NEWTON'S LAWS

Newton's laws: Relations between **motions** of bodies and the **forces** acting on them.

Newton's first law: A body at rest remains at rest, and a body in motion remains in motion at the same velocity in a straight path when the net force acting on it is zero.

Therefore, a body tends to preserve its state of inertia.

Newton's second law: The a**cceleration** of a body is **proportional** to the net **force** acting on it and is **inversely** proportional to its mass.

Newton's third law: When a body exerts a force on a second body, the second body exerts an equal and opposite force on the first.

Therefore, the direction of an exposed reaction force depends on the body taken as the system.

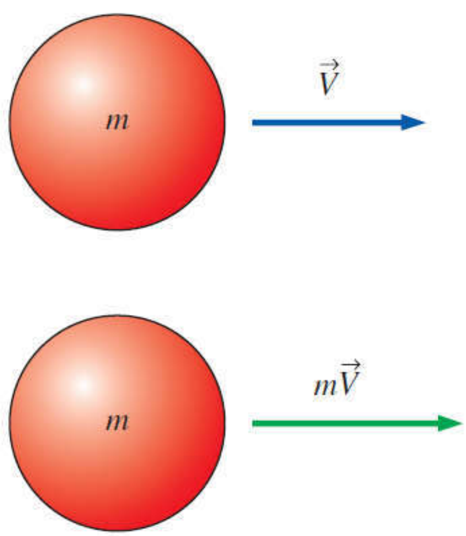
Newton's second law: $\vec{F} = m\vec{a}$

Newton's second law: $\vec{F} = m\vec{a}$:

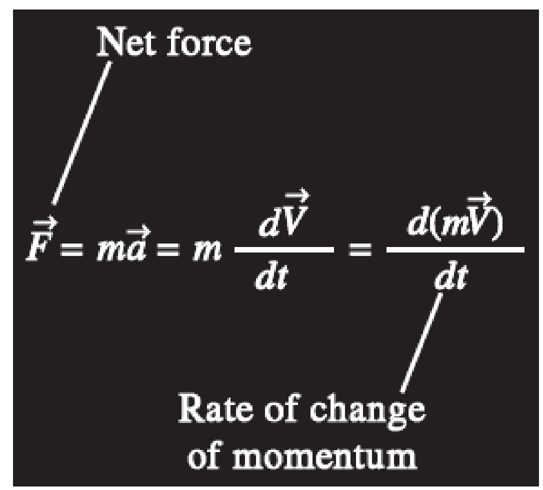
The product of the mass and the velocity of a body is known as (**Linear momentum** (momentum of the body))

In Fluid Mechanics Newton's second law is usually referred to as the *linear momentum equation.*

Conservation of momentum principle: The momentum of a system remains constant only when the net force acting on it is zero.



Linear momentum is the product of mass and velocity, and its direction is the direction of velocity.



* Newton's second law is also expressed as *the rate of change of the momentum of a body is equal to the net force acting on it*.

The counterpart of **Newton's second law for rotating rigid bodies** is expressed as $\vec{M} = I\vec{\alpha}$,
Where

$\vec{M}$ = the net moment or torque applied on the body,
I = the moment of inertia of the body about the axis of rotation
$\vec{\alpha}$ = the angular acceleration, (The rate of change of angular momentum $d\vec{H}/dt$

Angular momentum equation: $$\vec{M} = I\vec{\alpha} = I\frac{d\vec{\omega}}{dt} = \frac{d(I\vec{\omega})}{dt} = \frac{d\vec{H}}{dt}$$

Angular momentum about x-axis: $$M_x = I_x\frac{d\omega_x}{dt} = \frac{dH_x}{dt}$$

The rate of change of the angular momentum of a body is equal to the net torque acting on it.

$$\underbrace{\vec{M}}_{\text{Net torque}} = I\vec{\alpha} = I\frac{d\vec{\omega}}{dt} = \frac{d(I\vec{\omega})}{dt} = \underbrace{\frac{d\vec{H}}{dt}}_{\text{Rate of change of angular momentum}}$$

The conservation of angular momentum Principle: The total angular momentum of a rotating body remains constant when the net torque acting on it is zero, and thus the angular momentum of such systems is conserved.

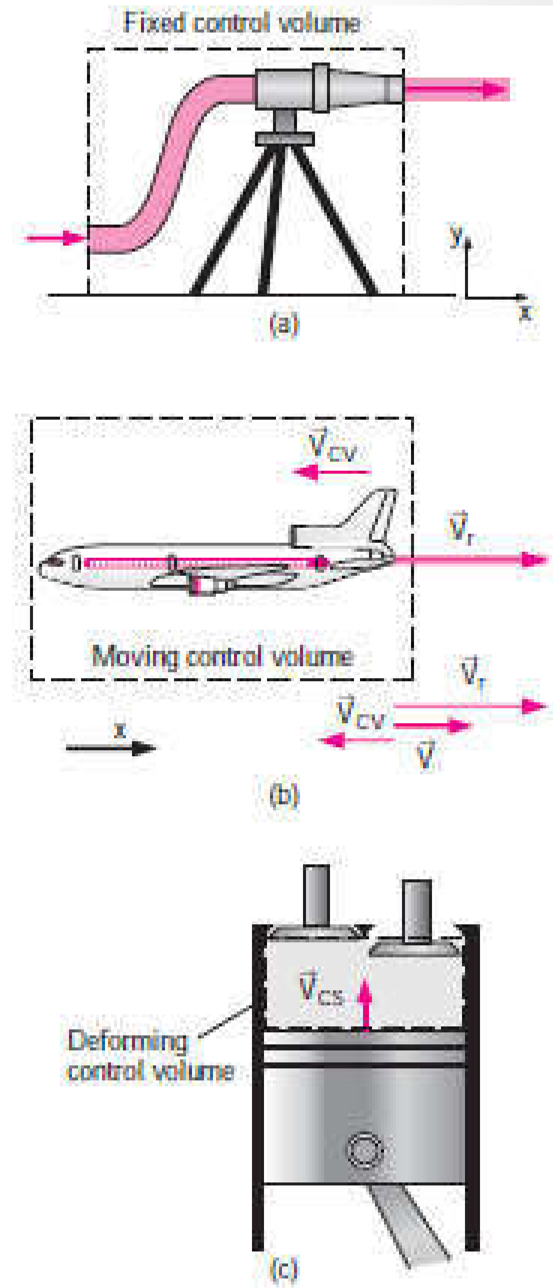
CHOOSING A CONTROL VOLUME

A control volume can be **selected as** any arbitrary region in space through which fluid flows, and its **bounding** control surface can be fixed, moving, and even deforming during flow.

Many **flow systems** involve stationary hardware firmly **fixed to a stationary surface,** and such systems are best analyzed using ***fixed*** control volumes.

When analyzing **flow systems that are moving or deforming**, it is usually more convenient to allow the control volume to ***move*** or ***deform***.

In ***deforming*** control volume, part of the control surface moves relative to other parts.



Examples of (*a*) fixed, (*b*) moving, (*c*) deforming control volumes.

Determining the **thrust** developed by the jet engine of an airplane cruising at constant velocity (CV(airplane + nozzle exit))

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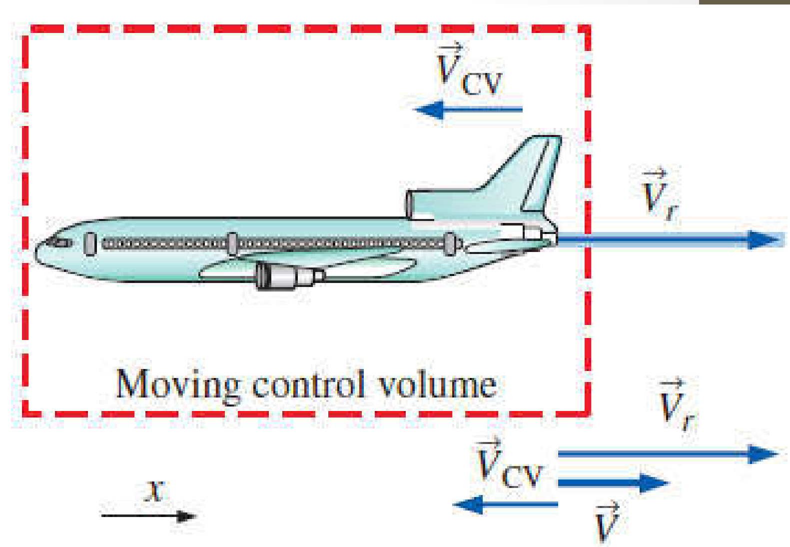


The **control volume** moves with velocity $\overrightarrow{V_{cr}}$, (identical to the cruising velocity of the airplane)

When determining **the flow rate of exhaust gases leaving the nozzle**,

the proper velocity to use is **the velocity of the exhaust gases** relative to the **nozzle exit plane**, that is, the ***relative velocity*** $\overrightarrow{V_r}$.

$$\vec{V}_r = \vec{V} - \vec{V}_{CV}$$

$\vec{V}_{airplane}$ = 500 km/h (to the left)

$$\vec{V}_{gases\,(nozzle)} = 800\,\frac{km}{h}\ (to\ the\ right)$$

The velocity of the exhaust gases relative to the nozzle exit is

$$\vec{V}_r = \vec{V} - \vec{V}_{CV} = 800\vec{i} - (-500\vec{i}) = 1300\vec{i}\ \text{km/h}$$

FORCES ACTING ON A CONTROL VOLUME

The forces acting on a control volume consist of

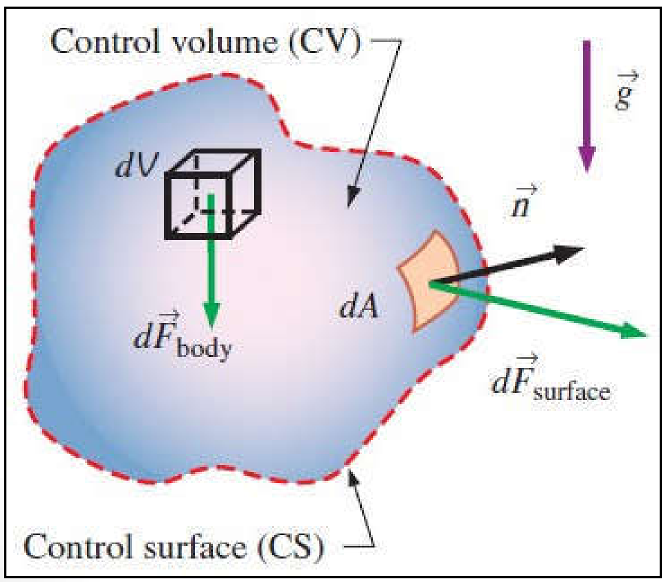
Body forces that **act throughout the entire body** of the control volume (such as **gravity**, electric, and magnetic forces)

Surface forces that **act on the control surface** (such as **pressure** and **viscous** forces and **reaction forces** at points of contact).

Only external forces are considered in the analysis.

$$\sum \vec{F} = \sum \vec{F}_{\text{body}} + \sum \vec{F}_{\text{surface}}$$

→ *Total force acting on control volume*

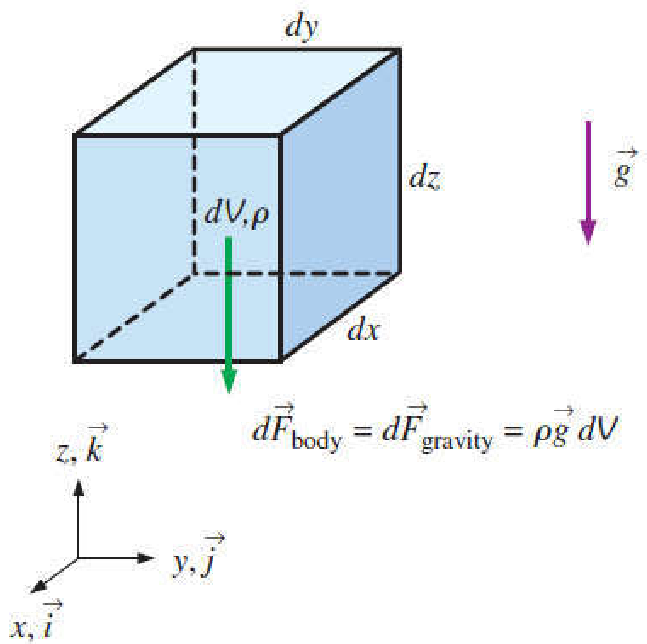


The total force acting on a **control volume** is composed of **body forces** and **surface forces**

- Body force is shown on a differential volume element
- Surface force is shown on a differential surface element.

The most common body force is that of **gravity**, which exerts a downward force on every differential element of the control volume.

$$d\vec{F}_{body} = d\vec{F}_{gravity}$$ → The differential body force



The gravitational force acting on a differential volume element of fluid is equal to its weight

The axes have been rotated so that the gravity vector acts *downward* in the negative *z*-direction.

Gravitational force acting on a fluid element: $d\vec{F}_{gravity} = \rho\vec{g}\, dV$

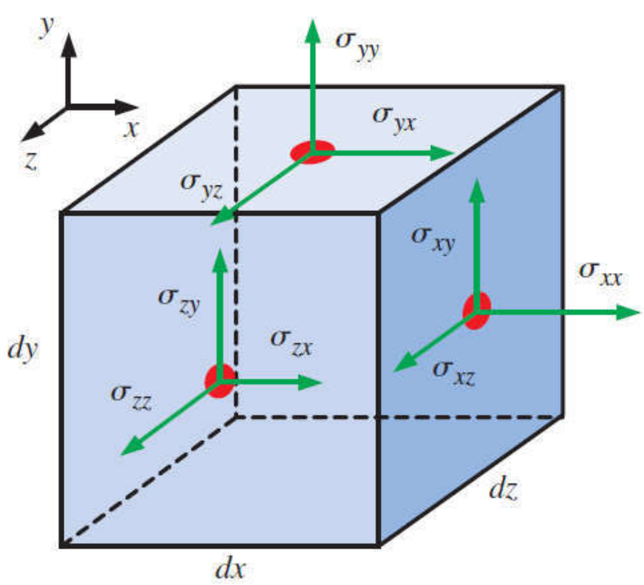
Gravitational vector in Cartesian coordinates: $\vec{g} = -g\vec{k}$

Total body force acting on control volume: $\sum \vec{F}_{body} = \int_{CV} \rho\vec{g}\, dV = m_{CV}\vec{g}$

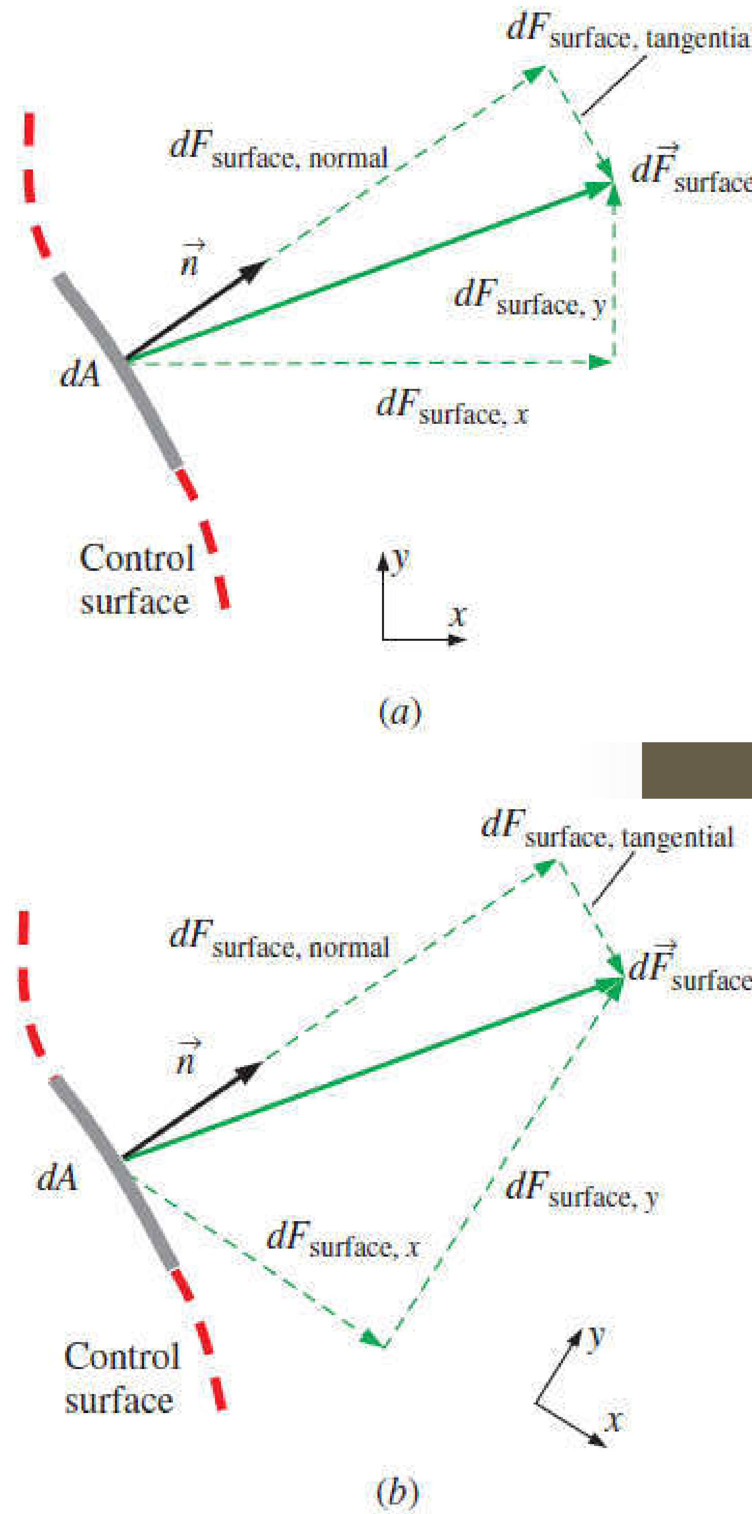
Surface Forces are not as simple to analyze:

1- The *description* of the force in terms of its coordinate components changes with orientation.

2- stress tensor σ_{ij} used in order to describe the surface stresses at a point in the flow



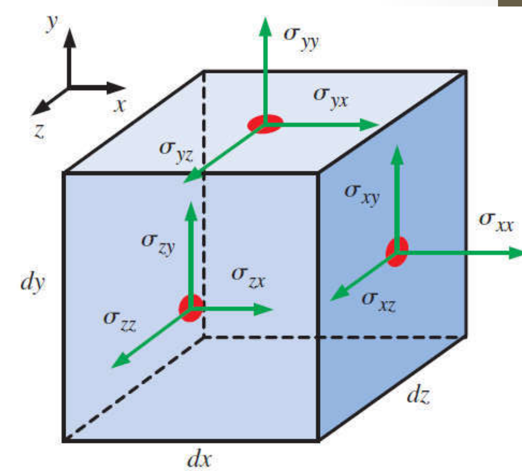
Stress tensor in Cartesian coordinates: $$\sigma_{ij} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{pmatrix}$$



Surface forces are not as simple to analyze, they **consist** of both ***normal*** and ***tangential*** components.

Normal stresses are composed of pressure and viscous stresses. $(\sigma_{xx}, \sigma_{yy}, \sigma_{zz})$
Shear stresses are composed entirely of viscous stresses. $(\sigma_{xy}, \sigma_{zx}, \ldots etc)$

The inner product of the **stress tensor** $\boldsymbol{\sigma_{ij}}$ and the unit outward **normal vector** $\vec{n}$ = a vector whose magnitude is the **force per unit area**, and whose direction is the **direction of the surface force** itself



$$d\vec{F}_{\text{surface}} = \sigma_{ij} \cdot \vec{n}\, dA$$

Surface force acting on a differential surface element:

$$\sum \vec{F}_{\text{surface}} = \int_{CS} \sigma_{ij} \cdot \vec{n}\, dA$$

Total surface force acting on control surface:

$$\sum \vec{F} = \sum \vec{F}_{\text{body}} + \sum \vec{F}_{\text{surface}} = \int_{CV} \rho \vec{g}\, dV + \int_{CS} \sigma_{ij} \cdot \vec{n}\, dA$$

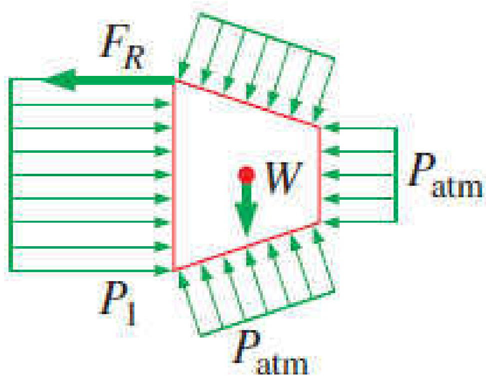
Total force:

$$\underbrace{\sum \vec{F}}_{\text{total force}} = \underbrace{\sum \vec{F}_{\text{gravity}}}_{\text{body force}} + \underbrace{\sum \vec{F}_{\text{pressure}} + \sum \vec{F}_{\text{viscous}} + \sum \vec{F}_{\text{other}}}_{\text{surface forces}}$$

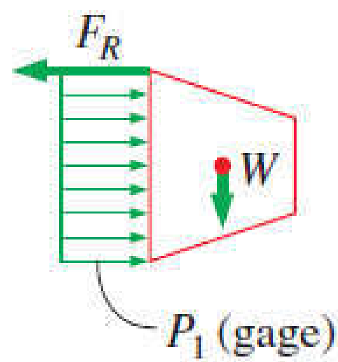
A common simplification in the application of Newton's laws of motion is to subtract the *atmospheric pressure* and **work with gage pressures**.

This is because atmospheric pressure acts in all directions, and its effect cancels out in every direction.

This means we can also **ignore the pressure forces at outlet sections where the fluid is discharged to the atmosphere** since the discharge pressure in such cases is very near atmospheric pressure at subsonic velocities.



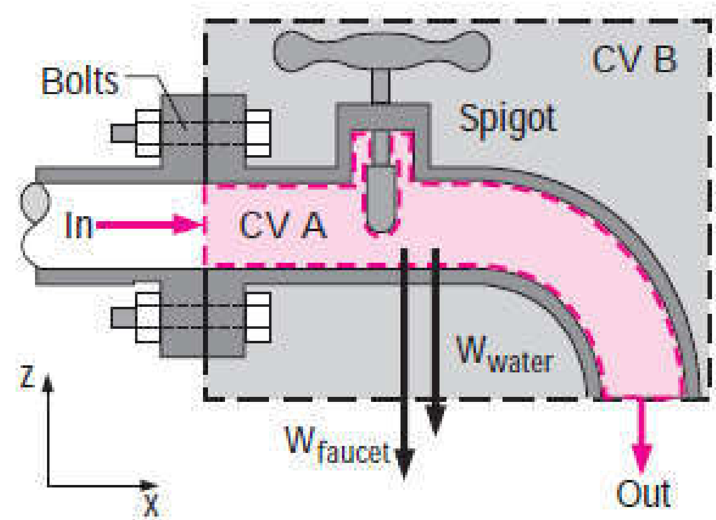
With atmospheric pressure considered



With atmospheric pressure cancelled out

Atmospheric pressure acts in all directions, and thus it can be ignored when performing force balances since its effect cancels out in every direction.

Example of how to choose a control volume



It is desired to calculate the net force on the flange to ensure that the flange bolts are strong enough.

CV A, With this control volume there are:

- **Pressure forces** along control surface
- **Viscous forces** along the pipe wall and at locations inside the valve
- **Body force** (weight of the water in the control volume).

Net force on the flange (reaction force (the net force of the walls on the water) plus the weight of the faucet and the water)

CV B, With this control volume there are:

CV B, (not concerned with the flow or geometry inside the control volume).

- **Net reaction force (**acting at the portions of the control surface that slice through the flange.)
- **The gage pressure of the water** at the flange (the inlet to the control volume)
- **The weights of the water and the faucet assembly**.
- **The pressure** along the control surface is atmospheric (zero gage pressure) and cancels out.

CV B is much easier to work with than CV A.

THE LINEAR MOMENTUM EQUATION

Newton's second law for a system of mass m subjected to net force $\Sigma\vec{F}$ is expressed as

$$\sum \vec{F} = m\vec{a} = m\frac{d\vec{V}}{dt} = \frac{d}{dt}(m\vec{V}) \qquad \text{(6–13)}$$

where $m\vec{V}$ is the **linear momentum** of the system. Noting that both the density and velocity may change from point to point within the system, Newton's second law can be expressed more generally as

$$\sum \vec{F} = \frac{d}{dt}\int_{sys} \rho\vec{V}\, dV \qquad \text{(6–14)}$$

where $\rho\vec{V}\, dV$ is the momentum of a differential element dV, which has mass $\delta m = \rho dV$.

Newton's second law can be stated as

***The sum of all external forces acting on a system is equal to the time rate of change of linear momentum of the system*.**

This statement is valid for a coordinate system that is at rest or moves with a constant velocity, called an ***inertial coordinate system*** or *inertial reference frame*.

$$\sum \vec{F} = m\vec{a} = m\frac{d\vec{V}}{dt} = \frac{d}{dt}(m\vec{V})$$

$$\sum \vec{F} = \frac{d}{dt}\int_{sys} \rho \vec{V}\, dV$$

$$\frac{d(m\vec{V})_{sys}}{dt} = \frac{d}{dt}\int_{CV} \rho \vec{V}\, dV + \int_{CS} \rho \vec{V}\,(\vec{V}_r \cdot \vec{n})\, dA$$

General: $$\sum \vec{F} = \frac{d}{dt}\int_{CV} \rho \vec{V}\, dV + \int_{CS} \rho \vec{V}(\vec{V}_r \cdot \vec{n})\, dA$$

$$\vec{V}_r = \vec{V} - \vec{V}_{CS}$$

(The sum of all external forces acting on a CV) = (The time rate of change of the linear momentum of the contents of the CV) + (The net flow rate of linear momentum out of the control surface by mass flow)

Fixed CV: $$\sum \vec{F} = \frac{d}{dt}\int_{CV} \rho \vec{V}\, dV + \int_{CS} \rho \vec{V}(\vec{V} \cdot \vec{n})\, dA$$

$$\frac{dB_{sys}}{dt} = \frac{d}{dt}\int_{CV} \rho b\, dV + \int_{CS} \rho b(\vec{V}_r \cdot \vec{n})\, dA$$

$B = m\vec{V}$, $b = \vec{V}$, $b = \vec{V}$

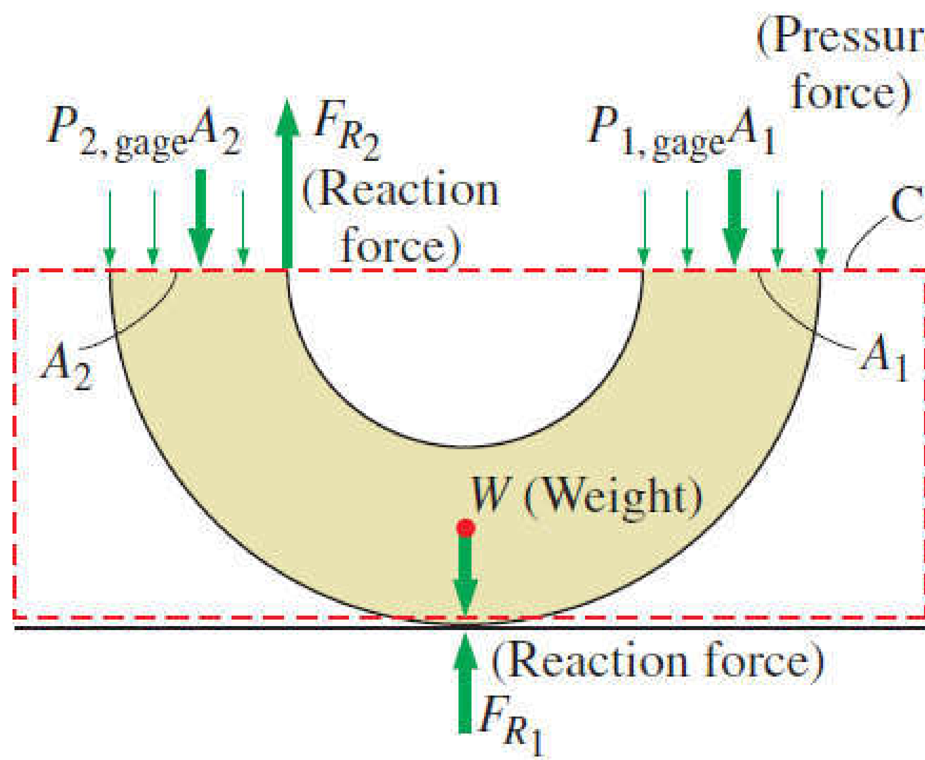
$$\frac{d(m\vec{V})_{sys}}{dt} = \frac{d}{dt}\int_{CV} \rho \vec{V}\, dV + \int_{CS} \rho \vec{V}(\vec{V}_r \cdot \vec{n})\, dA$$

The linear momentum equation is obtained by replacing B in the Reynolds transport theorem by the momentum $m\vec{V}$, and b by the momentum per unit mass $\vec{V}$.

The momentum equation is commonly used to calculate the forces (usually on support systems or connectors) induced by the flow.

$$\sum \vec{F} = \frac{d}{dt}\int_{CV} \rho \vec{V}\, dV + \int_{CS} \rho \vec{V}(\vec{V}\cdot\vec{n})\, dA$$

The product $\boldsymbol{\rho\ (\vec{V}.\vec{n})\ dA}$ represents the **mass flow rate** through area element dA into or out of the control volume.



An 180° elbow supported by the ground

In most flow systems, the sum of forces $\Sigma\vec{F}$ consists of weights, pressure forces, and reaction forces. Gage pressures are used here since atmospheric pressure cancels out on all sides of the control surface.

Special Cases
1- Steady Flow

$$\sum \vec{F} = \frac{d}{dt}\int_{CV} \rho \vec{V}\, dV + \int_{CS} \rho \vec{V}(\vec{V}\cdot\vec{n})\, dA$$

Steady flow

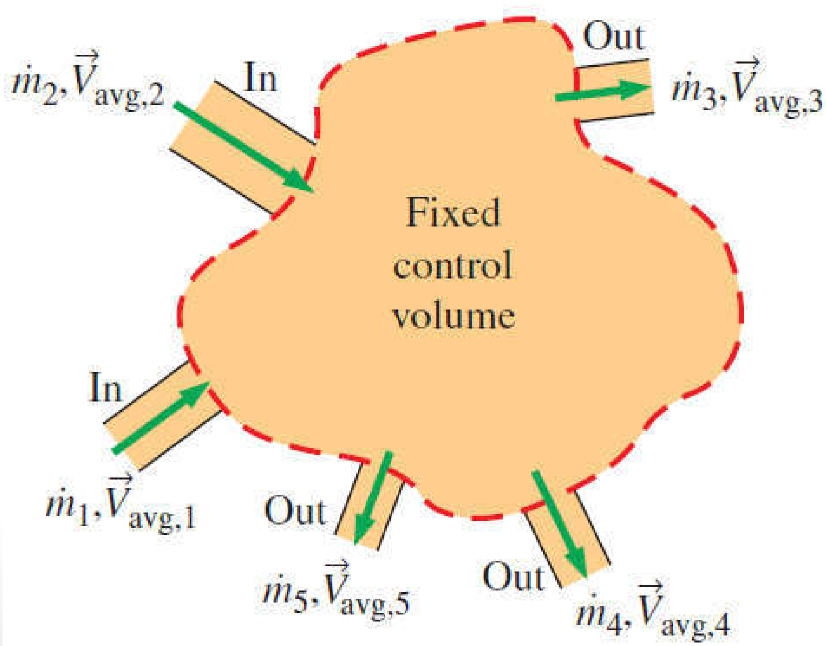
$$\sum \vec{F} = \int_{CS} \rho \vec{V}\,(\vec{V}_r \cdot \vec{n})\, dA$$

Mass flow rate across an inlet or outlet

$$\dot{m} = \int_{A_c} \rho(\vec{V}\cdot\vec{n})\, dA_c = \rho V_{avg} A_c$$

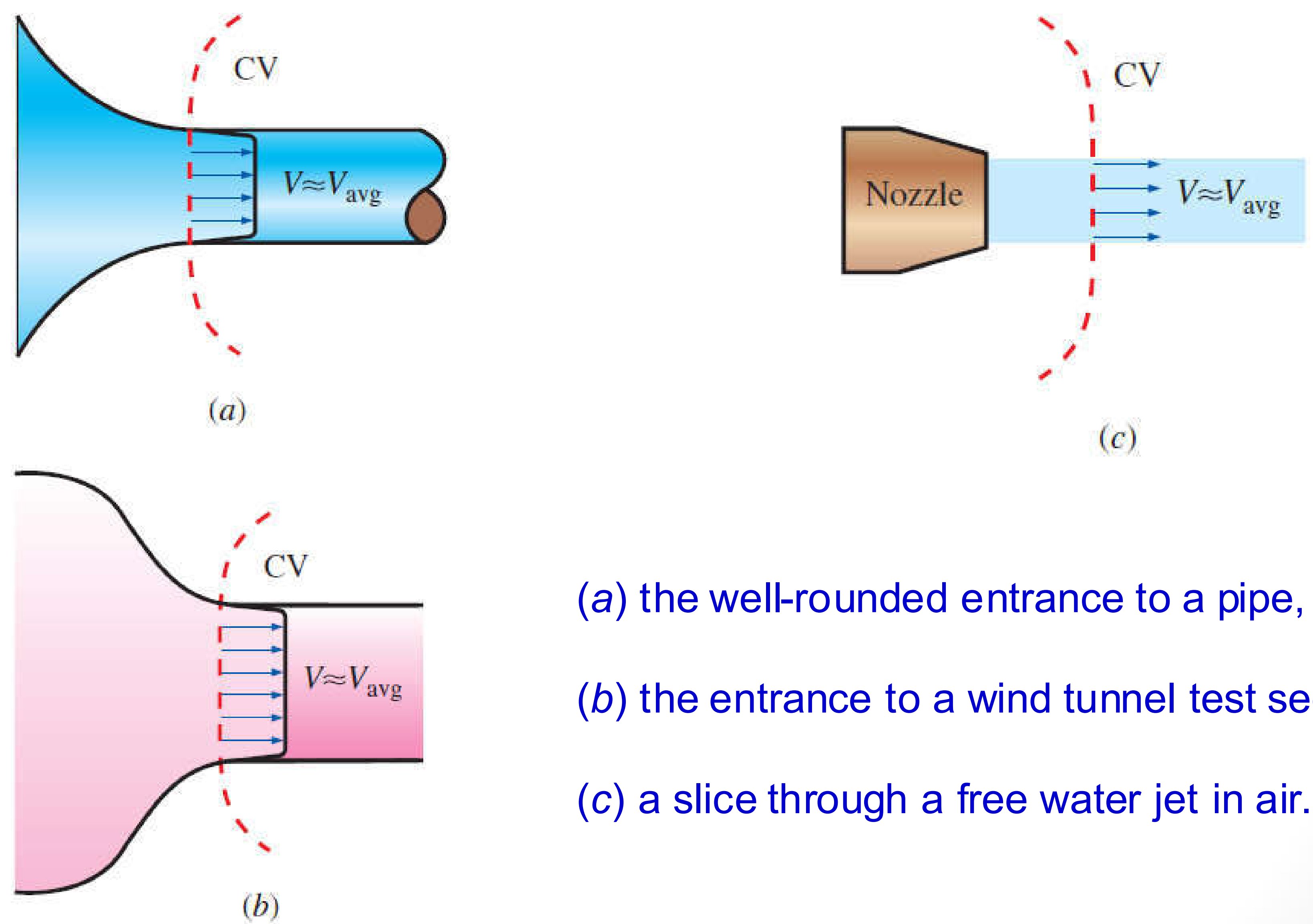
Momentum flow rate across a uniform inlet or outlet:

$$\int_{A_c} \rho \vec{V}(\vec{V}\cdot\vec{n})\, dA_c = \rho V_{avg} A_c \vec{V}_{avg} = \dot{m}\vec{V}_{avg}$$



In a typical engineering problem, the control volume may contain many inlets and outlets; at each inlet or outlet we define the mass flow rate and the average velocity.

Examples of **inlets** or **outlets** in which the **uniform flow approximation** is reasonable



(*a*) the well-rounded entrance to a pipe,

(*b*) the entrance to a wind tunnel test section

(*c*) a slice through a free water jet in air.

Momentum-Flux Correction Factor, β

The velocity across most inlets and outlets is *not* uniform.

The control surface integral of Eq. 6–17 may be converted into algebraic form using a dimensionless correction factor β, called the **momentum-flux correction factor**.

$$\sum \vec{F} = \frac{d}{dt}\int_{CV} \rho\vec{V}\, dV + \int_{CS} \rho\vec{V}(\vec{V}\cdot\vec{n})\, dA \qquad (6\text{-}17)$$

$$\sum \vec{F} = \frac{d}{dt}\int_{CV} \rho\vec{V}\, dV + \sum_{out} \beta\dot{m}\vec{V}_{avg} - \sum_{in} \beta\dot{m}\vec{V}_{avg}$$

Momentum flux across an inlet or outlet: $\int_{A_c} \rho\vec{V}(\vec{V}\cdot\vec{n})\, dA_c = \beta\dot{m}\vec{V}_{avg}$

$$\beta = \frac{\int_{A_c} \rho V(\vec{V}\cdot\vec{n})\, dA_c}{\dot{m}V_{avg}} = \frac{\int_{A_c} \rho V(\vec{V}\cdot\vec{n})\, dA_c}{\rho V_{avg} A_c V_{avg}}$$

β is always greater than or equal to 1.

β is close to 1 for turbulent flow and not very close to 1 for fully developed laminar flow.

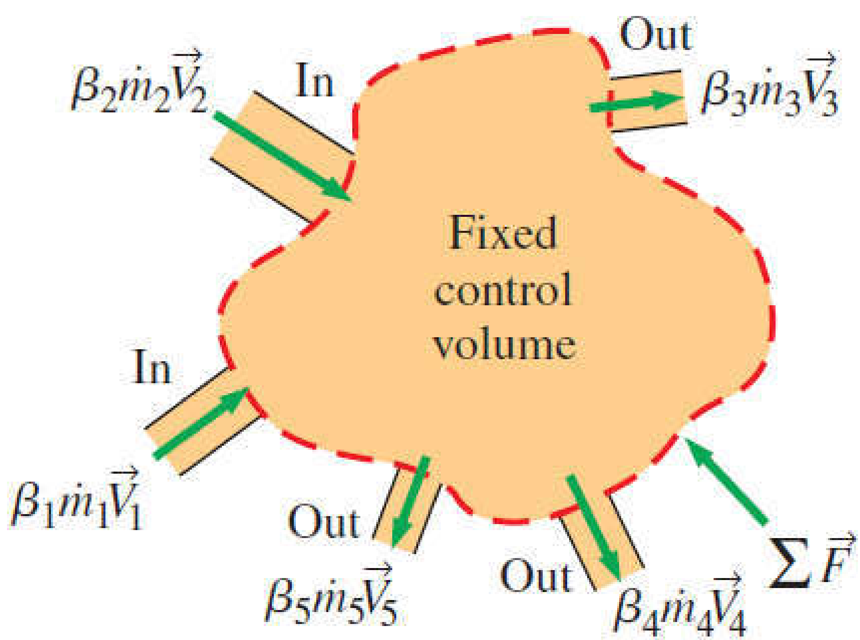
Momentum-flux correction factor: $\beta = \frac{1}{A_c}\int_{A_c} \left(\frac{V}{V_{avg}}\right)^2 dA_c$

Steady Flow

Steady linear momentum equation:

$$\sum \vec{F} = \sum_{\text{out}} \beta \dot{m} \vec{V} - \sum_{\text{in}} \beta \dot{m} \vec{V}$$

The net force acting on the control volume during steady flow is equal to the difference between the rates of outgoing and incoming momentum flows.



$$\sum \vec{F} = \sum_{\text{out}} \beta \dot{m} \vec{V} - \sum_{\text{in}} \beta \dot{m} \vec{V}$$

The net force acting on the control volume during steady flow is equal to the difference between the outgoing and the incoming momentum fluxes.

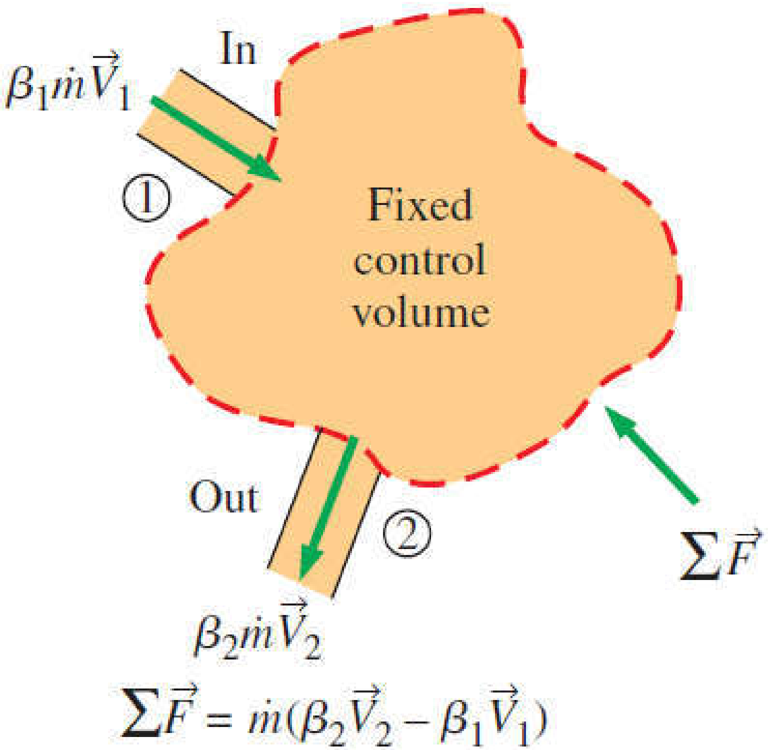
Steady Flow with One Inlet and One Outlet

$$\sum \vec{F} = \dot{m}\,(\beta_2 \vec{V}_2 - \beta_1 \vec{V}_1)$$

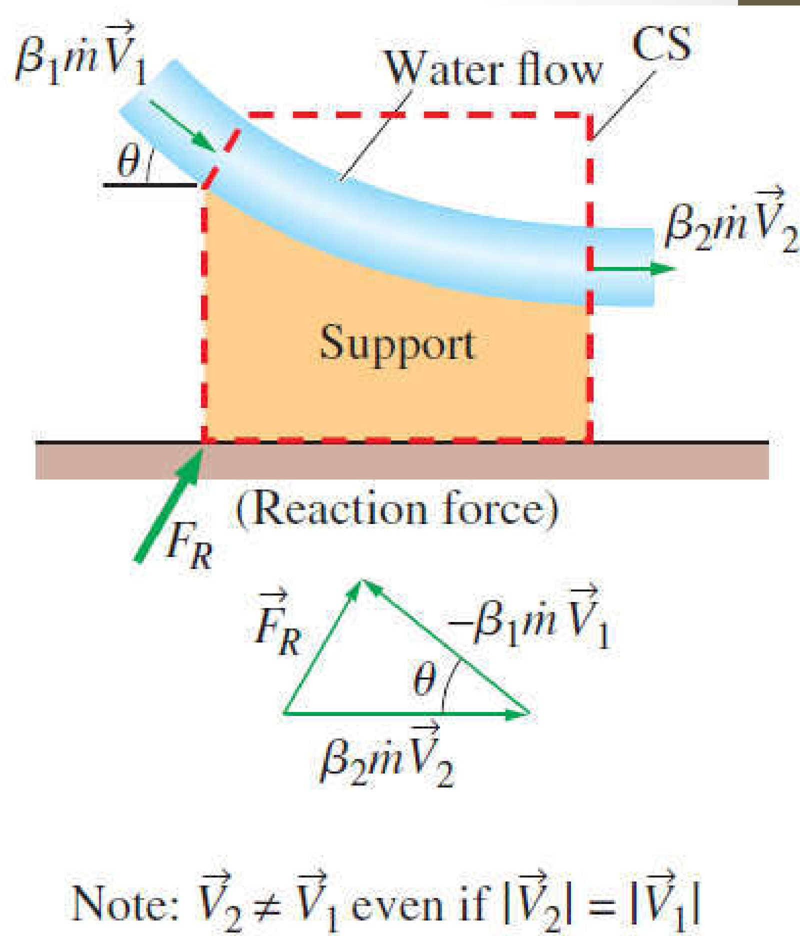
One inlet and one outlet

$$\sum F_x = \dot{m}(\beta_2 V_{2,x} - \beta_1 V_{1,x})$$

Along x-coordinate



A control volume with only one inlet and one outlet.



The determination by vector addition of the reaction force on the support caused by a change of direction of water.

Flow with No External Forces

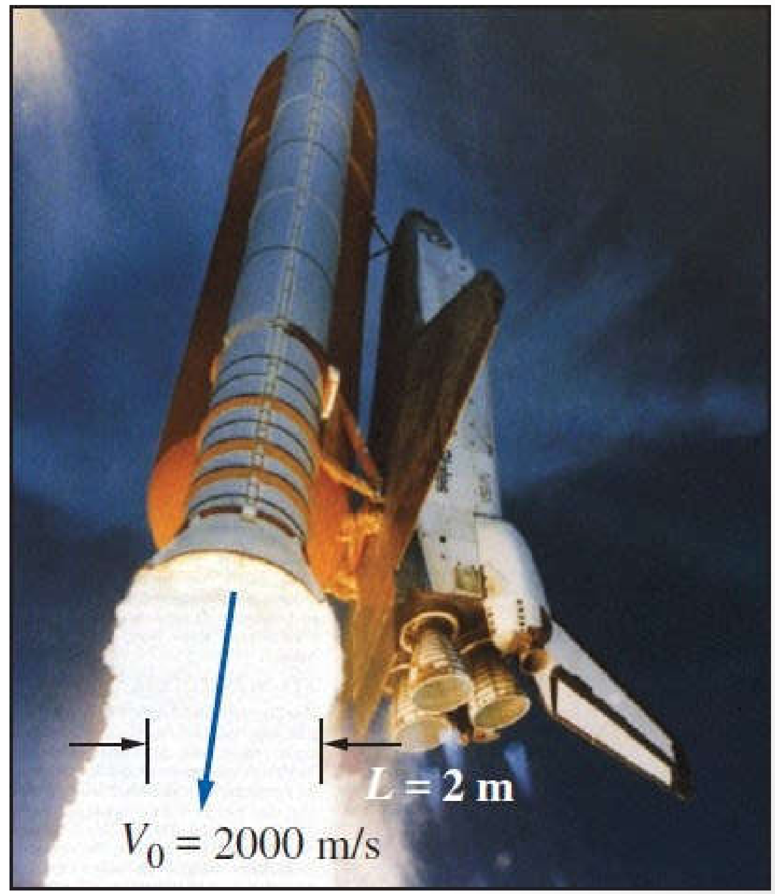
$$\textit{No external forces:} \quad 0 = \frac{d(m\vec{V})_{CV}}{dt} + \sum_{out} \beta \dot{m} \vec{V} - \sum_{in} \beta \dot{m} \vec{V}$$

In the ***absence of external forces****, the rate of* ***change of the momentum*** *of a control volume* ***is equal*** *to the* ***difference between the rates of incoming and outgoing*** *momentum flow rates.*

$$\frac{d(m\vec{V})_{CV}}{dt} = m_{CV}\frac{d\vec{V}_{CV}}{dt} = (m\vec{a})_{CV} = m_{CV}\vec{a}$$

$$\vec{F}_{thrust} = m_{CV}\vec{a} = \sum_{in} \beta \dot{m} \vec{V} - \sum_{out} \beta \dot{m} \vec{V}$$

The thrust needed to lift the space shuttle is generated by the rocket engines as a result of momentum change of the fuel as it is accelerated from about zero to an exit speed of about 2000 m/s after combustion.

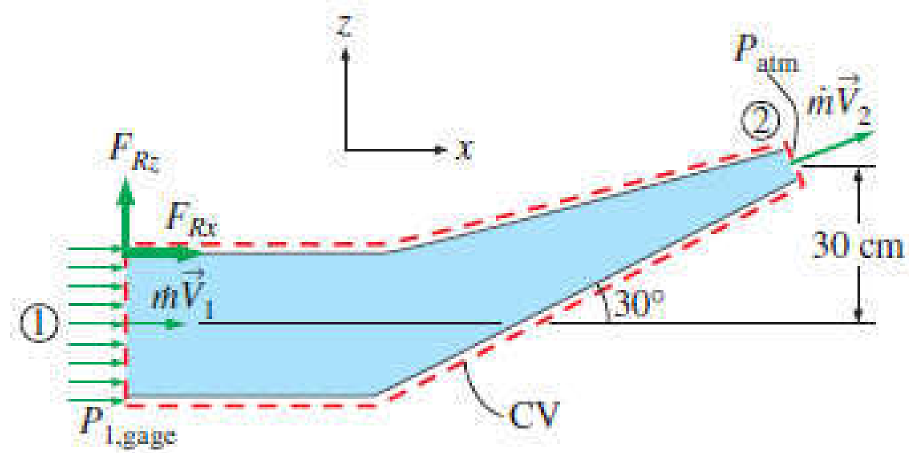


EXAMPLE 6–2 The Force to Hold a Deflector Elbow in Place

A reducing elbow is used to deflect water flow at a rate of 14 kg/s in a horizontal pipe upward 30° while accelerating it (Fig. 6–20). The elbow discharges water into the atmosphere. The cross-sectional area of the elbow is 113 cm^2 at the inlet and 7 cm^2 at the outlet. The elevation difference between the centers of the outlet and the inlet is 30 cm. The weight of the elbow and the water in it is considered to be negligible. Determine (*a*) the gage pressure at the center of the inlet of the elbow and (*b*) the anchoring force needed to hold the elbow in place.

Take the momentum-flux correction factor to be β = 1.03, at inlet and outlet

$$\sum \vec{F} = \sum_{\text{out}} \beta \dot{m} \vec{V} - \sum_{\text{in}} \beta \dot{m} \vec{V}$$



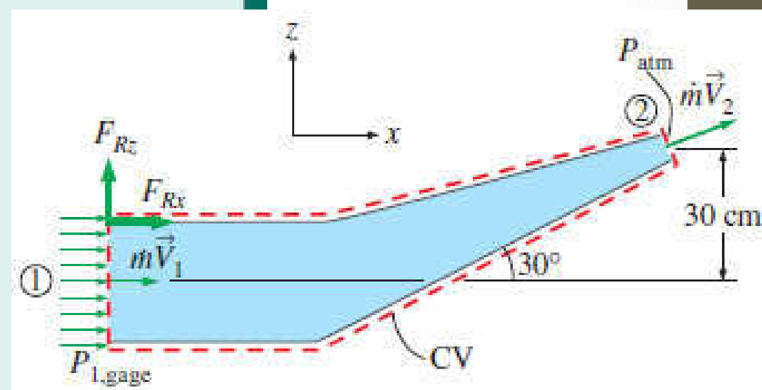
Properties We take the density of water to be 1000 kg/m^3.

Analysis (*a*) We take the elbow as the control volume and designate the inlet by 1 and the outlet by 2. We also take the *x*- and *z*-coordinates as shown. The continuity equation for this one-inlet, one-outlet, steady-flow system is $\dot{m}_1 = \dot{m}_2 = \dot{m} = 14$ kg/s. Noting that $\dot{m} = \rho AV$, the inlet and outlet velocities of water are

$$V_1 = \frac{\dot{m}}{\rho A_1} = \frac{14 \text{ kg/s}}{(1000 \text{ kg/m}^3)(0.0113 \text{ m}^2)} = 1.24 \text{ m/s}$$

$$V_2 = \frac{\dot{m}}{\rho A_2} = \frac{14 \text{ kg/s}}{(1000 \text{ kg/m}^3)(7 \times 10^{-4} \text{ m}^2)} = 20.0 \text{ m/s}$$

We use the Bernoulli equation (Chap. 5) as a first approximation to calculate the pressure. In Chap. 8 we will learn how to account for frictional losses along the walls. Taking the center of the inlet cross section as the reference level ($z_1 = 0$) and noting that $P_2 = P_{atm}$, the Bernoulli equation for a streamline going through the center of the elbow is expressed as



$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

$$P_1 - P_2 = \rho g\left(\frac{V_2^2 - V_1^2}{2g} + z_2 - z_1\right)$$

$$P_1 - P_{atm} = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)$$

$$\times\left(\frac{(20 \text{ m/s})^2 - (1.24 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)} + 0.3 - 0\right)\left(\frac{1 \text{ kN}}{1000 \text{ kg·m/s}^2}\right)$$

$$P_{1,\text{ gage}} = 202.2 \text{ kN/m}^2 = \mathbf{202.2 \text{ kPa}} \quad \text{(gage)}$$

(*b*) The momentum equation for steady flow is

$$\sum \vec{F} = \sum_{\text{out}} \beta \dot{m} \vec{V} - \sum_{\text{in}} \beta \dot{m} \vec{V}$$

We let the *x*- and *z*-components of the anchoring force of the elbow be F_{Rx} and F_{Rz}, and assume them to be in the positive direction. We also use gage pressure since the atmospheric pressure acts on the entire control surface. Then the momentum equations along the *x*- and *z*-axes become

$$F_{Rx} + P_{1,\,\text{gage}} A_1 = \beta \dot{m} V_2 \cos\theta - \beta \dot{m} V_1$$
$$F_{Rz} = \beta \dot{m} V_2 \sin\theta$$

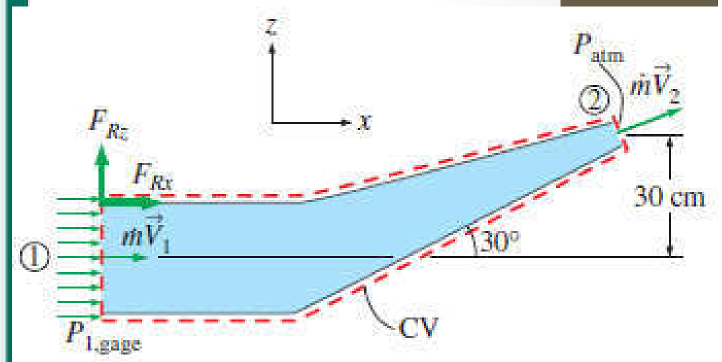
where we have set $\beta = \beta_1 = \beta_2$. Solving for F_{Rx} and F_{Rz}, and substituting the given values,

$$F_{Rx} = \beta \dot{m}(V_2 \cos\theta - V_1) - P_{1,\,\text{gage}} A_1$$
$$= 1.03(14\text{ kg/s})[(20 \cos 30^\circ - 1.24)\text{ m/s}]\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right)$$
$$- (202{,}200\text{ N/m}^2)(0.0113\text{ m}^2)$$
$$= 232 - 2285 = \mathbf{-2053\text{ N}}$$

$$F_{Rz} = \beta \dot{m} V_2 \sin\theta = (1.03)(14\text{ kg/s})(20 \sin 30^\circ\text{ m/s})\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right) = \mathbf{144\text{ N}}$$

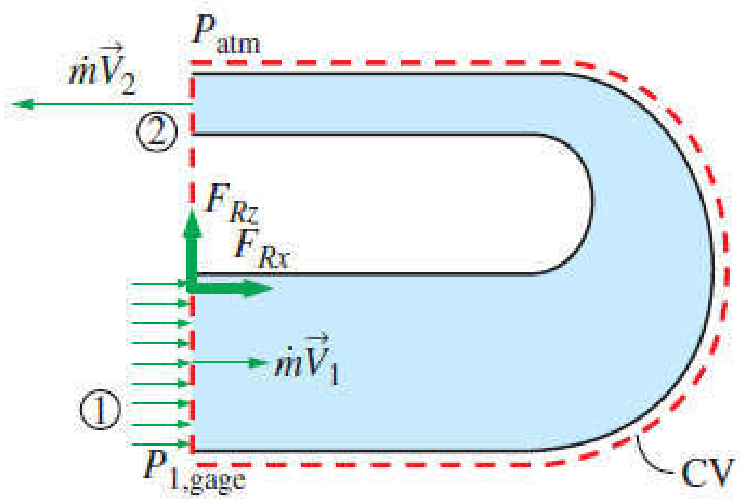
The negative result for F_{Rx} indicates that the assumed direction is wrong, and it should be reversed. Therefore, F_{Rx} acts in the negative *x*-direction.

Discussion There is a nonzero pressure distribution along the inside walls of the elbow, but since the control volume is outside the elbow, these pressures do not appear in our analysis. The weight of the elbow and the water in it could be added to the vertical force for better accuracy. The actual value of $P_{1,\,\text{gage}}$ will be higher than that calculated here because of frictional and other irreversible losses in the elbow.



EXAMPLE 6–3 The Force to Hold a Reversing Elbow in Place

The deflector elbow in Example 6–2 is replaced by a reversing elbow such that the fluid makes a 180° U-turn before it is discharged, as shown in Fig. 6–21. The elevation difference between the centers of the inlet and the exit sections is still 0.3 m. Determine the anchoring force needed to hold the elbow in place.



$$\sum \vec{F} = \sum_{out} \beta \dot{m} \vec{V} - \sum_{in} \beta \dot{m} \vec{V}$$

$$P_{1,\,gage} = 202.2\ \text{kN/m}^2 = \mathbf{202.2\ kPa}\quad \text{(gage)}$$

$$V_1 = \frac{\dot{m}}{\rho A_1} = \frac{14\ \text{kg/s}}{(1000\ \text{kg/m}^3)(0.0113\ \text{m}^2)} = 1.24\ \text{m/s}$$

$$V_2 = \frac{\dot{m}}{\rho A_2} = \frac{14\ \text{kg/s}}{(1000\ \text{kg/m}^3)(7 \times 10^{-4}\ \text{m}^2)} = 20.0\ \text{m/s}$$

SOLUTION The inlet and the outlet velocities and the pressure at the inlet of the elbow remain the same, but the vertical component of the anchoring force at the connection of the elbow to the pipe is zero in this case ($F_{Rz} = 0$) since there is no other force or momentum flux in the vertical direction (we are neglecting the weight of the elbow and the water). The horizontal component of the anchoring force is determined from the momentum equation written in the *x*-direction. Noting that the outlet velocity is negative since it is in the negative *x*-direction, we have

$$F_{Rx} + P_{1,\,gage}A_1 = \beta_2\dot{m}(-V_2) - \beta_1\dot{m}V_1 = -\beta\dot{m}(V_2 + V_1)$$

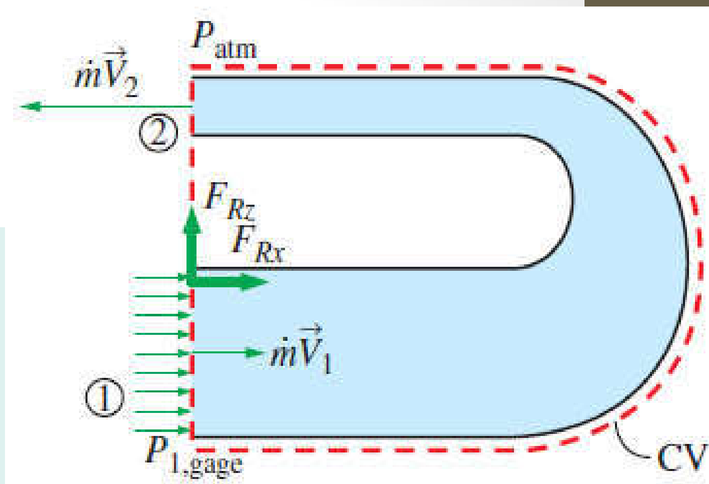
Solving for F_{Rx} and substituting the known values,

$$F_{Rx} = -\beta\dot{m}(V_2 + V_1) - P_{1,\,gage}A_1$$

$$= -(1.03)(14 \text{ kg/s})[(20 + 1.24) \text{ m/s}]\left(\frac{1 \text{ N}}{1 \text{ kg·m/s}^2}\right) - (202{,}200 \text{ N/m}^2)(0.0113 \text{ m}^2)$$

$$= -306 - 2285 = \mathbf{-2591 \text{ N}}$$

Therefore, the horizontal force on the flange is 2591 N acting in the negative *x*-direction (the elbow is trying to separate from the pipe). This force is equivalent to the weight of about 260 kg mass, and thus the connectors (such as bolts) used must be strong enough to withstand this force.



$$P_{1,\,gage} = 202.2 \text{ kN/m}^2 = \mathbf{202.2 \text{ kPa}} \quad \text{(gage)}$$

$$V_1 = \frac{\dot{m}}{\rho A_1} = \frac{14 \text{ kg/s}}{(1000 \text{ kg/m}^3)(0.0113 \text{ m}^2)} = 1.24 \text{ m/s}$$

$$V_2 = \frac{\dot{m}}{\rho A_2} = \frac{14 \text{ kg/s}}{(1000 \text{ kg/m}^3)(7 \times 10^{-4} \text{ m}^2)} = 20.0 \text{ m/s}$$

EXAMPLE 6–4 Water Jet Striking a Stationary Plate

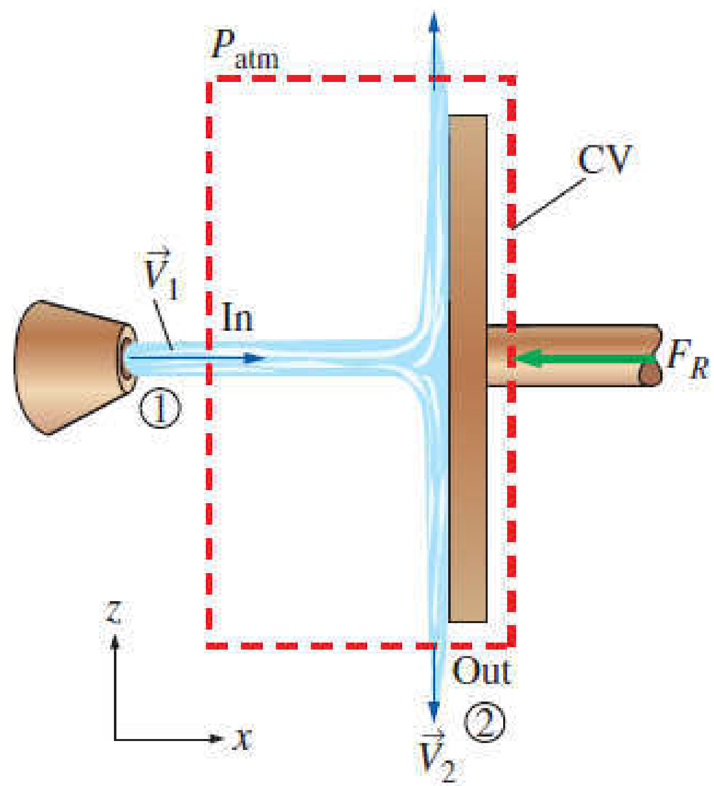
Water is accelerated by a nozzle to an average speed of 20 m/s, and strikes a stationary vertical plate at a rate of 10 kg/s with a normal velocity of 20 m/s (Fig. 6–22). After the strike, the water stream splatters off in all directions in the plane of the plate. Determine the force needed to prevent the plate from moving horizontally due to the water stream.

SOLUTION A water jet strikes a vertical stationary plate normally. The force needed to hold the plate in place is to be determined.

Assumptions **1** The flow of water at the nozzle outlet is steady. **2** The water splatters in directions normal to the approach direction of the water jet.

Take the momentum-flux correction factor to be $\beta = 1$.

$$\sum \vec{F} = \sum_{\text{out}} \beta \dot{m} \vec{V} - \sum_{\text{in}} \beta \dot{m} \vec{V}$$



3 The water jet is exposed to the atmosphere, and thus the pressure of the water jet and the splattered water leaving the control volume is atmospheric pressure, which is disregarded since it acts on the entire system. **4** The vertical forces and momentum fluxes are not considered since they have no effect on the horizontal reaction force. **5** The effect of the momentum-flux correction factor is negligible, and thus $\beta \cong 1$ at the inlet.

Analysis We draw the control volume for this problem such that it contains the entire plate and cuts through the water jet and the support bar normally. The momentum equation for steady flow is given as

$$\sum \vec{F} = \sum_{\text{out}} \beta \dot{m} \vec{V} - \sum_{\text{in}} \beta \dot{m} \vec{V} \qquad (1)$$

Writing Eq. 1 for this problem along the *x*-direction (without forgetting the negative sign for forces and velocities in the negative *x*-direction) and noting that $V_{1,x} = V_1$ and $V_{2,x} = 0$ gives

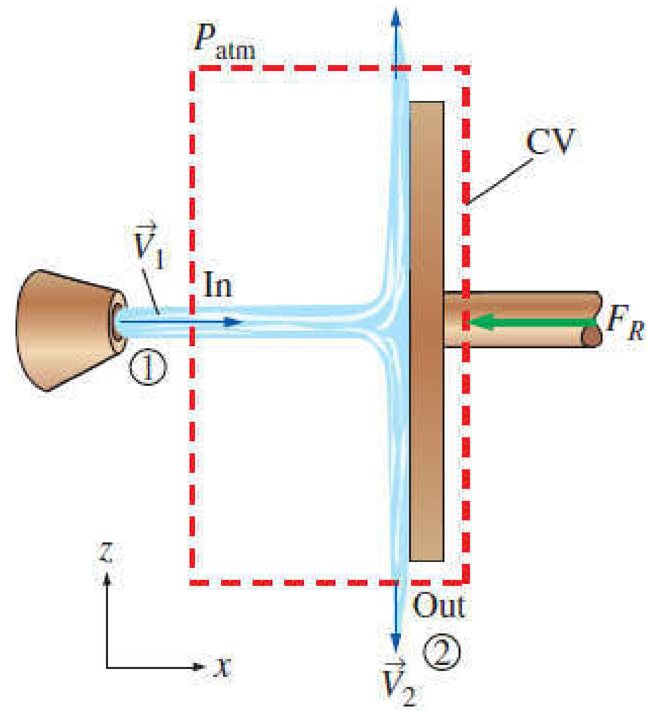
$$-F_R = 0 - \beta \dot{m} V_1$$

Substituting the given values,

$$F_R = \beta \dot{m} V_1 = (1)(10\ \text{kg/s})(20\ \text{m/s})\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right) = \mathbf{200\ N}$$

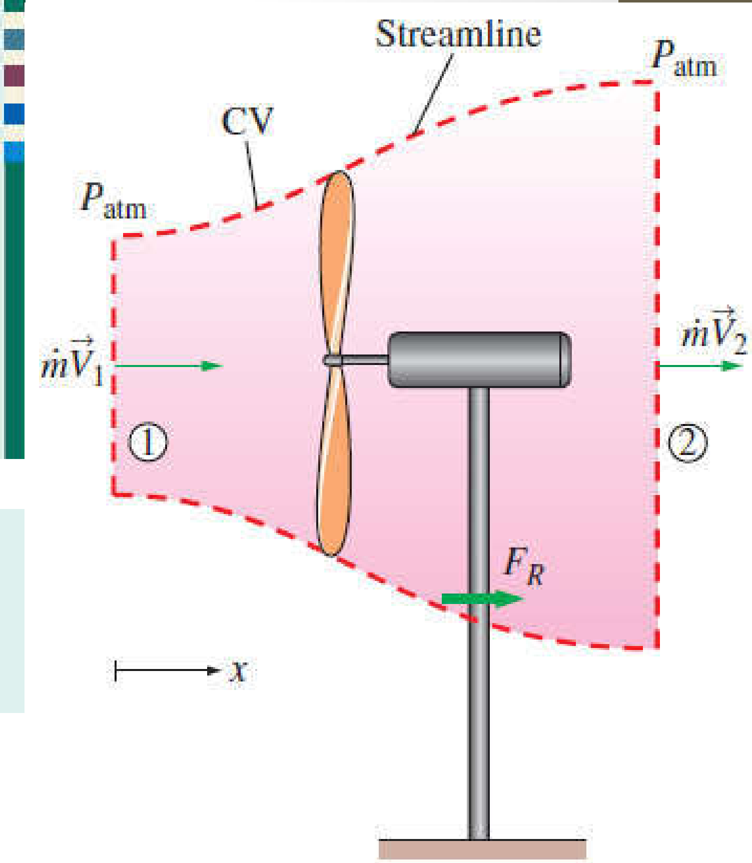
Therefore, the support must apply a 200-N horizontal force (equivalent to the weight of about a 20-kg mass) in the negative *x*-direction (the opposite direction of the water jet) to hold the plate in place. A similar situation occurs in the downwash of a helicopter (Fig. 6–23).

Discussion The plate absorbs the full brunt of the momentum of the water jet since the *x*-direction momentum at the outlet of the control volume is zero. If the control volume were drawn instead along the interface between the water and the plate, there would be additional (unknown) pressure forces in the analysis. By cutting the control volume through the support, we avoid having to deal with this additional complexity. This is an example of a "wise" choice of control volume.



EXAMPLE 6–5 Power Generation and Wind Loading of a Wind Turbine

A wind generator with a 30-ft-diameter blade span has a cut-in wind speed (minimum speed for power generation) of 7 mph, at which velocity the turbine generates 0.4 kW of electric power (Fig. 6–24). Determine (*a*) the efficiency of the wind turbine–generator unit and (*b*) the horizontal force exerted by the wind on the supporting mast of the wind turbine. What is the effect of doubling the wind velocity to 14 mph on power generation and the force exerted? Assume the efficiency remains the same, and take the density of air to be 0.076 lbm/ft^3.



Analysis Kinetic energy is a mechanical form of energy, and thus it can be converted to work entirely. Therefore, the power potential of the wind is proportional to its kinetic energy, which is $V^2/2$ per unit mass, and thus the maximum power is $\dot{m}V^2/2$ for a given mass flow rate:

$$\eta_{turbine}\eta_{generator} = \frac{\dot{W}_{elect,out}}{\dot{W}_{turbine,e}} = \frac{\dot{W}_{elect,out}}{|\Delta \dot{E}_{mech,fluid}|}$$

$$\sum \vec{F} = \sum_{out} \beta \dot{m}\vec{V} - \sum_{in} \beta \dot{m}\vec{V}$$

Analysis Kinetic energy is a mechanical form of energy, and thus it can be converted to work entirely. Therefore, the power potential of the wind is proportional to its kinetic energy, which is $V^2/2$ per unit mass, and thus the maximum power is $\dot{m}V^2/2$ for a given mass flow rate:

$$V_1 = (7\text{ mph})\left(\frac{1.4667\text{ ft/s}}{1\text{ mph}}\right) = 10.27\text{ ft/s}$$

$$\dot{m} = \rho_1 V_1 A_1 = \rho_1 V_1 \frac{\pi D^2}{4} = (0.076\text{ lbm/ft}^3)(10.27\text{ ft/s})\frac{\pi(30\text{ ft})^2}{4} = 551.7\text{ lbm/s}$$

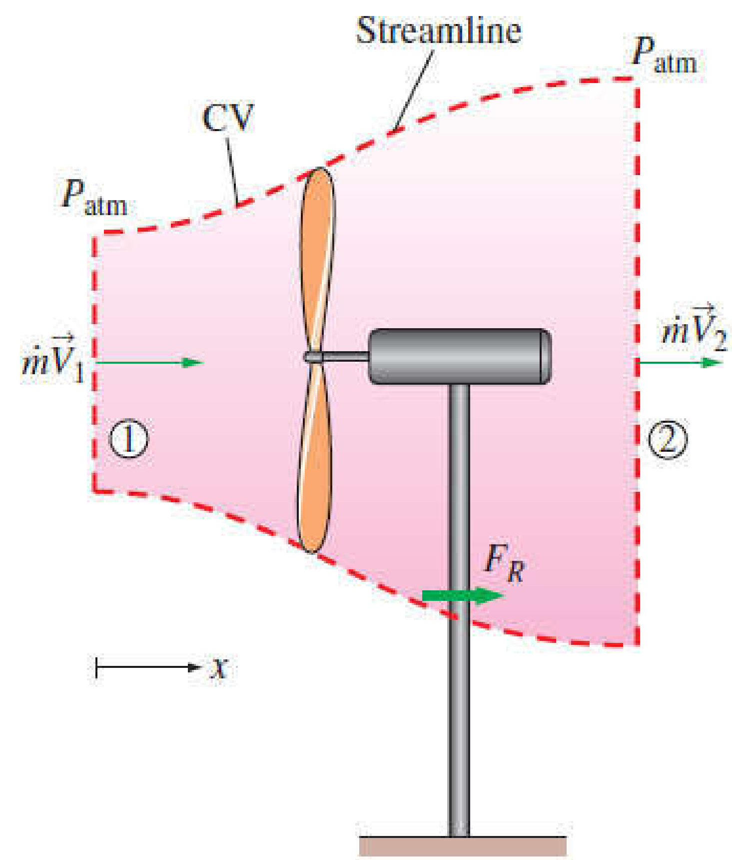
$$\dot{W}_{max} = \dot{m}\text{ke}_1 = \dot{m}\frac{V_1^2}{2}$$

$$= (551.7\text{ lbm/s})\frac{(10.27\text{ ft/s})^2}{2}\left(\frac{1\text{ lbf}}{32.2\text{ lbm}\cdot\text{ft/s}^2}\right)\left(\frac{1\text{ kW}}{737.56\text{ lbf}\cdot\text{ft/s}}\right)$$

$$= 1.225\text{ kW}$$

Therefore, the available power to the wind turbine is 1.225 kW at the wind velocity of 7 mph. Then the turbine–generator efficiency becomes

$$\eta_{\text{wind turbine}} = \frac{\dot{W}_{act}}{\dot{W}_{max}} = \frac{0.4\text{ kW}}{1.225\text{ kW}} = 0.327 \qquad (\text{or } 32.7\%)$$



$$\eta_{turbine}\eta_{generator} = \frac{\dot{W}_{elect,out}}{\dot{W}_{turbine,e}} = \frac{\dot{W}_{elect,out}}{|\Delta \dot{E}_{mech,fluid}|}$$

(*b*) The frictional effects are assumed to be negligible, and thus the portion of incoming kinetic energy not converted to electric power leaves the wind turbine as outgoing kinetic energy. Noting that the mass flow rate remains constant, the exit velocity is determined to be

$$\dot{m}\mathrm{ke}_2 = \dot{m}\mathrm{ke}_1(1 - \eta_{\text{wind turbine}}) \rightarrow \dot{m}\frac{V_2^2}{2} = \dot{m}\frac{V_1^2}{2}(1 - \eta_{\text{wind turbine}}) \quad (1)$$

or

$$V_2 = V_1\sqrt{1 - \eta_{\text{wind turbine}}} = (10.27 \text{ ft/s})\sqrt{1 - 0.327} = 8.43 \text{ ft/s}$$

To determine the force on the mast (Fig. 6–25), we draw a control volume around the wind turbine such that the wind is normal to the control surface at the inlet and the outlet and the entire control surface is at atmospheric pressure (Fig. 6–23). The momentum equation for steady flow is given as

$$\sum \vec{F} = \sum_{\text{out}} \beta \dot{m}\vec{V} - \sum_{\text{in}} \beta \dot{m}\vec{V} \quad (2)$$

Writing Eq. 2 along the *x*-direction and noting that $\beta = 1$, $V_{1,x} = V_1$, and $V_{2,x} = V_2$ give

$$F_R = \dot{m}V_2 - \dot{m}V_1 = \dot{m}(V_2 - V_1) \quad (3)$$

Substituting the known values into Eq. 3 gives

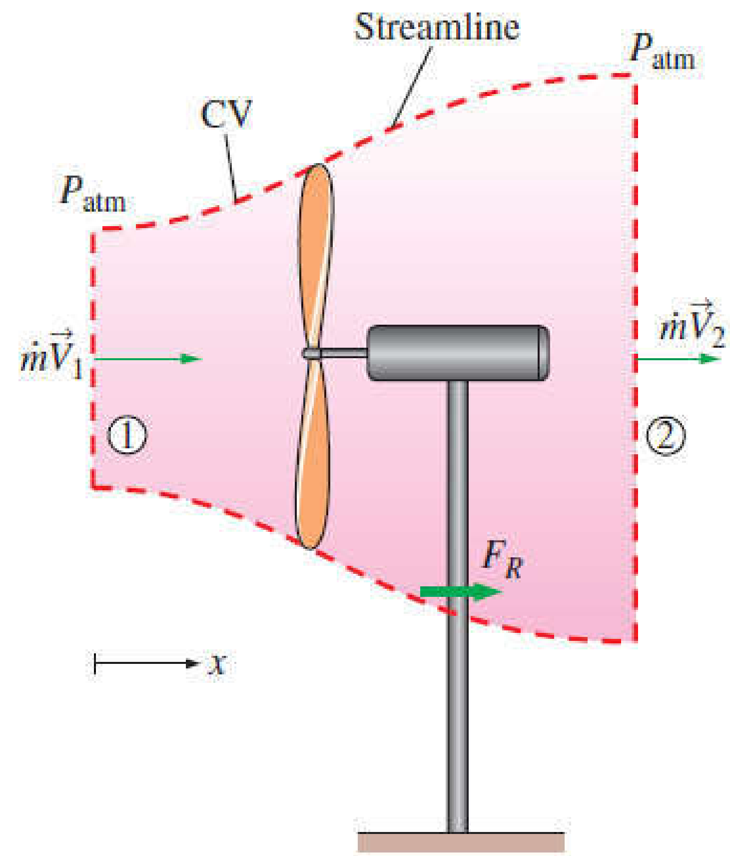
$$F_R = \dot{m}(V_2 - V_1) = (551.7 \text{ lbm/s})(8.43 - 10.27 \text{ ft/s})\left(\frac{1 \text{ lbf}}{32.2 \text{ lbm·ft/s}^2}\right)$$

$$= -31.5 \text{ lbf}$$

The negative sign indicates that the reaction force acts in the negative *x*-direction, as expected. Then the force exerted by the wind on the mast becomes $F_{\text{mast}} = -F_R =$ **31.5 lbf.**

The power generated is proportional to V^3 since the mass flow rate is proportional to V and the kinetic energy to V^2. Therefore, doubling the wind velocity to 14 mph will increase the power generation by a factor of $2^3 = 8$ to $0.4 \times 8 = 3.2$ kW. The force exerted by the wind on the support mast is proportional to V^2. Therefore, doubling the wind velocity to 14 mph will increase the wind force by a factor of $2^2 = 4$ to $31.5 \times 4 = 126$ lbf.

Discussion Wind turbines are treated in more detail in Chap. 14.



$$\sum \vec{F} = \sum_{\text{out}} \beta \dot{m}\vec{V} - \sum_{\text{in}} \beta \dot{m}\vec{V}$$

EXAMPLE 6–1 Momentum-Flux Correction Factor for Laminar Pipe Flow

Consider laminar flow through a very long straight section of round pipe. It is shown in Chap. 8 that the velocity profile through a cross-sectional area of the pipe is parabolic (Fig. 6–15), with the axial velocity component given by

$$V = 2V_{avg}\left(1 - \frac{r^2}{R^2}\right) \quad (1)$$

where R is the radius of the inner wall of the pipe and V_{avg} is the average velocity. Calculate the momentum-flux correction factor through a cross section of the pipe for the case in which the pipe flow represents an outlet of the control volume, as sketched in Fig. 6–15.

Momentum-flux correction factor: $\beta = \frac{1}{A_c}\int_{A_c}\left(\frac{V}{V_{avg}}\right)^2 dA_c$

Analysis We substitute the given velocity profile for V in Eq. 6–24 and integrate, noting that $dA_c = 2\pi r\,dr$,

$$\beta = \frac{1}{A_c}\int_{A_c}\left(\frac{V}{V_{avg}}\right)^2 dA_c = \frac{4}{\pi R^2}\int_0^R\left(1 - \frac{r^2}{R^2}\right)^2 2\pi r\,dr \quad (2)$$

Defining a new integration variable $y = 1 - r^2/R^2$ and thus $dy = -2r\,dr/R^2$ (also, $y = 1$ at $r = 0$, and $y = 0$ at $r = R$) and performing the integration, the momentum-flux correction factor for fully developed laminar flow becomes

Laminar flow: $$\beta = -4\int_1^0 y^2\,dy = -4\left[\frac{y^3}{3}\right]_1^0 = \frac{4}{3} \quad (3)$$

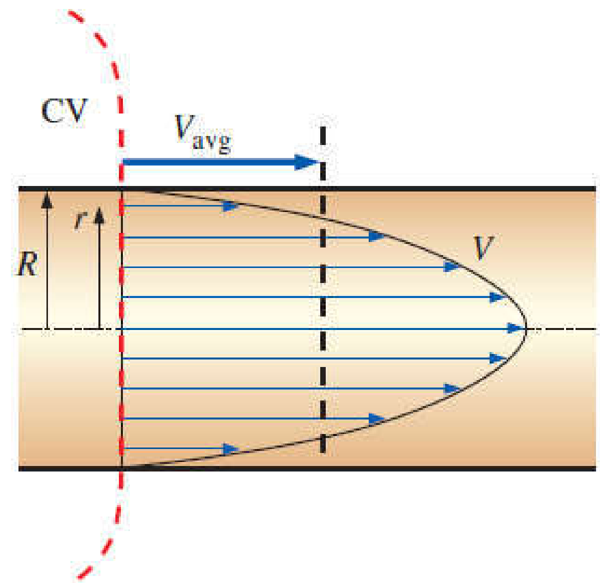
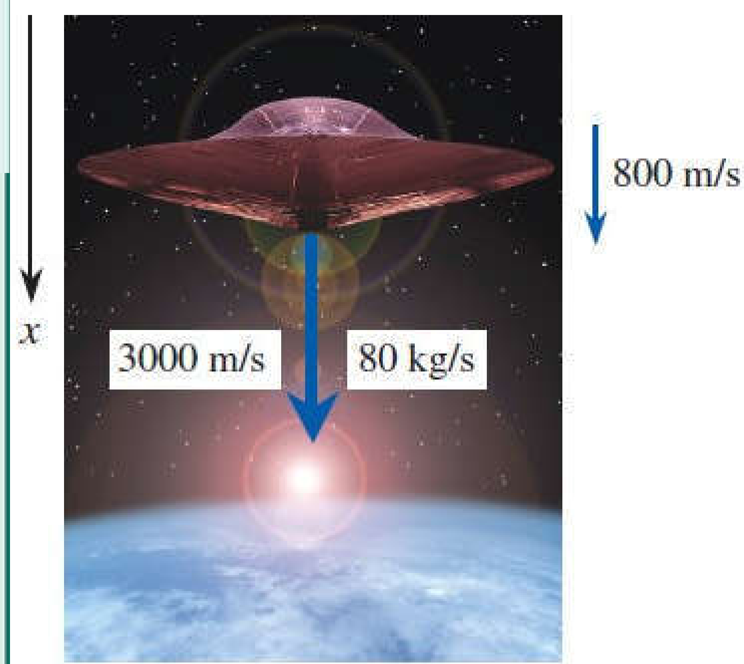


FIGURE 6–15
Velocity profile over a cross section of a pipe in which the flow is fully developed and laminar.

EXAMPLE 6–6 Deceleration of a Spacecraft

A spacecraft with a mass of 12,000 kg is dropping vertically towards a planet at a constant speed of 800 m/s (Fig. 6–26). To slow down the spacecraft, a solid-fuel rocket at the bottom is fired, and combustion gases leave the rocket at a constant rate of 80 kg/s and at a velocity of 3000 m/s relative to the spacecraft in the direction of motion of the spacecraft for a period of 5 s. Disregarding the small change in the mass of the spacecraft, determine (*a*) the deceleration of the spacecraft during this period, (*b*) the change of velocity of the spacecraft, and (*c*) the thrust exerted on the spacecraft.



Analysis (*a*) For convenience, we choose an inertial reference frame that moves with the spacecraft at the same initial velocity. Then the velocities of the fluid stream relative to an inertial reference frame become simply the velocities relative to the spacecraft. We take the direction of motion of the spacecraft as the positive direction along the *x*-axis. There are no external forces acting on the spacecraft, and its mass is essentially constant. Therefore, the spacecraft can be treated as a solid body with constant mass, and the momentum equation in this case is, from Eq. 6–29,

$$\vec{F}_{\text{thrust}} = m_{\text{spacecraft}}\,\vec{a}_{\text{spacecraft}} = \sum_{\text{in}} \beta \dot{m} \vec{V} - \sum_{\text{out}} \beta \dot{m} \vec{V}$$

where the fluid stream velocities relative to the inertial reference frame in this case are identical to the velocities relative to the spacecraft. Noting that the motion is on a straight line and the discharged gases move in the positive *x*-direction, we write the momentum equation using magnitudes as

$$m_{\text{spacecraft}} a_{\text{spacecraft}} = m_{\text{spacecraft}} \frac{dV_{\text{spacecraft}}}{dt} = -\dot{m}_{\text{gas}} V_{\text{gas}}$$

Noting that gases leave in the positive *x*-direction and substituting, the acceleration of the spacecraft during the first 5 seconds is determined to be

$$a_{\text{spacecraft}} = \frac{dV_{\text{spacecraft}}}{dt} = -\frac{\dot{m}_{\text{gas}}}{m_{\text{spacecraft}}} V_{\text{gas}} = -\frac{80 \text{ kg/s}}{12{,}000 \text{ kg}}(+3000 \text{ m/s}) = \mathbf{-20 \text{ m/s}^2}$$

The negative value confirms that the spacecraft is decelerating in the positive *x* direction at a rate of 20 m/s^2.

(*b*) Knowing the deceleration, which is constant, the velocity change of the spacecraft during the first 5 seconds is determined from the definition of acceleration to be

$$dV_{\text{spacecraft}} = a_{\text{spacecraft}} dt \rightarrow \Delta V_{\text{spacecraft}} = a_{\text{spacecraft}} \Delta t = (-20 \text{ m/s}^2)(5 \text{ s})$$
$$= \mathbf{-100 \text{ m/s}}$$

(*c*) The thrusting force exerted on the space aircraft is, from Eq. 6-29,

$$F_{\text{thrust}} = 0 - \dot{m}_{\text{gas}} V_{\text{gas}} = 0 - (80 \text{ kg/s})(+3000 \text{ m/s})\left(\frac{1 \text{ kN}}{1000 \text{ kg}\cdot\text{m/s}^2}\right) = \mathbf{-240 \text{ kN}}$$

The negative sign indicates that the trusting force due to firing of the rocket acts on the aircraft in the negative *x*-direction.

Discussion Note that if this fired rocket were attached somewhere on a test stand, it would exert a force of 240 kN (equivalent to the weight of about 24 tons of mass) to its support in the opposite direction of the discharged gases.

EXAMPLE 6–7 Net Force on a Flange

Water flows at a rate of 18.5 gal/min through a flanged faucet with a partially closed gate valve spigot (Fig. 6–27). The inner diameter of the pipe at the location of the flange is 0.780 in (= 0.0650 ft), and the pressure at that location is measured to be 13.0 psig. The total weight of the faucet assembly plus the water within it is 12.8 lbf. Calculate the net force on the flange.

Analysis We choose the faucet and its immediate surroundings as the control volume, as shown in Fig. 6–27 along with all the forces acting on it. These forces include the weight of the water and the weight of the faucet assembly, the gage pressure force at the inlet to the control volume, and the net force of the flange on the control volume, which we call $\vec{F}_R$. We use gage pressure for convenience since the gage pressure on the rest of the control surface is zero (atmospheric pressure). Note that the pressure through the outlet of the control volume is also atmospheric since we are assuming incompressible flow; hence, the gage pressure is also zero through the outlet.

We now apply the control volume conservation laws. Conservation of mass is trivial here since there is only one inlet and one outlet; namely, the mass flow rate into the control volume is equal to the mass flow rate out of the control volume. Also, the outflow and inflow average velocities are identical since the inner diameter is constant and the water is incompressible, and are determined to be

$$V_2 = V_1 = V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} = \frac{18.5 \text{ gal/min}}{\pi(0.065 \text{ ft})^2/4}\left(\frac{0.1337 \text{ ft}^3}{1 \text{ gal}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 12.42 \text{ ft/s}$$

Also,

$$\dot{m} = \rho\dot{V} = (62.3 \text{ lbm/ft}^3)(18.5 \text{ gal/min})\left(\frac{0.1337 \text{ ft}^3}{1 \text{ gal}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 2.568 \text{ lbm/s}$$

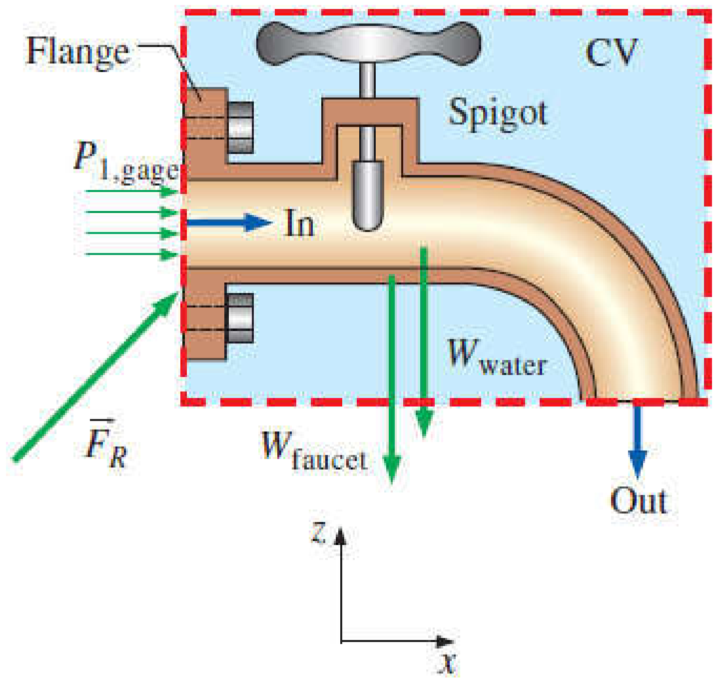


FIGURE 6–27
Control volume for Example 6–7 with all forces shown; gage pressure is used for convenience.

Next we apply the momentum equation for steady flow,

$$\sum \vec{F} = \sum_{\text{out}} \beta \dot{m} \vec{V} - \sum_{\text{in}} \beta \dot{m} \vec{V} \qquad (1)$$

We let the x- and z-components of the force acting on the flange be F_{Rx} and F_{Rz}, and assume them to be in the positive directions. The magnitude of the velocity in the x-direction is $+V_1$ at the inlet, but zero at the outlet. The magnitude of the velocity in the z-direction is zero at the inlet, but $-V_2$ at the outlet. Also, the weight of the faucet assembly and the water within it acts in the $-z$-direction as a body force. No pressure or viscous forces act on the chosen (wise) control volume in the z-direction.

The components of Eq. 1 along the x- and z-directions become

$$F_{Rx} + P_{1,\text{ gage}} A_1 = 0 - \dot{m}(+V_1)$$

$$F_{Rz} - W_{\text{faucet}} - W_{\text{water}} = \dot{m}(-V_2) - 0$$

Solving for F_{Rx} and F_{Rz}, and substituting the given values,

$$\begin{aligned} F_{Rx} &= -\dot{m}V_1 - P_{1,\text{ gage}} A_1 \\ &= -(2.568 \text{ lbm/s})(12.42 \text{ ft/s})\left(\frac{1 \text{ lbf}}{32.2 \text{ lbm}\cdot\text{ft/s}^2}\right) - (13 \text{ lbf/in}^2)\,\frac{\pi(0.780 \text{ in})^2}{4} \\ &= -7.20 \text{ lbf} \end{aligned}$$

$$\begin{aligned} F_{Rz} &= -\dot{m}V_2 + W_{\text{faucet + water}} \\ &= -(2.568 \text{ lbm/s})(12.42 \text{ ft/s})\left(\frac{1 \text{ lbf}}{32.2 \text{ lbm}\cdot\text{ft/s}^2}\right) + 12.8 \text{ lbf} = 11.8 \text{ lbf} \end{aligned}$$

Then the net force of the flange on the control volume is expressed in vector form as

$$\vec{F}_R = F_{Rx}\vec{i} + F_{Rz}\vec{k} = -7.20\vec{i} + 11.8\vec{k} \text{ lbf}$$

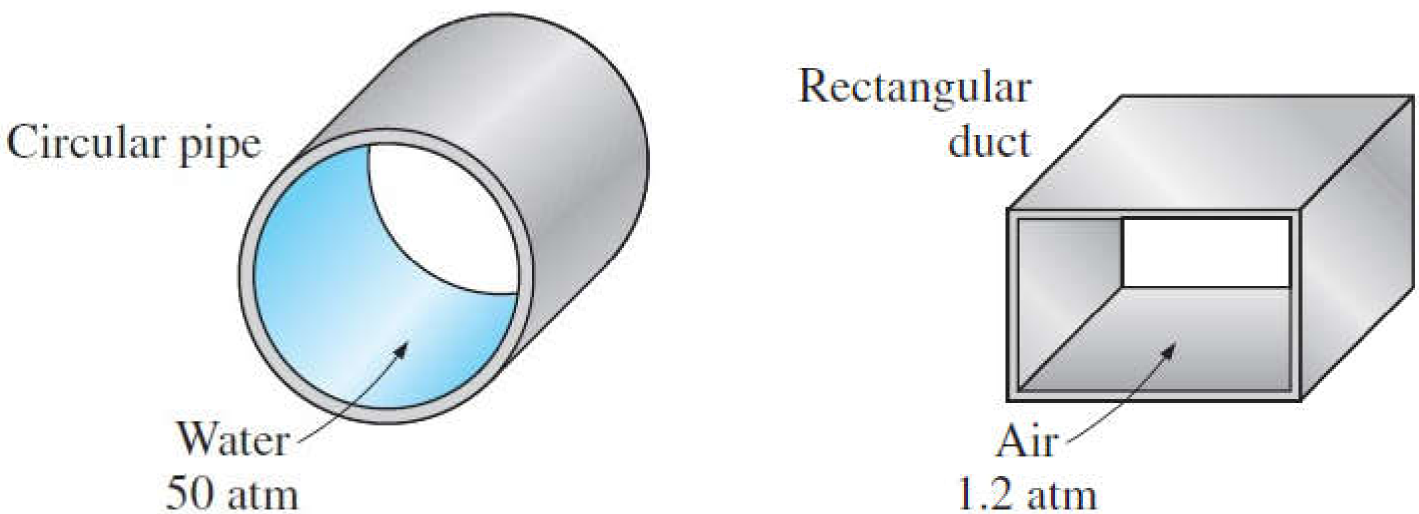
From Newton's third law, the force the faucet assembly exerts on the flange is the negative of $\vec{F}_R$,

$$\vec{F}_{\text{faucet on flange}} = -\vec{F}_R = \mathbf{7.20\vec{i} - 11.8\vec{k}} \textbf{ lbf}$$

Lecture 2

Introduction

- Liquid or gas flow through *pipes* or *ducts* is commonly used in heating and cooling applications and fluid distribution networks.
- The fluid in such applications is usually forced to flow by a fan or pump through a flow section.
- We pay particular attention to *friction*, which is directly related to the *pressure drop* and *head loss* during flow through pipes and ducts.
- The pressure drop is then used to determine the *pumping power requirement*.



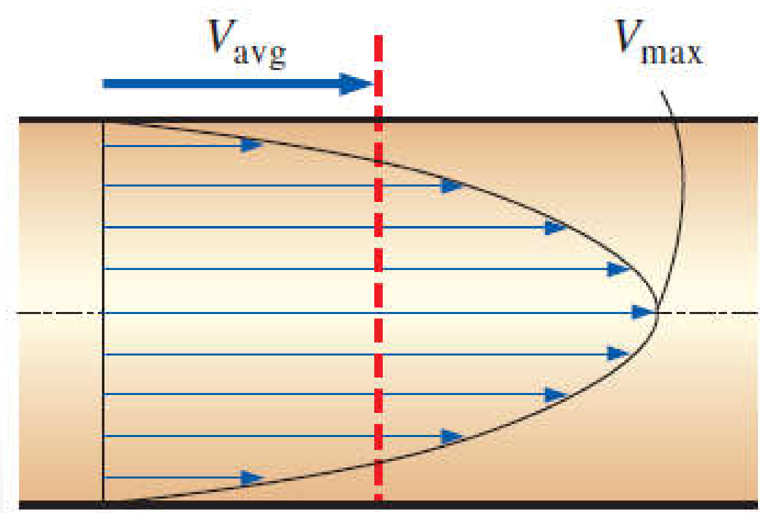
Circular pipes can withstand large pressure differences between the inside and the outside without undergoing any significant distortion, but noncircular pipes cannot.

$$\dot{m} = \rho V_{avg} A_c = \int_{A_c} \rho u(r)\, dA_c$$

The value of the average velocity V_{avg} at some streamwise cross-section is determined from the requirement that the *conservation of mass* principle be satisfied

$$V_{avg} = \frac{\int_{A_c} \rho u(r)\, dA_c}{\rho A_c} = \frac{\int_0^R \rho u(r) 2\pi r\, dr}{\rho \pi R^2} = \frac{2}{R^2} \int_0^R u(r) r\, dr$$

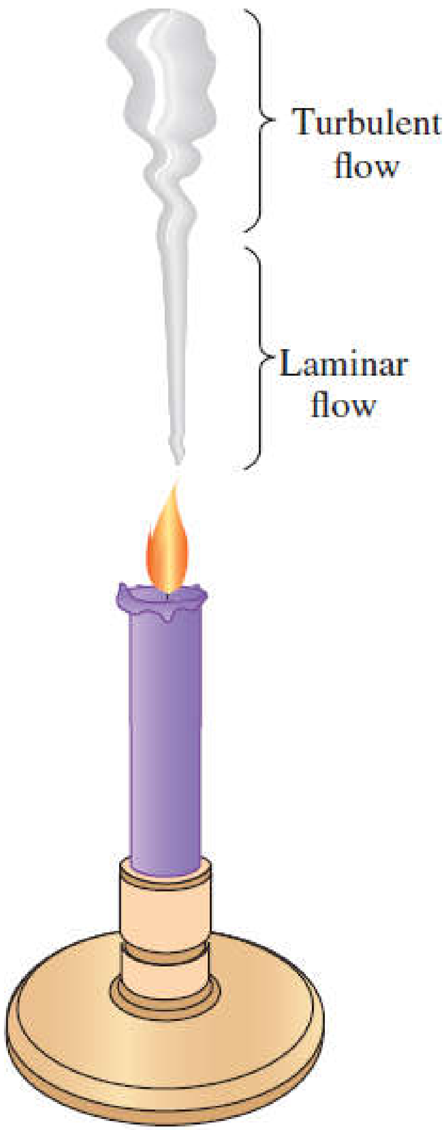
The average velocity for incompressible flow in a circular pipe of radius *R*



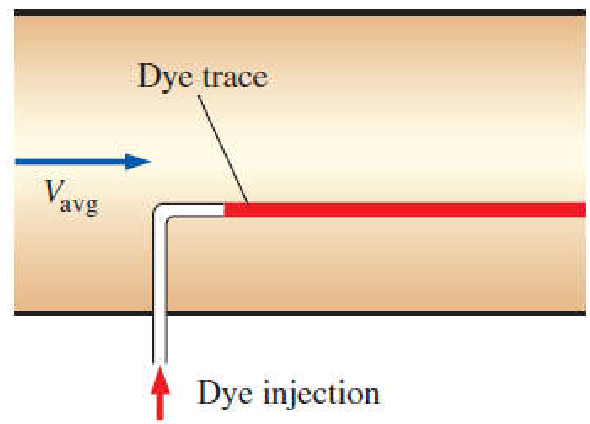
Average velocity V_{avg} is defined as the average speed through a cross section. For fully developed laminar pipe flow, V_{avg} is half of the maximum velocity.

LAMINAR AND TURBULENT FLOWS

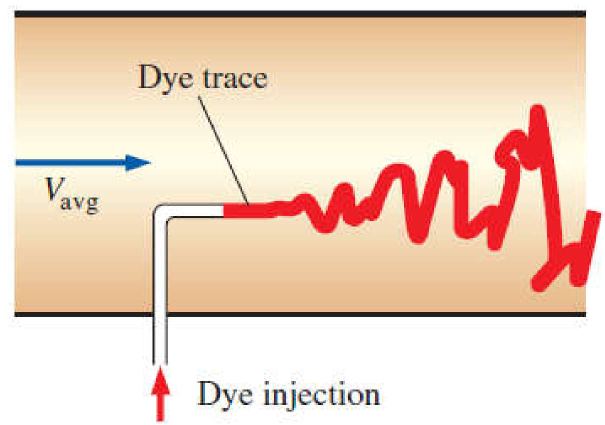
Laminar flow is encountered when highly viscous fluids such as oils flow in small pipes or narrow passages.



Laminar: Smooth streamlines and highly ordered motion.
Turbulent: Velocity fluctuations and highly disordered motion.
Transition: The flow fluctuates between laminar and turbulent flows.
Most flows encountered in practice are turbulent.



(*a*) Laminar flow



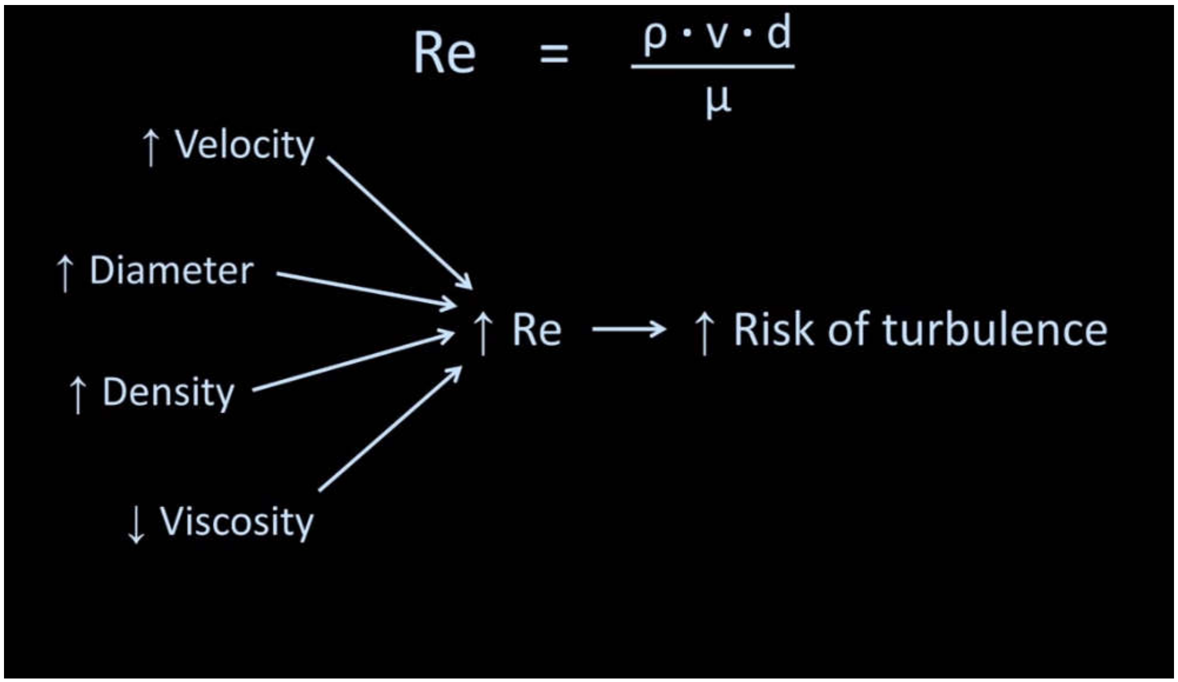
(*b*) Turbulent flow

The **intense mixing** of the fluid in turbulent flow as a result of rapid fluctuations **enhances** momentum transfer between fluid particles, **which** increases the friction force on the surface and thus the required pumping power.

Reynolds Number

The transition from laminar to turbulent flow depends on the *geometry*, *surface roughness*, *flow velocity*, *surface temperature*, and *type of fluid*.

The flow regime depends mainly on the ratio of *inertial forces* to *viscous forces* (**Reynolds number**).



Promotes Turbulent flow

$$\mathrm{Re} = \frac{\text{Inertial forces}}{\text{Viscous forces}} = \frac{V_{avg} D}{\nu} = \frac{\rho V_{avg} D}{\mu}$$

Promotes Laminar flow

D is characteristic length of the Geometry.

Critical Reynolds number, Re_{cr}: The Reynolds number at which the flow becomes turbulent.

The value of the critical Reynolds number is different for different geometries and flow conditions.

For flow through noncircular pipes, the Reynolds number is based on the **hydraulic diameter,**

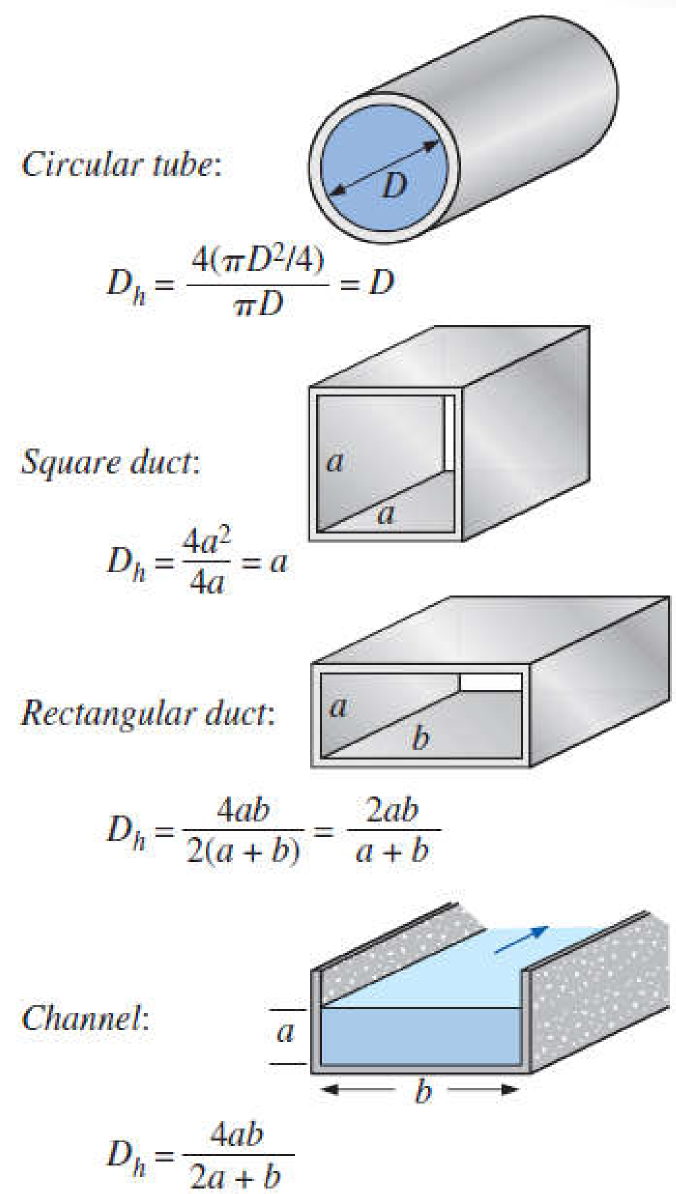
$$D_h = \frac{4A_c}{p}$$

A_c is the cross-sectional area of the pipe and p is its wetted perimeter

For flow in a circular pipe:

$$\begin{aligned} \text{Re} &\lesssim 2300 && \text{laminar flow} \\ 2300 \lesssim \text{Re} &\lesssim 4000 && \text{transitional flow} \\ \text{Re} &\gtrsim 4000 && \text{turbulent flow} \end{aligned}$$

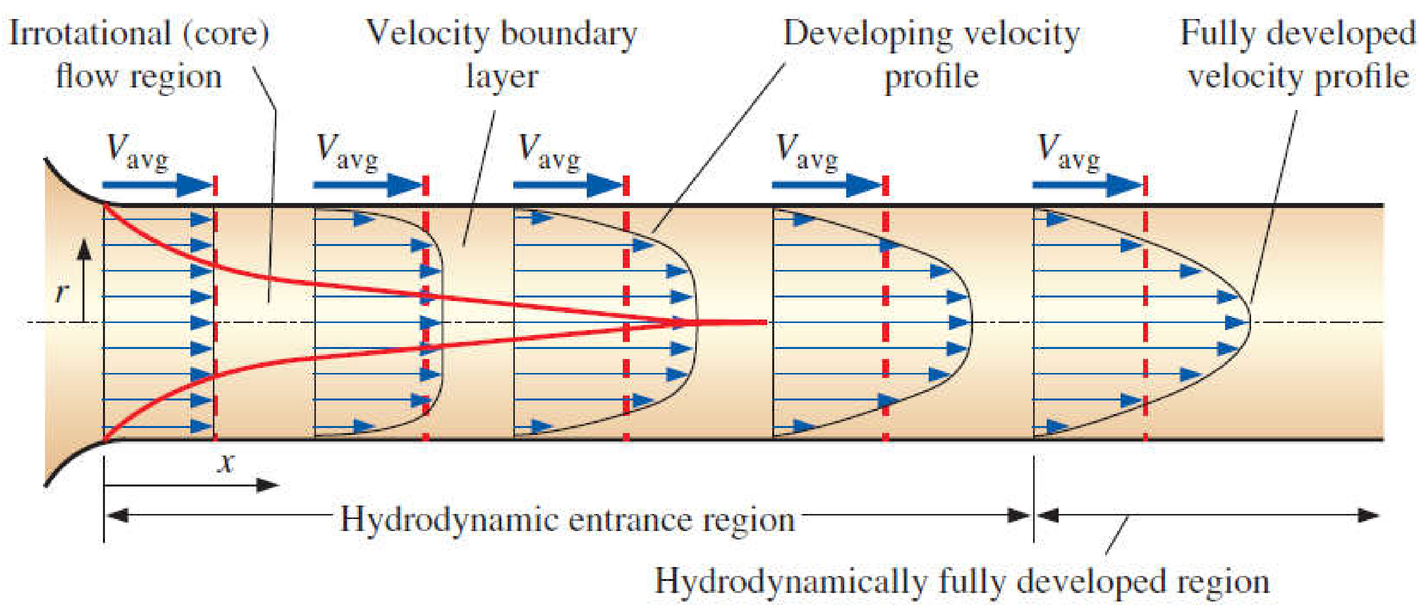
The hydraulic diameter $D_h = 4A_c/p$ is defined such that it reduces to ordinary diameter for circular tubes.



THE ENTRANCE REGION

- Fluid entering a circular pipe at a uniform velocity.
- Because of the no-slip condition, the fluid particles in the layer in contact with the surface of the pipe come to a complete stop.
- This layer also causes the fluid particles in the adjacent layers to slow down gradually as a result of friction.
- To make up for this velocity reduction, the velocity of the fluid at the midsection of the pipe has to increase to keep the mass flow rate through the pipe constant.

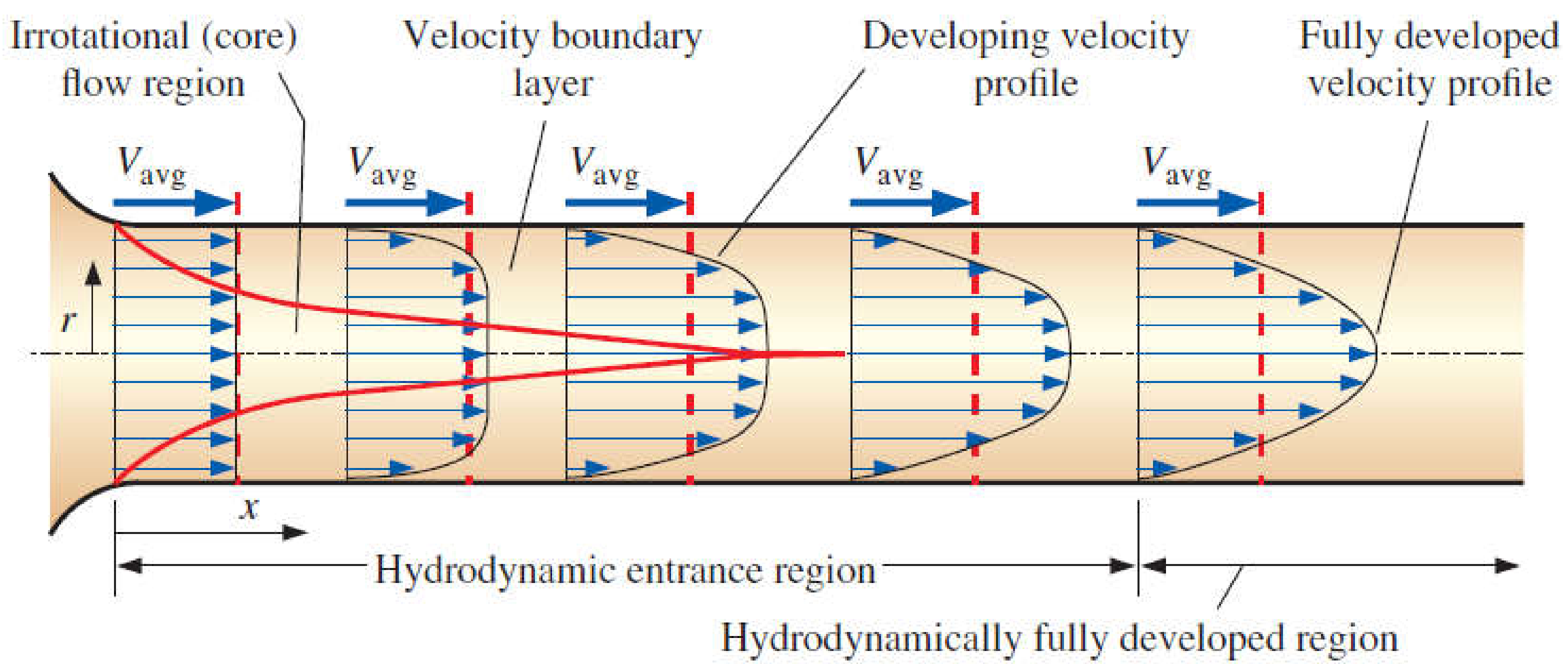
As a result, a velocity gradient develops along the pipe.



THE ENTRANCE REGION

Velocity boundary layer: The region of the flow in which the effects of the viscous shearing forces caused by fluid viscosity are felt. **Or Boundary layer.**

Irrotational (core) flow region: The frictional effects are negligible and the velocity remains essentially constant in the radial direction.



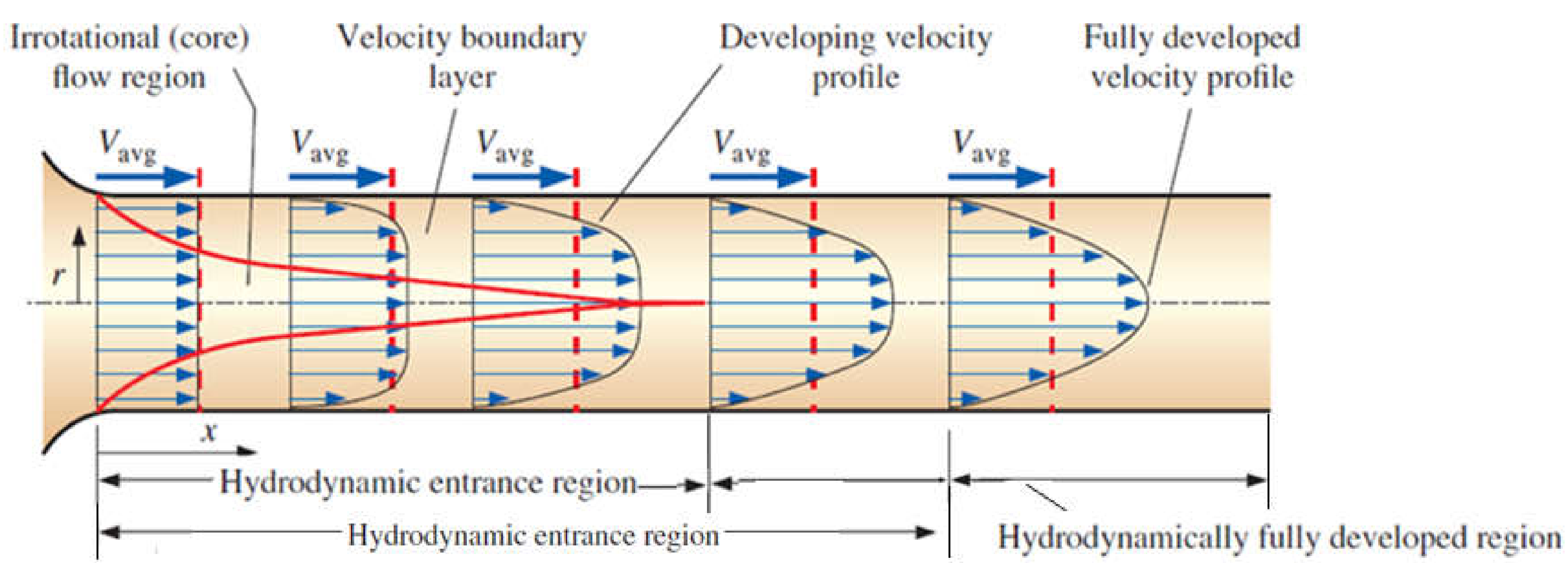
The thickness of this boundary layer increases in the flow direction until the boundary layer reaches the pipe center and thus fills the entire pipe

The region from the pipe inlet to the point at which the boundary layer merges at the centerline **(Hydrodynamic entrance region), and t**he length of this region **Hydrodynamic entry length L_h**

Flow in the entrance region. This is the region where the velocity profile develops (**Hydrodynamically developing flow)**

Hydrodynamically fully developed region: The region beyond the entrance region in which the velocity profile is fully developed and remains unchanged.

Fully developed: When both the velocity profile the normalized temperature profile remain unchanged.



Entry Lengths

The hydrodynamic entry length is usually taken to be the distance from the pipe entrance **to** where the **wall shear stress** (and thus the friction factor) reaches within about 2 percent of the fully developed value.

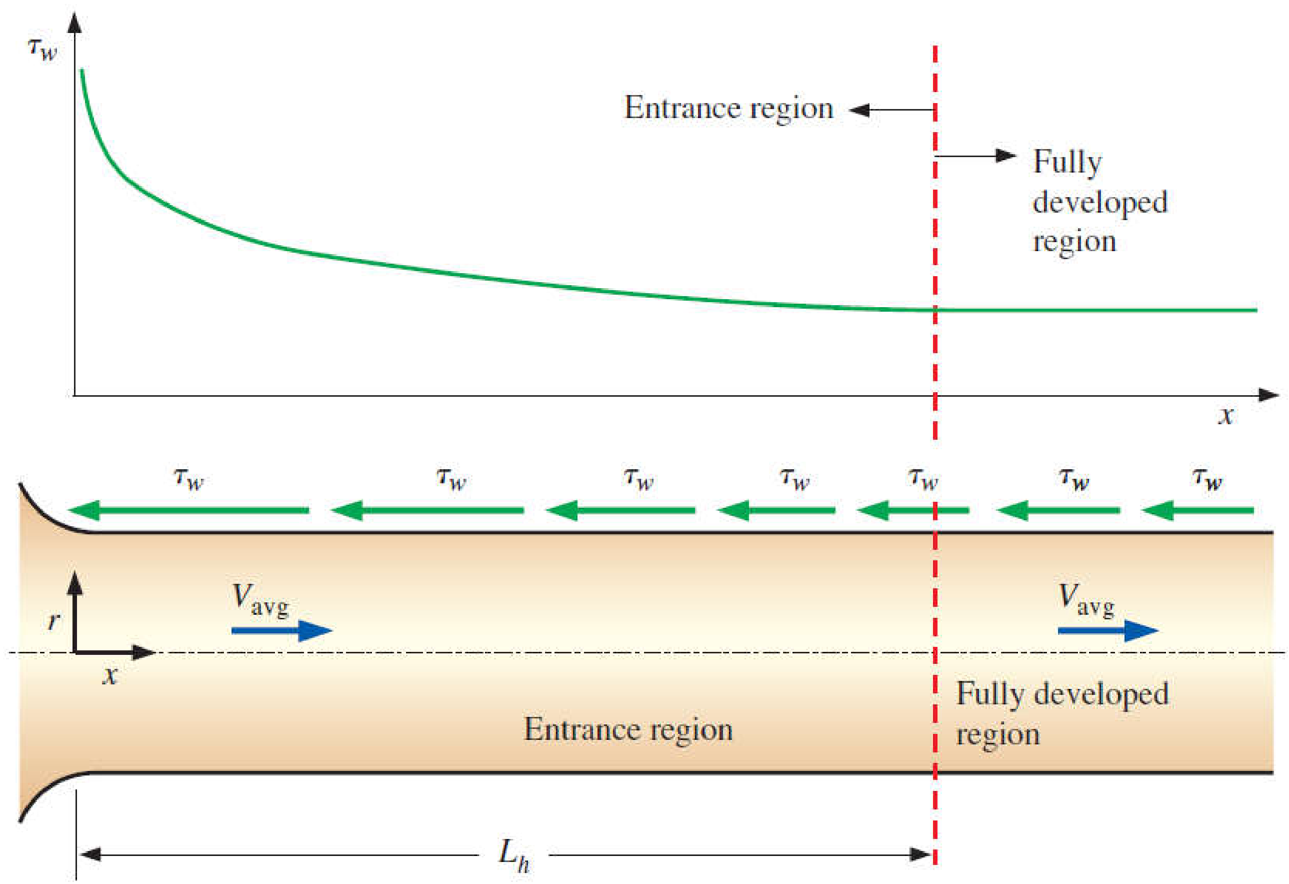
$$\frac{L_{h,\,\text{laminar}}}{D} \cong 0.05\text{Re}$$

hydrodynamic entry length for laminar flow

Re = 20, L = D,   Re = 2300, L = 115D

$$\frac{L_{h,\,\text{turbulent}}}{D} = 1.359\text{Re}^{1/4}$$

hydrodynamic entry length for turbulent flow



Entry length is much shorter in Turbulent flow

$$\frac{L_{h,\,\text{turbulent}}}{D} \approx 10$$

hydrodynamic entry length for turbulent flow, an approximation

LAMINAR FLOW IN PIPES

We consider steady, laminar, incompressible flow of a fluid with constant properties in the fully developed region of a straight circular pipe.

In fully developed laminar flow,

Each fluid particle moves at a constant axial velocity along a streamline, the velocity profile $u(r)$ remains unchanged in the flow direction.

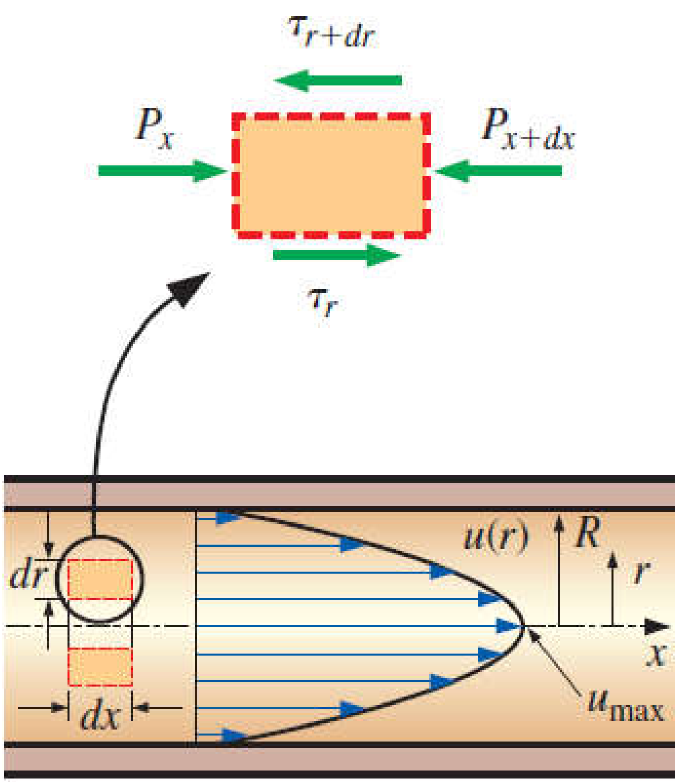
There is no motion in the radial direction, and thus the velocity component in the direction normal to the pipe axis is everywhere zero.

There is no acceleration since the flow is steady and fully developed.

To obtain velocity profile

LAMINAR FLOW IN PIPES

A force balance on the volume element of (radius r, thickness dr, and length dx) in the flow direction gives



To obtain velocity profile

$$u(r) = 2V_{avg}\left(1 - \frac{r^2}{R^2}\right)$$

Velocity profile

$$u_{max} = 2V_{avg}$$

Maximim velocity at centerline

$$(2\pi r\, dr\, P)_x - (2\pi r\, dr\, P)_{x+dx} + (2\pi r\, dx\, \tau)_r - (2\pi r\, dx\, \tau)_{r+dr} = 0$$

Pressure Drop and Head Loss

A pressure drop due to viscous effects represents an irreversible pressure loss, and it is called **pressure loss ΔP_L**.

$$\Delta P_L = f\frac{L}{D}\frac{\rho V_{avg}^2}{2}$$

pressure loss for all types of fully developed internal flows

$$f = \frac{64\mu}{\rho D V_{avg}} = \frac{64}{Re}$$

Circular pipe, laminar

$\rho V_{avg}^2/2$ dynamic pressure

$$f = \frac{8\tau_w}{\rho V_{avg}^2}$$

Darcy friction factor

$$h_L = \frac{\Delta P_L}{\rho g} = f\frac{L}{D}\frac{V_{avg}^2}{2g}$$

Head loss

In laminar flow, the friction factor is a function of the Reynolds number only and is independent of the roughness of the pipe surface.

The head loss represents the additional height that the fluid needs to be raised by a pump in order to overcome the frictional losses in the pipe.

Once the pressure loss (or head loss) is known, the required pumping power *to overcome the pressure loss* is determined from

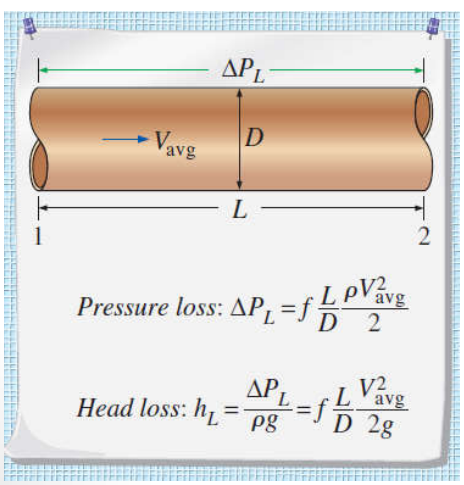
$$\dot{W}_{pump,L} = \dot{V}\,\Delta P_L = \dot{V}\rho g h_L = \dot{m} g h_L$$

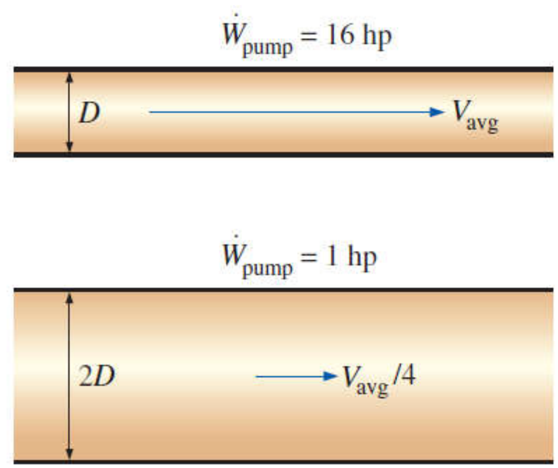
$$\Delta P_L = f\frac{L}{D}\frac{\rho V_{avg}^2}{2}$$

$$V_{avg} = \frac{(P_1 - P_2)R^2}{8\mu L} = \frac{(P_1 - P_2)D^2}{32\mu L} = \frac{\Delta P\, D^2}{32\mu L}$$ → V_{av} Horizontal pipe

$$\dot{V} = V_{avg} A_c = \frac{(P_1 - P_2)R^2}{8\mu L}\pi R^2 = \frac{(P_1 - P_2)\pi D^4}{128\mu L} = \frac{\Delta P\, \pi D^4}{128\mu L}$$ → Poiseuille's law

For a specified flow rate, the pressure drop and thus the required pumping power is **proportional** to the length of the pipe and the viscosity of the fluid, but it is inversely proportional to the fourth power of the diameter of the pipe.





The pressure drop **ΔP equals the pressure loss ΔP_L in the case of a horizontal pipe**, but this is not the case for inclined pipes or pipes with variable cross-sectional area.

This can be demonstrated by writing the energy equation for steady, incompressible one-dimensional flow in terms of heads as

$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_{pump,\,u} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_{turbine,\,e} + h_L$$

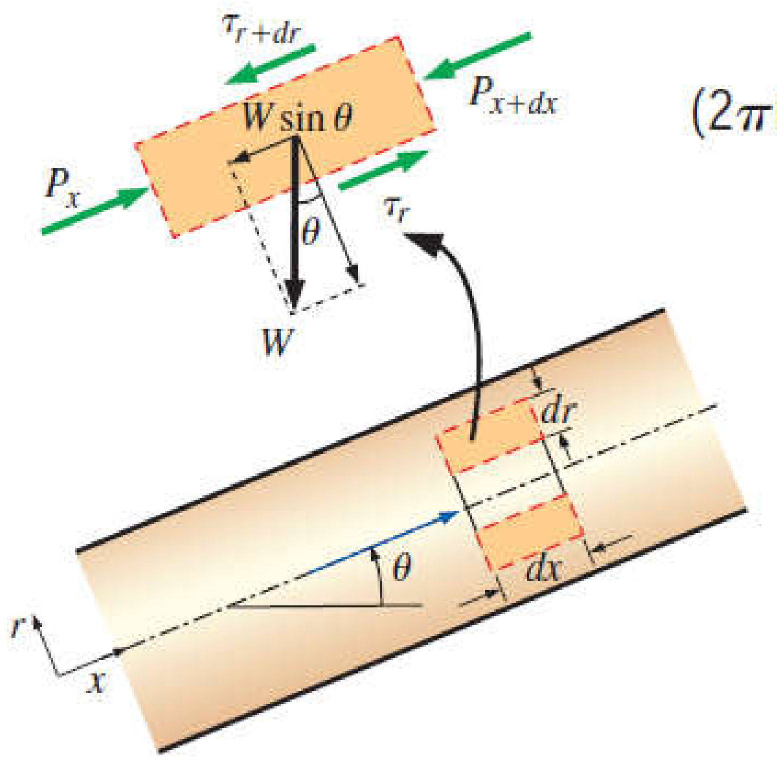
$$P_1 - P_2 = \rho(\alpha_2 V_2^2 - \alpha_1 V_1^2)/2 + \rho g[(z_2 - z_1) + h_{turbine,\,e} - h_{pump,\,u} + h_L]$$

Therefore, the pressure drop $\Delta P = P_1 - P_2$ and pressure loss $\Delta P_L = \rho g h_L$ for a given flow section are equivalent if (1) the flow section is horizontal so that there are no hydrostatic or gravity effects ($z_1 = z_2$), (2) the flow section does not involve any work devices such as a pump or a turbine since they change the fluid pressure ($h_{pump,\,u} = h_{turbine,\,e} = 0$), (3) the cross-sectional area of the flow section is constant and thus the average flow velocity is constant ($V_1 = V_2$), and (4) the velocity profiles at sections 1 and 2 are the same shape ($\alpha_1 = \alpha_2$).

Effect of Gravity on Velocity and Flow Rate in Laminar Flow

$$(2\pi r\, dr\, P)_x - (2\pi r\, dr\, P)_{x+dx} + (2\pi r\, dx\, \tau)_r - (2\pi r\, dx\, \tau)_{r+dr} = 0$$

$$W_x = W \sin\theta = \rho g V_{element} \sin\theta = \rho g (2\pi r\, dr\, dx) \sin\theta$$



$$(2\pi r\, dr\, P)_x - (2\pi r\, dr\, P)_{x+dx} + (2\pi r\, dx\, \tau)_r - (2\pi r\, dx\, \tau)_{r+dr} - \rho g (2\pi r\, dr\, dx) \sin\theta = 0$$

$$\frac{\mu}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right) = \frac{dP}{dx} + \rho g \sin\theta$$

$$u(r) = -\frac{R^2}{4\mu}\left(\frac{dP}{dx} + \rho g \sin\theta\right)\left(1 - \frac{r^2}{R^2}\right)$$

$$V_{avg} = \frac{(\Delta P - \rho g L \sin\theta) D^2}{32\mu L}$$

$$\dot{V} = \frac{(\Delta P - \rho g L \sin\theta)\pi D^4}{128\mu L}$$

Free-body diagram of a ring-shaped differential fluid element of radius *r*, thickness *dr*, and length *dx* oriented coaxially with an inclined pipe in fully developed laminar flow.

Laminar Flow in Circular Pipes

(Fully developed flow with no pump or turbine in the flow section, and

$$\Delta P = P_1 - P_2)$$

Horizontal pipe: $\dot{V} = \dfrac{\Delta P\, \pi D^4}{128\mu L}$

Inclined pipe: $\dot{V} = \dfrac{(\Delta P - \rho g L \sin\theta)\pi D^4}{128\mu L}$

Uphill flow: $\theta > 0$ and $\sin\theta > 0$

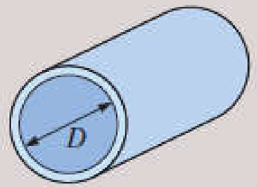
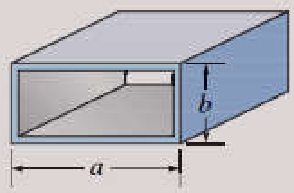
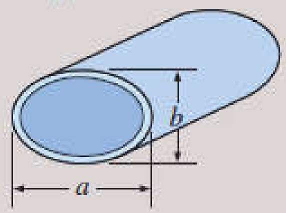
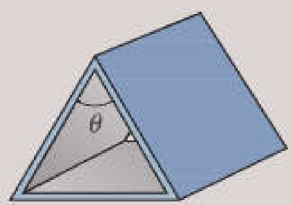
Downhill flow: $\theta < 0$ and $\sin\theta < 0$

The relations developed for fully developed laminar flow through horizontal pipes can also be used for inclined pipes by replacing ΔP with $\Delta P - \rho g L \sin\theta$.

Laminar Flow in Noncircular Pipes

The friction factor *f* relations are given in Table 8–1 for *fully developed laminar flow* in pipes of various cross sections. The Reynolds number for flow in these pipes is based on the hydraulic diameter $D_h = 4A_c/p$, where A_c is the cross-sectional area of the pipe and p is its wetted perimeter

Friction factor for fully developed *laminar flow* in pipes of various cross sections ($D_h = 4A_c/p$ and $\text{Re} = V_{avg} D_h/\nu$)

Tube Geometry	a/b or $\theta°$	Friction Factor f
Circle 	—	64.00/Re
Rectangle 	a/b	
	1	56.92/Re
	2	62.20/Re
	3	68.36/Re
	4	72.92/Re
	6	78.80/Re
	8	82.32/Re
	∞	96.00/Re
Ellipse 	a/b	
	1	64.00/Re
	2	67.28/Re
	4	72.96/Re
	8	76.60/Re
	16	78.16/Re
Isosceles triangle 	θ	
	10°	50.80/Re
	30°	52.28/Re
	60°	53.32/Re
	90°	52.60/Re
	120°	50.96/Re

Summary for Laminar flow

$$u(r) = 2V_{avg}\left(1 - \frac{r^2}{R^2}\right)$$

$$u_{max} = 2V_{avg}$$

$$\Delta P_L = f\frac{L}{D}\frac{\rho V_{avg}^2}{2}$$

$$f = \frac{64\mu}{\rho D V_{avg}} = \frac{64}{\text{Re}}$$

$$h_L = \frac{\Delta P_L}{\rho g} = f\frac{L}{D}\frac{V_{avg}^2}{2g}$$

$$V_{avg} = \frac{\Delta P\, D^2}{32\mu L}$$

$$\dot{V} = V_{avg} A_c = \frac{\Delta P\, \pi D^4}{128\mu L}$$

For Horizontal pipe

$$V_{avg} = \frac{(\Delta P - \rho g L \sin\theta) D^2}{32\mu L}$$

$$\dot{V} = \frac{(\Delta P - \rho g L \sin\theta)\pi D^4}{128\mu L}$$

With gravity effect

$$\dot{W}_{pump,\,L} = \dot{V}\,\Delta P_L = \dot{V}\rho g h_L = \dot{m} g h_L$$

EXAMPLE 8–1 **Laminar Flow in Horizontal and Inclined Pipes**

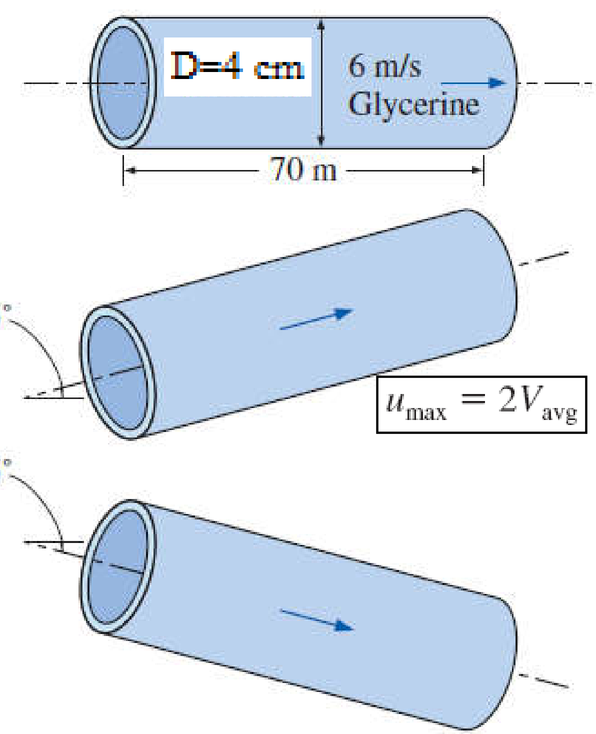
Consider the fully developed flow of glycerin at 40°C through a 70-m-long, 4-cm-diameter, horizontal, circular pipe. If the flow velocity at the centerline is measured to be 6 m/s, determine the velocity profile and the pressure difference across this 70-m-long section of the pipe, and the useful pumping power required to maintain this flow. For the same useful pumping power input, determine the percent increase of the flow rate if the pipe is inclined 15° downward and the percent decrease if it is inclined 15° upward. The pump is located outside this pipe section.

Properties The density and dynamic viscosity of glycerin at 40°C are $\rho = 1252\ kg/m^3$ and $\mu = 0.3073\ kg/m\cdot s$, respectively.

$$u(r) = 2V_{avg}\left(1 - \frac{r^2}{R^2}\right)$$

$$h_L = \frac{\Delta P_L}{\rho g} = f\frac{L}{D}\frac{V_{avg}^2}{2g}$$

$$f = \frac{64\mu}{\rho D V_{avg}} = \frac{64}{Re}$$



$$\frac{P_1}{\rho g} + \alpha_1\frac{V_1^2}{2g} + z_1 + h_{pump,\,u} = \frac{P_2}{\rho g} + \alpha_2\frac{V_2^2}{2g} + z_2 + h_{turbine,\,e} + h_L$$

$$\dot{W}_{pump,\,L} = \dot{V}\Delta P_L = \dot{V}\rho g h_L = \dot{m} g h_L$$

$$\dot{V} = V_{avg} A_c$$

$$u(r) = u_{max}\left(1 - \frac{r^2}{R^2}\right)$$

Substituting, the velocity profile is determined to be

$$u(r) = (6\text{ m/s})\left(1 - \frac{r^2}{(0.02\text{ m})^2}\right) = \mathbf{6(1 - 2500r^2)}$$

where u is in m/s and r is in m. The average velocity, the flow rate, and the Reynolds number are

$$V = V_{avg} = \frac{u_{max}}{2} = \frac{6\text{ m/s}}{2} = 3\text{ m/s}$$

$$\dot{V} = V_{avg}A_c = V(\pi D^2/4) = (3\text{ m/s})[\pi(0.04\text{ m})^2/4] = 3.77 \times 10^{-3}\text{ m}^3\text{/s}$$

$$\text{Re} = \frac{\rho VD}{\mu} = \frac{(1252\text{ kg/m}^3)(3\text{ m/s})(0.04\text{ m})}{0.3073\text{ kg/m}\cdot\text{s}} = 488.9$$

which is less than 2300. Therefore, the flow is indeed laminar. Then the friction factor and the head loss become

$$f = \frac{64}{\text{Re}} = \frac{64}{488.9} = 0.1309$$

$$h_L = f\frac{LV^2}{D\,2g} = 0.1309\frac{(70\text{ m})}{(0.04\text{ m})}\frac{(3\text{ m/s})^2}{2(9.81\text{ m/s}^2)} = 105.1\text{m}$$

The energy balance for steady, incompressible one-dimensional flow is given by Eq. 8–28 as

$$\frac{P_1}{\rho g} + \alpha_1\frac{V_1^{\,2}}{2g} + z_1 + h_{\text{pump, u}} = \frac{P_2}{\rho g} + \alpha_2\frac{V_2^{\,2}}{2g} + z_2 + h_{\text{turbine, e}} + h_L$$

For fully developed flow in a constant diameter pipe with no pumps or turbines, it reduces to

$$\Delta P = P_1 - P_2 = \rho g(z_2 - z_1 + h_L)$$

Then the pressure difference and the required useful pumping power for the horizontal case become

$$\Delta P = \rho g(z_2 - z_1 + h_L)$$
$$= (1252\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0 + 105.1\ \text{m})\left(\frac{1\ \text{kPa}}{1000\ \text{kg/m}\cdot\text{s}^2}\right)$$
$$= \mathbf{1291\ kPa}$$

$$\dot{W}_{\text{pump, u}} = \dot{V}\Delta P = (3.77 \times 10^3\ \text{m}^3/\text{s})(1291\ \text{kPa})\left(\frac{1\ \text{kW}}{\text{kPa}\cdot\text{m}^3/\text{s}}\right) = \mathbf{4.87\ kW}$$

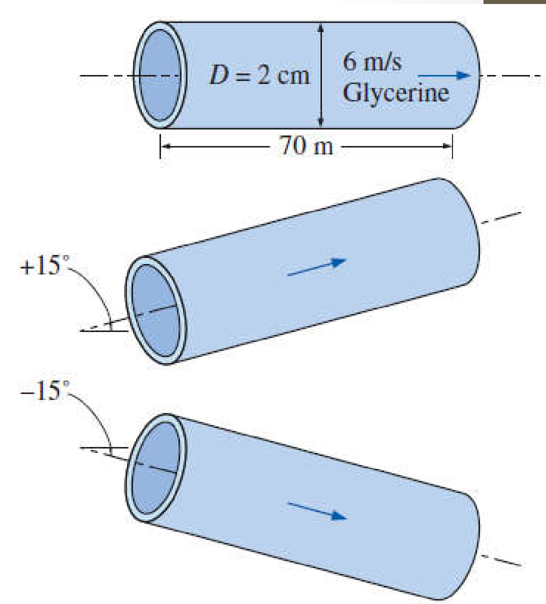
The elevation difference and the pressure difference for a pipe inclined upwards 15° is

$$\Delta z = z_2 - z_1 = L\sin 15° = (70\ \text{m})\sin 15° = 18.1\ \text{m}$$

$$\Delta P_{\text{upward}} = (1252\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(18.1\ \text{m} + 105.1\ \text{m})\left(\frac{1\ \text{kPa}}{1000\ \text{kg/m}\cdot\text{s}^2}\right)$$
$$= 1366\ \text{kPa}$$

Then the flow rate through the upward inclined pipe becomes

$$\dot{V}_{\text{upward}} = \frac{\dot{W}_{\text{pump, }u}}{\Delta P_{\text{upward}}} = \frac{4.87\ \text{kW}}{1366\ \text{kPa}}\left(\frac{1\ \text{kPa}\cdot\text{m}^3/\text{s}}{1\ \text{kW}}\right) = 3.57 \times 10^{-3}\ \text{m}^3/\text{s}$$



$$\dot{V} = V_{avg}A_c = V(\pi D^2/4) = (3\ \text{m/s})[\pi(0.04\ \text{m})^2/4] = 3.77 \times 10^{-3}\ \text{m}^3/\text{s}$$

Then the flow rate through the upward inclined pipe becomes

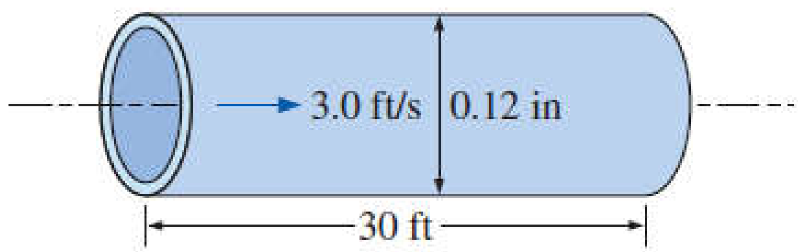
$$\dot{V}_{upward} = \frac{\dot{W}_{pump,\,u}}{\Delta P_{upward}} = \frac{4.87\ \text{kW}}{1366\ \text{kPa}}\left(\frac{1\ \text{kPa·m}^3/\text{s}}{1\ \text{kW}}\right) = 3.57 \times 10^{-3}\ \text{m}^3/\text{s}$$

which is a decrease of **5.6 percent** in flow rate. It can be shown similarly that when the pipe is inclined 15° downward from the horizontal, the flow rate will increase by **5.6 percent.**

Discussion Note that the flow is driven by the combined effect of pumping power and gravity. As expected, gravity opposes uphill flow, enhances downhill flow, and has no effect on horizontal flow. Downhill flow can occur even in the absence of a pressure difference applied by a pump. For the case of $P_1 = P_2$ (i.e., no applied pressure difference), the pressure throughout the entire pipe would remain constant, and the fluid would flow through the pipe under the influence of gravity at a rate that depends on the angle of inclination, reaching its maximum value when the pipe is vertical. When solving pipe flow problems, it is always a good idea to calculate the Reynolds number to verify the flow regime—laminar or turbulent.

EXAMPLE 8–2 **Pressure Drop and Head Loss in a Pipe**

Water at 40°F (ρ = 62.42 lbm/ft^3 and μ = 1.038 × 10^{-3} lbm/ft·s) is flowing steadily through a 0.12-in- (= 0.010 ft) diameter 30-ft-long horizontal pipe at an average velocity of 3.0 ft/s (Fig. 8–18). Determine (*a*) the head loss, (*b*) the pressure drop, and (*c*) the pumping power requirement to overcome this pressure drop.



$$h_L = \frac{\Delta P_L}{\rho g} = f \frac{L}{D} \frac{V_{avg}^2}{2g}$$

$$f = \frac{64\mu}{\rho D V_{avg}} = \frac{64}{Re}$$

$$\dot{W}_{pump,\,L} = \dot{V}\,\Delta P_L$$

Properties The density and dynamic viscosity of water are given to be ρ = 62.42 lbm/ft^3 and μ = 1.038 × 10^{-3} lbm/ft·s, respectively.

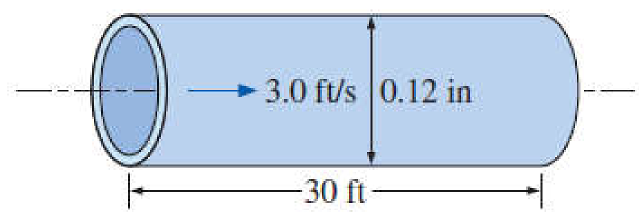
Analysis (*a*) First we need to determine the flow regime. The Reynolds number is

$$\text{Re} = \frac{\rho V_{avg} D}{\mu} = \frac{(62.42 \text{ lbm/ft}^3)(3 \text{ ft/s})(0.01 \text{ ft})}{1.038 \times 10^{-3} \text{ lbm/ft·s}} = 1803$$

which is less than 2300. Therefore, the flow is laminar. Then the friction factor and the head loss become

$$f = \frac{64}{\text{Re}} = \frac{64}{1803} = 0.0355$$

$$h_L = f\frac{L}{D}\frac{V_{avg}^2}{2g} = 0.0355\frac{30 \text{ ft}}{0.01 \text{ ft}}\frac{(3 \text{ ft/s})^2}{2(32.2 \text{ ft/s}^2)} = \mathbf{14.9 \text{ ft}}$$



(*b*) Noting that the pipe is horizontal and its diameter is constant, the pressure drop in the pipe is due entirely to the frictional losses and is equivalent to the pressure loss,

$$\Delta P = \Delta P_L = f\frac{L}{D}\frac{\rho V_{avg}^2}{2} = 0.0355\frac{30 \text{ ft}}{0.01 \text{ ft}}\frac{(62.42 \text{ lbm/ft}^3)(3 \text{ ft/s})^2}{2}\left(\frac{1 \text{ lbf}}{32.2 \text{ lbm·ft/s}^2}\right)$$

$$= \mathbf{929 \text{ lbf/ft}^2 = 6.45 \text{ psi}}$$

(*c*) The volume flow rate and the pumping power requirements are

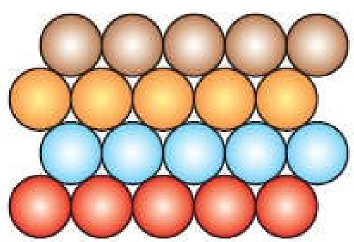
$$\dot{V} = V_{avg}A_c = V_{avg}(\pi D^2/4) = (3 \text{ ft/s})[\pi(0.01 \text{ ft})^2/4] = 0.000236 \text{ ft}^3\text{/s}$$

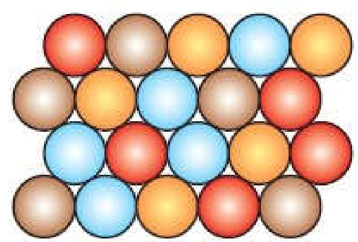
$$\dot{W}_{pump} = \dot{V}\,\Delta P = (0.000236 \text{ ft}^3\text{/s})(929 \text{ lbf/ft}^2)\left(\frac{1 \text{ W}}{0.737 \text{ lbf·ft/s}}\right) = \mathbf{0.30 \text{ W}}$$

TURBULENT FLOW IN PIPES

Turbulent flow is a complex mechanism dominated by fluctuations, and it is still not fully understood.

We must rely on experiments and the empirical or semi-empirical correlations developed for various situations.


(*a*) Before turbulence


(*b*) After turbulence

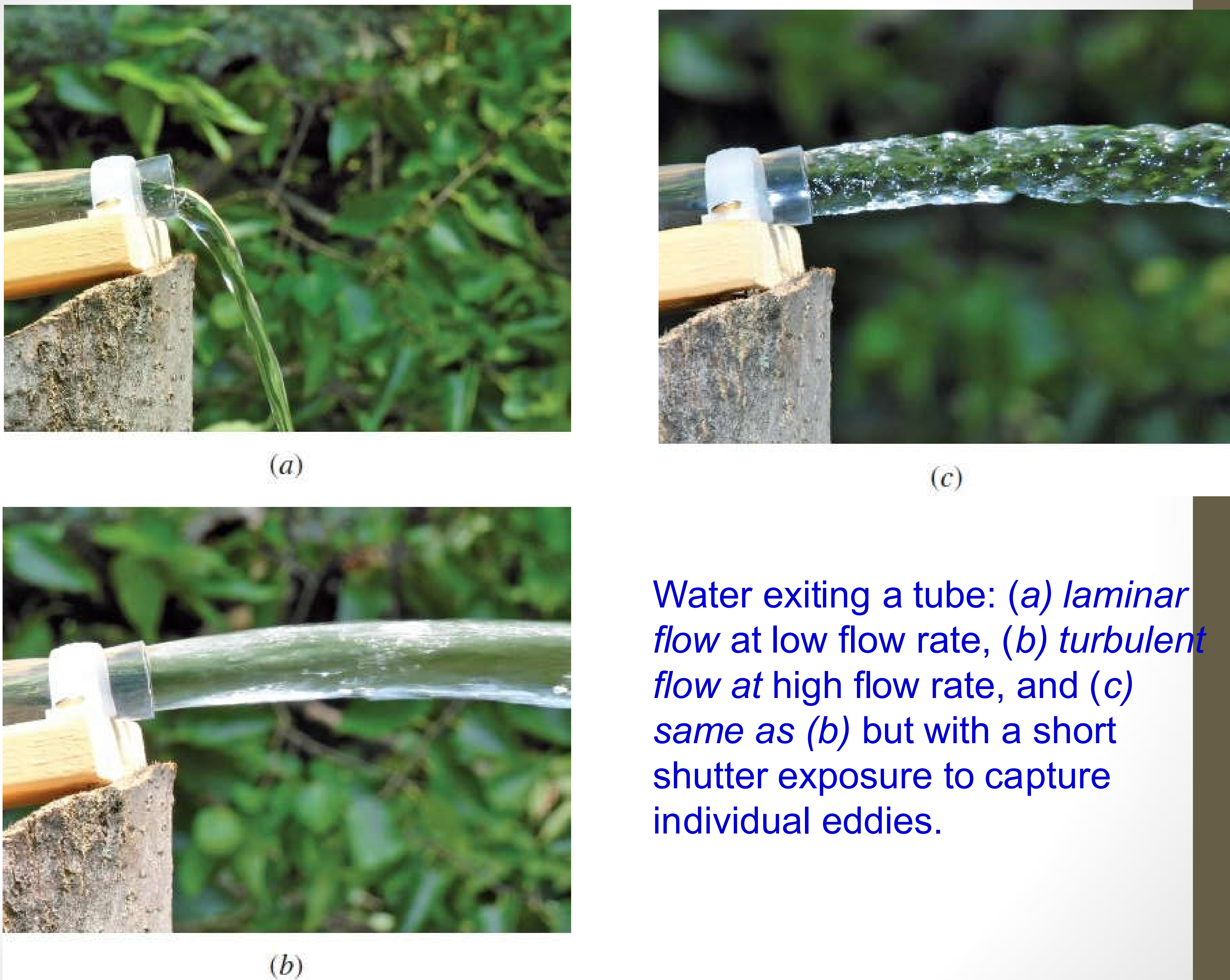
The intense mixing in turbulent flow brings fluid particles at different momentums into close contact and thus enhances momentum transfer.

Turbulent flow is characterized by **disorderly and rapid fluctuations of swirling regions of fluid, called eddies**, throughout the flow.

These fluctuations **provide an additional mechanism for momentum and energy transfer**.

In turbulent flow, **the swirling eddies transport mass, momentum, and energy to other regions of flow** much more rapidly than molecular diffusion, greatly **enhancing mass, momentum, and heat transfer**.

As a result, **turbulent flow is associated with much higher values of friction**, heat transfer, and mass transfer coefficients



Water exiting a tube: (*a) laminar flow* at low flow rate, (*b) turbulent flow at* high flow rate, and (*c) same as (b)* but with a short shutter exposure to capture individual eddies.

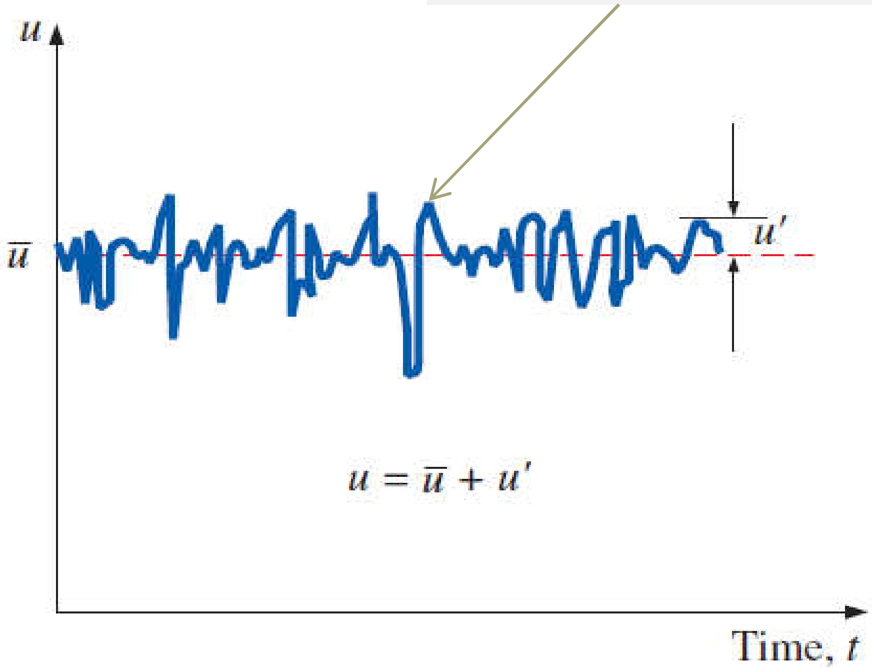
Even when the average flow is steady, the eddy motion in turbulent flow causes fluctuations in the values of velocity, temperature, pressure, and even density

$$v = \bar{v} + v', P = \bar{P} + P' \quad T = \bar{T} + T'$$

- values of the velocity fluctuate about an average value,
- The **velocity** can be expressed as the sum of an *average value* u^{-} and a *fluctuating component* u'

$$u = \bar{u} + u'$$

Variation of u with time



The magnitude of u' is usually just a few percent of u^{-} but the high frequencies of eddies makes them very effective for the transport of momentum, thermal energy, and mass.

To determine the shear stress the experimental studies show that the shear stress is much larger than that in laminar flow due to the turbulent fluctuations.

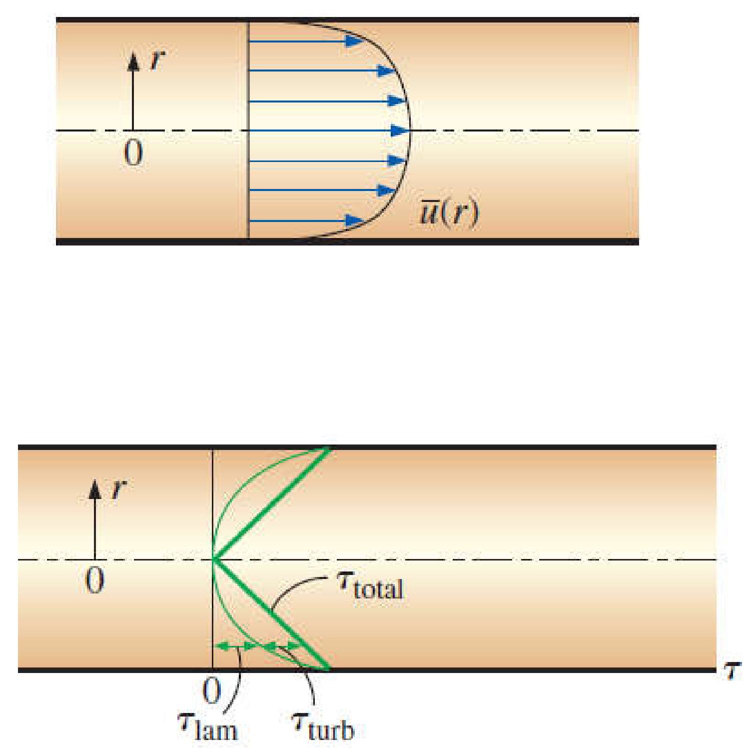
The turbulent shear stress consist of two parts

The *laminar component:* accounts for the friction between layers in the flow direction

The *turbulent component:* accounts for the friction between the fluctuating fluid particles and the fluid body (related to the fluctuation components of velocity).

Then the *total shear stress* in turbulent flow can be expressed as

$$\tau_{total} = \tau_{lam} + \tau_{turb}$$



The velocity profile and the variation of shear stress with radial distance for turbulent flow in a pipe.

Turbulent Shear Stress

- Turbulent flow in a horizontal pipe, the eddies results in velocity fluctuation *(v')*
- *Mass rising* $\rho v' dA$,
- Reduction in average flow velocity because of momentum transfer to the fluid particles with lower average flow velocity.

The shear force per unit area due to the eddy motion of fluid particles can be viewed as the instantaneous turbulent shear stress.

$$\delta F/dA = -\rho u'v'$$

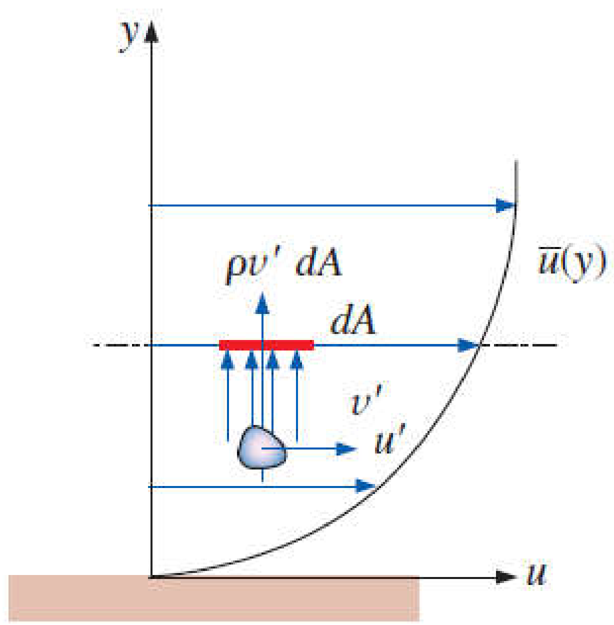
$$\tau_{turb} = -\rho\overline{u'v'}$$

turbulent shear stress

Terms such as $-\rho\overline{u'v'}$ or $-\rho\overline{u'^2}$ are called **Reynolds stresses** or **turbulent stresses**.

$$\tau_{turb} = -\rho\overline{u'v'} = \mu_t \frac{\partial \bar{u}}{\partial y}$$

Turbulent shear stress



μ_t **eddy viscosity** or **turbulent viscosity**: accounts for momentum transport by turbulent eddies.

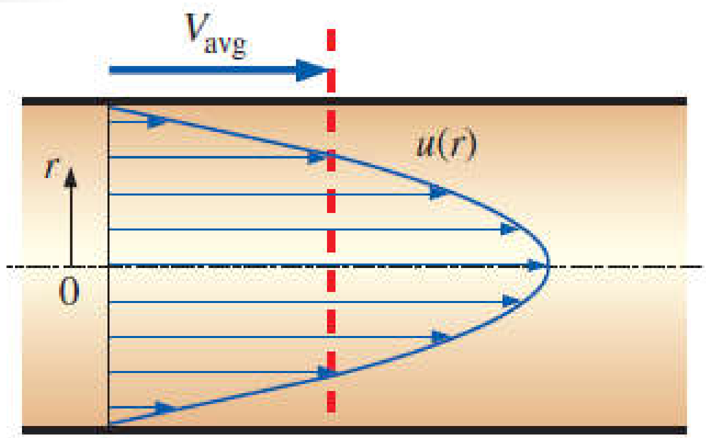
$$\tau_{total} = (\mu + \mu_t)\frac{\partial \bar{u}}{\partial y} = \rho(\nu + \nu_t)\frac{\partial \bar{u}}{\partial y}$$

Total shear stress

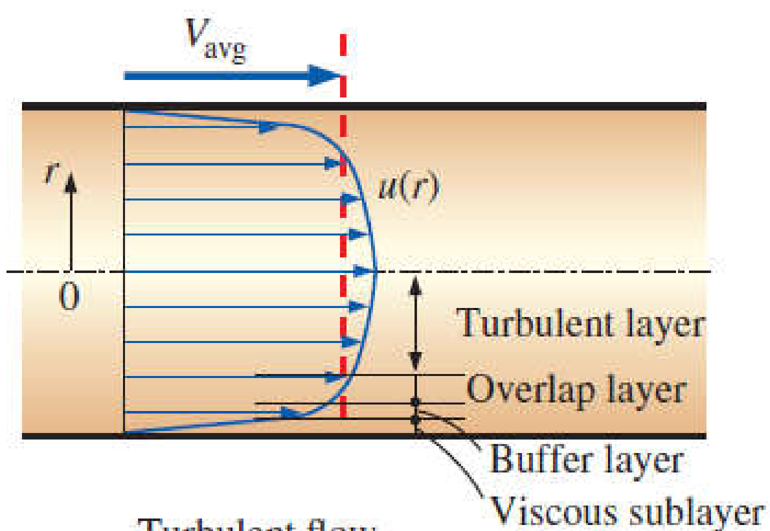
$$\nu_t = \mu_t/\rho$$

kinematic eddy viscosity or **kinematic turbulent viscosity** (also called the *eddy diffusivity of momentum*).

Turbulent Velocity Profile



Laminar flow



Turbulent flow

the **viscous** (or **laminar** or **linear** or **wall**) sublayer. The velocity profile in this layer is very nearly *linear*, and the flow is streamlined.

the **buffer layer**, in which turbulent effects are becoming significant, but the flow is still dominated by viscous effects.

the **overlap** (or **transition**) **layer**, also called the **inertial sublayer**, in which the turbulent effects are much more significant, but still not dominant.

the **outer** (or **turbulent**) **layer** in the remaining part of the flow in which turbulent effects dominate over molecular diffusion (viscous) effects.

The velocity profile in fully developed pipe flow is parabolic in laminar flow, but much fuller in turbulent flow. Note that $u(r)$ in the turbulent case is the *time-averaged* velocity component in the axial direction (the overbar on u has been dropped for simplicity).

Then the velocity gradient in the viscous sublayer remains nearly constant at $du/dy = u/y$, and the wall shear stress

$$\tau_w = \mu \frac{u}{y} = \rho \nu \frac{u}{y} \qquad \text{or} \qquad \frac{\tau_w}{\rho} = \frac{\nu u}{y}$$

$$u_* = \sqrt{\tau_w / \rho}$$ friction velocity

Viscous sublayer: $$\frac{u}{u_*} = \frac{y u_*}{\nu}$$ law of the wall

for smooth surfaces for $0 \le y u_*/\nu \le 5$.

Thickness of viscous sublayer: $$y = \delta_{\text{sublayer}} = \frac{5\nu}{u_*} = \frac{25\nu}{u_\delta}$$

where u_δ is the flow velocity at the edge of the viscous sublayer,

The thickness of the viscous sublayer is proportional to the kinematic viscosity and inversely proportional to the average flow velocity.

ν/u_* Viscous length; it is used to nondimensionalize the distance *y* from the surface.

In the overlap layer, the experimental data for velocity are observed to line up on a straight line when plotted against the logarithm of distance from the wall. (**velocity in the overlap layer is proportional to the logarithm of distance**)

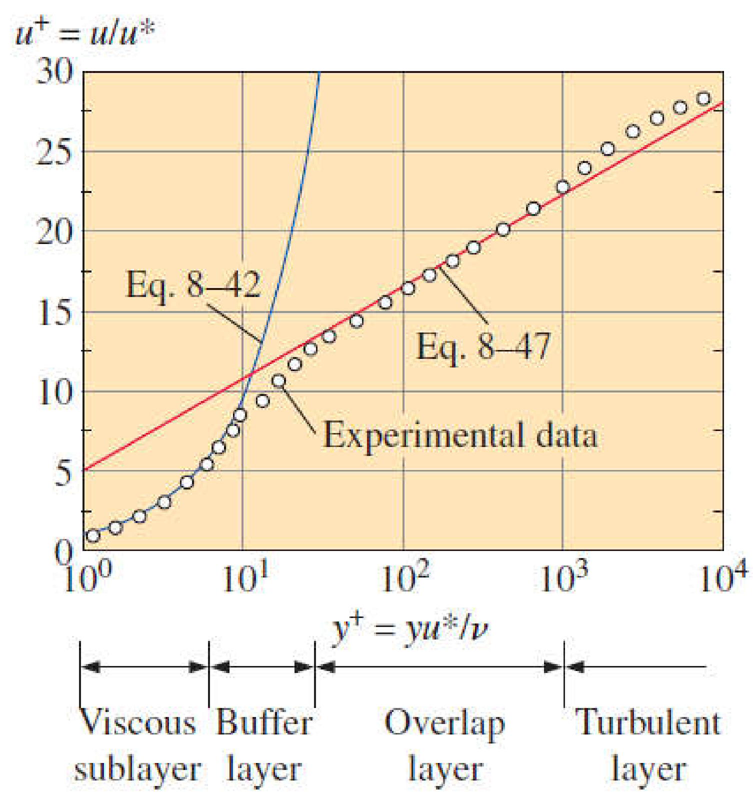
The logarithmic law:

$$\frac{u}{u_*} = \frac{1}{\kappa} \ln \frac{y u_*}{\nu} + B$$

where k and *B* are constants values are determined experimentally to be (0.40 and 5.0) respectively

Overlap layer: $\frac{u}{u_*} = 2.5 \ln \frac{y u_*}{\nu} + 5.0$ or $u^+ = 2.5 \ln y^+ + 5.0$

Where $y^+ = \frac{y u_*}{\nu}$



Comparison of the law of the wall and the logarithmic-law velocity profiles with experimental data for fully developed turbulent flow in a pipe.

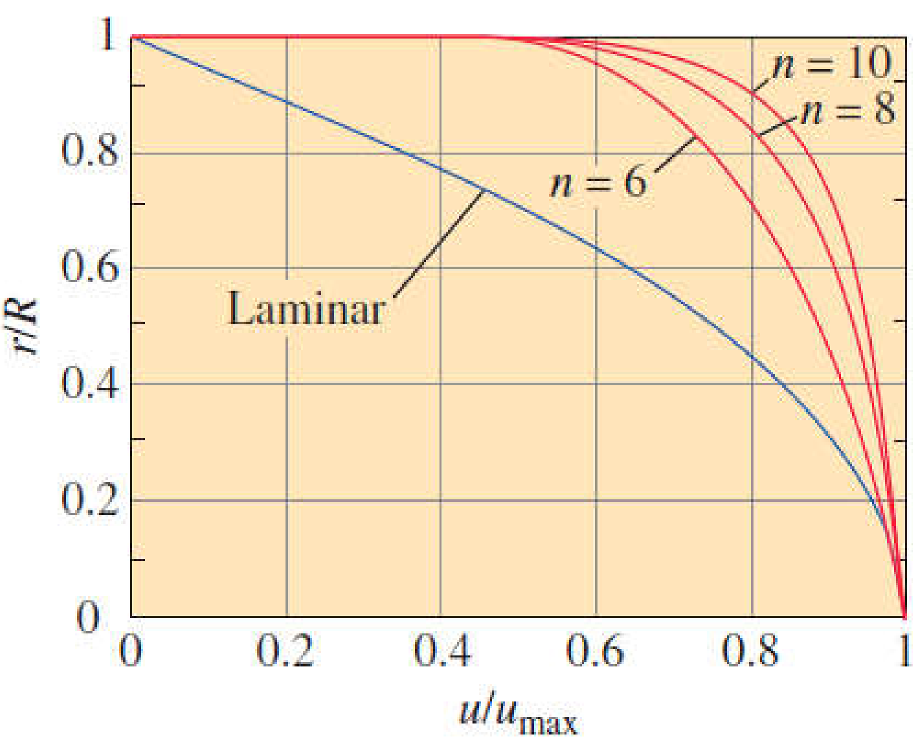
Outer turbulent layer: $$\frac{u_{max} - u}{u_*} = 2.5 \ln \frac{R}{R - r}$$

Velocity defect law

The deviation of velocity from the centerline value u_{max} - u is called the velocity defect.

Numerous other empirical velocity profiles exist for turbulent pipe flow. Among those, the simplest and the best known is the **power-law velocity profile** expressed as

Power-law velocity profile: $$\frac{u}{u_{max}} = \left(\frac{y}{R}\right)^{1/n} \quad \text{or} \quad \frac{u}{u_{max}} = \left(1 - \frac{r}{R}\right)^{1/n}$$



The value n = 7 generally approximates many flows in practice, giving rise to the term *one-seventh power-law velocity profile*.

Power-law velocity profiles for fully developed turbulent flow in a pipe for different exponents, and its comparison with the laminar velocity profile.

The Moody Chart and the Colebrook Equation

The friction factor in fully developed turbulent pipe flow depends on the Reynolds number and the **relative roughness** ε/D.

The experimental results obtained are presented in tabular, graphical, and functional forms obtained by curve-fitting experimental data.

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) \qquad \text{(turbulent flow)}$$

Colebrook equation (for smooth and rough pipes)

It presents the **Darcy friction** factor for pipe flow as a function of the Reynolds number and ε/D over a wide range.

Equivalent roughness values for some commercial pipes are given as well as on the Moody chart

Equivalent roughness values for new commercial pipes*

Material	Roughness, ε ft	Roughness, ε mm
Glass, plastic	0 (smooth)	
Concrete	0.003–0.03	0.9–9
Wood stave	0.0016	0.5
Rubber, smoothed	0.000033	0.01
Copper or brass tubing	0.000005	0.0015
Cast iron	0.00085	0.26
Galvanized iron	0.0005	0.15
Wrought iron	0.00015	0.046
Stainless steel	0.000007	0.002
Commercial steel	0.00015	0.045

The Moody Chart and the Colebrook Equation

The results obtained should not be treated as "exact." It is usually considered to be accurate to ±15%.

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{Re\sqrt{f}}\right)$$

The Colebrook equation is *implicit* in f since f appears on both sides of the equation. It must be solved iteratively.

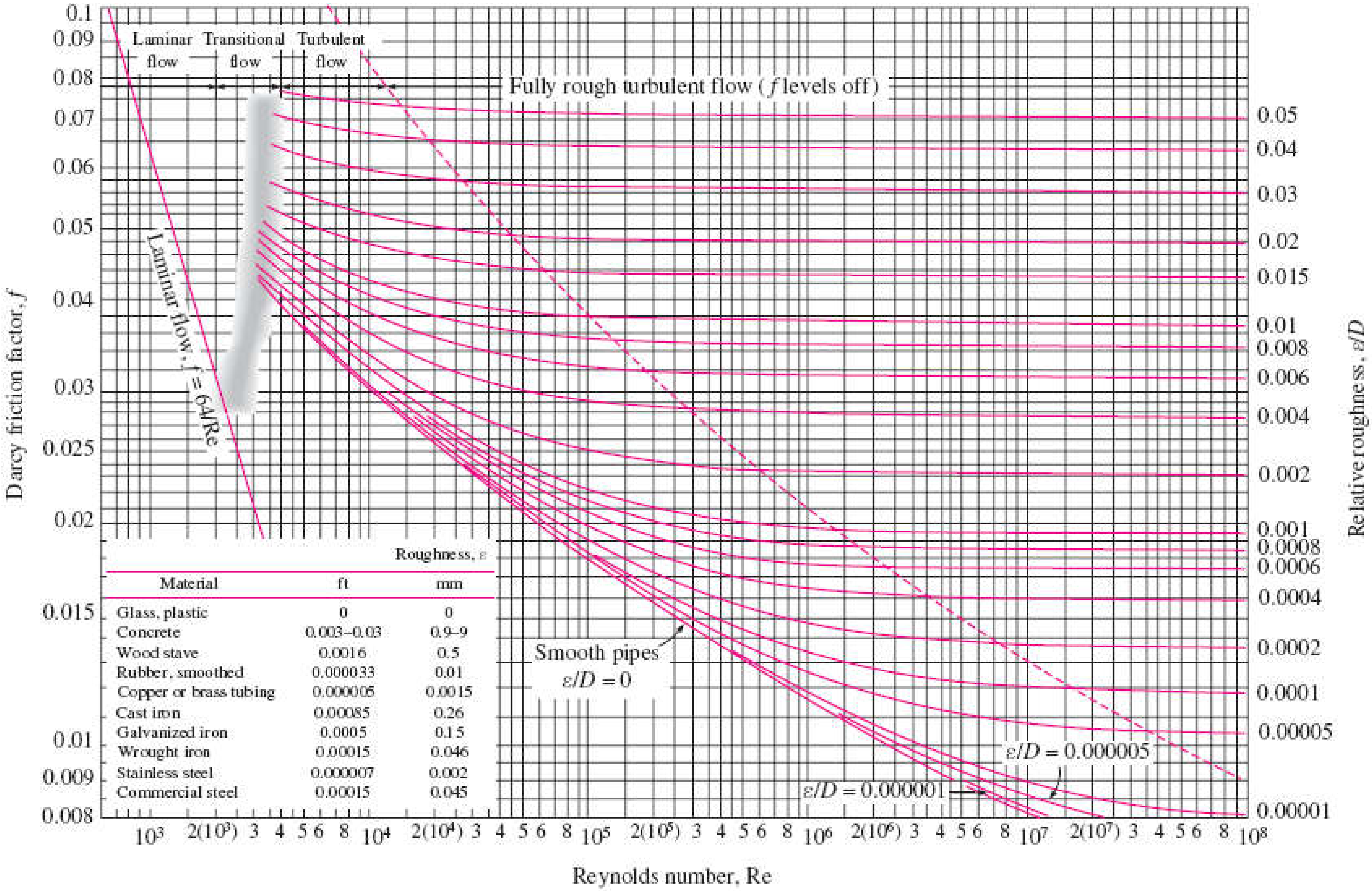
$$\frac{1}{\sqrt{f}} \cong -1.8 \log\left[\frac{6.9}{Re} + \left(\frac{\varepsilon/D}{3.7}\right)^{1.11}\right]$$

Explicit Haaland equation,
For more accurate results

Relative Roughness, ε/D	Friction Factor, f
0.0*	0.0119
0.00001	0.0119
0.0001	0.0134
0.0005	0.0172
0.001	0.0199
0.005	0.0305
0.01	0.0380
0.05	0.0716

*Smooth surface. All values are for Re = 10^6 and are calculated from the Colebrook equation.

The friction factor is minimum for a smooth pipe and increases with roughness.



	Roughness, ε	
Material	ft	mm
Glass, plastic	0	0
Concrete	0.003–0.03	0.9–9
Wood stave	0.0016	0.5
Rubber, smoothed	0.000033	0.01
Copper or brass tubing	0.000005	0.0015
Cast iron	0.00085	0.26
Galvanized iron	0.0005	0.15
Wrought iron	0.00015	0.046
Stainless steel	0.000007	0.002
Commercial steel	0.00015	0.045

The Moody Chart

Observations from the Moody chart

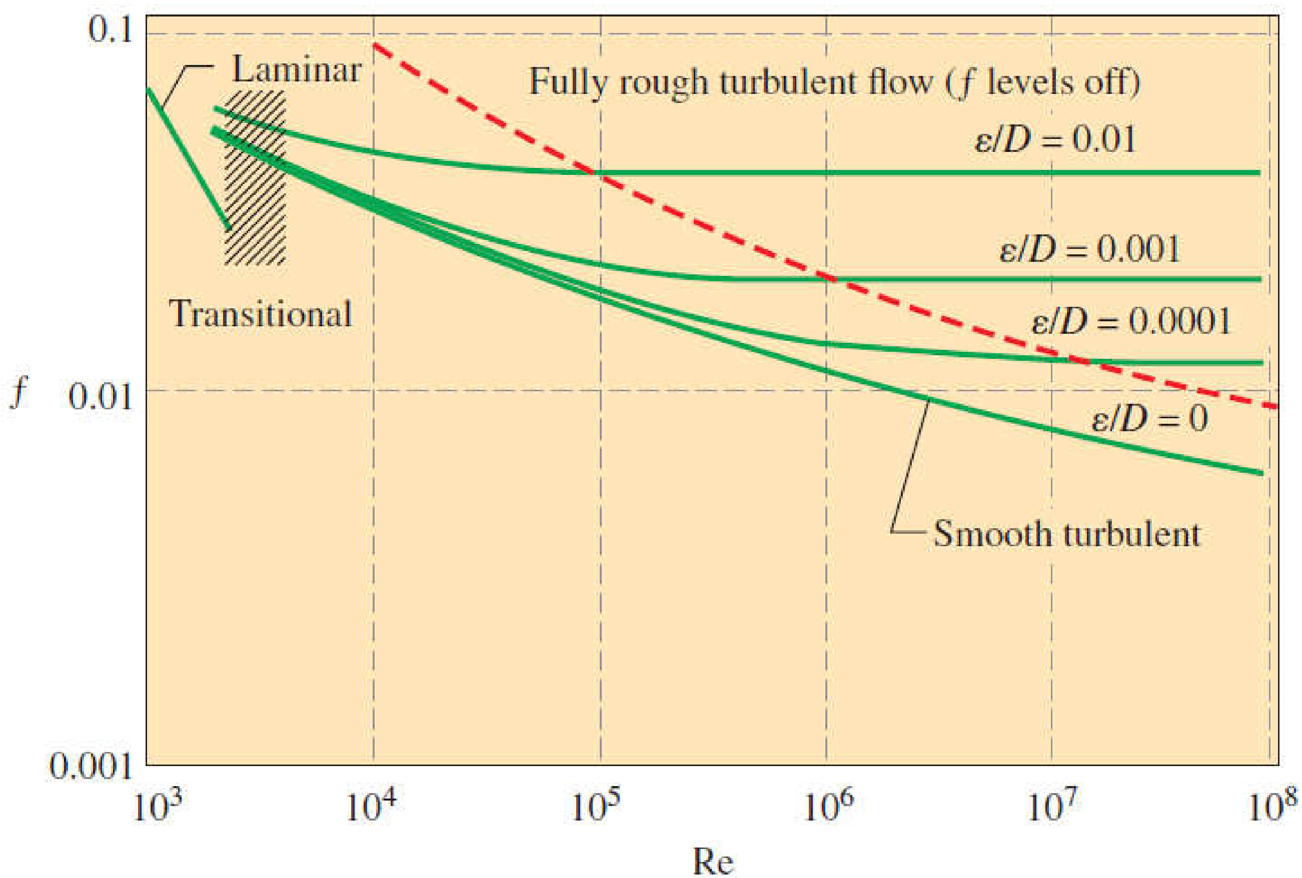
- For laminar flow, the friction factor decreases with increasing Reynolds number, and it is independent of surface roughness.
- The friction factor is a minimum for a smooth pipe and increases with roughness. The Colebrook equation in this case ($\varepsilon = 0$) reduces to the Prandtl equation.

$$1/\sqrt{f} = 2.0 \log(\mathrm{Re}\sqrt{f}) - 0.8$$

- At small relative roughnesses, the friction factor increases in the transition region and approaches the value for smooth pipes.
- At very large Reynolds numbers the friction factor curves corresponding to specified relative roughness curves are nearly **horizontal,** and thus the friction factors are independent of the Reynolds number.
- The flow in Large Re no. region is called *fully rough turbulent flow* because the thickness of the viscous sublayer decreases with increasing Reynolds number, and it becomes so thin that it is negligibly small compared to the surface roughness height. The Colebrook equation in the *fully rough* zone reduces to the von Kármán equation.

$$1/\sqrt{f} = -2.0 \log[(\varepsilon/D)/3.7]$$

In calculations, we should make sure that we use the actual internal diameter of the pipe, which may be different than the nominal diameter.



At very large Reynolds numbers, the friction factor curves on the Moody chart are nearly horizontal, and thus the friction factors are independent of the Reynolds number.

Standard sizes for Schedule 40 steel pipes

Nominal Size, in	Actual Inside Diameter, in
$\frac{1}{8}$	0.269
$\frac{1}{4}$	0.364
$\frac{3}{8}$	0.493
$\frac{1}{2}$	0.622
$\frac{3}{4}$	0.824
1	1.049
$1\frac{1}{2}$	1.610
2	2.067
$2\frac{1}{2}$	2.469
3	3.068
5	5.047
10	10.02

Types of Fluid Flow Problems

1. Determining the **pressure drop** (or head loss) when the pipe length and diameter are given for a specified flow rate (or velocity)
2. Determining the **flow rate** when the pipe length and diameter are given for a specified pressure drop (or head loss)
3. Determining the **pipe diameter** when the pipe length and flow rate are given for a specified pressure drop (or head loss)

Problem type	Given	Find
1	$L, D, \dot{V}$	ΔP (or h_L)
2	$L, D, \Delta P$	$\dot{V}$
3	$L, \Delta P, \dot{V}$	D

The three types of problems encountered in pipe flow.

$$h_L = 1.07\frac{\dot{V}^2L}{gD^5}\left\{\ln\left[\frac{\varepsilon}{3.7D} + 4.62\left(\frac{\nu D}{\dot{V}}\right)^{0.9}\right]\right\}^{-2} \qquad \begin{array}{c} 10^{-6} < \varepsilon/D < 10^{-2} \\ 3000 < \text{Re} < 3 \times 10^8 \end{array}$$

$$\dot{V} = -0.965\left(\frac{gD^5h_L}{L}\right)^{0.5}\ln\left[\frac{\varepsilon}{3.7D} + \left(\frac{3.17\nu^2L}{gD^3h_L}\right)^{0.5}\right] \qquad \text{Re} > 2000$$

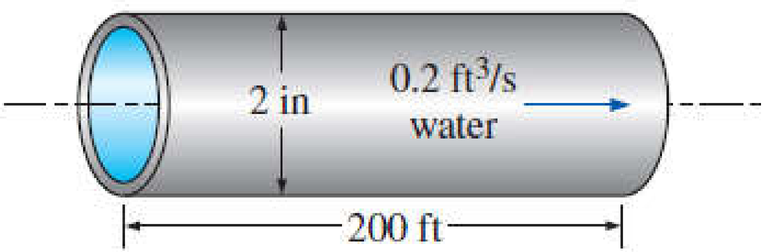
$$D = 0.66\left[\varepsilon^{1.25}\left(\frac{L\dot{V}^2}{gh_L}\right)^{4.75} + \nu\dot{V}^{9.4}\left(\frac{L}{gh_L}\right)^{5.2}\right]^{0.04} \qquad \begin{array}{c} 10^{-6} < \varepsilon/D < 10^{-2} \\ 5000 < \text{Re} < 3 \times 10^8 \end{array}$$

To avoid tedious iterations in head loss, flow rate, and diameter calculations, these explicit relations that are accurate to within 2 percent of the Moody chart may be used.

EXAMPLE 8–3 Determining the Head Loss in a Water Pipe

Water at 60°F (ρ = 62.36 lbm/ft^3 and μ = 7.536 × 10^{-4} lbm/ft·s) is flowing steadily in a 2-in-diameter horizontal pipe made of stainless steel at a rate of 0.2 ft^3/s (Fig. 8–32). Determine the pressure drop, the head loss, and the required pumping power input for flow over a 200-ft-long section of the pipe.

Properties The density and dynamic viscosity of water are given to be ρ = 62.36 lbm/ft^3 and μ = 7.536 × 10^{-4} lbm/ft·s, respectively.



$$\Delta P_L = f\frac{L}{D}\frac{\rho V_{avg}^2}{2}$$

$$\dot{W}_{pump,\,L} = \dot{V}\,\Delta P_L$$

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{Re\sqrt{f}}\right) \quad \text{(turbulent flow)}$$

Equivalent roughness values for new commercial pipes*

Material	Roughness, ε ft	Roughness, ε mm
Glass, plastic	0 (smooth)	
Concrete	0.003–0.03	0.9–9
Wood stave	0.0016	0.5
Rubber, smoothed	0.000033	0.01
Copper or brass tubing	0.000005	0.0015
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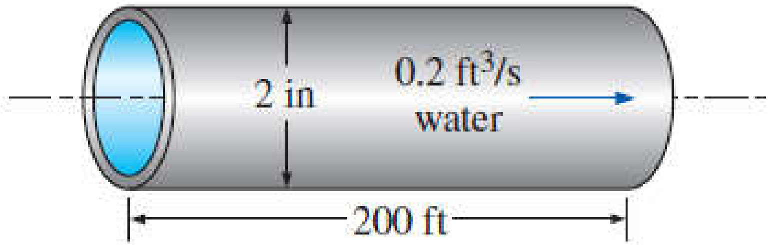
Stainless steel	0.000007	0.002
Commercial steel	0.00015	0.045



$$\Delta P_L = f\frac{L}{D}\frac{\rho V_{avg}^2}{2}$$

$$\dot{W}_{pump,\,L} = \dot{V}\,\Delta P_L$$

Analysis We recognize this as a problem of the first type, since flow rate, pipe length, and pipe diameter are known. First we calculate the average velocity and the Reynolds number to determine the flow regime:

$$V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} = \frac{0.2\ \text{ft}^3/\text{s}}{\pi(2/12\ \text{ft})^2/4} = 9.17\ \text{ft/s}$$

$$\text{Re} = \frac{\rho V D}{\mu} = \frac{(62.36\ \text{lbm/ft}^3)(9.17\ \text{ft/s})(2/12\ \text{ft})}{7.536 \times 10^{-4}\ \text{lbm/ft·s}} = 126{,}400$$

Since Re is greater than 4000, the flow is turbulent. The relative roughness of the pipe is estimated using Table 8–2

$$\varepsilon/D = \frac{0.000007\ \text{ft}}{2/12\ \text{ft}} = 0.000042$$

The friction factor corresponding to this relative roughness and Reynolds number is determined from the Moody chart. To avoid any reading error, we determine f from the Colebrook equation on which the Moody chart is based:

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) \rightarrow \frac{1}{\sqrt{f}} = -2.0\log\left(\frac{0.000042}{3.7} + \frac{2.51}{126{,}400\sqrt{f}}\right)$$

Using an equation solver or an iterative scheme, the friction factor is determined to be $f = 0.0174$. Then the pressure drop (which is equivalent to pressure loss in this case), head loss, and the required power input become

$$\Delta P = \Delta P_L = f\frac{L}{D}\frac{\rho V^2}{2} = 0.0174\,\frac{200\text{ ft}}{2/12\text{ ft}}\,\frac{(62.36\text{ lbm/ft}^3)(9.17\text{ ft/s})^2}{2}\left(\frac{1\text{ lbf}}{32.2\text{ lbm}\cdot\text{ft/s}^2}\right)$$

$$= \mathbf{1700\ lbf/ft^2} = \mathbf{11.8\ psi}$$

$$h_L = \frac{\Delta P_L}{\rho g} = f\frac{L}{D}\frac{V^2}{2g} = 0.0174\,\frac{200\text{ ft}}{2/12\text{ ft}}\,\frac{(9.17\text{ ft/s})^2}{2(32.2\text{ ft/s}^2)} = \mathbf{27.3\ ft}$$

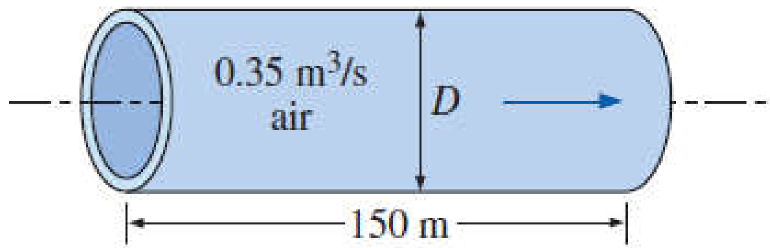
$$\dot{W}_{\text{pump}} = \dot{V}\,\Delta P = (0.2\text{ ft}^3/\text{s})(1700\text{ lbf/ft}^2)\left(\frac{1\text{ W}}{0.737\text{ lbf}\cdot\text{ft/s}}\right) = \mathbf{461\ W}$$

Therefore, power input in the amount of 461 W is needed to overcome the frictional losses in the pipe.

EXAMPLE 8–4 Determining the Diameter of an Air Duct

Heated air at 1 atm and 35°C is to be transported in a 150-m-long circular plastic duct at a rate of 0.35 m³/s (Fig. 8–33). If the head loss in the pipe is not to exceed 20 m, determine the minimum diameter of the duct.

$$\Delta P_L = f\frac{L}{D}\frac{\rho V_{avg}^2}{2}$$



$$\dot{W}_{pump,\,L} = \dot{V}\,\Delta P_L$$

Properties The density, dynamic viscosity, and kinematic viscosity of air at 35°C are $\rho = 1.145$ kg/m³, $\mu = 1.895 \times 10^{-5}$ kg/m·s, and $\nu = 1.655 \times 10^{-5}$ m²/s.

Analysis This is a problem of the third type since it involves the determination of diameter for specified flow rate and head loss. We can solve this problem using three different approaches: (1) an iterative approach by assuming a pipe diameter, calculating the head loss, comparing the result to the specified head loss, and repeating calculations until the calculated head loss matches the specified value; (2) writing all the relevant equations (leaving the diameter as an unknown) and solving them simultaneously using an equation solver; and (3) using the third Swamee–Jain formula. We will demonstrate the use of the last two approaches.

The average velocity, the Reynolds number, the friction factor, and the head loss relations are expressed as (*D* is in m, *V* is in m/s, and Re and *f* are dimensionless)

Equivalent roughness values for new commercial pipes*

Material	Roughness, ε ft	Roughness, ε mm
Glass, plastic	0 (smooth)	
Concrete	0.003–0.03	0.9–9
Wood stave	0.0016	0.5
Rubber, smoothed	0.000033	0.01
Copper or brass tubing	0.000005	0.0015
Cast iron	0.00085	0.26
Galvanized iron	0.0005	0.15
Wrought iron	0.00015	0.046
Stainless steel	0.000007	0.002
Commercial steel	0.00015	0.045

$$V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} = \frac{0.35\ \text{m}^3/\text{s}}{\pi D^2/4}$$

$$\text{Re} = \frac{VD}{\nu} = \frac{VD}{1.655 \times 10^{-5}\ \text{m}^2/\text{s}}$$

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) = -2.0 \log\left(\frac{2.51}{\text{Re}\sqrt{f}}\right)$$

$$h_L = f\frac{L}{D}\frac{V^2}{2g} \quad \rightarrow \quad 20\ \text{m} = f\frac{150\ \text{m}}{D}\frac{V^2}{2(9.81\ \text{m/s}^2)}$$

The roughness is approximately zero for a plastic pipe (Table 8–2). Therefore, this is a set of four equations and four unknowns, and solving them with an equation solver such as EES gives

$$D = \mathbf{0.267\ m}, \quad f = 0.0180, \quad V = 6.24\ \text{m/s}, \quad \text{and} \quad \text{Re} = 100{,}800$$

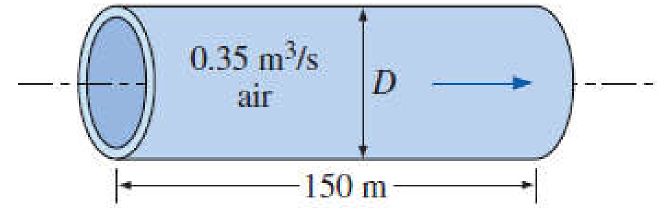
Therefore, the diameter of the duct should be more than 26.7 cm if the head loss is not to exceed 20 m. Note that Re > 4000, and thus the turbulent flow assumption is verified.

The diameter can also be determined directly from the third Swamee–Jain formula to be

$$D = 0.66\left[\varepsilon^{1.25}\left(\frac{L\dot{V}^2}{gh_L}\right)^{4.75} + \nu\dot{V}^{9.4}\left(\frac{L}{gh_L}\right)^{5.2}\right]^{0.04}$$

$$= 0.66\left[0 + (1.655 \times 10^{-5}\ \text{m}^2/\text{s})(0.35\ \text{m}^3/\text{s})^{9.4}\left(\frac{150\ \text{m}}{(9.81\ \text{m/s}^2)(20\ \text{m})}\right)^{5.2}\right]^{0.04}$$

$$= \mathbf{0.271\ m}$$



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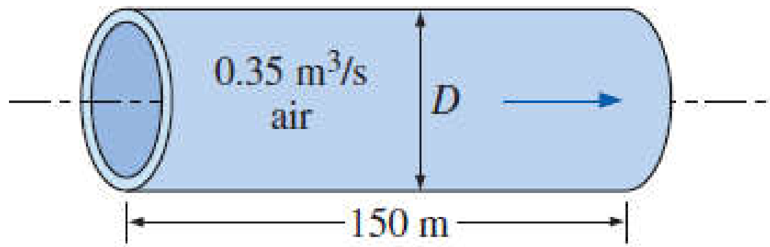
Material	Roughness, ε ft	Roughness, ε mm
Glass, plastic	0 (smooth)	
Concrete	0.003–0.03	0.9–9
Wood stave	0.0016	0.5
Rubber, smoothed	0.000033	0.01
Copper or brass tubing	0.000005	0.0015
Cast iron	0.00085	0.26
Galvanized iron	0.0005	0.15
Wrought iron	0.00015	0.046
Stainless steel	0.000007	0.002
Commercial steel	0.00015	0.045

EXAMPLE 8–5 Determining the Flow Rate of Air in a Duct

Reconsider Example 8–4. Now the duct length is doubled while its diameter is maintained constant. If the total head loss is to remain constant, determine the drop in the flow rate through the duct.

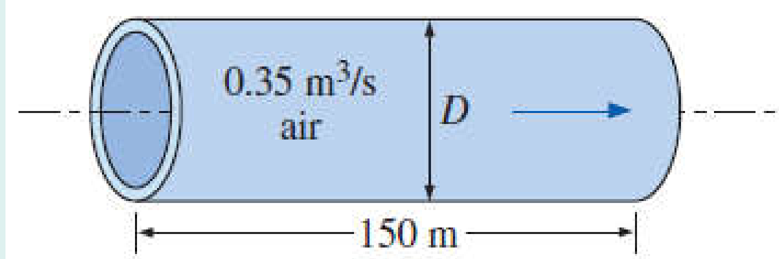


SOLUTION The diameter and the head loss in an air duct are given. The drop in the flow rate is to be determined.

Analysis This is a problem of the second type since it involves the determination of the flow rate for a specified pipe diameter and head loss. The solution involves an iterative approach since the flow rate (and thus the flow velocity) is not known.

The average velocity, Reynolds number, friction factor, and the head loss relations are expressed as (D is in m, V is in m/s, and Re and f are dimensionless)

The average velocity, Reynolds number, friction factor, and the head loss relations are expressed as (D is in m, V is in m/s, and Re and f are dimensionless)



$$V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} \quad \rightarrow \quad V = \frac{\dot{V}}{\pi (0.267\text{ m})^2/4}$$

$$\text{Re} = \frac{VD}{\nu} \quad \rightarrow \quad \text{Re} = \frac{V(0.267\text{ m})}{1.655 \times 10^{-5}\text{ m}^2/\text{s}}$$

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) \quad \rightarrow \quad \frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{2.51}{\text{Re}\sqrt{f}}\right)$$

$$h_L = f\frac{L}{D}\frac{V^2}{2g} \quad \rightarrow \quad 20\text{ m} = f\frac{300\text{ m}}{0.267\text{ m}}\frac{V^2}{2(9.81\text{ m/s}^2)}$$

This is a set of four equations in four unknowns and solving them with an equation solver such as EES (Fig. 8–34) gives

$$\dot{V} = 0.24\text{ m}^3/\text{s}, \quad f = 0.0195, \quad V = 4.23\text{ m/s}, \quad \text{and} \quad \text{Re} = 68{,}300$$

Then the drop in the flow rate becomes

$$\dot{V}_{drop} = \dot{V}_{old} - \dot{V}_{new} = 0.35 - 0.24 = \mathbf{0.11\text{ m}^3/\text{s}} \quad \text{(a drop of 31 percent)}$$

Therefore, for a specified head loss (or available head or fan pumping power), the flow rate drops by about 31 percent from 0.35 to 0.24 m³/s when the duct length doubles.

Alternative Solution If a computer is not available (as in an exam situation), another option is to set up a *manual iteration loop*. We have found that the best convergence is usually realized by first guessing the friction factor f, and then solving for the velocity V. The equation for V as a function of f is

Average velocity through the pipe: $$V = \sqrt{\frac{2gh_L}{fL/D}}$$

Once V is calculated, the Reynolds number can be calculated, from which a *corrected* friction factor is obtained from the Moody chart or the Colebrook equation. We repeat the calculations with the corrected value of f until convergence. We guess $f = 0.04$ for illustration:

Iteration	f (guess)	V, m/s	Re	Corrected f
1	0.04	2.955	4.724×10^4	0.0212
2	0.0212	4.059	6.489×10^4	0.01973
3	0.01973	4.207	6.727×10^4	0.01957
4	0.01957	4.224	6.754×10^4	0.01956
5	0.01956	4.225	6.756×10^4	0.01956

Notice that the iteration has converged to three digits in only three iterations and to four digits in only four iterations. The final results are identical to those obtained with EES, yet do not require a computer.

Discussion The new flow rate can also be determined directly from the second Swamee–Jain formula to be

$$\dot{V} = -0.965\left(\frac{gD^5h_L}{L}\right)^{0.5}\ln\left[\frac{\varepsilon}{3.7D} + \left(\frac{3.17\nu^2L}{gD^3h_L}\right)^{0.5}\right]$$

$$= -0.965\left(\frac{(9.81\text{ m/s}^2)(0.267\text{ m})^5(20\text{ m})}{300\text{ m}}\right)^{0.5}$$

$$\times \ln\left[0 + \left(\frac{3.17(1.655\times10^{-5}\text{ m}^2\text{/s})^2(300\text{ m})}{(9.81\text{ m/s}^2)(0.267\text{ m})^3(20\text{ m})}\right)^{0.5}\right]$$

$$= 0.24\text{ m}^3\text{/s}$$

Note that the result from the Swamee–Jain relation is the same (to two significant digits) as that obtained with the Colebrook equation using EES or using our manual iteration technique. Therefore, the simple Swamee–Jain relation can be used with confidence.

Lecture 4

MINOR LOSSES

The fluid in a typical piping system passes through various fittings, valves, bends, elbows, tees, inlets, exits, enlargements, and contractions in addition to the pipes.

These components interrupt the smooth flow of the fluid and cause additional losses because of the flow separation and mixing they induce.

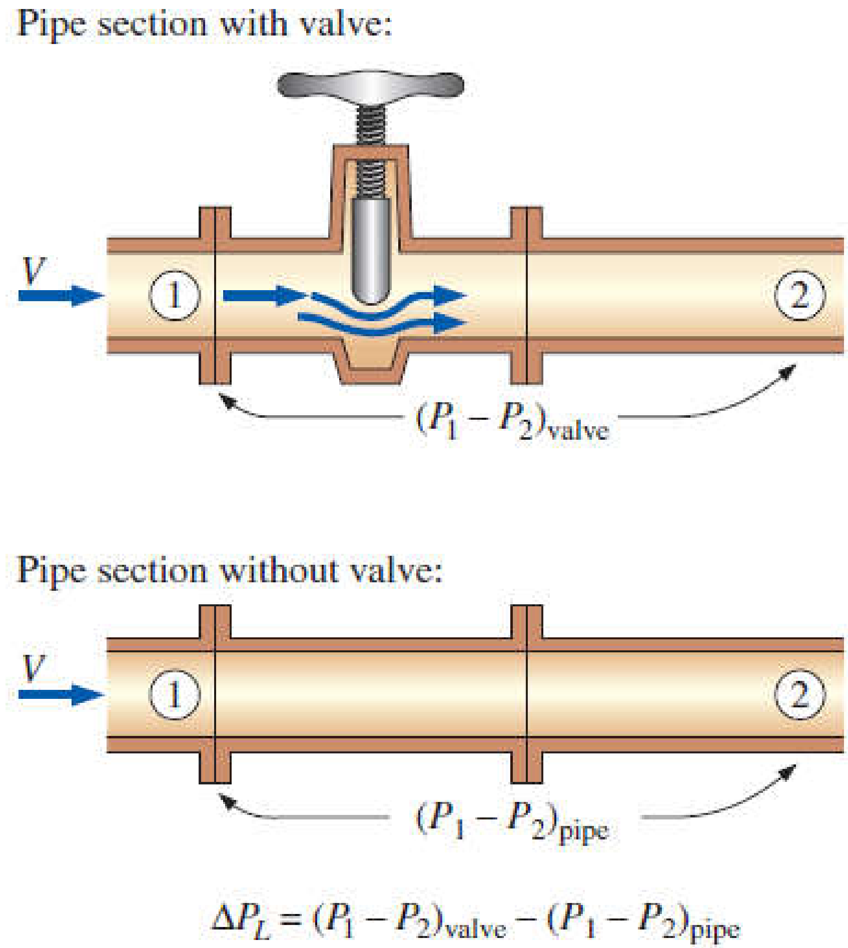
In a typical system with long pipes, these losses are called **minor losses** because they are minor compared **to the total head loss in the pipes (the major losses).**

Minor losses are usually expressed in terms of the **loss coefficient K_L**.

$$K_L = \frac{h_L}{V^2/(2g)}$$

$$h_L = \Delta P_L / \rho g$$

Head loss due to component



For a constant-diameter section of a pipe with a minor loss component, the loss coefficient of the component (such as the gate valve shown) is determined by measuring the additional pressure loss it causes and dividing it by the dynamic pressure in the pipe.

When the **inlet diameter equals outlet diameter**, the loss coefficient of a component can also be determined by measuring the **pressure loss across** the component and **dividing** it by **the dynamic pressure**:

$$K_L = \Delta P_L / (\rho V^2/2).$$

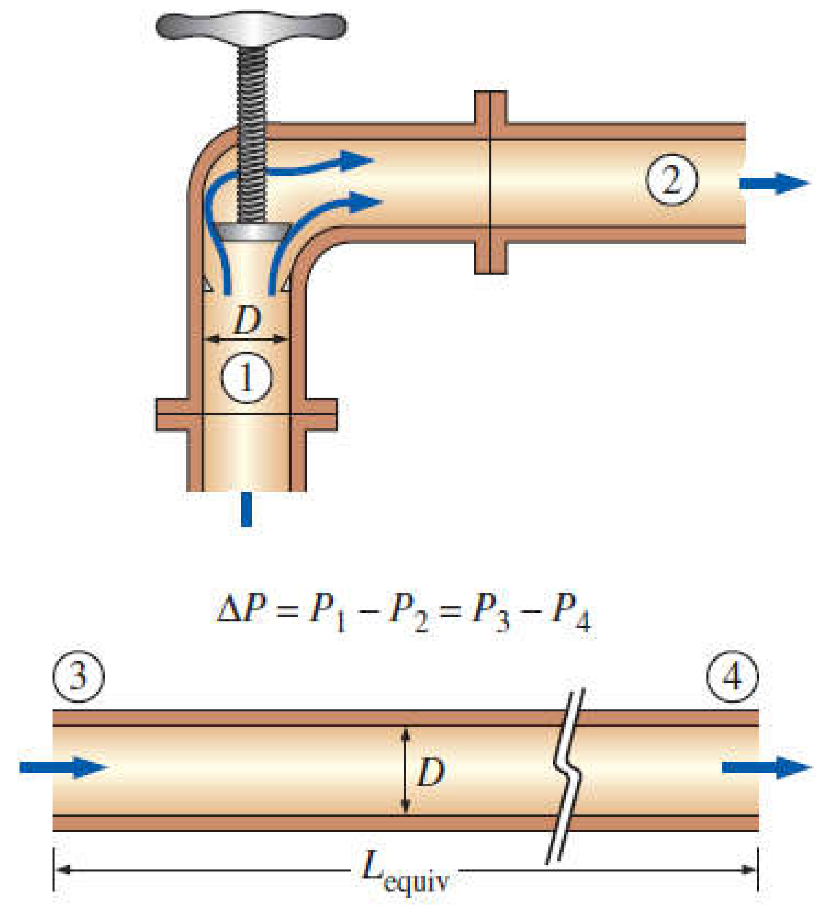
When the **loss coefficient** for a component is available, the head loss for that component is

$$h_L = K_L \frac{V^2}{2g}$$

Minor loss

Minor losses are also expressed in terms of the **equivalent length L_{equiv}**.

$$h_L = K_L \frac{V^2}{2g} = f \frac{L_{equiv}}{D} \frac{V^2}{2g} \rightarrow L_{equiv} = \frac{D}{f} K_L$$



The head loss caused by a component (such as the angle valve shown) is equivalent to the head loss caused by a section of the pipe whose length is the equivalent length.

Total head loss (general)

$$h_{L,\text{total}} = h_{L,\text{major}} + h_{L,\text{minor}} = \sum_i f_i \frac{L_i}{D_i}\frac{V_i^2}{2g} + \sum_j K_{L,j}\frac{V_j^2}{2g}$$

Total head loss (D = constant)

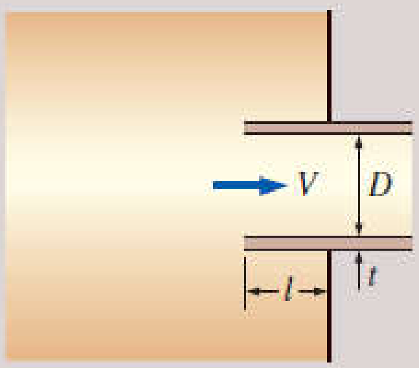
$$h_{L,\text{total}} = \left(f\frac{L}{D} + \sum K_L\right)\frac{V^2}{2g}$$

The head loss at the inlet of a pipe is almost negligible for well-rounded inlets (K_L = 0.03 for $r/D > 0.2$) but increases to about 0.50 for sharp-edged inlets.

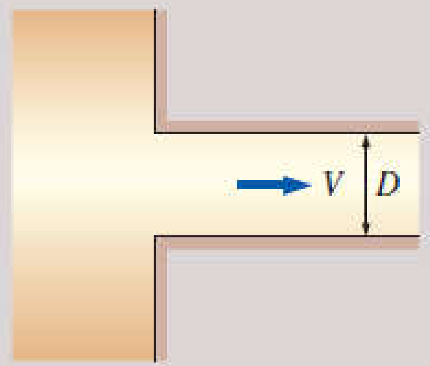
Loss coefficients K_L of various pipe components for turbulent flow (for use in the relation $h_L = K_L V^2/(2g)$, where V is the average velocity in the pipe that contains the component)*

Pipe Inlet

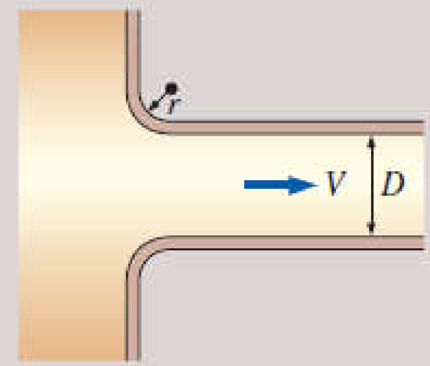
Reentrant: $K_L = 0.80$
($t \ll D$ and $l \approx 0.1D$)



Sharp-edged: $K_L = 0.50$

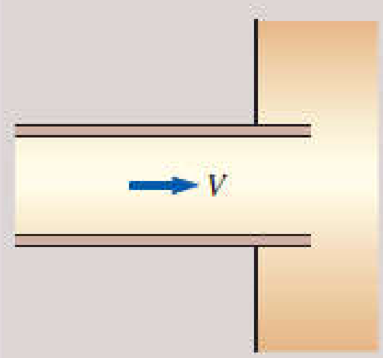


Well-rounded ($r/D > 0.2$): $K_L = 0.03$
Slightly rounded ($r/D = 0.1$): $K_L = 0.12$
(see Fig. 8–39)

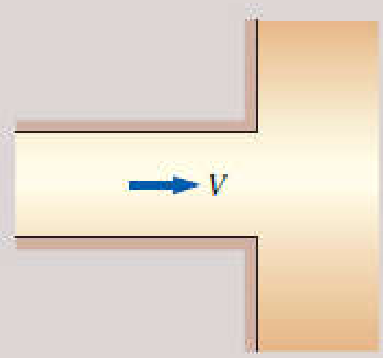


Pipe Exit

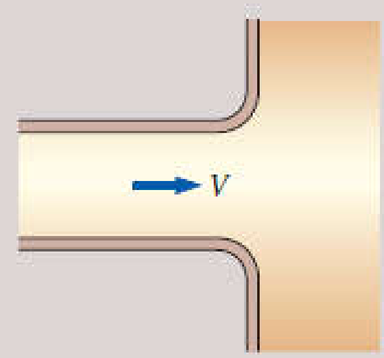
Reentrant: $K_L = \alpha$



Sharp-edged: $K_L = \alpha$



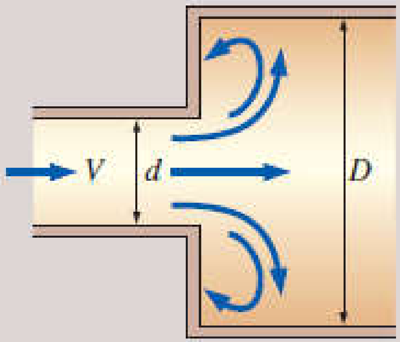
Rounded: $K_L = \alpha$



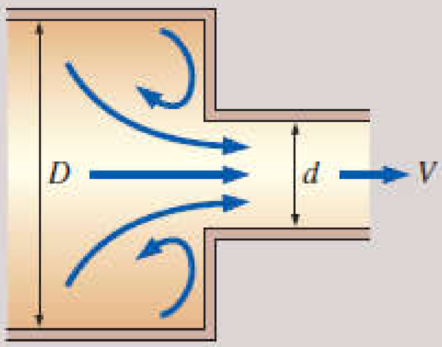
Note: The kinetic energy correction factor is $\alpha = 2$ for fully developed laminar flow, and $\alpha \approx 1.05$ for fully developed turbulent flow.

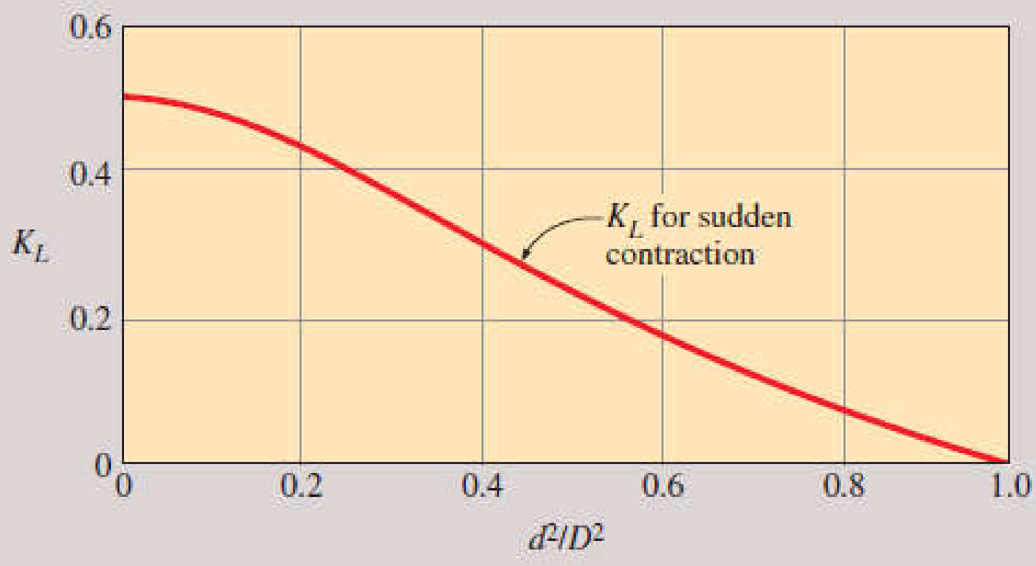
Sudden Expansion and Contraction (based on the velocity in the smaller-diameter pipe)

Sudden expansion: $K_L = \alpha\left(1 - \frac{d^2}{D^2}\right)^2$



Sudden contraction: See chart.

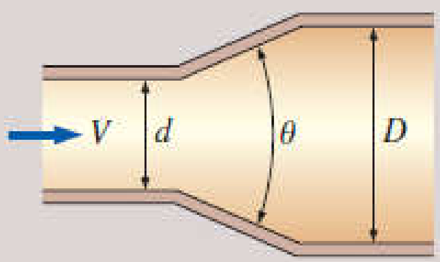




Gradual Expansion and Contraction (based on the velocity in the smaller-diameter pipe)

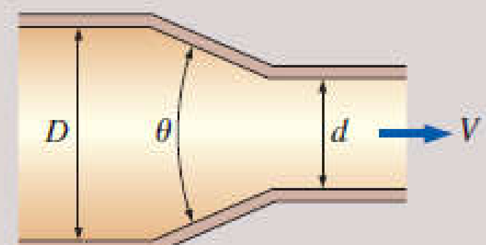
Expansion (for $\theta = 20°$):

$K_L = 0.30$ for $d/D = 0.2$
$K_L = 0.25$ for $d/D = 0.4$
$K_L = 0.15$ for $d/D = 0.6$
$K_L = 0.10$ for $d/D = 0.8$



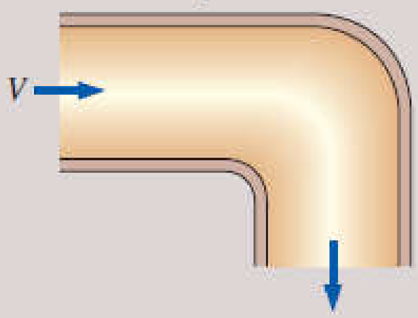
Contraction:

$K_L = 0.02$ for $\theta = 30°$
$K_L = 0.04$ for $\theta = 45°$
$K_L = 0.07$ for $\theta = 60°$

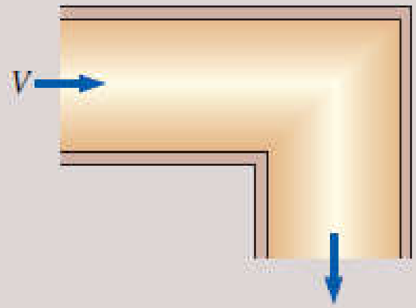


Bends and Branches

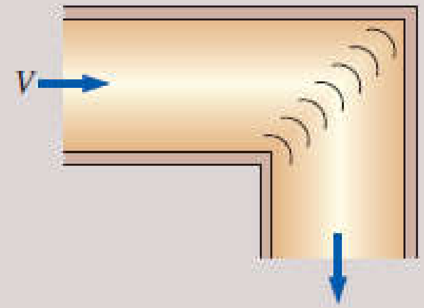
90° smooth bend:
Flanged: $K_L = 0.3$
Threaded: $K_L = 0.9$



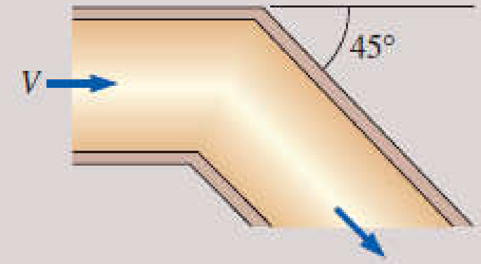
90° miter bend
(without vanes): $K_L = 1.1$



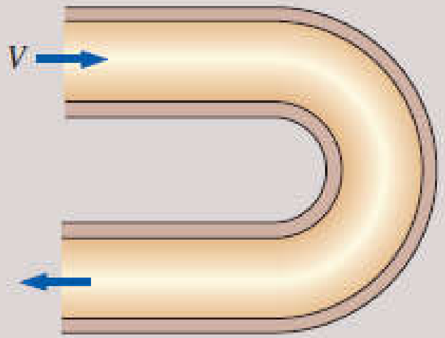
90° miter bend
(with vanes): $K_L = 0.2$



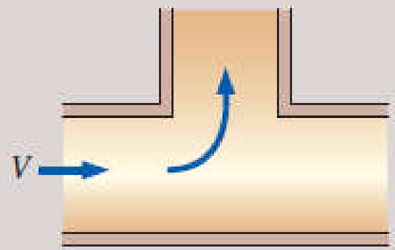
45° threaded elbow:
$K_L = 0.4$



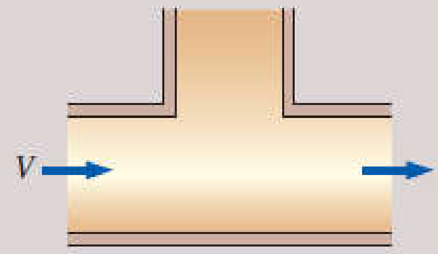
180° return bend:
Flanged: $K_L = 0.2$
Threaded: $K_L = 1.5$



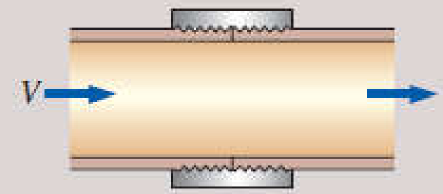
Tee (branch flow):
Flanged: $K_L = 1.0$
Threaded: $K_L = 2.0$



Tee (line flow):
Flanged: $K_L = 0.2$
Threaded: $K_L = 0.9$



Threaded union:
$K_L = 0.08$



Valves

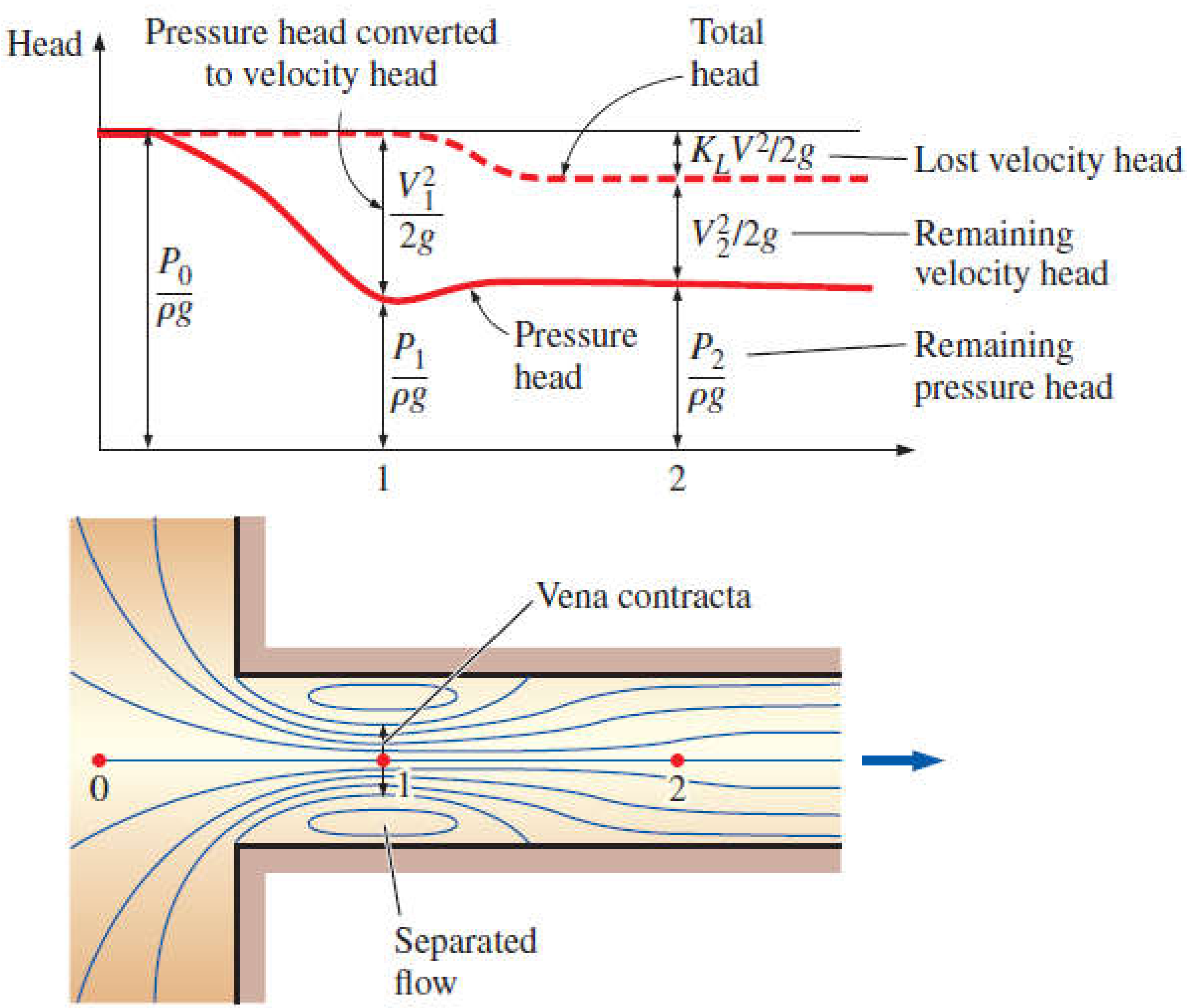
Globe valve, fully open: $K_L = 10$
Angle valve, fully open: $K_L = 5$
Ball valve, fully open: $K_L = 0.05$
Swing check valve: $K_L = 2$

Gate valve, fully open: $K_L = 0.2$
$\frac{1}{4}$ closed: $K_L = 0.3$
$\frac{1}{2}$ closed: $K_L = 2.1$
$\frac{3}{4}$ closed: $K_L = 17$

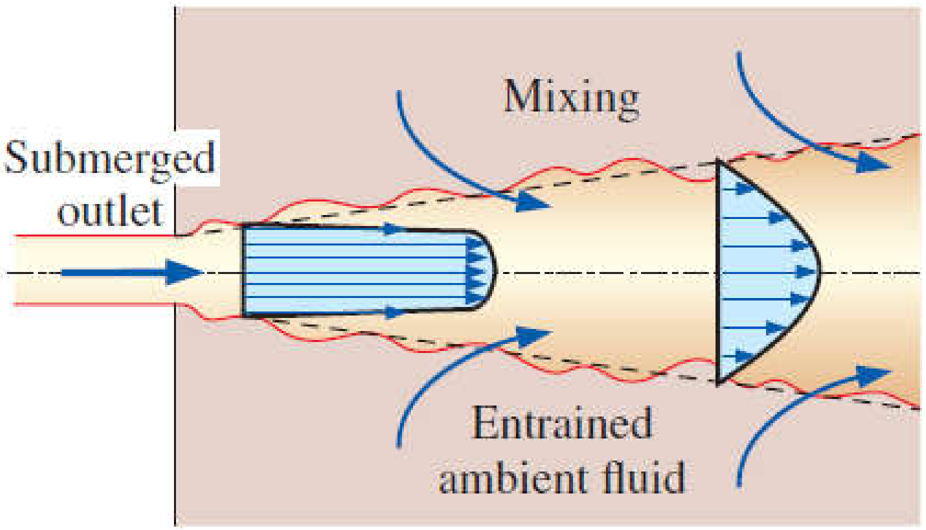
* These are representative values for loss coefficients. Actual values strongly depend on the design and manufacture of the components and may differ from the given values considerably (especially for valves). Actual manufacturer's data should be used in the final design.

A sharp edged inlet causes half of the velocity head to be lost as the fluid enters the pipe. This is because the fluid cannot make sharp 90° turns easily, especially at high velocities. As a result, the flow separates at the corners, and the flow is constricted into the **vena contracta** region formed in the midsection of the pipe

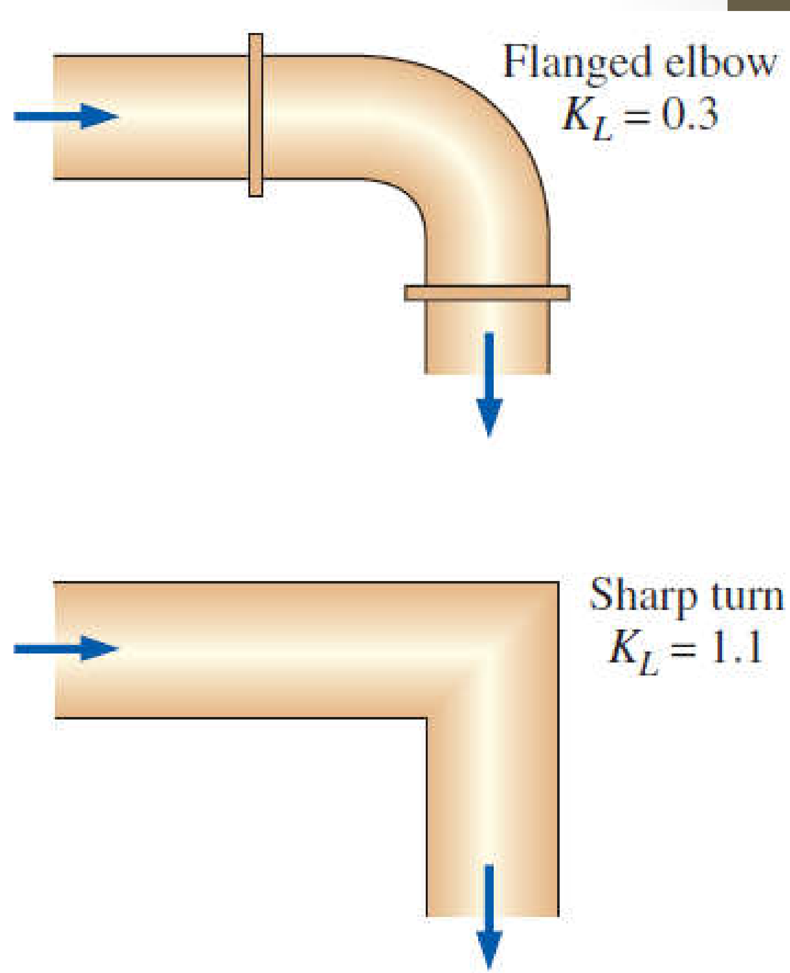
$$K_L = \alpha\left(1 - \frac{A_{small}}{A_{large}}\right)^2 \quad \text{(sudden expansion)}$$



Graphical representation of flow contraction and the associated head loss at a sharp-edged pipe inlet.

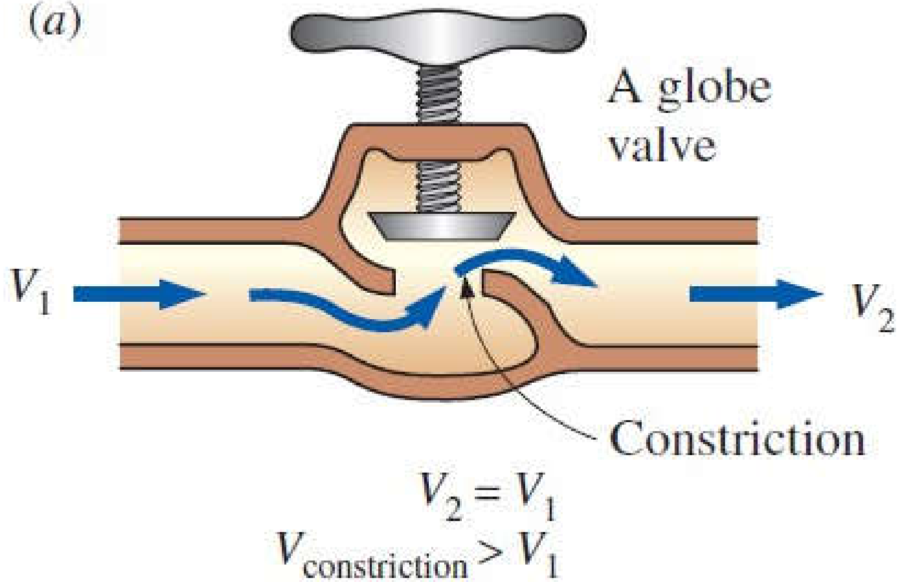


All the kinetic energy of the flow is “lost” (turned into thermal energy) through friction as the jet decelerates and mixes with ambient fluid downstream of a submerged outlet.



The losses during changes of direction can be minimized by making the turn “easy” on the fluid by using circular arcs instead of sharp turns.

(*a*)



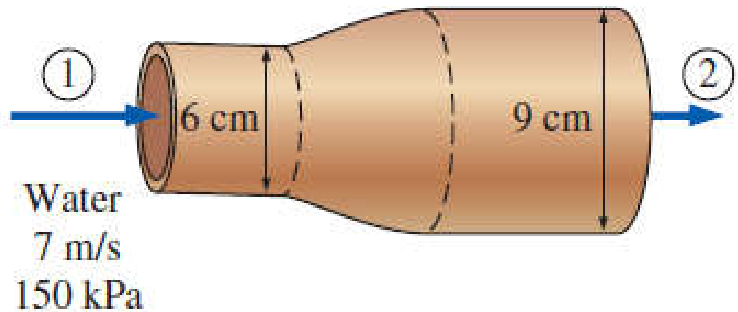
(*b*)

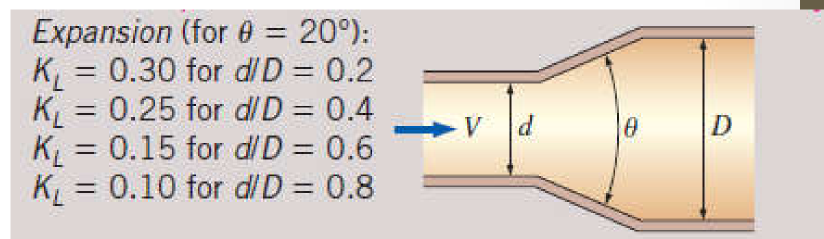
(*a*) The large head loss in a partially closed valve is due to irreversible deceleration, flow separation, and mixing of high-velocity fluid coming from the narrow valve passage.

(*b*) The head loss through a fully-open ball valve, on the other hand, is quite small.

EXAMPLE 8–6 Head Loss and Pressure Rise during Gradual Expansion

A 6-cm-diameter horizontal water pipe expands gradually to a 9-cm-diameter pipe (Fig. 8–43). The walls of the expansion section are angled 10° from the axis. The average velocity and pressure of water before the expansion section are 7 m/s and 150 kPa, respectively. Determine the head loss in the expansion section and the pressure in the larger-diameter pipe.





Assumptions **1** The flow is steady and incompressible. **2** The flow at sections 1 and 2 is fully developed and turbulent with $\alpha_1 = \alpha_2 \cong 1.06$.

Properties We take the density of water to be $\rho = 1000\ \text{kg/m}^3$. The loss coefficient for a gradual expansion of total included angle $\theta = 20°$ and diameter ratio $d/D = 6/9$ is $K_L = 0.133$ (by interpolation using Table 8–4).

Analysis Noting that the density of water remains constant, the downstream velocity of water is determined from conservation of mass to be

$$\dot{m}_1 = \dot{m}_2 \quad \rightarrow \quad \rho V_1 A_1 = \rho V_2 A_2 \quad \rightarrow \quad V_2 = \frac{A_1}{A_2} V_1 = \frac{D_1^2}{D_2^2} V_1$$

$$V_2 = \frac{(0.06\ \text{m})^2}{(0.09\ \text{m})^2}(7\ \text{m/s}) = 3.11\ \text{m/s}$$

Then the irreversible head loss in the expansion section becomes

$$h_L = K_L \frac{V_1^2}{2g} = (0.133)\frac{(7\text{ m/s})^2}{2(9.81\text{ m/s}^2)} = \mathbf{0.333\ m}$$

Noting that $z_1 = z_2$ and there are no pumps or turbines involved, the energy equation for the expansion section is expressed in terms of heads as

$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + \cancel{z_1} + h_{\text{pump},u}^{\nearrow 0} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + \cancel{z_2} + h_{\text{turbine},e}^{\nearrow 0} + h_L$$

or

$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + h_L$$

Solving for P_2 and substituting,

$$P_2 = P_1 + \rho\left\{\frac{\alpha_1 V_1^2 - \alpha_2 V_2^2}{2} - gh_L\right\} = (150\text{ kPa}) + (1000\text{ kg/m}^3)$$

$$\times\left\{\frac{1.06(7\text{ m/s})^2 - 1.06(3.11\text{ m/s})^2}{2} - (9.81\text{ m/s}^2)(0.333\text{ m})\right\}$$

$$\times\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right)\left(\frac{1\text{ kPa}}{1\text{ kN/m}^2}\right)$$

$$= \mathbf{168\ kPa}$$

Therefore, despite the head (and pressure) loss, the pressure *increases* from 150 to 168 kPa after the expansion. This is due to the conversion of dynamic pressure to static pressure when the average flow velocity is decreased in the larger pipe.

PIPING NETWORKS AND PUMP SELECTION

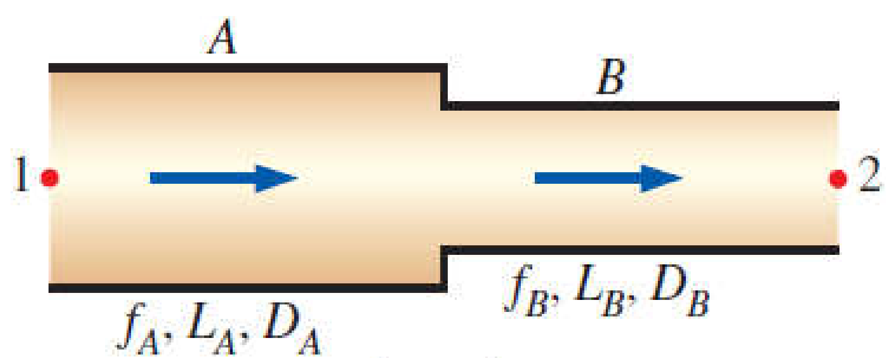
Most piping systems encountered in practice such as the water distribution systems in cities or commercial or residential establishments involve: **Parallel** and **Series connections** (supply of fluid into the system , discharges of fluid from the system)

A piping project may involve the **design of a new system** or the **expansion of an existing system**.

PIPING NETWORKS AND PUMP SELECTION

When the pipes are connected in series:

- The **flow rate** through the entire system **remains constant** (Conservation of mass) **regardless of the diameters of the individual pipes** in the system.
- The **total head loss is equal to the sum of the head losses** in individual pipes in the system, including the minor losses.
- The **expansion or contraction losses** at connections are considered to **belong to the smaller-diameter** pipe.



$$\dot{V}_A = \dot{V}_B$$

$$h_{L,\,1\text{-}2} = h_{L,\,A} + h_{L,\,B}$$

PIPING NETWORKS AND PUMP SELECTION

For a pipe branches out into two (or more) **parallel pipes** and then rejoins at a junction downstream:

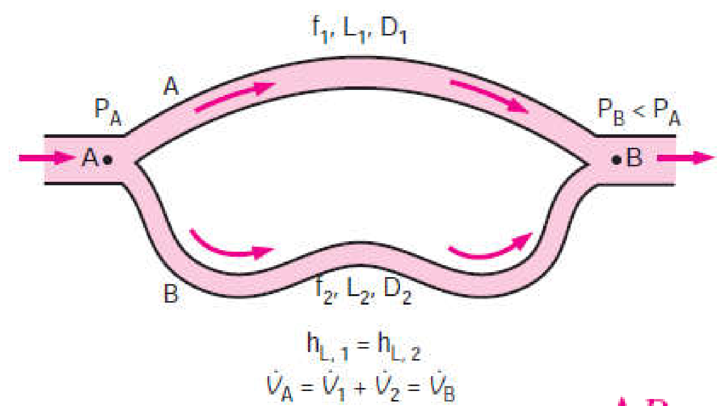
- The total flow rate is the sum of the flow rates in the individual pipes.

- The pressure drop (or head loss) in each individual pipe connected in parallel must be the same since ($\Delta P = P_A - P_B$).

Head Loss

For a system of two parallel pipes 1 and 2 between junctions *A* and *B* with negligible minor losses:

$$h_{L,1} = h_{L,2} \quad \rightarrow \quad f_1 \frac{L_1}{D_1}\frac{V_1^2}{2g} = f_2 \frac{L_2}{D_2}\frac{V_2^2}{2g}$$



$$h_L = \frac{\Delta P_L}{\rho g}$$

Velocity and Flow rates:

The ratio of the average velocities and the flow rates in the two parallel pipes become

$$\frac{V_1}{V_2} = \left(\frac{f_2}{f_1}\frac{L_2}{L_1}\frac{D_1}{D_2}\right)^{1/2} \quad \text{and} \quad \frac{\dot{V}_1}{\dot{V}_2} = \frac{A_{c,1}V_1}{A_{c,2}V_2} = \frac{D_1^2}{D_2^2}\left(\frac{f_2}{f_1}\frac{L_2}{L_1}\frac{D_1}{D_2}\right)^{1/2}$$

$$h_{L,1} = h_{L,2} \quad \rightarrow \quad f_1 \frac{L_1}{D_1}\frac{V_1^2}{2g} = f_2 \frac{L_2}{D_2}\frac{V_2^2}{2g}$$

$$\frac{V_1}{V_2} = \left(\frac{f_2}{f_1}\frac{L_2}{L_1}\frac{D_1}{D_2}\right)^{1/2} \quad \text{and} \quad \frac{\dot{V}_1}{\dot{V}_2} = \frac{A_{c,1}V_1}{A_{c,2}V_2} = \frac{D_1^2}{D_2^2}\left(\frac{f_2}{f_1}\frac{L_2}{L_1}\frac{D_1}{D_2}\right)^{1/2}$$

The flow rate in one of the parallel branches is proportional to its diameter to the power 5/2 and is inversely proportional to the square root of its length and friction factor.

The analysis of piping networks is based on two simple principles:

1. ***Conservation of mass throughout the system must be satisfied*.** This is done by requiring the total flow into a junction to be equal to the total flow out of the junction for all junctions in the system.
2. ***Pressure drop (and thus head loss) between two junctions must be the same for all paths between the two junctions*.** This is because pressure is a point function and it cannot have two values at a specified point. In practice this rule is used by requiring that the algebraic sum of head losses in a loop (for all loops) be equal to zero.

Piping Systems with Pumps and Turbines

Useful pump head delivered to the fluid

Turbine head extracted from the fluid

Total head losses in the piping

$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_{pump,u} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_{turbine,e} + h_L$$

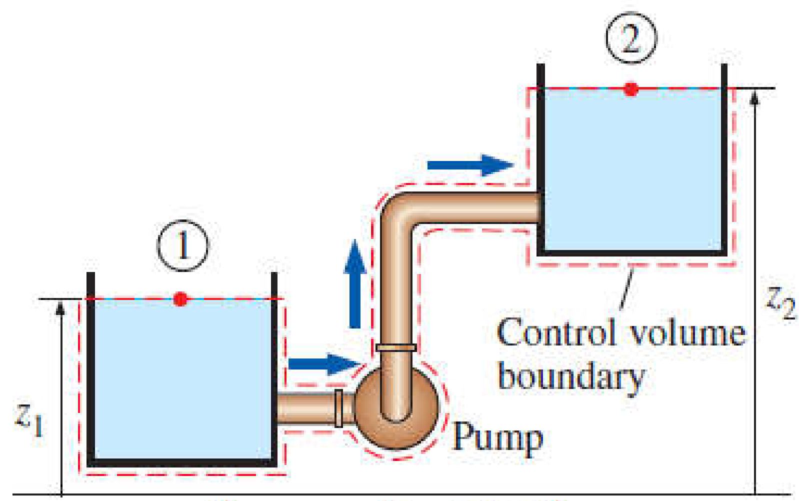
The steady-flow energy equation

Many practical piping systems involve a pump to move a fluid from one reservoir to another.

Taking points 1 and 2 to be at the *free surfaces* of the reservoirs, the energy equation in this case reduces for the useful pump head required to

$$h_{pump,u} = (z_2 - z_1) + h_L$$

When a pump moves a fluid from one reservoir to another, the useful pump head requirement is equal to the elevation difference between the two reservoirs plus the head loss.



$$h_{pump,u} = (z_2 - z_1) + h_L$$
$$\dot{W}_{pump,u} = \rho \dot{V} g h_{pump,u}$$

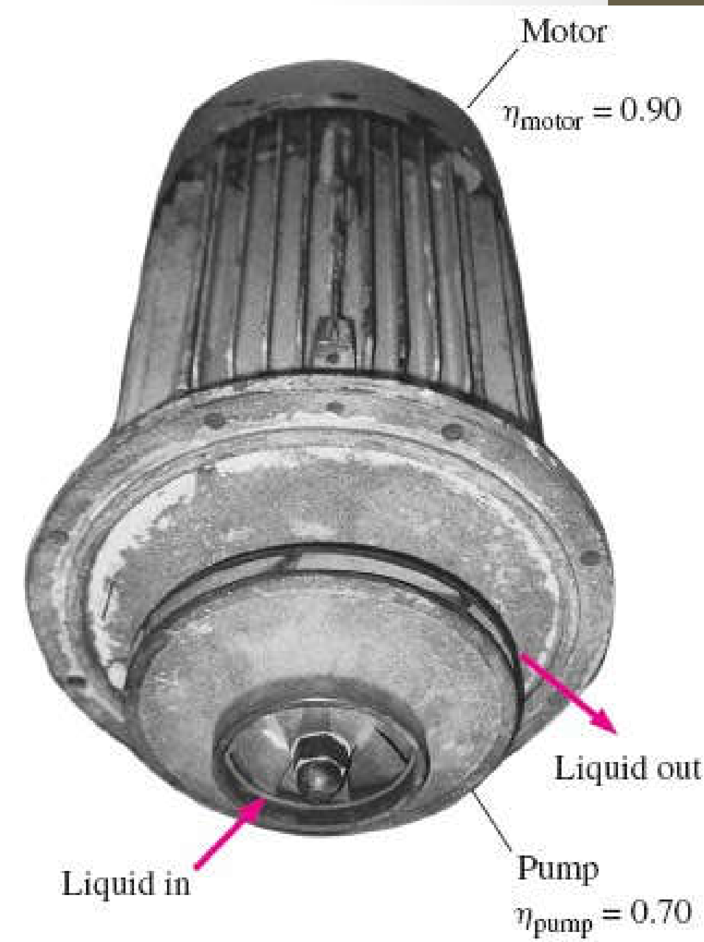
Piping Systems with Pumps and Turbines

Once the useful pump head is known:

- The **mechanical power** that needs to be delivered by the pump to the fluid can be determined

- The **electric power** consumed by the motor of the pump for a specified flow rate is determined

$$\dot{W}_{pump,\,shaft} = \frac{\rho \dot{V} g h_{pump,\,u}}{\eta_{pump}} \quad \text{and} \quad \dot{W}_{elect} = \frac{\rho \dot{V} g h_{pump,\,u}}{\eta_{pump\text{–}motor}}$$



$$\eta_{pump\text{–}motor} = \eta_{pump}\eta_{motor} = 0.70 \times 0.90 = 0.63$$

The efficiency of the pump–motor combination is the product of the pump and the motor efficiencies.

Example(1)

Water flowing through a pipe of 20 cm I.D. at section 1 and 10 cm at section 2 The discharge through the pipe is 35 lit/s. The section 1 is 6 m above the datum line and section 2 is 2 m above it. If the pressure at section 1 is 245 kPa, find the intensity of pressure at section 2. Given that ρ = 1000 kg/m^3, μ= 1.0 mPa.s.

Example (2)

A conical tube of 4 m length is fixed at an inclined angle of 30° with the horizontal line and its small diameter upwards. The velocity at smaller end is ($u_1 = 5$ m/s), while ($u_2 = 2$ m/s) at other end. The head losses in the tub is [0.35 $(u_1-u_2)^2/2g$]. Determine the pressure head at lower end if the flow takes place in down direction and the pressure head at smaller end is 2 m of liquid.

Example (3)

Water with density ρ = 998 kg/m3, is flowing at steady mass flow rate through a uniform-diameter pipe. The entrance pressure of the fluid is 68.9 kPa in the pipe, which connects to a pump, which actually supplies 155.4 J/kg of fluid flowing in the pipe. The exit pipe from the pump is the same diameter as the inlet pipe. The exit section of the pipe is 3.05 m higher than the entrance, and the exit pressure is 137.8 kPa. The Reynolds number in the pipe is above 4,000 in this system. Calculate the frictional loss (F) in the pipe system.

Example (4)

A pump draws 69.1 gal/min of liquid solution having a density of 114.8 lb/ft^3 from an open storage feed tank of large cross-sectional area through a 3.068″I.D. suction pipe. The pump discharges its flow through a 2.067″I.D. line to an open over head tank. The end of the discharge line is 50′ above the level of the liquid in the feed tank. The friction losses in the piping system are F = 10 ft lb$_f$/lb. what pressure must the pump develop and what is the horsepower of the pump if its efficiency is η=0.65.

Example (5)

98% H_2So_4 is pumped at 1.25 kg/s through a 25 mm inside diameter pipe, 30 m long, to a reservoir 12 m higher than the feed point. Calculate the pressure drop in the pipeline. Take that ρ = 1840 kg/m^3, μ= 25 mPa.s, e = 0.05 mm.

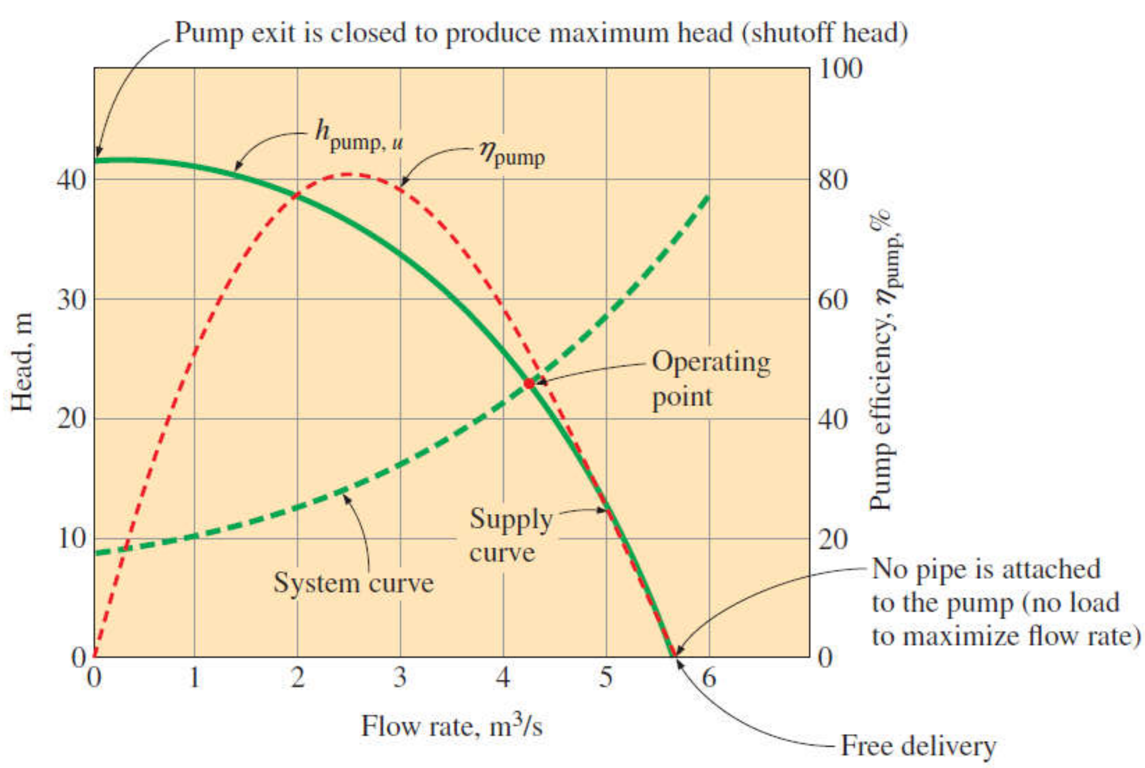
Lecture 5

The head loss of a piping system increases with the flow rate.

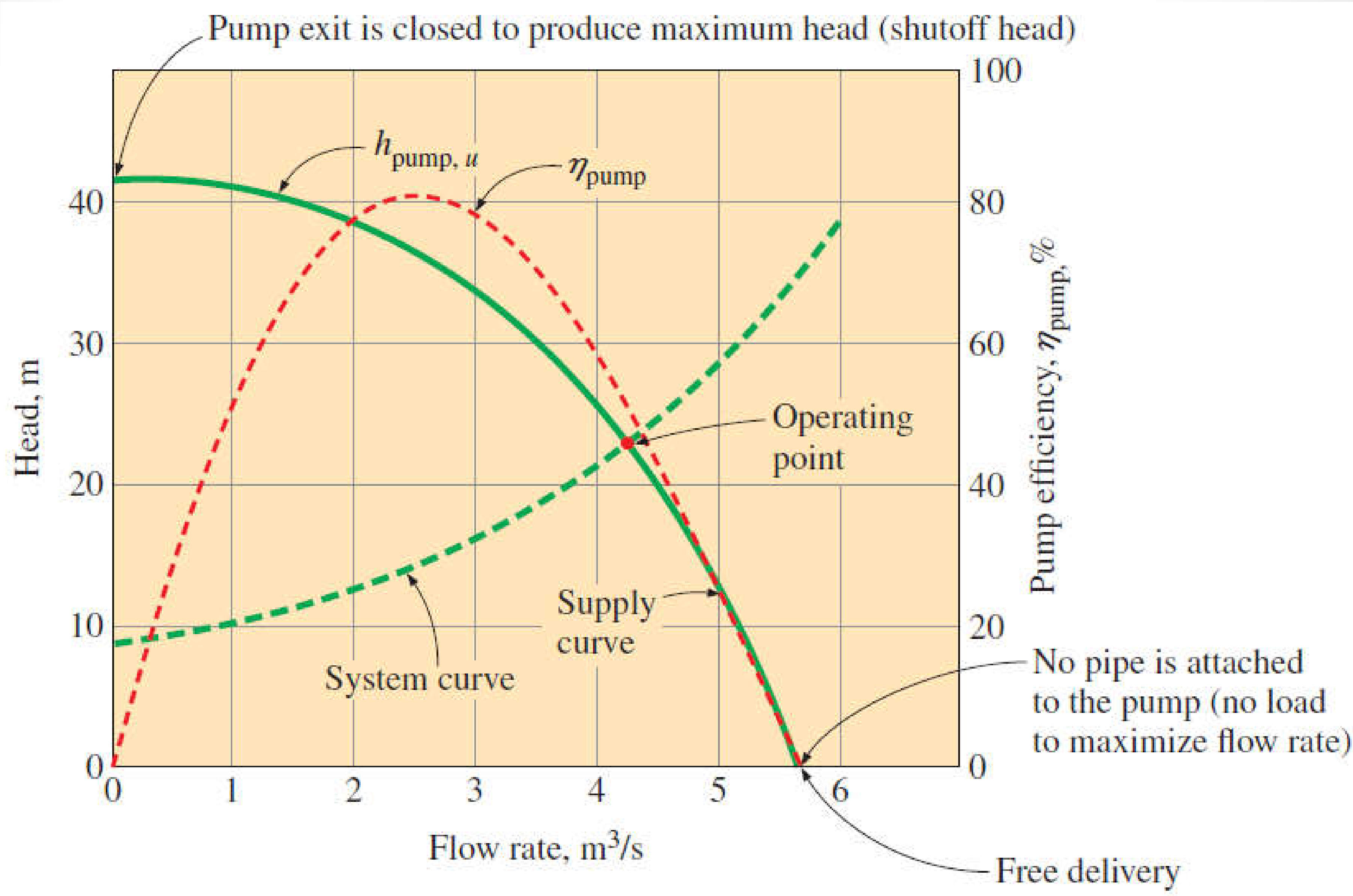
system (or demand) curve. plot of required useful pump head $h_{pump,u}$ as a function of flow rate

Both the pump head and the pump efficiency vary with the flow rate

characteristic (or supply of performance) curves, Experimentally determined $h_{pump,u}$ and $\eta_{pump,u}$ versus *Flow rate* curves.



The pump installed in a piping system will operate at the point where the *system curve* and the *characteristic curve* intersect. This point of intersection is called the **operating point.**



Characteristic pump curves for centrifugal pumps, the system curve for a piping system, and the operating point.

EXAMPLE 8–7 Pumping Water through Two Parallel Pipes

Water at 20°C is to be pumped from a reservoir (z_A = 5 m) to another reservoir at a higher elevation (z_B = 13 m) through two 36-m-long pipes connected in parallel, as shown in Fig. 8–50. The pipes are made of commercial steel, and the diameters of the two pipes are 4 and 8 cm. Water is to be pumped by a 70 percent efficient motor–pump combination that draws 8 kW of electric power during operation. The minor losses and the head loss in pipes that connect the parallel pipes to the two reservoirs are considered to be negligible. Determine the total flow rate between the reservoirs and the flow rate through each of the parallel pipes.

$$\dot{W}_{elect} = \frac{\rho \dot{V} g h_{pump,\,u}}{\eta_{pump\text{–}motor}}$$

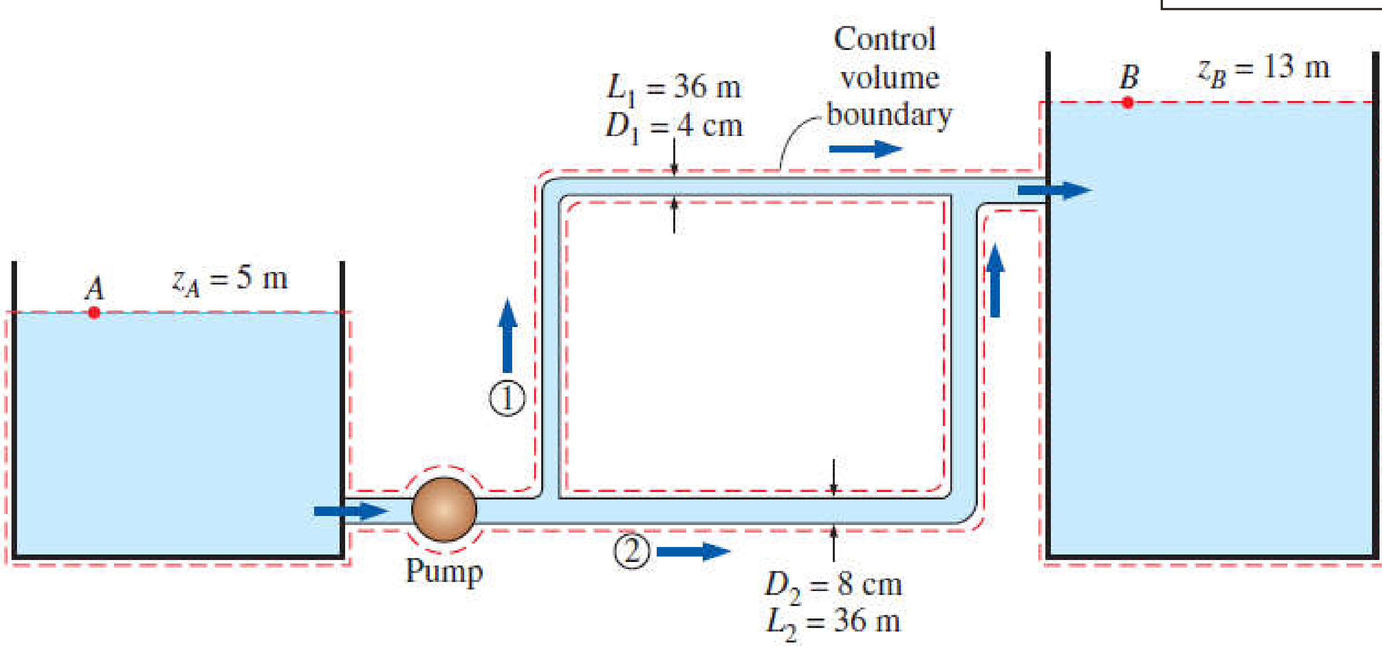
$$h_{pump,\,u} = (z_2 - z_1) + h_L$$

Properties The density and dynamic viscosity of water at 20°C are ρ = 998 kg/m^3 and $\mu = 1.002 \times 10^{-3}$ kg/m·s. The roughness of commercial steel pipe is ε = 0.000045 m (Table 8–2).

$$h_{L,1} = h_{L,2} \quad \rightarrow \quad f_1 \frac{L_1}{D_1}\frac{V_1^2}{2g} = f_2 \frac{L_2}{D_2}\frac{V_2^2}{2g}$$

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{Re\sqrt{f}}\right)$$

$$Re = \frac{\rho V D}{\mu}$$



$$\dot{W}_{elect} = \frac{\rho \dot{V} g h_{pump,u}}{\eta_{pump-motor}} \rightarrow 8000 \text{ W} = \frac{(998 \text{ kg/m}^3)\dot{V}(9.81 \text{ m/s}^2) h_{pump,u}}{0.70} \quad (1)$$

We choose points A and B at the free surfaces of the two reservoirs. Noting that the fluid at both points is open to the atmosphere (and thus $P_A = P_B = P_{atm}$) and that the fluid velocities at both points are nearly zero ($V_A \approx V_B \approx 0$) since the reservoirs are large, the energy equation for a control volume between these two points simplifies to

$$\frac{P_A}{\rho g} + \alpha_A \frac{V_A^2}{2g}^{\nearrow 0} + z_A + h_{pump,u} = \frac{P_B}{\rho g} + \alpha_B \frac{V_B^2}{2g}^{\nearrow 0} + z_B + h_L$$

or

$$h_{pump,u} = (z_B - z_A) + h_L$$

or

$$h_{pump,u} = (13 \text{ m} - 5 \text{ m}) + h_L \quad (2)$$

where

$$h_L = h_{L,1} = h_{L,2} \quad (3)(4)$$

We designate the 4-cm-diameter pipe by 1 and the 8-cm-diameter pipe by 2. Equations for the average velocity, the Reynolds number, the friction factor, and the head loss in each pipe are

$$V_1 = \frac{\dot{V}_1}{A_{c,1}} = \frac{\dot{V}_1}{\pi D_1^2/4} \quad \rightarrow \quad V_1 = \frac{\dot{V}_1}{\pi(0.04\text{ m})^2/4} \tag{5}$$

$$V_2 = \frac{\dot{V}_2}{A_{c,2}} = \frac{\dot{V}_2}{\pi D_2^2/4} \quad \rightarrow \quad V_2 = \frac{\dot{V}_2}{\pi(0.08\text{ m})^2/4} \tag{6}$$

$$\text{Re}_1 = \frac{\rho V_1 D_1}{\mu} \quad \rightarrow \quad \text{Re}_1 = \frac{(998\text{ kg/m}^3)V_1(0.04\text{ m})}{1.002 \times 10^{-3}\text{ kg/m·s}} \tag{7}$$

$$\text{Re}_2 = \frac{\rho V_2 D_2}{\mu} \quad \rightarrow \quad \text{Re}_2 = \frac{(998\text{ kg/m}^3)V_2(0.08\text{ m})}{1.002 \times 10^{-3}\text{ kg/m·s}} \tag{8}$$

$$\frac{1}{\sqrt{f_1}} = -2.0\log\left(\frac{\varepsilon/D_1}{3.7} + \frac{2.51}{\text{Re}_1\sqrt{f_1}}\right)$$

$$\rightarrow \quad \frac{1}{\sqrt{f_1}} = -2.0\log\left(\frac{0.000045}{3.7 \times 0.04} + \frac{2.51}{\text{Re}_1\sqrt{f_1}}\right) \tag{9}$$

$$\frac{1}{\sqrt{f_2}} = -2.0\log\left(\frac{\varepsilon/D_2}{3.7} + \frac{2.51}{\text{Re}_2\sqrt{f_2}}\right)$$

$$\rightarrow \quad \frac{1}{\sqrt{f_2}} = -2.0\log\left(\frac{0.000045}{3.7 \times 0.08} + \frac{2.51}{\text{Re}_2\sqrt{f_2}}\right) \tag{10}$$

$$h_{L,1} = f_1\frac{L_1}{D_1}\frac{V_1^2}{2g} \quad \rightarrow \quad h_{L,1} = f_1\frac{36\text{ m}}{0.04\text{ m}}\frac{V_1^2}{2(9.81\text{ m/s}^2)} \tag{11}$$

$$h_{L,2} = f_2\frac{L_2}{D_2}\frac{V_2^2}{2g} \quad \rightarrow \quad h_{L,2} = f_2\frac{36\text{ m}}{0.08\text{ m}}\frac{V_2^2}{2(9.81\text{ m/s}^2)} \tag{12}$$

$$\dot{V} = \dot{V}_1 + \dot{V}_2 \tag{13}$$

This is a system of 13 equations in 13 unknowns, and their simultaneous solution by an equation solver gives

$$\dot{V} = \mathbf{0.0300\ m^3/s}, \quad \dot{V}_1 = \mathbf{0.00415\ m^3/s}, \quad \dot{V}_2 = \mathbf{0.0259\ m^3/s}$$

$$V_1 = 3.30\ \text{m/s}, \ V_2 = 5.15\ \text{m/s}, \ h_L = h_{L,1} = h_{L,2} = 11.1\ \text{m}, \ h_{\text{pump}} = 19.1\ \text{m}$$

$$\text{Re}_1 = 131{,}600, \quad \text{Re}_2 = 410{,}000, \quad f_1 = 0.0221, \quad f_2 = 0.0182$$

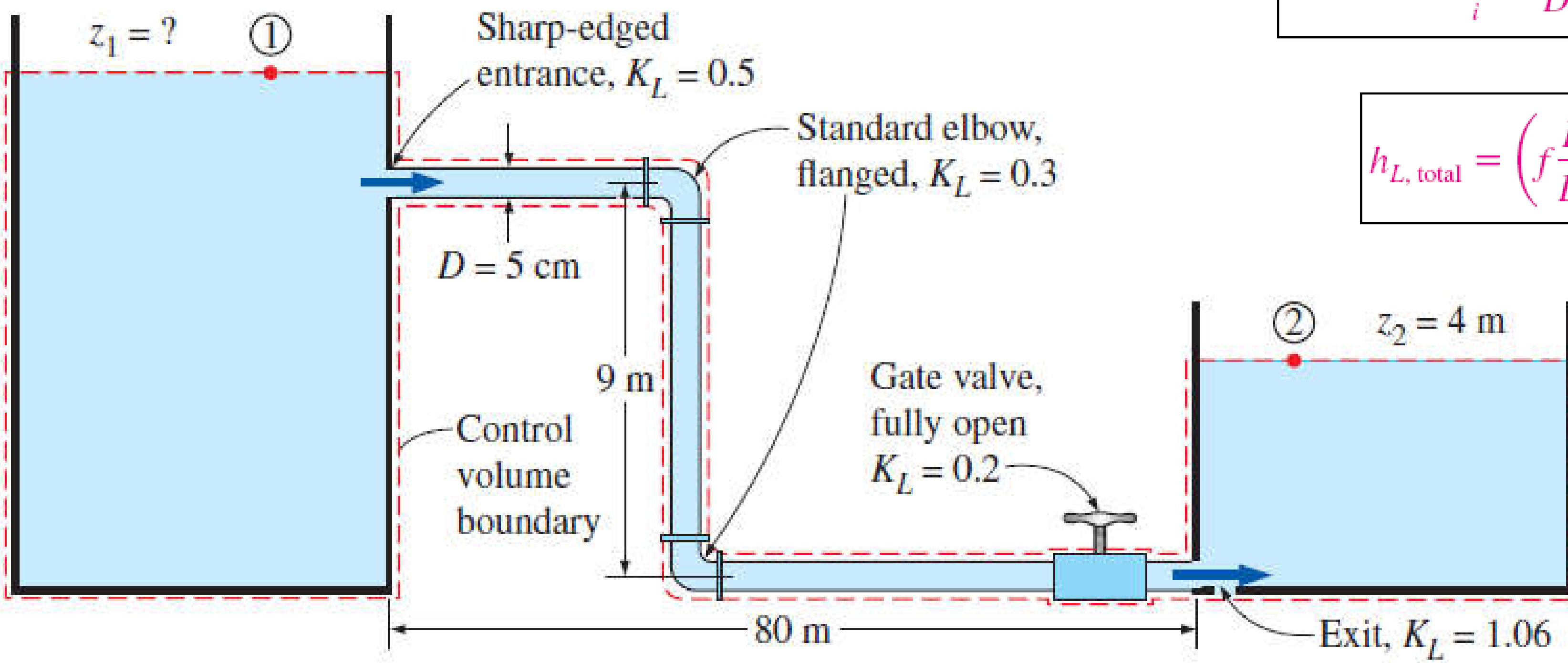
Note that Re > 4000 for both pipes, and thus the assumption of turbulent flow is verified.

EXAMPLE 8–8 Gravity-Driven Water Flow in a Pipe

Water at 10°C flows from a large reservoir to a smaller one through a 5-cm-diameter cast iron piping system, as shown in Fig. 8–51. Determine the elevation z_1 for a flow rate of 6 L/s.

$$h_{L,\text{ total}} = h_{L,\text{ major}} + h_{L,\text{ minor}} = \sum_i f_i \frac{L_i}{D_i}\frac{V_i^2}{2g} + \sum_j K_{L,j}\frac{V_j^2}{2g}$$

$$h_{L,\text{ total}} = \left(f\frac{L}{D} + \sum K_L\right)\frac{V^2}{2g}$$



$$\text{Re} = \frac{\rho V D}{\mu}$$

Properties The density and dynamic viscosity of water at 10°C are ρ = 999.7 kg/m³ and μ = 1.307 × 10⁻³ kg/m·s. The roughness of cast iron pipe is ε = 0.00026 m (Table 8–2).

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right)$$

$$\frac{1}{\sqrt{f}} \cong -1.8\log\left[\frac{6.9}{\text{Re}} + \left(\frac{\varepsilon/D}{3.7}\right)^{1.11}\right]$$

Analysis The piping system involves 89 m of piping, a sharp-edged entrance ($K_L = 0.5$), two standard flanged elbows ($K_L = 0.3$ each), a fully open gate valve ($K_L = 0.2$), and a submerged exit ($K_L = 1.06$). We choose points 1 and 2 at the free surfaces of the two reservoirs. Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and that the fluid velocities at both points are nearly zero ($V_1 \approx V_2 \approx 0$), the energy equation for a control volume between these two points simplifies to

$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_L \quad \rightarrow \quad z_1 = z_2 + h_L$$

where

$$h_L = h_{L,\text{total}} = h_{L,\text{major}} + h_{L,\text{minor}} = \left(f\frac{L}{D} + \sum K_L\right)\frac{V^2}{2g}$$

since the diameter of the piping system is constant. The average velocity in the pipe and the Reynolds number are

$$V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} = \frac{0.006\ \text{m}^3/\text{s}}{\pi(0.05\ \text{m})^2/4} = 3.06\ \text{m/s}$$

$$\text{Re} = \frac{\rho V D}{\mu} = \frac{(999.7\ \text{kg/m}^3)(3.06\ \text{m/s})(0.05\ \text{m})}{1.307 \times 10^{-3}\ \text{kg/m·s}} = 117{,}000$$

The flow is turbulent since Re > 4000. Noting that $\varepsilon/D = 0.00026/0.05 = 0.0052$, the friction factor is determined from the Colebrook equation (or the Moody chart),

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right) \rightarrow \frac{1}{\sqrt{f}} = -2.0\log\left(\frac{0.0052}{3.7} + \frac{2.51}{117{,}000\sqrt{f}}\right)$$

It gives $f = 0.0315$. The sum of the loss coefficients is

$$\sum K_L = K_{L,\,\text{entrance}} + 2K_{L,\,\text{elbow}} + K_{L,\,\text{valve}} + K_{L,\,\text{exit}}$$

$$= 0.5 + 2 \times 0.3 + 0.2 + 1.06 = 2.36$$

Then the total head loss and the elevation of the source become

$$h_L = \left(f\frac{L}{D} + \sum K_L\right)\frac{V^2}{2g} = \left(0.0315\frac{89\text{ m}}{0.05\text{ m}} + 2.36\right)\frac{(3.06\text{ m/s})^2}{2(9.81\text{ m/s}^2)} = 27.9\text{ m}$$

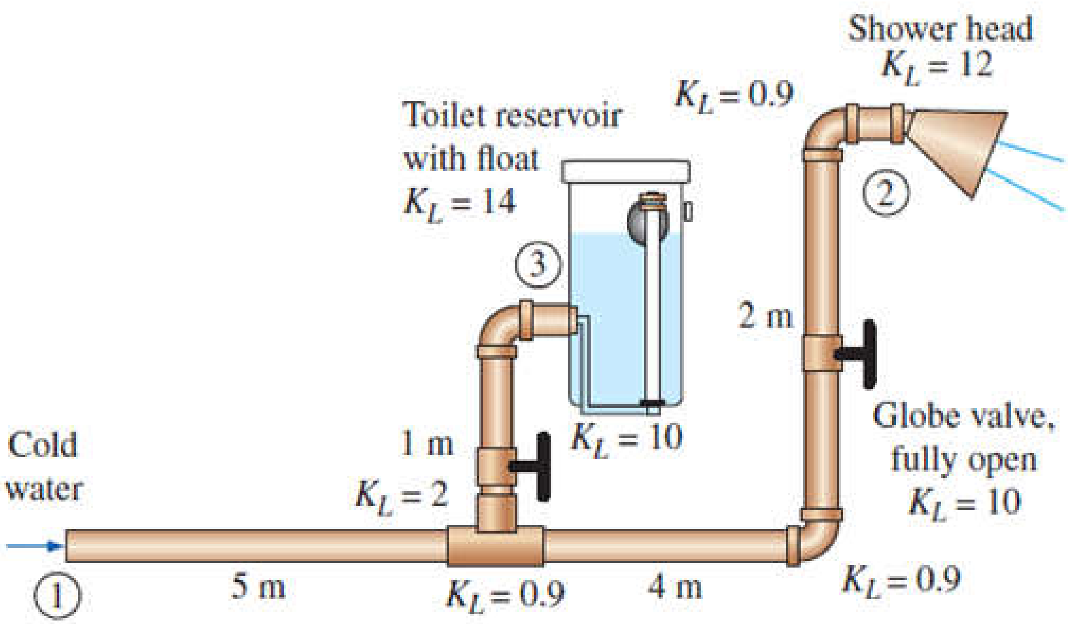
$$z_1 = z_2 + h_L = 4 + 27.9 = \mathbf{31.9\ m}$$

Therefore, the free surface of the first reservoir must be 31.9 m above the ground level to ensure water flow between the two reservoirs at the specified rate.

Discussion Note that $fL/D = 56.1$ in this case, which is about 24 times the total minor loss coefficient. Therefore, ignoring the sources of minor losses in this case would result in about 4 percent error. It can be shown that at the same flow rate, the total head loss would be 35.9 m (instead of 27.9 m) if the valve were three-fourths closed, and it would drop to 24.8 m if the pipe between the two reservoirs were straight at the ground level (thus eliminating the elbows and the vertical section of the pipe). The head loss could be reduced further (from 24.8 to 24.6 m) by rounding the entrance. The head loss can be reduced significantly (from 27.9 to 16.0 m) by replacing the cast iron pipes by smooth pipes such as those made of plastic.

EXAMPLE 8–9 Effect of Flushing on Flow Rate from a Shower

The bathroom plumbing of a building consists of 1.5-cm-diameter copper pipes with threaded connectors, as shown in Fig. 8–52. (*a*) If the gage pressure at the inlet of the system is 200 kPa during a shower and the toilet reservoir is full (no flow in that branch), determine the flow rate of water through the shower head. (*b*) Determine the effect of flushing of the toilet on the flow rate through the shower head. Take the loss coefficients of the shower head and the reservoir to be 12 and 14, respectively.



$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_{pump,\,u} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_{turbine,\,e} + h_L$$

$$h_{L,\,total} = \left(f\frac{L}{D} + \sum K_L\right)\frac{V^2}{2g}$$

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{Re\sqrt{f}}\right)$$

$$Re = \frac{\rho V D}{\mu}$$

Properties The properties of water at 20°C are $\rho = 998$ kg/m^3, $\mu = 1.002 \times 10^{-3}$ kg/m·s, and $\nu = \mu/\rho = 1.004 \times 10^{-6}$ m^2/s. The roughness of copper pipes is $\varepsilon = 1.5 \times 10^{-6}$ m.

(*a*) The piping system of the shower alone involves 11 m of piping, a tee with line flow ($K_L = 0.9$), two standard elbows ($K_L = 0.9$ each), a fully open globe valve ($K_L = 10$), and a shower head ($K_L = 12$). Therefore, $\sum K_L = 0.9 + 2 \times 0.9 + 10 + 12 = 24.7$. Noting that the shower head is open to the atmosphere, and the velocity heads are negligible, the energy equation for a control volume between points 1 and 2 simplifies to

$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_{\text{pump}, u} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_{\text{turbine}, e} + h_L$$

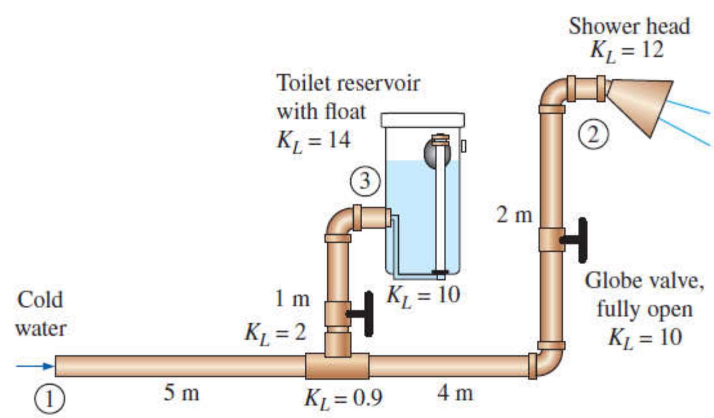
$$\rightarrow \quad \frac{P_{1,\text{ gage}}}{\rho g} = (z_2 - z_1) + h_L$$

Therefore, the head loss is

$$h_L = \frac{200{,}000 \text{ N/m}^2}{(998 \text{ kg/m}^3)(9.81 \text{ m/s}^2)} - 2 \text{ m} = 18.4 \text{ m}$$

Also,

$$h_L = \left(f\frac{L}{D} + \sum K_L\right)\frac{V^2}{2g} \quad \rightarrow \quad 18.4 = \left(f\frac{11 \text{ m}}{0.015 \text{ m}} + 24.7\right)\frac{V^2}{2(9.81 \text{ m/s}^2)}$$



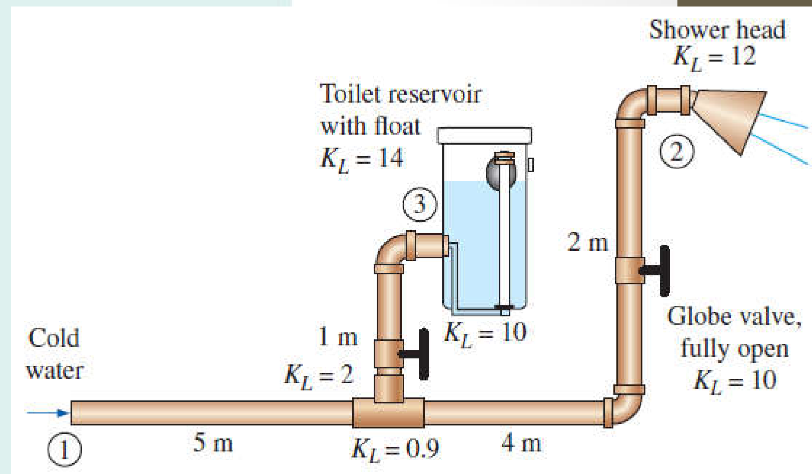
since the diameter of the piping system is constant. Equations for the average velocity in the pipe, the Reynolds number, and the friction factor are

$$V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} \quad \rightarrow \quad V = \frac{\dot{V}}{\pi (0.015 \text{ m})^2/4}$$

$$\text{Re} = \frac{VD}{\nu} \quad \rightarrow \quad \text{Re} = \frac{V(0.015 \text{ m})}{1.004 \times 10^{-6} \text{ m}^2/\text{s}}$$

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right)$$

$$\rightarrow \quad \frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{1.5 \times 10^{-6} \text{ m}}{3.7(0.015 \text{ m})} + \frac{2.51}{\text{Re}\sqrt{f}}\right)$$



This is a set of four equations with four unknowns, and solving them with an equation solver such as EES gives

$$\dot{V} = 0.00053 \text{ m}^3/\text{s}, \quad f = 0.0218, \quad V = 2.98 \text{ m/s}, \quad \text{and} \quad \text{Re} = 44{,}550$$

Therefore, the flow rate of water through the shower head is **0.53 L/s.**

(*b*) When the toilet is flushed, the float moves and opens the valve. The discharged water starts to refill the reservoir, resulting in parallel flow after the tee connection. The head loss and minor loss coefficients for the shower branch were determined in (*a*) to be $h_{L,2} = 18.4$ m and $\sum K_{L,2} = 24.7$, respectively. The corresponding quantities for the reservoir branch can be determined similarly to be

$$h_{L,3} = \frac{200{,}000\ \text{N/m}^2}{(998\ \text{kg/m}^3)(9.81\ \text{m/s}^2)} - 1\ \text{m} = 19.4\ \text{m}$$

$$\sum K_{L,3} = 2 + 10 + 0.9 + 14 = 26.9$$

The relevant equations in this case are

$$\dot{V}_1 = \dot{V}_2 + \dot{V}_3$$

$$h_{L,2} = f_1 \frac{5\ \text{m}}{0.015\ \text{m}} \frac{V_1^2}{2(9.81\ \text{m/s}^2)} + \left(f_2 \frac{6\ \text{m}}{0.015\ \text{m}} + 24.7\right) \frac{V_2^2}{2(9.81\ \text{m/s}^2)} = 18.4$$

$$h_{L,3} = f_1 \frac{5\ \text{m}}{0.015\ \text{m}} \frac{V_1^2}{2(9.81\ \text{m/s}^2)} + \left(f_3 \frac{1\ \text{m}}{0.015\ \text{m}} + 26.9\right) \frac{V_3^2}{2(9.81\ \text{m/s}^2)} = 19.4$$

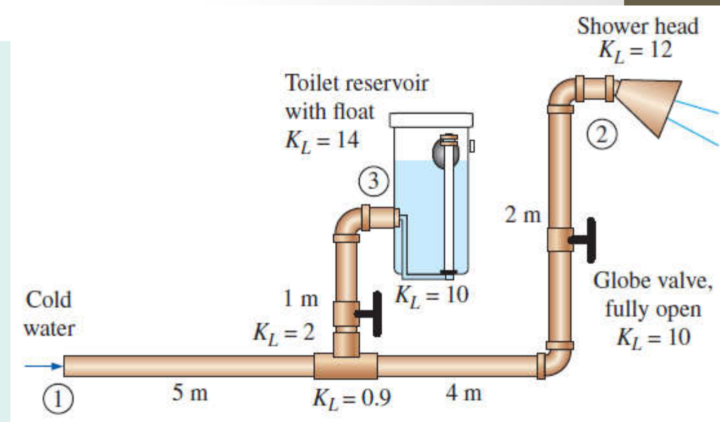
$$V_1 = \frac{\dot{V}_1}{\pi(0.015\ \text{m})^2/4},\quad V_2 = \frac{\dot{V}_2}{\pi(0.015\ \text{m})^2/4},\quad V_3 = \frac{\dot{V}_3}{\pi(0.015\ \text{m})^2/4}$$

$$\text{Re}_1 = \frac{V_1(0.015\ \text{m})}{1.004 \times 10^{-6}\text{m}^2/\text{s}},\ \text{Re}_2 = \frac{V_2(0.015\ \text{m})}{1.004 \times 10^{-6}\text{m}^2/\text{s}},\ \text{Re}_3 = \frac{V_3(0.015\ \text{m})}{1.004 \times 10^{-6}\text{m}^2/\text{s}}$$

$$\frac{1}{\sqrt{f_1}} = -2.0 \log\left(\frac{1.5 \times 10^{-6}\ \text{m}}{3.7(0.015\ \text{m})} + \frac{2.51}{\text{Re}_1\sqrt{f_1}}\right)$$

$$\frac{1}{\sqrt{f_2}} = -2.0 \log\left(\frac{1.5 \times 10^{-6}\ \text{m}}{3.7(0.015\ \text{m})} + \frac{2.51}{\text{Re}_2\sqrt{f_2}}\right)$$

$$\frac{1}{\sqrt{f_3}} = -2.0 \log\left(\frac{1.5 \times 10^{-6}\ \text{m}}{3.7(0.015\ \text{m})} + \frac{2.51}{\text{Re}_3\sqrt{f_3}}\right)$$

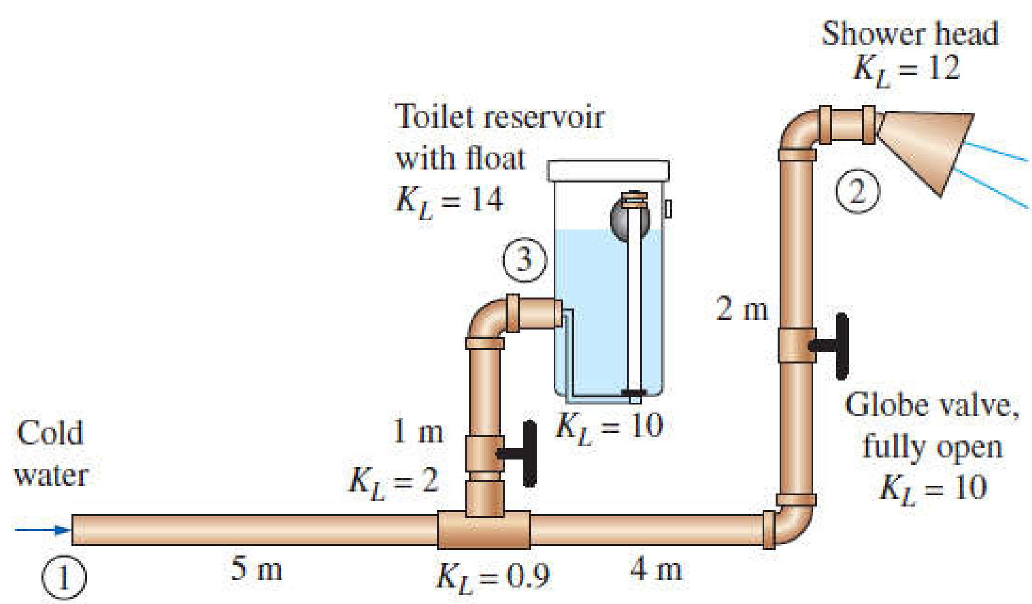


Solving these 12 equations in 12 unknowns simultaneously using an equation solver, the flow rates are determined to be

$$\dot{V}_1 = 0.00090\ \mathrm{m^3/s}, \quad \dot{V}_2 = 0.00042\ \mathrm{m^3/s}, \quad \text{and} \quad \dot{V}_3 = 0.00048\ \mathrm{m^3/s}$$

Therefore, the flushing of the toilet **reduces the flow rate of cold water through the shower by 21 percent** from 0.53 to 0.42 L/s, causing the shower water to suddenly get very hot (Fig. 8–53).

Discussion If the velocity heads were considered, the flow rate through the shower would be 0.43 instead of 0.42 L/s. Therefore, the assumption of negligible velocity heads is reasonable in this case. Note that a leak in a piping system would cause the same effect, and thus an unexplained drop in flow rate at an end point may signal a leak in the system.



Lecture 6

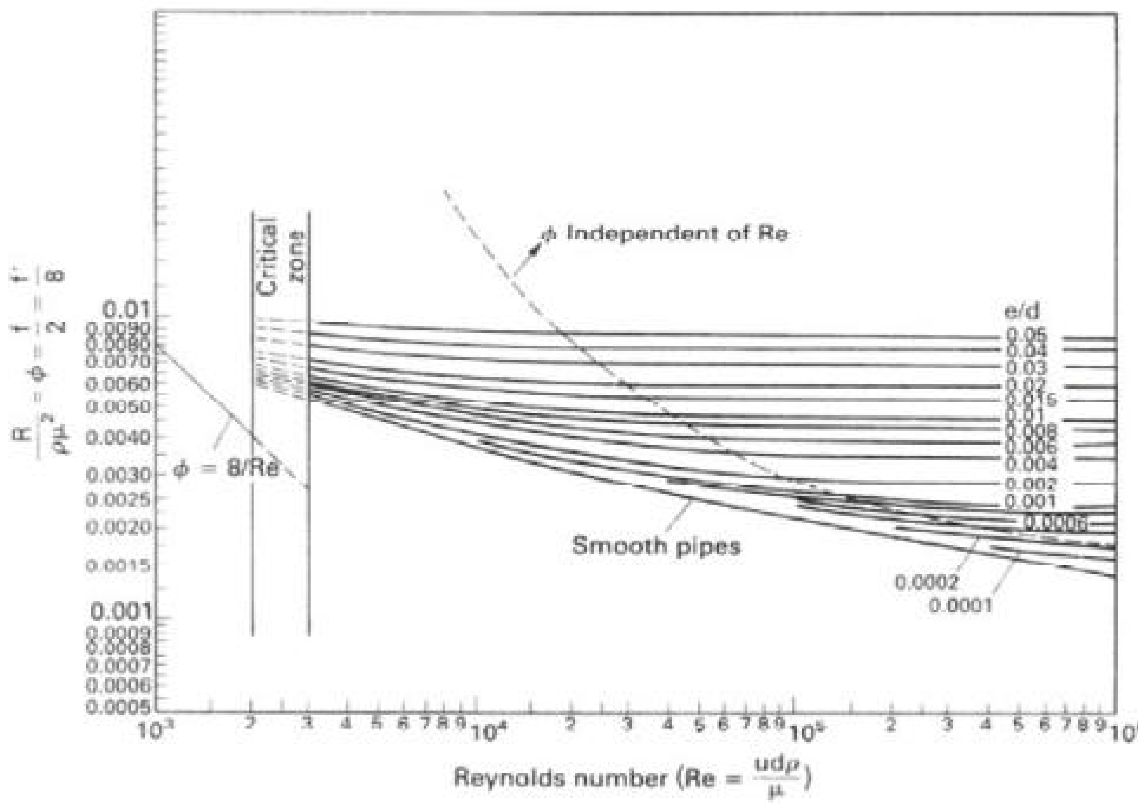


Fig.1 Pipe friction chart vs.Re

For turbulent flow, it is not possible to determine directly the fluid flow rate through a pipe from Figure 1 For a known pressure drop, the method of solution to this problem is as follows:

$\Phi = J_f = \tau/\rho u^2 \quad \Rightarrow \tau = \Phi\ \rho u^2$ -------------------------------(1)

But from force balance for fluid flow through horizontal pipe

$\tau\ \pi\ dL = -\Delta P_{fs}\ (\pi/4\ d^2)$

$\Rightarrow \tau = -\Delta P_{fs}/L\ (d/4)$---(2)

By equalization of equations (1) and (2)

$\Rightarrow \Phi\rho u^2 = -\Delta P_{fs}/L\ (d/4)$ -------------- x $\rho d^2/\mu^2$

$\Rightarrow \Phi\rho^2 u^2 d^2/\mu^2 = -\Delta P_{fs}/L\,(d^3\rho/4\mu^2)$

or $\boxed{\Phi\, Re^2 = -\Delta P_{fs}/L\,(\rho\, d^3/4\mu^2)}$ ------------------------(3)

This equation dose not contains the mean linear velocity (u) of fluid. This can be determine through using Figure 2 as follows:

1- Calculate the value of ΦRe^2 from equation (3) of (ΔP_{fs}, ρ, d, L, and μ).

2- Read the corresponding value of Re from Figure 2 for a known value of (e/d).

3- Calculate U from the extracted value of Re.

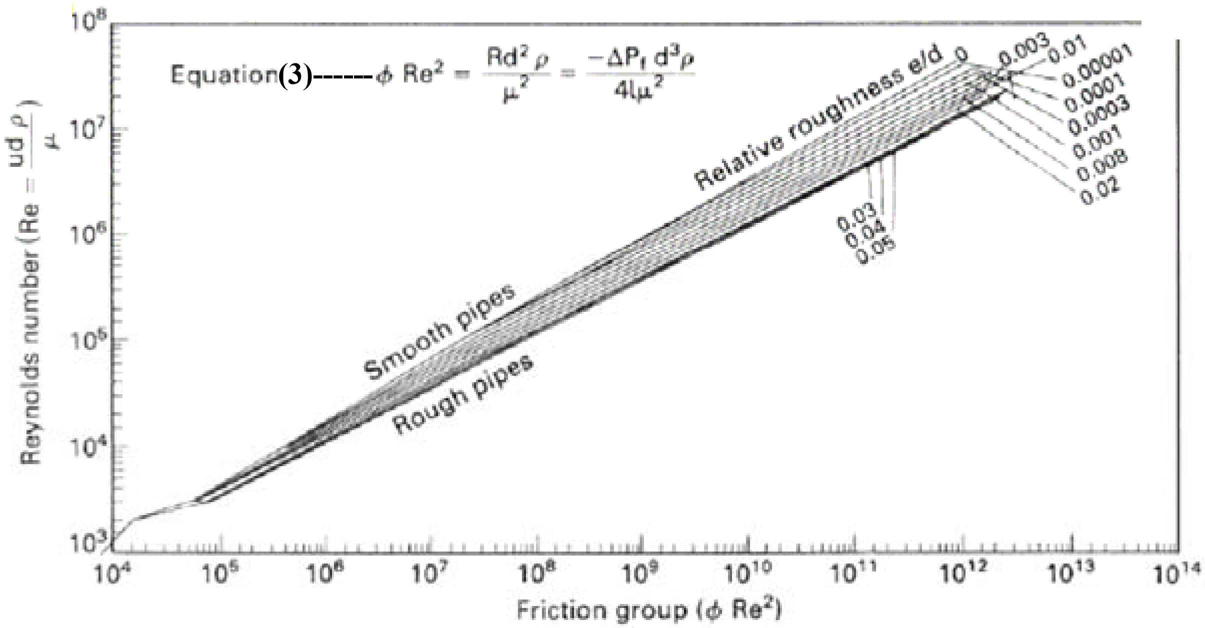


Fig.2 Pipe friction chart vs. Re for various values of e/d

Example

Calculate the pressure drop in Pa for a fluid flowing through a 30.48 m long commercial steel pipe of I.D. 0.0526 m and a pipe roughness (e = 0.045 mm). The fluid flows at steady transfer rate of 9.085 m^3/h. Take that ρ = 1200 kg/m^3, μ= 0.01 Pa.s.

Solution:

$Q = 9.085\ m^3/h \times h/3600s = 2.524 \times 10^{-3}\ m^3/s$

$u = Q/A = (2.524 \times 10^{-3}\ m^3/s) / (\pi/4 \times 0.0526^2) = 1.16\ m/s$

$Re = (1200 \times 1.16 \times 0.0526)/0.01 = 7322$

$e/d = 0.000045/0.0526 = 0.000856$

Figure 1 $\Phi = 0.0042 \Rightarrow f = 2\,\Phi = 0.084$

$$\frac{\Delta P}{\rho} + \cancel{g\Delta z} + \cancel{\frac{\Delta u^2}{2\alpha}} - \eta\, \cancel{Ws} + F = 0$$

$\Rightarrow -\Delta P_{fs} = \rho\, F_s = 4\, f\,(L/d)(\rho u^2/2) = 4\,(0.0084)\,(30.48/0.0526)\,(1200 \times 1.16^2/2)$

$= 15719$ Pa

Example

Repeat the previous example with the following conditions, the volumetric flow rate (i.e. the velocity) is unknown and the pressure drop in the pipe is 15.72 kPa.

Solution:

$\Phi Re^2 = (-\Delta P_{fs}/L)(\rho d^3/4\mu^2) = [(15720)/(30.48)][(1200)(0.0526)^3/(4)(0.01)^2] = 2.252 \times 10^5$

$e/d = 0.000856 \Rightarrow$ Figure 2 $Re = 7200 \Rightarrow u = 7200(0.01)/(1200 \times 0.0526) = 1.141 m/s$

Example

Repeat the previous example with the following conditions, the diameter of the pipe is unknown and the pressure drop in the pipe is 15.72 kPa and the velocity of the liquid is 1.15 m/s. Estimate the diameter of the pipe.

Solution:

$-\Delta P_{fs} = \rho F_s = 4 f (L/d)(\rho u^2/2) \Rightarrow d = (4 \rho f L u^2/2)/ -\Delta P_{fs}$

$\Rightarrow d = 6.154 f$ --------------------------------(1)

$Re = \rho u d/\mu = 138000 d$ ----------------------(2)

$e/d = 0.000045/d$ -----------------------------(3)

	Φ	f=2Φ	Eq.(1)	Eq.(2)	Eq.(3)
	Figure 1	f	d	Re	e/d
Assumed →		0.001	0.006154	849	0.0073
	0.01	0.02	0.123	16985	0.00036
	0.0037	0.0074	0.045	6284	0.00099
	0.0045	0.009	0.0554	7643	0.0008
	0.0043	0.0086	0.0529	7303	0.00085
$\Rightarrow d = 0.0529$ m	0.0043	0.0086			

Example

Sulfuric acid is pumped at 3 kg/s through a 60 m length of smooth 25 mm pipe. Calculate the drop in pressure. If the pressure drop falls to one half, what will new flow rate be? Take that $\rho = 1840 kg/m^3$, $\mu = 25$ mPa.s.

Solution:

$$u = \frac{Q}{A} = \frac{\dot{m}}{\rho A} = \frac{3 \, kg/s}{(1840 \, kg/m^3)(\pi/4 \times 0.025^2) m^2} = 3.32 \, m/s$$

$Re = (1840 \times 3.32 \times 0.025)/0.025 = 6111$

Figure 1 for smooth pipe $\Phi = 0.0043 \Rightarrow f = 0.0086$

$-\Delta P_{fs} = \rho F_s = 4 f (L/d)(\rho u^2/2) = 4(0.0086)(60/0.025)(1840 \times 3.32^2)/2 = 837.209$ kPa

The pressure drop falls to one half (i.e. $-\Delta P_{fs}$ = 837.209 kPa/2 = 418.604kPa

$\Phi Re^2 = (-\Delta P_{fs}/L)(\rho d^3/4\mu^2)$=[(418604)/(60)][(1840)(0.025)^3/(4)(0.025)^2]= 8.02 x 10^4

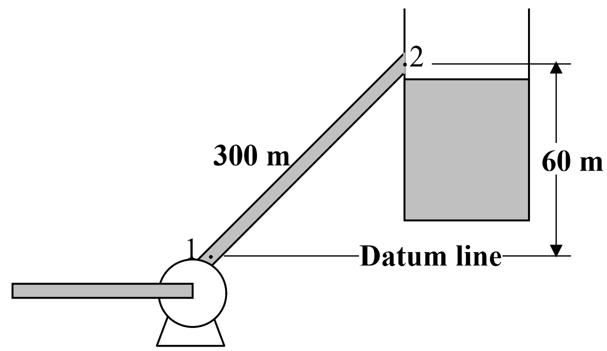
From Figure 2 for smooth pipe Re = 3800 $\Rightarrow$u = 2.06m/s

$\dot{m} = \rho\, u\, A$ = 1.865 kg/s.

Example

A pump developing a pressure of 800 kPa is used to pump water through a 150 mm pipe, 300 m long to a reservoir 60 m higher. With the valves fully open, the flow rate obtained is 0.05 m^3/s. As a result of corrosion and scalling the effective absolute roughness of the pipe surface increases by a factor of 10 by what percentage is the flow rate reduced. μ= 1 mPa.s

<u>**Solution:**</u>



The total head of pump developing =($\Delta P/\rho g$) = 800,000/(1000 x 9.81)= 81.55 mH_2o

The head of potential energy = 60 m

Neglecting the kinetic energy losses (same diameter)

$$\frac{\Delta P}{\rho} + g\Delta z + \frac{\Delta u^2}{2\alpha} - \eta\, Ws + F = 0$$

$\Rightarrow \Delta P/\rho g + \Delta z + h_F = 0$

$\Rightarrow h_F = -\Delta P/\rho g - \Delta z$ = 81.55 – 60 = 21.55 m

u = Q/A = (0.05 m^3/s)/(π/4 0.15^2) =2.83 m/s

$h_{Fs} = (-\Delta P_{fs}/\rho g) = 4f\,(L/d)\,(u^2/2g)$

$\Rightarrow f = h_{Fs}\, d\, 2g/(4Lu^2)$ = (21.55) (0.15)(9.81)/(2 x 300 x 2.83^2) = 0.0066

Φ = 0.0033, Re = (1000 x 2.83 x 0.15)/0.001 = 4.23 x10^5

From Figure 1 e/d = 0.003

Due to corrosion and scalling the roughness increase by factor 10

i.e. $(e/d)_{new}$ = 10 $(e/d)_{old}$ = 0.03

The pump head that supplied is the same

$(-\Delta P_{fs}) = h_{Fs}\, \rho g$ = 21.55 (1000) 9.81 = 211.41 kPa

$\Phi Re^2 = (-\Delta P_{fs}/L)(\rho d^3/4\mu^2)$ = [(211410)/(300)][(1000)(0.15)^3/(4)(0.01)^2] = 6 x 10^8

From Figure 2 Re = 2.95 x10^5 $\Rightarrow$u = 1.97m/s

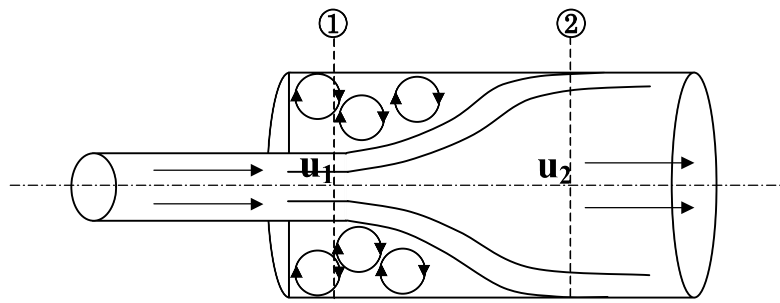
The percentage reduced in flow rate = (2.83 - 1.97)/ 2.83 x 100 % = 30.1 %.

<u>Form Friction</u>

Skin friction loss in flow straight pipe is calculated by using the Fanning friction factor (f). However, if the velocity of the fluid is changed in *direction* <u>or</u> *magnitude*, additional friction losses occur. This results from additional turbulence, which develops because of vertices and other factors.

1- Sudden Expansion (Enlargement) Losses

If the cross section of a pipe enlarges gradually, very little or no extra losses are incurred. If the change is sudden, as that in Figure, it results in additional losses due to eddies formed by the jet expanding in the enlarged suction. This friction loss can be calculated by the following for laminar or turbulent flow in both sections, as:



Continuity equation $u_1A_1 = u_2A_2 \quad \Rightarrow u_2 = u_1(A_1/A_2)$

Momentum balance $F_e = \frac{u_1^2}{2\alpha_1} - \frac{u_2^2}{2\alpha_2} + u_2^2 - u_1u_2$

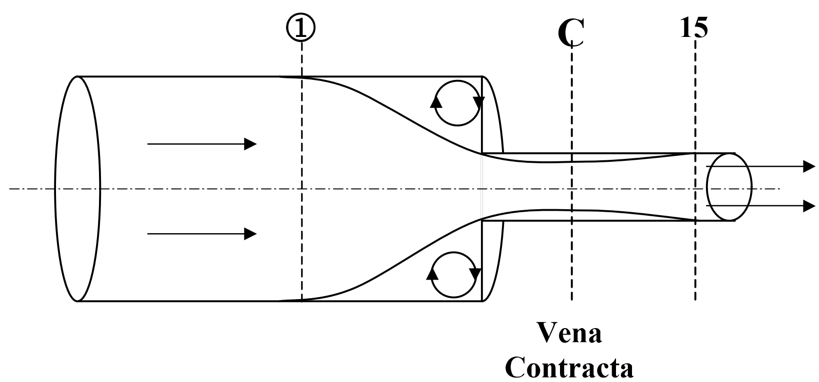
For fully turbulent flow in both sections

$$F_e = \frac{(u_1 - u_2)^2}{2} = \frac{u_1^2}{2}\left[1 - \left(\frac{A_1}{A_2}\right)\right]^2$$

$$\boxed{\therefore F_e = K_e \frac{u_1^2}{2}}; \text{ where } \boxed{K_e = \left[1 - \left(\frac{A_1}{A_2}\right)\right]^2}$$

2- Sudden Contraction Losses

The effective area for the flow gradually decreases as the sudden contraction is approached and then continues to decrease, for a short distance, to what is known as the "Vena Contacta". After the Vena Contracta the flow area gradually approaches that of the smaller pipe, as shown in Figure. When the cross section of the pipe is suddenly reduced, the stream cannot follow around the sharp corner, and additional loses due to eddies occur.



$$\boxed{F_c = K_c \frac{u_2^2}{2}}; \text{ where } \boxed{K_c = 0.55\left[1 - \frac{A_2}{A_1}\right]}$$

3- Losses in Fittings and Valves

Pipe fittings and valves also disturb the normal flow lines in a pipe and cause additional friction losses. In a short pipe with many fittings, the friction losses from these fittings could be greater than in the straight pipe. The friction loss for fittings and valves is:

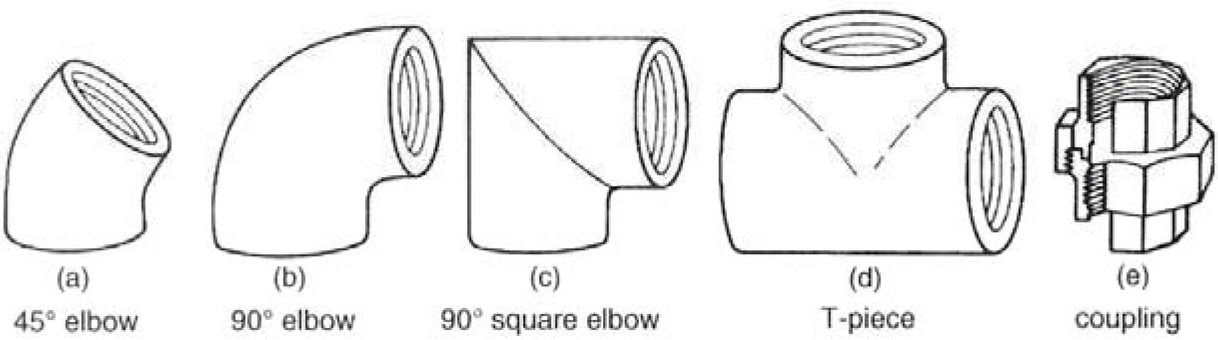
$F_f = K_f \frac{u_2}{2}$; where K_f as in table below.

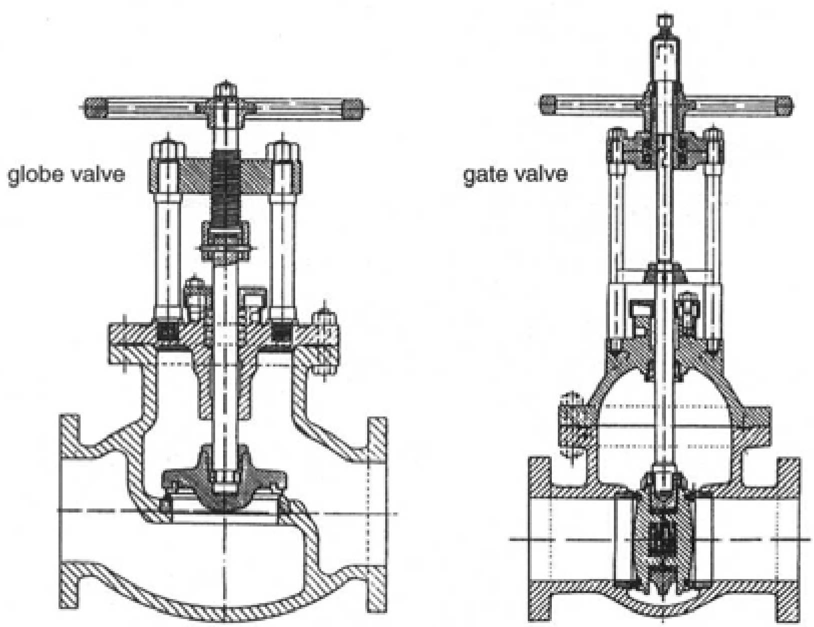
$F_f = 4f \frac{Le}{d}\frac{u_2}{2}$; where $\frac{Le}{d}$ as in table below.

Table of the Friction losses in pipe fittings

Fittings	K_f	Le/d
45° elbows (a)*	15	0.3
90° elbows (standard radius) (b)	30-40	0.6-0.8
90° square elbows (c)	60	1.2
Entry from leg of T-piece (d)	60	1.2
Entry into leg of T-piece (d)	90	1.8
Unions and couplings (e)	Very small	Very small
Globe valves fully open	60-300	1.2-6.0
Gate valves: fully open	7	0.15
3/4 open	40	1
1/2 open	200	4
1/4 open	800	16

*** See Figure below**





Figures of standard pipe fittings and standard valves

Total Friction Losses

The frictional losses from the friction in the straight pipe (skin friction), enlargement losses, contraction losses, and losses in fittings and valves are all incorporated in F term in mechanical energy balance equation (modified Bernoulli's equation), so that,

$$F = 4f\frac{L}{d}\frac{u^2}{2} + K_e\frac{u_1^2}{2} + K_c\frac{u_2^2}{2} + K_f\frac{u^2}{2}$$

If all the velocity u, u_1, and u_2 are the same, then this equation becomes, for this special case;

$$F = \left[4f\frac{L}{d} + K_e + K_c + K_f\right]\frac{u^2}{2}$$

If equivalent length of the straight pipe for the losses in fittings and/or valves, then this equation becomes;

$$F = \left[4f\left(\frac{L}{d} + \sum\frac{Le}{d}\right) + K_e + K_c\right]\frac{u^2}{2}$$

Example

630 cm^3/s water at 320 K is pumped in a 40 mm I.D. pipe through a length of 150 m in horizontal direction and up through a vertical height of 10 m. In the pipe there is a control valve which may be taken as equivalent to 200 pipe diameters and also other fittings equivalent to 60 pipe diameters. Also other pipe fittings equivalent to 60 pipe diameters. Also in the line there is a heat exchanger across which there is a loss in head of 1.5 m H_2o. If the main pipe has a roughness of 0.0002 m, what power must supplied to the pump if η = 60%, μ= 0.65 mPa.s.

<u>Solution:</u>

Q = 630 cc/s (m/100 cm)^3 = 6.3 x 10^{-4} m^3/s

u = (6.3 x 10^{-4} m^3/s)/(π/4 0.04^2) = 0.5 m/s

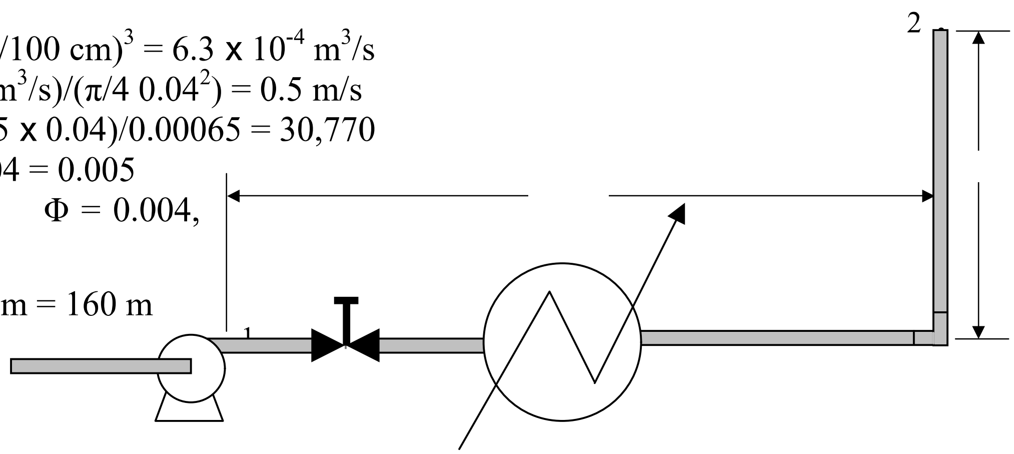
Re = (1000 x 0.5 x 0.04)/0.00065 = 30,770

e/d = 0.0002/0.04 = 0.005

From Figure 1 Φ = 0.004,

$\Rightarrow f$ = 0.008

L = 150 m + 10 m = 160 m



$$\frac{\Delta P}{\rho g} + \Delta z + \frac{\Delta u^2}{2\alpha g} - \frac{\eta Ws}{g} + h_F + (h)_{H.Ex.} = 0$$

$$h_F = \left[4f\left(\frac{L}{d} + \sum\frac{Le}{d}\right)\right]\frac{u^2}{2g} = 4\ (0.008)\ (160/0.04 + 200 + 60) \times 0.5^2/(2 \times 9.81) = 1.74\ \text{m}$$

$\Rightarrow$(-ΔP/ρg) = Δh =10 + 1.74 + 1.5 = 13.24 m

⇒ The head required (that must be supplied to water by the pump) is $\Delta h = 13.24$ m and the power required for the water is $\eta Ws = Q\,\Delta P = Q\,(\Delta h\,\rho g)$

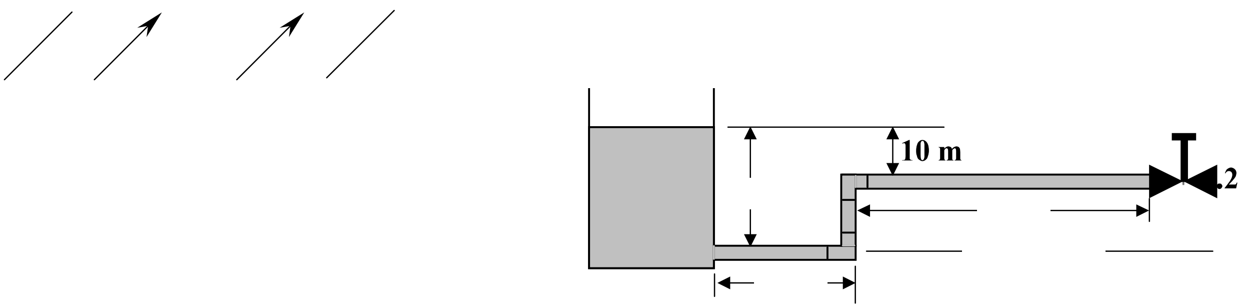
⇒ $\eta Ws = 6.3 \times 10^{-4}\ m^3/s\ (13.24 \times 1000 \times 9.81) = 81.8$ (N.m/s ≡ J/s ≡ W)
The power required for the pump is $(Ws) = \eta Ws / \eta = 81.8 / 0.6 = 136.4$ W.

Example

Water in a tank flows through an outlet 25 m below the water level into a 0.15 m I.D. horizontal pipe 30 m long, with 90° elbow at the end leading to vertical pipe of the same diameter 15 m long. This is connected to a second 90° elbow which leads to a horizontal pipe of the same diameter, 60 m long, containing a fully open globe valve and discharge to atmosphere 10 m below the level of the water in the tank. Calculate *the initial rate*. Take that μ= 1 mPa.s, e/d =0.01

Solution:

L = 30 + 15 + 60 = 105 m



$$\frac{P_1}{\rho g} + \frac{u_1^2}{2\alpha_1 g} + z_1 + \frac{\eta Ws}{g} = \frac{P_2}{\rho g} + \frac{u_2^2}{2\alpha_2 g} + z_2 + h_F$$

$$\Rightarrow z_1 - z_2 = \frac{u_2^2}{2g\alpha_2} + h_F$$

$$h_F = \left[4f\left(\frac{L}{d} + \sum\frac{Le}{d}\right)\right]\frac{u^2}{2g}$$

$4f\,[(105/0.15)+2(40)+250]\ u_2^2/(2 \times 9.81)$
$= 210\,f\,u_2^2$

Assume turbulent flow (α_2 =1.0)

⇒ $(25\text{-}15) = u_2^2/(2 \times 9.81) + 210\ f\ u_2^2$ $\qquad \Rightarrow u_2 = \sqrt{\dfrac{10}{0.05 + 210 f}}$ ---(*)

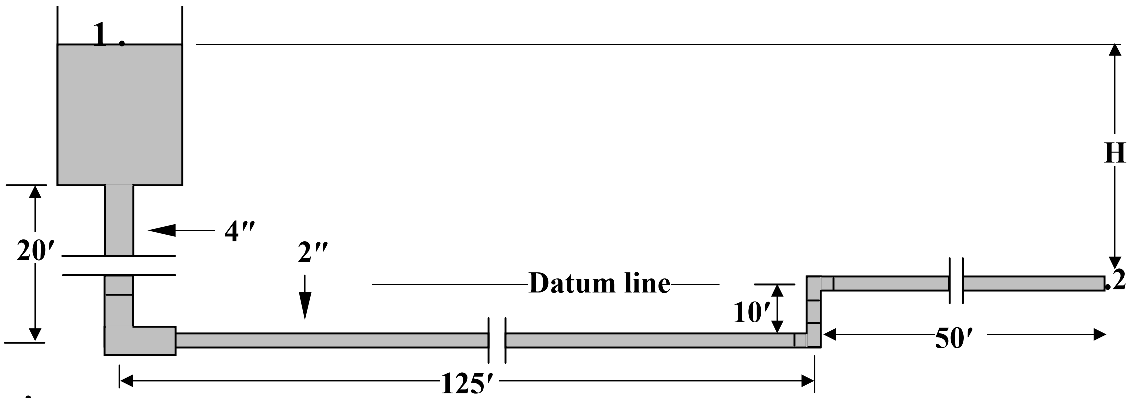
This equation solved by trial and error

	Eq.(*)		Figure 1
f	u_2	Re	Φ
0.01	2.16	3.235×10^5	0.0046
0.0092	2.246	3.37×10^5	0.0046

⇒ $u_2(t=0) = 2.246$ m/s, Re = 3.37 x105 (turbulent) ⇒Q=0.04 m^3/s ; m = 40 kg/s

Example

An elevated storage tank contains water at 82.2°C as shown in Figure below. It is desired to have a discharge rate at point 2 of 0.223 ft^3/s. What must be the height H in ft of the surface of the water in the tank relative to discharge point? The pipe is schedule 40, e = 1.5 x10^{-4} ft. Take that ρ = 60.52 lb/ft^3, μ= 2.33 x10^{-4} lb/ft.s.



Solution:

$$\frac{P_1}{\rho}+\frac{u_1^2}{2\alpha_1 g_c}+\frac{g\,z_1}{g_c}+\eta W_s=\frac{P_2}{\rho}+\frac{u_2^2}{2\alpha_2 g_c}+\frac{g\,z_2}{g_c}+F$$

$$\Rightarrow \frac{g}{g_c}z_1=\frac{u_2^2}{2\alpha_2 g_c}+F$$, where z_1 = H

for schedule 40

$d_{4''}$ = 4.026/12 = 0.3353 ft, $A_{4''}$ = 0.0884 ft^2,

$d_{2''}$ = 2.067/12 = 0.1722 ft, $A_{4''}$ = 0.0233 ft^2,

$u_{4''}$ = (0.223 ft^3/s) / (0.0884 ft^2) = 2.523 ft, $u_{2''}$ = (0.223 ft^3/s) / (0.0233 ft^2) = 9.57 ft,

The F-term for friction losses in the system includes the followings:

1- Contraction losses at tank exit. 2-Friction in 4″ straight pipe.

3- Friction in 4″ elbow. 4-Contraction losses in 4″ to 2″ pipe.

5- Friction in 2″ straight pipe. 6-Friction in the two 2″ elbows.

1- Contraction losses at tank exit. (let tank area =A_1, 4″ pipe area = A_3)

$$F_c = K_c\frac{u_2^2}{2}$$; where $$K_c = 0.55\left[1-\frac{A_2}{A_1}\right]\approx 0.5$$

$\Rightarrow$ F_c = 0.55 (2.523^2 /2 x 32.174) = 0.054 ft.lb_f/lb.

2- Friction in 4″ straight pipe.

Re = (60.52 x 2.523 x 0.3353)/ 2.33 x10^{-4} = 2.193 x10^5

e/d = 0.000448 $\Rightarrow$ Figure 1 f = 0.0047

$$F_{Fs} = 4f\frac{L}{d}\frac{u^2}{2g_c}$$ =4 (0.0047) (20/0.3353) x 2.523^2/(2 x 32.174) = 0.111 ft.lb_f/lb.

3- Friction in 4″ elbow.

$$F_f = K_f\frac{u_2}{2}$$; where K_f = 0.75 $\Rightarrow$ F_f = 0.074 ft.lb_f/lb.

4- Contraction losses in 4″ to 2″ pipe.

$\boxed{F_c = K_c \frac{u_2^2}{2}}$; where $\boxed{K_c = 0.55\left[1 - \frac{A_2}{A_1}\right]}$ = 0.55 (1- 0.0233/0.0884) = 0.405

⇒ F_c = 0.405 (9.57^2 /2 x 32.174) = 0.575 ft.lb$_f$/lb.

5- Friction in 2″ straight pipe.
Re = (60.52 x 9.57 x 0.1722)/ 2.33 x10^{-4} = 4.28 x10^5

e/d = 0.00087 ⇒ Figure 1   *f* = 0.0048

$F_{Fs} = 4f \frac{L}{d} \frac{u^2}{2g_c}$ =4 (0.0048) (185/0.3353) x 9.57^2/(2 x 32.174) = 29.4 ft.lb$_f$/lb.

6- Friction in the two 2″ elbow.

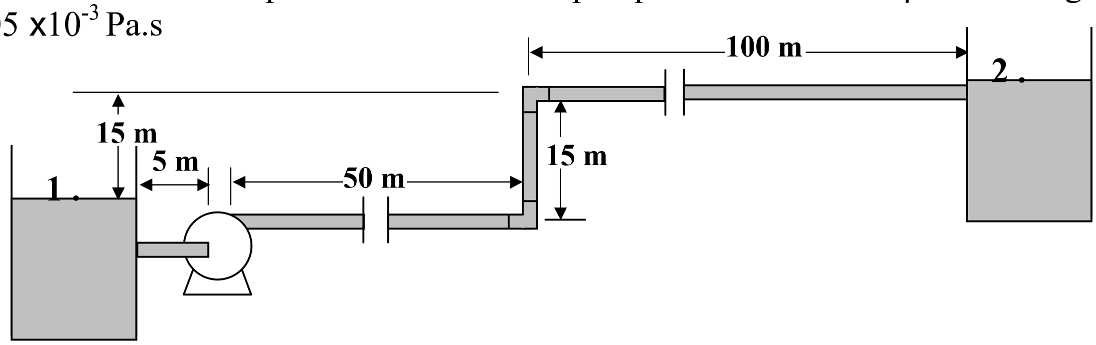
$\boxed{F_f = 2(K_f \frac{u_2}{2})}$; where K_f = 0.75 ⇒ F_f = 2.136 ft.lb$_f$/lb.

F (total frictional losses) = 0.054 + 0.111 + 0.575 + 29.4 + 2.136 =32.35 ft.lb$_f$/lb

⇒ H g/g$_c$ = (9.57^2 /2 x 32.174) + 32.35= 33.77 ft.lb$_f$/lb
H = 33.77 ft ≈ 10.3m (height of water level above the discharge outlet)

Example

Water at 20°C being pumped from a tank to an elevated tank at the rate of 0.005 m^3/s. All the piping in the Figure below is 4″ Schedule 40 pipe. The pump has an efficiency of η= 0.65. calculate the kW power needed for the pump. e = 4.6 x10^{-5} m ρ = 998.2 kg/m^3, μ= 1.005 x10^{-3} Pa.s



Solution:

For 4″ Schedule 40 pipe   d = 0.1023 m, A = 8.219 x10^{-3} m^2
u = Q/A = (5 x10^{-3} m^3/s)/ 8.219 x10^{-3} m^2 = 0.6083 m/s

$\boxed{\frac{\Delta P}{\rho} + g\Delta z + \frac{\Delta u^2}{2\alpha} - \eta\, Ws + F = 0}$ ⇒ ηWs = F +gΔz

The F-term for friction losses in the system includes the followings:
1- Contraction losses at tank exit.   2-Friction loss in straight pipe.
3- Friction in the two elbows.   4- Expansion loss at the tank entrance.

1- Contraction losses at tank exit.

$\boxed{F_c = K_c \frac{u_2^2}{2}}$; where $\boxed{K_c = 0.55\left[1 - \frac{A_2}{A_1}\right]}$ ≈ 0.5

⇒ F_c = 0.55 (0.6083^2 /2) = 0.102 J/kg or m^2/s^2.

2- Friction loss in straight pipe.
Re = (998.2 x 0.6083 x 0.1023)/ 1.005x10^{-3} = 6.181 x10^4

$e/d = 0.00045 \Rightarrow$ Figure (3.7) $f = 0.0051$
L = 5+50+15+100 = 170 m

$F_{Fs} = 4f \frac{L}{d}\frac{u^2}{2}$ =4 (0.0051) (170/0.1023) x ($0.6083^2/2$) = 6.272 J/kg or m^2/s^2.

3- Friction in the two elbows.

$F_f = 2(K_f \frac{u_2}{2})$; where $K_f = 0.75 \Rightarrow F_f = 0.278$ J/kg or m^2/s^2.

4- Expansion loss at the tank entrance.

$F_e = K_e \frac{u_1^2}{2}$; where $K_e = \left[1 - \left(\frac{A_1}{A_2}\right)\right]^2 \approx 1.0 \Rightarrow F_e = 0.185$ J/kg or m^2/s^2.

F (total frictional losses) = 0.102 + 6.272 + 0.278 + 0.185 = 6.837 J/kg or m^2/s^2.

$\Rightarrow \eta Ws = 6.837 + 9.81(15) = 153.93$ J/kg or m^2/s^2

The power required for pump (Ws) = ηWs/Ws = 153.93/0.65 = 236.8 J/kg or m^2/s^2

The total power required for pump ($\dot{m}$ Ws) = Q ρ Ws

= (5 x10^{-3} m^3/s) 998.2 kg/m^3 (236.8 J/kg) = 1.182 kW.

Example

Water at 4.4°C is to flow through a horizontal commercial steel pipe having a length of 305 m at the rate of 150 gal/min. A head of water of 6.1 m is available to overcome the skin friction losses (h_{Fs}). Calculate the pipe diameter. e = 4.6 x10^{-5} m ρ = 1000 kg/m^3, μ= 1.55 x10^{-3} Pa.s.

Solution:

$h_{Fs} = \left[4f\left(\frac{L}{d}\right)\right]\frac{u^2}{2g}$ = 6.1 m

Q = 150 gal/min (ft^3/ 7.481gal)(min/60s) (m/3.28 ft)^3 = 9.64 x10^{-3} m^3/s

u = Q/A = (9.64 x10^{-3} m^3/s)/(π/4 d^2) ⇒ u = 0.01204 d^{-2}.

⇒ 6.1 = 4f (305/d)(0.01204 d^{-2})/(2 x9.81)

⇒ f = 676.73 d^5 ⇒ d = (f/676.73)$^{1/5}$ ----------------------(1)

Re = (1000 x (0.01204 d^{-2}) x d)/ 1.55x10^{-3} = 7769.74 d^{-1} ----------------------(2)

e/d = 4.6 x10^{-5} d^{-1} ----------------------(3)

solution by trial and error

		Eq.(1)	Eq.(2)	Fq. (3)	Figure 1
	f	d	Re	e/d	$f = 2\Phi$
Assumed →	0.00378	0.089	8.73 x10^4	0.00052	0.0052
	0.0052	0.095	8.176 x10^4	0.000484	0.0051
	0.0051	0.0945	8.22 x10^4	0.00049	0.0051

⇒ d = 0.0945 m.

Example

A petroleum fraction is pumped 2 km from a distillation plant to storage take through a mild steel pipeline, 150 mm I.D. at 0.04 m^3/s rate. What is the pressure drop along the pipe and the power supplied to the pumping unit if it has an efficiency of 50%. The pump impeller is eroded and the pressure at its delivery falls to one half. By how much is the flow rate reduced? Take that: sp.gr. = 0.705, μ= 0.5 m Pa.s e = 0.004 mm.

Solution:

$u = Q/A = (0.04\ m^3/s)/(\pi/4 \times 0.15^2) \Rightarrow u = 2.26\ m/s$

$Re = (705 \times 2.26 \times 0.15)/\ 0.5\times10^{-3} = 4.78\times10^5$

$e/d = 0.000027 \Rightarrow$ Figure 1 $f = 2\ \Phi \Rightarrow f = 0.0033$

$$\boxed{-\Delta P_{Fs} = \left[4f\left(\frac{L}{d}\right)\right]\frac{\rho\, u^2}{2}} = 4\ (0.0033)\ (2000/0.15)\ (705 \times 2.26^2/2) = 316876\ Pa.$$

$$Power = \frac{Q(-\Delta P)}{\eta} = (0.04\ m^3/s)(316876\ Pa)/0.5 = 25.35\ kW$$

Due to impeller erosion $(-\Delta P)_{new} = (-\Delta P)_{old}/2 = 316876\ Pa/2 = 158438\ Pa$

$\Phi Re^2 = (-\Delta P_{fs}/L)(\rho d^3/4\mu^2) = [(158438)/(2000)][(1000)(0.15)^3/(4)(0.5\times10^{-3})^2] = 1.885\times10^8$

$e/d = 0.000027 \Rightarrow$ From Figure 2 $Re = 3\times10^5 \Rightarrow u = 1.42 m/s$

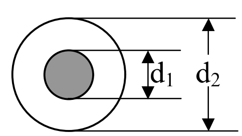
The new volumetric flow rate is now $Q = 1.42\ (\pi/4 \times 0.15^2) = 0.025\ m^3/s$.

Friction Losses in Noncircular Conduits

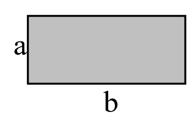
The friction loss in long straight channels or conduits of noncircular cross-section can be estimated by using the same equations employed for circular pipes if the diameter in the Reynolds number and in the friction factor equation is taken as equivalent diameter. The equivalent diameter De or hydraulic diameter defined as four times the cross-sectional area divided by the wetted perimeter of the conduit.

$$De = 4\frac{\text{Cross - sectional area of channel}}{\text{Wetted perimeter of channel}}$$

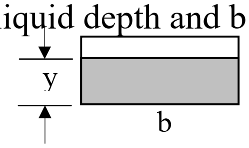
- For circular cross section.
  $De = 4\ (\pi/4 \times d^2)/\ \pi\ d = d$
- For an annular space with outside diameter d_1 and inside d_2.
  $De = 4\ [\pi/4 \times (d_1^2 - d_2^2)]/\ \pi\ (d_1 + d_2) = d_1 + d_2$



- For a rectangular duct of sides a and b.
  $De = 4\ (a.b)\ /\ 2(a + b) = 2ab\ /\ (a + b)$



- For open channels and pattly filled ducts of y-liquid depth and b width
  $De = 4\ (b.y)\ /\ (b + 2y)$



Selection of Pipe Sizes

In large or complex piping systems, the optimum size of pipe to use for a specific situation depends upon *the relative costs of capital investment*, *power*, *maintenance*, and so on. Charts are available for determining these optimum sizes. However, for small installations approximations are usually sufficient accurate. A table of representative values of ranges of velocity in pipes is shown in the following table: -

Type of fluid	Type of flow	Velocity	
		ft/s	m/s
Nonviscous liquid	Inlet to pump	2 - 3	0.6 – 0.9
	Process line or Pump discharge	5 - 8	1.5 – 2.5
Viscous liquid	Inlet to pump	0.2 – 0.8	0.06 – 0.25
	Process line or Pump discharge	0.5 - 2	0.15 – 0.6
Gas		30 - 120	9 – 36
Steam		30 - 75	9 – 23

Lecture 7

Pumping of Liquids

5.1 Introduction

Pupmps are devices for supplying *energy* or *head* to a flowing liquid in order to overcome head losses due to friction and also if necessary, *to raise* liquid to a higher level.

For the pumping of liquids or gases from one vessel to another or through long pipes, some form of mechanical pump is usually employed**. The energy required by the pump** will depend on the height through which the fluid is raised, the pressure required at delivery point, the length and diameter of the pipe, the rate of flow, together with the physical properties of the fluid, particularly its *viscosity* and *density*. The pumping of liquids such as sulphuric acid or petroleum products from bulk store to process buildings, or the pumping of fluids round reaction units and through heat exchangers, are typical illustrations of the use of pumps in the process industries. On the one hand, it may be necessary to inject reactants or catalyst into a reactor at a low, but accurately controlled rate, and on the other to pump cooling water to a power station or refinery at a very high rate. The fluid may be a gas or liquid of low viscosity, or it may be a highly viscous liquid, possibly with non-Newtonian characteristics. It may be clean, or it may contain suspended particles and be very corrosive. All these factors influence the choice of pump.

Because of the wide variety of requirements, many different types are in use including centrifugal, piston, gear, screw, and peristaltic pumps, though in the chemical and petroleum industries the centrifugal type is by far the most important.

5.2 The Total Head (Δh)

The head imparted to a flowing liquid by a pump is known as the total head (Δh). If a pump is placed between points ① and ② in a pipeline, the head for steady flow are related by: -

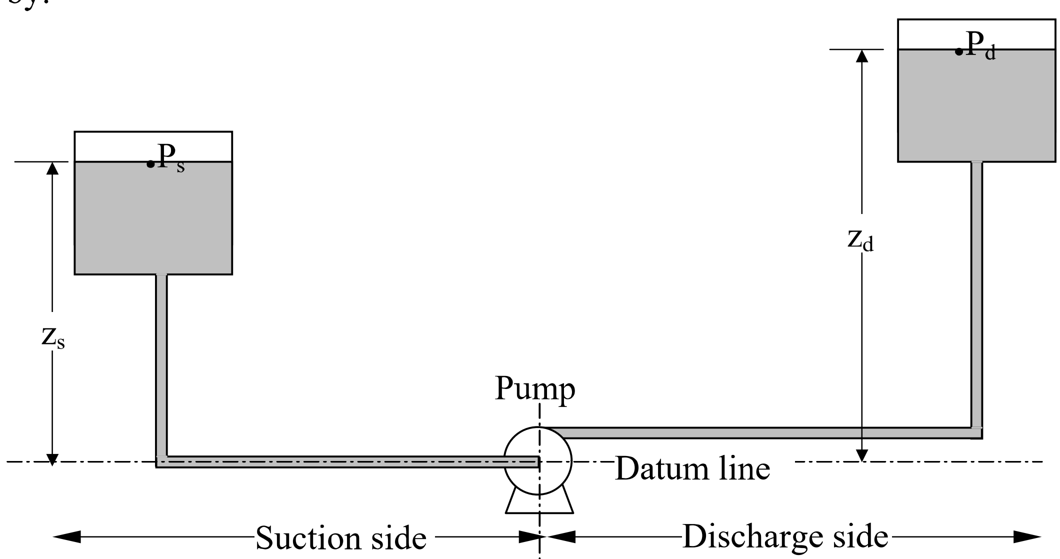


Figure (1) Typical pumping system.

$$\Delta h = \frac{\eta\, Ws}{g} = \left(\frac{P_2}{\rho\, g} + \frac{u_2^2}{2\alpha_2\, g} + z_2 \right) - \left(\frac{P_1}{\rho\, g} + \frac{u_1^2}{2\alpha_1\, g} + z_1 \right) - h_F$$

$$\Rightarrow \Delta h = \frac{\Delta P}{\rho\, g} + \frac{\Delta\, u^2}{2\alpha\, g} + \Delta z + h_F$$

5.3 System Heads

The important heads to consider in a pumping system are: -

1- Suction head
2- Discharge head
3- Total head
4- Net positive suction head (NPSH)

The following definitions are given in reference to typical pumping system shown in preceding Figure, where the datum line is the centerline of the pump

1- Suction head (h_s)

$$h_s = z_s + \frac{P_s}{\rho\, g} - (h_F)_s$$

2- Discharge head (h_d)

$$h_d = z_d + \frac{P_d}{\rho\, g} + (h_F)_d$$

3- Total head (Δh)

The total head (Δh), which is required to impart to the flowing liquid is the difference between the discharge and suction heads. Thus,

$$\Delta h = h_d - h_s$$

$$\Rightarrow \Delta h = (z_d - z_s) + (\frac{P_d - P_s}{\rho\, g}) + [(h_F)_d + (h_F)_s]$$

where,

$$(h_F)_d = 4 f_d \left(\frac{L}{d} + \sum \frac{Le}{d} \right)_d \frac{u_d^2}{2\, g}$$

$$(h_F)_s = 4 f_s \left(\frac{L}{d} + \sum \frac{Le}{d} \right)_s \frac{u_s^2}{2\, g}$$

The suction head (hs) decreases and the discharge head (hd) increases with increasing liquid flow rate because of the increasing value of the friction head loss terms (hF)s and (hF)d. Thus the total; head (Δh) which the pump is required to impart to the flowing liquid increases with increasing the liquid pumping rate.

Note:

If the liquid level on the suction side is below the centerline of the pump, z_s is negative.

4- Net positive suction head (NPSH)

Available net positive suction head

$$NPSH = z_s + \left(\frac{P_s - P_v}{\rho\, g} \right) - (h_F)_s$$

This equation gives the head available to get the liquid through the suction piping.

P_v is the vapor pressure of the liquid being pumped at the particular temperature in question.

The available net positive suction head (NPSH) can also be written as:

$$NPSH = h_s - \frac{P_v}{\rho\, g}$$

The available net positive suction head (NPSH) in a system should always be positive i.e. the suction head always be capable of overcoming the vapor pressure (Pv) since the frictional head loss (hF)s increases with increasing pumping rate.

At the boiling temperature of the liquid Ps and Pv are equal and the available NPSH becomes [zs-(hF)s]. In this case no suction lift is possible since zs must be positive. If the term (Ps-Pv) is sufficiently large, liquid can b lifted from below the centerline of the pump. In this case zs is negative.

From energy consideration it is immaterial whether the suction pressure is below atmospheric pressure or well above it, as long as the fluid remains liquid. However, if the suction pressure is only slightly greater than the vapor pressure, some liquid may flash to vapor inside the pump, a process called "*Cavitation*", which greatly reduces the pump capacity and severe erosion.

If the suction pressure is actually less than the vapor pressure, there will be vaporization in the suction line, and no liquid can be drawn into the pump.

To avoid cavitation, the pressure at the pump inlet must exceed the vapor pressure by a certain value, called the " net positive suction head (NPSH)". The required values of NPSH is about 2-3 m H_2o for small pump; but it increases with pump capacity and values up to 15 m H2o are recommended for very large pump.

5.4 Power Requirement

The power requirement to the pump drive from an external source is denoted by (P). It is calculated from Ws by:

$$P = \dot{m} Ws = \frac{Q\Delta P}{\eta} = \frac{Q\Delta h\ \rho\, g}{\eta} = \frac{\dot{m}\Delta hg}{\eta}$$

The mechanical efficiency (η) decreases as the liquid viscosity and hence the frictional losses increase. The mechanical efficiency is also decreased by power losses in gear, Bering, seals, etc.

These losses are not proportional to pump size. Relatively large pumps tend to have the best efficiency whilst small pumps usually have low efficiencies. Furthermore high-speed pumps tend to be more efficient than low-speed pumps. In general, high efficiency pumps have high NPSH requirements.

5.5 Types of Pumps

Pumps can be classified into: -

1- Centrifugal pumps.
2- Positive displacement pumps.

1- Centrifugal pumps

This type depends on giving the liquid a high kinetic energy, which is then converted as efficiently as possible into pressure energy. It used for liquid with very wide ranging properties and suspensions with high solid content including, for example, cement slurries, and may be constructed from a very wide range of corrosion resistant materials. Process industries commonly use centrifugal pumps. The whole pump casing may be constructed from plastics such as polypropylene or it may be fitted with a corrosion resistant lining. Because it operates at high speed, it may be directly coupled to an electric motor and it will give a high flow rate for its size. They are available in sizes about 0.004 to 380 m3/min [1-100,000 gal/min] and for discharge pressures from a few m H2o head to 5,000 kPa.

In this type of pump (Figure 2), the fluid is fed to the center of a rotating impeller and is thrown outward by centrifugal action. As a result of the high speed of rotation the liquid acquires a high kinetic energy and the pressure difference between the suction and delivery sides arises from the interconversion of kinetic and pressure energy.

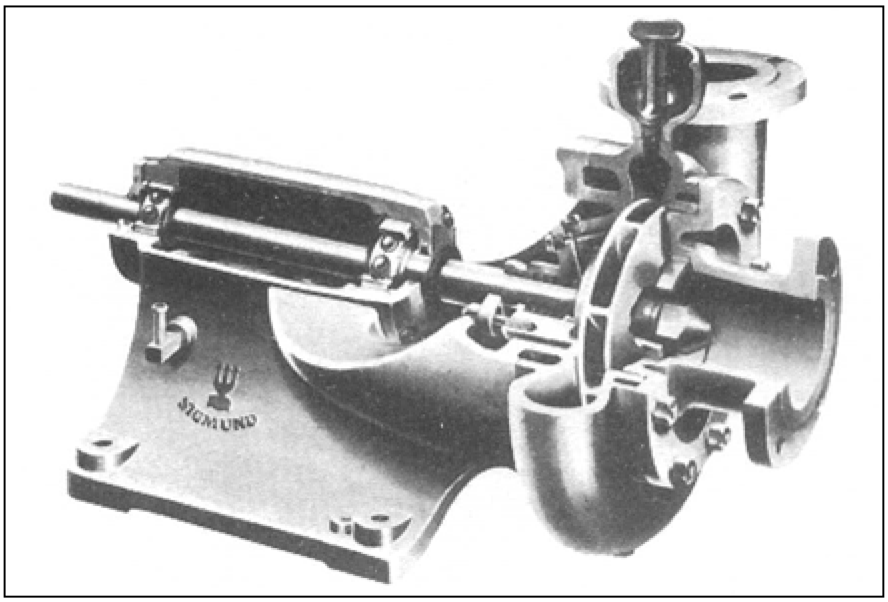


Figure (2) Section of centrifugal pump

The impeller (Figure 3) consists of a series of curved vanes so shaped that the flow within the pump is as smooth as possible. The greater the number of vanes on the impeller, the greater is the control over the direction of motion of the liquid and hence the smaller are the losses due to turbulence and circulation between the vanes. In the open impeller, the vanes are fixed to a central hub, whereas in the closed type the vanes are held between two supporting plates and leakage across the impeller is reduced.

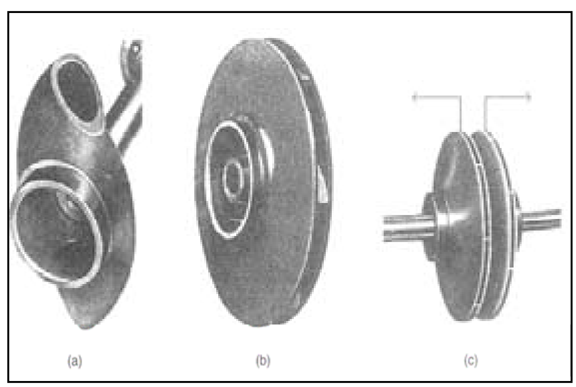


Figure (3) Types of impeller
(a) for pumping suspensions (b) standard closed impeller (c) double impeller

The liquid enters the casing of the pump, normally in an axial direction, and is picked up by the vanes of the impeller. In the simple type of centrifugal pump, the liquid discharges into a volute, a chamber of gradually increasing cross-section with a tangential outlet. A volute type of pump is shown in Figure 4. In the turbine pump

(Figure 4(b)) the liquid flows from the moving vanes of the impeller through a series of fixed vanes forming a diffusion ring. This gives a more gradual change in direction to the fluid and more efficient conversion of kinetic energy into pressure energy than is obtained with the volute type.

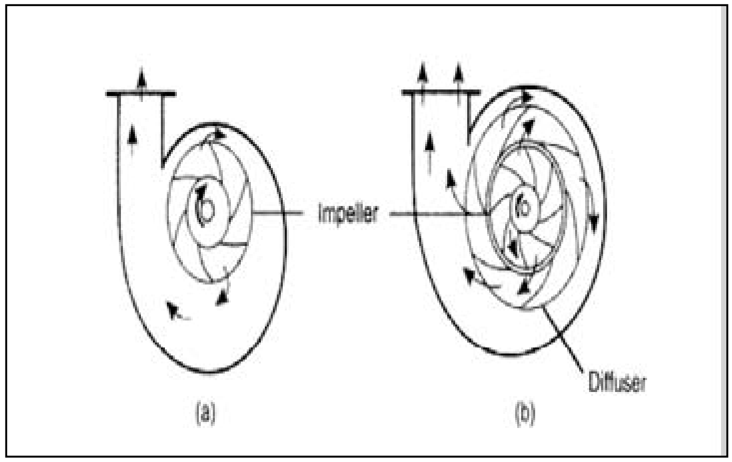


Figure (4) Radial flow pumps
(a) with volute (b) with diffuser vanes

2- Positive Displacement Pumps

In this type, the volume of liquid delivered is directly related to the displacement of the piston and therefore, increases directly with speed and is not appreciably influenced by the pressure. It used for *high pressure and constant rates* this type can be classified into: -

2.1-Reciprocating Pumps, such as

a- The Piston Pump

This pump may be single-acting, with the liquid admitted only to the portion of the cylinder in front of the piston or double-acting, in which case the feed is admitted to both sides of the piston. The majority of pumps are of the single-acting type typically giving a low flow rate of say 0.02 m^3/s at a high pressure of up to 100 Mpa.

b- The Plunger (or Ram) Pump

This pump is the same in principle as the piston type but differs in that the gland is at one end of the cylinder making its replacement easier than with the standard piston type. The piston or ram pump may be used for injections of small quantities of inhibitors to polymerization units or of corrosion inhibitors to high-pressure systems, and also for boiler feed water applications.

c- The Diaphragm Pump

The diaphragm pump has been developed for handling corrosive liquids and those containing suspensions of abrasive solids. It is in two sections separated by a diaphragm of rubber, leather, or plastics material. In one section a plunger or piston operates in a cylinder in which a non-corrosive fluid is displaced. The particularly simple and inexpensive pump results, capable of operating up to 0.2 Mpa.

d- The Metering (or Dosing) Pump

Metering pumps are driven by constant speed electric motors. They are used where *a constant and accurately controlled rate of delivery of a liquid is required*, and they will maintain this constant rate irrespective of changes in the pressure against which they operate. The pumps are usually of the plunger type for low throughput and high-pressure applications; for large volumes and lower pressures a diaphragm is used. In either case, the rate of delivery is controlled by adjusting the stroke of the piston element, and this can be done whilst the pump is in operation. A single-motor driver may operate several individual pumps and in this way give control of the actual flows and of the flow ratio of several streams at the same time. The output may be controlled from zero to maximum flow rate, either manually on the pump or remotely. These pumps may be used for the dosing of works effluents and water supplies, and the feeding of reactants, catalysts, or inhibitors to reactors at controlled rates, and although a simple method for controlling flow rate is provided, high precision standards of construction are required.

2.2-Rotary Pumps, such as

a- The Gear Pump

Gear and lobe pumps operate on the principle of using mechanical means to transfer small elements or "packages" of fluid from the low pressure (inlet) side to the high pressure (delivery) side. There is a wide range of designs available for achieving this end. The general characteristics of the pumps are similar to those of reciprocating piston pumps, but the delivery is more even because the fluid stream is broken down into so much smaller elements. The pumps are capable of delivering to a high pressure, and the pumping rate is approximately proportional to the speed of the pump and is not greatly influenced by the pressure against which it is delivering. Again, it is necessary to provide a pressure relief system to ensure that the safe operating pressure is not exceeded.

b- The Cam Pump

A rotating cam is mounted eccentrically in a cylindrical casing and a very small clearance is maintained between the outer edge of the cam and the casing. As the cam rotates it expels liquid from the space ahead of it and sucks in liquid behind it. The delivery and suction sides of the pump are separated by a sliding valve, which rides on the cam. The characteristics again are similar to those of the gear pump.

c- The Vane Pump

The rotor of the vane pump is mounted off centre in a cylindrical casing. It carries rectangular vanes in a series of slots arranged at intervals round the curved surface of the rotor. The vanes are thrown outwards by centrifugal action and the fluid is carried in the spaces bounded by adjacent vanes, the rotor, and the casing. Most of the wear is on the vanes and these can readily be replaced.

d- The Flexible Vane Pump

The pumps described above will not handle liquids containing solid particles in suspension, and the flexible vane pumps has been developed to overcome this disadvantage. In this case, the rotor (Figure 8.10) is an integral elasomer moulding of a hub with flexible vanes which rotates in a cylindrical casing containing a crescent-shaped block, as in the case of the internal gear pump.

e- The Flow Inducer or Peristaltic Pump

This is a special form of pump in which a length of silicone rubber or other elastic tubing, typically of 3 to 25 mm diameter, is compressed in stages by means of a rotor as shown in Figure 8.11. The tubing is fitted to a curved track mounted concentrically with a rotor carrying three rollers. As the rollers rotate, they flatten the tube against the track at the points of contact. These "flats" move the fluid by positive displacement, and the flow can be precisely controlled by the speed of the motor. These pumps have been particularly useful for biological fluids where all forms of contact must be avoided. They are being increasingly used and are suitable for pumping emulsions, creams, and similar fluids in laboratories and small plants where the freedom from glands, avoidance of aeration, and corrosion resistance are valuable, if not essential. Recent developments^ have produced thick-wall, reinforced moulded tubes which give a pumping performance of up to 0.02 m^3/s at 1 MN/m^2. The control is such that these pumps may conveniently be used as metering pumps for dosage processes.

f- The Mono pump

Another example of a positive acting rotary pump is the single screw-extruder pump typified by the Mono pump, in which a specially shaped helical metal rotor revolves eccentrically within a double-helix, resilient rubber stator of twice the pitch length of the metal rotor. A continuous forming cavity is created as the rotor turns — the cavity progressing towards the discharge, advancing in front of a continuously forming seal line and thus carrying the pumped material with it. The Mono pump gives a uniform flow and is quiet in operation. It will pump against high pressures; the higher the required pressure, the longer are the stator and the rotor and the greater the number of turns. The pump can handle corrosive and gritty liquids and is extensively used for feeding slurries to filter presses. It must never be run dry. The Mono Merlin Wide Throut pump is used for highly viscous liquids.

g- The Screw pumps

A most important class of pump for dealing with highly viscous material is represented by the screw extruder used in the polymer industry. The screw pump is of more general application and will be considered first. The fluid is sheared in the channel between the screw and the wall of the barrel. The mechanism that generates the pressure can be visualized in terms of a model consisting of an open channel covered by a moving plane surface. If a detailed analysis of the flow in a screw pump is to be carried out, then it is also necessary to consider the small but finite leakage flow that can occur between the flight and the wall. With the large pressure generation in a polymer extruder, commonly 100 bar (107 N/m2), the flow through this gap, which is typically about 2 per cent of the barrel internal diameter, can be significant. The pressure drop over a single pitch length may be of the order of 10 bar (106 N/m2), and this will force fluid through the gap. Once in this region the viscous fluid is subject to a high rate of shear (the rotation speed of the screw is often about 2 Hz), and an appreciable part of the total viscous heat generation occurs in this region of an extruder.

5.6 The advantages and disadvantages of the centrifugal pump

The main advantages are:

(1) It is simple in construction and can, therefore, be made in a wide range of materials.
(2) There is a complete absence of valves.
(3) It operates at high speed (up to 100 Hz) and, therefore, can be coupled directly to an electric motor. In general, the higher the speed the smaller the pump and motor for a given duty.
(4) It gives a steady delivery.
(5) Maintenance costs are lower than for any other type of pump.
(6) No damage is done to the pump if the delivery line becomes blocked, provided it is not ran in this condition for a prolonged period.
(7) It is much smaller than other pumps of equal capacity. It can, therefore, be made into a sealed unit with the driving motor, and immersed in the suction tank.
(8) Liquids containing high proportions of suspended solids are readily handled.

The main disadvantages are:

(1) The single-stage pump will not develop a high pressure. Multistage pumps will develop greater heads but they are very much more expensive and cannot readily be made in corrosion-resistant material because of their greater complexity. It is generally better to use very high speeds in order to reduce the number of stages required.
(2) It operates at a high efficiency over only a limited range of conditions: this applies especially to turbine pumps.
(3) It is not usually self-priming.
(4) If a non-return valve is not incorporated in the delivery or suction line, the liquid will run back into the suction tank as soon as the pump stops.
(5) Very viscous liquids cannot be handled efficiently.

5.7 Priming The Pump

The theoretical head developed by a centrifugal pump depends on *the impeller speed*, *the radius of the impeller*, and *the velocity of the fluid leaving the impeller*. If these factors are constant, the developed head is the same for fluids of all densities and is the same for liquids and gases. A centrifugal pump trying to operate on air, then can neither draw liquid upward from an initially empty suction line nor force liquid a full discharge line. <u>Air can be displaced by priming the pump</u>.

For example, if a pump develops a head of 100 ft and is full of water, the increase in pressure is [100 ft (62.3lb/ft^3) (ft^2 / 144 in^2)] = 43 psi (2.9 atm). If full of air the pressure increase is about 0.05 psi (0.0035 atm).

5.8 Operating Characteristics

The operating characteristics of a pump are conveniently shown by plotting the head (h), power (P), efficiency (η), and sometimes required NPSH against the flow (or capacity) (Q) as shown in Figure (5). Theses are known as characteristic curves of the pump. It is important to note that the efficiency reaches a maximum and then falls, whilst the head at first falls slowly with Q but eventually falls off rapidly. The optimum conditions for operation are shown as the duty point, i.e. the point where the head curve cuts the ordinate through the point of maximum efficiency.

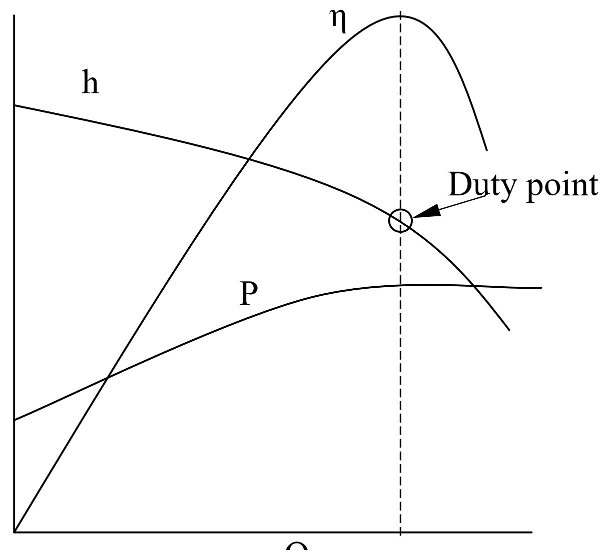


Figure (5) Radial flow pump characteristics

Characteristic curves have a variety of shapes depending on *the geometry of the impeller* and *pump casing*. Pump manufactures normally supply the curves only for operation with water.

In a particular system, a centrifugal pump can only operate at one point on the Δh against Q curve and that is the point where the Δh against Q curve of the pump intersect with the Δh against Q curve of the system as shown in Figure.

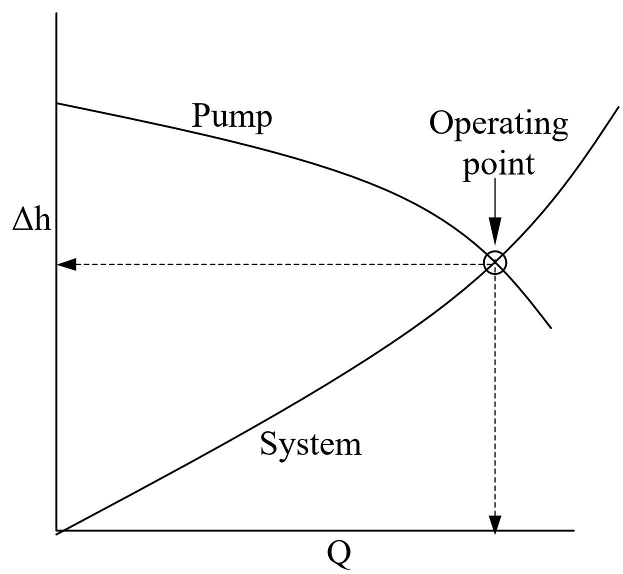


Figure (6) System and pump total head against capacity

The system total head at a particular liquid flow rate

$$\Delta h = (z_d - z_s) + \left(\frac{P_d - P_s}{\rho\, g}\right) + [(h_F)_d + (h_F)_s]$$

where,

$$(h_F)_d = 4 f_d \left[\frac{L}{d} \sum \frac{Le}{d}\right]_d \frac{u_d^2}{2g}$$

$$(h_F)_s = 4 f_s \left[\frac{L}{d} \sum \frac{Le}{d}\right]_s \frac{u_s^2}{2g}$$

For the same pipe <u>type</u> and <u>diameter</u> for suction and discharge lines: -

$$\Delta h = \Delta z + \frac{\Delta P}{\rho\, g} + 4 f \left[\left(\frac{L}{d} + \sum \frac{Le}{d}\right)_d + \left(\frac{L}{d} + \sum \frac{Le}{d}\right)_s\right] \frac{u^2}{2g}$$

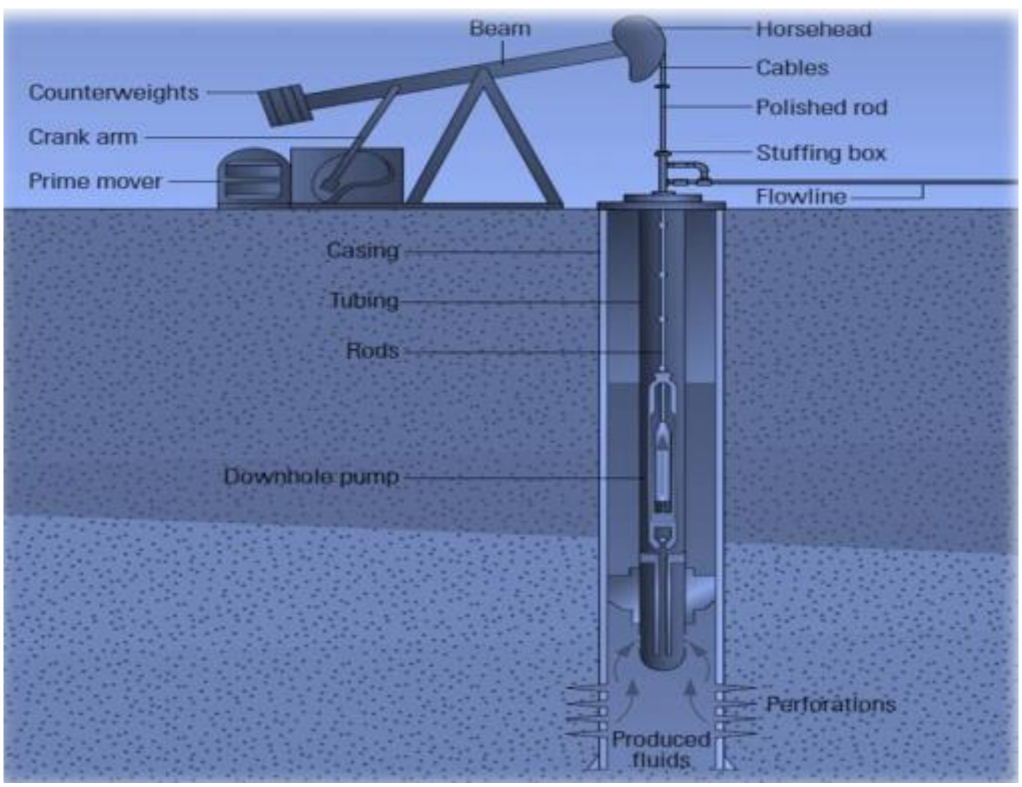
but $u = \dfrac{Q}{(\pi / 4\, d^2)}$

$$\Rightarrow \Delta h = \Delta z + \frac{\Delta P}{\rho\, g} + \frac{4f}{2g}\left[\left(\frac{L}{d} + \sum \frac{Le}{d}\right)_d + \left(\frac{L}{d} + \sum \frac{Le}{d}\right)_s\right]\left(\frac{Q}{(\pi / 4\, d^2)}\right)^2$$

Special types of pumps:

1-Peripheral

2-Gas Lift Pump



3-Jet Pumps

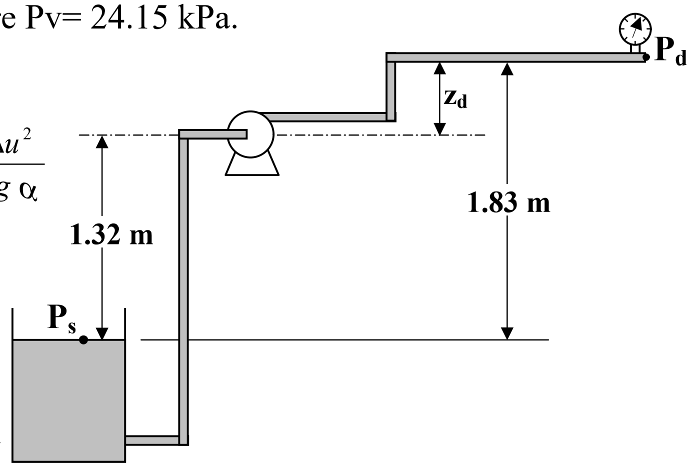
Lecture 8

Example -5.1-

A petroleum product is pumped at a rate of 2.525 x 10-3 m^3/s from a reservoir under atmospheric pressure to 1.83 m height. If the pump 1.32 m height from the reservoir, the discharge line diameter is 4 cm and the pressure drop along its length 3.45 kPa. The gauge pressure reading at the end of the discharge line 345 kPa. The pressure drop along suction line is 3.45 kPa and pump efficiency η=0.6 calculate:-

(i) The total head of the system Δh. **(ii)** The power required for pump. **(iii)** The NPSH

Take that: the density of this petroleum product ρ=879 kg/m^3, the dynamic viscosity μ=6.47 x 10-4 Pa.s, and the vapor pressure Pv= 24.15 kPa.



Solution:

(i)

$$\Delta h = (z_d - z_s) + \left(\frac{P_d - P_s}{\rho\, g}\right) + [(h_F)_d + (h_F)_s] + \frac{\Delta u^2}{2g\,\alpha}$$

$u_s = 0$

$u_d = (2.525 \times 10^{-3}\ m^3/s)/(\pi/4\ 0.04^2)$

$= 2$ m/s

$Re_d = (879 \times 2 \times 0.04)/\ 6.47 \times 10^{-4}$

$= 1.087 \times 10^5$

The pressure drop in suction line 3.45kPa

$\Rightarrow (h_F)_s = 3.45 \times 10^3/(879 \times 9.81)$

$= 0.4$ m

And in discharge line is also 3.45 kPa $\Rightarrow (h_F)_d = 0.4$ m

The kinetic energy term = $2^2/(2 \times 9.81)$ =0.2 m

The pressure at discharge point = gauge + atmospheric pressure = 345 + 101.325

= 446.325 kPa

The difference in pressure head between discharge and suction points is

$(446.325 - 101.325) \times 10^3 /(879 \times 9.81) = 40$ m

Δz = 1.83 m

$\Rightarrow$ Δh = 40 m + 1.83 m + 0.2 m + 0.4 m + 0.4 m = 42.83 m

(ii)

$$P = \frac{Q\Delta P}{\eta} = \frac{Q\Delta h\ \rho\ g}{\eta} = [(2.525 \times 10^{-3}\ m^3/s)(42.83\ m)(879\ kg/m^3)(9.81\ m/s^2)]/0.6$$

$\Rightarrow$ P = 1.555 kW

(iii)

$$NPSH = z_s + \left(\frac{P_s - P_v}{\rho\ g}\right) - (h_F)_s$$

$= (-\ 1.32) + (1.01325 \times 10^5\ - 24150)/\ (879 \times 9.81) - 0.4$ m

$= 7.23$ m

Example -5.2-

It is required to pump cooling water from storage pond to a condenser in a process plant situated 10 m above the level of the pond. 200 m of 74.2 mm i.d. pipe is available and the pump has the characteristics given below. The head loss in the condenser is equivalent to 16 velocity heads based on the flow in the 74.2 mm pipe. If the friction factor Φ = 0.003, estimate the rate of flow and the power to be supplied to the pump assuming η = 0.5

$Q\ (m^3/s)$	0.0028	0.0039	0.005	0.0056	0.0059
Δh (m)	23.2	21.3	18.9	15.2	11.0

Solution:

$$\Delta h = \Delta z + \cancel{\frac{\Delta P}{\rho g}} + \cancel{\frac{\Delta u^2}{2g\alpha}} + \left[(h_F)_d + (h_F)_s + (h_F)_{condenser}\right]$$

$$(h_F)_{d+s} = 4f\frac{L}{d}\frac{u^2}{2g} = 4(0.006)(200/0.0742)(u^2/2g) = 3.3\ u^2$$

$$(h_F)_{condenser} = 16\frac{u^2}{2g} = 0.815\ u^2$$

$$u = Q/A = 321.26\ Q$$

$$\Rightarrow \Delta h = 10 + (0.815 + 3.3)(321.26\ Q)^2 = 10 + 2.2 \times 10^5\ Q^2$$

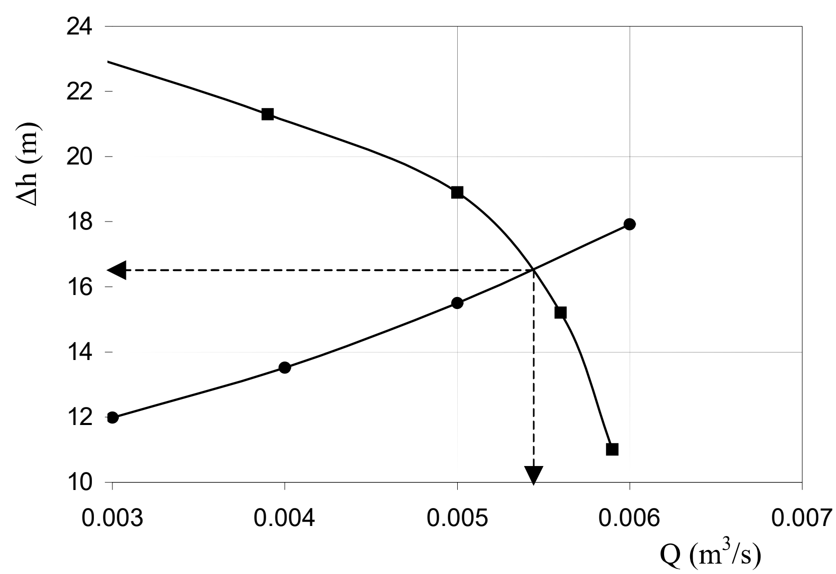
To draw the system curve

$Q\ (m^3/s)$	0.003	0.004	0.005	0.006
Δh (m)	11.98	13.52	15.5	17.92

From Figure

$Q = 0.0054\ m^3/s$

$\Delta h = 16.4$ m



$$\text{Power required for pump} = \frac{Q\Delta h\ \rho\ g}{\eta} = (0.0054)(16.4)(1000)(9.81)/0.5$$

$$= 17.375\ \text{kW}$$

Example -5.3-

A centrifugal pump used to take water from reservoir to another through 800 m length and 0.15 m i.d. if the difference in two tank is 8 m, calculate the flow rate of the water and the power required, assume f=0.004.

Q (m^3/h)	0	23	46	69	92	115
Δh (m)	17	16	13.5	10.5	6.6	2.0
η	0	0.495	0.61	0.63	0.53	0.1

Solution:

$$\Delta h = \Delta z + \frac{\Delta P}{\rho g} + \frac{\Delta u^2}{2g\alpha} [(h_F)_d + (h_F)_s]$$

u = Q/A = 56.59 Q

$$(h_F)_{d+s} = 4f\frac{L}{d}\frac{u^2}{2g} = 4(0.004)(800/0.15)(56.59\ Q(h/3600\ s))^2/2g$$

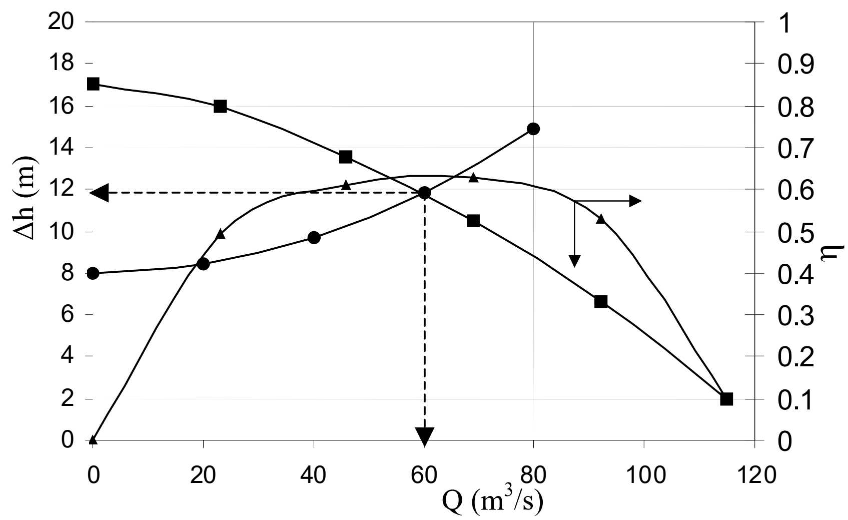
$$= 1.0747 \times 10^{-3} Q^2 \text{ ----------------------- } (Q \text{ in } m^3/h)$$

$$\Rightarrow \Delta h = 8 + 1.0747 \times 10^{-3} Q^2$$

To draw the system curve

Q (m^3/h)	0	20	40	60	80
Δh (m)	8.0	8.43	9.72	11.87	14.88

From Figure
Q = 60 m^3/h
Δh = 11.8 m
η = 0.64



$$\text{Power required for pump} = \frac{Q\Delta h\ \rho\ g}{\eta} = (60)(1\ h/3600\ s)(11.8)(1000)(9.81)/0.64$$

$$= 3.014\ kW$$

Example -5.4-

A pump take brine solution at a tank and transport it to another in a process plant situated 12 m above the level in the first tank. 250 m of 100 mm i.d. pipe is available sp.gr. of brine is 1.2 and μ = 1.2 cp. The absolute roughness of pipe is 0.04 mm and f = 0.0065. Calculate (i) the rate of flow for the pump (ii) the power required for pump if η = 0.65. (iii) if the vapor pressure of water over the brine solution at 86°F is 0.6 psia, calculate the NPSH available, if suction line length is 30 m.

Q (m^3/s)	0.0056	0.0076	0.01	0.012	0.013
Δh (m)	25	24	22	17	13

Solution:

(i) $\Delta h = \Delta z + \cancel{\frac{\Delta P}{\rho g}} + \cancel{\frac{\Delta u^2}{2g\alpha}} (h_F)_{d+s}$

u = Q/A = 127.33 Q

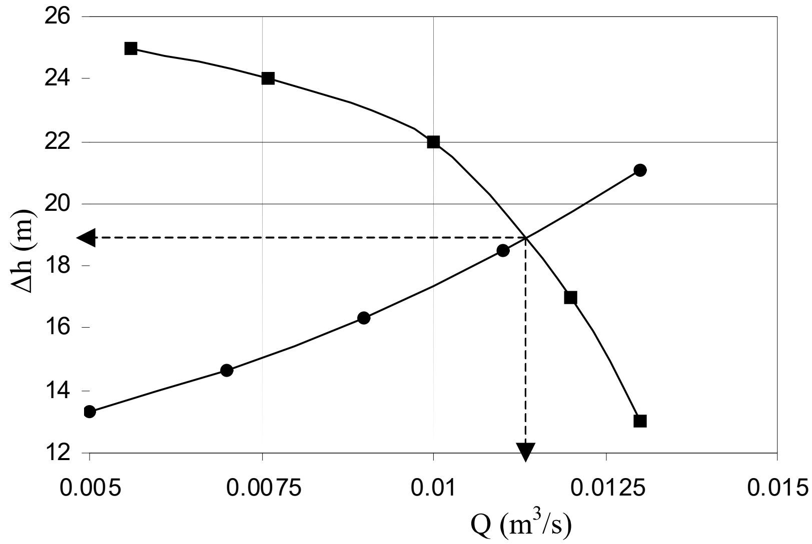
$(h_F)_{d+s} = 4f\frac{L}{d}\frac{u^2}{2g}$ = 4(0.0065)(250/0.1)(127.33 Q)^2/2g

= 53.707 x 10^3 Q^2

⇒ Δh = 12 + 53.707 x 10^3 Q^2

To draw the system curve

Q (m^3/h)	0.005	0.007	0.009	0.011	0.013
Δh (m)	13.34	14.63	16.35	18.5	21.08



From Figure
Q = 0.0114 m^3/s
Δh = 18.9 m

(ii)

Power required for pump = $\frac{Q\Delta h\ \rho\ g}{\eta}$=(00.0114)(18.9)(1200)(9.81)/0.65=3.9 kW

(iii)

$NPSH = z_s + \left(\frac{P_s - P_v}{\rho\ g}\right) - (h_F)_s$

u = Q/A = 0.0114/(π/4 0.1^2) =1.45 m/s

For datum line passes through the centerline of the pump (z_s = 0)

$(h_F)_s = 4f\frac{L_s}{d}\frac{u^2}{2g}$ = 4(0.0065)(30/0.1)(1.45)^2/2g = 0.84 m

⇒ NPSH = (101.325 x 10^3 - 0.6psi 101.325 x 10^{3}Pa/14.7psi)/(1200 x 9.81) – 0.84
= 7.416 m

5.9 Centrifugal Pump Relations

The power (P_E) required in ***an ideal centrifugal pump*** can be expected to be a function of the liquid density (ρ), the impeller diameter (D), and the rotational speed of the impeller (N). If the relationship is assumed to be given by the equation,

$P_E = c\ \rho^a\ N^b\ D^c$ ---------------------------------(1)

then it can be shown by dimensional analysis that

$P_E = c_1\ \rho\ N^3\ D^5$ ---------------------------------(2)

where, c_1 is a constant which depends on the geometry of the system.

The power (P_E) is also proportional to the product of the volumetric flow rate (Q) and the total head (Δh) developed by the pump.

$P_E = c_2 Q \Delta h$ ---------------------------------(3)

where, c_2 is a constant.

The volumetric flow rate (Q) and the total head (Δh) developed by the pump are: -

$Q = c_3 N D^3$ ---------------------------------(4)

$\Delta h = c_4 N^2 D^2$ ---------------------------------(5)

where, c_3 and c_4 are constants.

Equation (5) could be written in the following form,

$\Delta h^{3/2} = c_4^{3/2} N^3 D^3$ ---------------------------------(6)

Combine equations (4) and (6) [eq. (4) divided by eq. (6)] to give;

$$\frac{Q}{\Delta h^{\frac{3}{2}}} = \frac{c_3}{c_4^{\frac{3}{2}}}\frac{1}{N} \Rightarrow \frac{QN^2}{\Delta h^{\frac{3}{2}}} = const. \quad \text{------------------------------(7)}$$

or, $$\frac{N\sqrt{Q}}{\Delta h^{\frac{3}{4}}} = const. = N_s \quad \text{---------------------------------(8)}$$

When the rotational speed of the impeller N is (rpm), the volumetric flow rate Q in (USgalpm) and the total head Δh developed by the pump is in (ft), the constant N_s in equation (8) is known as ***the specific speed of the pump***. The specific speed is used as an index of pump types and always evaluated at the best efficiency point (bep) of the pump. Specific speed vary in the range (400 – 10,000) depends on the impeller type, and has the dimensions of $(L/T^2)^{3/4}$. [British gal=1.2USgal, ft^3=7.48USgal, m^3=264USgal]

5.9.1 Homologous Centrifugal Pumps

Two different size pumps are said to be geometrically similar when the ratios of corresponding dimensions in one pump are equal to those of the other pump. Geometrically similar pumps are said to be homologous. A sets of equations known as the ***affinity laws*** govern the performance of homologous centrifugal pumps at various impeller speeds.

For the tow homologous pumps, equations (4), and (5) are given

$$\frac{Q_1}{Q_2} = \left(\frac{N_1}{N_2}\right)\left(\frac{D_1}{D_2}\right)^3 \quad \text{---------------------------------(9)}$$

$$\frac{\Delta h_1}{\Delta h_2} = \left(\frac{N_1}{N_2}\right)^2\left(\frac{D_1}{D_2}\right)^2 \quad \text{---------------------------------(10)}$$

Similarly for the tow homologous pumps equation (2)can be written in the form;

$$\frac{P_{E1}}{P_{E2}} = \left(\frac{N_1}{N_2}\right)^3\left(\frac{D_1}{D_2}\right)^5 \quad \text{---------------------------------(11)}$$

And by analogy with equation (10),

$$\frac{NPSH_1}{NPSH_2} = \left(\frac{N_1}{N_2}\right)^2\left(\frac{D_1}{D_2}\right)^2 \text{---------------------------------(12)}$$

Equations (9), (10), (11), and (12) are the affinity law for homologous centrifugal pumps.

For a particular pump where the impeller of diameter D_1, is replaced by an impeller with a slightly different diameter D_2 the following equations hold

$$\frac{Q_1}{Q_2} = \left(\frac{N_1}{N_2}\right)\left(\frac{D_1}{D_2}\right) \qquad \text{---------------------------------(13)}$$

$$\frac{\Delta h_1}{\Delta h_2} = \left(\frac{N_1}{N_2}\right)^2\left(\frac{D_1}{D_2}\right)^2 \qquad \text{---------------------------------(14)}$$

$$\frac{P_{E1}}{P_{E2}} = \left(\frac{N_1}{N_2}\right)^3\left(\frac{D_1}{D_2}\right)^3 \qquad \text{---------------------------------(15)}$$

The characteristic performance curves are available for a centrifugal pump operating at a given rotation speed, equations (13), (14), and (15) enable the characteristic performance curves to be plotted for other operating speeds and for other slightly impeller diameters.

Example -5.5-

A volute centrifugal pump with an impeller diameter of 0.02 m has the following performance data when pumping water at the best efficiency point (bep). Impeller speed N = 58.3 rev/s capacity Q = 0.012 m^3/s, total head Δh = 70 m, required NPSH = 18 m, and power = 12,000 W. Evaluate the performance data of an homologous pump with twice the impeller diameter operating at half the impeller speed.

Solution:

Let subscripts 1 and 2 refer to the first and second pump respectively,

$N_1/N_2 = 2$, $D_1/D_2 = 1/2$

Ratio of capacities

$$\frac{Q_1}{Q_2} = \left(\frac{N_1}{N_2}\right)\left(\frac{D_1}{D_2}\right)^3 = 2\ (1/8) = 1/4$$

⇒ Capacity of the second pump $Q_2 = 4\ Q_1 = 4(0.012) = 0.048\ m^3/s$

Ratio of total heads

$$\frac{\Delta h_1}{\Delta h_2} = \left(\frac{N_1}{N_2}\right)^2\left(\frac{D_1}{D_2}\right)^2 = 4\ (1/4) = 1$$

⇒ Total head of the second pump $\Delta h_2 = \Delta h_1 = 70$ m

Ratio of powers

$$\frac{P_{E1}}{P_{E2}} = \left(\frac{N_1}{N_2}\right)^3\left(\frac{D_1}{D_2}\right)^5 = 8\ (1/32) = 1/4$$

assume $\frac{P_{B1}}{P_{B2}} = \frac{P_{E1}}{P_{E2}} = \frac{1}{4}$

⇒ Break power of the second pump $P_{B2} = 4\ P_{B1} = 4(12{,}000) = 48{,}000$ W

$$\frac{NPSH_1}{NPSH_2} = \left(\frac{N_1}{N_2}\right)^2\left(\frac{D_1}{D_2}\right)^2 = 4\ (1/4) = 1$$

⇒ NPSH of the second pump $NPSH_2 = NPSH_1 = 18$ m

Note: -

The break power P_B can be defined as the actual power delivered to the pump by prime mover. It is the sum of liquid power and friction power and is given by the equation, $P_B = \frac{P_E}{\eta}$

Example -5.6-

A centrifugal pump was manufactured to couple directly to a 15 hp electric motor running at 1450 rpm delivering 50 liter/min against a total head 20 m. It is desired to replace the motor by a diesel engine with 1,000 rpm speed and couple it directly to the pump. Find the probable discharge and head developed by the pump. Also find the hp of the engine that would be employed.

Solution:

With the same impeller $D_1 = D_2$,

then $Q_1/Q_2 = N_1/N_2$

$\Rightarrow Q_2 = 50\ (1000 / 1450) = 34.5$ liter/min

and $\Delta h_2 = \Delta h_1\ (N_2/N_1)^2 = 20\ (1000/1450)^2 = 9.5$ m

$P_{E2} = P_{E1}\ (N_2/N_1)^3 = 15\ (1000/1450)^3 = 4.9$ hp

5.10 Centrifugal Pumps in Series and in Parallel

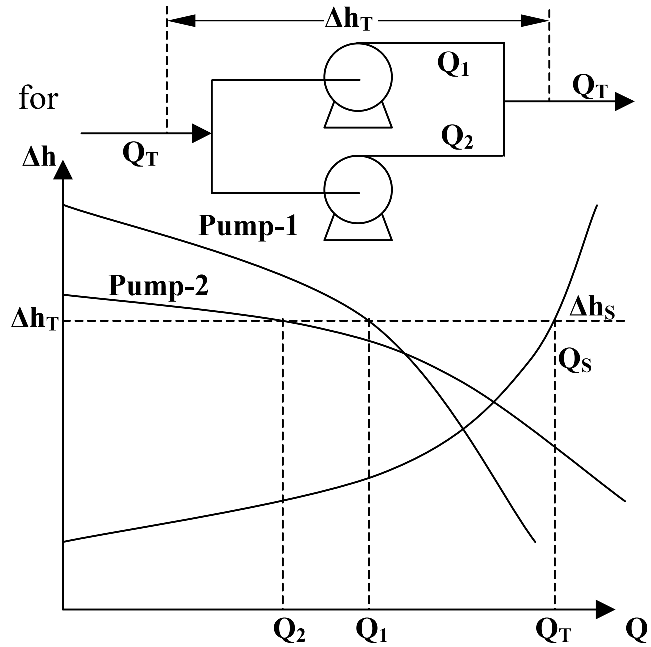
5.10.1 Centrifugal Pumps in Parallel

Consider two centrifugal pumps in ***parallel***. The total head for the pump combination (Δh_T) is the same as the total head for each pump,

$\Delta h_T = \Delta h_1 = \Delta h_2$

$Q_T = Q_1 + Q_2$

The operating characteristics curves for two pumps in parallel are: -



Solution by trail and error

1- Draw Δh versus Q for the two pumps and the system.
2- Draw horizontal Δh_T line and determine Q_1, Q_2, and Q_S.
3- Q_T (Total) = $Q_1 + Q_2 = Q_S$ (system).
4- If $Q_T \neq Q_S$ repeat steps 2, 3, and 4 until $Q_T = Q_S$.

Another procedure for solution

1- The same as above.
2- Draw several horizontal lines (4 to 6) for Δh_T and determine their Q_T.
3- Draw Δh_T versus Q_T.
4- The duty point is the intersection of Δh_T curve with Δh_S curve.

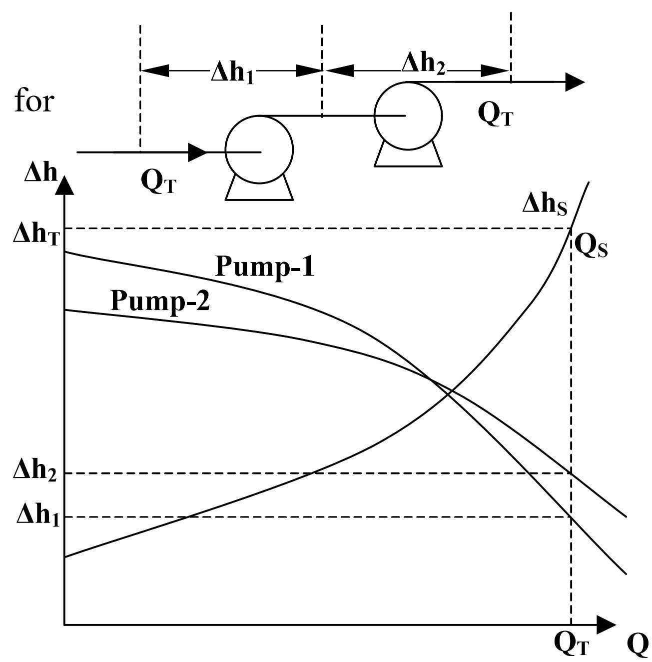
5.10.2 Centrifugal Pumps in Series

Consider two centrifugal pumps in ***series***. The total head for the pump combination (Δh_T) is the sum of the total heads for the two pumps,

$\Delta h_T = \Delta h_1 + \Delta h_2$

$Q_T = Q_1 = Q_2$

The operating characteristics curves for two pumps in series are: -



Solution by trail and error

1- Draw Δh versus Q for the two pumps and the system.
2- Draw vertical Q_T line and determine Δh_1, Δh_2, and Δh_S.
3- Q_T (Total) = $Q_1 + Q_2 = Q_S$ (system).
4- If $\Delta h_T \neq \Delta h_S$ repeat steps 2, 3, and 4 until $\Delta h_T = \Delta h_S$.

Another procedure for solution

1- The same as above.
2- Draw several Vertical lines (4 to 6) for Q_T and determine their Δh_T.
3- Draw Δh_T versus Q_T.
4- The duty point is the intersection of Δh_T curve with Δh_S curve.