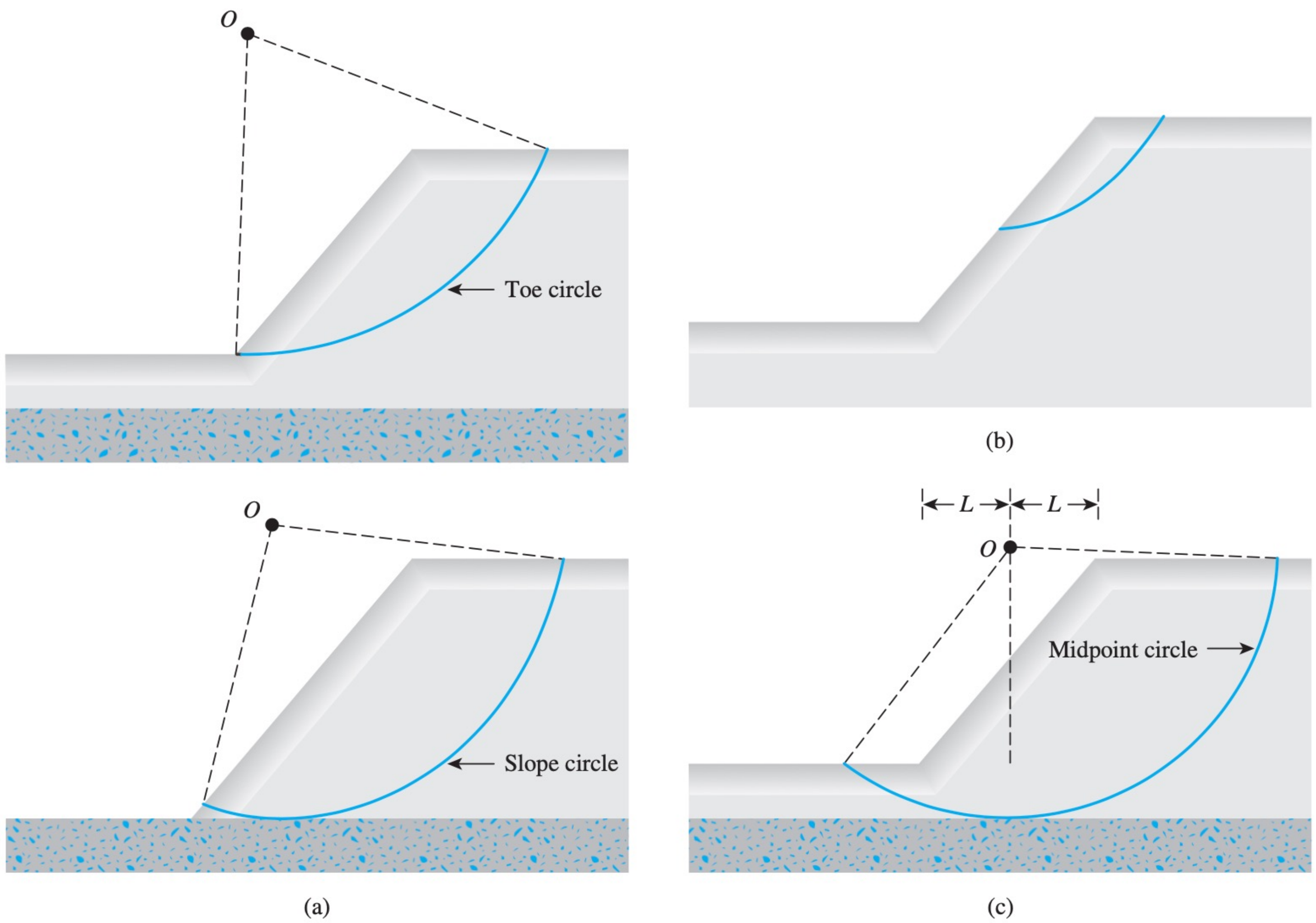
Analysis of Finite Slopes with Plane Failure Surfaces

- Finite slope analysis— Cullman's method



Stability Slopes, Cont'd

- **Analysis of Finite Slopes with Circular Failure Surfaces**



Modes of failure of finite slope: (a) slope failure; (b) shallow slope failure; (c) base failure

Stability Slopes, Cont'd

- **Analysis of Finite Slopes with Circular Failure Surfaces**

Types of Stability Analysis Procedures

- ***Mass procedure***: In this case, the mass of the soil above the surface of sliding is taken as a unit. This procedure is useful when the soil that forms the slope is assumed to be homogeneous, although this is not the case in most natural slopes.

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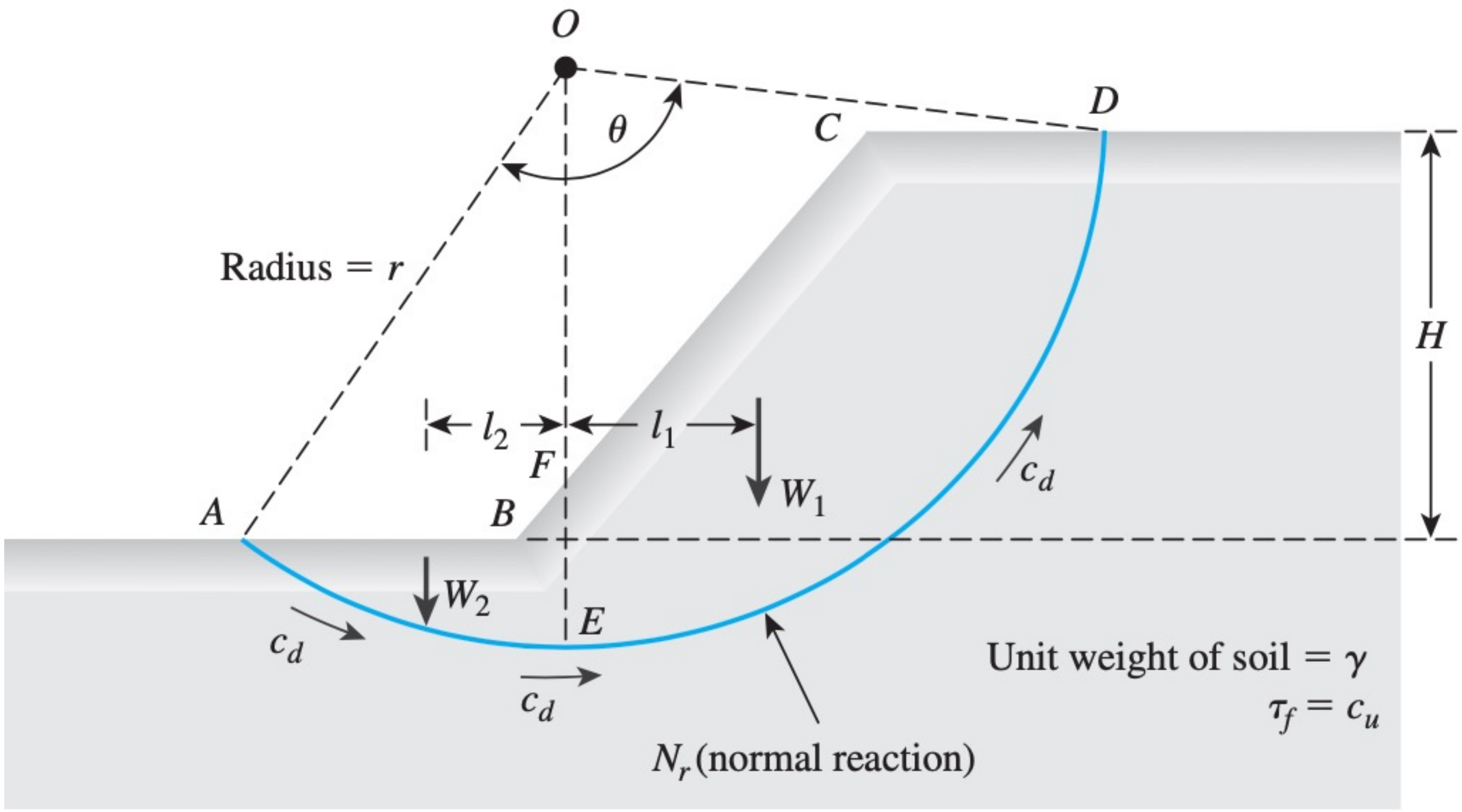
Stability Slopes, Cont'd

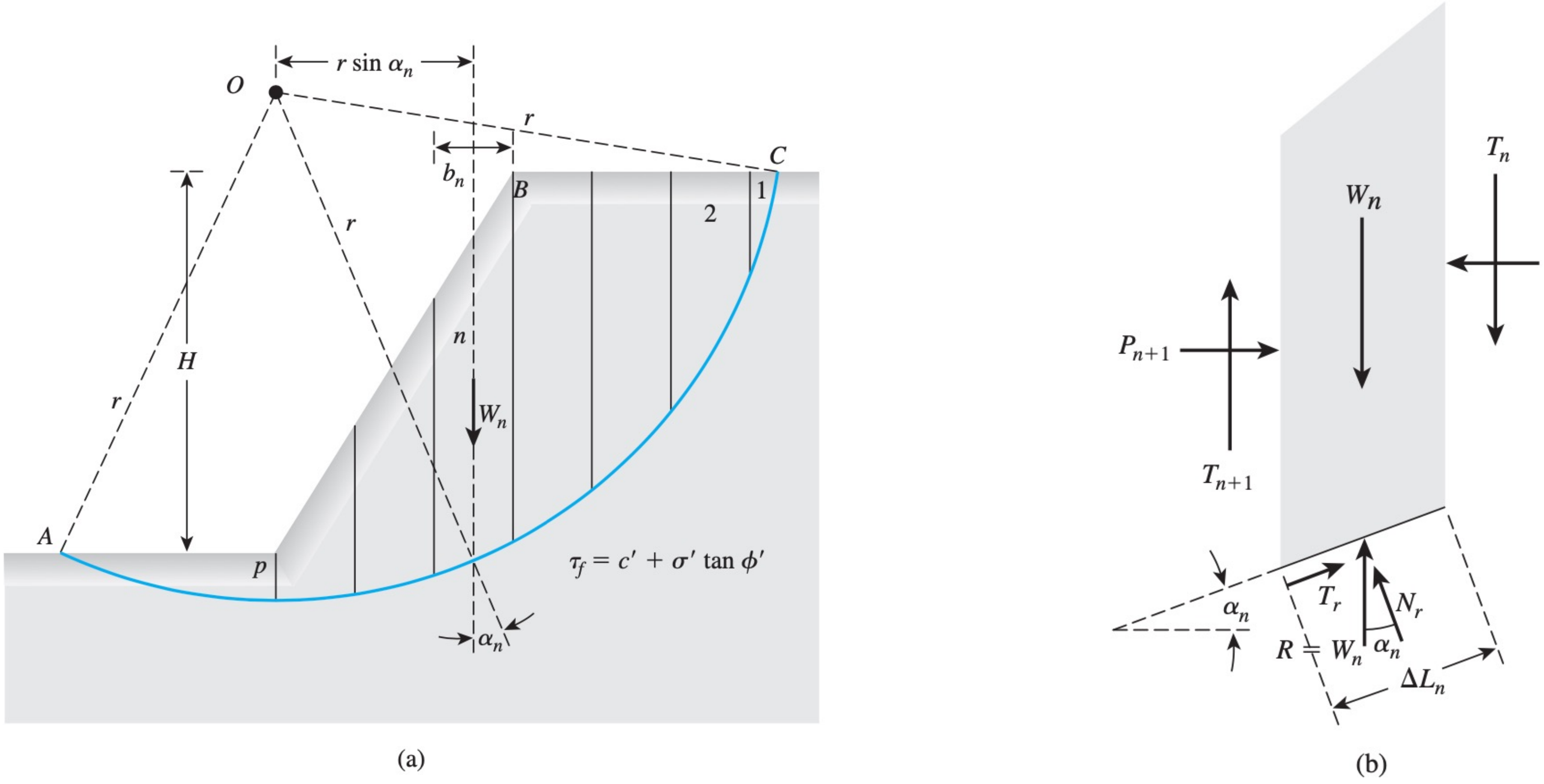
- **Analysis of Finite Slopes with Circular Failure Surfaces, Mass procedure**



Stability analysis of slope in homogeneous saturated clay soil

Stability Slopes, Cont'd

- **Analysis of Finite Slopes with Circular Failure Surfaces, Slices procedure**
  - ✓ **Ordinary Method of Slices**



Stability analysis by ordinary method of slices: (a) trial failure surface; (b) forces acting on *n*th slice

Stability Slopes, Cont'd

✓ Ordinary Method of Slices, Cont'd

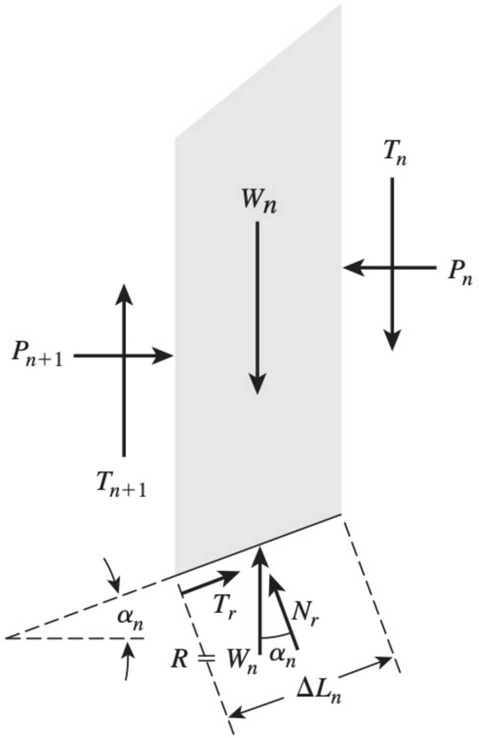
For equilibrium consideration,

$$N_r = W_n \cos \alpha_n$$

The resisting shear force can be expressed as

$$T_r = \tau_d(\Delta L_n) = \frac{\tau_f(\Delta L_n)}{F_s} = \frac{1}{F_s}[c' + \sigma' \tan \phi']\Delta L_n$$

The normal stress, σ', in the above Eq. is equal to: $\dfrac{N_r}{\Delta L_n} = \dfrac{W_n \cos \alpha_n}{\Delta L_n}$

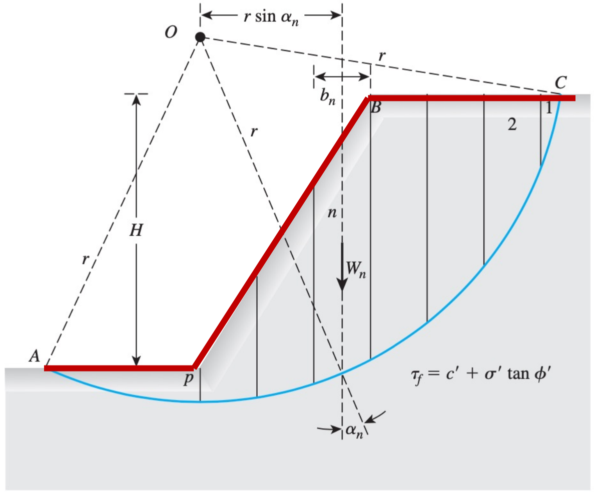


For equilibrium of the trial wedge *ABC*, the moment of the driving force about *O* equals the moment of the resisting force about *O*, or:

$$\sum_{n=1}^{n=p} W_n r \sin \alpha_n = \sum_{n=1}^{n=p} \frac{1}{F_s}\left(c' + \frac{W_n \cos \alpha_n}{\Delta L_n} \tan \phi'\right)(\Delta L_n)(r)$$

or

$$F_s = \frac{\displaystyle\sum_{n=1}^{n=p} (c'\Delta L_n + W_n \cos \alpha_n \tan \phi')}{\displaystyle\sum_{n=1}^{n=p} W_n \sin \alpha_n}$$

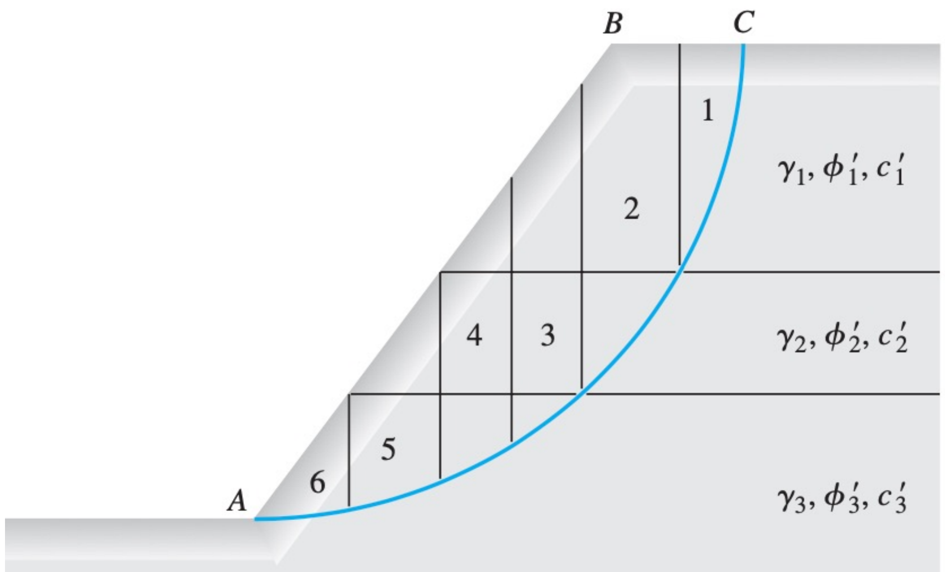


Stability Slopes, Cont’d

✓ Ordinary Method of Slices, Cont’d

Notes:

1) ΔL_n in the above Eq. is approximately equal to $(b_n)/(\cos\alpha_n)$, where b_n = the width of the nth slice.
2) Note that the value of α_n may be either positive or negative. The value of α_n is positive when the slope of the arc is in the same quadrant as the ground slope. To find the minimum factor of safety; that is, the factor of safety for the critical circle—one must make several trials by changing the center of the trial circle. This method generally is referred to as the *ordinary method of slices*.
3) The method of slices can be extended to slopes with layered soil, as shown in the Figure. The general procedure of stability analysis is the same. However, some minor points should be kept in mind. When the previous Eq. is used for the factor of safety calculation, the values of ϕ’ and c’ will not be the same for all slices.



Stability analysis, by ordinary method of slices, for slope in layered soils

Stability Slopes, Cont'd

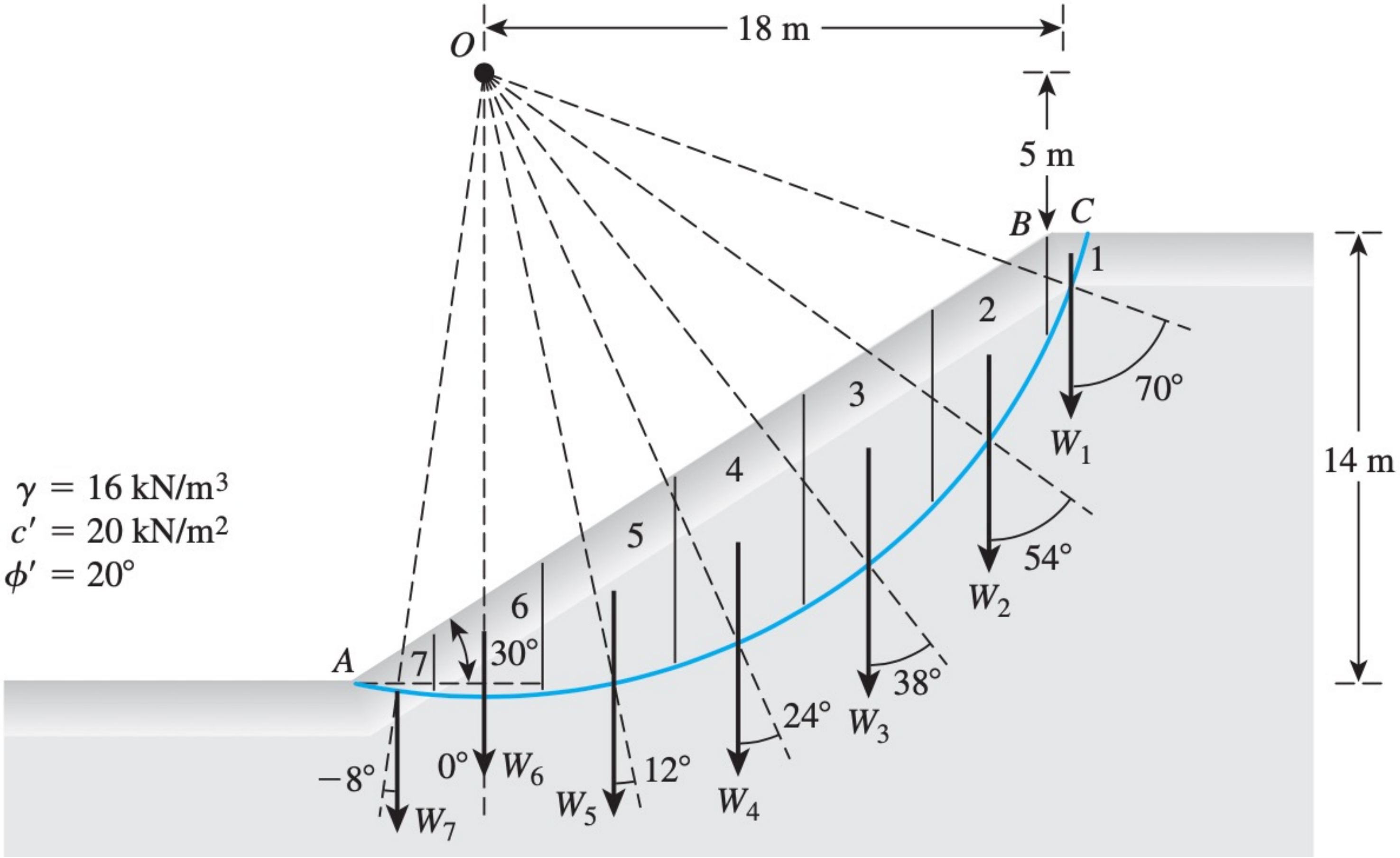
✓ Ordinary Method of Slices, Cont'd

If total shear strength parameters (that is, $\tau_f = c + tan\ \phi$) were used, Eq. of the factor of safety would be as in the form:

$$F_s = \frac{\sum_{n=1}^{n=p} (c\Delta L_n + W_n \cos \alpha_n \tan \phi)}{\sum_{n=1}^{n=p} W_n \sin \alpha_n}$$

Example

For the slope shown in Figure shown below, find the factor of safety against sliding for the trial slip surface *AC*. Use the ordinary method of slices.



Stability analysis of a slope by ordinary method of slices

Solution:

The sliding wedge is divided into seven slices, as shown in the Fig., then following table presents the results.

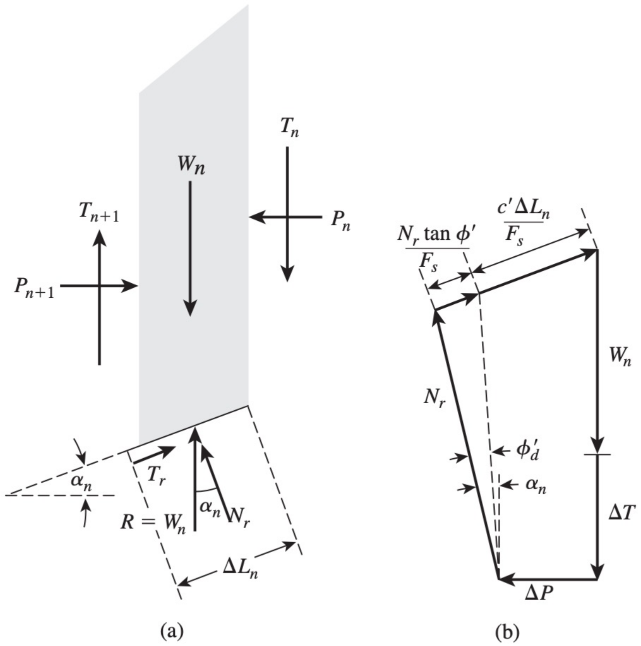
Slice no. (1)	W (kN/m) (2)	α_n (deg) (3)	$\sin\alpha_n$ (4)	$\cos\alpha_n$ (5)	ΔL_n (m) (6)	$W_n \sin\alpha_n$ (kN/m) (7)	$W_n \cos\alpha_n$ (kN/m) (8)
1	22.4	70	0.94	0.342	2.924	21.1	7.66
2	294.4	54	0.81	0.588	6.803	238.5	173.1
3	435.2	38	0.616	0.788	5.076	268.1	342.94
4	435.2	24	0.407	0.914	4.376	177.1	397.8
5	390.4	12	0.208	0.978	4.09	81.2	381.8
6	268.8	0	0	1	4	0	268.8
7	66.58	−8	−0.139	0.990	3.232	−9.25	65.9
					Σ Col. 6 = 30.501 m	Σ Col. 7 = 776.75 kN/m	Σ Col. 8 = 1638 kN/m

$$F_s = \frac{(\Sigma \text{ Col. } 6)(c') + (\Sigma \text{ Col. } 8)\tan\phi'}{\Sigma \text{ Col. } 7}$$

$$= \frac{(30.501)(20) + (1638)(\tan 20)}{776.75} = \mathbf{1.55}$$

$$F_s = \frac{\sum_{n=1}^{n=p} (c'\Delta L_n + W_n \cos\alpha_n \tan\phi')}{\sum_{n=1}^{n=p} W_n \sin\alpha_n}$$

Bishop's Simplified Method of Slices

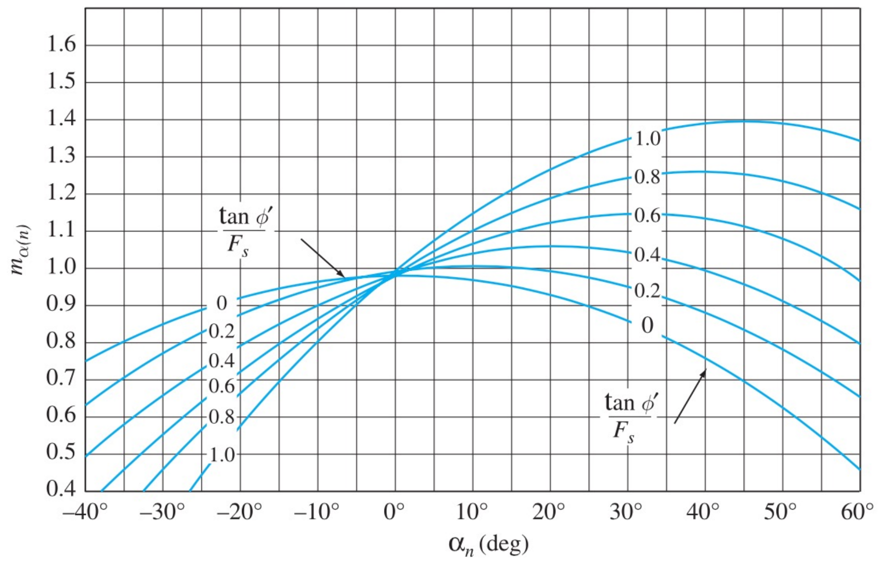


Bishop's simplified method of slices: (a) forces acting on the *n*th slice; (b) force polygon for equilibrium

$$F_s = \frac{\sum_{n=1}^{n=p} (c'b_n + W_n \tan \phi' + \Delta T \tan \phi') \frac{1}{m_{\alpha(n)}}}{\sum_{n=1}^{n=p} W_n \sin \alpha_n}$$

Where:

$$m_{\alpha(n)} = \cos \alpha_n + \frac{\tan \phi' \sin \alpha_n}{F_s}$$



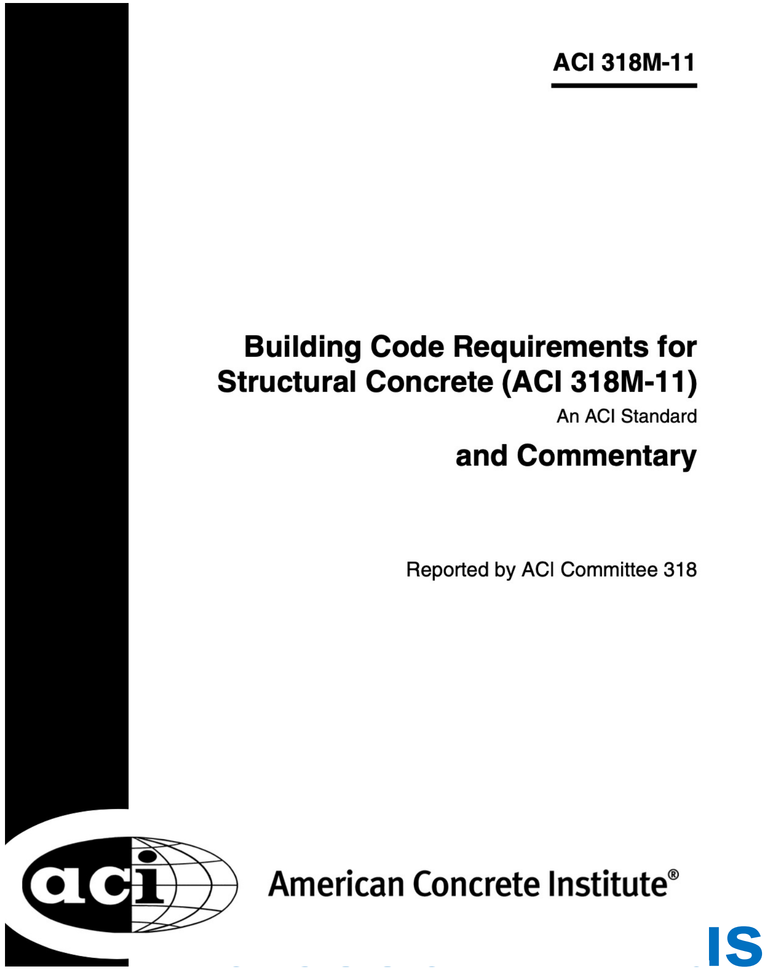
Variation of $m_{\alpha(n)}$ with α_n and $\tan \phi'/F_s$

For simplicity, if we let $\Delta T = 0$, the Eq. becomes

$$F_s = \frac{\sum_{n=1}^{n=p} (c'b_n + W_n \tan \phi') \frac{1}{m_{\alpha(n)}}}{\sum_{n=1}^{n=p} W_n \sin \alpha_n}$$

The End

Foundation Engineering (2):
Structural Design of Reinforced Concrete of Shallow Foundations



ACI 318M-11

Building Code Requirements for Structural Concrete (ACI 318M-11)

An ACI Standard

and Commentary

Reported by ACI Committee 318

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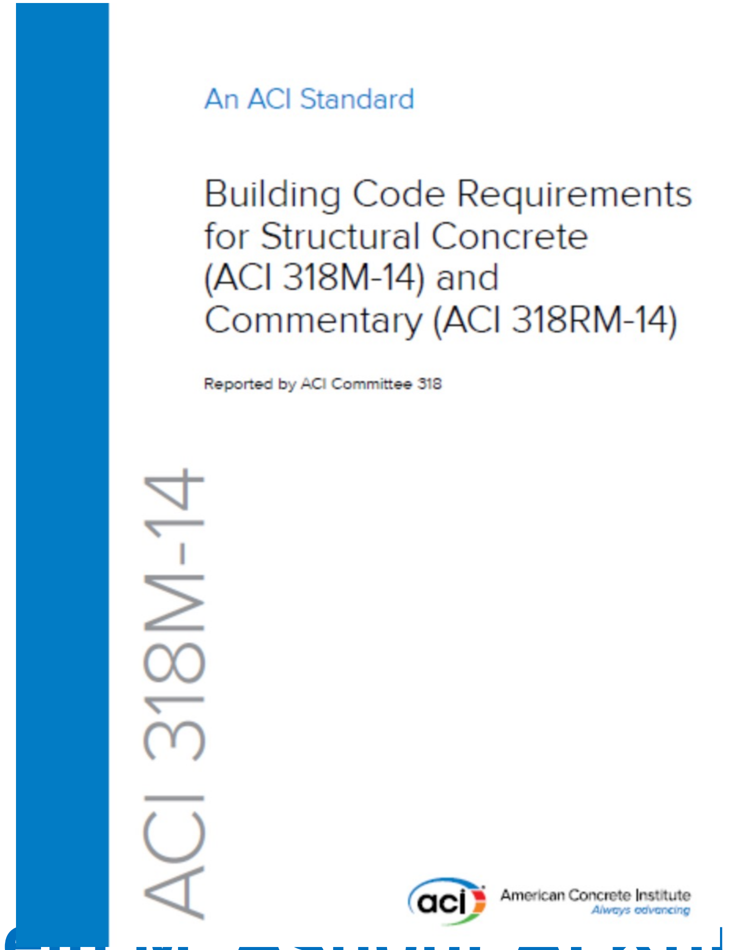

An ACI Standard

Building Code Requirements for Structural Concrete (ACI 318M-14) and Commentary (ACI 318RM-14)

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ACI 318M-14

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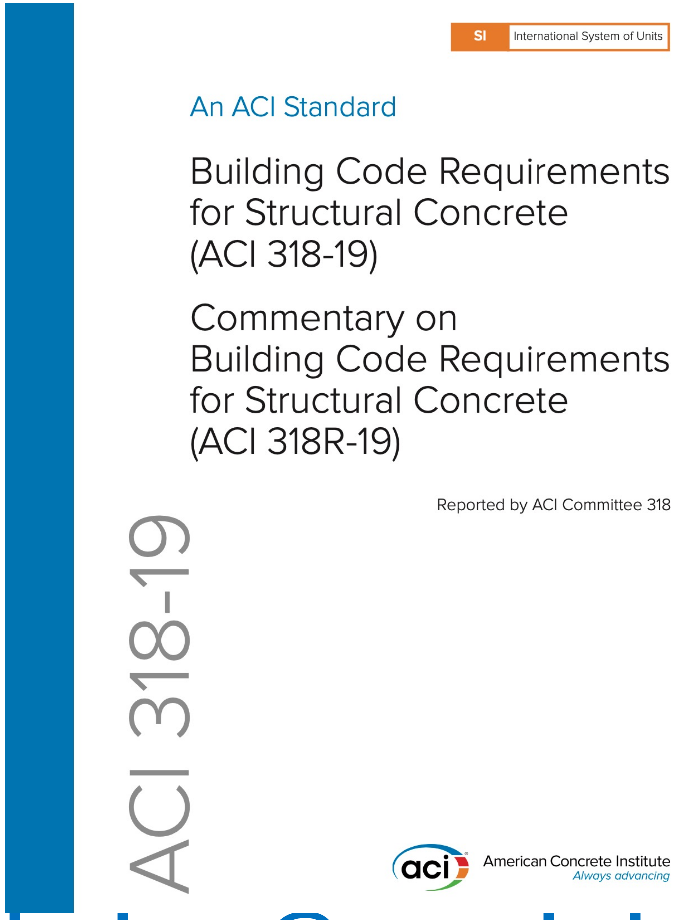

SI International System of Units

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Building Code Requirements for Structural Concrete (ACI 318-19)

Commentary on Building Code Requirements for Structural Concrete (ACI 318R-19)

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ACI 318-19

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Load Factors

According to ACI Code 318-11 Section 9.2, depending on the type, the ultimate load-carrying capacity of a structural member should be one of the following:

$$U = 1.4D$$

$$U = 1.2D + 1.6L + 0.5(L_r \text{ or } S \text{ or } R)$$

$$U = 1.2D + 1.6(L_r \text{ or } S \text{ or } R) + (1.0L \text{ or } 0.5W)$$

$$U = 1.2D + 1.0W + 1.0L + 0.5(L_r \text{ or } S \text{ or } R)$$

$$U = 1.2D + 1.0E + 1.0L + 0.2S$$

$$U = 0.9D + 1.0W$$

$$U = 0.9D + 1.0E$$

9.2 — Required strength

9.2.1 — Required strength ***U*** shall be at least equal to the effects of factored loads in Eq. (9-1) through (9-7). The effect of one or more loads not acting simultaneously shall be investigated.

where

U = ultimate load-carrying capacity of a member
D = dead loads
E = effects of earthquake
L = live loads
L_r = roof live loads
R = rain load
S = snow load
W = wind load

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Strength Reduction Factor

The design strength provided by a structural member is equal to the nominal strength times a strength reduction factor, ϕ, or

$$\text{Design strength} = \phi\ (\text{nominal strength})$$

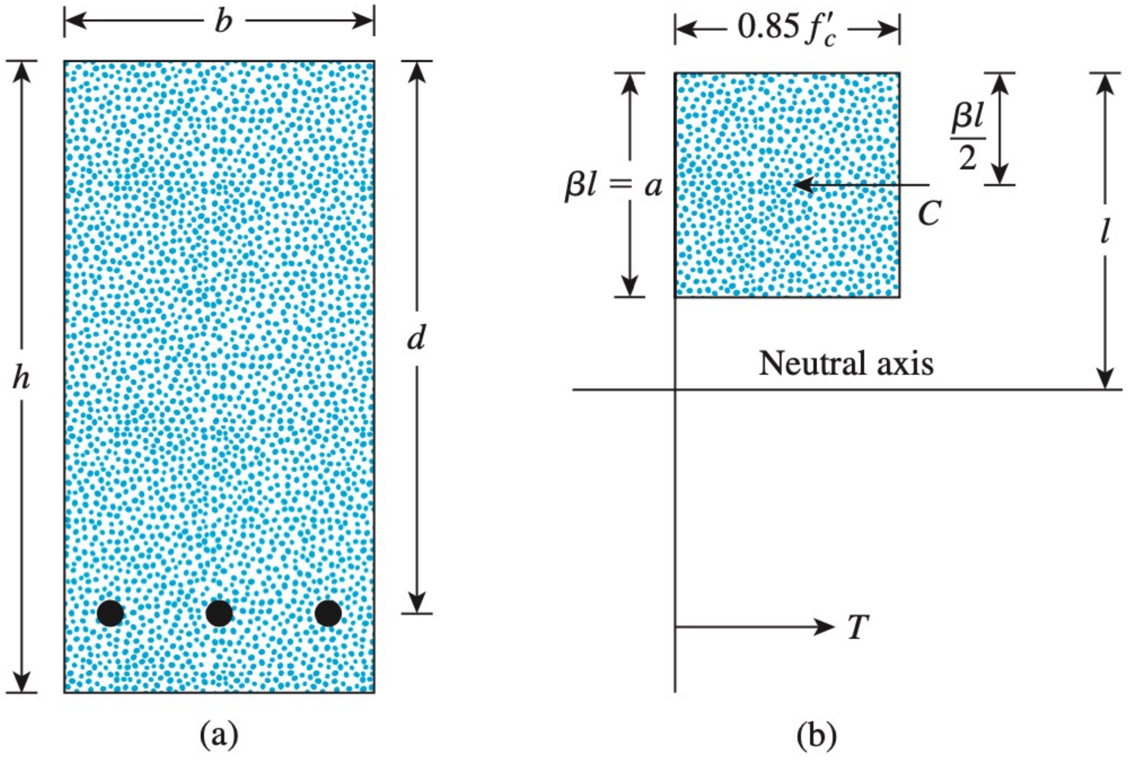
The reduction factor, ϕ, takes into account the inaccuracies in the design assumptions, changes in property or strength of the construction materials, and so on. Following are some of the recommended values of ϕ (ACI Code Section 9.3):

Condition	Value of ϕ
a. Axial tension; flexure with or without axial tension	0.9
b. Shear or torsion	0.75
c. Axial compression with spiral reinforcement	0.75
d. Axial compression without spiral reinforcement	0.65
e. Bearing on concrete	0.65
f. Flexure in plain concrete	0.65

Review of Design Concepts for a Rectangular Section in Bending

A section of a concrete beam having a width ***b*** and a depth ***h***. The assumed stress distribution across the section at ultimate load is shown in the Figure. The following notations have been used in this figure:

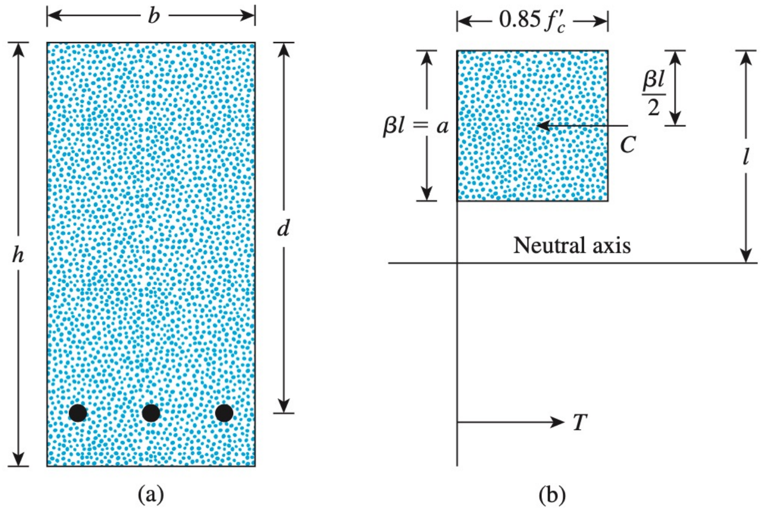
f'_c = compressive strength of concrete at 28 days
A_s = area of steel tension reinforcement
f_y = yield stress of reinforcement in tension
d = effective depth
l = location of the neutral axis measured from the top of the compression face
$a = \beta l$



β = 0.85 for f'_c of 28 MN/m^2 (4000 lb/in.^{2}) of less and decreases at the rate of 0.05 for every 7 MN/m^2 (1000 lb/in.^{2}) increase of f'_c. However, it cannot be less than 0.65 in any case (ACI Code Section 10.2.7).

By applying equilibrium principles, so:

$$\Sigma \text{ compressive force, } C = \Sigma \text{ tensile force, } T$$



Thus,

$$0.85 f'_c ab = A_s f_y$$

or

$$a = \frac{A_s f_y}{0.85 f'_c b}$$

Also, for the beam section, the nominal ultimate moment can be given as

$$M_n = A_s f_y \left(d - \frac{a}{2} \right)$$

where M_n = theoretical ultimate moment.

The design ultimate moment, M_u, can be given as

$$M_n = A_s f_y \left(d - \frac{a}{2} \right)$$

$M_u = \emptyset M_n$

Where:

M_u is the ultimate design moment

$\emptyset$ is reduction factor for flexural stress

Substituting a in the M_n equation and applying reduction factor, so the design moment M_u will be as :

$$M_u = \phi A_s f_y \left[d - \left(\frac{1}{2}\right) \frac{A_s f_y}{0.85 f'_c b} \right] = \phi A_s f_y \left(d - \frac{0.59\, A_s f_y}{f'_c b} \right)$$

The steel percentage is defined by the equation: $\boldsymbol{s = \frac{A_s}{bd}}$

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In a balanced beam, failure would occur by sudden simultaneous yielding of tensile steel and crushing of concrete. The balanced percentage of steel (for Young's modulus (Modulus of Elasticity)) of steel, Es = 200 MN/m^2) can be given as:

$$s_b = \frac{0.85 f_c'}{f_y}(\beta)\left(\frac{600}{600 + f_y}\right)$$

where f_c' and f_y are in MN/m^2.

In conventional English units (with $E_s = 29 \times 10^6$ lb/in.^{2})

$$s_b = \frac{0.85 f_c'}{f_y}(\beta)\left(\frac{87{,}000}{87{,}000 + f_y}\right)$$

where f_c' and f_y and in lb/in.^{2}

To avoid sudden failure without warning, ACI Code Section 10.3.5 recommends that the maximum steel percentage ($s_{\max}$) should be limited to a net tensile strain (ϵ_t) of 0.004. For all practical purposes,

$$S_{max} \approx 0.75\, s_b$$

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Shear Concept

The nominal or theoretical shear strength of a section, V_n, can be given as

$$V_n = V_c + V_s$$

where V_c = nominal shear strength of concrete
V_s = nominal shear strength of reinforcement

The permissible shear strength, V_u, can be given by

$$V_u = \phi V_n = \phi(V_c + V_s)$$

The values of V_c can be given by the following equations (ACI Code Sections 11.2 and

$$V_c = 0.17\lambda\sqrt{f_c'}\,bd \quad \text{(for member subjected to shear and flexure)}$$

and

$$V_c = 0.33\lambda\sqrt{f_c'}\,bd \quad \text{(for member subjected to diagonal tension)}$$

where f_c' is in MN/m^2, V_c is in MN, b and d are in m, and $\lambda = 1$ for normal weight concrete.

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In conventional English units:

$$V_c = 2\lambda \sqrt{f_c'}\, bd$$

$$V_c = 4\lambda \sqrt{f_c'}\, bd$$

where V_c is in lb, f_c' is in lb/in.^{2}, and b and d are in inches.

Note that

$$v_c = \frac{V_c}{bd}$$

where v_c is the shear stress.

Now, combining Eqs. one obtains

$$\text{Permissible shear stress} = v_u = \frac{V_u}{bd} = 0.17\,\phi\lambda\sqrt{f_c'}$$

Similarly,

$$v_u = 0.33\lambda\phi\sqrt{f_c'}$$

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Reinforcing Bars

The nominal sizes of reinforcing bars commonly used in the United States are given in the Table

Bar No.	Diameter (mm)	Diameter (in.)	Area of cross section (mm^2)	Area of cross section ($in.^2$)
3	9.52	0.375	71	0.11
4	12.70	0.500	129	0.20
5	15.88	0.625	200	0.31
6	19.05	0.750	284	0.44
7	22.22	0.875	387	0.60
8	25.40	1.000	510	0.79
9	28.65	1.128	645	1.00
10	32.26	1.270	819	1.27
11	35.81	1.410	1006	1.56
14	43.00	1.693	1452	2.25
18	57.33	2.257	2580	4.00

Development Length

The development length, L_d, is the length of embedment required to develop the yield stress in the tension reinforcement for a section in flexure. ACI Code Section 12.2 lists the basic development lengths for tension reinforcement.

CHAPTER 12 — DEVELOPMENT AND SPLICES OF REINFORCEMENT

CODE

12.1 — Development of reinforcement — General

12.1.1 — Calculated tension or compression in reinforcement at each section of structural concrete members shall be developed on each side of that section by embedment length, hook, headed deformed bar or mechanical device, or a combination thereof. Hooks and heads shall not be used to develop bars in compression.

12.1.2 — The values of $\sqrt{f'_c}$ used in this chapter shall not exceed 8.3 MPa.

12.1.3 — In addition to requirements in this chapter that affect detailing of reinforcement, structural integrity requirements of 7.13 shall be satisfied.

COMMENTARY

R12.1 — Development of reinforcement — General

The development length concept for anchorage of reinforcement was first introduced in the 1971 Code, to replace the dual requirements for flexural bond and anchorage bond contained in earlier editions. It is no longer necessary to consider the flexural bond concept, which placed emphasis on the computation of nominal peak bond stresses. Consideration of an average bond resistance over a full development length of the reinforcement is more meaningful, partially because all bond tests consider an average bond resistance over a length of embedment of the reinforcement, and partially because uncalculated extreme variations in local bond stresses exist near flexural cracks.[12.1]

The development length concept is based on the attainable average bond stress over the length of embedment of the reinforcement. Development lengths are required because of the tendency of highly stressed bars to split relatively thin sections of restraining concrete. A single bar embedded in a mass of concrete should not require as great a development length; although a row of bars, even in mass concrete, can create a weakened plane with longitudinal splitting along the plane of the bars.

In application, the development length concept requires minimum lengths or extensions of reinforcement beyond all points of peak stress in the reinforcement. Such peak stresses generally occur at the points in 12.10.2.

Structural integrity requirements of 7.13 may control detailing of reinforcement at splices and terminations.

The strength reduction factor ϕ is not used in the development length and lap splice equations. An allowance for strength reduction is already included in the expressions for determining development and splice lengths. Units of measurement are given in the Notation to assist the user and are not intended to preclude the use of other correctly applied units for the same symbol, such as m or kN.

From a point of peak stress in reinforcement, some length of reinforcement or anchorage is necessary to develop the stress. This development length or anchorage is necessary on both sides of such peak stress points. Often the reinforcement continues for a considerable distance on one side of a critical stress point so that calculations need involve only the other side, for example, the negative moment reinforcement continuing through a support to the middle of the next span.

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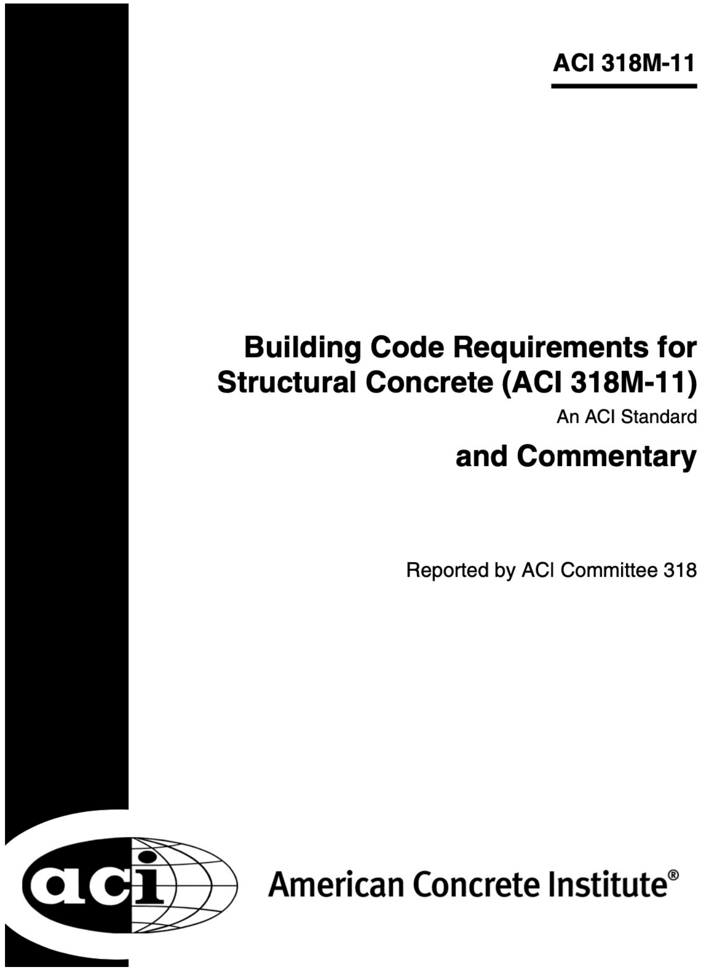
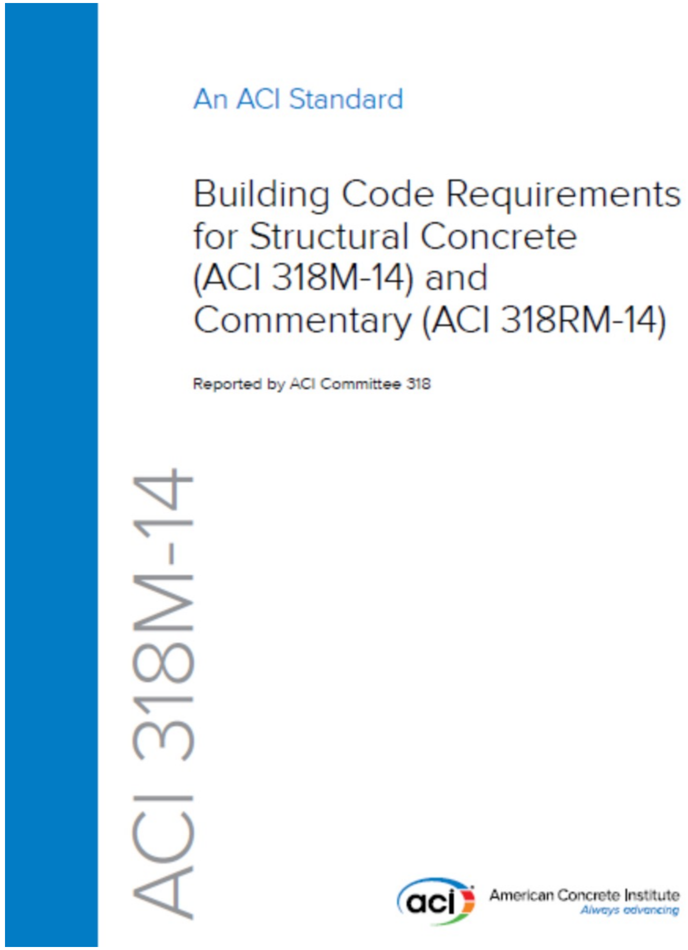
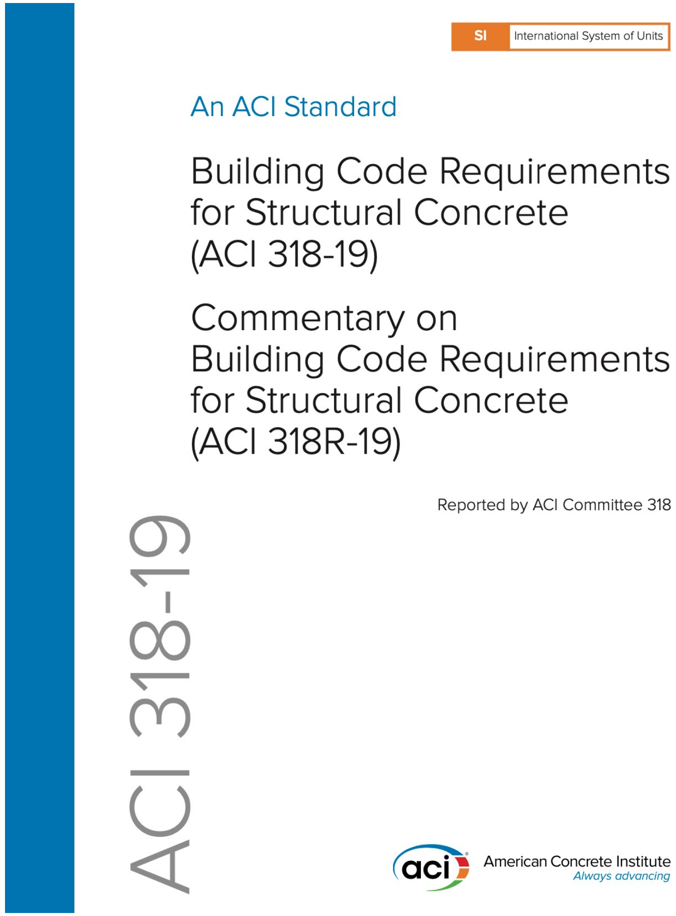
Any Question?

Go a head for the next week

Foundation Engineering (2):
Structural Design of Reinforced Concrete of Shallow Foundations (examples), part -2-







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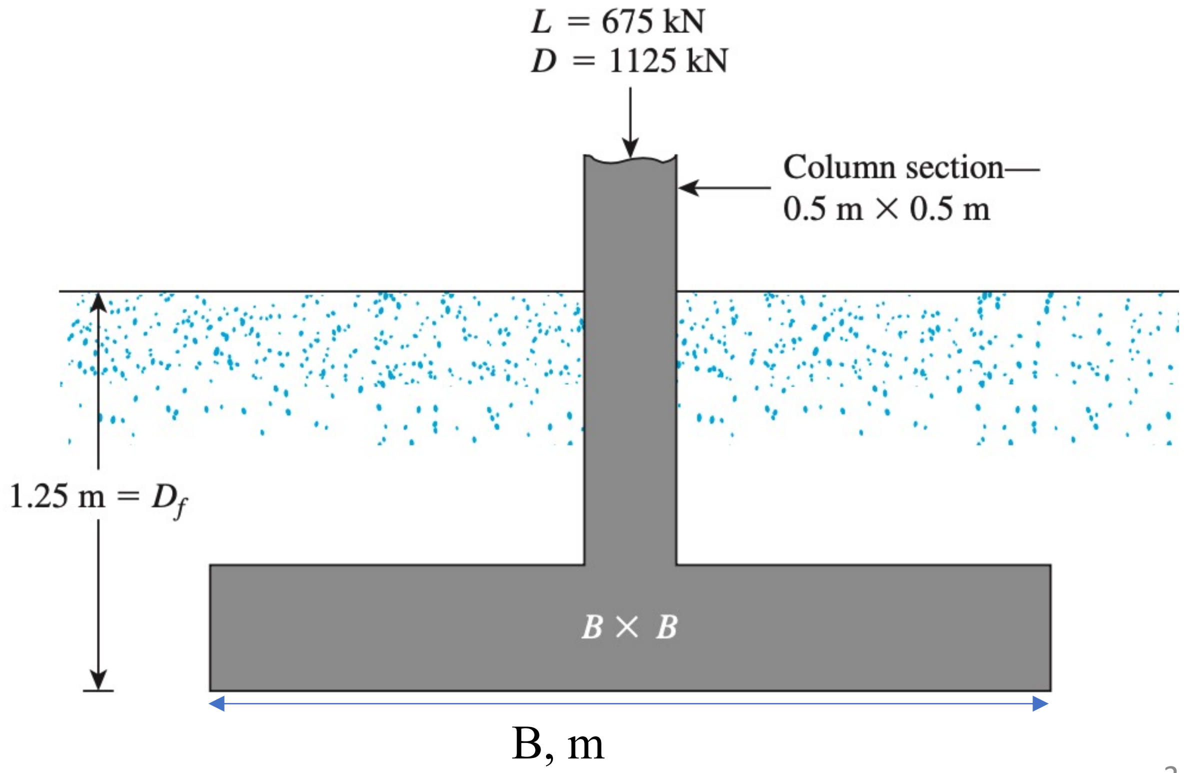
Design Example of a Square Foundation for a Column

As shown in the Figure a square column foundation with the following conditions:
Live load = L = 675 kN
Dead load = D = 1125 kN
Allowable gross soil-bearing capacity = q_{all} = 145 kN/m^2 Column size = 0.5 m X 0.5 m
$f'c$ = 20.68 MN/m^2 fy = 413.7 MN/m^2
Let it be required to design the column foundation.



General Considerations

Let the average unit weight of concrete and soil above the base of the foundation be 21.97 kN/m^3. So, the net allowable soil-bearing capacity

$$q_{\text{all(net)}} = 145 - (D_f)(21.97) = 145 - (1.25)(21.97) = 117.54 \text{ kN/m}^2$$

Hence, the required foundation area is

$$A = B^2 = \frac{D + L}{q_{\text{all(net)}}} = \frac{675 + 1125}{117.54} = 15.31 \text{ m}^2$$

Use a foundation with dimensions (B) of 4 m × 4 m.

The factored load for the foundation is

$$U = 1.2D + 1.6L = (1.2)(1125) + (1.6)(675) = 2430 \text{ kN}$$

Hence, the factored soil pressure is

$$q_s = \frac{U}{B^2} = \frac{2430}{16} = 151.88\,\text{kN/m}^2$$

Assume the thickness of the foundation to be equal to 0.75 m. With a clear cover of 76 mm over the steel bars and an assumed bar diameter of 25 mm, we have

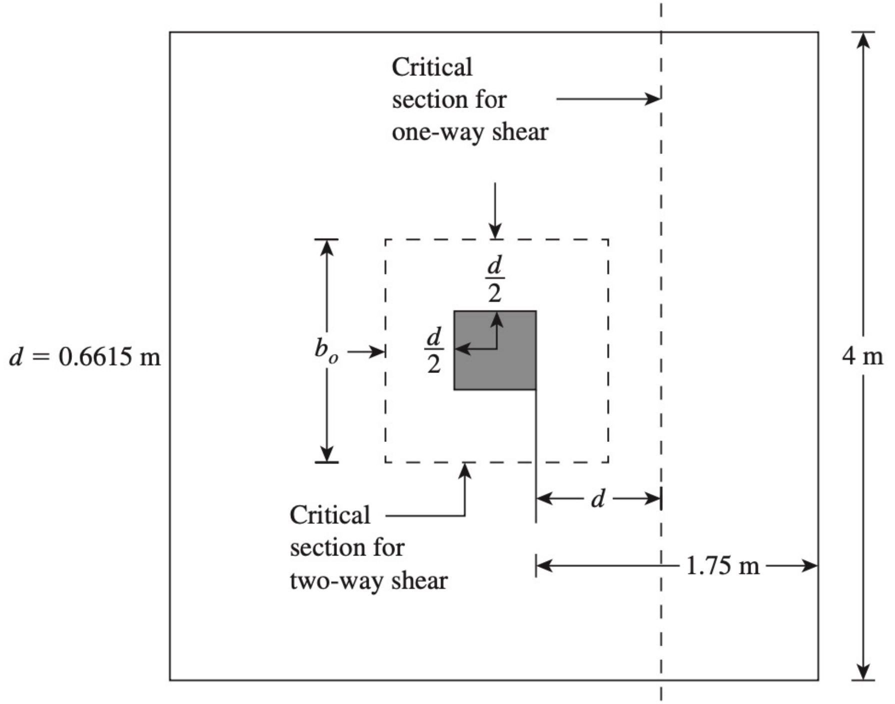
$$d = 0.75 - 0.076 - \frac{0.025}{2} = 0.6615\,\text{m}$$

Check for Shear

As we have seen previously . V_u should be equal to or less than ϕV_c. For one-way shear [with $\lambda = 1$ in Eq.

$$V_u \leq \phi(0.17)\sqrt{f_c'}\,bd$$

The critical section for one-way shear is located at a distance d from the edge of the column (ACI Code Section 11.1.3) as shown in Figure



$$V_u = q_s \times \text{critical area} = (151.88)(4)(1.75 - 0.6615) = 661.3\ \text{kN}$$

Also (with $\lambda = 1$),

$$\phi V_c = (0.75)(0.17)(\sqrt{20.68})\,(4)(0.6615)(1000) = 1534.2\ \text{kN}$$

So,

$$V_u = 661.3\ \text{kN} \leq \phi V_c = 1534.2\ \text{kN—O.K.}$$

The assumed depth of foundation is more than adequate.

For two-way shear, the critical section is located at a distance of $d/2$ from the edge of the column (ACI Code Section 11.11.1.2). This is shown in Figure . For this case, [with $\lambda = 1$ in Eq.

$$\phi V_c = \phi(0.33)\sqrt{f_c'}b_o d$$

The term b_o is the perimeter of the critical section for two-way shear. Or for this design,

$$b_o = 4[0.5 + 2(d/2)] = 4[0.5 + 2(0.3308)] = 4.65 \text{ m}$$

Hence,

$$\phi V_c = (0.75)(0.33)(\sqrt{20.68})(4.65)(0.6615) = 3.462 \text{ MN} = 3462 \text{ kN}$$

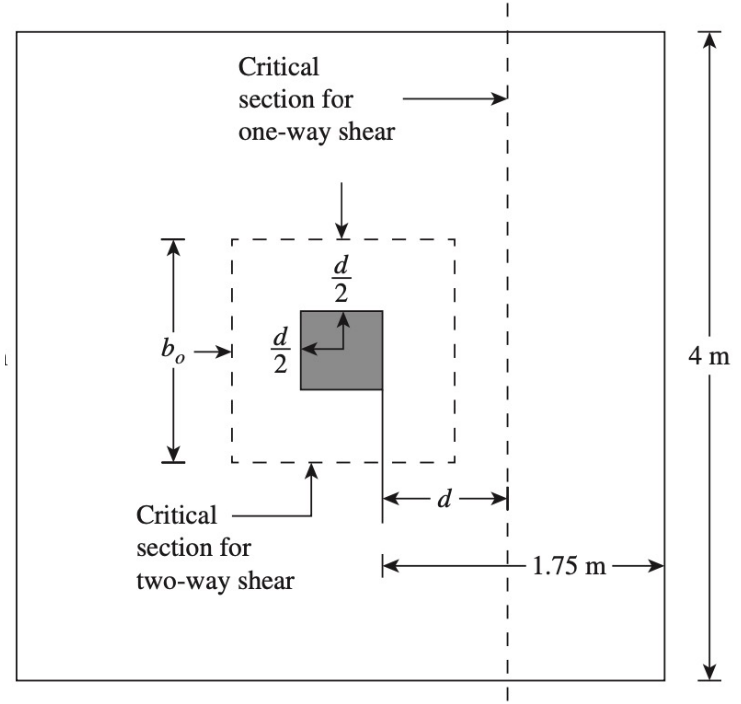
Also,

$$V_u = (q_s)(\text{critical area})$$
$$\text{Critical area} = (4 \times 4) - (0.5 + 0.6615)^2 = 14.65 \text{ m}^2$$

So,

$$V_u = (151.88)(14.65) = 2225.18 \text{ kN}$$
$$V_u = 2225.18 \text{ kN} < \phi V_c = 3462 \text{ kN—O.K.}$$

The assumed depth of foundation is more than adequate.



Flexural Reinforcement

According to Figure , the moment at critical section (ACI Code Section 15.4.2) is

$$M_u = (q_s B)\left(\frac{1.75}{2}\right)^2 = \frac{[(151.88)(4)](1.75)^2}{2} = 930.27 \text{ kN-m}$$

From Eq.

$$a = \frac{A_s f_v}{0.85 f_o' b} \qquad (\textit{Note: } b = B)$$

or

$$A_s = \frac{0.85 f_c' B a}{f_y} = \frac{(0.85)(20.68)(4)a}{413.7} = 0.17a$$

From Eq.

$$M_u \leq \phi A_s f_y \left(d - \frac{a}{2}\right)$$

With $\phi = 0.9$ and $A_s = 0.17a$,

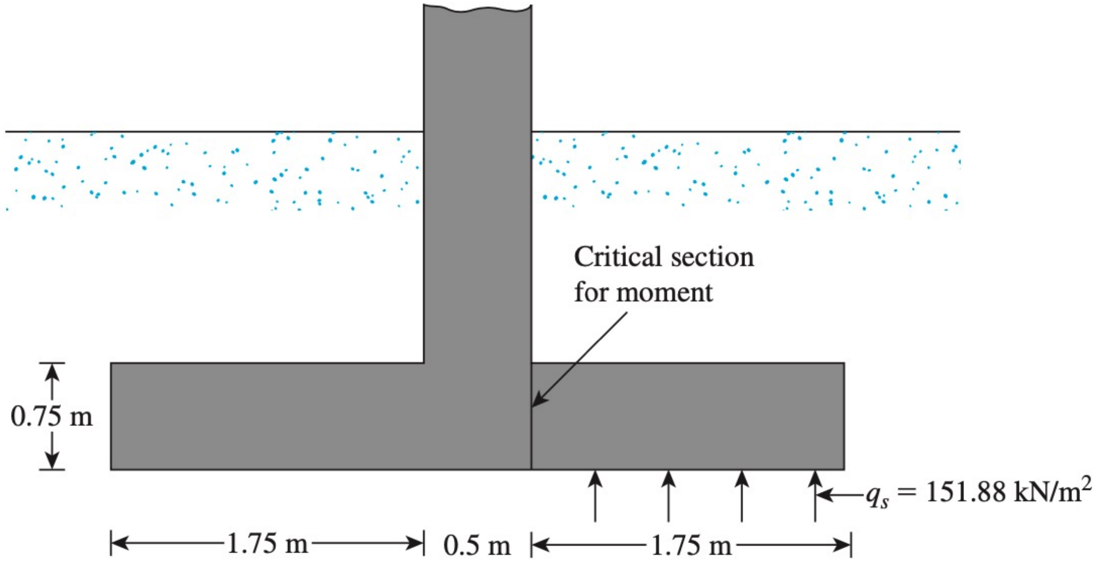
$$M_u = 930.27 = (0.9)(0.17a)(413700)\left(0.6615 - \frac{a}{2}\right)$$

Solution of the preceding equation given $a = 0.0226$ m. Hence,

$$A_s = 0.17a = (0.17)(0.0226) = 0.0038 \text{ m}^2$$

The percentage of steel is

$$s = \frac{A_s}{bd} = \frac{A_s}{Bd} = \frac{0.0038}{(4)(0.6615)} = 0.0015 < s_{\min}$$
$$= 0.0018 \text{ (ACI Code Section 7.12)}$$



So,

$$A_{s(\min)} = (0.0018)(B)(d) = (0.0018)(4)(0.6615)$$
$$= 0.004762 \text{ m}^2 = 47.62 \text{ cm}^2$$

Provide 10 × 25-mm diameter bars each way [$A_s = (4.91)(10) = 49.1 \text{ cm}^2$].

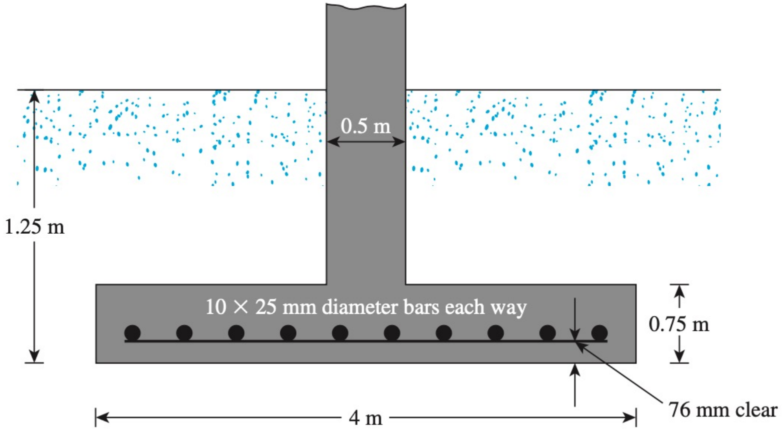
Check for Development Length (L_d)

From ACI Code Section 12.2.2, for 25 mm diameter bars, $L_d \approx 1338$ mm. Actual L_d provided is (4000 -500)/2 -76) = 1674 mm is more than the required, so, it is OK

Check for Bearing Strength

ACI Code Section 10.14 indicates that the bearing strength should be at least $0.85\,\phi f'_c A_1 \sqrt{A_2/A_1}$ with a limit of $\sqrt{A_2/A_1} \le 2$. For this problem, $\sqrt{A_2/A_1} = \sqrt{(4 \times 4)/(0.5 \times 0.5)} = 8$. So, use $\sqrt{A_2/A_1} = 2$. Also $\phi = 0.7$ 65 Hence, the design bearing strength = (0.85)(0.65)(20.68)(0.5 × 0.5)(2) = 5.713 MN = 5713 kN. However, the factored column load $U = 2430 \text{ kN} < 5713 \text{ kN}$—O.K.

The final design section is shown in Figure



Design Example of a Rectangular Foundation for a Column

This section describes the design of a rectangular foundation to support a column having dimensions of 0.4 m x 0.4 m in cross section. Other details are as follows:

Dead load = D = 290 kN

Live load = L = 110 kN

Depth from the ground surface to the top of the foundation = 1.2 m.

Allowable gross soil-bearing capacity = 120 kN/m^2

Maximum width of foundation = B = 1.5 m

f_y = 413.7 MN/m^2

f'_c = 20.68 MN/m^2

Unit weight of soil = $\gamma = 17.27$ kN/m^3

Unit weight of concrete = $\gamma_c = 22.97$ kN/m^3

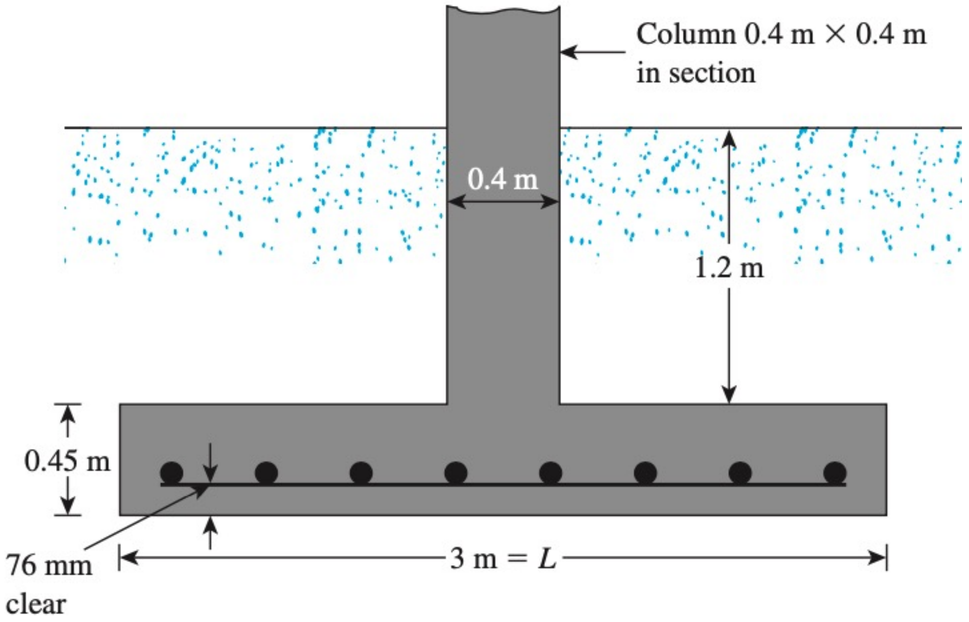
General Considerations

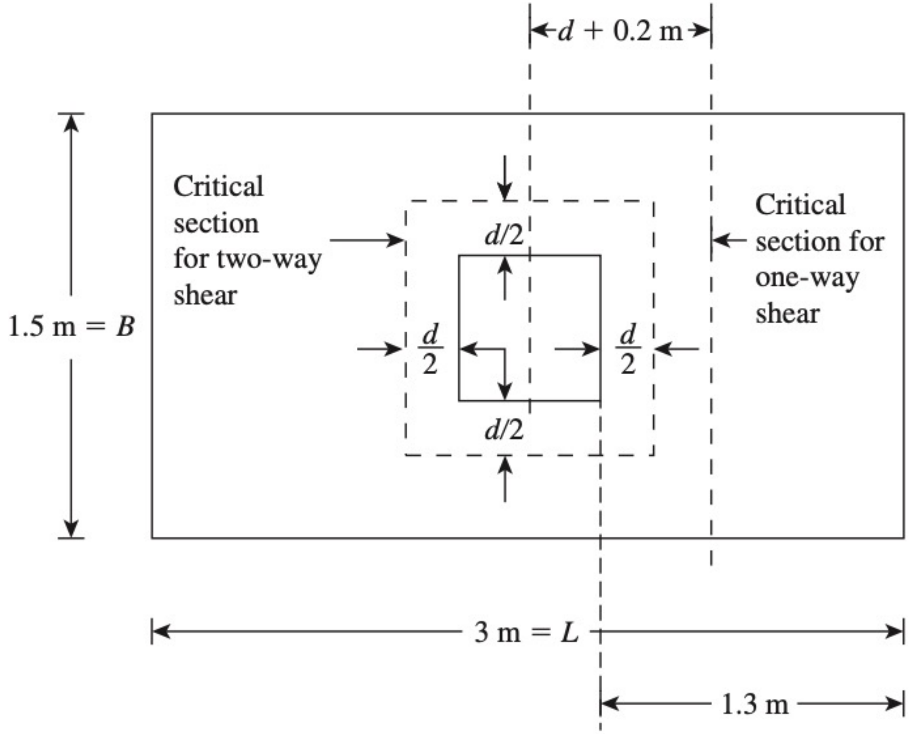
For this design, let us assume a foundation thickness of 0.45 m (Figure). The weight of foundation/m^2 = 0.45 γ_c = (0.45) (22.97) = 10.34 kN/m^2, and the weight of soil above the foundation/m^2 = (1.2)γ = (1.2) (17.27) = 20.72 kN/m^2. Hence, the net allowable soil-bearing capacity $[q_{\text{net(all)}}] = 120 - 10.34 - 20.72 = 88.94$ kN/m^2.

The required area of the foundation = $(D + L)/q_{\text{net(all)}} = (290 + 110)/88.94 = 4.5$ m^2. Hence, the length of the foundation is 4.5 m^2/B = 4.5/1.5 = 3 m.

The factored column load = 1.2D + 1.6L = 1.2(290) + 1.6(110) = 524 kN.

The factored soil-bearing capacity, q_s = factored load/foundation area = 524/4.5 = 116.44 kN/m^2.





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Shear Strength of Foundation

Assume that the steel bars to be used have a diameter of 16 mm. So, the effective depth $d = 450 - 76 - 16/2 = 366$ mm. (Note that the assumed clear cover is 76 mm.

Figure A.4a shows the critical section for one-way shear (AC… …de Section 11.11.1.1). According to this figure

$$V_u = \left(1.5 - \frac{0.4}{2} - 0.366\right)Bq_s = (0.934)(1.5)(116.44) = 163.13 \text{ kN}$$

The nominal shear capacity of concrete for one-way beam action [with $\lambda = 1$ in Eq. (11.a)]

$$V_c = 0.17\sqrt{f'_o}Bd = 0.17(\sqrt{20.68})(1.5)(0.366) = 0.4244 \text{ MN} = 424.4 \text{ kN}$$

Now

$$V_u = 163.13 \leq \phi V_c = (0.75)(424.4) = 318.3 \text{ kN—O.K.}$$

The critical section for two-way shear is also shown in Figure A.4a. This is based on the recommendations given by ACI Code Section 11.11.1.2. For this section

$$V_u = q_s[(1.5)(3) - 0.766^2] = 455.66 \text{ kN}$$

The nominal shear capacity of the foundation can be given as (ACI Code Section 11.11.2)

$$V_c = v_c b_o d = 0.33\lambda\sqrt{f'_c}\,b_o d$$

where b_o = perimeter of the critical section

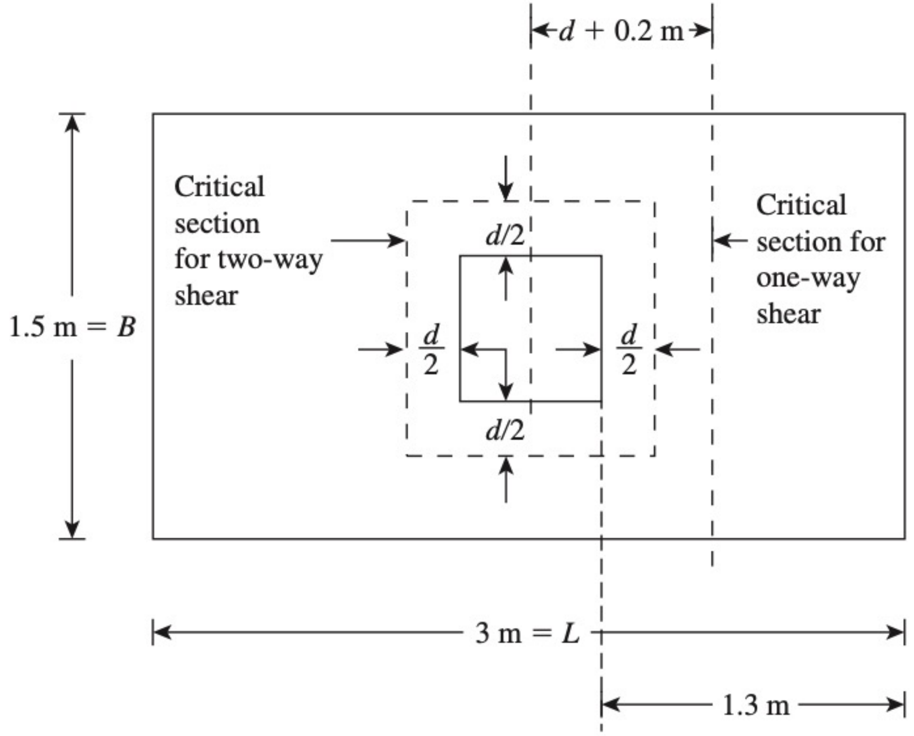
or

$$V_c = (0.33)(1)(\sqrt{20.68})(4 \times 0.766)(0.366) = 1.683 \text{ MN}$$

So, for two-way shear condition

$$V_u = 455.66 \text{ kN} < \phi V_c = (0.75)(1683) = 1262.25 \text{ kN}$$

Therefore, the section is adequate.



Check for Bearing Capacity of Concrete Column at the interface with Foundation

According to ACI Code Section 10.14.1, the bearing strength is equal to $0.85\ \phi f_c' A_1$ ($\phi = 0.65$). For this problem, $U = 524$ kN < bearing strength $= (0.85)(0.65)(20.68)(0.4)^2 = 1.828$ MN.

So, a minimum area of dowels should be provided across the interface of the column and the foundation (ACI Code Section 15.8.2). Based on ACI Code Section 15.8.2.1

$$\begin{aligned}\text{Minimum area of steel} &= (0.005)\ (\text{area of column})\\ &= (0.005)\ (400^2) = 800\ \text{mm}^2\end{aligned}$$

So use 4×16-mm diameter bars as dowels.

The minimum required length of development (L_d) of dowels in the foundation is $(0.24 f_y d_b)/\lambda\sqrt{f_c'}$, but not less than $0.043\ f_y d_b$ (ACI Code Section 12.3.2). So,

$$L_d = \frac{0.24 f_y d_b}{\lambda\sqrt{f_c'}} = \frac{(0.24)(413.7)(16)}{(1)(\sqrt{20.68})} = 349.33\ \text{mm}$$

Also,

$$L_d = 0.043\ f_y d_b = (0.043)(413.7)(16) = 284.6\ \text{mm}$$

Hence, $L_d = 349.33$-mm controls.

Available depth for the dowels (Figure A.4a) is $450 - 76 - 16 - 16 = 342$ mm. Since hooks cannot be used, the foundation depth must be increased. Let the new depth be equal to 480 mm to accommodate the required $L_d = 349.33$ mm. Hence, the new value of d is equal to $480 - 76 - 16 - 16 = 372$ mm.

Flexural Reinforcement in the Long Direction

According to Figure A.4a, the design moment about the column face is

$$M_u = \frac{(q_s B)1.3^2}{2} = \frac{(116.44)(1.5)(1.3)^2}{2} = 147.59 \text{ kN-m}$$

From Eq. (A.2),

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{(A_s)(413.7)}{(0.85)(20.68)(1.5)} = 15.69 A_s$$

Again, from Eq. (A.4),

$$M_u = \phi M_n = \phi A_s f_y \left(d - \frac{a}{2}\right)$$

or

$$147.59 = (0.9)(A_s)(413.7 \times 10^3)\left[0.396 - \frac{15.69}{2}(A_s)\right]$$

$$147.59 = 147{,}444.7\, A_s - 2{,}920{,}928 A_s^2$$

(*Note:* $d = 0.396$ m, assuming that these bars are placed as the bottom layer.)

The solution of the preceding equation gives

$$A_s = 0.00102 \text{ m}^2 \left[\text{that is, steel percentage} = \frac{A_s}{Bd} = \frac{0.00102}{(1.5)(0.396)} = 0.0017\right]$$

Also, from ACI Code Section 7.12.2, $s_{\min} = 0.0018$. Hence, provide 7 $\times$ 16-mm diameter bars (A_s provided is 0.001407 m^2).

Flexural Reinforcement in the Short Direction

According to Figure A.4a, the moment at the face of the column is

$$M_u = \frac{(q_s L)(0.55)^2}{2} = \frac{(116.44)(3)(0.55)^2}{2} = 52.83\,\text{kN-m}$$

From Eq. (A.2),

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{(A_s)(413.7)}{(0.85)(20.68)(3)} = 7.845 A_s$$

From Eq. (A.4),

$$M_u = \phi A_s f_y \left(d - \frac{a}{2} \right)$$

or

$$52.83 = (0.9)(A_s)\,(413.7 \times 10^3)\left[0.380 - \frac{7.845}{2}(A_s) \right]$$

$$\left(\textit{Note: } d = 480 - 76 - 16 - \frac{16}{2} = 380 \text{ mm for short bars in the upper layer.} \right)$$

The solution of the preceding equation gives

$$A_s = 0.0004 \text{ m}^2 \qquad (\text{thus } s < s_{\min})$$

So, use $s = s_{\min}$, or

$$A_s = s_{\min}\, bd = (0.0018)(3)(0.48) \approx 0.0026 \text{ m}^2$$

(*Note* : Use gross area when $s_{\min} = 0.0018$ is used.)

Use 13 $\times$ 16-mm diameter bars.

According to ACI Code Section 12.2, the development length L_d for 16 mm diameter bars is about 693 mm. For such a case, the footing width needs to be [2(0.693 + 0.076) + 0.4] = 1.938 m. Since the footing width is limited to 1.5 m, we should use 12-mm diameter bars.

So, use 23 $\times$ 12 mm diameter bars.

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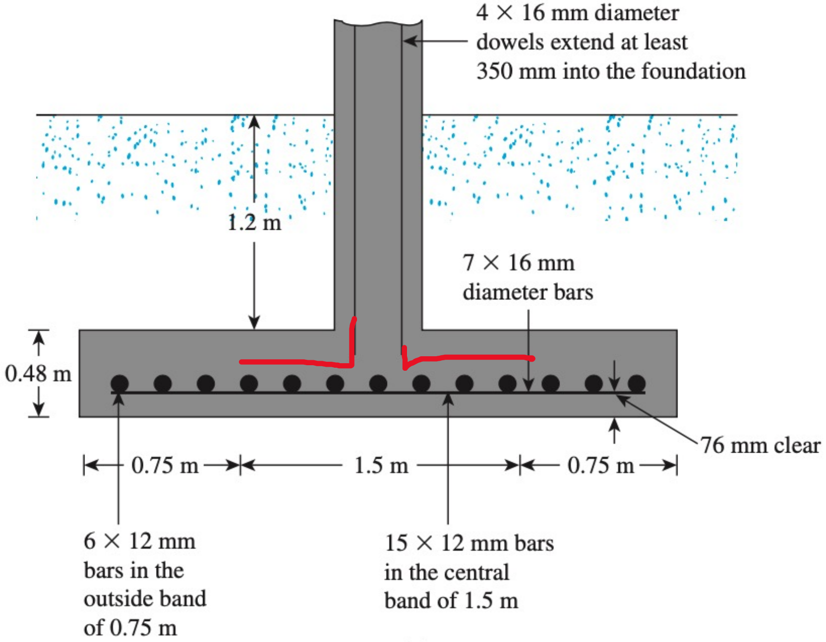
Final Design Sketch

According to ACI Code Section 15.4.4, a portion of the reinforcement in the short direction shall be distributed uniformly over a bandwidth equal to the smallest dimension of the foundation. The remainder of the reinforcement should be distributed uniformly outside the central band of the foundation. The reinforcement in the central band can be given to the equal to $2/(\beta_c + 1)$ (where $\beta_c = L/B$). For this problem, $\beta_c = 2$. Hence, 2/3 of the reinforcing bars (that is, 15 bars) should be placed in the center band of the foundation. The remaining bars should be placed outside the central band. However, one needs to check the steel percentage in the outside band, or

$$s = \frac{A_s}{bd} = \frac{(2)(113\ \text{mm}^2)}{\left(\dfrac{3000 - 1500}{2}\right)(380)} = 0.00079 < s_{\min} = 0.0018$$

So, use $A_s = (s_{\min})(b)(d) = (0.0018)(750)(480) = 648\ \text{mm}^2$. Hence, 6 × 12-mm diameter bars on each side of the central band will be sufficient.



Thank you

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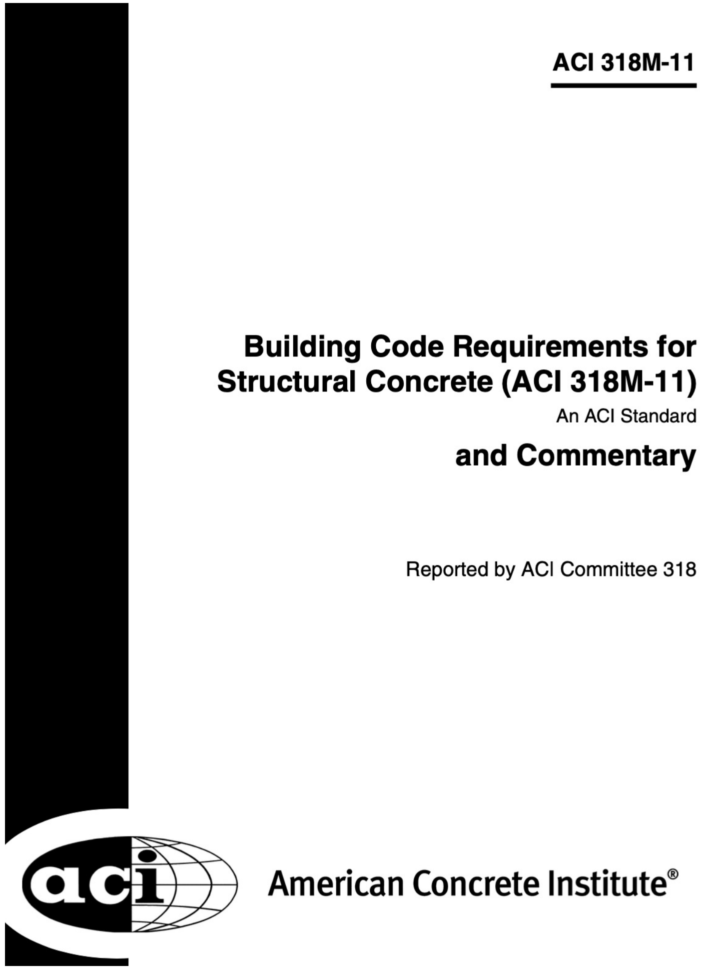
Structural Design of Reinforced Concrete of Shallow Foundations (examples)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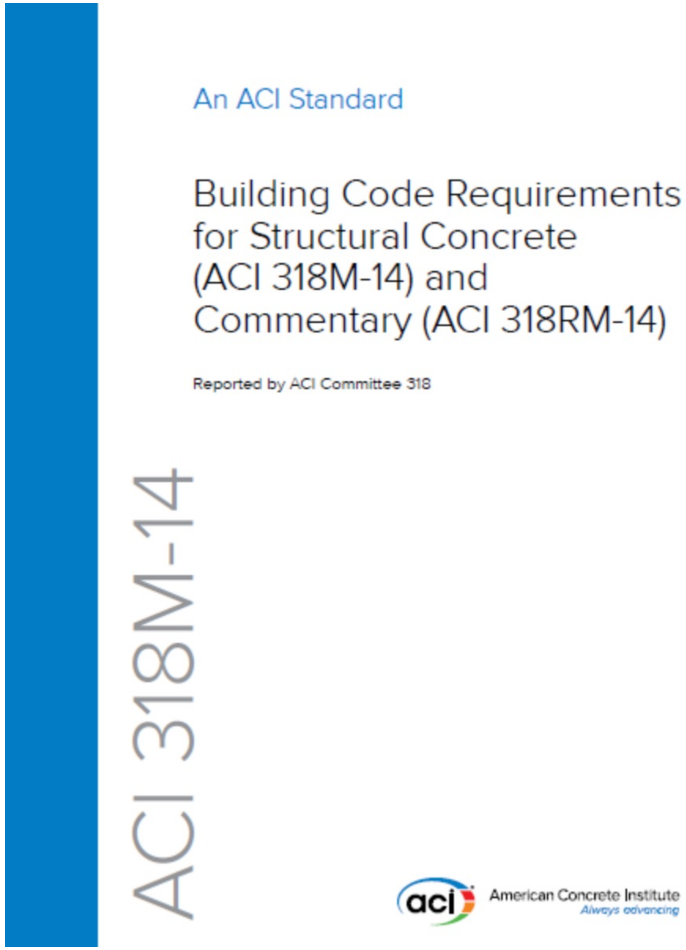
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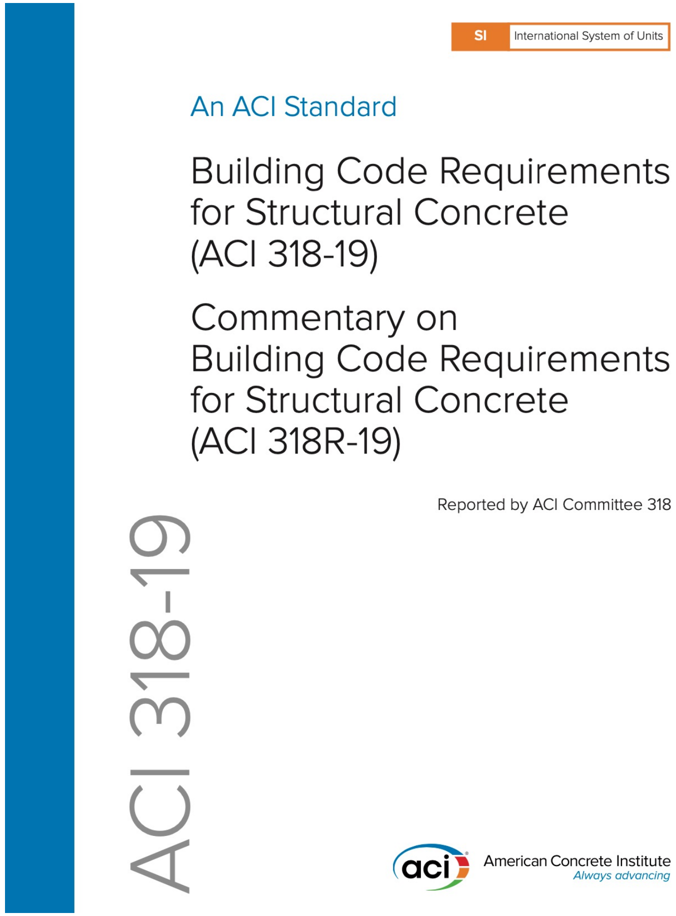
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Example (1): Design of a Continuous (Wall) Foundation

It is required to design a load-bearing wall with the following data:

Dead load $= D = 43.8$ kN/m

Live load $= L = 17.5$ kN/m

Gross allowable bearing capacity of soil $= 94.9$ kN/m^2

Depth of the top of foundation from the ground surface $= 1.2$ m

$f_y = 413.7$ MN/m^2

$f'_c = 20.68$ MN/m^2

Unit weight of soil $= \gamma = 17.27$ kN/m^3

Unit weight of concrete $= \gamma_c = 22.97$ kN/m^3

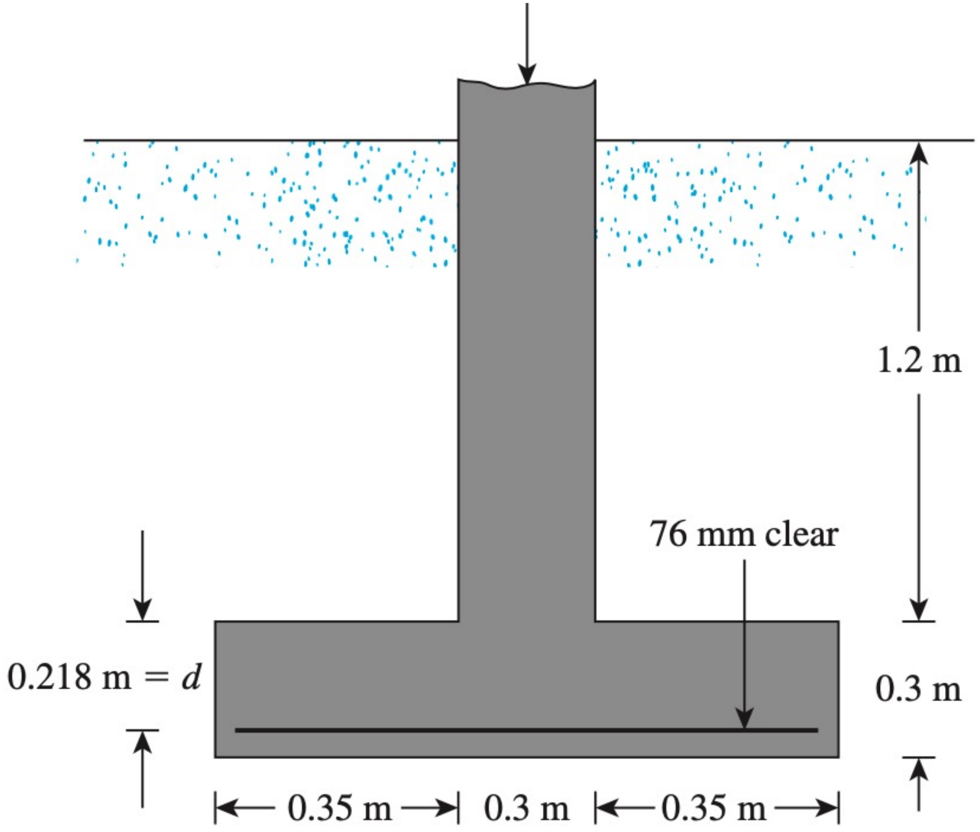
Solution

Code Requirements (ACI 318M -11) 15.7 — Minimum footing depth:

Depth of footing above bottom reinforcement shall not be less than 150 mm for footings on soil, or less than 300 mm for footings on piles.

For this design, assume the foundation thickness to be 0.3 m. Refer to (ACI 318M-11) Code, Section 7.7.1, which recommends a minimum cover of 76 mm over steel reinforcement, and assume that the steel bars to be used are 12 mm in diameter. Thus:

$$d = 300 - 76 - \frac{12}{2} = 218 \text{ mm}$$



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Also,

$$\text{Weight of the foundation} = (0.3)\gamma_c = (0.3)(22.97) = 6.89\ \text{kN/m}^2$$

$$\text{Weight of soil above the foundation} = (1.2)\gamma = (1.2)(17.27) = 20.72\ \text{kN/m}^2$$

So, the net allowable soil bearing capacity is

$$q_{\text{net(all)}} = 94.9 - 6.89 - 20.72 = 67.29\ \text{kN/m}^2$$

Hence, the required width of foundation is

$$B = \frac{D + L}{q_{\text{net(all)}}} = \frac{43.8 + 17.5}{67.29} = 0.91\ \text{m}$$

So, assume $B = 1$ m.
According to ACI Code Section 9.2,

$$U = 1.2D + 1.6L = (1.2)(43.8) + (1.6)(17.5) = 80.56 \text{ kN/m}$$

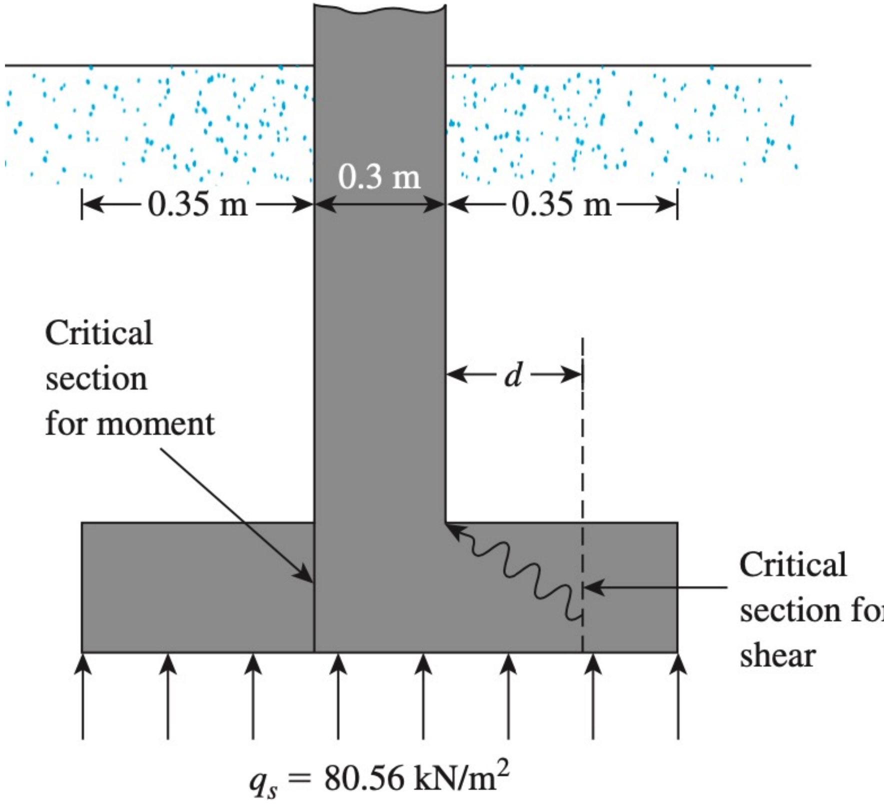
Converting the net allowable soil pressure to an ultimate (factored) value,

$$q_s = \frac{U}{(B)(1)} = \frac{80.56}{(1)(1)} = 80.56 \text{ kN/m}^2$$

Investigation of Shear Strength of the Foundation

The critical section for shear occurs at a distance d from the face of the wall (ACI Code Section 11.11.3), as shown in Figure . So, shear at critical section

$$V_u = (0.35 - d)q_s = (0.35 - 0.218)(80.56) = 10.63 \text{ kN/m}$$



with $\lambda = 1$

$$V_c = 0.17\sqrt{f_c'}\,bd = 0.17\sqrt{20.68}\,(1)(0.218) = 0.1685\,\text{MN/m} \approx 168\ \text{kN/m}$$

Also,

$$\phi V_c = (0.75)(168) = 126\ \text{kN/m} > V_u = 10.63\ \text{kN/m—O.K.}$$

(*Note:* $\phi = 0.75$ for shear—ACI Code Section 9.3.2.3.)

Because $V_u < \phi V_c$, the total thickness of the foundations could be reduced to 250 mm. So, the modified

$$d = 250 - 76 - \frac{12}{2} = 168\,\text{mm} > 152\,\text{mm} = d_{\text{min}}\ \text{(ACI Code Section 15.7)}$$

Neglecting the small difference in footing weight, if $d = 168$ mm,

$$\phi V_c = (0.75)(0.17)\sqrt{20.68}\,(1)(0.168) = 0.0974\ \text{MN/m} = 97.4\ \text{kN/m}$$

$$= 97.4\ \text{kN/m} > V_u\text{—O.K.}$$

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Flexural Reinforcement

For steel reinforcement, factored moment at the face of the wall has to be determined (ACI Code Section 15.4.2). The bending of the foundation will be in one direction only. So, according to Figure , the design ultimate moment

$$M_u = \frac{q_s l^2}{2}$$

$$l = 0.35 \text{ m}$$

So,

$$M_u = \frac{(80.56)(0.35)^2}{2} = 4.93 \text{ kN-m/m}$$

15.4 — Moment in footings

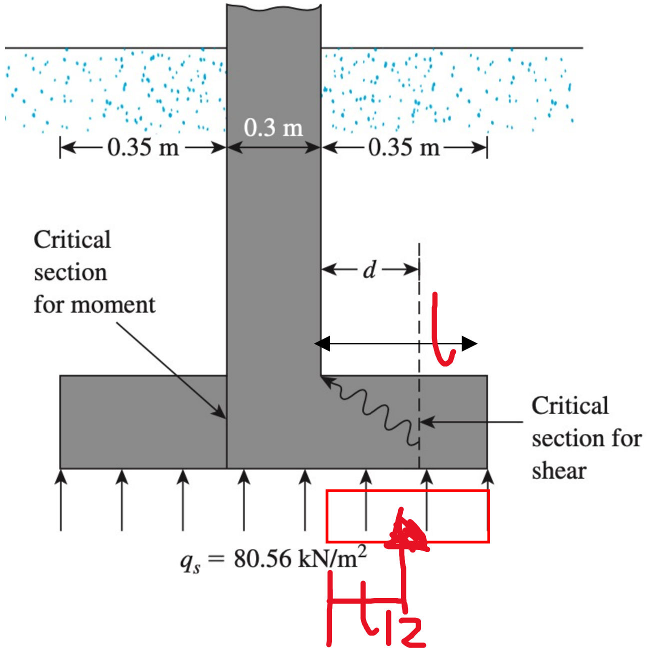
15.4.1 — External moment on any section of a footing shall be determined by passing a vertical plane through the footing, and computing the moment of the forces acting over entire area of footing on one side of that vertical plane.

15.4.2 — Maximum factored moment, M_u, for an isolated footing shall be computed as prescribed in 15.4.1 at critical sections located as follows:

(a) At face of column, pedestal, or wall, for footings supporting a concrete column, pedestal, or wall;

(b) Halfway between middle and edge of wall, for footings supporting a masonry wall;

(c) Halfway between face of column and edge of steel base plate, for footings supporting a column with steel base plate.



$$M_n = A_s f_y\left(d - \frac{a}{2}\right)$$

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{(A_s)(413.7)}{(0.85)(20.68)(1)} = 23.5351 A_s$$

Thus,

$$M_n = (A_s)(413.7)\left(0.168 - \frac{23.5351}{2} A_s\right)$$

or

$$M_n(\text{MN-m/m}) = 69.5A_s - 4868.24\text{A}_s^2$$

Again,

$$M_u \leqslant \phi M_n$$

where $\phi = 0.9$.

Thus,

$$4.93 \times 10^{-3}(\text{MN-m/m}) = 0.9\,(69.5A_s - 4868.24A_s^2)$$

Solving for A_s, one gets

$$A_{s(1)} = 0.0128\ \text{m}^2;\ A_{s(2)} = 0.0001\ \text{m}^2$$

Hence, steel percentage with $A_{s(1)}$ is

$$s_1 = \frac{A_{s(1)}}{bd} = \frac{0.0128}{(1)(0.168)} = 0.0762$$

Similarly, steel percentage with $A_{s(2)}$ is

$$s_2 = \frac{A_{s(2)}}{bd} = \frac{0.0001}{(1)(0.168)} = 0.0006 < s_{\min} = 0.0018\,(\text{ACI Code Section 7.12.2.1})$$

The maximum steel percentage that can be provided is given in Eq
Thus,

$$s_{\max} = (0.75)(0.85)\frac{f'_c}{f_y}\beta\left(\frac{600}{600 + f_y}\right)$$

Note that $\beta = 0.85$. Substituting the proper values of β, f'_c, and f_y in the preceding equation, one obtains

$$s_{\max} = 0.016$$

Note that $s_1 = 0.0762 > s_{\max} = 0.016$. So use $s = s_{\min} = 0.0018$. So,

$$A_s = (s_{\min})(b)(d) = (0.0018)(1)(0.168) = 0.000302\ \text{m}^2 = 302\ \text{mm}^2$$

Use 12-mm diameter bars @ 350 mm c/c. Hence,

$$A_s\,(\text{provided}) = \frac{1000}{350}\left(\frac{\pi}{4}\right)(12)^2 = 323\,\text{mm}^2$$

Development Length of Reinforcement Bars (*Ld*)

According to ACI Code Section 12.2, the minimum development length L_d for 12 mm diameter bars is about 558 mm . Assuming a 76-mm cover to be on both sides of the footing, the minimum footing width should be [2(558 + 76) + 300] mm = 1568 mm = 1.568 m. Hence, the revised calculations are:

$$q_s = \frac{U}{(B)(1)} = \frac{80.56}{1.568} = 51.38\,\text{kN/m}^2$$

$$M_u = \frac{q_s l^2}{2} = \frac{1}{2}(51.38)(0.558 + 0.076)^2$$

$$= 10.326\,\text{kN}\cdot\text{m/m} = 10.326 \times 10^{-3}\,\text{MN}\cdot\text{m/m}$$

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{A_s(413.7)}{(0.85)(20.68)(1.568)} = 15.01 A_s$$

$$M_n = A_s f_y\left(d - \frac{a}{2}\right) = A_s(413.7)\left(0.168 - \frac{15.01 A_s}{2}\right)$$

$$\phi M_n \geqslant M_u$$

$$10.326 \times 10^{-3} = 0.9 A_s(413.7)\left(0.168 - \frac{15.01 A_s}{2}\right)$$

$$A_s = 0.00016\,\text{m}^2$$

The steel percentage is $s = \dfrac{A_s}{bd} = \dfrac{0.00016}{(1.568)(0.25)} < 0.0018.$

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(*Note:* Use gross area when $s_{\min} = 0.0018$ is used.)

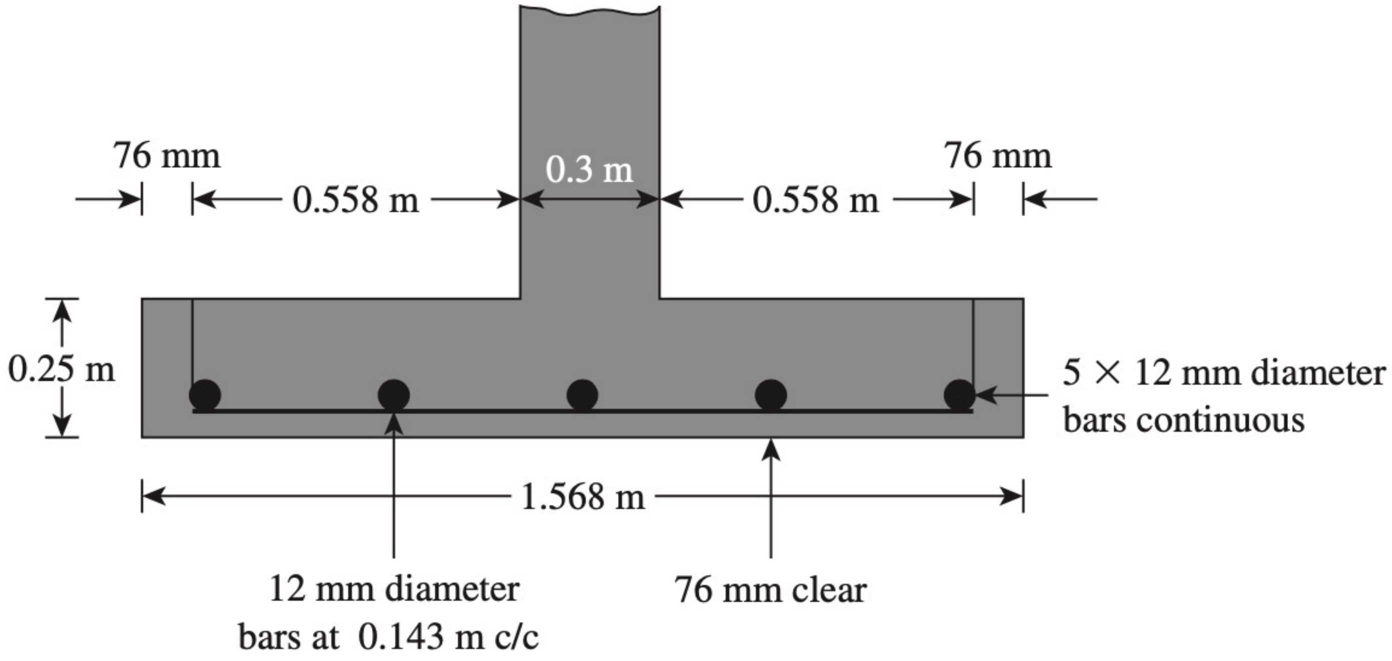
Use $A_s = (0.0018)(1.568)(0.25) = 0.000706\ m^2 = 706\ mm^2$. Provide 7 × 12 mm bars (A_s = 791 mm^2).

Minimum reinforcement should be furnished in the long direction to offset shrinkage and temperature effects (ACI Code Section 7.12.). So,

$$A_s = (0.0018)(b)(d) = (0.0018)\,[(0.558 + 0.076)(2) + 0.3](0.168)$$
$$= 0.000474\ m^2 = 474\ mm^2$$

Provide 5 × 12 mm bars ($A_s = 565\ mm^2$).

The final design sketch is shown in Figure:



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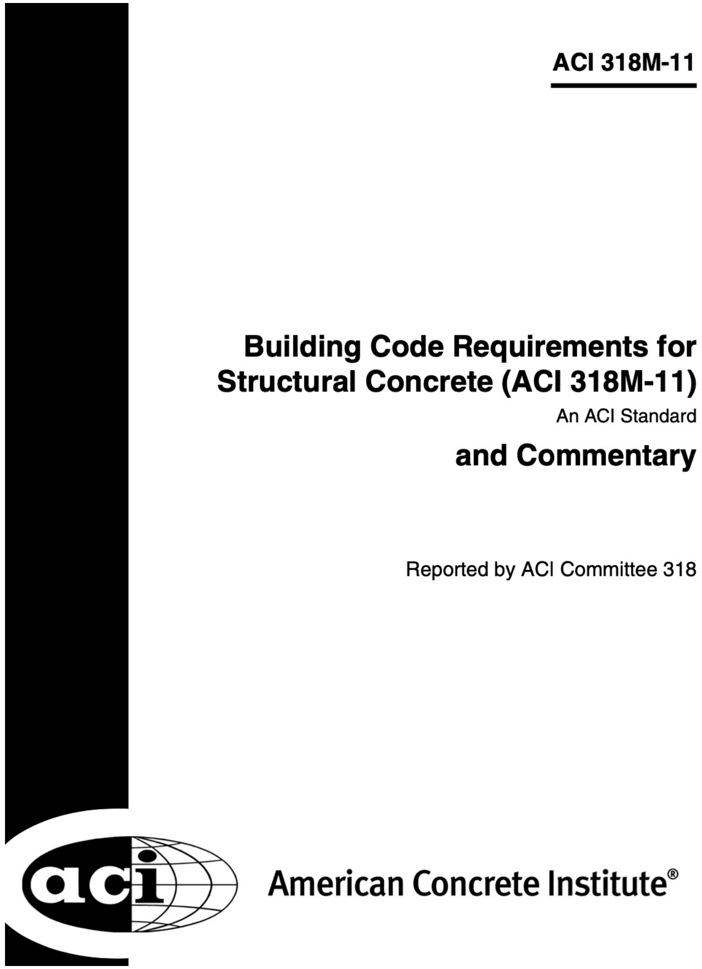
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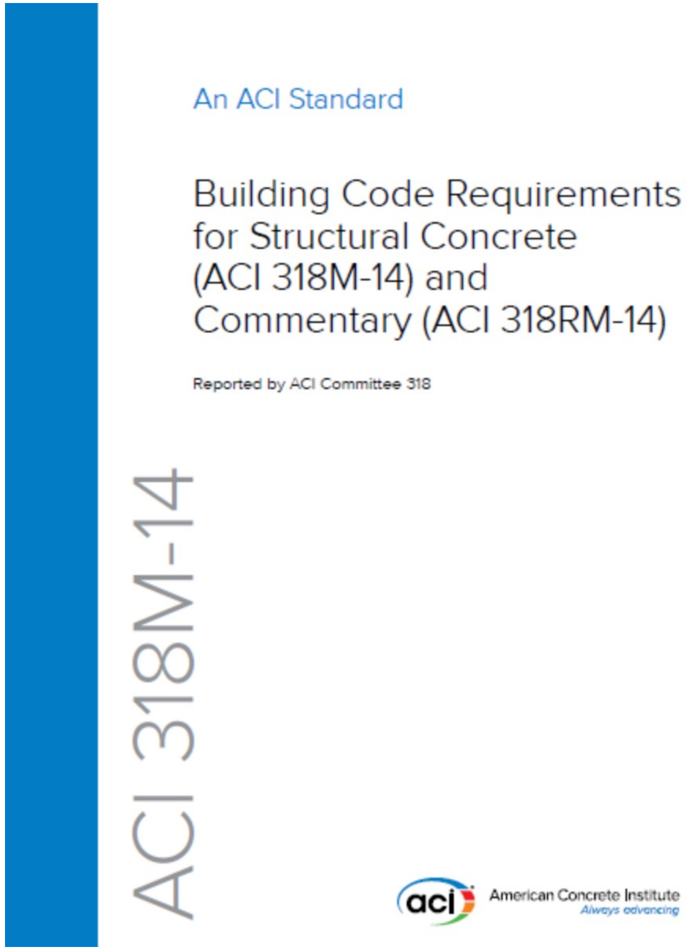
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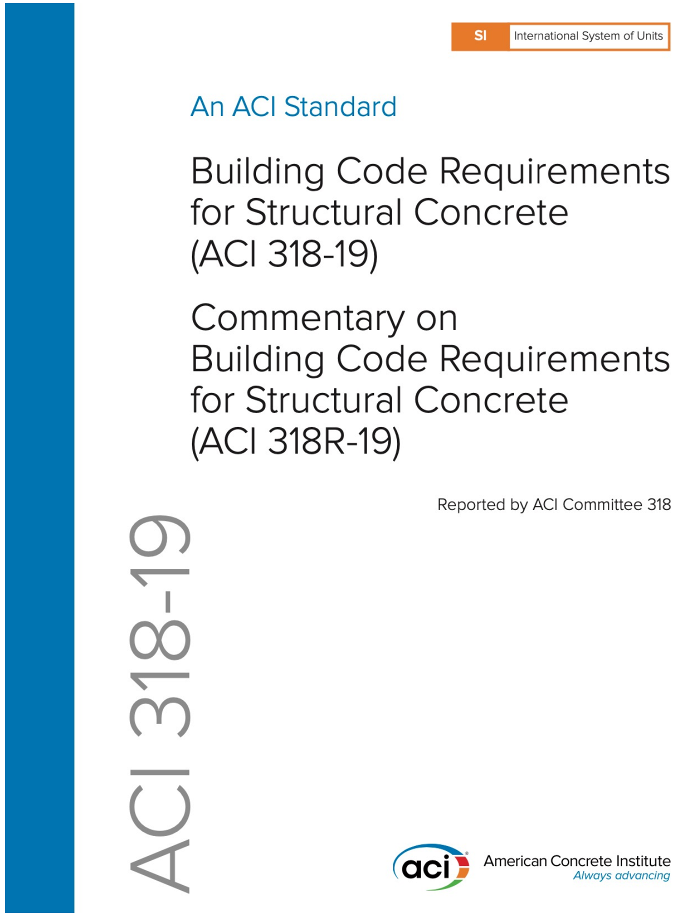
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Gross allowable bearing capacity of soil $= 94.9$ kN/m^2

Depth of the top of foundation from the ground surface $= 1.2$ m

$f_y = 413.7$ MN/m^2

$f'_c = 20.68$ MN/m^2

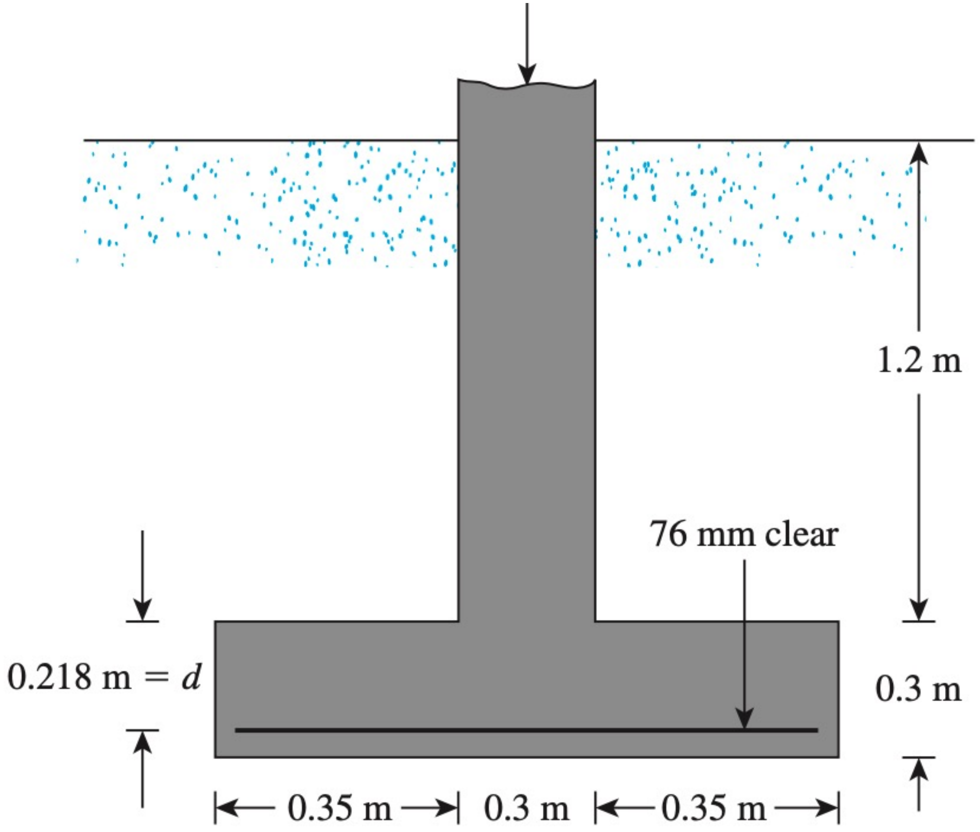
Unit weight of soil $= \gamma = 17.27$ kN/m^3

Unit weight of concrete $= \gamma_c = 22.97$ kN/m^3

Solution

Code Requirements (ACI 318M -11) 15.7 — Minimum footing depth:
Depth of footing above bottom reinforcement shall not be less than 150 mm for footings on soil, or less than 300 mm for footings on piles.

For this design, assume the foundation thickness to be 0.3 m. Refer to (ACI 318M-11) Code, Section 7.7.1, which recommends a minimum cover of 76 mm over steel reinforcement, and assume that the steel bars to be used are 12 mm in diameter. Thus:



$$d = 300 - 76 - \frac{12}{2} = 218 \text{ mm}$$

Also,

$$\text{Weight of the foundation} = (0.3)\gamma_c = (0.3)(22.97) = 6.89 \text{ kN/m}^2$$

$$\begin{aligned}\text{Weight of soil above the foundation} &= (1.2)\gamma = (1.2)(17.27) \\ &= 20.72 \text{ kN/m}^2\end{aligned}$$

So, the net allowable soil bearing capacity is

$$q_{\text{net(all)}} = 94.9 - 6.89 - 20.72 = 67.29 \text{ kN/m}^2$$

Hence, the required width of foundation is

$$B = \frac{D + L}{q_{\text{net(all)}}} = \frac{43.8 + 17.5}{67.29} = 0.91 \text{ m}$$

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So, assume $B = 1$ m.
According to ACI Code Section 9.2,

$$U = 1.2D + 1.6L = (1.2)(43.8) + (1.6)(17.5) = 80.56 \text{ kN/m}$$

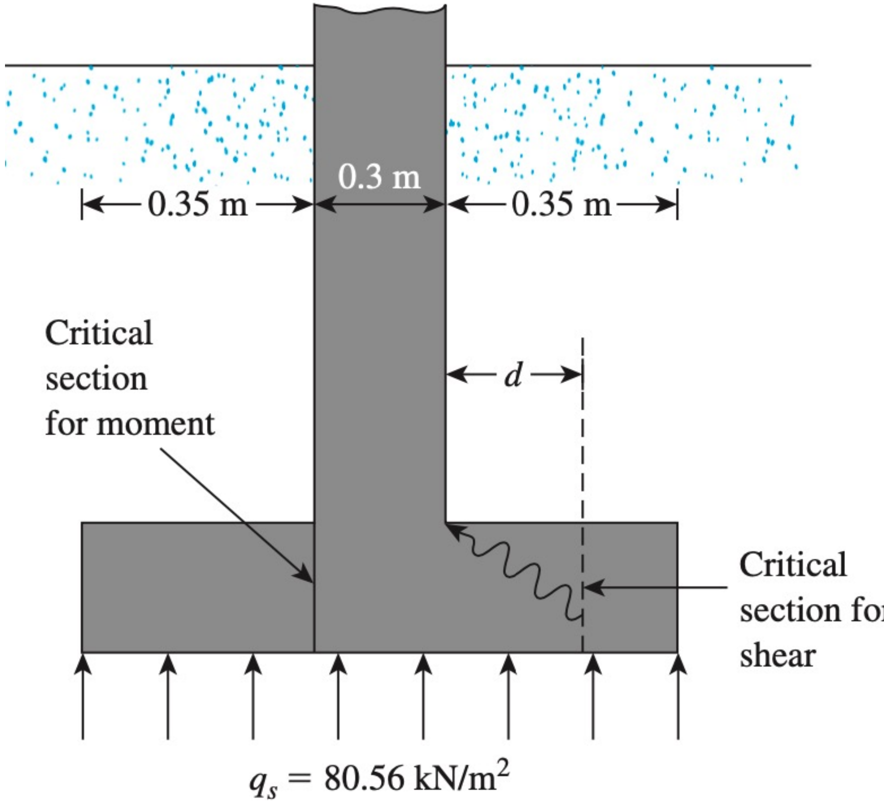
Converting the net allowable soil pressure to an ultimate (factored) value,

$$q_s = \frac{U}{(B)(1)} = \frac{80.56}{(1)(1)} = 80.56 \text{ kN/m}^2$$

Investigation of Shear Strength of the Foundation

The critical section for shear occurs at a distance d from the face of the wall (ACI Code Section 11.11.3), as shown in Figure . So, shear at critical section

$$V_u = (0.35 - d)q_s = (0.35 - 0.218)(80.56) = 10.63 \text{ kN/m}$$



with $\lambda = 1$

$$V_c = 0.17\sqrt{f_c'}\,bd = 0.17\sqrt{20.68}\,(1)(0.218) = 0.1685\,\text{MN/m} \approx 168\ \text{kN/m}$$

Also,

$$\phi V_c = (0.75)(168) = 126\ \text{kN/m} > V_u = 10.63\ \text{kN/m—O.K.}$$

(*Note:* $\phi = 0.75$ for shear—ACI Code Section 9.3.2.3.)

Because $V_u < \phi V_c$, the total thickness of the foundations could be reduced to 250 mm. So, the modified

$$d = 250 - 76 - \frac{12}{2} = 168\,\text{mm} > 152\,\text{mm} = d_{\text{min}}\ \text{(ACI Code Section 15.7)}$$

Neglecting the small difference in footing weight, if $d = 168$ mm,

$$\phi V_c = (0.75)(0.17)\sqrt{20.68}\,(1)(0.168) = 0.0974\ \text{MN/m} = 97.4\ \text{kN/m}$$

$$= 97.4\ \text{kN/m} > V_u\text{—O.K.}$$

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Flexural Reinforcement

For steel reinforcement, factored moment at the face of the wall has to be determined (ACI Code Section 15.4.2). The bending of the foundation will be in one direction only. So, according to Figure , the design ultimate moment

$$M_u = \frac{q_s l^2}{2}$$

$$l = 0.35 \text{ m}$$

So,

$$M_u = \frac{(80.56)(0.35)^2}{2} = 4.93 \text{ kN-m/m}$$

15.4 — Moment in footings

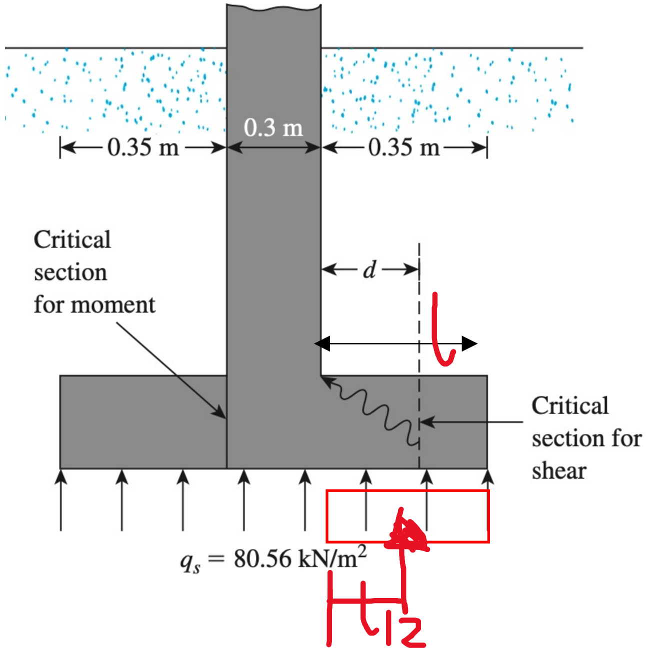
15.4.1 — External moment on any section of a footing shall be determined by passing a vertical plane through the footing, and computing the moment of the forces acting over entire area of footing on one side of that vertical plane.

15.4.2 — Maximum factored moment, M_u, for an isolated footing shall be computed as prescribed in 15.4.1 at critical sections located as follows:

(a) At face of column, pedestal, or wall, for footings supporting a concrete column, pedestal, or wall;

(b) Halfway between middle and edge of wall, for footings supporting a masonry wall;

(c) Halfway between face of column and edge of steel base plate, for footings supporting a column with steel base plate.



$$M_n = A_s f_y\left(d - \frac{a}{2}\right)$$

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{(A_s)(413.7)}{(0.85)(20.68)(1)} = 23.5351 A_s$$

Thus,

$$M_n = (A_s)(413.7)\left(0.168 - \frac{23.5351}{2} A_s\right)$$

or

$$M_n(\text{MN-m/m}) = 69.5 A_s - 4868.24 A_s^2$$

Again,

$$M_u \leqslant \phi M_n$$

where $\phi = 0.9$.

Thus,

$$4.93 \times 10^{-3}(\text{MN-m/m}) = 0.9\,(69.5 A_s - 4868.24 A_s^2)$$

Solving for A_s, one gets

$$A_{s(1)} = 0.0128 \text{ m}^2;\ A_{s(2)} = 0.0001 \text{ m}^2$$

Hence, steel percentage with $A_{s(1)}$ is

$$s_1 = \frac{A_{s(1)}}{bd} = \frac{0.0128}{(1)(0.168)} = 0.0762$$

Similarly, steel percentage with $A_{s(2)}$ is

$$s_2 = \frac{A_{s(2)}}{bd} = \frac{0.0001}{(1)(0.168)} = 0.0006 < s_{\min} = 0.0018 \text{ (ACI Code Section 7.12.2.1)}$$

The maximum steel percentage that can be provided is given in Eq
Thus,

$$s_{\max} = (0.75)(0.85)\frac{f'_c}{f_y}\beta\left(\frac{600}{600 + f_y}\right)$$

Note that $\beta = 0.85$. Substituting the proper values of β, f'_c, and f_y in the preceding equation, one obtains

$$s_{\max} = 0.016$$

Note that $s_1 = 0.0762 > s_{\max} = 0.016$. So use $s = s_{\min} = 0.0018$. So,

$$A_s = (s_{\min})(b)(d) = (0.0018)(1)(0.168) = 0.000302 \text{ m}^2 = 302 \text{ mm}^2$$

Use 12-mm diameter bars @ 350 mm c/c. Hence,

$$A_s\text{(provided)} = \frac{1000}{350}\left(\frac{\pi}{4}\right)(12)^2 = 323 \text{ mm}^2$$

Development Length of Reinforcement Bars (*Ld*)

According to ACI Code Section 12.2, the minimum development length L_d for 12 mm diameter bars is about 558 mm . Assuming a 76-mm cover to be on both sides of the footing, the minimum footing width should be [2(558 + 76) + 300] mm = 1568 mm = 1.568 m. Hence, the revised calculations are:

$$q_s = \frac{U}{(B)(1)} = \frac{80.56}{1.568} = 51.38\,\text{kN/m}^2$$

$$M_u = \frac{q_s l^2}{2} = \frac{1}{2}(51.38)(0.558 + 0.076)^2$$

$$= 10.326\ \text{kN}\cdot\text{m/m} = 10.326 \times 10^{-3}\,\text{MN}\cdot\text{m/m}$$

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{A_s(413.7)}{(0.85)(20.68)(1.568)} = 15.01 A_s$$

$$M_n = A_s f_y\left(d - \frac{a}{2}\right) = A_s(413.7)\left(0.168 - \frac{15.01 A_s}{2}\right)$$

$$\phi M_n \geqslant M_u$$

$$10.326 \times 10^{-3} = 0.9 A_s (413.7)\left(0.168 - \frac{15.01 A_s}{2}\right)$$

$$A_s = 0.00016\ \text{m}^2$$

The steel percentage is $s = \dfrac{A_s}{bd} = \dfrac{0.00016}{(1.568)(0.25)} < 0.0018.$

(*Note:* Use gross area when $s_{min} = 0.0018$ is used.)

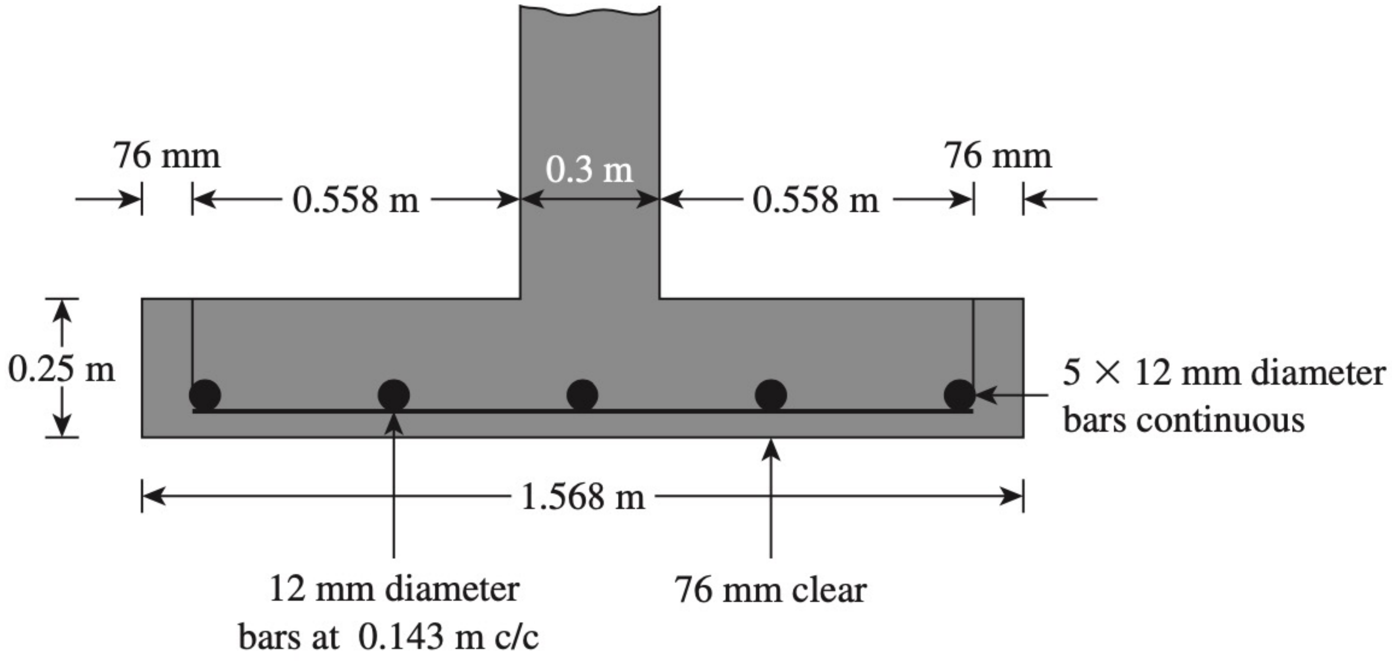
Use $A_s = (0.0018)(1.568)(0.25) = 0.000706\ m^2 = 706\ mm^2$. Provide 7 × 12 mm bars (A_s = 791 mm^2).

Minimum reinforcement should be furnished in the long direction to offset shrinkage and temperature effects (ACI Code Section 7.12.). So,

$$A_s = (0.0018)(b)(d) = (0.0018)\,[(0.558 + 0.076)(2) + 0.3](0.168)$$
$$= 0.000474\ m^2 = 474\ mm^2$$

Provide 5 × 12 mm bars ($A_s = 565\ mm^2$).

The final design sketch is shown in Figure:



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