COMPRESSIBILITY AND CONSOLIDATION OF SOILS

When a structure is placed on a foundation consisting of soil, the loads from the structure cause the soil to be stressed. The two most important requirements for the stability and safety of the structure are

- **COMPRESSIBILITY OF SOILS**

The volume decrease of a soil under stress might be conceivably attributed to:

1. Compression of the solid grains;
2. Compression of pore water or pore air;
3. Expulsion of pore water or pore air from the voids, thus decreasing the void ratio or porosity.

Specifically, the compressibility of a soil depends on the structural arrangement of the soil particles, and in fine-grained soils, the degree to which adjacent particles are bonded together. A structure which is more porous, such as a honey-combed structure, is more compressible than a dense structure.

When the pressure is increased, volume decrease occurs for a soil. If the pressure is later decreased some expansion will take place, but the rebound or recovery will not occur to the full extent.

In sands, consolidation may be generally considered to keep pace with construction; while, in clays, the process of consolidation proceeds long after the construction has been completed and thus needs greater attention.

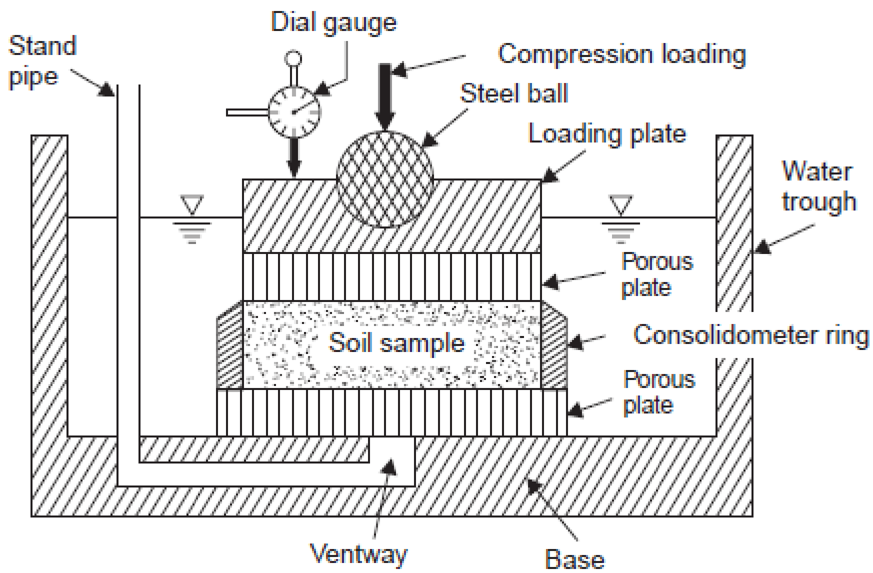
- **One-dimensional Compression and Consolidation**

'Consolidation', may be defined as the gradual and time-dependent process involving expulsion of pore water from a saturated soil mass, compression and stress transfer. This definition is valid for the one-dimensional as well as the general three-dimensional case.

1- Escape of pore water must occur during the compression or one-dimensional consolidation of a saturated soil; this escape takes place according to Darcy's law.
2- The time required for the compression or consolidation is dependent upon the coefficient of permeability of the soil and may be quite long if the permeability is low.

- **Compressibility and Consolidation Test—Oedometer**

The apparatus developed by Terzaghi for the determination of compressibility characteristics including the time-rate of compression is called the Oedometer, it was later improved by Casagrande and G. Gilboy and referred to as the Consolidometer.



(a) Fixed ring type

1- There are two types: ***The fixed ring type and the floating ring type***. In the fixed ring type, the top porous plate along is permitted to move downwards for compressing the specimen.
2- But, in the floating ring type, both the top and bottom porous plates are free to move to compress the soil sample. Direct measurement of the permeability of the sample at any stage of the test is possible only with the fixed ring type.

3- However, the effect of side friction on the soil sample is smaller in the floating type, while lateral confinement of the sample is available in both to simulate a soil mass in-situ.

4- The consolidation test consists in placing a representative undisturbed sample of the soil in a consolidometer ring, subjecting the sample to normal stress in predetermined stress increments through a loading machine and during each stress increment, observing the reduction in the height of the sample at different elapsed times after the application of the load.

5- The time-rate of volume change differs significantly for cohesionless soils and cohesive soils. Cohesionless soils experience compression relatively quickly, often instantaneously, after the load is imposed. But clay soils require a significant period before full compression occurs under an applied loading.

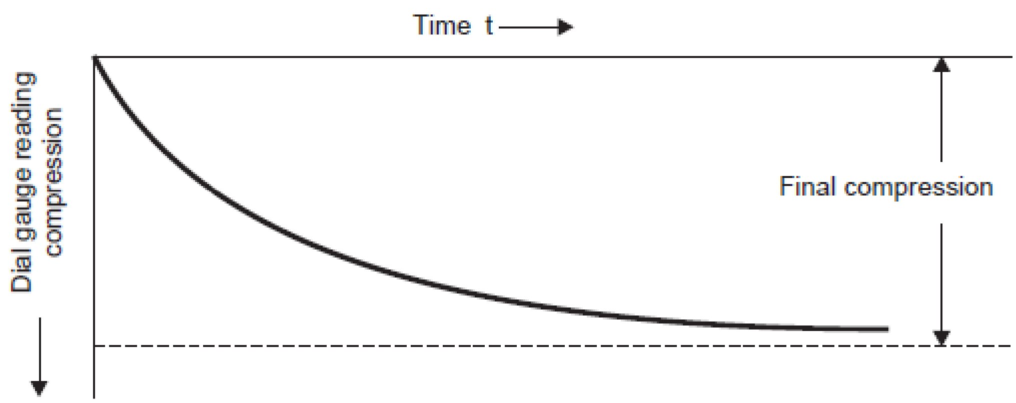
6- An initial setting load of 5 kN/m^2, which may be as low as 2.5 kN/m^2 for very soft soils, shall be applied until there is no change in the dial gauge reading for two consecutive hours or for a maximum of 24 hours. A normal load to give the desired pressure intensity shall be applied to the soil, a stopwatch being started simultaneously with loading. The dial gauge reading shall be recorded after various intervals of time—0.25, 1, 2.25, 4, 6.25, to 1440 minutes.

7- Throughout the test, the container shall be kept filled with water in order to prevent desiccation and to provide water for rebound expansion. After the final reading has been taken for 10 kN/m^2 the load shall be reduced to the initial setting load, kept for 24 hours and the final reading of the dial gauge noted.

- **Presentation and Analysis of Compression Test Data**

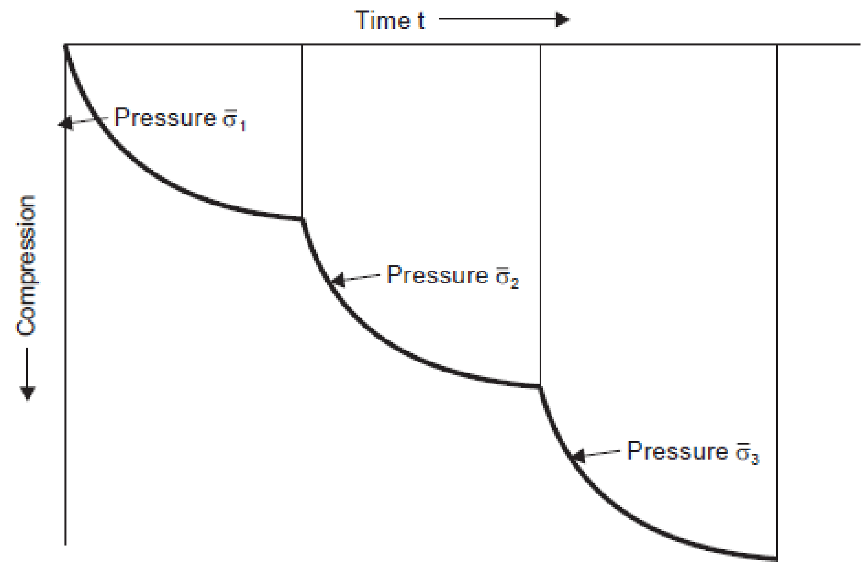
- The consolidation is rapid at first, but the rate gradually decreases. After a time, the dial reading becomes practically steady, and the soil sample may be assumed to have reached a condition of equilibrium.

- For the common size of the soil sample, this condition is generally attained in about twenty-four hours, although, theoretically speaking, the time required for complete consolidation is infinite.



Typical time-compression curve for a stress increment on clay

- The time-compression curves for consecutive increments of stress appear somewhat as shown in Fig. below.



Time-compression curve for successive increments of stress

- Since compression is due to decrease in void spaces of the soil, it is commonly indicated as a change in the void ratio.

$$e = \frac{V}{V_s} - 1$$

$$V_s = \frac{W_s}{G \cdot \gamma_w}$$

$$V = A \cdot H \qquad V = (1+e)\, v_s$$

Here, A = area of cross-section of the sample;

H = height of the sample at any stage of the test;

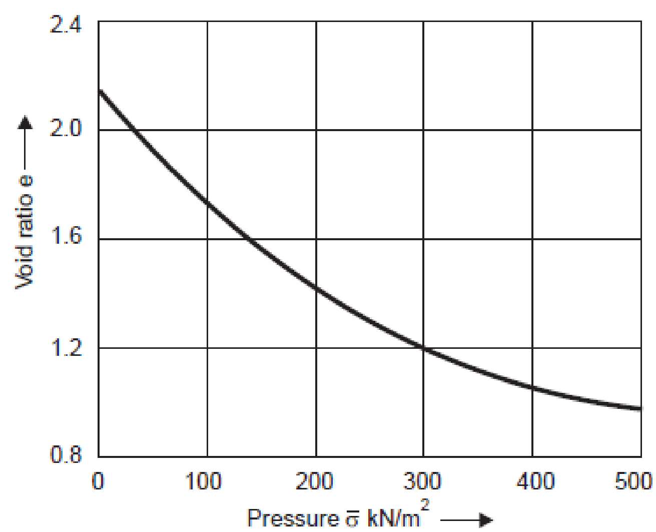
W_s = weight of solids or dry soil, obtained by drying and weighing the sample at the end of the test;

G = specific gravity of solids, found separately for the soil sample.

- At any stage of the test, the height of the sample may be obtained by deducting the reduction in thickness. $e = w.\ G$, --- $V = A.H = Vs(1 + e)$

$$\frac{\Delta e}{1+e_0} = \frac{\frac{\Delta H}{H_s}}{1+\frac{H_v}{H_s}} = \frac{\frac{\Delta H}{H_s}}{\frac{H_{s+}H_v}{H_s}} = \frac{\Delta H}{H_{s+}H_v} = \frac{\Delta H}{H}$$

$$\frac{\Delta H}{H} = \frac{\Delta e}{(1+e)} \qquad \Delta e = [(1+e)/H].\Delta H$$



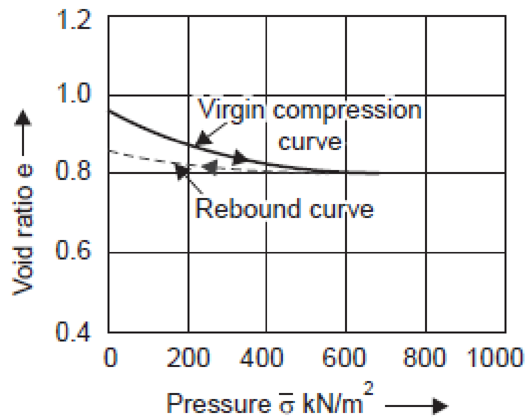
Pressure-void ratio relationship

The slope of this curve at any point is defined as the coefficient of compressibility, *av*.

$$a_v = -\frac{\Delta e}{\Delta \bar{\sigma}}$$

- **Compressibility of Sands**

The pressure-void ratio relationship for typical sand under one-dimensional compression is shown in Fig. 1. A typical time-compression curve for an increment of stress in Fig. 2.



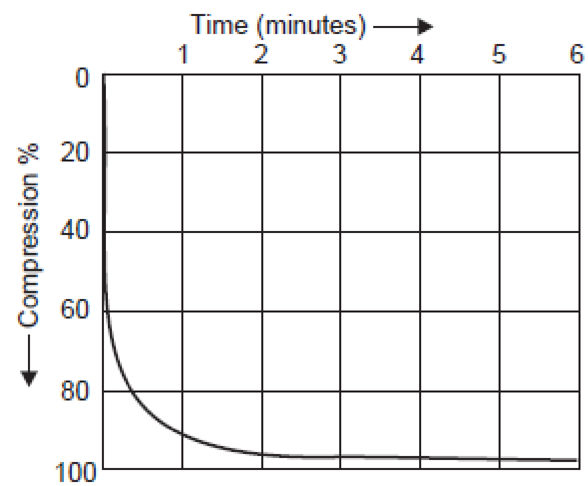


Fig. 1 Pressure void-ratio relationship for a typical sand **Fig. 2** Typical time-compression curve for a sand

- In about one minute about 95% of the compression has occurred in this particular case.

- In clean sands, it is about the same whether it is saturated or dry.

- **Compressibility and Consolidation of Clays**

A typical pressure versus void ratio curve for clay to natural pressure scale is shown in Fig1 below, and to the logarithmic pressure scale in Fig.2

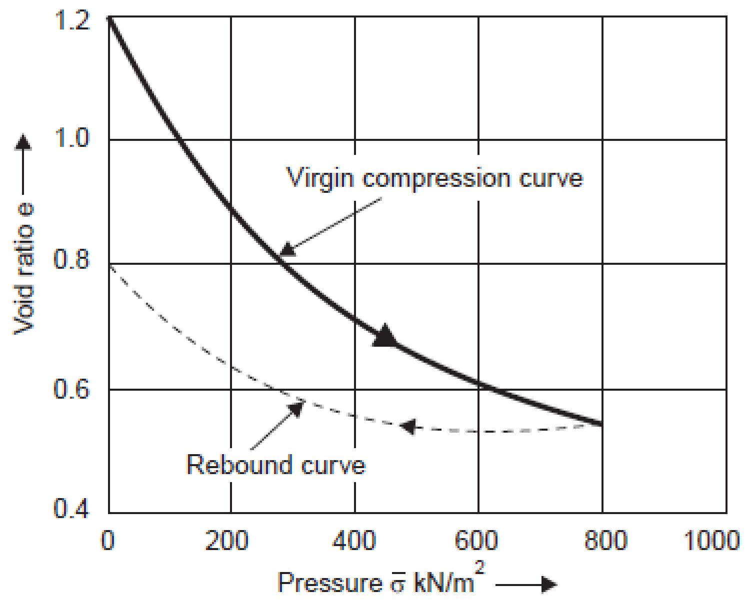


Fig 1 Pressure-void ratio relationship for a typical clay
(Natural or arithmetic scale)

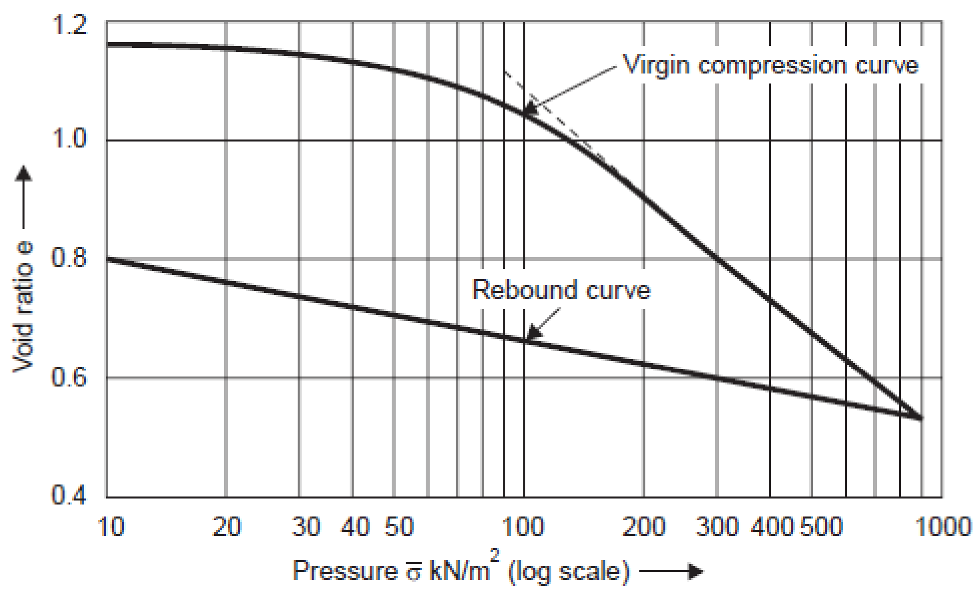


Fig. 2 Pressure-void ratio relationship for a typical clay
(Pressure to logarithmic scale)

In the semi-logarithmic plot, it can be seen that the virgin compression curve in this case approximates a straight line from about 200 kN/m^2 pressure. The equation of this straight line portion may be written in the following form:

$$e = e_0 - C_c \log_{10} \frac{\bar{\sigma}}{\bar{\sigma}_0}$$

Where *e* corresponds to σ and e_0 corresponds to σ_0, The numerical value of the slope of this straight line, Cc.

- Skempton established a relationship between the compressibility of a clay, as indicated by its compression index, and the liquid limit Cc = 0.009 (wL – 10).

$$C_c = \frac{(e - e_0)}{\log_{10} \frac{\bar{\sigma}}{\bar{\sigma}_0}}$$

The rebound curve obtained during unloading may be similarly expressed with C_e designating what is called the 'Expansion index':

$$e = e_0 - C_e \log_{10} \frac{\bar{\sigma}}{\bar{\sigma}_0}$$

- If, after complete removal of all loads, the sample is reloaded with the same series of loads as in the initial cycle, a different curve, called the 'recompression curve' is obtained. - Some of the volume change due to external loading is permanent.

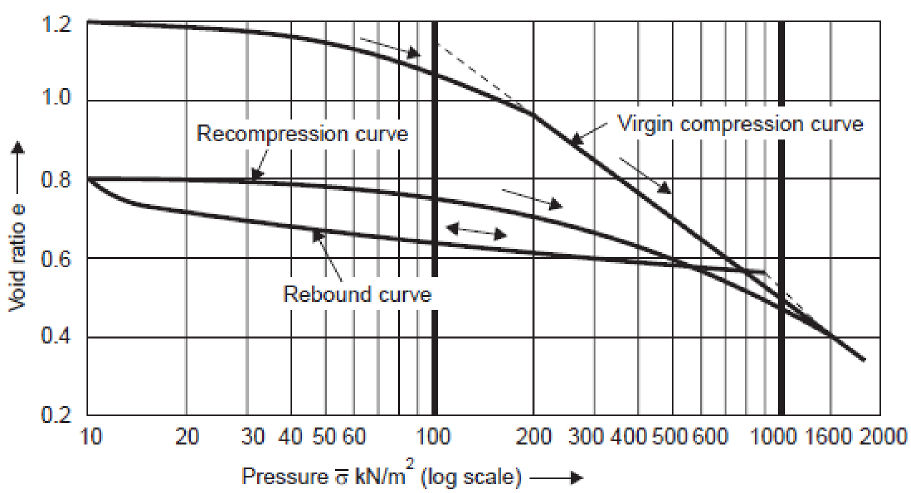
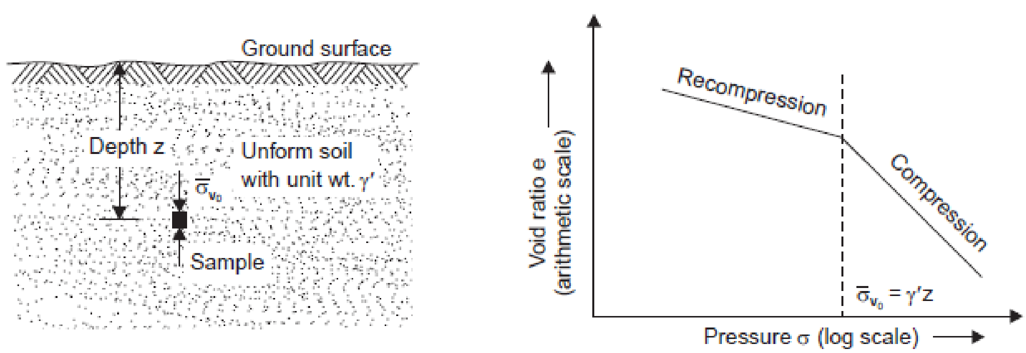


Fig. 3 Virgin compression, rebound and recompression

Curves for a clay (Pressure to logarithmic scale)

- may be noted from **Fig.** 3 that the curvature of the virgin compression curve at pressures smaller than about 200 kN/m^2 resembles the curvature of the recompression curve at pressures smaller than about 800 kN/m^2 from which the rebound occurred. This resemblance indicates that the specimen was probably subjected to a pressure of about 150 to 200 kN/m^2 at some time before its removal from the ground.

- As an example let us consider a soil sample obtained from a site from a depth *z* as shown in Fig. 4 (*a*). The ground surface has never been above the existing level and there never was extra external loading acting on the area. Thus, the maximum stress to which the soil sample was ever subjected is the current over burden pressure σ_{v0} ($= \gamma'.z$).



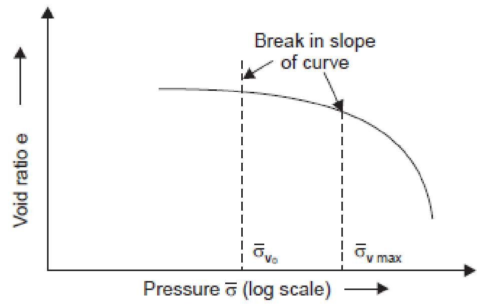
(a) Location of sample taken for compression test

(b) Compression test results for the sample

Fig. 4 Conditions applying to compression test sample

- The portion of the curve prior to pressure (σ_{v0}) represents a recompression curve, while that at greater pressures than (σ_{v0}) represents the virgin compression curve as shown in Fig.4 (*b*).

- If the ground surface had at some time is past history been above the existing surface and had been eroded away, or if any other external load acted earlier and got released, so the existing over-burden pressure, (σ_{v0}), would not be the maximum pressure. If this greatest past pressure is σ_{vmax} , greater than σ_{v0} , compression test would be as shown in Fig. 5.



- **Normally Consolidated Soil and Over consolidated Soil**

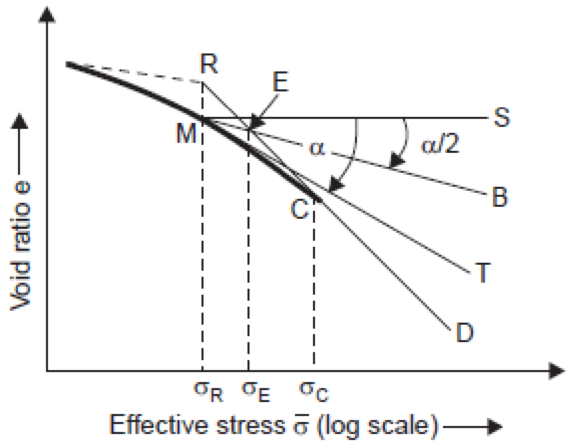
A quantitative measure of the degree of over consolidation is what is known as the ‘Overconsolidation Ratio’, OCR. It is defined as follows:

$$\text{OCR} = \frac{\text{Maximum effective stress to which the soil has been subjected in its stress history}}{\text{Existing effective stress in the soil}}$$

Thus, the maximum OCR of normally consolidated soil equals 1.

In this connection, it is of considerable engineering interest to be able to determine the past maximum effective stress (σ_E).

- Casagrande (1936) proposed a geometrical technique to evaluate past maximum effective stress or pre consolidation pressure from the *e* versus log σ plot obtained by loading a sample in the laboratory.



1. The point of maximum curvature **M** on the curved portion of the *e* vs. log σ plot is located.

2. A horizontal line **MS** is drawn through **M**.

3. A tangent **MT** to the curved portion is drawn through **M**.

4. The angle **SMT** is bisected, **MB** being the bisector.

5. The straight portion **DC** of the plot is extended backward to meet **MB** in **E**.

6. The pressure corresponding to the point **E,** (σ_E)**,** is the most probable past maximum effective stress or the preconsolidation pressure.

* If $\sigma_E = \sigma_o$ Normally consildation clay.

* If $\sigma_E > \sigma_o$ over consolidated clay or pre consolidated clay .

* If $\sigma_E < \sigma_o$ under consolidated clay.

- **Modulus of Volume Change and Consolidation Settlement**

The 'modulus of volume change is defined as the change in volume of a soil per unit initial volume due to a unit increase in effective stress

$$m_v = -\frac{\Delta e}{(1+e_0)} \cdot \frac{1}{\Delta \bar{\sigma}} \qquad -\frac{\Delta e}{\Delta \bar{\sigma}} = a_v, \text{ the coefficient of compressibility.}$$

and

$$m_v = \frac{a_v}{(1+e_0)}$$

but $m_v = -\frac{\Delta H}{H_0}.\frac{1}{\Delta\overline{\sigma}}$ so $\Delta H = m_v.H_0.\Delta\overline{\sigma}$

$$m_v = \frac{e}{(1+e_0)}.\frac{1}{\Delta\overline{\sigma}}, \text{ ignoring sign.}$$

$$\frac{\Delta H}{H_0} = \frac{\Delta e}{(1+e_0)}$$

$$S_c = \Delta H = \frac{\Delta e}{(1+e_0)}.H_0$$

Substituting for Δ*e* in terms of the compression index, *Cc,* recognizing (e – e_0) as Δe, we have:

$$S_c = \Delta H = H_0.\frac{C_c}{(1+e_0)}.\log_{10}\frac{\overline{\sigma}}{\overline{\sigma}_0}$$

$$S_c = H_0\cdot\frac{C_c}{(1+e_0)}.\log_{10}\left(\frac{\overline{\sigma}_0+\overline{\sigma}}{\overline{\sigma}_0}\right)$$

- **TERZAGHI'S THEORY OF ONE-DIMENSIONAL CONSOLIDATION**

Now let us see the derivation of Terzaghi's theory with respect to the laboratory oedometer sample with double drainage as shown in Fig. 5.

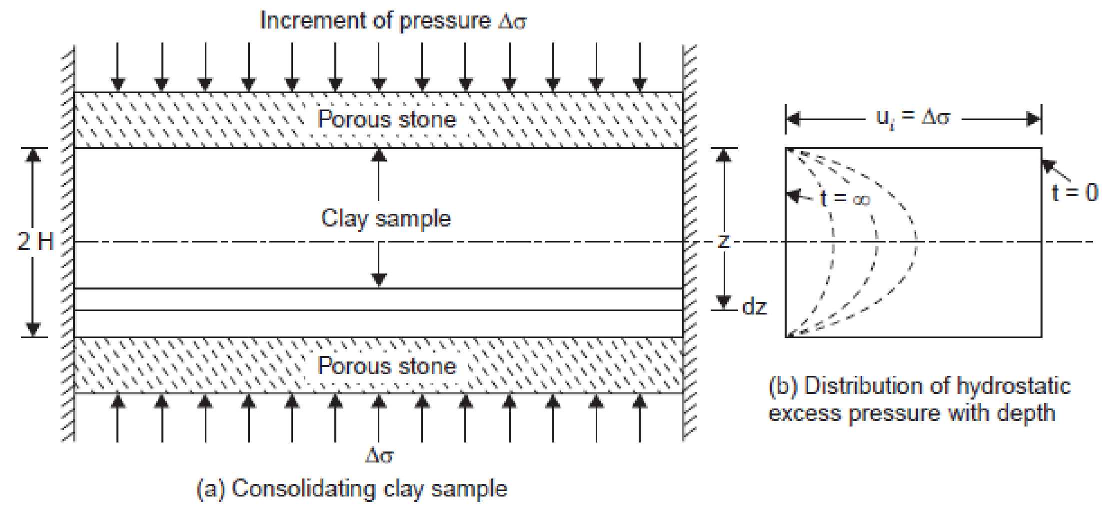


Fig. 5 Consolidation of a clay sample with double drainage

where $c_v = \dfrac{k}{\gamma_w \cdot m_v}$ Then $\dfrac{\partial u}{\partial t} = c_v \cdot \dfrac{\partial^2 u}{\partial z^2}$

C_v is known as the "Coefficient of consolidation". *u* represents the hydrostatic excess pressure at a depth *z* from the drainage face at time *t* from the start of the process of consolidation.

$$c_v = \frac{k}{\gamma_w m_v} = \frac{k(1+e_0)}{a_v \gamma_w}$$

$$U_z = (u_i - u)/u_i = \left(1 - \frac{u}{u_i}\right)$$

U_z = degree of consolidation, u_i = initial pressure, u pressure at any moment.

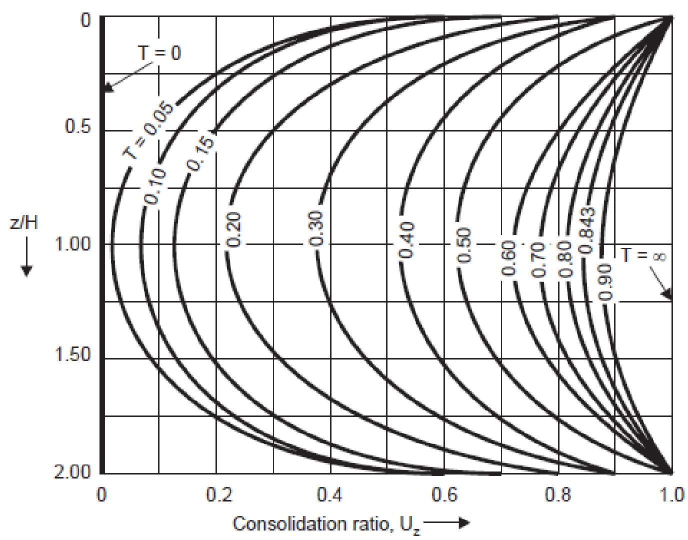
- The third dimensionless parameter, relating to time, and called 'Time-factor', *T*, is defined as follows:

$$T = \frac{c_v t}{H^2}$$

where C_v is the coefficient of consolidation, *H* is the length drainage path, and *t* is the elapsed time from the start of consolidation process.

- **GRAPHICAL PRESENTATION OF CONSOLIDATION RELATIONSHIPS**

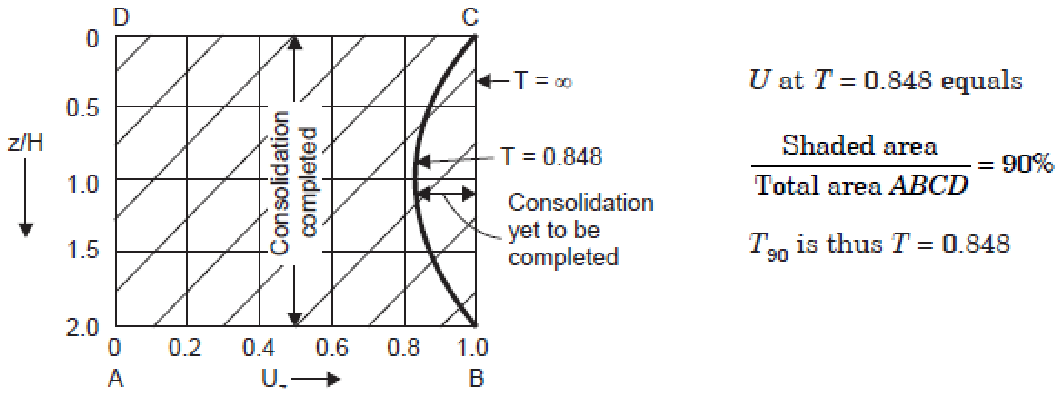
A graphical presentation of the results indicated by assigning different values of z/H and T, different values of U_z are solved and plotted to obtain the family of curves shown.



At the start of the process, $t = 0$ and $T = 0$, and U_z is zero for all depths. At any finite time factor, the consolidation ratio is 1 at drainage faces and is minimum at the middle of the layer. For example, for $T = 0.20$: $Uz = 0.23$ at $z/H = 1$; $Uz = 0.46$ at $z/H = 0.25$ and 1.75; and $Uz = 0.70$ at $z/H = 0.125$ and 1.875. This indicates that at a depth of one-eighth of the layer, consolidation is 70% complete; at a depth of one-fourth of the layer, consolidation is 46% complete; while, at the middle of the layer, consolidation is just 23% complete

Figure above does not depict how much consolidation occurs *as a whole* in the entire stratum. This information is of primary concern to the geotechnical engineer and may be deduced from Fig. above by the following procedure:

The relation between U_z and z/H for a time factor, $T = 0.848$, is reproduced in Fig. below.



Average consolidation at time factor 0.848

When $U < 60\%$, $T = (\pi/4)\, U^2$

When $U > 60\%$, $T = -\,0.9332 \log_{10} (1 - U) - 0.0851$.

- **EVALUATION OF COEFFICIENT OF CONSOLIDATION FROM OEDOMETER TEST DATA**

The coefficient of consolidation, C_v, in any stress range of interest, may be evaluated from its definition, by experimentally determining the parameters k, a_v and e_0 for the stress range under consideration. k may be got from a permeability test conducted on the oedometer sample itself, after complete consolidation under the particular stress increment.

a_v and e_0 may be obtained from the oedometer test data, by plotting the $e - \sigma$ curve.

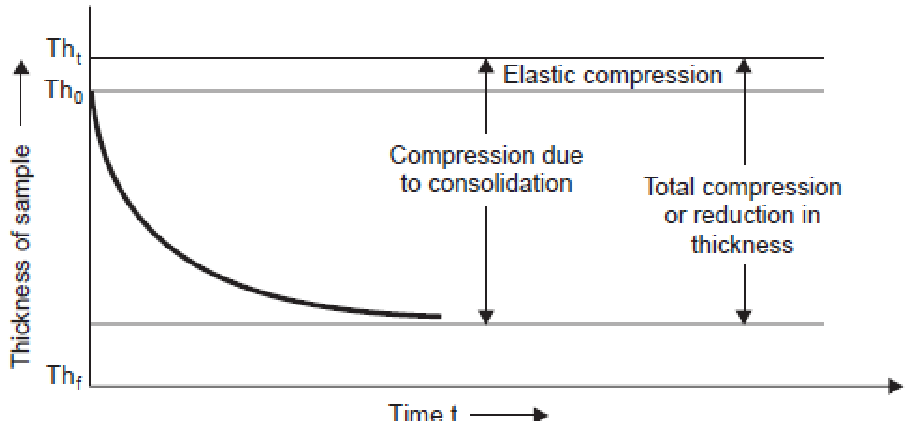
The more generally used fitting methods are the following:

(*a*) The square root of time fitting method

(*b*) The logarithm of time fitting method

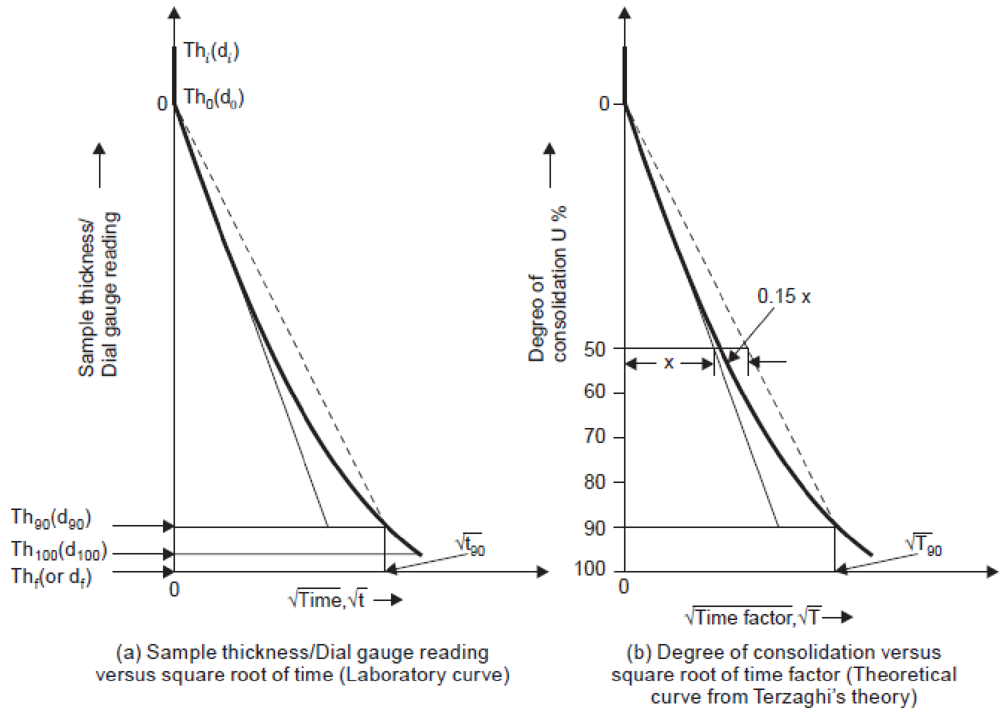
The Square Root of Time Fitting Method

The relation between the sample thickness and elapsed time since the application of the load increment is obtainable from an oedometer test and is somewhat as shown in Fig. below for a typical load-increment.



Time versus reduction in sample thickness for a load-increment

The theoretical curve on the square root plot is straight line up to about 60% consolidation with a gentle concave upwards curve thereafter. If another straight line, shown dotted, is drawn such that the abscissa of this line are 1.15 times those of the straight line portion of the theoretical curve, it can be shown to cut the theoretical curve at 90% consolidation.



Square root of time fitting method (After Taylor, 1948)

$$c_v = \frac{T_{90} H^2}{t_{90}}$$

The coefficient of consolidation, c_v, may be obtained from

where t_{90} is read off from Fig. above

T_{90} is 0.848 from Terzaghi's theory

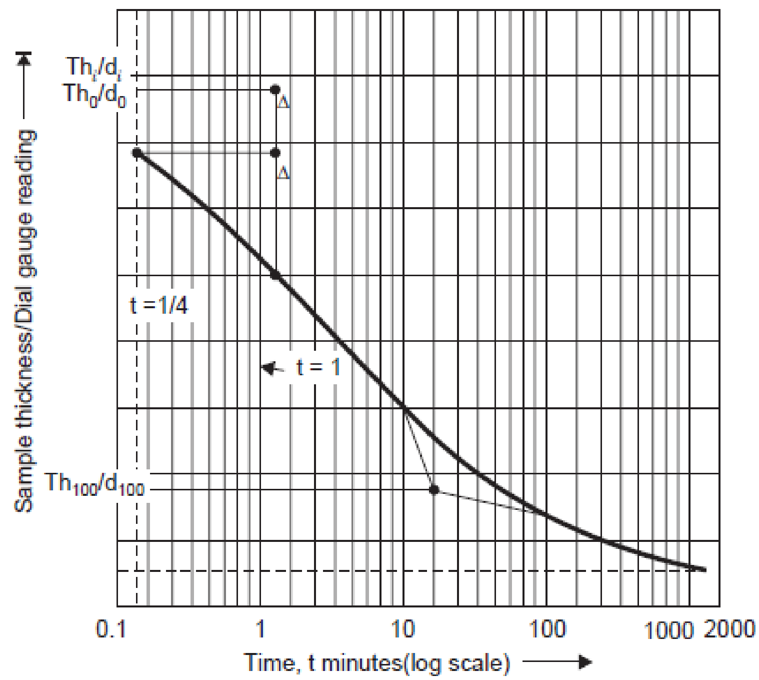
H is the drainage path, which may be taken as half the thickness of the sample for

Double drainage conditions, or as $(Th_0 + Th_f)/4$ in terms of the sample thickness

The Logarithm of Time Fitting Method

The point corresponding to 100 percent consolidation curve is plotted on a semi-logarithmic scale, with time factor on a logarithmic scale and degree of consolidation on arithmetic scale, the intersection of the tangent and asymptote is at the ordinate of 100% consolidation.

The difference in ordinates between two points with times in the ratio of 4 to 1 is marked off; then a distance equal to this difference may be stepped off above the upper points to obtain the corrected zero point. This point may be checked by more trials, with different pairs of points on the curve.



Sample thickness/Dial gauge reading
versus logarithm of time (Laboratory curve)

$$C_v = \frac{T_{50} H^2}{t_{50}}$$

where t_{50} is read off from Fig, T_{50} = 0.197 from Terzaghi's theory, and

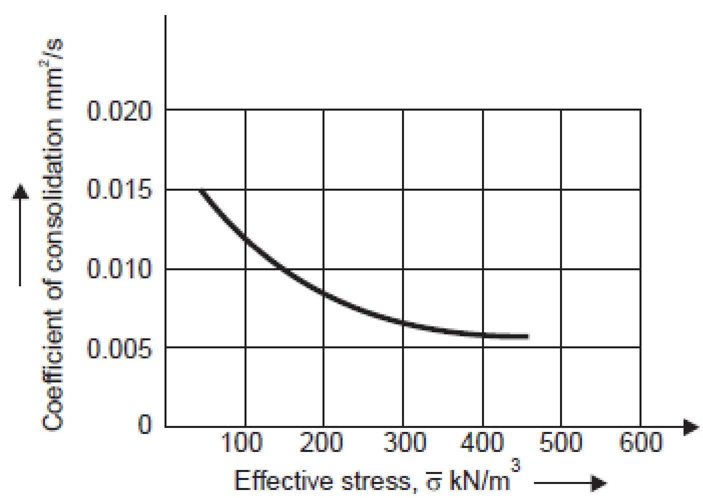
H is the drainage path as stated in the previous subsection.

Typical Values of Coefficient of Consolidation

7.7.3 Typical Values of Coefficient of Consolidation

The process of applying one of the fitting methods may be repeated for different increments of pressure using the time-compression curves obtained in each case. This is the reason for the caution that, for problems in the field involving settlement analysis, the coefficient of consolidation should be evaluated in the laboratory for the particular range of stress likely to exist in the field.

The range of values for C_v is rather wide from 5×10^{-4} mm^2/s to 2×10^{-2} mm^2/s. Further, it is also found that the value of C_v decreases as the liquid limit of the clay increases.

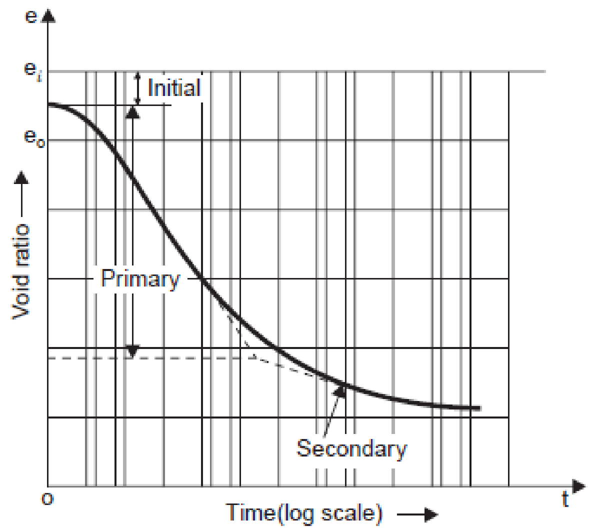


Variation of coefficient of consolidation with effective stress

SECONDARY CONSOLIDATION

When the hydrostatic excess pressure is fully dissipated, no more consolidation should be expected. However, in practice, the decrease in void ratio continues, though very slowly, for a long time after this stage, called 'Primary Consolidation'.

The effect or the phenomenon of continued consolidation after the complete dissipation of excess pore water pressure is termed 'Secondary Consolidation' and the resulting compression is called 'Secondary Compression'.



Secondary compression appears as a straight line sloping downward or, in some cases, as a straight line followed by a second straight line with a flatter slope. The void ratio, e_f, at the end of primary consolidation can be found from the intersection of the backward extension of the secondary line with a tangent drawn to the curve of primary compression, as shown in the figure. The rate of secondary compression, depends upon the increment of stress and the characteristics of the soil.

$$\Delta e = -\alpha \,.\, \log_{10}(t_2/t_1)$$

$$\varepsilon = \frac{\Delta H}{H} = -C_\alpha \log_{10}\left(\frac{t_2}{t_1}\right)$$

In other words, C_α may be taken to be the slope of the straight line representing the secondary compression on a plot of strain versus logarithm of time.

$$C_\alpha = \frac{\alpha}{(1+e)}$$

Sl. No	*Nature of Soil*	C_α *– Value*
1.	Over consolidated days	0.0005 to 0.0015
2.	Normally consolidated days	0.005 to 0.030
3.	Organic soils, peats	0.04 to 0.10

Example 27: In a consolidation test the following results have been obtained. When the load was changed from 50 kN/m^2 to 100 kN/m^2, the void ratio changed from 0.70 to 0.65. Determine the coefficient of volume decrease, *mv* and the compression index, *Cc*.

$e_0 = 0.70$ $\sigma_0 = 50$ kN/m^2

$e_1 = 0.65$ $\sigma = 100$ kN/m^2

Coefficient of compressibility, $a_v = \frac{\Delta e}{\Delta \bar{\sigma}}$, ignoring sign.

$$= \frac{(0.70 - 0.65)}{(100 - 50)} \text{ m}^2/\text{kN} = 0.05/50 \text{ m}^2/\text{kN} = 0.001 \text{ m}^2/\text{kN}.$$

Modulus of volume change, or coefficient of volume decrease,

$$m_v = \frac{a_v}{(1 + e_0)} = \frac{0.001}{(1 + 0.70)} = \frac{0.001}{1.7} \text{ m}^2/\text{kN}.$$

$$= 5.88 \times 10^{-4} \text{ m}^2/\text{kN}$$

Compression index, $C_c = \frac{\Delta e}{\Delta(\log \bar{\sigma})} = \frac{(0.70 - 0.65)}{(\log_{10} 100 - \log_{10} 50)}$

$$= \frac{0.05}{\log_{10} \frac{100}{50}} = \frac{0.05}{\log_{10} 2} = \frac{0.050}{0.301} = \mathbf{0.166.}$$

Example 28: A sand fill compacted to a bulk density of 18.84 kN/m^3 is to be placed on a compressible saturated marsh deposit 3.5 m thick. The height of the sand fill is to be 3 m. If the volume compressibility m_v of the deposit is 7×10^{-4} m^2/kN, estimate the final settlement of the fill.

H_t. of sand fill = 3 m

Bulk unit weight of fill = 18.84 kN/m^3

Increment of the pressure on top of marsh deposit $\Delta\sigma = 3 \times 18.84 = 56.52$ kN/m^2

Thickness of marsh deposit, $H0 = 3.5$ m

Volume compressibility $mv = 7 \times 10^{-4}$ m^2/kN

Final settlement of the marsh deposit, $\Delta H = mv.H_0.\Delta\sigma$

$= 7 \times 10^{-4} \times 3500 \times 56.52$ mm = **138.5 mm**.

Example 29: A layer of soft clay is 6 m thick and lies under a newly constructed building. The weight of sand overlying and clayey layer produces a pressure 260 kN/m^2 and the new construction increases the pressure 100 kN/m^2. If the compression index is 0.5, compute the settlement. Water content is 40% and specific gravity of grains is 2.65.

Initial pressure, $\sigma_0 = 260$ kN/m^2

Increment of pressure, $\Delta\sigma = 100$ kN/m^2

Thickness of clay layer, $H = 6$ m = 600 cm.

Compression index, $Cc = 0.5$, Water content, $w = 40\%$

Specific gravity of grains, $G = 2.65$

Void ratio, $e_0 = wG$, (since the soil is saturated) = 0.40 × 2.65 = 1.06

$$S = \frac{H.C_c.}{(1+e_0)} \log_{10}\left(\frac{\bar{\sigma}_0 + \Delta\bar{\sigma}}{\bar{\sigma}_0}\right)$$

$$= \frac{600 \times 0.5}{(1+1.06)} \log_{10}\left(\frac{260+100}{260}\right) \text{ cm}$$

$$= \frac{300}{2.06} \log_{10}\left(\frac{360}{260}\right) \text{ cm}$$

$$= \mathbf{21.3\ cm}.$$

Example 30: There is a bed of compressible clay of 4 m thickness with pervious sand on top and impervious rock at the bottom. In a consolidation test on an undisturbed specimen of clay from this deposit 90% settlement was reached in 4 hours. The specimen was 20 mm thick. Estimate the time in years for the building founded over this deposit to reach 90% of its final settlement.

This is a case of one-way drainage in the field.

∴ Drainage path for the field deposit, H_f = 4 m = 4000 mm.

In the laboratory consolidation test, commonly it is a case of two-way drainage.

∴ Drainage path for the laboratory sample, H_1 = 20/2 = 10 mm

Time for 90% settlement of laboratory sample = 4 hrs.

Time factor for 90% settlement, T_{90} = 0.848

$$\therefore \quad T_{90} = \frac{C_v t_{90_f}}{H_f^2} = \frac{C_v t_{90_l}}{H_l^2}$$

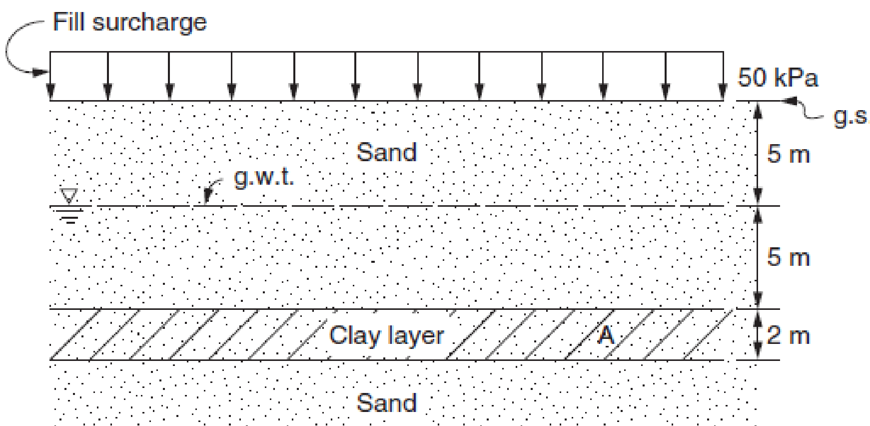
$$\text{or} \quad \frac{t_{90_f}}{H_f^2} = \frac{t_{90_l}}{H_l^2}$$

$$\therefore \quad t_{90f} = \frac{t_{90_l}}{H_l} \times H_f = \frac{4 \times (4000)^2}{10^2} \text{ hrs}$$

$$= \frac{4 \times 400}{24 \times 365} \text{ years}$$

$$\approx \mathbf{73\ years.}$$

Example 31: A site has a level ground surface and a level groundwater table located 5 m below the ground surface. As shown in Fig. below, subsurface exploration has discovered that the site is underlain with sand, except for a uniform and continuous clay layer that is located at a depth of 10 to 12 m below ground surface. Below the groundwater table, the pore water pressures are hydrostatic in the sand layers (i.e., no artesian pressures). The average void ratio e_o of the clay layer is 1.10 and the buoyant unit weight γ_b of the clay layer = 7.9 kN/m^3. The total unit weight γ_t of the sand above the groundwater table = 18.7 kN/m^3 and the total unit weight γ_t of the sand below the groundwater table = 19.7 kN/m^3. The compression index C_c = 0.83 and recompression index C_r = 0.05. Determine the primary consolidation settlement S_c of the 2-m-thick clay layer if a uniform fill surcharge of 50 kPa is applied over a very large area at ground surface. If a laboratory consolidation test performed on an undisturbed specimen obtained from the center of the clay layer (Point A) indicates the maximum past pressure σ_E = 150 kPa and σ_E= 175 kPa and σ_E=250 kPa



Solution The first step is to determine the vertical effective stress σ_o at the center of the clay layer or:

σ_o = (5 m)(18.7 kN/m^3) + (5 m)(19.7 – 9.81 kN/m^3) + (1 m)(7.9 kN/m^3) = 150 kPa.

1) Since σ_o is equal to σ_E, the clay is normally consolidated (OCR = 1).

$$Sc = C_c\,[H_o /(1 + e_o)] \log [(\sigma_o + \Delta\sigma_v)/ \sigma_o]$$

$$Sc = (0.83)[(2\text{ m})/(1 + 1.1)] \log [(150\text{ kPa} + 50\text{ kPa})/150\text{ kPa}] = 0.10\text{ m}$$

2) Since σ_o is less than σ_E, the clay layer is overconsolidated (OCR > 1).

Since $\sigma_o + \Delta\sigma_v$ = 200 kPa > σ_E

$$S_c = Cr\,[Ho /(1 + e_o)] \log (\sigma_E / \sigma_o) + C_c\,[Ho /(1 + e_o)] \log [(\sigma_o + \Delta\sigma_v)/ \sigma_E]$$

S_c = (0.05)[(2 m)/(1 + 1.1)] log (175 kPa/150 kPa) + (0.83)[(2 m)/(1 + 1.1)] log [(150 kPa + 50 kPa)/175 kPa] = 0.049 m

3) Since $\sigma_o + \Delta\sigma_v$ = 200 kPa < σ_E

$S_c = C_r\,[Ho/(1 + e_o)]\,\log\,[(\sigma_o + \Delta\sigma_v)/\,\sigma_o]$

S_c = (0.05)[(2 m)/(1 + 1.1)] log [(150 kPa + 50 kPa)/150 kPa] = 0.006 m

COMPRESSIBILITY AND CONSOLIDATION OF SOILS

When a structure is placed on a foundation consisting of soil, the loads from the structure cause the soil to be stressed. The two most important requirements for the stability and safety of the structure are

- **COMPRESSIBILITY OF SOILS**

The volume decrease of a soil under stress might be conceivably attributed to:

1. Compression of the solid grains;
2. Compression of pore water or pore air;
3. Expulsion of pore water or pore air from the voids, thus decreasing the void ratio or porosity.

Specifically, the compressibility of a soil depends on the structural arrangement of the soil particles, and in fine-grained soils, the degree to which adjacent particles are bonded together. A structure which is more porous, such as a honey-combed structure, is more compressible than a dense structure.

When the pressure is increased, volume decrease occurs for a soil. If the pressure is later decreased some expansion will take place, but the rebound or recovery will not occur to the full extent.

In sands, consolidation may be generally considered to keep pace with construction; while, in clays, the process of consolidation proceeds long after the construction has been completed and thus needs greater attention.

- **One-dimensional Compression and Consolidation**

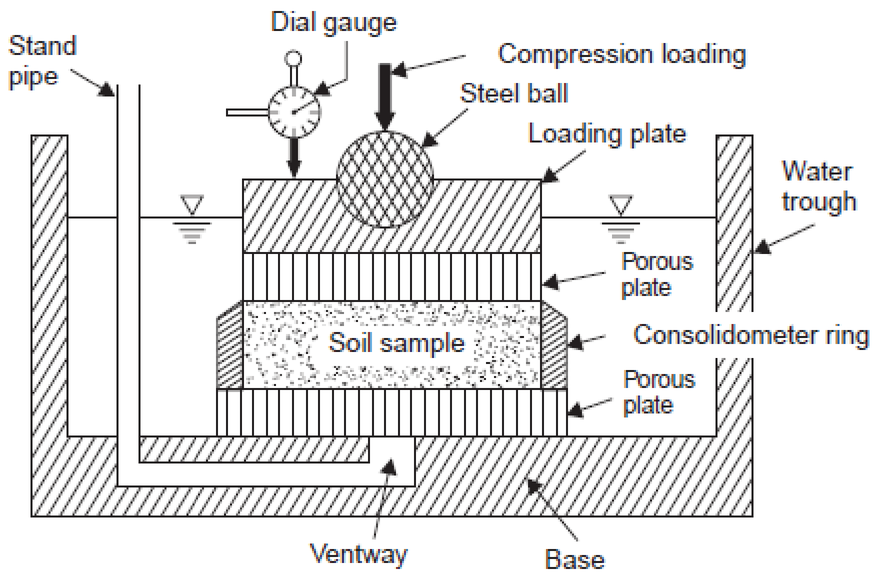
'Consolidation', may be defined as the gradual and time-dependent process involving expulsion of pore water from a saturated soil mass, compression and stress transfer. This definition is valid for the one-dimensional as well as the general three-dimensional case.

1- Escape of pore water must occur during the compression or one-dimensional consolidation of a saturated soil; this escape takes place according to Darcy's law.

2- The time required for the compression or consolidation is dependent upon the coefficient of permeability of the soil and may be quite long if the permeability is low.

- **Compressibility and Consolidation Test—Oedometer**

The apparatus developed by Terzaghi for the determination of compressibility characteristics including the time-rate of compression is called the Oedometer, it was later improved by Casagrande and G. Gilboy and referred to as the Consolidometer.



(a) Fixed ring type

1- There are two types: ***The fixed ring type and the floating ring type***. In the fixed ring type, the top porous plate along is permitted to move downwards for compressing the specimen.

2- But, in the floating ring type, both the top and bottom porous plates are free to move to compress the soil sample. Direct measurement of the permeability of the sample at any stage of the test is possible only with the fixed ring type.

3- However, the effect of side friction on the soil sample is smaller in the floating type, while lateral confinement of the sample is available in both to simulate a soil mass in-situ.

4- The consolidation test consists in placing a representative undisturbed sample of the soil in a consolidometer ring, subjecting the sample to normal stress in predetermined stress increments through a loading machine and during each stress increment, observing the reduction in the height of the sample at different elapsed times after the application of the load.

5- The time-rate of volume change differs significantly for cohesionless soils and cohesive soils. Cohesionless soils experience compression relatively quickly, often instantaneously, after the load is imposed. But clay soils require a significant period before full compression occurs under an applied loading.

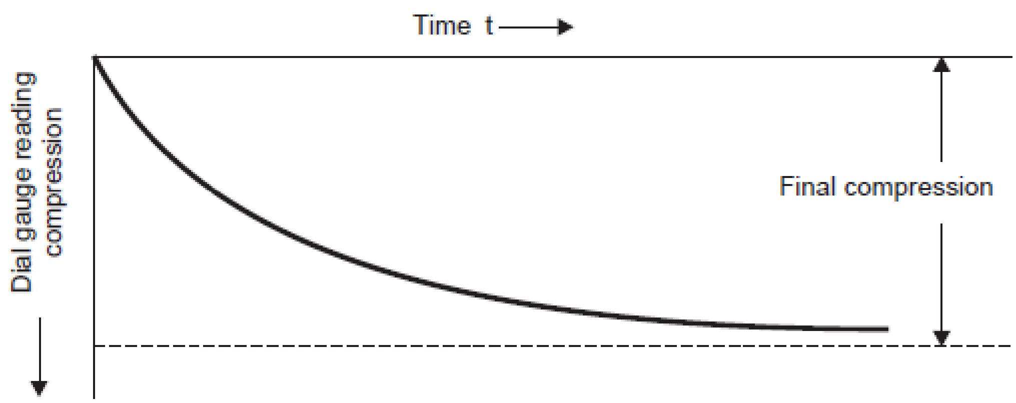
6- An initial setting load of 5 kN/m^2, which may be as low as 2.5 kN/m^2 for very soft soils, shall be applied until there is no change in the dial gauge reading for two consecutive hours or for a maximum of 24 hours. A normal load to give the desired pressure intensity shall be applied to the soil, a stopwatch being started simultaneously with loading. The dial gauge reading shall be recorded after various intervals of time—0.25, 1, 2.25, 4, 6.25, to 1440 minutes.

7- Throughout the test, the container shall be kept filled with water in order to prevent desiccation and to provide water for rebound expansion. After the final reading has been taken for 10 kN/m^2 the load shall be reduced to the initial setting load, kept for 24 hours and the final reading of the dial gauge noted.

- **Presentation and Analysis of Compression Test Data**

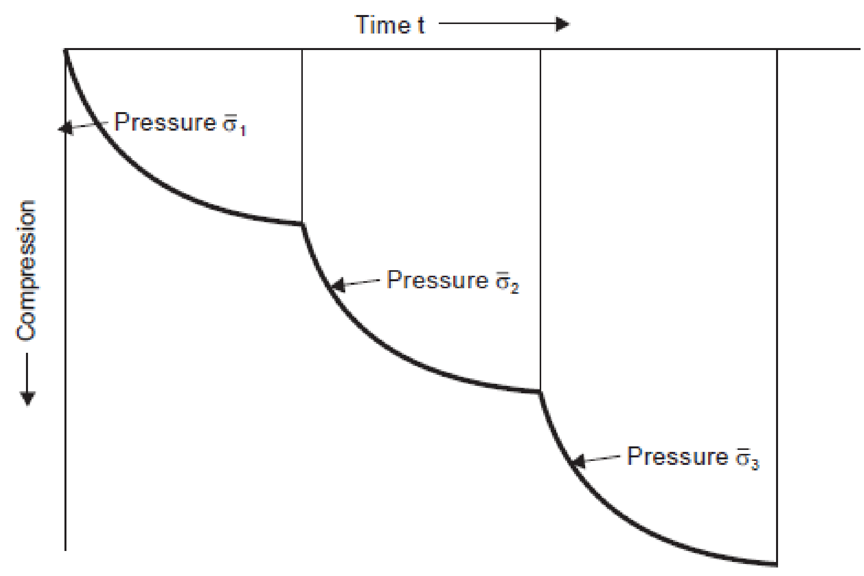
- The consolidation is rapid at first, but the rate gradually decreases. After a time, the dial reading becomes practically steady, and the soil sample may be assumed to have reached a condition of equilibrium.

- For the common size of the soil sample, this condition is generally attained in about twenty-four hours, although, theoretically speaking, the time required for complete consolidation is infinite.



Typical time-compression curve for a stress increment on clay

- The time-compression curves for consecutive increments of stress appear somewhat as shown in Fig. below.



Time-compression curve for successive increments of stress

- Since compression is due to decrease in void spaces of the soil, it is commonly indicated as a change in the void ratio.

$$e = \frac{V}{V_s} - 1$$

$$V_s = \frac{W_s}{G \cdot \gamma_w}$$

$$V = A \cdot H \qquad V = (1+e)\, v_s$$

Here, A = area of cross-section of the sample;

H = height of the sample at any stage of the test;

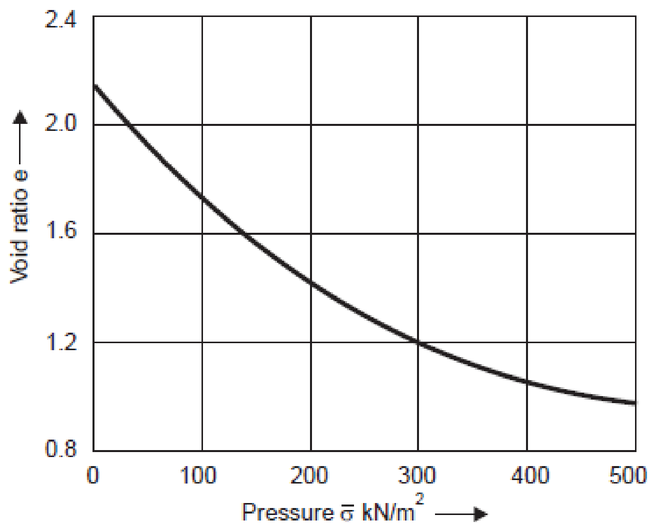
W_s = weight of solids or dry soil, obtained by drying and weighing the sample at the end of the test;

G = specific gravity of solids, found separately for the soil sample.

- At any stage of the test, the height of the sample may be obtained by deducting the reduction in thickness. $e = w.\ G$, --- $V = A.H = Vs(1 + e)$

$$\frac{\Delta e}{1+e_0} = \frac{\frac{\Delta H}{H_s}}{1+\frac{H_v}{H_s}} = \frac{\frac{\Delta H}{H_s}}{\frac{H_{s+}H_v}{H_s}} = \frac{\Delta H}{H_{s+}H_v} = \frac{\Delta H}{H}$$

$$\frac{\Delta H}{H} = \frac{\Delta e}{(1+e)} \qquad \Delta e = [(1+e)/H].\Delta H$$



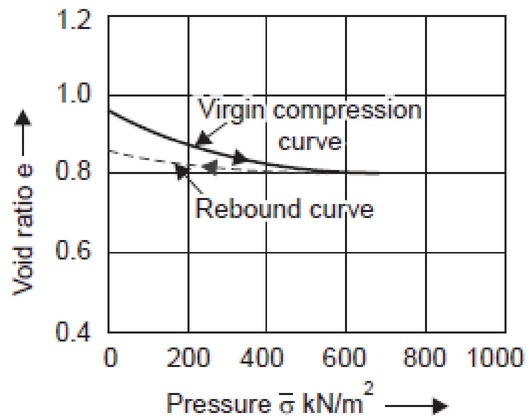
Pressure-void ratio relationship

The slope of this curve at any point is defined as the coefficient of compressibility, *av*.

$$a_v = -\frac{\Delta e}{\Delta \bar{\sigma}}$$

- **Compressibility of Sands**

The pressure-void ratio relationship for typical sand under one-dimensional compression is shown in Fig. 1. A typical time-compression curve for an increment of stress in Fig. 2.



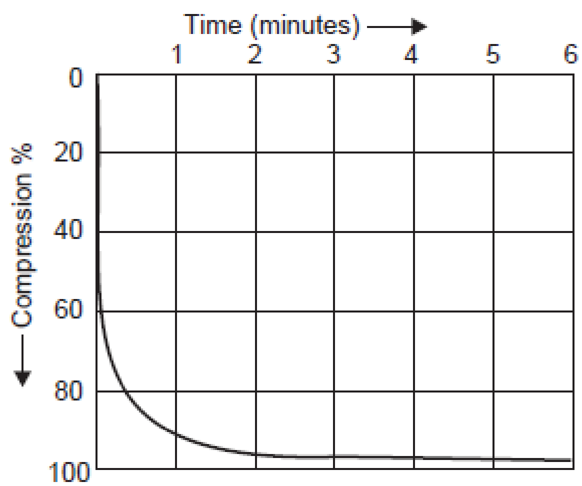


Fig. 1 Pressure void-ratio relationship for a typical sand

Fig. 2 Typical time-compression curve for a sand

- In about one minute about 95% of the compression has occurred in this particular case.

- In clean sands, it is about the same whether it is saturated or dry.

- **Compressibility and Consolidation of Clays**

A typical pressure versus void ratio curve for clay to natural pressure scale is shown in Fig1 below, and to the logarithmic pressure scale in Fig.2

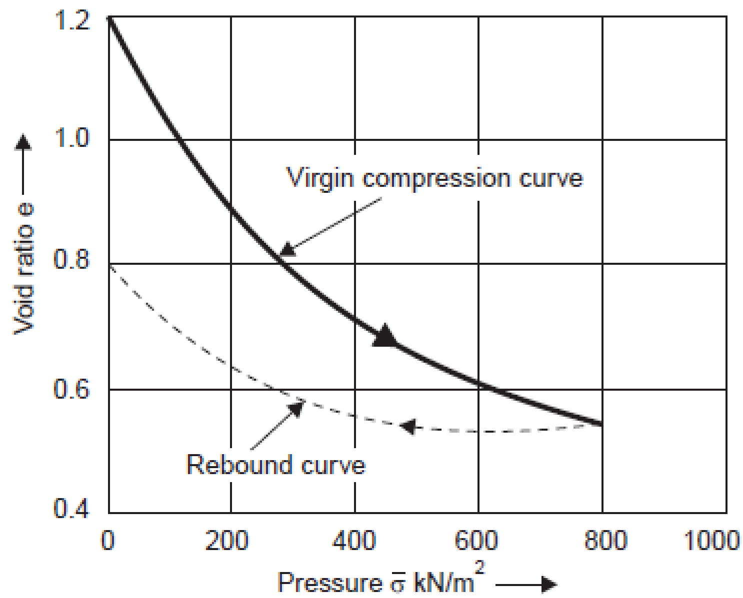


Fig 1 Pressure-void ratio relationship for a typical clay
(Natural or arithmetic scale)

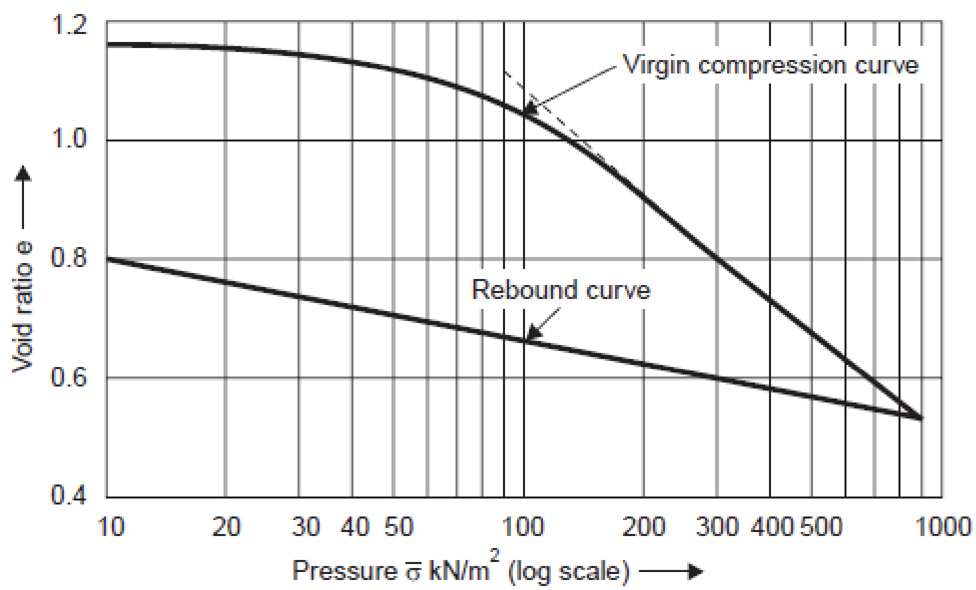


Fig. 2 Pressure-void ratio relationship for a typical clay
(Pressure to logarithmic scale)

In the semi-logarithmic plot, it can be seen that the virgin compression curve in this case approximates a straight line from about 200 kN/m^2 pressure. The equation of this straight line portion may be written in the following form:

$$e = e_0 - C_c \log_{10} \frac{\bar{\sigma}}{\bar{\sigma}_0}$$

Where *e* corresponds to σ and e_0 corresponds to σ_0, The numerical value of the slope of this straight line, Cc.

- Skempton established a relationship between the compressibility of a clay, as indicated by its compression index, and the liquid limit Cc = 0.009 (wL – 10).

$$C_c = \frac{(e - e_0)}{\log_{10} \frac{\bar{\sigma}}{\bar{\sigma}_0}}$$

The rebound curve obtained during unloading may be similarly expressed with C_e designating what is called the 'Expansion index':

$$e = e_0 - C_e \log_{10} \frac{\bar{\sigma}}{\bar{\sigma}_0}$$

- If, after complete removal of all loads, the sample is reloaded with the same series of loads as in the initial cycle, a different curve, called the 'recompression curve' is obtained. - Some of the volume change due to external loading is permanent.

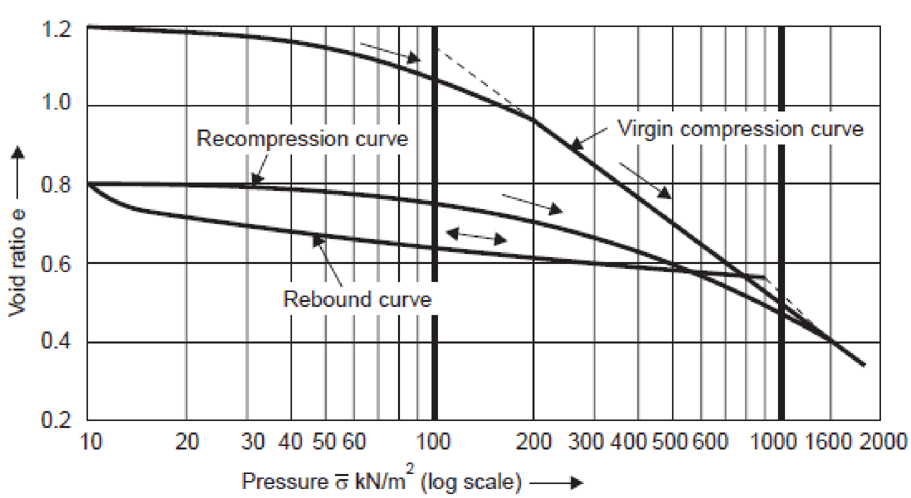
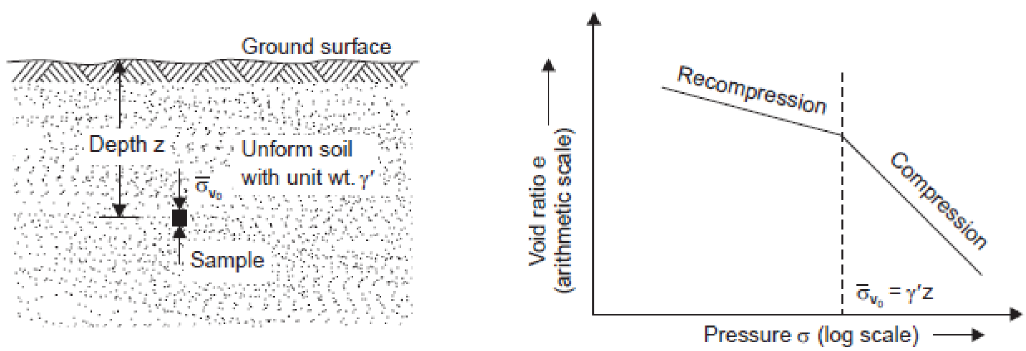


Fig. 3 Virgin compression, rebound and recompression

Curves for a clay (Pressure to logarithmic scale)

- may be noted from **Fig.** 3 that the curvature of the virgin compression curve at pressures smaller than about 200 kN/m^2 resembles the curvature of the recompression curve at pressures smaller than about 800 kN/m^2 from which the rebound occurred. This resemblance indicates that the specimen was probably subjected to a pressure of about 150 to 200 kN/m^2 at some time before its removal from the ground.

- As an example let us consider a soil sample obtained from a site from a depth *z* as shown in Fig. 4 (*a*). The ground surface has never been above the existing level and there never was extra external loading acting on the area. Thus, the maximum stress to which the soil sample was ever subjected is the current over burden pressure σ_{v0} ($= \gamma'.z$).



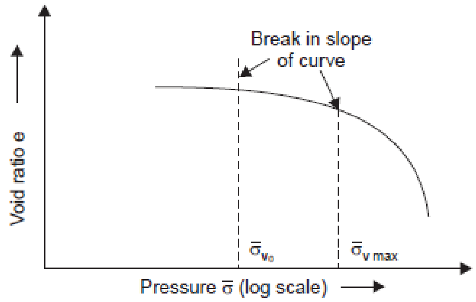
(a) Location of sample taken for compression test

(b) Compression test results for the sample

Fig. 4 Conditions applying to compression test sample

- The portion of the curve prior to pressure (σ_{v0}) represents a recompression curve, while that at greater pressures than (σ_{v0}) represents the virgin compression curve as shown in Fig.4 (*b*).

- If the ground surface had at some time is past history been above the existing surface and had been eroded away, or if any other external load acted earlier and got released, so the existing over-burden pressure, (σ_{v0}), would not be the maximum pressure. If this greatest past pressure is σ_{vmax} , greater than σ_{v0} , compression test would be as shown in Fig. 5.



- **Normally Consolidated Soil and Over consolidated Soil**

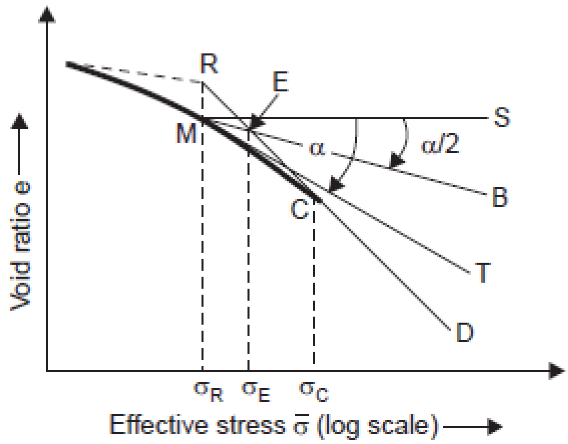
A quantitative measure of the degree of over consolidation is what is known as the 'Overconsolidation Ratio', OCR. It is defined as follows:

$$OCR = \frac{\text{Maximum effective stress to which the soil has been subjected in its stress history}}{\text{Existing effective stress in the soil}}$$

Thus, the maximum OCR of normally consolidated soil equals 1.

In this connection, it is of considerable engineering interest to be able to determine the past maximum effective stress (σ_E).

- Casagrande (1936) proposed a geometrical technique to evaluate past maximum effective stress or pre consolidation pressure from the *e* versus log σ plot obtained by loading a sample in the laboratory.



1. The point of maximum curvature **M** on the curved portion of the *e* vs. log σ plot is located.

2. A horizontal line **MS** is drawn through **M**.

3. A tangent **MT** to the curved portion is drawn through **M**.

4. The angle **SMT** is bisected, **MB** being the bisector.

5. The straight portion **DC** of the plot is extended backward to meet **MB** in **E**.

6. The pressure corresponding to the point **E,** (σ_E)**,** is the most probable past maximum effective stress or the preconsolidation pressure.

* If $\sigma_E = \sigma_o$ Normally consildation clay.

* If $\sigma_E > \sigma_o$ over consolidated clay or pre consolidated clay .

* If $\sigma_E < \sigma_o$ under consolidated clay.

- **Modulus of Volume Change and Consolidation Settlement**

The ‘modulus of volume change is defined as the change in volume of a soil per unit initial volume due to a unit increase in effective stress

$m_v = -\dfrac{\Delta e}{(1+e_0)} \cdot \dfrac{1}{\Delta\bar{\sigma}}$ $-\dfrac{\Delta e}{\Delta\bar{\sigma}} = a_v$, the coefficient of compressibility.

and

$$m_v = \frac{a_v}{(1+e_0)}$$

but $m_v = -\frac{\Delta H}{H_0}.\frac{1}{\Delta\overline{\sigma}}$ so $\Delta H = m_v.H_0.\Delta\overline{\sigma}$

$$m_v = \frac{e}{(1+e_0)}.\frac{1}{\Delta\overline{\sigma}}, \text{ ignoring sign.}$$

$$\frac{\Delta H}{H_0} = \frac{\Delta e}{(1+e_0)}$$

$$S_c = \Delta H = \frac{\Delta e}{(1+e_0)}.H_0$$

Substituting for Δ*e* in terms of the compression index, *Cc,* recognizing ($e - e_0$) as Δe, we have:

$$S_c = \Delta H = H_0.\frac{C_c}{(1+e_0)}.\log_{10}\frac{\overline{\sigma}}{\overline{\sigma}_0}$$

$$S_c = H_0.\frac{C_c}{(1+e_0)}.\log_{10}\left(\frac{\overline{\sigma}_0+\overline{\sigma}}{\overline{\sigma}_0}\right)$$

- **TERZAGHI'S THEORY OF ONE-DIMENSIONAL CONSOLIDATION**

Now let us see the derivation of Terzaghi's theory with respect to the laboratory oedometer sample with double drainage as shown in Fig. 5.

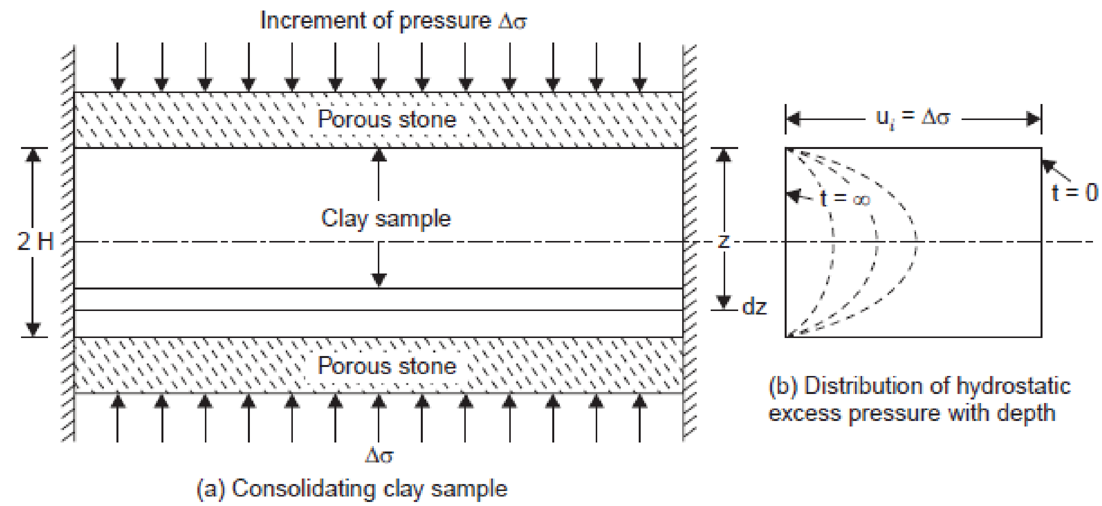


Fig. 5 Consolidation of a clay sample with double drainage

where $c_v = \dfrac{k}{\gamma_w \cdot m_v}$ Then $\dfrac{\partial u}{\partial t} = c_v \cdot \dfrac{\partial^2 u}{\partial z^2}$

C_v is known as the "Coefficient of consolidation". *u* represents the hydrostatic excess pressure at a depth *z* from the drainage face at time *t* from the start of the process of consolidation.

$$c_v = \frac{k}{\gamma_w m_v} = \frac{k(1+e_0)}{a_v \gamma_w}$$

$$U_z = (u_i - u)/u_i = \left(1 - \frac{u}{u_i}\right)$$

U_z = degree of consolidation, u_i = initial pressure, u pressure at any moment.

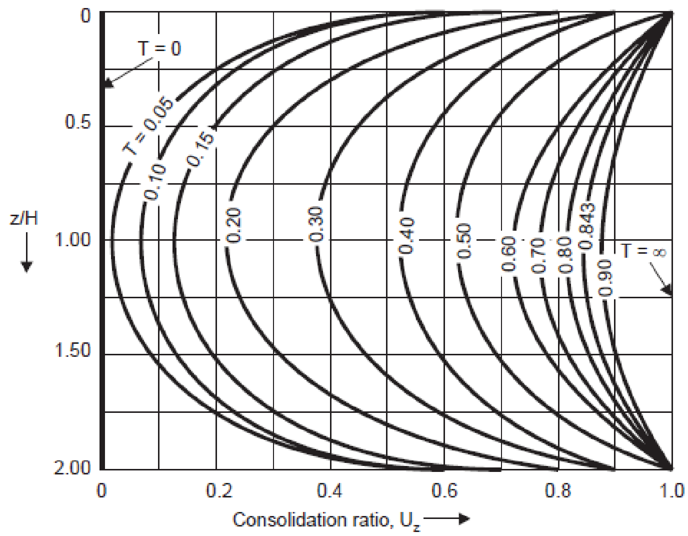
- The third dimensionless parameter, relating to time, and called 'Time-factor', *T*, is defined as follows:

$$T = \frac{c_v t}{H^2}$$

where C_v is the coefficient of consolidation, *H* is the length drainage path, and *t* is the elapsed time from the start of consolidation process.

- **GRAPHICAL PRESENTATION OF CONSOLIDATION RELATIONSHIPS**

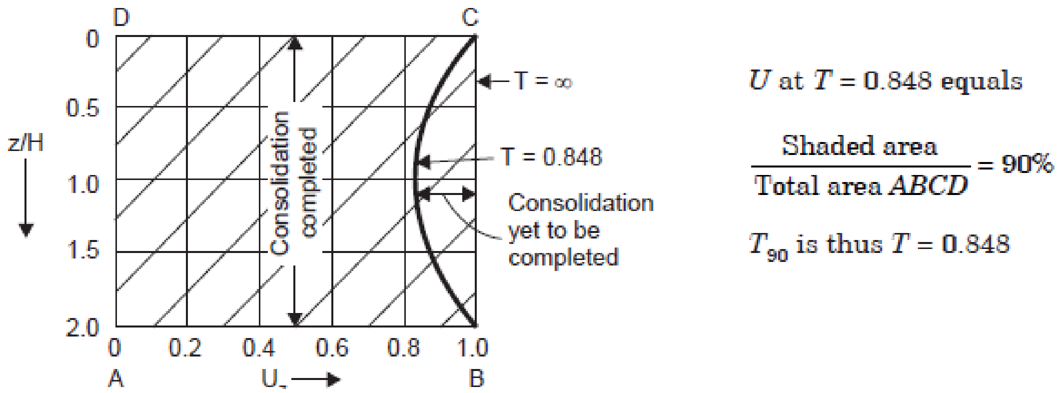
A graphical presentation of the results indicated by assigning different values of z/H and T, different values of U_z are solved and plotted to obtain the family of curves shown.



At the start of the process, $t = 0$ and $T = 0$, and U_z is zero for all depths. At any finite time factor, the consolidation ratio is 1 at drainage faces and is minimum at the middle of the layer. For example, for $T = 0.20$: $Uz = 0.23$ at $z/H = 1$; $Uz = 0.46$ at $z/H = 0.25$ and 1.75; and $Uz = 0.70$ at $z/H = 0.125$ and 1.875. This indicates that at a depth of one-eighth of the layer, consolidation is 70% complete; at a depth of one-fourth of the layer, consolidation is 46% complete; while, at the middle of the layer, consolidation is just 23% complete

Figure above does not depict how much consolidation occurs *as a whole* in the entire stratum. This information is of primary concern to the geotechnical engineer and may be deduced from Fig. above by the following procedure:

The relation between U_z and z/H for a time factor, $T = 0.848$, is reproduced in Fig. below.



Average consolidation at time factor 0.848

When $U < 60\%$, $T = (\pi/4)\, U^2$

When $U > 60\%$, $T = -\,0.9332 \log_{10} (1 - U) - 0.0851$.

- **EVALUATION OF COEFFICIENT OF CONSOLIDATION FROM OEDOMETER TEST DATA**

The coefficient of consolidation, C_v, in any stress range of interest, may be evaluated from its definition, by experimentally determining the parameters k, a_v and e_0 for the stress range under consideration. k may be got from a permeability test conducted on the oedometer sample itself, after complete consolidation under the particular stress increment.

a_v and e_0 may be obtained from the oedometer test data, by plotting the $e - \sigma$ curve.

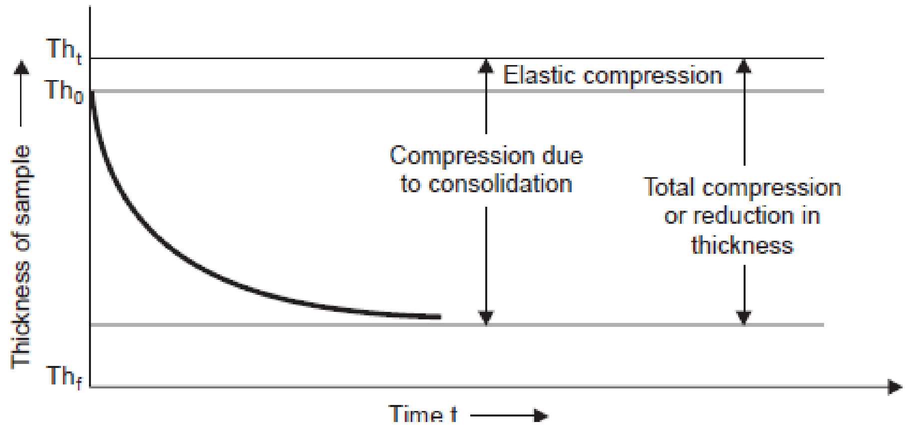
The more generally used fitting methods are the following:

(*a*) The square root of time fitting method

(*b*) The logarithm of time fitting method

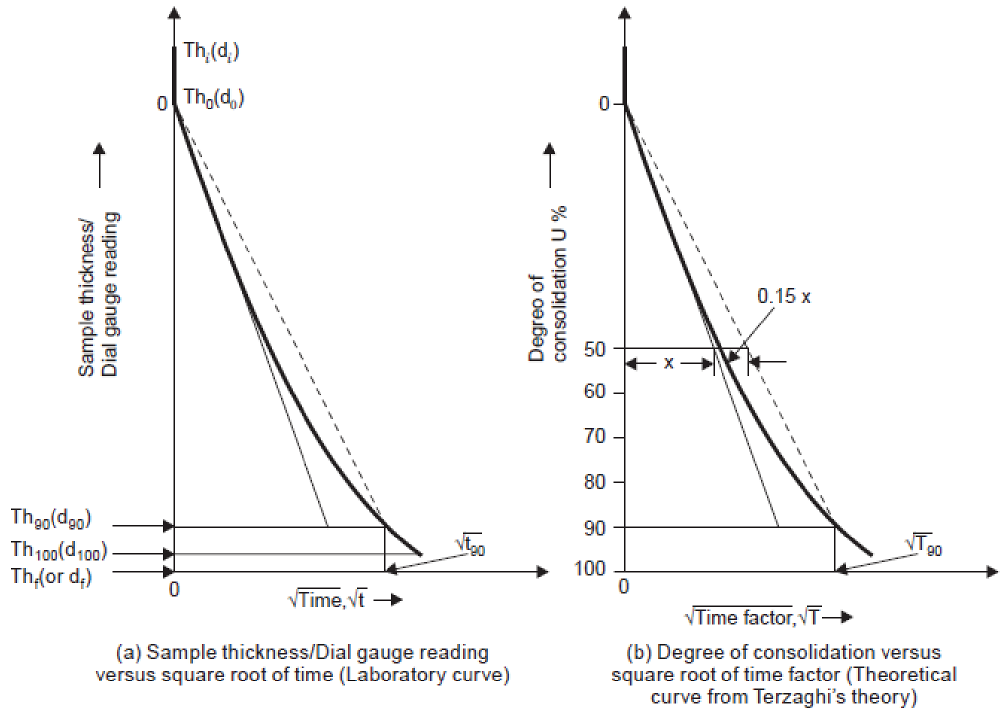
The Square Root of Time Fitting Method

The relation between the sample thickness and elapsed time since the application of the load increment is obtainable from an oedometer test and is somewhat as shown in Fig. below for a typical load-increment.



Time versus reduction in sample thickness for a load-increment

The theoretical curve on the square root plot is straight line up to about 60% consolidation with a gentle concave upwards curve thereafter. If another straight line, shown dotted, is drawn such that the abscissa of this line are 1.15 times those of the straight line portion of the theoretical curve, it can be shown to cut the theoretical curve at 90% consolidation.



Square root of time fitting method (After Taylor, 1948)

$$c_v = \frac{T_{90} H^2}{t_{90}}$$

The coefficient of consolidation, c_v, may be obtained from

where t_{90} is read off from Fig. above

T_{90} is 0.848 from Terzaghi's theory

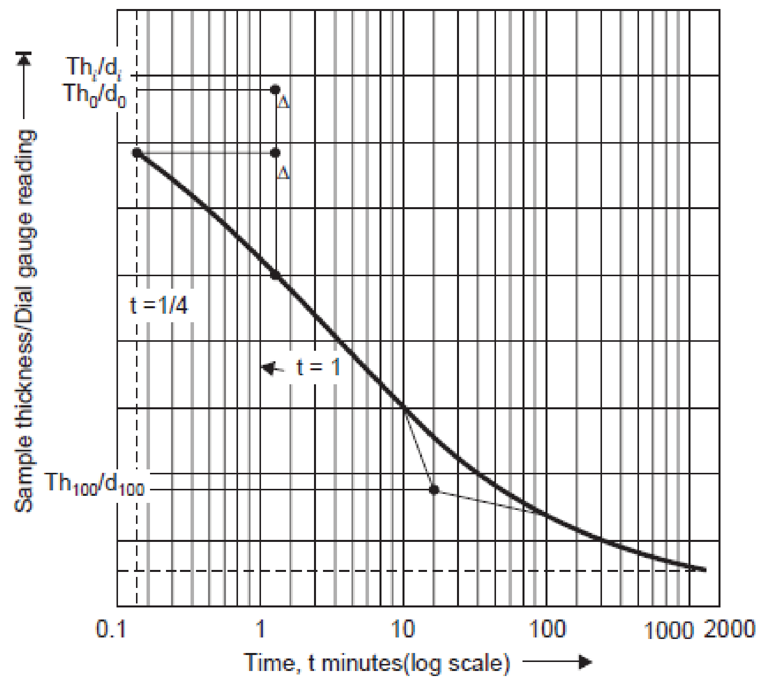
H is the drainage path, which may be taken as half the thickness of the sample for

Double drainage conditions, or as $(Th_0 + Th_f)/4$ in terms of the sample thickness

The Logarithm of Time Fitting Method

The point corresponding to 100 percent consolidation curve is plotted on a semi-logarithmic scale, with time factor on a logarithmic scale and degree of consolidation on arithmetic scale, the intersection of the tangent and asymptote is at the ordinate of 100% consolidation.

The difference in ordinates between two points with times in the ratio of 4 to 1 is marked off; then a distance equal to this difference may be stepped off above the upper points to obtain the corrected zero point. This point may be checked by more trials, with different pairs of points on the curve.



Sample thickness/Dial gauge reading
versus logarithm of time (Laboratory curve)

$$C_v = \frac{T_{50} H^2}{t_{50}}$$

where t_{50} is read off from Fig, T_{50} = 0.197 from Terzaghi's theory, and

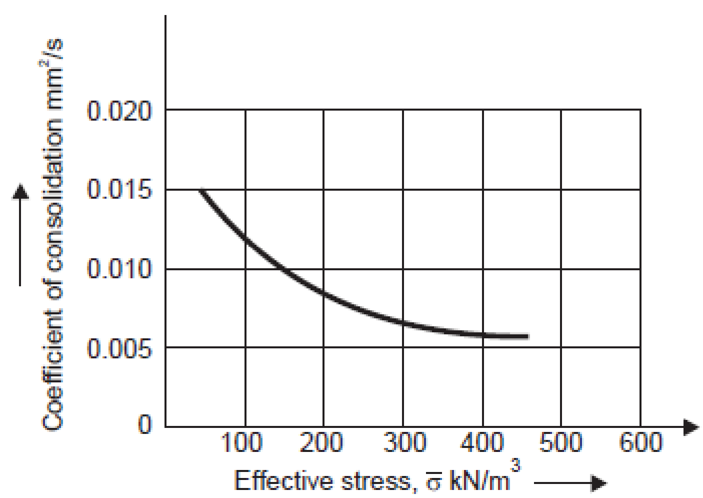
H is the drainage path as stated in the previous subsection.

Typical Values of Coefficient of Consolidation

7.7.3 Typical Values of Coefficient of Consolidation

The process of applying one of the fitting methods may be repeated for different increments of pressure using the time-compression curves obtained in each case. This is the reason for the caution that, for problems in the field involving settlement analysis, the coefficient of consolidation should be evaluated in the laboratory for the particular range of stress likely to exist in the field.

The range of values for C_v is rather wide from 5×10^{-4} mm^2/s to 2×10^{-2} mm^2/s. Further, it is also found that the value of C_v decreases as the liquid limit of the clay increases.



Variation of coefficient of consolidation with effective stress

SECONDARY CONSOLIDATION

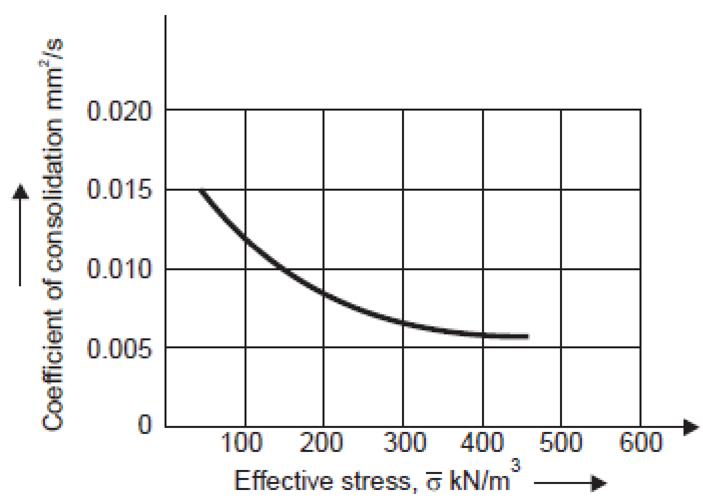
When the hydrostatic excess pressure is fully dissipated, no more consolidation should be expected. However, in practice, the decrease in void ratio continues, though very slowly, for a long time after this stage, called 'Primary Consolidation'.

Typical Values of Coefficient of Consolidation

7.7.3 Typical Values of Coefficient of Consolidation

The process of applying one of the fitting methods may be repeated for different increments of pressure using the time-compression curves obtained in each case. This is the reason for the caution that, for problems in the field involving settlement analysis, the coefficient of consolidation should be evaluated in the laboratory for the particular range of stress likely to exist in the field.

The range of values for C_v is rather wide from 5×10^{-4} mm^2/s to 2×10^{-2} mm^2/s. Further, it is also found that the value of C_v decreases as the liquid limit of the clay increases.

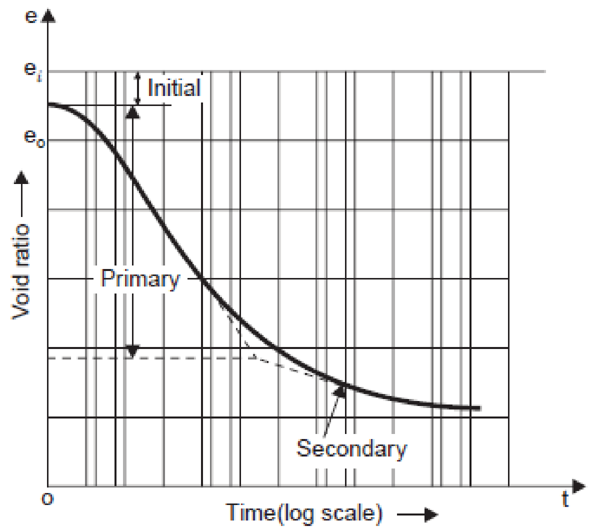


Variation of coefficient of consolidation with effective stress

SECONDARY CONSOLIDATION

When the hydrostatic excess pressure is fully dissipated, no more consolidation should be expected. However, in practice, the decrease in void ratio continues, though very slowly, for a long time after this stage, called 'Primary Consolidation'.

The effect or the phenomenon of continued consolidation after the complete dissipation of excess pore water pressure is termed 'Secondary Consolidation' and the resulting compression is called 'Secondary Compression'.



Secondary compression appears as a straight line sloping downward or, in some cases, as a straight line followed by a second straight line with a flatter slope. The void ratio, e_f, at the end of primary consolidation can be found from the intersection of the backward extension of the secondary line with a tangent drawn to the curve of primary compression, as shown in the figure. The rate of secondary compression, depends upon the increment of stress and the characteristics of the soil.

$\Delta e = -\alpha \cdot \log_{10}(t_2/t_1)$

$$\varepsilon = \frac{\Delta H}{H} = -C_\alpha \log_{10}\left(\frac{t_2}{t_1}\right)$$

In other words, C_α may be taken to be the slope of the straight line representing the secondary compression on a plot of strain versus logarithm of time.

$$C_\alpha = \frac{\alpha}{(1+e)}$$

Sl. No	Nature of Soil	C_α – Value
1.	Over consolidated days	0.0005 to 0.0015
2.	Normally consolidated days	0.005 to 0.030
3.	Organic soils, peats	0.04 to 0.10

Example 27: In a consolidation test the following results have been obtained. When the load was changed from 50 kN/m^2 to 100 kN/m^2, the void ratio changed from 0.70 to 0.65. Determine the coefficient of volume decrease, *mv* and the compression index, *Cc*.

$e_0 = 0.70$ $\sigma_0 = 50$ kN/m^2

$e_1 = 0.65$ $\sigma = 100$ kN/m^2

Coefficient of compressibility, $a_v = \frac{\Delta e}{\Delta \bar{\sigma}}$, ignoring sign.

$$= \frac{(0.70 - 0.65)}{(100 - 50)} \text{ m}^2/\text{kN} = 0.05/50 \text{ m}^2/\text{kN} = 0.001 \text{ m}^2/\text{kN}.$$

Modulus of volume change, or coefficient of volume decrease,

$$m_v = \frac{a_v}{(1 + e_0)} = \frac{0.001}{(1 + 0.70)} = \frac{0.001}{1.7} \text{ m}^2/\text{kN}.$$

$$= 5.88 \times 10^{-4} \text{ m}^2/\text{kN}$$

Compression index, $C_c = \frac{\Delta e}{\Delta(\log \bar{\sigma})} = \frac{(0.70 - 0.65)}{(\log_{10} 100 - \log_{10} 50)}$

$$= \frac{0.05}{\log_{10} \frac{100}{50}} = \frac{0.05}{\log_{10} 2} = \frac{0.050}{0.301} = \mathbf{0.166.}$$

Example 28: A sand fill compacted to a bulk density of 18.84 kN/m^3 is to be placed on a compressible saturated marsh deposit 3.5 m thick. The height of the sand fill is to be 3 m. If the volume compressibility m_v of the deposit is 7×10^{-4} m^2/kN, estimate the final settlement of the fill.

H_t. of sand fill = 3 m

Bulk unit weight of fill = 18.84 kN/m^3

Increment of the pressure on top of marsh deposit $\Delta\sigma = 3 \times 18.84 = 56.52$ kN/m^2

Thickness of marsh deposit, $H0$ = 3.5 m

Volume compressibility $mv = 7 \times 10^{-4}$ m^2/kN

Final settlement of the marsh deposit, $\Delta H = mv.H_0.\Delta\sigma$

$= 7 \times 10^{-4} \times 3500 \times 56.52$ mm = **138.5 mm**.

Example 29: A layer of soft clay is 6 m thick and lies under a newly constructed building. The weight of sand overlying and clayey layer produces a pressure 260 kN/m^2 and the new construction increases the pressure 100 kN/m^2. If the compression index is 0.5, compute the settlement. Water content is 40% and specific gravity of grains is 2.65.

Initial pressure, $\sigma_0 = 260$ kN/m^2

Increment of pressure, $\Delta\sigma = 100$ kN/m^2

Thickness of clay layer, H = 6 m = 600 cm.

Compression index, Cc = 0.5, Water content, w = 40%

Specific gravity of grains, G = 2.65

Void ratio, $e_0 = wG$, (since the soil is saturated) = 0.40 × 2.65 = 1.06

$$S = \frac{H.C_c.}{(1+e_0)} \log_{10}\left(\frac{\bar{\sigma}_0 + \Delta\bar{\sigma}}{\bar{\sigma}_0}\right)$$

$$= \frac{600 \times 0.5}{(1+1.06)} \log_{10}\left(\frac{260+100}{260}\right) \text{ cm}$$

$$= \frac{300}{2.06} \log_{10}\left(\frac{360}{260}\right) \text{ cm}$$

$$= \mathbf{21.3\ cm}.$$

Example 30: There is a bed of compressible clay of 4 m thickness with pervious sand on top and impervious rock at the bottom. In a consolidation test on an undisturbed specimen of clay from this deposit 90% settlement was reached in 4 hours. The specimen was 20 mm thick. Estimate the time in years for the building founded over this deposit to reach 90% of its final settlement.

This is a case of one-way drainage in the field.

∴ Drainage path for the field deposit, H_f = 4 m = 4000 mm.

In the laboratory consolidation test, commonly it is a case of two-way drainage.

∴ Drainage path for the laboratory sample, H_1 = 20/2 = 10 mm

Time for 90% settlement of laboratory sample = 4 hrs.

Time factor for 90% settlement, T_{90} = 0.848

$$\therefore \quad T_{90} = \frac{C_v t_{90_f}}{H_f^2} = \frac{C_v t_{90_l}}{H_l^2}$$

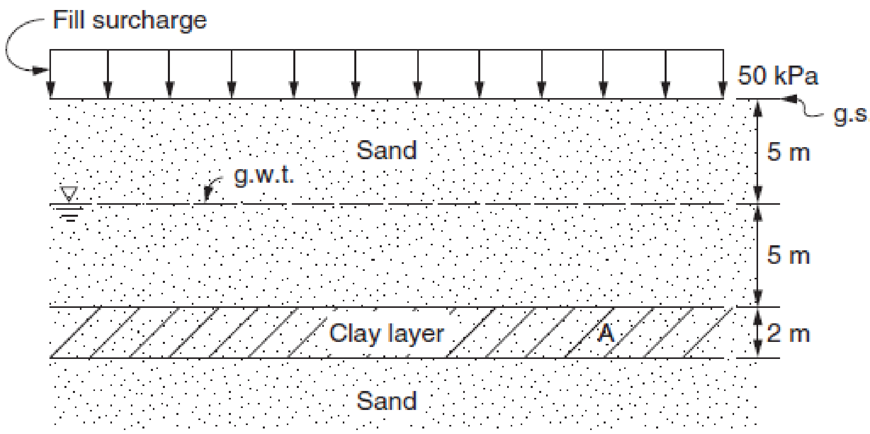
$$\text{or} \quad \frac{t_{90_f}}{H_f^2} = \frac{t_{90_l}}{H_l^2}$$

$$\therefore \quad t_{90f} = \frac{t_{90_l}}{H_l} \times H_f = \frac{4 \times (4000)^2}{10^2} \text{ hrs}$$

$$= \frac{4 \times 400}{24 \times 365} \text{ years}$$

$$\approx \mathbf{73\ years.}$$

Example 31: A site has a level ground surface and a level groundwater table located 5 m below the ground surface. As shown in Fig. below, subsurface exploration has discovered that the site is underlain with sand, except for a uniform and continuous clay layer that is located at a depth of 10 to 12 m below ground surface. Below the groundwater table, the pore water pressures are hydrostatic in the sand layers (i.e., no artesian pressures). The average void ratio e_o of the clay layer is 1.10 and the buoyant unit weight γ_b of the clay layer = 7.9 kN/m^3. The total unit weight γ_t of the sand above the groundwater table = 18.7 kN/m^3 and the total unit weight γ_t of the sand below the groundwater table = 19.7 kN/m^3. The compression index C_c = 0.83 and recompression index C_r = 0.05. Determine the primary consolidation settlement S_c of the 2-m-thick clay layer if a uniform fill surcharge of 50 kPa is applied over a very large area at ground surface. If a laboratory consolidation test performed on an undisturbed specimen obtained from the center of the clay layer (Point A) indicates the maximum past pressure σ_E = 150 kPa and σ_E= 175 kPa and σ_E=250 kPa



Solution The first step is to determine the vertical effective stress σ_o at the center of the clay layer or:

σ_o = (5 m)(18.7 kN/m^3) + (5 m)(19.7 – 9.81 kN/m^3) + (1 m)(7.9 kN/m^3) = 150 kPa.

1) Since σ_o is equal to σ_E, the clay is normally consolidated (OCR = 1).

$Sc = C_c\,[H_o/(1 + e_o)] \log [(\sigma_o + \Delta\sigma_v)/\sigma_o]$

Sc = (0.83)[(2 m)/(1 + 1.1)] log [(150 kPa + 50 kPa)/150 kPa] = 0.10 m

2) Since σ_o is less than σ_E, the clay layer is overconsolidated (OCR > 1).

Since $\sigma_o + \Delta\sigma_v$ = 200 kPa > σ_E

$S_c = Cr\,[Ho/(1 + e_o)] \log (\sigma_E/\sigma_o) + C_c\,[Ho/(1 + e_o)] \log [(\sigma_o + \Delta\sigma_v)/\sigma_E]$

S_c = (0.05)[(2 m)/(1 + 1.1)] log (175 kPa/150 kPa) + (0.83)[(2 m)/(1 + 1.1)] log [(150 kPa + 50 kPa)/175 kPa] = 0.049 m

3) Since $\sigma_o + \Delta\sigma_v$ = 200 kPa < σ_E

$S_c = C_r\,[H_o/(1 + e_o)] \log [(\sigma_o + \Delta\sigma_v)/\sigma_o]$

S_c = (0.05)[(2 m)/(1 + 1.1)] log [(150 kPa + 50 kPa)/150 kPa] = 0.006 m

SHEARING STRENGTH OF SOILS

‘Shearing Strength’ of a soil is perhaps the most important of its engineering properties. This is because all stability analyses in the field of geotechnical engineering, whether they relate to foundation, slopes of cuts or earth dams, involve a basic knowledge of this engineering property of the soil. ‘Shearing strength’ or merely ‘**Shear strength**’ may be defined as the resistance to shearing stresses and a consequent tendency for shear deformation.

Shearing strength comes from the following:

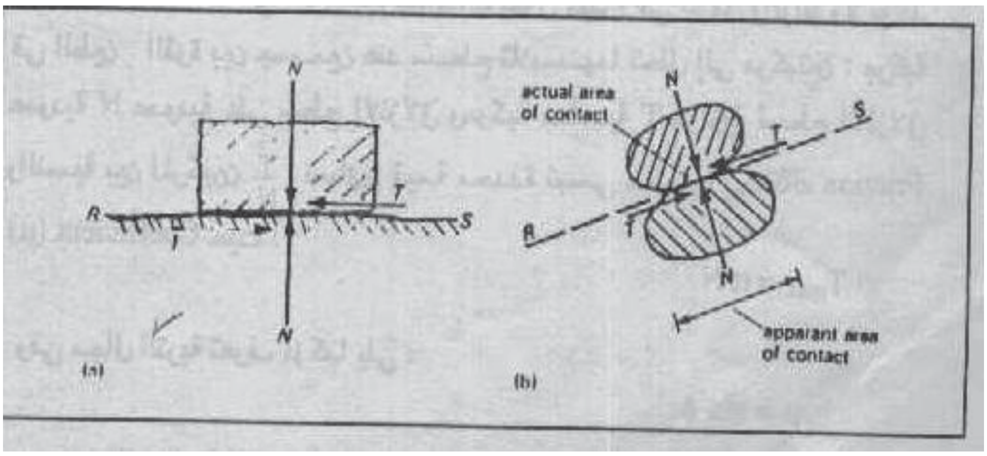
(1) Resistance due to the interlocking of particles.

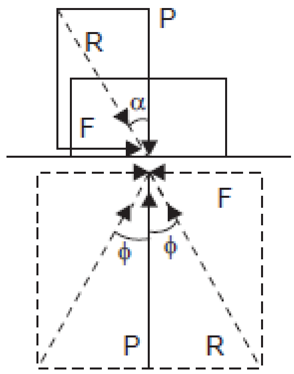
(2) Frictional resistance between the individual soil grains, which may be sliding friction, rolling friction, or both.

(3) Adhesion between soil particles or ‘cohesion’.

Granular soils of sands (1, 2) while cohesive soils or clays (2, 3) highly plastic clays (3)

- **Friction between Solid Bodies (Internal Friction within Granular Soil Masses)**





Full friction
α = ϕ
Applied shear force just equals maximum available friction : maximum frictional force is mobilised. Slip imminent to the right (critical condition)

When two solid bodies are in contact with each other, the frictional resistance available is dependent upon the normal force between the two. A shearing force equal to the maximum available frictional resistance is applied. The entire frictional resistance available will get mobilized now to resist the applied

Coefficient of friction (μ) = $\frac{F}{P}$ or $F = P.\ \mu = P.\ \tan\varphi$ or $F/P = \tan\varphi = \mu$

In granular or cohesionless soil masses, the resistance to sliding on any plane through the point within the mass is similar to that discussed in the previous sub-section; the friction angle in this case is called the 'angle of internal friction' Ø

- **PRINCIPAL PLANES AND PRINCIPAL STRESSES—MOHR'S CIRCLE**

At a point in a stressed material, every plane will be subjected, in general, to a normal or direct stress and a shearing stress.

A 'Principal plane' is defined as a plane on which the stress is wholly normal, or one which does not carry shearing stress.

Principal plane is divided to the 'major principal stress', the 'intermediate principal stress' and the 'minor principal stress',

Let us consider an element of soil whose sides are chosen as the principal planes, the major and the minor, as shown in Fig. below

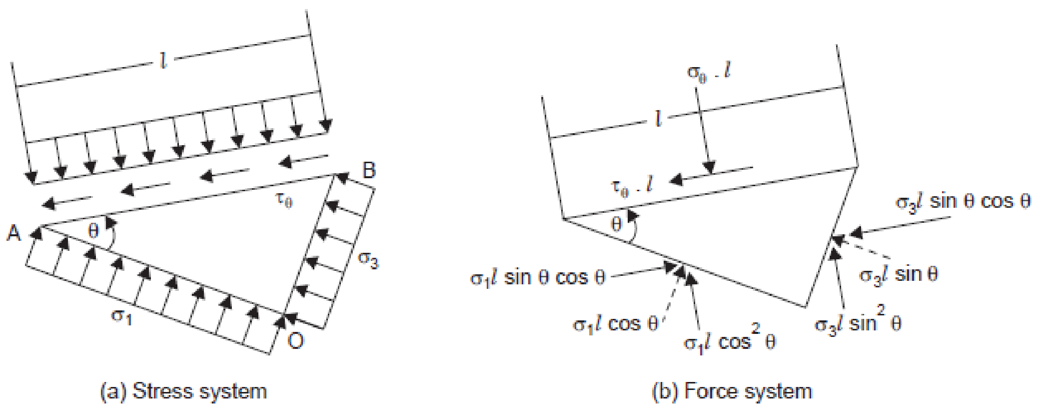


Fig 2 Stresses on a plane inclined to the principal planes

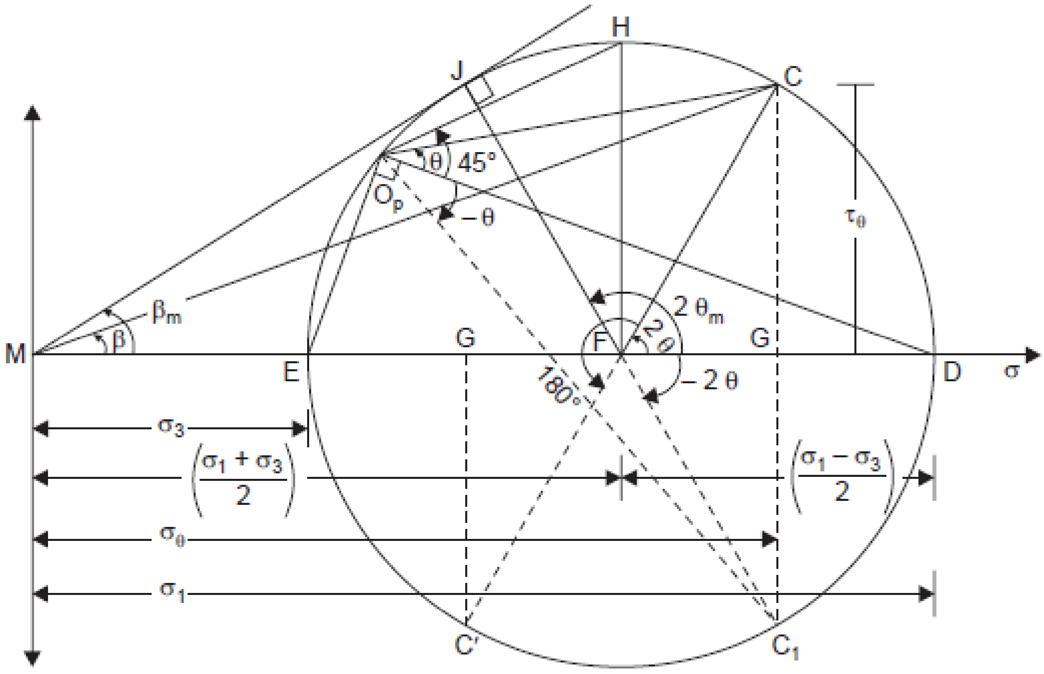
Let *O* be any point in the stressed medium and *OA* and *OB* be the major and minor principal planes, with the corresponding principal stresses σ_1 and σ_3, and inclined at an angle θ to the major principal plane, considered positive when measured counter-clockwise. Let us consider the element to be of unit thickness perpendicular to the plane of the figure, *AB* being *l*. considering the equilibrium of the element and resolving all forces in the directions parallel and perpendicular to *AB*,

$$\sigma_\theta = \sigma_1 \cos^2\theta + \sigma_3 \sin^2\theta = \sigma_3 + (\sigma_1 - \sigma_3)\cos^2\theta$$

$$= \frac{(\sigma_1 + \sigma_3)}{2} + \frac{(\sigma_1 - \sigma_3)}{2} . \cos 2\theta$$

$$\tau_\theta = \frac{(\sigma_1 - \sigma_3)}{2} . \sin 2\theta$$

Otto Mohr (1882) represented these results graphical in a circle diagram, which is called Mohr's circle. Normal stresses are represented as abscissae and shear stresses as ordinates.



Mohr's circle for the stress conditions illustrated in Fig.2

Let a line be drawn parallel to the major principal plane through *D*, the coordinate of which is the major principal stress. The intersection of this line with the Mohr's circle, O_p is called the 'Origin of planes'. If a line parallel to the minor principal plane is drawn through *E*, the co-ordinate of which is the minor principal stress, it will also be observed to pass through *Op;* the angle between these two lines is a right angle from the properties of the circle. The angle between these two lines is a right angle from the properties of the circle. any line through *Op*, parallel to any arbitrarily chosen plane, intersects the Mohr's circle at a point the co-ordinates of which represent the normal and shear stresses on that plane Since angle CO_pD = θ, angle *CFD* = 2θ, from the properties of the circle. From the geometry of the figure, the co-ordiantes of the point *C*, are established as follows:

$$\sigma_\theta = MG = MF + FG$$

$$= \frac{(\sigma_1 + \sigma_3)}{2} + \frac{(\sigma_1 - \sigma_3)}{2} \cdot \cos 2\theta$$

$$\tau_\theta = CG = \frac{(\sigma_1 - \sigma_3)}{2} \cdot \sin 2\theta$$

A few important basic facts and relationships may be directly obtained from the Mohr's circle:

1. The only planes free from shear are the given sides of the element which are the principal planes. The stresses on these are the greatest and smallest normal stresses.

2. The maximum or principal shearing stress is equal to the radius of the Mohr's circle, and it occurs on planes inclined at 45° to the principal planes.

$$\tau_{max} = (\sigma_1 - \sigma_3)/2$$

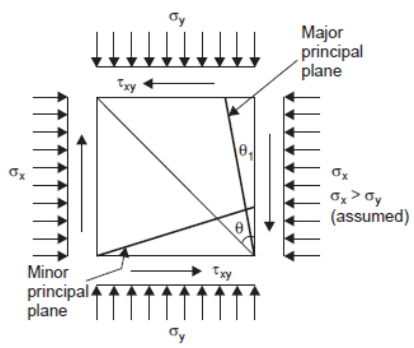
3. The normal stresses on planes of maximum shear are equal to each other and are equal to half the sum of the principal stresses.

$$\sigma_c = (\sigma_1 + \sigma_3)/2$$

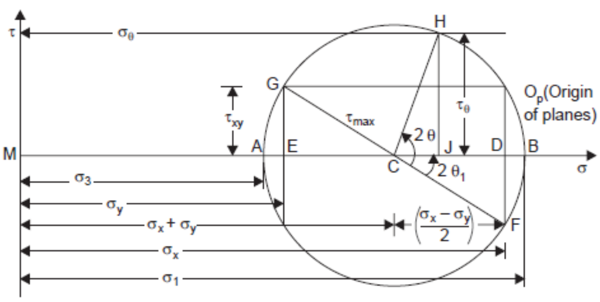
4. Shearing stresses on planes at right angles to each other are numerically equal and are of an opposite sign. These are called conjugate shearing stresses.

5. The sum of the normal stresses on mutually perpendicular planes is a constant ($MG' + MG = 2MF = \sigma 1 + \sigma 3$). If we designate the normal stress on a plane perpendicular to the plane on which it is σ_θ as σ_θ' : $\sigma_\theta + \sigma_\theta' = \sigma_1 + \sigma_3$

In case the normal and shearing stresses on two mutually perpendicular planes are known, the principal planes and principal stresses may be determined with the aid of the Mohr's circle diagram.



(a) General two-dimensional stress system



(b) Mohr's circle for general two-dimensional stress system normal stresses σ_x and σ_y on mutually perpendicular planes and shear stresses τ_{xy} on these planes, the normal and shearing stress components, σ_θ and τ_θ, respectively, on a plane inclined at an angle θ, measured counter-clockwise with respect to the plane on which σ*x* acts

$$\sigma_\theta = \frac{(\sigma_x + \sigma_y)}{2} + \frac{(\sigma_x - \sigma_y)}{2} \cdot \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\tau_\theta = \frac{(\sigma_x - \sigma_y)}{2} \cdot \sin 2\theta - \tau_{xy} \cdot \cos 2\theta$$

$$\left[\sigma_\theta - \frac{(\sigma_x + \sigma_y)}{2}\right]^2 + \tau_\theta{}^2 = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2$$

This represents a circle with centre $\left[\frac{(\sigma_x + \sigma_y)}{2}, 0\right]$, and radius

$$\sqrt{\left(\frac{(\sigma_x - \sigma_y)}{2}\right)^2 + \tau_{xy}^2}.$$

The following relationships are also easily obtained:

$$\sigma_1 = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

$$\sigma_3 = \left(\frac{\sigma_x + \sigma_y}{2}\right) - \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

$$\tan 2\theta_{1,3} = 2\tau_{xy}/(\sigma_x - \sigma_y)$$

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

STRENGTH THEORIES FOR SOILS

- **Mohr's Strength Theory**

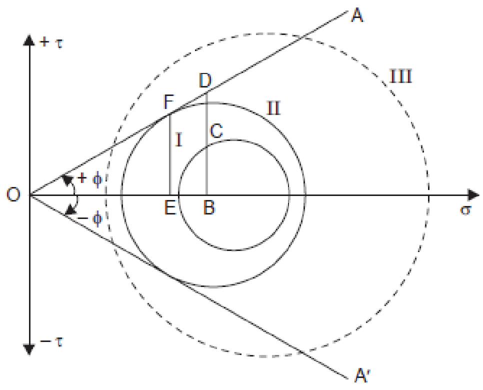
We have seen that the shearing stress may be expressed as $\tau = \sigma \tan \beta$ on any plane, where β is the angle of obliquity, If the obliquity angle is the maximum or has limiting value φ, the shearing stress is also at its limiting value and it is called the shearing strength, *s*

$$s = \sigma \tan \varphi$$

If the angle of internal friction φ is assumed to be a constant, the shearing strength may be represented by a pair of straight lines at inclinations of + φ and – φ with the σ-axis and passing through the origin of the Mohr's circle diagram. If the stress conditions at a point are represented by Mohr's circle I, the shear stress on any plane through the point is less than the shearing strength, as indicated by the line *BCD;*

BC represents the shear stress on a plane on which the normal stress is given by *OD.*

BD, representing the shearing strength for this normal stress, is greater than *BC*.



Mohr's strength theory—Mohr envelopes for cohesionless soil

Mohr's Circle II, which is tangential to the Mohr's envelope at *F*, are such that the shearing stress, *EF*,

$$s = \sigma_f \tan \phi = \sigma_3 \tan \phi \,(1 + \sin \phi)$$

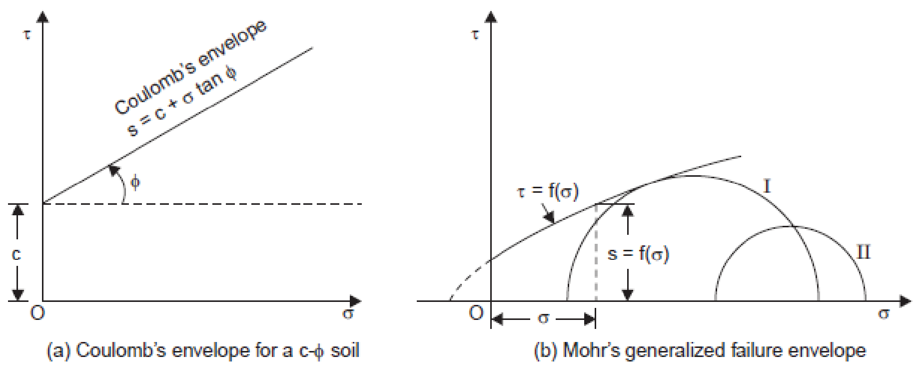
$$= \sigma_1 \tan \phi \,(1 - \sin \phi) = \left(\frac{\sigma_1 - \sigma_3}{2}\right) . \cos \phi$$

- **Mohr-Coulomb Theory**

The functional relationship between the normal stress on any plane and the shearing strength available on that plane was assumed to be linear by Coulomb; thus the following is usually known as Coulomb's law:

$$s = c + \sigma \tan \varphi$$

where c and φ are empirical parameters, known as the 'apparent cohesion' and 'angle of shearing resistance'

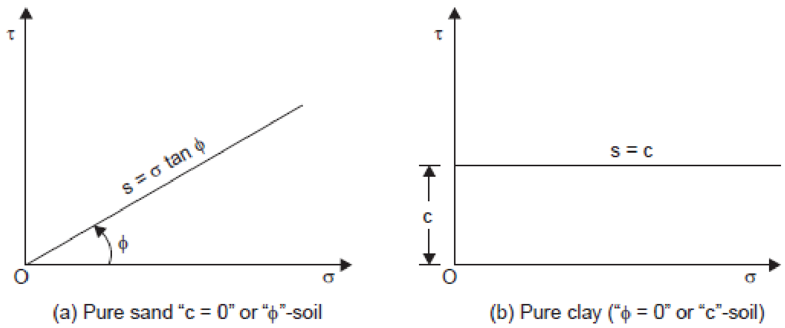


Mohr-Coulomb Theory—failure envelopes

The envelopes are called 'strength envelopes' or 'failure envelopes'. The meaning of an envelope has already been given in the previous section; if the normal and shear stress components on a plane plot on to the failure envelope, failure is supposed to be incipient and if the stresses plot below the envelope, the condition represents stability. And, it is impossible that these plot above the envelope, since failure should have occurred previously. Coulomb's law is also written as follows to indicate that the stress condition refers to

that on the plane of failure:

$$s = c + \sigma_f \tan \varphi$$



Coulomb envelopes for pure sand and for pure clay

SHEARING STRENGTH—A FUNCTION OF EFFECTIVE STRESS

The density of a soil increase when subjected to shearing action, drainage being allowed simultaneously. Therefore, even if two soils are equally dense on having been consolidated to the same effective stress, they will exhibit different shearing strengths if drainage is permitted during shear for one, while it is not for the other.

These ideas lead to a statement that ''the strength of a soil is a unique function of the effective stress acting on the failure plane''.

$$s = c' + \sigma_f \tan \varphi'$$

where c' and φ' are called the effective cohesion and effective angle of internal friction. Collectively, they are called 'effective stress parameters', while c and φ are called ''total stress parameters''.

TYPES OF SHEAR TESTS BASED ON DRAINAGE CONDITIONS

A cohesionless or a coarse-grained soil may be tested for shearing strength either in the dry condition or in the saturated condition.

12 Shear Strength of Soil

The *shear strength* of a soil mass is the internal resistance per unit area that the soil mass can offer to resist failure and sliding along any plane inside it. One must understand the nature of shearing resistance in order to analyze soil stability problems, such as bearing capacity, slope stability, and lateral pressure on earth-retaining structures.

12.1 Mohr–Coulomb Failure Criterion

Mohr (1900) presented a theory for rupture in materials that contended that a material fails because of a critical combination of normal stress and shearing stress and not from either maximum normal or shear stress alone. Thus, the functional relationship between normal stress and shear stress on a failure plane can be expressed in the following form:

$$\tau_f = f(\sigma) \tag{12.1}$$

The failure envelope defined by Eq. (12.1) is a curved line. For most soil mechanics problems, it is sufficient to approximate the shear stress on the failure plane as a linear function of the normal stress (Coulomb, 1776). This linear function can be written as

$$\tau_f = c + \sigma \tan \phi \tag{12.2}$$

where c = cohesion
ϕ = angle of internal friction
σ = normal stress on the failure plane
τ_f = shear strength

The preceding equation is called the *Mohr–Coulomb failure criterion.*

In saturated soil, the total normal stress at a point is the sum of the effective stress (σ') and pore water pressure (u), or

$$\sigma = \sigma' + u$$

The effective stress σ' is carried by the soil solids. The Mohr–Coulomb failure criterion, expressed in terms of effective stress, will be of the form

$$\tau_f = c' + \sigma' \tan \phi' \tag{12.3}$$

where c' = cohesion and ϕ' = friction angle, based on effective stress.

Thus, Eqs. (12.2) and (12.3) are expressions of shear strength based on total stress and effective stress. The value of c' for sand and inorganic silt is 0. For normally consolidated clays, c' can be approximated at 0. Overconsolidated clays have values of c' that are greater than 0. The angle of friction, ϕ', is sometimes referred to as the *drained angle of friction*. Typical values of ϕ' for some granular soils are given in Table 12.1.

The significance of Eq. (12.3) can be explained by referring to Fig. 12.1, which shows an elemental soil mass. Let the effective normal stress and the shear stress on the

Table 12.1 Typical Values of Drained Angle of Friction for Sands and Silts

Soil type	ϕ' (deg)
Sand: Rounded grains	
Loose	27–30
Medium	30–35
Dense	35–38
Sand: Angular grains	
Loose	30–35
Medium	35–40
Dense	40–45
Gravel with some sand	34–48
Silts	26–35

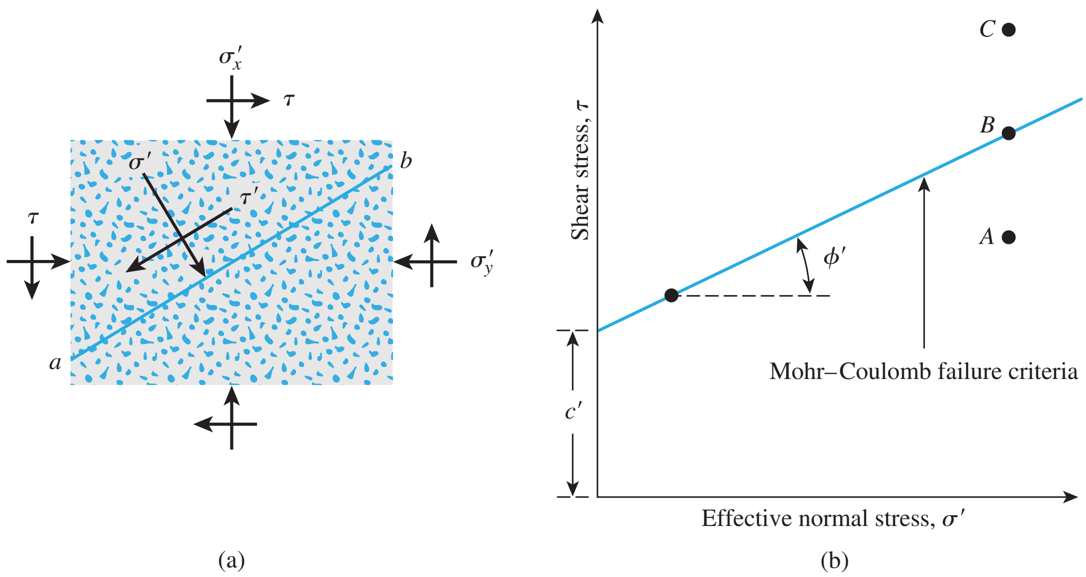


Figure 12.1 Mohr–Coulomb failure criterion

plane *ab* be σ' and τ, respectively. Figure 12.1b shows the plot of the failure envelope defined by Eq. (12.3). If the magnitudes of σ' and τ on plane *ab* are such that they plot as point *A* in Figure 12.1b, shear failure will not occur along the plane. If the effective normal stress and the shear stress on plane *ab* plot as point *B* (which falls on the failure envelope), shear failure will occur along that plane. A state of stress on a plane represented by point *C* cannot exist, because it plots above the failure envelope, and shear failure in a soil would have occurred already.

12.2 Inclination of the Plane of Failure Caused by Shear

As stated by the Mohr–Coulomb failure criterion, failure from shear will occur when the shear stress on a plane reaches a value given by Eq. (12.3). To determine the inclination of the failure plane with the major principal plane, refer to Figure 12.2, where σ'_1 and σ'_3 are, respectively, the major and minor effective principal stresses. The failure plane *EF* makes an angle θ with the major principal plane. To determine the angle θ and the relationship between σ'_1 and σ'_3, refer to Figure 12.3, which is a plot of the Mohr's circle for the state of stress shown in Figure 12.2 (see Chapter 10). In Figure 12.3, *fgh* is the failure envelope defined by the relationship $\tau_f = c' + \sigma' \tan \phi'$. The radial line *ab* defines the major principal plane (*CD* in Figure 12.2), and the radial line *ad* defines the failure plane (*EF* in Figure 12.2). It can be shown that $\angle bad = 2\theta = 90 + \phi'$, or

$$\theta = 45 + \frac{\phi'}{2} \tag{12.4}$$

Again, from Figure 12.3,

$$\frac{\overline{ad}}{\overline{fa}} = \sin \phi' \tag{12.5}$$

$$\overline{fa} = fO + Oa = c' \cot \phi' + \frac{\sigma'_1 + \sigma'_3}{2} \tag{12.6a}$$

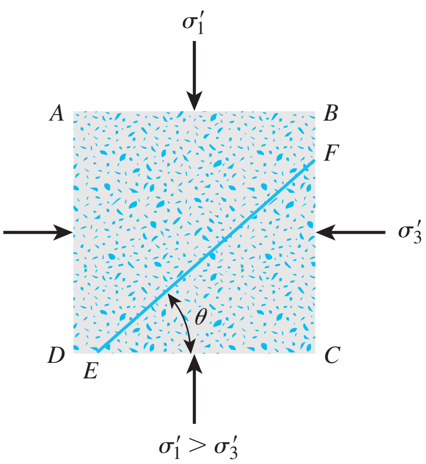


Figure 12.2 Inclination of failure plane in soil with major principal plane

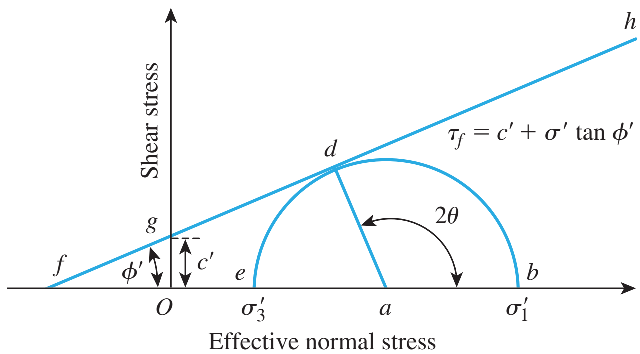


Figure 12.3 Mohr's circle and failure envelope

Also,

$$\overline{ad} = \frac{\sigma_1' - \sigma_3'}{2} \tag{12.6b}$$

Substituting Eqs. (12.6a) and (12.6b) into Eq. (12.5), we obtain

$$\sin\phi' = \frac{\dfrac{\sigma_1' - \sigma_3'}{2}}{c'\cot\phi' + \dfrac{\sigma_1' + \sigma_3'}{2}}$$

or

$$\sigma_1' = \sigma_3'\left(\frac{1 + \sin\phi'}{1 - \sin\phi'}\right) + 2c\left(\frac{\cos\phi'}{1 - \sin\phi'}\right) \tag{12.7}$$

However,

$$\frac{1 + \sin\phi'}{1 - \sin\phi'} = \tan^2\left(45 + \frac{\phi'}{2}\right)$$

and

$$\frac{\cos\phi'}{1 - \sin\phi'} = \tan\left(45 + \frac{\phi'}{2}\right)$$

Thus,

$$\sigma_1' = \sigma_3'\tan^2\left(45 + \frac{\phi'}{2}\right) + 2c'\tan\left(45 + \frac{\phi'}{2}\right) \tag{12.8}$$

An expression similar to Eq. (12.8) could also be derived using Eq. (12.2) (that is, total stress parameters c and ϕ), or

$$\sigma_1 = \sigma_3\tan^2\left(45 + \frac{\phi}{2}\right) + 2c\tan\left(45 + \frac{\phi}{2}\right) \tag{12.9}$$

12.3 Laboratory Test for Determination of Shear Strength Parameters

There are several laboratory methods now available to determine the shear strength parameters (i.e., c, ϕ, c', ϕ') of various soil specimens in the laboratory. They are as follows:

- Direct shear test
- Triaxial test
- Direct simple shear test
- Plane strain triaxial test
- Torsional ring shear test

The direct shear test and the triaxial test are the two commonly used techniques for determining the shear strength parameters. These two tests will be described in detail in the sections that follow.

12.4 Direct Shear Test

The direct shear test is the oldest and simplest form of shear test arrangement. A diagram of the direct shear test apparatus is shown in Figure 12.4. The test equipment consists of a metal shear box in which the soil specimen is placed. The soil specimens may be square or circular in plan. The size of the specimens generally used is about 51 mm × 51 mm or 102 mm × 102 mm (2 in. × 2 in. or 4 in. × 4 in.) across and about 25 mm (1 in.) high. The box is split horizontally into halves. Normal force on the specimen is applied from the top of the shear box. The normal stress on the specimens can be as great as 1050 kN/m^2 (150 lb/in.^{2}). Shear force is applied by moving one-half of the box relative to the other to cause failure in the soil specimen.

Depending on the equipment, the shear test can be either stress controlled or strain controlled. In stress-controlled tests, the shear force is applied in equal increments until the specimen fails. The failure occurs along the plane of split of the shear box. After the application of each incremental load, the shear displacement of the top half of the box is measured by a horizontal dial gauge. The change in the height of the specimen (and thus the volume change of the specimen) during the test can be obtained from the readings of a dial gauge that measures the vertical movement of the upper loading plate.

In strain-controlled tests, a constant rate of shear displacement is applied to one-half of the box by a motor that acts through gears. The constant rate of shear displacement is measured by a horizontal dial gauge. The resisting shear force of the soil corresponding to any shear displacement can be measured by a horizontal proving ring or load cell. The volume change of the specimen during the test is obtained in a manner

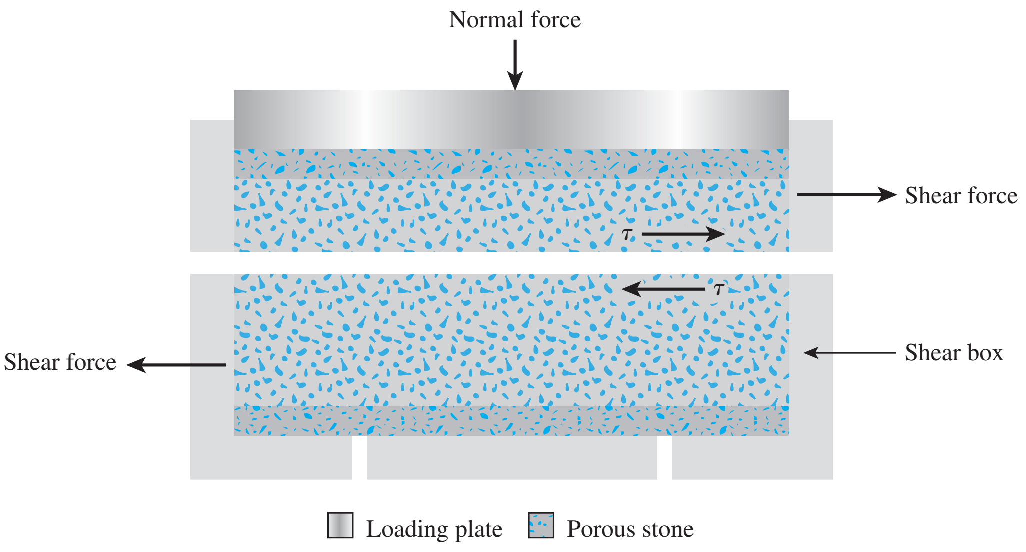


Figure 12.4 Diagram of direct shear test arrangement

Figure 12.5 Strain-controlled direct shear equipment (*Courtesy of Braja M. Das, Henderson, Nevada*)

similar to that in the stress-controlled tests. Figure 12.5 shows a photograph of strain-controlled direct shear test equipment. Figure 12.6 shows a photograph taken from the top of the direct shear test equipment with the dial gages and proving ring in place.

The advantage of the strain-controlled tests is that in the case of dense sand, peak shear resistance (that is, at failure) as well as lesser shear resistance (that is, at a point after failure called *ultimate strength*) can be observed and plotted. In stress-controlled tests, only the peak shear resistance can be observed and plotted. Note that the peak shear resistance in stress-controlled tests can be only approximated because failure occurs at a stress level somewhere between the prefailure load increment and the failure load increment. Nevertheless, compared with strain-controlled tests, stress-controlled tests probably model real field situations better.

For a given test, the normal stress can be calculated as

$$\sigma = \text{Normal stress} = \frac{\text{Normal force}}{\text{Cross-sectional area of the specimen}} \tag{12.10}$$

The resisting shear stress for any shear displacement can be calculated as

$$\tau = \text{Shear stress} = \frac{\text{Resisting shear force}}{\text{Cross-sectional area of the specimen}} \tag{12.11}$$

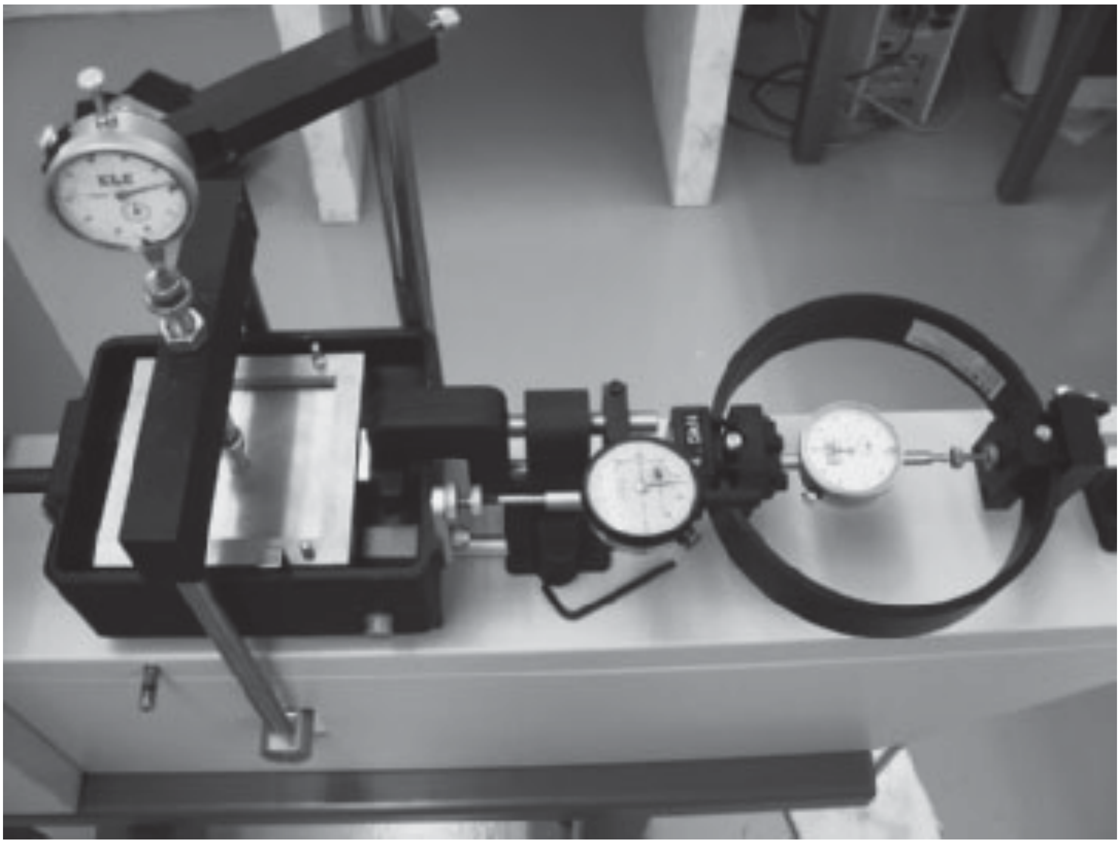


Figure 12.6 A photograph showing the dial gauges and proving ring in place (*Courtesy of Braja M. Das, Henderson, Nevada*)

Figure 12.7 shows a typical plot of shear stress and change in the height of the specimen against shear displacement for dry loose and dense sands. These observations were obtained from a strain-controlled test. The following generalizations can be developed from Figure 12.7 regarding the variation of resisting shear stress with shear displacement:

1. In loose sand, the resisting shear stress increases with shear displacement until a failure shear stress of τ_f is reached. After that, the shear resistance remains approximately constant for any further increase in the shear displacement.
2. In dense sand, the resisting shear stress increases with shear displacement until it reaches a failure stress of τ_f. This τ_f is called the *peak shear strength.* After failure stress is attained, the resisting shear stress gradually decreases as shear displacement increases until it finally reaches a constant value called the *ultimate shear strength.*

Since the height of the specimen changes during the application of the shear force (as shown in Figure 12.7), it is obvious that the void ratio of the sand changes (at least in the vicinity of the split of the shear box). Figure 12.8 shows the nature of variation of the void ratio for loose and dense sands with shear displacement. At large shear displacement, the void ratios of loose and dense sands become practically the same, and this is termed the *critical void ratio.*

It is important to note that, in dry sand,

$$\sigma = \sigma'$$

and

$$c' = 0$$

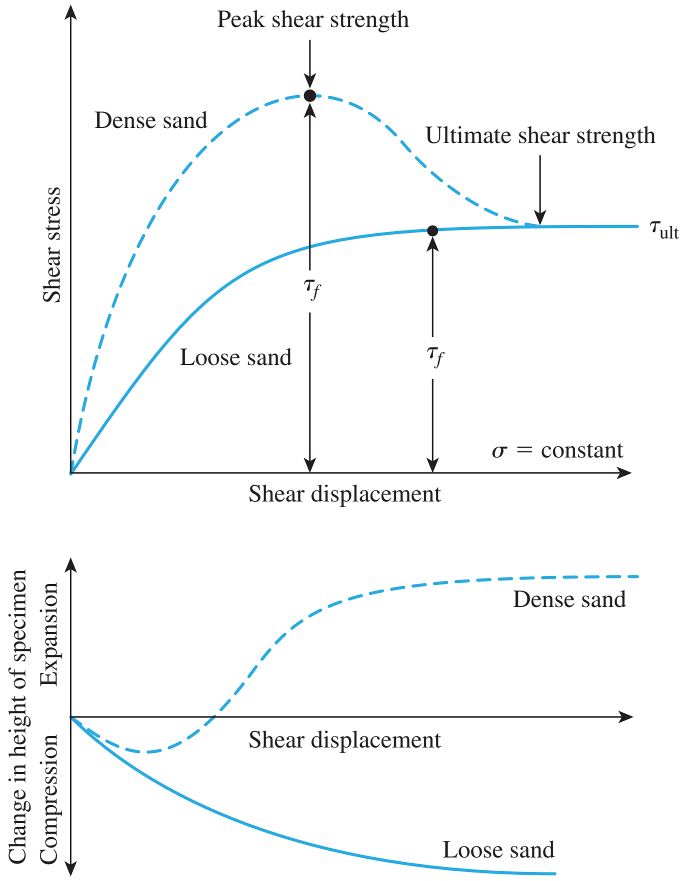


Figure 12.7 Plot of shear stress and change in height of specimen against shear displacement for loose and dense dry sand (direct shear test)

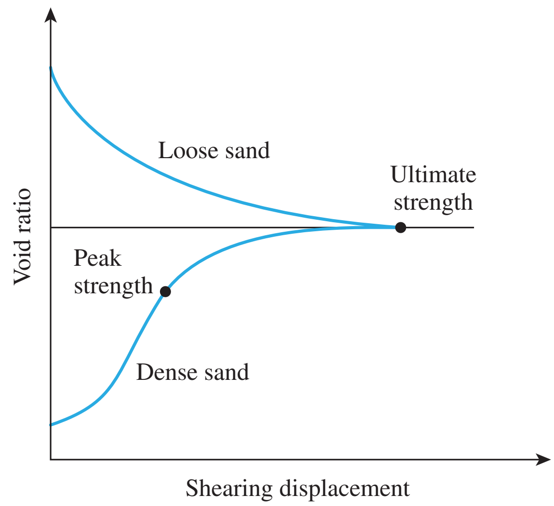


Figure 12.8 Nature of variation of void ratio with shearing displacement

Direct shear tests are repeated on similar specimens at various normal stresses. The normal stresses and the corresponding values of τ_f obtained from a number of tests are plotted on a graph from which the shear strength parameters are determined. Figure 12.9 shows such a plot for tests on a dry sand. The equation for the average line obtained from experimental results is

$$\tau_f = \sigma' \tan \phi' \tag{12.12}$$

So, the friction angle can be determined as follows:

$$\phi' = \tan^{-1}\left(\frac{\tau_f}{\sigma'}\right) \tag{12.13}$$

It is important to note that *in situ* cemented sands may show a c' intercept.

If the variation of the ultimate shear strength (τ_{ult}) with normal stress is known, it can be plotted as shown in Figure 12.9. The average plot can be expressed as

$$\tau_{ult} = \sigma' \tan \phi'_{ult} \tag{12.14}$$

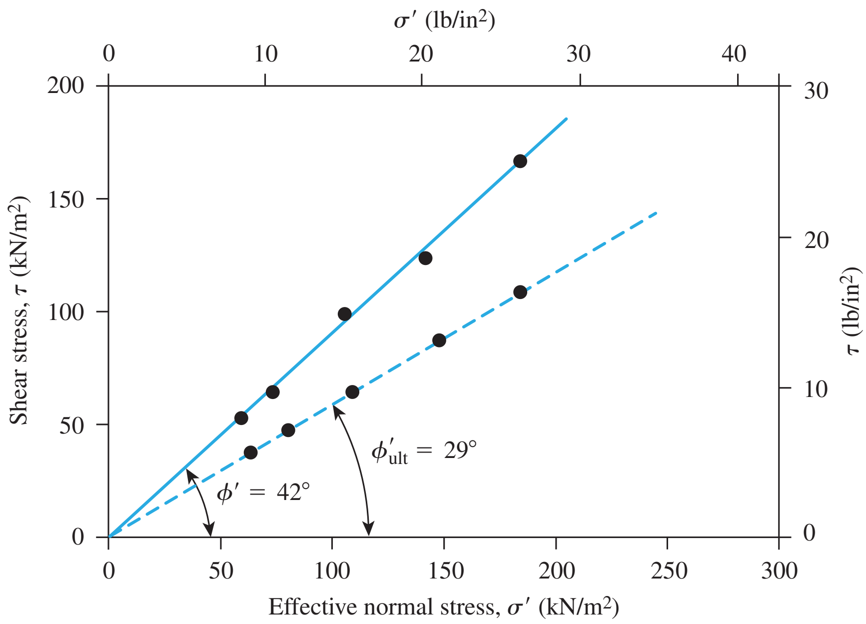


Figure 12.9 Determination of shear strength parameters for a dry sand using the results of direct shear tests

or

$$\phi'_{ult} = \tan^{-1}\left(\frac{\tau_{ult}}{\sigma'}\right) \tag{12.15}$$

12.5 Drained Direct Shear Test on Saturated Sand and Clay

In the direct shear test arrangement, the shear box that contains the soil specimen is generally kept inside a container that can be filled with water to saturate the specimen. A *drained test* is made on a saturated soil specimen by keeping the rate of loading slow enough so that the excess pore water pressure generated in the soil is dissipated completely by drainage. Pore water from the specimen is drained through two porous stones. (See Figure 12.4.)

Because the hydraulic conductivity of sand is high, the excess pore water pressure generated due to loading (normal and shear) is dissipated quickly. Hence, for an ordinary loading rate, essentially full drainage conditions exist. The friction angle, ϕ', obtained from a drained direct shear test of saturated sand will be the same as that for a similar specimen of dry sand.

The hydraulic conductivity of clay is very small compared with that of sand. When a normal load is applied to a clay soil specimen, a sufficient length of time must elapse for full consolidation—that is, for dissipation of excess pore water pressure. For this reason, the shearing load must be applied very slowly. The test may last from two to five days. Figure 12.10 shows the results of a drained direct shear test on an overconsolidated clay. Figure 12.11 shows the plot of τ_f against σ' obtained from a number of drained direct shear tests on a normally consolidated clay and an overconsolidated clay. Note that the value of $c' \simeq 0$ for a normally consolidated clay.

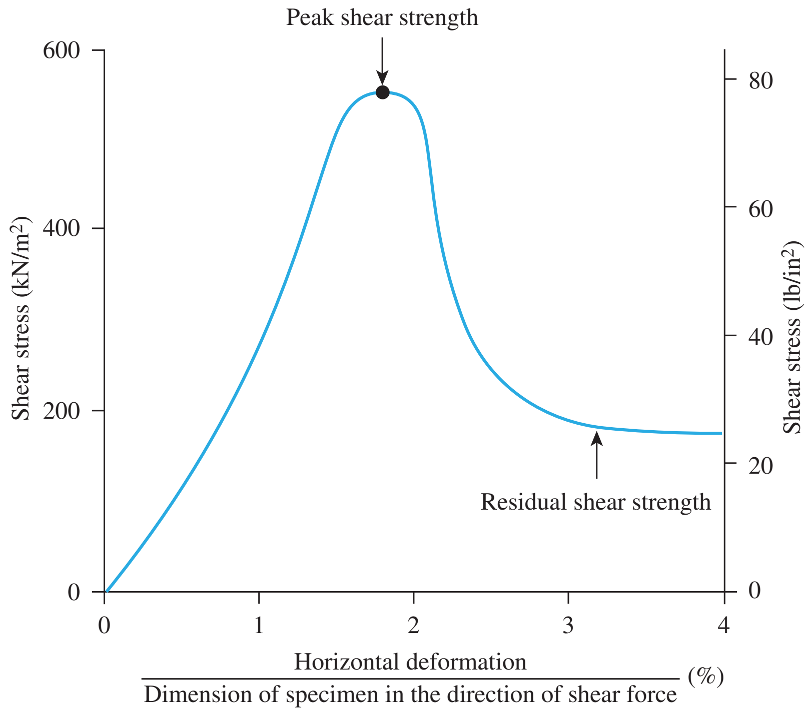


Figure 12.10 Results of a drained direct shear test on an overconsolidated clay [*Note:* Residual shear strength in clay is similar to ultimate shear strength in sand (see Figure 12.7)]

Similar to the ultimate shear strength in the case of sand (Figure 12.8), at large shearing displacements, we can obtain the *residual shear strength* of clay (τ_r) in a drained test. This is shown in Figure 12.10. Figure 12.11 shows the plot of τ_r versus σ'. The average plot *will pass through the origin* and can be expressed as

$$\tau_r = \sigma' \tan \phi'_r$$

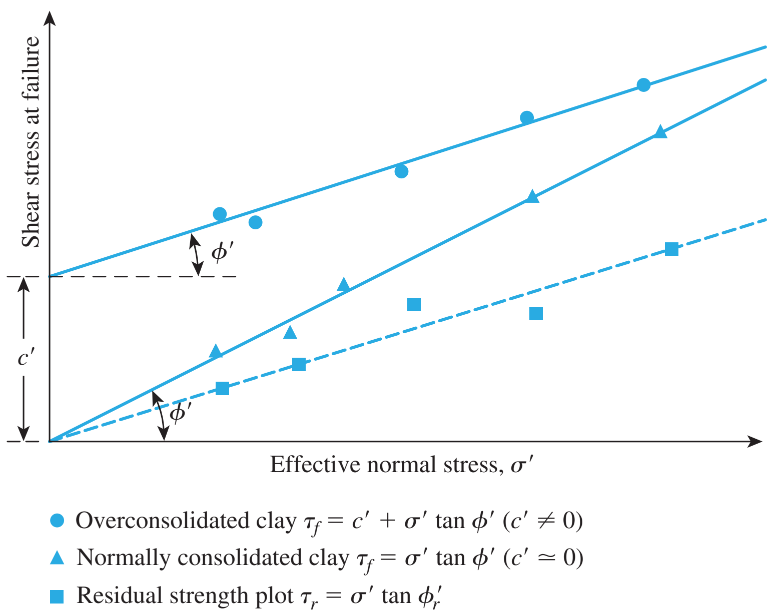


Figure 12.11 Failure envelope for clay obtained from drained direct shear tests

or

$$\phi'_r = \tan^{-1}\left(\frac{\tau_r}{\sigma'}\right) \tag{12.16}$$

The drained angle of friction, ϕ', of normally consolidated clays generally decreases with the plasticity index of soil. This fact is illustrated in Figure 12.12 for a number of clays from data reported by Kenney (1959). Although the data are scattered considerably, the general pattern seems to hold.

Skempton (1964) provided the results of the variation of the residual angle of friction, ϕ'_r, of a number of clayey soils with the clay-size fraction ($\leq 2\ \mu$m) present. The following table shows a summary of these results.

Soil	**Clay-size fraction (%)**	**Residual friction angle, ϕ'_r (deg)**
Selset	17.7	29.8
Wiener Tegel	22.8	25.1
Jackfield	35.4	19.1
Oxford clay	41.9	16.3
Jari	46.5	18.6
London clay	54.9	16.3
Walton's Wood	67	13.2
Weser-Elbe	63.2	9.3
Little Belt	77.2	11.2
Biotite	100	7.5

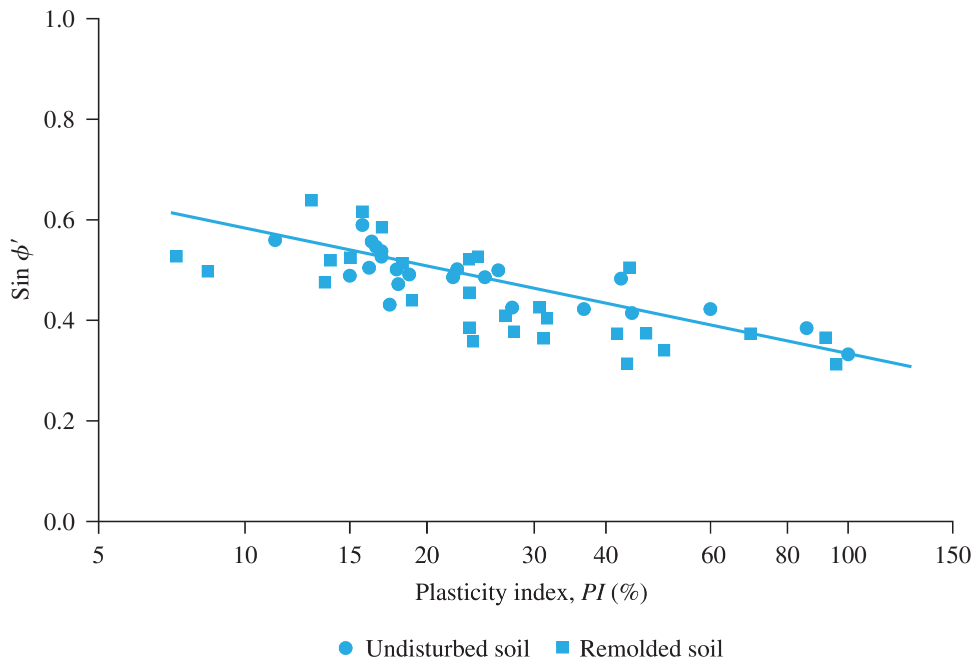


Figure 12.12 Variation of sin ϕ' with plasticity index for a number of soils (*After Kenney, 1959. With permission from ASCE.*)

12.6 General Comments on Direct Shear Test

The direct shear test is simple to perform, but it has some inherent shortcomings. The reliability of the results may be questioned because the soil is not allowed to fail along the weakest plane but is forced to fail along the plane of split of the shear box. Also, the shear stress distribution over the shear surface of the specimen is not uniform. Despite these shortcomings, the direct shear test is the simplest and most economical for a dry or saturated sandy soil.

In many foundation design problems, one must determine the angle of friction between the soil and the material in which the foundation is constructed (Figure 12.13). The foundation material may be concrete, steel, or wood. The shear strength along the surface of contact of the soil and the foundation can be given as

$$\tau_f = c'_a + \sigma' \tan \delta' \tag{12.17}$$

where c'_a = adhesion
δ' = effective angle of friction between the soil and the foundation material

Note that the preceding equation is similar in form to Eq. (12.3). The shear strength parameters between a soil and a foundation material can be conveniently determined by a direct shear test. This is a great advantage of the direct shear test. The foundation material can be placed in the bottom part of the direct shear test box and then the soil can be placed above it (that is, in the top part of the box), as shown in Figure 12.14, and the test can be conducted in the usual manner.

Figure 12.15 shows the results of direct shear tests conducted in this manner with a quartz sand and concrete, wood, and steel as foundation materials, with $\sigma' = 100\ \text{kN/m}^2$ ($14.5\ \text{lb/in.}^2$).

It was mentioned briefly in Section 12.1 [related to Eq. (12.1)] that Mohr's failure envelope is curvilinear in nature, and Eq. (12.2) is only an approximation. This fact should be kept in mind when considering problems at higher confining pressures. Figure 12.16 shows the decrease of ϕ' and δ' with the increase of normal stress (σ') for the same materials discussed in Figure 12.15. This can be explained by referring to Figure 12.17, which shows a curved Mohr's failure envelope. If a direct shear test is conducted with $\sigma' = \sigma'_{(1)}$, the shear strength will be $\tau_{f(1)}$. So,

$$\delta'_1 = \tan^{-1}\left[\frac{\tau_{f(1)}}{\sigma'_{(1)}}\right]$$

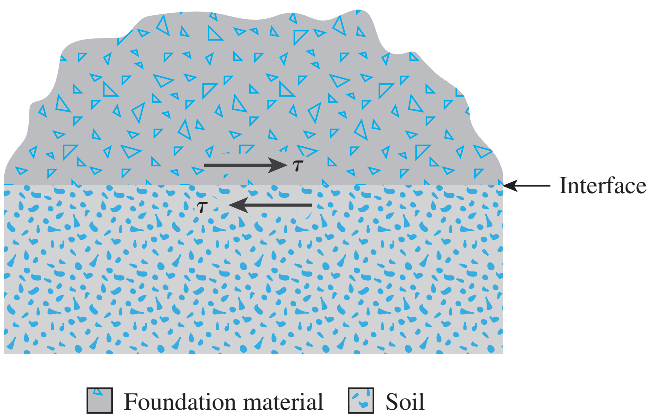


Figure 12.13
Interface of a foundation material and soil

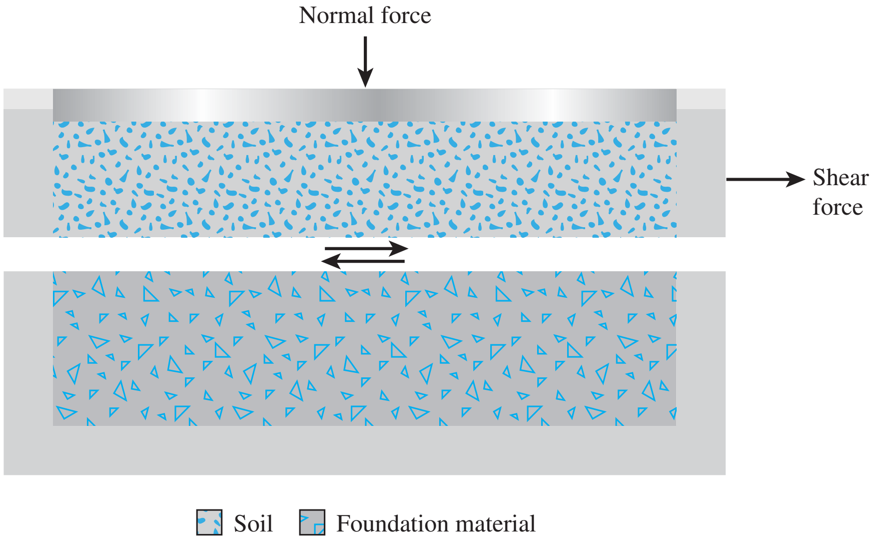


Figure 12.14 Direct shear test to determine interface friction angle

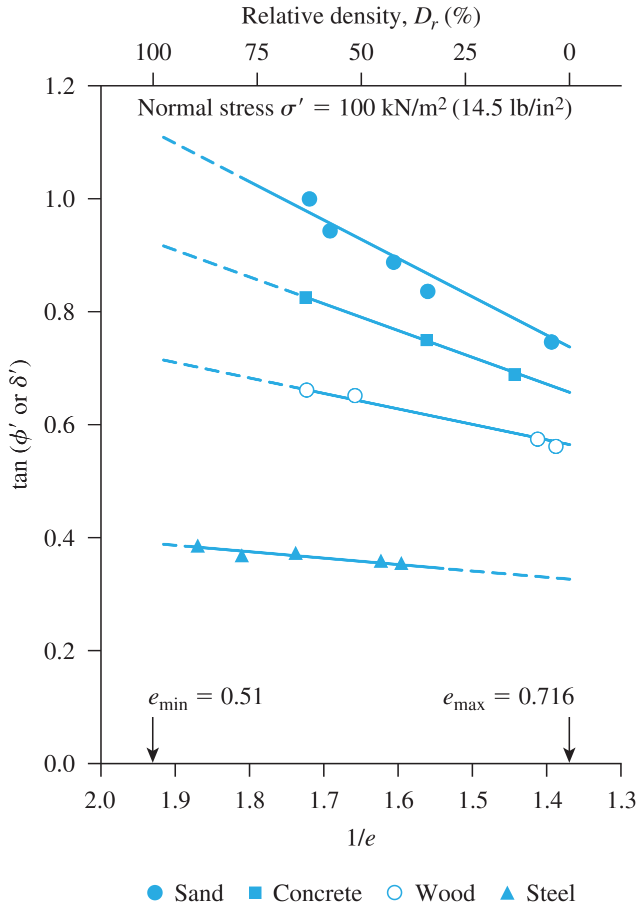


Figure 12.15 Variation of tan ϕ' and tan δ' with $1/e$ [*Note: e* = void ratio, $\sigma' = 100$ kN/m^2 (14.5 lb/in.^{2}), quartz sand] (*After Acar, Durgunoglu, and Tumay, 1982. With permission from ASCE.*)

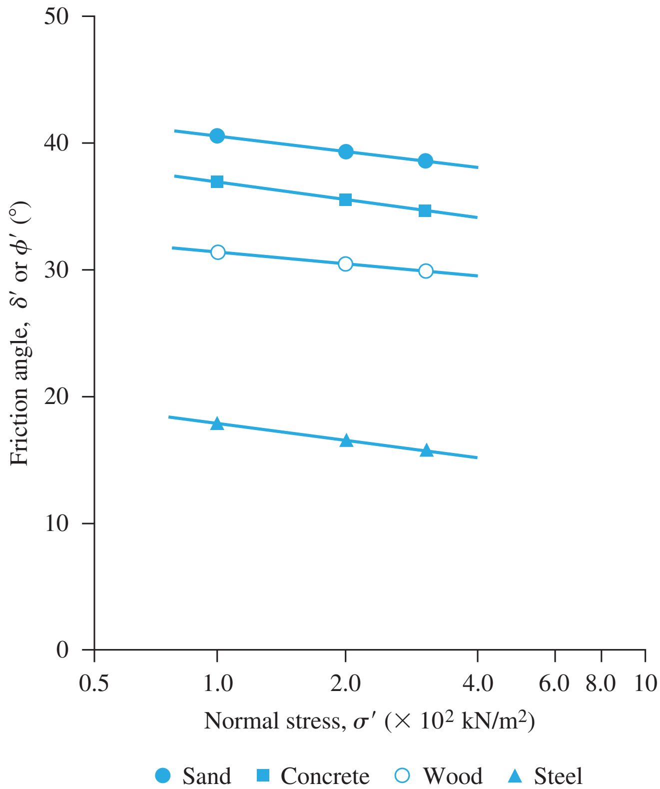


Figure 12.16 Variation of ϕ' and δ' with σ' (*Note:* Relative density = 45%; quartz sand) (*After Acar, Durgunoglu, and Tumay, 1982. With permission from ASCE.*)

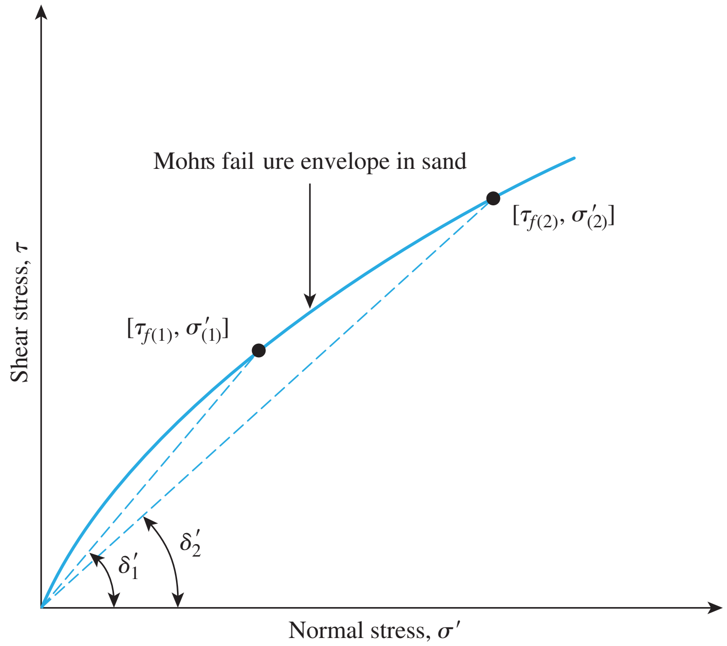


Figure 12.17 Curvilinear nature of Mohr's failure envelope in sand

This is shown in Figure 12.17. In a similar manner, if the test is conducted with $\sigma' = \sigma'_{(2)}$, then

$$\delta' = \delta'_2 = \tan^{-1}\left[\frac{\tau_{f(2)}}{\sigma'_{(2)}}\right]$$

As can be seen from Figure 12.17, $\delta'_2 < \delta'_1$ since $\sigma'_2 > \sigma'_{(1)}$. Keeping this in mind, it must be realized that the values of ϕ' given in Table 12.1 are only the average values.

Example 12.1

Following are the results of four drained direct shear tests on an *overconsolidated clay:*

- Diameter of specimen = 50 mm
- Height of specimen = 25 mm

Test no.	Normal force, N (N)	Shear force at failure, S_{peak} (N)	Residual shear force, $S_{residual}$ (N)
1	150	157.5	44.2
2	250	199.9	56.6
3	350	257.6	102.9
4	550	363.4	144.5

Determine the relationships for *peak shear strength* (τ_f) and *residual shear strength* (τ_r).

Solution

Area of the specimen $(A) = (\pi/4)\left(\frac{50}{1000}\right)^2 = 0.0019634\ \text{m}^2$. Now the following table can be prepared.

Test no.	Normal force, N (N)	Normal stress, σ' (kN/m²)	Peak shear force, S_{peak} (N)	$\tau_f = \frac{S_{peak}}{A}$ (kN/m²)	Residual shear force, $S_{residual}$ (N)	$\tau_r = \frac{S_{residual}}{A}$ (kN/m²)
1	150	76.4	157.5	80.2	44.2	22.5
2	250	127.3	199.9	101.8	56.6	28.8
3	350	178.3	257.6	131.2	102.9	52.4
4	550	280.1	363.4	185.1	144.5	73.6

The variations of τ_f and τ_r with σ' are plotted in Figure 12.18. From the plots, we find that

Peak strength: $\tau_f(\text{kN/m}^2) = \mathbf{40 + \sigma' \tan 27}$

Residual strength: $\tau_r(\text{kN/m}^2) = \mathbf{\sigma' \tan 14.6}$

(*Note:* For all *overconsolidated clays,* the residual shear strength can be expressed as

$$\tau_r = \sigma' \tan \phi'_r$$

where ϕ'_r = effective residual friction angle.)

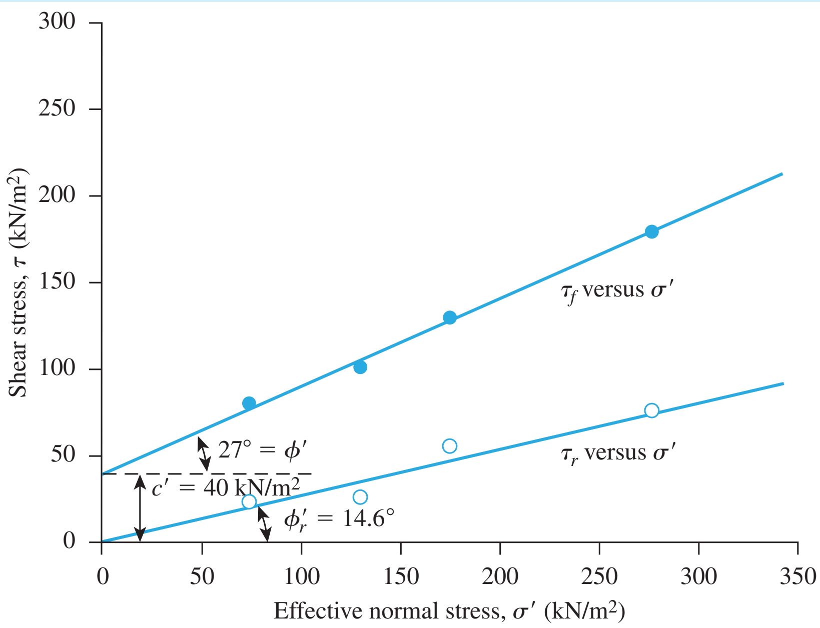


Figure 12.18 Variations of τ_f and τ_r with σ' ■

12.7 Triaxial Shear Test (General)

The triaxial shear test is one of the most reliable methods available for determining shear strength parameters. It is used widely for research and conventional testing. A diagram of the triaxial test layout is shown in Figure 12.19.

In this test, a soil specimen about 36 mm (1.4 in.) in diameter and 76 mm (3 in.) long generally is used. The specimen is encased by a thin rubber membrane and placed inside a plastic cylindrical chamber that usually is filled with water or glycerine. The specimen is subjected to a confining pressure by compression of the fluid in the chamber. (*Note:* Air is sometimes used as a compression medium.) To cause shear failure in the specimen, one must apply axial stress through a vertical loading ram (sometimes called *deviator stress*). This stress can be applied in one of two ways:

1. Application of dead weights or hydraulic pressure in equal increments until the specimen fails. (Axial deformation of the specimen resulting from the load applied through the ram is measured by a dial gauge.)
2. Application of axial deformation at a constant rate by means of a geared or hydraulic loading press. This is a strain-controlled test.

The axial load applied by the loading ram corresponding to a given axial deformation is measured by a proving ring or load cell attached to the ram.

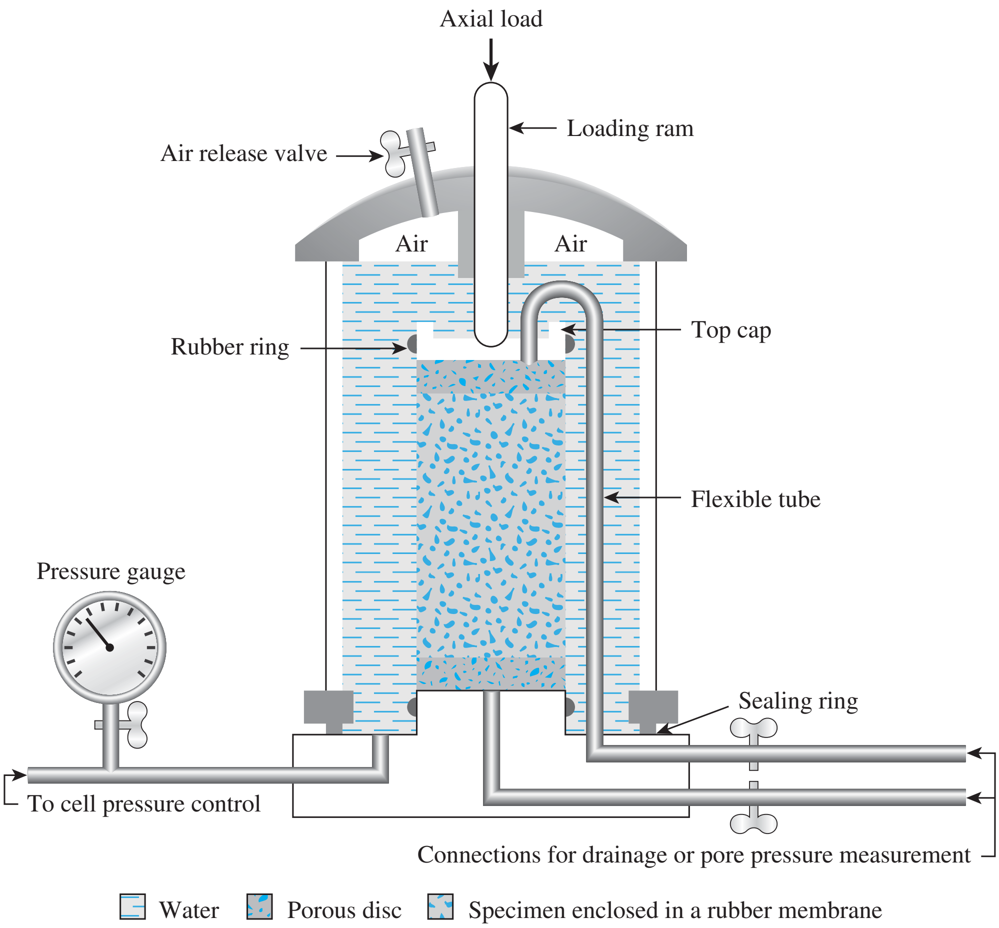


Figure 12.19 Diagram of triaxial test equipment (*After Bishop and Bjerrum, 1960. With permission from ASCE.*)

Connections to measure drainage into or out of the specimen, or to measure pressure in the pore water (as per the test conditions), also are provided. The following three standard types of triaxial tests generally are conducted:

1. Consolidated-drained test or drained test (CD test)
2. Consolidated-undrained test (CU test)
3. Unconsolidated-undrained test or undrained test (UU test)

The general procedures and implications for each of the tests in *saturated soils* are described in the following sections.

12.8 Consolidated-Drained Triaxial Test

In the CD test, the saturated specimen first is subjected to an all around confining pressure, σ_3, by compression of the chamber fluid (Figure 12.20a). As confining pressure is applied, the pore water pressure of the specimen increases by u_c (if drainage is prevented). This increase in the pore water pressure can be expressed as a nondimensional parameter in the form

$$B = \frac{u_c}{\sigma_3} \tag{12.18}$$

where B = Skempton's pore pressure parameter (Skempton, 1954).

For saturated soft soils, B is approximately equal to 1; however, for saturated stiff soils, the magnitude of B can be less than 1. Black and Lee (1973) gave the theoretical values of B for various soils at complete saturation. These values are listed in Table 12.2.

Now, if the connection to drainage is opened, dissipation of the excess pore water pressure, and thus consolidation, will occur. With time, u_c will become equal to 0. In saturated soil, the change in the volume of the specimen (ΔV_c) that takes place during consolidation can be obtained from the volume of pore water drained (Figure 12.21a). Next, the deviator stress, $\Delta\sigma_d$, on the specimen is increased very slowly (Figure 12.20b). The drainage connection is kept open, and the slow rate of deviator stress application allows complete dissipation of any pore water pressure that developed as a result ($\Delta u_d = 0$).

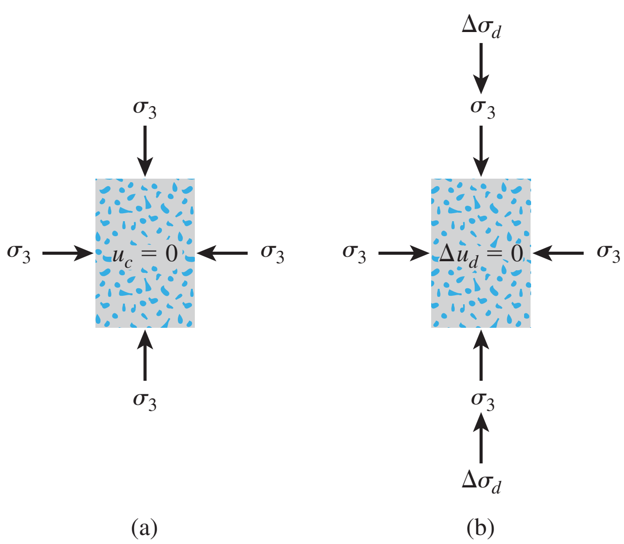


Figure 12.20 Consolidated-drained triaxial test: (a) specimen under chamber-confining pressure; (b) deviator stress application

Table 12.2 Theoretical Values of B at Complete Saturation

Type of soil	Theoretical value
Normally consolidated soft clay	0.9998
Lightly overconsolidated soft clays and silts	0.9988
Overconsolidated stiff clays and sands	0.9877
Very dense sands and very stiff clays at high confining pressures	0.9130

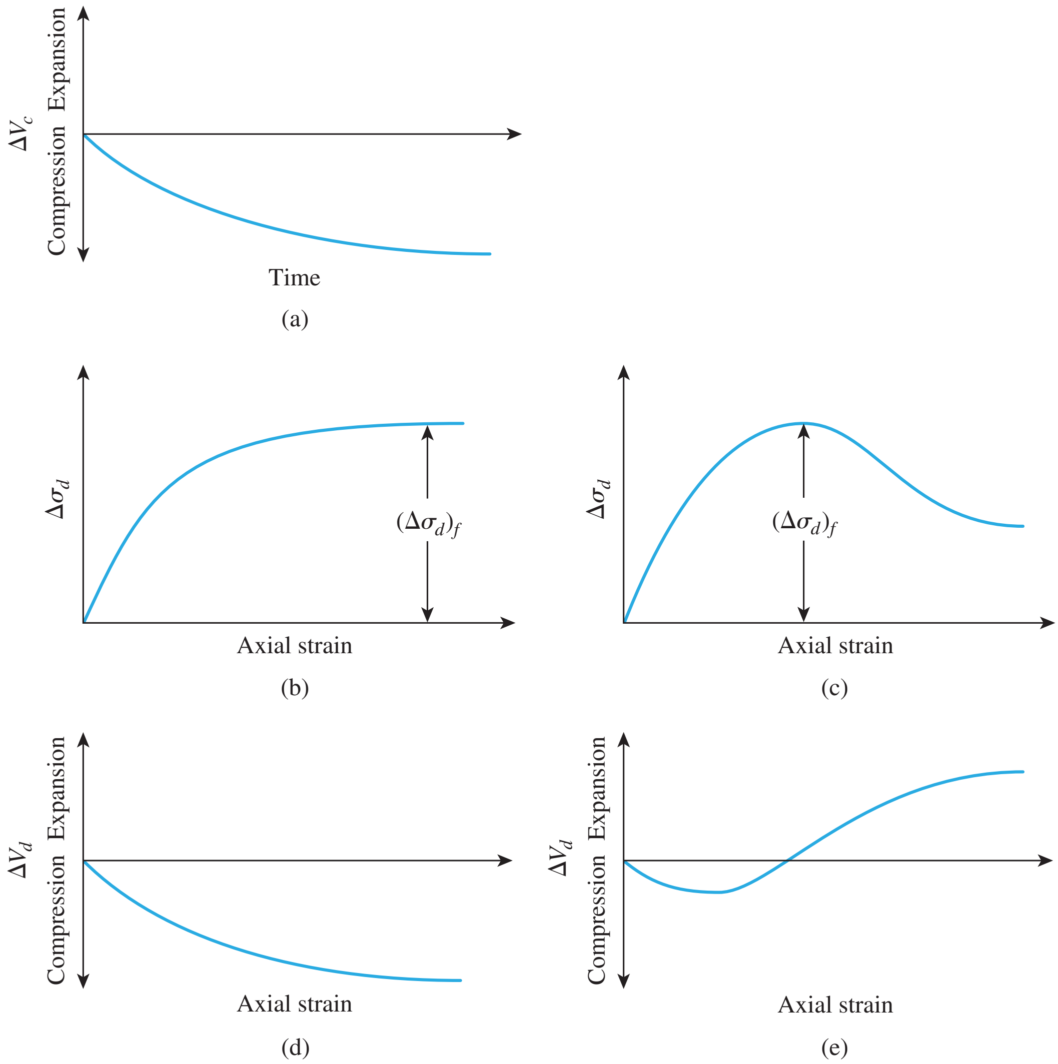


Figure 12.21 Consolidated-drained triaxial test: (a) volume change of specimen caused by chamber-confining pressure; (b) plot of deviator stress against strain in the vertical direction for loose sand and normally consolidated clay; (c) plot of deviator stress against strain in the vertical direction for dense sand and overconsolidated clay; (d) volume change in loose sand and normally consolidated clay during deviator stress application; (e) volume change in dense sand and overconsolidated clay during deviator stress application

A typical plot of the variation of deviator stress against strain in loose sand and normally consolidated clay is shown in Figure 12.21b. Figure 12.21c shows a similar plot for dense sand and overconsolidated clay. The volume change, ΔV_d, of specimens that occurs because of the application of deviator stress in various soils is also shown in Figures 12.21d and 12.21e.

Because the pore water pressure developed during the test is completely dissipated, we have

$$\text{Total and effective confining stress} = \sigma_3 = \sigma'_3$$

and

$$\text{Total and effective axial stress at failure} = \sigma_3 + (\Delta\sigma_d)_f = \sigma_1 = \sigma'_1$$

In a triaxial test, σ'_1 is the major principal effective stress at failure and σ'_3 is the minor principal effective stress at failure.

Several tests on similar specimens can be conducted by varying the confining pressure. With the major and minor principal stresses at failure for each test the Mohr's circles can be drawn and the failure envelopes can be obtained. Figure 12.22 shows the type of effective stress failure envelope obtained for tests on sand and normally consolidated clay. The coordinates of the point of tangency of the failure envelope with a Mohr's circle (that is, point A) give the stresses (normal and shear) on the failure plane of that test specimen.

For normally consolidated clay, referring to Figure 12.22

$$\sin\phi' = \frac{AO'}{OO'}$$

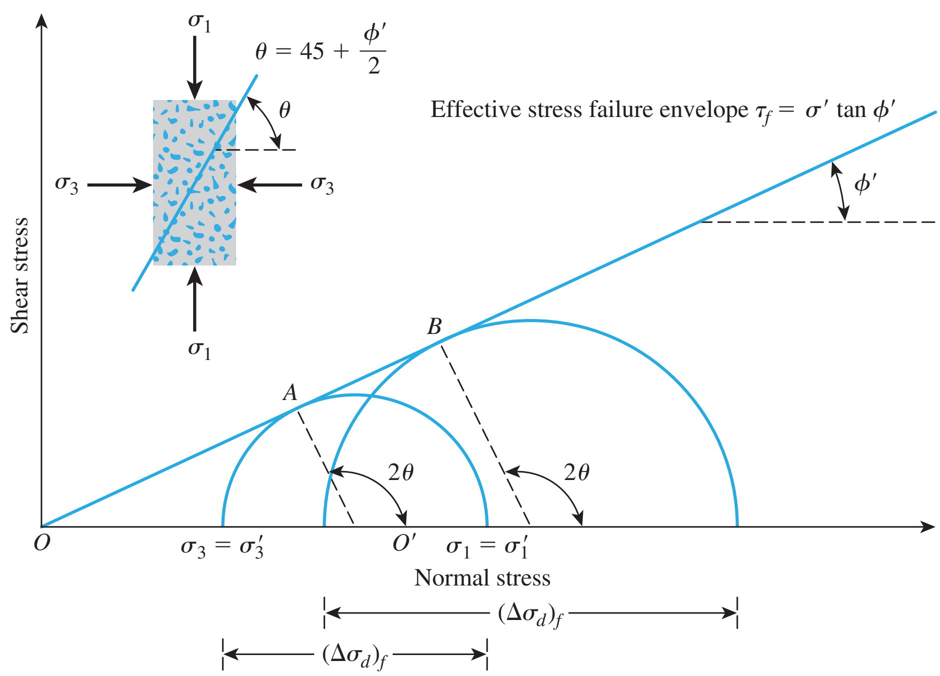


Figure 12.22 Effective stress failure envelope from drained tests on sand and normally consolidated clay

or

$$\sin\phi' = \frac{\left(\dfrac{\sigma'_1 - \sigma'_3}{2}\right)}{\left(\dfrac{\sigma'_1 + \sigma'_3}{2}\right)}$$

$$\phi' = \sin^{-1}\left(\frac{\sigma'_1 - \sigma'_3}{\sigma'_1 + \sigma'_3}\right) \tag{12.19}$$

Also, the failure plane will be inclined at an angle of $\theta = 45 + \phi'/2$ to the major principal plane, as shown in Figure 12.22.

Overconsolidation results when a clay initially is consolidated under an all-around chamber pressure of σ_c ($= \sigma'_c$) and is allowed to swell by reducing the chamber pressure to σ_3 ($= \sigma'_3$). The failure envelope obtained from drained triaxial tests of such overconsolidated clay specimens shows two distinct branches (*ab* and *bc* in Figure 12.23). The portion *ab* has a flatter slope with a cohesion intercept, and the shear strength equation for this branch can be written as

$$\tau_f = c' + \sigma' \tan\phi'_1 \tag{12.20}$$

The portion *bc* of the failure envelope represents a normally consolidated stage of soil and follows the equation $\tau_f = \sigma' \tan\phi'$.

If the triaxial test results of two overconsolidated soil specimens are known, the magnitudes of ϕ'_1 and c' can be determined as follows. From Eq. (12.8), for Specimen 1:

$$\sigma'_{1(1)} = \sigma'_{3(1)} \tan^2(45 + \phi'_1/2) + 2c' \tan(45 + \phi'_1/2) \tag{12.21}$$

And, for Specimen 2:

$$\sigma'_{1(2)} = \sigma'_{3(2)} \tan^2(45 + \phi'_1/2) + 2c' \tan(45 + \phi'_1/2) \tag{12.22}$$

or

$$\sigma'_{1(1)} - \sigma'_{1(2)} = [\sigma'_{3(1)} - \sigma'_{3(2)}] \tan^2(45 + \phi'_1/2)$$

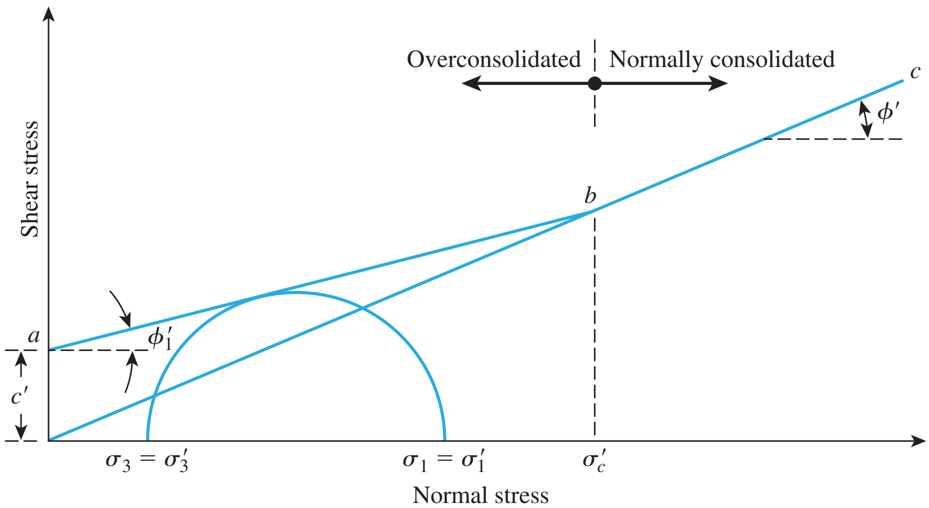


Figure 12.23 Effective stress failure envelope for overconsolidated clay

Hence,

$$\phi'_1 = 2\left\{\tan^{-1}\left[\frac{\sigma'_{1(1)} - \sigma'_{1(2)}}{\sigma'_{3(1)} - \sigma'_{3(2)}}\right]^{0.5} - 45^\circ\right\} \tag{12.23}$$

Once the value of ϕ'_1 is known, we can obtain c' as

$$c' = \frac{\sigma'_{1(1)} - \sigma'_{3(1)}\tan^2\left(45 + \frac{\phi'_1}{2}\right)}{2\tan\left(45 + \frac{\phi'_1}{2}\right)} \tag{12.24}$$

A consolidated-drained triaxial test on a clayey soil may take several days to complete. This amount of time is required because deviator stress must be applied very slowly to ensure full drainage from the soil specimen. For this reason, the CD type of triaxial test is uncommon.

Example 12.2

A consolidated-drained triaxial test was conducted on a normally consolidated clay. The results are as follows:

- $\sigma_3 = 16$ lb/in.^{2}
- $(\Delta\sigma_d)_f = 25$ lb/in.^{2}

Determine

a. Angle of friction, ϕ'
b. Angle θ that the failure plane makes with the major principal plane

Solution

For normally consolidated soil, the failure envelope equation is

$$\tau_f = \sigma' \tan\phi' \qquad (\text{because } c' = 0)$$

For the triaxial test, the effective major and minor principal stresses at failure are as follows:

$$\sigma'_1 = \sigma_1 = \sigma_3 + (\Delta\sigma_d)_f = 16 + 25 = 41 \text{ lb/in.}^2$$

and

$$\sigma'_3 = \sigma_3 = 16 \text{ lb/in.}^2$$

Part a

The Mohr's circle and the failure envelope are shown in Figure 12.24. From Eq. (12.19),

$$\sin\phi' = \frac{\sigma'_1 - \sigma'_3}{\sigma'_1 + \sigma'_3} = \frac{41 - 16}{41 + 16} = 0.438$$

or

$$\phi' = \mathbf{26^\circ}$$

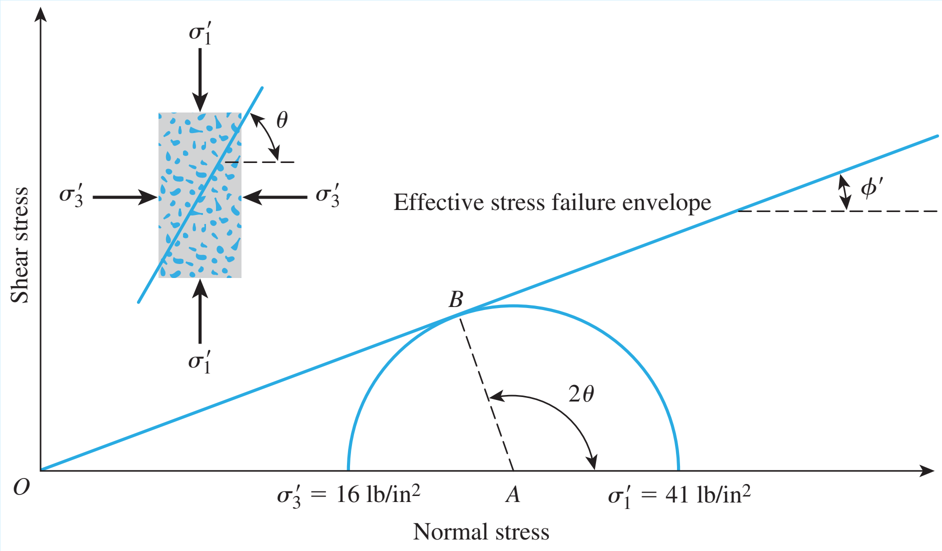


Figure 12.24 Mohr's circle and failure envelope for a normally consolidated clay

Part b

From Eq. (12.4),

$$\theta = 45 + \frac{\phi'}{2} = 45° + \frac{26°}{2} = \mathbf{58°}$$

■

Example 12.3

Refer to Example 12.2.

a. Find the normal stress σ' and the shear stress τ_f on the failure plane.
b. Determine the effective normal stress on the plane of maximum shear stress.

Solution

Part a

From Eqs. (10.8) and (10.9),

$$\sigma'\,(\text{on the failure plane}) = \frac{\sigma'_1 + \sigma'_3}{2} + \frac{\sigma'_1 - \sigma'_3}{2}\cos 2\theta$$

and

$$\tau_f = \frac{\sigma'_1 - \sigma'_3}{2}\sin 2\theta$$

Substituting the values of $\sigma'_1 = 41$ lb/in.^{2}, $\sigma'_3 = 16$ lb/in.^{2}, and $\theta = 58°$ into the preceding equations, we get

$$\sigma' = \frac{41 + 16}{2} + \frac{41 - 16}{2}\cos(2 \times 58) = \mathbf{23.0\ lb/in.^2}$$

and

$$\tau_f = \frac{41 - 16}{2} \sin(2 \times 58) = \mathbf{11.2\ lb/in.^2}$$

Part b

From Eq. (10.9), it can be seen that the maximum shear stress will occur on the plane with $\theta = 45°$. From Eq. (10.8),

$$\sigma' = \frac{\sigma'_1 + \sigma'_3}{2} + \frac{\sigma'_1 - \sigma'_3}{2} \cos 2\theta$$

Substituting $\theta = 45°$ into the preceding equation gives

$$\sigma' = \frac{41 + 16}{2} + \frac{41 - 16}{2} \cos 90 = \mathbf{28.5\ lb/in.^2}$$

■

Example 12.4

The equation of the effective stress failure envelope for normally consolidated clayey soil is $\tau_f = \sigma' \tan 30°$. A drained triaxial test was conducted with the same soil at a chamber-confining pressure of 10 lb/in.^{2} Calculate the deviator stress at failure.

Solution

For normally consolidated clay, $c' = 0$. Thus, from Eq. (12.8),

$$\sigma'_1 = \sigma'_3 \tan^2\left(45 + \frac{\phi'}{2}\right)$$

$$\phi' = 30°$$

$$\sigma'_1 = 10 \tan^2\left(45 + \frac{30}{2}\right) = 30\ \text{lb/in.}^2$$

So,

$$(\Delta\sigma_d)_f = \sigma'_1 - \sigma'_3 = 30 - 10 = \mathbf{20\ lb/in.^2}$$

■

Example 12.5

The results of two drained triaxial tests on a saturated clay follow:

Specimen I:

$$\sigma_3 = 70\ \text{kN/m}^2$$

$$(\Delta\sigma_d)_f = 130\ \text{kN/m}^2$$

Specimen II:

$$\sigma_3 = 160 \text{ kN/m}^2$$

$$(\Delta\sigma_d)_f = 223.5 \text{ kN/m}^2$$

Determine the shear strength parameters.

Solution

Refer to Figure 12.25. For Specimen I, the principal stresses at failure are

$$\sigma'_3 = \sigma_3 = 70 \text{ kN/m}^2$$

and

$$\sigma'_1 = \sigma_1 = \sigma_3 + (\Delta\sigma_d)_f = 70 + 130 = 200 \text{ kN/m}^2$$

Similarly, the principal stresses at failure for Specimen II are

$$\sigma'_3 = \sigma_3 = 160 \text{ kN/m}^2$$

and

$$\sigma'_1 = \sigma_1 = \sigma_3 + (\Delta\sigma_d)_f = 160 + 223.5 = 383.5 \text{ kN/m}^2$$

Now, from Eq. (12.23),

$$\phi'_1 = 2\left\{\tan^{-1}\left[\frac{\sigma'_{1(\text{I})} - \sigma'_{1(\text{II})}}{\sigma'_{3(\text{I})} - \sigma'_{3(\text{II})}}\right]^{0.5} - 45^\circ\right\} = 2\left\{\tan^{-1}\left[\frac{200 - 383.5}{70 - 160}\right]^{0.5} - 45^\circ\right\} = \mathbf{20^\circ}$$

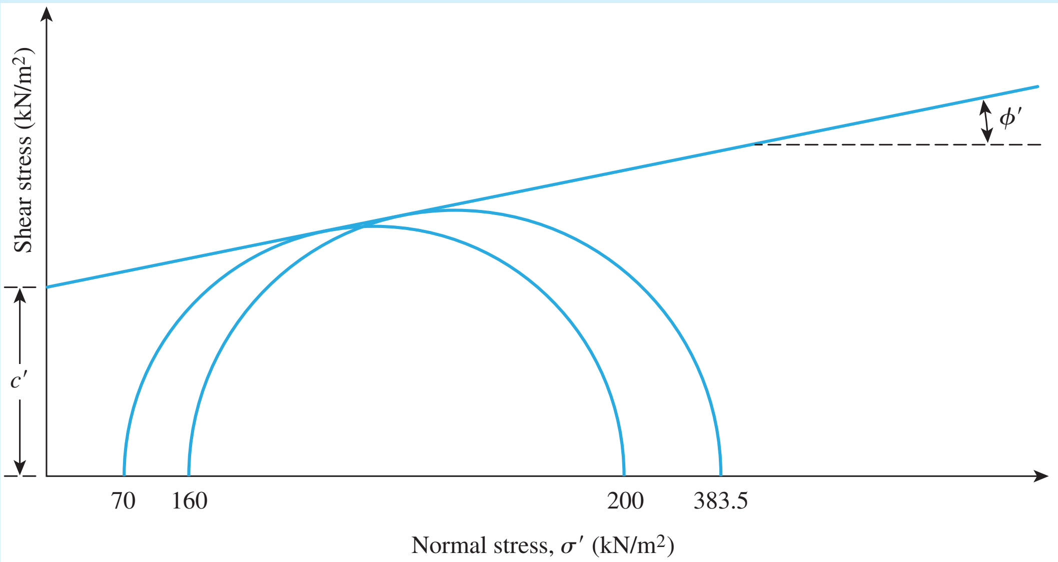


Figure 12.25 Effective stress failure envelope and Mohr's circles for Specimens I and II

Again, from Eq. (12.24),

$$c' = \frac{\sigma'_{1(\text{I})} - \sigma'_{3(\text{I})} \tan^2\left(45 + \frac{\phi'_1}{2}\right)}{2 \tan\left(45 + \frac{\phi'_1}{2}\right)} = \frac{200 - 70 \tan^2\left(45 + \frac{20}{2}\right)}{2 \tan\left(45 + \frac{20}{2}\right)} = \mathbf{20\ kN/m^2}$$ ■

12.9 Consolidated-Undrained Triaxial Test

The consolidated-undrained test is the most common type of triaxial test. In this test, the saturated soil specimen is first consolidated by an all-around chamber fluid pressure, σ_3, that results in drainage (Figures 12.26a and 12.26b). After the pore water pressure generated by the application of confining pressure is dissipated, the deviator stress, $\Delta\sigma_d$, on the specimen is increased to cause shear failure (Figure 12.26c). During this phase of the test, the drainage line from the specimen is kept closed. Because drainage is not permitted, the pore water pressure, Δu_d, will increase. During the test, simultaneous measurements of $\Delta\sigma_d$ and Δu_d are made. The increase in the pore water pressure, Δu_d, can be expressed in a nondimensional form as

$$\overline{A} = \frac{\Delta u_d}{\Delta \sigma_d} \tag{12.25}$$

where $\overline{A}$ = Skempton's pore pressure parameter (Skempton, 1954).

The general patterns of variation of $\Delta\sigma_d$ and Δu_d with axial strain for sand and clay soils are shown in Figures 12.26d through 12.26g. In loose sand and normally consolidated clay, the pore water pressure increases with strain. In dense sand and overconsolidated clay, the pore water pressure increases with strain to a certain limit, beyond which it decreases and becomes negative (with respect to the atmospheric pressure). This decrease is because of a tendency of the soil to dilate.

Unlike the consolidated-drained test, the total and effective principal stresses are not the same in the consolidated-undrained test. Because the pore water pressure at failure is measured in this test, the principal stresses may be analyzed as follows:

- Major principal stress at failure (total): $\sigma_3 + (\Delta\sigma_d)_f = \sigma_1$
- Major principal stress at failure (effective): $\sigma_1 - (\Delta u_d)_f = \sigma'_1$
- Minor principal stress at failure (total): σ_3
- Minor principal stress at failure (effective): $\sigma_3 - (\Delta u_d)_f = \sigma'_3$

In these equations, $(\Delta u_d)_f$ = pore water pressure at failure. The preceding derivations show that

$$\sigma_1 - \sigma_3 = \sigma'_1 - \sigma'_3$$

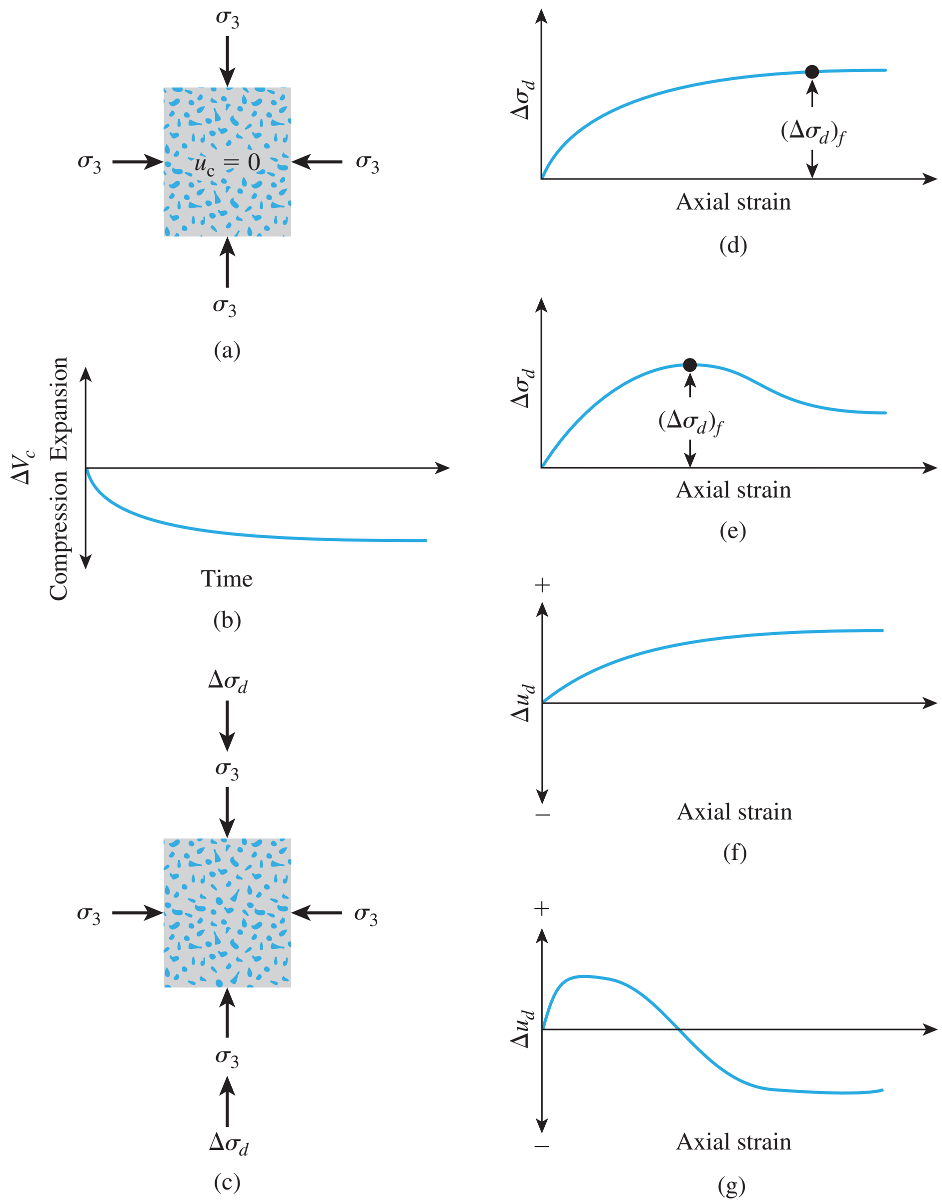


Figure 12.26 Consolidated undrained test: (a) specimen under chamber confining pressure; (b) volume change in specimen caused by confining pressure; (c) deviator stress application; (d) deviator stress against axial strain for loose sand and normally consolidated clay; (e) deviator stress against axial strain for dense sand and overconsolidated clay; (f) variation of pore water pressure with axial strain for loose sand and normally consolidated clay; (g) variation of pore water pressure with axial strain for dense sand and overconsolidated clay

Tests on several similar specimens with varying confining pressures may be conducted to determine the shear strength parameters. Figure 12.27 shows the total and effective stress Mohr's circles at failure obtained from consolidated-undrained triaxial tests in sand and normally consolidated clay. Note that *A* and *B* are two total stress Mohr's circles obtained from two tests. *C* and *D* are the effective stress Mohr's circles corresponding to total stress circles *A* and *B,* respectively. The diameters of circles *A* and *C* are the same; similarly, the diameters of circles *B* and *D* are the same.

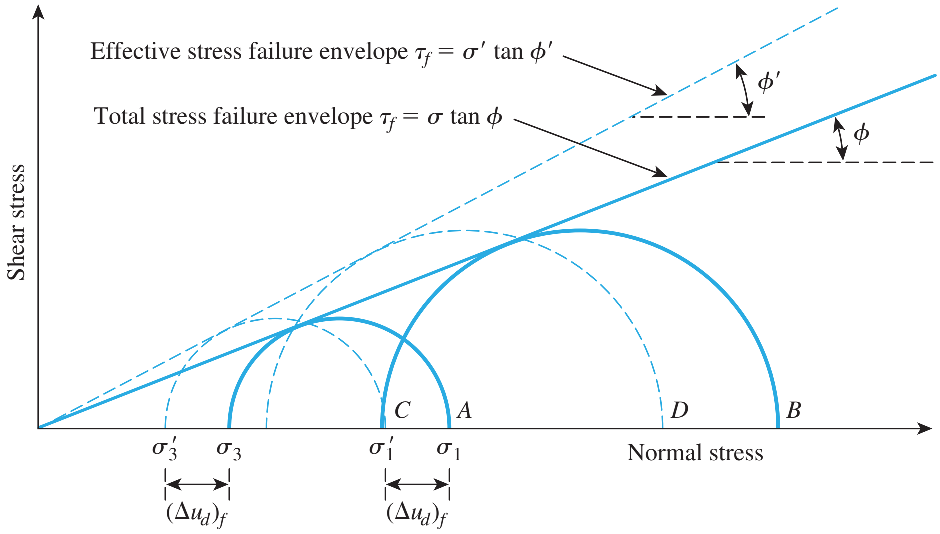


Figure 12.27 Total and effective stress failure envelopes for consolidated undrained triaxial tests. (*Note:* The figure assumes that no back pressure is applied.)

In Figure 12.27, the total stress failure envelope can be obtained by drawing a line that touches all the total stress Mohr's circles. For sand and normally consolidated clays, this will be approximately a straight line passing through the origin and may be expressed by the equation

$$\tau_f = \sigma \tan \phi \tag{12.26}$$

where σ = total stress
ϕ = the angle that the total stress failure envelope makes with the normal stress axis, also known as the *consolidated-undrained angle of shearing resistance*

Equation (12.26) is seldom used for practical considerations. Similar to Eq. (12.19), for sand and normally consolidated clay, we can write

$$\phi = \sin^{-1}\left(\frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3}\right) \tag{12.27}$$

and

$$\begin{aligned}\phi' &= \sin^{-1}\left(\frac{\sigma'_1 - \sigma'_3}{\sigma'_1 + \sigma'_3}\right)\\ &= \sin^{-1}\left\{\frac{[\sigma_1 - (\Delta u_d)_f] - [\sigma_3 - (\Delta u_d)_f]}{[\sigma_1 - (\Delta u_d)_f] + [\sigma_3 - (\Delta u_d)_f]}\right\}\\ &= \sin^{-1}\left[\frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3 - 2(\Delta u_d)_f}\right]\end{aligned} \tag{12.28}$$

Again referring to Figure 12.27, we see that the failure envelope that is tangent to all the effective stress Mohr's circles can be represented by the equation $\tau_f = \sigma' \tan \phi'$, which is the same as that obtained from consolidated-drained tests (see Figure 12.22).

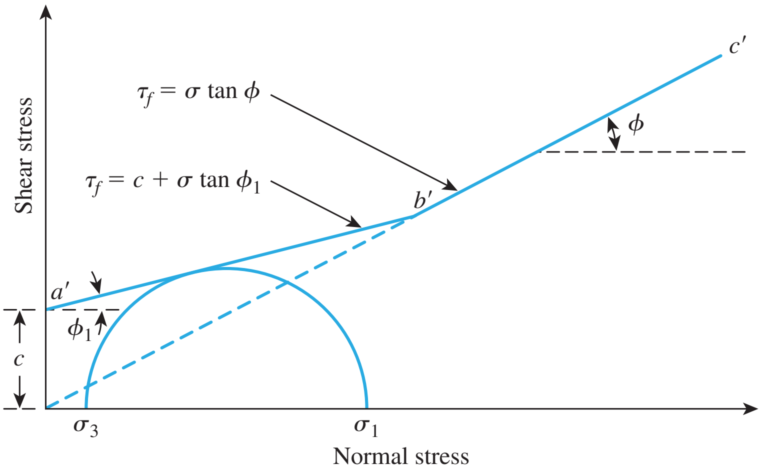


Figure 12.28 Total stress failure envelope obtained from consolidated-undrained tests in over-consolidated clay

In overconsolidated clays, the total stress failure envelope obtained from consolidated-undrained tests will take the shape shown in Figure 12.28. The straight line $a'b'$ is represented by the equation

$$\tau_f = c + \sigma \tan \phi_1 \tag{12.29}$$

and the straight line $b'c'$ follows the relationship given by Eq. (12.26). The effective stress failure envelope drawn from the effective stress Mohr's circles will be similar to that shown in Figure 12.23.

Consolidated-drained tests on clay soils take considerable time. For this reason, consolidated-undrained tests can be conducted on such soils with pore pressure measurements to obtain the drained shear strength parameters. Because drainage is not allowed in these tests during the application of deviator stress, they can be performed quickly.

Skempton's pore water pressure parameter $\overline{A}$ was defined in Eq. (12.25). At failure, the parameter $\overline{A}$ can be written as

$$\overline{A} = \overline{A}_f = \frac{(\Delta u_d)_f}{(\Delta \sigma_d)_f} \tag{12.30}$$

The general range of $\overline{A}_f$ values in most clay soils is as follows:

- Normally consolidated clays: 0.5 to 1
- Overconsolidated clays: −0.5 to 0

Table 12.3 gives the values of $\overline{A}_f$ for some normally consolidated clays as obtained by the Norwegian Geotechnical Institute.

Laboratory triaxial tests of Simons (1960) on Oslo clay, Weald clay, and London clay showed that $\overline{A}_f$ becomes approximately zero at an overconsolidation value of about 3 or 4 (Figure 12.29).

Table 12.3 Triaxial Test Results for Some Normally Consolidated Clays Obtained by the Norwegian Geotechnical Institute*

Location	Liquid limit	Plastic limit	Liquidity index	Sensitivity[a]	Drained friction angle, ϕ' (deg)	$\bar{A}_f$
Seven Sisters, Canada	127	35	0.28		19	0.72
Sarpborg	69	28	0.68	5	25.5	1.03
Lilla Edet, Sweden	68	30	1.32	50	26	1.10
Fredrikstad	59	22	0.58	5	28.5	0.87
Fredrikstad	57	22	0.63	6	27	1.00
Lilla Edet, Sweden	63	30	1.58	50	23	1.02
Göta River, Sweden	60	27	1.30	12	28.5	1.05
Göta River, Sweden	60	30	1.50	40	24	1.05
Oslo	48	25	0.87	4	31.5	1.00
Trondheim	36	20	0.50	2	34	0.75
Drammen	33	18	1.08	8	28	1.18

* *After Bjerrum and Simons, 1960. With permission from ASCE.*
[a] See Section 12.13 for the definition of sensitivity

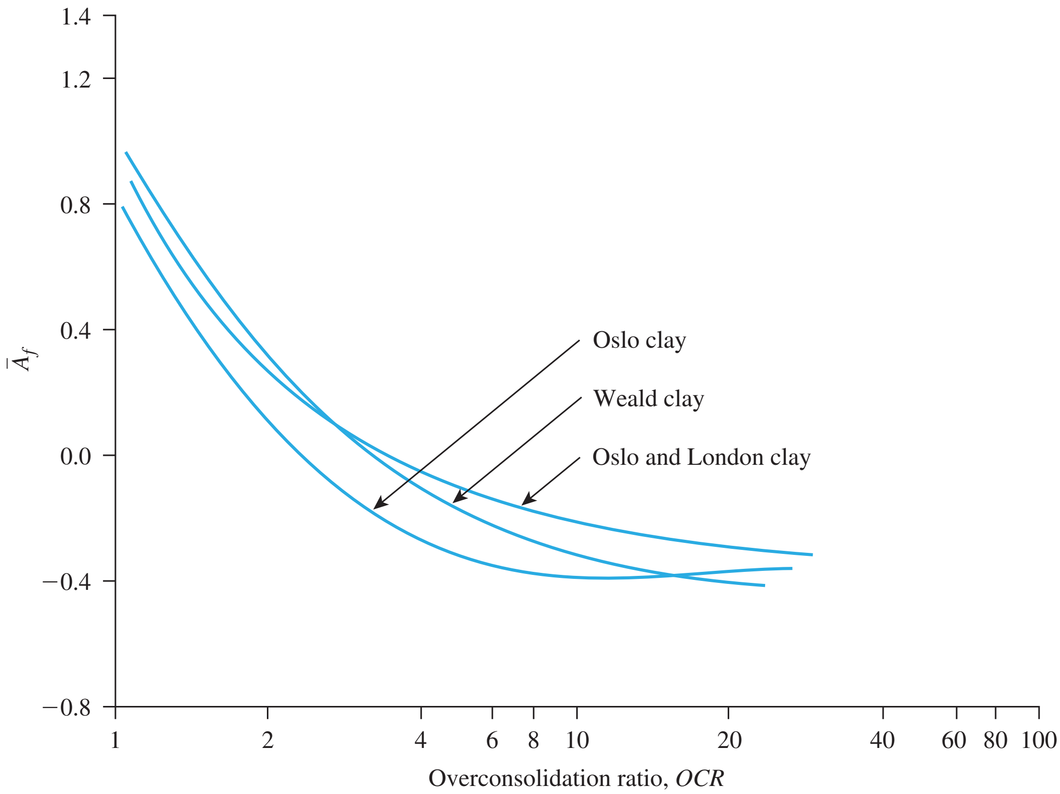


Figure 12.29 Variation of $\bar{A}_f$ with overconsolidation ratio for three clays (*Based on Simon, 1960*)

Example 12.6

A specimen of saturated sand was consolidated under an all-around pressure of 12 lb/in.^{2} The axial stress was then increased and drainage was prevented. The specimen failed when the axial deviator stress reached 9.1 lb/in.^{2} The pore water pressure at failure was 6.8 lb/in.^{2} Determine

a. Consolidated-undrained angle of shearing resistance, ϕ
b. Drained friction angle, ϕ'

Solution

Part a

For this case, $\sigma_3 = 12$ lb/in.^{2}, $\sigma_1 = 12 + 9.1 = 21.1$ lb/in.^{2}, and $(\Delta u_d)_f = 6.8$ lb/in.^{2}. The total and effective stress failure envelopes are shown in Figure 12.30. From Eq. (12.27),

$$\phi = \sin^{-1}\left(\frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3}\right) = \sin^{-1}\left(\frac{21.1 - 12}{21.1 + 12}\right) \approx \mathbf{16^\circ}$$

Part b

From Eq. (12.28),

$$\phi' = \sin^{-1}\left[\frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3 - 2(\Delta u_d)_f}\right] = \sin^{-1}\left[\frac{21.1 - 12}{21.1 + 12 - (2)(6.8)}\right] = \mathbf{27.8^\circ}$$

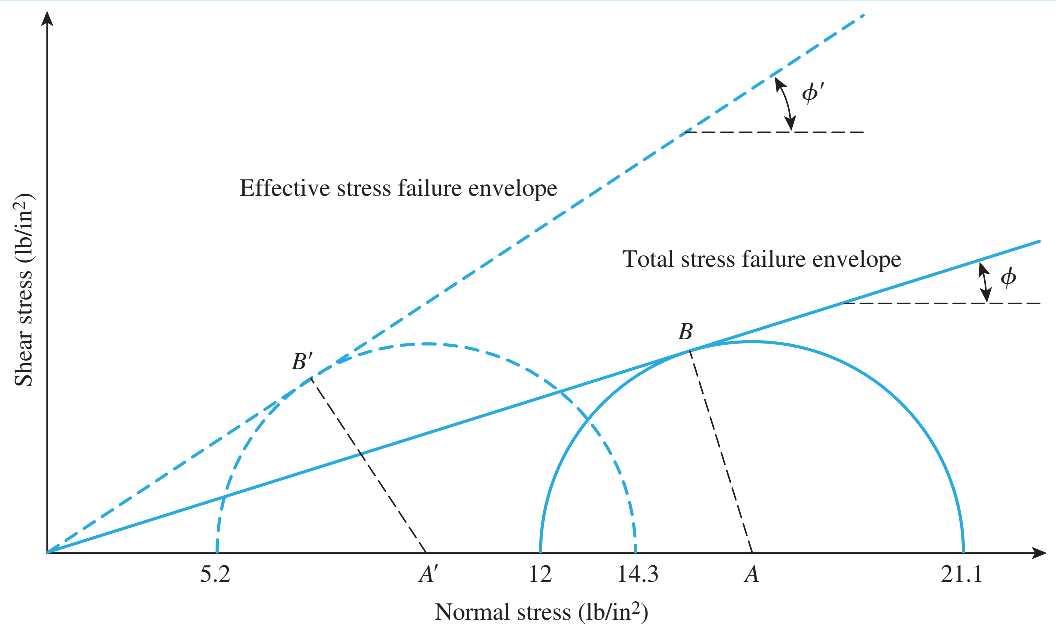


Figure 12.30 Failure envelopes and Mohr's circles for a saturated sand ■

Example 12.7

Refer to the soil specimen described in Example 12.6. What would be the deviator stress at failure, $(\Delta\sigma_d)_f$, if a drained test was conducted with the same chamber all-around pressure (that is, 12 lb/in.^{2})?

Solution

From Eq. (12.8) (with $c' = 0$),

$$\sigma'_1 = \sigma'_3 \tan^2\left(45 + \frac{\phi'}{2}\right)$$

$\sigma'_3 = 12$ lb/in.^{2} and $\phi' = 27.8°$ (from Example 12.6). So,

$$\sigma'_1 = 12 \tan^2\left(45 + \frac{27.8}{2}\right) \approx 33 \text{ lb/in.}^2$$

$$(\Delta\sigma_d)_f = \sigma'_1 - \sigma'_3 = 33 - 12 = \mathbf{21\ lb/in.^2}$$

■

12.10 Unconsolidated-Undrained Triaxial Test

In unconsolidated-undrained tests, drainage from the soil specimen is not permitted during the application of chamber pressure σ_3. The test specimen is sheared to failure by the application of deviator stress, $\Delta\sigma_d$, and drainage is prevented. Because drainage is not allowed at any stage, the test can be performed quickly. Because of the application of chamber confining pressure σ_3, the pore water pressure in the soil specimen will increase by u_c. A further increase in the pore water pressure (Δu_d) will occur because of the deviator stress application. Hence, the total pore water pressure u in the specimen at any stage of deviator stress application can be given as

$$u = u_c + \Delta u_d \tag{12.31}$$

From Eqs. (12.18) and (12.25), $u_c = B\sigma_3$ and $\Delta u_d = \overline{A}\Delta\sigma_d$, so

$$u = B\sigma_3 + \overline{A}\Delta\sigma_d = B\sigma_3 + \overline{A}(\sigma_1 - \sigma_3) \tag{12.32}$$

This test usually is conducted on clay specimens and depends on a very important strength concept for cohesive soils if the soil is fully saturated. The added axial stress at failure $(\Delta\sigma_d)_f$ is practically the same regardless of the chamber confining pressure. This property is shown in Figure 12.31. The failure envelope for the total stress Mohr's circles becomes a horizontal line and hence is called a $\phi = 0$ condition. From Eq. (12.9) with $\phi = 0$, we get

$$\tau_f = c = c_u \tag{12.33}$$

where c_u is the undrained shear strength and is equal to the radius of the Mohr's circles. Note that the $\phi = 0$ concept is applicable to only saturated clays and silts.

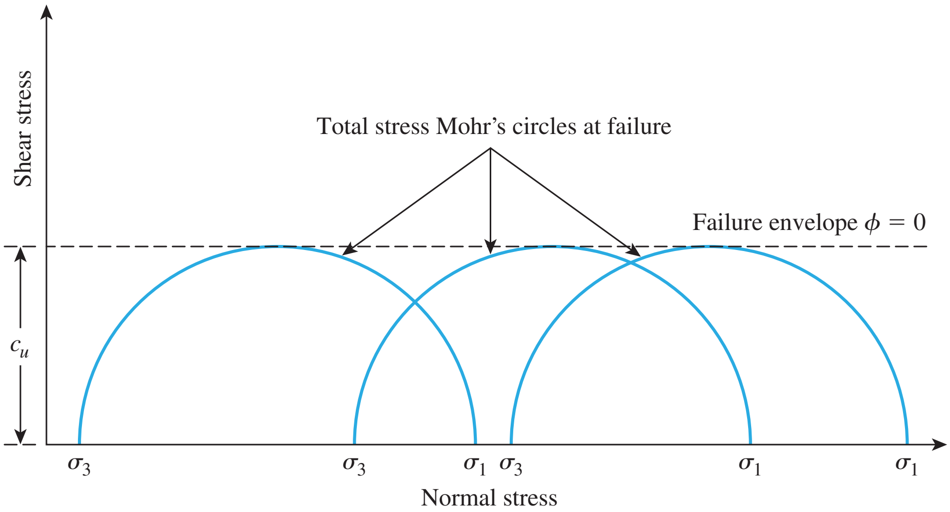


Figure 12.31 Total stress Mohr's circles and failure envelope ($\phi = 0$) obtained from unconsolidated-undrained triaxial tests on fully saturated cohesive soil

The reason for obtaining the same added axial stress $(\Delta\sigma_d)_f$ regardless of the confining pressure can be explained as follows. If a clay specimen (No. I) is consolidated at a chamber pressure σ_3 and then sheared to failure without drainage, the total stress conditions at failure can be represented by the Mohr's circle P in Figure 12.32. The pore pressure developed in the specimen at failure is equal to $(\Delta u_d)_f$. Thus, the major and minor principal effective stresses at failure are, respectively,

$$\sigma_1' = [\sigma_3 + (\Delta\sigma_d)_f] - (\Delta u_d)_f = \sigma_1 - (\Delta u_d)_f$$

and

$$\sigma_3' = \sigma_3 - (\Delta u_d)_f$$

Q is the effective stress Mohr's circle drawn with the preceding principal stresses. Note that the diameters of circles P and Q are the same.

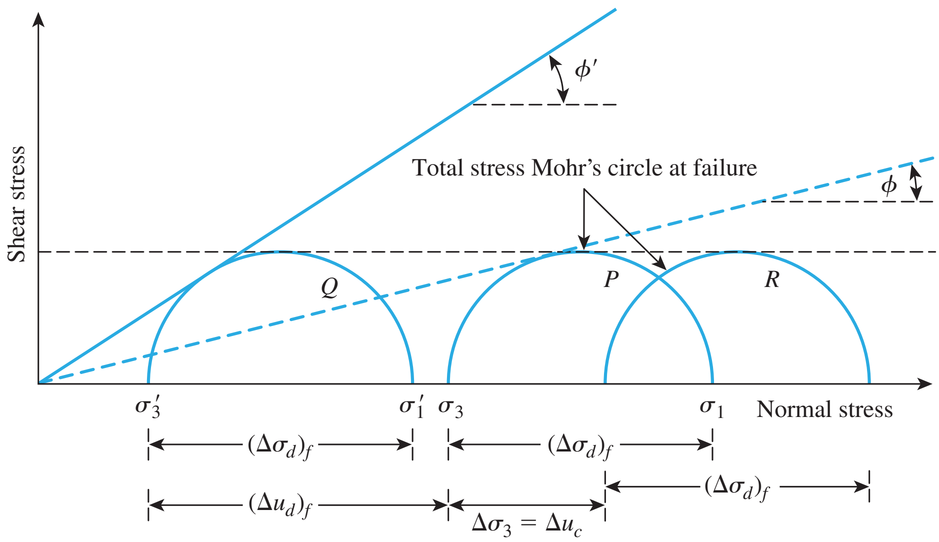


Figure 12.32 The $\phi = 0$ concept

Now let us consider another similar clay specimen (No. II) that has been consolidated under a chamber pressure σ_3 with initial pore pressure equal to zero. If the chamber pressure is increased by $\Delta\sigma_3$ without drainage, the pore water pressure will increase by an amount Δu_c. For saturated soils under isotropic stresses, the pore water pressure increase is equal to the total stress increase, so $\Delta u_c = \Delta\sigma_3$ ($B = 1$). At this time, the effective confining pressure is equal to $\sigma_3 + \Delta\sigma_3 - \Delta u_c = \sigma_3 + \Delta\sigma_3 - \Delta\sigma_3 = \sigma_3$. This is the same as the effective confining pressure of Specimen I before the application of deviator stress. Hence, if Specimen II is sheared to failure by increasing the axial stress, it should fail at the same deviator stress $(\Delta\sigma_d)_f$ that was obtained for Specimen I. The total stress Mohr's circle at failure will be R (see Figure 12.32). The added pore pressure increase caused by the application of $(\Delta\sigma_d)_f$ will be $(\Delta u_d)_f$.

At failure, the minor principal effective stress is

$$[(\sigma_3 + \Delta\sigma_3)] - [\Delta u_c + (\Delta u_d)_f] = \sigma_3 - (\Delta u_d)_f = \sigma_3'$$

and the major principal effective stress is

$$[\sigma_3 + \Delta\sigma_3 + (\Delta\sigma_d)_f] - [\Delta u_c + (\Delta u_d)_f] = [\sigma_3 + (\Delta\sigma_d)_f] - (\Delta u_d)_f$$
$$= \sigma_1 - (\Delta u_d)_f = \sigma_1'$$

Thus, the effective stress Mohr's circle will still be Q because strength is a function of effective stress. Note that the diameters of circles P, Q, and R are all the same.

Any value of $\Delta\sigma_3$ could have been chosen for testing Specimen II. In any case, the deviator stress $(\Delta\sigma_d)_f$ to cause failure would have been the same as long as the soil was fully saturated and fully undrained during both stages of the test.

12.11 Unconfined Compression Test on Saturated Clay

The unconfined compression test is a special type of unconsolidated-undrained test that is commonly used for clay specimens. In this test, the confining pressure σ_3 is 0. An axial load is rapidly applied to the specimen to cause failure. At failure, the total minor principal stress is zero and the total major principal stress is σ_1 (Figure 12.33). Because the undrained shear strength is independent of the confining pressure as long as the soil is fully saturated and fully undrained, we have

$$\tau_f = \frac{\sigma_1}{2} = \frac{q_u}{2} = c_u \qquad (12.34)$$

where q_u is the *unconfined compression strength.* Table 12.4 gives the approximate consistencies of clays on the basis of their unconfined compression strength. A photograph of

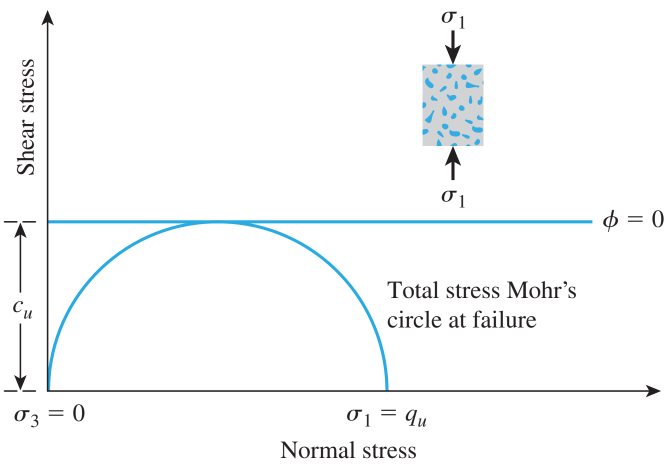


Figure 12.33 Unconfined compression test

Table 12.4 General Relationship of Consistency and Unconfined Compression Strength of Clays

	q_u	
Consistency	**kN/m²**	**ton/ft²**
Very soft	0–25	0–0.25
Soft	25–50	0.25–0.5
Medium	50–100	0.5–1
Stiff	100–200	1–2
Very stiff	200–400	2–4
Hard	>400	>4

unconfined compression test equipment is shown in Figure 12.34. Figures 12.35 and 12.36 show the failure in two specimens—one by shear and one by bulging—at the end of unconfined compression tests.

Theoretically, for similar saturated clay specimens, the unconfined compression tests and the unconsolidated-undrained triaxial tests should yield the same values of c_u. In practice, however, unconfined compression tests on saturated clays yield slightly lower values of c_u than those obtained from unconsolidated-undrained tests.

12.12 Empirical Relationships Between Undrained Cohesion (c_u) and Effective Overburden Pressure (σ'_O)

Several empirical relationships have been proposed between c_u and the effective overburden pressure σ'_o. The most commonly cited relationship is that given by Skempton (1957) which can be expressed as

$$\frac{c_{u(\mathrm{VST})}}{\sigma'_o} = 0.11 + 0.0037(PI) \quad \text{(for normally consolidated clay)} \tag{12.35}$$

Figure 12.34 Unconfined compression test equipment (*Courtesy of ELE International*)

Figure 12.35 Failure by shear of an unconfined compression test specimen (*Courtesy of Braja M. Das, Henderson, Nevada*)

where $c_{u(\mathrm{VST})}$ = undrained shear strength from vane shear test (see Section 12.15)
PI = plasticity index (%)

Chandler (1988) suggested that the preceding relationship will hold good for overconsolidated soil with an accuracy of ±25%. This does not include sensitive and fissured clays. Ladd, *et al*. (1977) proposed that

$$\frac{\left(\dfrac{c_u}{\sigma'_o}\right)_{\text{overconsolidated}}}{\left(\dfrac{c_u}{\sigma'_o}\right)_{\text{normally consolidated}}} = (\mathrm{OCR})^{0.8} \tag{12.36}$$

where OCR = overconsolidation ratio.

Figure 12.36
Failure by bulging of an unconfined compression test specimen (*Courtesy of Braja M. Das, Henderson, Nevada*)

Example 12.8

An overconsolidated clay deposit located below the groundwater table has the following:

- Average present effective overburden pressure = 160 kN/m^2
- Overconsolidation ratio = 3.2
- Plasticity index = 28

Estimate the average undrained shear strength of the clay [that is, $c_{u(\text{VST})}$].

Solution

From Eq. (12.35),

$$\left[\frac{c_{u(\text{VST})}}{\sigma'_o}\right]_{\text{normally consolidated}} = 0.11 + 0.0037(PI) = 0.11 + (0.0037)(28) = 0.2136$$

From Eq. (12.36),

$$\left[\frac{c_{u(\text{VST})}}{\sigma'_o}\right]_{\text{overconsolidated}} = (\text{OCR})^{0.8}\left[\frac{c_{u(\text{VST})}}{\sigma'_o}\right]_{\text{normally consolidated}} = (3.2)^{0.8}(0.2136) = 0.542$$

Thus,

$$c_{u(\text{VST})} = 0.542\sigma'_o = (0.542)(160) = \mathbf{86.7\ kN/m^2}$$

■

12.13 Sensitivity and Thixotropy of Clay

For many naturally deposited clay soils, the unconfined compression strength is reduced greatly when the soils are tested after remolding without any change in the moisture content, as shown in Figure 12.37. This property of clay soils is called *sensitivity.* The degree of sensitivity may be defined as the ratio of the unconfined compression strength in an undisturbed state to that in a remolded state, or

$$S_t = \frac{q_{u(\text{undisturbed})}}{q_{u(\text{remolded})}} \tag{12.37}$$

The sensitivity ratio of most clays ranges from about 1 to 8; however, highly flocculent marine clay deposits may have sensitivity ratios ranging from about 10 to 80. Some clays turn to viscous fluids upon remolding. These clays are found mostly in the previously glaciated areas of North America and Scandinavia. Such clays are referred to as *quick* clays. Rosenqvist (1953) classified clays on the basis of their sensitivity. This general classification is shown in Figure 12.38.

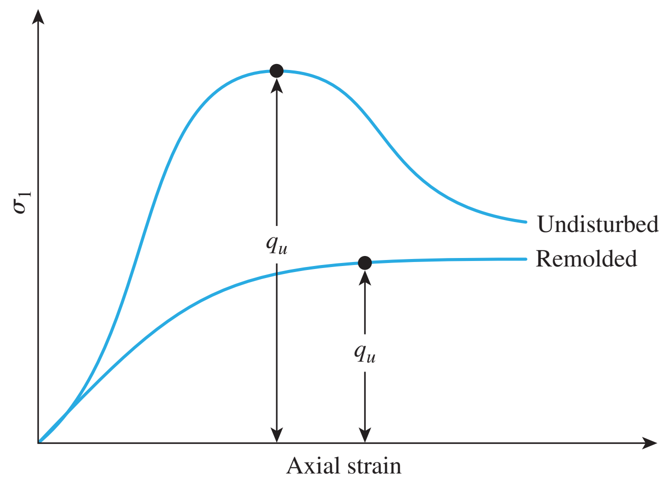


Figure 12.37 Unconfined compression strength for undisturbed and remolded clay

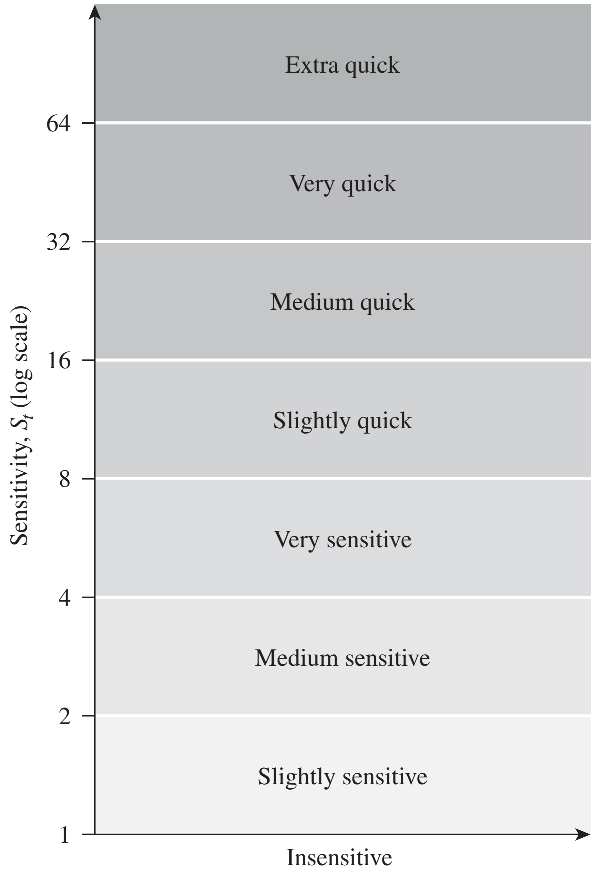


Figure 12.38 Classification of clays based on sensitivity

The loss of strength of clay soils from remolding is caused primarily by the destruction of the clay particle structure that was developed during the original process of sedimentation.

If, however, after remolding, a soil specimen is kept in an undisturbed state (that is, without any change in the moisture content), it will continue to gain strength with time. This phenomenon is referred to as *thixotropy*. Thixotropy is a time-dependent, reversible process in which materials under constant composition and volume soften when remolded. This loss of strength is gradually regained with time when the materials are allowed to rest. This phenomenon is illustrated in Figure 12.39a.

Most soils, however, are partially thixotropic—that is, part of the strength loss caused by remolding is never regained with time. The nature of the strength-time variation for partially thixotropic materials is shown in Figure 12.39b. For soils, the difference between the undisturbed strength and the strength after thixotropic hardening can be attributed to the destruction of the clay-particle structure that was developed during the original process of sedimentation.

Seed and Chan (1959) conducted several tests on three compacted clays with a water content near or below the plastic limit to study the thixotropic strength regain

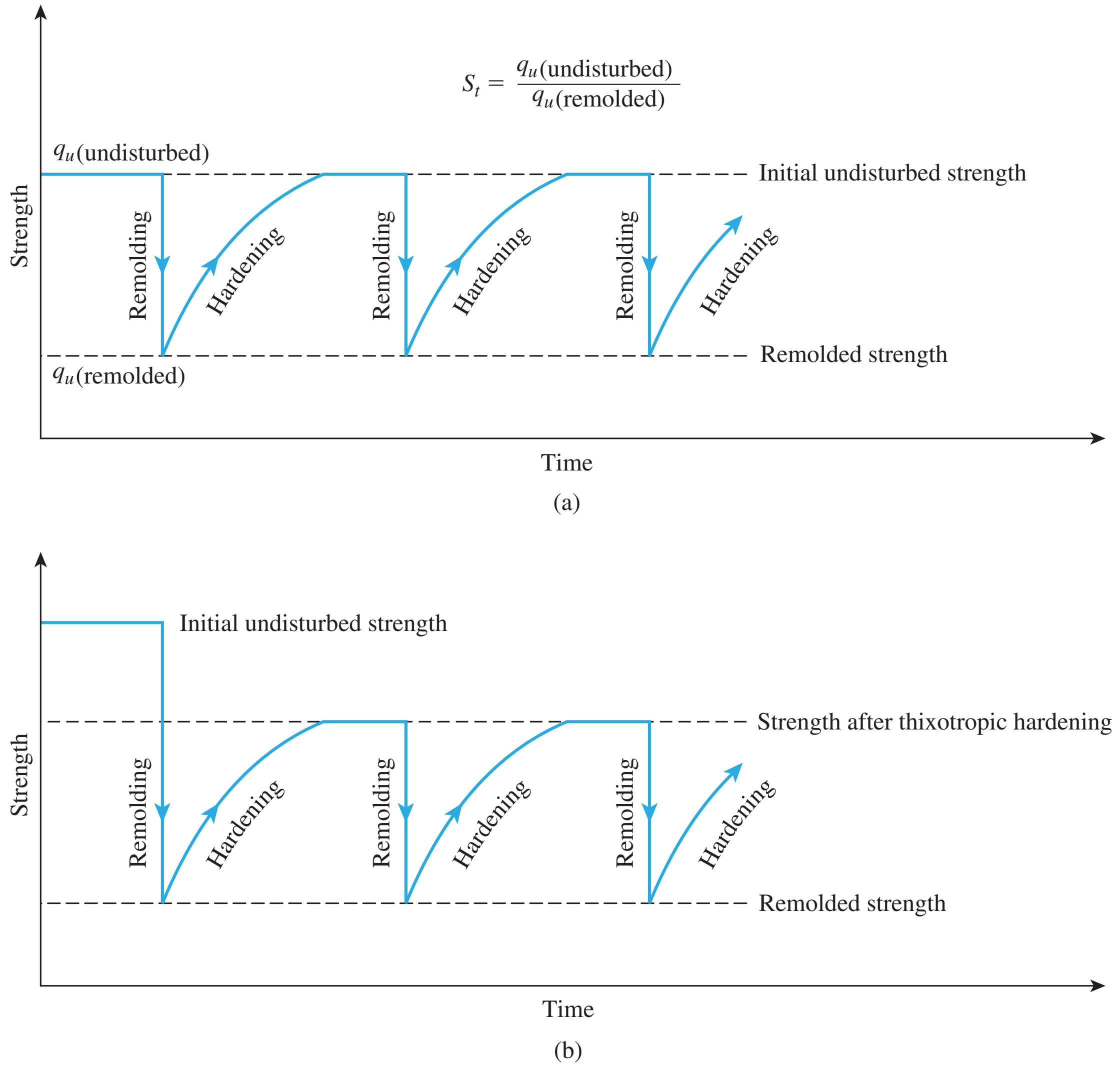


Figure 12.39 Behavior of (a) thixotropic material; (b) partially thixotropic material

characteristics of the clays. The results of these tests are shown in Figure 12.40. Note that in Figure 12.40,

$$\text{Thixotropic strength ratio} = \frac{c_{u(\text{at time } t \text{ after compaction})}}{c_{u(\text{at time } t=0 \text{ after compaction})}} \tag{12.38}$$

12.14 Strength Anisotropy in Clay

The unconsolidated-undrained shear strength of some saturated clays can vary, depending on the direction of load application; this variation is referred to as *anisotropy with respect to strength.* Anisotropy is caused primarily by the nature of the deposition of the cohesive soils, and subsequent consolidation makes the clay particles orient perpendicular to the

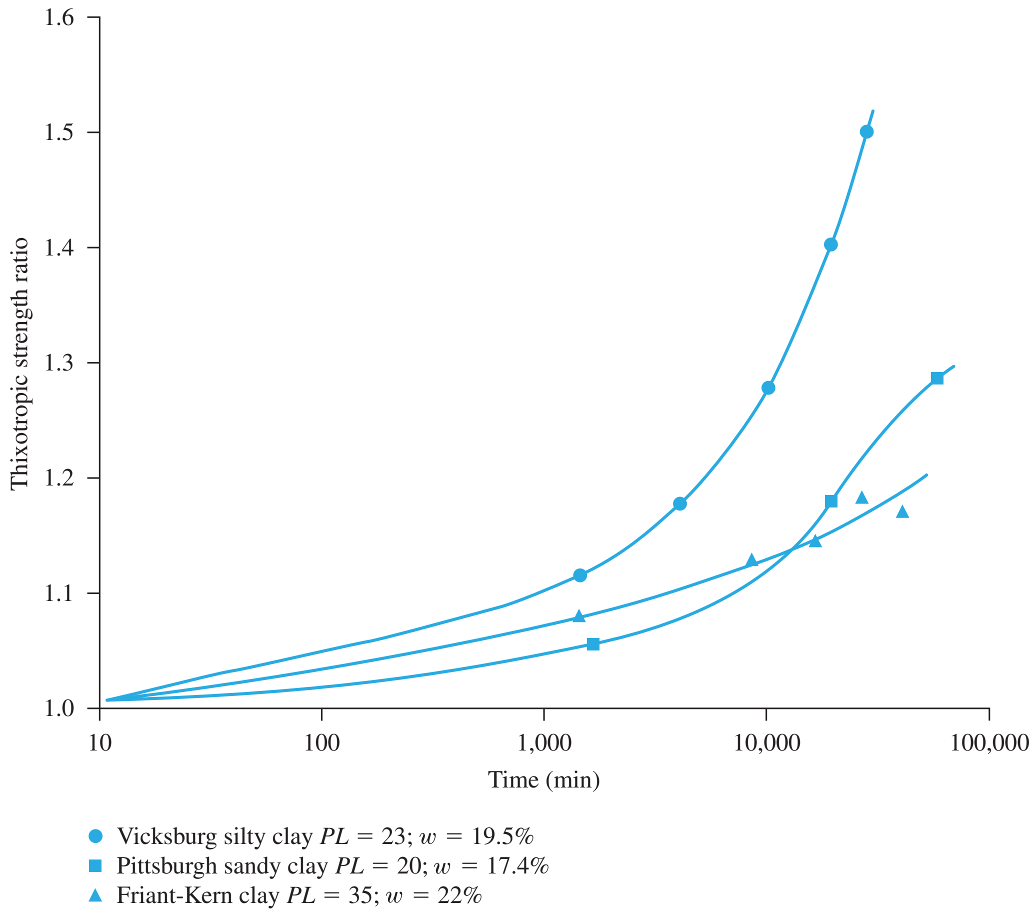


Figure 12.40 Thixotropic strength increase with time for three clays (*Based on Seed and Chan, 1959*)

direction of the major principal stress. Parallel orientation of the clay particles can cause the strength of clay to vary with direction. Figure 12.41 shows an element of saturated clay in a deposit with the major principal stress making an angle α with respect to the horizontal. For anisotropic clays, the magnitude of c_u is a function of α.

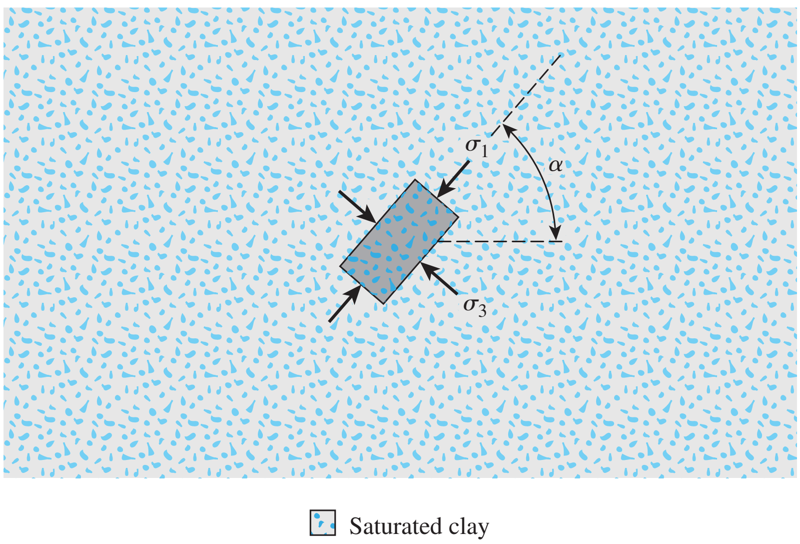


Figure 12.41 Strength anisotropy in clay

As an example, the variation of c_u with α for undisturbed specimens of Winnipeg Upper Brown clay (Loh and Holt, 1974) is shown in Figure 12.42. Based on several laboratory test results, Casagrande and Carrillo (1944) proposed the following relationship for the directional variation of undrained shear strength:

$$c_{u(\alpha)} = c_{u(\alpha=0°)} + [c_{u(\alpha=90°)} - c_{u(\alpha=0°)}]\sin^2 \alpha \tag{12.39}$$

For normally consolidated clays, $c_{u(\alpha=90°)} > c_{u(\alpha=0°)}$; for overconsolidated clays, $c_{u(\alpha=90°)} < c_{u(\alpha=0°)}$. Figure 12.43 shows the directional variation for $c_{u(\alpha)}$ based on Eq. (12.39). The anisotropy with respect to strength for clays can have an important effect on various stability calculations.

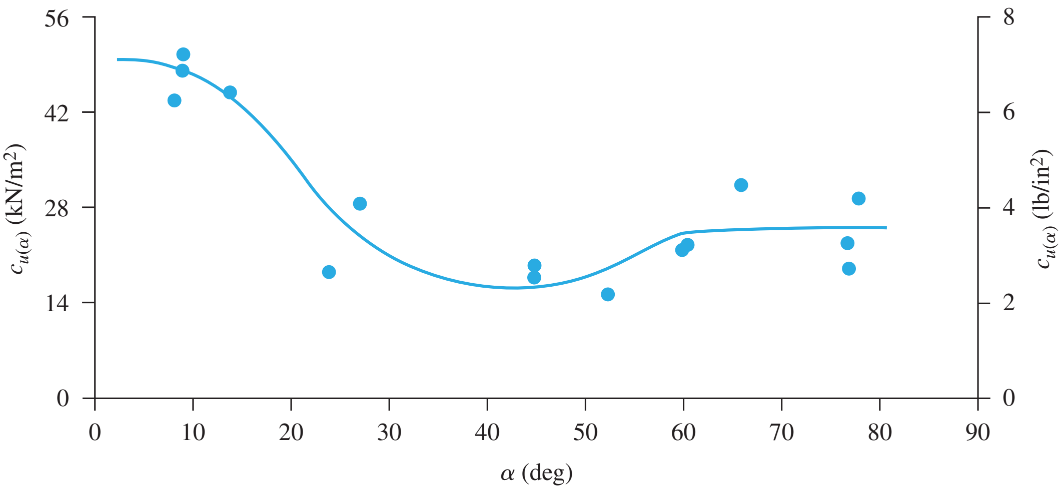


Figure 12.42 Directional variation of c_u for undisturbed Winnipeg Upper Brown clay (*Based on Loh and Holt, 1974*)

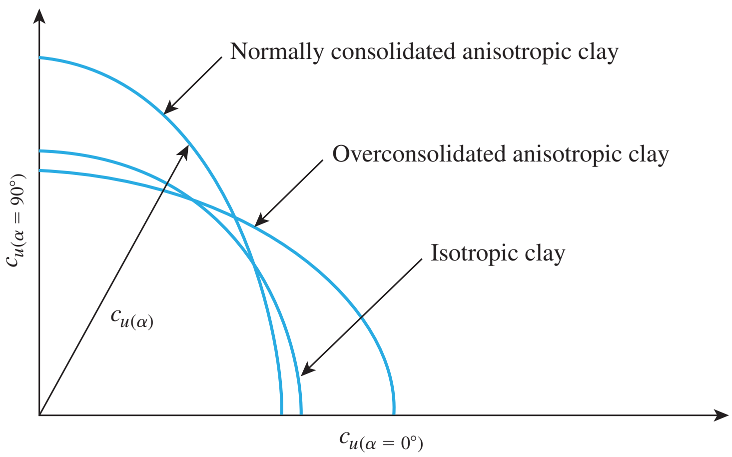


Figure 12.43 Graphical representation of Eq. (12.39)

12.15 Vane Shear Test

Fairly reliable results for the undrained shear strength, c_u ($\phi = 0$ concept), of very soft to medium cohesive soils may be obtained directly from vane shear tests. The shear vane usually consists of four thin, equal-sized steel plates welded to a steel torque rod (Figure 12.44). First, the vane is pushed into the soil. Then torque is applied at the top of the torque rod to rotate the vane at a uniform speed. A cylinder of soil of height h and diameter d will resist the torque until the soil fails. The undrained shear strength of the soil can be calculated as follows.

If T is the maximum torque applied at the head of the torque rod to cause failure, it should be equal to the sum of the resisting moment of the shear force along the side surface of the soil cylinder (M_s) and the resisting moment of the shear force at each end (M_e) (Figure 12.45):

$$T = M_s + \underbrace{M_e + M_e}_{\text{Two ends}} \tag{12.40}$$

The resisting moment can be given as

$$M_s = \underbrace{(\pi d h)c_u}_{\substack{\text{Surface}\\\text{area}}} \quad \underbrace{(d/2)}_{\substack{\text{Moment}\\\text{arm}}} \tag{12.41}$$

where d = diameter of the shear vane
h = height of the shear vane

For the calculation of M_e, investigators have assumed several types of distribution of shear strength mobilization at the ends of the soil cylinder:

1. *Triangular.* Shear strength mobilization is c_u at the periphery of the soil cylinder and decreases linearly to zero at the center.
2. *Uniform.* Shear strength mobilization is constant (that is, c_u) from the periphery to the center of the soil cylinder.
3. *Parabolic.* Shear strength mobilization is c_u at the periphery of the soil cylinder and decreases parabolically to zero at the center.

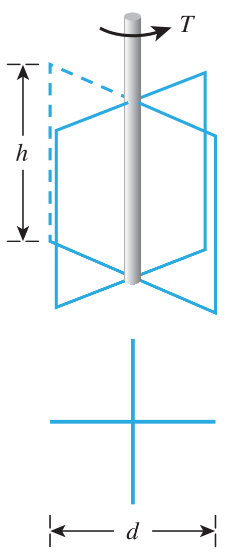


Figure 12.44 Diagram of vane shear test equipment

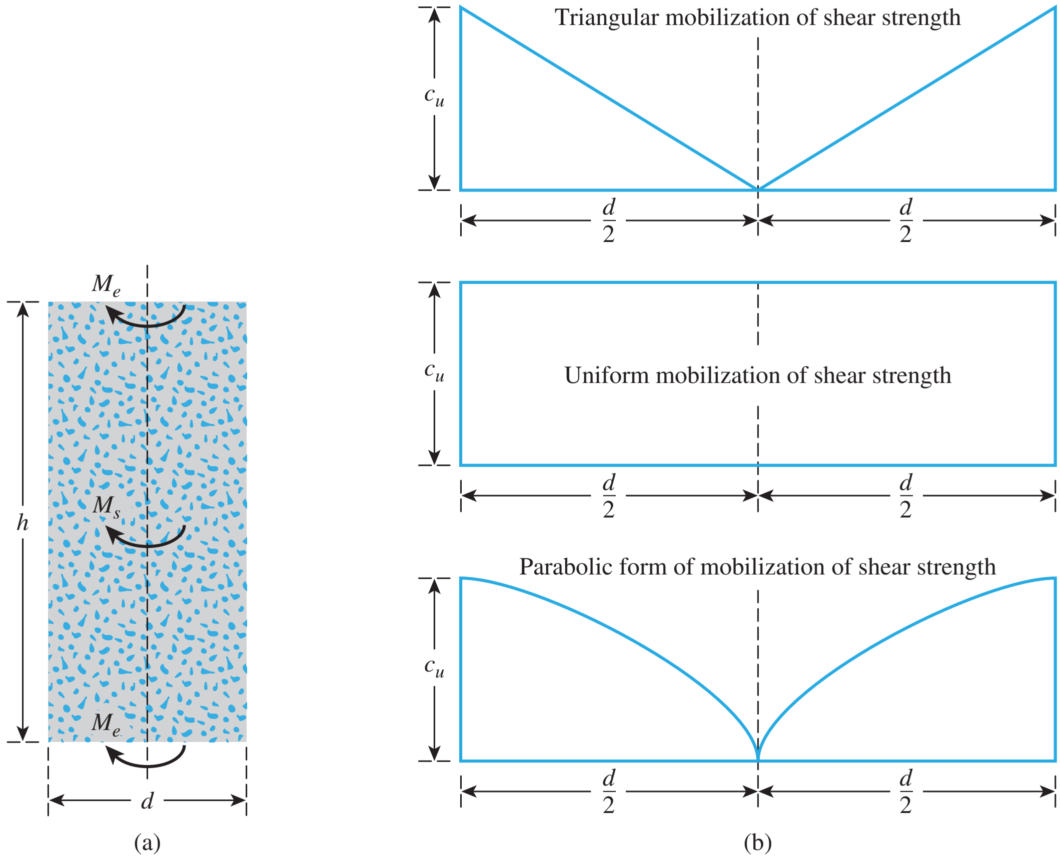


Figure 12.45 Derivation of Eq. (12.43): (a) resisting moment of shear force; (b) variations in shear strength-mobilization

These variations in shear strength mobilization are shown in Figure 12.45b. In general, the torque, T, at failure can be expressed as

$$T = \pi c_u \left[\frac{d^2 h}{2} + \beta \frac{d^3}{4} \right] \tag{12.42}$$

or

$$c_u = \frac{T}{\pi \left[\frac{d^2 h}{2} + \beta \frac{d^3}{4} \right]} \tag{12.43}$$

where $\beta = \frac{1}{2}$ for triangular mobilization of undrained shear strength
$\beta = \frac{2}{3}$ for uniform mobilization of undrained shear strength
$\beta = \frac{3}{5}$ for parabolic mobilization of undrained shear strength

Note that Eq. (12.43) usually is referred to as *Calding's equation.*

Vane shear tests can be conducted in the laboratory and in the field during soil exploration. The laboratory shear vane has dimensions of about 13 mm ($\frac{1}{2}$ in.) in diameter and 25 mm (1 in.) in height. Figure 12.46 shows a photograph of laboratory vane shear test equipment. Figure 12.47 shows the field vanes recommended by ASTM (2004). Table 12.5 gives the ASTM recommended dimensions of field vanes.

SHEARING STRENGTH OF SOILS

'Shearing Strength' of a soil is perhaps the most important of its engineering properties. This is because all stability analyses in the field of geotechnical engineering, whether they relate to foundation, slopes of cuts or earth dams, involve a basic knowledge of this engineering property of the soil. 'Shearing strength' or merely '**Shear strength**' may be defined as the resistance to shearing stresses and a consequent tendency for shear deformation.

Shearing strength comes from the following:

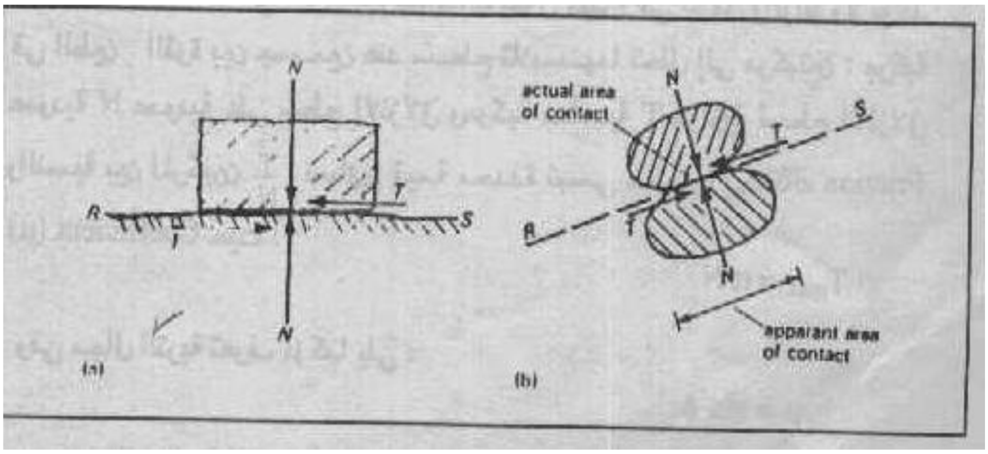
(1) Resistance due to the interlocking of particles.

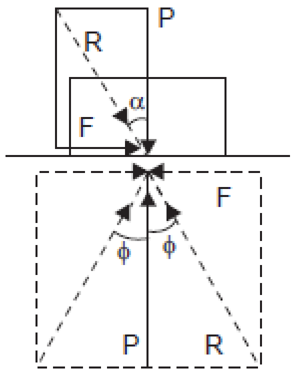
(2) Frictional resistance between the individual soil grains, which may be sliding friction, rolling friction, or both.

(3) Adhesion between soil particles or 'cohesion'.

Granular soils of sands (1, 2) while cohesive soils or clays (2, 3) highly plastic clays (3)

- **Friction between Solid Bodies (Internal Friction within Granular Soil Masses)**





Full friction
$\alpha = \phi$
Applied shear force just equals maximum available friction : maximum frictional force is mobilised. Slip imminent to the right (critical condition)

When two solid bodies are in contact with each other, the frictional resistance available is dependent upon the normal force between the two. A shearing force equal to the maximum available frictional resistance is applied. The entire frictional resistance available will get mobilized now to resist the applied

Coefficient of friction $(\mu) = \frac{F}{P}$ or $F = P.\ \mu = P.\ \tan\varphi$ or $F/P = \tan\varphi = \mu$

In granular or cohesionless soil masses, the resistance to sliding on any plane through the point within the mass is similar to that discussed in the previous sub-section; the friction angle in this case is called the 'angle of internal friction' Ø

- **PRINCIPAL PLANES AND PRINCIPAL STRESSES—MOHR'S CIRCLE**

At a point in a stressed material, every plane will be subjected, in general, to a normal or direct stress and a shearing stress.

A 'Principal plane' is defined as a plane on which the stress is wholly normal, or one which does not carry shearing stress.

Principal plane is divided to the 'major principal stress', the 'intermediate principal stress' and the 'minor principal stress',

Let us consider an element of soil whose sides are chosen as the principal planes, the major and the minor, as shown in Fig. below

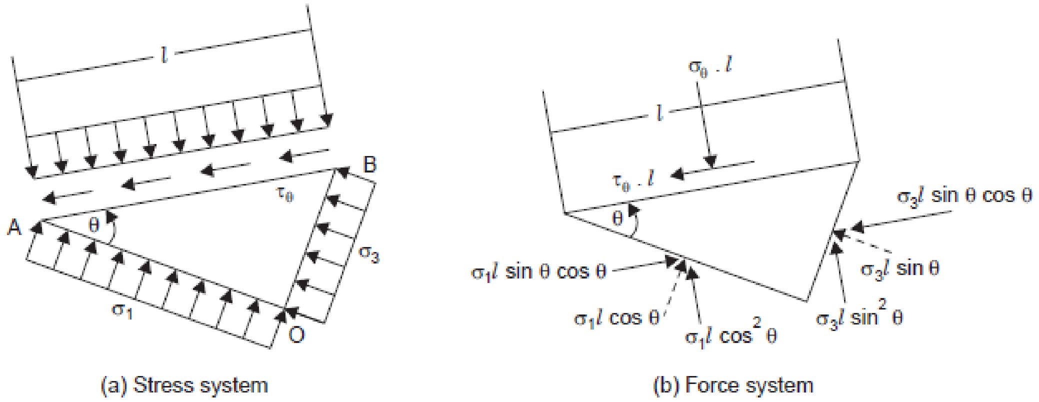


Fig 2 Stresses on a plane inclined to the principal planes

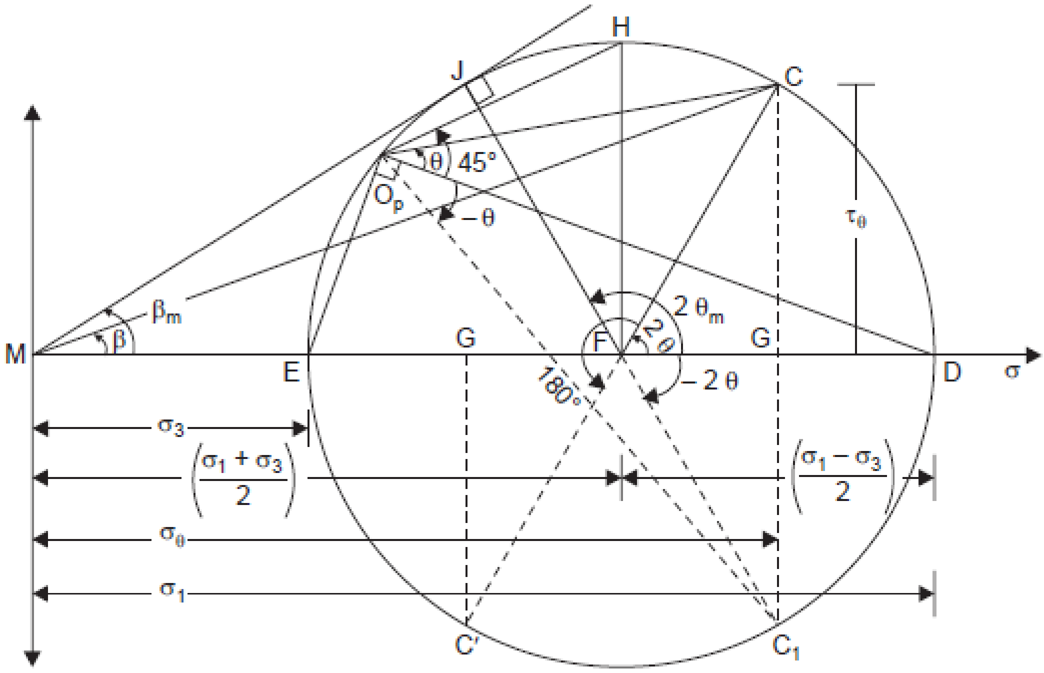
Let *O* be any point in the stressed medium and *OA* and *OB* be the major and minor principal planes, with the corresponding principal stresses σ_1 and σ_3, and inclined at an angle θ to the major principal plane, considered positive when measured counter-clockwise. Let us consider the element to be of unit thickness perpendicular to the plane of the figure, *AB* being *l*. considering the equilibrium of the element and resolving all forces in the directions parallel and perpendicular to *AB*,

$$\sigma_\theta = \sigma_1 \cos^2\theta + \sigma_3 \sin^2\theta = \sigma_3 + (\sigma_1 - \sigma_3)\cos^2\theta$$

$$= \frac{(\sigma_1 + \sigma_3)}{2} + \frac{(\sigma_1 - \sigma_3)}{2} \,.\, \cos 2\theta$$

$$\tau_\theta = \frac{(\sigma_1 - \sigma_3)}{2} \,.\, \sin 2\theta$$

Otto Mohr (1882) represented these results graphical in a circle diagram, which is called Mohr's circle. Normal stresses are represented as abscissae and shear stresses as ordinates.



Mohr's circle for the stress conditions illustrated in Fig.2

Let a line be drawn parallel to the major principal plane through *D*, the coordinate of which is the major principal stress. The intersection of this line with the Mohr's circle, O_p is called the 'Origin of planes'. If a line parallel to the minor principal plane is drawn through *E*, the co-ordinate of which is the minor principal stress, it will also be observed to pass through *Op;* the angle between these two lines is a right angle from the properties of the circle. The angle between these two lines is a right angle from the properties of the circle. any line through *Op*, parallel to any arbitrarily chosen plane, intersects the Mohr's circle at a point the co-ordinates of which represent the normal and shear stresses on that plane Since angle $CO_pD = \theta$, angle $CFD = 2\theta$, from the properties of the circle. From the geometry of the figure, the co-ordiantes of the point *C*, are established as follows:

$$\sigma_\theta = MG = MF + FG$$

$$= \frac{(\sigma_1 + \sigma_3)}{2} + \frac{(\sigma_1 - \sigma_3)}{2} \,.\, \cos 2\theta$$

$$\tau_\theta = CG = \frac{(\sigma_1 - \sigma_3)}{2} \,.\, \sin 2\theta$$

A few important basic facts and relationships may be directly obtained from the Mohr's circle:

1. The only planes free from shear are the given sides of the element which are the principal planes. The stresses on these are the greatest and smallest normal stresses.

2. The maximum or principal shearing stress is equal to the radius of the Mohr's circle, and it occurs on planes inclined at 45° to the principal planes.

$$\tau_{max} = (\sigma_1 - \sigma_3)/2$$

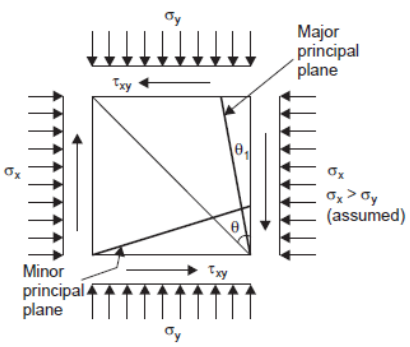
3. The normal stresses on planes of maximum shear are equal to each other and are equal to half the sum of the principal stresses.

$$\sigma_c = (\sigma_1 + \sigma_3)/2$$

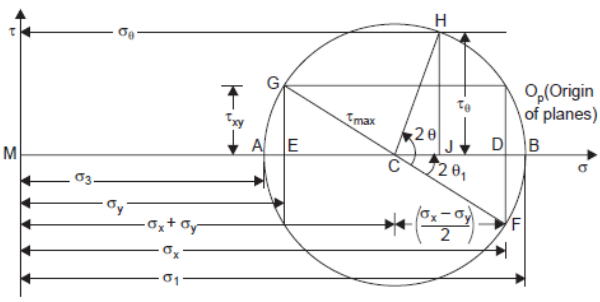
4. Shearing stresses on planes at right angles to each other are numerically equal and are of an opposite sign. These are called conjugate shearing stresses.

5. The sum of the normal stresses on mutually perpendicular planes is a constant ($MG' + MG = 2MF = \sigma 1 + \sigma 3$). If we designate the normal stress on a plane perpendicular to the plane on which it is σ_θ as σ_θ' : $\sigma_\theta + \sigma_\theta' = \sigma_1 + \sigma_3$

In case the normal and shearing stresses on two mutually perpendicular planes are known, the principal planes and principal stresses may be determined with the aid of the Mohr's circle diagram.



(a) General two-dimensional stress system



(b) Mohr's circle for general two-dimensional stress system normal stresses σ_x and σ_y on mutually perpendicular planes and shear stresses τ_{xy} on these planes, the normal and shearing stress components, σ_θ and τ_θ, respectively, on a plane inclined at an angle θ, measured counter-clockwise with respect to the plane on which σ*x* acts

$$\sigma_\theta = \frac{(\sigma_x + \sigma_y)}{2} + \frac{(\sigma_x - \sigma_y)}{2} \,.\, \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\tau_\theta = \frac{(\sigma_x - \sigma_y)}{2} \,.\, \sin 2\theta - \tau_{xy} \,.\, \cos 2\theta$$

$$\left[\sigma_\theta - \frac{(\sigma_x + \sigma_y)}{2}\right]^2 + {\tau_\theta}^2 = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2$$

This represents a circle with centre $\left[\frac{(\sigma_x + \sigma_y)}{2}, 0\right]$, and radius

$$\sqrt{\left(\frac{(\sigma_x - \sigma_y)}{2}\right)^2 + \tau_{xy}^2} \,.$$

The following relationships are also easily obtained:

$$\sigma_1 = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

$$\sigma_3 = \left(\frac{\sigma_x + \sigma_y}{2}\right) - \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

$$\tan 2\theta_{1,3} = 2\tau_{xy}/(\sigma_x - \sigma_y)$$

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

STRENGTH THEORIES FOR SOILS

- **Mohr's Strength Theory**

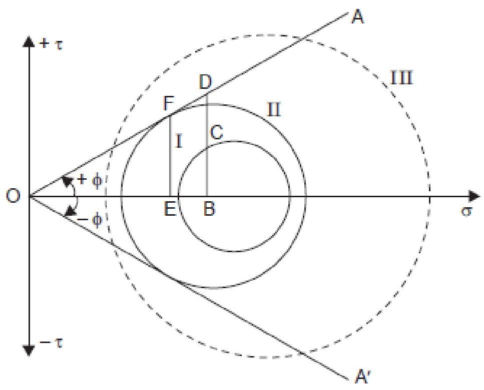
We have seen that the shearing stress may be expressed as $\tau = \sigma \tan \beta$ on any plane, where β is the angle of obliquity, If the obliquity angle is the maximum or has limiting value φ, the shearing stress is also at its limiting value and it is called the shearing strength, *s*

$$s = \sigma \tan \varphi$$

If the angle of internal friction φ is assumed to be a constant, the shearing strength may be represented by a pair of straight lines at inclinations of + φ and – φ with the σ-axis and passing through the origin of the Mohr's circle diagram. If the stress conditions at a point are represented by Mohr's circle I, the shear stress on any plane through the point is less than the shearing strength, as indicated by the line *BCD;*

BC represents the shear stress on a plane on which the normal stress is given by *OD.*

BD, representing the shearing strength for this normal stress, is greater than *BC*.



Mohr's strength theory—Mohr envelopes for cohesionless soil

Mohr's Circle II, which is tangential to the Mohr's envelope at *F*, are such that the shearing stress, *EF*,

$$s = \sigma_f \tan\phi = \sigma_3 \tan\phi\,(1 + \sin\phi)$$

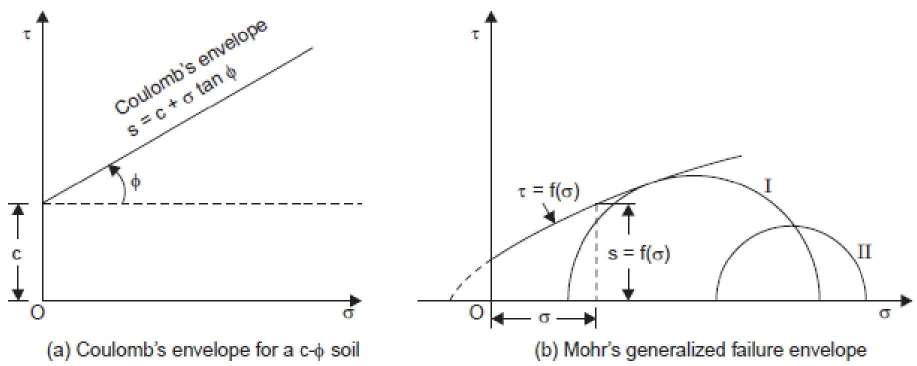
$$= \sigma_1 \tan\phi\,(1 - \sin\phi) = \left(\frac{\sigma_1 - \sigma_3}{2}\right) . \cos\phi$$

- **Mohr-Coulomb Theory**

The functional relationship between the normal stress on any plane and the shearing strength available on that plane was assumed to be linear by Coulomb; thus the following is usually known as Coulomb's law:

$$s = c + \sigma \tan\varphi$$

where c and φ are empirical parameters, known as the 'apparent cohesion' and 'angle of shearing resistance'

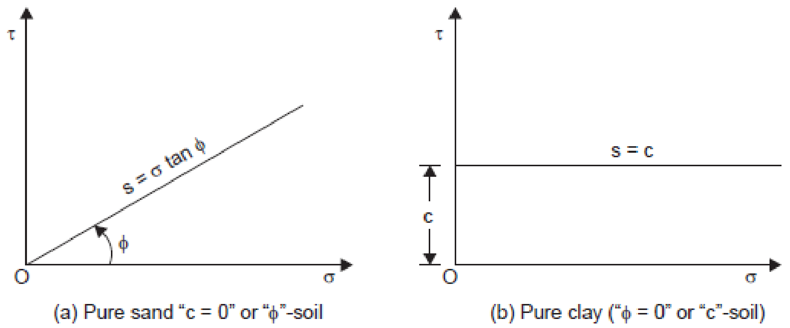


Mohr-Coulomb Theory—failure envelopes

The envelopes are called 'strength envelopes' or 'failure envelopes'. The meaning of an envelope has already been given in the previous section; if the normal and shear stress components on a plane plot on to the failure envelope, failure is supposed to be incipient and if the stresses plot below the envelope, the condition represents stability. And, it is impossible that these plot above the envelope, since failure should have occurred previously. Coulomb's law is also written as follows to indicate that the stress condition refers to

that on the plane of failure:

$$s = c + \sigma_f \tan \varphi$$



Coulomb envelopes for pure sand and for pure clay

SHEARING STRENGTH—A FUNCTION OF EFFECTIVE STRESS

The density of a soil increase when subjected to shearing action, drainage being allowed simultaneously. Therefore, even if two soils are equally dense on having been consolidated to the same effective stress, they will exhibit different shearing strengths if drainage is permitted during shear for one, while it is not for the other.

These ideas lead to a statement that ''the strength of a soil is a unique function of the effective stress acting on the failure plane''.

$$s = c' + \sigma_f \tan \varphi'$$

where c' and φ' are called the effective cohesion and effective angle of internal friction. Collectively, they are called 'effective stress parameters', while c and φ are called ''total stress parameters''.

TYPES OF SHEAR TESTS BASED ON DRAINAGE CONDITIONS

A cohesionless or a coarse-grained soil may be tested for shearing strength either in the dry condition or in the saturated condition.

A cohesive or fine-grained soil is usually tested in the saturated condition. Depending upon whether drainage is permitted before and during the test, shear tests on such saturated soils are classified as follows:

1- Unconsolidated Undrained Test (UU)

Drainage is not permitted at any stage of the test, that is, either before the test during the application of the normal stress or during the test when the shear stress is applied. Hence no time is allowed for dissipation of pore water pressure and consequent consolidation of the soil; also, no significant volume changes are expected. Usually, 5 to 10 minutes may be adequate for the whole test, because of the shortness of drainage path. However, undrained tests are often performed only on soils of low permeability.

2- Consolidated Undrained Test (CU)

Drainage is permitted fully in this type of test during the application of the normal stress and no drainage is permitted during the application of the shear stress. Thus volume changes do not take place during shear and excess pore pressure develops. 5 to 10 minutes may be adequate for the test.

3- Consolidated Drained Test (CD)

Drainage is permitted fully before and during the test, at every stage. The soil is consolidated under the applied normal stress and is tested for shear by applying the shear stress also very slowly while drainage is permitted at every stage. Practically no excess pore pressure develops at any stage and volume changes take place. It may require 4 to 6 weeks to complete a single test of this kind in the case of cohesive soils

The shear parameters *c* and φ vary with the type of test or drainage conditions. The suffixes *u*, *cu*, and *d* are used for the parameters obtained from the *UU*, *CU* and *CD*-tests respectively. For problems of short-term stability of foundations, excavations and earth dams *UU*-tests are appropriate. For problems of long-term stability, either *CU*-test or *CD* tests are appropriate, depending upon the drainage conditions in the field.

SHEARING STRENGTH TESTS

Laboratory Tests

1. Direct Shear Test
2. Triaxial Compression Test

3. Unconfined Compression Test

4. Laboratory Vane Shear Test

5. Torsion Test

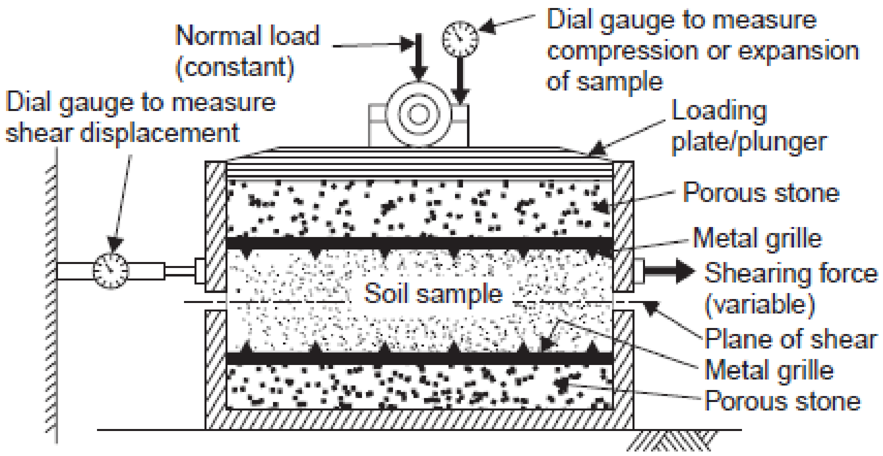
6. Ring Shear Tests

Field Tests

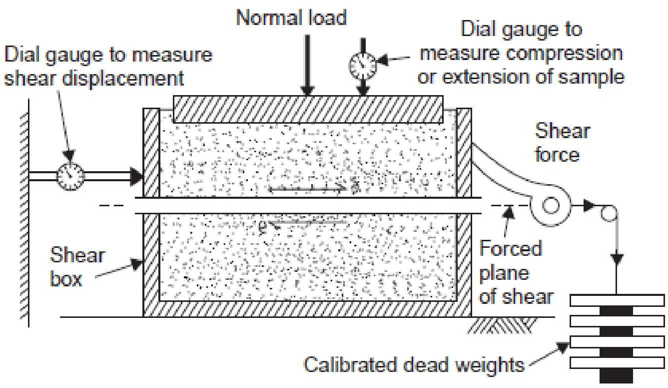
1. Vane Shear Test

2. Penetration Test

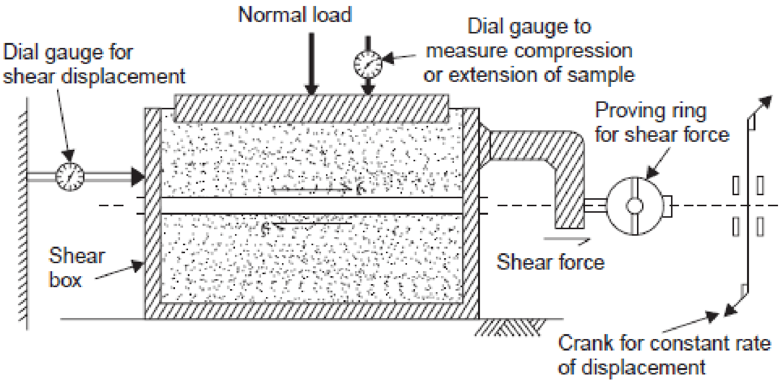
Direct Shear Test



(a) Schematic Diagram



(b) Stress-control type

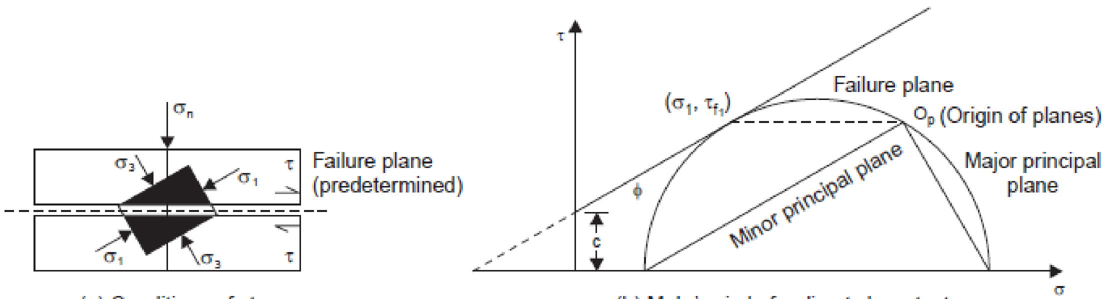


(c) Strain-control type

Direct shear device

- Two types of application of shear are possible—one in which the shear stress is controlled and the other in which the shear strain is controlled. The principles of these two types of devices are illustrated schematically in Fig above. (*b*) and (*c*), respectively.
- The shear strain may be plotted against the shear stress;

The stress-conditions on the failure plane and the corresponding Mohr's circle for direct shear test are shown in Fig. below (*a*) and (*b*) respectively.

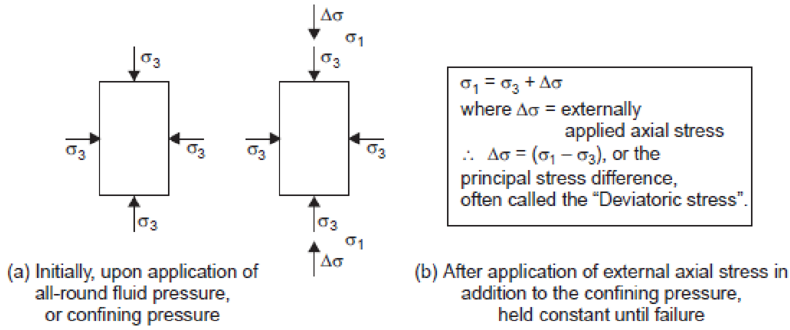


(a) Conditions of stress in the shear box

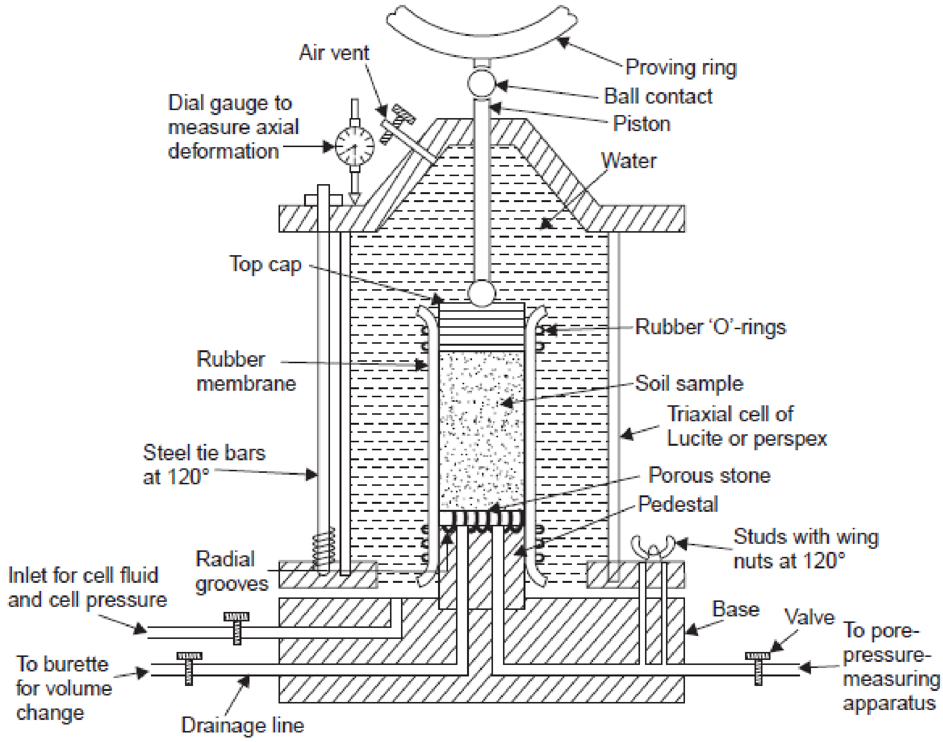
(b) Mohr's circle for direct shear test

Triaxial Compression Test

The soil specimen is subjected to three compressive stresses in mutually perpendicular directions, one of the three stresses being increased until the specimen fails in shear. Usually a cylindrical specimen with a height equal to twice its diameter is used. The desired three-dimensional stress system is achieved by an initial application of all-round fluid pressure or confining pressure through water. While this confining pressure is kept constant throughout the test, axial or vertical loading is increased gradually and at a uniform rate.



Principle and stress conditions of triaxial compression test

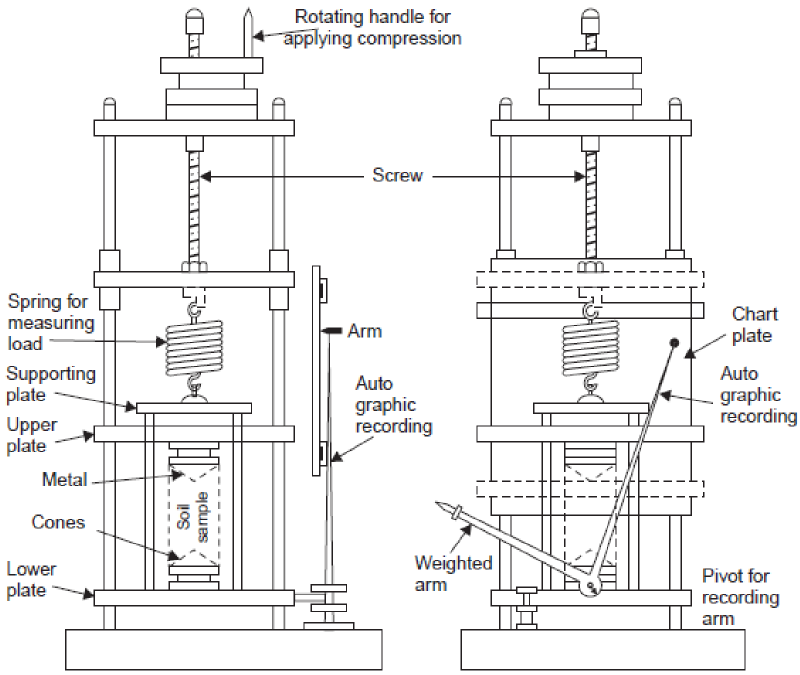


Triaxial cell with accessories

Unconfined Compression Test

This is a special case of a triaxial compression test; the confining pressure being zero. A cylindrical soil specimen, usually of the same standard size as that for the triaxial compression, is loaded axially by a compressive force until failure takes place. Since the specimen is laterally unconfined, the test is known as 'unconfined compression test'. No rubber membrane is necessary to encase the specimen. The axial or vertical compressive stress is the major principal stress and the other two principal stresses are zero.

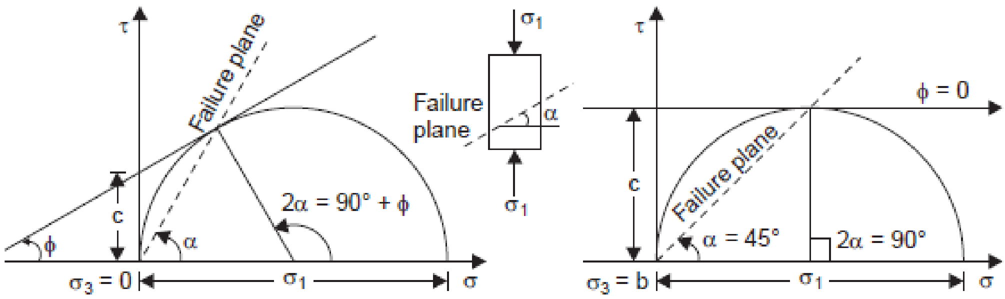
This test may be conducted on undisturbed or remoulded cohesive soils.



(a) Side view (b) Front view

The Mohr's circles for the unconfined compression test are shown in the following Fig. From
Eq., recognizing that $\sigma_3 = 0$

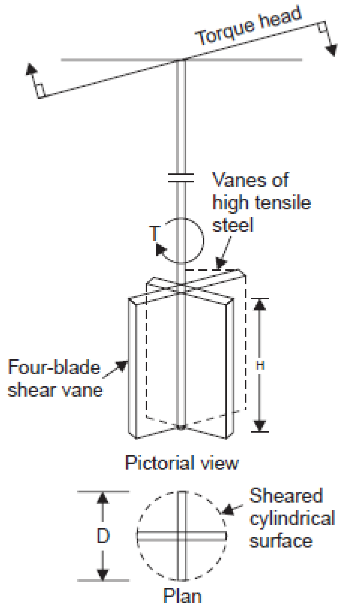
$$\sigma_1 = 2c \tan (45° + \varphi/2)$$



(a) For a c-ϕ soil (b) For a pure clay (ϕ = 0)

Vane Shear Test

If suitable undisturbed for remoulded samples cannot be got for conducting triaxial or unconfined compression tests, the shear strength is determined by a device called the Shear Vane



Laboratory vane:
H = 20 mm
D = 12 mm
t = 0.5 to 1 mm

Field vane:
H = 10 to 20 cm
D = 5 to 10 cm
t = 2.5 cm

The applied torque is measured by a calibrated torsion spring, the angle of twist being read on a special gauge. A uniform rotation of about 1° per minute is used.

$$s = \frac{T}{\pi D^2 (H/2 + D/6)}$$

If only one end of the vane partakes in shearing the soil, then

$$s = \frac{T}{\pi D^2 (H/2 + D/12)}$$

Here, T = torque
D = diameter of the vane
H = height of the vane
The shearing force at the cylindrical surface = π.*D.H.S.*,
where *S* is the shearing strength of the soil.
The moment of this force about the axis of the vane contributes to the torque and is given by $\pi DH.S.\ D/2$ or $\pi.S.\ H\ .\ D^2/2$
For the circular faces at top or bottom, considering the shearing strength of a ring of thickness *dr* at a radius *r*, the elementary torque is $(2\pi\ r\ dr)\ .\ s\ .\ r$
and the total for one face is

$$\int_0^{D/2} 2\pi s r^2\, dr = \frac{2\pi s}{3} \cdot \frac{D^3}{8} = \frac{\pi s}{12} \cdot D^3$$

If we add these contributions considering both the top and bottom faces and equate to the torque T at failure, we get Eqs above.

PORE PRESSURE PARAMETERS

Pore water pressures play an important role in determining the strength of soil. The change in pore water pressure due to change in applied stress is characterised by dimensionless coefficients, called 'Pore pressure coefficients' or 'Pore pressure parameters' *A* and *B*

In an undrained triaxial compression test, pore water pressures develop in the first stage of application of cell pressure or confining pressure, as also in the second stage of application of additional axial stress or deviator stress.

The ratio of the pore water pressure developed to the applied confining pressure is called the *B*-parameter:

$$B = \frac{\Delta u_c}{\Delta \sigma_c} = \frac{\Delta u_c}{\Delta \sigma_3}$$

Since no drainage is permitted, the decrease in volume of soil skeleton is equal to that in the volume of pore water. Using this and the principles of theory of elasticity, it can be shown that

$$B = \frac{1}{1 + n \cdot \dfrac{C_v}{C_c}}$$

the pore pressure coefficient or parameter *A* is defined from *A* as follows :

$$\overline{A} = \frac{\Delta u_d}{(\Delta\sigma_1 - \Delta\sigma_3)}$$

where Δ_{ud} = port pressure developed due to an increase of deviator stress ($\Delta\sigma_1 - \Delta\sigma_3$), and *A* is the product of *A* and *B*.

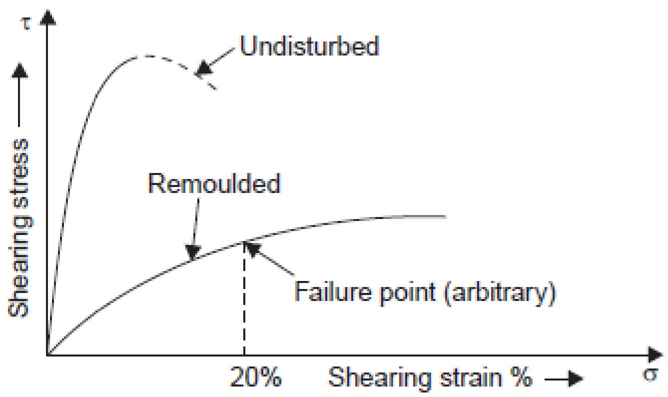
The general expression for the pore water pressure developed and changes in applied stresses is as follows:

$$\Delta u = B\,\{\Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3)\}$$

Sensitivity of clays

This is an important phenomenon which is quantitatively characterised by 'Sensitivity', defined as follows:

$$\text{Sensitivity,} \qquad S_t = \frac{\text{Unconfined compression strength, undisturbed}}{\text{Unconfined compression strength, remoulded}}$$



Stress-strain curves for a sensitive clay in the undisturbed and remoulded states

Sensitivity S_t	*Classification*
1	Insensitive
1–2	Low
2–4	Medium
4–8	Sensitive
8–16	Extra-sensitive
Greater than 16	Quick (S_t can be even up to 150)

Overconsolidated clays are rarely sensitive, although some quick clays have been found to be overconsolidated.

Example 32: The stresses at failure on the failure plane in a cohesionless soil mass were: Shear stress = 4 kN/m^2; normal stress = 10 kN/m^2. Determine the resultant stress on the failure plane, the angle of internal friction of the soil and the angle of inclination of the failure plane to the major principal plane.

$$\text{Resultant stress} = \sqrt{\sigma^2 + \tau^2}$$

$$= \sqrt{10^2 + 4^2} = \mathbf{10.77\ kN/m^2}$$

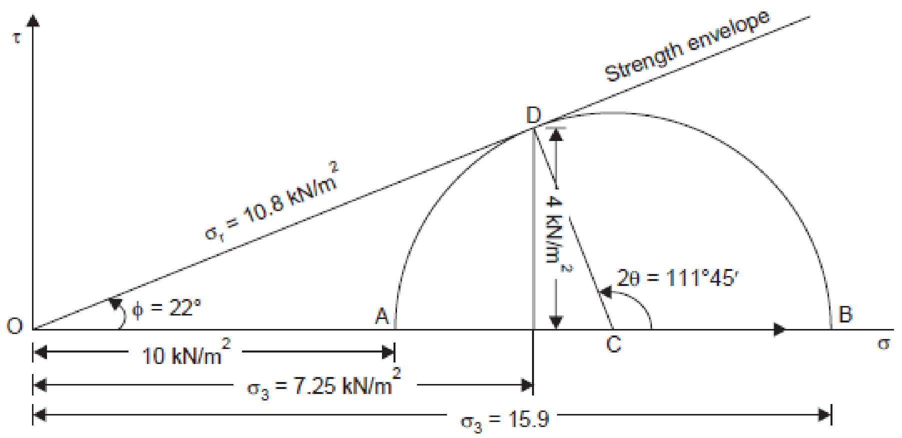
$$\tan\phi = \tau/\sigma = 4/10 = 0.4$$

$$\phi = \mathbf{21°\ 48'}$$

$$\theta = 45° + \phi/2 = 45° + \frac{21°\ 48'}{2} = \mathbf{55°54'}.$$

Graphical solution

The procedure is first to draw the σ-and τ-axes from an origin O and then, to a suitable scale, set-off point D with coordinates (10,4), Joining O to D, the strength envelope is got. The Mohr Circle should be tangential to OD to D. DC is drawn perpendicular to OD to cut OX in C, which is the centre of the circle. With C as the center and CD as radius, the circle is completed to cut OX in A and *B*.



By scaling, the resultant stress = *OD* = 10.8 kN/m^2. With protractor, φ = 22° and θ = 55°53′ .We also observe than σ3 = *OA* = 7.25 kN/m2 and σ1 = *OB* = 15.9 kN/m^2.

Example 33: Clean and dry sand samples were tested in a large shear box, 25 cm × 25 cm and the following results were obtained:

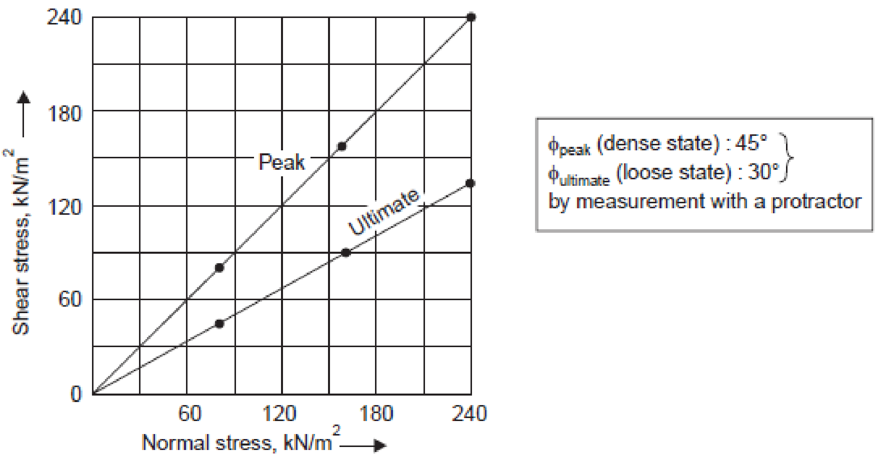
Normal load (kN)	5	10	15
Peak shear load (kN)	5	10	15
Ultimate shear load (kN)	2.9	5.8	8.7

Determine the angle of shearing resistance of the sand in the dense and loose states.
The value of φ obtained from the peak stress represents the angle of shearing resistance of the sand in its initial compacted state; that from the ultimate stress corresponds to the sand when loosened by the shearing action.
The area of the shear box = 25 × 25 = 625 cm^2= 0.0625 m^2.
Normal stress in the first test = 5/0.0625 kN/m^2 = 80 kN/m^2

Similarly the other normal stresses and shear stresses are obtained by dividing by the area of the box and are as follows in kN/m^2 :

Normal stress, σ	80	160	240
Peak shear stress, τ_{max}	80	160	240
Ultimate shear stress, τ_f	46.4	92.8	139.2

Since more than one set of values are available, graphical method is better



Failure envelopes

Example 34: Calculate the potential shear strength on a horizontal plane at a depth of 3 m below the surface in a formation of cohesionless soil when the water table is at a depth of 3.5m. The degree of saturation may be taken as 0.5 on the average. Void ratio = 0.50; grain specific gravity = 2.70; angle of internal friction = 30°. What will be the modified value of shear strength if the water table reaches the ground surface?

Effective unit weight $\gamma' = \frac{(G-1)}{(1+e)} \cdot \gamma_w$

$$\approx \frac{(2.70-1)}{(1+0.5)} \times 10 = 11.33 \text{ kN/m}^3$$

Unit weight, γ, at 50% saturation

$$= \frac{(G+S.e)}{(1+e)} \cdot \gamma_w = \frac{(2.70+0.5\times 0.5)}{(1+0.5)} \times 10 = 19.667 \text{ kN/m}^3$$

(*a*) When the water table is at 3.5 m below the surface:

Normal stress at 3 m depth, $\sigma = 19.67 \times 3 = 59$ kN/m^2

Shear strength, $s = \sigma \tan \varphi$ for a sand

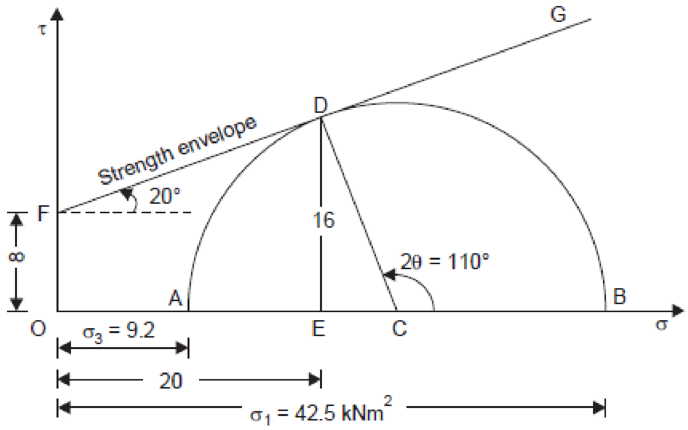
$= 59 \tan 30° =$ **34 kN/m^2 (nearly).**

(*b*) When water table reaches the ground surface:

Effective Normal stress at 3 m depth = $\sigma = \gamma' . h = 11.33 \times 3 = 34$ kN/m^2

Shear strength, $s = \sigma \tan \varphi =$ 34 tan 30°= **19.6 kN/m2 (nearly).**

Example 35: The following data were obtained in a direct shear test. Normal pressure = 20 kN/m^2, tangential pressure = 16 kN/m^2. Angle of internal friction = 20°, cohesion = 8 kN/m^2. Represent the data by Mohr's Circle and compute the principal stresses and the direction of the principal planes.

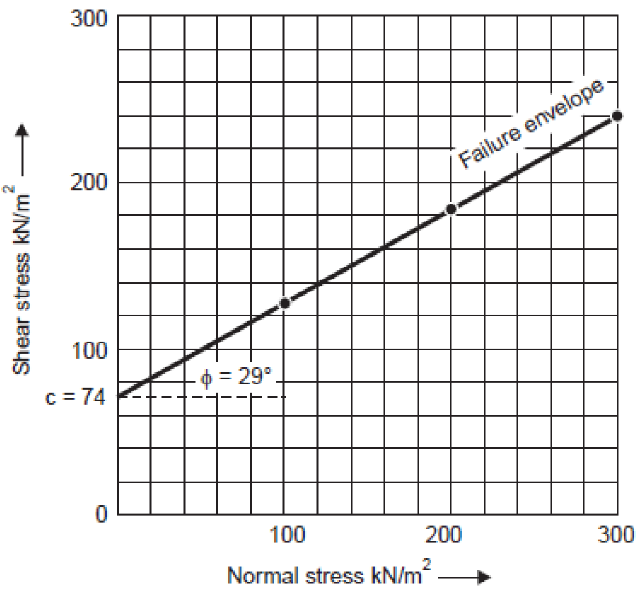


1- The strength envelope FG is located since both c and φ are given.
2- Draw Point D with co-ordiantes (20, 16); it should fall on the envelope.
3- DC is drawn perpendicular to FD to meet the σ-axis in C.
4- With C as centre and CD as radius, the Mohr's circle is completed.
5- The principal stresses σ_3 (OA) and σ_1 (OB) are found to be 9.2 kN/m^2 and 42.5 kN/m^2.
6- scale angle BCD and found to be 110°. Hence the major principal plane is inclined at 55° (clockwise) and the minor principal plane at 35° (counter clockwise) to the plane of shear (horizontal plane, in this case).

Example 36: The following results were obtained in a shear box text. Determine the angle of shearing resistance and cohesion intercept:

Normal stress (kN/m^2)	100	200	300
Shear stress (kN/m^2)	130	185	240

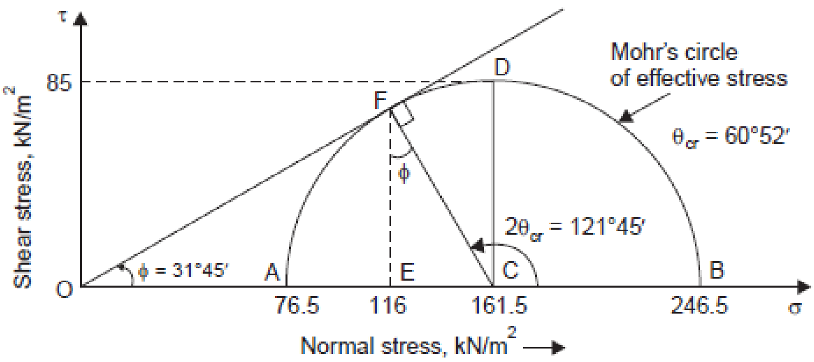
The normal and shear stresses on the failure plane are plotted as shown:



Failure envelope (Ex. 36)

c = 74 kN/m^2 φ = 29°

Example 37: The stresses acting on the plane of maximum shearing stress through a given point in sand are as follows: total normal stress = 250 kN/m^2; pore-water pressure = 88.5 kN/m^2; shearing stress = 85 kN/m^2. Failure is occurring in the region surrounding the point. Determine the major and minor principal effective stresses, the normal effective stress and the shearing stress on the plane of failure and the friction angle of the sand. Define clearly the terms 'plane of maximum shearing stress' and 'plane of failure' in relation to the Mohr's rupture diagram.



Total normal stress = 250 kN/m^2

Pore water pressure = 88.5 kN/m^2

Effective normal stress on the plane of maximum shear = (250 – 88.5) = 161.5 kN/m^2

Max. Shear stress = 85 kN/m^2

Graphical solution:

1- Draw Point D with co-ordiantes is (161.5, 85).
2- CD is plotted perpendicular to σ -axis as the maximum shear stress.
3- With C as center and CD as radius, the Mohr's circle is drawn.
4- A tangent drawn to the circle from the origin O and cut Mohr's circle in F.
5- The perpendicular line from F cut σ -axis in E.
The results are: Major effective principal stress = (OB) = 246.5 kN/m^2

Minor effective principal stress (OA) = 76.5 kN/m2

Angle of internal friction, φ (angle FOB) = 31°45′

Normal effective stress on plane of failure (OE) = 116 kN/m^2

Shearing stress on the plane of failure (EF) = 72 kN/m^2

Analytical solution:

$$\left(\frac{\bar{\sigma}_1+\bar{\sigma}_3}{2}\right) = \text{Normal stress on the plane of maximum shear} = 161.5$$

$$\left(\frac{\bar{\sigma}_1-\bar{\sigma}_3}{2}\right) = \text{Maximum shear stress} = 85$$

$$\bar{\sigma}_1 = \mathbf{246.5\ kN/m^2}\ \text{(Major principal effective stress)}$$

$$\bar{\sigma}_3 = \mathbf{76.5\ kN/m^2}\ \text{(Minor principal effective stress)}$$

$$\sin\phi = \frac{(\bar{\sigma}_1-\bar{\sigma}_3)/2}{(\bar{\sigma}_1+\bar{\sigma}_3)/2} = \frac{85}{161.5} = 0.526$$

∴ Angle of internal friction φ = 31°45′ nearly.

Normal stress on the failure plane

$$= \left(\frac{\bar{\sigma}_1+\bar{\sigma}_3}{2}\right) - \left(\frac{\bar{\sigma}_1-\bar{\sigma}_3}{2}\right).\sin\phi$$

$$= 161.5 - \frac{85\times 85}{161.5} = \mathbf{116.76\ kN/m^2}$$

Shear stress on the failure plane

$$= \left(\frac{\bar{\sigma}_1 - \bar{\sigma}_3}{2}\right) . \cos \phi$$

$$= 85 \times \cos 31°45' = \mathbf{72.27\ kN/m^2}$$

Example 7-18 : A shear vane used to test in-situ a soft clay soil had a diameter of 75 mm and a length of 150 mm. An average torque was recorded of 25 Nm. Determine the undrained shear strength of the clay.

$$C_u = \frac{25 \times 10^{-3}}{\frac{1}{2} \times \pi \times 0.075^2 \left(0.150 + \frac{1}{3} \times 0.075\right)} = 16.2\ kN/m^2$$

Example 7-13 : Using the triaxial test apparatus, a soil sample was first of all consolidated at a cell pressure of 600 kN/m^2 under a maintained back pressure of 300 kN/m^2. Then with the drains closed the cell pressure was raised to 720 kN/m^2, resulting in the pore pressure increasing to 415 kN/m^2. Following this the axial lood was raised to give an increase in deviator stress of 550 kN/m^2, while the cell pressure remained constant. Now the pore pressure reading was 562 kN/m^2. Calculate the pore pressure coefficients B, A and B. Following an increase in isotropic stress (Cell pressure) of $\Delta\sigma_3 = 720 - 600$ the pore pressure change was $\Delta u_0 = 415 - 300$.

Therefore $B = \frac{\Delta u_0}{\Delta\sigma_3} = \frac{415 - 300}{720 - 600} = \underline{0.958}$

Then an increase in deviator stress of $(\Delta\sigma_1 - \Delta\sigma_3) = 550$ produced a further change in pore pressure of $\Delta u_1 = 562 - 415$.

Therefore $AB = \frac{\Delta u_1}{\Delta\sigma_1 - \Delta\sigma_3} = \frac{562 - 415}{550} = 0.267$ giving $A = \frac{0.267}{0.958} = \underline{0.279}$

The overall coefficient $\bar{B}$ is the ratio of the overall change in pore pressure to the change in the major principal stress.

Therefore $\overline{B} = \frac{\Delta u}{\Delta\sigma_1} = \frac{\Delta u_0 + \Delta u_1}{(\Delta\sigma_1 - \Delta\sigma_3) + \Delta\sigma_3} = \frac{562 + 300}{550 + 120} = \underline{0.391}$

TYPES OF SHEAR TESTS BASED ON DRAINAGE CONDITIONS

A cohesionless or a coarse-grained soil may be tested for shearing strength either in the dry condition or in the saturated condition.

A cohesive or fine-grained soil is usually tested in the saturated condition. Depending upon whether drainage is permitted before and during the test, shear tests on such saturated soils are classified as follows:

1- Unconsolidated Undrained Test (UU)

Drainage is not permitted at any stage of the test, that is, either before the test during the application of the normal stress or during the test when the shear stress is applied. Hence no time is allowed for dissipation of pore water pressure and consequent consolidation of the soil; also, no significant volume changes are expected. Usually, 5 to 10 minutes may be adequate for the whole test, because of the shortness of drainage path. However, undrained tests are often performed only on soils of low permeability.

2- Consolidated Undrained Test (CU)

Drainage is permitted fully in this type of test during the application of the normal stress and no drainage is permitted during the application of the shear stress. Thus volume changes do not take place during shear and excess pore pressure develops. 5 to 10 minutes may be adequate for the test.

3- Consolidated Drained Test (CD)

Drainage is permitted fully before and during the test, at every stage. The soil is consolidated under the applied normal stress and is tested for shear by applying the shear stress also very slowly while drainage is permitted at every stage. Practically no excess pore pressure develops at any stage and volume changes take place. It may require 4 to 6 weeks to complete a single test of this kind in the case of cohesive soils

The shear parameters *c* and φ vary with the type of test or drainage conditions. The suffixes *u*, *cu*, and *d* are used for the parameters obtained from the *UU*, *CU* and *CD*-tests respectively. For problems of short-term stability of foundations, excavations and earth dams *UU*-tests are appropriate. For problems of long-term stability, either *CU*-test or *CD* tests are appropriate, depending upon the drainage conditions in the field.

SHEARING STRENGTH TESTS

Laboratory Tests

1. Direct Shear Test

2. Triaxial Compression Test

3. Unconfined Compression Test

4. Laboratory Vane Shear Test

5. Torsion Test

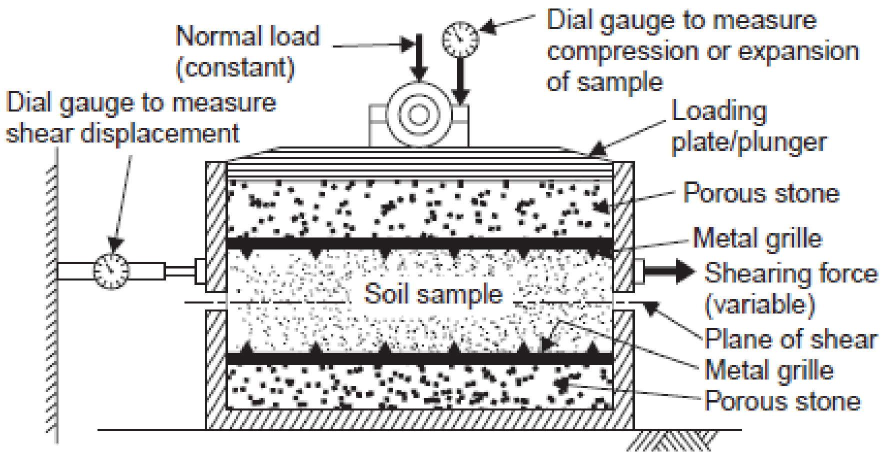
6. Ring Shear Tests

Field Tests

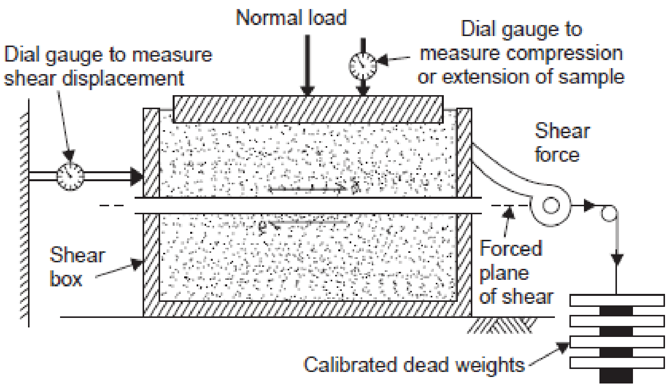
1. Vane Shear Test

2. Penetration Test

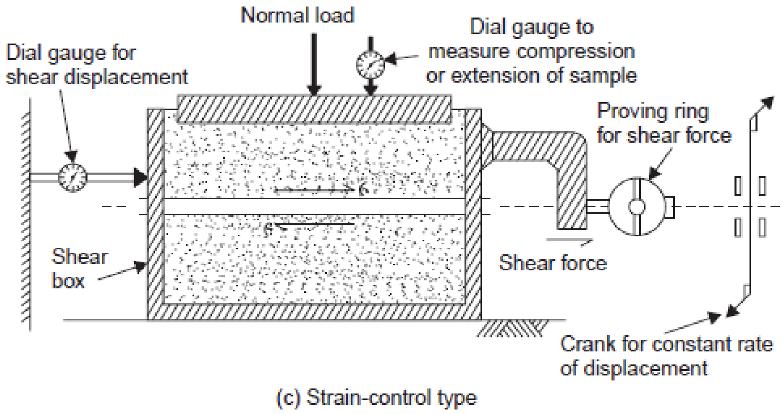
Direct Shear Test



(a) Schematic Diagram



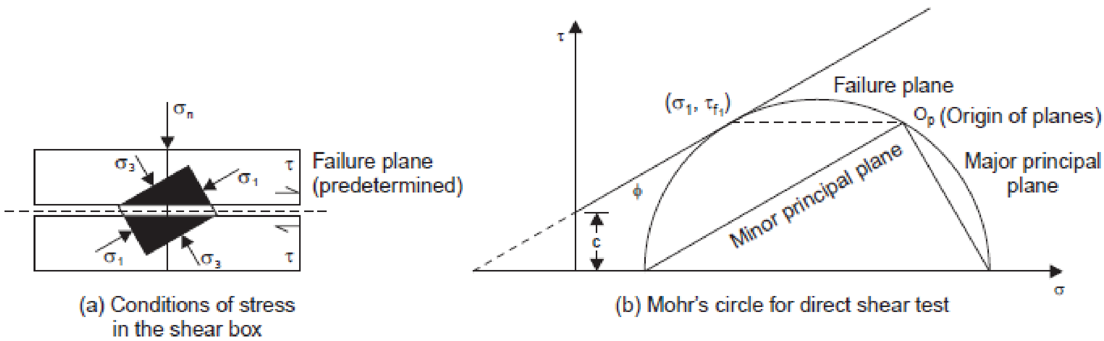
(b) Stress-control type



Direct shear device

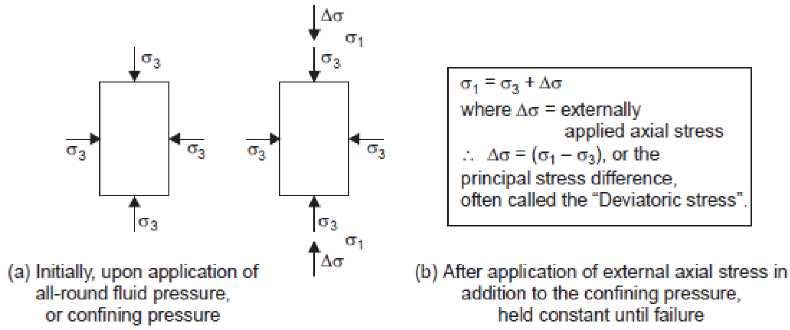
- Two types of application of shear are possible—one in which the shear stress is controlled and the other in which the shear strain is controlled. The principles of these two types of devices are illustrated schematically in Fig above. (*b*) and (*c*), respectively.
- The shear strain may be plotted against the shear stress;

The stress-conditions on the failure plane and the corresponding Mohr's circle for direct shear test are shown in Fig. below (*a*) and (*b*) respectively.

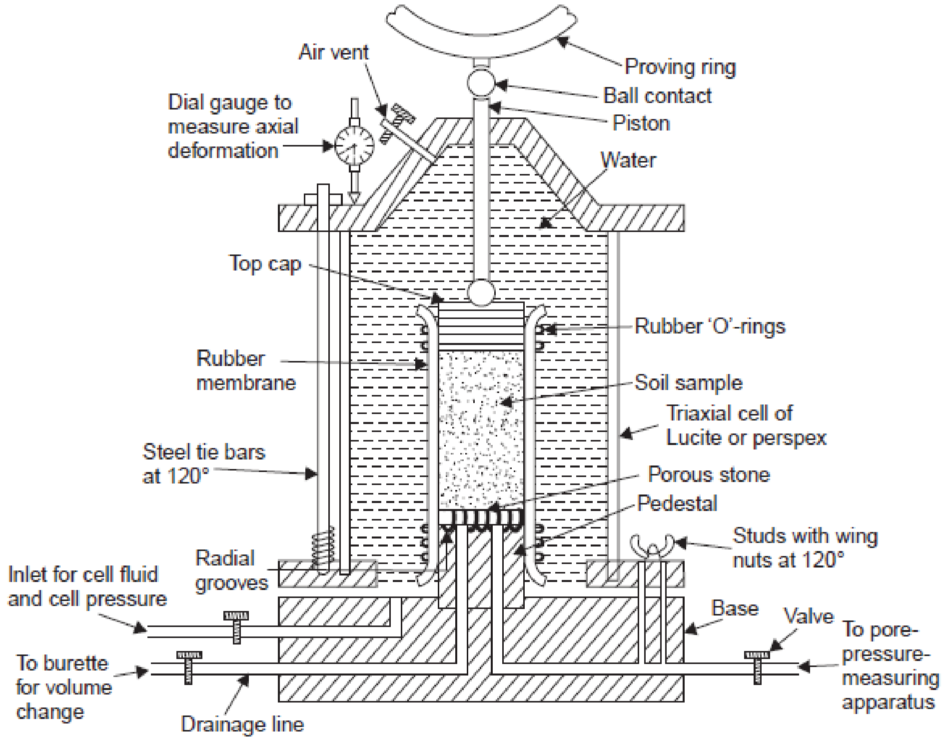


Triaxial Compression Test

The soil specimen is subjected to three compressive stresses in mutually perpendicular directions, one of the three stresses being increased until the specimen fails in shear. Usually a cylindrical specimen with a height equal to twice its diameter is used. The desired three-dimensional stress system is achieved by an initial application of all-round fluid pressure or confining pressure through water. While this confining pressure is kept constant throughout the test, axial or vertical loading is increased gradually and at a uniform rate.



Principle and stress conditions of triaxial compression test

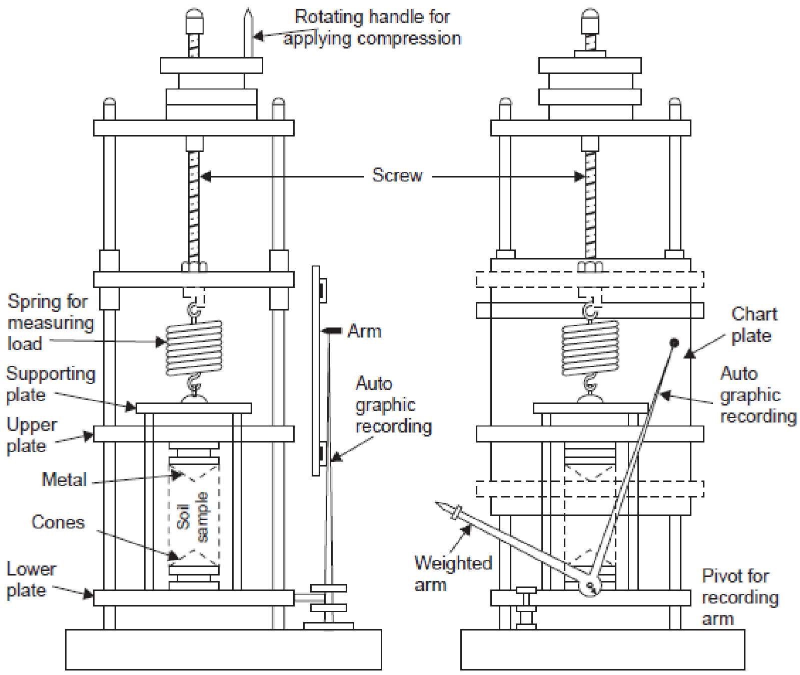


Triaxial cell with accessories

Unconfined Compression Test

This is a special case of a triaxial compression test; the confining pressure being zero. A cylindrical soil specimen, usually of the same standard size as that for the triaxial compression, is loaded axially by a compressive force until failure takes place. Since the specimen is laterally unconfined, the test is known as 'unconfined compression test'. No rubber membrane is necessary to encase the specimen. The axial or vertical compressive stress is the major principal stress and the other two principal stresses are zero.

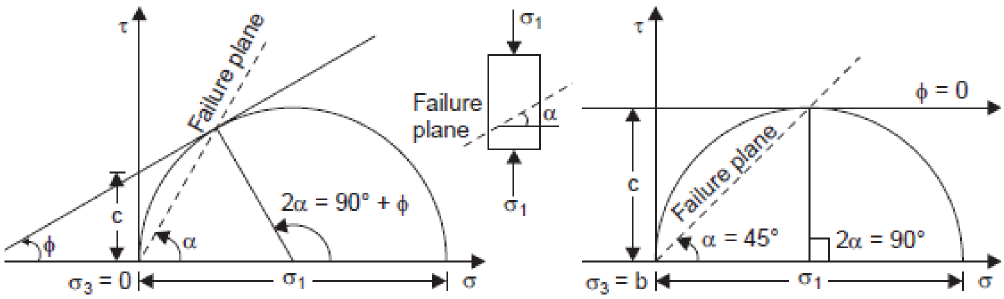
This test may be conducted on undisturbed or remoulded cohesive soils.



(a) Side view (b) Front view

The Mohr's circles for the unconfined compression test are shown in the following Fig. From
Eq., recognizing that $\sigma_3 = 0$

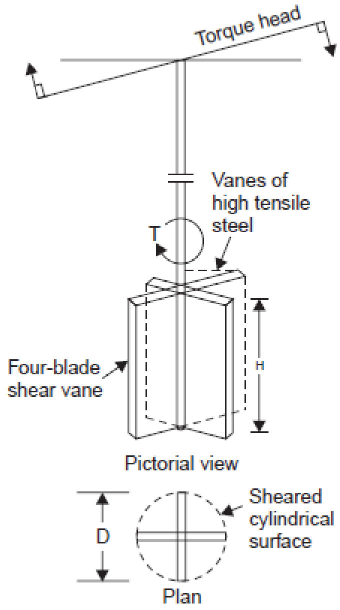
$$\sigma_1 = 2c \tan(45° + \varphi/2)$$



(a) For a c-ϕ soil (b) For a pure clay (ϕ = 0)

Vane Shear Test

If suitable undisturbed for remoulded samples cannot be got for conducting triaxial or unconfined compression tests, the shear strength is determined by a device called the Shear Vane



Laboratory vane:
H = 20 mm
D = 12 mm
t = 0.5 to 1 mm

Field vane:
H = 10 to 20 cm
D = 5 to 10 cm
t = 2.5 cm

The applied torque is measured by a calibrated torsion spring, the angle of twist being read on a special gauge. A uniform rotation of about 1° per minute is used.

$$s = \frac{T}{\pi D^2 (H/2 + D/6)}$$

If only one end of the vane partakes in shearing the soil, then

$$s = \frac{T}{\pi D^2 (H/2 + D/12)}$$

Here, T = torque
D = diameter of the vane
H = height of the vane
The shearing force at the cylindrical surface = $\pi.D.H.S.$,
where S is the shearing strength of the soil.
The moment of this force about the axis of the vane contributes to the torque and is given by $\pi DH.S.\ D/2$ or $\pi.S.\ H\ .\ D^2/2$
For the circular faces at top or bottom, considering the shearing strength of a ring of thickness dr at a radius r, the elementary torque is $(2\pi\ r\ dr)\ .\ s\ .\ r$
and the total for one face is

$$\int_0^{D/2} 2\pi s r^2\, dr = \frac{2\pi s}{3} . \frac{D^3}{8} = \frac{\pi s}{12} . D^3$$

If we add these contributions considering both the top and bottom faces and equate to the torque T at failure, we get Eqs above.

PORE PRESSURE PARAMETERS

CHAPTER 7

STRESSES, STRAINS, AND ELASTIC DEFORMATIONS OF SOILS

7.0 INTRODUCTION

In this chapter, we will review some fundamental principles of mechanics and strength of materials and apply these principles to soils treated as elastic porous materials. This chapter contains a catalog of a large number of equations for soil stresses and strains. You may become weary of these equations, but they are necessary for analyses of the mechanical behavior of soils. You do not have to memorize these equations except the fundamental ones.

When you complete this chapter, you should be able to:

- Calculate stresses and strains in soils (assuming elastic behavior) from external loads.
- Calculate elastic settlement.
- Calculate stress states.
- Calculate effective stresses.

You will use the following principles learned from statics and strength of materials:

- Stresses and strains
- Mohr's circle
- Elasticity—Hooke's law

Importance

You would have studied in mechanics the stresses imposed on homogeneous, elastic, rigid bodies by external forces. Soils are not homogeneous, elastic, rigid bodies, so the determination of stresses and strains in soils is a particularly difficult task. You may ask: "If soils are not elastic materials, then why do I have to study elastic methods of analysis?" Here are some reasons why a knowledge of elastic analysis is advantageous.

An elastic analysis of an isotropic material involves only two constants—Young's modulus and Poisson's ratio—and thus if we assume that soils are isotropic elastic materials, then we have a powerful, but simple, analytical tool to predict a soil's response under loading. We will have to determine only the two elastic constants from our laboratory or field tests.

A geotechnical engineer must ensure that a geotechnical structure must not collapse under any anticipated loading condition and that settlement under working load (a fraction of the collapse load) must be within tolerable limits. We would prefer the settlement under working loads to be elastic so that no permanent settlement would occur. To calculate the elastic settlement, we have to use an elastic analysis. For example, in designing foundations on coarse-grained soils, we normally assume that the settlement is elastic, and we then use elastic analysis to calculate the settlement.

An important task of a geotechnical engineer is to determine the stresses and strains that are imposed on a soil mass by external loads. It is customary to assume that the strains in the soils are small, and this assumption allows us to apply our knowledge of mechanics of elastic bodies to soils. Small strains mean infinitesimal strains. For a realistic description of soils, elastic analysis is not satisfactory. We need soil models that can duplicate the complexity of soil behavior. However, even for complex soil models, an elastic analysis is a first step.

Various types of surface loads or stresses are applied to soils. For example, an oil tank will impose a uniform, circular, vertical stress on the surface of the soil while an unsymmetrical building may impose a nonuniform vertical stress. We would like to know how the surface stresses are distributed within the soil mass and the resulting deformations. The induced stresses can lead to soil failure, or the deformations may be intolerable. Here is a sample practical situation. Two storage tanks are to be founded on a deep layer of stiff, saturated clay. Your client and the mechanical engineer who is designing the pipe works need an estimate of the settlement of the tanks when they are completely filled. Because of land restrictions, your client desires that the tanks be as close as possible to each other. If two separate foundations are placed too close to each other, the stresses in the soil induced by each foundation will overlap and cause intolerable tilting of the structures and their foundations. An example of tilting of structures caused by stress overlap is shown in Figure 7.1.

The settlement of soils is caused by the stress transmitted to the soil particles. This stress is called effective stress. It is important that you know how to calculate effective stress in soils.

FIGURE 7.1 The "kissing" silos. (Bozozuk, 1976, permission from National Research Council of Canada.) These silos tilt toward each other at the top because stresses in the soil overlap at and near the internal edges of their foundations. The foundations are too close to each other.

7.1 DEFINITIONS OF KEY TERMS

Stress, or intensity of loading, is the load per unit area. The fundamental definition of stress is the ratio of the force ΔP acting on a plane ΔS to the area of the plane ΔS when ΔS tends to zero; Δ denotes a small quantity.

Effective stress **(σ')** is the stress carried by the soil particles.

Total stress **(σ)** is the stress carried by the soil particles and the liquids and gases in the voids.

Strain, or intensity of deformation, is the ratio of the change in a dimension to the original dimension or the ratio of change in length to the original length.

Stress **(*strain*)** ***state*** at a point is a set of stress (strain) vectors corresponding to all planes passing through that point. Mohr's circle is used to graphically represent stress (strain) state for two-dimensional bodies.

Porewater pressure**, u,** is the pressure of the water held in the soil pores.

Isotropic means the material properties are the same in all directions, and also the loadings are the same in all directions.

Anisotropic means the material properties are different in different directions, and also the loadings are different in different directions.

Elastic materials are materials that return to their original configuration on unloading and obey Hooke's law.

Plastic materials do not return to their original configuration on unloading.

7.2 QUESTIONS TO GUIDE YOUR READING

1. What are normal and shear stresses?
2. What is stress state and how is it determined?
3. Is soil an elastic material?
4. What are the limitations in analyzing soils based on the assumption that they (soils) are elastic materials?
5. What are shear strains, vertical strains, volumetric strains, and deviatoric strains?
6. How do I use elastic analysis to estimate the elastic settlement of soils, and what are the limitations?
7. What are the differences between plane strain and axisymmetric conditions?
8. How do I determine the stresses and strains/displacements imposed on a soil mass by external loads?
9. What is effective stress?
10. Is deformation a function of effective or total stress?

7.3 STRESSES AND STRAINS

7.3.1 Normal Stresses and Strains

Consider a cube of dimensions $x = y = z$ that is subjected to forces P_x, P_y, P_z, normal to three adjacent sides, as shown in Figure 7.2. The normal stresses are

$$\sigma_z = \frac{P_z}{xy}, \quad \sigma_x = \frac{P_x}{yz}, \quad \sigma_y = \frac{P_y}{xz} \tag{7.1}$$

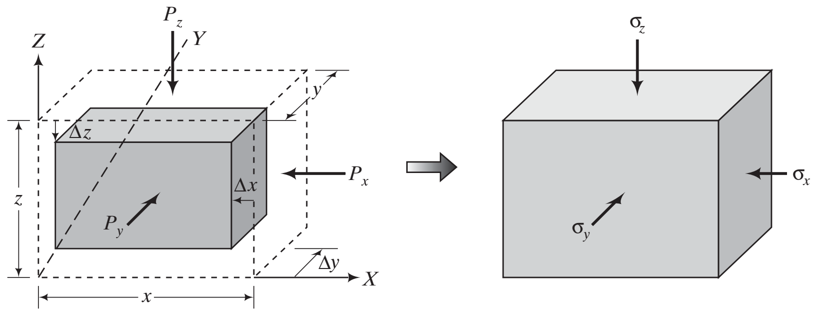


FIGURE 7.2 Stresses and displacements due to applied loads.

Let us assume that under these forces the cube compressed by Δx, Δy, and Δz in the X, Y, and Z directions. The strains in these directions, assuming they are small (infinitesimal), are

$$\varepsilon_z = \frac{\Delta z}{z}, \quad \varepsilon_x = \frac{\Delta x}{x}, \quad \varepsilon_y = \frac{\Delta y}{y} \tag{7.2}$$

7.3.2 Volumetric Strain

The volumetric strain is

$$\varepsilon_p = \varepsilon_x + \varepsilon_y + \varepsilon_z \tag{7.3}$$

7.3.3 Shear Stresses and Shear Strains

Let us consider, for simplicity, the XZ plane and apply a force F that causes the square to distort into a parallelogram, as shown in Figure 7.3. The force F is a shearing force, and the shear stress is

$$\tau = \frac{F}{xy} \tag{7.4}$$

Simple shear strain is a measure of the angular distortion of a body by shearing forces. If the horizontal displacement is Δx, the shear strain or simple shear strain, γ_{zx}, is

$$\gamma_{zx} = \tan^{-1}\frac{\Delta x}{z}$$

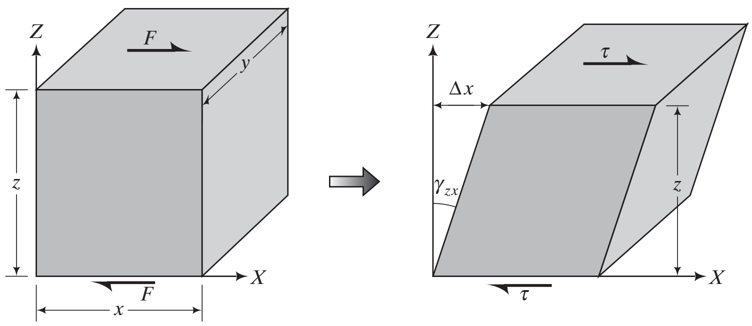


FIGURE 7.3 Shear stresses and shear strains.

For small strains, tan $\gamma_{zx} = \gamma_{zx}$, and therefore

$$\gamma_{zx} = \frac{\Delta x}{z} \tag{7.5}$$

If the shear stress on a plane is zero, the normal stress on that plane is called a principal stress. We will discuss principal stresses later. In geotechnical engineering, compressive stresses in soils are assumed to be positive. Soils cannot sustain any appreciable tensile stresses, and we normally assume that the tensile strength of soils is negligible. Strains can be compressive or tensile.

THE *ESSENTIAL POINTS* ARE:

1. **A normal stress is the load per unit area on a plane normal to the direction of the load.**
2. **A shear stress is the load per unit area on a plane parallel to the direction of the shear force.**
3. **Normal stresses compress or elongate a material; shear stresses distort a material.**
4. **A normal strain is the change in length divided by the original length in the direction of the original length.**
5. **Principal stresses are normal stresses on planes of zero shear stress.**
6. **Soils can only sustain compressive stresses.**

What's next . . . What happens when we apply stresses to a deformable material? From the last section, you may answer that the material deforms, and you are absolutely correct. Different materials respond differently to applied loads. Next, we will examine some typical responses of deformable materials to applied loads to serve as a base for characterizing the loading responses of soils.

7.4 IDEALIZED STRESS–STRAIN RESPONSE AND YIELDING

7.4.1 Material Responses to Normal Loading and Unloading

If we apply an incremental vertical load, ΔP, to a deformable cylinder (Figure 7.4) of cross-sectional area A, the cylinder will compress by, say, Δz and the radius will increase by Δr. The loading condition we apply here is called uniaxial loading. The change in vertical stress is

$$\Delta\sigma_z = \frac{\Delta P}{A} \tag{7.6}$$

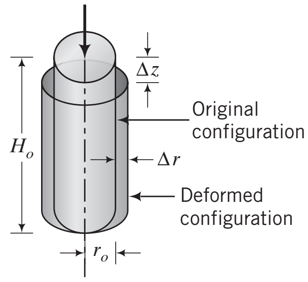


FIGURE 7.4
Forces and displacements on a cylinder.

TABLE 7.1 Typical Values of Poisson's Ratio

Soil type	Description	ν[a]
Clay	Soft	0.35–0.40
	Medium	0.30–0.35
	Stiff	0.20–0.30
Sand	Loose	0.15–0.25
	Medium	0.25–0.30
	Dense	0.25–0.35

[a]These values are effective values, ν'.

The vertical and radial strains are, respectively,

$$\Delta\varepsilon_z = \frac{\Delta z}{H_o} \tag{7.7}$$

$$\Delta\varepsilon_r = \frac{\Delta r}{r_o} \tag{7.8}$$

where H_o is the original length and r_o is the original radius. In Equations (7.7) and (7.8), a negative sign should be inserted for expansion and a positive sign for compression. Thus, for radial expansion, Equation (7.8) should have a negative sign. It is not included here for generality. The ratio of the radial (or lateral) strain to the vertical strain is called Poisson's ratio, ν, defined as

$$\nu = \frac{-\Delta\varepsilon_r}{\Delta\varepsilon_z} \tag{7.9}$$

Typical values of Poisson's ratio for soil are listed in Table 7.1.

We can plot a graph of $\sigma_z = \Sigma\Delta\sigma_z$ versus $\varepsilon_z = \Sigma\Delta\varepsilon_z$. If, for equal increments of ΔP, we get the same value of Δz, then we will get a straight line in the graph of σ_z versus ε_z, as shown by *OA* in Figure 7.5. If at some stress point, say, at *A* (Figure 7.5), we unload the cylinder and it returns to its original configuration, the material comprising the cylinder is called a *linearly elastic* material. Suppose for equal increments of ΔP we get different values of Δz, but on unloading the cylinder it returns to its original configuration. Then a plot of the stress–strain relationship will be a curve, as illustrated by *OB* in Figure 7.5. In this case, the material comprising the cylinder is called a *nonlinearly elastic* material. If we apply a load P_1 that causes a displacement Δz_1 on an elastic material and a second load P_2 that causes a displacement Δz_2,

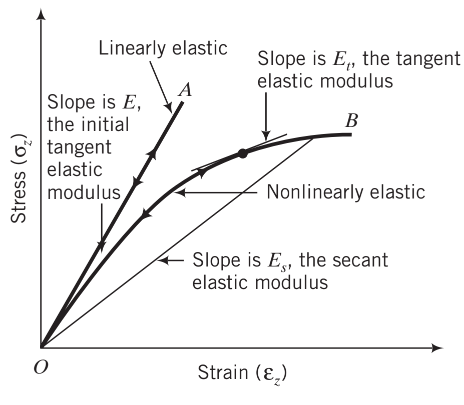


FIGURE 7.5
Linear and nonlinear stress–strain curves of an elastic material.

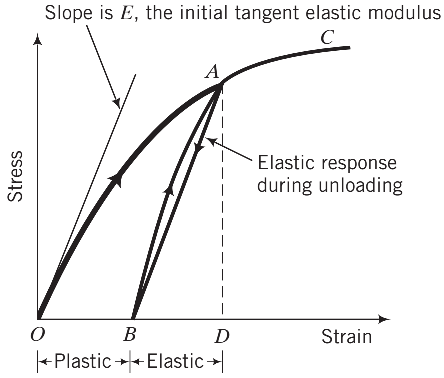


FIGURE 7.6
Idealized stress–strain curves of an elastoplastic material.

then the total displacement is $\Delta z = \Delta z_1 + \Delta z_2$. Elastic materials obey the principle of superposition. The order in which the load is applied is not important; we could apply P_2 first and then P_1, but the final displacement would be the same.

Some materials—soil is one of them—do not return to their original configurations after unloading. They exhibit a stress–strain relationship similar to that depicted in Figure 7.6, where OA is the loading response, AB the unloading response, and BC the reloading response. The strains that occur during loading, OA, consist of two parts—an elastic or recoverable part, BD, and a plastic or unrecoverable part, OB. Such material behavior is called *elastoplastic*. Part of the loading response is elastic, the other plastic.

As engineers, we are particularly interested in the plastic strains since these are the result of permanent deformations of the material. But to calculate the permanent deformation, we must know the elastic deformation. Here, elastic analyses become useful. The stress at which permanent deformation initiates is called the yield stress.

The *elastic modulus* or *initial tangent elastic modulus* (E) is the slope of the stress–strain line for linear isotropic material (Figure 7.5). For a nonlinear elastic material, either the tangent modulus (E_t) or the secant modulus (E_s) or both is determined from the stress–strain relationship (Figure 7.5). The *tangent elastic modulus* is the slope of the tangent to the stress–strain point under consideration. The *secant elastic modulus* is the slope of the line joining the origin (0, 0) to some desired stress–strain point. For example, some engineers prefer to determine the secant modulus by using a point on the stress–strain curve corresponding to the maximum stress, while others prefer to use a point on the stress–strain curve corresponding to a certain level of strain, for example, 1% or one-half the maximum stress (the corresponding secant elastic modulus is normally denoted by E_{50}). The tangent elastic modulus and the secant elastic modulus are not constants. These moduli tend to decrease as shear strains increase. It is customary to determine the *initial tangent elastic modulus* for an elastoplastic material by unloading it and calculating the initial slope of the unloading line as the initial tangent elastic modulus (Figure 7.6).

Strictly speaking, these moduli determined as indicated are not true elastic moduli. The true elastic moduli are determined by small, incremental loading and unloading of the soil. If the stress–strain path followed during the loading is the same as the path followed during unloading, then the slope gives the true elastic modulus.

7.4.2 Material Response to Shear Forces

Shear forces distort materials. A typical response of an elastoplastic material to simple shear is shown in Figure 7.7. The initial shear modulus (G_i) is the slope of the initial straight portion of the τ_{zx} versus γ_{zx} curve. The secant shear modulus (G) is the slope of a line from the desired shear stress–shear strain point to the origin of the τ_{zx} versus γ_{zx} plot (Figure 7.7).

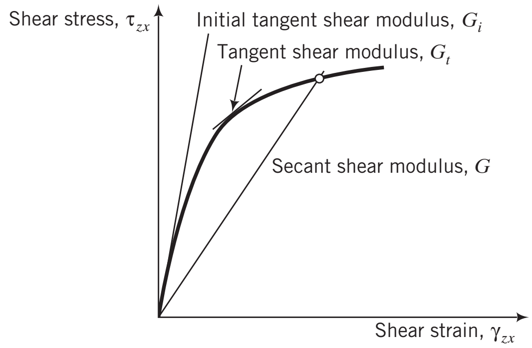


FIGURE 7.7
Shear stress–shear strain response of elastoplastic material.

7.4.3 Yield Surface

Let us consider a more complex situation than the uniaxial loading of a cylinder (Figure 7.8a). In this case, we are going to apply increments of vertical and radial stresses. Since we are not applying any shear stresses, the axial stresses and radial stresses are principal stresses: $\sigma_z = \sigma_1 = \Sigma\Delta\sigma_z$ and $\sigma_r = \sigma_3 = \Sigma\Delta\sigma_r$, respectively. Let us, for example, set σ_3 to zero and increase σ_1. The material will yield at some value of σ_1, which we will call $(\sigma_1)_y$, and plots as point *A* in Figure 7.8b. If, alternatively, we set $\sigma_1 = 0$ and increase σ_3, the material will yield at $(\sigma_3)_y$ and is represented by point *B* in Figure 7.8b. We can then subject the cylinder to various combinations of σ_1 and σ_3 and plot the resulting yield points. Linking the yield points results in a curve, *AB*, which is called the *yield curve* or *yield surface*, as shown in Figure 7.8b. A material subjected to a combination of stresses that lies below this curve will respond elastically (recoverable deformation). If loading is continued beyond the yield stress, the material will respond elastoplastically (irrecoverable or permanent deformations occur). If the material is isotropic, the yield surface will be symmetrical about the σ_1 and σ_3 axes.

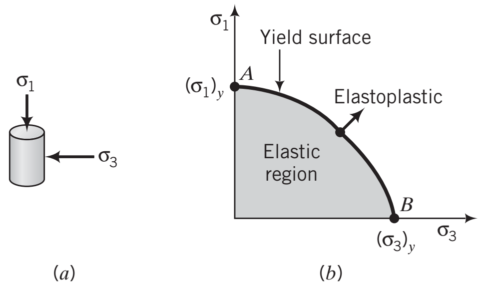


FIGURE 7.8
Elastic, yield, and elastoplastic stress states.

THE *ESSENTIAL POINTS* ARE:

1. **An elastic material recovers its original configuration on unloading; an elastoplastic material undergoes both elastic (recoverable) and plastic (permanent) deformation during loading.**
2. **Soils are elastoplastic materials.**
3. **At small strains soils behave like an elastic material, and thereafter like an elastoplastic material.**
4. **The locus of the stresses at which a soil yields is called a yield surface. Stresses below the yield stress cause the soil to respond elastically; stresses beyond the yield stress cause the soil to respond elastoplastically.**

What's next . . . In the next two sections, we will write the general expression for Hooke's law, which is the fundamental law for linear elastic materials, and then consider two loading cases appropriate to soils.

7.5 HOOKE'S LAW

Access www.wiley.com/college/budhu, and click Chapter 7 and then elastic.xls for a spreadsheet to calculate stresses and strains using Hooke's law.

7.5.1 General State of Stress

Stresses and strains for a linear, isotropic, elastic soil are related through Hooke's law. For a general state of stress (Figure 7.9), Hooke's law is

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix} = \frac{1}{E} \begin{bmatrix} 1 & -\nu & -\nu & 0 & 0 & 0 \\ -\nu & 1 & -\nu & 0 & 0 & 0 \\ -\nu & -\nu & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2(1+\nu) & 0 & 0 \\ 0 & 0 & 0 & 0 & 2(1+\nu) & 0 \\ 0 & 0 & 0 & 0 & 0 & 2(1+\nu) \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix} \tag{7.10}$$

where E is the elastic (or Young's) modulus and ν is Poisson's ratio. Equation (7.10) is called the elastic equation or elastic stress–strain constitutive equation. From Equation (7.10), we have, for example,

$$\gamma_{zx} = \frac{2(1+\nu)}{E}\tau_{zx} = \frac{\tau_{zx}}{G} \tag{7.11}$$

where

$$G = \frac{E}{2(1+\nu)} \tag{7.12}$$

is the shear modulus. We will call E, G, and ν the elastic parameters. Only two of these parameters—either E or G and ν—are required to solve problems dealing with isotropic, elastic materials. We can calculate G from Equation (7.12), if E and ν are known. Poisson's ratio for soils is not easy to determine, and a direct way to obtain G is to subject the material to shearing forces, as described in Section 7.4.2. For nonlinear elastic materials, the tangent modulus or the secant modulus is used in Equation (7.10) and the calculations are done incrementally for small increments of stress.

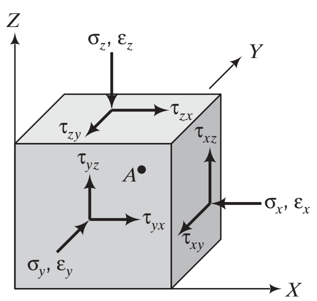


FIGURE 7.9
General state of stress.

TABLE 7.2 Typical Values of *E* and *G*

Soil type	Description	E^a (MPa)	G^a (MPa)
Clay	Soft	1–15	0.5–5
	Medium	15–30	5–15
	Stiff	30–100	15–40
Sand	Loose	10–20	5–10
	Medium	20–40	10–15
	Dense	40–80	15–35

[a]These are average secant elastic moduli for drained condition (see Chapter 10).

The elastic and shear moduli for soils depend on the stress history, the direction of loading, and the magnitude of the applied strains. In Chapter 10 we will study a few tests that are used to determine E and G, and in Chapter 11 we will explore the details of the use of E and G in the mechanical analyses of soils. Typical values of E and G are shown in Table 7.2.

7.5.2 Principal Stresses

If the stresses applied to a soil are principal stresses, then Hooke's law reduces to

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{Bmatrix} = \frac{1}{E}\begin{bmatrix} 1 & -\nu & -\nu \\ -\nu & 1 & -\nu \\ -\nu & -\nu & 1 \end{bmatrix}\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{Bmatrix} \tag{7.13}$$

The matrix on the right-hand side of Equation (7.13) is called the compliance matrix. The inverse of Equation (7.13) is

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{Bmatrix} = \frac{E}{(1+\nu)(1-2\nu)}\begin{bmatrix} 1-\nu & \nu & \nu \\ \nu & 1-\nu & \nu \\ \nu & \nu & 1-\nu \end{bmatrix}\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{Bmatrix} \tag{7.14}$$

The matrix on the right-hand side of Equation (7.14) is called the stiffness matrix. If you know the stresses and the material parameters E and ν, you can use Equation (7.13) to calculate the strains; or if you know the strains, E, and ν, you can use Equation (7.14) to calculate the stresses.

7.5.3 Displacements from Strains and Forces from Stresses

The displacements and forces are obtained by integration. For example, the vertical displacement, Δz, is

$$\Delta z = \int \varepsilon_z dz \tag{7.15}$$

and the axial force is

$$P_z = \int \Delta\sigma_z dA \tag{7.16}$$

where dz is the height or thickness of the element and dA is the elemental area.

THE *ESSENTIAL POINTS* ARE:

1. **Hooke's law applies to a linearly elastic material.**
2. **As a first approximation, you can use Hooke's law to calculate stresses, strains, and elastic settlement of soils.**
3. **For nonlinear materials, Hooke's law is used with an approximate elastic modulus (tangent modulus or secant modulus) and the calculations are done for incremental increases in stresses or strains.**

What's next . . . The stresses and strains in three dimensions become complicated when applied to real problems. For practical purposes, many geotechnical problems can be solved using two-dimensional stress and strain parameters. In the next section, we will discuss two conditions that simplify the stress and strain states of soils.

7.6 PLANE STRAIN AND AXIAL SYMMETRIC CONDITIONS

7.6.1 Plane Strain Condition

There are two conditions of stresses and strains that are common in geotechnical engineering. One is the *plane strain* condition in which the strain in one direction is zero. As an example of a plane strain condition, let us consider an element of soil, A, behind a retaining wall (Figure 7.10). Because the displacement that is likely to occur in the Y direction (Δy) is small compared with the length in this direction, the strain tends to zero; that is, $\varepsilon_y = \Delta y/y \cong 0$. We can then assume that soil element A is under a plane strain condition. Since we are considering principal stresses, we will map the X, Y, and Z directions as 3, 2, and 1 directions. In the case of the retaining wall, the Y direction (2 direction) is the zero strain direction, and therefore $\varepsilon_2 = 0$ in Equation (7.13).

Hooke's law for a plane strain condition is

$$\varepsilon_1 = \frac{1+\nu}{E}[(1-\nu)\sigma_1 - \nu\sigma_3] \tag{7.17}$$

$$\varepsilon_3 = \frac{1+\nu}{E}[(1-\nu)\sigma_3 - \nu\sigma_1] \tag{7.18}$$

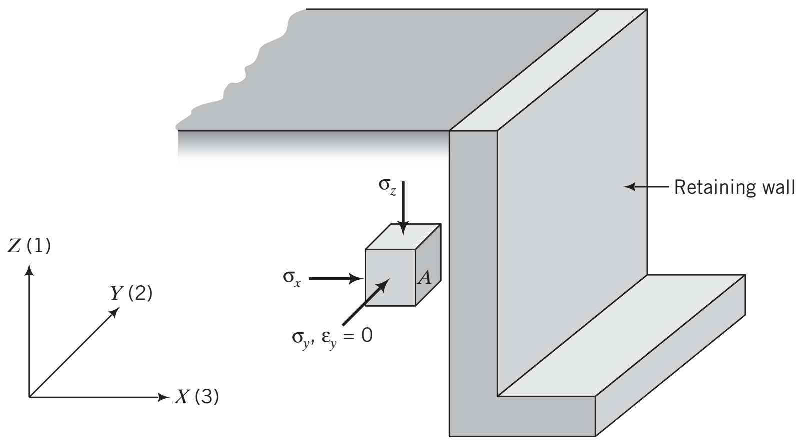


FIGURE 7.10 Plane strain condition in a soil element behind a retaining wall.

and

$$\sigma_2 = \nu(\sigma_1 + \sigma_3) \tag{7.19}$$

In matrix form, Equations (7.17) and (7.18) become

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_3 \end{Bmatrix} = \frac{1+\nu}{E}\begin{bmatrix} 1-\nu & -\nu \\ -\nu & 1-\nu \end{bmatrix}\begin{Bmatrix} \sigma_1 \\ \sigma_3 \end{Bmatrix} \tag{7.20}$$

The inverse of Equation (7.20) gives

$$\begin{Bmatrix} \sigma_1 \\ \sigma_3 \end{Bmatrix} = \frac{E}{(1+\nu)(1-2\nu)}\begin{bmatrix} 1-\nu & \nu \\ \nu & 1-\nu \end{bmatrix}\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_3 \end{Bmatrix} \tag{7.21}$$

7.6.2 Axisymmetric Condition

The other condition that occurs in practical problems is *axial symmetry,* or the *axisymmetric* condition, where two stresses are equal. Let us consider a water tank or an oil tank founded on a soil mass, as illustrated in Figure 7.11.

The radial stresses (σ_r) and circumferential stresses (σ_θ) on a cylindrical element of soil directly under the center of the tank are equal because of axial symmetry. The oil tank will apply a uniform vertical (axial) stress at the soil surface and the soil element will be subjected to an increase in axial stress, $\Delta\sigma_z = \Delta\sigma_1$, and an increase in radial stress, $\Delta\sigma_r = \Delta\sigma_\theta = \Delta\sigma_3$. Will a soil element under the edge of the tank be under an axisymmetric condition? The answer is no, since the stresses at the edge of the tank are all different; there is no symmetry.

Hooke's law for the axisymmetric condition is

$$\varepsilon_1 = \frac{1}{E}[\sigma_1 - 2\nu\sigma_3] \tag{7.22}$$

$$\varepsilon_3 = \frac{1}{E}[(1-\nu)\sigma_3 - \nu\sigma_1] \tag{7.23}$$

or, in matrix form,

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_3 \end{Bmatrix} = \frac{1}{E}\begin{bmatrix} 1 & -2\nu \\ -\nu & 1-\nu \end{bmatrix}\begin{Bmatrix} \sigma_1 \\ \sigma_3 \end{Bmatrix} \tag{7.24}$$

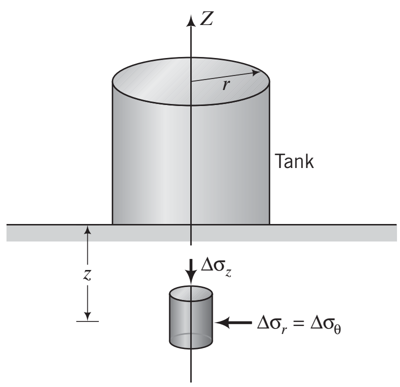


FIGURE 7.11
Axisymmetric condition on a soil element under the center of a tank.

The inverse of Equation (7.24) gives

$$\begin{Bmatrix} \sigma_1 \\ \sigma_3 \end{Bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & 2\nu \\ \nu & 1 \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_3 \end{Bmatrix} \tag{7.25}$$

Plane strain and axisymmetric stress conditions are ideal conditions. In reality, the stress conditions imposed on soils are much more complicated.

THE *ESSENTIAL POINTS* ARE:

1. **A plane strain condition is one in which the strain in one or more directions is zero or small enough to be neglected.**
2. **An axisymmetric condition is one in which two stresses are equal.**

EXAMPLE 7.1 *Application of Hooke's Law for Plane Strain Condition*

A retaining wall moves outward, causing a lateral strain of 0.1% and a vertical strain of 0.05% on a soil element located 3 m below ground level. Assuming the soil is a linear, isotropic, elastic material with $E = 5000$ kPa and $\nu = 0.3$, calculate the increase in stresses imposed. If the retaining wall is 6 m high and the stresses you calculate are the average stresses, determine the lateral force increase per unit length of wall.

Strategy You will have to make a decision whether to use the plane strain or axisymmetric condition and then use the appropriate equation. You are asked to find the increase in stresses, so it is best to write the elastic equations in terms of increment. The retaining wall moves outward, so the lateral strain is tensile (−) while the vertical strain is compressive (+). The increase in lateral force is found by integration of the average lateral stress increase.

Solution 7.1

Step 1: Determine the appropriate stress condition and write the appropriate equation.

The soil element is likely to be under the plane strain condition ($\varepsilon_2 = 0$); use Equation (7.21).

$$\begin{Bmatrix} \Delta\sigma_1 \\ \Delta\sigma_3 \end{Bmatrix} = \frac{5000}{(1+0.3)(1-2\times 0.3)} \begin{bmatrix} 1-0.3 & 0.3 \\ 0.3 & 1-0.3 \end{bmatrix} \begin{Bmatrix} 0.0005 \\ -0.001 \end{Bmatrix}$$

Step 2: Solve the equation.

$$\Delta\sigma_1 = 9615.4\{(0.7 \times 0.0005) + [0.3 \times (-0.001)]\} = 0.5 \text{ kPa}$$

$$\Delta\sigma_3 = 9615.4\{(0.3 \times 0.0005) + [0.7 \times (-0.001)]\} = -5.3 \text{ kPa}$$

The negative sign means reduction.

Step 3: Calculate the lateral force per unit length.

$$\Delta\sigma_3 = \Delta\sigma_x$$

$$\Delta P_x = \int_0^6 \Delta\sigma_x \, dA = -\int_0^6 5.3(dx \times 1) = -[5.3x]_0^6 = -31.8 \text{ kN/m}$$

EXAMPLE 7.2 *Application of Hooke's Law for Axisymmetric Condition*

An oil tank is founded on a layer of medium sand 5 m thick underlain by a deep deposit of dense sand. The geotechnical engineer assumed, based on experience, that the settlement of the tank would occur from settlement in the medium sand. The vertical and lateral stresses at the middle of the medium sand directly under the center

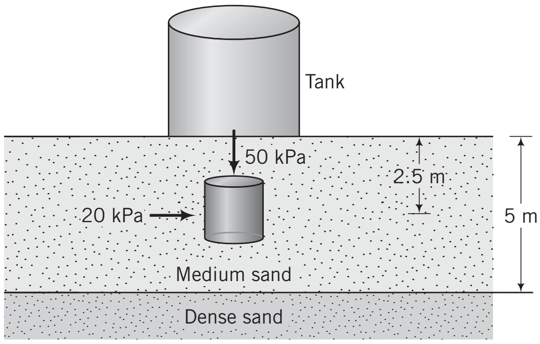


FIGURE E7.2

of the tank are 50 kPa and 20 kPa, respectively. The values of E and v are 20 MPa and 0.3, respectively. Assuming a linear, isotropic, elastic material behavior, calculate the strains imposed in the medium sand and the vertical settlement.

Strategy You have to decide on the stress conditions on the soil element directly under the center of the tank. Once you make your decision, use the appropriate equations to find the strains and then integrate the vertical strains to calculate the settlement. Draw a diagram illustrating the problem.

Solution 7.2

Step 1: Draw a diagram of the problem—see Figure E7.2.

Step 2: Decide on a stress condition.

The element is directly under the center of the tank, so the axisymmetric condition prevails.

Step 3: Choose the appropriate equations and solve.

Use Equation (7.24).

$$\begin{Bmatrix} \Delta\varepsilon_1 \\ \Delta\varepsilon_3 \end{Bmatrix} = \frac{1}{20 \times 10^3} \begin{bmatrix} 1 & -0.6 \\ -0.3 & 0.7 \end{bmatrix} \begin{Bmatrix} 50 \\ 20 \end{Bmatrix}$$

Using algebra, we get

$$\Delta\varepsilon_1 = \frac{1}{20 \times 10^3}[1 \times 50 - 0.6 \times 20] = 1.9 \times 10^{-3}$$

$$\Delta\varepsilon_3 = \frac{1}{20 \times 10^3}[-0.3 \times 50 + 0.7 \times 20] = -5 \times 10^{-5}$$

Step 4: Calculate vertical displacement.

$$\Delta\varepsilon_1 = \Delta\varepsilon_z$$

$$\Delta z = \int_0^5 \Delta\varepsilon_z dz = [1.9 \times 10^{-3} z]_0^5 = 9.5 \times 10^{-3}\text{m} = 9.5 \text{ mm}$$

What's next . . . We have used the elastic equations to calculate stresses, strains, and displacements in soils assuming that soils are linear, isotropic, elastic materials. Soils, in general, are not linear, isotropic, elastic materials. We will briefly discuss anisotropic, elastic materials in the next section.

7.7 ANISOTROPIC, ELASTIC STATES

Anisotropic materials have different elastic parameters in different directions. Anisotropy in soils results from essentially two causes.

1. The manner in which the soil is deposited. This is called structural anisotropy and it is the result of the kind of soil fabric that is formed during deposition. You should recall (Chapter 2) that the soil fabric produced is related to the history of the environment in which the soil is formed. A special form of structural anisotropy occurs when the horizontal plane is a plane of isotropy. We call this form of structural anisotropy transverse anisotropy.
2. The difference in stresses in the different directions. This is known as stress-induced anisotropy.

Transverse anisotropy, also called cross anisotropy, is the most prevalent type of anisotropy in soils. If we were to load the soil in the vertical direction (Z direction) and repeat the same loading in the horizontal direction, say, the X direction, the soil would respond differently; its stress–strain characteristics and strength would be different in these directions. However, if we were to load the soil in the Y direction, the soil's response would be similar to the response obtained in the X direction. The implication is that a soil mass will, in general, respond differently depending on the direction of the load. For transverse anisotropy, the elastic parameters are the same in the lateral directions (X and Y directions) but are different from the vertical direction.

To fully describe anisotropic soil behavior we need 21 elastic constants (Love, 1927), but for transverse anisotropy we need only five elastic constants; these are E_z, E_x, ν_{xx}, ν_{zx}, and ν_{zz}. The first letter in the double subscripts denotes the direction of loading and the second letter denotes the direction of measurement. For example, ν_{zx} means Poisson's ratio determined from the ratio of the strain in the lateral direction (X direction) to the strain in the vertical direction (Z direction) with the load applied in the vertical direction (Z direction).

In the laboratory, the direction of loading of soil samples taken from the field is invariably vertical. Consequently, we cannot determine the five desired elastic parameters from conventional laboratory tests. Graham and Houlsby (1983) suggested a method to overcome the lack of knowledge of the five desired elastic parameters in solving problems on transverse anisotropy. However, their method is beyond the scope of this book.

For axisymmetric conditions, the transverse anisotropic, elastic equations are

$$\begin{Bmatrix} \Delta\varepsilon_z \\ \Delta\varepsilon_r \end{Bmatrix} = \begin{bmatrix} \dfrac{1}{E_z} & \dfrac{-2\nu_{rz}}{E_r} \\ \dfrac{-\nu_{zr}}{E_z} & \dfrac{(1-\nu_{rr})}{E_r} \end{bmatrix} \begin{Bmatrix} \Delta\sigma_z \\ \Delta\sigma_r \end{Bmatrix} \tag{7.26}$$

where the subscript z denotes vertical and r denotes radial. By superposition, $\nu_{rz}/\nu_{zr} = E_r/E_z$.

THE *ESSENTIAL POINTS* ARE:

1. **Two forms of anisotropy are present in soils. One is structural anisotropy, which is related to the history of loading and environmental conditions during deposition, and the other is stress-induced anisotropy, which results from differences in stresses in different directions.**
2. **The prevalent form of structural anisotropy in soils is transverse anisotropy; the soil properties and the soil response in the lateral directions are the same but are different from those in the vertical direction.**
3. **You need to find the elastic parameters in different directions of a soil mass to determine elastic stresses, strains, and displacements.**

EXAMPLE 7.3 *Application of Hooke's Law for Transverse Anisotropic Soils*

Redo Example 7.2, but now the soil under the oil tank is an anisotropic, elastic material with $E_z = 20$ MPa, $E_r = 25$ MPa, $\nu_{rz} = 0.15$, and $\nu_{rr} = 0.3$.

Strategy The solution of this problem is a straightforward application of Equation (7.26).

Solution 7.3

Step 1: Determine ν_{zr} (by superposition).

$$\frac{\nu_{rz}}{\nu_{zr}} = \frac{E_r}{E_z}$$

$$\nu_{zr} = \frac{20}{25} \times 0.15 = 0.12$$

Step 2: Find the strains.

Use Equation (7.26).

$$\begin{Bmatrix} \Delta\varepsilon_z \\ \Delta\varepsilon_r \end{Bmatrix} = 10^{-3} \begin{bmatrix} \dfrac{1}{20} & \dfrac{-2 \times 0.15}{25} \\ \dfrac{-0.12}{20} & \dfrac{(1 - 0.3)}{25} \end{bmatrix} \begin{Bmatrix} 50 \\ 20 \end{Bmatrix}$$

The solution is $\varepsilon_z = 2.26 \times 10^{-3} = 0.23\%$ and $\varepsilon_r = 0.26 \times 10^{-3} = 0.03\%$.

Step 3: Determine vertical displacement.

$$\Delta z = \int_0^5 \varepsilon_z dz = [2.26 \times 10^{-3} z]_0^5 = 11.3 \times 10^{-3}\,\text{m} = 11.3\,\text{mm}$$

The vertical displacement in the anisotropic case is about 19% more than in the isotropic case (Example 7.2). Also, the radial strain is tensile for the isotropic case but compressive in the anisotropic case for this problem.

What's next . . . We now know how to calculate stresses and strains in soils if we assume soils are elastic, homogeneous materials. One of the important tasks for engineering works is to determine strength or failure of materials. We can draw an analogy of the strength of materials with the strength of a chain. The chain is only as strong as its weakest link. For soils, failure may be initiated at a point within a soil mass and then propagate through it; this is known as progressive failure. The stress state at a point in a soil mass due to applied boundary forces may be equal to the strength of the soil, thereby initiating failure. Therefore, as engineers, we need to know the stress state at a point due to applied loads. We can use Equation (7.10) to find stress states, but geoengineers have been using a two-dimensional stress system based on Mohr's circle. We will discuss stress and strain states next using your knowledge of Mohr's circle in strength of materials.

7.8 STRESS AND STRAIN STATES

Access www.wiley.com/college/budhu, and click Chapter 7 and then Mohrcircle.zip to download an application to plot, interpret, and explore a variety of stress states interactively.

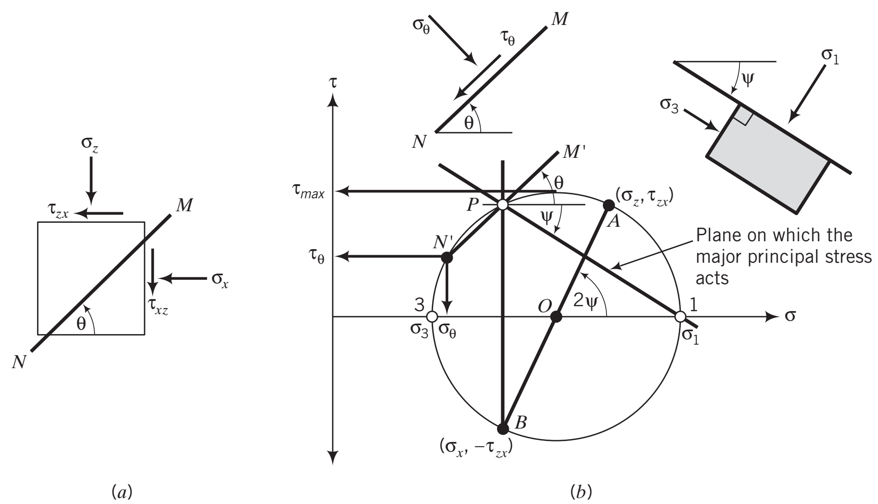


FIGURE 7.12 Stresses on a two-dimensional element and Mohr's circle.

7.8.1 Mohr's Circle for Stress States

Suppose a cuboidal sample of soil is subjected to the stresses shown in Figure 7.9. We would like to know what the stresses are at a point, say, A, within the sample due to the applied stresses. One approach to find the stresses at A, called the stress state at A, is to use Mohr's circle. The stress state at a point is the set of stress vectors corresponding to all planes passing through that point. For simplicity, we will consider a two-dimensional element with stresses, as shown in Figure 7.12a. Let us draw Mohr's circle. First, we have to choose a sign convention. We have already decided that compressive stresses are positive for soils. We will assume counterclockwise shear is positive and $\sigma_z > \sigma_x$. The two coordinates of the circle are (σ_z, τ_{zx}) and (σ_x, τ_{xz}). Recall from your strength of materials course that, for equilibrium, $\tau_{xz} = -\tau_{zx}$; these are called complementary shear stresses and are orthogonal to each other. Plot these two coordinates on a graph of shear stress (ordinate) and normal stress (abscissa), as shown by A and B in Figure 7.12b. Draw a circle with AB as the diameter. The circle crosses the normal stress axis at 1 and 3. The stresses at these points are the major principal stress, σ_1, and the minor principal stress, σ_3.

The principal stresses are related to the stress components $\sigma_z, \sigma_x, \tau_{zx}$ by

$$\sigma_1 = \frac{\sigma_z + \sigma_x}{2} + \sqrt{\left(\frac{\sigma_z - \sigma_x}{2}\right)^2 + \tau_{zx}^2} \tag{7.27}$$

$$\sigma_3 = \frac{\sigma_z + \sigma_x}{2} - \sqrt{\left(\frac{\sigma_z - \sigma_x}{2}\right)^2 + \tau_{zx}^2} \tag{7.28}$$

The angle between the major principal stress plane and the horizontal plane (ψ) is

$$\tan \psi = \frac{\tau_{zx}}{\sigma_1 - \sigma_x} \tag{7.29}$$

The stresses on a plane oriented at an angle θ from the major principal stress plane are

$$\sigma_\theta = \frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2} \cos 2\theta \tag{7.30}$$

$$\tau_\theta = \frac{\sigma_1 - \sigma_3}{2} \sin 2\theta \tag{7.31}$$

The stresses on a plane oriented at an angle θ from the horizontal plane are

$$\sigma_\theta = \frac{\sigma_z + \sigma_x}{2} + \frac{\sigma_z - \sigma_x}{2}\cos 2\theta + \tau_{zx}\sin 2\theta \tag{7.32}$$

$$\tau_\theta = \tau_{zx}\cos 2\theta - \frac{\sigma_z - \sigma_x}{2}\sin 2\theta \tag{7.33}$$

In the above equations, θ is positive for counterclockwise orientation.

The maximum (principal) shear stress is at the top of the circle with magnitude

$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} \tag{7.34}$$

For the stresses shown in Figure 7.9 we would get three circles, but we have simplified the problem by plotting one circle for stresses on all planes perpendicular to one principal direction.

The stress σ_z acts on the horizontal plane and the stress σ_x acts on the vertical plane for our case. If we draw these planes in Mohr's circle, they intersect at a point, *P*. Point *P* is called the pole of the stress circle. It is a special point because any line passing through the pole will intersect Mohr's circle at a point that represents the stresses on a plane parallel to the line. Let us see how this works. Suppose we want to find the stresses on a plane inclined at an angle θ from the horizontal plane, as depicted by *MN* in Figure 7.12a. Once we locate the pole, *P*, we can draw a line parallel to *MN* through *P* as shown by *M′N′* in Figure 7.12b. The line *M′N′* intersects the circle at *N′* and the coordinates of *N′*, $(\sigma_\theta, \tau_\theta)$, represent the normal and shear stresses on *MN*.

7.8.2 Mohr's Circle for Strain States

So far, we have studied stress states. The strain state is found in a similar manner to the stress state. With reference to Figure 7.13, the principal strains are

$$\text{Major principal strain:}\quad \varepsilon_1 = \frac{\varepsilon_z + \varepsilon_x}{2} + \sqrt{\left(\frac{\varepsilon_z - \varepsilon_x}{2}\right)^2 + \left(\frac{\gamma_{zx}}{2}\right)^2} \tag{7.35}$$

$$\text{Major principal strain:}\quad \varepsilon_3 = \frac{\varepsilon_z + \varepsilon_x}{2} - \sqrt{\left(\frac{\varepsilon_z - \varepsilon_x}{2}\right)^2 + \left(\frac{\gamma_{zx}}{2}\right)^2} \tag{7.36}$$

where γ_{zx} is called the engineering shear strain or simple shear strain.

The maximum simple shear strain is

$$\gamma_{max} = \varepsilon_1 - \varepsilon_3 \tag{7.37}$$

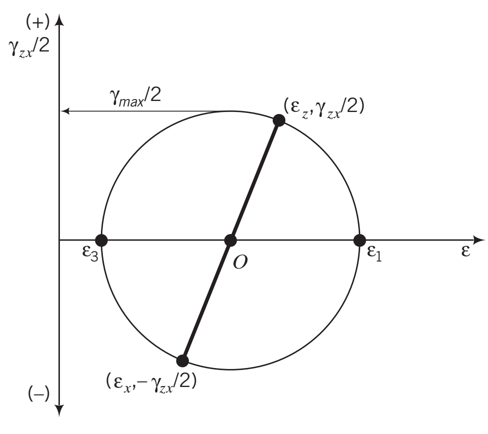


FIGURE 7.13
Mohr's circle of strain.

In soils, strains can be compressive or tensile. There is no absolute reference strain. For stresses, we can select atmospheric pressure as the reference, but not so for strains. Usually, we deal with changes or increments of strains resulting from stress changes.

THE *ESSENTIAL POINTS* ARE:

1. **Mohr's circle is used to find the stress state or strain state from a two-dimensional set of stresses or strains on a soil.**
2. **The pole on a Mohr's circle identifies a point through which any plane passing through it will intersect the Mohr's circle at a point that represents the stresses on that plane.**

EXAMPLE 7.4 *Mohr's Circle for Stress State*

A sample of soil (0.1 m × 0.1 m) is subjected to the forces shown in Figure E7.4a. Determine (a) σ_1, σ_3, and Ψ; (b) the maximum shear stress; and (c) the stresses on a plane oriented at 30° counterclockwise from the major principal stress plane.

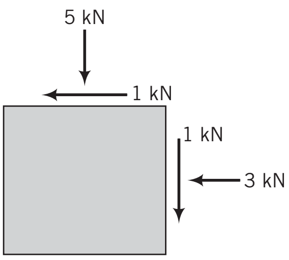


FIGURE E7.4a

Strategy There are two approaches to solve this problem. You can use either Mohr's circle or the appropriate equations. Both approaches will be used here.

Solution 7.4

Step 1: Find the area.

$$\text{Area:} \quad A = 0.1 \times 0.1 = 10^{-2}\ \text{m}^2$$

Step 2: Calculate the stresses.

$$\sigma_z = \frac{\text{Force}}{\text{Area}} = \frac{5}{10^{-2}} = 500\ \text{kPa}$$

$$\sigma_x = \frac{3}{10^{-2}} = 300\ \text{kPa}$$

$$\tau_{zx} = \frac{1}{10^{-2}} = 100\ \text{kPa}; \quad \tau_{xz} = -\tau_{zx} = -100\ \text{kPa}$$

Step 3: Draw Mohr's circle and extract σ_1, σ_3, and τ_{max}.

Mohr's circle is shown in Figure E7.4b.

$$\sigma_1 = 540\ \text{kPa}, \quad \sigma_3 = 260\ \text{kPa}, \quad \tau_{max} = 140\ \text{kPa}$$

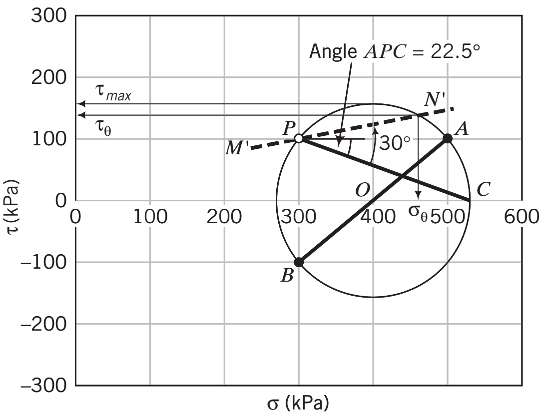


FIGURE E7.4b

Step 4: Draw the pole on Mohr's circle. The pole of Mohr's circle is shown by point P in Figure E7.4b.

Step 5: Determine ψ.

Draw a line from P to σ_1 and measure the angle between the horizontal plane and this line.

$$\psi = 22.5°$$

Alternatively, the angle $AOC = 2\psi = 45°$.

$$\therefore \psi = 22.5°$$

Step 6: Determine the stresses on a plane inclined at 30° from the major principal stress plane.

Draw a line M^1N^1 through P with an inclination of 30° from the major principal stress plane, angle CPN'. The coordinate at point N' is (470, 120).

Alternatively:

Step 1: Use Equations (7.27) to (7.29) and (7.34) to find σ_1, σ_3, ψ, and τ_{max}.

$$\sigma_1 = \frac{500 + 300}{2} + \sqrt{\left(\frac{500 - 300}{2}\right)^2 + 100^2} = 541.4 \text{ kPa}$$

$$\sigma_3 = \frac{500 + 300}{2} - \sqrt{\left(\frac{500 - 300}{2}\right)^2 + 100^2} = 258.6 \text{ kPa}$$

$$\tan \psi = \frac{\tau_{yx}}{\sigma_1 - \sigma_x} = \frac{100}{541.4 - 300} = 0.414$$

$$\therefore \psi = 22.5°$$

$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} = \frac{541.4 - 258.6}{2} = 141.4 \text{ kPa}$$

Check Equilibrium

$$\text{Length of } 2 - 3 = 0.1 \text{ m}$$

$$\text{Length of } 3 - 1 = 0.1 \times (\tan 22.5°) = 0.0414 \text{ m}$$

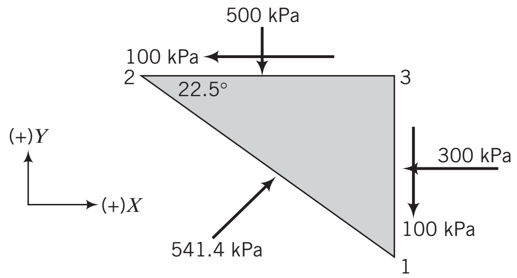


FIGURE E7.4c

Length of $1 - 2 = 0.1/(\cos 22.5°) = 0.1082$ m

$$\Sigma F_x = 0: \; -300 \times 0.0414 - 100 \times 0.1 + 541.4 \times 0.1082 \times \cos(22.5°) = 0$$

$$\Sigma F_y = 0: \; -100 \times 0.0414 - 500 \times 0.1 + 541.4 \times 0.1082 \times \sin(22.5°) = 0$$

Step 2: Use Equations (7.30) and (7.31) to find σ_θ and τ_θ.

$$\sigma_\theta = \frac{541.4 + 258.6}{2} + \frac{541.4 - 258.6}{2}\cos(2 \times 30) = 470.7 \text{ kPa}$$

$$\tau_\theta = \frac{541.4 - 258.6}{2}\sin(2 \times 30) = 122.5 \text{ kPa}$$

What's next . . . The stresses we have calculated are for soils as solid elastic materials. We have not accounted for the pressure within the soil pore spaces. In the next section, we will discuss the principle of effective stresses that accounts for the pressures within the soil pores. This principle is the most important principle in soil mechanics.

7.9 TOTAL AND EFFECTIVE STRESSES

7.9.1 The Principle of Effective Stress

The deformations of soils are similar to the deformations of structural framework such as a truss. The truss deforms from changes in loads carried by each member. If the truss is loaded in air or submerged in water, the deformations under a given load will remain unchanged. Deformations of the truss are independent of hydrostatic pressure. The same is true for soils.

Let us consider an element of a *saturated soil* subjected to a normal stress, σ, applied on the horizontal boundary, as shown in Figure 7.14. The stress s is called the *total stress,* and for equilibrium (Newton's third law) the stresses in the soil must be equal and opposite to σ. The resistance or reaction to σ is provided by a combination of the stresses from the solids, called *effective stress* (σ'), and from water in the pores, called *porewater pressure* (u). We will denote effective stresses by a prime ($'$) following the symbol for normal stress, usually σ. The equilibrium equation is

$$\sigma = \sigma' + u \tag{7.38}$$

so that

$$\sigma' = \sigma - u \tag{7.39}$$

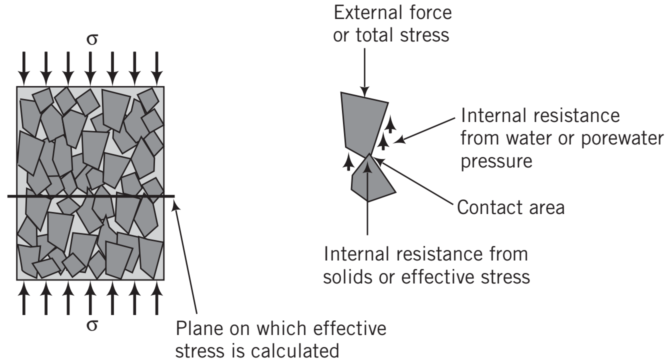


FIGURE 7.14
Effective stress.

Equation (7.39) is called the *principle of effective stress* and was first recognized by Terzaghi (1883–1963) in the mid-1920s during his research into soil consolidation (Chapter 9). ***The principle of effective stress is the most important principle in soil mechanics. Deformations of soils are a function of effective stresses, not total stresses. The principle of effective stresses applies only to normal stresses and not to shear stresses***. The porewater cannot sustain shear stresses, and therefore the soil solids must resist the shear forces. Thus $\tau = \tau'$, where τ is the total shear stress and τ' is the effective shear stress. The effective stress is not the contact stress between the soil solids. Rather, it is the average stress on a plane through the soil mass.

Soils cannot sustain tension. Consequently, the effective stress cannot be less than zero. Porewater pressures can be positive or negative. The latter are sometimes called suction or suction pressure.

For unsaturated soils, the effective stress (Bishop et al., 1960) is

$$\sigma' = \sigma - u_a + \chi(u_a - u) \tag{7.40}$$

where u_a is the pore air pressure, u is the porewater pressure, and χ is a factor depending on the degree of saturation. For dry soil, $\chi = 0$; for saturated soil, $\chi = 1$. Values of χ for a silt are shown in Figure 7.15.

7.9.2 Effective Stresses Due to Geostatic Stress Fields

The effective stress in a soil mass not subjected to external loads is found from the unit weight of the soil and the depth of groundwater. Consider a soil element at a depth z below the ground surface, with the groundwater level (GWL) at ground surface (Figure 7.16a). The total vertical stress is

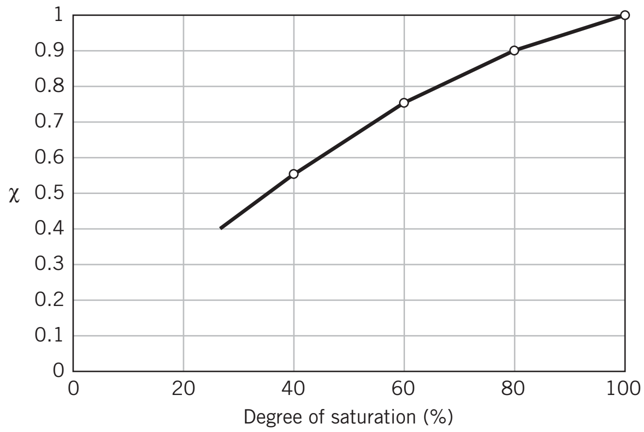


FIGURE 7.15
Values of χ for a silt at different degrees of saturation.

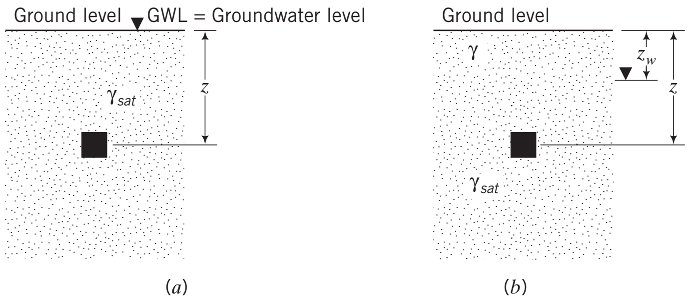


FIGURE 7.16 Soil element at a depth *z* with groundwater level (a) at ground level and (b) below ground level.

$$\sigma = \gamma_{sat} z \tag{7.41}$$

The porewater pressure is

$$u = \gamma_w z \tag{7.42}$$

and the effective stress is

$$\sigma' = \sigma - u = \gamma_{sat} z - \gamma_w z = (\gamma_{sat} - \gamma_w) z = \gamma' z \tag{7.43}$$

If the GWL is at a depth z_w below ground level (Fig. 7.16b), then

$$\sigma = \gamma z_w + \gamma_{sat}(z - z_w) \quad \text{and} \quad u = \gamma_w (z - z_w)$$

The effective stress is

$$\sigma' = \sigma - u = \gamma z_w + \gamma_{sat}(z - z_w) - \gamma_w(z - z_w)$$

$$= \gamma z_w + (\gamma_{sat} - \gamma_w)(z - z_w) = \gamma z_w + \gamma'(z - z_w)$$

7.9.3 Effects of Capillarity

In silts and fine sands, the soil above the groundwater can be saturated by capillary action. You would have encountered capillary action in your physics course when you studied menisci. We can get an understanding of capillarity in soils by idealizing the continuous void spaces as capillary tubes. Consider a single idealized tube, as shown in Figure 7.17. The height at which water will rise in the tube can be found from statics. Summing forces vertically (upward forces are negative), we get

$$\Sigma F_z = \text{weight of water} - \text{the tension forces from capillary action}$$

that is,

$$\frac{\pi d^2}{4} z_c \gamma_w - \pi d T \cos\theta = 0 \tag{7.44}$$

Solving for z_c, we get

$$z_c = \frac{4T\cos\theta}{d\gamma_w} \tag{7.45}$$

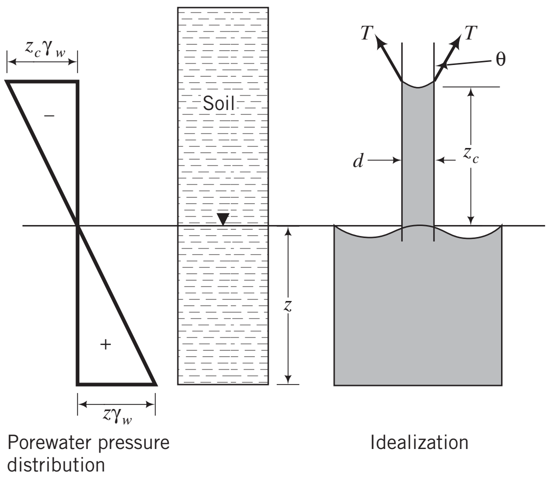


FIGURE 7.17 Capillary simulation in soils.

where T is the surface tension (force per unit length), θ is the contact angle, z_c is the height of capillary rise, and d is the diameter of the tube representing the diameter of the void space. The surface tension of water is 0.073 N/m and the contact angle of water with a clean glass surface is 0. Since T, θ, and γ_w are constants,

$$z_c \propto \frac{1}{d} \tag{7.46}$$

For soils, d is assumed to be equivalent to 0.1 D_{10} where D_{10} is the effective size. The interpretation of Equation (7.46) is that the smaller the soil pores, the higher the capillary zone. The capillary zone in fine sands will be larger than for medium or coarse sands.

The porewater pressure due to capillarity is negative (suction), as shown in Figure 7.17, and is a function of the size of the soil pores and the water content. At the groundwater level, the porewater pressure is zero and decreases (becomes negative) as you move up the capillary zone. The effective stress increases because the porewater pressure is negative. For example, for the capillary zone, z_c, the porewater pressure at the top is $-z_c\gamma_w$ and the effective stress is $\sigma' = \sigma - (-z_c\gamma_w) = \sigma + z_c\gamma_w$.

The approach we have taken to interpret capillary action in soils is simple, but it is sufficient for most geotechnical applications. For a comprehensive treatment of capillary action, you can refer to Adamson (1982).

7.9.4 Effects of Seepage

In Chapter 6, we discussed one-dimensional flow of water through soils. As water flows through soil it exerts a frictional drag on the soil particles, resulting in head losses. The frictional drag is called seepage force in soil mechanics. It is often convenient to define seepage as the seepage force per unit volume (it has units similar to unit weight), which we will denote by j_s. If the head loss over a flow distance, L, is Δh, the seepage force is

$$j_s = \frac{\Delta h \gamma_w}{L} = i\gamma_w \tag{7.47}$$

If seepage occurs downward (Figure 7.18a), then the seepage stresses are in the same direction as the gravitational effective stresses. From static equilibrium, the resultant vertical effective stress is

$$\sigma'_z = \gamma' z + iz\gamma_w = \gamma' z + j_s z \tag{7.48}$$

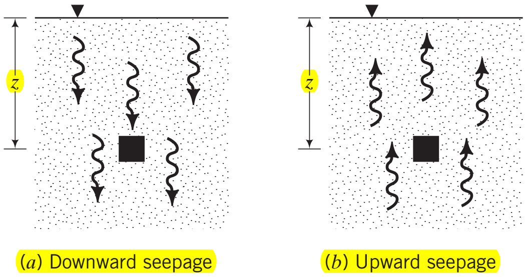


FIGURE 7.18
Seepage in soils.

If seepage occurs upward (Figure 7.18b), then the seepage stresses are in the opposite direction to the gravitational effective stresses. From static equilibrium, the resultant vertical effective stress is

$$\sigma'_z = \gamma' z - iz\gamma_w = \gamma' z - j_s z \qquad (7.49)$$

Seepage forces play a very important role in destabilizing geotechnical structures. For example, a cantilever retaining wall, shown in Figure 7.19, depends on the depth of embedment for its stability. The retained soil (left side of wall) applies an outward lateral pressure to the wall, which is resisted by an inward lateral resistance from the soil on the right side of the wall. If a steady quantity of water is available on the left side of the wall, for example, from a broken water pipe, then water will flow from the left side to the right side of the wall. The path followed by a particle of water is depicted by *AB* in Figure 7.19, and as water flows from *A* to *B*, head loss occurs. The seepage stresses on the left side of the wall are in the direction of the gravitational stresses. The effective stress increases and, consequently, an additional outward lateral force is applied on the left side of the wall. On the right side of the wall, the seepage stresses are upward and the effective stress decreases. The lateral resistance provided by the embedment is reduced. Seepage stresses in this problem play a double role (increase the lateral disturbing force and reduce the lateral resistance) in reducing the stability of a geotechnical structure. In Chapters 14 through 15, you will study the effects of seepage on the stability of several types of geotechnical structures.

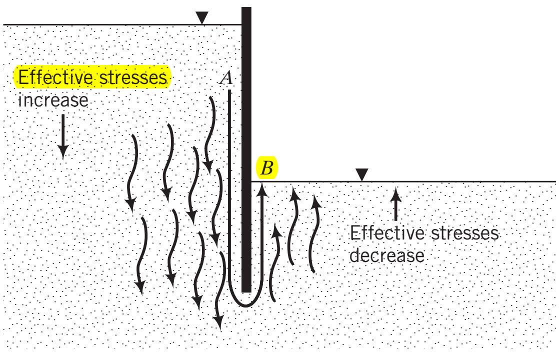


FIGURE 7.19
Effects of seepage on the effective stresses near a retaining wall.

THE *ESSENTIAL POINTS* ARE:

1. **The effective stress represents the average stress carried by the soil solids and is the difference between the total stress and the porewater pressure.**
2. **The effective stress principle applies only to normal stresses and not to shear stresses.**
3. **Deformations of soils are due to effective, not total, stress.**
4. **Soils, especially silts and fine sands, can be affected by capillary action.**
5. **Capillary action results in negative porewater pressures and increases the effective stresses.**
6. **Downward seepage increases the resultant effective stress; upward seepage decreases the resultant effective stress.**

EXAMPLE 7.5 *Calculating Vertical Effective Stress*

Calculate the effective stress for a soil element at depth 5 m in a uniform deposit of soil, as shown in Figure E7.5. Assume that the pore air pressure is zero.

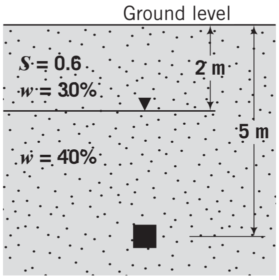


FIGURE E7.5

Strategy You need to get unit weights from the given data, and you should note that the soil above the groundwater level is not saturated.

Solution 7.5

Step 1: Calculate unit weights.

Above groundwater level

$$\gamma = \left(\frac{G_s + Se}{1 + e}\right)\gamma_w = \frac{G_s(1 + w)}{1 + e}\gamma_w$$

$$Se = wG_s, \quad \therefore e = \frac{0.3 \times 2.7}{0.6} = 1.35$$

$$\gamma = \frac{2.7(1 + 0.3)}{1 + 1.35} \times 9.8 = 14.6 \text{ kN/m}^3$$

Below groundwater level

Soil is saturated, $S = 1$.

$$e = wG_s = 0.4 \times 2.7 = 1.08$$

$$\gamma_{sat} = \left(\frac{G_s + e}{1 + e}\right)\gamma_w = \left(\frac{2.7 + 1.08}{1 + 1.08}\right)9.8 = 17.8 \text{ kN/m}^3$$

Step 2: Calculate the effective stress.

Total stress: $\sigma_z = 2\gamma + 3\gamma_{sat} = 2 \times 14.6 + 3 \times 17.8 = 82.6$ kPa

Porewater pressure: $u = 3\gamma_w = 3 \times 9.8 = 29.4$ kPa

Effective stress: $\sigma'_z = \sigma_z - u = 82.6 - 29.4 = 53.2$ kPa

Alternatively:

$$\sigma'_z = 2\gamma + 3(\gamma_{sat} - \gamma_w) = 2\gamma + 3\gamma' = 2 \times 14.6 + 3(17.8 - 9.8) = 53.2 \text{ kPa}$$

EXAMPLE 7.6 *Calculating and Plotting Vertical Effective Stress Distribution*

A borehole at a site reveals the soil profile shown in Figure E7.6a. Plot the distribution of vertical total and effective stresses with depth. Assume pore air pressure is zero.

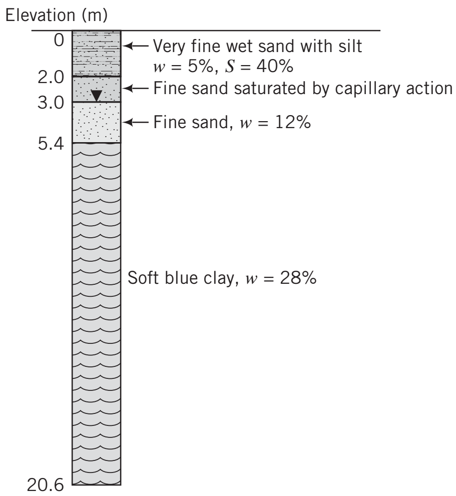


FIGURE E7.6a

Strategy From the data given, you will have to find the unit weight of each soil layer to calculate the stresses. You are given that the 1.0 m of fine sand above the groundwater level is saturated by capillary action. Therefore, the porewater pressure in this 1.0 m zone is negative.

Solution 7.6

Step 1: Calculate the unit weights.

0–2 m

$$S = 40\% = 0.4; \quad w = 0.05$$

$$e = \frac{wG_s}{S} = \frac{0.05 \times 2.7}{0.4} = 0.34$$

$$\gamma = \frac{G_s(1 + w)}{1 + e}\gamma_w = \frac{2.7(1 + 0.05)}{1 + 0.34}9.8 = 20.7 \text{ kN/m}^3$$

2–5.4 m

$$S = 1; \quad w = 0.12$$

$$e = wG_s = 0.12 \times 2.7 = 0.32$$

$$\gamma_{sat} = \left(\frac{G_s + e}{1 + e}\right)\gamma_w = \left(\frac{2.7 + 0.32}{1 + 0.32}\right)9.8 = 22.4 \text{ kN/m}^3$$

5.4–20.6 m

$$S = 1; \quad w = 0.28$$

$$e = wG_s = 0.28 \times 2.7 = 0.76$$

$$\gamma_{sat} = \left(\frac{2.7 + 0.76}{1 + 0.76}\right)9.8 = 19.3 \text{ kN/m}^3$$

Step 2: Calculate the stresses using a table or a spreadsheet program.

Depth (m)	Thickness (m)	σ_z (kPa)	u (kPa)	$\sigma'_z = \sigma - u$ (kPa)
0	0	0	0	0
2	2	$20.7 \times 2 = 41.4$	$-1 \times 9.8 = -9.8$	51.2
3	1	$41.4 + 22.4 \times 1 = 63.8$	0	63.8
5.4	2.4	$63.8 + 22.4 \times 2.4 = 117.6$	$2.4 \times 9.8 = 23.5$	94.1
20.6	15.2	$117.6 + 19.3 \times 15.2 = 411$	$23.5 + 15.2 \times 9.8 = 172.5$ or $17.6 \times 9.8 = 172.5$	238.5

Step 3: Plot the stresses versus depth—see Figure E7.6b.

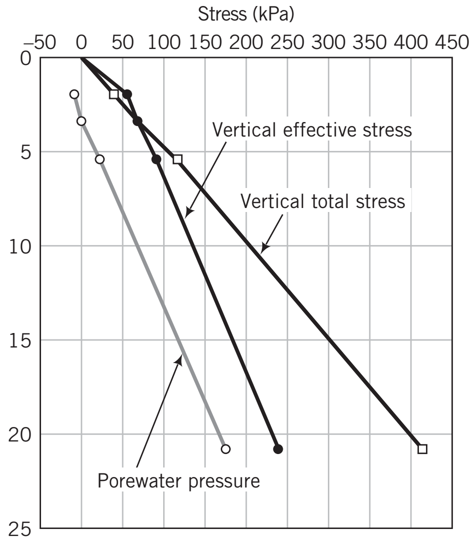


FIGURE E7.6b

EXAMPLE 7.7 *Effects of Seepage on Effective Stress*

Water is seeping downward through a soil layer, as shown in Figure E7.7. Two piezometers (A and B) located 2 m apart (vertically) showed a head loss of 0.2 m. Calculate the resultant vertical effective stress for a soil element at a depth of 6 m.

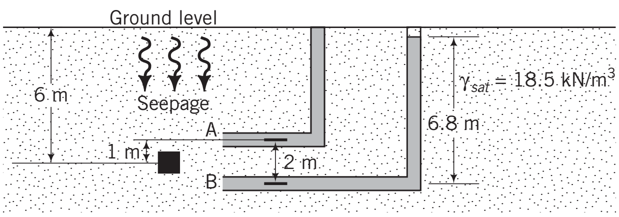


FIGURE E7.7

Strategy You have to calculate the seepage stress. But to obtain this you must know the hydraulic gradient, which you can find from the data given.

Solution 7.7

Step 1: Find the hydraulic gradient.

$$\Delta H = 0.2 \text{ m}; \quad L = 2 \text{ m}; \quad i = \frac{\Delta H}{L} = \frac{0.2}{2} = 0.1$$

Step 2: Determine the effective stress.

Assume the hydraulic gradient is the average for the soil mass; then

$$\sigma'_z = (\gamma_{sat} - \gamma_w)z + i\gamma_w z = (18.5 - 9.8)6 + 0.1 \times 9.8 \times 6 = 58.1 \text{ kPa}$$

EXAMPLE 7.8 *Effects of Groundwater Condition on Effective Stress*

(a) Plot the total and effective stresses and porewater pressure with depth for the soil profile shown in Figure E7.8a for steady-state seepage condition. A porewater pressure transducer installed at the top of the sand layer gives a pressure of 58.8 kPa. Assume $G_s = 2.7$ and neglect pore air pressure.

(b) If a borehole were to penetrate the sand layer, how far would the water rise above the groundwater level?

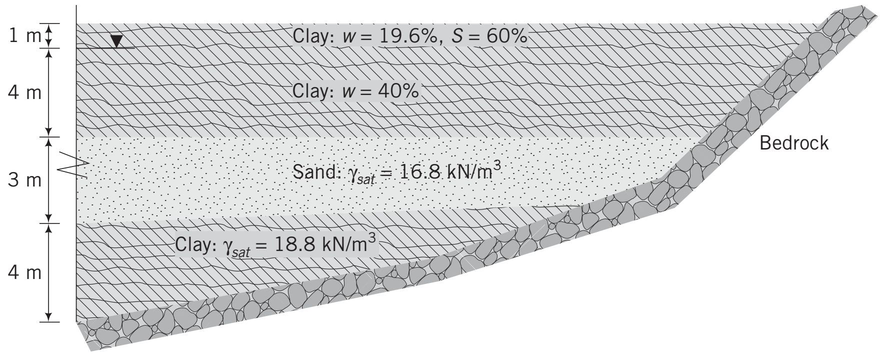


FIGURE E7.8a

Strategy You have to calculate the unit weight of the top layer of clay. From the soil profile, the groundwater appears to be under artesian condition, so the effective stress would change sharply at the interface of the top clay layer and the sand. It is best to separate the soil above the groundwater from the soil below the groundwater. So, divide the soil profile into artificial layers.

Solution 7.8

Step 1: Divide the soil profile into artificial layers.

See Figure E7.8b.

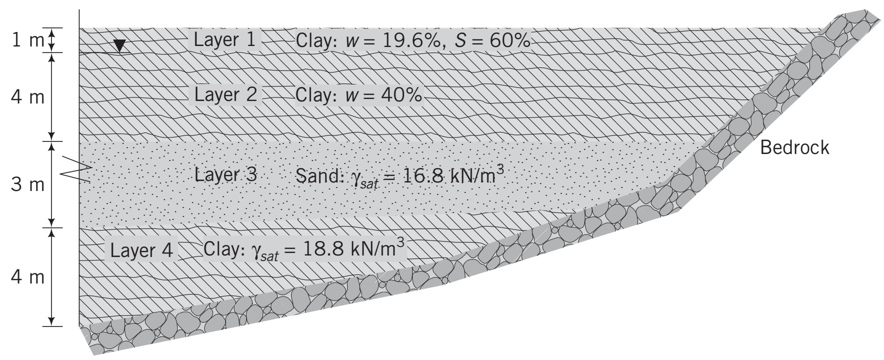


FIGURE E7.8b

Step 2: Find the unit weight of the top clay layers.

Above groundwater level: $$\gamma = \frac{G_s + Se}{1 + e}\gamma_w = \frac{G_s(1 + w)}{1 + \dfrac{wG_s}{S}}\gamma_w = \frac{2.7(1 + 0.196)}{1 + \dfrac{0.196 \times 2.7}{0.6}} \times 9.8 = 16.8 \text{ kN/m}^3$$

Below groundwater level: $$\gamma_{sat} = \frac{G_s + e}{1 + e}\gamma_w = \frac{G_s(1 + w)}{1 + wG_s}\gamma_w = \frac{2.7(1 + 0.12)}{1 + 0.12 \times 2.7} \times 9.8 = 17.8 \text{ kN/m}^3$$

Step 3: Determine the effective stress.

See spreadsheet. Note: The porewater pressure at the top of the sand is 58.8 kPa.

Layer	Depth (m)	Thickness (m)	γ (kN/m^3)	σ_z (kPa)	u (kPa)	σ'_z (kPa)
1 - top	0	1	16.8	0	0	0
1 - bottom	1			16.8	0.0	16.8
2 - top	1	4	17.8	16.8	0.0	16.8
2 - bottom	5			88.0	39.2	48.8
3 - top	5	3	16.8	88.0	58.8	29.2
3 - bottom	8			138.4	88.2	50.2
4 - top	8	4	18.8	138.4	88.2	50.2
4 - bottom	12			213.6	127.4	86.2

Step 4: Plot vertical stress and porewater pressure distributions with depth.

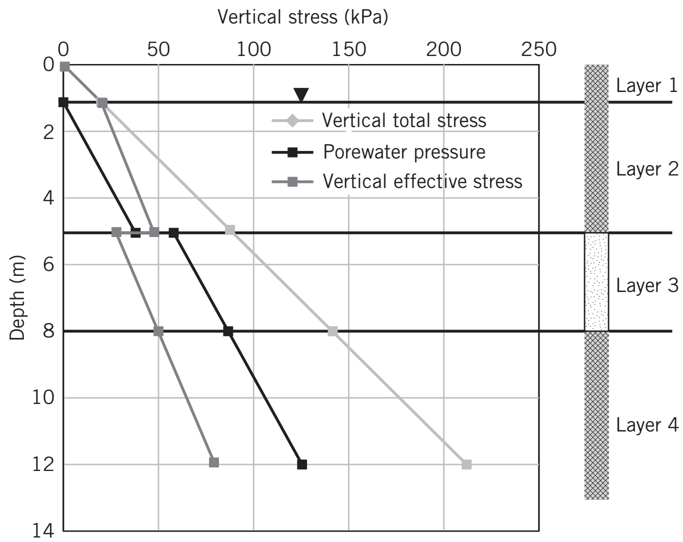


FIGURE E7.8c

Note:

(1) The vertical effective stress changes abruptly at the top of the sand layer due to the artesian condition.

(2) For each layer or change in condition (groundwater or unit weight), the vertical stress at the bottom of the preceding layer acts a surcharge, transmitting a uniform vertical stress of equal magnitude to all subsequent layers. As an example, the vertical total stress at the bottom of layer 2 is 88 kPa. This stress is transferred to both layers 3 and 4. Thus, the vertical total stress at the bottom of layer 3 from its own weight is $3 \times 16.8 = 50.4$ kPa, and adding the vertical total stress from the layers above gives $88 + 50.4 = 138.4$ kPa.

Step 5: Calculate the height of water.

$$h = \frac{58.8}{9.8} = 6 \text{ m}$$

Height above existing groundwater level $= 6 - 4 = 2$ m, or 1 m above ground level.

What's next . . . We have only considered vertical stresses. But an element of soil in the ground is also subjected to lateral stresses. Next, we will introduce an equation that relates the vertical and lateral effective stresses.

7.10 LATERAL EARTH PRESSURE AT REST

The ratio of the horizontal principal effective stress to the vertical principal effective stress is called the lateral earth pressure coefficient at rest (K_o), that is,

$$K_o = \frac{\sigma'_3}{\sigma'_1} \tag{7.50}$$

The at-rest condition implies that no deformation occurs. We will revisit the at-rest coefficient in later chapters. You must remember that K_o applies only to effective principal, not total principal, stresses. To find the lateral total stress, you must add the porewater pressure. Remember that the porewater pressure is hydrostatic and, at any given depth, the porewater pressures in all directions are equal.

For a soil that was never subjected to effective stresses higher than its current effective stress (normally consolidated soil), $K_o = K_o^{nc}$ is reasonably predicted by an equation suggested by Jaky (1944) as

$$K_o^{nc} \approx 1 - \sin \phi'_{cs} \tag{7.51}$$

where ϕ'_{cs} is a fundamental frictional soil constant that will be discussed in Chapter 10.

The value of K_o^{nc} is constant. During unloading or reloading, the soil stresses must adjust to be in equilibrium with the applied stress. This means that stress changes take place not only vertically but also horizontally. For a given surface stress, the changes in horizontal total stresses and vertical total stresses are different, but the porewater pressure changes in every direction are the same. Therefore, the current effective stresses are different in different directions. A soil in which the current effective stress is lower than the past maximum stress is called an overconsolidated soil (to be discussed further in Chapter 9). The K_o values for overconsolidated soils are not constants. We will denote K_o for overconsolidated soils as K_o^{oc}. Various equations have been suggested linking K_o^{oc} to K_o^{nc}. One equation that is popular and found to match test data reasonably well is an equation proposed by Meyerhof (1976) as

$$K_o^{oc} = K_o^{nc}(\text{OCR})^{1/2} = (1 - \sin \phi'_{cs})(\text{OCR})^{1/2} \tag{7.52}$$

where OCR is the overconsolidation ratio (see Chapter 9 for more information), defined as the ratio of the past vertical effective stress to the current vertical effective stress.

EXAMPLE 7.9 *Calculating Horizontal Effective and Total Stresses*

Calculate the horizontal effective stress and the horizontal total stress for the soil element at 5 m in Example 7.5 if $K_o = 0.5$.

Strategy The stresses on the horizontal and vertical planes on the soil element are principal stresses (no shear stress occurs on these planes). You need to apply K_o to the effective principal stress and then add the porewater pressure to get the lateral total principal stress.

Solution 7.9

Step 1: Calculate the horizontal effective stress.

$$K_o = \frac{\sigma'_3}{\sigma'_1} = \frac{\sigma'_x}{\sigma'_z}; \quad \sigma'_x = K_o\sigma'_z = 0.5 \times 53.2 = 26.6 \text{ kPa}$$

Step 2: Calculate the horizontal total stress.

$$\sigma_x = \sigma'_x + u = 26.6 + 29.4 = 56 \text{ kPa}$$

EXAMPLE 7.10 ***Calculating Horizontal Total and Effective Stresses from Dissipation of Excess Porewater Pressure***

Determine the horizontal effective and total stresses on a normally consolidated soil sample for:

(a) time: $t = t_1, \Delta u = 20$ kPa

(b) time: $t \rightarrow \infty, \Delta u = 0$

The vertical total stress is 100 kPa and the frictional constant $\phi'_{cs} = 30°$.

Strategy The horizontal earth pressure coefficient must be applied to the vertical effective stress, not the vertical total stress. You need to calculate the vertical effective stress, then the horizontal effective stress. Add the excess porewater pressure to the horizontal effective stress to find the horizontal total stress.

Solution 7.10

Step 1: Calculate the vertical effective stresses.

$$\sigma'_z = \sigma_z - \Delta u$$

(a) $\sigma'_z = 100 - 20 = 80$ kPa

(b) $\sigma'_z = 100 - 0 = 100$ kPa

Step 2: Calculate the horizontal effective stress.

$$K_o^{nc} = 1 - \sin\phi'_{cs} = 1 - \sin 30° = 0.5$$

$$\sigma'_x = K_o^{nc}\sigma'_z$$

(a) $\sigma'_x = 0.5 \times 80 = 40$ kPa

(b) $\sigma'_x = 0.5 \times 100 = 50$ kPa

Step 3: Calculate the total horizontal stresses.

$$\sigma_x = \sigma'_x + \Delta u$$

(a) $\sigma_x = 40 + 20 = 60$ kPa

(b) $\sigma_x = 50 + 0 = 50$ kPa

What's next . . . The stresses we have considered so far are called geostatic stresses, and when we considered elastic deformation of soils, the additional stresses imposed on the soil were given. But in practice, we have to find these additional stresses from applied loads located either on the ground surface or within the soil mass. We will use elastic analysis to find these additional stresses. Next, we will consider increases in stresses from a number of common surface loads. You will encounter myriad equations. You are not expected to remember these equations, but you are expected to know how to use them.

7.11 STRESSES IN SOIL FROM SURFACE LOADS

Computer Program Utility

Access www.wiley.com/college/budhu, and click on Chapter 7 and then STRESS.zip to download and run a computer application to obtain the stress increases and displacements due to surface loads. You can use this program to explore stress changes due to different types of loads, and prepare and print Newmark charts for vertical stresses beneath arbitrarily shaped loads (described in Section 7.11.8). This computer program will also be helpful in solving problems in later chapters.

The distribution of stresses within a soil from applied surface loads or stresses is determined by assuming that the soil is a semi-infinite, homogeneous, linear, isotropic, elastic material. A semi-infinite mass is bounded on one side and extends infinitely in all other directions; this is also called an "elastic half-space." For soils, the horizontal surface is the bounding side. Because of the assumption of a linear elastic soil mass, we can use the principle of superposition. That is, the stress increase at a given point in a soil mass in a certain direction from different loads can be added together.

Surface loads are divided into two general classes, finite and infinite. However, these are qualitative classes and are subject to interpretation. Examples of finite loads are point loads, circular loads, and rectangular loads. Examples of infinite loads are fills and surcharges. The relative rigidity of the foundation (a system that transfers the load to the soil) to the soil mass influences the stress distribution within the soil. The elastic solutions presented are for flexible loads and do not account for the relative rigidity of the soil foundation system. If the foundation is rigid, the stress increases are generally lower (15% to 30% less for clays and 20% to 30% less for sands) than those calculated from the elastic solutions presented in this section. Traditionally, the stress increases from the elastic solutions are not adjusted because soil behavior is nonlinear and it is better to err on the conservative side. The increases in soil stresses from surface loads are total stresses. These increases in stresses are resisted initially by both the porewater and the soil particles.

Equations and charts for several types of flexible surface loads based on the above assumptions are presented. Most soils exist in layers with finite thicknesses. The solution based on a semi-infinite soil mass will not be accurate for these layered soils. In Appendix C, you will find selected graphs and tables for vertical stress increases in one-layer and two-layer soils. A comprehensive set of equations for a variety of loading situations is available in Poulos and Davis (1974).

7.11.1 Point Load

Boussinesq (1885) presented a solution for the distribution of stresses for a point load applied on the soil surface. An example of a point load is the vertical load transferred to the soil from an electric power line pole.

The increases in stresses on a soil element located at point A (Figure 7.20a) due to a point load, Q, are

$$\Delta\sigma_z = \frac{3Q}{2\pi z^2\left[1 + \left(\frac{r}{z}\right)^2\right]^{5/2}} \tag{7.53}$$

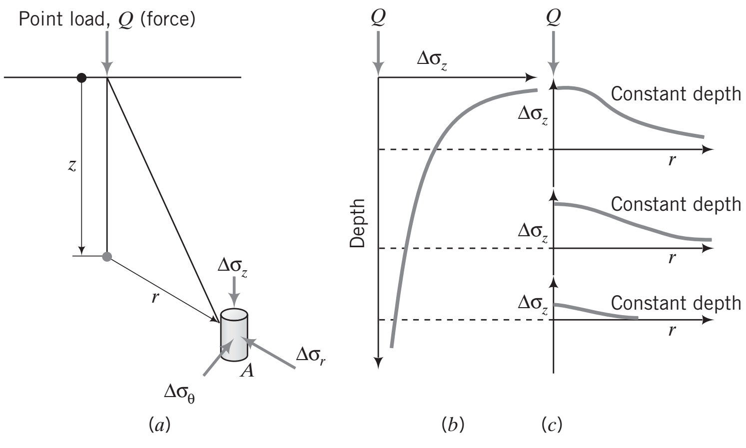


FIGURE 7.20 Point load and vertical stress distribution with depth and radial distance.

$$\Delta\sigma_r = \frac{Q}{2\pi}\left(\frac{3r^2 z}{(r^2 + z^2)^{5/2}} - \frac{1 - 2\nu}{r^2 + z^2 + z(r^2 + z^2)^{1/2}}\right) \tag{7.54}$$

$$\Delta\sigma_\theta = \frac{Q}{2\pi}(1 - 2\nu)\left(\frac{z}{(r^2 + z^2)^{3/2}} - \frac{1}{r^2 + z^2 + z(r^2 + z^2)^{1/2}}\right) \tag{7.55}$$

$$\Delta\tau_{rz} = \frac{3Q}{2\pi}\left[\frac{rz^2}{(r^2 + z^2)^{5/2}}\right] \tag{7.56}$$

where ν is Poisson's ratio. Most often, the increase in vertical stress is needed in practice. Equation (7.53) can be written as

$$\Delta\sigma_z = \frac{Q}{z^2}I \tag{7.57}$$

where I is an influence factor, and

$$I = \frac{3}{2\pi}\frac{1}{\left[1 + \left(\frac{r}{z}\right)^2\right]^{5/2}} \tag{7.58}$$

The distributions of the increase in vertical stress from Equations (7.57) and (7.58) reveal that the increase in vertical stress decreases with depth (Figure 7.20b) and radial distance (Figure 7.20c).

The vertical displacement is

$$\Delta z = \frac{Q(1 + \nu)}{2\pi E z\left[1 + \left(\frac{r}{z}\right)^2\right]^{1/2}}\left[2(1 - \nu) + \frac{1}{1 + \left(\frac{r}{z}\right)^2}\right] \tag{7.59}$$

and the radial displacement is

$$\Delta r = \frac{Q(1 + \nu)}{2\pi E z\left[1 + \left(\frac{r}{z}\right)^2\right]^{1/2}}\left[\frac{\left(\frac{r}{z}\right)}{\left\{1 + \left(\frac{r}{z}\right)^2\right\}} - \frac{(1 - 2\nu)\left(\frac{r}{z}\right)}{\left\{1 + \left(\frac{r}{z}\right)^2\right\}^{1/2} + 1}\right] \tag{7.60}$$

where E is Young's modulus.

EXAMPLE 7.11 *Vertical Stress Increase Due to a Point Load*

A pole carries a vertical load of 200 kN. Determine the vertical total stress increase at a depth 5 m (a) directly below the pole and (b) at a radial distance of 2 m.

Strategy The first step is to determine the type of surface load. The load carried by the pole can be approximated to a point load. You can then use the equation for the vertical stress increase for a point load.

Solution 7.11

Step 1: Determine the load type.

Assume the load from the pole can be approximated by a point load.

Step 2: Use the equation for a point load. Use Equation (7.57):

$$z = 5\text{ m},\quad Q = 200\text{ kN};\quad \text{Under load, } r = 0,\quad \therefore \frac{r}{z} = 0$$

$$\text{From Equation (7.58):}\quad \frac{r}{z} = 0,\quad I = \frac{3}{2\pi} = 0.48$$

$$\Delta\sigma_z = \frac{Q}{z^2}I = \frac{200}{5^2} \times 0.48 = 3.8\text{ kPa}$$

Step 3: Determine the vertical stress at the radial distance.

$$r = 2\text{ m},\quad \frac{r}{z} = \frac{2}{5} = 0.4,\quad I = \frac{3}{2\pi}\frac{1}{[1 + (0.4)^2]^{5/2}} = 0.33$$

$$\Delta\sigma_z = \frac{200}{5^2} \times 0.33 = 2.6\text{ kPa}$$

7.11.2 Line Load

With reference to Figure 7.21a, the increases in stresses due to a line load, Q (force/length), are

$$\Delta\sigma_z = \frac{2Qz^3}{\pi(x^2 + z^2)^2} \tag{7.61}$$

$$\Delta\sigma_x = \frac{2Qx^2z}{\pi(x^2 + z^2)^2} \tag{7.62}$$

$$\Delta\tau_{zx} = \frac{2Qxz^2}{\pi(x^2 + z^2)^2} \tag{7.63}$$

A practical example of a line load is the load from a long brick wall.

7.11.3 Line Load Near a Buried Earth-Retaining Structure

The increase in lateral stress on a buried earth-retaining structure (Figure 7.21b) due to a line load of intensity Q (force/length) is

$$\Delta\sigma_x = \frac{4Qa^2b}{\pi H_o(a^2 + b^2)^2} \tag{7.64}$$

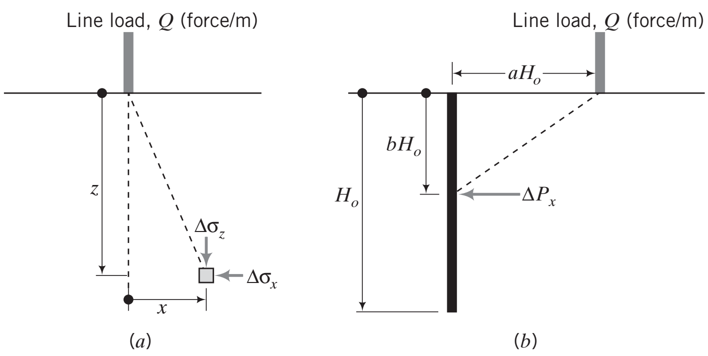


FIGURE 7.21 (a) Line load and (b) line load near a retaining wall.

The increase in lateral force is

$$\Delta P_x = \frac{2Q}{\pi(a^2 + 1)} \tag{7.65}$$

7.11.4 Strip Load

A strip load is the load transmitted by a structure of finite width and infinite length on a soil surface. Two types of strip loads are common in geotechnical engineering. One is a load that imposes a uniform stress on the soil, for example, the middle section of a long embankment (Figure 7.22a). The other is a load that induces a triangular stress distribution over an area of width B (Figure 7.22b). An example of a strip load with a triangular stress distribution is the stress under the side of an embankment.

The increases in stresses due to a surface stress q_s (force/area) are as follows:

(a) Area transmitting a uniform stress (Figure 7.22a)

$$\Delta\sigma_z = \frac{q_s}{\pi}[\alpha + \sin\alpha\cos(\alpha + 2\beta)] \tag{7.66}$$

$$\Delta\sigma_x = \frac{q_s}{\pi}[\alpha - \sin\alpha\cos(\alpha + 2\beta)] \tag{7.67}$$

$$\Delta\tau_{zx} = \frac{q_s}{\pi}[\sin\alpha\sin(\alpha + 2\beta)] \tag{7.68}$$

where q_s is the applied surface stress.

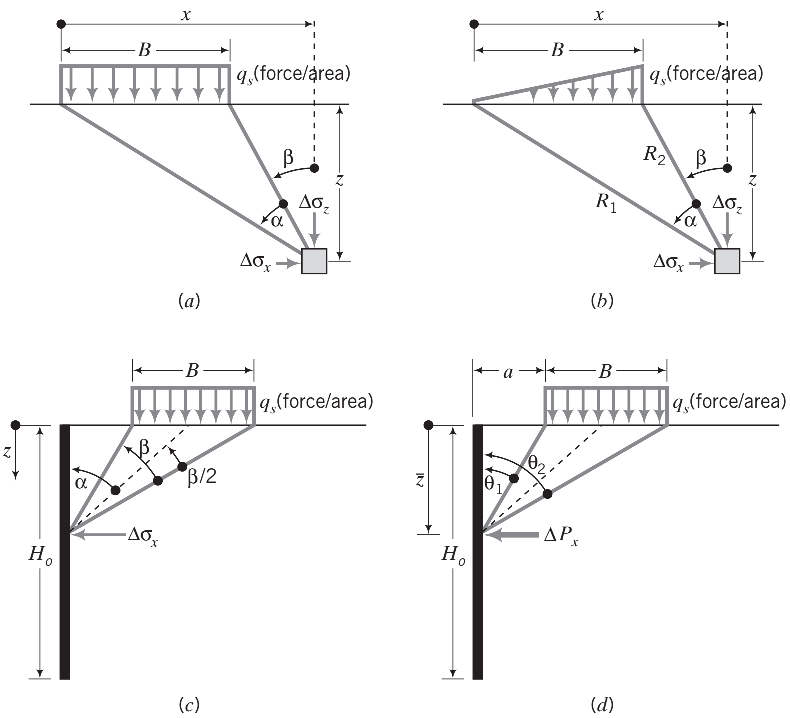


FIGURE 7.22 Strip load imposing (a) a uniform surface stress and (b) a linear varying surface stress. (c) Strip load imposing a uniform surface stress near a retaining wall and (d) lateral force on a retaining wall from a strip load imposing a uniform surface stress.

The vertical displacement due to a strip loading is useful only as relative displacement between two points not located at infinity. The relative vertical displacement between the center of the strip load (0, 0) and a point at the surface (*x*, 0) is

$$\Delta z(x,0) - \Delta z(0,0) = \frac{2q_s(1-\nu^2)}{\pi E}\left\{\left(x - \frac{B}{2}\right)\ln\left|x - \frac{B}{2}\right| - \left(x + \frac{B}{2}\right)\ln\left|x + \frac{B}{2}\right| + B\ln\left(\frac{B}{2}\right)\right\} \quad (7.69)$$

(b) Area transmitting a triangular stress (Figure 7.22b)

$$\Delta\sigma_z = \frac{q_s}{\pi}\left(\frac{x}{B}\alpha - \frac{1}{2}\sin 2\beta\right) \quad (7.70)$$

$$\Delta\sigma_x = \frac{q_s}{\pi}\left(\frac{x}{B}\alpha - \frac{z}{B}\ln\frac{R_1^2}{R_2^2} + \frac{1}{2}\sin 2\beta\right) \quad (7.71)$$

$$\Delta\tau_{zx} = \frac{q_s}{2\pi}\left(1 + \cos 2\beta - 2\frac{z}{B}\alpha\right) \quad (7.72)$$

The relative vertical displacement between the center of the strip load (0, 0) and a point at the surface (*x*, 0) is

$$\Delta z(x,0) - \Delta z(0,0) = \frac{q_s(1-\nu^2)}{\pi E\left(\frac{B}{2}\right)}\left\{\frac{B^2}{2}\ln B - \frac{x^2}{2}\ln x + \left(\frac{x^2}{2} - \frac{B^2}{2}\right)\ln|B - x| + \frac{B}{2}x\right\} \quad (7.73)$$

(c) Area transmitting a uniform stress near a retaining wall (Figure 7.22c, d)

$$\Delta\sigma_x = \frac{2q_s}{\pi}(\beta - \sin\beta\cos 2\alpha) \quad (7.74)$$

The lateral force and its location were derived by Jarquio (1981) and are

$$\Delta P_x = \frac{q_s}{90}[H_o(\theta_2 - \theta_1)] \quad (7.75)$$

$$\bar{z} = \frac{H_o^2(\theta_2 - \theta_1) - (R_1 - R_2) + 57.3BH_o}{2H_o(\theta_2 - \theta_1)} \quad (7.76)$$

where

$$\theta_1 = \tan^{-1}\left(\frac{a}{H_o}\right), \quad \theta_2 = \tan^{-1}\left(\frac{a+B}{H_o}\right)$$

$$R_1 = (a+B)^2(90 - \theta_2), \quad \text{and} \quad R_2 = a^2(90 - \theta_1)$$

7.11.5 Uniformly Loaded Circular Area

An example of a circular area that transmits stresses to a soil mass is a circular foundation of an oil or water tank. The increases of vertical and radial stresses under the center of a circular area of radius r_o are

$$\Delta\sigma_z = q_s\left[1 - \left(\frac{1}{1 + (r_o/z)^2}\right)^{3/2}\right] = q_s I_c \quad (7.77)$$

where

$$I_c = \left[1 - \left(\frac{1}{1 + (r_o/z)^2}\right)^{3/2}\right]$$

is an influence factor and

$$\Delta\sigma_r = \Delta\sigma_\theta = \frac{q_s}{2}\left[(1 + 2\nu) - \frac{4(1 + \nu)}{[1 + (r_o/z)^2]^{1/2}} + \frac{1}{[1 + (r_o/z)^2]^{3/2}}\right] \quad (7.78)$$

The vertical elastic settlement at the surface due to a circular flexible loaded area is

$$\text{Below center ofloaded area:} \quad \Delta z = \frac{q_s D(1 - \nu^2)}{E} \quad (7.79)$$

$$\text{Below edge:} \quad \Delta z = \frac{2}{\pi}\frac{q_s D(1 - \nu^2)}{E} \quad (7.80)$$

where $D = 2r_o$ is the diameter of the loaded area. The vertical stress increases and vertical elastic settlements at all points in the soil mass from a circular loaded area are shown in Appendix B.

EXAMPLE 7.12 *Vertical Stress Increase Due to a Ring Load*

A silo is supported on a ring foundation, as shown in Figure E7.12a. The total vertical load is 4 MN. (a) Plot the vertical stress increase with depth up to 8 m under the center of the ring (point O, Figure E7.12a). (b) Determine the maximum vertical stress increase and its location.

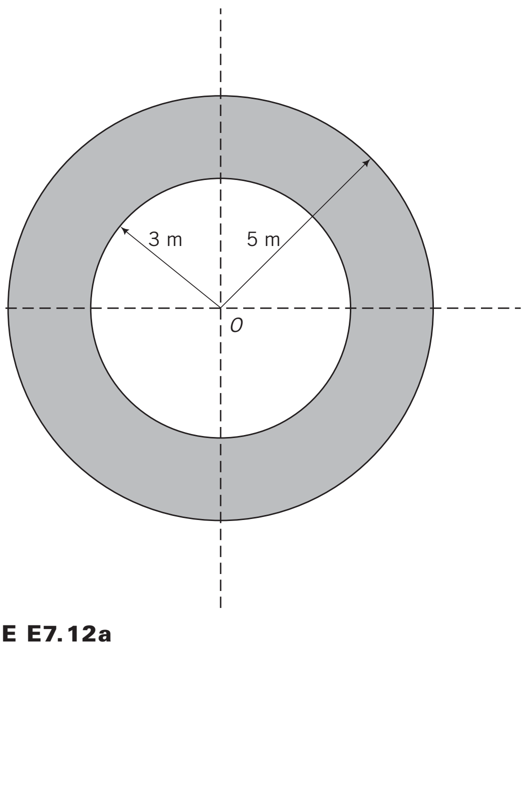


FIGURE E7.12a

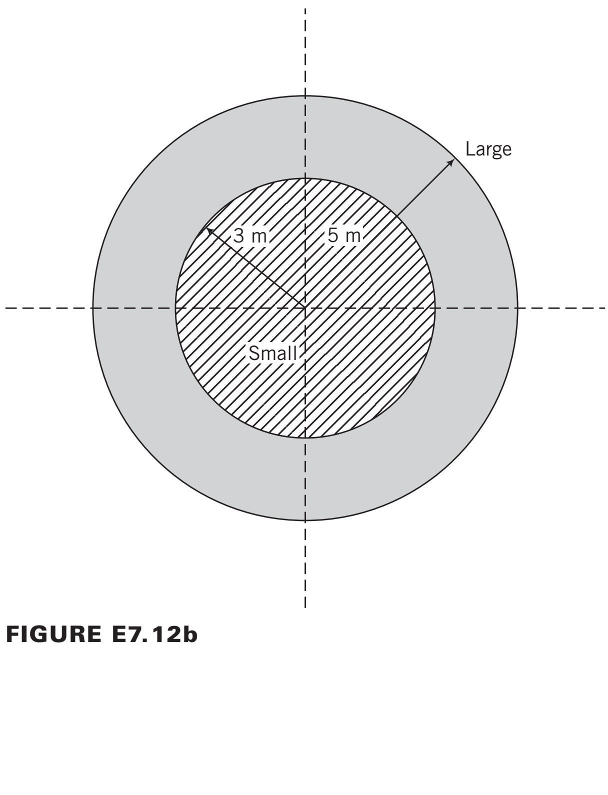


FIGURE E7.12b

Strategy To use the equation for a uniform circular area to simulate the ring foundation, you need to create two artificial circular foundations, one with a radius of 5 m and the other with a radius of 3 m. Both foundations must be fully loaded with the applied uniform, vertical stress. By subtracting the vertical stress increase of the smaller foundation from the larger foundation, you would obtain the vertical stress increase from the ring foundation. You are applying here the principle of superposition.

Solution 7.12

Step 1: Identify the loading type.

It is a uniformly loaded ring foundation.

Step 2: Calculate the imposed surface stress.

$$r_2 = 5 \text{ m}, \quad r_1 = 3 \text{ m}$$

$$\text{Area} = \pi(r_2^2 - r_1^2) = \pi(5^2 - 3^2) = 16\pi \text{ m}^2$$

$$q_s = \frac{Q}{A} = \frac{4000}{16\pi} = 79.6 \text{ kPa}$$

Step 3: Create two solid circular foundations of radii 5 m and 3 m.

See Figure E7.12b. Let "large" denotes the foundation of radius 5 m and "small" denotes the foundation of radius 3 m.

Step 4: Create a spreadsheet to do the calculations.

Ring load	
Load	4000 kN
Outer radius	5 m
Inner radius	3 m
Area	50.3 m^2
q_s	79.6 kPa

	Large		**Small**		I_{diff}	$\Delta\sigma_z$ **(kPa)**
z	***r/z***	$(I_c)_{large}$	r_o/z	$(I_c)_{small}$	$(I_c)_{large} - (I_c)_{small}$	$q_s \times I_{diff}$
1	7.00	0.992	3.00	0.968	0.024	1.9
2	2.50	0.949	1.50	0.829	0.119	9.5
3	1.67	0.864	1.00	0.646	0.217	17.3
4	1.25	0.756	0.75	0.488	0.268	21.3
5	1.00	0.646	0.60	0.369	0.277	22.0
6	0.83	0.547	0.50	0.284	0.262	20.9
7	0.71	0.461	0.43	0.223	0.238	18.9
8	0.63	0.390	0.38	0.179	0.211	16.8

The cofficients I_c were obtained from the application program, STRESS.zip. You can download this application from www.wiley.com/college/budhu.

Step 5: Plot the vertical stress increase variation with depth.

See Figure E7.12c.

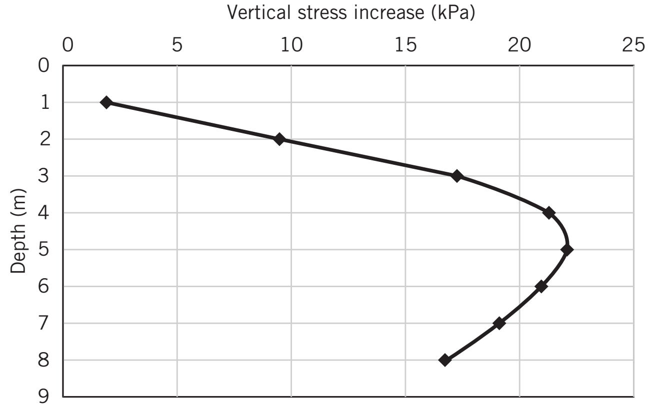


FIGURE E7.12c

Step 6: Determine the maximum vertical stress increase and depth of occurrence.

From Figure E7.12c, the maximum vertical stress increase is 22 kPa and the depth of occurrence is 5 m from the surface.

7.11.6 Uniformly Loaded Rectangular Area

Many structural foundations are rectangular or approximately rectangular in shape. The increases in stresses below the *corner* of a rectangular area of width B and length L are

$$\Delta\sigma_z = \frac{q_s}{2\pi}\left[\tan^{-1}\frac{LB}{zR_3} + \frac{LBz}{R_3}\left(\frac{1}{R_1^2} + \frac{1}{R_2^2}\right)\right] \tag{7.81}$$

$$\Delta\sigma_x = \frac{q_s}{2\pi}\left[\tan^{-1}\frac{LB}{zR_3} - \frac{LBz}{R_1^2 R_3}\right] \tag{7.82}$$

$$\Delta\sigma_y = \frac{q_s}{2\pi}\left[\tan^{-1}\frac{LB}{zR_3} - \frac{LBz}{R_2^2 R_3}\right] \tag{7.83}$$

$$\Delta\tau_{zx} = \frac{q_s}{2\pi}\left[\frac{B}{R_2} - \frac{z^2 B}{R_1^2 R_3}\right] \tag{7.84}$$

where $R_1 = (L^2 + z^2)^{1/2}$, $R_2 = (B^2 + z^2)^{1/2}$, and $R_3 = (L^2 + B^2 + z^2)^{1/2}$.

These equations can be written as

$$\Delta\sigma_z = q_s I_z \tag{7.85}$$

$$\Delta\sigma_x = q_s I_x \tag{7.86}$$

$$\Delta\sigma_y = q_s I_y \tag{7.87}$$

$$\tau_{zx} = q_s I_\tau \tag{7.88}$$

where I denotes the influence factor. The influence factor for the vertical stress is

$$I_z = \frac{1}{4\pi}\left[\frac{2mn\sqrt{m^2 + n^2 + 1}}{m^2 + n^2 + m^2n^2 + 1}\left(\frac{m^2 + n^2 + 2}{m^2 + n^2 + 1}\right) + \tan^{-1}\left(\frac{2mn\sqrt{m^2 + n^2 + 1}}{m^2 + n^2 - m^2n^2 + 1}\right)\right] \tag{7.89}$$

where $m = B/z$ and $n = L/z$. You can program your calculator or use a spreadsheet program to find I_z. You must be careful in the last term ($\tan^{-1}$) in programming. If $m^2 + n^2 + 1 < m^2n^2$, then you have to add π to the bracketed quantity in the last term. The distribution of vertical stress below a uniformly loaded square foundation is shown in Figure 7.23. The increase in vertical stress is about 10% below a depth of

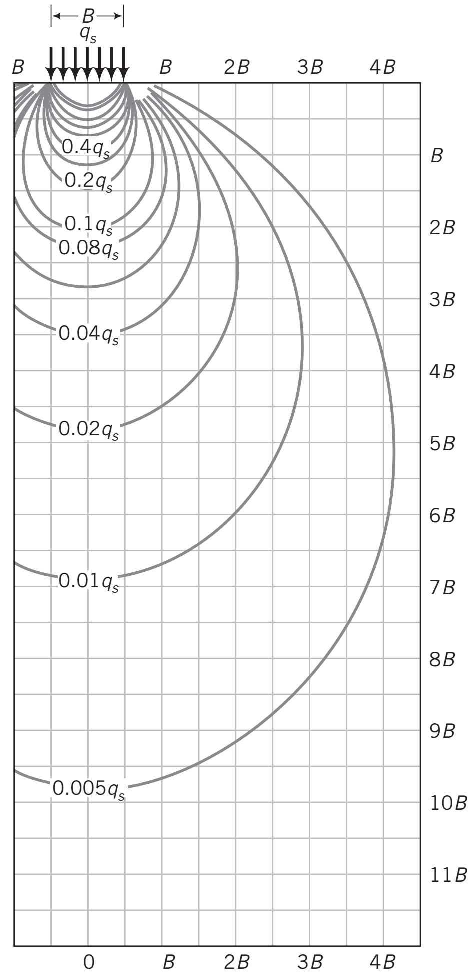


FIGURE 7.23 Vertical stress contour below a square foundation.

$2B$; B is the diameter of the foundation. The vertical stress decreases from the center of the foundation outward, reaching a value of about 10% at a horizontal distance of $B/2$ from the edge at a depth of B. A chart for I_z for the corner of rectangular loaded area is shown in Figure 7.24 on page 172. You would have to calculate $m = B/z$ and $n = L/z$ and read I_z from the chart; m and n are interchangeable. In general, the vertical stress increase is less than 10% of the surface stress when $z > 2B$.

The vertical elastic settlement at the ground surface under a rectangular flexible surface load is

$$\Delta z = \frac{q_s B(1 - \nu^2)}{E} I_s \tag{7.90}$$

where I_s is a settlement influence factor that is a function of the L/B ratio (L is length and B is width). Setting $\xi_s = L/B$, the equations for I_s are

At center of a rectangle (Giroud, 1968):

$$I_s = \frac{2}{\pi}\left[\ln\left(\xi_s + \sqrt{1 + \xi_s^2}\right) + \xi_s \ln\frac{1 + \sqrt{1 + \xi_s^2}}{\xi_s}\right] \tag{7.91}$$

At corner of a rectangle (Giroud, 1968):

$$I_s = \frac{1}{\pi}\left[\ln\left(\xi_s + \sqrt{1 + \xi_s^2}\right) + \xi_s \ln\frac{1 + \sqrt{1 + \xi_s^2}}{\xi_s}\right] \tag{7.92}$$

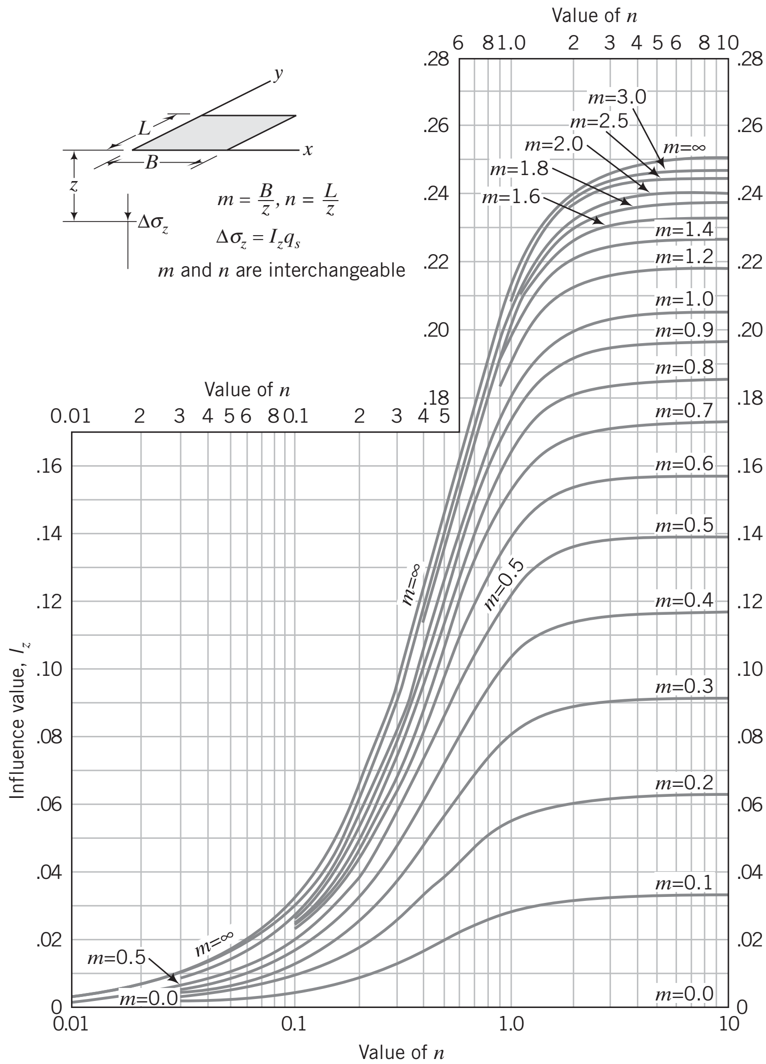


FIGURE 7.24 Influence factor for calculating the vertical stress increase under the corner of a rectangle. (Source: NAV-FAC-DM 7.1.)

The above equations can be simplified to the following for $\xi_s \geq 1$:

$$\text{At center of a rectangle:} \quad I_s \cong 0.62 \ln(\xi_s) + 1.12 \tag{7.93}$$

$$\text{At corner of a rectangle:} \quad I_s \cong 0.31 \ln(\xi_s) + 0.56 \tag{7.94}$$

7.11.7 Approximate Method for Rectangular Loads

In preliminary analyses of vertical stress increases under the center of rectangular loads, geotechnical engineers often use an approximate method (sometimes called the 2:1 method). The surface load on an area $B \times L$ is dispersed at a depth z over an area $(B + z) \times (L + z)$, as illustrated in Figure 7.25. The vertical stress increase under the center of the load is

$$\Delta\sigma_z = \frac{Q}{(B + z)(L + z)} = \frac{q_s BL}{(B + z)(L + z)} \tag{7.95}$$

The approximate method is reasonably accurate (compared with Boussinesq's elastic solution) when $z > B$.

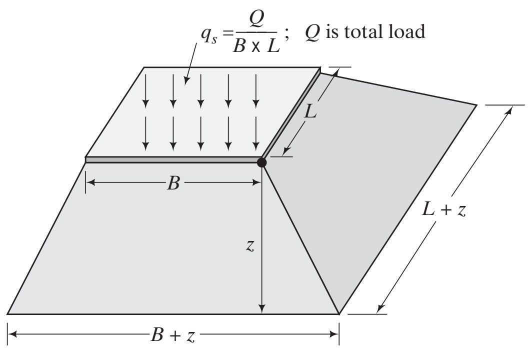


FIGURE 7.25
Dispersion of load for approximate increase in vertical stress under a rectangular loaded area.

EXAMPLE 7.13 *Vertical Stress Increase Due to a Rectangular Load*

A rectangular concrete slab, 3 m × 4.5 m, rests on the surface of a soil mass. The load on the slab is 2025 kN. Determine the vertical stress increase at a depth of 3 m (a) under the center of the slab, point A (Figure E7.13a); (b) under point B (Figure E7.13a); and (c) at a distance of 1.5 m from a corner, point C (Figure E7.13a).

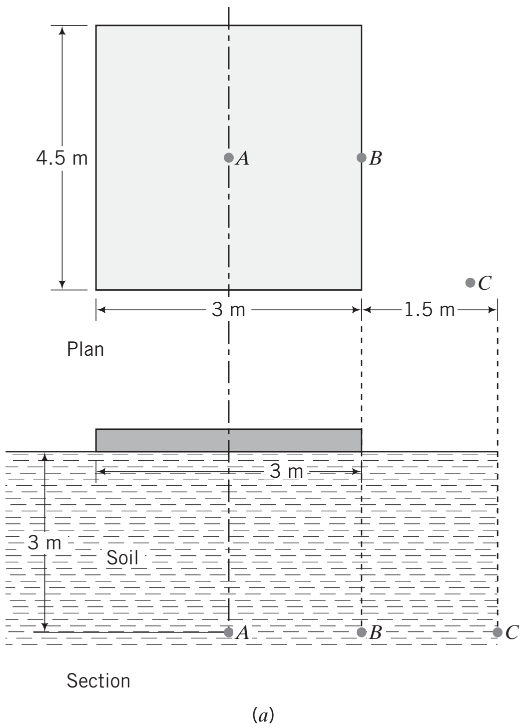


FIGURE E7.13a

Strategy The slab is rectangular and the equations for a uniformly loaded rectangular area are for the corner of the area. You should divide the area so that the point of interest is a corner of a rectangle(s). You may have to extend the loaded area if the point of interest is outside it (loaded area). The extension is fictitious, so you have to subtract the fictitious increase in vertical stress for the extended area.

Solution 7.13

Step 1: Identify the loading type.

It is a uniformly loaded rectangle.

Step 2: Divide the rectangle so that the center is a corner.

In this problem, all four rectangles, after the subdivision, are equal (Figure E7.13b; point *C* is excluded for simplicity), so you only need to find the vertical stress increase for one rectangle of size $B = 1.5$ m, $L = 2.25$ m and multiply the results by 4.

$$m = \frac{B}{z} = \frac{1.5}{3} = 0.5; \quad n = \frac{L}{z} = \frac{2.25}{3} = 0.75$$

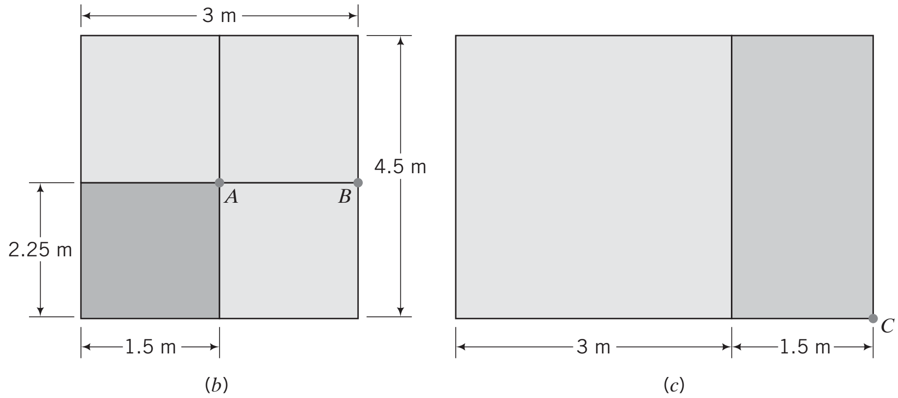


FIGURE E7.13b, c

From the chart in Figure 7.24, $I_z = 0.107$.

Step 3: Find the vertical stress increase at the center of the slab (point *A*, Figure E7.13b).

$$q_s = \frac{Q}{A} = \frac{2025}{3 \times 4.5} = 150 \text{ kPa}$$

$$\Delta\sigma_z = 4q_s I_z = 4 \times 150 \times 0.105 = 63 \text{ kPa}$$

Note: The approximate method [Equation (7.95)] gives

$$\Delta\sigma_z = \frac{Q}{(B + z)(L + z)} = \frac{2025}{(3 + 3)(4.5 + 3)} = 45 \text{ kPa}$$

which is about 30% less than the elastic solution.

Step 4: Find the vertical stress increase for point *B*.

Point *B* is at the corner of two rectangles, each of width 3 m and length 2.25 m. You need to find the vertical stress increase for one rectangle and multiply the result by 2.

$$m = \frac{3}{3} = 1; \quad n = \frac{2.25}{3} = 0.75$$

From the chart in Figure 7.24, $I_z = 0.158$.

$$\Delta\sigma_z = 2q_s I_z = 2 \times 150 \times 0.158 = 47.4 \text{ kPa}$$

You should note that the vertical stress increase at *B* is lower than at *A*, as expected.

Step 5: Find the stress increase for point *C*.

Stress point *C* is outside the rectangular slab. You have to extend the rectangle to *C* (Figure E7.13c) and find the stress increase for the large rectangle of width $B = 4.5$ m, length $L = 4.5$ m and then subtract the stress increase for the smaller rectangle of width $B = 1.5$ m and length $L = 4.5$ m.

Large rectangle

$$m = \frac{4.5}{3} = 1.5, \quad n = \frac{4.5}{3} = 1.5; \quad \text{from chart in Figure 7.24, } I_z = 0.22$$

Small rectangle

$$m = \frac{1.5}{3} = 0.5, \quad n = \frac{4.5}{3} = 1.5; \quad \text{from chart in Figure 7.24, } I_z = 0.13$$

$$\Delta\sigma_z = q_s \Delta I_z = 150 \times (0.22 - 0.13) = 13.5 \text{ kPa}$$

7.11.8 Vertical Stress Below Arbitrarily Shaped Areas

Newmark (1942) developed a chart to determine the increase in vertical stress due to a uniformly loaded area of any shape. The chart consists of concentric circles divided by radial lines (Figure 7.26). The area of each segment represents an equal proportion of the applied surface stress at a depth z below the surface. If there are 10 concentric circles and 20 radial lines, the stress on each circle is $q_s/10$ and on each segment is $q_s/(10 \times 20)$. The radius-to-depth ratio of the first (inner) circle is found by setting $\Delta\sigma_z = 0.1q_s$ in Equation (7.77), that is,

$$0.1q_s = q_s\left[1 - \left\{\frac{1}{1 + (r_o/z)^2}\right\}^{3/2}\right]$$

from which $r/z = 0.27$. For the other circles, substitute the appropriate value for $\Delta\sigma_z$; for example, for the second circle $\Delta\sigma_z = 0.2q_s$, and find r/z. The chart is normalized to the depth; that is, all dimensions are scaled by a factor initially determined for the depth. Every chart should show a scale and an influence factor I_N. The influence factor for Figure 7.26 is 0.001.

The procedure for using Newmark's chart is as follows:

1. Set the scale, shown on the chart, equal to the depth at which the increase in vertical stress is required. We will call this the depth scale.

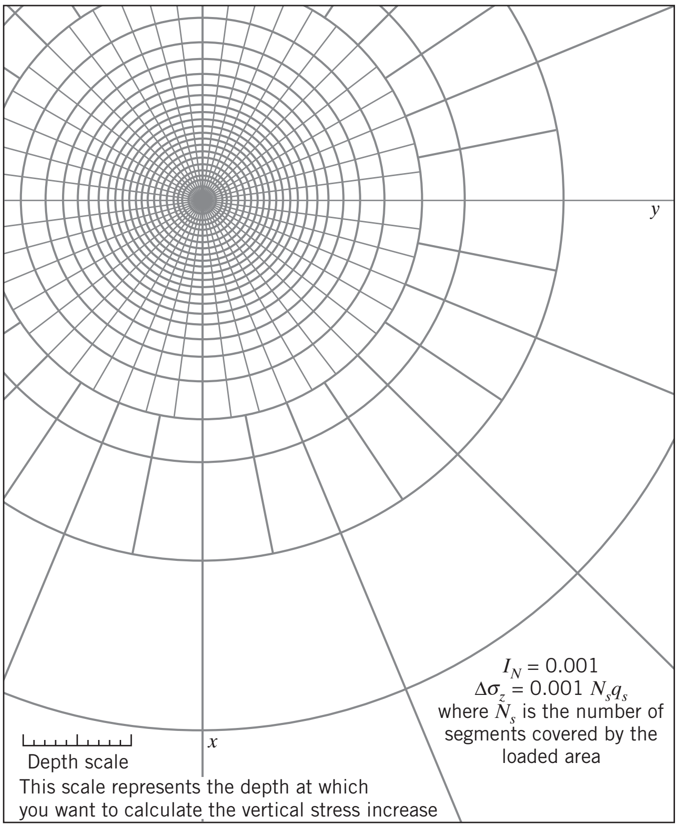


FIGURE 7.26
Newmark's chart for increase in vertical stress.

2. Identify the point below the loaded area where the stress is required. Let us say this point is A.

3. Plot the loaded area, scaling its plan dimension using the depth scale with point A at the center of the chart.

4. Count the number of segments (N_s) covered by the scaled loaded area. If certain segments are not fully covered, you can estimate what fraction is covered.

5. Calculate the increase in vertical stress as $\Delta\sigma_z = q_s I_N N_s$.

EXAMPLE 7.14 *Vertical Stress Increase Due to an Irregular Loaded Area*

The plan of a foundation of uniform thickness for a building is shown in Figure E7.14a. Determine the vertical stress increase at a depth of 4 m below the centroid. The foundation applies a vertical stress of 200 kPa on the soil surface.

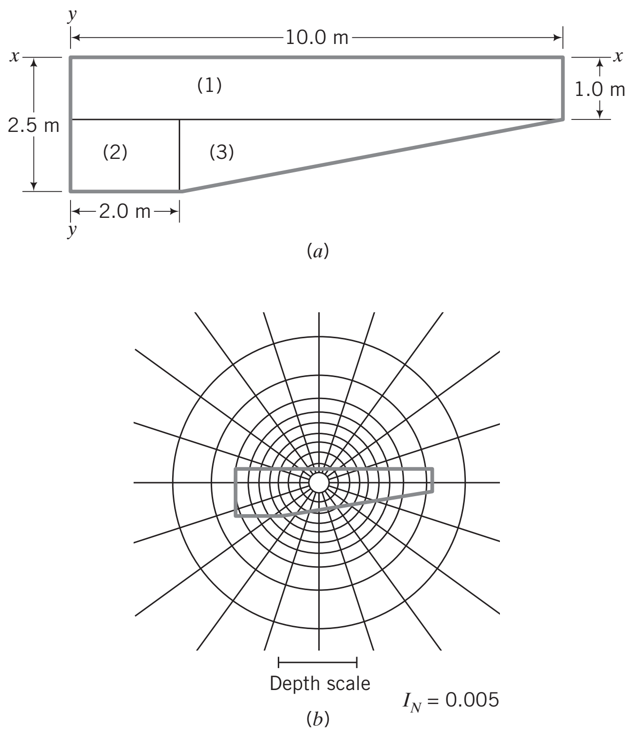


FIGURE E7.14a, b

Strategy You need to locate the centroid of the foundation, which you can find using the given dimensions. The shape of the foundation does not fit neatly into one of the standard shapes (e.g., rectangles or circles) discussed. The convenient method to use for this (odd) shape foundation is Newmark's chart.

Solution 7.14

Step 1: Find the centroid.

Divide the loaded area into a number of regular shapes. In this example, we have three. Take the sum of moments of the areas about y-y (Figure E7.14a) and divide by the sum of the areas to get $\bar{x}$. Take moments about x-x (Figure E7.14a) to get $\bar{y}$.

$$\bar{x} = \frac{(1.0 \times 10.0 \times 5.0) + (1.5 \times 2.0 \times 1.0) + \left[\frac{1}{2} \times 8.0 \times 1.5 \times \left(2 + \frac{1}{3} \times 8.0\right)\right]}{(1.0 \times 10.0) + (1.5 \times 2.0) + \frac{1}{2} \times 8.0 \times 1.5} = \frac{81}{19} = 4.26 \text{ m}$$

$$\bar{y} = \frac{(1 \times 10 \times 0.5) + (1.5 \times 2 \times 1.75) + \left[\frac{1}{2} \times 8.0 \times 1.5 \times \left(1.0 + \frac{1.5}{3}\right)\right]}{(1.0 \times 10.0) + (1.5 \times 2.0) + \frac{1}{2} \times 8.0 \times 1.5} = \frac{19.25}{19} \approx 1 \text{ m}$$

Step 2: Scale and plot the foundation on a Newmark's chart.

The scale on the chart is set equal to the depth. The centroid is located at the center of the chart and the foundation is scaled using the depth scale (Figure E7.14b).

Step 3: Count the number of segments covered by the foundation.

$$N_s = 61$$

Step 4: Calculate the vertical stress increase.

$$\Delta\sigma_z = q_s I_N N_s = 200 \times 0.005 \times 61 = 61 \text{ kPa}$$

7.11.9 Embankment Loads

Loads from an embankment can be considered as a combination of a rectangle and two triangular strip loads. The vertical stress increase due to an embankment load is shown in Figure 7.27. The applied vertical, surface stress is the height of the embankment multiplied by the unit weight of the embankment (fill) material.

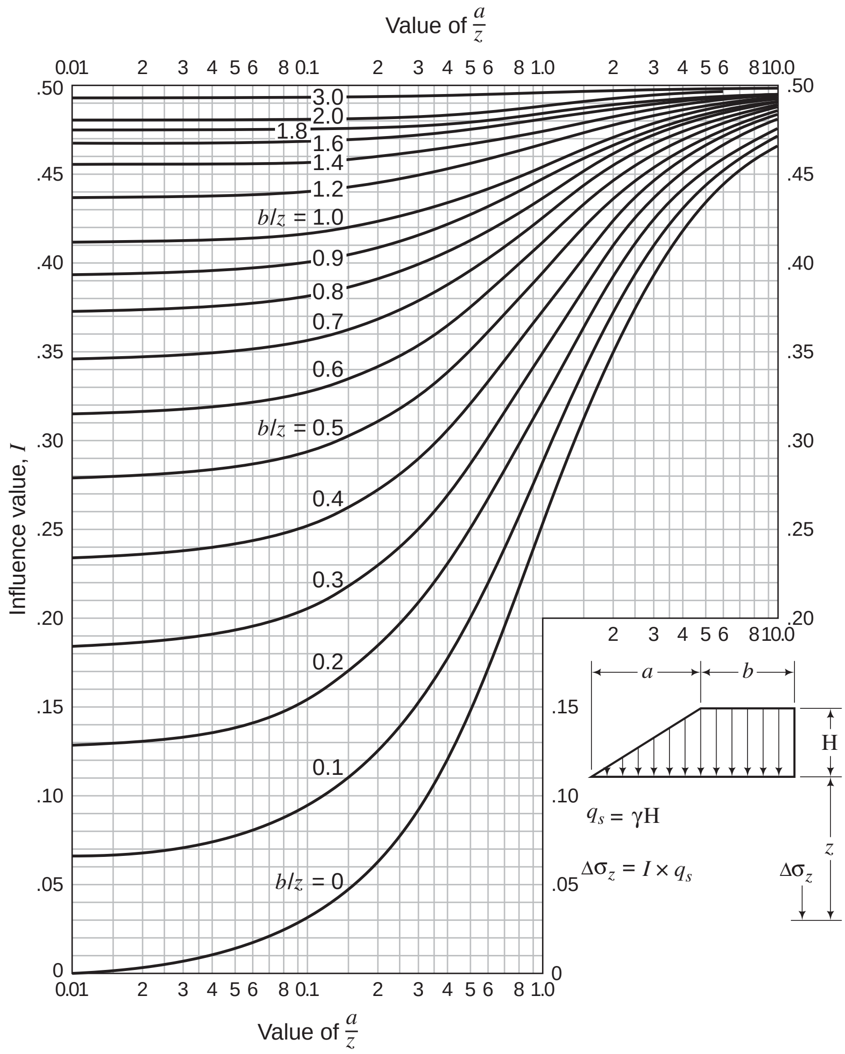


FIGURE 7.27
Vertical stress increase due to an embankment.

7.11.10 Infinite Loads

Uniform loads of large lateral extent such as fills and surcharges are assumed to be transferred to the soil as a uniformly distributed vertical stress throughout the depth. For example, if a fill of unit weight 15 kN/m^3 and height 2 m is placed on the surface of a soil, then the vertical stress at any depth below the surface is $2 \times 15 = 30$ kPa.

THE *ESSENTIAL POINTS* ARE:

1. **The increases in stresses below a surface load are found by assuming the soil is an elastic, semi-infinite mass.**
2. **Various equations are available for the increases in stresses from surface loading.**
3. **The stress increase at any depth depends on the shape and distribution of the surface load.**
4. **A stress applied at the surface of a soil mass by a loaded area decreases with depth and lateral distance away from the center of the loaded area.**
5. **The vertical stress increases are generally less than 10% of the surface stress when the depth-to-width ratio is greater than 2.**

7.12 SUMMARY

Elastic theory provides a simple, first approximation to calculate the deformation of soils at small strains. You are cautioned that the elastic theory cannot adequately describe the behavior of most soils, and more involved theories are required. The most important principle in soil mechanics is the principle of effective stress. Soil deformation is due to effective, not total, stresses. Applied surface stresses are distributed such that their magnitudes decrease with depth and distance away from their points of application.

Self-Assessment

Access Chapter 7 at http://www.wiley.com/college/budhu to take the end-of-chapter quiz to test your understanding of this chapter.

Practical Examples

EXAMPLE 7.15 ***Vertical Stress Increase Due to an Electric Power Transmission Pole***

A Douglas fir electric power transmission pole is 12 m above ground level and embedded 2 m into the ground. The butt diameter is 450 mm and the tip diameter (the top of the pole) is 320 mm. The weight of the pole, cross arms, and wires is 33 kN. Assuming the pole transmits the load as a point load, plot the vertical stress increase with depth up to a depth where the stress increase is less than 5 kPa along the center of the pole.

Strategy This is a straightforward application of Boussinesq's equation.

Solution 7.15

Step 1: Calculate vertical stress increase.

At center of pole, $r = 0$, $r/z = 0$.

Equation (7.58): $I = \dfrac{3}{2\pi} = 0.477$

The vertical stress increase with depth is shown in the table below.

z (m)	r/z	I	$\Delta\sigma_z$ Equation (7.57) (kPa)
0.1	0.00	0.477	1577.6
0.2	0.00	0.477	393.9
0.5	0.00	0.477	63.0
1	0.00	0.477	17.8
2	0.00	0.477	3.9

Step 2: Plot the vertical stress distribution with depth.
See Figure E7.15.

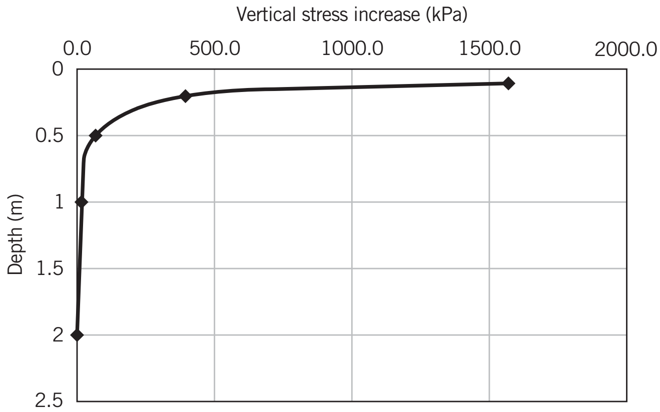


FIGURE E7.15

EXAMPLE 7.16 *Height of Embankment to Obtain a Desired Vertical Stress Increase*

A route for a proposed highway passes through a 2-km stretch of soft clay (ASTM-CS: CH) approximately 4 m thick underlain by poorly graded gravel with clay (ASTM-CS: GP-GC). The geotechnical engineer estimated the settlement (Chapter 9) of the soft clay due to the pavement and traffic loads and found that it is intolerable. One solution is to preload the soft clay by constructing a temporary embankment in stages. Each loading stage will remain on the soft clay for about 6 months to allow the porewater to drain and to cause the clay to settle. The loading must be of such a magnitude that the soft clay would not fail. The estimated maximum vertical stress increase at the center of the soil clay layer along a vertical line through the center of the embankment for the first stage of the loading is 20 kPa. Calculate the height of embankment required if the pavement width is 8 m and the embankment slope cannot exceed 1 (V): 1.5 (H). The unit weight of the fill is 16 kN/m^3.

Strategy The solution of this type of problem may require iteration. The constraints on the problem are the maximum vertical stress increase and the slope of the embankment. Since you are given the maximum vertical stress increase, you need to find a (Figure 7.27) and use the maximum slope of the embankment to find H.

Solution 7.16

Step 1: Make a sketch of the problem.
See Figure E7.16.

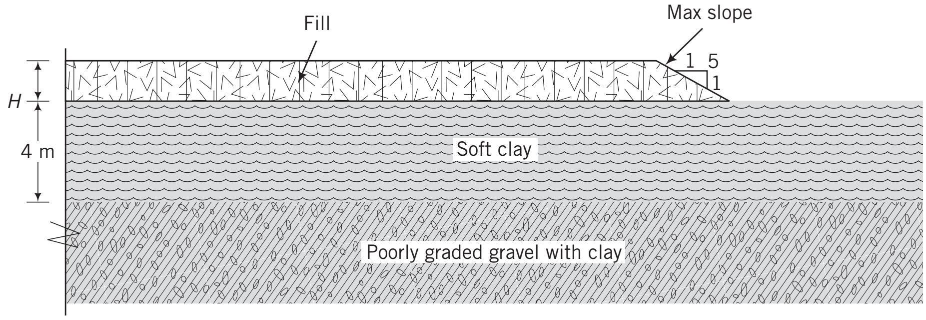


FIGURE E7.16

Step 2: Calculate $\frac{b}{z}$ ratio.

$\frac{b}{z} = \frac{4}{2} = 2$; $\frac{a}{z}$ ratio has to be determined.

Step 3: Determine I required.

From Figure 7.27, $I = 0.48$ for $\frac{b}{z} = 2$ and $\frac{a}{z} = 0.001$ to 2.

Step 4: Determine a/z ratio required.

Since Figure 7.27 only gives the vertical stress increase for one half the embankment load, you have to divide the desired vertical stress increase by 2.

$$\therefore \frac{\Delta\sigma_z}{2} = q_s I = \gamma H I = 16HI$$

$$I = \frac{20}{2 \times 16H} = \frac{1}{1.6H}$$

Since the minimum value of a is $1.5H$, then

$$I = \frac{1}{1.6H} = \frac{1}{1.6\frac{a}{1.5}} = \frac{0.94}{a}$$

$a = \frac{0.94}{I} = \frac{0.94}{0.48} = 1.96$ m; $\frac{a}{z} = \frac{1.96}{2} = 0.88$, which lies within the range 0.001 to 2.

Therefore, $I = 0.48$.

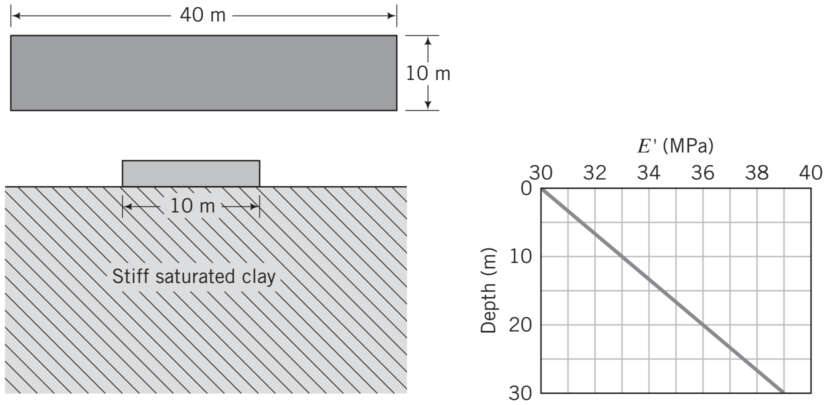
If $\frac{a}{z}$ were not within the range 0.001 to 2, then you would have to do iterations by choosing a value of I for $\frac{b}{z} = 2$ and then check that $\frac{a}{z}$ corresponds to that value of I.

Step 5: Determine H required.

$$H = \frac{a}{1.5} = \frac{1.96}{1.5} = 1.3 \text{ m}$$

EXAMPLE 7.17 *Vertical Stress Increase Due to a Foundation*

A building foundation of width 10 m and length 40 m transmits a load of 80 MN to a deep deposit of stiff saturated clay (Figure E7.17a). The elastic modulus of the clay varies with depth (Figure E7.17b) and $\nu = 0.32$. Estimate the elastic settlement of the clay under the center of the foundation.



(*a*) Foundation plan and section (*b*) Variation of elastic modulus with depth

FIGURE E7.17

Strategy The major decision in this problem is what depth to use to determine an appropriate elastic modulus. One option is to use an average elastic modulus over a depth of $2B$ or $3B$. Beyond a depth of about $2B$, the vertical stress increase is less than 10%. Let us use a depth of $3B$.

Solution 7.17

Step 1: Find the applied vertical surface stress.

$$q_s = \frac{Q}{A} = \frac{80 \times 10^3}{10 \times 40} = 200 \text{ kPa}$$

Step 2: Determine the elastic modulus.

Assume an effective depth of $3B = 3 \times 10 = 30$ m.
The average value of E is 34.5 MPa.

Step 3: Calculate the vertical settlement.

Use Equation(7.90): $\Delta z = \dfrac{q_s B(1 - \nu^2)}{E} I_s$

$$\frac{L}{B} = \frac{40}{10} = 4, \quad I_s = 0.62 \ln\left(\frac{L}{B}\right) + 1.12 = 0.62 \ln(4) + 1.12 = 1.98$$

$$\Delta z = \frac{200 \times 5 \times (1 - 0.32^2)}{34.5 \times 10^6} 1.98 = 51.5 \times 10^{-6} \text{ m} = 51.5 \times 10^{-3} \text{ mm}$$

EXERCISES

Theory

7.1 An elastic soil is confined laterally and is axially compressed under drained conditions. In soil mechanics, the loading imposed on the soil is called K_o compression or consolidation. Show that under the K_o condition,

$$\frac{\sigma'_x}{\sigma'_z} = \frac{\nu'}{1 - \nu'}$$

where ν' is Poisson's ratio for drained condition.

7.2 Show that if an elastic, cylindrical soil is confined in the lateral directions, the constrained elastic modulus is

$$E_c = \frac{E'(1 - \nu')}{(1 + \nu')(1 - 2\nu')}$$

where E' = Young's modulus and ν' is Poisson's ratio for drained condition.

7.3 The increase in porewater pressure in a saturated soil is given by $\Delta u = \Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3)$. Show that if the soil is a linear, isotropic, elastic material, $A = \frac{1}{3}$ for the axisymmetric condition.

Problem Solving

Stresses and strains

7.4 A cylindrical soil, 75 mm in diameter and 150 mm long, is axially compressed. The length decreases to 147 mm and the radius increases by 0.3 mm. Calculate:

(a) The axial and radial strains

(b) The volumetric strains

(c) Poisson's ratio

7.5 A cylindrical soil, 75 mm in diameter and 150 mm long, is radially compressed. The length increases to 153 mm and the radius decreases to 37.2 mm. Calculate:

(a) The axial and radial strains

(b) The volumetric strains

7.6 A soil, 100 mm × 150 mm × 20 mm high, is subjected to simple shear deformation (see Figure 7.3). The normal force in the Z direction is 1 kN and the shear force is 0.5 kN. The displacements at the top of the soil in the X and Z directions are $\Delta x = 1$ mm and $\Delta z = 1$ mm. Calculate:

(a) The shear and normal stresses

(b) The axial and shear strains

Elastic deformation

7.7 A long embankment is located on a soil profile consisting of 4 m of medium clay followed by 8 m of medium-to-dense sand on top of bedrock. A vertical settlement of 5 mm at the center of the embankment was measured during construction. Assuming all the settlement is elastic and occurs in the medium clay, determine the average stresses imposed on the medium clay under the center of the embankment using the elastic equations. The elastic parameters are $E = 15$ MPa and $\nu = 0.3$. (*Hint*: Assume the lateral strain is zero.)

7.8 An element of soil (sand) behind a retaining wall is subjected to an increase in vertical stress of 5 kPa and a decrease in lateral stress of 25 kPa. Determine the change in vertical and lateral strains, assuming the soil is a linearly elastic material with $E = 20$ MPa and $\nu = 0.3$.

Stress state using Mohr's circle

7.9 A cylindrical specimen of soil is compressed by an axial principal stress of 500 kPa and a radial principal stress of 200 kPa. Plot Mohr's circle of stress and determine (a) the maximum shear stress and (b) the normal and shear stresses on a plane inclined at 30° counterclockwise from the horizontal.

7.10 A soil specimen (100 mm × 100 mm × 100 mm) is subjected to the forces shown in Figure P7.10. Determine (a) the magnitude of the principal stresses, (b) the orientation of the principal stress plane to the horizontal, (c) the maximum shear stress, and (d) the normal and shear stresses on a plane inclined at 20° clockwise to the horizontal.

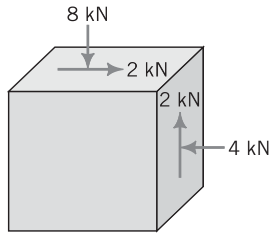


FIGURE P7.10

7.11 The initial principal stresses at a certain depth in a clay soil are 100 kPa on the horizontal plane and 50 kPa on the vertical plane. Construction of a surface foundation induces additional stresses consisting of a vertical stress of 45 kPa, a lateral stress of 20 kPa, and a counterclockwise (with respect to the horizontal plane) shear stress of 40 kPa. Plot Mohr's circle (1) for the initial state of the soil and (2) after construction of the foundation. Determine (a) the change in magnitude of the principal stresses, (b) the change in maximum shear stress, and (c) the change in orientation of the principal stress plane resulting from the construction of the foundation.

Effective stress

7.12 Plot the distribution of total stress, effective stress, and porewater pressure with depth for the soil profile shown in Figure P7.12. Neglect capillary action and pore air pressure.

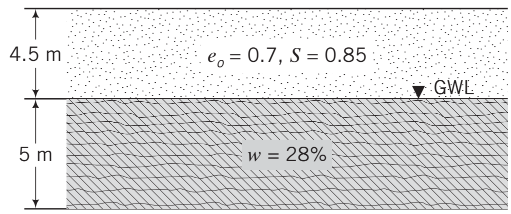


FIGURE P7.12

7.13 If the groundwater in problem 7.12 were (a) to rise to the surface, (b) to rise 2 m above the surface, and (c) to rapidly decrease from 2 m above the surface to 1 m below its present level, determine and plot the distributions of vertical effective and total stresses and porewater pressure with depth.

7.14 At what depth would the vertical effective stress in a deep deposit of clay be 100 kPa, if $e = 1.1$? The groundwater level is at 1 m below ground surface and $S = 95\%$ above the groundwater level. Neglect pore air pressure.

7.15 A culvert is to be constructed in a bed of sand ($e = 0.5$) for drainage purposes. The roof of the culvert will be

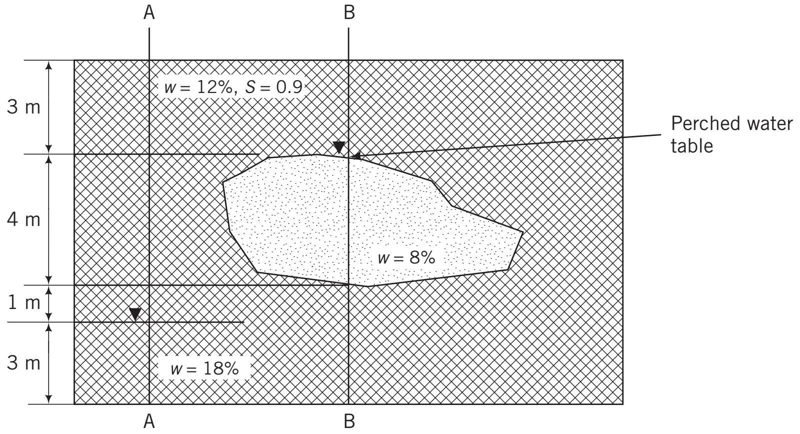


FIGURE P7.18

located 3 m below ground surface. Currently, the groundwater level is at ground surface. But, after installation of the culvert, the groundwater level is expected to drop to 2 m below ground surface. Calculate the change in vertical effective stress on the roof of the culvert after installation. You can assume the sand above the groundwater level is saturated.

7.16 A soil profile consists of 10-m-thick fine sand of effective size 0.15 mm above a very thick layer of clay. Groundwater level is at 3 m below the ground surface. (a) Determine the height of capillary rise, assuming that the equivalent capillary tube diameter is 10% of the effective size and the sand surface is similar to smooth glass. (b) Plot the distribution of vertical effective stress and porewater pressure with depth if the void ratio of the sand is 0.6 and the degree of saturation is 90%. Neglect pore air pressure.

7.17 A soil profile consists of a clay layer underlain by a sand layer, as shown in Figure P7.17. If a tube is inserted into the bottom sand layer and the water level rises to 1 m above the ground surface, determine the vertical effective stresses and porewater pressures at A, B, and C. If K_o is 0.5, determine the lateral effective and lateral total stresses at A, B, and C. What is the value of the porewater pressure at A to cause the vertical effective stress there to be zero?

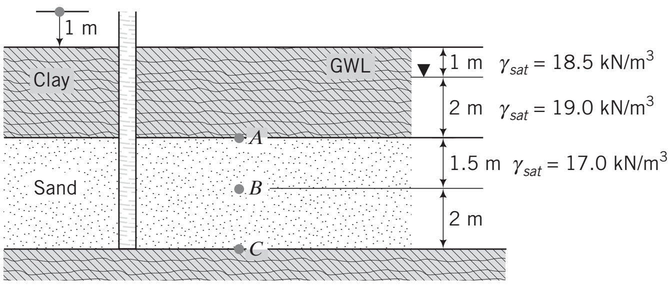


FIGURE P7.17

7.18 A soil section, as shown in Figure P7.18, has a perched groundwater level. Plot the vertical total and effective stresses and porewater pressures with depth along sections A-A and B-B. Neglect pore air pressure.

Stresses in soil from surface loads

7.19 A pole is held vertically on a soil surface by three equally spaced wires tied to the top of the pole. Each wire has a tension of 1 kN and is inclined at 45° to the vertical. Calculate:

(a) The increase in vertical stress at a depth 1 m below the surface

(b) The amount of elastic settlement below the axis of the pole if $E = 40$ MPa and $\nu = 0.45$

7.20 A rectangular foundation 4 m × 6 m (Figure P7.20) transmits a stress of 100 kPa on the surface of a soil deposit. Plot the distribution of increases of vertical stresses with depth under points A, B, and C up to a depth of 20 m. At what depth is the increase in vertical stress below A less than 10% of the surface stress?

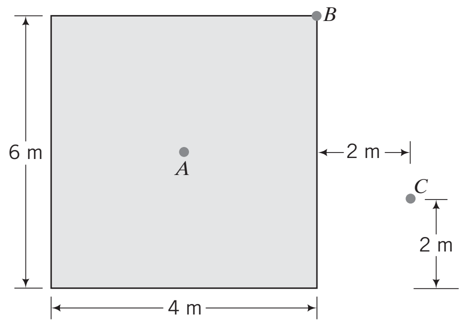


FIGURE P7.20

7.21 Determine the increase in vertical stress at a depth of 5 m below the centroid of the foundation shown in Figure P7.21.

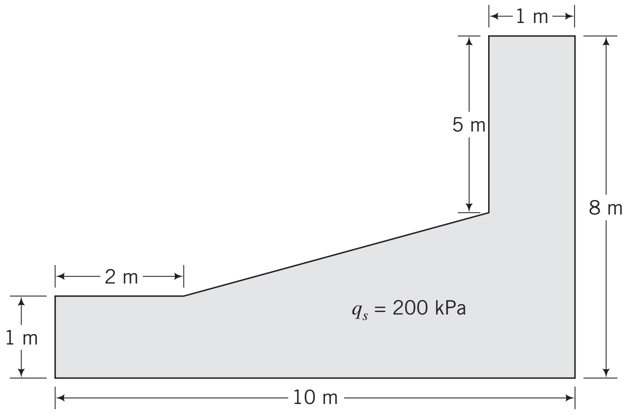


FIGURE P7.21

7.22 Three foundations are located next to each other (Figure P7.22). Determine the stress increases at A, B, and C at a depth of 2 m below the ground surface.

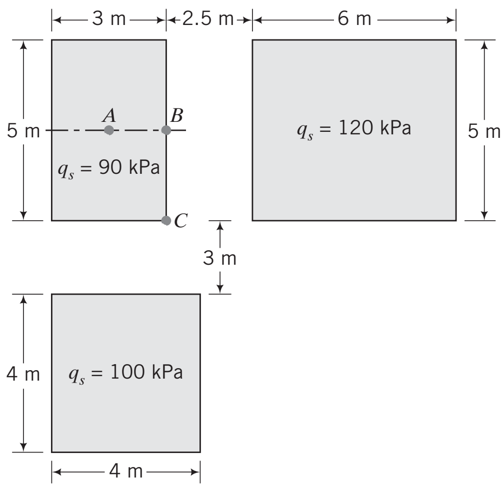


FIGURE P7.22

Practical

7.23 You are the geotechnical engineer for a proposed office building in a densely clustered city. The office building will be constructed adjacent to an existing office complex. The soil at the site is a deposit of very dense sand with $E = E' = 45$ MPa and $\nu = \nu' = 0.3$. The sand rests on a deep deposit of dense gravel. The existing high-rise complex is founded on a concrete slab, 100 m $\times$ 120 m, located at 2 m below ground surface, and transmits a load of 2400 MN to the soil. Your office foundation is 50 m $\times$ 80 m and transmits a load of 1000 MN. You also intend to locate your foundation at 2 m below ground level. The front of your building is aligned with the existing office complex, and the side distance is 0.5 m. The lesser dimension of each building is the frontal dimension. The owners of the existing building are concerned about possible settlement of their building due to your building. You are invited to a meeting with your client, the owners of the existing building, and their technical staff. You are expected to determine what effects your office building would have on the existing building. You only have one hour to make the preliminary calculations. You are expected to present the estimated increase in stresses and settlement of the existing office complex will due to the construction of your office building. Prepare your analysis and presentation.

7.24 A house (plan dimension: 10 m $\times$ 15 m) is located on a deep deposit of sand mixed with some clays and silts. The groundwater at the time the house was completed was 0.5 m below the surface. A utility trench, 4 m deep, was later dug on one side along the length of the house. Any water that accumulated in the trench was pumped out so that the trench remained dry. Because of a labor dispute, work on laying the utility in the trench ceased, but the open trench was continuously pumped. Sometime during the dispute, the owners noticed cracking of the walls in the house. Assuming $S = 0.9$ for the soil above the groundwater level and a void ratio of 0.7, write a short, preliminary technical report (not more than a page) to the owner explaining why the cracks developed. The walls of the trench did not move laterally. The hydraulic conductivities of the soil in the vertical and horizontal directions are 0.5×10^{-4} cm/sec and 2.3×10^{-4} cm/sec, respectively. The calculations should be in an appendix to the report. Neglect pore air pressure.

7.25 A farmer requires two steel silos to store wheat. Each silo is 8 m in external diameter and 10 m high. The foundation for each silo is a circular concrete slab thickened at the edge. The total load of each silo filled with wheat is 9552 kN. The soil consists of a 30 m thick deposit of medium clay above a deep deposit of very stiff clay. The farmer desires that the silos be a distance of 2 m apart and hires you to recommend whether this distance is satisfactory. The area is subjected to a gust wind speed of 100 kilometers per hour.

(a) Plot the distribution of vertical stress increase at the edges and at the center of one of the silos up to a depth of 16 m. Assume the soft clay layer is semi-infinite and the concrete slab is flexible. Use a spreadsheet to tabulate and plot your results.

(b) Calculate the elastic settlement at the surface of one of the silos at the edges and at the center, assuming $E = 30$ MPa and $\nu = 0.7$.

(c) Calculate the elastic tilt of the foundation of one of the silos and sketch the deformed shape of the foundation slab.

(d) Would the tops of the silos touch each other based on the elastic tilt? Show calculations in support of your answer.

(e) What minimum separation distance would you recommend? Make clear sketches to explain your recommendation to the owner.

(f) Explain how the wind would alter the stress distribution below the silos. (*Hint:* Use the charts in Appendix B.)

7.26 A water tank, 15 m in diameter and 10 m high, is proposed for a site where there is an existing pipeline (Figure P7.26). Plot the distribution of vertical and lateral stress increases imposed by the water tank on the pipeline along one-half the circumference nearest to the tank. The empty tank's weight (deal load) is 350 kN. Assume the water tank is filled to its capacity.

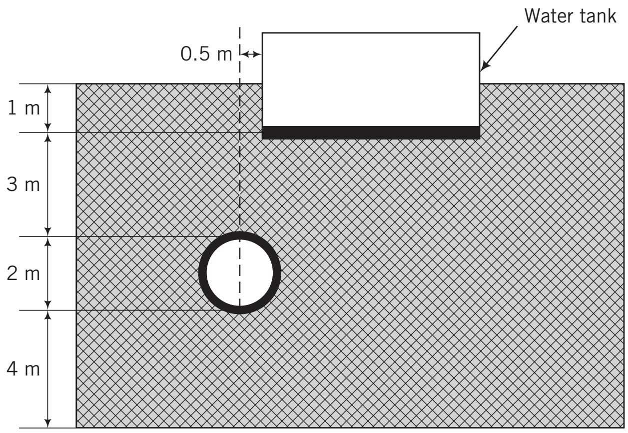


FIGURE P7.26

7.27 A developer proposes to construct an apartment building near an existing retaining wall (Figure P7.27). The building of width 12 m and length 300 m (parallel to the retaining wall) will impose a surface stress of 150 kPa. In the preliminary design, the long edge of the building is located 1 m from the wall. Assume the building load can be treated as a strip load.

(a) Plot the distribution of the lateral force increase with depth up to a depth of 4 m.

(b) What is the maximum value of the lateral force increase, and where does it occur?

(c) If the embedment depth of the retaining wall is 4 m, calculate the maximum additional moment about the base of the wall (point *O* in Figure P7.27) from constructing the building.

(d) What advice would you give to the developer regarding how far the apartment should be located from the existing retaining wall?

7.28 A 10-m-thick, water-bearing sand layer (permeable), called an aquifer, is sandwiched between a 6-m clay layer (impermeable) at the top and bedrock (impermeable) at the bottom. The groundwater level is at the ground surface. An open pipe is placed at the top of the sand layer. Water in the pipe rises to a height of 5 m above the groundwater level. The water contents of the clay and sand are 52% and 8%, respectively.

(a) Does an artesian condition exist? Why?

(b) Plot the distribution of vertical total and effective stresses, and porewater pressure with depth up to a depth of 10 m.

(c) If K_o of the clay is 0.5 and K_o of the sand is 0.45, plot the distribution of lateral total and effective stresses.

(d) An invert (surface of the bottom arc) level of 4 m from the ground surface is proposed for a water pipe 2 m in diameter. Draw the soil profile and locate the water pipe. Explain any issue (justify with calculations) with locating the water pipe at the proposed invert level.

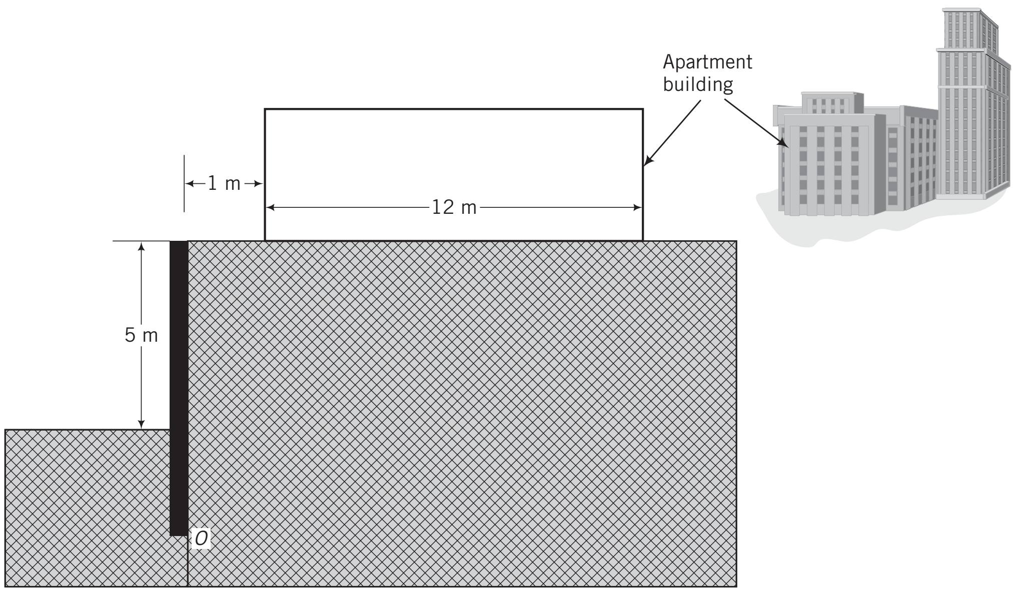


FIGURE P7.27

CHAPTER 8

STRESS PATH

8.0 INTRODUCTION

In this chapter, you will learn about stress paths and their importance in understanding soil behavior under loads. When you complete this chapter, you should be able to:

- Calculate stresses and strains invariants.
- Plot stress paths for common soil loadings.
- Understand the difference between total and effective stress paths.

Importance

The stresses and strains discussed in Chapter 7 are all dependent on the axis system chosen. We have arbitrarily chosen the Cartesian coordinate and the cylindrical coordinate systems. We could, however, define a set of stresses and strains that are independent of the axis system. Such a system, which we will discuss in this chapter, will allow us to use generalized stress and strain parameters to analyze and interpret soil behavior. In particular, we will be able to represent a three-dimensional system of stresses and strains by a two-dimensional system.

We have examined how applied surface stresses are distributed in soils as if soils were linear, isotropic, elastic materials. Different structures will impose different stresses and cause the soil to respond differently. For example, an element of soil under the center of an oil tank will experience a continuous increase or decrease in vertical stress while the tank is being filled or emptied. However, the soil near a retaining earth structure will suffer a reduction in lateral stress if the wall moves out. These different loading conditions would cause the soil to respond differently. Therefore, we need to trace the history of stress increases/decreases in soils to evaluate possible soil responses, and to conduct tests that replicate the loading history of the in situ soil. Figure 8.1 shows an excavation near a high-rise building. The

FIGURE 8.1 An excavation near a high-rise building. The applied loading history of soil elements at the same depth at the edge of the excavation and at, say, the center of the building will be different.