Al-Muthanna University

College of Engineering

Department of Civil Engineering

CE301- Theory of Structures

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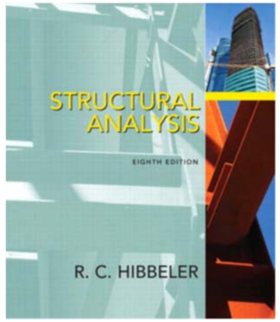
References



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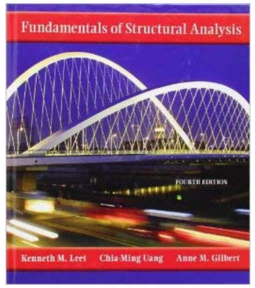

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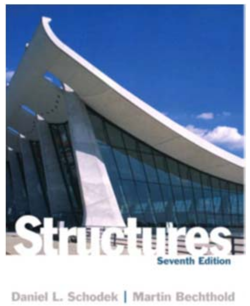

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Syllabus

Theory of Structures-I

Part One: Fundamentals of Structures

1. Introduction
2. Loads on Structures
3. Stability and Determinacy of Structures

Part Two: Analysis of Statically Determinate Structures

4. Analysis of Truss Structures
5. Analysis of Frame Structures
6. Analysis of Arch Structures
7. Moving Loads and Influence Lines of Statically Determinate Structures

Part Three: Deflection of Statically Determinate Structures

8. Unit Load Method
9. Least Work Method
10. Conjugate-Beam Method

Introduction

What is a STRUCTURE?

A structure has been defined as:

"**any assemblage of materials which is intended to sustain loads**"
(i.e. sustain mechanical forces without breaking)

These loads may be due to:

- Gravity
- Wind
- Ground shaking
- Impact
- Temperature, or other environmental sources

Introduction

Where do Loads Come from?

Applications of structures In Civil Engineering?

Structures include a wide variety of systems that are built to serve some specific human needs (for example, habitation, transportation, storage, etc, such as:

- Buildings
- Bridges
- Dams
- Highways
- Etc.

Selected Applications of structures in Civil Engineering

(a) Building

(b) Bridge

(c) Dams

(d) Industrial sheds

(e) Cable ways

(f) Chimneys

Structural Engineering

Structural engineering is the discipline which is concerned with:

1. identifying the loads that a structure may experience over its expected life
2. determining a suitable arrangement of structural members
3. selecting the material and dimensions of the members
4. defining the assembly process, and lastly
5. monitoring the structure as it is being assembled and possibly also over its life.

Factors to be considered in a structure?

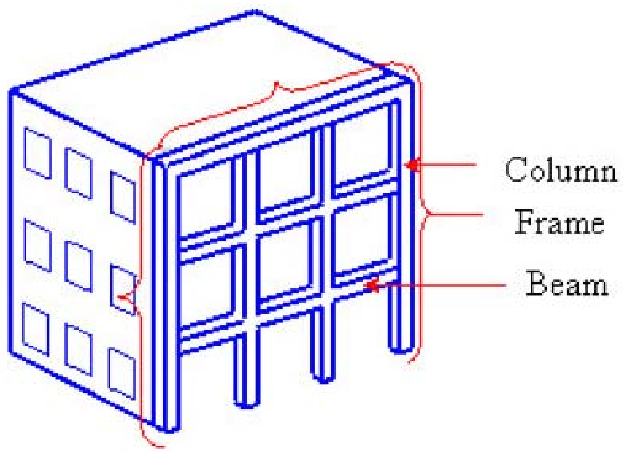
- Safety
- Esthetics
- Serviceability
- Economic & environmental constraints

Points to be understood in a structure?

- Behaviour of a structure while it's subjected to a load
- What is the load itself?
- Space and dimensionality of the structure (size and scale)
- What is the device that channels loads to the ground?

What is the functionality of a structure?

1. Majorly, a structure must functions as a whole unit
2. Secondarily, a structure must function as a group of discrete elements in which each element perform a separate function (Slab, Beam, column, etc.).



Note:

If these elements are located or attached to each other in such a way that the resultant configuration of all elements **DOES NOT function as a whole unit** in channeling the applied load to the ground, **the configuration CANNOT be said a structure**.

What is the Structural Design?

Structural design includes the arrangement and proportioning of structures and their parts so they will satisfactorily support the loads to which they may be subjected.

More specifically,

- ✓ structural design involves the following:
- ✓ the general layout of the structural system;
- ✓ studies of alternative structural configurations that may provide feasible solutions; consideration of loading conditions;
- ✓ preliminary structural analyses and design of the possible solutions;
- ✓ the selection of a solution;
- ✓ and the final structural analysis and design of the structure.

Structural design also includes the preparation of design drawings.

Structural Design?

This design process is both creative and technical and requires a fundamental knowledge of material properties and the laws of mechanics which govern material response.

Once a preliminary design of a structure is proposed, the structure must then be ***analyzed*** to ensure that it has its ***required stiffness and strength***

Structural Analysis?

To analyze a structure properly

1. idealizations must be made as to how the members are supported and connected together.
2. The loadings are determined from codes and local specifications.
3. The forces in the members and their displacements are found using the theory of structural analysis.

The results of this analysis then can be used to:

1. redesign the structure
2. accounting for a more accurate determination of the weight of the members and their size

Structural Analysis?

The fundamental principles used in structural analysis are Newton's laws of inertia and motion, which are:

1. A body will exist in a state of rest or in a state of uniform motion in a straight
line unless it is forced to change that state by forces imposed on it.

2. The rate of change of momentum of a body is equal to the net applied force.

3. For every action there is an equal and opposite reaction.

These laws of motion can be expressed by the equation ***F = ma***

Where,
F is the summation of all the forces that are acting on the body
m is the mass of the body
a is acceleration of the body

Static Equilibrium?

the system is not accelerating.
The equation of equilibrium thus becomes F = 0

These structures are not moving, as is the case for most civil engineering structures.

Using the principle of static equilibrium, we will study:
1. the forces that act on structures
2. methods to determine the response of structures to these forces.

Response means the displacement of the system and the forces that occur in each component of the system. (i.e. Displacement response and Force response).

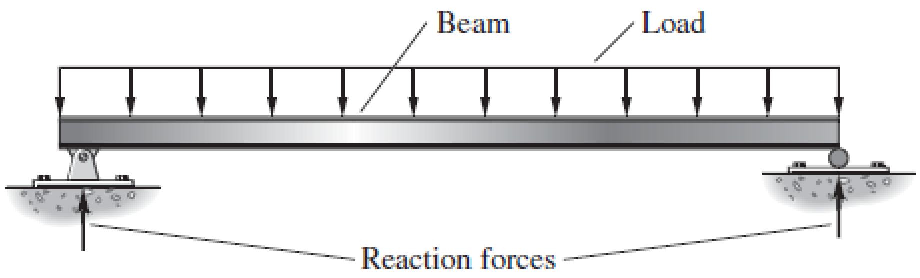
Structural Forces

- The applied loads are the known loads that act on a structure.
- They can be the result of the structure's own weight, occupancy loads, environmental loads, and so on.

- The reactions are the forces that the supports exert on a structure.
- They are considered to be part of the external forces applied and are in equilibrium with the other external loads on the structure.

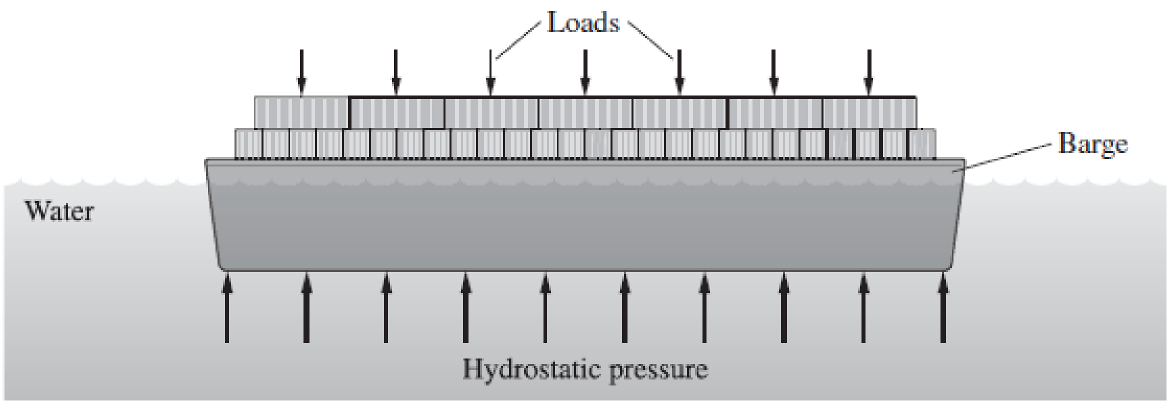
Structural Forces

The beam shown below is supporting a uniformly distributed gravity load and is itself supported by upward reactions at its ends.



(a) A simple beam

The barge below is carrying a group of containers on its deck. It is in turn supported by a uniformly distributed hydrostatic pressure provided by the water beneath.



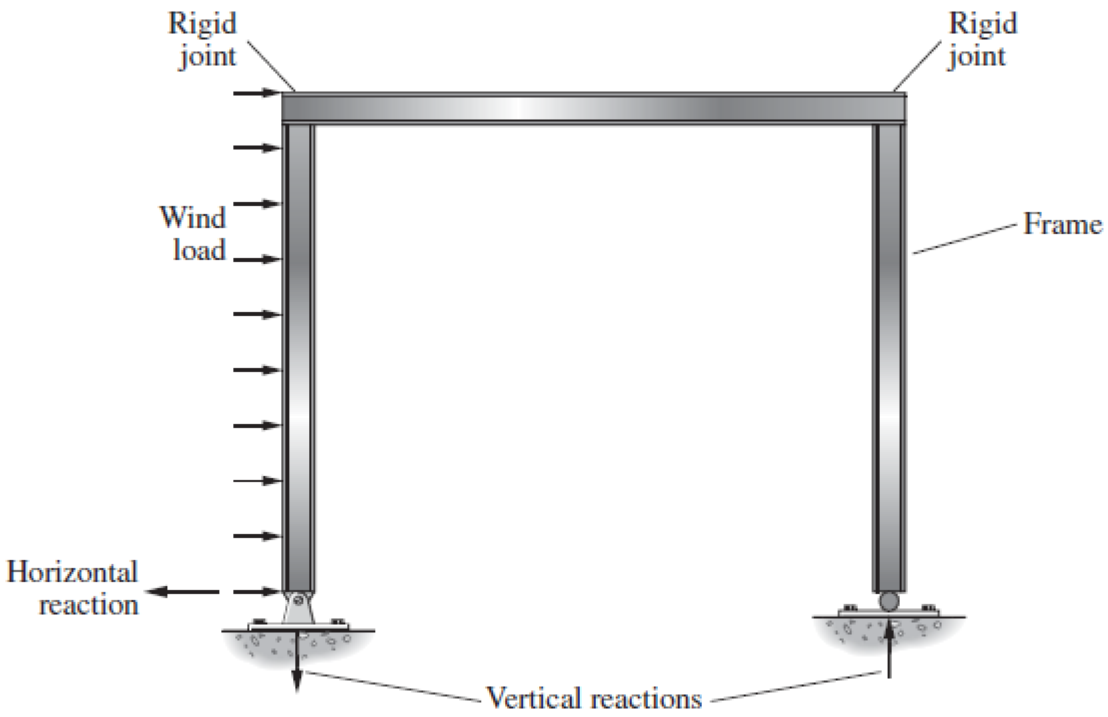
(b) Forces on a barge

Structural Forces

The Figure below shows a building frame subjected to a lateral wind load.

This load tends to overturn the structure, thus requiring an upward reaction at the right-hand support and a downward one at the left-hand support.

These forces create a couple that offsets the effect of the wind force.



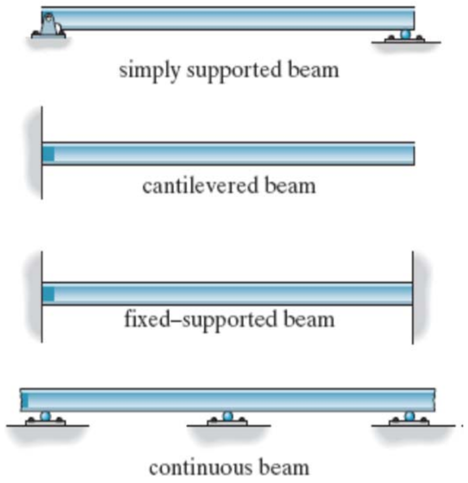
(c) A portal frame

Types of Structural Elements

It is important for a structural engineer to recognize the various types of elements composing a structure and to be able to classify structures as to their form and function.

Beams:

- Beams are usually straight horizontal members used primarily to carry vertical loads.
- Beams pick up the loads that are applied transverse to their length and transfer the loads to the supporting vertical members (Columns or walls).



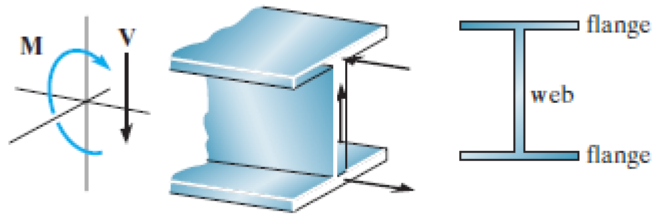
- **<u>BEAMS CARRY LOADS BY BENDING</u>, hence** Beams are primarily designed to resist bending moment.

- If beams **are short and carry large loads**, the internal shear force may become large and this force may govern their design.

Types of Structural Elements, **Metal Beams**

For Metal Beams

The efficient cross section is:



Wide Flange Section

Flanges resist the applied moment **M**
Web in resists the applied shear **V**.

For Short Beam

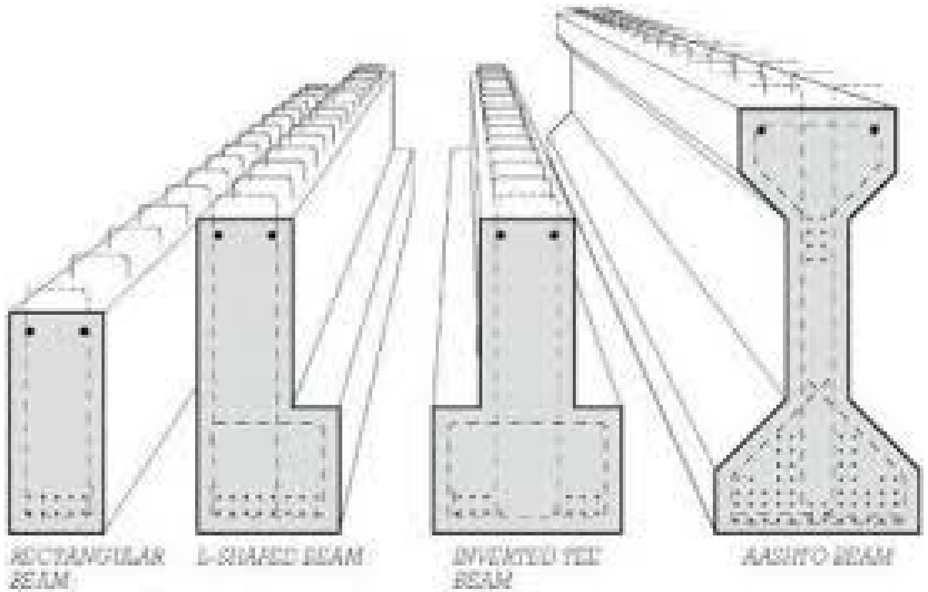
A cross section having tapered flanges is sometimes selected.

For Very Large Span Beam and the Large Applied Load

Plate girder section is used. This member is fabricated by using a large plate for the web and welding or bolting plates to its ends for flanges

Types of Structural Elements, **Concrete Beams**

Common cross sections of Concrete Beams



Note: Concrete beams generally have rectangular cross sections.

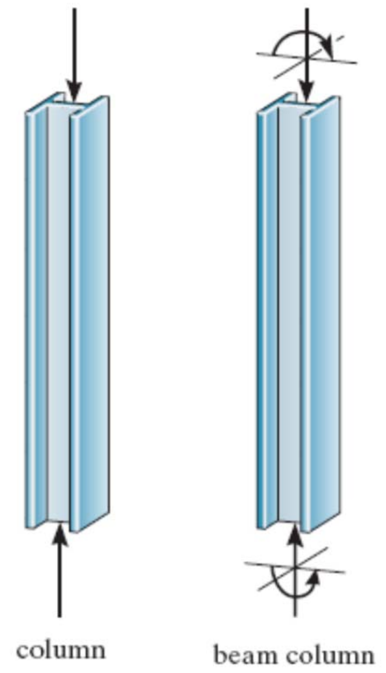
Precast concrete beams or girders:

Fabricated at a shop and then transported to the job site

Types of Structural Elements

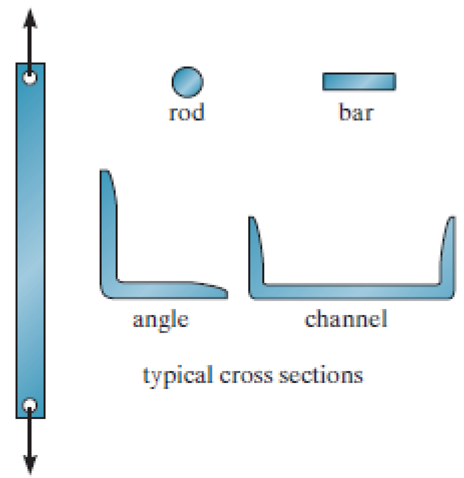
Columns:

Carry, majorly, axial load from beam or slab to the foundation. Columns may also carry moments (Beam-Column).



Tie Rods:

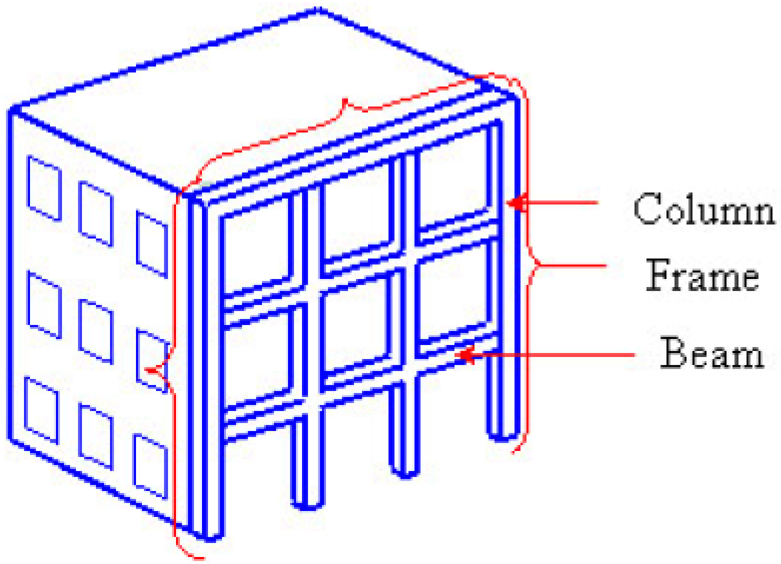
Structural members subjected to a tensile force. Due to the nature of this load (Tensile Force), these members are rather slender, and are often chosen from rods, bars,
angles, or channels

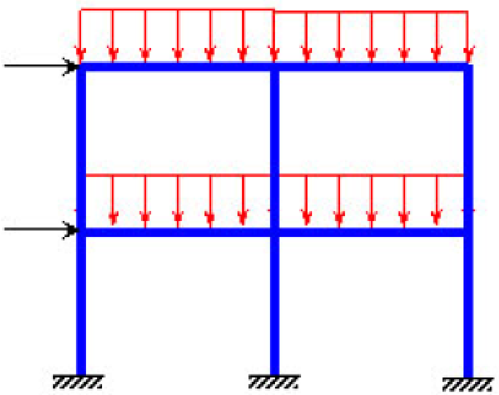


Types of Structures

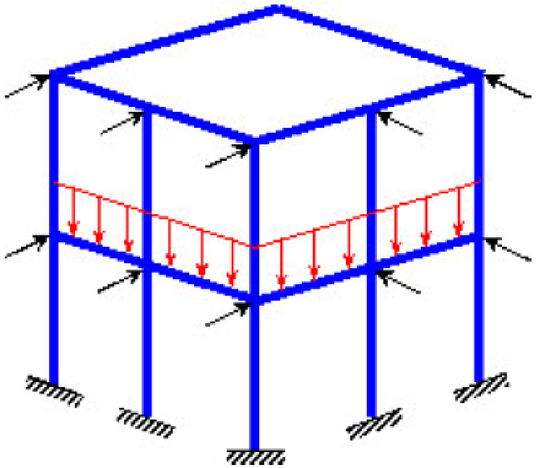
Frames:

Similar to Beam-Column members BUT has a different structural action due to rigid joints.





(a) 2-dimensional structure



(b) 3-dimensional structure

Types of Structures

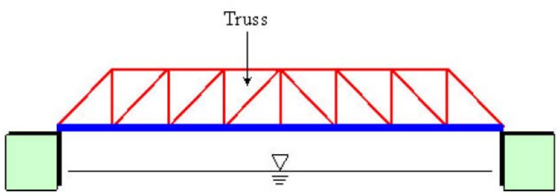
Trusses:

Used when:

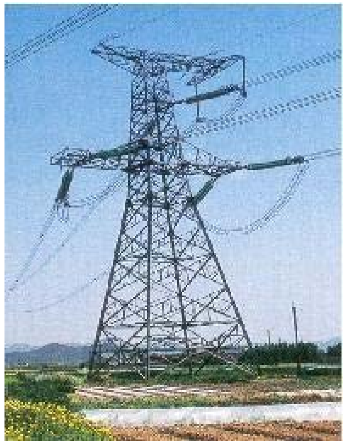
- the span of a structure is required to be large
- its depth is not an important criterion for design

➢ Trusses consist of slender elements, usually arranged in triangular fashion.

Planar Trusses are composed of members that lie in the same plane and
are frequently used for bridge and roof support,



space Trusses have members extending in three dimensions and are suitable for derricks and towers.



Types of Structures

Cables:

Used to span long distance

Cables are usually flexible and carry their loads in tension. They are commonly used to support bridges

Cables support their loads in tension.

Arches:

Used to span long distance

Arches are usually rigid and carry their loads in compression. They are commonly used to support bridges.

Arches support their loads in compression.

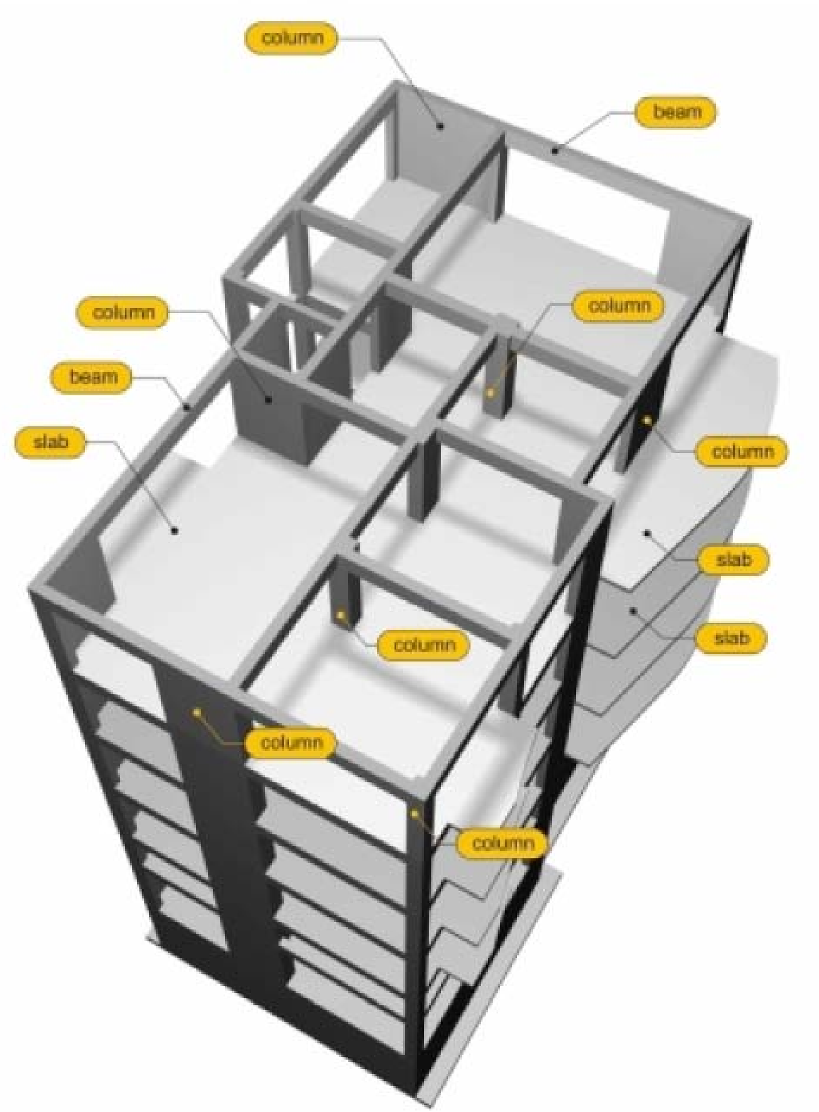
Types of Structures

Surface Structures:

Are made from a material having a very small thickness compared to its other dimensions.

- ***Flexible surface structures*** made from flexible materials like tent or air-inflated structure.
- ***Flexible surface structures*** act as membrane that is subjected to pure tension.

- ***Rigid surface structures*** is made from rigid material such as reinforced concrete.
- ***Rigid surface structures*** may be shaped as folded plates or cylinders, and are referred to as *thin plates* or *shells.*



Building Structure is a group of connected structural elements

THANK YOU

Design Loads on Structures

Gravity Loads

Design Process

Loads

↓

Structural forms

↓

Elements carrying primary loads

↓

Various supporting members

↓

Foundation

Loads

In order to design a structure, it is necessary to first specify the loads that act on it

How to specify the design loads?

The design loading for a structure is often specified in codes.

What is a code?

The code is a set of technical specifications and standards to control major details of analysis, design and construction of the structure.

Codes

In general, the structural engineer works with two types of codes:

- **General Building Codes:**

Specify the requirements of governmental bodies for minimum design loads on structures and minimum standards for construction.

- **Design Codes:**

Provide detailed technical standards and are used to establish the requirements for the actual structural design.

Note:

It should be realized that codes provide only a general guide for design. The ultimate responsibility for the design lies with the structural engineer

Codes

TABLE 1–1 Codes

General Building Codes
Minimum Design Loads for Buildings and Other Structures, ASCE/SEI 7-10, American Society of Civil Engineers
International Building Code
Design Codes
Building Code Requirements for Reinforced Concrete, Am. Conc. Inst. (ACI)
Manual of Steel Construction, American Institute of Steel Construction (AISC)
Standard Specifications for Highway Bridges, American Association of State Highway and Transportation Officials (AASHTO)
National Design Specification for Wood Construction, American Forest and Paper Association (AFPA)
Manual for Railway Engineering, American Railway Engineering Association (AREA)

Loads, Dead Load

- **Dead loads** consist of the weights of the various structural members and the weights of any objects that are ***permanently attached to the structure***.

- ***For a building, the dead loads include the weights of:***

✓ Columns
✓ Beams
✓ Girders
✓ floor slab
✓ Roofing
✓ Walls
✓ Windows
✓ Plumbing
✓ Electrical fixtures
✓ Miscellaneous attachments.

Note:
In some cases, a structural dead load can be estimated through experienced dealing with existing structures. For example:

-Steel Framed Structures : 2.9–3.6 kN/m^2
-Reinforced Concrete Buildings: 5.3–6.2 kN/m^2

Loads, Dead Load

TABLE 1–2 Minimum Densities for Design Loads from Materials*

	lb/ft^3	kN/m^3
Aluminum	170	26.7
Concrete, plain cinder	108	17.0
Concrete, plain stone	144	22.6
Concrete, reinforced cinder	111	17.4
Concrete, reinforced stone	150	23.6
Clay, dry	63	9.9
Clay, damp	110	17.3
Sand and gravel, dry, loose	100	15.7
Sand and gravel, wet	120	18.9
Masonry, lightweight solid concrete	105	16.5
Masonry, normal weight	135	21.2
Plywood	36	5.7
Steel, cold-drawn	492	77.3
Wood, Douglas Fir	34	5.3
Wood, Southern Pine	37	5.8
Wood, spruce	29	4.5

*Reproduced with permission from American Society of Civil Engineers *Minimum Design Loads for Buildings and Other Structures*, ASCE/SEI 7-10. Copies of this standard may be purchased from ASCE at www.pubs.asce.org.

Loads, Live Load

- **Live Loads** may be caused by the weights of objects temporarily placed on a structure, moving vehicles, or natural forces.
- **Live Loads** can vary both in their magnitude and location (Moving Loads).

Types of Live Loads

1. Building Loads:

The floors of buildings are assumed to be subjected to uniform live loads, which depend on the purpose for which the building is designed.

TABLE 1–4 Minimum Live Loads*

Occupancy or Use	Live Load psf	Live Load kN/m²	Occupancy or Use	Live Load psf	Live Load kN/m²
Assembly areas and theaters			Residential		
Fixed seats	60	2.87	Dwellings (one- and two-family)	40	1.92
Movable seats	100	4.79	Hotels and multifamily houses		
Garages (passenger cars only)	50	2.40	Private rooms and corridors	40	1.92
Office buildings			Public rooms and corridors	100	4.79
Lobbies	100	4.79	Schools		
Offices	50	2.40	Classrooms	40	1.92
Storage warehouse			Corridors above first floor	80	3.83
Light	125	6.00			
Heavy	250	11.97			

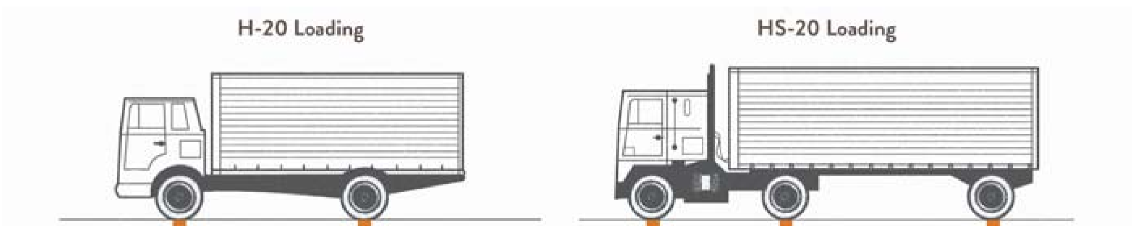
Loads, Types of Live Loads

2. Highway Bridge Loads:

The primary live loads on bridge spans are those due to traffic, and the heaviest vehicle loading encountered is that caused by a series of trucks.

- Specifications for truck loadings on highway bridges are reported in the LRFD Bridge Design Specifications of the American Association of State and Highway Transportation Officials (AASHTO).

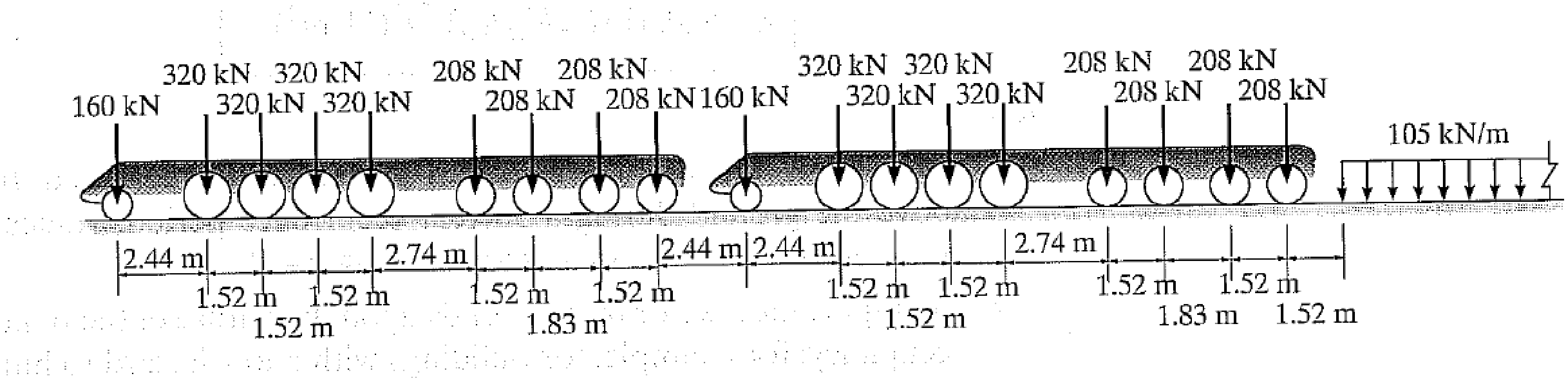
- For two-axle trucks, these loads are designated with an H, followed by the weight of the truck in tons and another number which gives the year of the specifications.
- H15-44: Two-axle truck loading case of 15 ton weight in AASHTO 1944.
- Two-axle trucks plus a one-axle semitrailer (HS)



Loads, Types of Live Loads

3. Railroad Bridge Loads:

- The loadings on railroad bridges are specified in the Specifications for Steel Railway Bridges published by the American Railroad Engineers Association (AREA).
- E loads, as originally devised by Theodore Cooper in 1894, were used for design.
- M loads, devised by B. Steinmann are currently acceptable for design.



E-72 loading

Idealized Structure

Why Idealization?

Engineer will have to analyse a structure lies in a plane and is subjected to a force system that lies in the same plane.

Structural engineer can perform a practical force analysis of the members.

How to idealized a structure?

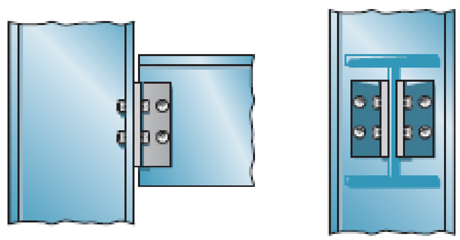
1. Choosing an appropriate analytical model for a structure so that the forces in the structure may be determined with reasonable accuracy.
2. Investigate the structural stability.

Idealized Structure

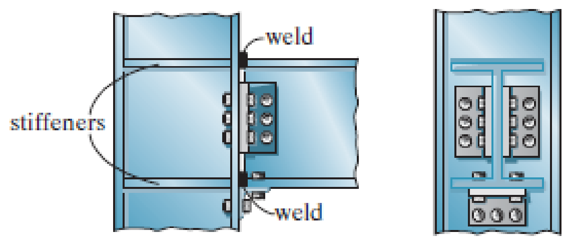
Points to be considered for proper idealization:

- **Support Connections**

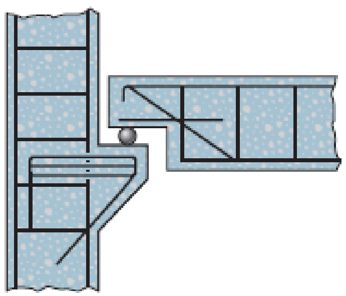
  Pin connection (allows some freedom for slight rotation)
  Roller support (allows some freedom for slight rotation)
  Fixed joint (allows no relative rotation)



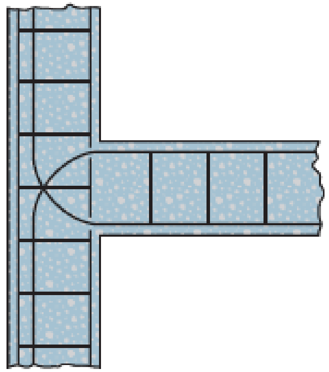
typical "pin-supported" connection (metal)
(a)



typical "fixed-supported" connection (metal)
(b)



typical "roller-supported" connection (concrete)
(a)



typical "fixed-supported" connection (concrete)
(b)

Idealized Structure

Points to be considered for proper idealization:

- **Tributary Loadings**

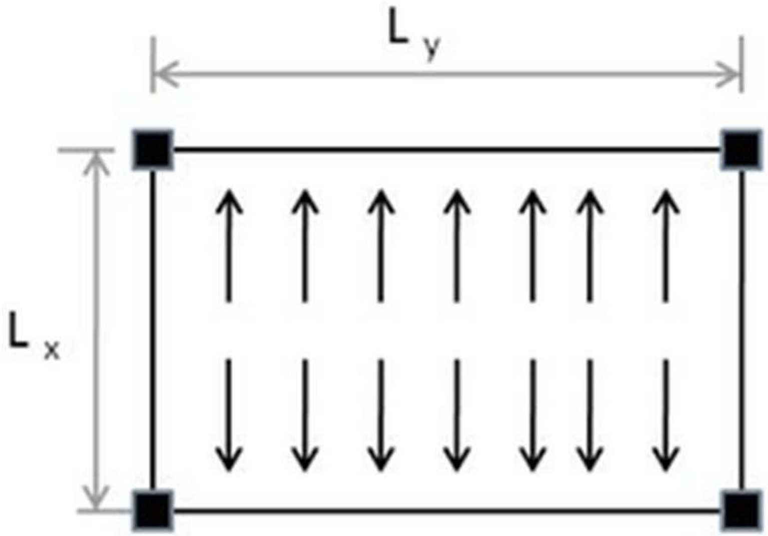
How the load applied on surfaces is transmitted to the various structural elements.

There are 2 ways in which the load on surfaces can transmit to various structural elements:

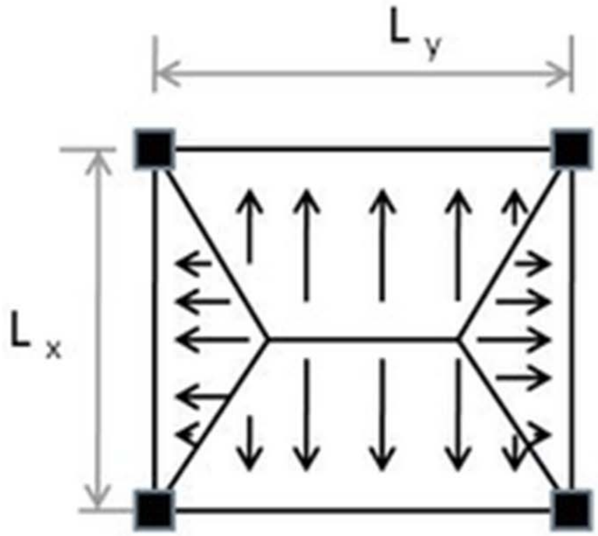
1-One way system

2-Two way system

Load Transfer Mechanism/ Load Path



One-way slab



Two-way slab

Load Transfer Mechanism

Load Path

Building floor slab

⬇

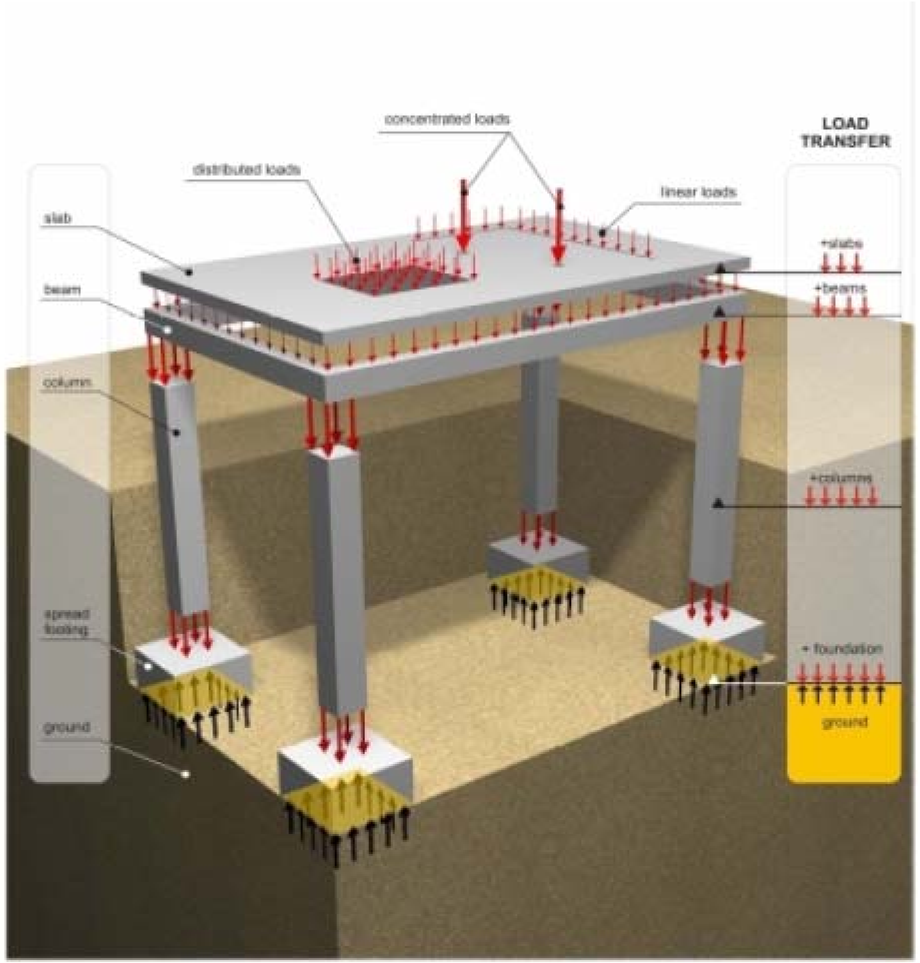
Supporting beams

⬇

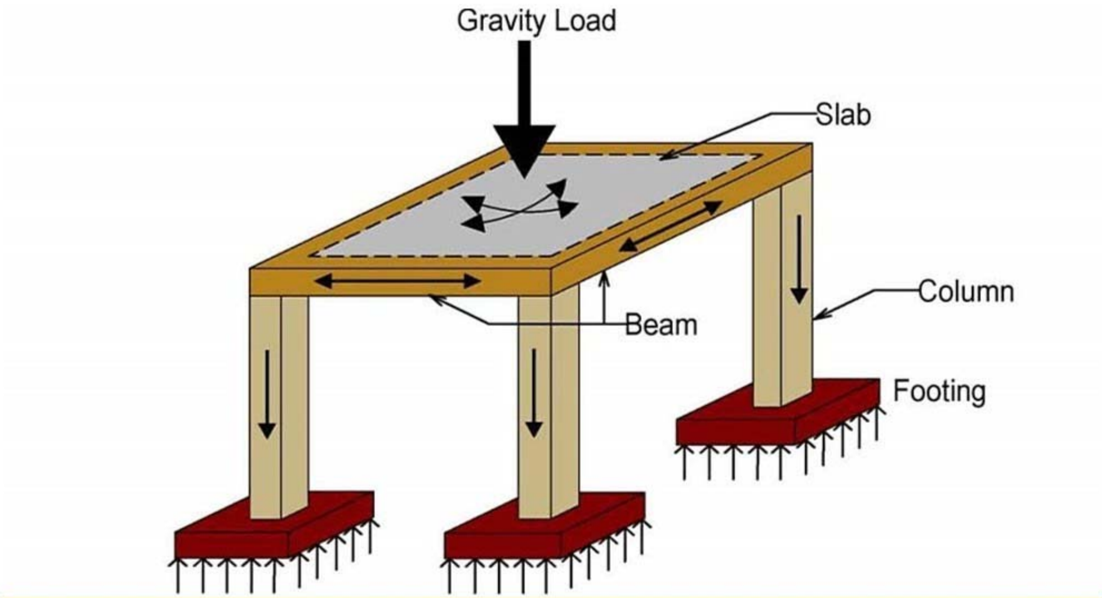
Columns

⬇

Foundation footings



Load path of building



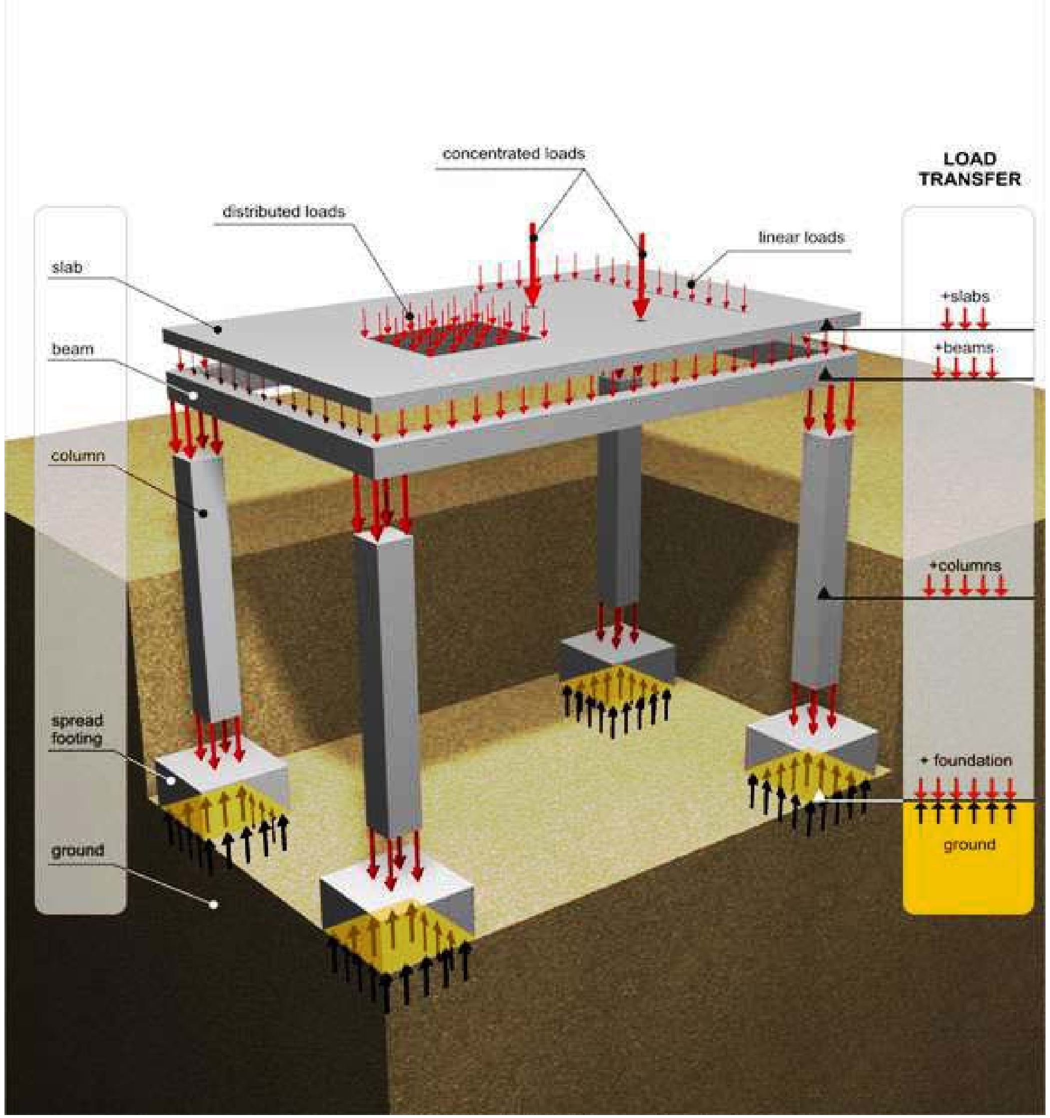
1.The Load Transfer On Building Is From Last Floor Of The building means From Terrace Floor

2.First Load Transfer from Slab

3.Then Slab Transfer Load To Beam

4.Then Beam Transfer Load To Footing

Load Transfer Mechanism/ Load Path



Tributary Loadings

When flat surfaces such as walls, floors, or roofs are supported by a structural frame, it is necessary to determine how the load on these surfaces is transmitted to the various structural elements used for their support.

There are generally two ways in which this can be done. The choice depends on the:

Geometry of the structural system

The material from which it is made

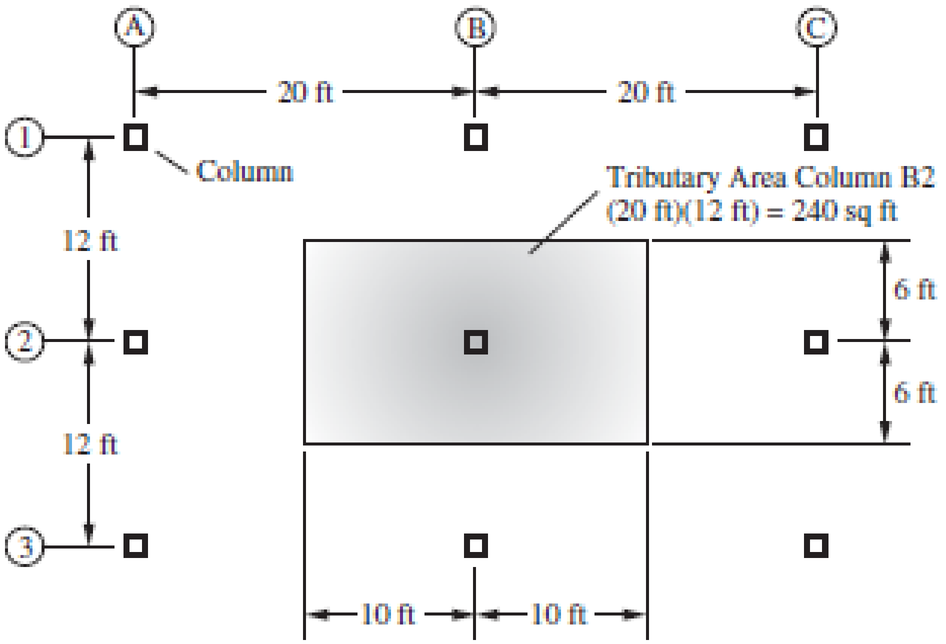
The method of its construction.

Tributary Area

Is the loaded area of a structure that directly contributes to the load applied to a particular member

The tributary area for a member is assumed to extend from the member in question halfway to the adjacent members in each direction.

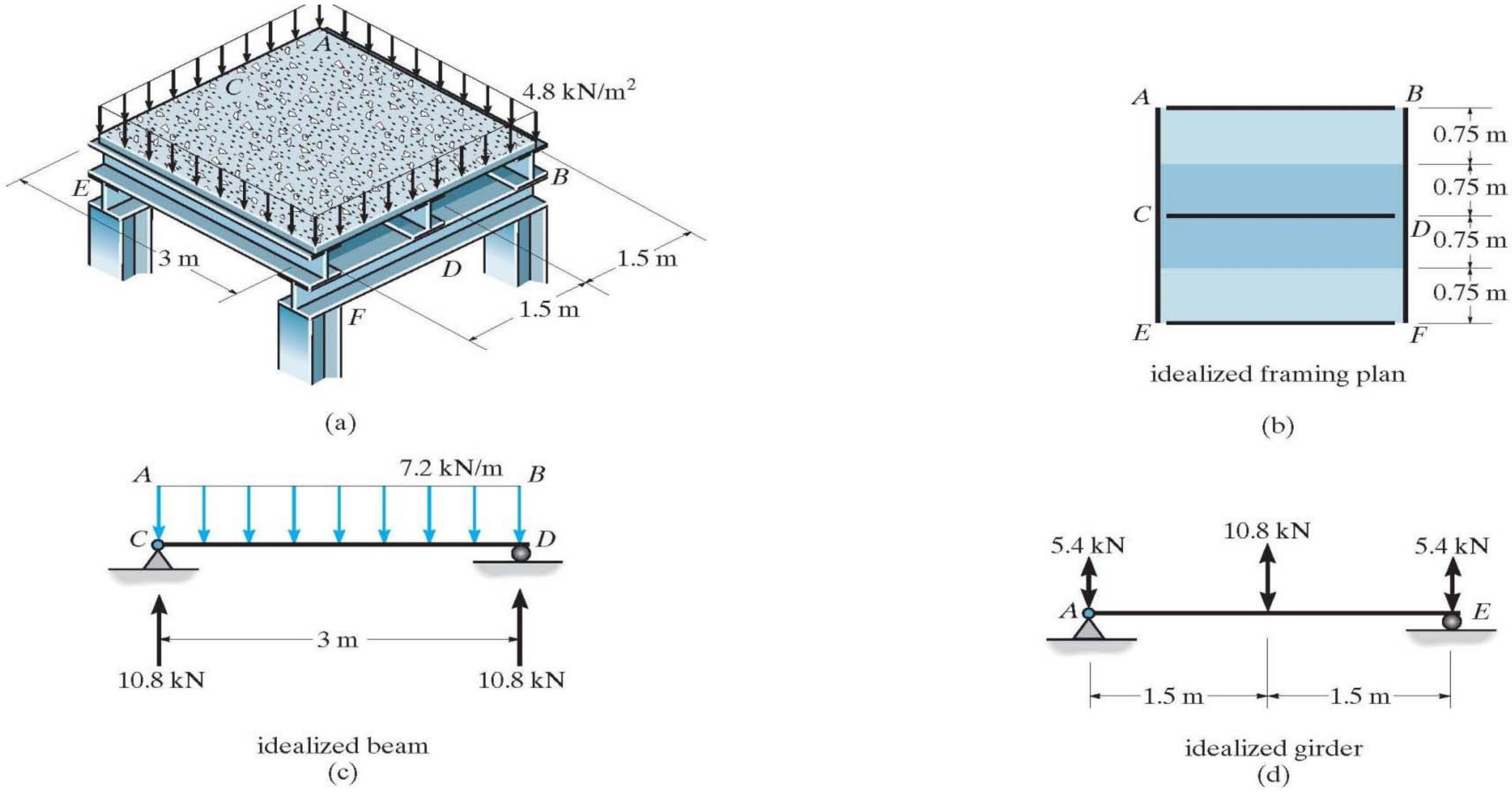
When a building is being analyzed, it is customary for the analyst to assume that the load supported by a member is the load that is applied to its tributary area.



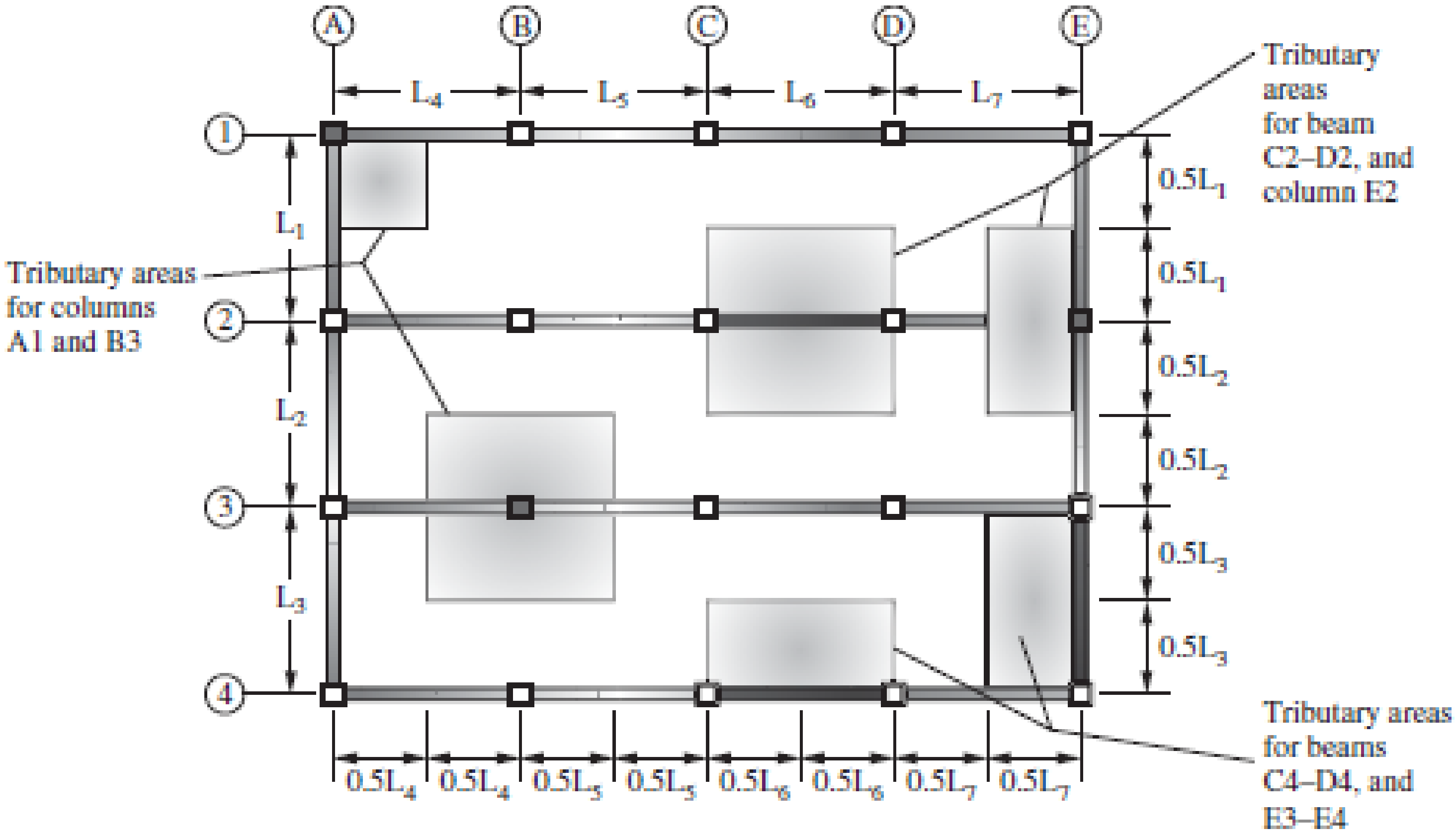
Idealized Structure, Tributary Loadings

One way system:

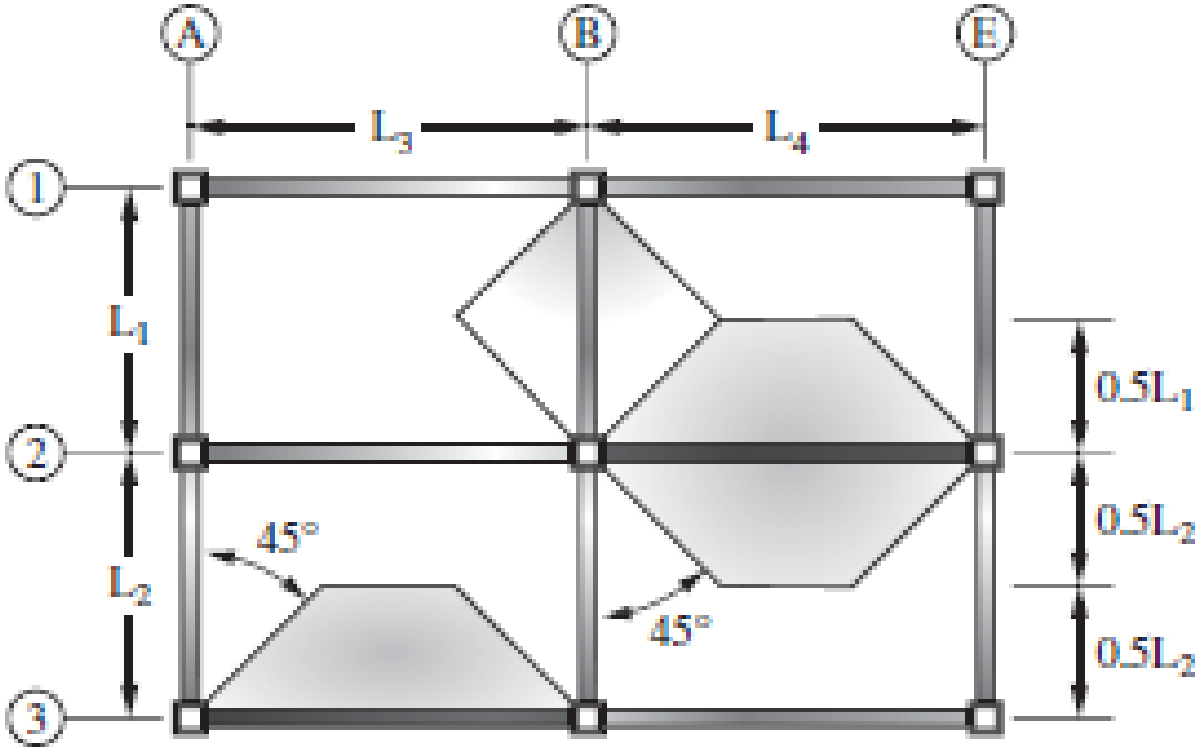
A slab or deck that is supported such that it delivers its load to the supporting members by one-way action.



Tributary Area/ One Way System



Tributary Area/ Two Way System

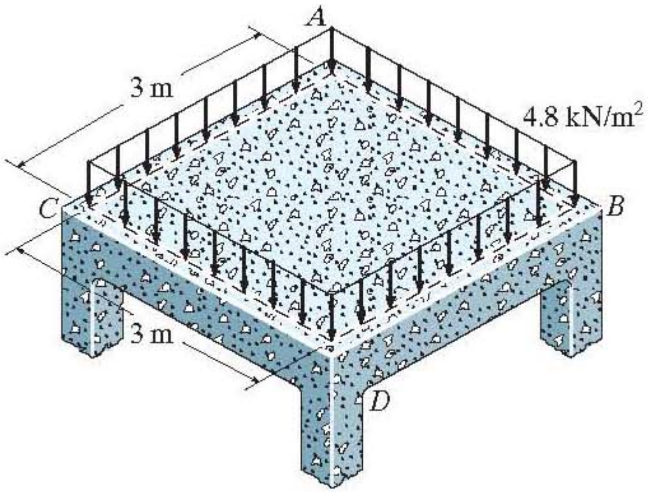


Idealized Structure, Tributary Loadings

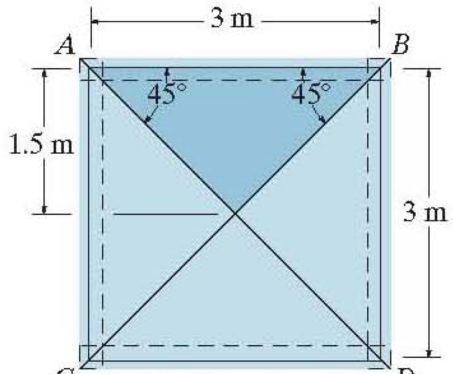
Two way system:

A slab or deck that is supported such that it delivers its load to the supporting members by one-way action.

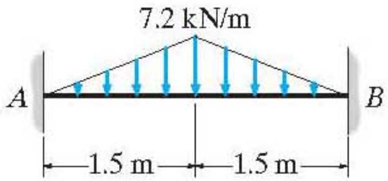
- According to the ACI 318 concrete code: If L2/L1 ≤ 2 the load is assumed to be delivered to the supporting beams and girders in two directions



(a)



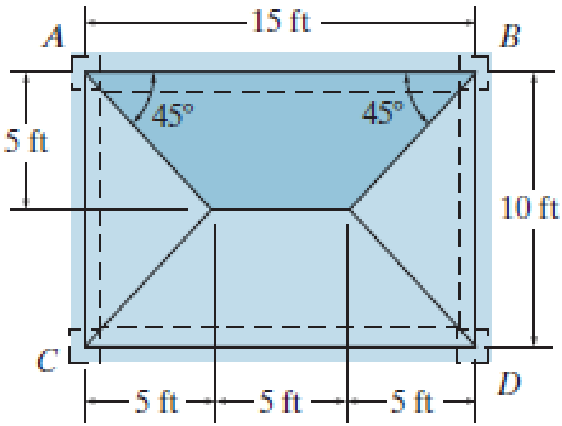
idealized framing plan
(b)



idealized beam
(c)

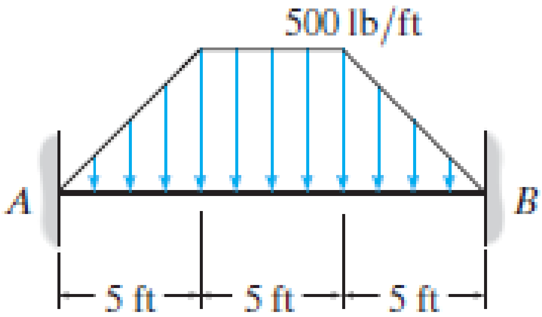
Idealized Structure, Tributary Loadings

Two way system:



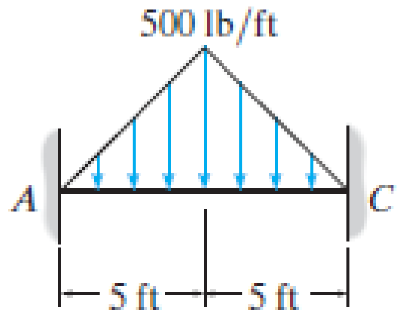
idealized framing plan

(a)



idealized beam

(b)



idealized beam

(c)

Design Loads on Structures

Lateral Loads

Lateral Loads, Wind Load

- In accordance with Bernoulli's theorem for ideal fluid striking an object, the increase in static pressure equals the decrease in dynamic pressure, or

$$q = \frac{1}{2}\rho V^2 \qquad (1)$$

- Where q is the dynamic pressure on the object, ρ is the mass density of air (specific weight w = 0.07651 pcf at sea level and 15^0 C), and V is the wind velocity.

Lateral Loads, Wind Load

- In terms of velocity V in miles per hour, the dynamic pressure q (psf) would be given by

$$q = \frac{1}{2}\rho V^2 = \frac{1}{2}\left(\frac{0.07651}{32.2}\right)\left(\frac{5280}{2600}\right) = 0.0026V^2 \qquad (2)$$

- In design of usual types of buildings, the dynamic pressure q is commonly converted into equivalent static pressure p, which may be expressed as

$$p = qC_eC_gC_p \qquad (3)$$

Lateral Loads, Wind Load

Where

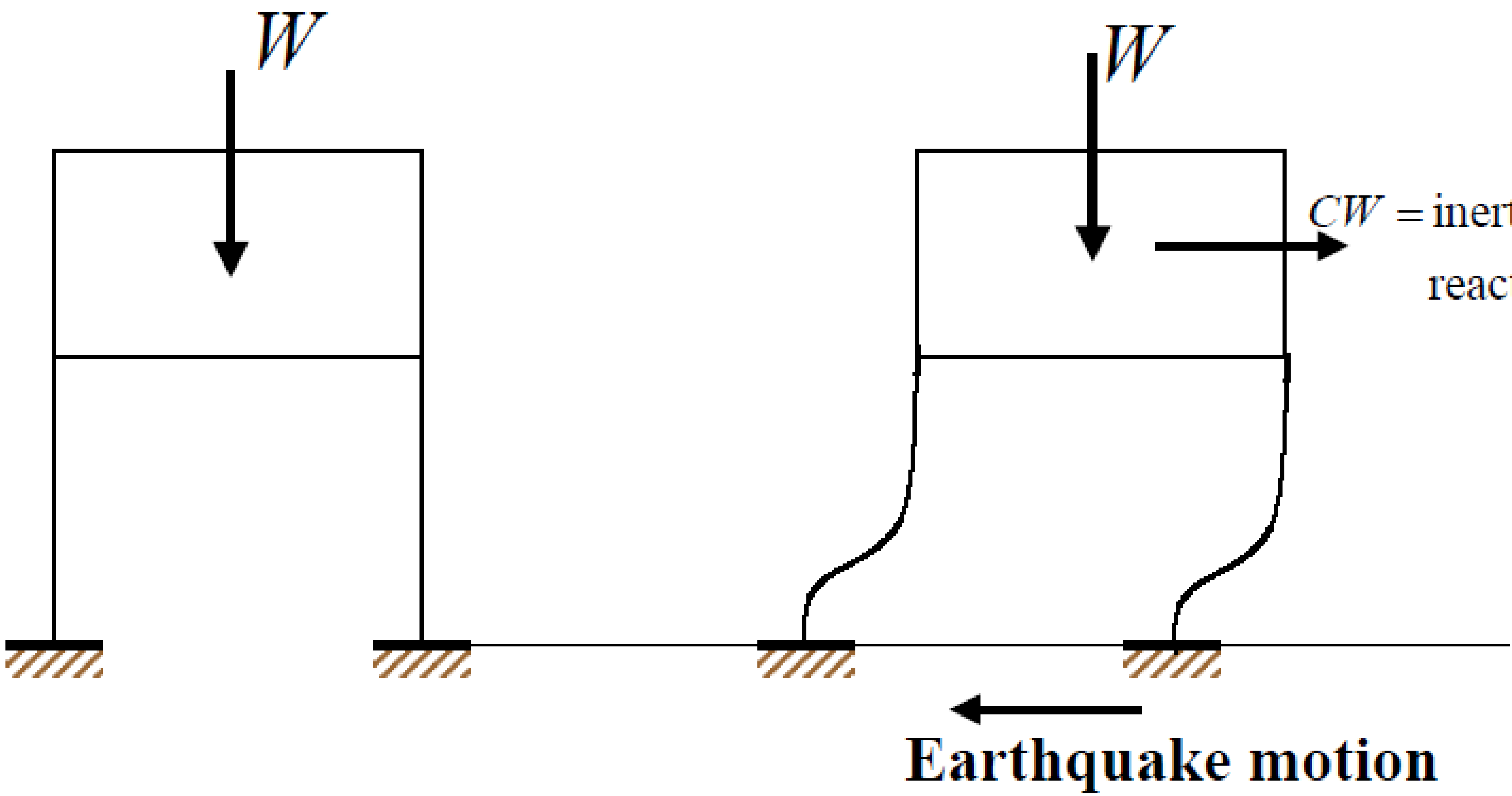
C_e = exposure factor that varies from 1.0 (for 0-40-ft height) to 2.0 (for 740-1200-ft height).

C_g = gust factor, such as 2.0 for structural members and 2.5 for small elements including cladding.

C_p = shape factor for the building as a whole.

- The commonly used wind pressure of 20 psf, as specified by many building codes, correspond to a velocity of 88 mph from Eq. 2.

Lateral Loads, Earthquake Load



(a) At rest

(b) Under horizontal motion from earthquake

Lateral Loads, Earthquake Load

- In the ANSI, the lateral seismic forces *V*, expressed as follows, are assumed to act non-concurrently in the direction of each of the main axes of the structure:

$$V = ZIKCSW \tag{4}$$

Z = seismic zone coefficient (varies from 1/8 to 1).
I = occupancy important factor (varies from 1.5 to 1.25).
K = horizontal force factor (varies from 0.67 to 2.5).
T = fundamental natural period.
S = soil profile coefficient (varies from 1.0 to 1.5).
W = total dead load of the building.

$$C = \frac{1}{15\sqrt{T}} \leq 0.12$$

Lateral Loads, Earthquake Load

- When the natural period *T* cannot be determined by rational means from technical data, it may be obtained as follows for shear walls or exterior concrete frames using deep beams or wide piers, or both:

$$T = \frac{0.05 h_n}{\sqrt{D}} \qquad (5)$$

D = dimension of the structure in the direction of the applied forces, in feet.

h_n = height of the building

THANK YOU

Design Loads on Structures

Lateral Loads

Lateral Loads, Wind Load

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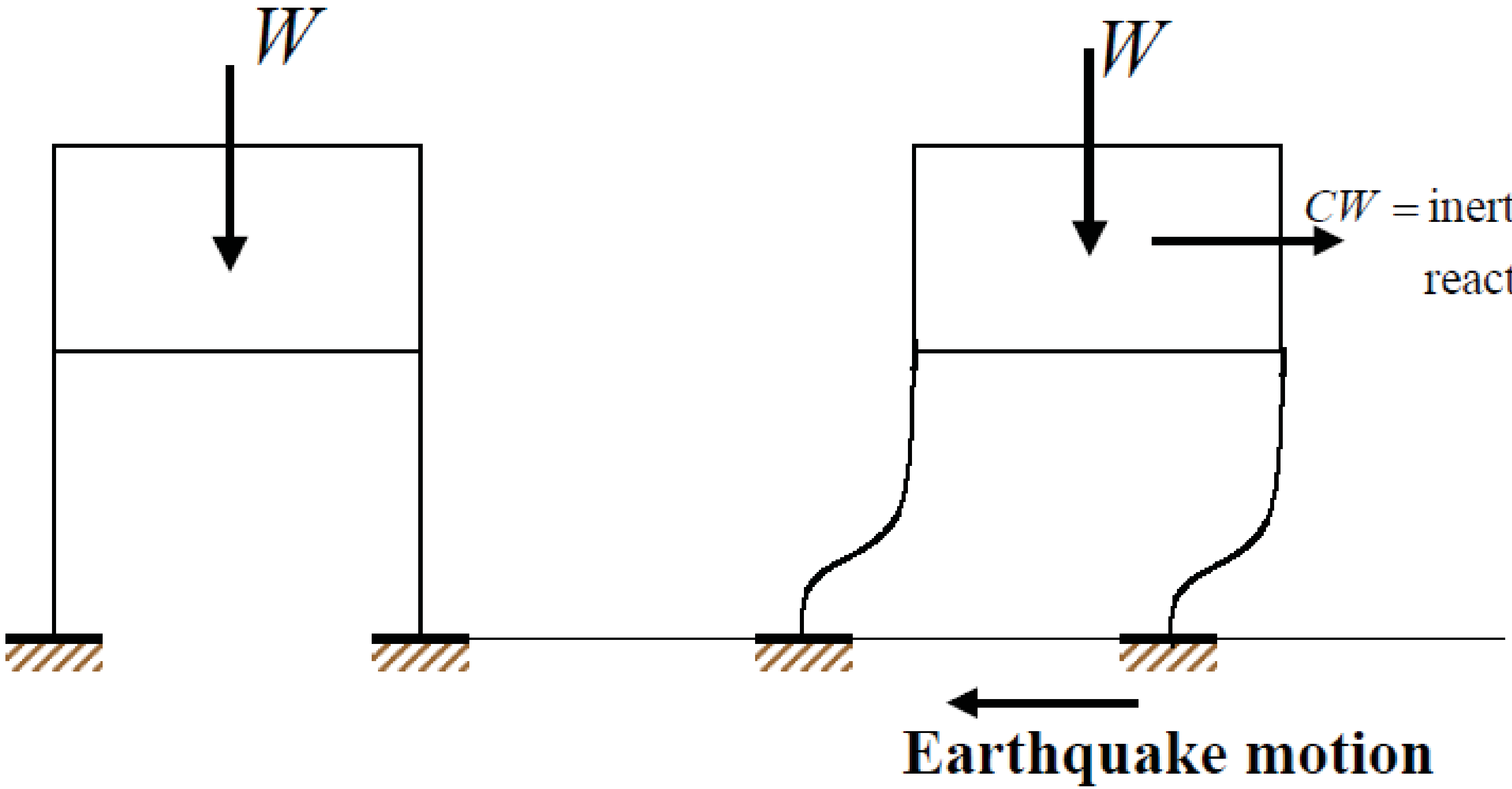
Lateral Loads, Earthquake Load



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Lateral Loads, Earthquake Load

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Lateral Loads, Earthquake Load

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D = dimension of the structure in the direction of the applied forces, in feet.

h_n = height of the building

THANK YOU

Analysis of Statically Determinate Structures

Principle of Superposition

- The total displacements or internal loadings (stress) at a point in a structure subjected to several external loadings can be determined by adding together the displacements or internal loadings (stress) caused by each of the external loads acting separately
- Linear relationship exist among loads, stresses & displacements

Principle of Superposition

- 2 requirements for the principle to apply:
  1. Material must behave in a linear-elastic manner, Hooke's Law is valid
  2. The geometry of the structure must not undergo significant change when the loads are applied, small displacement theory

Principle of Superposition

- For equilibrium:

$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum F_z = 0$$

$$\sum M_x = 0 \qquad \sum M_y = 0 \qquad \sum M_z = 0$$

- For most structures, it can be reduced to:

$$\sum F_x = 0$$

$$\sum F_y = 0$$

$$\sum M_o = 0$$

Determinacy and Stability of Structures

Determinacy and Stability

- Determinacy
  - Equilibrium equations provide sufficient conditions for equilibrium
  - All forces can be determined strictly from these equations
  - No. of unknown forces > equilibrium equations > statically indeterminate
  - This can be determined using a free body diagram

Determinacy and Stability

- Determinacy
  - For a coplanar structure

$$\begin{array}{ll} r = 3n, & \text{statically determinate} \\ r > 3n, & \text{statically indeterminate} \end{array}$$

  - The additional equations needed to solve for the unknown equations are referred to as compatibility equations

Example 2.3

Classify each of the beams as statically determinate or statically indeterminate. If statically indeterminate, report the no. of degree of indeterminacy. The beams are subjected to external loadings that are assumed to be known & can act anywhere on the beams.

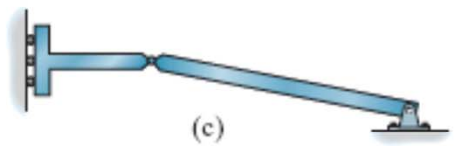
Solution

(a)

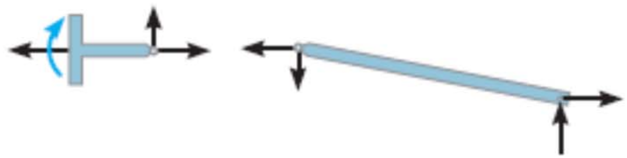
$r = 3,\ n = 1,\ 3 = 3(1)$ Statically determinate

(b)

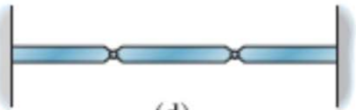
$r = 5,\ n = 1,\ 5 > 3(1)$ Statically indeterminate to the second degree



(c)



$r = 6,\ n = 2,\ 6 = 3(2)$ Statically determinate



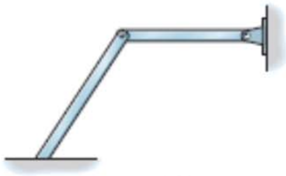
(d)

$r = 10,\ n = 3,\ 10 > 3(3)$ Statically indeterminate to the first degree

Example 2.4

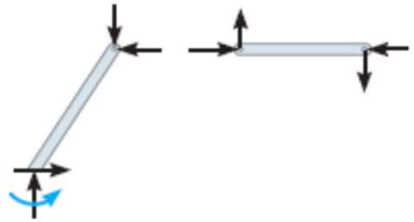
Classify each of the pin-connected structures as statically determinate or statically indeterminate. If statically indeterminate, report the no. of degree of indeterminacy. The structures are subjected to arbitrary external loadings that are assumed to be known & can act anywhere on the structures.

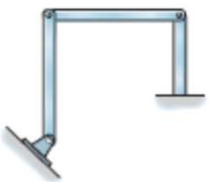
Solution



(a)

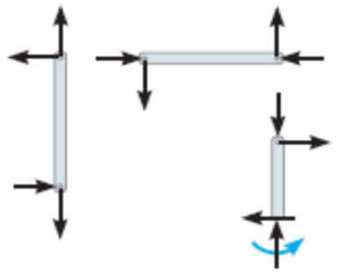
$r = 7,\ n = 2,\ 7 > 6$
Statically indeterminate to the first degree *Ans.*

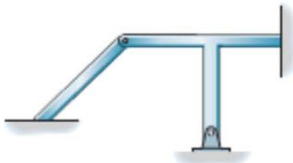




(b)

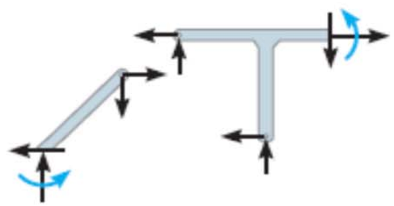
$r = 9,\ n = 3,\ 9 = 9,$
Statically determinate *Ans.*

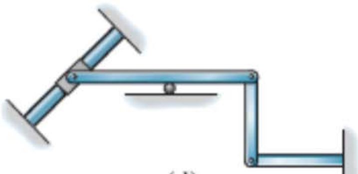




(c)

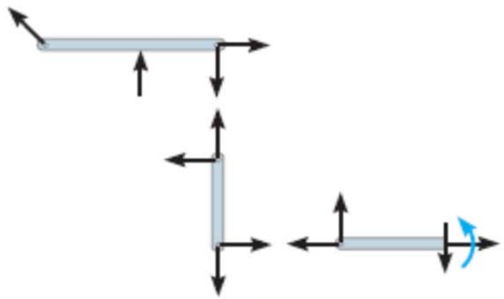
$r = 10,\ n = 2,\ 10 > 6,$
Statically indeterminate to the fourth degree *Ans.*





(d)

$r = 9,\ n = 3,\ 9 = 9,$
Statically determinate *Ans.*



THANK YOU

Stability of Structures

External and Internal Forces

External Forces

are the actions of other bodies on the structure under consideration. For the purposes of analysis, it is usually convenient to further classify these forces as **applied forces** and reaction forces.

Applied forces,
usually referred to as *loads* (e.g., live loads and wind loads), have a tendency to move the structure and are usually *known* in the analysis.

Reaction forces,
or *reactions*, are the forces exerted by supports on the structure and have a tendency to prevent its motion and keep it in equilibrium. The reactions are usually among the *unknowns* to be determined by the analysis. The state of equilibrium or motion of the structure as a whole is governed solely by the external forces acting on it.

External and Internal Forces

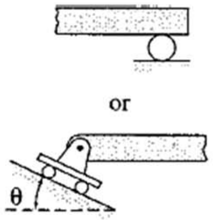
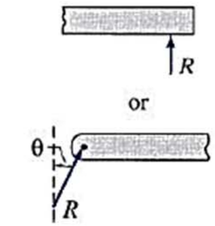
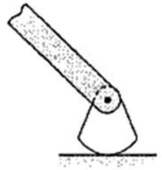
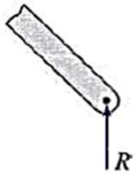
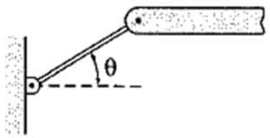
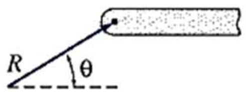
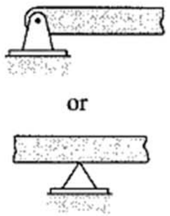
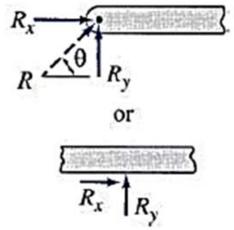
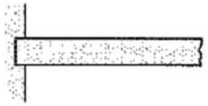
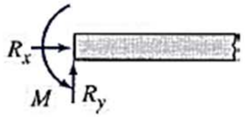
Internal Forces

are the forces and couples exerted on a member or portion of the structure by the rest of the structure. These forces develop within the structure and hold the various portions of it together.
The internal forces always occur in equal but opposite pairs, because each member or portion exerts back on the rest of the structure the same forces acting upon it but in opposite directions, according to Newton’s third law.

Because the internal forces cancel each other, they do not appear in the equations of equilibrium of the entire structure.

The internal forces are also among the unknowns in the analysis and are determined by applying the equations of equilibrium to the individual members or portions of the structure.

Types of Supports in Plane Structures

Category	Type of support	Symbolic representation	Reactions	Number of unkowns
I	Roller			**1** The reaction force R acts perpendicular to the supporting surface and may be directed either into or away from the structure. The magnitude of R is the unknown.
	Rocker			
	Link			**1** The reaction force R acts in the direction of the link and may be directed either into or away from the structure. The magnitude of R is the unknown.
II	Hinge			**2** The reaction force R may act in any direction. It is usually convenient to represent R by its rectangular components, R_x and R_y. The magnitudes of R_x and R_y are the two unknowns.
III	Fixed			**3** The reactions consist of two force components R_x and R_y and a couple of moment M. The magnitudes of R_x, R_y, and M are the three unknowns.

Types of Supports in Plane Structures

Roller Support

Types of Supports in Plane Structures

Rocker Support

Types of Supports in Plane Structures

Rocker Support

Types of Supports in Plane Structures

Hinged Support

Internal Stability of Structures

A structure is considered to be *internally stable, or rigid*,

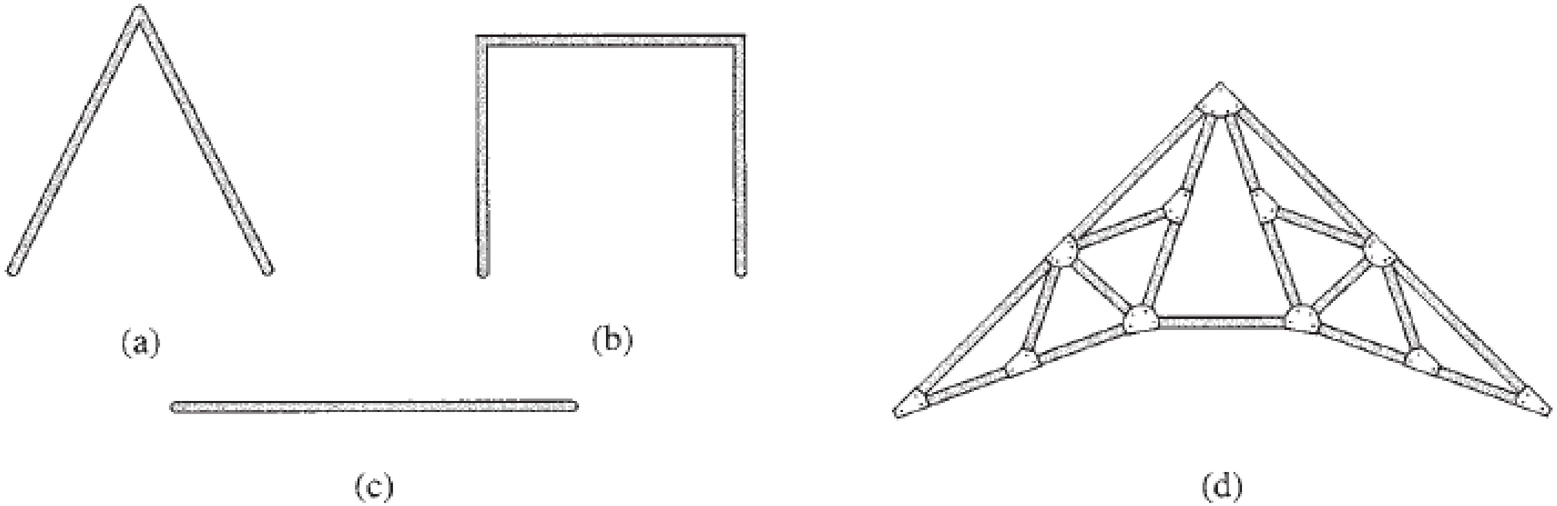
if it maintains its shape and remains a rigid body when detached from the supports.

Conversely,

a structure is termed *internally unstable* (or nonrigid)

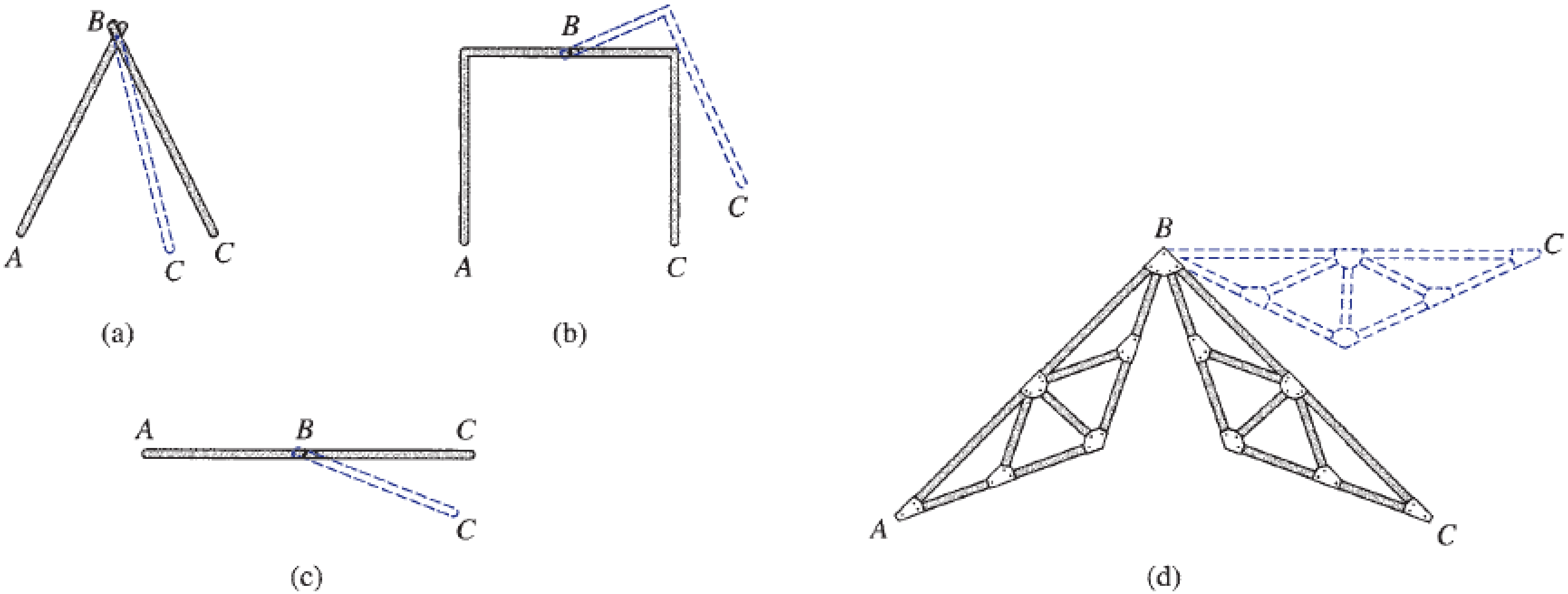
if it cannot maintain its shape and may undergo large displacements under small disturbances when not supported externally.

Internal Stability of Structures



Examples of Internally Stable Structures

Internal Stability of Structures



Examples of Internally Unstable Structures

Each structure is composed of two rigid parts, *AB* and *BC*, connected by a hinged joint *B*, which cannot prevent the rotation of one part with respect to the other.

Stable Structures

Principle :
stable structure can be treated as a plane rigid body.

- ***Stable structure must be supported by at least three reactions that satisfy the three equations of equilibrium.***

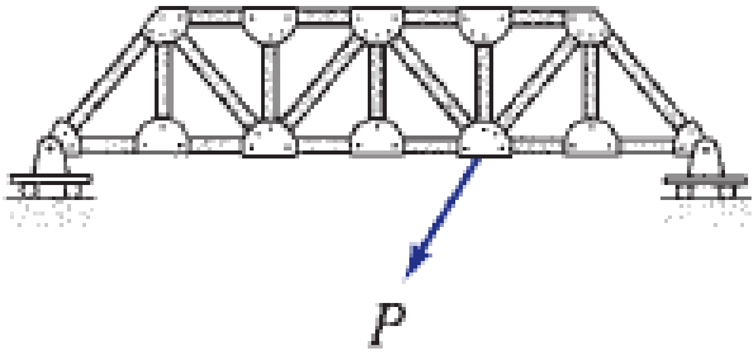
If a structure is supported by fewer than three support reactions, the reactions are not sufficient to prevent all possible movements of the structure in its plane. Such a structure cannot remain in equilibrium under a general system of loads and is, therefore, referred to as ***statically unstable***.

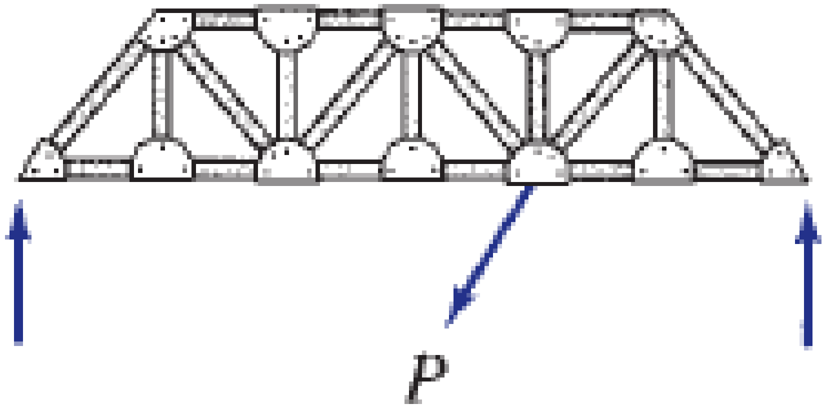
- ***Members must be properly held or constrained by their supports***

structure may be supported by a sufficient number of reactions but may still be unstable due to improper arrangement of supports. Such structures are referred to as ***geometrically unstable.***

Statically Unstable Structures

Partial constraints

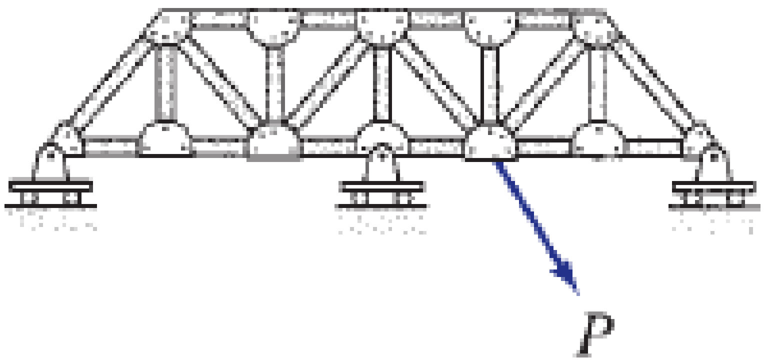


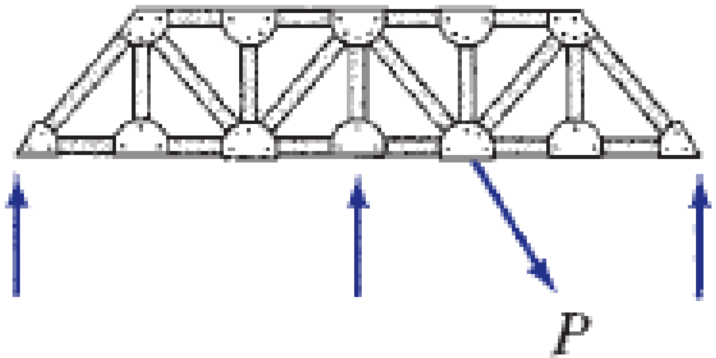


The truss shown in this figure is supported on only two rollers. It should be obvious that although the two reactions can prevent the truss from rotating and translating in the vertical direction, they cannot prevent its translation in the horizontal direction. Thus, the truss is not fully constrained and is statically unstable.

Geometrically Unstable Structures

Improper constraints

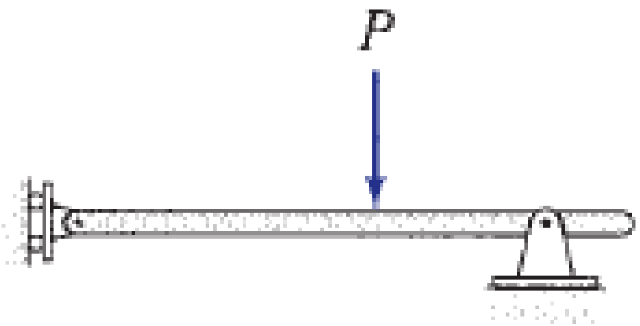


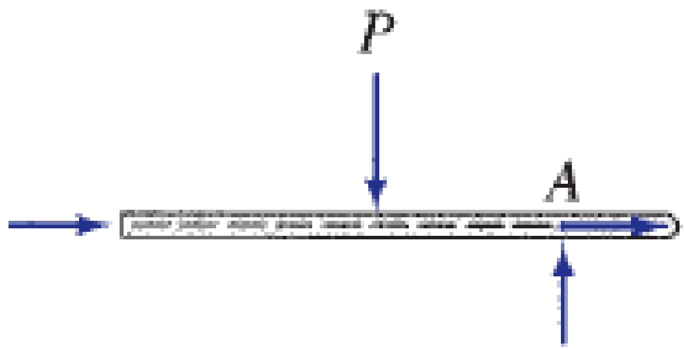


- The truss is supported by three parallel reactions.
- It can be seen that although there is a sufficient number of reactions, all of them are in the vertical direction, so they cannot prevent translation of the structure in the horizontal direction.
- The truss is, therefore, geometrically unstable.

Geometrically Unstable Structures

Improper constraints





- The beam is supported by three nonparallel reactions.
- However, since the lines of action of all three reaction forces are concurrent at the same point, *A*, they cannot prevent rotation of the beam about point *A*. In other words, the moment equilibrium equation $\sum M_A = 0$ cannot be satisfied for a general system of coplanar loads applied to the beam.
- The beam is, therefore, geometrically unstable.

Stable Structures

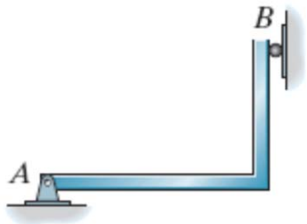
Summary:

in order for a plane internally stable structure to be geometrically stable so that it can remain in equilibrium under the action of any arbitrary coplanar loads, it must be supported by at least three reactions, ***all of which must be neither parallel nor concurrent.***

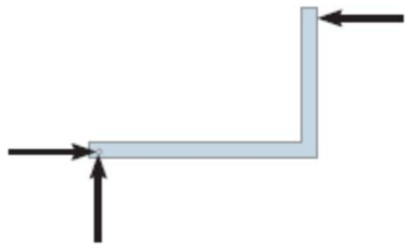
Example

Classify each of the structure as stable or unstable. The structures are subjected to arbitrary external loads that are assumed to be known.

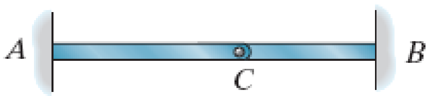
Solution



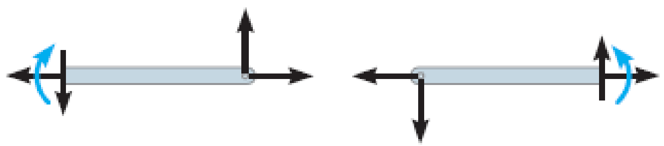
(a)



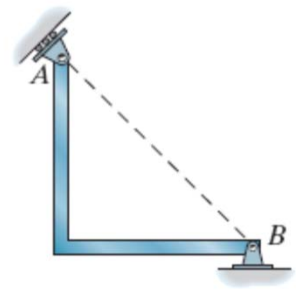
The member is *stable* since the reactions are nonconcurrent and nonparallel.
It is also statically determinate.



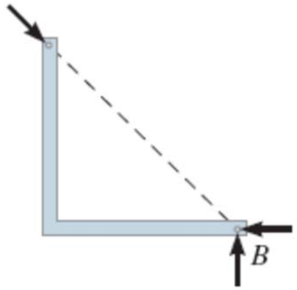
(b)



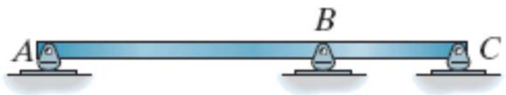
The compound beam is stable.
It is also indeterminate to the second degree.



(c)



The member is unstable since the three reactions are concurrent at B.

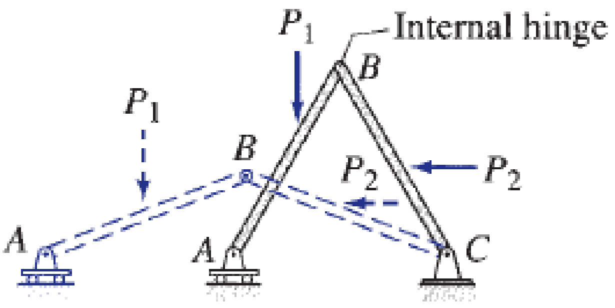


(d)

The beam is unstable since the three reactions are all parallel

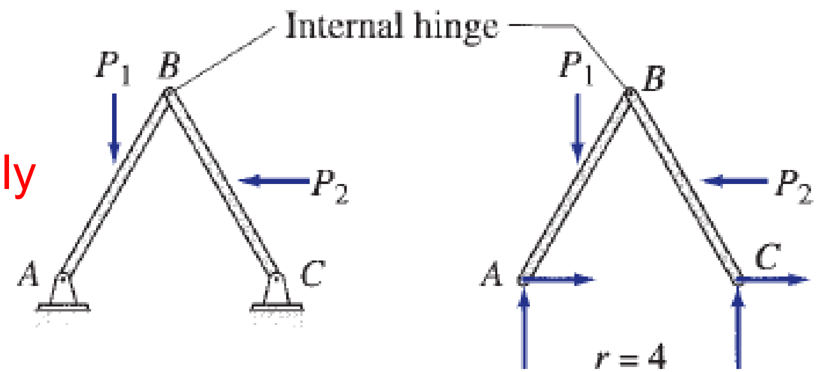
Equations of Condition

- The supports system provide three nonparallel non-concurrent external reactions.
- The reactions, which would have been sufficient to fully constrain an internally stable or rigid structure, are not sufficient for this structure.



(a)

The structure can, however, be made externally stable by replacing the roller support at *A* by a hinged support to prevent the horizontal movement of end *A* of the structure.



One equation of condition: $\Sigma M_B^{AB} = 0$ or $\Sigma M_B^{BC} = 0$

(b)

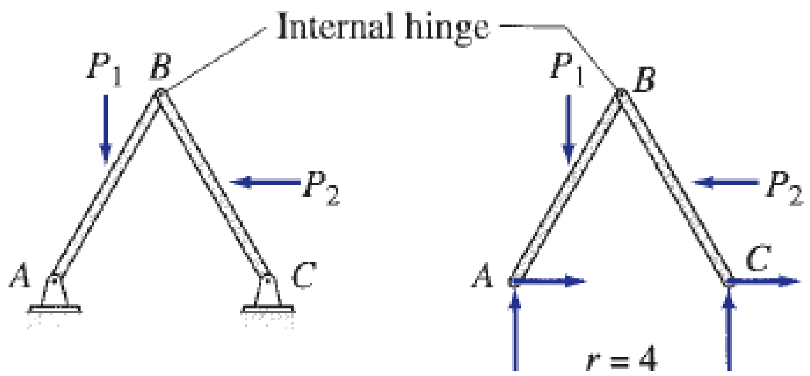
Equations of Condition

- The three equilibrium equations are not sufficient to determine the four unknown reactions at the supports for this structure.

- Presence of the internal hinge at *B* yields an additional equation that can be used with the three equilibrium equations to determine the four unknowns.

- The additional equation is based on the condition that an internal hinge cannot transmit moment; that is, the moments at the ends of the parts of the structure connected to a hinged joint are zero.

- Such equations are commonly referred to as the *equations of condition or construction*



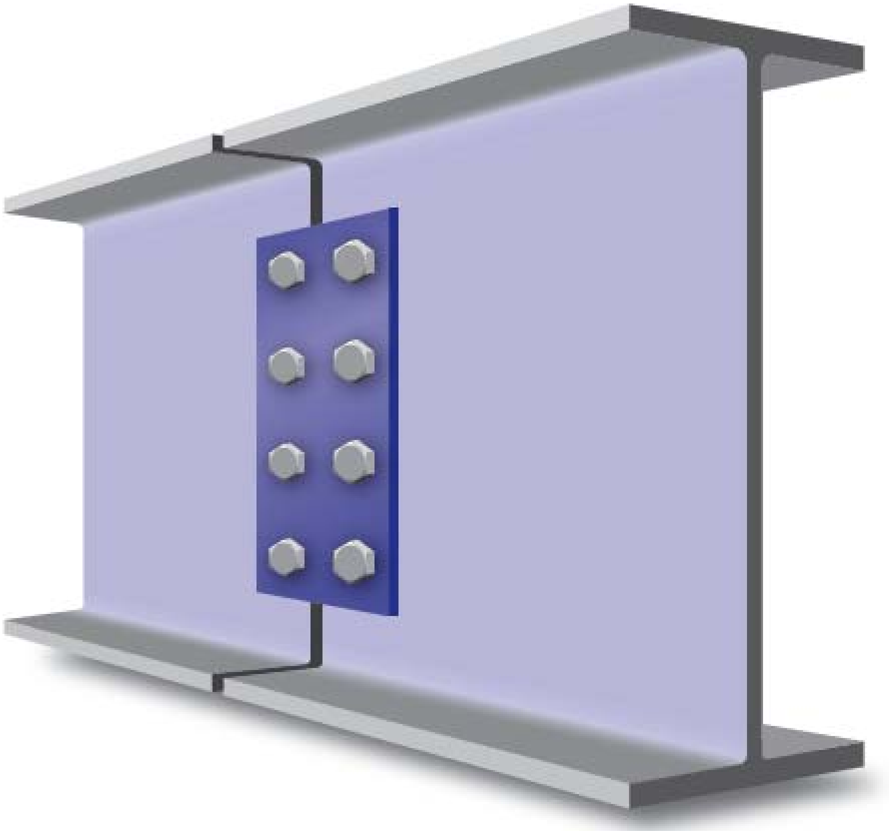
One equation of condition: $\Sigma M_B^{AB} = 0$ or $\Sigma M_B^{BC} = 0$

(b)

Internal Hinges

An internal hinge connecting two members or portions of a structure provides one independent equation of condition.

Shear splices are sometimes used to connect two beams into a longer one.



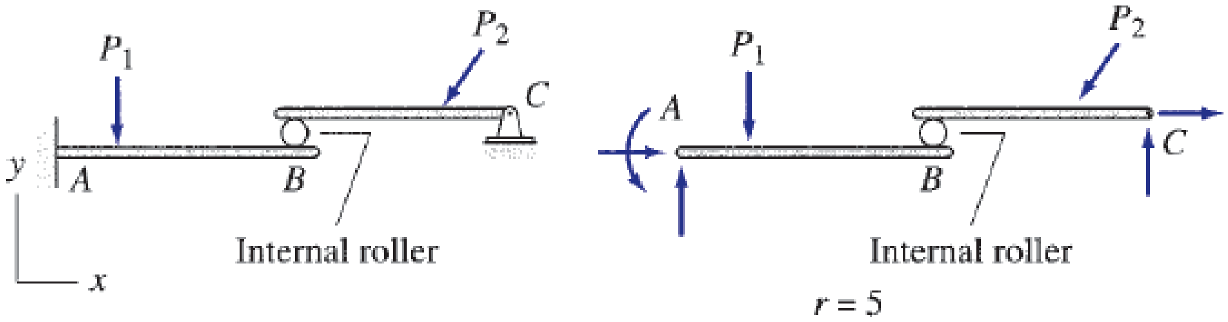
Such connections are designed to transfer (shear) forces but not (bending) moments, and are treated as internal hinges for analysis.

Internal Roller

connections are used in structures that permit not only relative rotations of the member ends but also relative translations in certain directions of the ends of the connected members. Such connections are modeled as internal roller joints for the purposes of analysis.

Since an internal roller can transmit neither moment nor force in the direction parallel to the supporting surface, it provides two equations of condition:

$$\sum F_x^{AB} = 0 \quad \text{or} \quad \sum F_x^{BC} = 0$$



Two equations of condition: $\Sigma F_x^{AB} = 0$ or $\Sigma F_x^{BC} = 0$

$\Sigma M_B^{AB} = 0$ or $\Sigma M_B^{BC} = 0$

Procedure for Analysis

The following procedure provides a method for determining the *joint reactions* for structures composed of pin-connected members.

Free-Body Diagrams

- Disassemble the structure and draw a free-body diagram of each member. Also, it may be convenient to supplement a member free-body diagram with a free-body diagram of the *entire structure*. Some or all of the support reactions can then be determined using this diagram.
- Recall that reactive forces common to two members act with equal magnitudes but opposite directions on the respective free-body diagrams of the members.
- All two-force members should be identified. These members, regardless of their shape, have no external loads on them, and therefore their free-body diagrams are represented with equal but opposite collinear forces acting on their ends.
- In many cases it is possible to tell by inspection the proper arrowhead sense of direction of an unknown force or couple moment; however, if this seems difficult, the directional sense can be assumed.

Equations of Equilibrium

- Count the total number of unknowns to make sure that an equivalent number of equilibrium equations can be written for solution. Except for two-force members, recall that in general three equilibrium equations can be written for each member.
- Many times, the solution for the unknowns will be straightforward if the moment equation $\Sigma M_O = 0$ is applied about a point (O) that lies at the intersection of the lines of action of as many unknown forces as possible.
- When applying the force equations $\Sigma F_x = 0$ and $\Sigma F_y = 0$, orient the x and y axes along lines that will provide the simplest reduction of the forces into their x and y components.
- If the solution of the equilibrium equations yields a *negative* magnitude for an unknown force or couple moment, it indicates that its arrowhead sense of direction is *opposite* to that which was assumed on the free-body diagram.

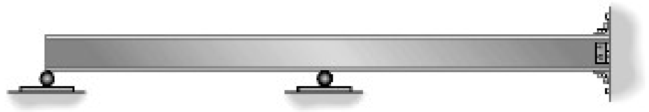
THANK YOU

Solved Problems

Determinacy and Stability of Structures

Examples, Reference: Hibbeler

Classify each of the structures as statically determinate, statically indeterminate, or unstable. If indeterminate, specify the degree of indeterminacy. The supports or connections are to be assumed as stated.



(a)

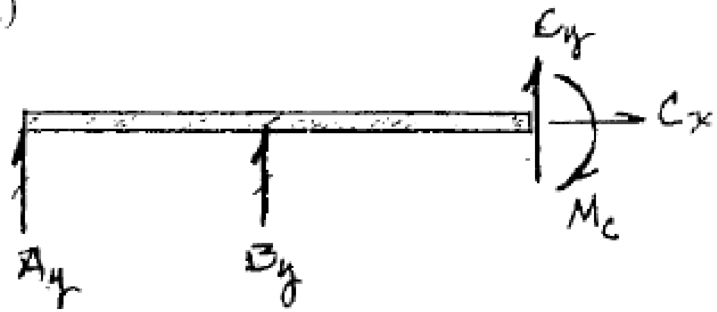
$r = 5, \quad n = 1$

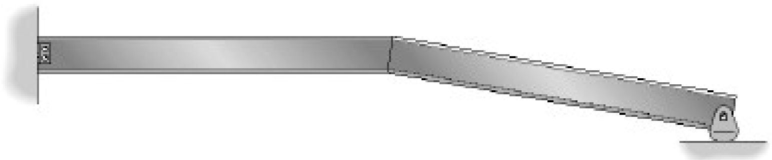
$r > 3n$

$5 > 3(1)$

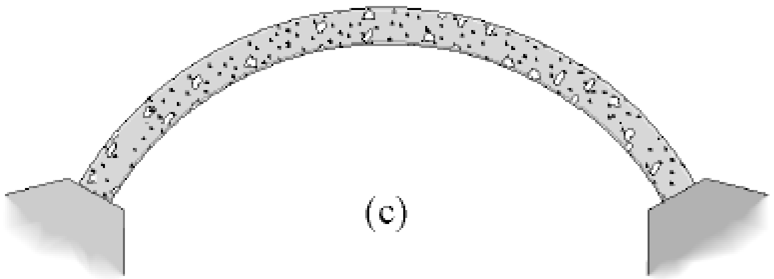
Indeterminate to 2°, Stable

(a)

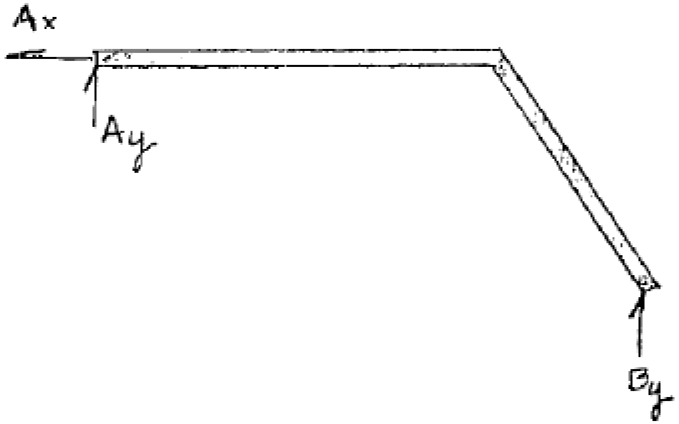




(b)



(c)



(b)

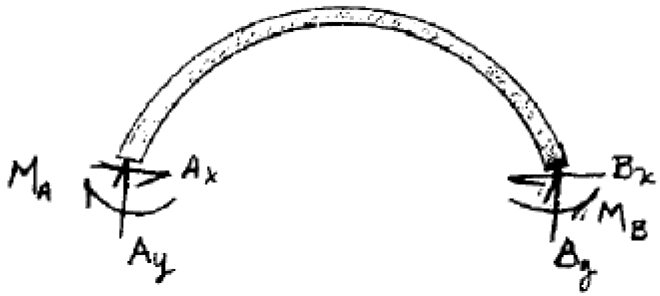
$r = 3, \quad n = 1$

$r = 3n$

$3 = 3(1)$

Determinate, Stable

(c)

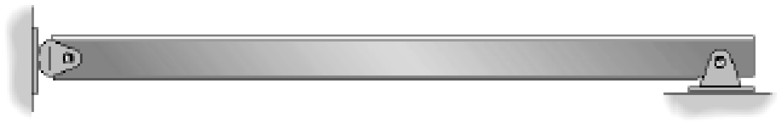


$r = 6, \quad n = 1$

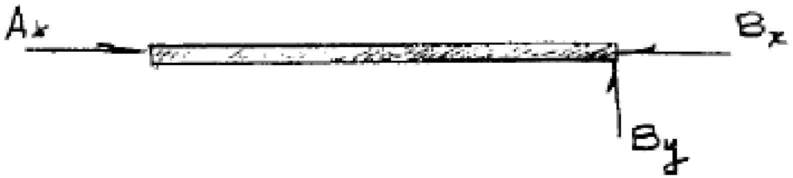
$r > 3n$

$6 > 3(1)$

Indeterminate to 3°, Stable

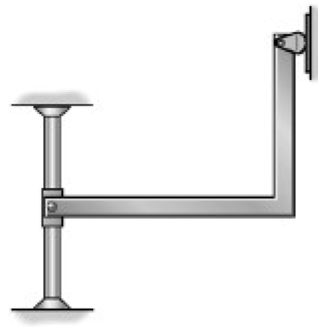


(d)

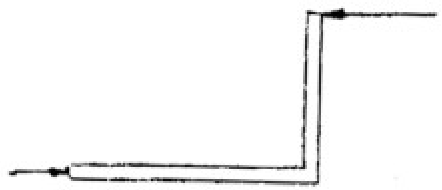


(d)

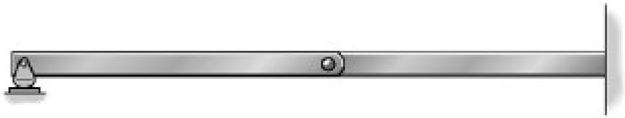
Unstable Concurrent Reactions



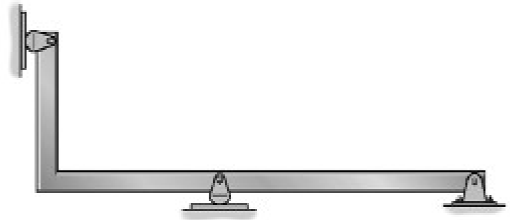
(a)



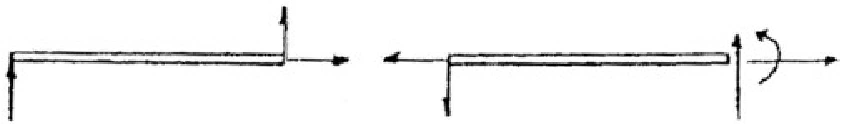
Parallel reactions
Unstable.

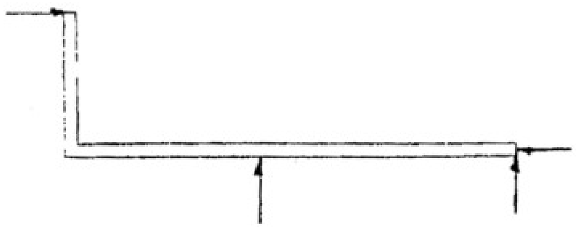


(b)



(c)





$r = 3n$

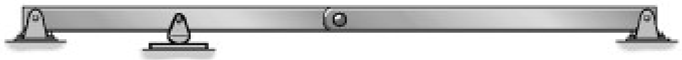
$6 = 3(2)$

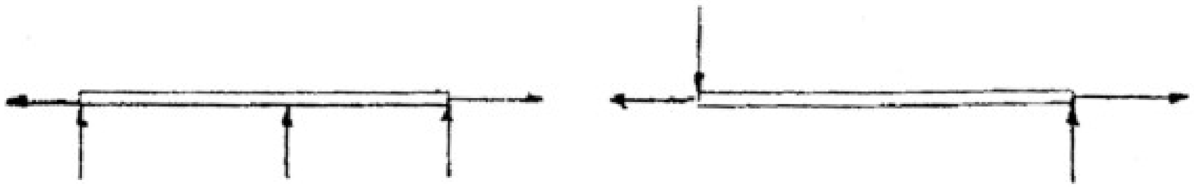
Statically determinate.

$r > 3n$

$4 > 3(1)$

Statically indeterminate to 1°





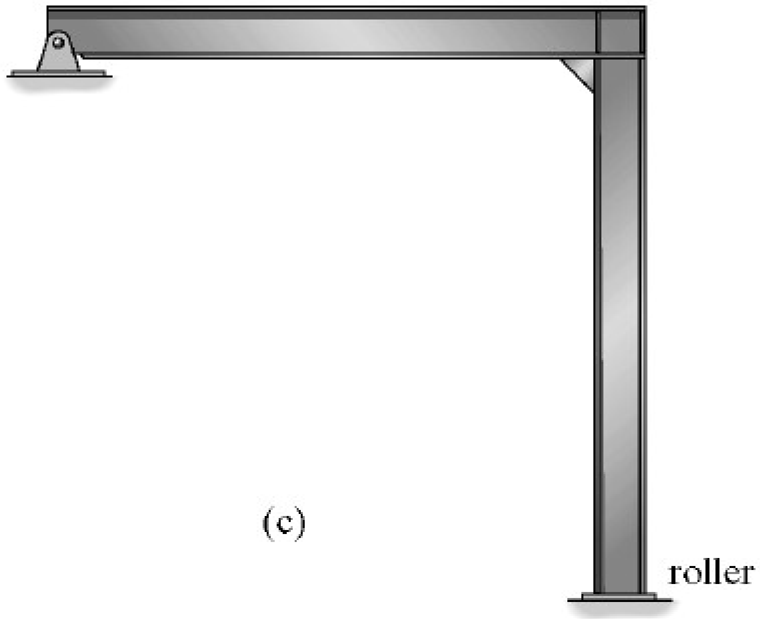
$r > 3n$

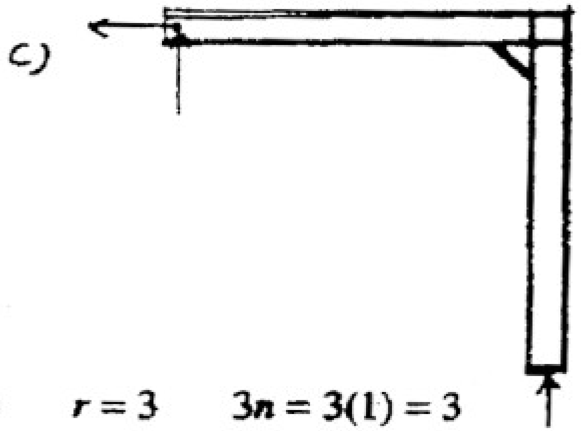
$7 > 3(2)$

Statically indeterminate to 1°.

$r = 6 \qquad 3n = 3(1) = 3 < 6$

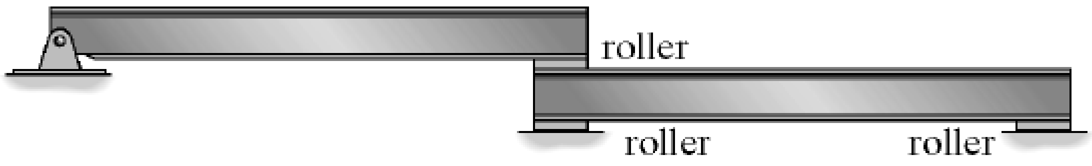
Indeterminate to 3°

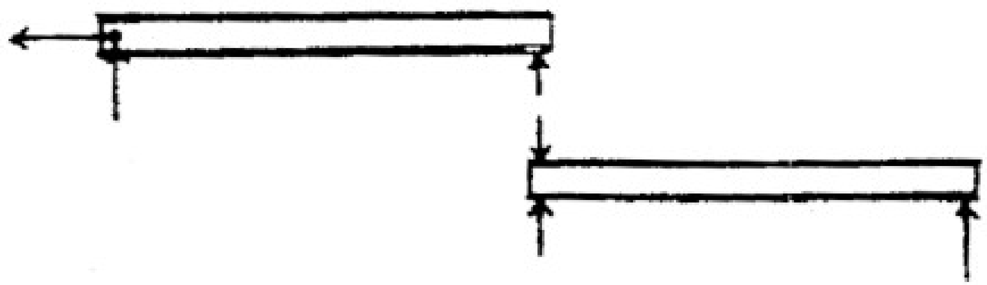




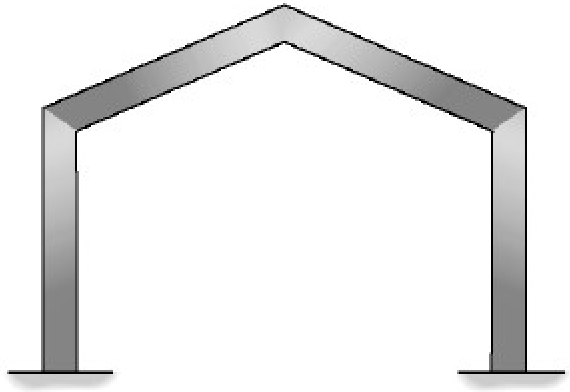
(c) $r = 3 \qquad 3n = 3(1) = 3$

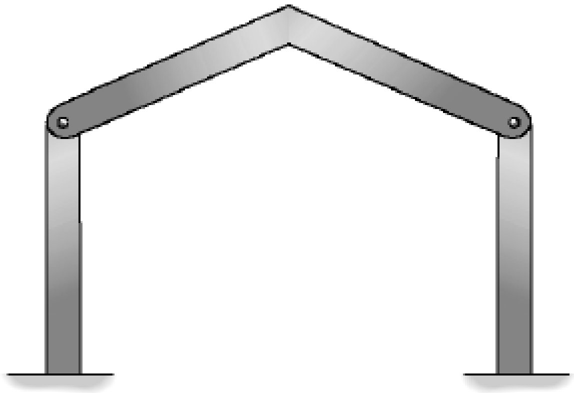
Statically determinate

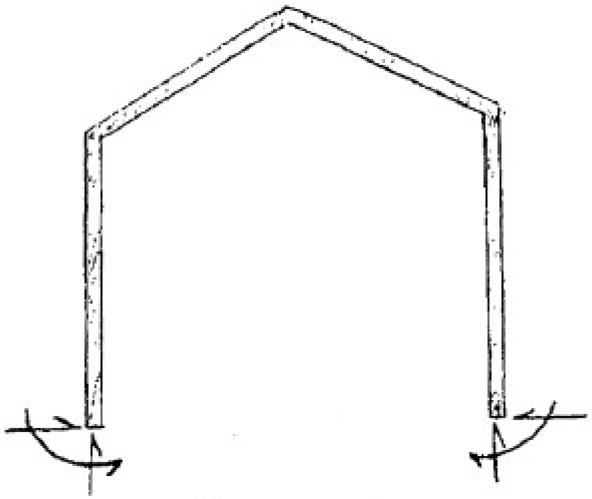




Parallel reactions on lower beam
Unstable





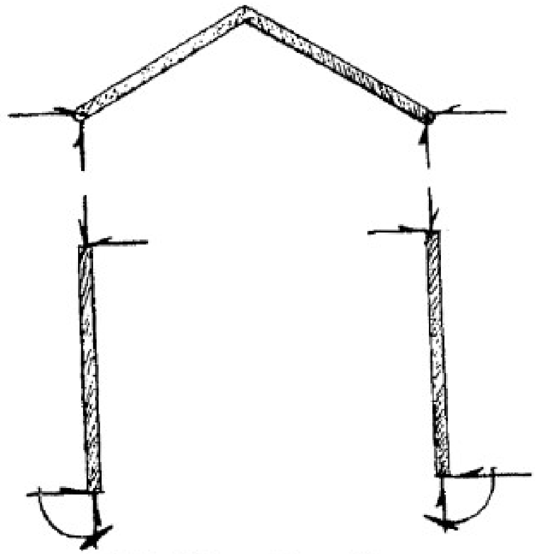


$r = 6, \quad n = 1$

$r > 3n$

$6 = 3(1)$

Indeterminanat to 3°

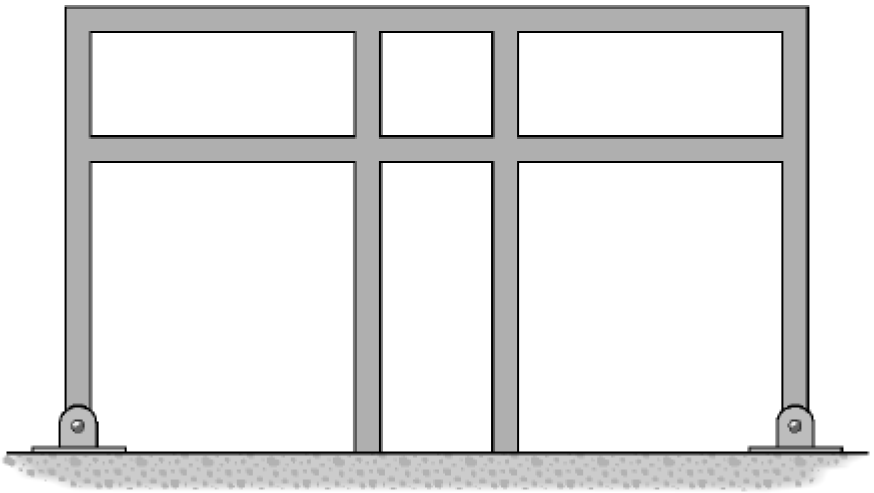


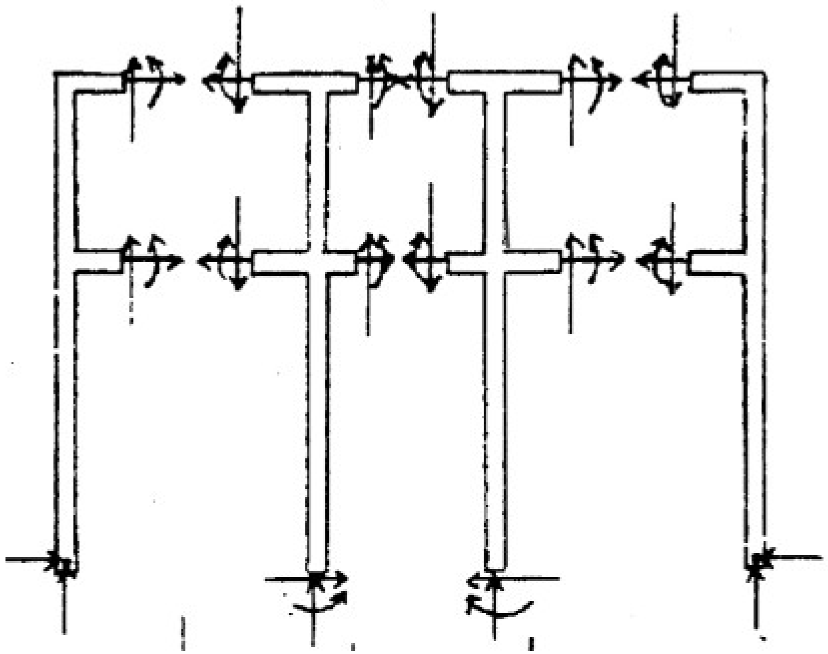
$r = 10, \quad n = 3$

$r > 3n$

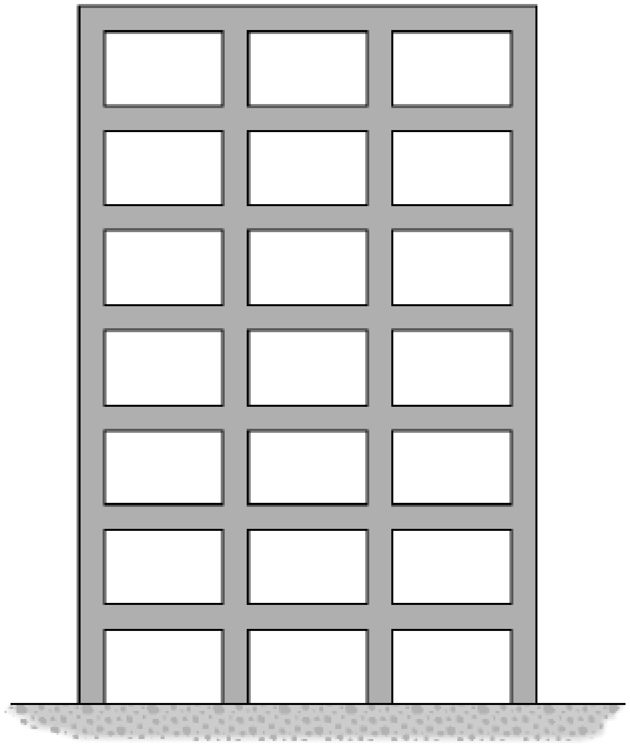
$10 = 3(3)$

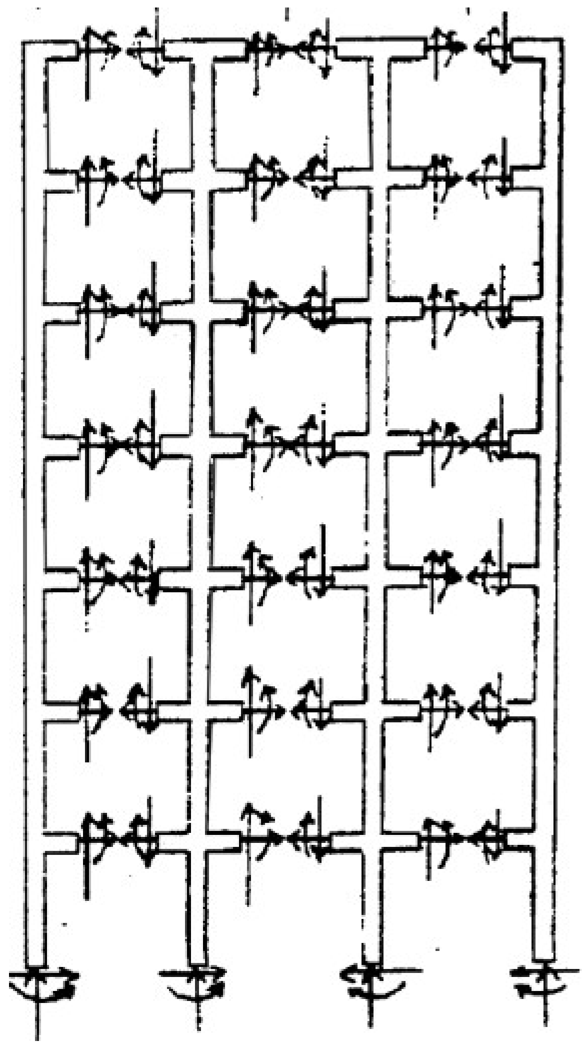
Statically indeterminate to the 1°





$28 - 12 = 16°$





$75 - 12 = 63°$

THANK YOU

Determinacy and Stability of Structures

Two-Dimensional Frames

Pin-Jointed 2D Frames

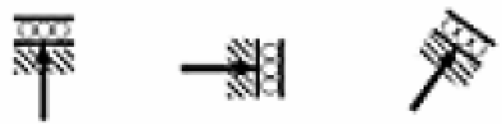
- A structure may be indeterminate due to redundant components of reaction and/or redundant members.

- A redundant reaction or member is one which is not essential to satisfy the minimum requirements of stability and static equilibrium, (Note: it is not necessarily a member with zero force).

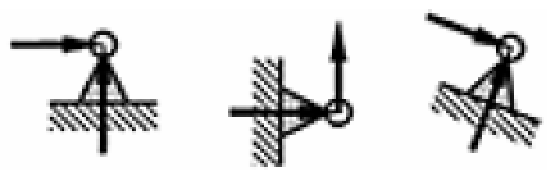
- The external components of reaction (r) in pin-jointed frames are normally one of two types:

i) a roller support providing one degree-of-restraint, i.e. perpendicular to the roller,

ii) ii) a pinned support providing two degrees-of-restraint, e.g. in the horizontal and vertical directions.



roller supports: providing one restraint perpendicular to the roller.



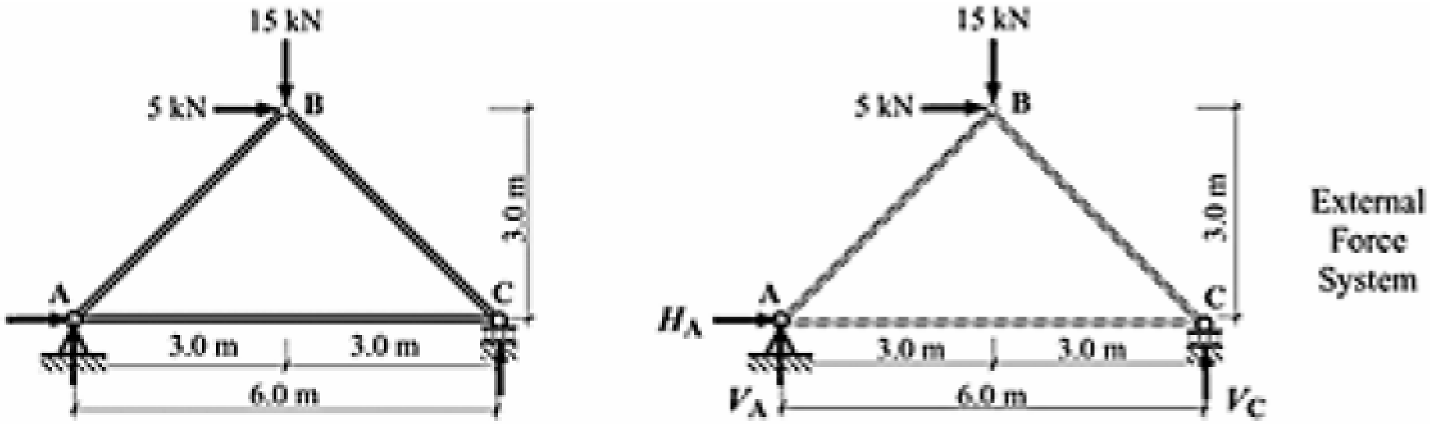
pinned supports: providing two mutually perpendicular restraints

Internal members of pin-jointed frames

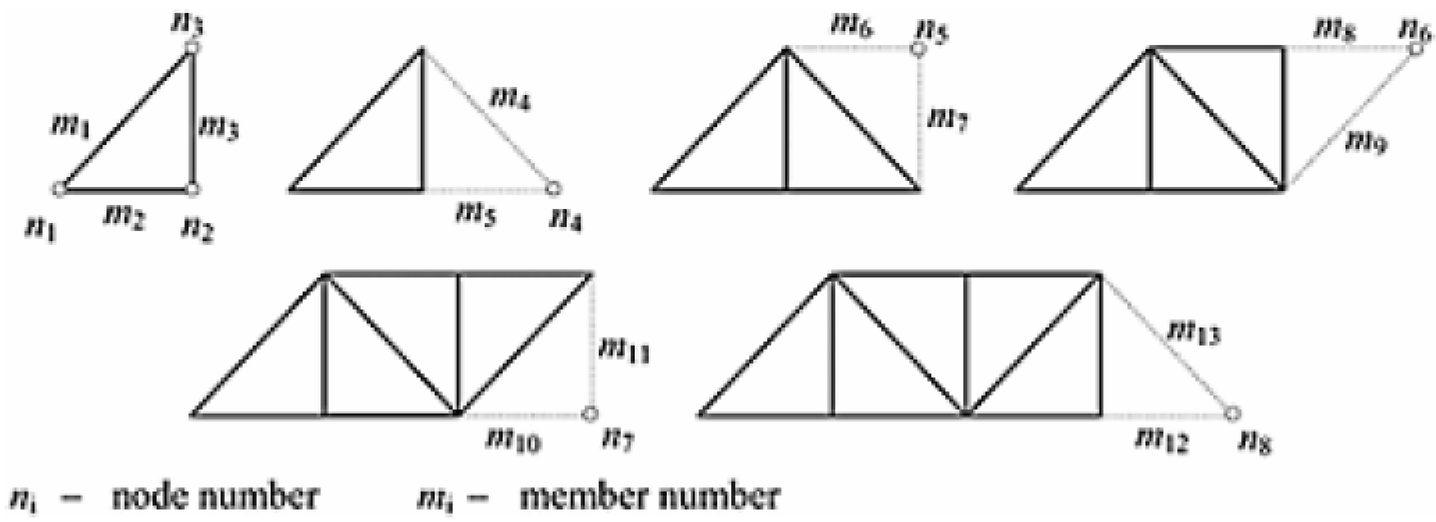
- The internal members of pin-jointed frames transfer either tensile or compressive axial loads through the nodes to the supports and hence reactions.

- A simple pin-jointed frame is one in which the minimum number of members is present to ensure stability and static equilibrium.

It is necessary to provide three non-parallel, non-concentric, components of reaction to satisfy the three equations of static equilibrium.

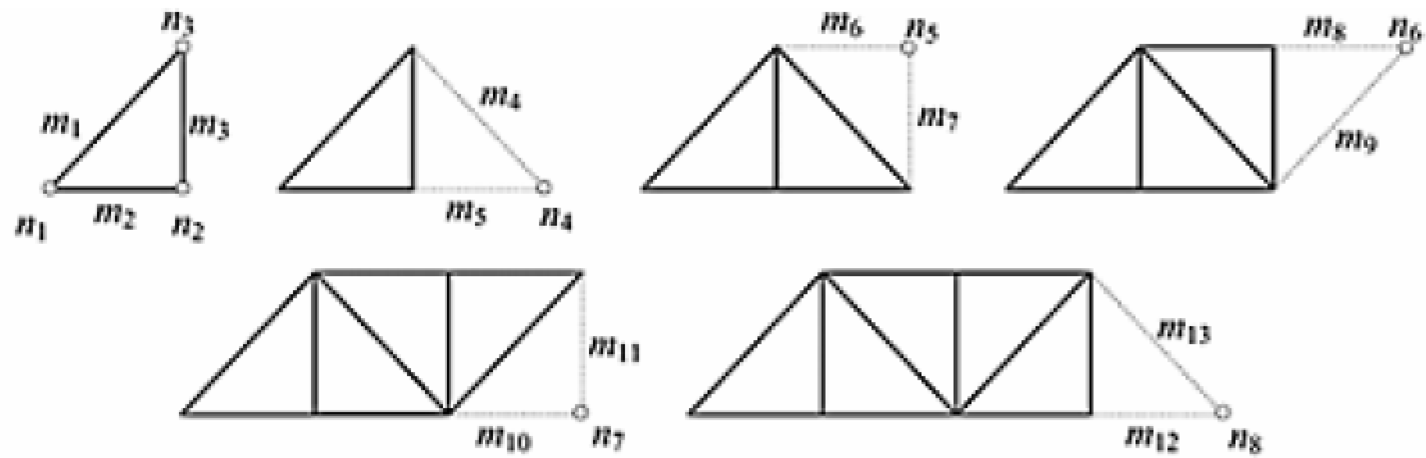


- The applied forces and the external components of reaction represent co-planar force systems which are in static equilibrium
- there are three unknowns, (HA, VA and VC), and three equations of equilibrium which can be used to determine their values
- There are no redundant components of reaction.



Consider the basic three member pinned-frame

- There are three nodes and three members.
- A triangle is the basis for the development of all pin-jointed frames since it is an inherently stable system
- i.e. only one configuration is possible for any given three lengths of the members.



n_i – node number m_i – member number

- Initially there are three nodes and three members.
- If the number of members in the frame is to be increased then for each node added, two members are required to maintain the triangulation.
- The minimum number of members required to create a simple frame can be determined as follows:

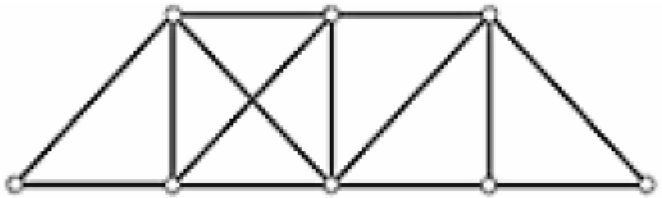
m = the initial three members + (2 × number of additional joints)

$= 3 + 2(n - 3)$ ⟶ $m = (2n - 3)$

e.g. in this case $n = 8$ and therefore the minimum number of members $= [(2 \times 8) - 3)]$

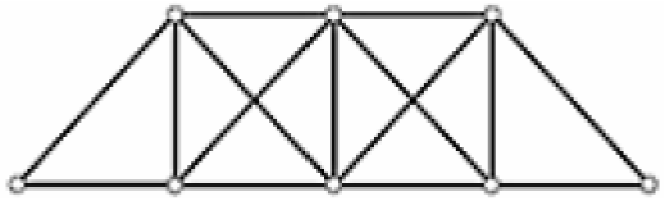
$\therefore m = 13$

Any members which are added to the frame in addition to this number are redundant members and make the frame statically indeterminate; e.g.



one redundant member

m= 14
n= 8
2n-3=13
One redundant member
Statically indeterminate to 1st degree

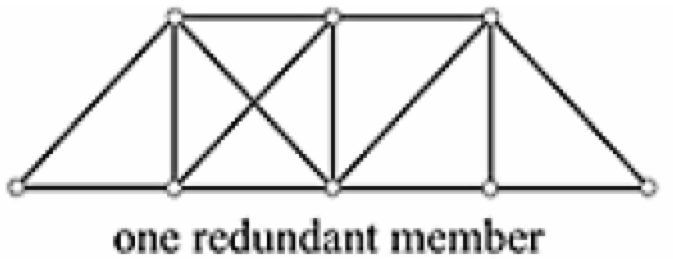


two redundant members

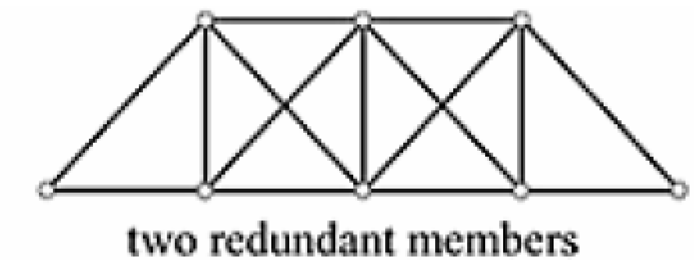
m= 15
n= 8
2n-3=13
Two redundant member
Statically indeterminate to 2nd degree

Any members which are added to the frame in addition to this number are redundant members and make the frame statically indeterminate; e.g.

For Example,



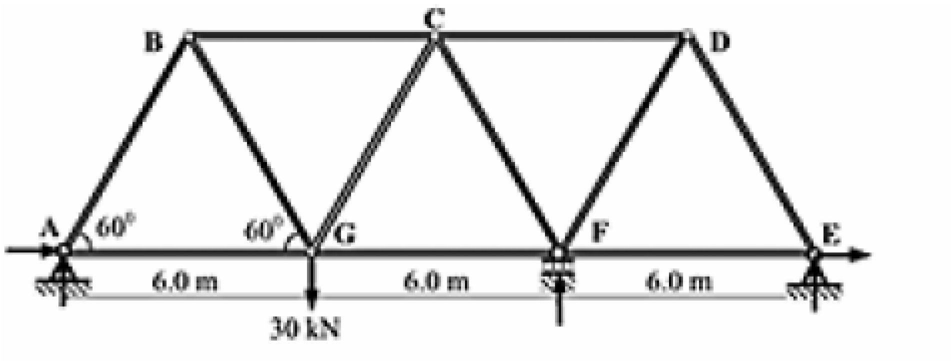
m= 14
n= 8
2n-3=13
One redundant member
Statically indeterminate to 1st degree



m= 15
n= 8
2n-3=13
Two redundant member
Statically indeterminate to 2nd degree

Consider Reactions,

Σ (number of members+support reactions)$=(m+r)=(2n-3)+3=2n$

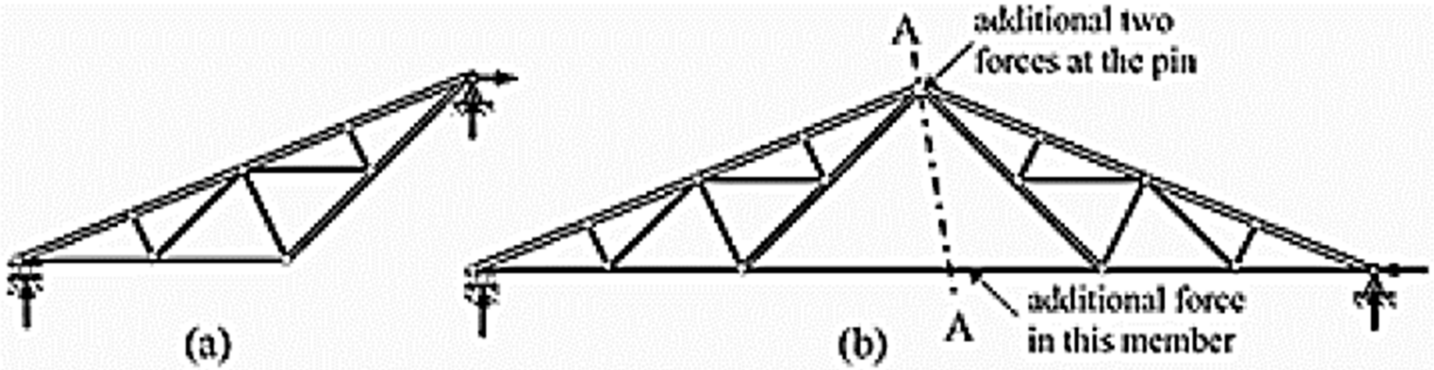


m= 11
r=5
n= 7
m+r= 16
2n= 14
Two redundant members
Statically indeterminate to 2nd degree

The degree of indeterminacy =(m+r)−2n

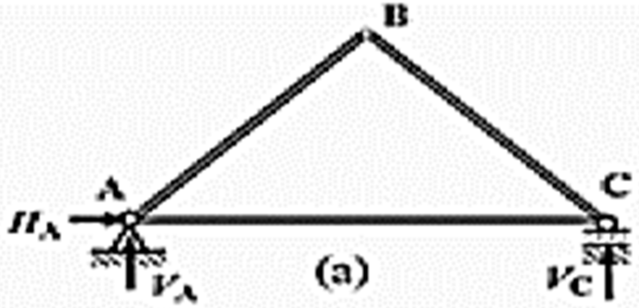
Compound trusses

- Compound trusses which are fabricated from two or more simple trusses by a structural system involving ***no more than three, non-parallel, non-concurrent, unknown forces***
- Compound trusses can also be stable and determinate.
- Simple truss can be connected to a similar one by a pin and an additional member to create a compound truss comprising two statically determinate trusses.
- Since only an additional three unknown forces have been generated the three equations of equilibrium can be used to solve these by considering a section A–A as shown
-



This compound truss satisfies the relationships $m=(2n-3)$ and $I_D=0$

Solved Problems



(a)

number of joints $n = 3$

number of members $m = 3$

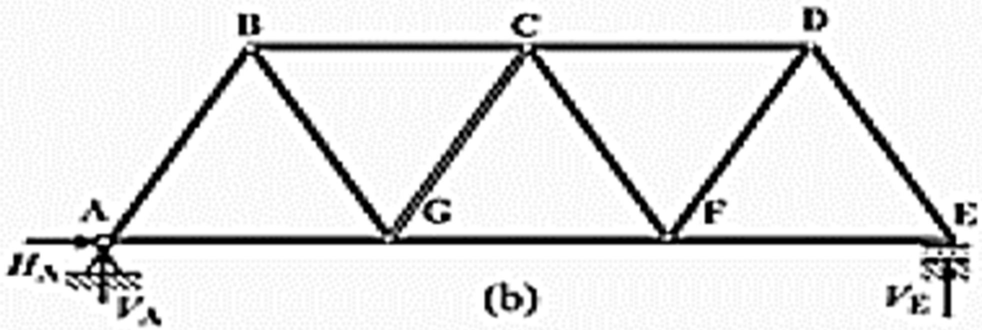
$(2n - 3) = 3$

number of support reactions $r = 3$

$(m + r) = 6 = 2n$

The frame is statically determinate

$I_D = 0$



(b)

number of joints $n = 7$

number of members $m = 11$

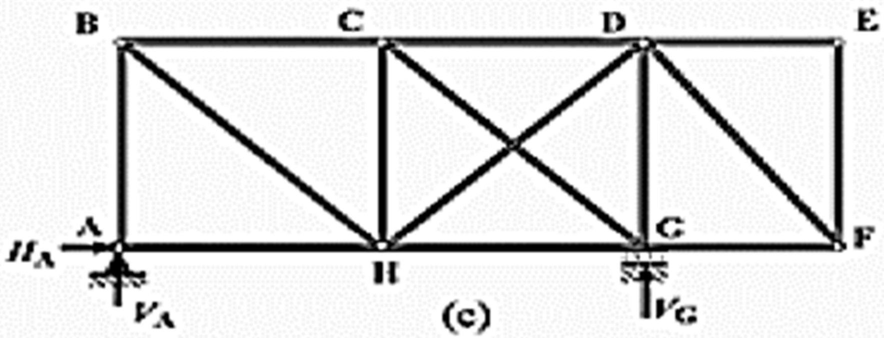
$(2n - 3) = 11$

number of support reactions $r = 3$

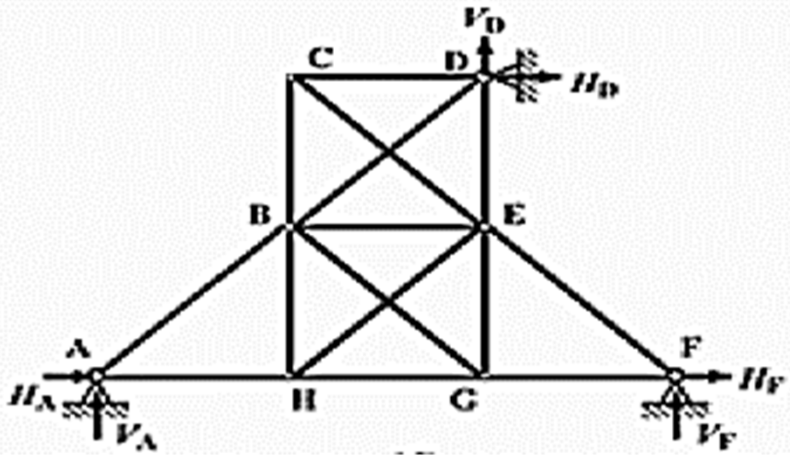
$(m + r) = 14 = 2n$

The frame is statically determinate

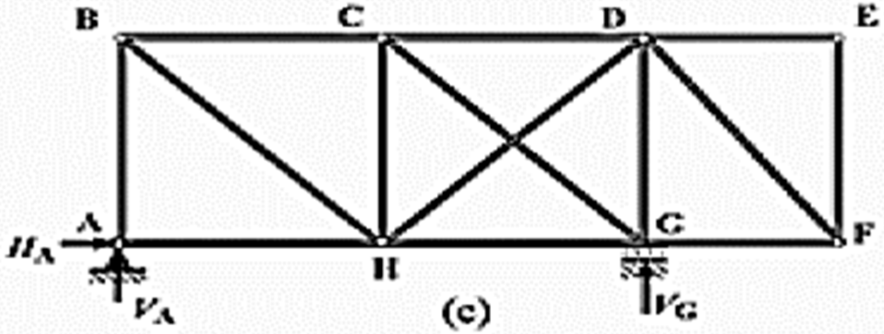
$I_D = 0$



number of joints $n = 8$
number of members $m = 14$
$(2n - 3) = 13$
number of support reactions $r = 3$
$(m + r) = 17 > 2n$
The frame is statically indeterminate with one redundant internal member
$I_D = 1$

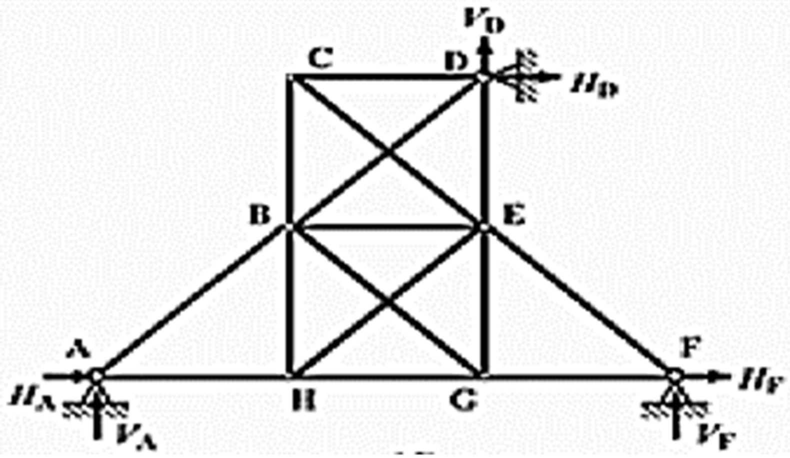


number of joints $n = 8$
number of members $m = 15$
$(2n - 3) = 13$
number of support reactions $r = 6$
$(m + r) = 21 > 2n$
The frame is statically indeterminate and has 5 redundancies:
(2 internal members + 3 external reactions)
$I_D = 5$



(c)

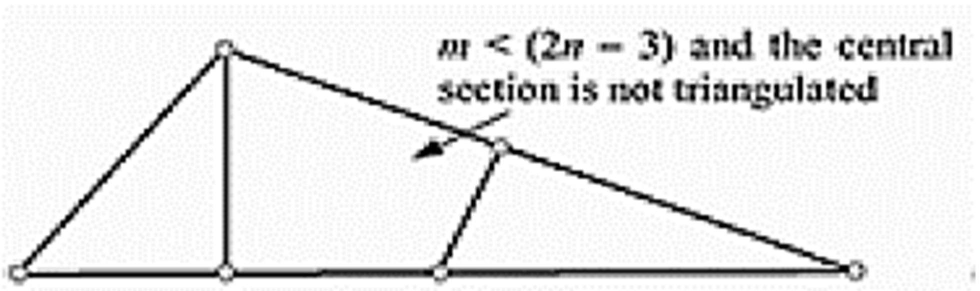
number of joints $n = 8$
number of members $m = 14$
$(2n - 3) = 13$
number of support reactions $r = 3$
$(m + r) = 17 > 2n$
The frame is statically indeterminate with one redundant internal member
$I_D = 1$

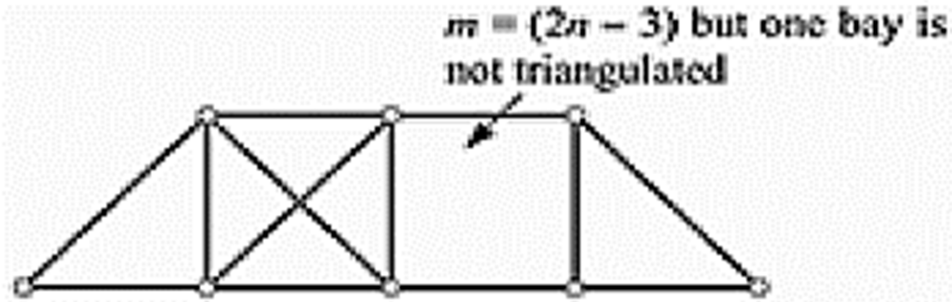


number of joints $n = 8$
number of members $m = 15$
$(2n - 3) = 13$
number of support reactions $r = 6$
$(m + r) = 21 > 2n$
The frame is statically indeterminate and has 5 redundancies:
(2 internal members + 3 external reactions)
$I_D = 5$

Note: Unstable truss

It is also essential to consider the configuration of the members in a frame to ensure that it is triangulated





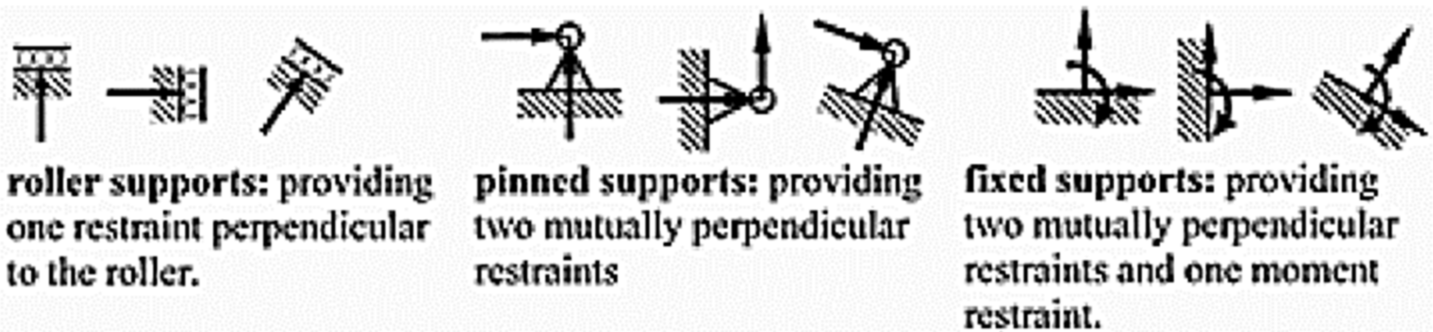
Rigid-Jointed 2D Frames

The external components of reaction (r) in rigid-jointed frames are normally one of three types:

i) a roller support providing one degree-of-restraint, i.e. perpendicular to the roller

ii) a pinned support providing two degrees-of-restraint, e.g. in the horizontal and vertical directions

iii) a fixed support providing three degrees-of-restraint, i.e. in the horizontal and vertical directions and a moment restraint

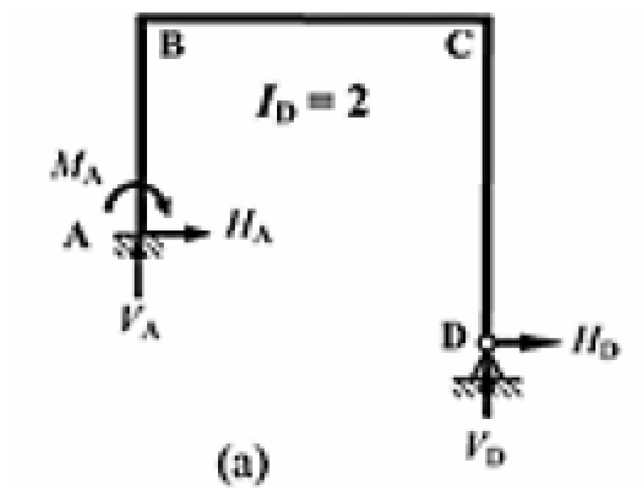


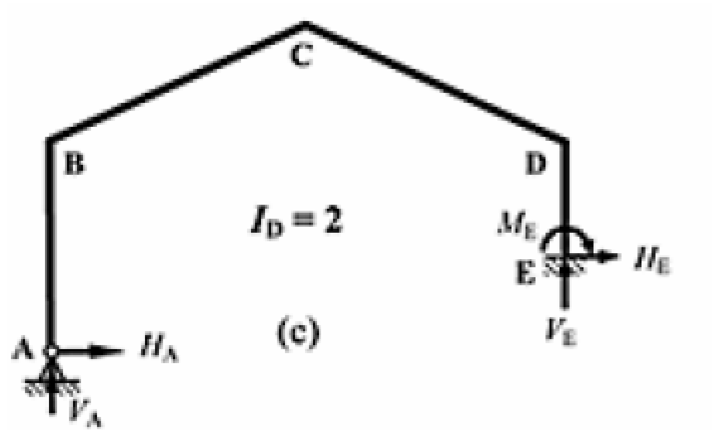
In rigid-jointed frames, the applied load system is transferred to the supports by inducing axial loads, shear forces and bending moments in the members

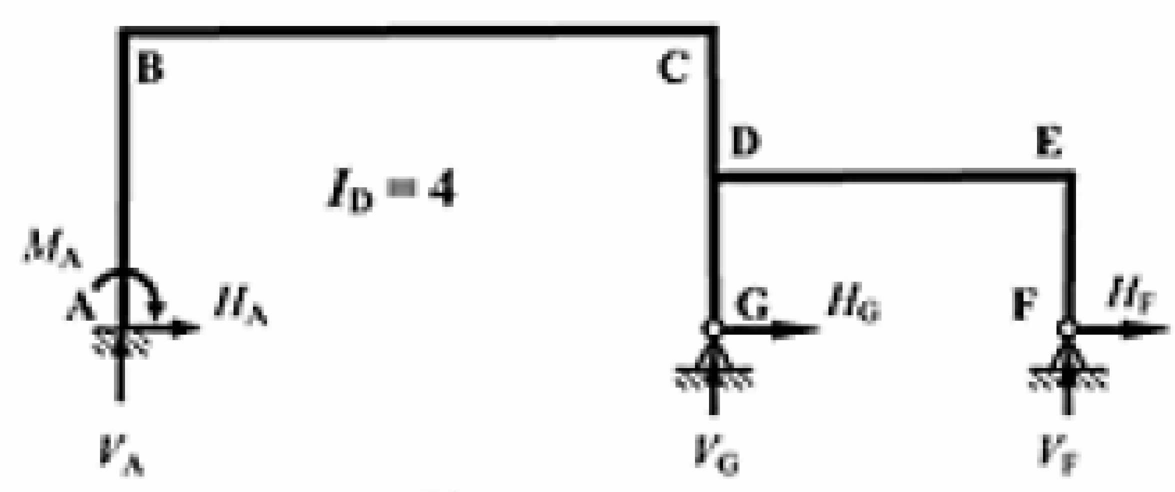
Since three components of reaction are required for static equilibrium the total number of unknowns is equal to:
$[(3\times m)+r]$.
At each node there are three equations of equilibrium, i.e.

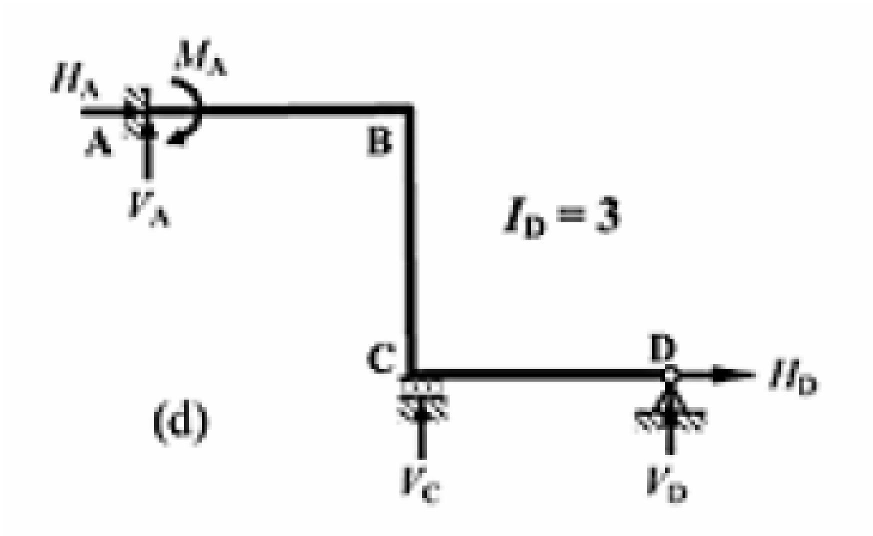
Σ the vertical forces Fy=0;
Σ the horizontal forces Fx=0;
Σ the moments M=0, providing $(3\times n)$ equations

The degree of indeterminacy $=[(3m)+r]-3n$

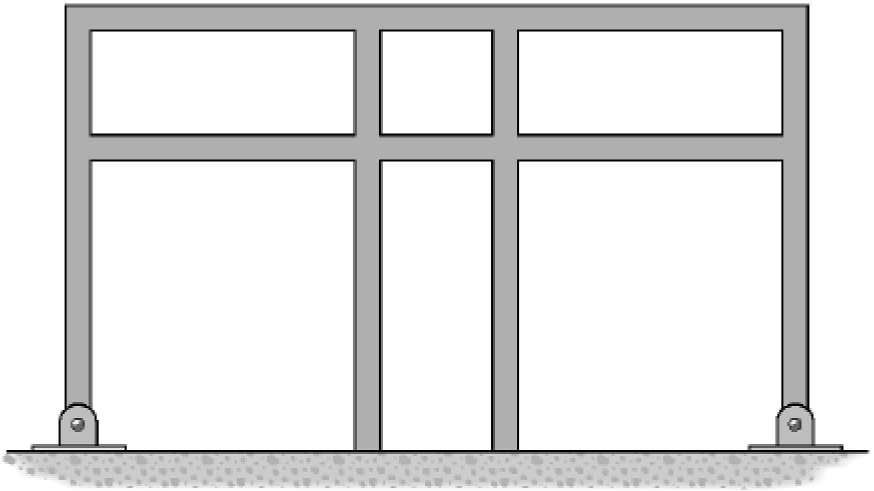




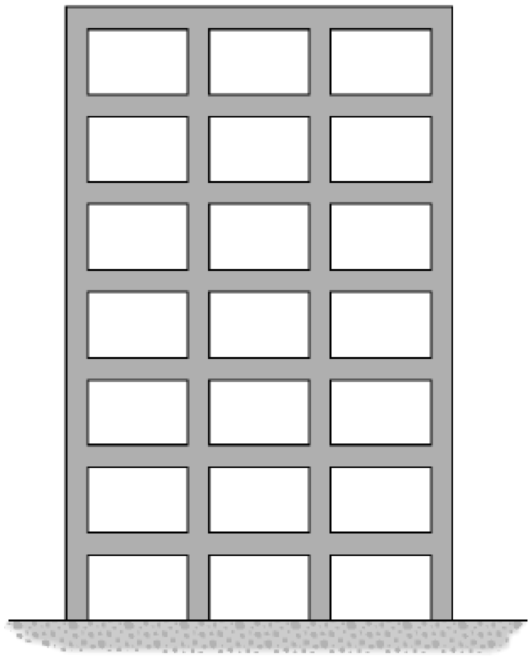




(d)



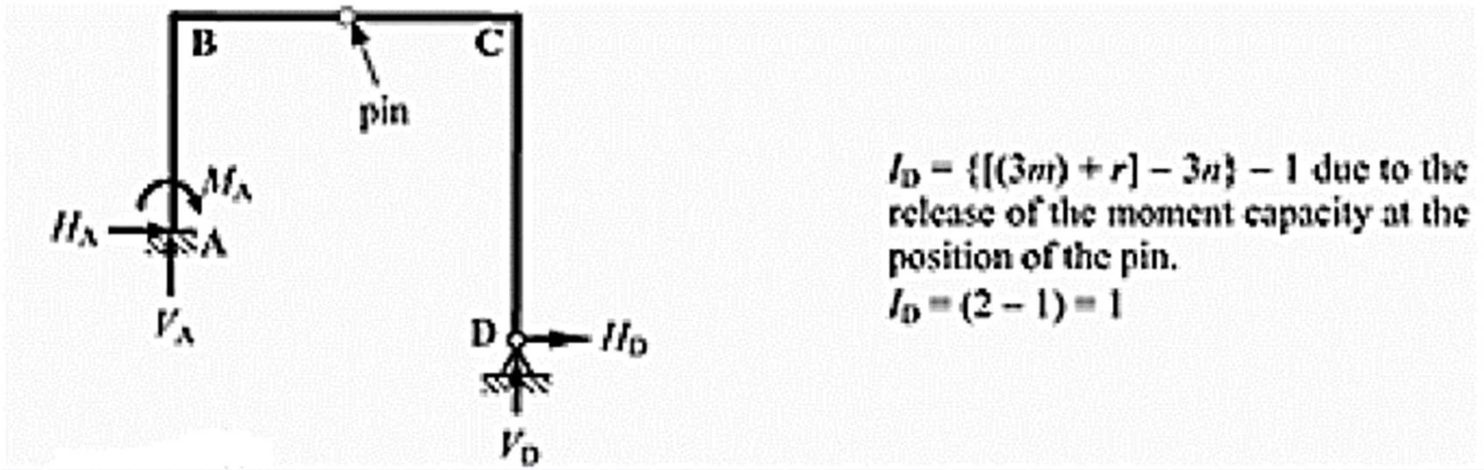
m= 14
r= 10
n= 12
3m+r=52
3n=36
ID=52-36=16



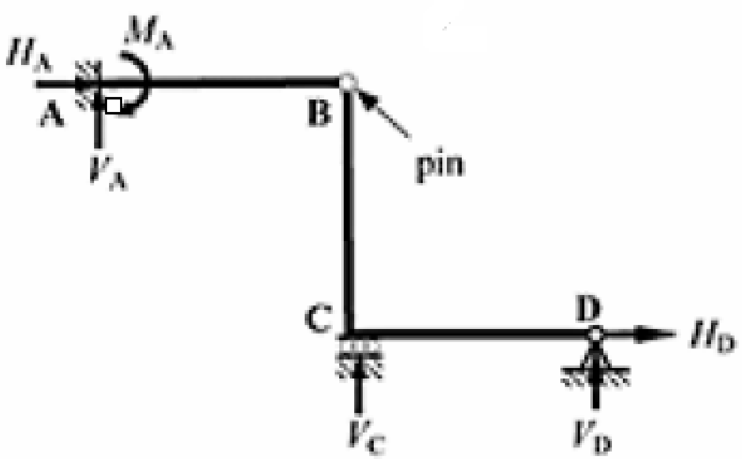
m= 49
r= 12
n= 32
3m+r=159
3n=96
ID=159-96=63

Rigid-Jointed Frame with internal pin

- The existence of an internal pin in a member in a rigid-frame results in only shear and axial loads being transferred through the frame at its location.
- This reduces the number of unknowns and hence redundancies, since an additional equation is available for solution
- Sum of the moments about the pin equals zero, i.e. Σ Mpin=0



Rigid-Jointed Frame with internal pin



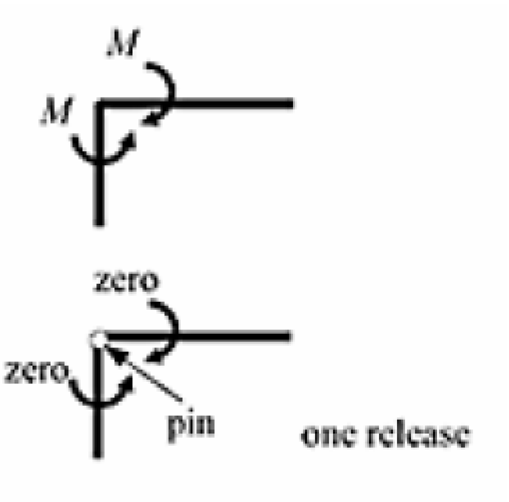
$I_D = \{[(3m) + r] - 3n\} - 1$ due to the release of the moment capacity at the position of the pin.

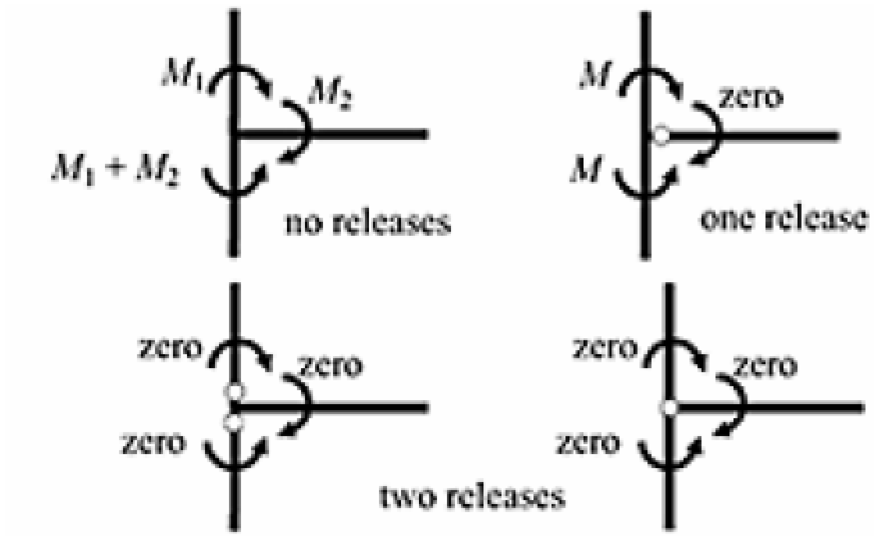
$I_D = (3 - 1) = 2$

Rigid-Jointed Frame with internal pin

- When there are three members meeting at the node then there are effectively two values of moment, i.e. M1 and M2 and in the third member M3=(M1+M2) .

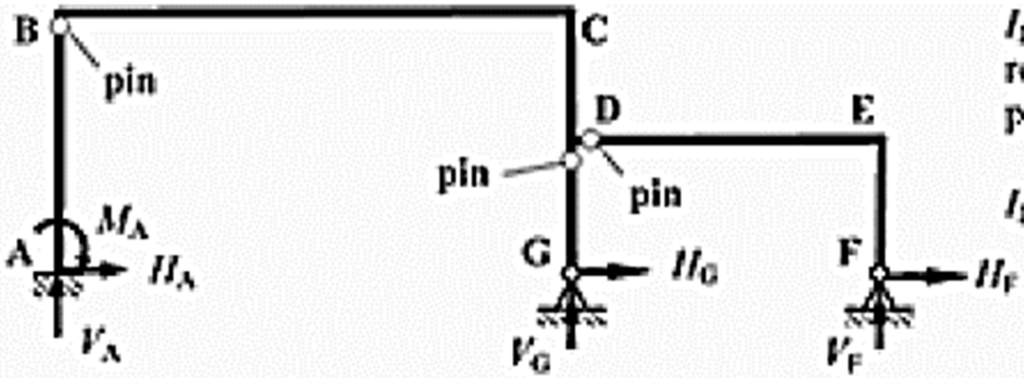
- The introduction of a pin in one of the members produces a single release and in two members produces two releases.





In general terms the introduction of ‘p’ pins at a joint introduces ‘p’ additional equations. When pins are introduced to all members at the joint the number of additional equations produced equals (number of members at the joint—1).

Rigid-Jointed Frame with internal pin

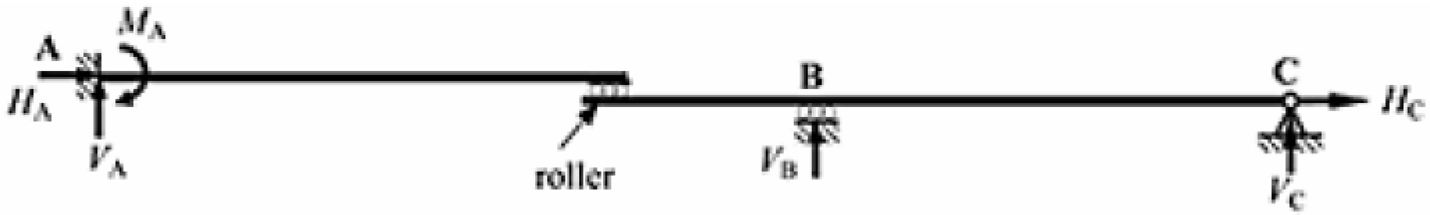


$I_D = \{[(3m) + r] - 3n\} - 3$ due to the release of the moment capacity at the positions of the pins.

$I_D = \{[(3 \times 6) + 7] - (3 \times 7)\} - 3 = 1$

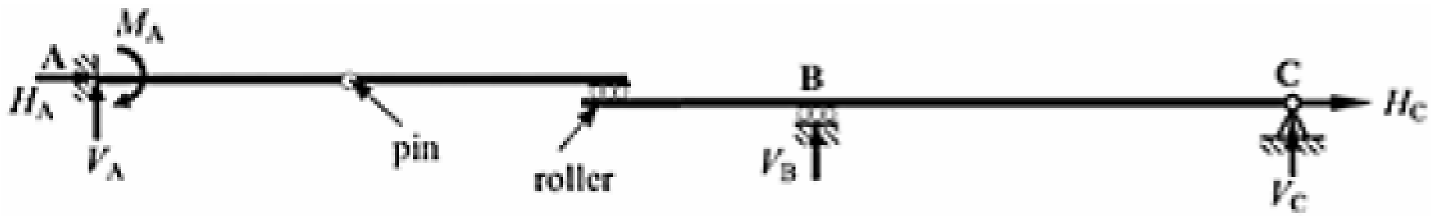
Rigid-Jointed members with internal roller

The inclusion of an internal roller within a member results in the release of the moment capacity and in addition the force parallel to the roller and **hence provides two additional equations**.



$I_D = \{[(3m)+r]-3n\}-2$ due to the release of the moment and axial load capacity at the roller

$\therefore I_D = \{[(3\times2)+6]-(3\times3)-2=1$



$I_D = \{[(3m)+r]-3n\}-3$ due to the release of the moment capacity at the position of the pin and the release of the moment and axial load capacity at the roller

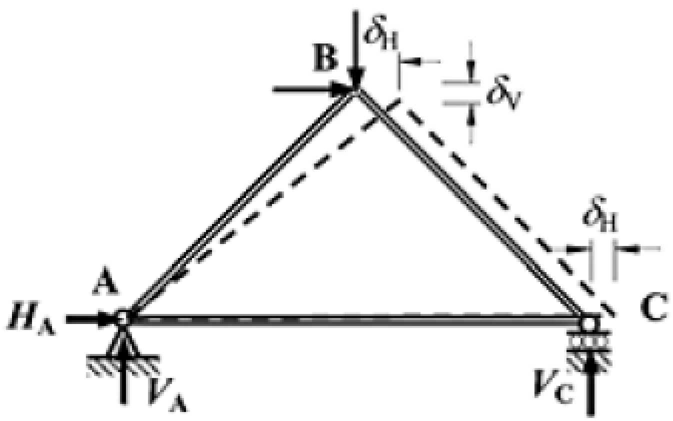
$I_D = \{[(3\times2)+6]-(3\times3)-3=0$ The structure is statically determinate.

Structural Degrees-of-Freedom (DOF)

DOF
Pin-Jointed
2D Frames

DOF:

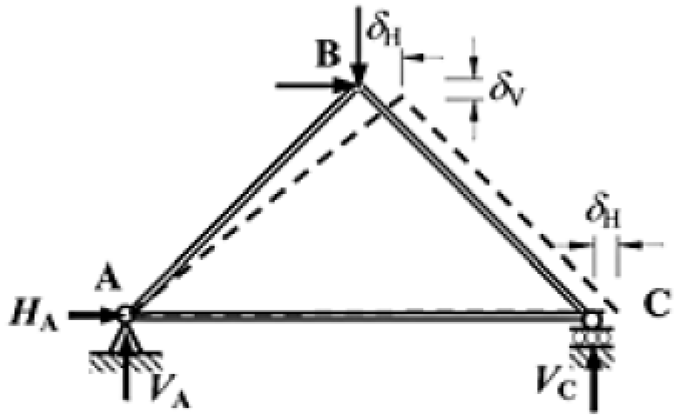
The degrees-of-freedom in a structure can be regarded as the possible components of displacements of the nodes including those at which some support conditions are provided.



In pin-jointed, plane-frames each node, unless restrained, can displace a small amount δ which can be resolved in to horizontal and vertical components δH and δV

Each component of displacement can be regarded as a separate degree-of-freedom and in this frame there is a total of three degrees-of-freedom:

The vertical and horizontal displacement of node B and the horizontal displacement of node C as indicated



- In a pin-jointed frame there are effectively two possible components of displacement for each node which does not constitute a support.

- At each roller support there is an additional degree-of-freedom due to the release of one restraint.

- In a simple, i.e. statically determinate frame, the number of degrees-of-freedom is equal to the number of members.
- In the case of indeterminate frames, the number of degrees-of-freedom is equal to the (number of members—I_D)

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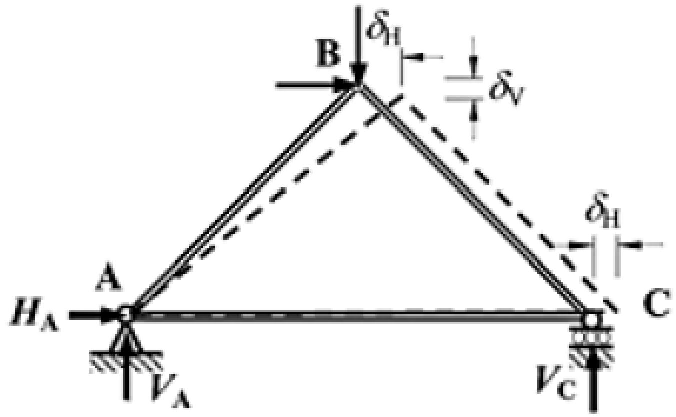
The vertical and horizontal displacement of node B and the horizontal displacement of node C as indicated



- In a pin-jointed frame there are effectively two possible components of displacement for each node which does not constitute a support.
- At each roller support there is an additional degree-of-freedom due to the release of one restraint.
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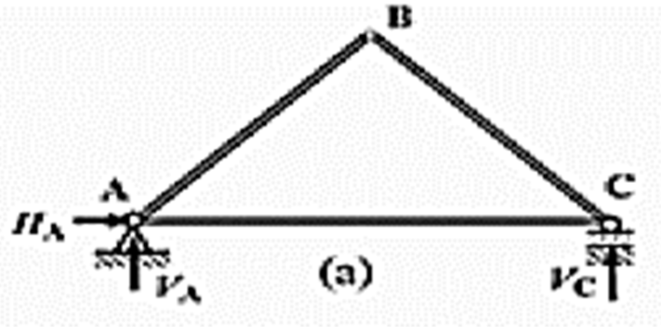
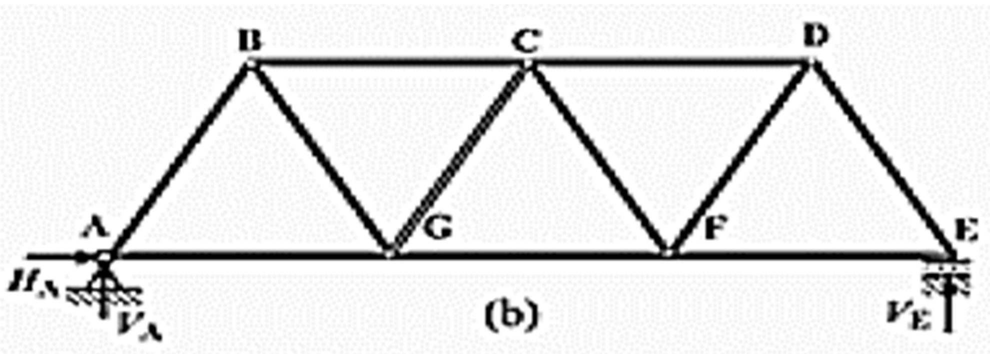
- In the case of indeterminate frames, the number of degrees-of-freedom is equal to the (number of members—I_D)

Examples

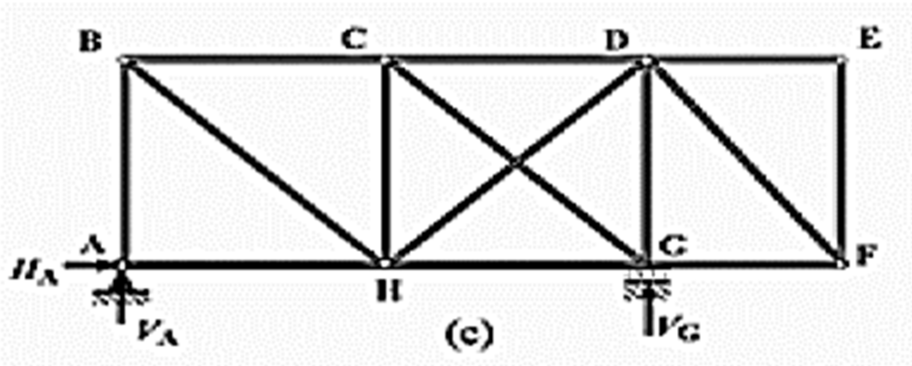


- the number of members m=3
- possible components of displacements at node B =2
- possible components of displacements at node support C =1
- **Total number of degrees-of-freedom** (=m)=3

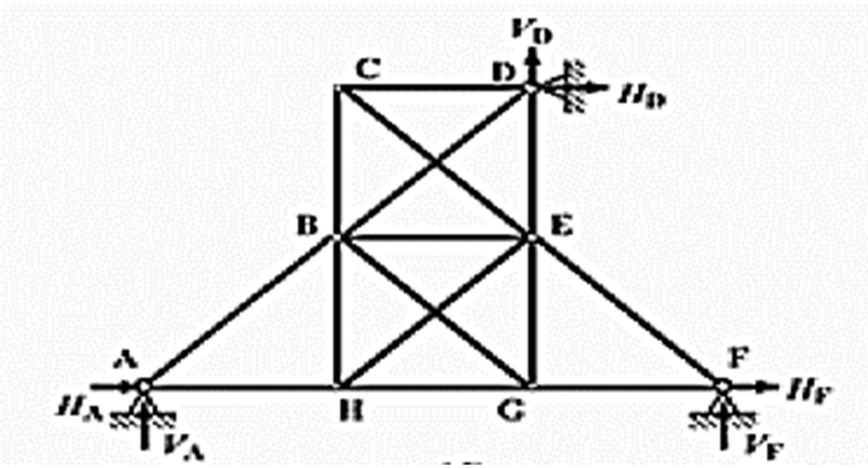


the number of members m=11
possible components of displacements at nodes =10
possible components of displacements at support E
=1
Total number of degrees-of-freedom (=m)=11

Examples



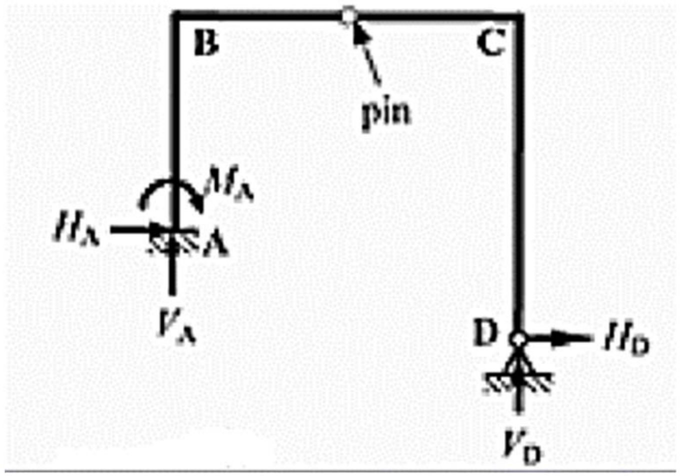
the number of members m=14
possible components of displacements at nodes =12
possible components of displacements at support G =1
degree-of-indeterminacy ID=1
Total number of degrees-of-freedom (m−ID)=13



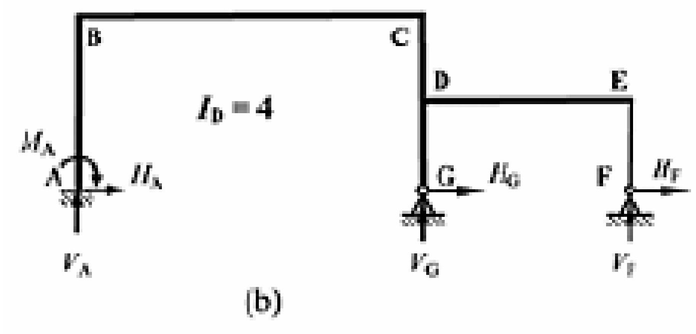
the number of members m=15
possible components of displacements at nodes =10
degree-of-indeterminacy I_D=5
Total number of degrees-of-freedom (m−I_D)=10

DOF
Rigid-Jointed
2D Frames

Examples

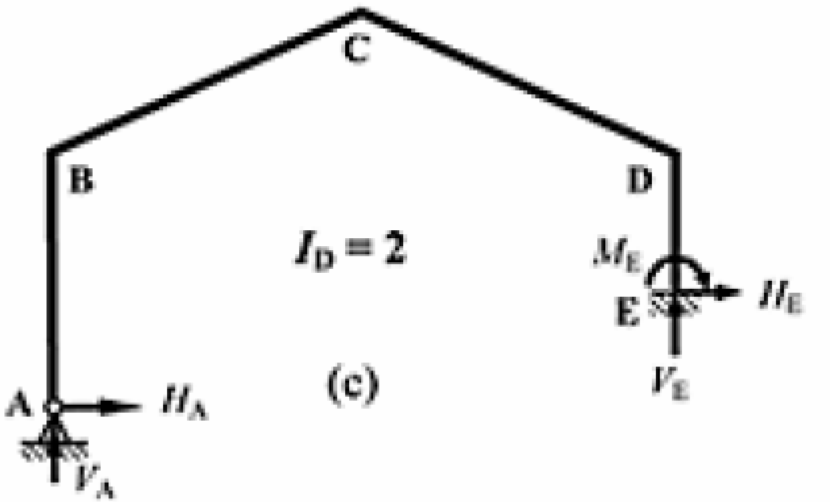


the number of nodes (excluding supports) =2
possible components of displacements at nodes =6
possible components of displacements at support D =1
Total number of degrees-of-freedom =7

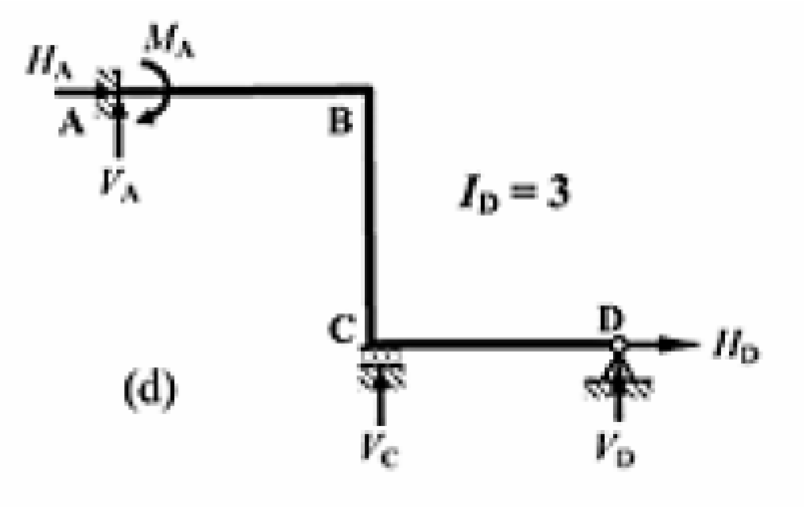


the number of nodes (excluding supports) =4
possible components of displacements at nodes =12
possible components of displacements at support G =1
possible components of displacements at support F =1
Total number of degrees-of-freedom =14

Examples

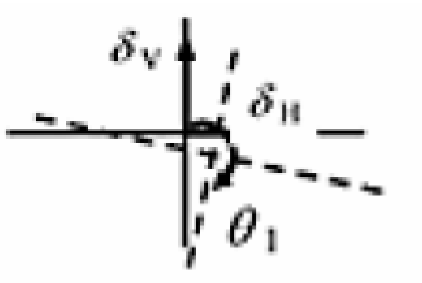


the number of nodes (excluding supports) =3
possible components of displacements at nodes =9
possible components of displacements at support A =1
Total number of degrees-of-freedom =10



the number of nodes (excluding supports) =1
possible components of displacements at nodes =3
possible components of displacements at support C =2
possible components of displacements at support D =1
Total number of degrees-of-freedom =6

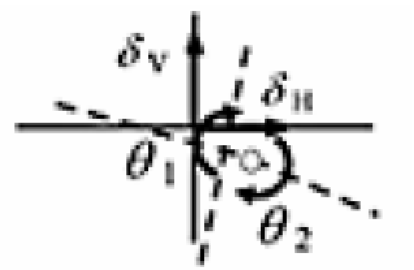
The introduction of a pin in a member at a node produces an additional degree-of-freedom.



The node is a rigid connection with no pins in any of the members and has the three degrees-of-freedom

total = 3 one of rotation - θ_1 two of translation - δ_H, δ_V

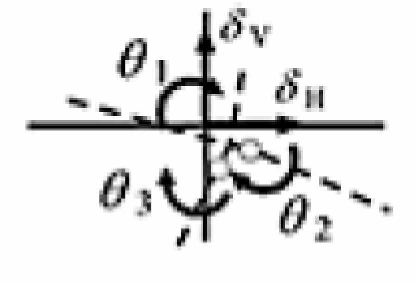
If a pin is present in one member, this produces an additional degrees-of-freedom since the rotation of this member can be different from the remaining three



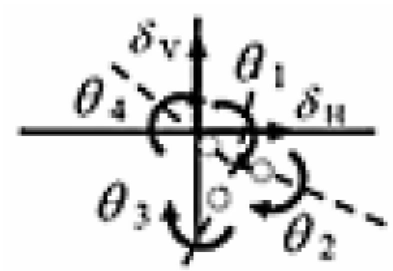
total = 4 two of rotation - θ_1, θ_2 two of translation - δ_H, δ_V

If a pin is present in one member, this produces an additional degrees-of-freedom since the rotation of this member can be different from the remaining three

total = 4 two of rotation - θ_1, θ_2 two of translation - δ_H, δ_V



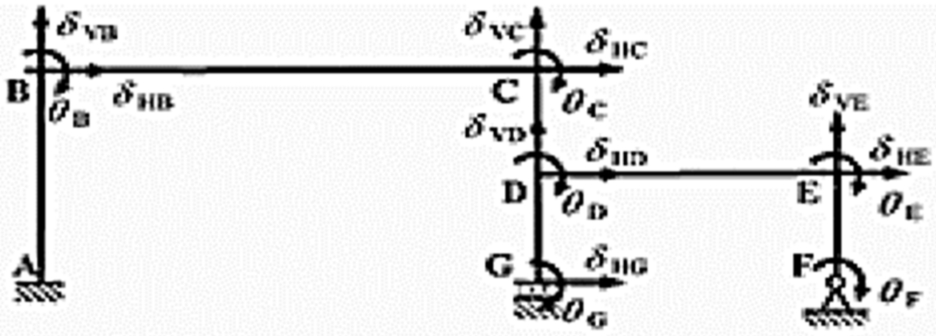
total = 5 three of rotation - $\theta_1, \theta_2, \theta_3$ two of translation - δ_H, δ_V



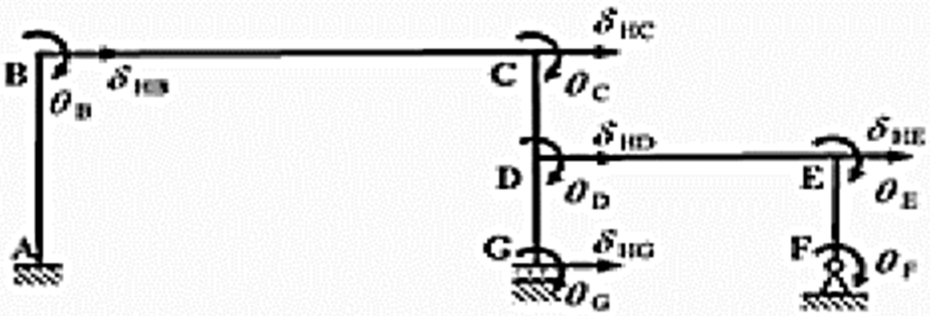
total = 6 four of rotation - $\theta_1, \theta_2, \theta_3, \theta_4$ two of translation - δ_H, δ_V

Note:

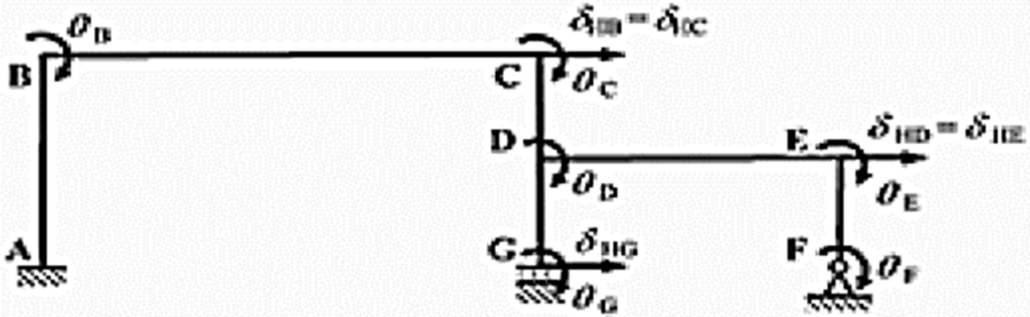
In many cases the effects due to axial deformations is significantly smaller than those due to the bending effect and **consequently an analysis assuming axial rigidity of members is acceptable**.



No axial rigidity
Degrees-of-freedom:
three at nodes B, C, D and E
one at node F
two at node G
Total = [(3 × 4) + 1 + 2] = 15



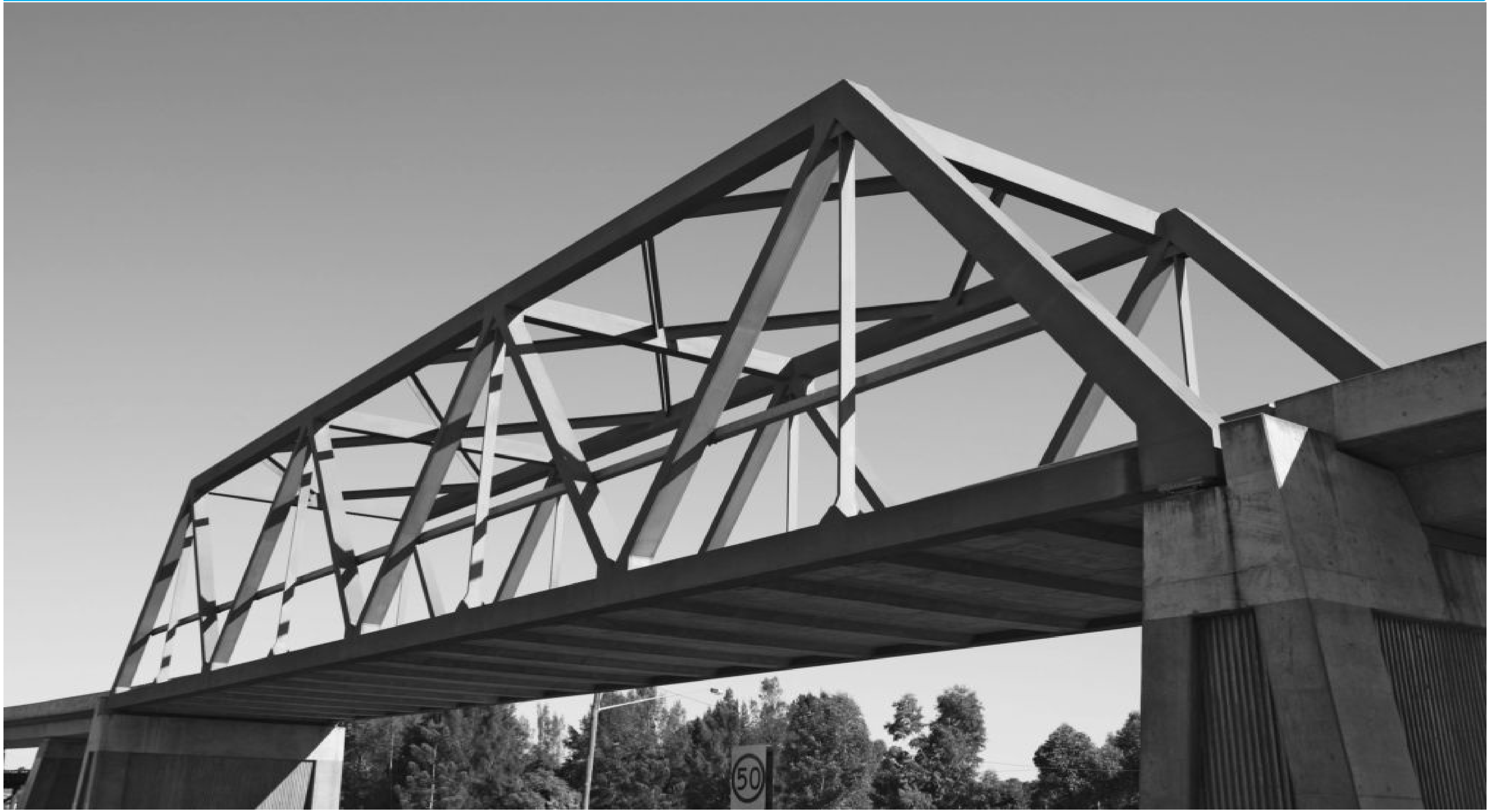
Assume all columns to be axial rigid
Degrees-of-freedom:
two at nodes B, C, D and E
one at node F
two at node G
Total = [(2 × 4) + 1 + 2] = 11



Assume all beams and columns to be axial rigid
Degrees-of-freedom:
one rotation at B, C, D, and E
one translation at levels BC and DE
one at node F
two at node G
Total = [(1 × 4) + 2 + 1 + 2] = 9

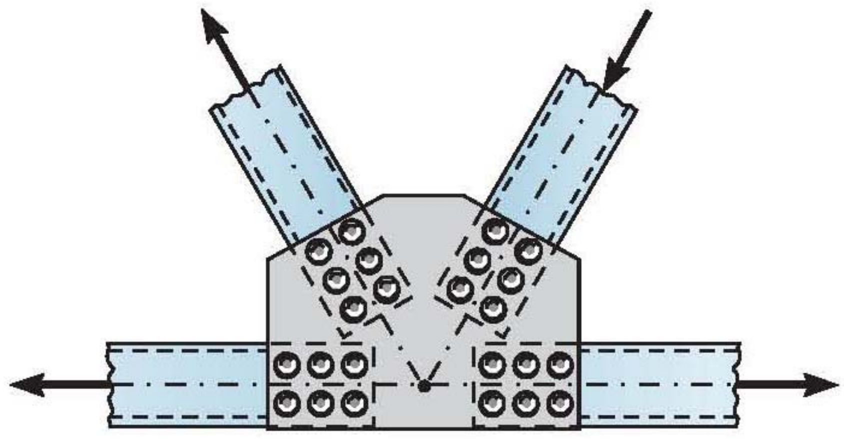
THANK YOU

Analysis of Statically Determinate Truss Structures



Common Types of Trusses

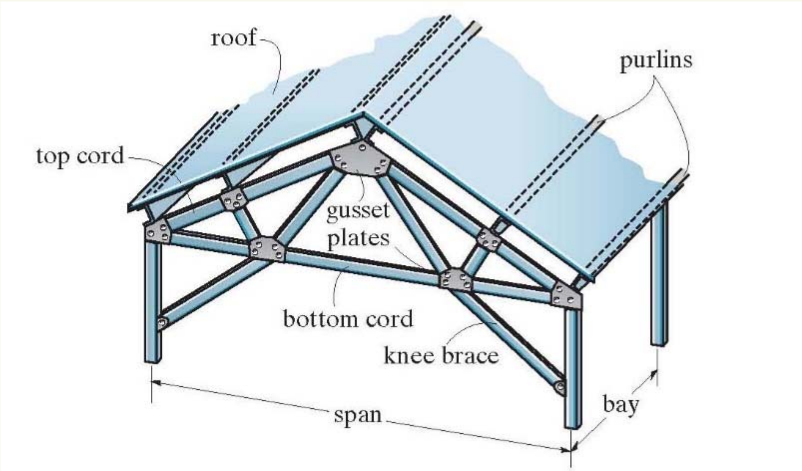
- A truss is a structure composed of slender members joined together at their end points
- The joint connections are usually formed by bolting or welding the ends of the members to a common plate called gusset
- Planar trusses lie in a single plane & is often used to support roof or bridges



gusset plate

Common Types of Trusses

- **Roof Trusses**
  - They are often used as part of an industrial building frame
  - Roof load is transmitted to the truss at the joints by means of a series of purlins
  - To keep the frame rigid & thereby capable of resisting horizontal wind forces, knee braces are sometimes used at the supporting column

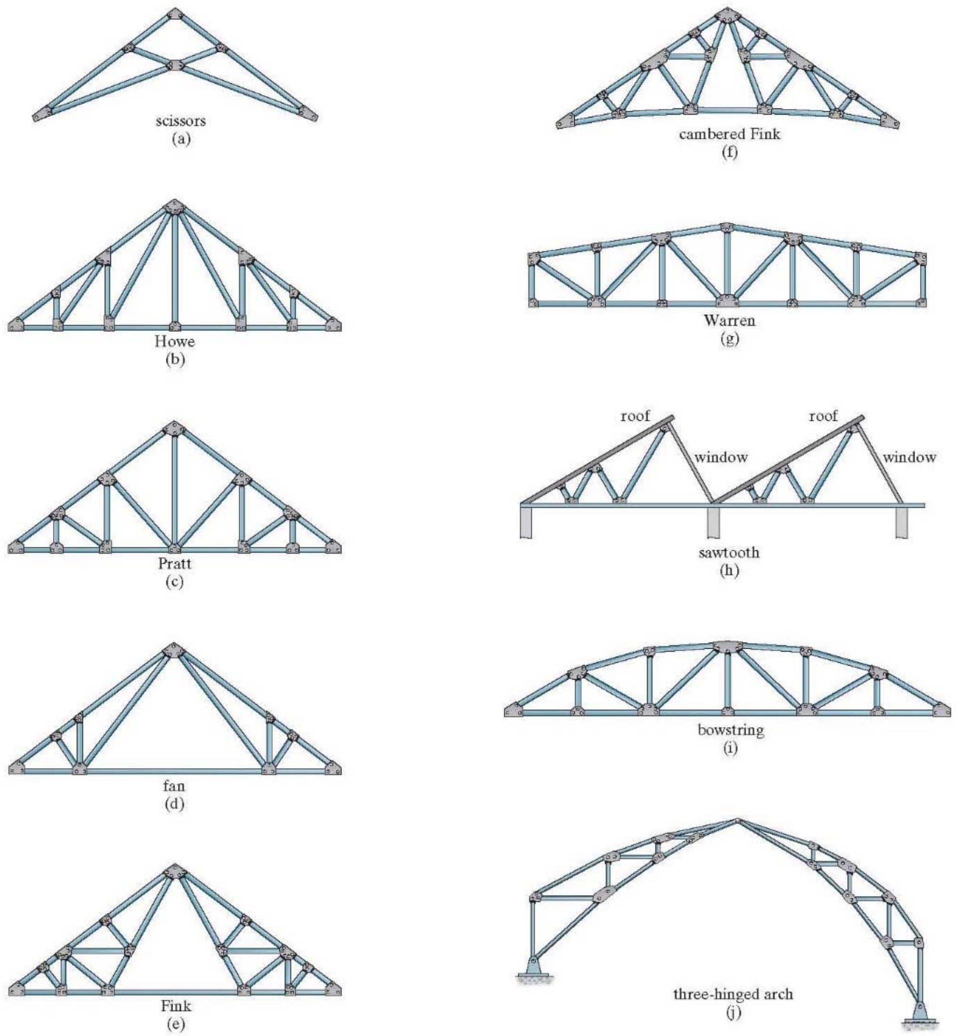


Common Types of Trusses

- **Roof Trusses**
  - The space between adjacent bents is called a *bay*
- Bays are economically spaced at about 15 ft (4.6 m) for spans around 60 ft
- (18 m) and about 20 ft (6.1 m) for spans of 100 ft (30 m).
- Bays are often tied together using diagonal bracing in order to maintain rigidity of the building's structure.

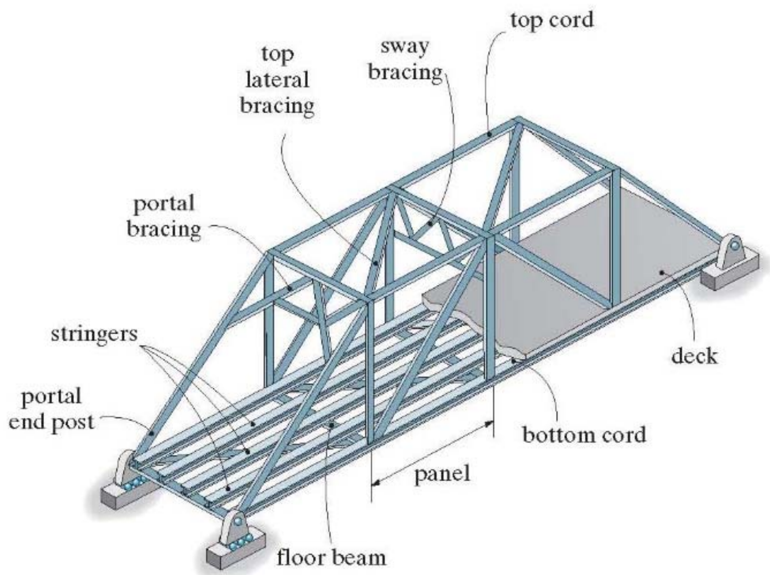
Common Types of Trusses

- Roof Trusses
- Trusses used to support roofs are selected on the basis of :
- the span,
- the slope, and
- the roof material.



Common Types of Trusses

- Bridge Trusses
  - The load on the deck is first transmitted to the stringers -> floor beams -> joints of supporting side truss
  - The top & bottom cords of these side trusses are connected by top & bottom lateral bracing resisting lateral forces

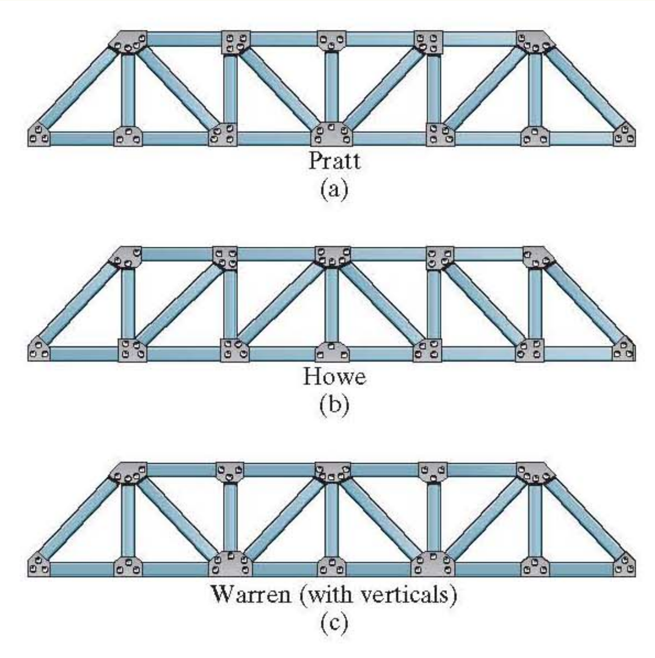


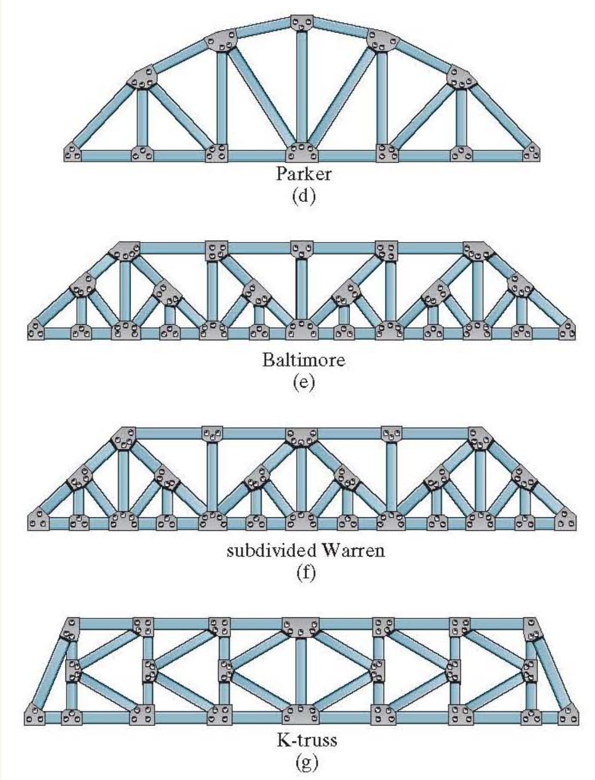
Common Types of Trusses

- Bridge Trusses

Common Types of Trusses

- Bridge Trusses
  - Additional stability is provided by the portal & sway bracing
  - In the case of a long span truss, a roller is provided at one end for thermal expansion



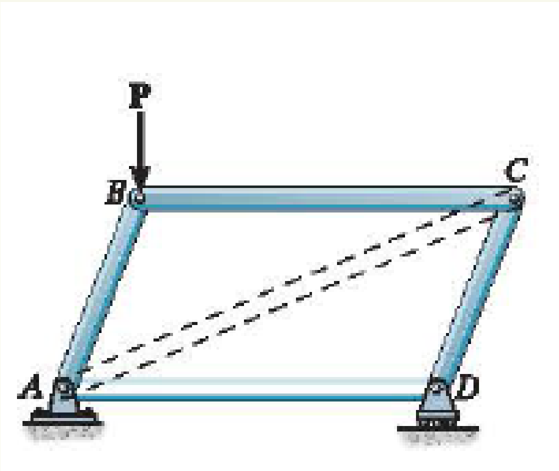


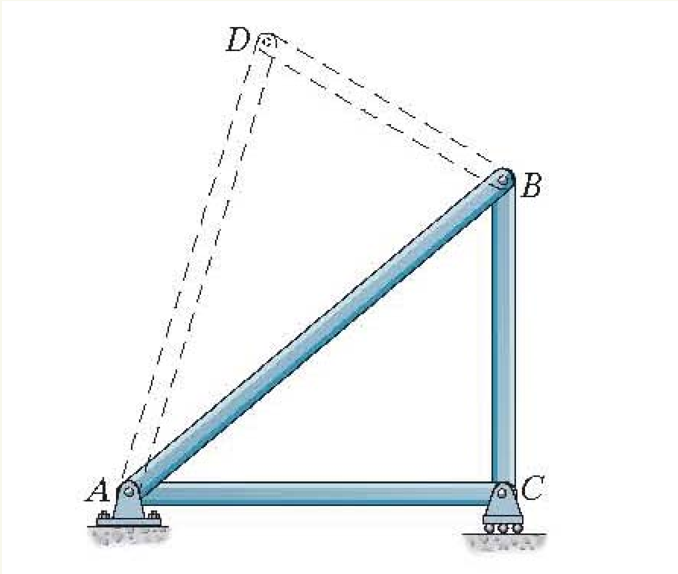
Common Types of Trusses

- Assumptions for Design
  - The members are joined together by smooth pins
  - All loadings are applied at the joints
- Due to the 2 assumptions, each truss member acts as an axial force member

Classification of Coplanar Trusses

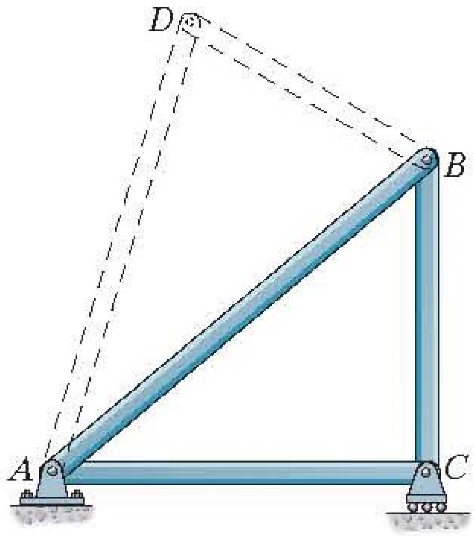
- Simple , Compound or Complex Truss
- Simple Truss
  - To prevent collapse, the framework of a truss must be rigid
  - The simplest framework that is rigid or stable is a triangle



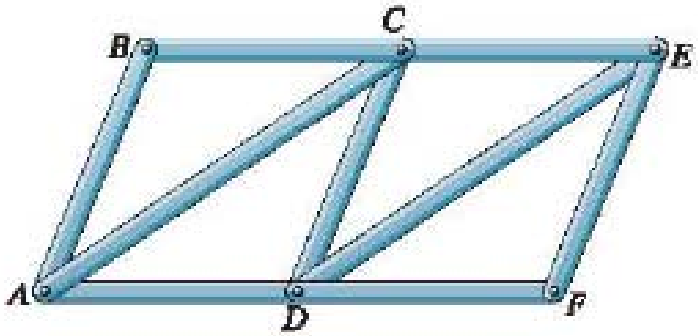


Classification of Coplanar Trusses

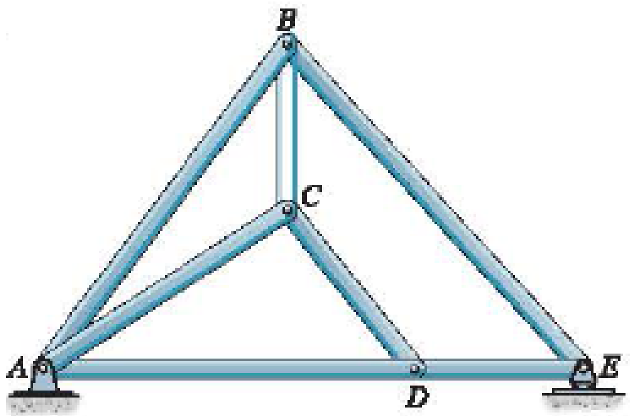
- Simple Truss
  - A simple truss is the basic “stable” triangle element is ABC
- connecting two members *(AD* and *BD)* to form an additional element
- Thus it is seen that as each additional element of two members is placed on the truss, the number of joints is increased by one.



Classification of Coplanar Trusses



simple truss



Classification of Coplanar Trusses

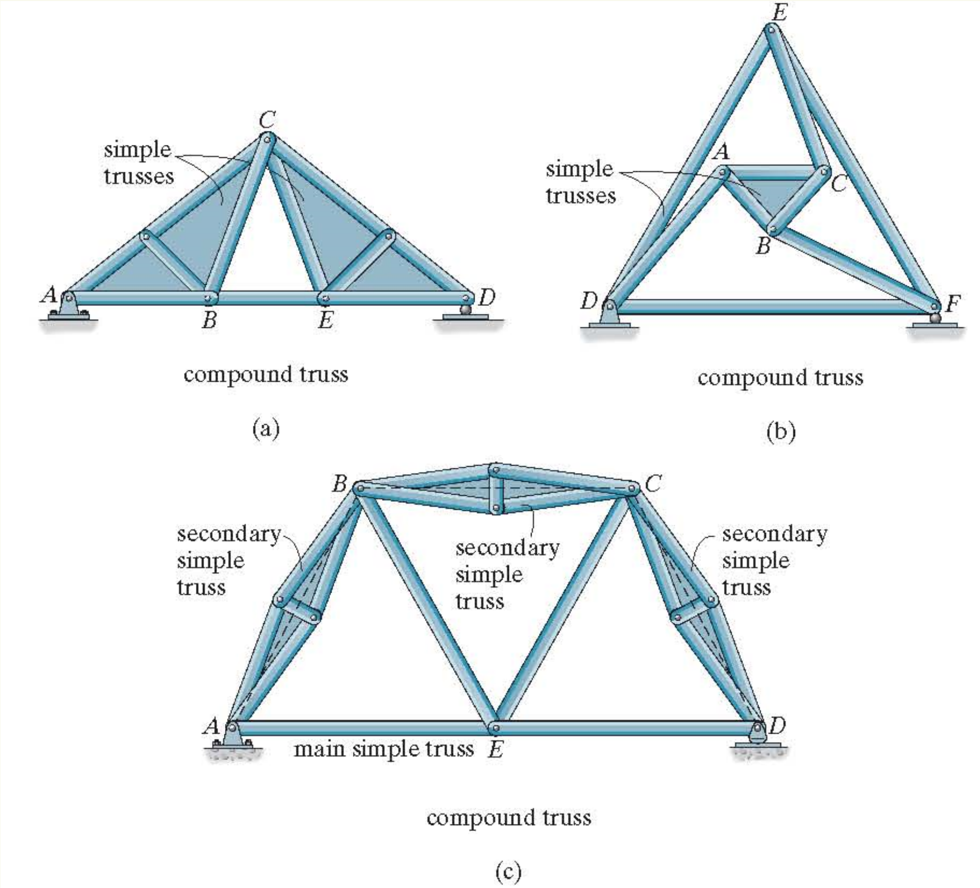
- Compound Truss
  - It is formed by connecting 2 or more simple truss together
  - Often, this type of truss is used to support loads acting over a larger span as it is cheaper to construct a lighter compound truss than a heavier simple truss

Classification of Coplanar Trusses

- Compound Truss
  - Type 1
    - The trusses may be connected by a common joint & bar
  - Type 2
    - The trusses may be joined by 3 bars
  - Type 3
    - The trusses may be joined where bars of a large simple truss, called the main truss, have been substituted by simple truss, called secondary trusses

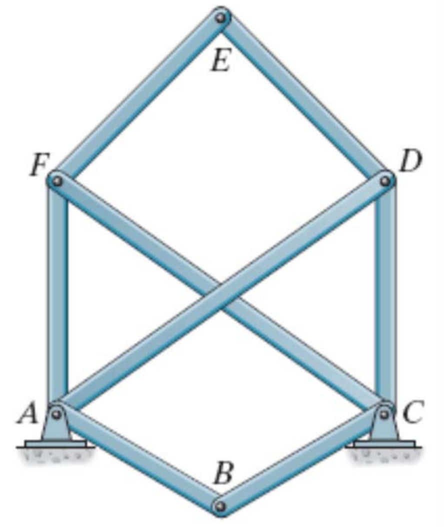
Classification of Coplanar Trusses

- Compound Truss



Classification of Coplanar Trusses

- Complex Truss
  - A complex truss is one that cannot be classified as being either simple or compound



complex truss

Classification of Coplanar Trusses

- Determinacy
  - Total unknowns = forces in no. of bars of the truss (b) + total no. of external support reactions (r)
  - Force system at each joint is coplanar & concurrent
  - Rotational or moment equilibrium is automatically satisfied

Classification of Coplanar Trusses

- Determinacy
  - Therefore only

$$\sum F_x = 0 \text{ and } \sum F_y = 0$$

  - By comparing the total unknowns with the total no. of available equilibrium eqn, we have:

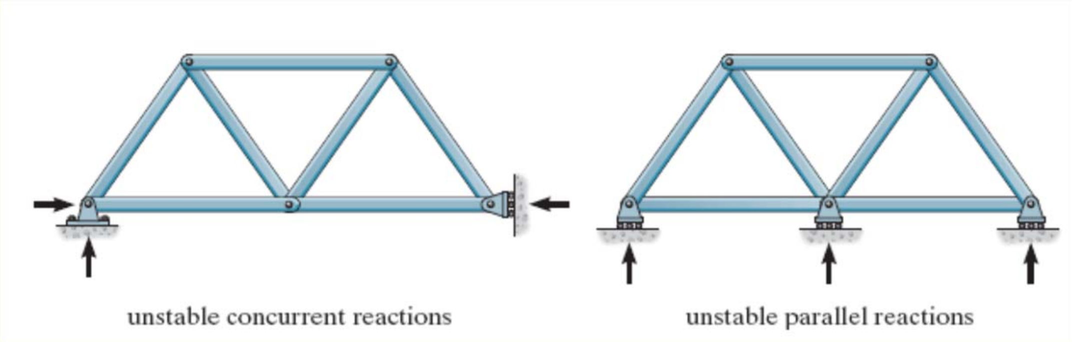
$$b + r = 2j \quad \text{statically determinate}$$
$$b + r > 2j \quad \text{statically indeterminate}$$

Classification of Coplanar Trusses

- Stability
  - If b + r < 2j => collapse
  - A truss can be unstable if it is statically determinate or statically indeterminate
  - Stability will have to be determined either through inspection or by force analysis

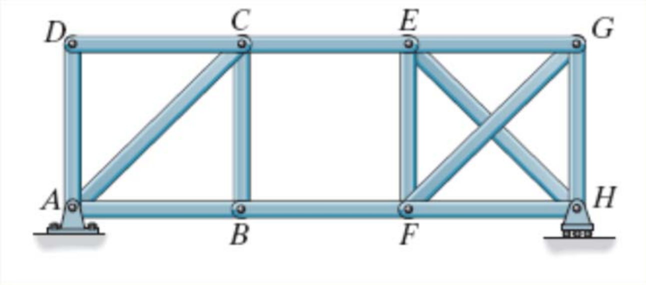
Classification of Coplanar Trusses

- Stability
  - External Stability
    - A structure is externally unstable if all of its reactions are concurrent or parallel
    - The trusses are externally unstable since the support reactions have lines of action that are either concurrent or parallel



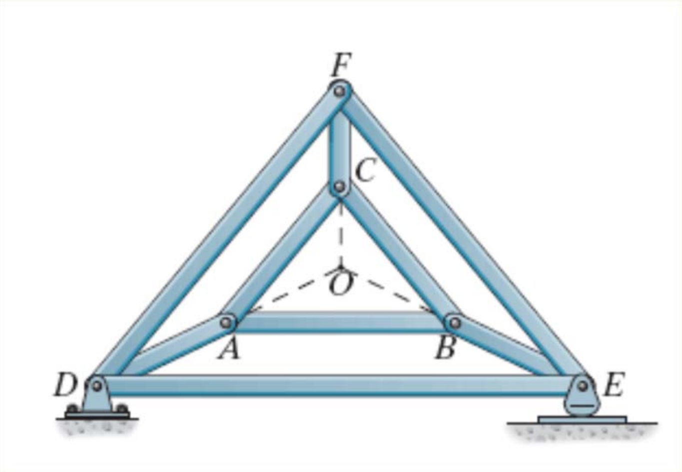
Classification of Coplanar Trusses

- Internal Stability
  - The internal stability can be checked by careful inspection of the arrangement of its members
  - If it can be determined that each joint is held fixed so that it cannot move in a "rigid body" sense wrt the other joints, then the truss will be stable
  - A simple truss will always be internally stable
  - If a truss is constructed so that it does not hold its joints in a fixed position, it will be unstable or have a "critical form"



Classification of Coplanar Trusses

- Internal Stability
  - To determine the internal stability of a compound truss, it is necessary to identify the way in which the simple truss are connected together
  - The truss shown is unstable since the inner simple truss ABC is connected to DEF using 3 bars which are concurrent at point O

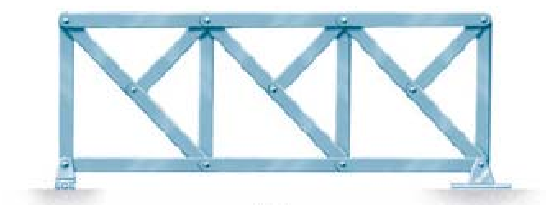


Classification of Coplanar Trusses

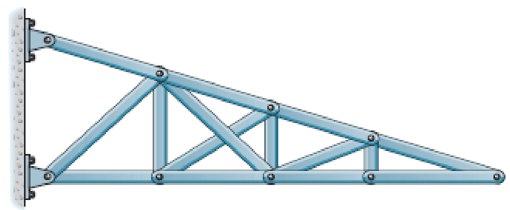
- Internal Stability
  - Thus an external load can be applied at A, B or C & cause the truss to rotate slightly
  - For complex truss, it may not be possible to tell by inspection if it is stable
  - The instability of any form of truss may also be noticed by using a computer to solve the 2j simultaneous eqns for the joints of the truss
  - If inconsistent results are obtained, the truss is unstable or have a critical form

Example 3.1

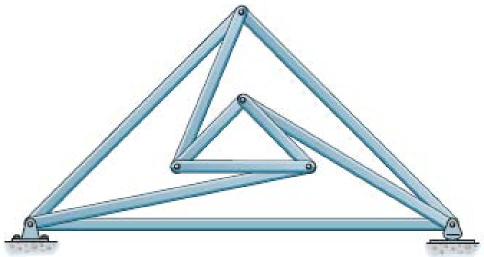
Classify each of the trusses as stable, unstable, statically determinate or statically indeterminate. The trusses are subjected to arbitrary external loadings that are assumed to be known & can act anywhere on the trusses.



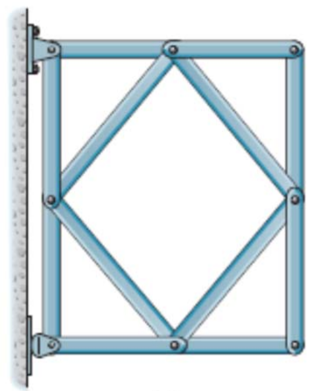
(a)



(b)



(c)

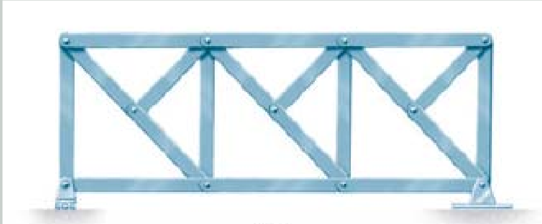


(d)

Solution

For (a),

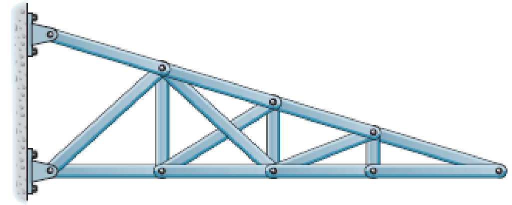
- Externally stable
- Reactions are not concurrent or parallel
- $b = 19$, $r = 3$, $j = 11$
- $b + r = 2j = 22$
- Truss is statically determinate
- By inspection, the truss is internally stable


(a)

Solution

For (b),

- Externally stable
- b = 15, r = 4, j = 9
- b + r = 19 >2j
- Truss is statically indeterminate
- By inspection, the truss is internally stable

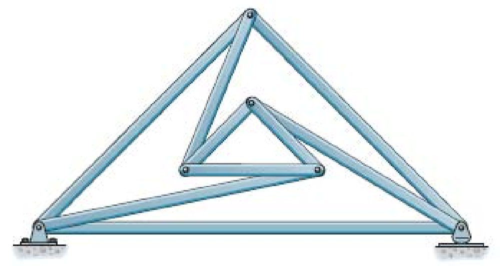


(b)

Solution

For (c),

- Externally stable
- $b = 9$, $r = 3$, $j = 6$
- $b + r = 12 = 2j$
- Truss is statically determinate
- By inspection, the truss is internally stable

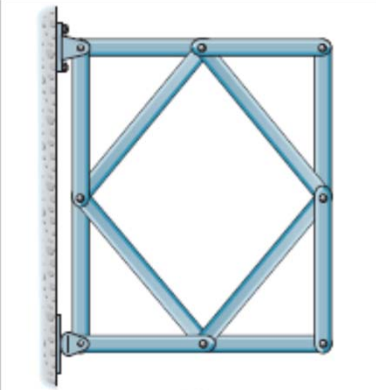


(c)

Solution

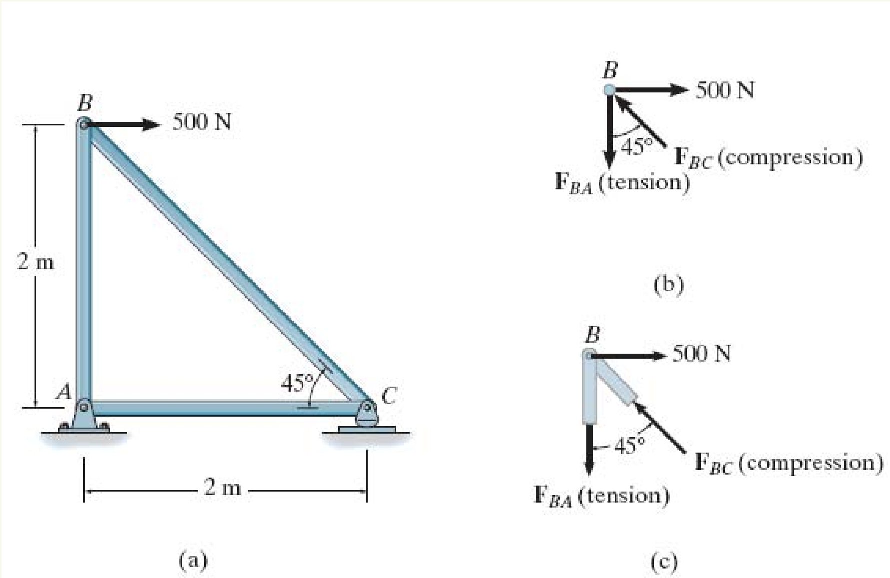
For (d),

- Externally stable
- $b = 12, r = 3, j = 8$
- $b + r = 15 < 2j$
- The truss is internally unstable


(d)

The Method of Joints

- Satisfying the equilibrium eqns for the forces exerted on the pin at each joint of the truss
- Applications of eqns yields 2 algebraic eqns that can be solved for the 2 unknowns



The Method of Joints

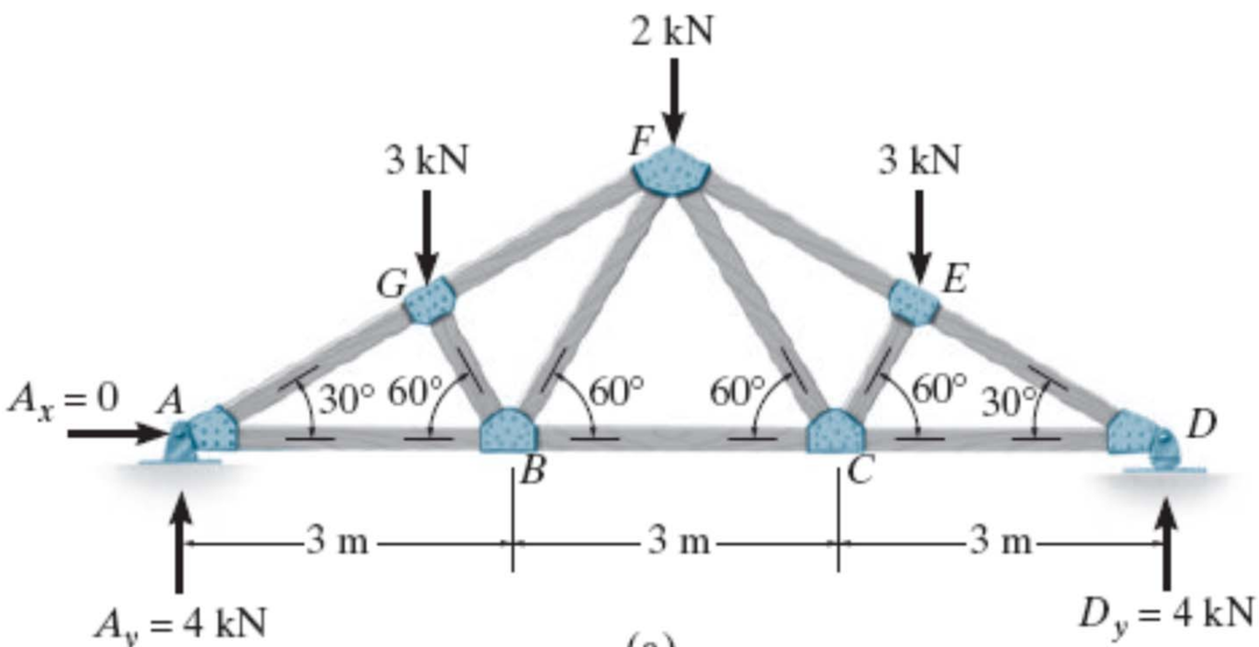
- Always assume the unknown member forces acting on the joint's free body diagram to be in tension
- Numerical solution of the equilibrium eqns will yield positive scalars for members in tension & negative for those in compression
- The correct sense of direction of an unknown member force can in many cases be determined by inspection

The Method of Joints

- A +ve answer indicates that the sense is correct, whereas a –ve answer indicates that the sense shown on the free-body diagram must be reversed

Example 3.2

Determine the force in each member of the roof truss as shown. State whether the members are in tension or compression. The reactions at the supports are given as shown.



(a)

Solution

Only the forces in half the members have to be determined as the truss is symmetric wrt both loading & geometry,

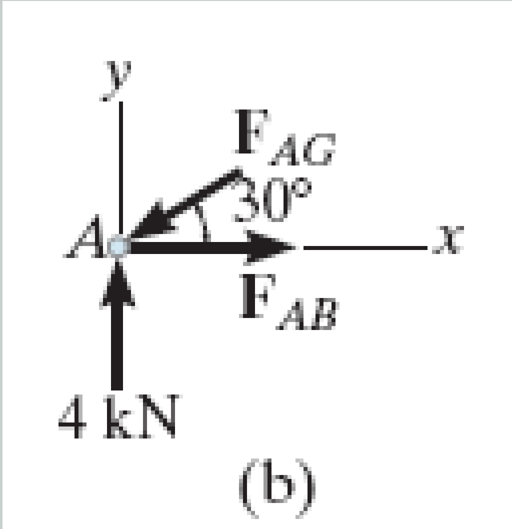
Joint A,

$$+\uparrow \sum F_y = 0; \quad 4 - F_{AG} \sin 30^0 = 0$$

$$F_{AG} = 8kN(C)$$

$$\underset{\rightarrow}{+} \sum F_x = 0; \quad F_{AB} - 8\cos 30^0 = 0$$

$$F_{AB} = 6.93kN(T)$$



Solution

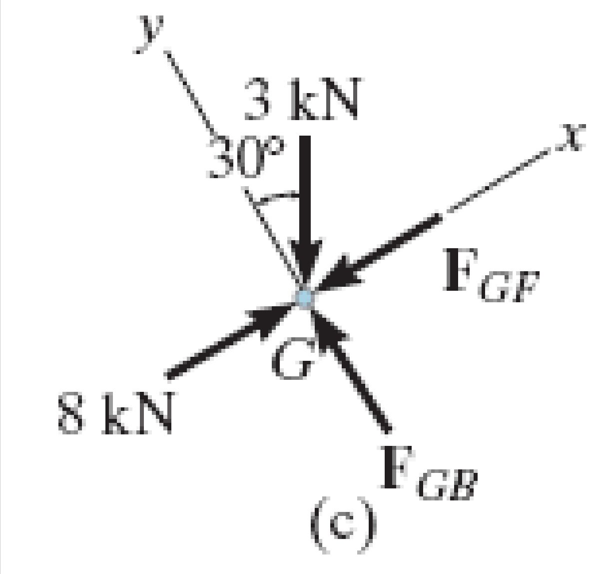
Joint G,

$$+\uparrow \sum F_y = 0; \; F_{GB} - 3\cos 30^0 = 0$$

$$F_{GB} = 2.60 kN (C)$$

$$\underset{\rightarrow}{+} \sum F_x = 0; \;\; 8 - 3\sin 30^0 - F_{GF} = 0$$

$$F_{GF} = 6.50 kN (C)$$



Solution

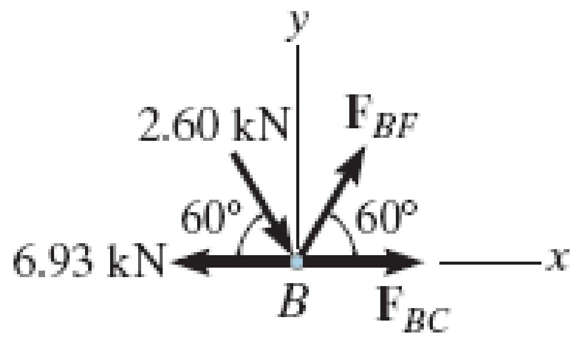
Joint B,

$$+\uparrow \sum F_y = 0; \quad F_{BF}\sin 60^0 - 2.60\sin 60^0 = 0$$

$$F_{BF} = 2.60kN(T)$$

$$\underrightarrow{+}\sum F_x = 0; \quad F_{BC} + 2.60\cos 60^0 + 2.60\cos 60^0 - 6.93 = 0$$

$$F_{BC} = 4.33kN(T)$$

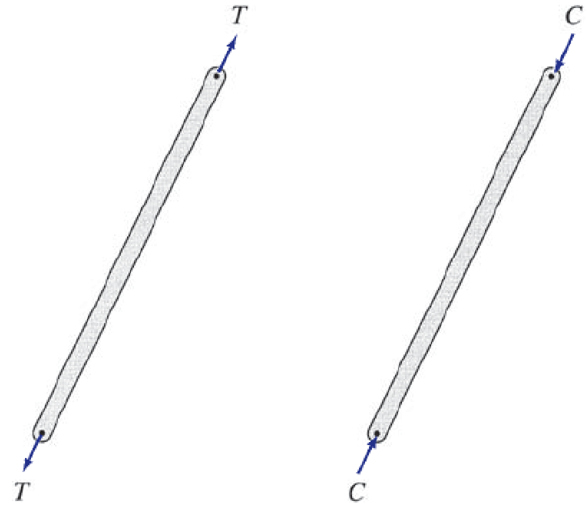


(d)

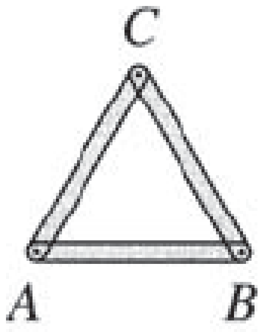
THANK YOU

Analysis of Truss Structures

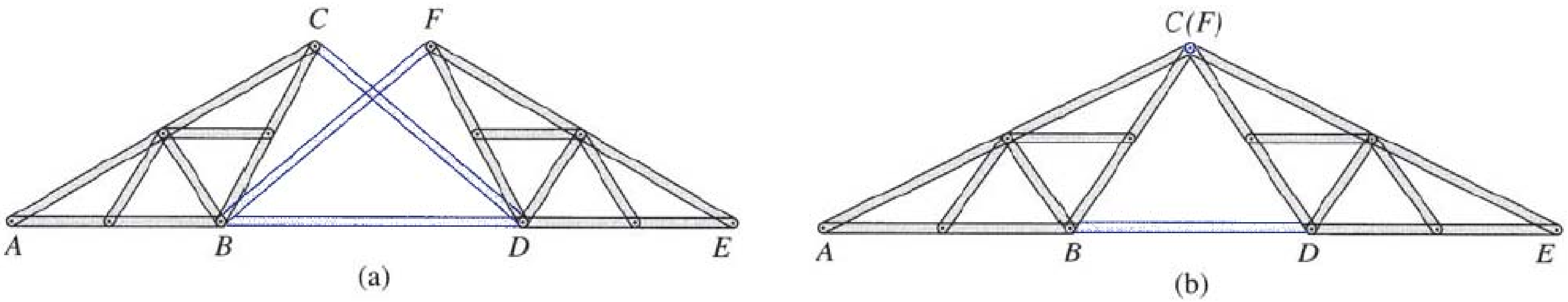
Idealized Truss Members



Types of Truss



Simple Truss



Compound Truss

Zero-Force Members

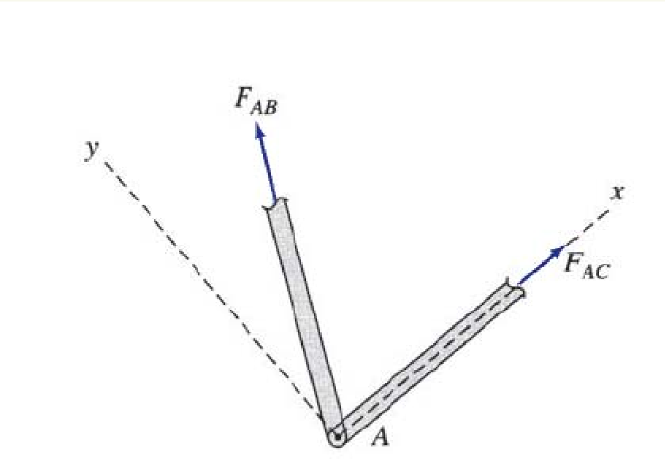
- Truss analysis using method of joints is greatly simplified if one is able to first determine those members that support no loading
- These zero-force members may be necessary for the stability of the truss during construction & to provide support if the applied loading is changed
- Zero-force members are also added to trusses to brace compression members against buckling and slender tension members against vibrating.
- The zero-force members of a truss can generally be determined by inspection of the joints & they occur in 2 cases.

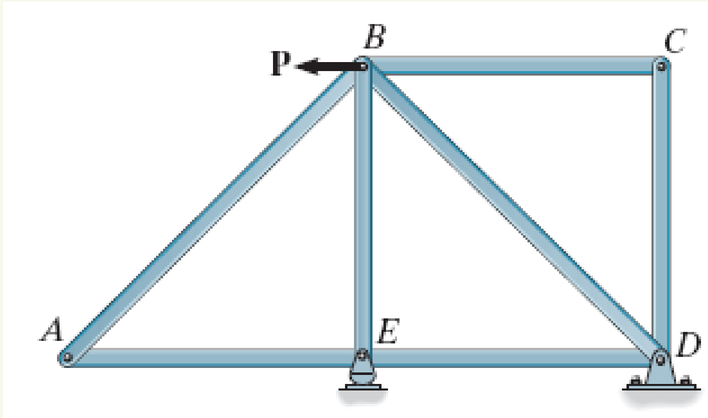
Zero-Force Members

- Case 1

If only two non-collinear members are connected to a joint that has no external loads or reactions applied to it, then the force in both members is zero.

  - The 2 members at joint C are connected together at a right angle & there is no external load on the joint
  - The free-body diagram of joint C indicates that the force in each member must be zero in order to maintain equilibrium





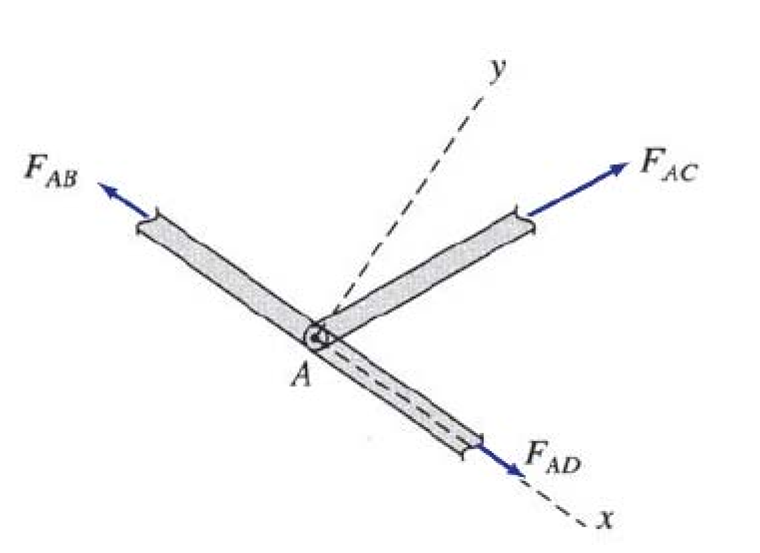
(a)

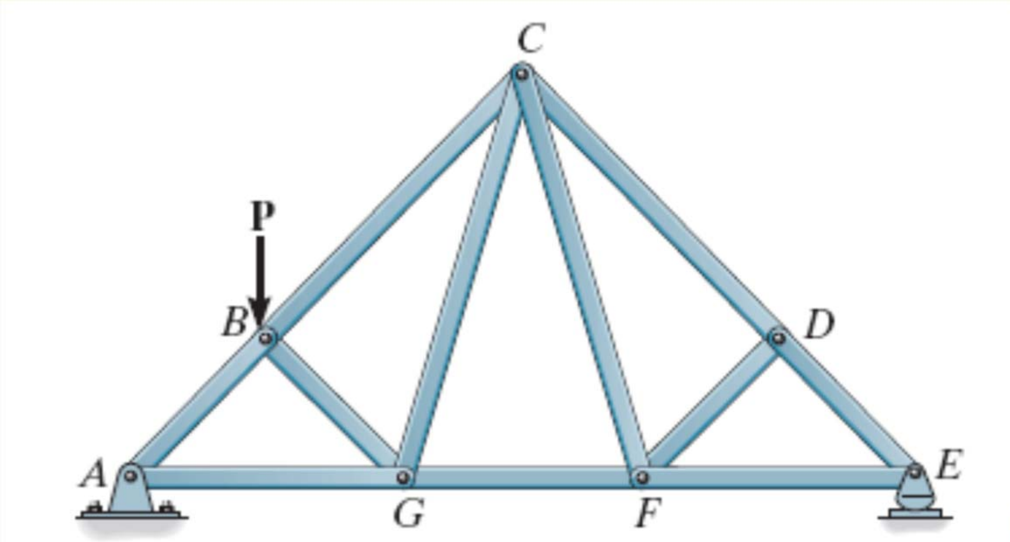
Zero-Force Members

- Case 2

If three members, two of which are collinear, are connected to a joint that has no external loads or reactions applied to it, then the force in the member that is not collinear is zero

  - Zero-force members also occur at joints having a geometry as joint D





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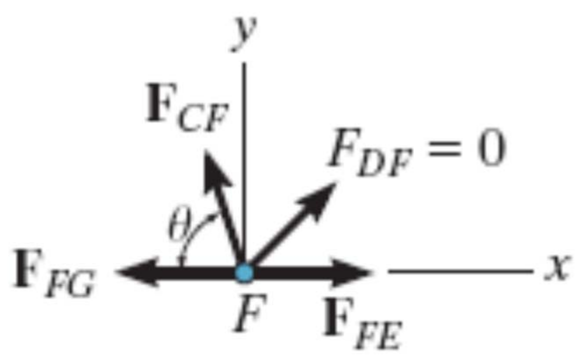
Zero-Force Members

- Case 2
  - No external load acts on the joint, so a force summation in the y-direction which is perpendicular to the 2 collinear members requires that $F_{DF} = 0$
  - Using this result, FC is also a zero-force member, as indicated by the force analysis of joint F



(c)

$$+\uparrow \Sigma F_y = 0;\ F_{CF} \sin\theta + 0 = 0$$
$$F_{CF} = 0 \text{ (since } \sin\theta \neq 0)$$

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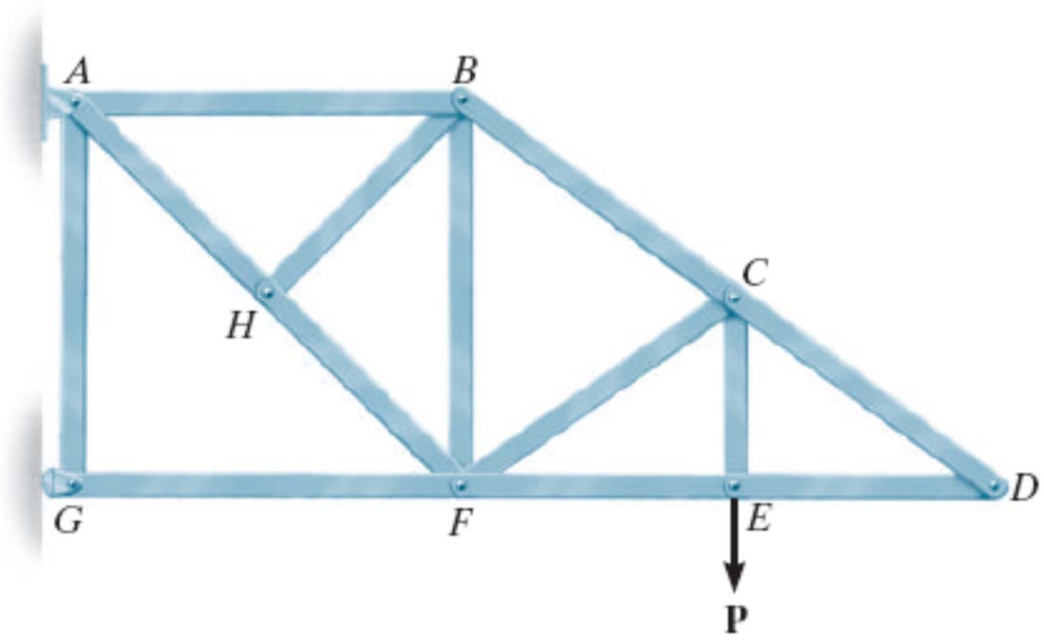
Example 3.4

Using the method of joints, indicate all the members of the truss that have zero force.



Solution

We have,

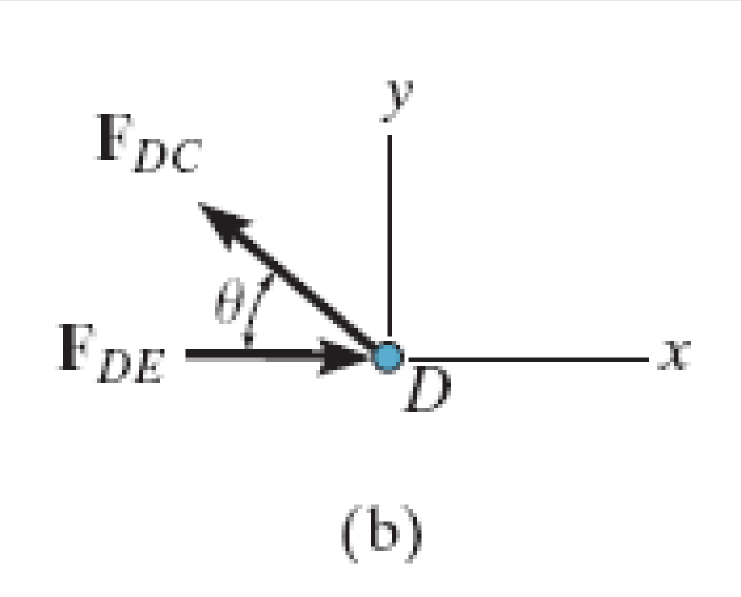
Joint D,

$$+\uparrow \sum F_y = 0; \quad F_{DC} \sin\theta = 0$$

$$F_{DC} = 0$$

$$\underset{\rightarrow}{+} \sum F_x = 0; \quad F_{DE} + 0 = 0$$

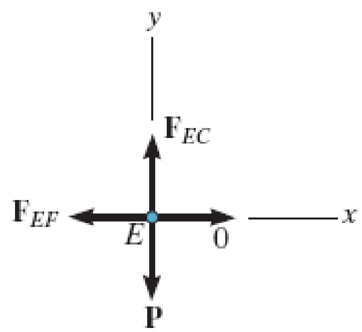
$$F_{DE} = 0$$



Solution

Joint E,

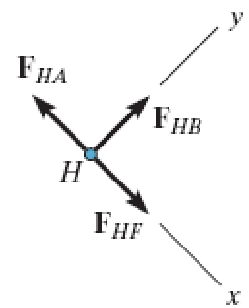
$$\underset{\leftarrow}{+}\sum F_x = 0; \quad F_{EF} = 0$$



(c)

Joint H,

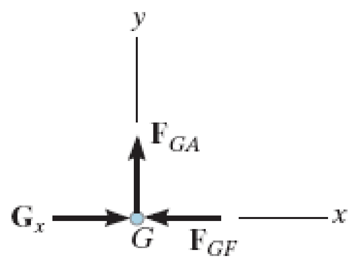
$$+\uparrow \sum F_y = 0; \quad F_{HB} = 0$$



(d)

Joint G,

$$+\uparrow \sum F_y = 0; \quad F_{GA} = 0$$



(e)

The Method of joints-Procedure for Analysis

The following step-by-step procedure can be used for the analysis of statically determinate simple plane trusses by the method of joints.

1. Check the truss for static determinacy, as discussed in the preceding section. If the truss is found to be statically determinate and stable, proceed to step 2. Otherwise, end the analysis at this stage.
2. Identify by inspection any zero-force members of the truss.
3. Determine the slopes of the inclined members (except the zero-force members) of the truss.
4. Draw a free-body diagram of the whole truss, showing all external loads and reactions. Write zeros by the members that have been identified as zero-force members.
5. Examine the free-body diagram of the truss to select a joint that has no more than two unknown forces (which must not be collinear) acting on it. If such a joint is found, then go directly to the next step. Otherwise, determine reactions by applying the three equations of equilibrium and the equations of condition (if any) to the free body of the whole truss; then select a joint with two or fewer unknowns, and go to the next step.
6. Draw a free-body diagram of the selected joint, showing tensile forces by arrows pulling away from the joint and compressive forces by arrows pushing into the joint. It is usually convenient to assume the unknown member forces to be tensile.
7. Determine the unknown forces by applying the two equilibrium equations. A positive answer for procedure for Analysis member force means that the member is in tension, as initially assumed, whereas a negative answer indicates that the member is in compression. If at least one of the unknown forces acting at the selected joint is in the horizontal or vertical direction, the unknowns can be conveniently determined by satisfying the two equilibrium equations by inspection of the joint on the free-body diagram of the truss.

The Method of joints-Procedure for Analysis

8. If all the desired member forces and reactions have been determined, then go to the next step. Otherwise, select another joint with no more than two unknowns, and return to step 7.

9. If the reactions were determined in step 5 by using the equations of equilibrium and condition of the whole truss, then apply the remaining joint equilibrium equations that have not been utilized so far to check the calculations. If the reactions were computed by applying the joint equilibrium equations, then use the equilibrium equations of the entire truss to check the calculations. If the analysis has been performed correctly, then these extra equilibrium equations must be satisfied.

The Method of Sections

- If the forces in only certain members of a truss are desired, the method of joints may not prove to be efficient, because it may involve calculation of forces in several other members of the truss before a joint is reached that can be analyzed for a desired member force.
- The 3 equations of equilibrium may be applied to either one of these 2 parts to determine the member forces at the "cut section"
- A decision must be made as to how to "cut" the truss
- **In general, the section should pass through not more than 3 members in which the forces are unknown**

The Method of Sections

- The method of sections involves cutting the truss into two portions by passing an imaginary section through the members whose forces are desired. The desired member forces are then determined by considering the equilibrium of one of the two portions of the truss.
- Each portion of the truss is treated as a rigid body in equilibrium, under the action of any applied loads and reactions and the forces in the members that have been cut by the section.
- The unknown member forces are determined by applying the three equations of equilibrium to one of the two portions of the truss.

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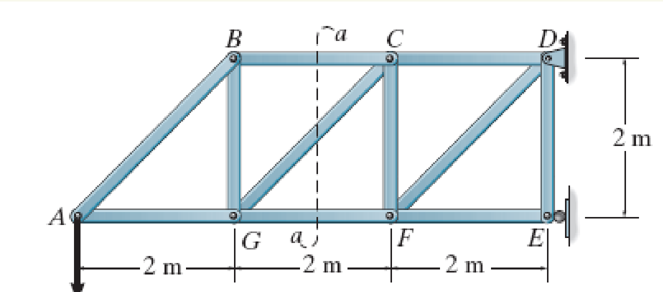
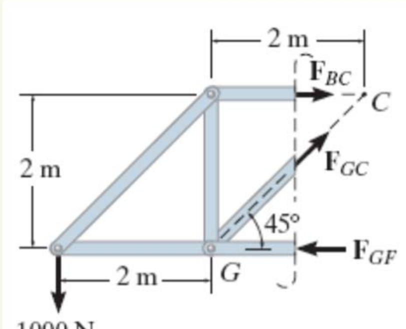
The Method of Sections

- If the force in GC is to be determined, section aa will be appropriate
- Also, the member forces acting on one part of the truss are equal but opposite
- The 3 unknown member forces, F_{BC}, F_{GC} & F_{GF} can be obtained by applying the 3 equilibrium eqns



(a)



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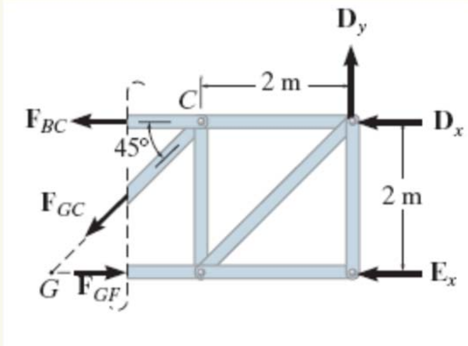


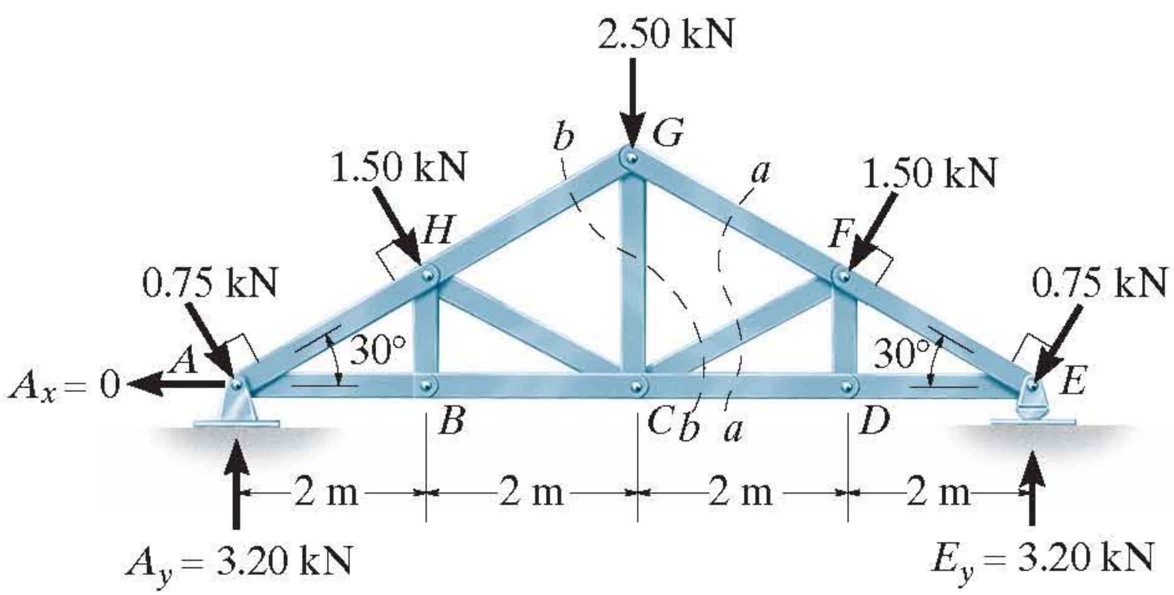
(c)

The Method of Sections

- When applying the equilibrium eqns, consider ways of writing the eqns to yield a direct solution for each of the unknown, rather than to solve simultaneous eqns

Example 3.5

Determine the force in members CF and GC of the roof truss. State whether the members are in tension or compression. The reactions at the supports have been calculated.



Solution

The free-body diagram of member CF can be obtained by considering the section aa,

A direct solution for F_{CF} can be obtained by applying $\sum M_E = 0$

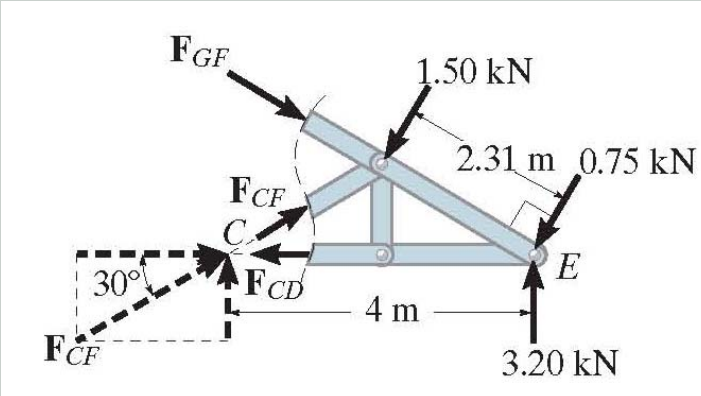
Applying Principal of transmissibility,

F_{CF} is slide to point C for simplicity.

With anti-clockwise moments as +ve, $\sum M_E = 0$

$$-F_{CF}\sin 30^o(4) + 1.50(2.31) = 0$$

$$F_{CF} = 1.73kN(C)$$



Solution

The free-body diagram of member GC can be obtained by considering the section bb,

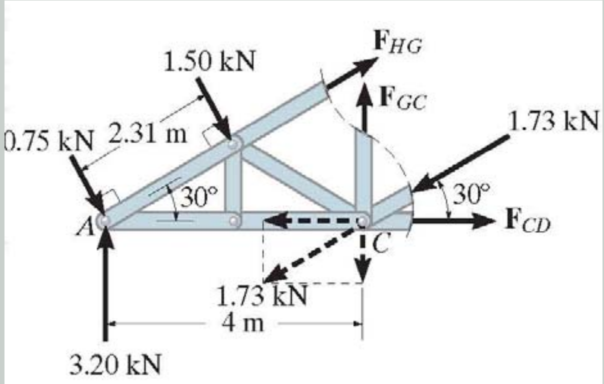
Moments will be summed about point A in order to eliminate the unknowns F_{HG} and F_{CD}. Sliding to F_{CF} point C, we have:

F_{CF} is slide to point C for simplicity.

With anti-clockwise moments as +ve, $\sum M_A = 0$

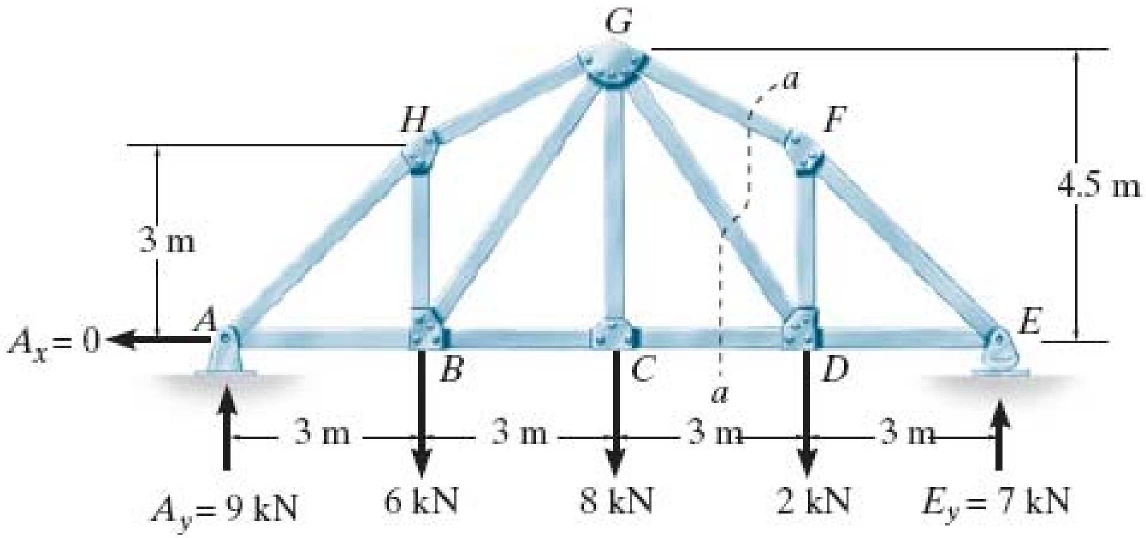
$$-1.50(2.31) + F_{GC}(4) - 1.73\sin 30^o(4) = 0$$

$$F_{GC} = 1.73kN(T)$$



Example 3.6

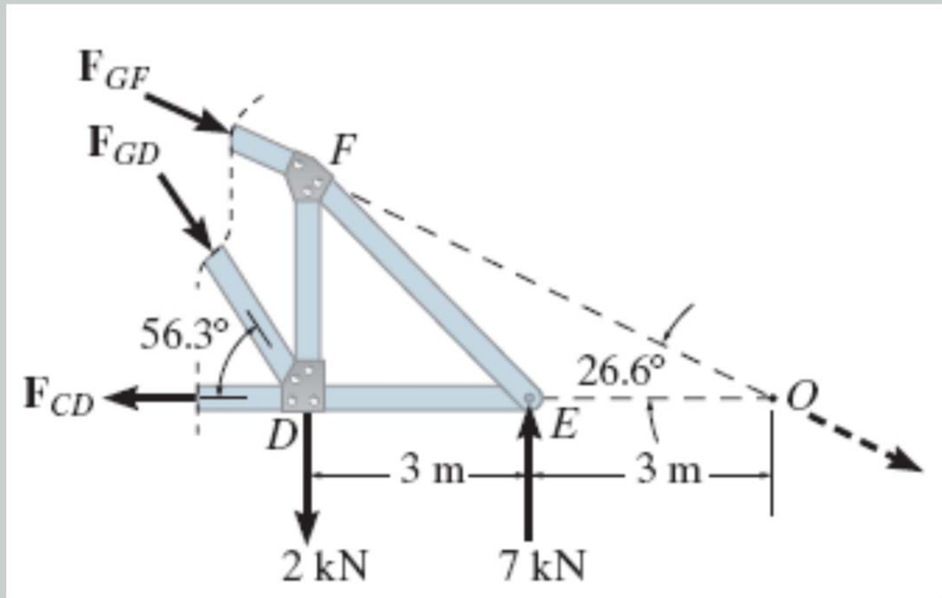
Determine the force in member GF and GD of the truss. State whether the members are in tension or compression. The reactions at the supports have been calculated.



Solution

The distance EO can be determined by proportional triangles or realizing that member GF drops vertically 4.5 – 3 = 1.5m in 3m.

Hence, to drop 4.5m from G the distance from C to O must be 9m



Solution

The angles F_{GD} and F_{GF} make with the horizontal are

$\tan^{-1}(4.5/3) = 56.3^o$

$\tan^{-1}(4.5/9) = 26.6^o$

The force in GF can be determined directly by applying

$$\sum M_D = 0$$

F_{GF} is slide to point O.

With anti-clockwise moments as +ve, $\sum M_D = 0$

$$-F_{GF} \sin 26.6^o (6) + 7(3) = 0$$

$$F_{GF} = 7.83kN(C)$$

Solution

The force in GD can be determined directly by applying

$\sum M_O = 0$

F_{GD} is slide to point D.

With anti-clockwise moments as +ve, $\sum M_O = 0$

$$-7(3) + 2(6) + F_{GD} \sin 56.3^o (6) = 0$$

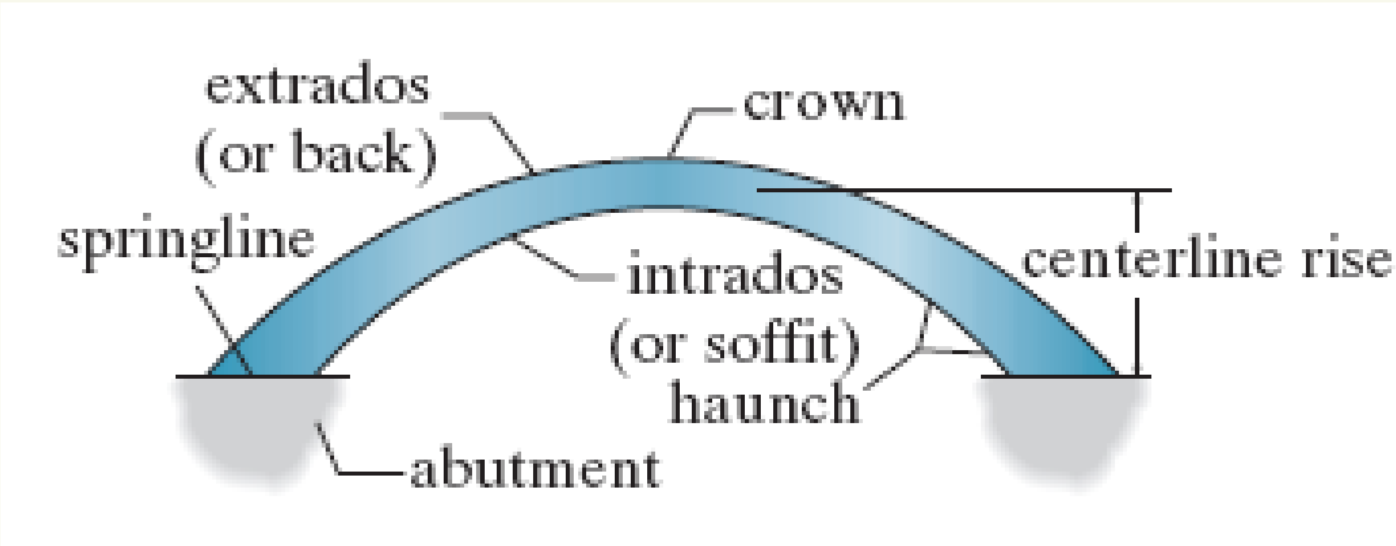
$$F_{GD} = 1.80 kN (C)$$

THANK YOU

Analysis of Arch Structures

Arches

- An arch acts as inverted cable so it receives loading in compression
- Because of its rigidity, it must also resist some bending and shear depending upon how it is loaded & shaped
- Arch is a compression member. It can span a large area by resolving forces into compressive stresses, and thereby eliminating tensile stresses.
- This is sometimes denominated "arch action”.



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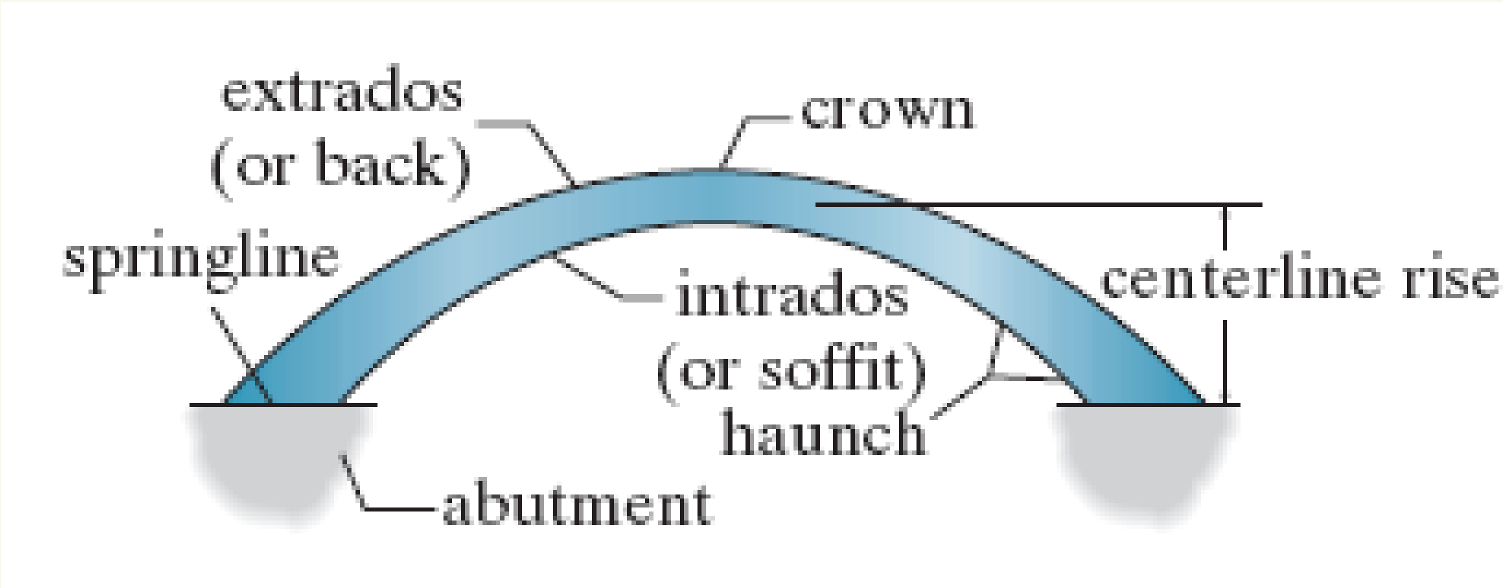
Arches

- As the forces in the arch are transferred to its base, the arch pushes outward at its base, denominated "thrust".
- As the rise, i. e. height, of the arch decreases the outward thrust increases. In order to preserve arch action and prevent collapse of the arch, the thrust must be restrained, either by internal ties or external bracing, such as abutments.



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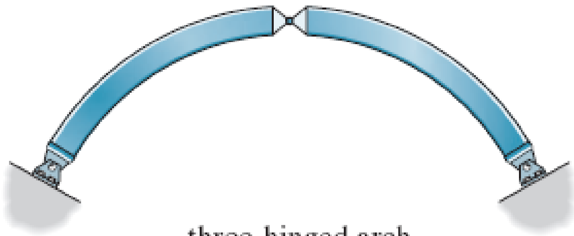
Arches

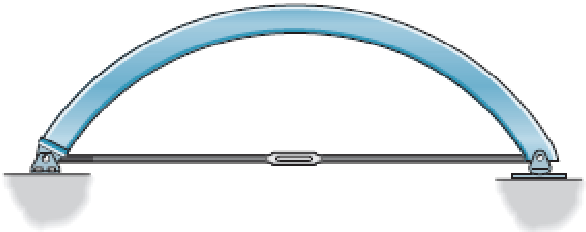
- Depending on its uses, several types of arches can be selected to support a loading

fixed arch
(a)



three-hinged arch
(c)

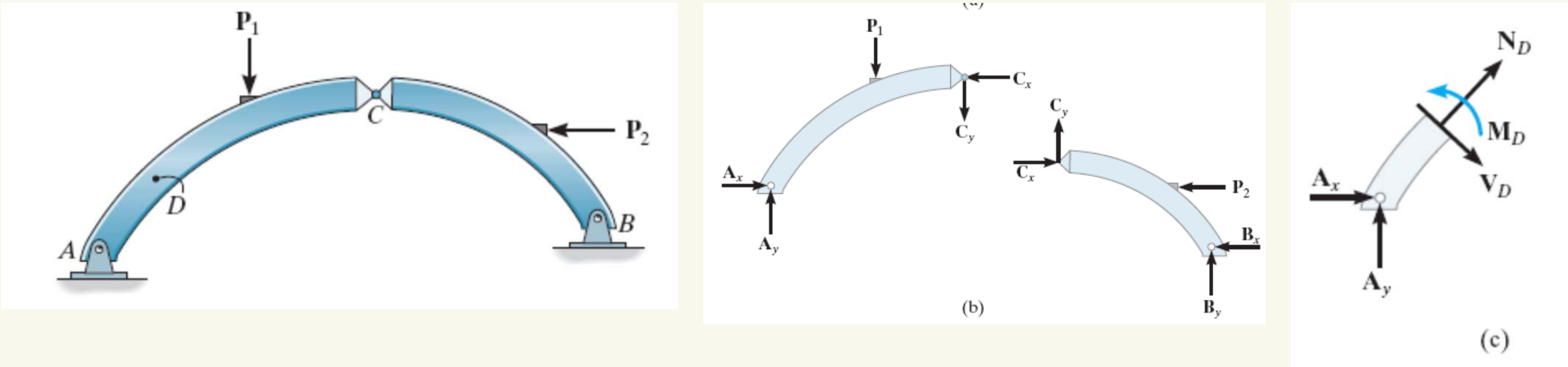
two-hinged arch
(b)



tied arch
(d)

Three-Hinged Arch

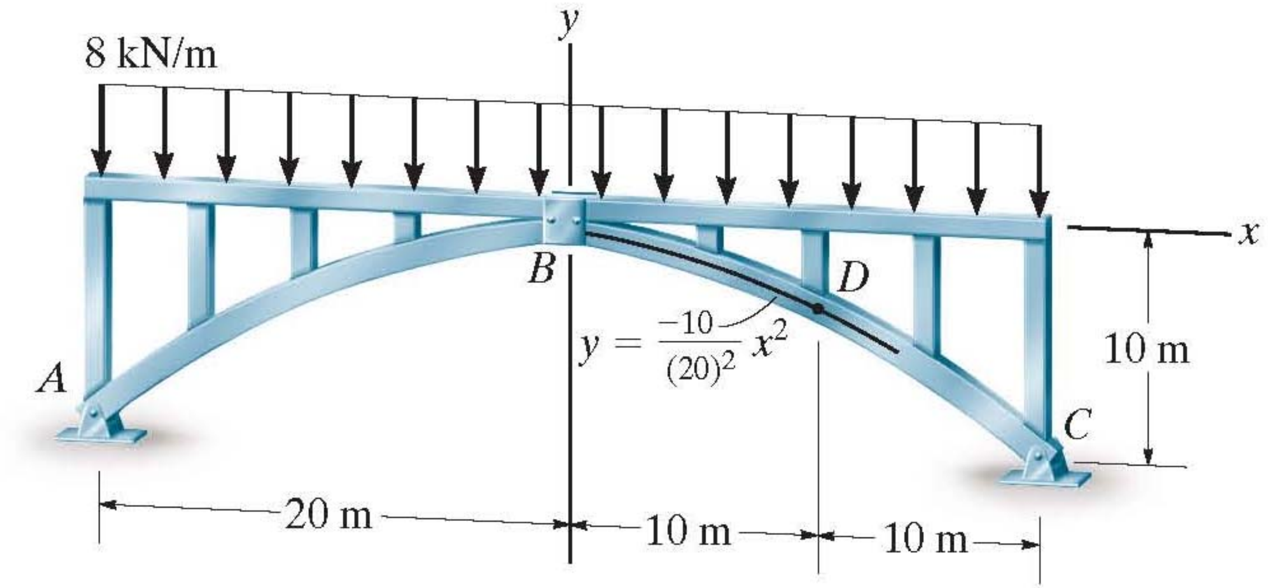
- The third hinge is located at the crown & the supports are located at different elevations
- To determine the reactions at the supports, the arch is disassembled



(b) (c)

Example 5.4

The three-hinged open-spandrel arch bridge has a parabolic shape and supports the uniform load. Show that the parabolic arch is subjected *only to axial compression* at an intermediate point such as point *D*. Assume the load is uniformly transmitted to the arch ribs.



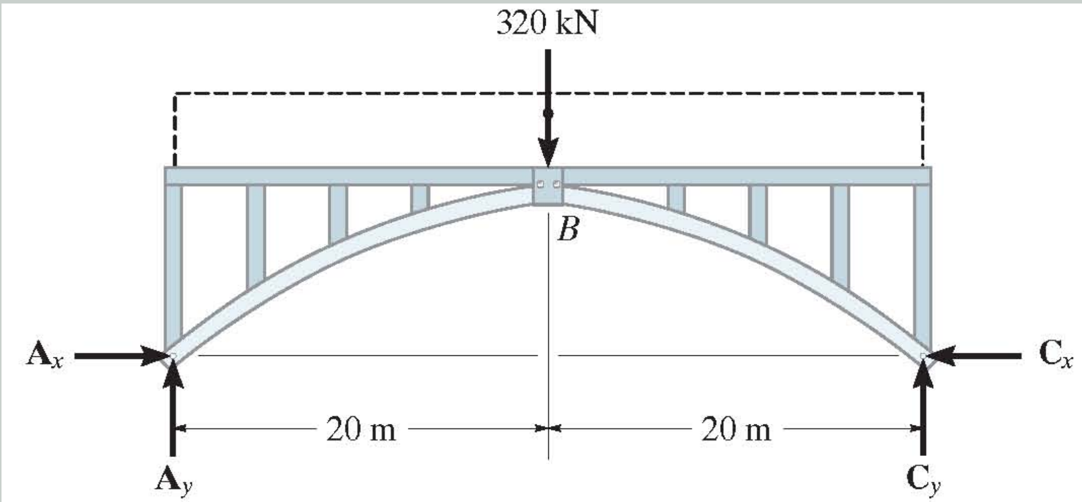
PEARSON

Solution

- Applying the eqn of equilibrium, we have:

Entire Arch :
With anti - clockwise direction moments as + ve,

$$\sum M_A = 0$$
$$C_y(40m) - 320kN(20m) = 0$$
$$C_y = 160kN$$



PEARSON

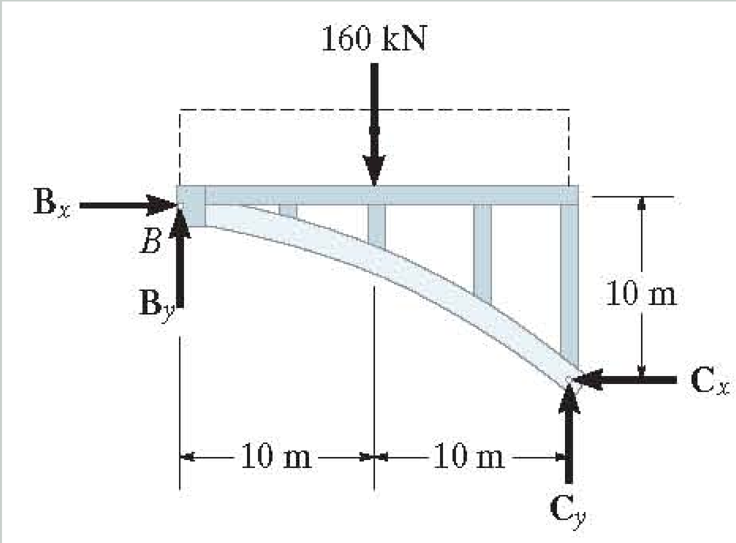
Solution

Arch segment BC :
With anti - clockwise direction moments as + ve,

$$\sum M_B = 0$$
$$-160kN(10m) + 160kN(20m) - C_x(10m) = 0$$
$$C_x = 160kN$$

$$\underset{\rightarrow}{+} \sum F_x = 0 \Rightarrow B_x = 160kN$$

$$+\uparrow \sum F_y = 0$$
$$B_y - 160kN + 160kN = 0$$
$$B_y = 0$$



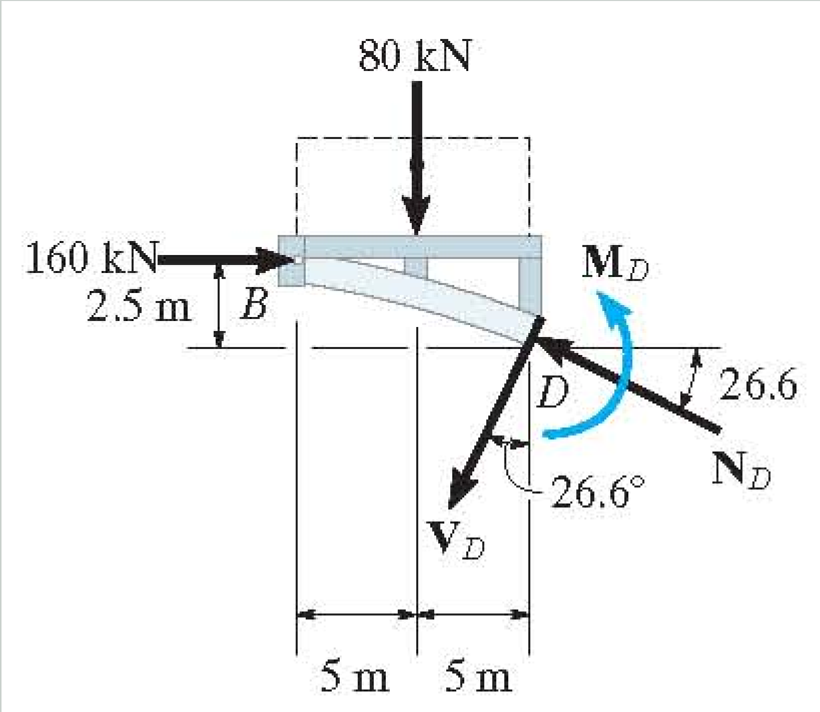
Solution

A section of the arch taken through point D

$$x = 10m$$
$$y = -10(10)^2/(20)^2 = -2.5m$$

The slope of the segment at D is:

$$\tan\theta = \frac{dy}{dx} = \frac{-20}{(20)^2}x\bigg|_{x=10m} = -0.5$$



Solution

Applying the eqn of equlibrium, Fig 5.10(d), we have :

$$\underset{\rightarrow}{+} \sum F_x = 0$$

$$160kN - N_D \cos 26.6^o - V_D \sin 26.6^o = 0$$

$$+\uparrow \sum F_y = 0$$

$$-80kN + N_D \sin 26.6^o - V_D \cos 26.6^o = 0$$

$$\Rightarrow N_D = 178.9kN$$

With anti-clockwise moments as +ve :

$$\sum M_D = 0$$

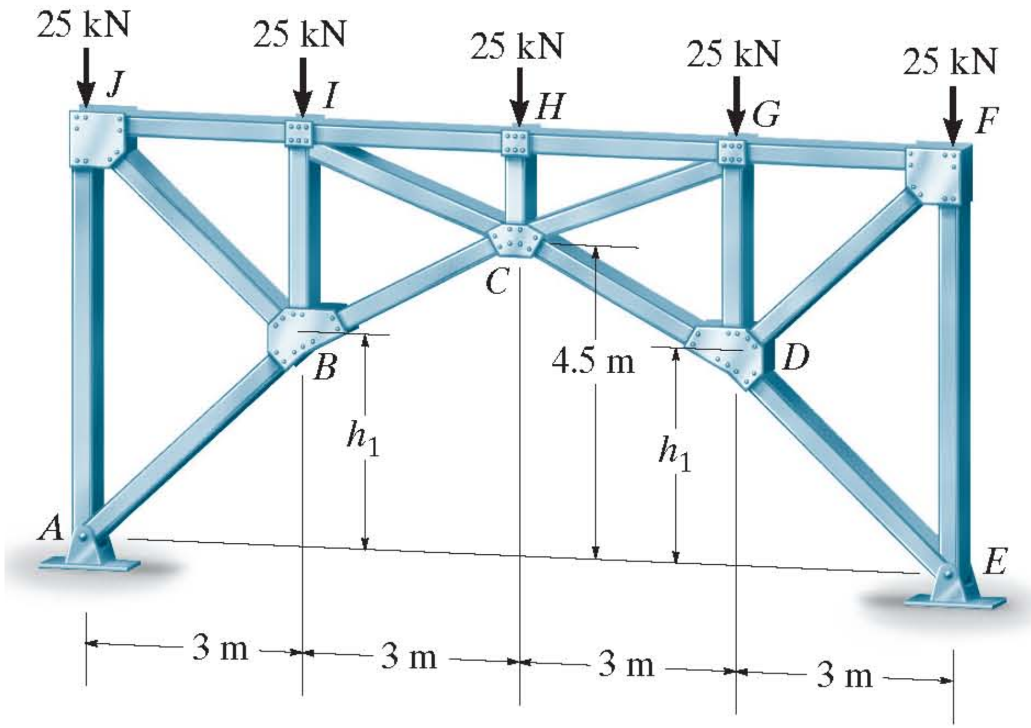
$$M_D + 80kN(5m) - 160kN(2.5m) = 0$$

$$\Rightarrow V_D = 0$$

$$\Rightarrow M_D = 0$$

Example 5.6

The three-hinged trussed arch supports the symmetric loading. Determine the required height of the joints *B* and *D*, so that the arch takes a funicular shape. Member *HG* is intended to carry no force.



Solution

For a symmetric loading, the funicular shape for the arch must be parabolic as indicated by the dashed line. Here we must find the eqn which fits this shape.

With the x, y axes having an origin at C, the eqn is of the form of $y = -cx^2$. To obtain the constant c, we require:

$$-(4.5m) = -c(6m)^2$$

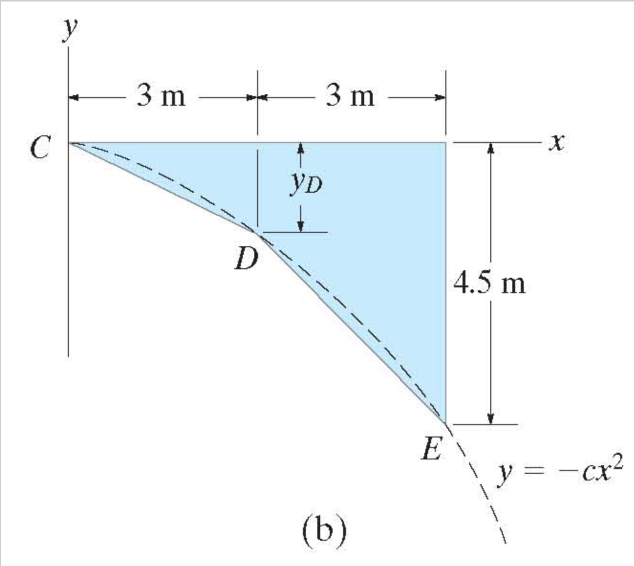
$$c = 0.125/m$$

Therefore,

$$y_D = -(0.125/m)(3m)^2 = -1.125m$$

From Fig 5.12(a)

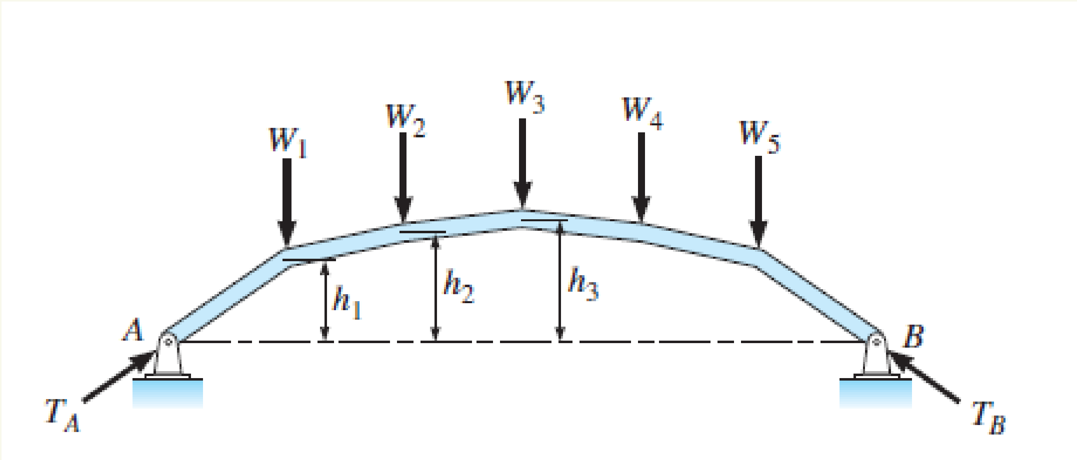
$$h_1 = 4.5m - 1.125m = 3.375m$$



THANK YOU

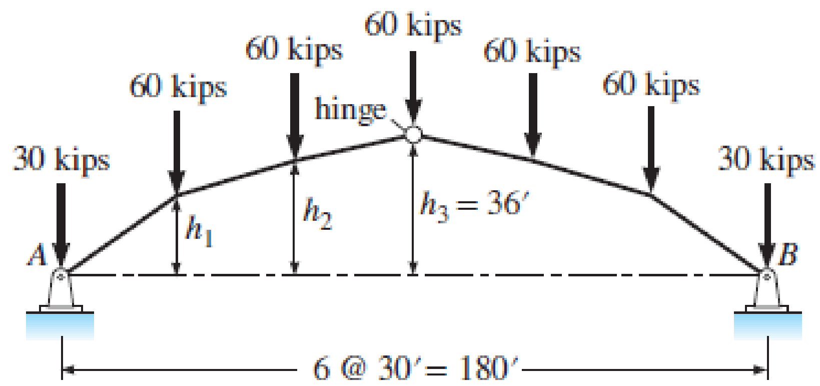
Funicular Shape of Arch

For a particular set of concentrated loads the arch profile in direct compression is called the **funicular arch**

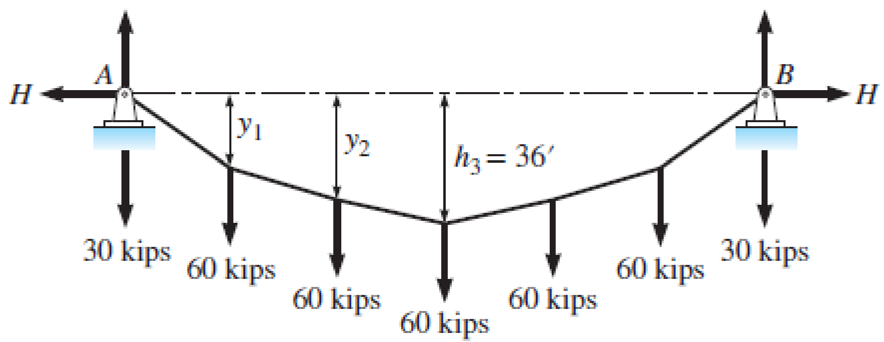


- Cable profile is always a funicular structure because a cable can only carry direct stress. If the cable profile is turned upside down, a funicular arch is produced.
- When the vertical loads acting on the cable are applied to the arch, they produce compression forces at all sections equal in magnitude to the tension forces in the cable at the corresponding sections

Example-Funicular Arch



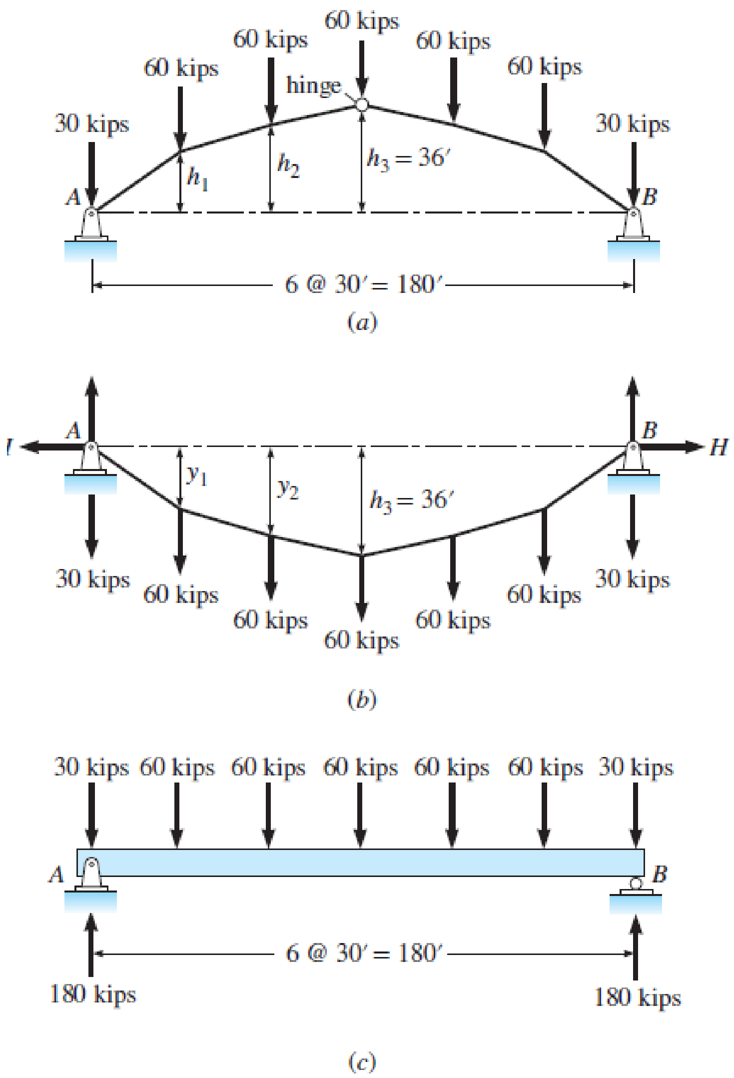
We imagine that the set of loads is applied to a cable that spans the same distance as the arch



The sag of the cable is set at 36 ft

the height of the arch at midspan.

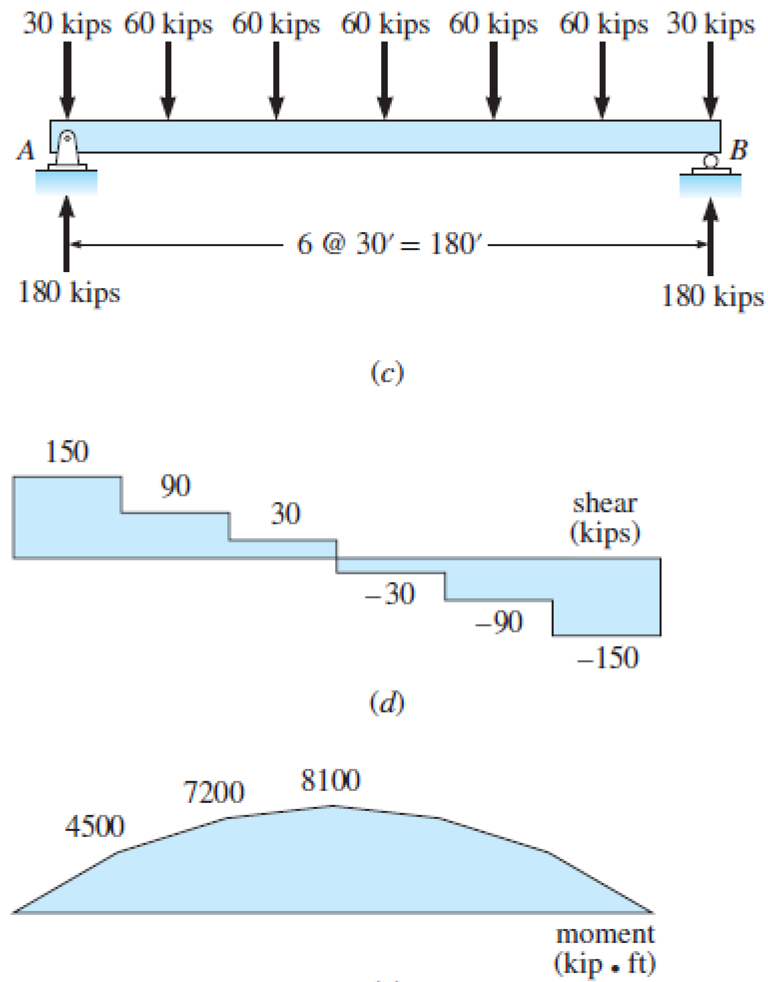
Example-Funicular Arch



Applying the general cable theory, we imagine that the loads supported by the cable are applied to an imaginary simply supported beam with a span equal to that of the cable

Example-Funicular Arch

We next construct the shear and moment curves.



(c) (d) (e)

Example-Funicular Arch

According to the general cable theorem at every point,

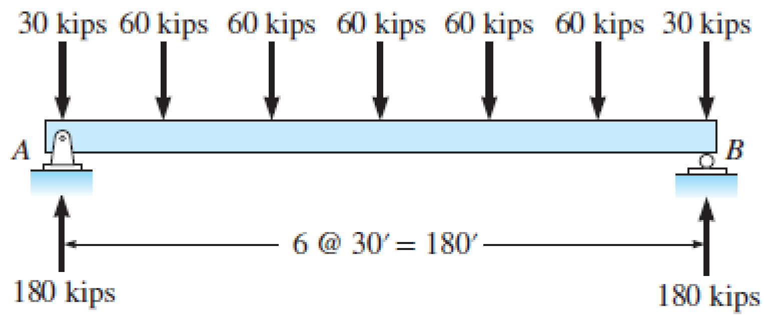
$M = h_z$

where

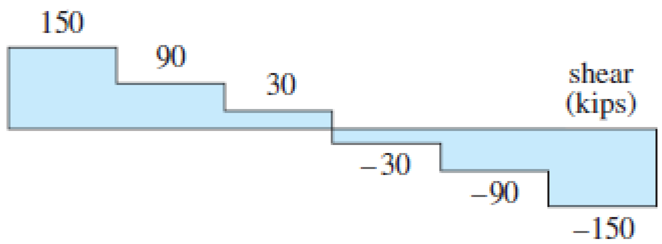
M = moment at an arbitrary point in the beam

H = horizontal component of support reaction

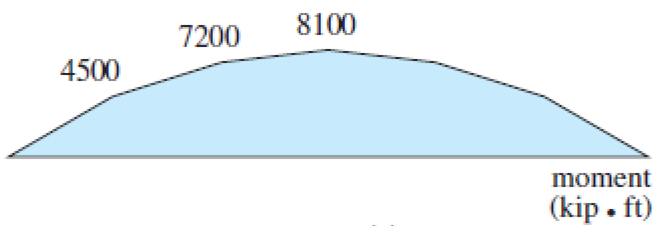
h_z = cable sag at an arbitrary point



(c)



(d)

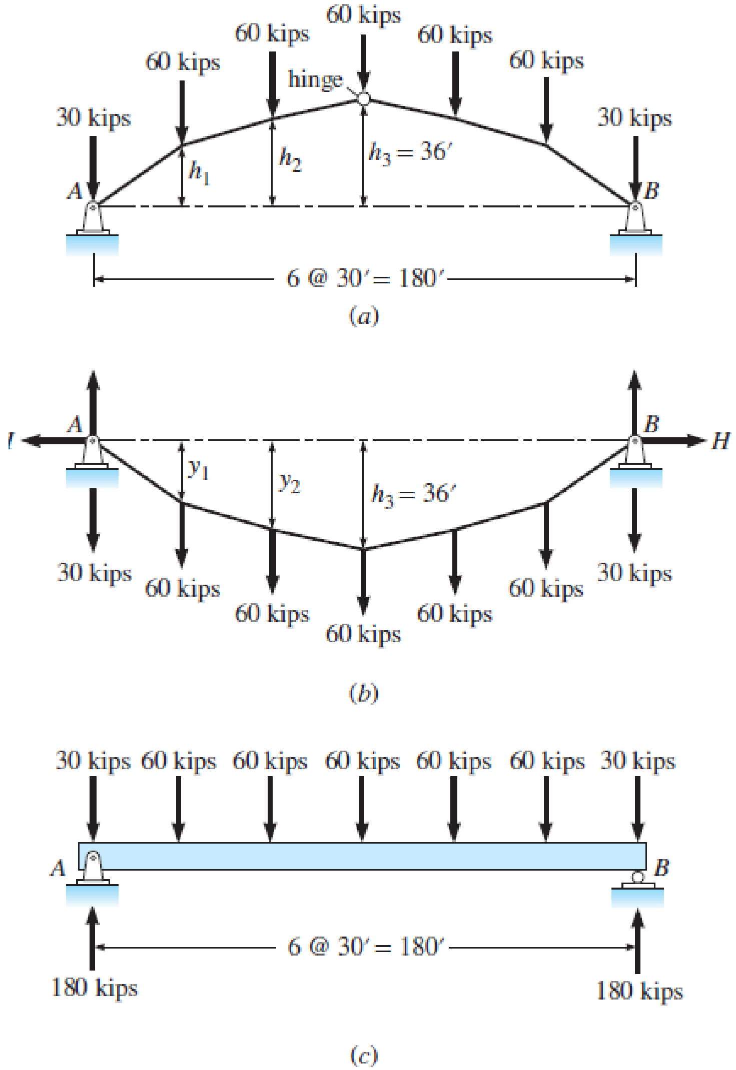


(e)

Example-Funicular Arch

Since h = 36 ft at midspan and M = 8100 kip·ft, we can apply Equation $\mathbf{M = h_z}$ at that point to establish H.

$$H = \frac{M}{h} = \frac{8100}{36} = 225 \text{ kips}$$



Example-Funicular Arch

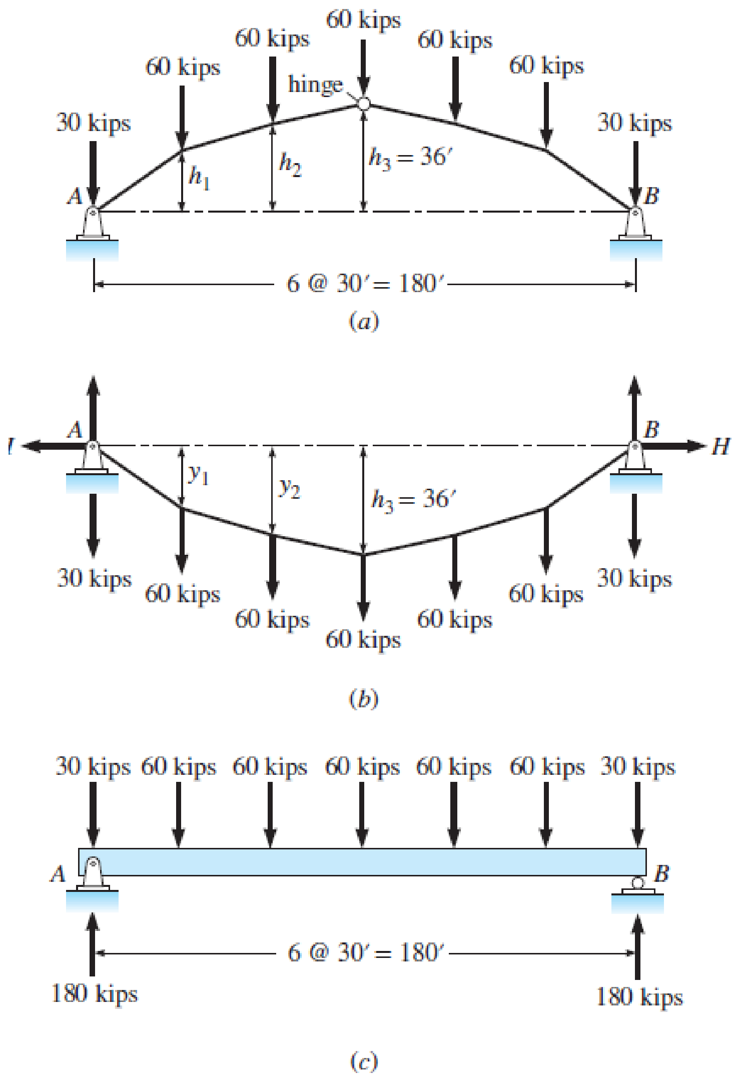
With H established we next apply Equation at 30 and 60 ft from the supports.

Compute h1 at 30 ft.

$$h_1 = \frac{M}{H} = \frac{4500}{225} = 20 \text{ ft}$$

Compute h_2 at 60 ft.

$$h_2 = \frac{M}{H} = \frac{7200}{225} = 32 \text{ ft}$$



THANK YOU

Influence Lines
for Statically Determinate Structures

Introduction

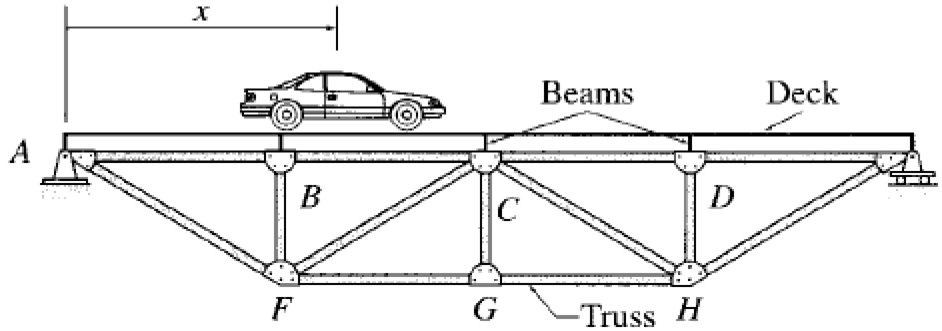
Structures may be subjected to loads:

- Loads whose positions were fixed on the structures (Stationary Loads). For Example:

Dead load due to the weight of the structure itself and of other material and equipment permanently attached to the structure.

- Loads whose positions may vary on the structure (Moving Loads). For Example:
  - Live loads
  - Environmental loads

Introduction



- In the bridge truss shown car moves across the bridge
- the forces in the members of the truss will vary with the position x of the car.
- It should be realized that the forces in different truss members will become maximum at different positions of the car.
- For example, if the force in member AB becomes maximum
- when the car is at a certain position x = x1,
- then the force in another member, for example member CH, may become maximum when the car is at a different position x =x2.

Introduction

- The design of each member of the truss must be based on the maximum force that develops in that member as the car moves across the bridge.

- Therefore, the analysis of the truss would involve, for each member, determining the position of the car at which the force in the member becomes maximum and then computing the value of the maximum member force.

Influence Lines

- If a structure is subjected to a moving load, the variation of shear & bending moment is best described using the influence line
- One can tell at a glance, where the moving load should be placed on the structure so that it creates the greatest influence at a specified point
- The magnitude of the associated shear, moment or deflection at the point can then be calculated using the ordinates of the influence-line diagram

Influence Lines

- One should be clear between the difference between Influence Lines & shear or moment diagram
- Influence line represent the effect of a moving load only at a specific point
- Shear or moment diagrams represent the effect of fixed loads at all points along the axis of the member

Influence Lines

- Procedure for Analysis
  - Tabulate Values
  - Influence-Line equations

Influence Lines

- Tabulate Values
  - Place a unit load at various locations, x, along the member
  - At each location use statics to determine the value of function at the specified point
  - If the influence line for a vertical force reaction at a point on a beam is to be constructed, consider the reaction to be +ve at the point when it acts upward on the beam

Influence Lines

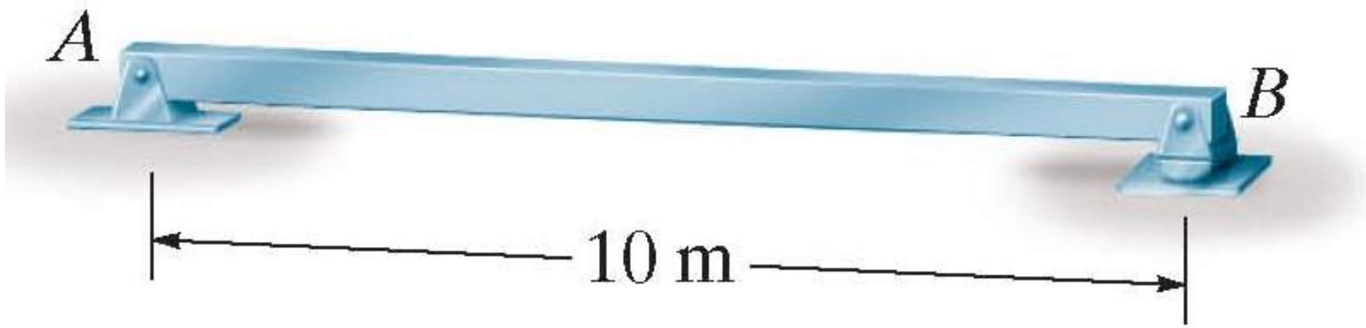
- Tabulate Values (cont'd)
  - If a shear or moment influence line is to be drawn for a point, take the shear or moment at the point as +ve according to the same sign convention used for drawing shear & moment diagram
  - All statically determinate beams will have influence lines that consists of straight line segments

Influence Lines

- Influence-Line Eqns
  - The influence line can also be constructed by placing the unit load at a variable position, x, on the member & then computing the value of R, V or M at the point as a function of x
  - The eqns of the various line segments composing the influence line can be determined & plotted

Example 6.1

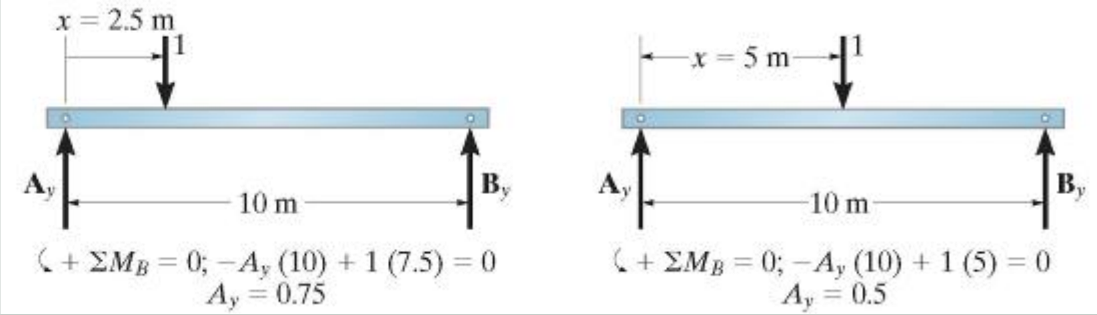
Construct the influence line for the vertical reaction at A of the beam.



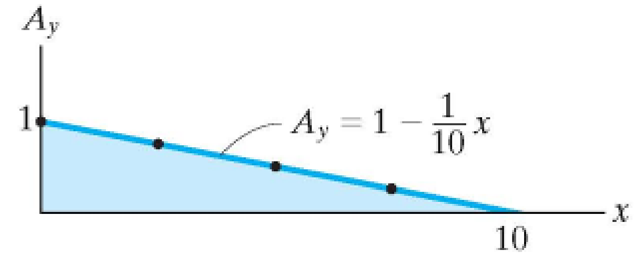
Solution

Tabulate Values

A unit load is placed on the beam at each selected point x & the value of A_y is calculated by summing moments about B.



x	A_y
0	1
2.5	0.75
5	0.5
7.5	0.25
10	0



influence line for A_y

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Solution

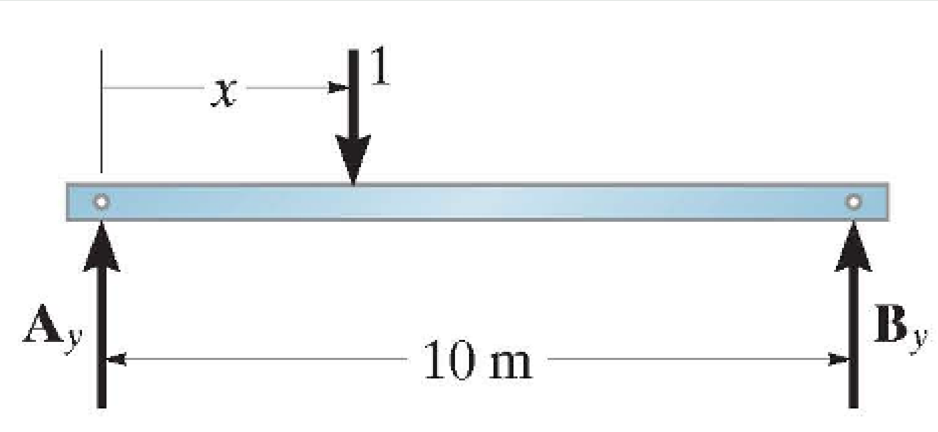
Influence-Line Equation

The reaction as a function of *x* can be determined from

$$\Sigma M_B = 0$$

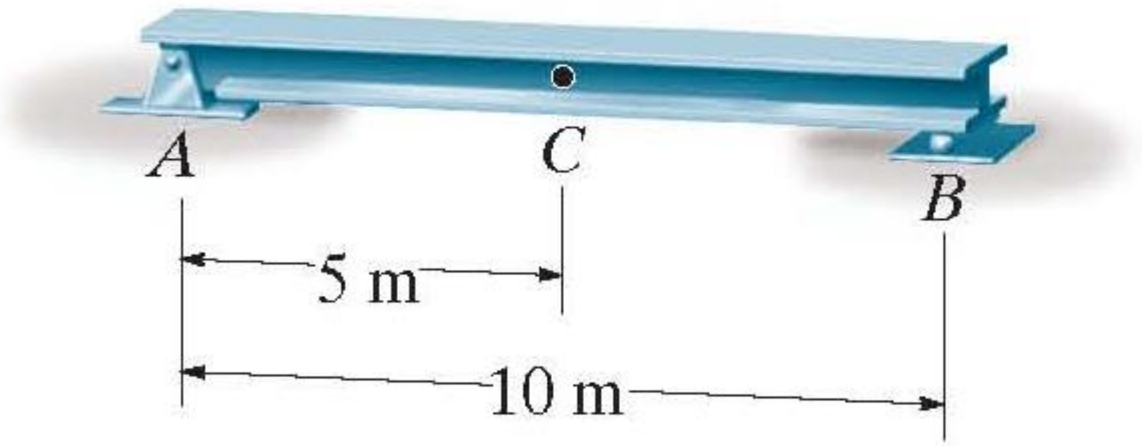
$$-A_y(10) + (10 - x)(1) = 0$$

$$A_y = 1 - \frac{1}{10}x$$



Example 6.5

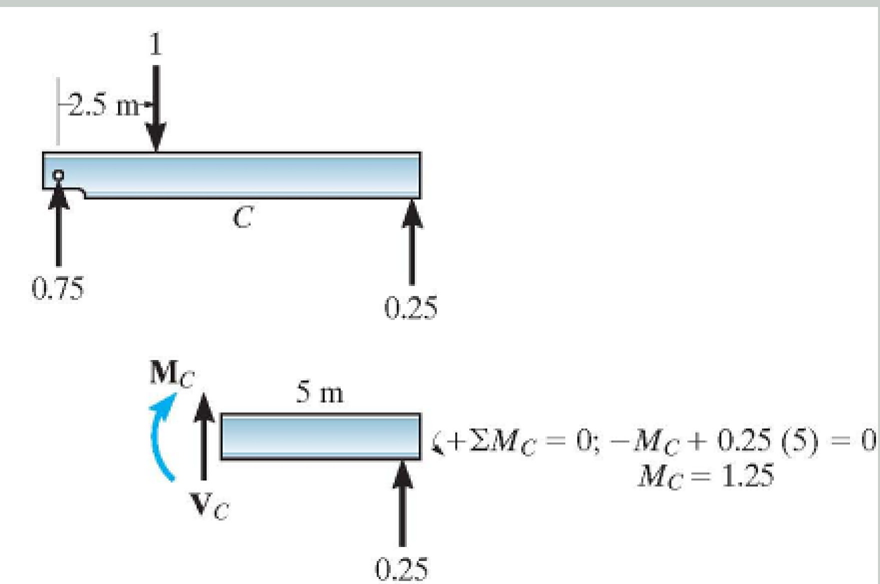
Construct the influence line for the moment at C of the beam.

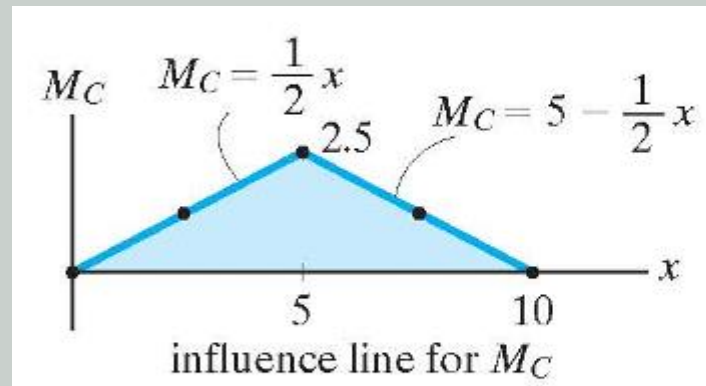


Solution

Tabulate Values

At each selected position of the unit load, the value of M_c is calculated using the method of sections.





x	M_C
0	0
2.5	1.25
5	2.5
7.5	1.25
10	0

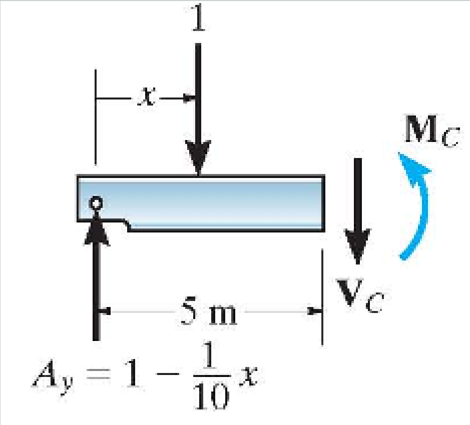
Solution

Influence-Line Equations

$$\Sigma M_C = 0$$

$$M_C + 1(5-x) - (1-\frac{1}{10}x)5 = 0$$

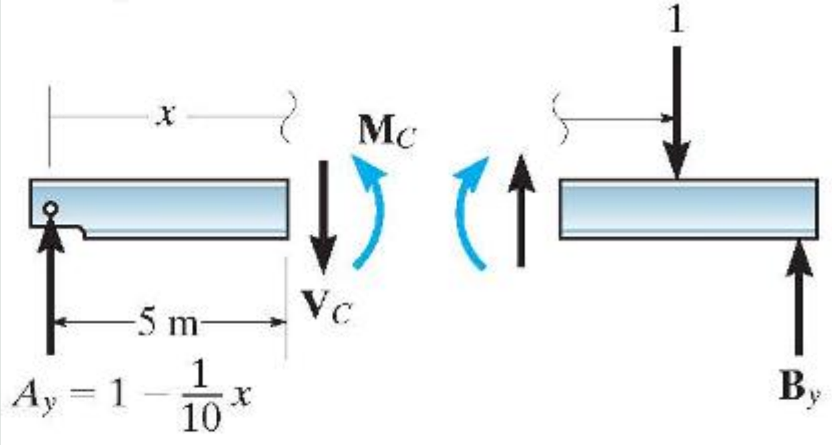
$$M_C = \frac{1}{2}x \quad \text{for } 0 \le x < 5m$$



$$\Sigma M_C = 0$$

$$M_C - (1-\frac{1}{10}x)5 = 0$$

$$M_C = 5 - \frac{1}{2}x \quad \text{for } 5m < x \le 10m$$



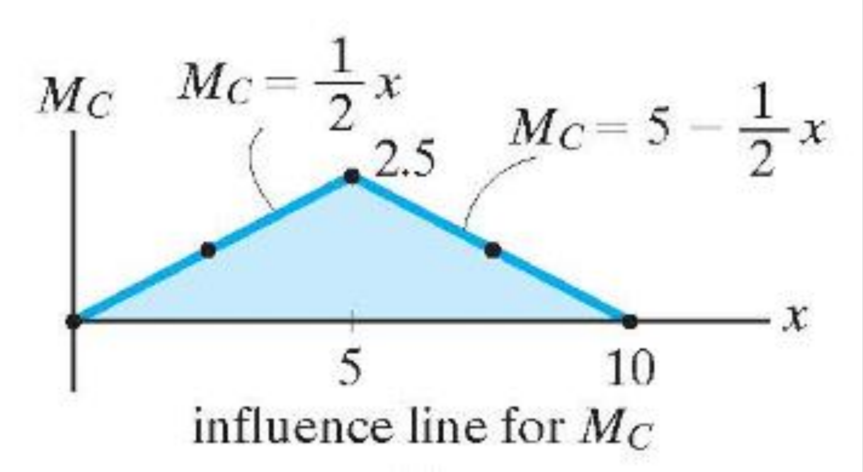
Solution

Influence-Line Equations

$$\Sigma M_C = 0$$

$$M_C + 1(5 - x) - (1 - \frac{1}{10}x)5 = 0$$

$$M_C = \frac{1}{2}x \quad \text{for } 0 \le x < 5m$$



$$\Sigma M_C = 0$$

$$M_C - (1 - \frac{1}{10}x)5 = 0$$

$$M_C = 5 - \frac{1}{2}x \quad \text{for } 5m < x \le 10m$$

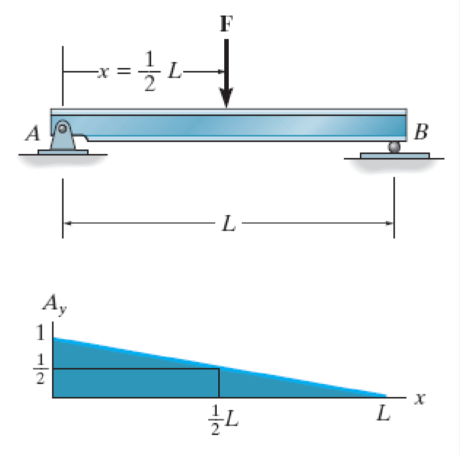
THANK YOU

Influence Lines for Beams

- Once the influence line for a function has been constructed, it will be possible to position live loads on the beam which will produce the max value of the function
- 2 types of loadings will be considered
  - Concentrated force
  - Uniform load

Influence Lines for Beams

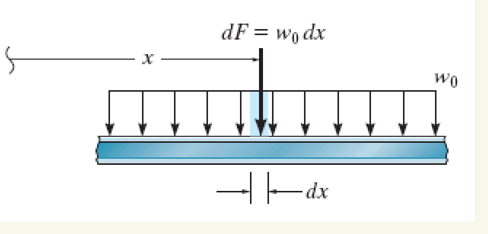
- Concentrated force
  - For any concentrated force, F acting on the beam, **the value of the function can be found by multiplying the ordinate of the influence line at position x by magnitude of F**
  - Consider Figure, influence line for A_y
  - For unit load, $A_y = ½$
  - For a force of F, $A_y = (½)\ F$

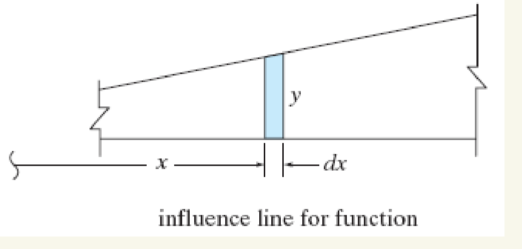


influence line for A_y

Influence Lines for Beams

- Uniform load
  - Each dx segment of this load creates a concentrated force of $dF = w_o dx$
  - If dF is located at x, where the influence line ordinate is y, the value of the function is $(dF)(y) = (w_o dx)y$
  - The effect of all concentrated force is determined by integrating over the entire length of the beam





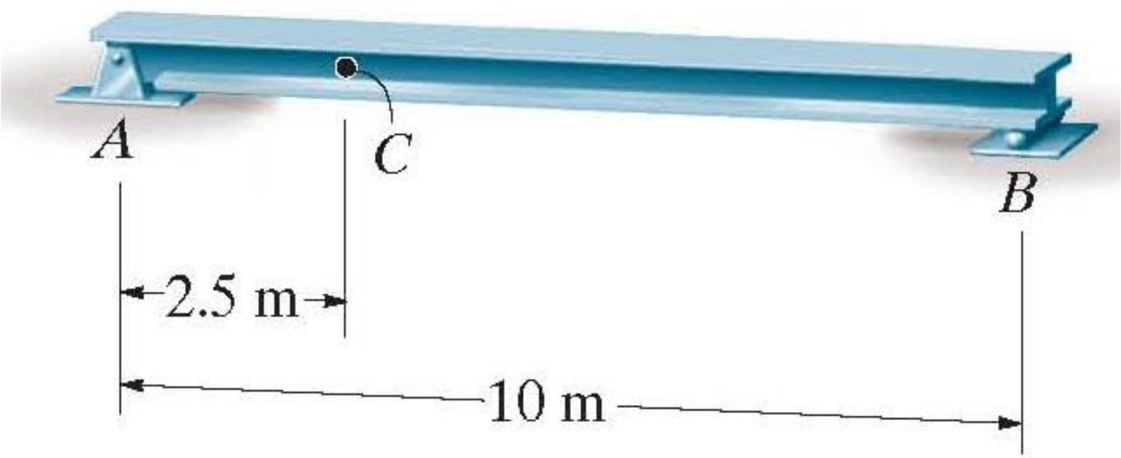
Influence Lines for Beams

- Uniform load (cont'd)
  - Since $\int w_o y dx = w_o \int y dx$ is equivalent to the area under the influence line, in general:
  - value of the function caused by a uniform load = the area under the influence line x intensity of the uniform load. $\int y dx$
  - then, in general, **the value of a function caused by a uniform distributed load is simply the area under the influence line for the function multiplied by the intensity of the uniform load**

Example 6.7

Determine the max +ve live shear that can be developed at point C in the beam due to:

- ➔A concentrated moving load of 4kN
- ➔And a uniform moving load of 2kN/m



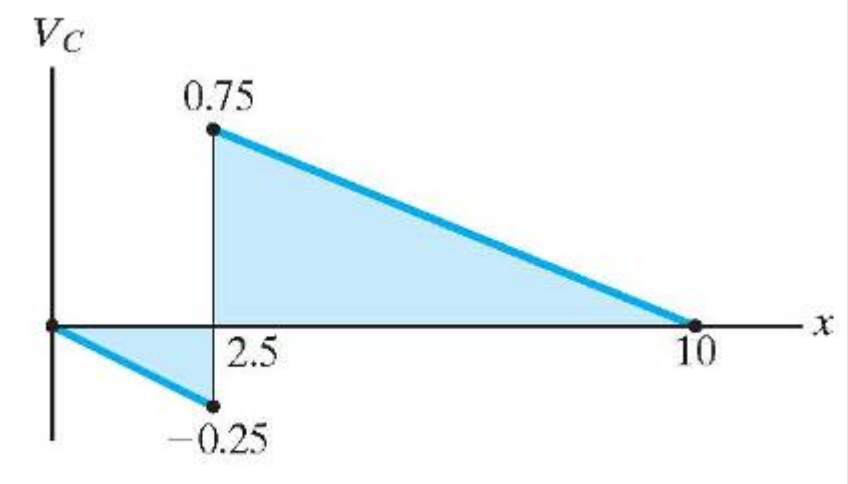
Solution

Concentrated force

The max +ve positive shear at C will occur when the 4kN force is located at x = 2.5m.

The ordinate at this peak is +0.75, hence:

$$V_C = 0.75(4kN) = 3kN$$



influence line for V_C

Solution

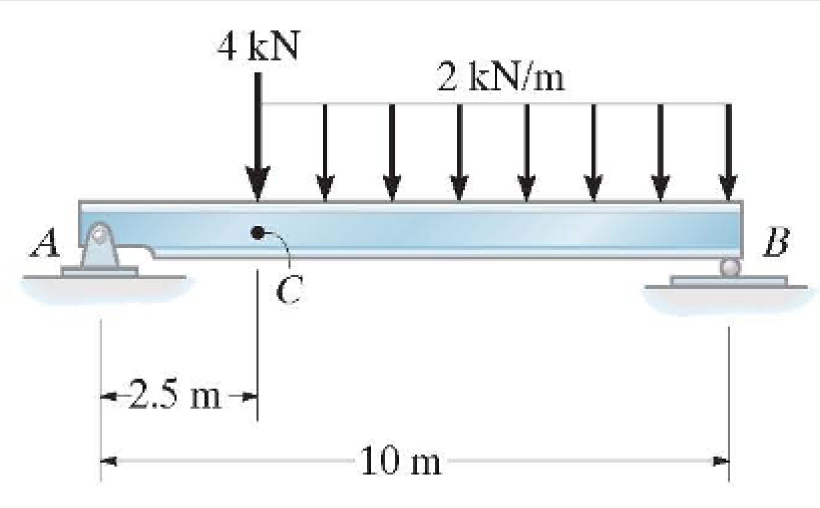
Uniform load

The uniform moving load creates the max +ve influence for V_C when the loads acts on the beam between x = 2.5m and x = 10m

The magnitude of V_C due to this loading is:

$$V_C = \left[\frac{1}{2}(10m - 2.5m)(0.75)\right](2kN/m)$$

$$= 5.625kN$$

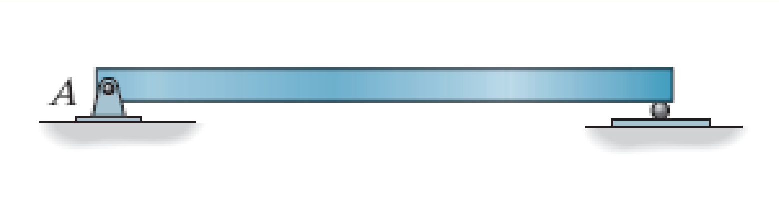


Total max shear at C:

$$(V_C)_{\max} = 3kN + 5.625kN = 8.625kN$$

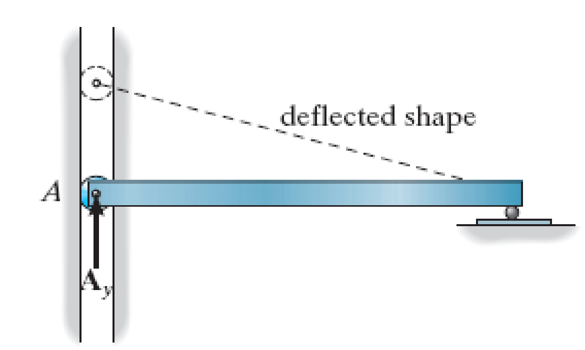
Qualitative Influence Lines

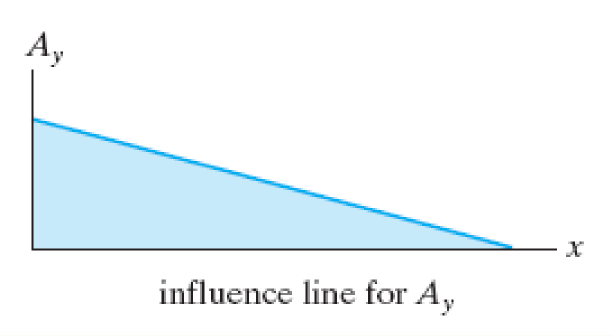
- The Muller-Breslau Principle states that the influence line for a function is to the same scale as the deflected shape of the beam when the beam is acted upon by the function
- If the shape of the influence line for the vertical reaction at A is to be determined, the pin is first replaced by a roller guide



Qualitative Influence Lines

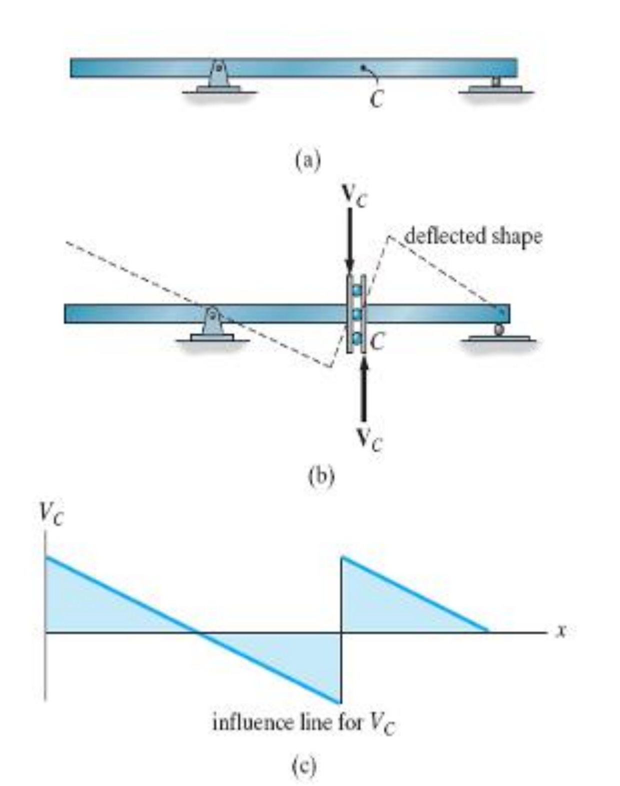
- When the +ve force A_y is applied at A, the beam deflects to the dashed position which rep the general shape of the influence line





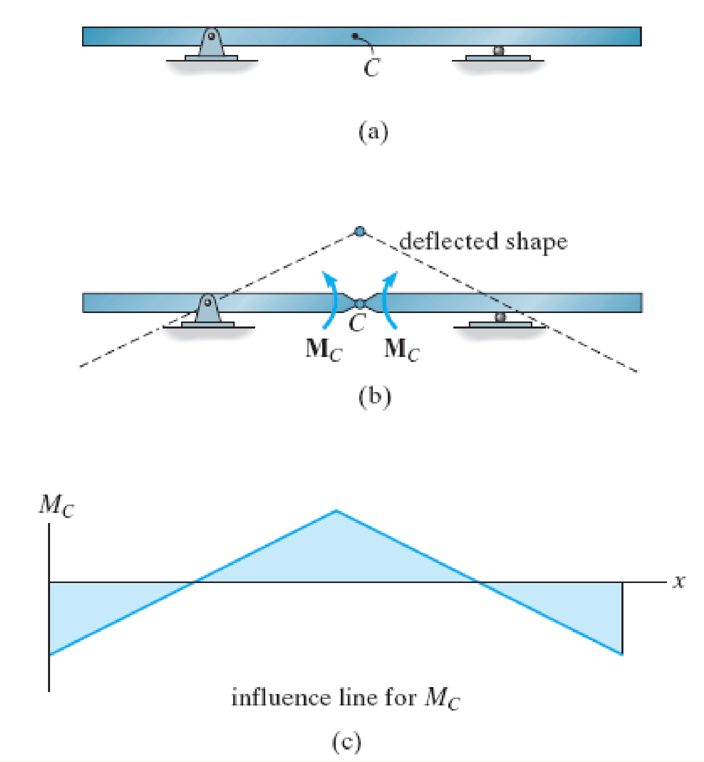
Qualitative Influence Lines

- If the shape of the influence line for shear at C is to be determined, the connection at C may be symbolized by a roller guide
- Applying a +ve shear force V_c to the beam at C & allowing the beam to deflect to the dashed position



Qualitative Influence Lines

- If the shape of influence line for the moment at C to be determined, an internal hinge or pin is placed at C
- Applying +ve moment M_c to the beam, the beam deflects to the dashed line

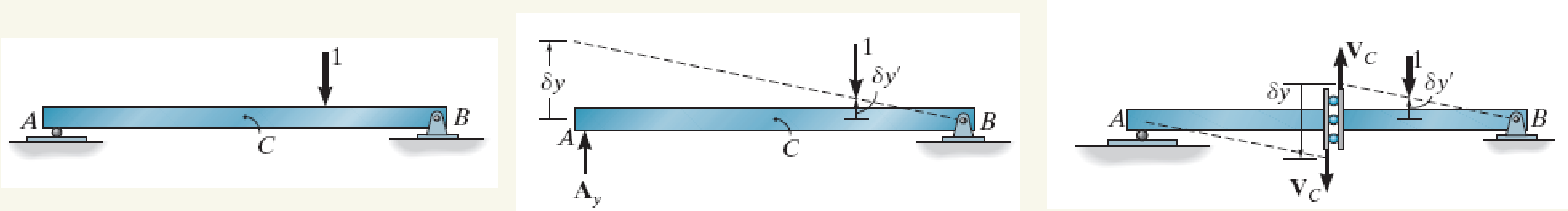


Qualitative Influence Lines

- The proof of the Muller-Breslau Principal can be established using the principle of virtual work
- Work = a linear displacement x force in the direction of displacement
- Or work = rotational displacement x moment if the direction of the displacement
- If a rigid body is in equilibrium, the sum of all the forces & moments on it must be equal to zero

Qualitative Influence Lines

- If the body is given an imaginary or virtual disp, work done by all these forces & couple moments must also be equal to zero
- If the beam is given a virtual disp δy at the support A, then only A_y & unit load do virtual work
- A_y does +ve work = A_y δy
- The unit load does –ve work = -1 δy



Qualitative Influence Lines

- Since the beam is in equilibrium, the virtual work sums to zero

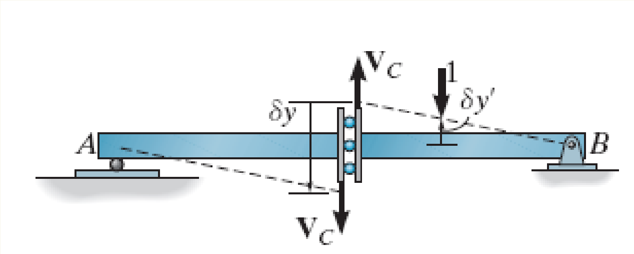
$$A_y \delta y - 1\delta y' = 0$$

$$\text{If } \delta y = 1, \text{ then} \Rightarrow A_y = \delta y'$$

- The value of A_y represents the ordinate of the influence line at the position of the unit load

Qualitative Influence Lines

- If the beam is sectioned at C, the beam undergoes a virtual disp δy then only the internal shear at C and the unit load do work
- The virtual work eqn is:



$$V_c \delta y - 1\delta y' = 0$$

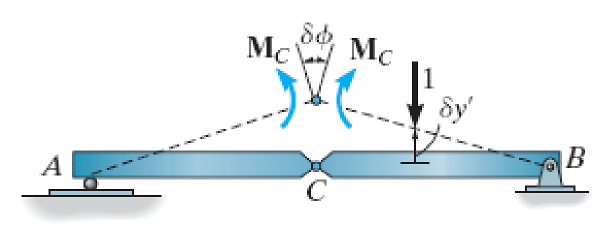
$$\text{If } \delta y = 1, \text{ then } \Rightarrow V_c = \delta y'$$

Qualitative Influence Lines

- The shape of the influence line for shear at C has been established
- If a virtual rotation $\delta\phi$ is introduced at the pin, virtual work will be done only by the internal moment & unit load

$$M_c \delta\phi - 1\delta y' = 0$$

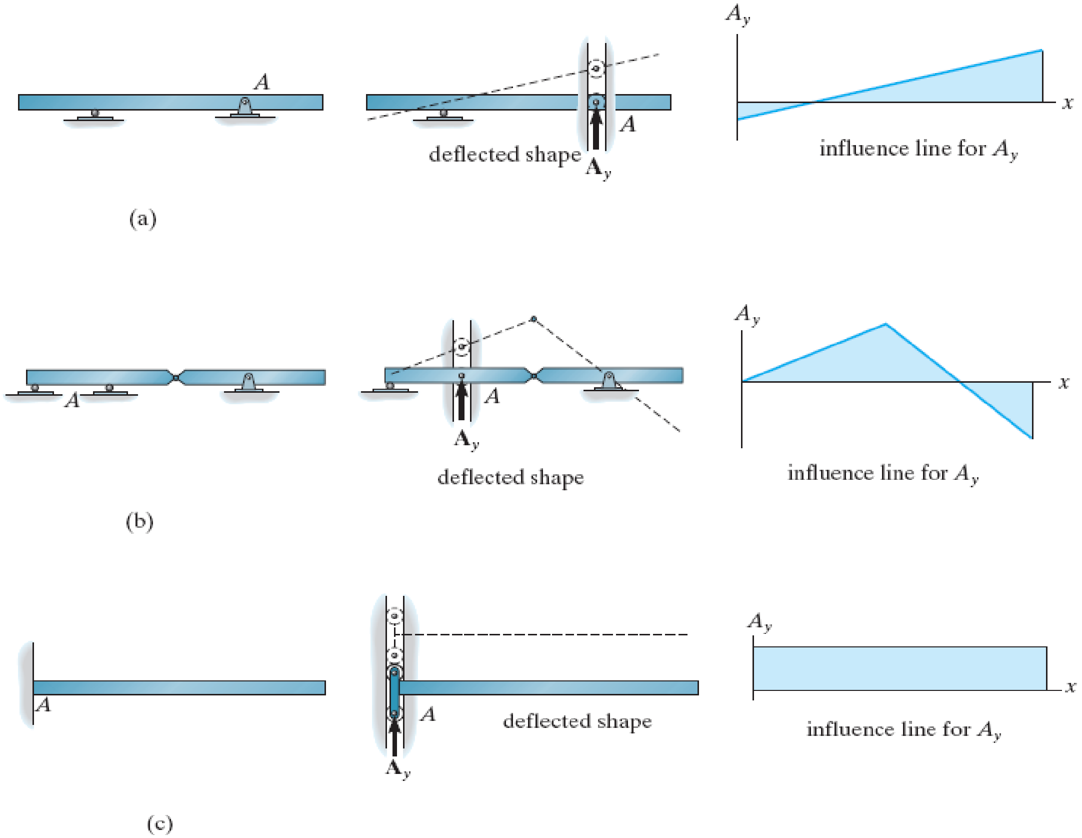
$$\text{If } \delta y = 1, \text{ then} \Rightarrow M_c = \delta y'$$



Example 6.9

For each beam sketch the influence line for the vertical reaction at *A*.

Solution:

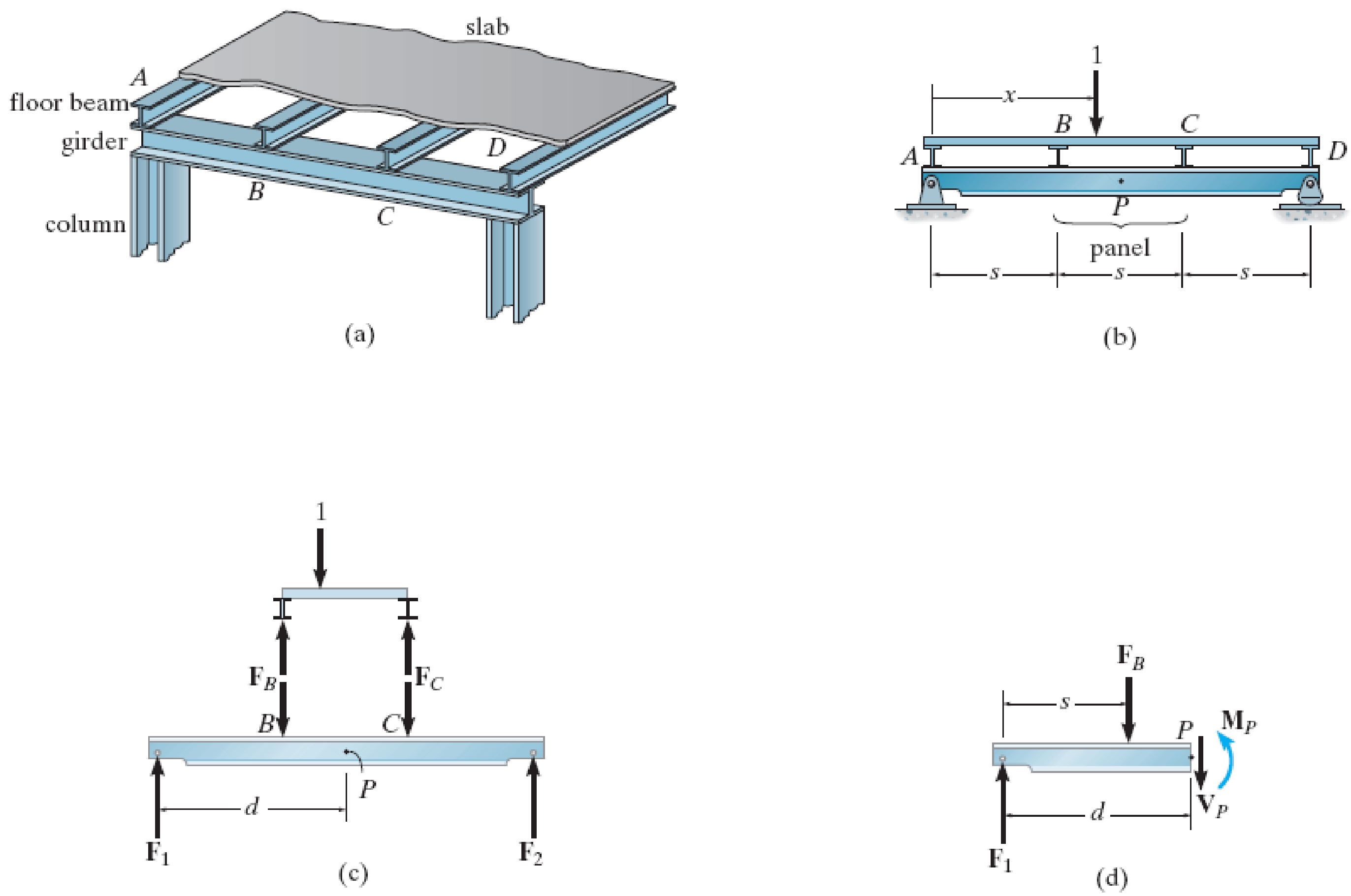


THANK YOU

Influence Lines for Floor Girders

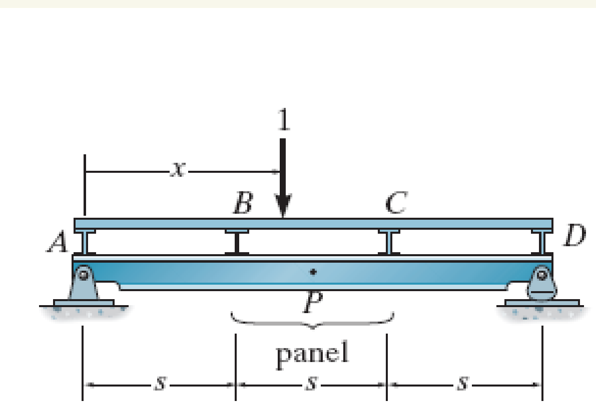
Floor loads are transmitted

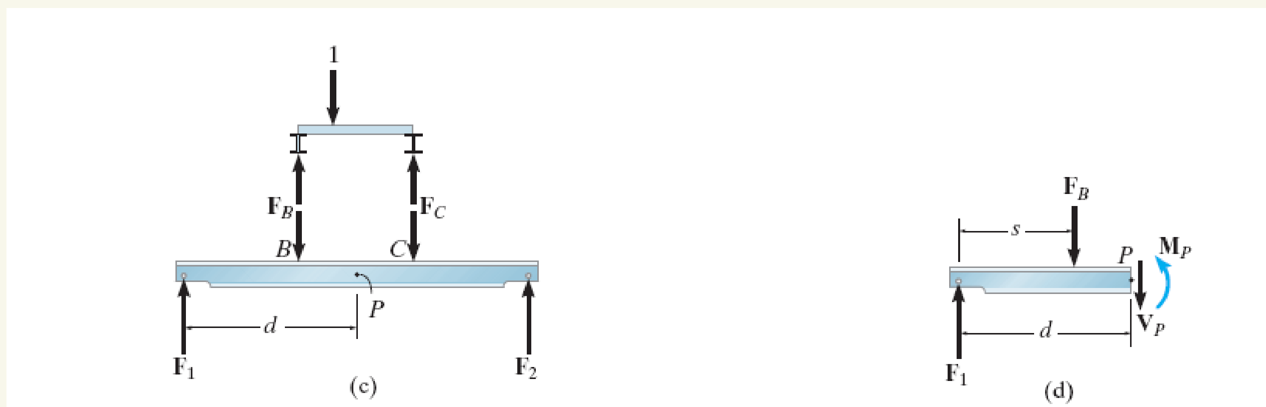
- From slabs to floor beams
- Then to side girders
- Finally to supporting columns



Influence Lines for Floor Girders

- The influence line for a specified point on the girder can be determined using the same statics procedure
- In particular, the value for the internal moment in a girder panel will depend upon where point P is chosen for the influence line
- Magnitude of M_P depends upon the point's location from end of the girder





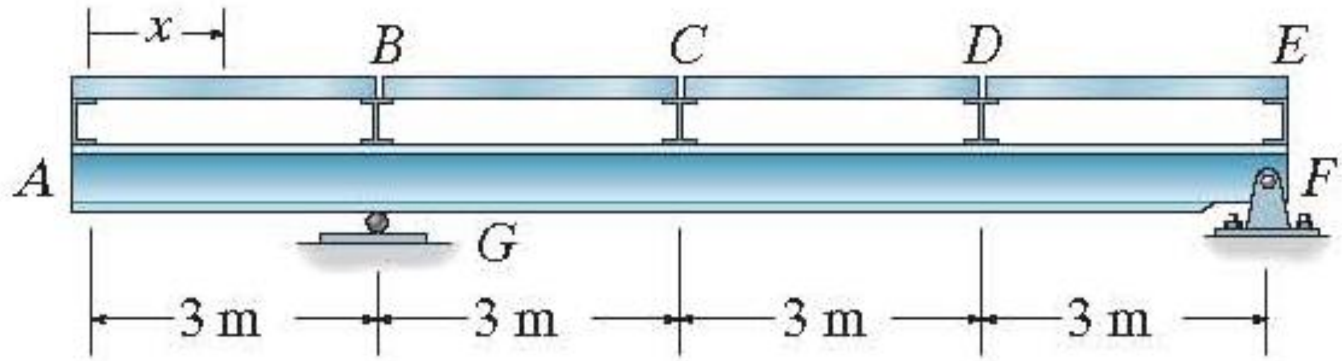
Influence Lines for Floor Girders

- **Influence lines for shear in floor girders are specified for panels in the girder and not specific points along the girder**

- This shear is known as **girder shear**

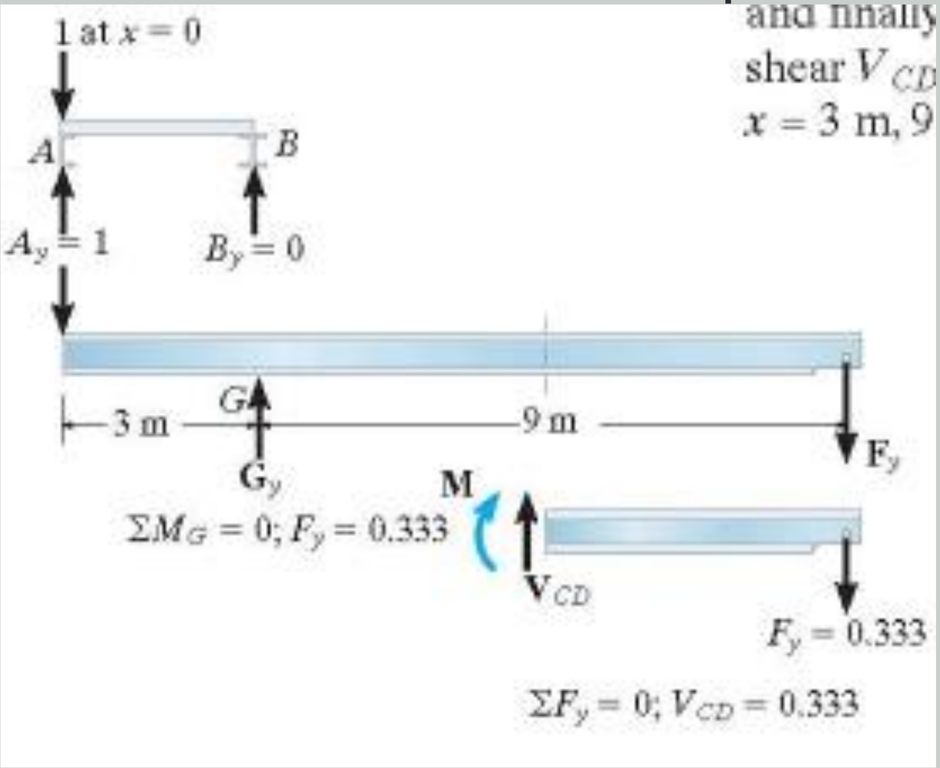
Example 6.13

Draw the influence line for the shear in panel *CD* of the floor girder.



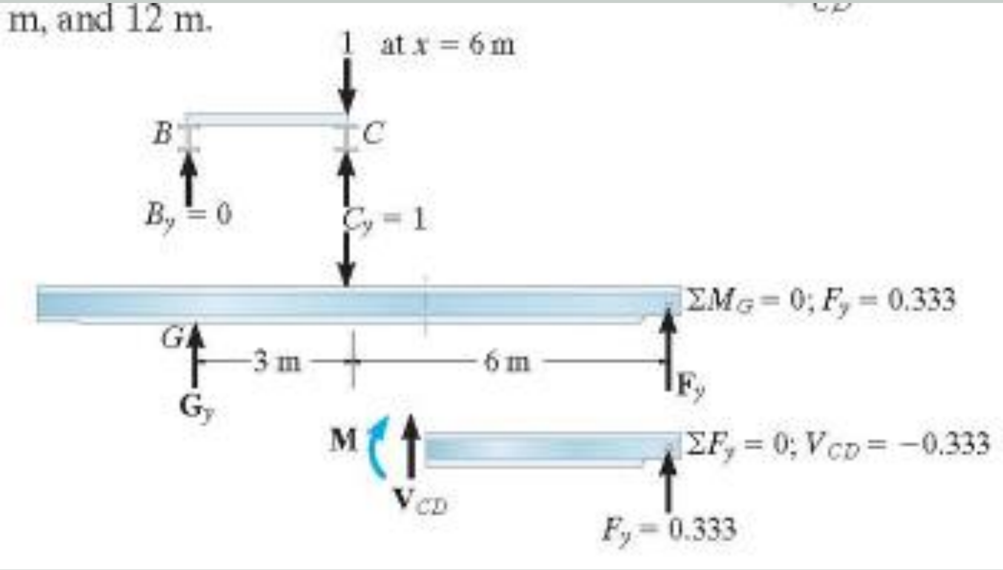
Solution

The unit load is placed at each floor beam location & the shear in panel CD is calculated. Finally a segment of the girder is considered & the internal panel shear V_{CD} is calculated.



Steps:
The reactions of the floor beams on the girder are calculated first,
followed by a determination of the girder support reaction at *F* (G_y is not needed), and finally, a segment of the girder is considered and the
internal panel shear V_{CD} is calculated.

The unit load is placed at each floor beam location & the shear in panel CD is calculated. Finally a segment of the girder is considered & the internal panel shear V_{CD} is calculated.



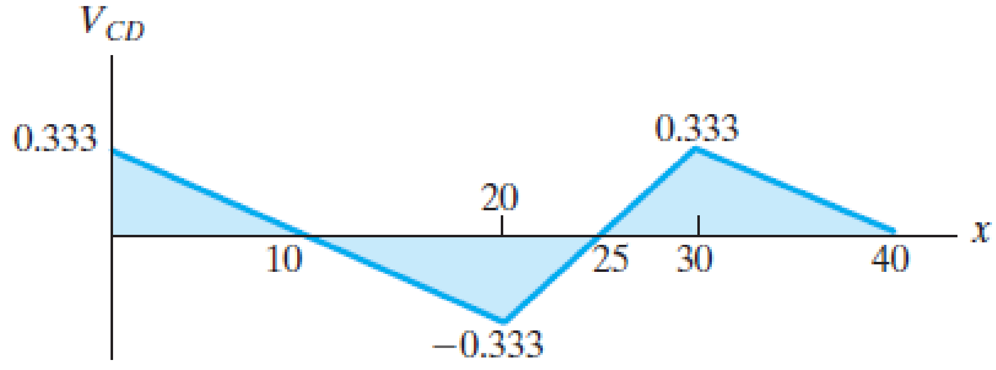
The reactions of the floor beams on the girder are calculated first,

followed by a determination of the girder support reaction at *F*

and finally, a segment of the girder is considered and the internal panel shear V_{CD} is calculated.

The unit load is placed at each floor beam location & the shear in panel CD is calculated. Finally a segment of the girder is considered & the internal panel shear V_{CD} is calculated.

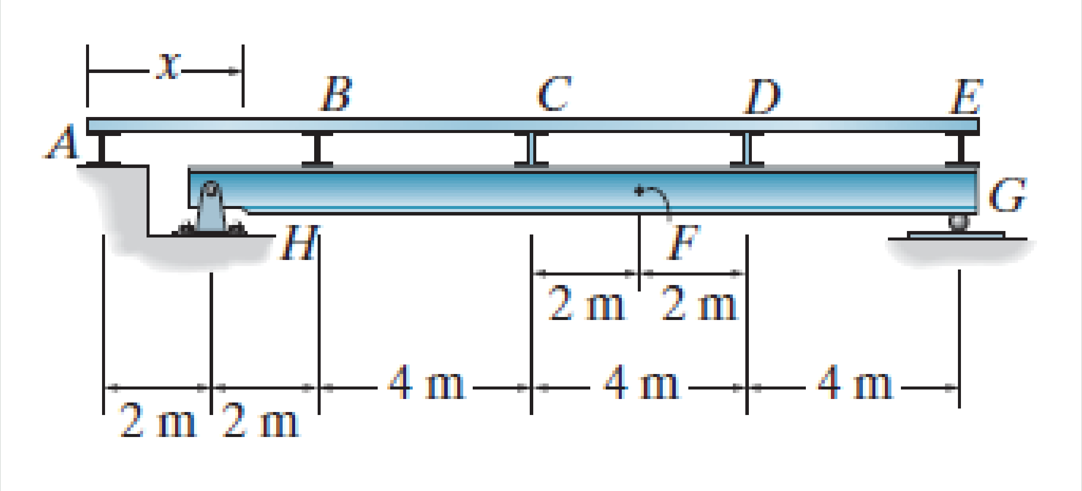
x	V_{CD}
0	0.333
10	0
20	−0.333
30	0.333
40	0



influence line for V_{CD}

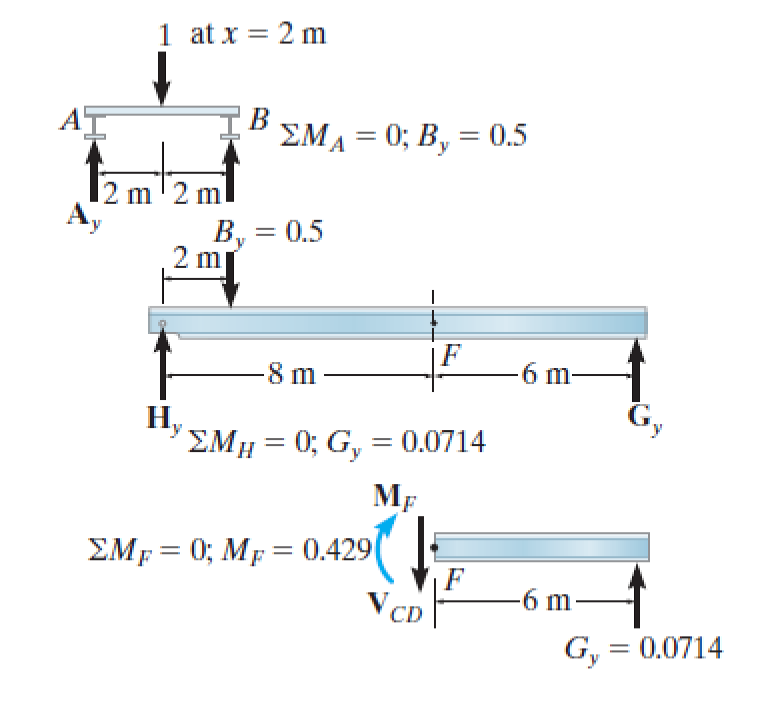
Example 6.14

Draw the influence line for the moment at point *F* for the floor girder



Solution

- The unit load is placed at x=0 and each panel point thereafter
- The corresponding values for M_F are calculated



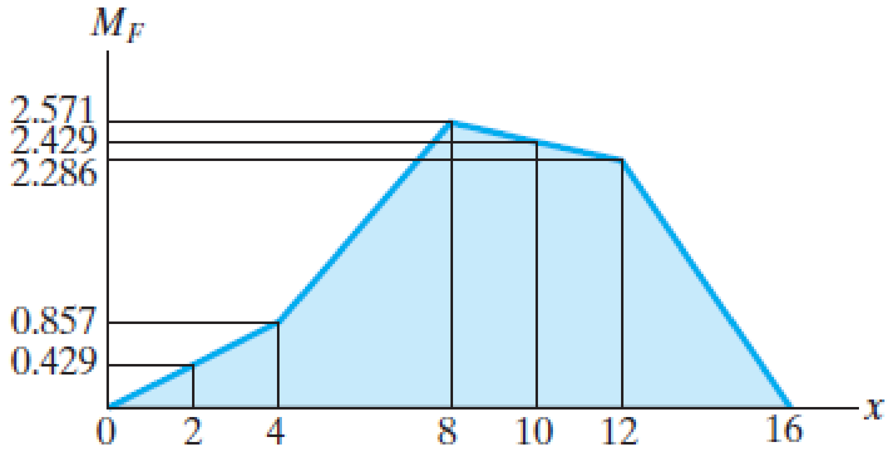
it is first necessary to determine the reactions of the floor beams on the girder,

followed by a determination of the girder support reaction G_Y

and finally segment G_F of the girder is considered and the internal moment M_F is calculated

Results

x	M_F
0	0
2	0.429
4	0.857
8	2.571
10	2.429
12	2.286
16	0

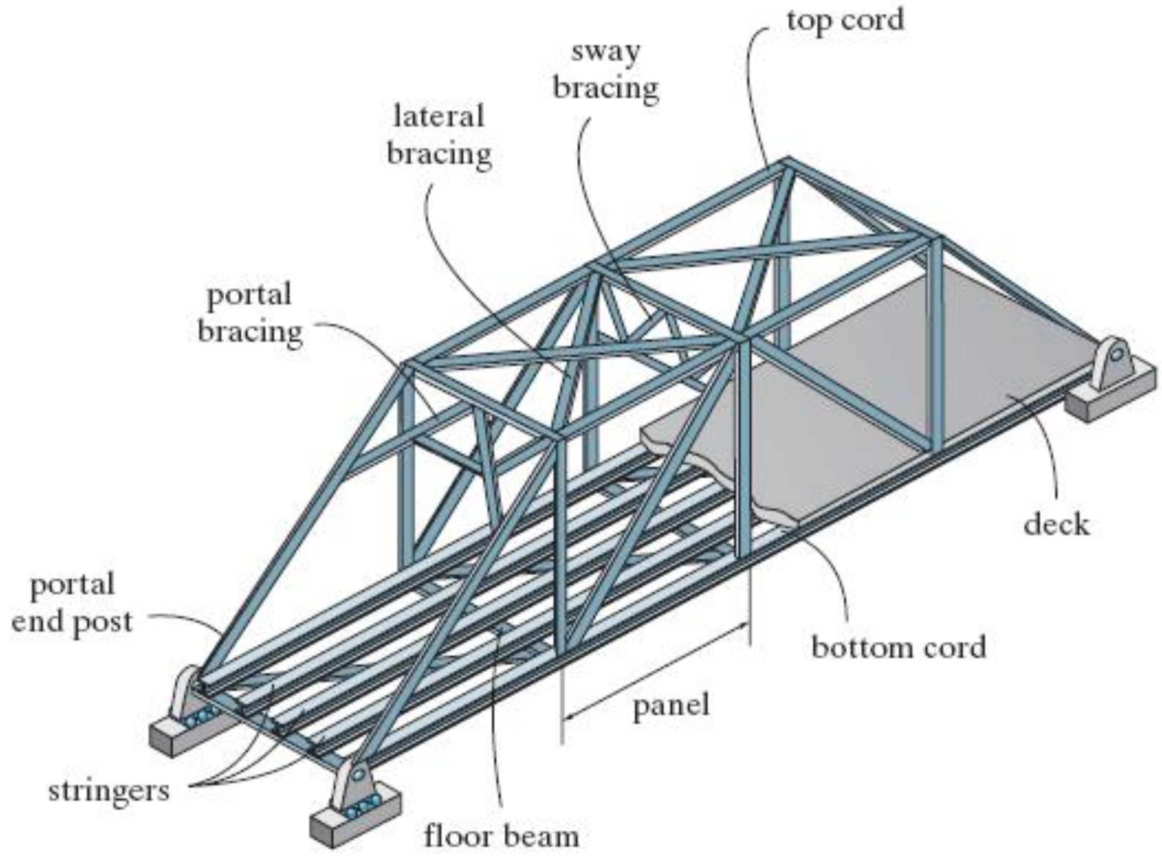


influence line for M_F

THANK YOU

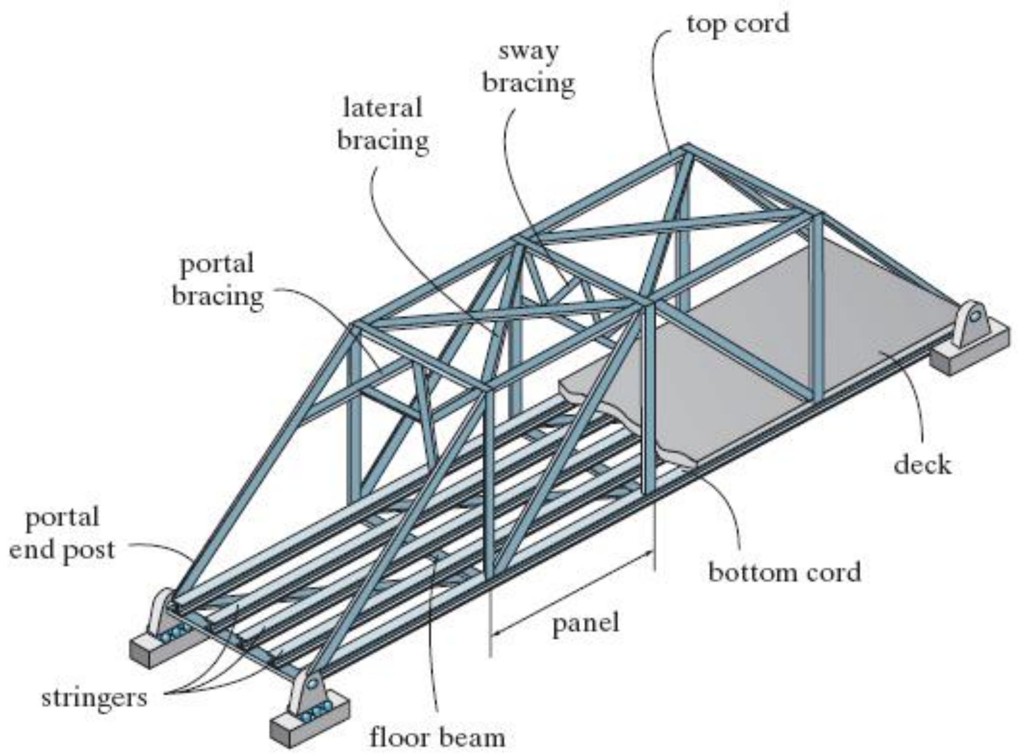
Influence Lines for Trusses

- The loading on the bridge deck is transmitted to stringers which in turn transmit the loading to floor beams and then to joints along the bottom cord



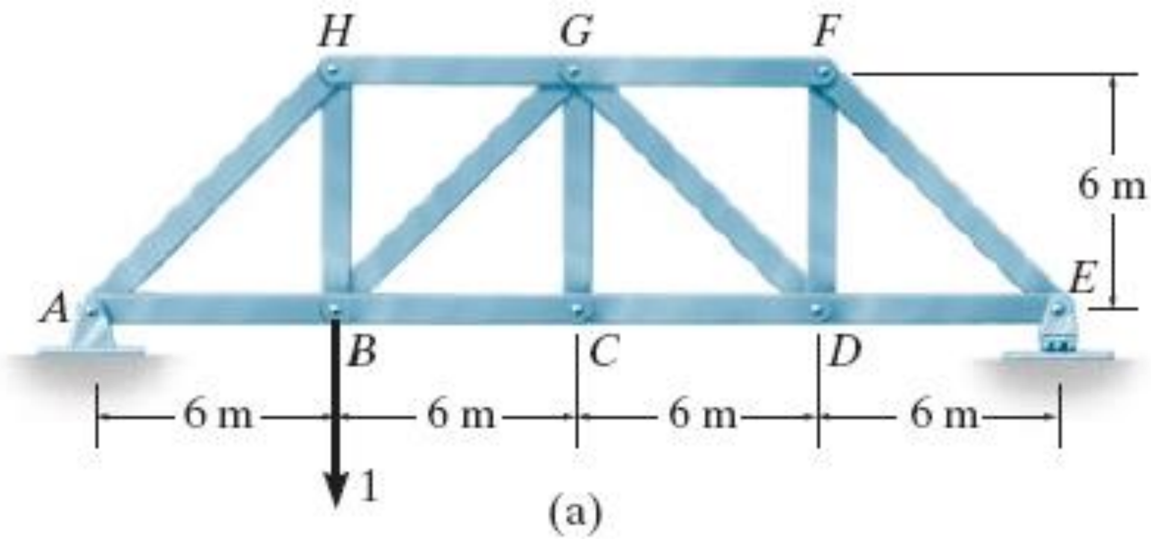
Influence Lines for Trusses

- We can obtain the ordinate values of the influence line for a member by loading each joint along the deck with a unit load and then use the method of joints or method of sections to calculate the force in the member



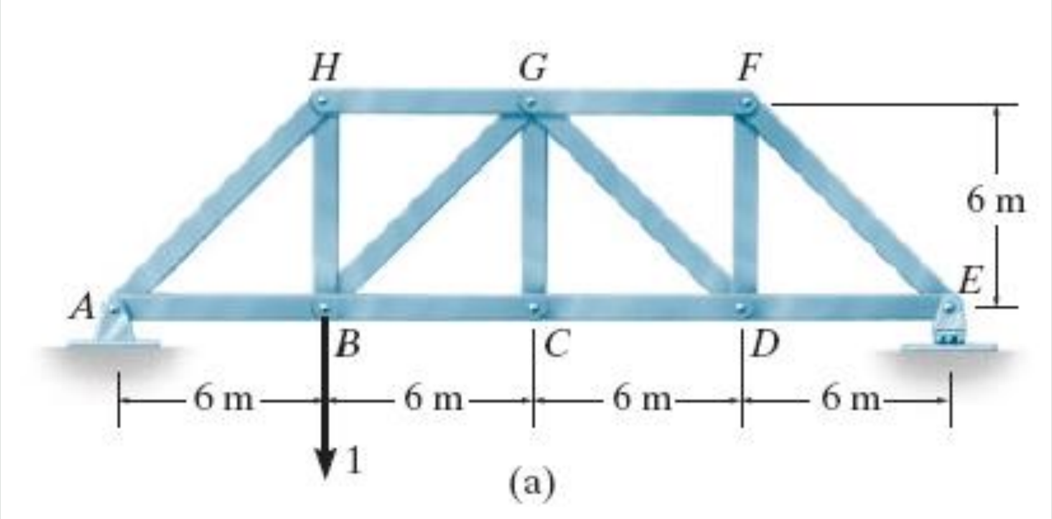
Example 6.15

Draw the influence line for the force in member *GB* of the bridge truss.

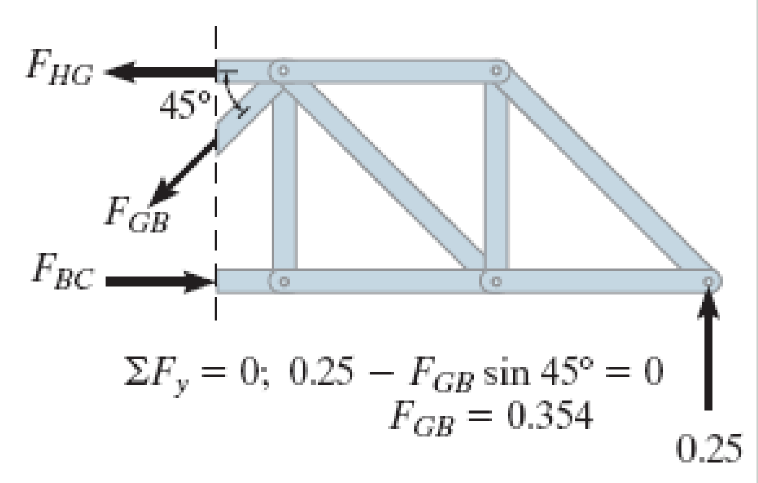


Solution

Each successive joint at the bottom cord is loaded with a unit load and the force in member GB is calculated using the method of sections.

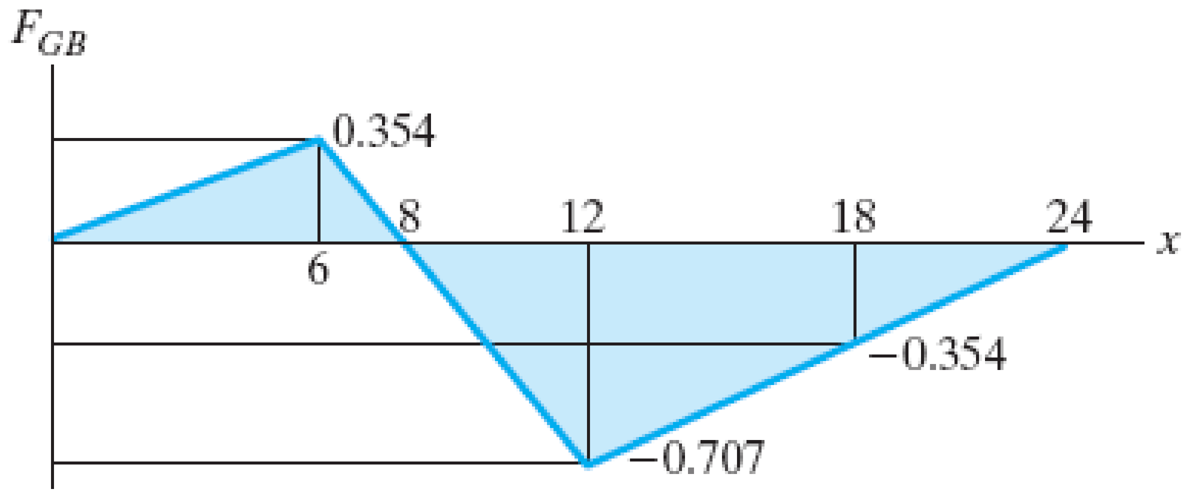


placing the unit load at x=6m



Solution

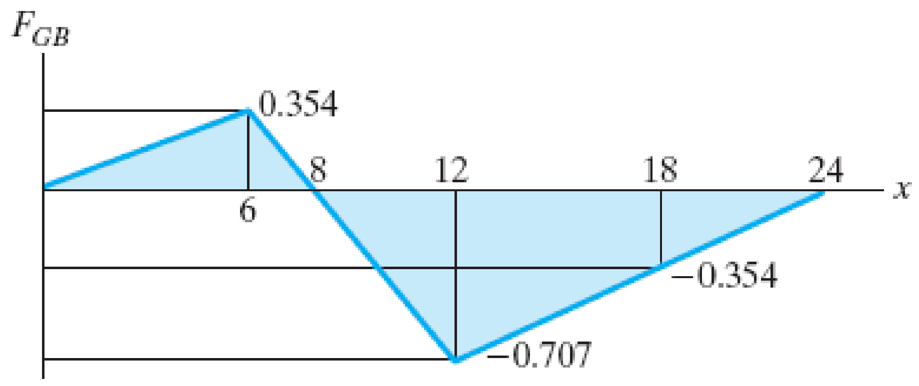
x	F_{GB}
0	0
6	0.354
12	−0.707
18	−0.354
24	0



influence line for F_{GB}

Solution

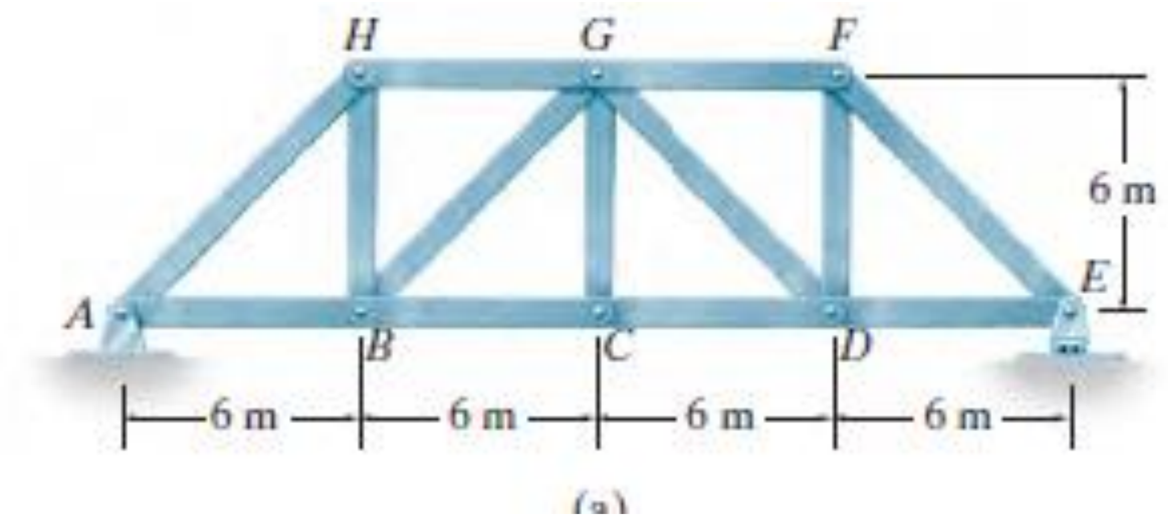
- Since the influence line extends over the entire span of truss, member GB is referred to as primary member.
- This means that GB is subjected to a force regardless of where the bridge deck is loaded.
- The point of zero force is determined by similar triangles.



influence line for F_{GB}

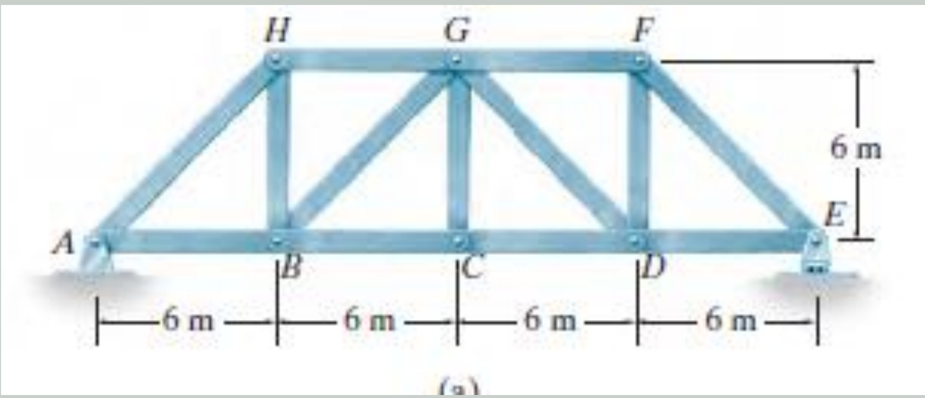
Example 6.16

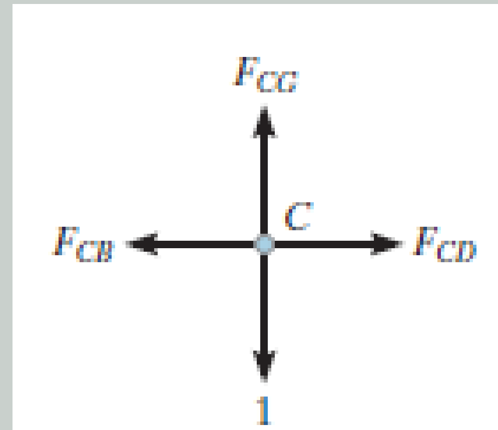
- Draw the influence line for the force in member *CG* of the bridge truss shown



Solution

- isolating joint *C*



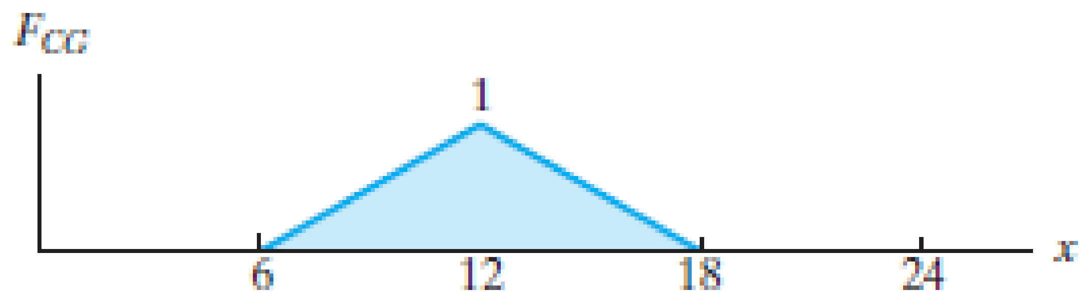


it is seen that CG is a zero-force member unless the unit load is applied at joint C, in which case

x	F_{GC}
0	0
6	0
12	1
18	0
24	0

Solution

x	F_{GC}
0	0
6	0
12	1
18	0
24	0

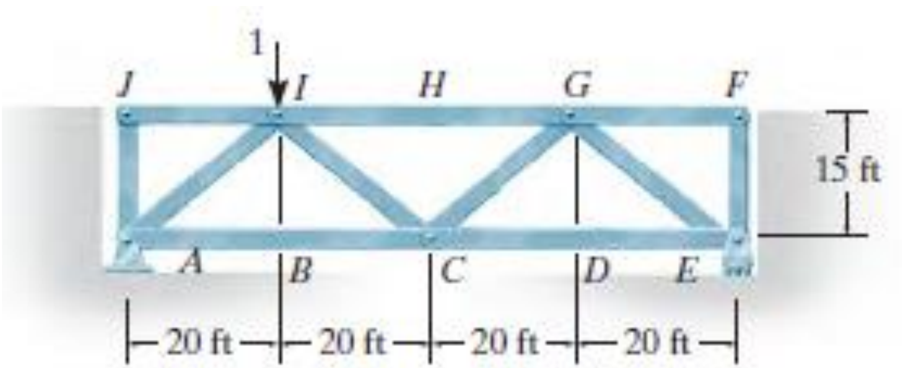


influence line for F_{CG}

- Since the influence line for *CG* does *not* extend over the entire span of the truss, member *CG* is referred to as a *secondary member*

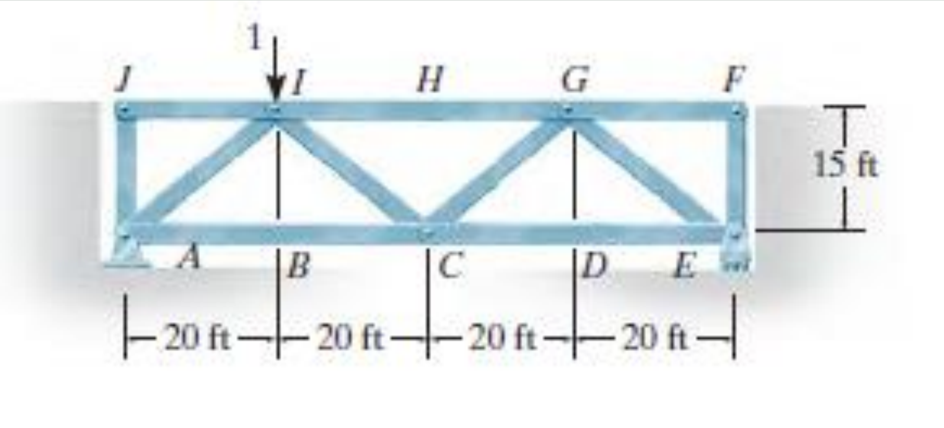
Example 6.16

- Determine the largest force that can be developed in member *BC* due to a moving force of 25 k and a moving distributed load of 0.6 k/ft.The loading is applied at the top cord

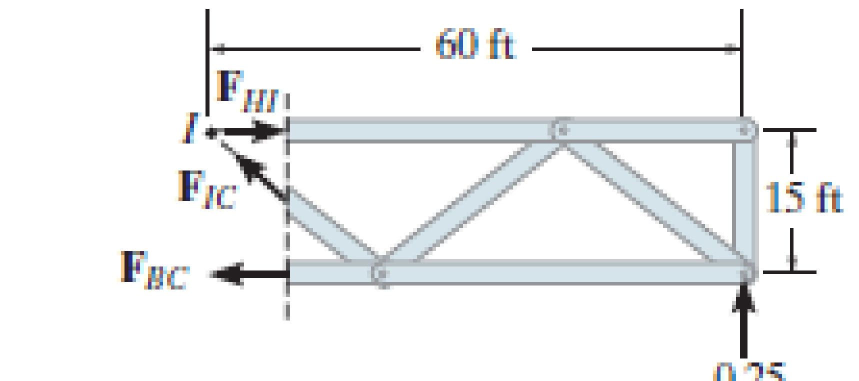


Warren truss

Solution



x	F_{BC}
0	0
20	1
40	0.667
60	0.333
80	0

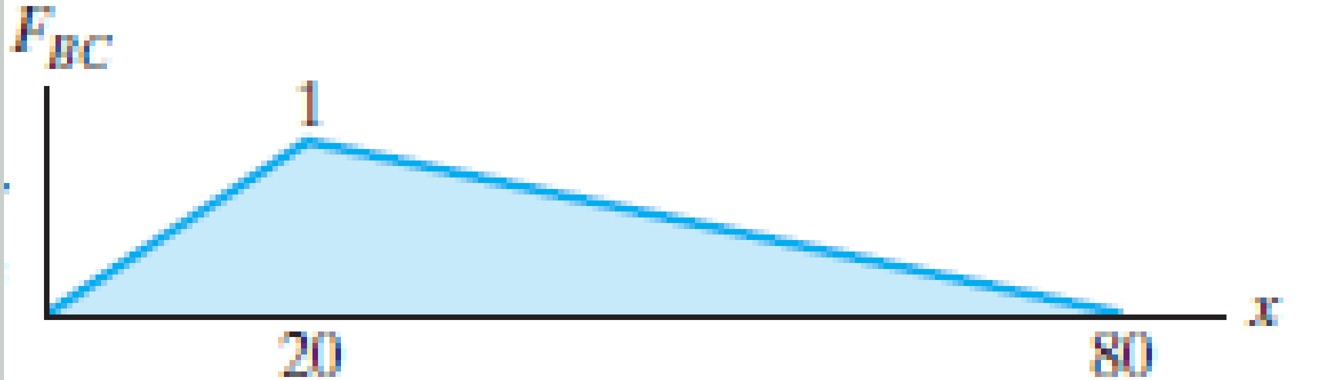


$$\circlearrowleft + \Sigma M_I = 0; \; -F_{BC}(15) + 0.25(60) = 0$$

$$F_{BC} = 1.00 \text{ (T)}$$

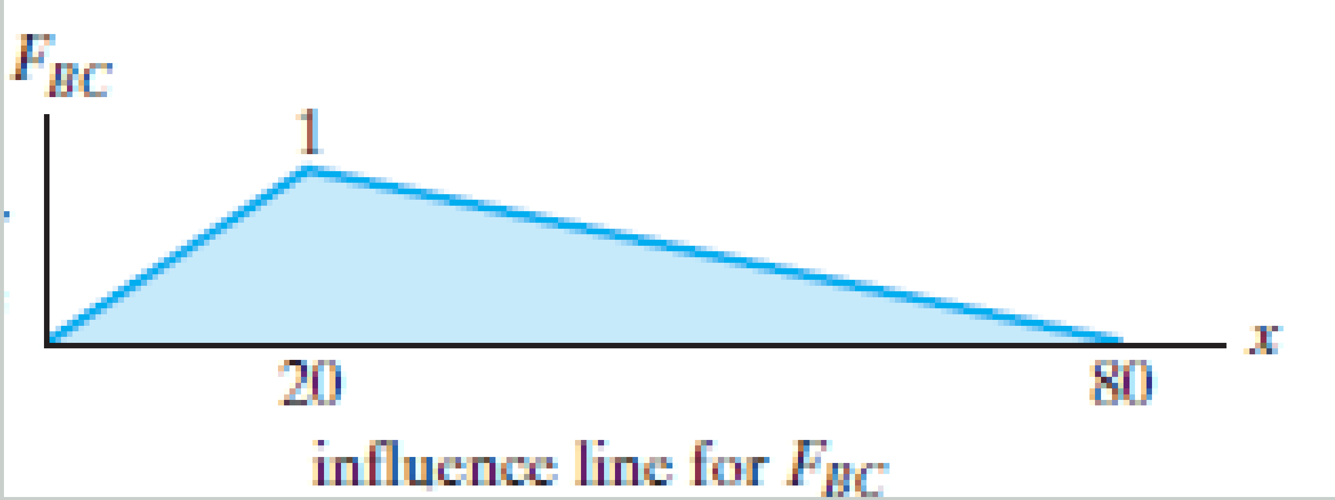
Solution

x	F_{BC}
0	0
20	1
40	0.667
60	0.333
80	0



influence line for F_{BC}

Solution

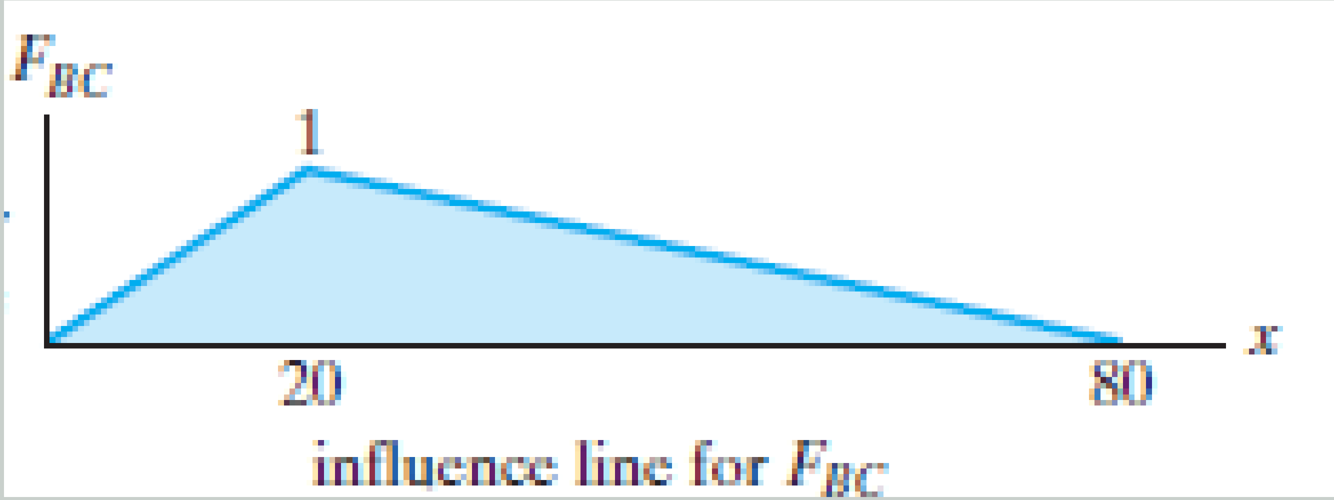


Concentrated Live Force.

The largest force in member BC occurs when the moving force of 25 k is placed at x = 20 ft, Thus,

$$F_{BC} = (1.00)(25) = 25.0 \text{ k}$$

Solution



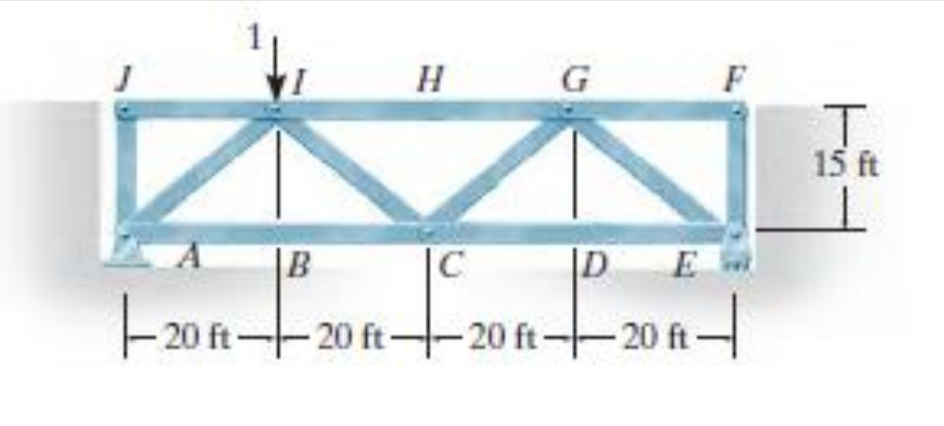
Distributed Live Load.

- The uniform live load must be placed over the entire deck of the truss to create the largest tensile force in *BC*. Thus,

$$F_{BC} = \left[\tfrac{1}{2}(80)(1.00)\right]0.6 = 24.0 \text{ k}$$

Solution

Total Maximum Force.



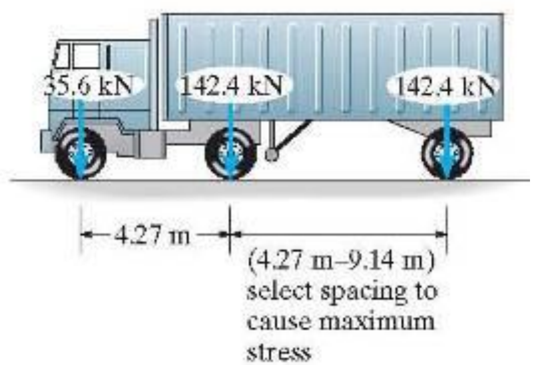
$$(F_{BC})_{max} = 25.0\text{ k} + 24.0\text{ k} = 49.0\text{ k}$$

THANK YOU

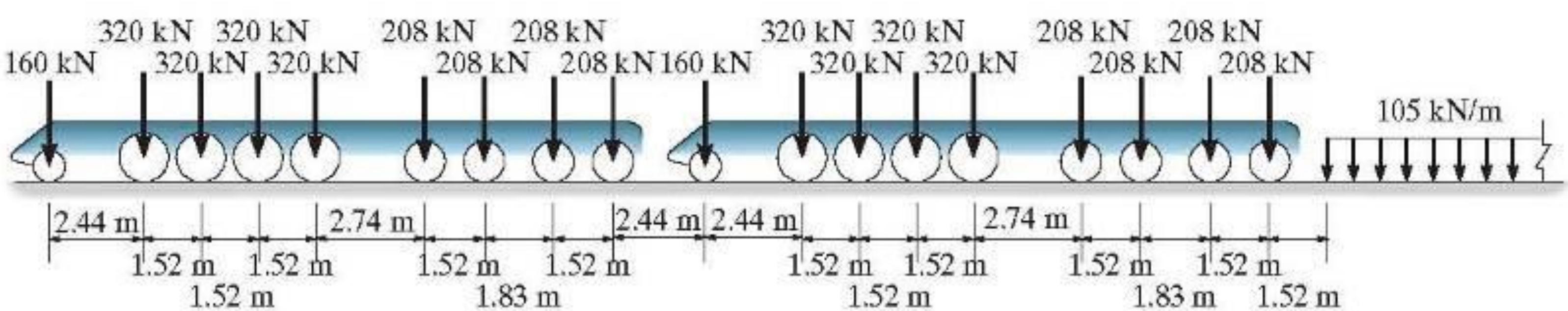
Applications of Influence Lines

Series of concentrated loads

Live loads due to vehicular traffic on highway and railway bridges are represented by a series of moving concentrated loads with specified spacing between the loads

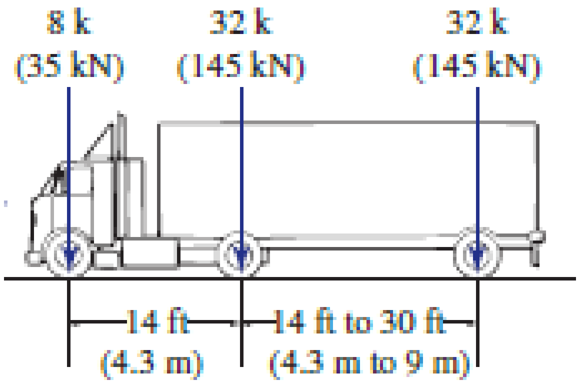


HS 20–44 loading

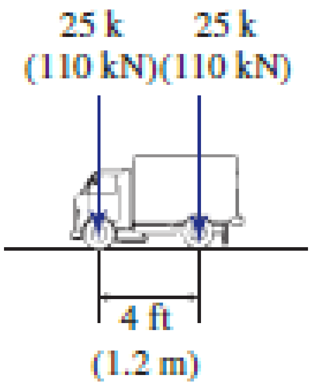


E-72 loading

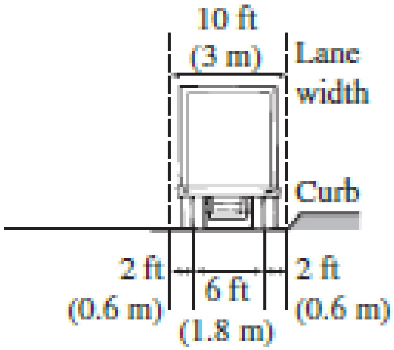
Series of concentrated loads



(a) HL-93 Truck



(b) HL-93 Tandem



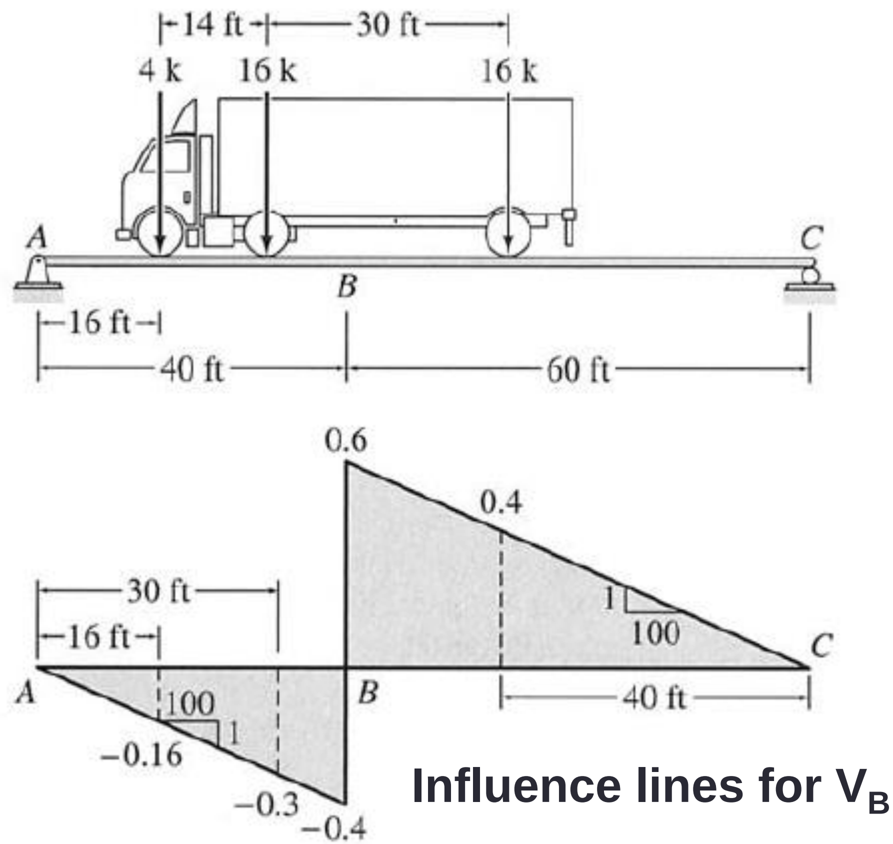
(c) Wheel Spacing for Truck/Tandem

Series of concentrated loads

- We discuss how the influence line for a response function can be used to determine:
- **(1) the value of the response function for a given position of a series of concentrated loads.**
- **(2) the maximum value of the response function due to a series of moving concentrated loads.**

Series of concentrated loads

- HS20-44 truck



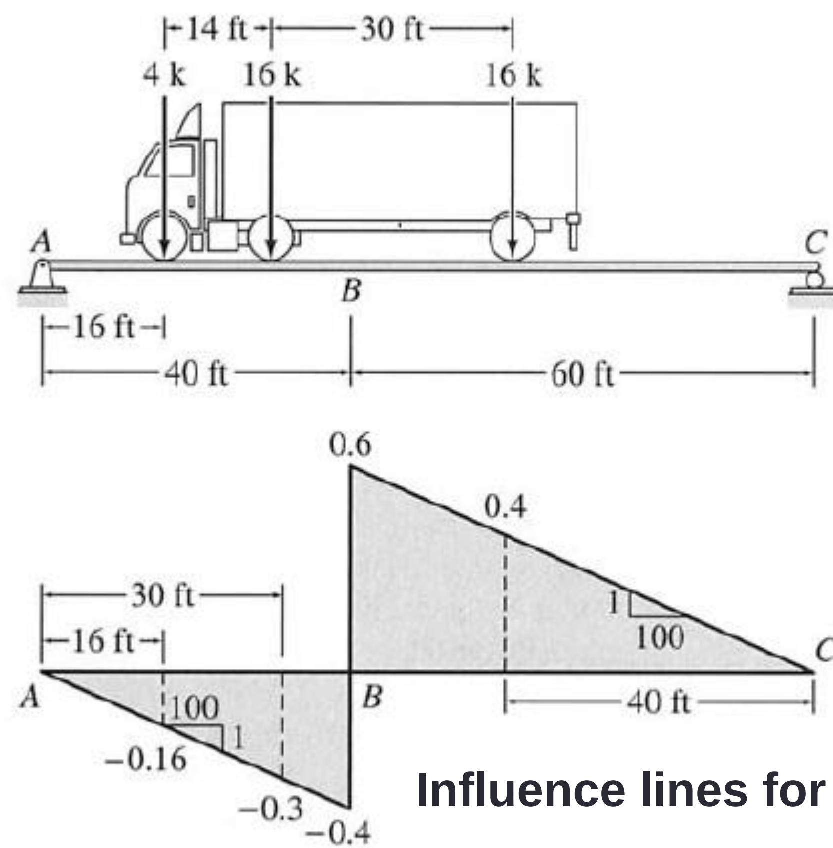
Influence lines for V_B

- The front axle of the truck is located at a distance of 16 ft from the left support A.
- The distances between the three loads as well as the location of the 4-k load are known, so the locations of the other two loads can be easily established.
- Although the influence-line ordinates corresponding to the loads can be obtained by using the properties of the similar triangles formed by the influence line,

Series of concentrated loads

it is usually convenient to evaluate such an ordinate by multiplying the slope of the segment of the influence line where the load is located by the distance of the load from the point at which the influence line segment intersects the horizontal axis (i.e., becomes zero). The sign (plus or minus) of the ordinate is obtained by inspection.

For example, the influence-line ordinate corresponding to the 4-k load can be computed by multiplying the slope (1:100) of the influence-line segment for the portion AB by the distance (16 ft) of the load from point A.



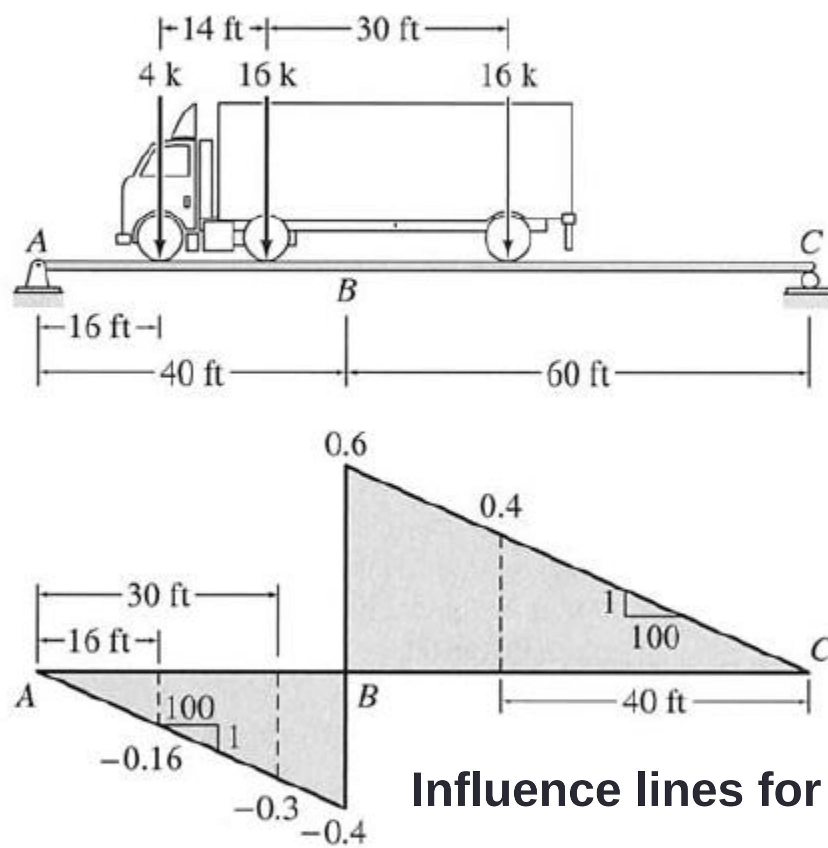
Influence lines for V_B

Series of concentrated loads

Thus the ordinate of the influence line for V_B corresponding to the 4k load equals

$$-(1/100)(16) = -0.16 \text{ k/k}.$$

The ordinates corresponding to the three loads thus obtained are :



Influence lines for V_B

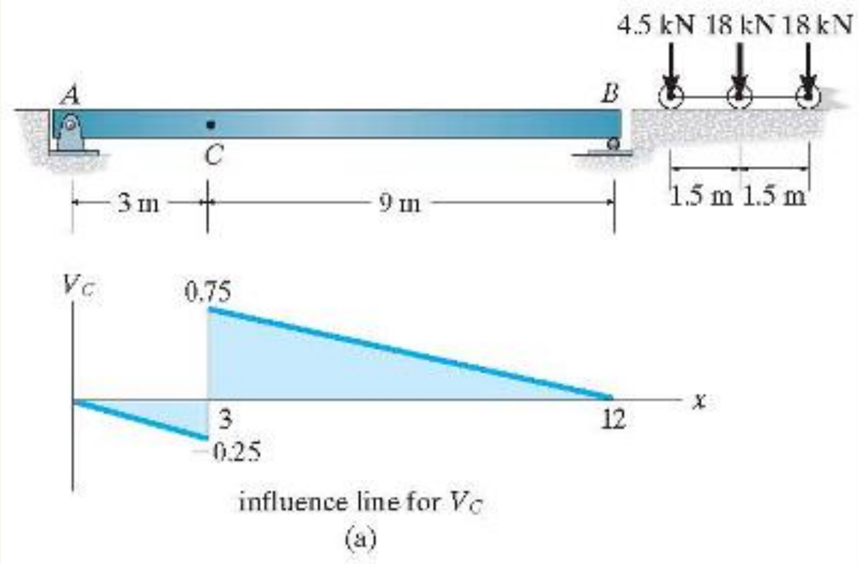
$$V_B = -4(0.16) - 16(0.3) + 16(0.4) = 0.96 \text{ k}$$

Max Influence at a Point due to a series of concentrated loads

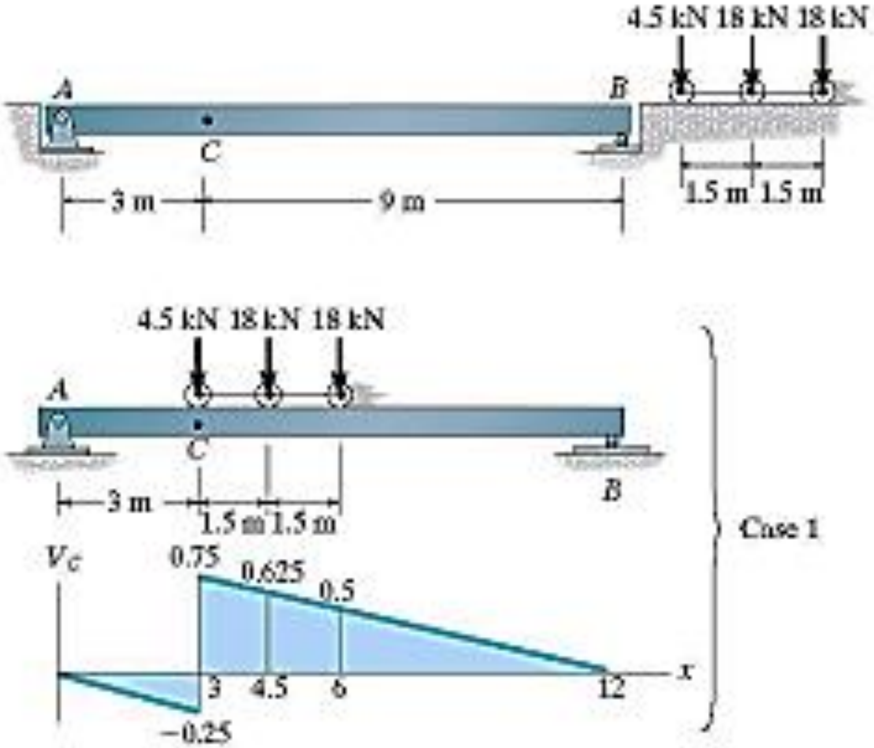
- The max effect caused by a live concentrated force is determined by multiplying the peak ordinate of the influence line by the magnitude of the force
- Trial-and-error procedure can be used or a method that is based on the change in function that takes place as the load is moved

Max Influence at a Point due to a series of concentrated loads

- **Shear**
  - Consider the simply supported beam with associated influence line for shear at point C
  - The max +ve shear at C is to be determined due to the series of concentrated loads moving from right to left
  - Critical loading occurs when one of the loads is placed just to the right of C

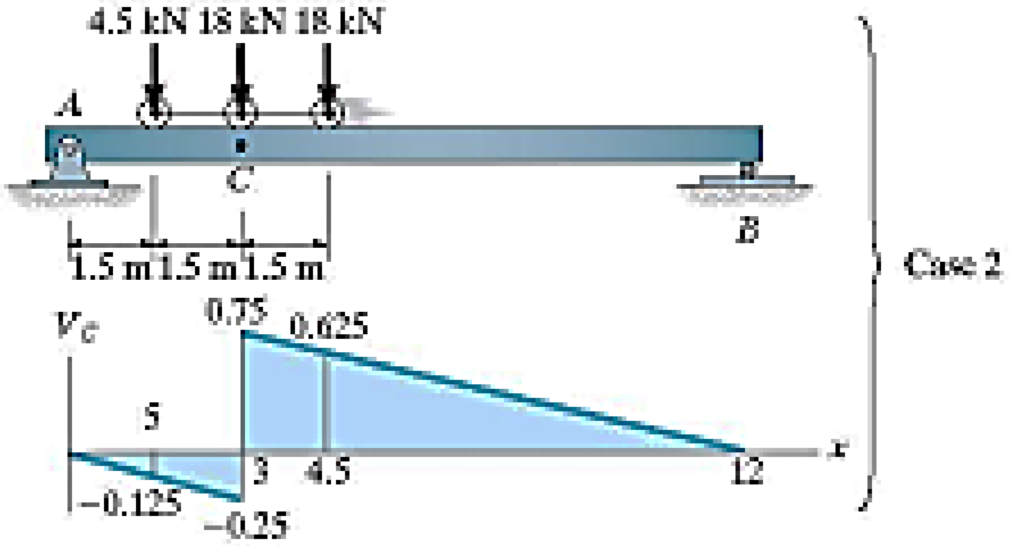


Max Influence at a Point due to a series of concentrated loads



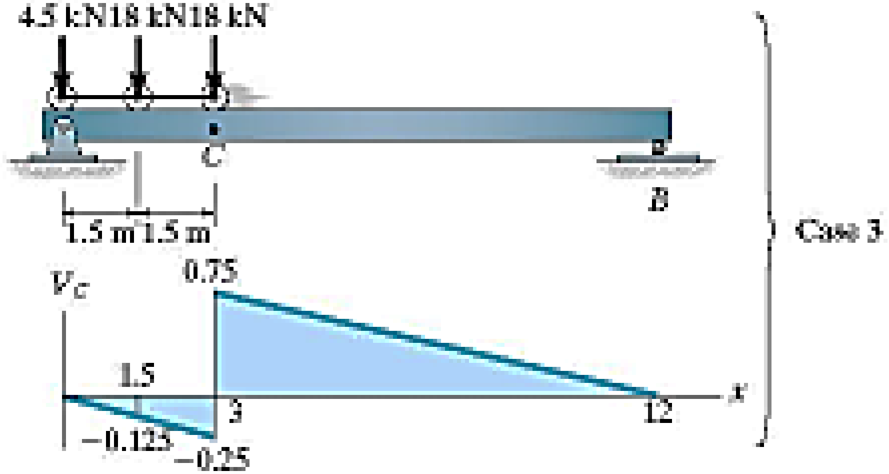
$$\text{Case 1}: (V_C)_1 = 4.5(0.75) + 18(0.625) + 18(0.5) = 23.63kN$$

Max Influence at a Point due to a series of concentrated loads



$$\text{Case 2}: (V_C)_2 = 4.5(-0.125) + 18(0.75) + 18(0.625) = 24.19kN$$

Max Influence at a Point due to a series of concentrated loads



$$\text{Case 3}: (V_C)_3 = 4.5(0) + 18(-0.125) + 18(0.75) = 11.25kN$$

Max Influence at a Point due to a series of concentrated loads

- Shear (cont'd)
  - By trial & error, each of the three possible cases can therefore be investigated

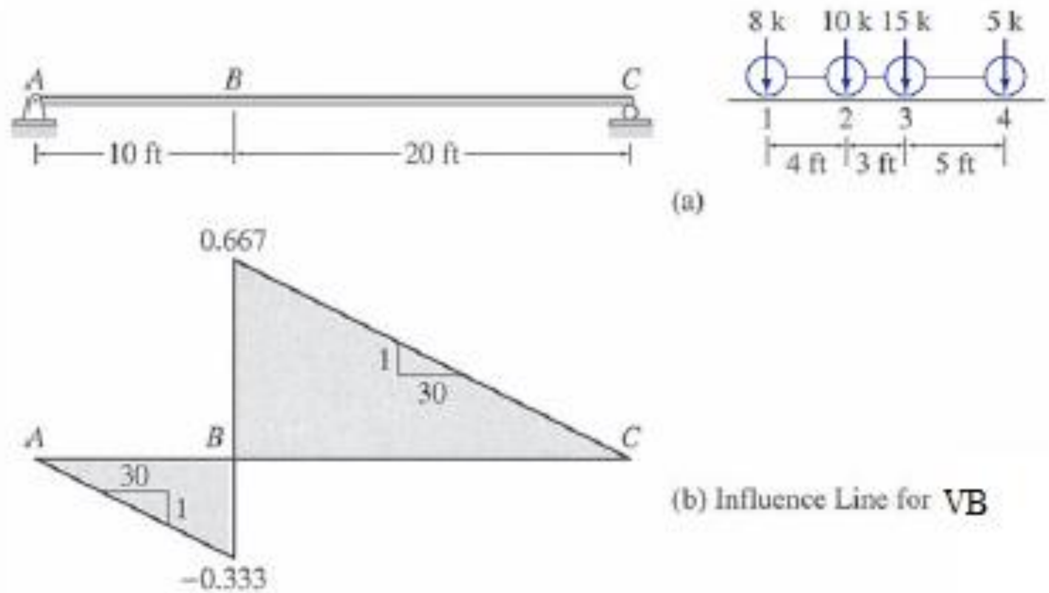
$$\text{Case 1}: (V_C)_1 = 4.5(0.75) + 18(0.625) + 18(0.5) = 23.63kN$$

$$\text{Case 2}: (V_C)_2 = 4.5(-0.125) + 18(0.75) + 18(0.625) = 24.19kN$$

$$\text{Case 3}: (V_C)_3 = 4.5(0) + 18(-0.125) + 18(0.75) = 11.25kN$$

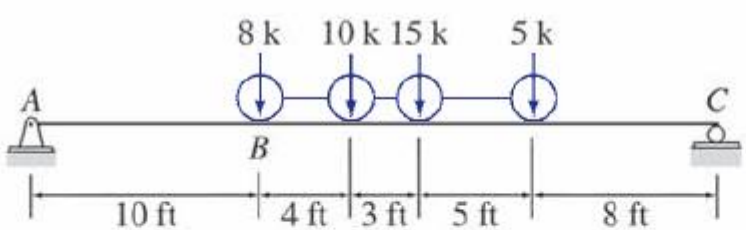
Case 2 yields the largest value for V_C and therefore represent the critical loading

Max Influence at a Point due to a series of concentrated loads



(a)

(b) Influence Line for VB

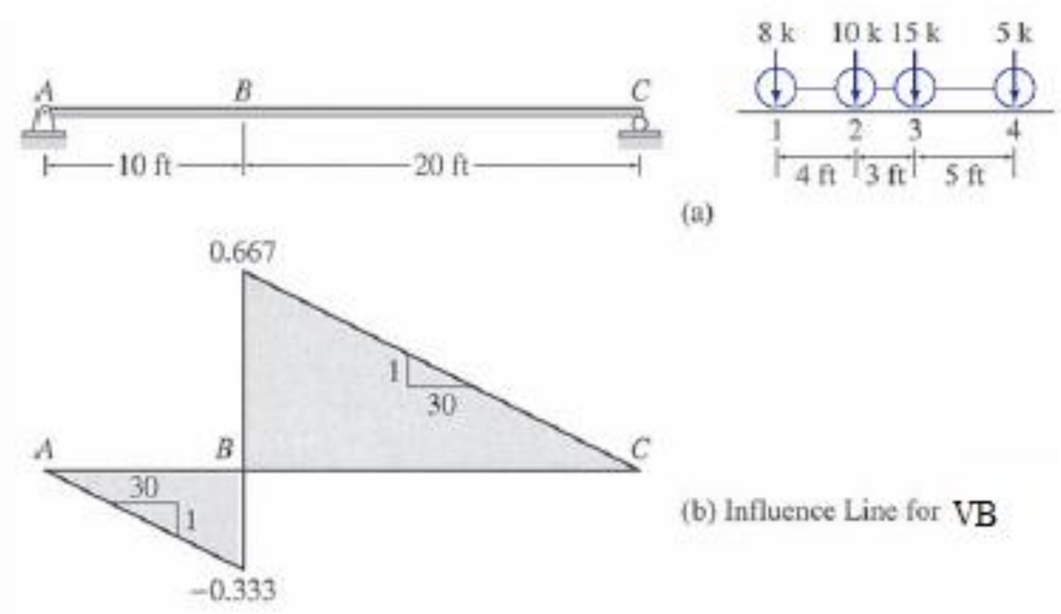


(c) Loading Position 1

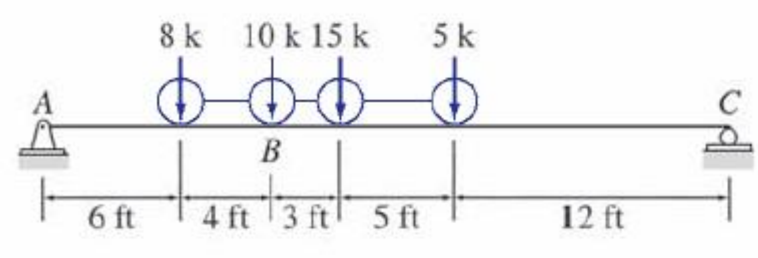
$$V_B = 8(20)\left(\frac{1}{30}\right) + 10(16)\left(\frac{1}{30}\right) + 15(13)\left(\frac{1}{30}\right) + 5(8)\left(\frac{1}{30}\right)$$

$$= 18.5 \text{ k}$$

Max Influence at a Point due to a series of concentrated loads



(b) Influence Line for VB

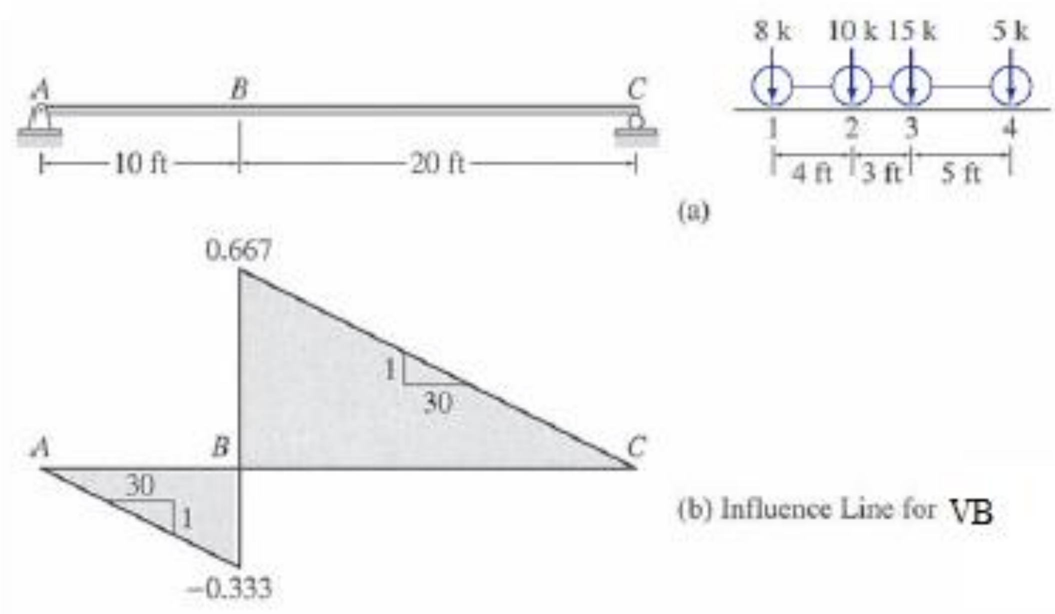


(d) Loading Position 2

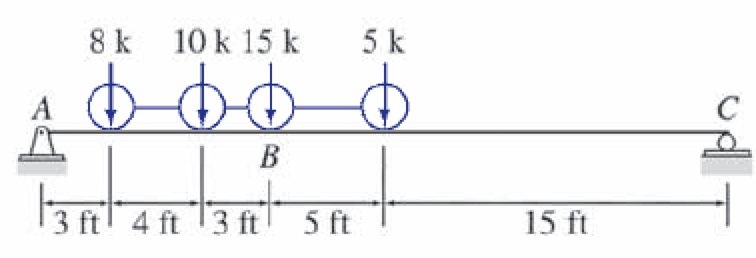
$$V_B = -8(6)\left(\frac{1}{30}\right) + 10(20)\left(\frac{1}{30}\right) + 15(17)\left(\frac{1}{30}\right) + 5(12)\left(\frac{1}{30}\right)$$

$$= 15.567 \text{ k}$$

Max Influence at a Point due to a series of concentrated loads



(b) Influence Line for VB



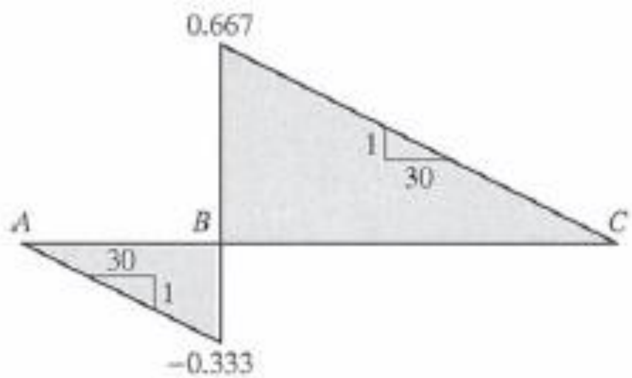
(e) Loading Position 3

$$V_B = -8(3)\left(\frac{1}{30}\right) - 10(7)\left(\frac{1}{30}\right) + 15(20)\left(\frac{1}{30}\right) + 5(15)\left(\frac{1}{30}\right)$$

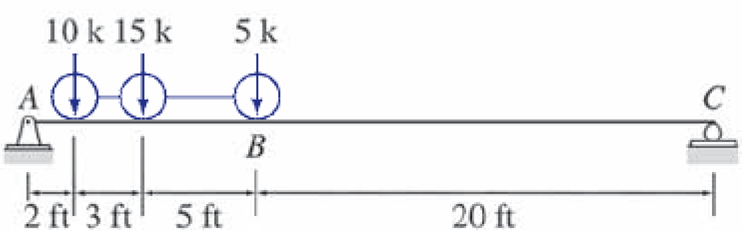
$$= 9.367 \text{ k}$$

Max Influence at a Point due to a series of concentrated loads

(a)



(b) Influence Line for VB



(f) Loading Position 4

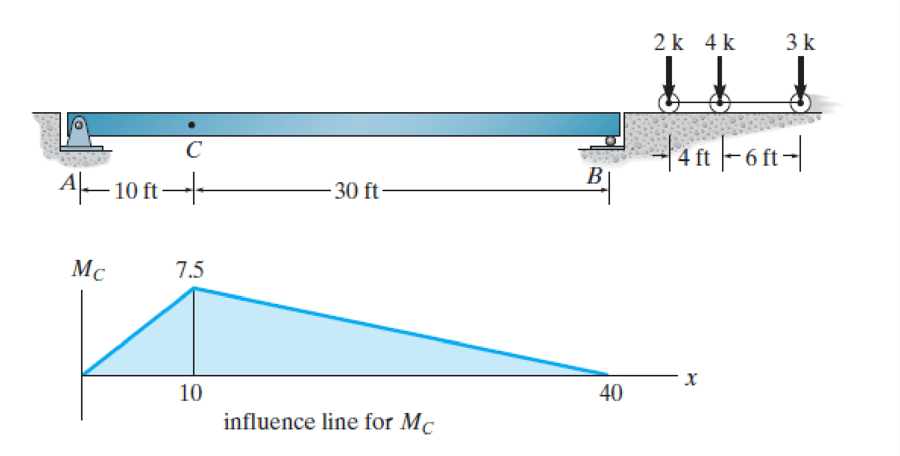
$$V_B = -10(2)\left(\frac{1}{30}\right) - 15(5)\left(\frac{1}{30}\right) + 5(20)\left(\frac{1}{30}\right) = 0.167 \text{ k}$$

Maximum +ve V_B =18.5 k

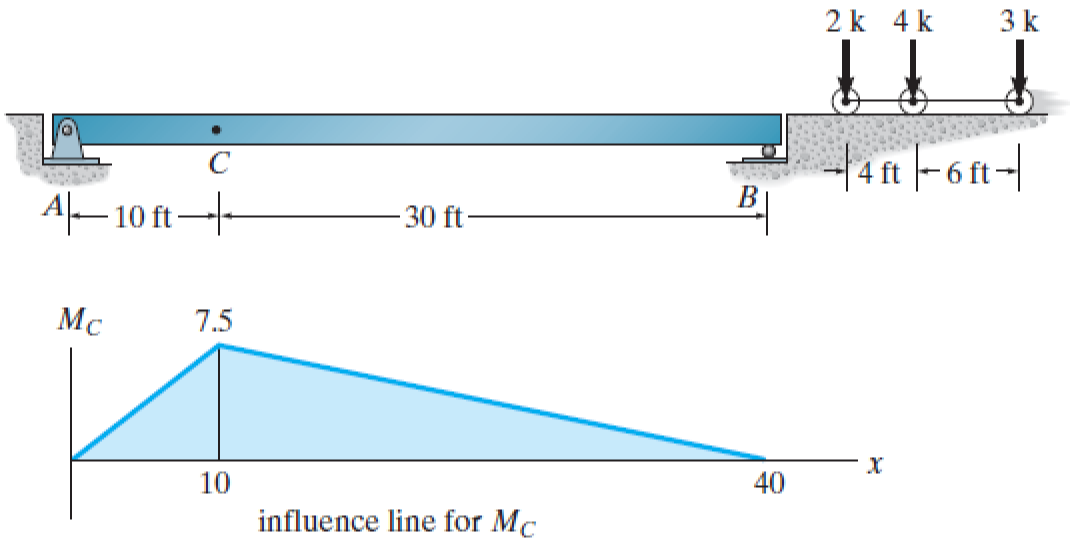
Max Influence at a Point due to a series of concentrated loads

- **Moment**
  - Use the foregoing methods to determine the critical position of a series of concentrated forces so that they create the largest internal moment at a specific point in a structure.

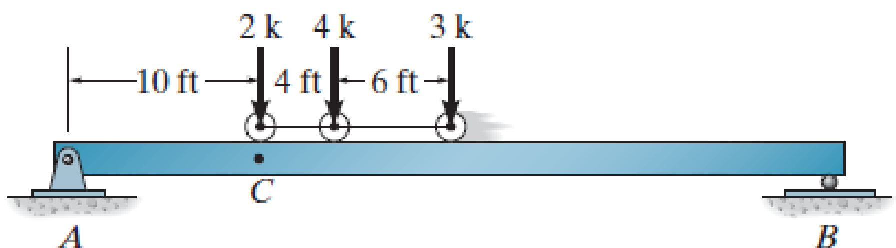
Of course, it is first necessary to draw the influence line for the moment at the point and determine the slopes of its line segments



Max Influence at a Point due to a series of concentrated loads

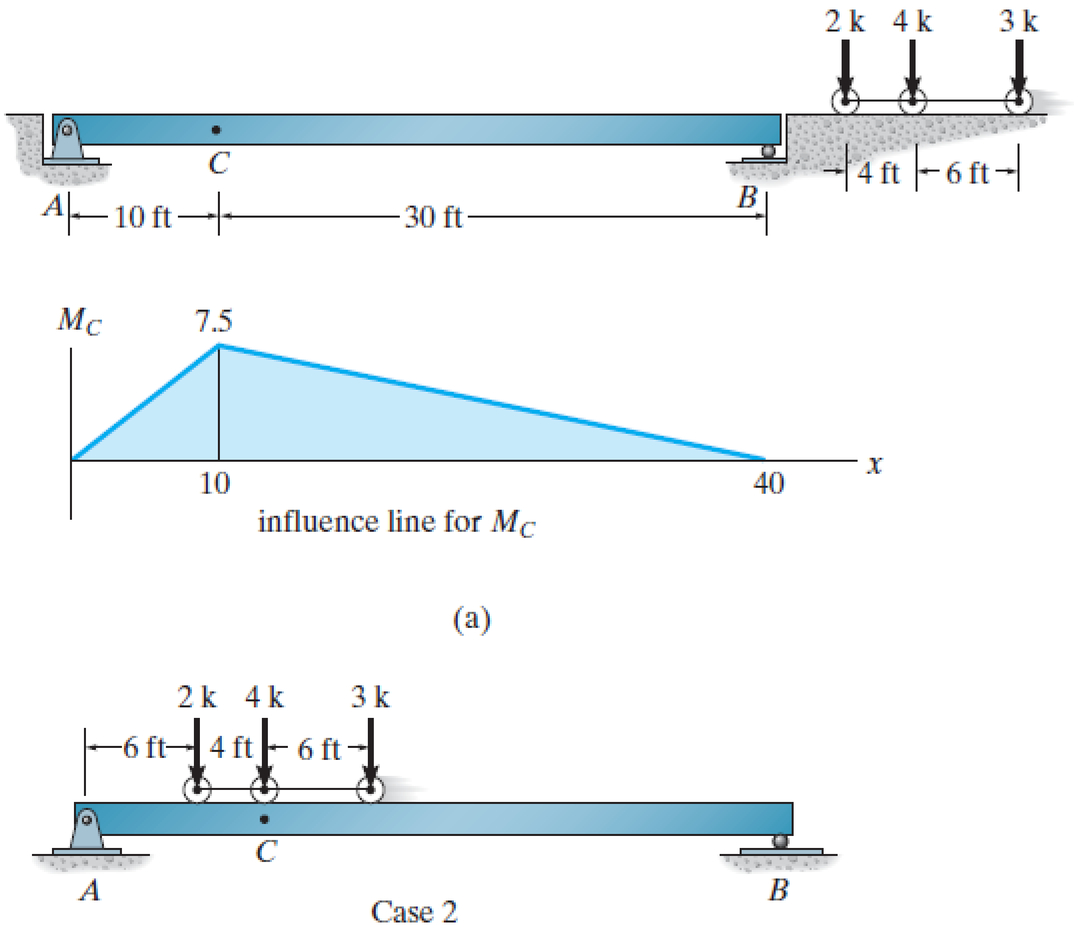


(a)

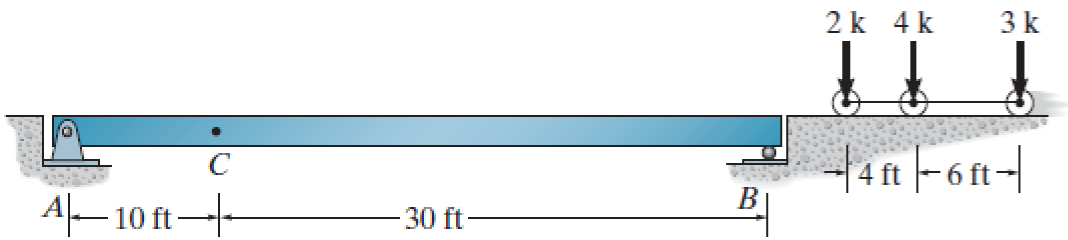


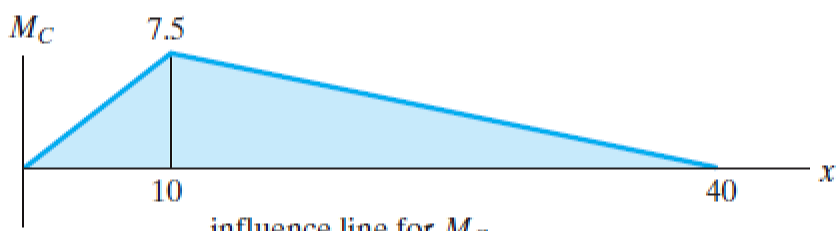
Case 1

Max Influence at a Point due to a series of concentrated loads

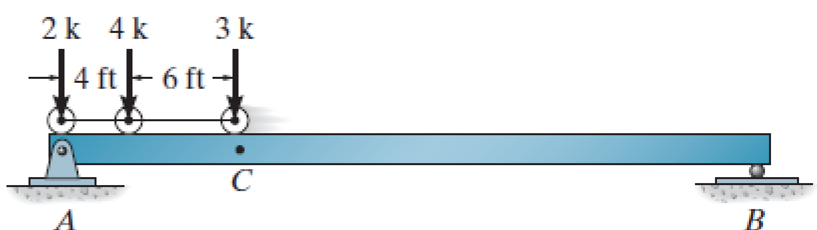


Max Influence at a Point due to a series of concentrated loads





influence line for M_C

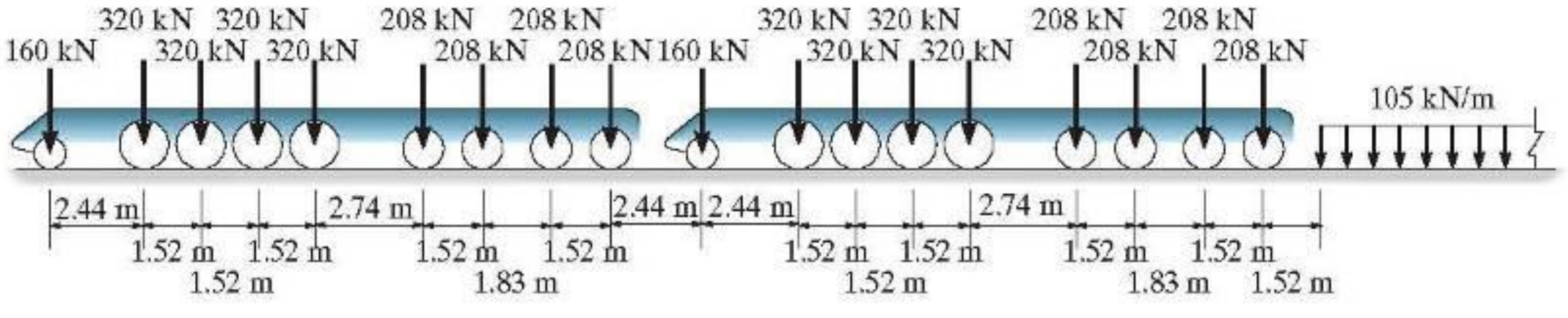


Case 3

Max Influence at a Point due to a series of concentrated loads

Q.

Is it possible to apply foregoing procedure for E-72 Loading?



E-72 loading

Think about it

THANK YOU

Absolute Max Shear & Moment

Absolute Max Shear

Single Concentrated Load

Single Concentrated Load

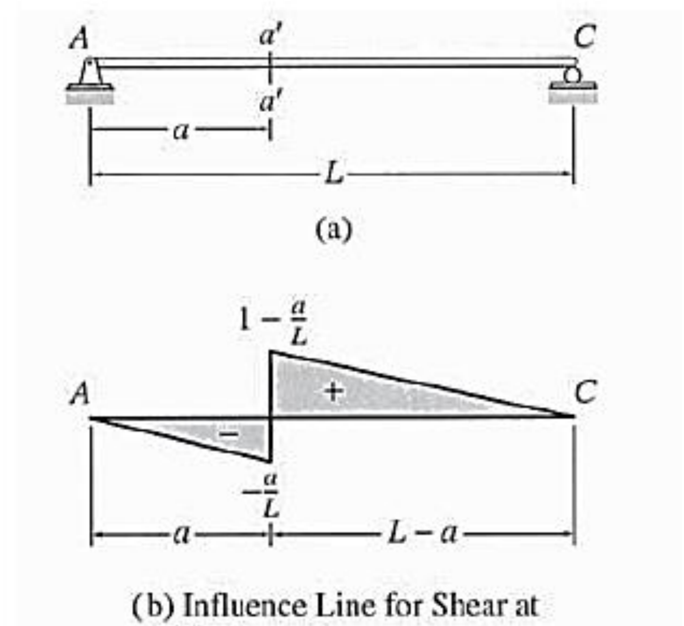
Suppose that we wish to determine the absolute maximum shear in the beam due to a single moving concentrated load of magnitude *P*

The maximum positive shear at the section a'a' is given by the product of the load magnitude, P and the maximum positive ordinate, 1- (a/L)

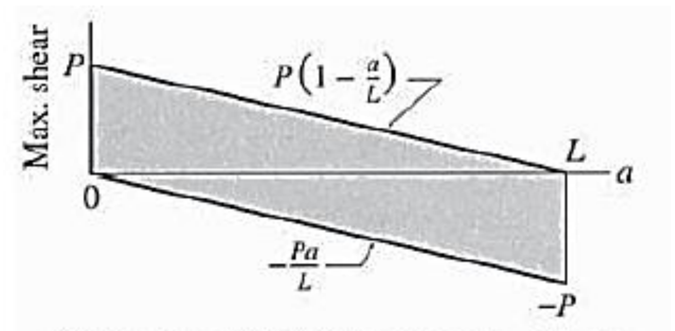
$$\text{maximum positive shear} = P\left(1 - \frac{a}{L}\right)$$

Similarly, the maximum negative shear at section aa is given by

$$\text{maximum negative shear} = -\frac{Pa}{L}$$



(b) Influence Line for Shear at Section a'a'



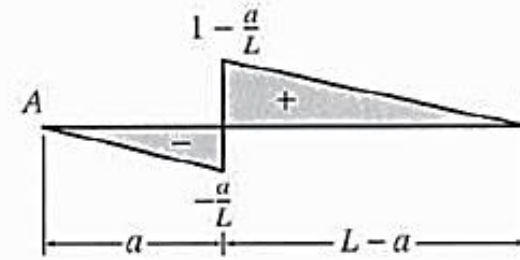
(d) Envelope of Maximum Shears — Single Concentrated Load

Single Concentrated Load

$$\text{maximum positive shear} = P\left(1 - \frac{a}{L}\right)$$

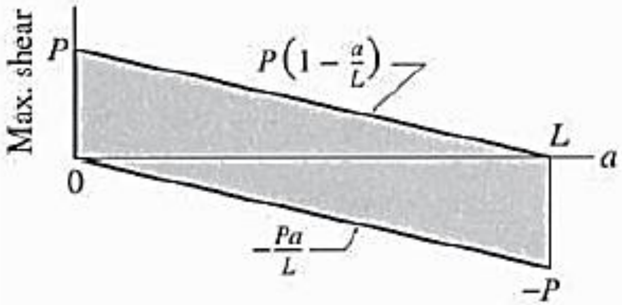
$$\text{maximum negative shear} = -\frac{Pa}{L}$$

These equations indicate that the maximum positive and maximum negative shears at a section due to a single moving concentrated load vary linearly with the distance **(a)** of the section from the left support A of the beam.



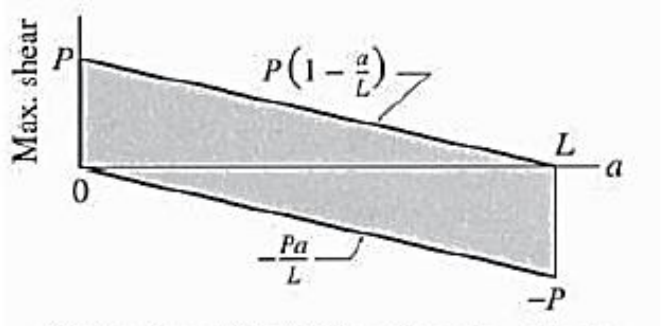
(b) Influence Line for Shear at Section a′a′

- A plot of above equations, with maximum shear as ordinate, against the location (a) of the section as abscissa is shown in Figure
- Such a graph, depicting the variation of the maximum value of a response function as a function of the location of the section, is referred to as the envelope of the maximum values of a response function.



(d) Envelope of Maximum Shears — Single Concentrated Load

Single Concentrated Load



(d) Envelope of Maximum Shears — Single Concentrated Load

An envelope of maximum values of a response function provides a convenient means of determining the absolute maximum value of the response function as well as its location.

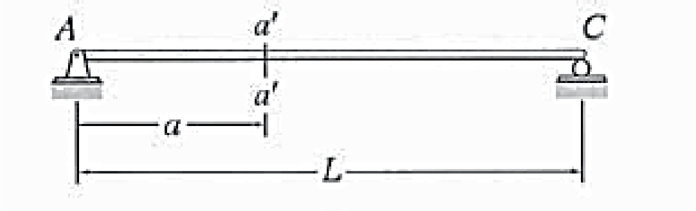
It can be seen from the envelope of maximum shears that in a simply supported beam subjected to a moving concentrated load P, the absolute maximum shear develops at sections just inside the supports and has the magnitude of P.

Absolute Max Moment

Single Concentrated Load

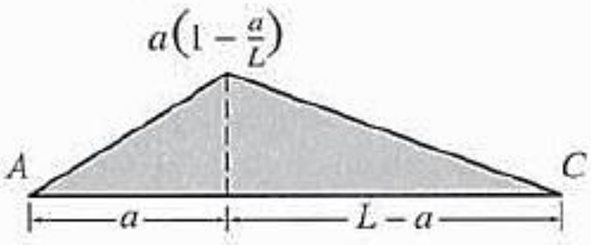
Single Concentrated Load

The envelope of maximum bending moments due to a single moving concentrated load *P* can be generated in a similar manner.



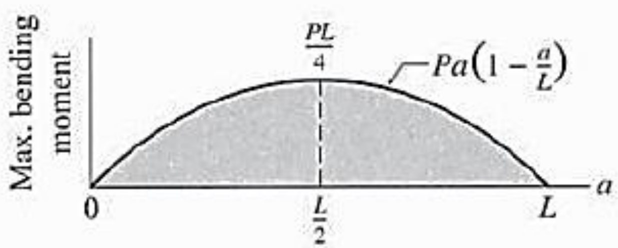
(a)

By using the influence line for bending moment at the arbitrary section a'a' given in Figure, we determine the expression for the maximum bending moment at the section a'a' as



(c) Influence Line for Bending Moment at Section a'a'

$$\text{maximum bending moment} = Pa\left(1-\frac{a}{L}\right)$$



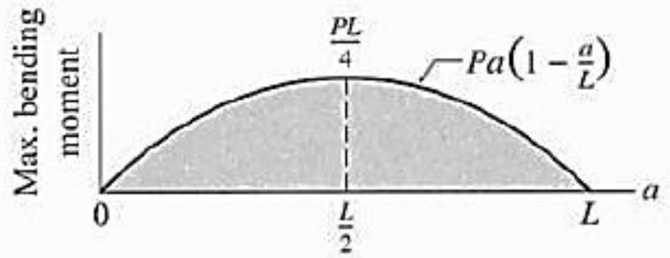
(e) Envelope of Maximum Bending Moments — Single Concentrated Load

Single Concentrated Load

The envelope of maximum bending moments constructed by plotting Equation

$$\text{maximum bending moment} = Pa\left(1 - \frac{a}{L}\right)$$

is shown in Figure.



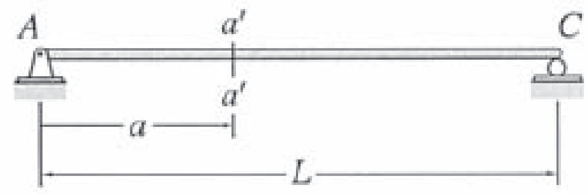
(e) Envelope of Maximum Bending Moments — Single Concentrated Load

It can be seen that the absolute maximum bending moment occurs at mid-span of the beam and has magnitude PL/4.

Absolute Max Shear

Uniformly Distributed Load

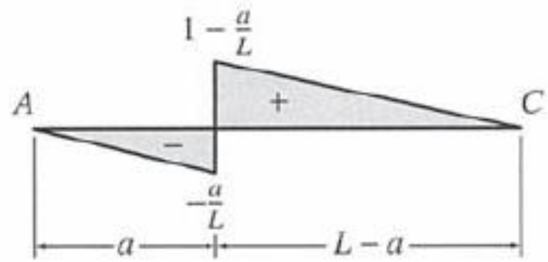
Uniformly Distributed Load



(a)

Due to a uniformly distributed live load of intensity , **w** .

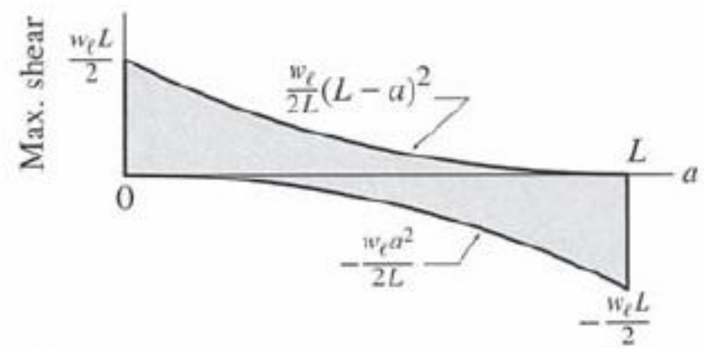
The maximum positive (or negative) shear at the section *a'a'* can be obtained by placing the load over the portion of the beam where the ordinates of the shear influence line are positive (or negative), and by multiplying the load intensity by the area of the influence line under the loaded portion of the beam. Thus,



(b) Influence Line for Shear at Section a'a'

$$\text{maximum positive shear} = \frac{w_\ell}{2L}(L-a)^2$$

$$\text{maximum negative shear} = -\frac{w_\ell a^2}{2L}$$



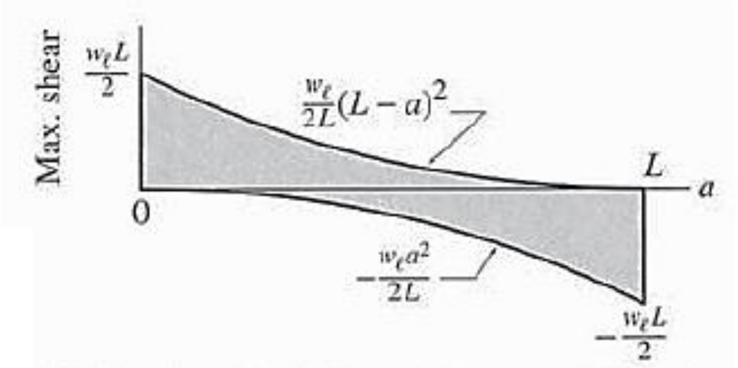
(f) Envelope of Maximum Shears — Uniformly Distributed Load

Uniformly Distributed Load

The envelope of maximum shears due to a uniformly distributed live load, constructed by plotting Eqs.

$$\text{maximum positive shear} = \frac{w_\ell}{2L}(L-a)^2$$

$$\text{maximum negative shear} = -\frac{w_\ell a^2}{2L}$$



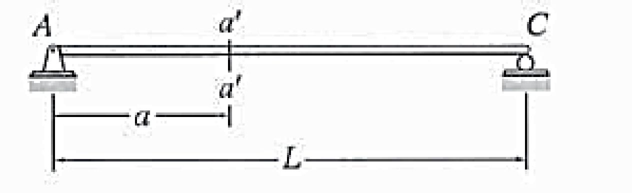
(f) Envelope of Maximum Shears — Uniformly Distributed Load

It can be seen that the absolute maximum shear develops at sections just inside the supports and has magnitude *WL/2*.

Absolute Max Moment

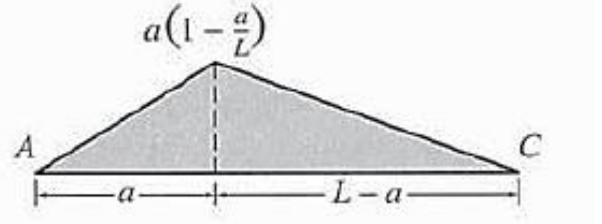
Uniformly Distributed Load

Uniformly Distributed Load



(a)

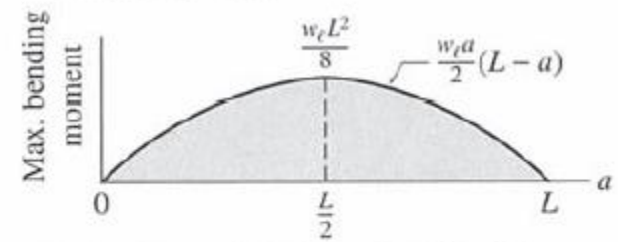
To determine the expression for the maximum bending moment at section *a'a'*, we multiply the load intensity, *w*, by the area of the bending moment influence line, to obtain



(c) Influence Line for Bending Moment at Section $a'a'$

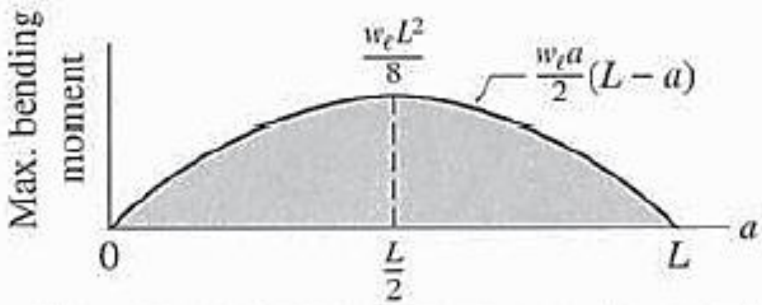
$$\text{maximum bending moment} = \frac{w_\ell a}{2}(L-a)$$

The envelope of maximum bending moments due to a uniformly distributed live load, constructed by plotting above equation



(g) Envelope of Maximum Bending Moments — Uniformly Distributed Load

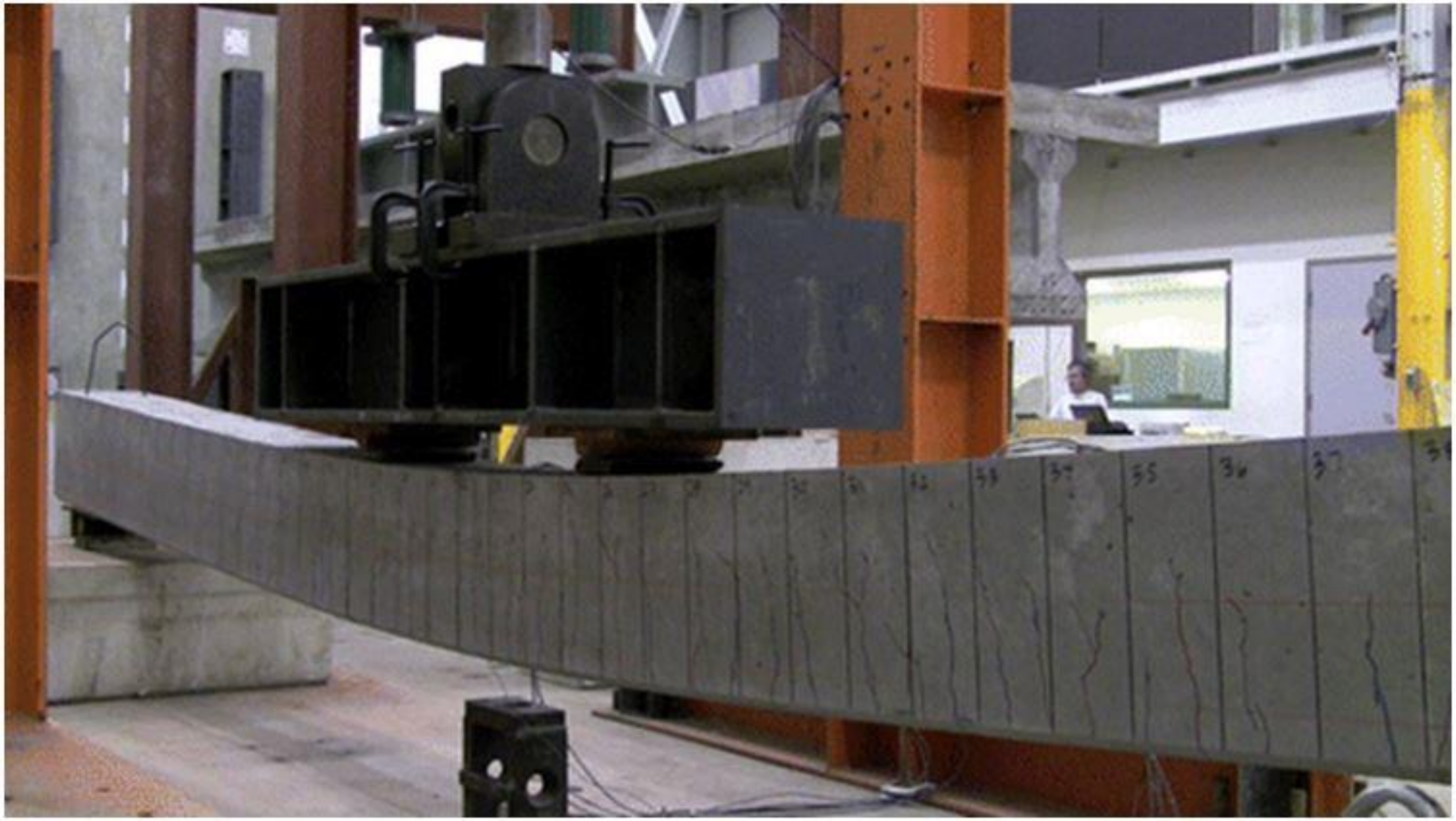
Uniformly Distributed Load



(g) Envelope of Maximum Bending Moments — Uniformly Distributed Load

It can be seen from this envelope that the absolute maximum bending moment occurs at mid-span of the beam and has magnitude $WL^2/8$.

THANK YOU



Deflection of Statically Determinate Structures

Limit State Method

✓**Limit states** design principles provide the boundaries of structural usefulness.

✓The term **limit state** is used to describe a condition at which a structure or part of a structure ceases to perform its intended function.

Methods of Design, Limit State Method

There are two categories of limit states:

1. **Limit State of Strength**

Define:

- ✓ Load-carrying capacity
- ✓ Fracture
- ✓ Buckling
- ✓ Fatigue
- ✓ Gross rigid body motion.

All limit states must be prevented

Methods of Design, Limit State Method

There are two categories of limit states:

1. **Limit State of Serviceability**

Define performance, including:

- ✓ Deflection
- ✓ Cracking
- ✓ Slipping
- ✓ Vibration
- ✓ Deterioration

All limit states must be prevented

Deformations

- Structures deform and change shape when subjected to:
  - loads
  - temperature
  - fabrication errors
  - settlement

- If the deformations disappear and the structure regains its original shape when the actions causing the deformations are removed, the deformations are termed elastic deformations.

- The permanent deformations of structures are referred to as inelastic, or plastic, deformations.
- In this course, we will focus our attention on linear elastic deformations.

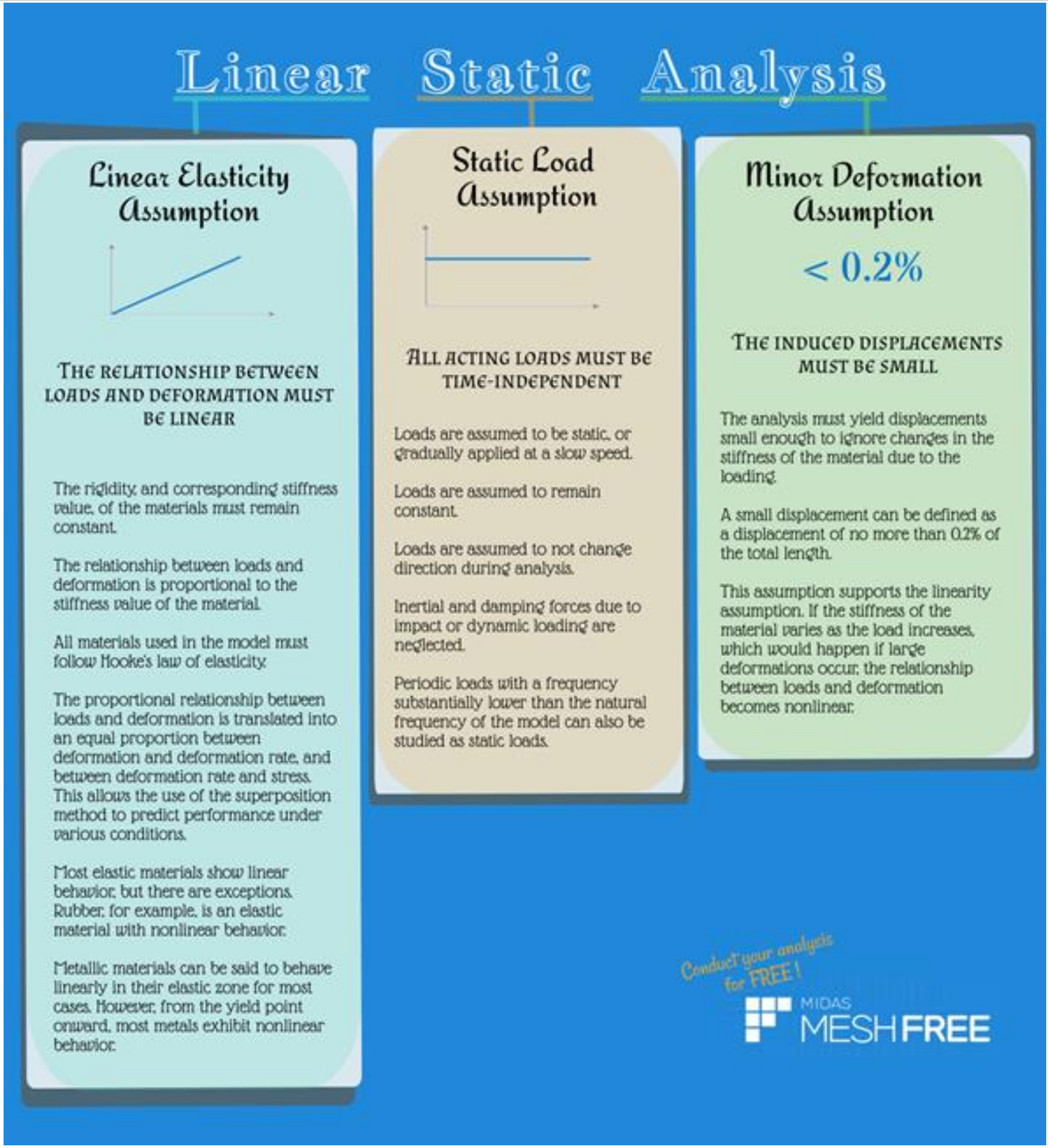
- Such deformations vary linearly with applied loads
- (for instance, if the magnitudes of the loads acting on the structure are doubled, its deformations are also doubled, and so forth).

In order for a structure to respond linearly to applied loads, it must be:

1. composed of linear elastic material

2. must undergo small deformations.

The principle of superposition is valid for such structures.



Effect of Deformations

- Excessive deformations are undesirable, as they may impair the structure's ability to serve its intended purpose.

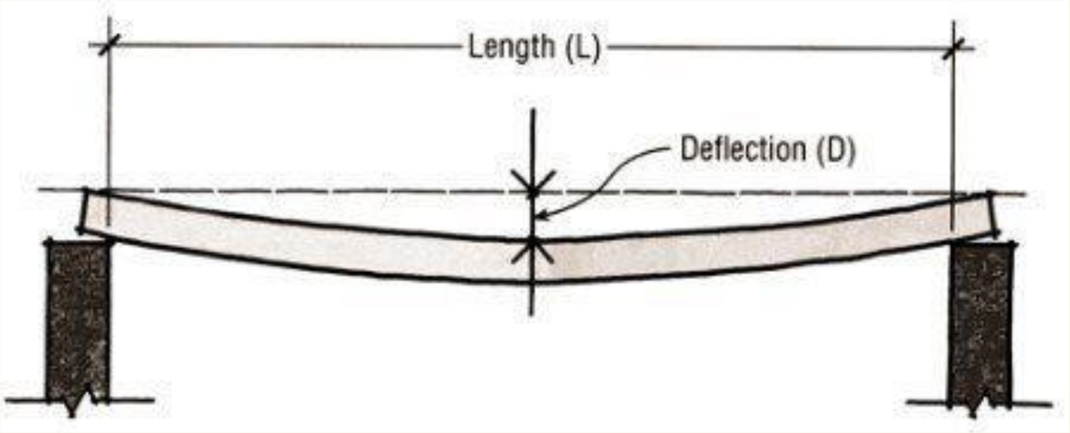
- For example, a high-rise building may be perfectly safe in the sense that the allowable stresses are not exceeded, yet useless (unoccupied) if it deflects excessively due to wind, causing cracks in the walls and windows.

- Structures are usually designed so that their deflections under normal service conditions will not exceed the allowable values specified in building codes.

Effect of Deformations

The computation of deflections forms an essential part of structural analysis.

Deflection calculations are also required in the determination of the reactions and stress resultants for statically indeterminate structures



Deflection diagrams & the elastic curve

- The computation of deflections forms an essential part of structural analysis.
- In designs, deflections must be limited in order to prevent cracking of attached brittle materials.
- A structure must not vibrate or deflect severely for the comfort of occupants
- Deflection calculations are also required in the determination of the reactions and stress resultants for statically indeterminate structures

Method to calculate deflections

- The methods that have been developed for computing deflections can be broadly classified into two categories:

- (1) Geometric methods

Based on a consideration of the geometry of the deflected shapes of structures

- (2) Work-energy methods.

Based on the basic principles of work and energy

Geometric methods

- Direct Integration Method
- Superposition Method
- Moment-Area Method
- Conjugate-Beam Method

Work-energy methods

- Virtual Work Methods
- Conservation of Energy and Strain Energy
- Castigliano’s Second Theorem
- Betti’s Law and Maxwell’s Law of Reciprocal Deflections

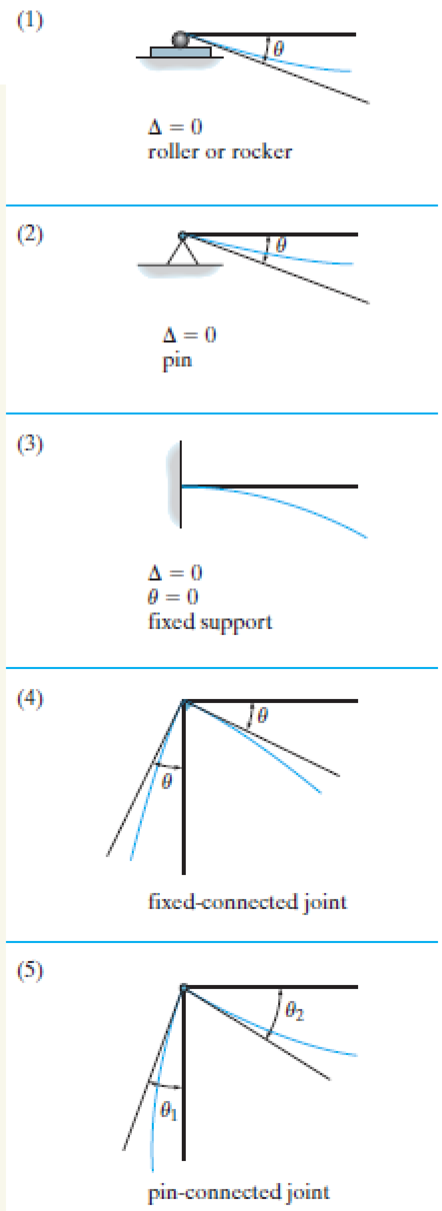
Deflection diagrams & the elastic curve

- Before the slope or displacement of a point on a beam or frame is determined, it is often helpful to sketch the deflected shape of the structure when it is loaded in order to partially check the results.

- This deflection diagram represents the elastic curve or locus of points which defines the displaced position of the centroid of the cross section along the members.

Deflection diagrams & the elastic curve

- For most problems the elastic curve can be sketched without much difficulty.
- When doing so, however, it is necessary to know the restrictions as to slope or displacement that often occur at a support or a connection
- If the elastic curve seems difficult to establish, it is suggested that the moment diagram be drawn first. From there, the curve can be constructed



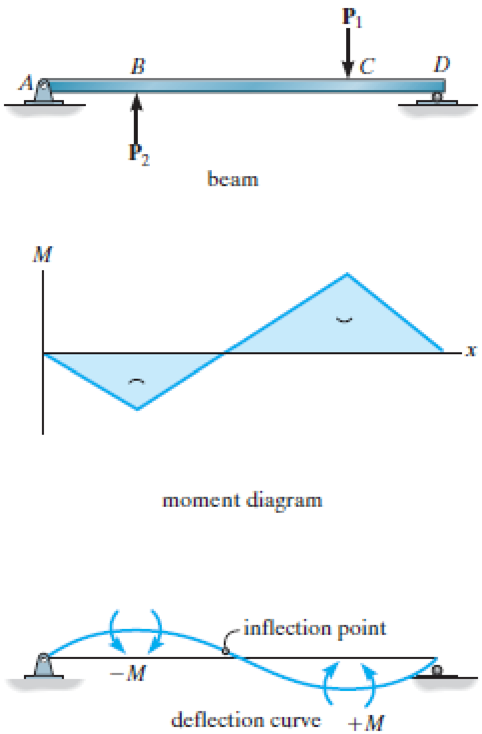
Deflection diagrams & the elastic curve

- Due to pin-and-roller support, the displacement at A & D must be zero
- Within the region of –ve moment, the elastic curve is concave downward
- Within the region of +ve moment, the elastic curve is concave upward
- There must be an inflection point where the curve changes from concave down to concave up

positive moment, concave upward

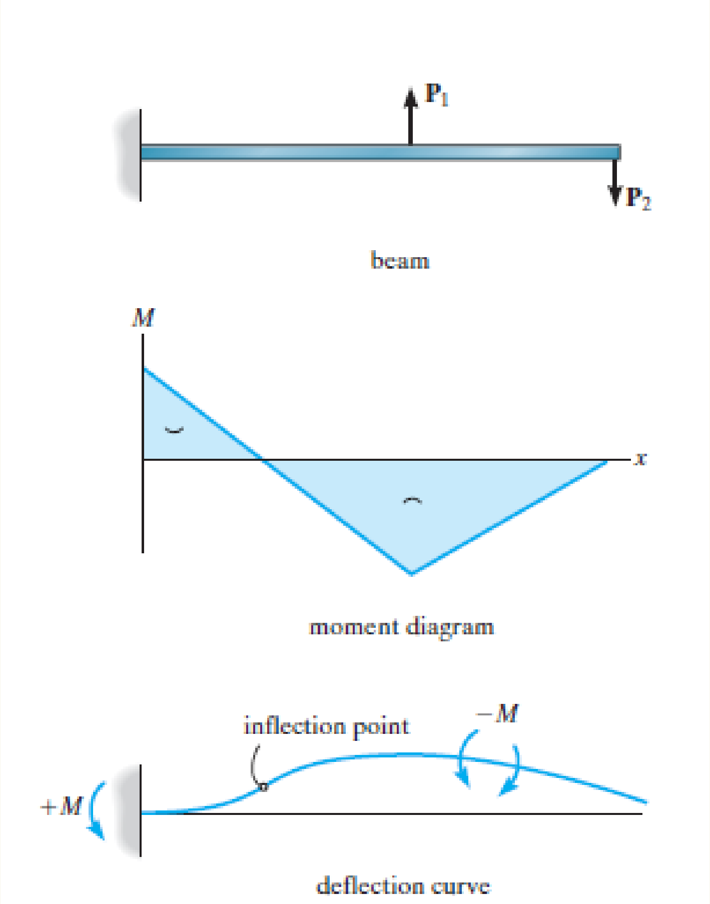
Fig. 8–1

negative moment, concave downward



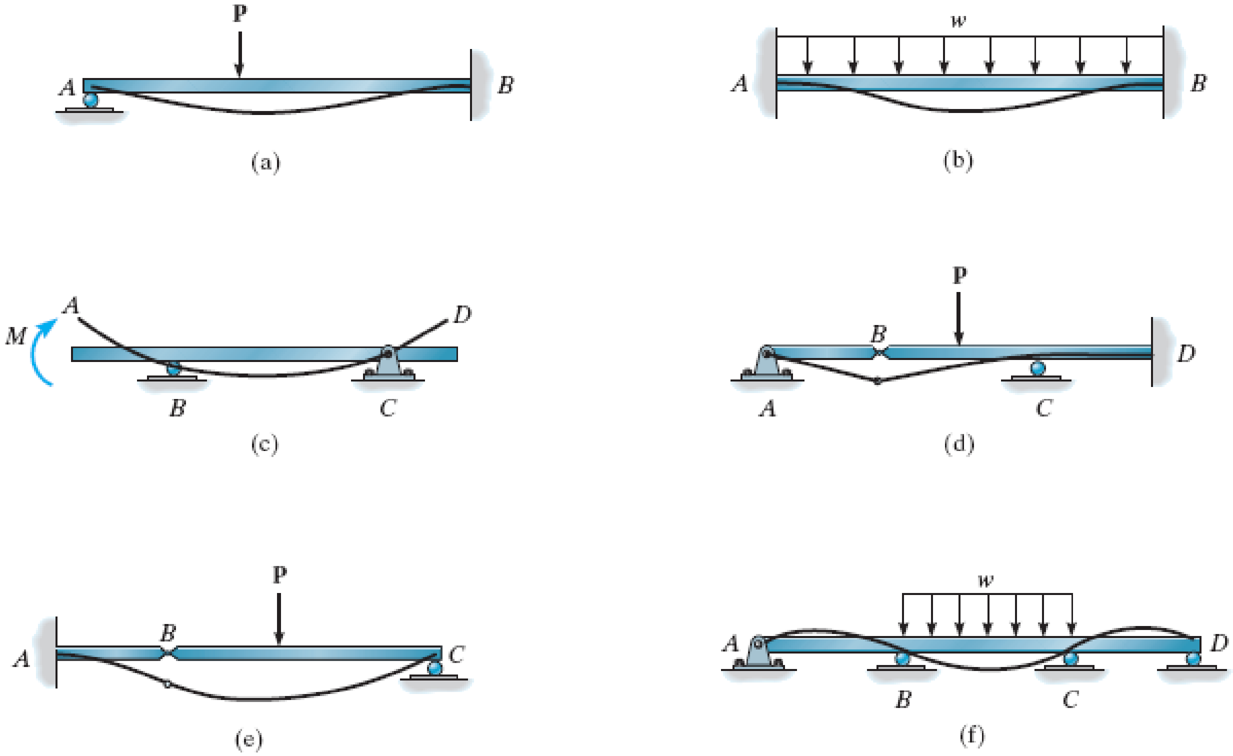
Deflection diagrams & the elastic curve

In particular, realize that the positive moment reaction from the wall keeps the initial slope of the beam horizontal



Example 8.1

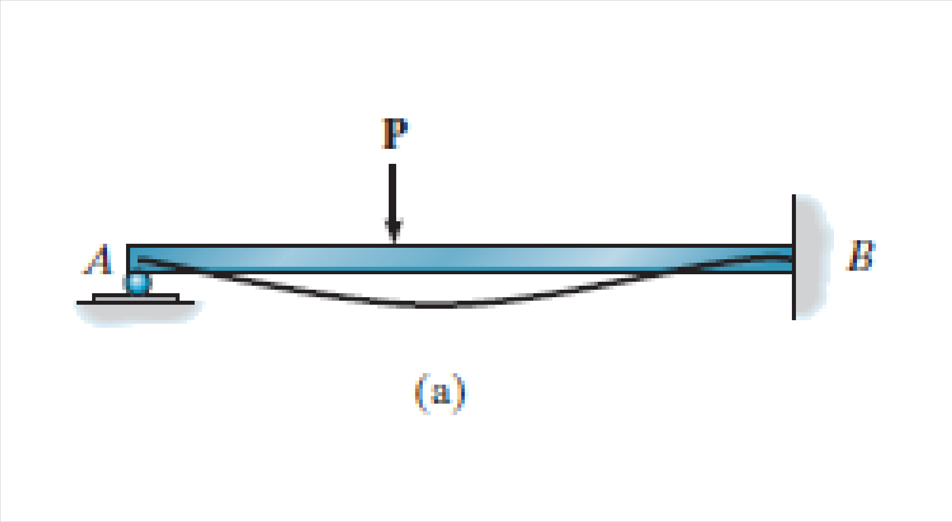
Draw the deflected shape of each of the beams.



Solution

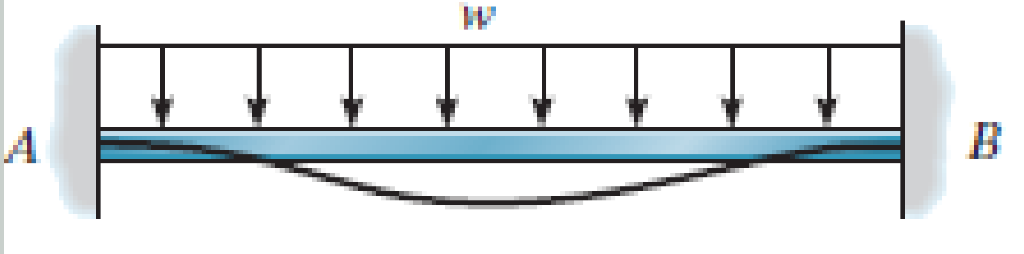
In (a), the roller at A allows free rotation with no deflection while the fixed wall at B prevents both rotation & deflection.

The deflected shape is shown by the bold line.



Solution

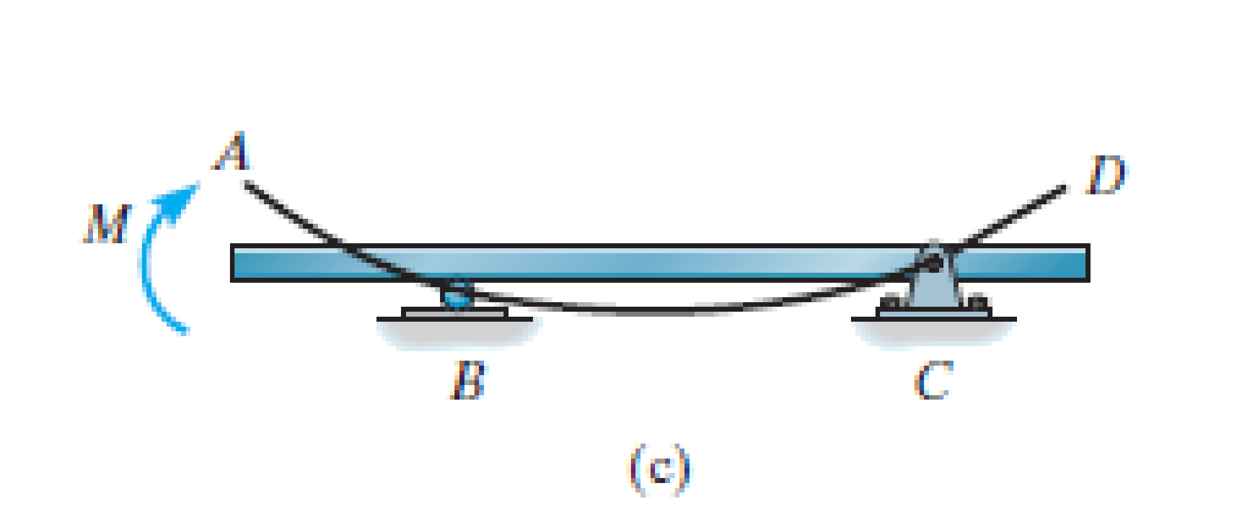
In (b), no rotation or deflection occur at A & B



(b)

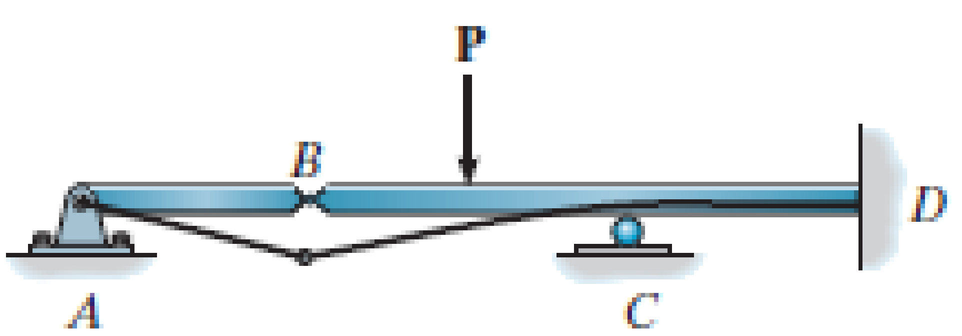
Solution

In (c), the couple moment will rotate end A. This will cause deflections at both ends of the beam since no deflection is possible at B & C. Notice that segment CD remains un-deformed since no internal load acts within.



Solution

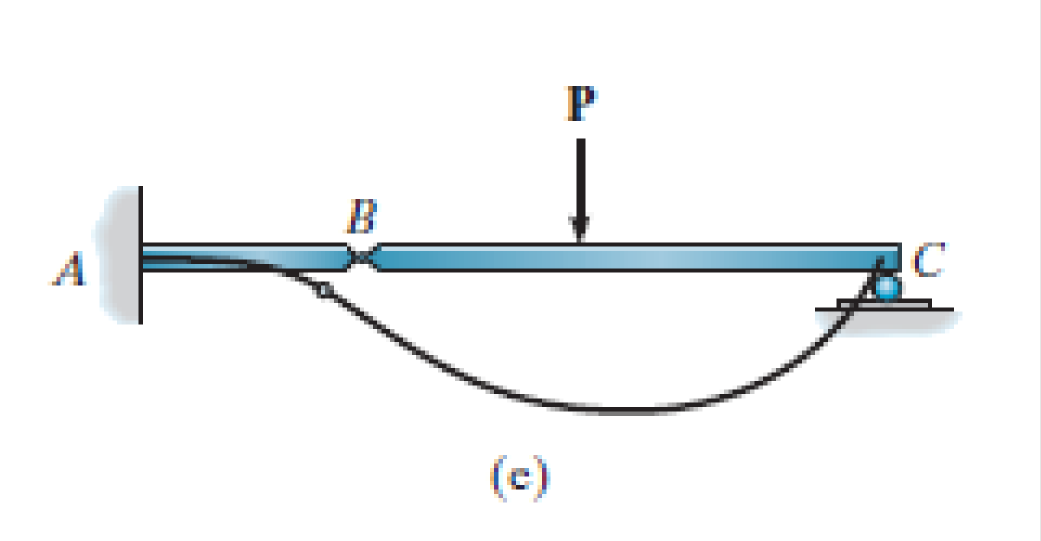
In (d), the pin at B allows rotation, so the slope of the deflection curve will suddenly change at this point while the beam is constrained by its support.



(d)

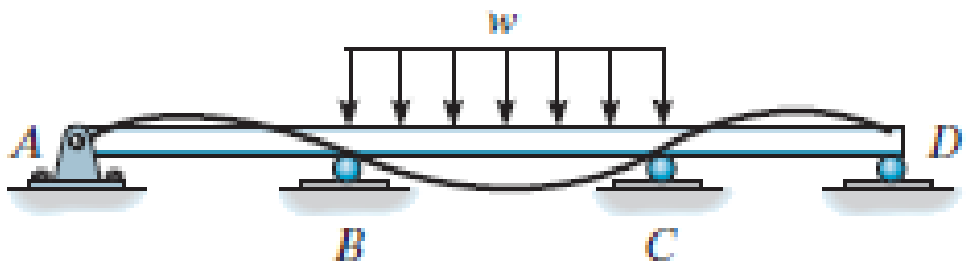
Solution

In (e), the compound beam deflects as shown. The slope changes abruptly on each side of B.



Solution

In (f), span BC will deflect concave upwards due to load. Since the beam is continuous, the end spans will deflect concave downwards.



(f)

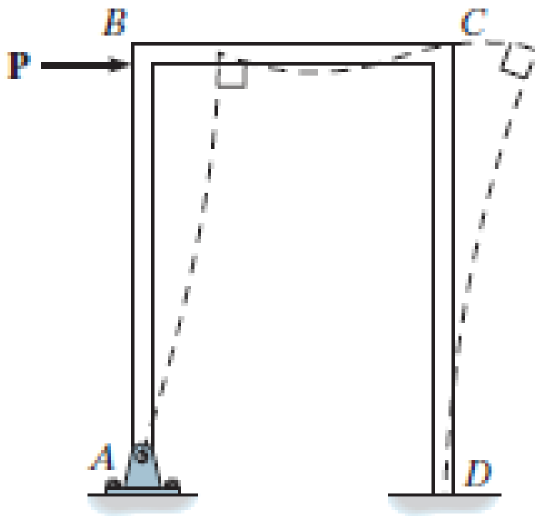
Example 8.2

Draw the deflected shape of each of the frames.

- when the load P pushes joints B and C to the right, it will cause clockwise rotation of each column as shown. As a result, joints B and C must rotate clockwise.

- Since the 90° angle between the connected members must be maintained at these joints, the beam BC will deform so that its curvature is reversed from concave up on the left to concave down on the right.

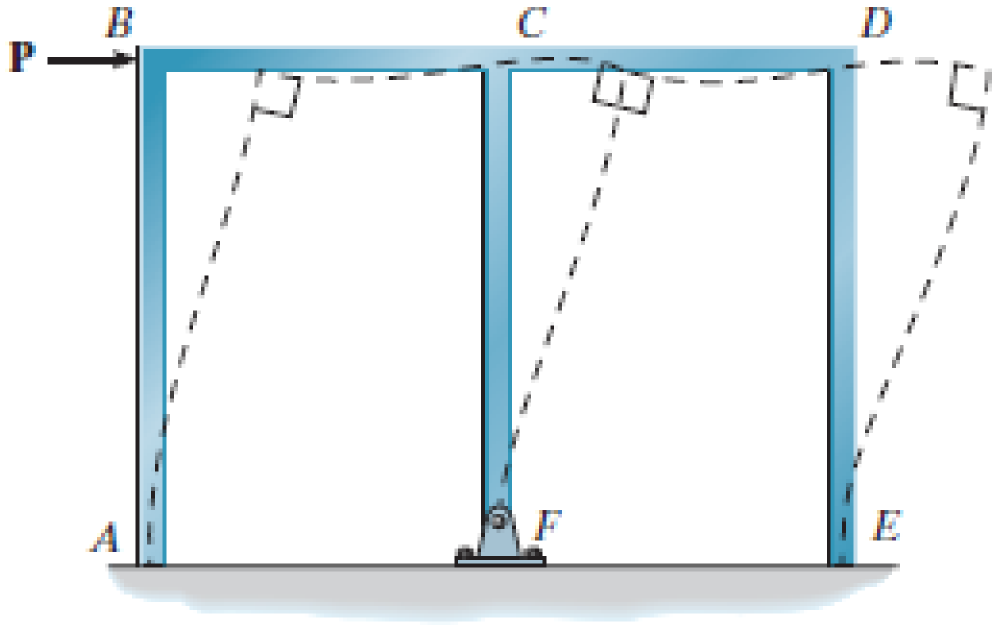
- Note that this produces a point of inflection within the beam.



(a)

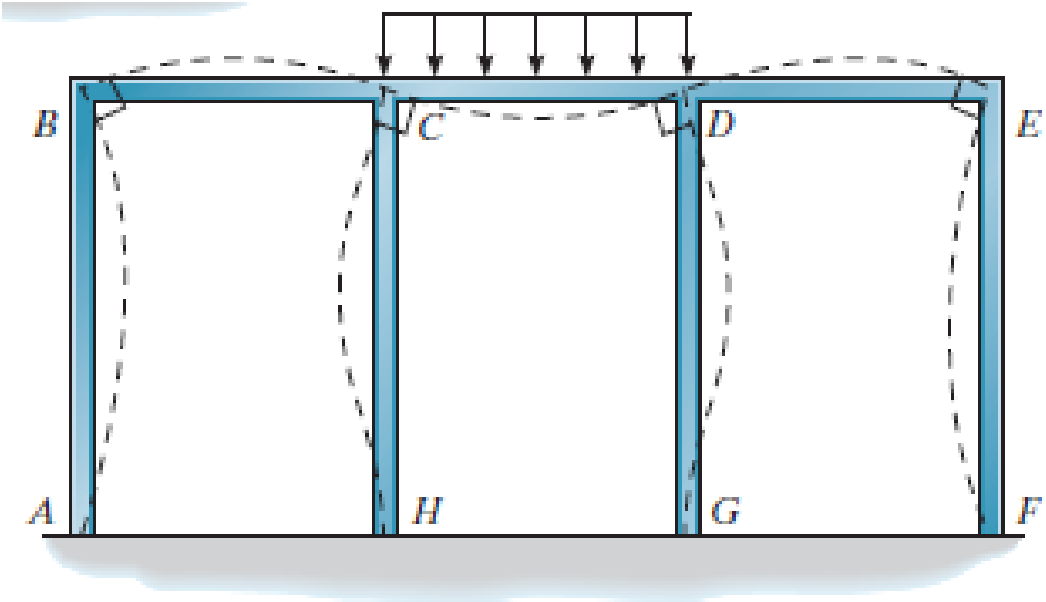
Draw the deflected shape of each of the frames.

- P displaces joints B, C, and D to the right, causing each column to bend as shown.
- The fixed joints must maintain their
- 90° angles, and so BC and CD must have a reversed curvature with an inflection point near their midpoint.



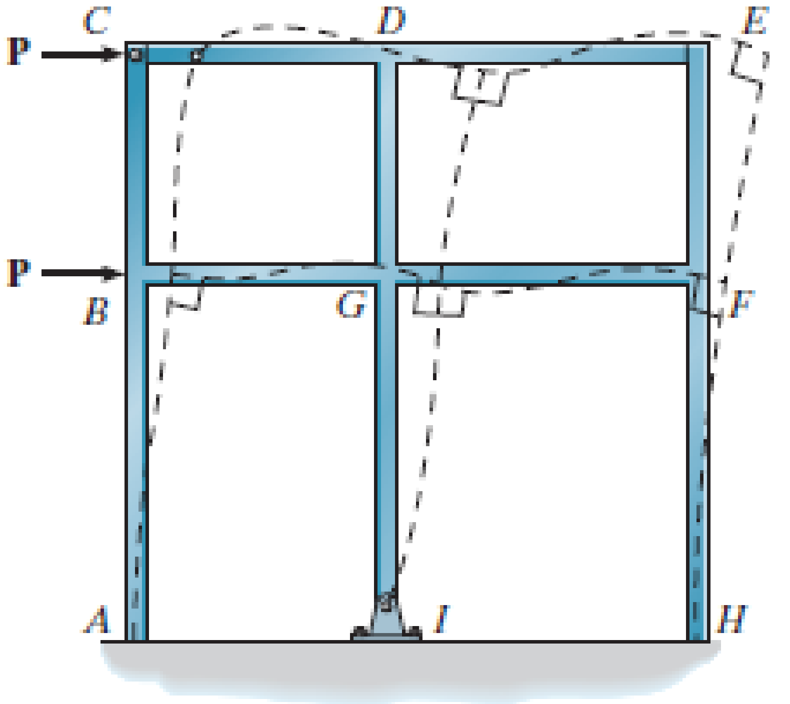
Draw the deflected shape of each of the frames.

- The vertical loading on this symmetric frame will bend beam CD concave upwards, causing clockwise rotation of joint C and counterclockwise rotation of joint D.
- Since the 90° angle at the joints must be maintained, the columns bend as shown.
- This causes spans BC and DE to be concave downwards, resulting in counterclockwise rotation at B and clockwise rotation at E.
- The columns therefore bend as shown.



Draw the deflected shape of each of the frames.

- The loads push joints B and C to the right, which bends the columns as shown.
- The fixed joint B maintains its 90° angle; however, no restriction on the relative rotation between the members at C is possible since the joint is a pin.
- Consequently, only beam CD does not have a reverse curvature

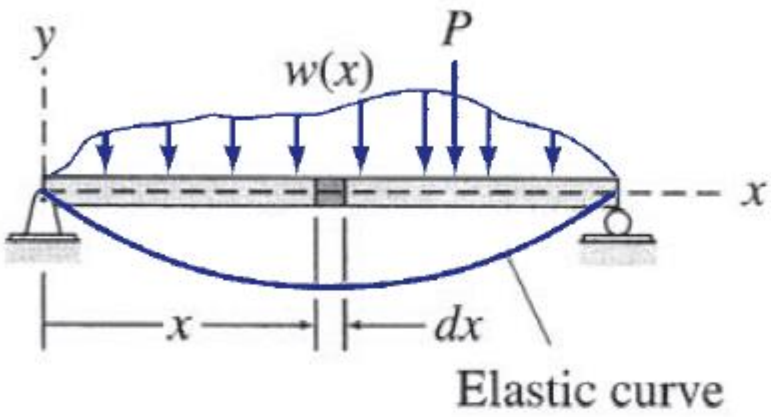


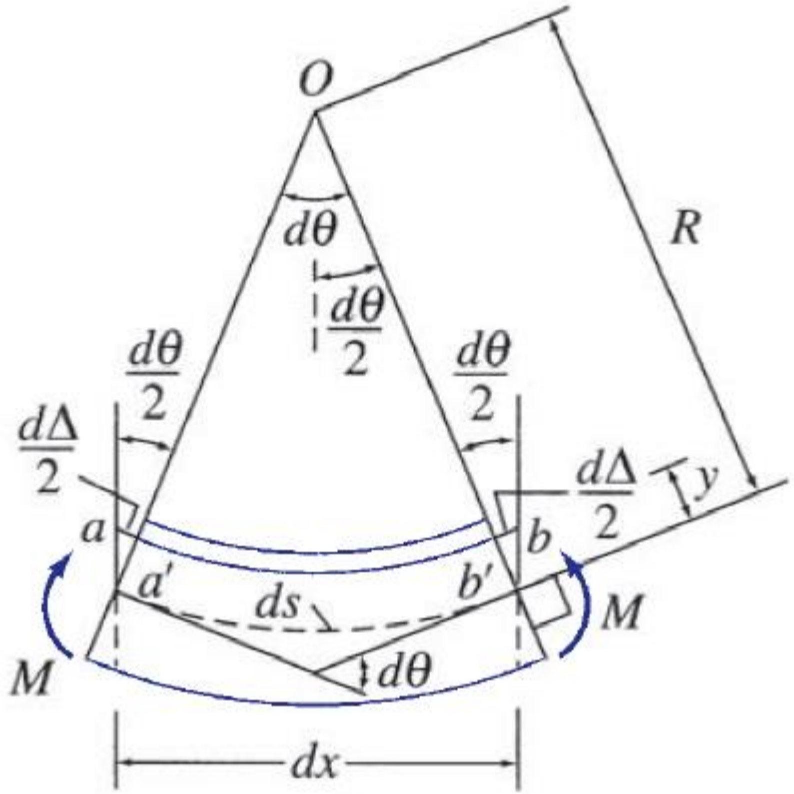
(d)

THANK YOU

Elastic Beam Theory

Elastic Beam Theory





R is the radius of curvature

Elastic Beam Theory

$$d\Delta = a'b' - ab = -2y\left(\frac{d\theta}{2}\right) = -y\,d\theta$$

$$\varepsilon = \frac{d\Delta}{dx} = \frac{d\Delta}{ds} = -\frac{y\,d\theta}{R\,d\theta} = -\frac{y}{R}$$

$$\varepsilon = \sigma/E \quad \Longrightarrow \quad \sigma = -\frac{Ey}{R}$$

$$\sigma = -\frac{Ey}{R} \quad \Bigg\| \quad \sigma = -\frac{My}{I} \quad \Bigg\| \quad \Longrightarrow \quad \frac{1}{R} = \frac{M}{EI}$$

Moment-Curvature Relationship

EI is commonly referred to as the **flexural rigidity**

Elastic Beam Theory

$$\frac{1}{R} = \frac{M}{EI}$$

Moment-Curvature Relationship

in Cartesian coordinates, we recall (from *calculus*) the relationship:

$$\frac{1}{R} = \frac{d^2y/dx^2}{[1+(dy/dx)^2]^{3/2}} \quad \Rightarrow \quad \frac{1}{R} = \frac{d^2y}{dx^2}$$

Then, the differential equation for the deflection of beams

$$\frac{d^2y}{dx^2} = \frac{M}{EI}$$

Bernoulli-Euler beam equation

Elastic Beam Theory

$$\frac{d^2y}{dx^2} = \frac{M}{EI}$$

Bernoulli-Euler beam equation

$$\theta = dy/dx$$

$$\frac{d\theta}{dx} = \frac{M}{EI}$$

THANK YOU