1. CONCENTRICALLY LOADED SHORT COLUMNS (JACKSON MORELAND)

$$\underset{\text{FACTORED}}{P_u} \leq \phi \underset{\text{NOMINAL}}{P_n} \qquad \text{— (I)}$$

$$\left.\begin{array}{l}\phi = 0.70 \text{ for SPIRALLY reinforced column} \\ \phi = 0.65 \text{ " TIED " }\end{array}\right] \quad \text{— (II)}$$

For <u>CONCENTRICALLY</u> loaded column (SPIRALLY REINFORCED):

$$P_{n,max} = 0.85\left[0.85 f_c' (A_g - A_{st}) + f_y A_{st}\right] \qquad \text{III}$$

For CONCENTRICALLY loaded TIED column:

$$P_{n,max} = 0.80\left[0.85 f_c' (A_g - A_{st}) + f_y A_{st}\right] \qquad \text{IV}$$

ACI code requirements for SPIRALLY reinf. columns:

f spiral size (diameter) $\geq$ 10 mm — V

ω 75 mm $\geq$ ℄ t. ℄ spiral spacing $\geq$ 25 mm — VI

CLEAR distance between spirals $\geq \frac{4}{3}$ Aggregate MAX SIZE — VII

▷ Spiral ANCHORAGE $\geq$ 1.5 extra turns @ end — VIII

5 Spiral LAP splice $\geq 48\ d_b$, ≥ 300 mm — IX

6 $\rho_s \geq 0.45\left(\frac{A_g}{A_c} - 1\right) f_c'/f_y$ — X

ρ_s = ratio of spiral reinforcement volume to total core volume (out-to-out of spirals)

A_c = core area of spirally reinforced member measured to outside diameter of spiral.

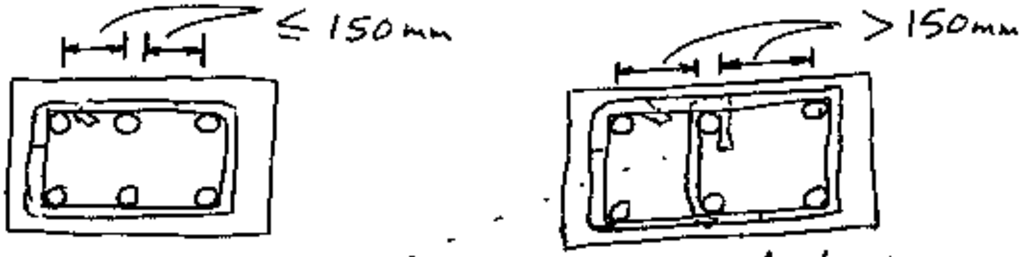
ACI code requirements for TIED columns:

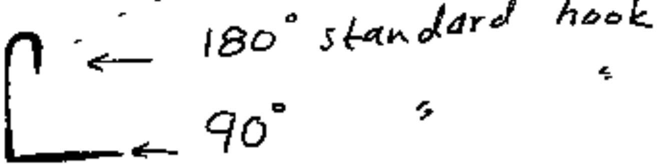
Tie bar size (diameter) ≥ 12 mm — XI

Vertical Tie spacing
- $\leq 16\times$ longitudinal bar diameter
- $\leq 48\times$ tie bar "
- $\leq$ least column dimension

— XII

TIE BAR ARRANGEMENT — XIII





Reinforcement (ρ_g) LIMITS: — XIV

$$0.01 \leq \frac{A_{st}}{A_g} \leq 0.08$$ — XV

$\rho_g = A_{st}/A_g$

For LAP spliced longitudinal bars: — XVI

$A_{st}/A_g \leq 0.04$

No. of LONGITUDINAL BARS
- ≥ 4 for bars in TIED cols. ▨ or ◍
- ≥ 6 " " " SPIRALLY reinforced cols.
- ≥ 3 " " " ▲ TIED cols.

Col. 108

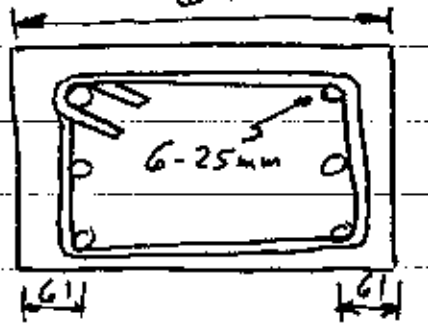
Ex.1 Calculate P_u

Concentric loading:

e = eccentricity = 0

$f'_c = 20.7\ N/mm^2$

$f_y = 345$ "



Solution:

$A_{st} = 6 \times 490 = 2940\ mm^2$

$\rho_g = \frac{A_{st}}{A_g} = \frac{2940}{610 \times 380} = 0.0127 > .01\ (min)$ $< .08\ (max)$ ∴ OK

"TIED" column

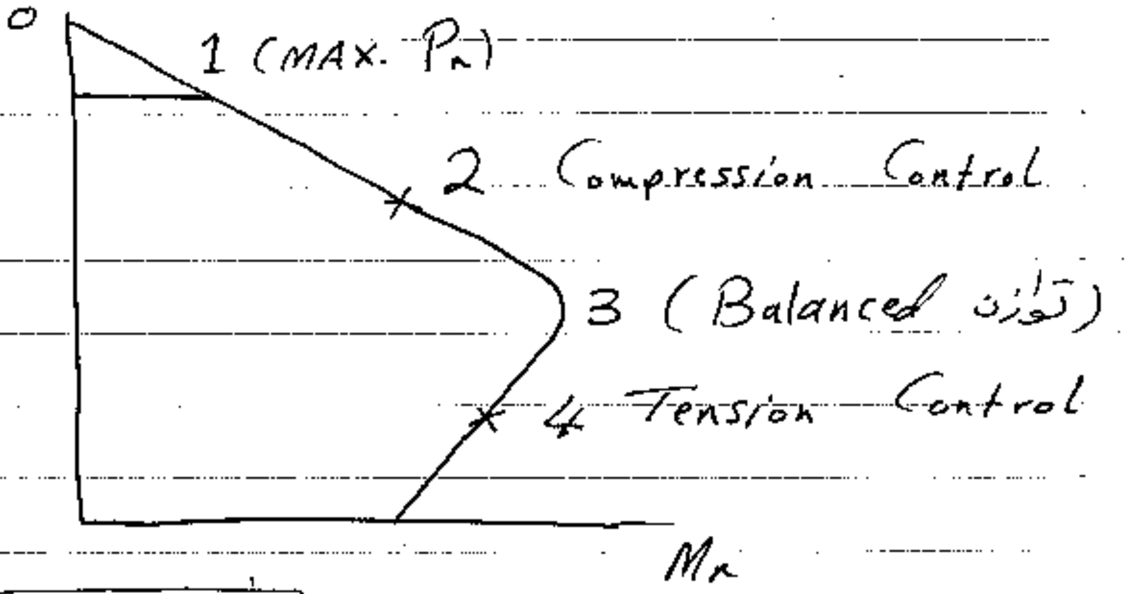
$P_{n,max} = 0.80\ [.85 f'_c (A_g - A_{st}) + f_y A_{st}]$

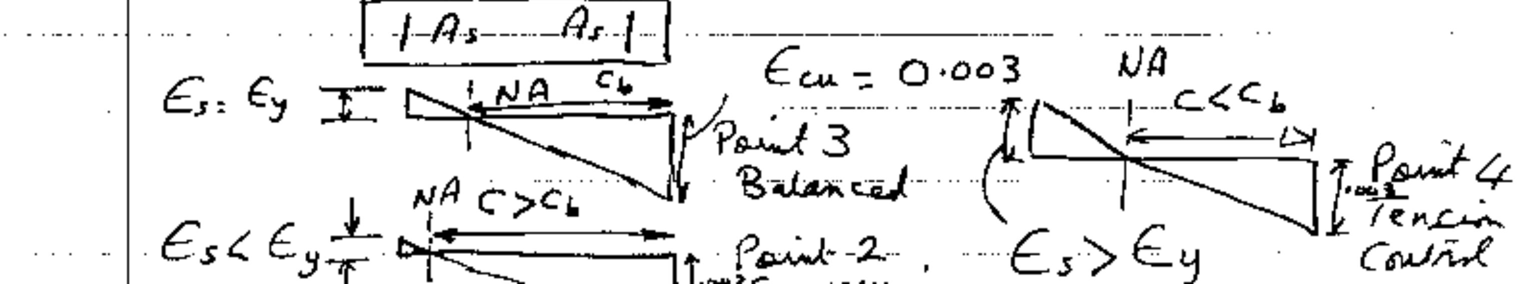
$= .8[.85 \times .0207 (610 \times 380 - 2940) + .345 \times 2940]$

$= 4033$ kN "NOMINAL" concentric load capacity

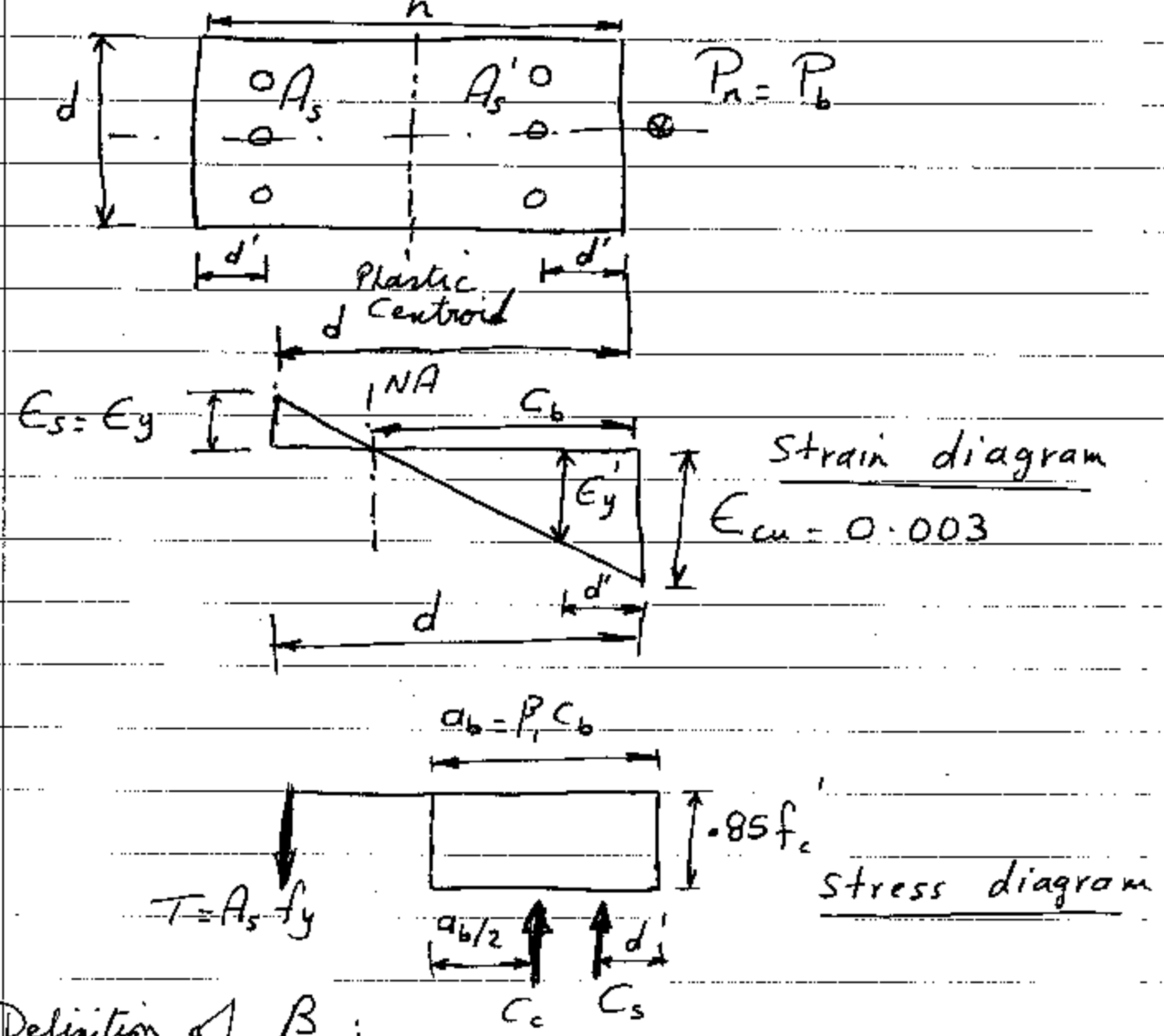
$P_u = \phi P_n = \overset{TIED}{.65} \times 4033 = 2621$ kN the max. allowable factored load for a "CONCENTRICALLY" ($e = 0$) loaded column.

2. "ECCENTRICALLY" loaded columns (SHORT قصيرة)





First case: Balanced Condition (Point 3)



Definition of β_1:

$\beta_1 = 0.85$ for $f_c' \leq 28\ N/mm^2$

$\beta_1 = 0.65$ " $f_c' \geq 56$ "

$\beta_1 = 0.85 - (.05/7)(f_c' - 28)$ for $28 < f_c' \leq 56\ N/mm^2$

Per ACI code: "Balanced Condition" occurs when simultaneously the tension steel A_s reaches ϵ_y ($\epsilon_y = f_y/E_s$), as the concrete fibre on the maximum compression side reaches a strain $\epsilon_{cu} = 0.003$

Similar Δ's:

$c_b/d = 0.003/(f_y/E_s + 0.003)$

ACI code gives $E_s = 200 \times 10^3 \ N/mm^2$

$$\therefore c_b = d \frac{600}{f_y + 600}$$

Two possibilities: (a) or (b)

(a) If compression steel yields: ($\epsilon_s' \geq \epsilon_y$)

$$C_s = A_s'(f_y - 0.85 f_c')$$

or (b) If compression steel does not yield: ($\epsilon_s' < \epsilon_y$)

$$C_s = A_s'(\epsilon_s' E_s - 0.85 f_c')$$

$$T = A_s f_y$$

$$C_c = 0.85 f_c' a_b b = 0.85 f_c' \beta_1 c_b b$$

Summing forces:

$$P_b \,[\text{NOMINAL}] = C_c + C_s - T$$

Summing moments about PC (Plastic Centroid)

$$P_b e_b = C_s(h/2 - d') + C_c(h/2 - a/2) + T(d - h/2)$$

Φ values for symmetrical sections ($A_s' = A_s = A_{st}/2$ & P.C is @ mid-depth) with $f_y \leq 420 \ N/mm^2$ and $(h - 2d')/h \geq .7$; then the following equations apply for "TENSION CONTROL":

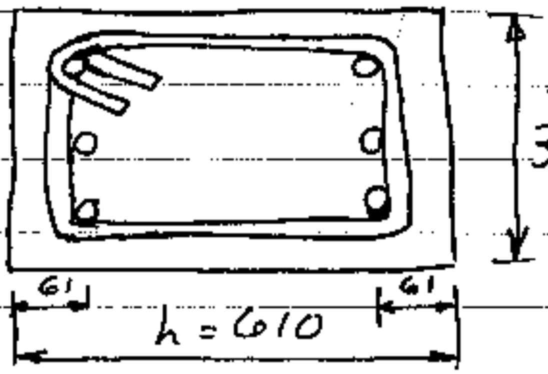
For a SPIRALLY reinforced column:

$$\phi = \frac{0.9}{1 + 2P_n/(f_c' A_g)} \geq 0.7 \Leftarrow \text{حد ادنى}$$

For a TIED column:

$$\phi = \frac{0.9}{1 + 2.5 P_n/(f_c' A_g)} \geq 0.65 \Leftarrow \text{حد ادنى}$$

EX5 Find the section capacity (tied column) when $e = 203$ mm; $f_c' = 20.7$ N/mm^2; $f_y = 400\ N/mm^2$; $A_s = A_s' = A_{st}/2 = 1470\ mm^2$.



Solution:

$\rho_t = A_{st}/(bh)$ (I)

$= 2940/(610 \times 380)$

$= 0.01268 \geq .01$ (OK) & $\leq .08$ (OK)

$\mu = f_y/(0.85 f_c')$ (II)

$= 400/(.85 \times 20.7)$

$\therefore \rho_t \mu = .01268 \times 400/(.85 \times 20.7) = 0.288$

$e/h = 203/610 = 0.333$

$\gamma = (h - 2d')/h$ (VI)

$= (610 - 2 \times 61)/610 = 0.8$; $\therefore$ GOTO p. 495 $\underline{\underline{F}}$.

with $e/h = 0.333$ & $\rho_t \mu = 0.288$:- $\alpha_F = 0.38$

$$\therefore 0.38 = \frac{P_{uF}}{f_c' bh} = \frac{P_{uF}}{.0207 \times 380 \times 610}$$

$\therefore P_{uF} = 1823$ kN (Based on OLD $\phi = 0.7$)

$\therefore P_u = P_{uF} \times \frac{.65 \ (\text{جديد})}{.7 \ (\text{قديم})} = \underline{\underline{1693}}$ kN

الجواب المطلوب ↑ الجواب النهائي

$\therefore M_u = .203 \times 1693 = \underline{\underline{343.6}}$ kNm

EX6: Find the column capacity with $e = 508$ mm.

Solution: $\rho_t \mu = 0.288$ (Ex. 5)

$e/h = 508/610 = 0.833$; goto p. 495 $\underline{\underline{F}}$:-

$\alpha_F = 0.16$; $\therefore .16 = \frac{P_{uF}}{.0207 \times 380 \times 610}$

$\therefore P_{uF} = 768$ kN

$P_u = (.65/.7)\ 768 = \underline{713}$ kN

$M_u = .508 \times 713 = \underline{\underline{362}}$ kNm

ϕ is assumed to be 0.65 ; check ϕ:

$\therefore P_n = 713/.65 = 1097$

$$\phi = \frac{.9}{1 + 2.5 P_n/(f_c' A_g)} \geq 0.65$$

$$= \frac{.9}{1 + 2.5 \times 1097/(.0207 \times 610 \times 380)} \geq .65$$

$$= 0.573 \geq 0.65$$

GOVERNS (GREATER)

$\therefore$ Final answer:

$$\left.\begin{array}{l} P_u = 713 \text{ kN} \\ M_u = 362 \text{ kNm} \end{array}\right] \leftarrow \text{COLUMN CAPACITY}$$

BIAXIAL MOMENTS IN COLUMNS

USE BRESLER RECIPROCAL method:

$$\frac{1}{P_u} = \frac{1}{P_{ux}} + \frac{1}{P_{uy}} \quad \frac{1}{P_{uo}}$$

P_u = Factored moment load capacity with e_x & e_y

P_{ux} = " " " " with e_x only ($e_y = 0$)

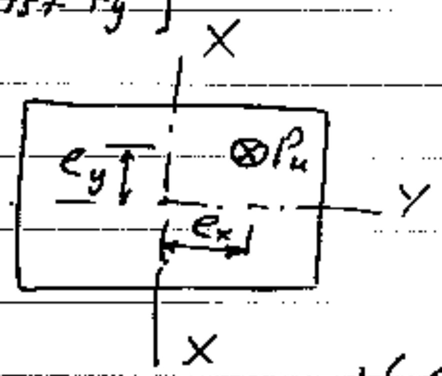
P_{uy} = " " " " " " e_y " ($e_x = 0$)

$P_{uo} = \phi\left[0.85 f_c'(A_g - A_{st}) + A_{st} f_y\right]$

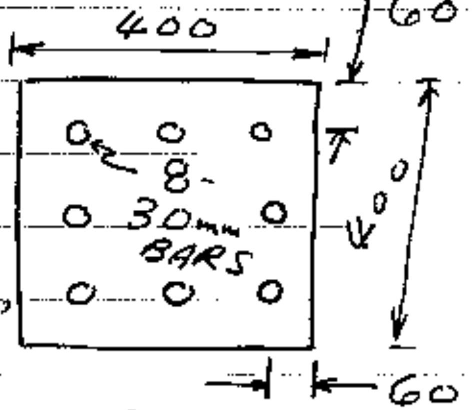
See diagram:

$e_x = \dfrac{M_{ux}}{P_u}$

$e_y = \dfrac{M_{uy}}{P_u}$



Ex: Calculate P_u with $M_{ux} = 151.4$ kNm & $M_{uy} = 67.8$ kNm. The tied column is 400x400 mm, $f_y = 400$ N/mm², $f_c' = 21$ N/mm².



Start ITERATION with P_{uF} = 640 kN.

Solution: Since the process involves ITERATION, it is better to TRANSFORM to P_{uF} & M_{uF}. At the end with the ratio of (.65/.7), P_u and M_u will go back to their required values:

$M_{uF} = (0.7/0.65) M_u$ & $P_{uF} = (.7/.65) P_u$

$\therefore M_{ux F} = (.7/.65)151.4 = 163$ kNm

$M_{uy F} =$ " $67.8 = 73$ "

→ Start with P_{uF} @ 640 kN

[illegible]

(11) Col. 70B

$e_x = M_{uxF}/P_{uF} = 163\times10^6/(640\times10^3) = 255$ mm

$e_y = M_{uyF}/P_{uF} = 73\times10^6/(640\times10^3) = 114$ mm

If we neglect (on the safe side) the two bars in the centre of the section, we obtain:

$A_{st} = 706\times 6 = 4236$ mm^2

$\rho_t = A_{st}/(bh) = 0.0265$

$\mu = f_y/(.85f_c') = 400/(.85\times21) = 22.41$

$\rho_t\mu = .0265\times 22.41 = 0.593$

$e_x/h = .638$; $e_y/h = .285$; $\gamma = 0.7$

GOTO p. 496 Ferguson:

$\alpha_x = .29$; $P_{uxF} = .29\times.021\times400^2 = 974$ kN

$\alpha_y = .50$; $P_{uyF} = .5\times.021\times400^2 = 1680$ 〃

$P_{no} = .85 f_c' (A_g - A_{st}) + A_{st} f_y$

$= .85\times.021(400^2 - 5668) + 5668\times.4 = 5014$ kN

$P_{uoF} = .7\times5014 = 3510$ kN Ferguson value.

$$\therefore \frac{1}{P_{uF}} = \frac{1}{P_{uyF}} + \frac{1}{P_{uxF}} - \frac{1}{P_{uoF}}$$

$$= \frac{1}{1680} + \frac{1}{974} - \frac{1}{3510} ; \therefore P_{uF} = 768 \text{ kN}$$

HOMEWORK: Try a few iterations, increasing P_{uF} in each case.

After a few cycles:

→ Try $P_{uF} = 1180$ kN (بداية الدورة)

$\left.\begin{array}{l} e_x = 163\times10^6/(1180\times10^3) = 138 \text{ mm}; e_x/h = .345 \\ e_y = 73\times10^6/\quad〃\quad = 61.9 〃 ; e_y/h = .155 \end{array}\right] \rho_t\mu = .593$

$\therefore \left.\begin{array}{l} \alpha_x = 0.44,\ P_{uxF} = 1478 \text{ kN} \\ \alpha_y = .66,\ P_{uyF} = 2218 〃 \end{array}\right] P_{uoF} = 3510$ kN

$$\frac{1}{P_{uF}} = \frac{1}{1478} + \frac{1}{2218} - \frac{1}{3510} \;;\; P_{uF} = \underline{1187}\ kN$$

قريب لك البداية في هذه الدور = $\underline{1180}$ kN

Use $P_{uF} = 1180$ kN with $M_{uxF} = 173$ kNm &
$M_{uyF} = 73$ kNm

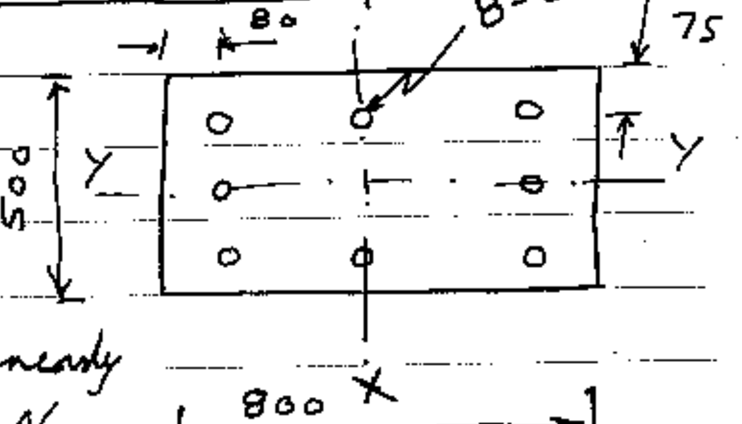
آخر الجواب

Final result [الجواب النهائي] ∴ use (.65/.7)
correction factor

$P_u = \underline{1096}$ kN & $M_{ux} = \underline{151.4}$ kNm &
$M_{uy} = 67.8$ kNm

ملاحظة: تكن تنظيم ملا بر تابع على الكاغد

Ex Ⓐ The short tied column is 500 x 800 mm. $P_u = 3990$ kN which acts simultaneously with $M_{ux} = 474$ kNm



(about major axis XX), and M_{uy} about minor axis YY. Calculate M_{uy}. $f_c' = 28\ N/mm^2$, $f_y = 400\ N/mm^2$

Solution: As before, it is easier to change to F values using the factor (.7/.65)

$\therefore P_{uF} = 4297$ kN & $M_{uxF} = 510$ kNm

$$\frac{1}{P_{uF}} = \frac{1}{P_{uxF}} + \frac{1}{P_{uyF}} - \frac{1}{P_{uoF}}$$

$e_x/h = (510/4297)/.8 = .148$; For M_x: $\gamma_x = 640/800 = 0.8$; $\rho_t = 6 \times 962/(\underline{800} \times 500) = 0.01443$; $\rho_t\,\mu = 0.01443 \times 400/(.85 \times 28) = .243$

GOTO p.495 F: $\alpha = .53$ & $\frac{\alpha e}{h} = .078$ (giving $e/h = .147$,

$\therefore .53 = P_{uxF} / (.028 \times 500 \times 800)$; $\therefore P_{uxF} = \underline{\underline{5936}}$ kN

$P_{uoF} = .7 P_{no} = .7 \left[.85 f_c' (A_g - A_{st}) + f_y A_{st} \right]$

$= .7 \left[.85 \times .028 (800 \times 500 - 8 \times 962) + .4 \times 8 \times 962 \right]$

$= 8961$ kN

$$\therefore \frac{1}{4297} = \frac{1}{5936} + \frac{1}{P_{uyF}} - \frac{1}{8961}$$

$\therefore P_{uyF} = 5687$ kN

$\alpha_y = P_{uyF} / (f_c' b h) = 5687 / (.028 \times 500 \times 800)$

$= 0.508$

$\gamma_y = 350 / 500 = 0.7$, $\therefore$ GOTO p. 494 $\underline{F}$:

$\alpha e/h = 0.084$

$\therefore e/h = .084 / .508 = .165$

$e_y = .165 \times 500 = 82.68$ mm

$\therefore M_{uyF} = 82.68 \times 4297 \times 10^{-3} = 355.3$ kN/m

ج : $M_{uy} = (.65/.7) 355.3 = \underline{\underline{329.9}}$ kNm

↑ الجواب المطلوب

<u>Ex.</u> Ⓑ For the same column of Ex. Ⓐ, when $P_u = 3250$ kN, $M_{uy} = 464$ kNm, find M_{ux} capacity

<u>Solution</u>: As before, it is easier to change values by multiplying with $.7/.65 \rightarrow P_{uF} = 3500$ kN & $M_{uyF} = 500$ kN

$e_y/h = (500/3500)/.5 = .286$; $\ell_{t\mu} = 0.243$ (Ex. Ⓐ)

& $\gamma_y = 0.7$; $\therefore \alpha_y = 0.37$ [p. 494 $\underline{F}$]

$\therefore P_{uy\underline{F}} = .37 \times .028 \times 500 \times 800 = 4144$ kN

$$\therefore \frac{1}{3500} = \frac{1}{P_{ux}} + \frac{1}{4144} - \frac{1}{8961}$$

$\therefore P_{ux\underline{F}} = 6410$ kN

$\therefore \alpha_x = 6410 / (.028 \times 500 \times 800) = .572$, $\gamma_x = .8$ & $\ell_{t\mu} = .24$

∴ p. 495 Ferguson : $\alpha_x e_x / h = \underline{0.07}$

∴ $e_x / h = .07 / .572 = .122$

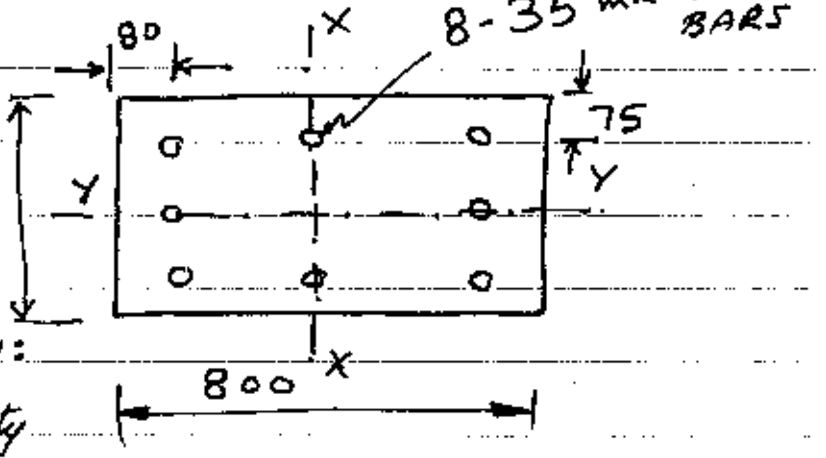
$M_{ux} F = .122 \times .8 \times 3500 = 342.7$ kNm

∴ $M_{ux} = (.65/.7)\,342.7 = 318.2$ kNm (الجواب المطلوب) the factored BM capacity w.r.t. major axis X-X.

مهم جداً

__Homework__ (__BIAXIAL BENDING__) Use the Bresler __RECIPROCAL__ method for the following, based on:

The __short__ __tied__ column is 500x800 mm. $f'_c = 28$ N/mm² & $f_y = 400$ "



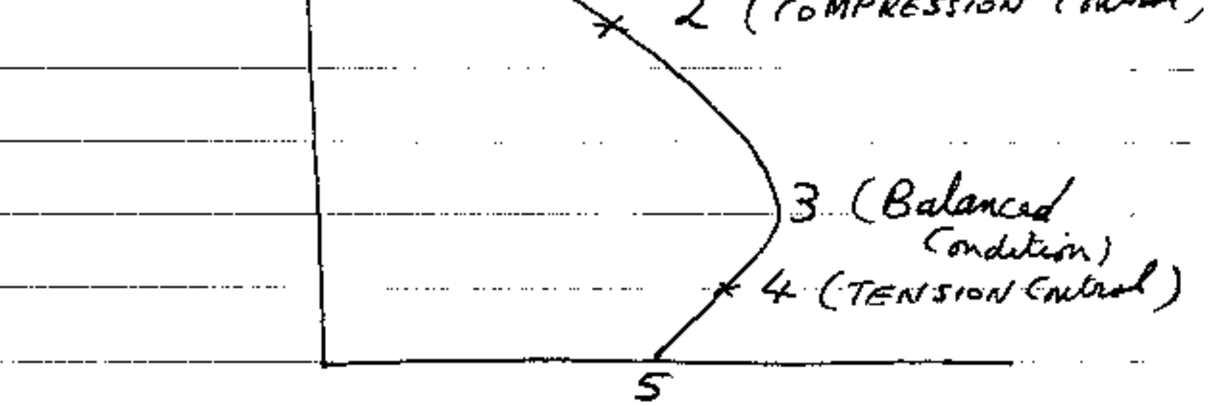
4-__HOMEWORK__ questions:

1. Calculate __P_u__ capacity when $M_{ux} = 650$ kNm & $M_{uy} = 200$ kNm
2. Calculate __P_u__ capacity when $M_{ux} = 300$ & $M_{uy} = 600$ kNm
3. With $P_u = 4640$ kN & $M_{ux} = 370$ kNm find M_{uy} capacity
4. " " 2800 " & $M_{uy} = 325$ " " M_{ux} "

__ملاحظة__: ان حل هذه الاسئلة مسبقاً للطالب بان يحل الاسئلة في الامتحان بوقت قصير

COLUMN INTERACTION DIAGRAM (CID)

Following is the CID for RC columns based on the latest ACI318 Code:



Important Points in "CID"

0: Top of curve لا نحتاجها

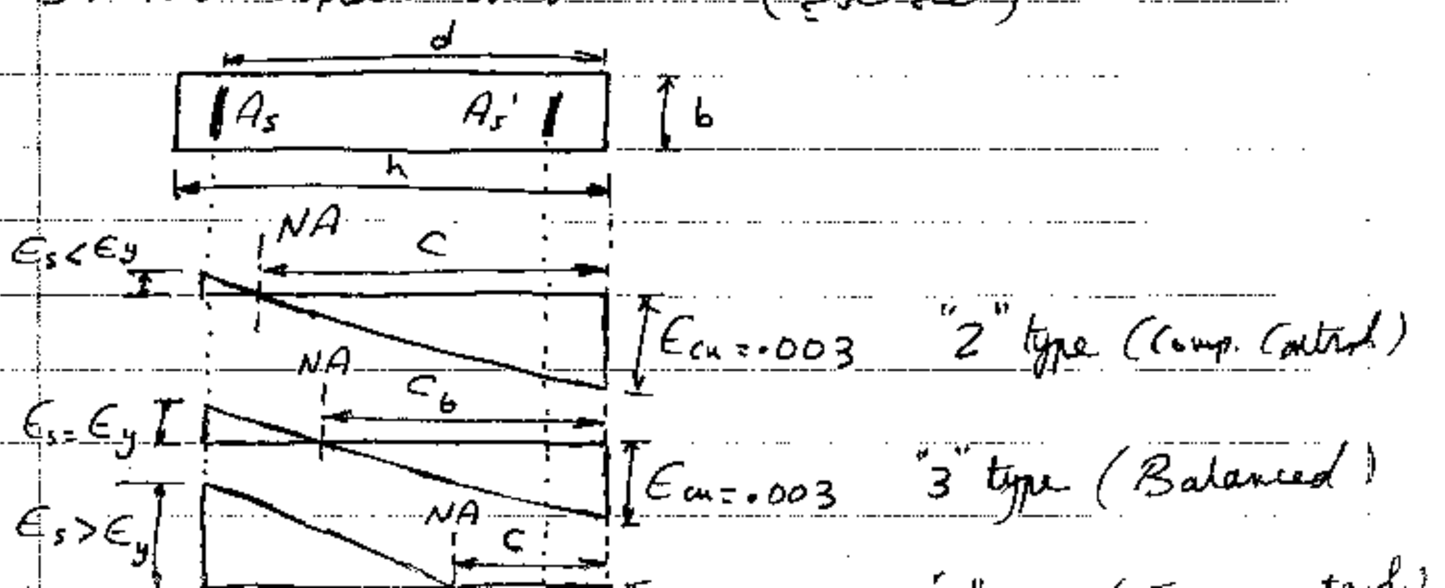
1: MAX P_n (Factor of 0.80 → TIED Columns)

2: COMPRESSION Control → $\epsilon_s < \epsilon_y$

3: Balanced Condition → $\epsilon_s = \epsilon_y$

4: TENSION Control → $\epsilon_s > \epsilon_y$

5: NO axial load (نظرياً)



⑪ COL. /08

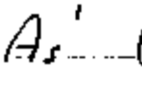
Use FERGUSON pp 493-496 interaction diagrams - depending on the value of $\underline{\underline{\gamma}}$.

Equations to be used:

$\rho_t = A_{st}/(bh)$ (I): $A_{st} = A_s + A_s'$ (مجموع حديد الوجهين)

$\mu = f_y/(0.85 f_c')$ (II)

$\rho_t \mu$ (III) (Use to find C/D curve



e/h (IV)

Correction Factor:

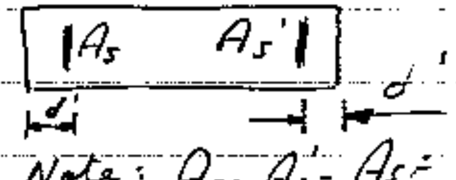
$$P_u = P_{uF} \times \boxed{\frac{0.65}{0.70}} \quad (\overline{V})$$

Factored (مطلوب ايجاده) ↑ من FERGUSON ↑ ← معامل التصحيح بسبب تغير

ϕ من (0.7) في (ACI) القديم الى (0.65) في (ACI) الجديد

يقرر $\underline{\underline{\gamma}}$ اي صفحة تستعمل من 493-496

$$\gamma = \frac{h - 2d'}{h} \quad (\overline{VI})$$



Note: $A_s = A_s' = \frac{A_{st}}{2}$

ملاحظة: الصفحات التالية (493-496) المرقمة
⑫ ، ⑬ ، ⑭ ، ⑮ تستعمل كل
امثلة لايجاد مقاومة العمود FACTORED
← "SHORT TIED COLUMN UNDER UNIAXIAL BENDING"

Note: To obtain P_{uF} (see pp 493-496 Ferguson):

$f_c' \leq 28$ N/mm² $\overline{VII}$ } ; then apply the correction of Eq. ($\overline{V}$)
$f_y = 400$ " $\overline{VIII}$ }

(12)

Col./08

28 MPa

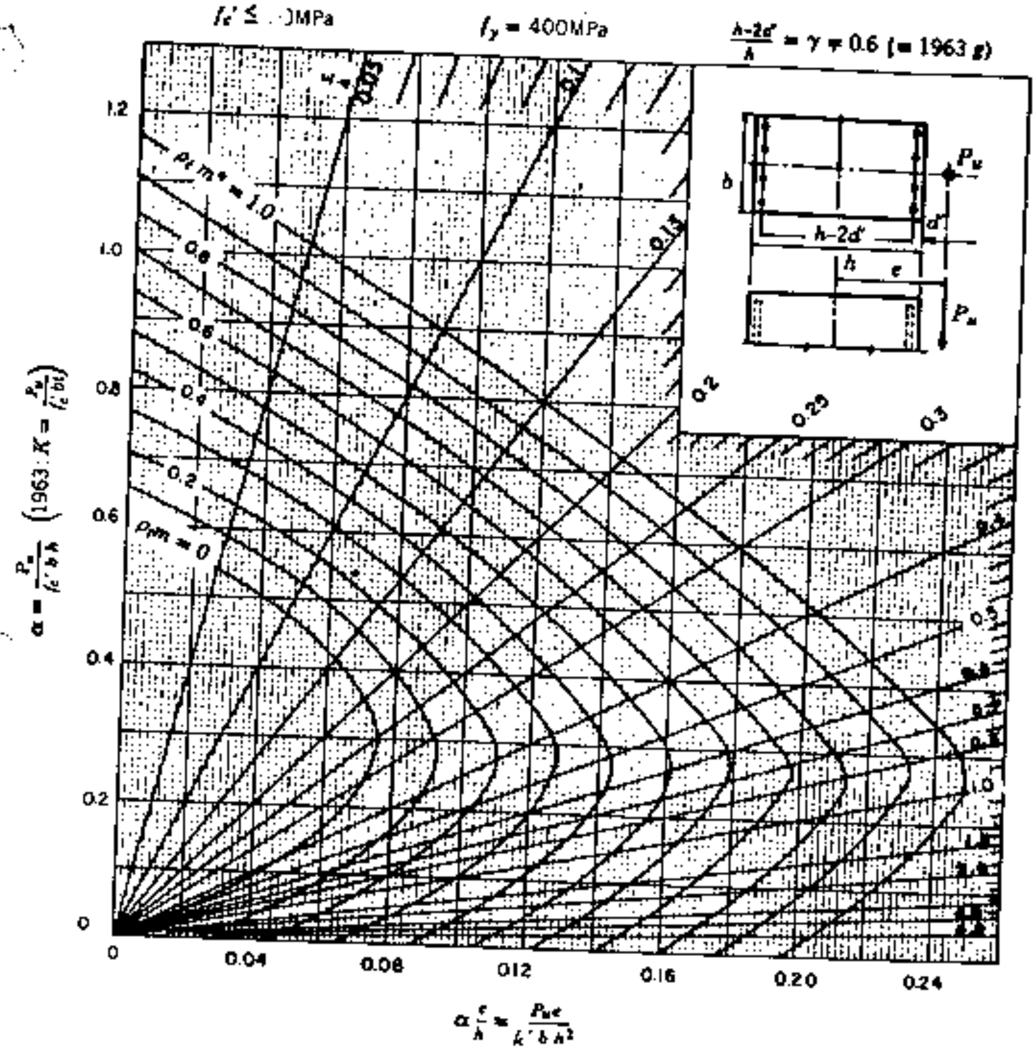


FIGURE 18.12a Column Interaction Chart. $\gamma = 0.6$. $m = f_y/0.85\ f_c'$ is 1963 notation. m now should become a lower-case Greek letter. Chart readings include the effect of ϕ, but not correctly below ordinate of 0.10.

lution

⑬ col./08

28 MPa

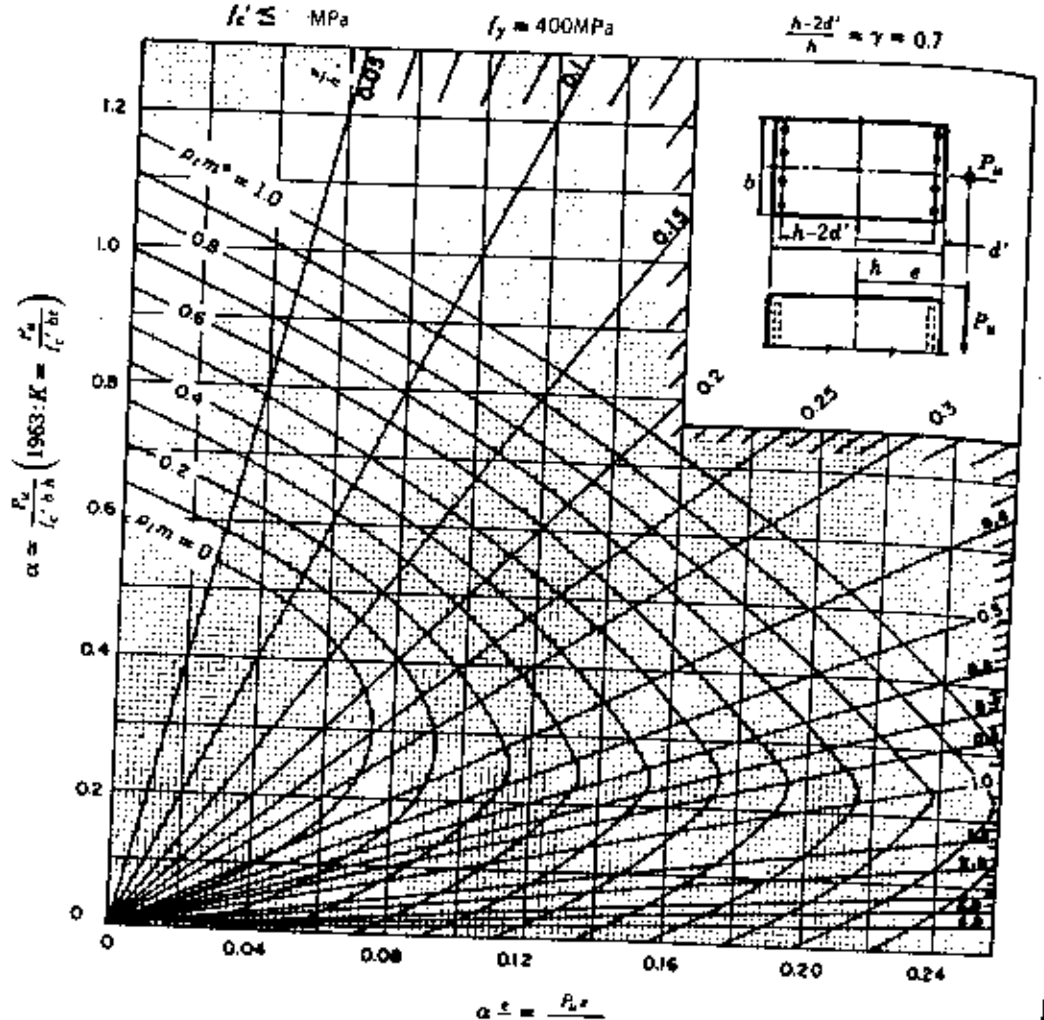


FIGURE 18.12*b* Column Interaction Chart. $\gamma = 0.7$. Chart readings include
effect of ϕ, but not correctly below ordinat

⑭ Col./08

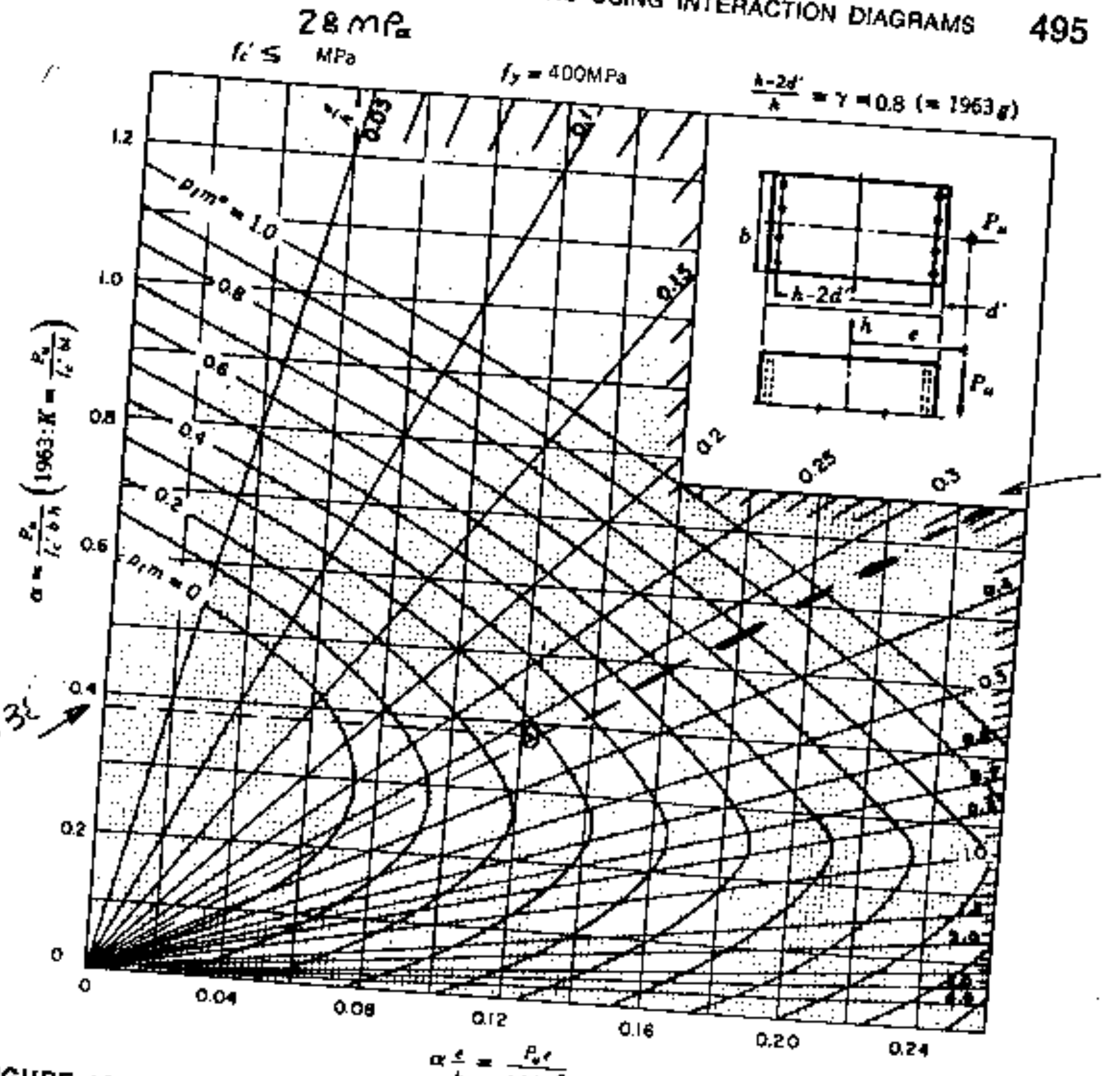


FIGURE 18.12c Column Interaction Chart. $\gamma = 0.8$. Chart readings include the effect of ϕ, but not correctly below ordinate of 0.10.

EX 5 solution:

$\alpha_F = 0.38$

$\therefore P_{uF} = .38 \times .0207 \times 380 \times 610 = 1823$ kN

P_u (حالة(1)) $= \frac{.65}{.7} \times \leftarrow = 1693$ kN

M_u (〃) $= .203 \times 1693 = 343.6$ kNm

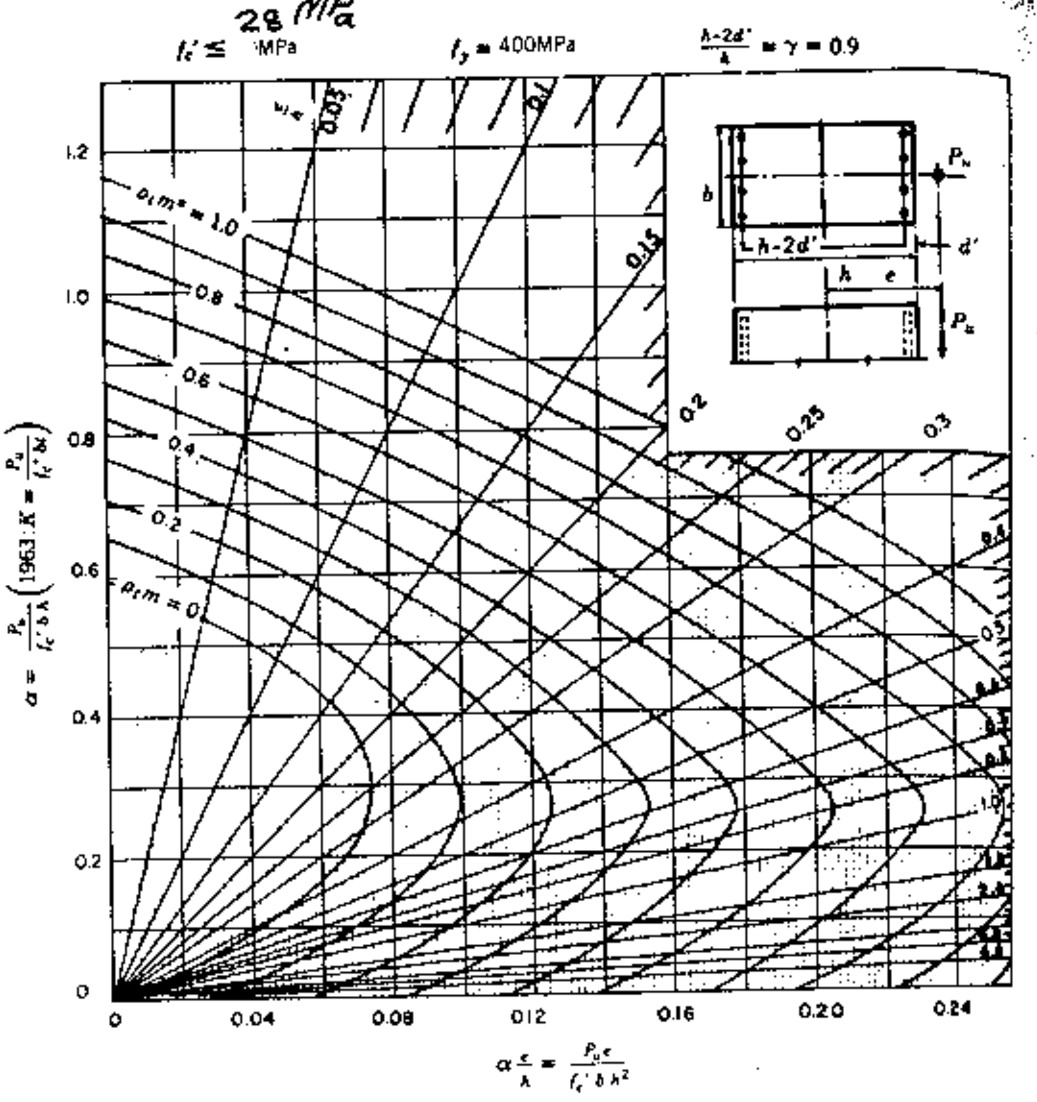


FIGURE 18.12*d* Column Interaction Chart. $\gamma = 0.9$. Chart readings include the effect of ϕ, but not correctly below ordinate of 0.10.

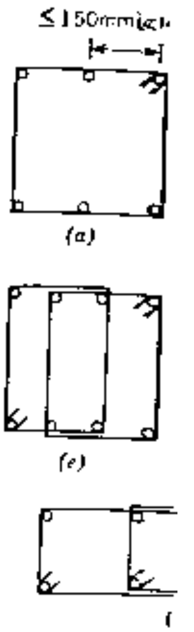


FIGURE 18.13

18.11 COLUMN T

Column ties n
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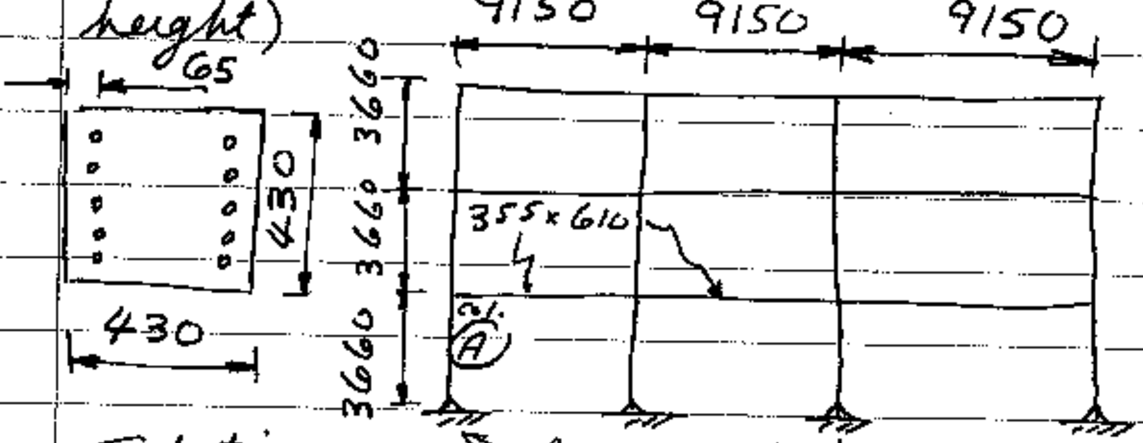
no bar more th
smallest of:

1. $16\,d_b$ for t
2. $48\,d_b$ for t
3. The minin

A few typica

EX The frame is NONSWAY. Find if column Ⓐ is safe. Column size is 430×430 mm $f'_c = 21\ N/mm^2$, $f_y = 400\ N/mm^2$ for the 10-25 mm longitudinal bars. $P_u = 2335$ kN and $M_u = 142$ kNm, column $l_u = 3050$ (clear height)



Solution ← Assume hinged

$M_u = 142\ kNm = M_2$ (GIVEN)

$\therefore e = 142\times10^3/2335 = 60.8$ mm

$e_{,min} = 15 + .03h = 15 + .03\times430 = 27.9$ mm

$\therefore e = 60.8$ mm governs (greater)

First step: Is column short or long:

NONSWAY column has $k \leq 1$

→ Start with $k = 1$ initially:

$kl_u = 1\times3050 = 3050$ mm

$kl_u/r = 3050/(0.3\times430) = 23.6$

Column is SHORT if Eq. (12 S') applies:

$$\frac{kl_u}{r} < 34 - 12\frac{M_1}{M_2} \leq 40 \quad M_1 = 0 \text{ (hinged)}$$

⓪③

$= 34$ (اكا الاكبر للعرض)

$kl_u/r = 23.6 < 34$; $\therefore$ Col. is SHORT

$\therefore$ NO MAGNIFICATION of moment M_2

$\gamma = (430 - 2\times65)/430 = 0.7$ — p. 494 F

$\rho_t = 10 \times 490 / 430^2 = 0.0265$ $\begin{matrix} \leq .08 \\ \geq .01 \end{matrix}\Big]$ ∴ OK per ACI code

$\mu = f_y / (.85 f_c') = 400 / (.85 \times 21) = 22.41$

$\rho_t \mu = 0.594$; $e/h = 60.8/430 = 0.141$

GOTO p. 494 Ferguson : $\alpha_{\underline{F}} = 0.67$

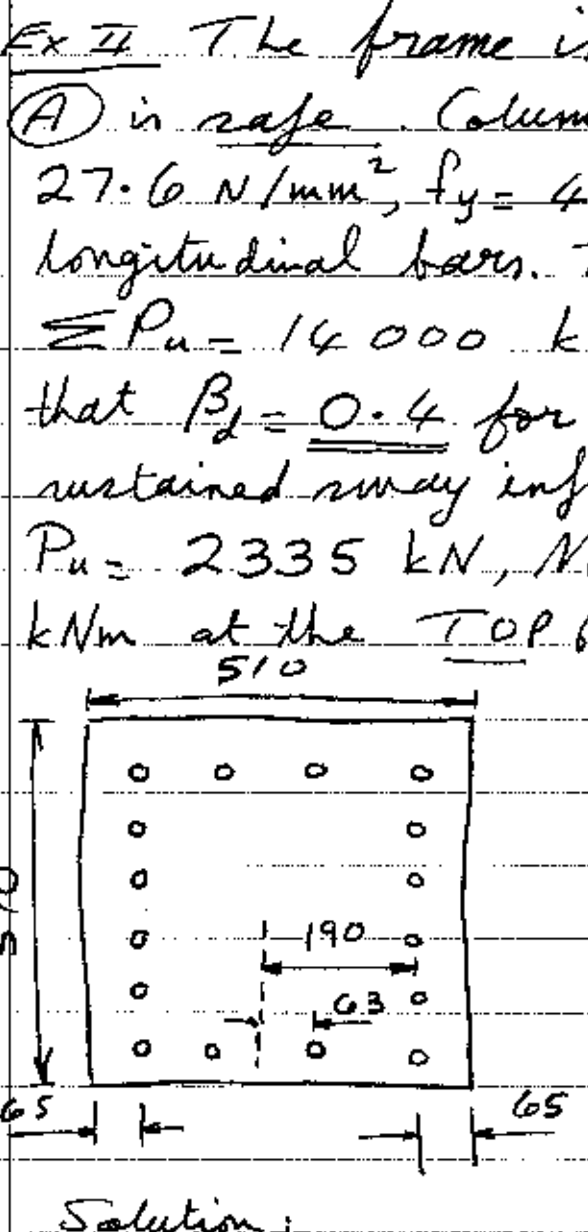
$\therefore P_{u\underline{F}} = \alpha f_c' bh = .67 \times .021 \times 430^2$

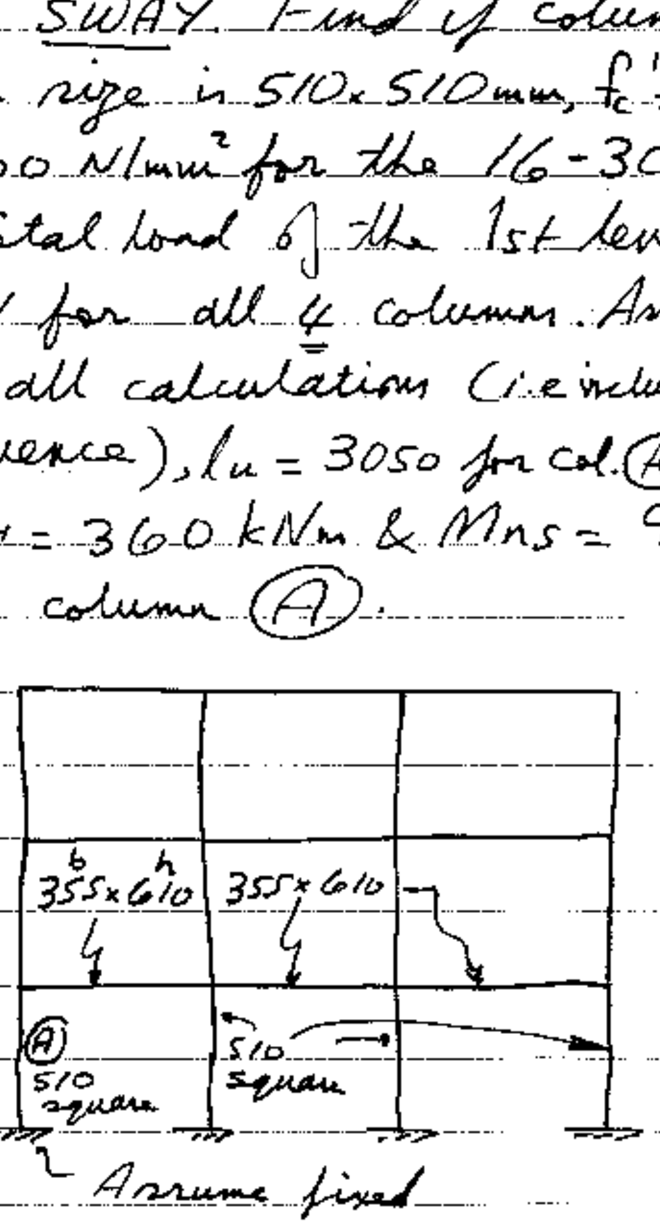
$= 2600$ kN

ولذا $\rightarrow P_u = P_{u\underline{F}} \times .65/.7 = 2600 \times .65/.7$

$= \underset{\text{مقاومة}}{\underset{\uparrow}{2414}}$ kN $> \underset{\text{موجود}}{\underset{\uparrow}{2335}}$ kN, ∴ Col. is SAFE

Ex II The frame is SWAY. Find if column (A) is safe. Column size is 510 × 510 mm, $f_c' = 27.6$ N/mm², $f_y = 400$ N/mm² for the 16-30 mm longitudinal bars. Total load of the 1st level $\Sigma P_u = 14\,000$ kN for all 4 columns. Assume that $\beta_d = 0.4$ for all calculations (i.e including sustained sway influence), $l_u = 3050$ for col. (A), $P_u = 2335$ kN, $M_s = 360$ kNm & $M_{ns} = 95$ kNm at the TOP of column (A).





Solution:

يقسم الحل لأكثر من فقرة للوضوح :

(a) Slenderness of "SWAY" Part

$l_u = 3050$ mm (معطيات السؤال)

To find $\underline{\Psi}$ apply Eqs. (75) & (85)

Eq.(75): $\Psi = \Sigma(EI/l_c)/\Sigma(EI/l_b)$

For column use $0.7 I_g = 0.7 \times 510^4/12 = 3.946 \times 10^9$ mm^4

" beam use $.35 I_g = .35 \times 355 \times 610^3/12 = 2.350 \times 10^9$ mm^4

$$\Psi_A \left(\begin{matrix} \text{TOP OF} \\ \text{EXERIOR} \\ \text{COLUMN} \end{matrix} \right) = \frac{\Sigma EI/l_c}{\Sigma EI/l_b} \qquad (75)$$

COLUMN خارجي

عدد ↓

$$= \frac{2 \times 3.946 \times 10^9/3660}{1 \times 2.350 \times 10^9/9150} = 8.40$$

عدد → 1 [خارجي]

$$\Psi_A \left(\begin{matrix} \text{TOP OF} \\ \text{INTERIOR} \\ \text{COLUMN} \end{matrix} \right) = \frac{2 \times 3.946 \times 10^9/3660}{2 \times 2.35 \times 10^9/9150} = 4.20$$

COLUMN داخلي ; عدد ↘ 2 ; عدد → 2 [داخلي]

Notes: (1) For l_c (& l_b) use ℄ to ℄ distance which contrasts with l_u — unsupported length

(2) At the FIXED end theoretically $\Psi = 0$; however the "Structural Stability Council" recommends that for practical purposes $\underline{\Psi}$ should not be < 1 (additional safety vs. $\Psi = 0$). ∴ Use $\Psi_B = 1$ at column bottom in all cases (داخلي، خارجي)

From the JACKSON-MORELAND Alignment chart (p. 531 FERGUSON): $k = \underline{\underline{1.85}}$ (external column UNBRACED) & $k = \underline{\underline{1.64}}$ (internal column UNBRACED)

For the exterior column: $\frac{kl_u}{r} = \frac{1.85 \times 3050}{.3 \times 510} = \underline{\underline{36.9}} > \underline{\underline{22}}$; ∴ Column is slender "SWAY" part

(b) "SWAY" Magnification

Because $\underline{Q}$ is unknown, we can only use Eq.(28)

$E_c = 4700\sqrt{f_c'} = 4.7\sqrt{27.6} = 24.7$ kN/m^2

$I_{se} = 12 \times 706 \times 190^2 + 4 \times 706 \times 63^2 = 3.17 \times 10^8 \text{ mm}^4$

Eq.(25): $EI = (0.2\, E_c I_g + E_s I_{se}) / (1 + \beta_d)$

$I_g = 510^4 / 12 = 5.638 \times 10^9 \text{ mm}^4$

$\therefore EI = (0.2 \times 24.7 \times 5.638 \times 10^9 + 200 \times 3.17 \times 10^8) / 1.$

$= 6.518 \times 10^{10} \text{ kN-mm}^2$

EXTERIOR col.

$\therefore P_c = \pi^2 \times 6.518 \times 10^{10} / (1.85 \times 3050)^2 = \underline{20206}$ k

For INTERIOR col.: $P_c = \pi^2 \times 6.518 \times 10^{10} / (1.64 \times 3050)^2 = \underline{25711}$ kN ← داخلية

$\therefore$ For the 4 - 1st level column: $\Sigma P_c = 2 \times 2571$
$+ 2 \times 20206 = \underline{91834}$ kN storey EULER capacity

Eq.(28): $$\delta_s = \frac{1}{1 - \frac{\Sigma P_u}{.75 \Sigma P_c}} = \frac{1}{1 - \frac{14000}{.75 \times 91834}} = 1.255$$

ملاحظة: الرقم 1.255 يعني بأنه لو عرفت Q لكان اسهل

حل كثير باستعمال $Eq.27 : \delta_s = \frac{1}{1-Q} \le 1.5$ +

Eq.(26): $M_2 = M_{2ns} + \delta_s M_{2s}$
$= 95 + 1.255 \times 360 = \underline{546.8 \text{ kN/m}}$

(c) "NONSWAY" Magnification

With $\left.\begin{array}{l}\Psi_A = 8.4 \\ \Psi_B = 1.0\end{array}\right\}$ $k_{ns} = 0.85$, From Jackson-Moreland (p. 531 FERG.)

For a "NONSWAY" case, apply Eq.(12 S):

Column is SHORT if $\frac{kl_u}{r} \le 34 - 12\, M_1/M_2 \le 40$

[illegible]

[illegible]

Actual $kl_u/r = .85 \times 3050 / (.3 \times 510)$
$= 16.9$

$\therefore$ "NONSWAY" part is "SHORT", since 16.9 < the minimum possible value of Eq.(12 S), which is 22.

Note: Even if the column is SLENDER for the NONSWAY part, magnification with δ_{ns} may not be applied unless Eq.(29.5r) requires that:

Eq.(29.5r): ONLY magnify 2nd time (مرة ثانية) if:

$$\frac{l_u}{r} > \frac{35}{\sqrt{\frac{P_u}{f_c' A_g}}} = \frac{35}{\sqrt{\frac{2335}{.0276 \times 510^2}}} = 61.4$$

$$l_u/r = \frac{3050}{.3 \times 510} = 19.9 < 61.4$$

∴ لا يكبر العزم مرة ثانية حتى لو كانت طويلة

(SLENDER) في حالة NONSWAY.

Taking only the OUTSIDE 12-30mm bars (on the SAFE side): $A_{st} = 706 \times 12 = 8472$ mm^2

$\rho_t = 8472/510^2 = 0.0326$

$\mu = f_y/(.85 f_c') = 17.05$; ∴ $\rho_t \mu = 0.555$

$e = \frac{M_c}{P_u} = \frac{546.8 \times 10^3}{2335} = 234.6$ mm $\gamma = 380/510 = .75$

$e/h = 234.6/510 = .459$

∴ Interpolating between pp 494 & 495 FERGUSON:

$\alpha = 0.37$

∴ $P_{uF} = \alpha f_c' bh = .37 \times .0276 \times 510^2 = 2656$ kN

CORRECTING : $P_u = (.65/.7) P_{uF}$

$= (.65/.7) 2656$

$= 2466$ kN $> \underset{\text{موجود}}{2335}$ OK

(المقاومة المتوفرة)

SLENDER (LONG) RC COLUMNS

The basic BUCKLING (EULER) equation is:

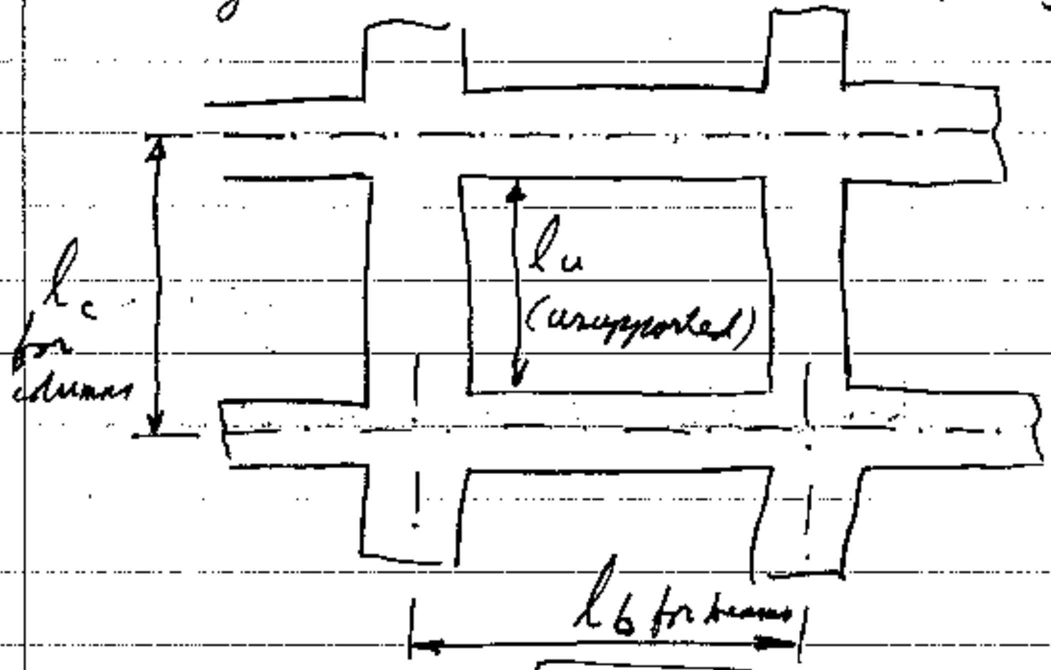
$$P_c = \frac{\pi^2 EI}{(k\,l_u)^2} \quad Ⓐ$$

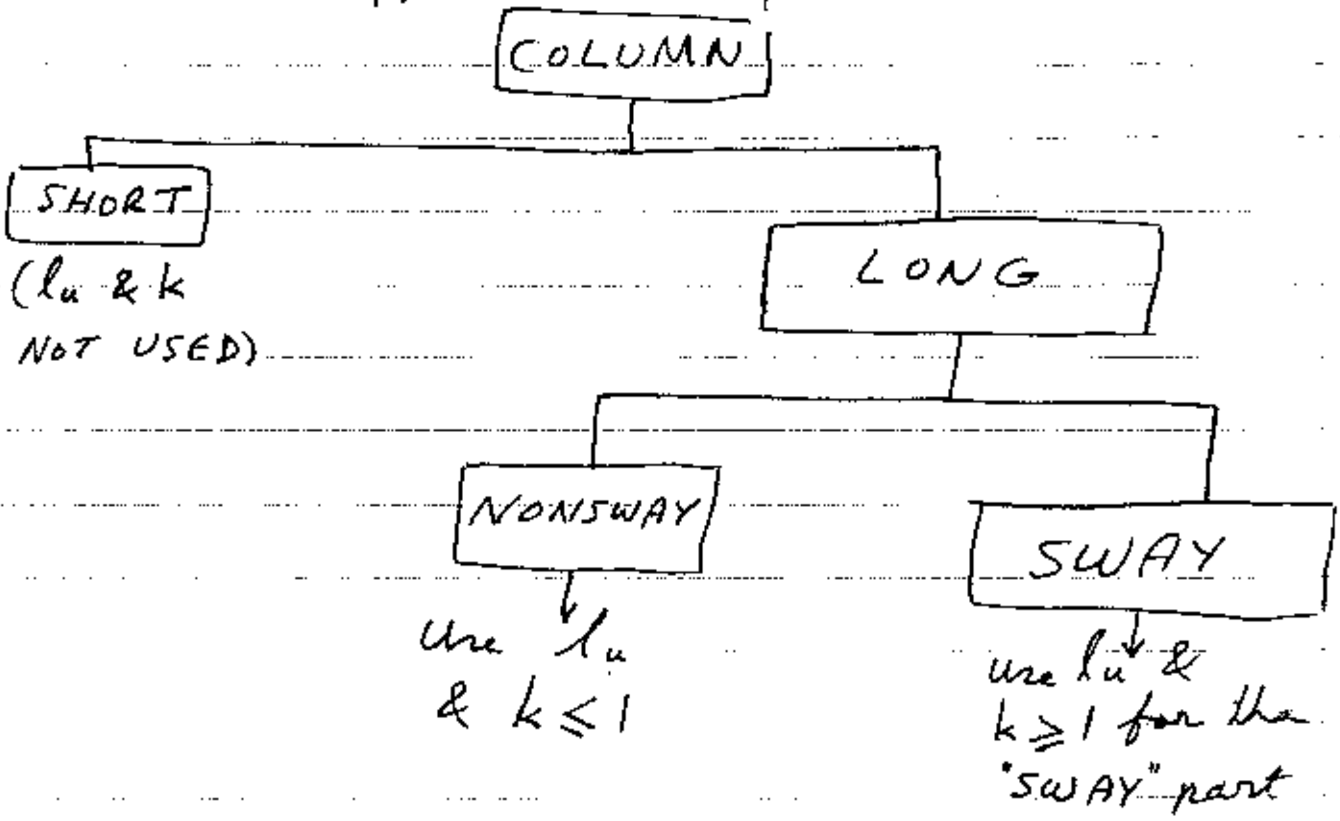
In Eq. Ⓐ: P_c = buckling load capacity

EI = value to be calculated for RC

l_u = column UNSUPPORTED length

k = factor to be calculated depending on end conditions.





LATEST ACI Column Design & Analysis

$P_c = \pi^2 EI / (k l_u)^2$ (1S)

For columns with known LONGITUDINAL steel:

$EI = (0.2 E_c I_g + E_s I_{se}) / (1 + \beta_d)$ (2S)

For preliminary calculations - Long. steel unknown:

$EI = (E_c I_g / 2.5) / (1 + \beta_d)$ (3S)

ملاحظة: لا يجوز استعمال المعادلة (3S) عند معرفة التسليح الطولي (Long. steel) يركز الكاتب على المعادلة (2S)

$E_c = 4700 \sqrt{f'_c}$ (4S)
قوة الاسطوانة

$E_s = 200\,000 \ N/mm^2$
$= 200 \ kN/mm^2$ (5S)

k = value per "JACKSON-MORELAND" Chart (6S)

$$\Psi = \frac{\Sigma(EI/l_c) \leftarrow \text{اعمدة}}{\Sigma(EI/l_b) \leftarrow \text{عتبات}} \quad \text{in a PLANE} \quad (7S)$$

To find Ψ, Eq. (8S) may be used:

For BEAMS, mom. of inertia $= 0.35\, I_g$
" COLUMNS " " " $= 0.70\, I_g$ (8S)

note: for ▨ $I_g = bh^3/12$

l_c = ℄ to ℄ length of column
l_b = " " " " beam
See diagram p. ㉓ col./07

Eq. (9S) gives upper limit for "MOM. MAGN." method:

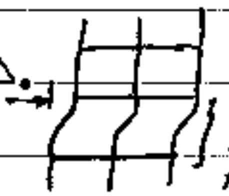
$kl_u / r \le 100$
$r = 0.3 \times$ overall column dimension ▨ [rectangular]
$r = 0.25 \times$ diameter of ◍ [circular] column (9S)

A storey may be assumed as "NONSWAY" if :

$$Q = \frac{\sum P_u \Delta_o}{V_u l_c} \leq 0.05 \qquad (10S)$$

where Q = storey stability index

Δ_o = 〃 lateral deflection:



V_u = factored horizontal shear in storey

With BIAXIAL moments, magnify each one separately. Then treat the column with the magnified moments as an equivalent "SHORT" column. (11S)

$\boxed{1}$ Magnified Moments – NONSWAY Frames

Ignore slenderness [i.e. column is SHORT] if:

$$(kl_u/r) \leq 34 - 12\,M_1/M_2 \leq 40 \quad - (12S)$$

↑ (08)

$$\text{where } (M_1/M_2) \geq -0.5 \quad - (13S)$$

$$M_c = \delta_{ns} M_2 \quad - (14S)$$

where M_c = Magnified moment to be used in DESIGN

δ_{ns} = NONSWAY moment magnification

M_2 = LARGER factored column end moment, always positive

M_1 = smaller column factored end moment, positive if member is bent in single curvature $\overset{\curvearrowright M_2}{\underset{\curvearrowleft M_1}{(}}$, negative if member is bent in double curvature $\overset{\curvearrowright M_2}{\underset{\curvearrowleft M_1}{S}}$

$$\delta_{ns} = \frac{C_m}{1 - \frac{P_u}{0.75 P_c}} \geq 1 \quad - (15S)$$

Col. 108

where P_u = factored axial load ($\leq \phi P_n$)

P_c = critical load = $\frac{\pi^2 EI}{(k l_u)^2}$ (16 S)

$C_m = (0.6 + 0.4 M_1/M_2) \geq 0.4$ (17 S)

$EI = (0.2 E_c I_g + E_s I_{se})/(1+\beta_d)$ (18 S)

ملاحظة: تستعمل هذه عند معرفة التسليح الطولي

$E_s = 200 \ kN/mm^2$ (19 S)

or $EI = (0.4 E_c I_g)/(1+\beta_d)$ (20 S)

ملاحظة: لا تستعمل هذه المعادلة عند معرفة التسليح الطولي

Note: When magnifying moment, use the greater of M_2 (إذا كان موجب) or $M_{2,min}$:

$M_{2,min} = P_u(\underset{\uparrow mm}{15} + 0.03h)$ - (21 S)

IF $M_2 < M_{2,min}$; then:

$\left.\begin{array}{l}\text{either } C_m = 1\\ \text{or base } C_m \text{ on actual } M_1 \& M_2\end{array}\right]$ (22 S)

$\boxed{2}$ Magnified Moments — SWAY Frames

For SWAY case, NEGLECT slenderness (i.e. column is SHORT) if $\frac{k l_u}{r} < 22$ - (23 S)

SWAY frame design consists of 3 steps:
(i) Calculate $\delta_s M_s$ per Eq. (27) or Eq. (28)
(ii) Add $\delta_s M_s$ to M_{ns} at each end to obtain M_1 & M_2
(iii) If the column is still SLENDER [now $k_{ns} < 1$], and Eq. (29 S) applies, then magnify again with Eq. (14 S)
(24 S)

End moments M_1 & M_2 shall be:

$M_1 = M_{1ns} + \delta_s M_{1s}$ — (25s)

$M_2 = M_{2ns} + \delta_s M_{2s}$ — (26s)

$$\delta_s M_s = \frac{M_s}{1-Q} \geqslant M_s$$ — (27s)

This may be used only if $\delta_s \leq 1.5$

where δ_s = SWAY magnification factor

Alternatively (أو بديل):

$$\delta_s M_s = \frac{M_s}{1-\dfrac{\Sigma P_u}{0.75\,\Sigma P_c}} \geq M_s$$ (28s)

ΣP_u = Total factored loads of all storey columns

ΣP_c = " critical " " " "

Note: For the SWAY case use $\beta_d = 0$ if the SWAY is caused by short term effects: WIND or EARTHQUAKE influence.

If a column has:

$$\frac{l_u}{r} > \frac{35}{\sqrt{\dfrac{P_u}{f_c' A_g}}}$$ (29s)

Then apply Eq.(14) $[M_c = \delta_{ns} M_2]$

If $\delta_s > 2.5$, then STIFFEN column — (30s)

ملاحظة: (١) اذا كان العمود NONSWAY وثبت انه طويل بموجب
[Eq. 12s] يستعمل δ_{ns} كما في Eq.[14s]

(٢) اذا كان العمود SWAY وثبت انه طويل بموجب [Eq. 23s] يستعمل
δ_s بعد حسابه [Eq. (27s)] او [Eq. (28s)] وينتج عن ذلك
M_1 [المعادلة (25s)] و M_2 [المعادلة (26s)]

(٣) بعد ذلك اذا كانت NONSWAY طويلة (k<1) وبنفس الوقت [Eq.29] تنطبق

Note: Even if the column is SLENDER for the NONSWAY part, magnification with δ_{ns} may not be applied unless Eq. (29.5') requires that:

Eq. (29.5'): ONLY magnify 2nd time (مرة ثانية) if:

$$\frac{l_u}{r} > \frac{35}{\sqrt{\frac{P_u}{f_c' A_g}}} = \frac{35}{\sqrt{\frac{2335}{.0276 \times 510^2}}} = 61.4$$

$$l_u/r = \frac{3050}{.3 \times 510} = 19.9 < 61.4$$

∴ لا يكبر العزم مرة ثانية حتى لو كانت طويلة

(SLENDER) في حالة NONSWAY.

Taking only the OUTSIDE 12-30mm bars (on the SAFE side): $A_{st} = 706 \times 12 = 8472 \text{ mm}^2$

$\rho_t = 8472/510^2 = 0.0326$

$\mu = f_y/(.85 f_c') = 17.05$; ∴ $\rho_t \mu = 0.555$

$e = \frac{M_c}{P_u} = \frac{546.8 \times 10^3}{2335} = 234.6 \text{ mm}$ $\gamma = 380/510 = .75$

$e/h = 234.6/510 = .459$

∴ Interpolating between pp 494 & 495 FERGUSON:

$\alpha = 0.37$

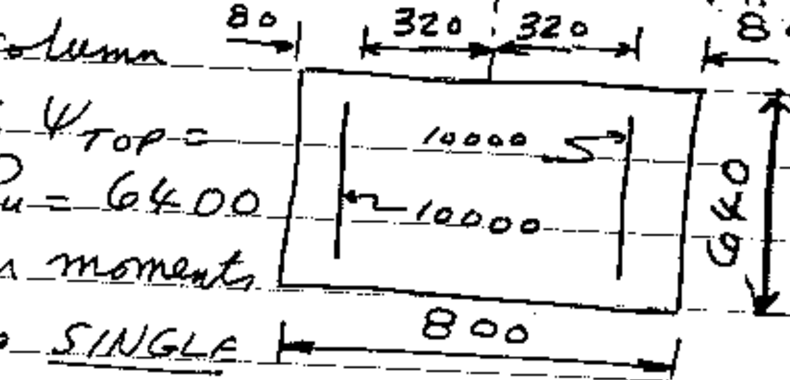
∴ $P_{uF} = \alpha f_c' bh = .37 \times .0276 \times 510^2 = 2656 \text{ kN}$

CORRECTING: $P_u = (.65/.7) P_{uF}$

$= (.65/.7) 2656$

$= 2466 \text{ kN} > 2335$ OK

(المقاومة المتوفرة) $\uparrow$ موجود



Ex. III The tied column has $l_u = 11$ m; $\Psi_{TOP} = \Psi_{BOTT} = 0{\cdot}9$; $P_u = 6400$ kN; Top & Bottom moments are equal causing SINGLE curvature: $M_{ns} = 200$ kNm & $M_s = 550$ kNm; $f'_c = 20$ N/mm^2; $f_y = 400$ N/mm^2; $\beta_d = 0{\cdot}4$ for the NONSWAY part. Assume that $\beta_d = 0$ for the SWAY part which is caused by short duration loading. $A_{st} = 20000$ mm^2. Assuming that all STOREY columns have the same P_u & P_c values, find if the column is SAFE per latest ACI 318 M code.

Solution:

$\Psi_{TOP} = \Psi_{BOTT}$; $\therefore k_s = 1{\cdot}26$ & $k_{ns} = 0{\cdot}76$ — Jackson-Moreland

$kl_u/r = 1{\cdot}26 \times 11/({\cdot}3 \times {\cdot}8) = 57{\cdot}8 > 22$: Slender SWAY

$E_c = 4{\cdot}7\sqrt{20} = 21{\cdot}02$ kN/mm^2; $I_g = 640 \times 800^3/12 = 2{\cdot}73 \times 10^{10}$ mm^4

$EI = ({\cdot}2 E_c I_g + E_s I_{se})/(1 + \beta_d)$

$$\therefore (EI)_{SWAY} = \frac{{\cdot}2 \times 21{\cdot}02 \times 2{\cdot}73 \times 10^{10} + 20000 \times 320^2 \times 200}{1 + 0} \leftarrow \beta_d(SWAY)$$

$= 5{\cdot}244 \times 10^{11}$ kN-mm^2

$P_c = \pi^2 EI/(kl_u)^2 = \pi^2 \times 5{\cdot}244 \times 10^{11}/(1{\cdot}26 \times 11000)^2$

$= 26940$ kN the CRITICAL (EULER) load for the "SWAY" part.

Eq.(28) : $\delta_s = 1/[1 - \Sigma P_u/{\cdot}75\Sigma P_c]$

$$\delta_s = \frac{1}{1 - \dfrac{6400}{{\cdot}75 \times 26940}} = 1{\cdot}464$$

Eq.(26S): $M_2 = M_{2ns} + \delta_s M_s$ single curvature

$M_2 = 200 + 1.464 \times 550 = 1005.2$ kNm $= \overset{\downarrow}{M_1}$

"NONSWAY" Part

$(k l_u / r)_{NONSWAY} = .76 \times 11 / (.3 \times .8) = 34.83$

Eq.(12S) Upper limit for SHORT column:

$k l_u / r \leq 34 - 12\, M_1/M_2 = 34 - 12 = 22$

$\therefore$ actual $(k l_u / r)_{NONSWAY} = 34.83 > 22$.

$\therefore$ NONSWAY column is slender

Eq.(29S): $35 / \sqrt{P_u / (f_c' A_g)} = \dfrac{35}{\sqrt{6400/(.02 \times 800 \times 640)}}$

$= 44.3$

$l_u / r = 11 / (.3 \times .8) = 45.83 > 44.3$

$\therefore$ Apply Eq. (14S) to magnify M_2:

Eq.(20S): $C_m = 0.6 + .4\, M_1/M_2$ $M_1 = M_2$

$\therefore C_m = .6 + .4 \times 1 = 1$

$EI = 5.244 \times 10^{11} / (1 + 0.4) = 3.746 \times 10^{11}$ kN-mm²

$P_c = \pi^2 EI / (k l_u)^2 = 3.746 \times 10^{11} \times \pi^2 / (.76 \times 11000)^2$

$= 52900$ kN CRITICAL (EULER) load

$\delta_{ns} = \dfrac{C_m}{1 - P_u / .75 P_c} = \dfrac{1}{1 - 6400/(.75 \times 52900)} = 1.192$

Eq.(14S): $M_c = \delta_{ns} M_2 = 1.192 \times 1005.2 = 1198$ kNm

$\rho_t \mu = \dfrac{20000}{800 \times 640} \times \dfrac{400}{.85 \times 20} = 0.919$; $\gamma = .8$ ↑ بعد التقريب

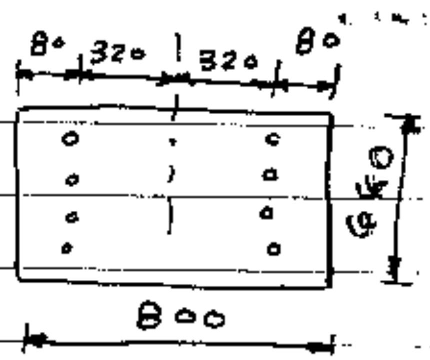
Go to p. 495 FERGUSON with ↗ $e/h = \dfrac{1198 \times 10^3}{800} \times \dfrac{1}{6400} = .23$

$\alpha_{FERG} = 0.69$; $\therefore P_{uF} = .69 \times .02 \times 800 \times 640 = 7066$ kN

$\underset{\substack{\uparrow \\ \text{مقاوم}}}{P_u} = \dfrac{.65}{.7} P_{uF} = \dfrac{.65}{.7} \times 7066$

$= \underset{\text{مقاوم}}{6561} > \underset{\text{موجود}}{6400}$ O.K.

$\therefore$ Column is SAFE



__EX. IV__ The tied col. has $l_u = 4.2\,m$

$\psi_{TOP} = \psi_{BOTT} = 2.75$; $P_u = 7000\,kN$;

Factored moments at the top are

$M_{ns} = 300\,kNm$ & $M_s = 370\,kNm$;

Factored moments at the bottom causing __SINGLE__ curvature are $M_{ns} = 300\,kNm$ & $M_s = 370\,kNm$.

$f_c' = 28\,N/mm^2$; $f_y = 400\,N/mm^2$ for the 8-30mm longitudinal bars. Storey computations indicate that the storey stability index $Q = 0.09$. Is the column safe?

__Solution__: Jackson-Moreland: $k_s = 1.8$ & $k_{ns} = 0.88$ $\left(\frac{k = 4.8}{= 2.75}\right)$

For the __SWAY__ part $kl_u/r = 1.8 \times 4.2/(.3 \times .8) = 31.5 > 22$

∴ "__SWAY__" column is __SLENDER__.

Eq.(27.5'): $\delta_s = \frac{1}{1-Q} \geq 1$; $\delta_s = \frac{1}{1-.09} = \underline{\underline{1.099}} < \overset{\overset{\text{الحد الاعلى}}{\downarrow}}{1.5}$, OK

$M_2 = M_{2ns} + \delta_s M_{2s} = 300 + 1.099 \times 370 = 706.6\,kNm$ __TOP__

$M_1 = M_{1ns} + \delta_s M_{1s} = 300 + 1.099 \times 370 = 706.6$ " __BOTT__

__NONSWAY__ part : $kl_u/r = .88 \times 4.2/(.3 \times .8) = 15.4$

upper limit for "__SHORT__" column : $34 - 12 \times 706.6/706.6 = 22$

$(kl_u/r)_{\text{المعززة}} = 15.4 < 22$ ∴ __NONSWAY__ part is short

$\rho_t \mu = [5655/(800 \times 640)] \times 400/(.85 \times 28) = \underline{\underline{0.186}}$

$\gamma = 0.8$; $e/h = (706.6/7000)/.8 = \underline{\underline{0.126}}$

GOTO p.495 Ferguson: $\alpha = 0.5$

∴ $P_{uF} = .53 \times .028 \times 800 \times 640 = 7598\,kN$

Now correct $P_u = (.65/.7)7598 = 7055\,kN$

$\underset{\text{مقاومة}}{7055\,kN} > \underset{\text{مجهود}}{7000\,kN}$, OK

∴ __Column__ is SAFE

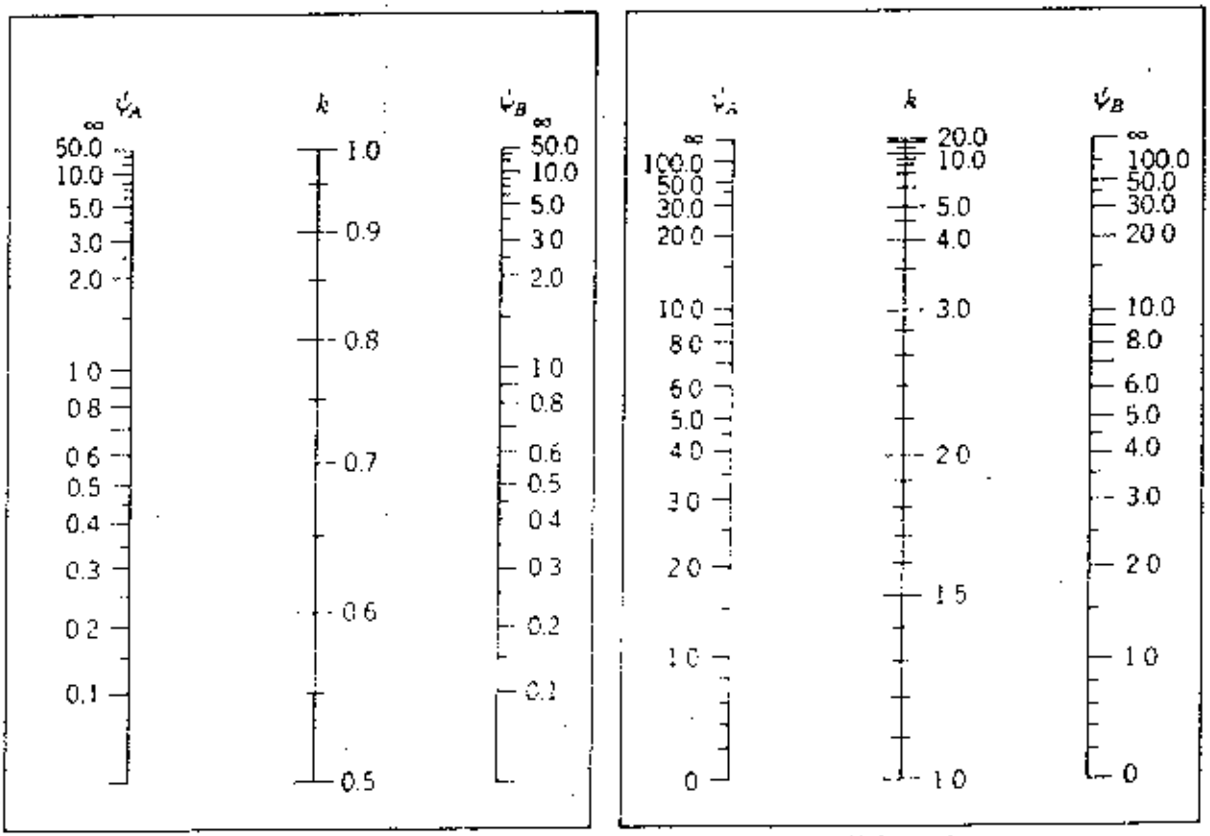
36 col 08

moments might be more appropriate, since creep is a function of sustained loading.) The author feels that this β_d correction may be unimportant in the assessment of an *EI* value that probably involves a typical error of at least 30%. Fortunately, the effect of β_d (and even of *EI*) is far removed from the end result of a design. Experience may show that it is only on very unusual long columns that β_d is really of significance.

19.8 EFFECTIVE COLUMN LENGTH, $k\ell_u$

In simple mechanics the concept of effective column length is well established; a fixed ended column has an effective length of half its overall height between fixed ends. If it were only a matter of the frame reaction to a statically determined moment applied directly as a load on the column, the effective length would be a relatively simple matter of frame or joint stiffness.

Jackson and Moreland[2] solved the $k\ell_u$ problem in terms of relative

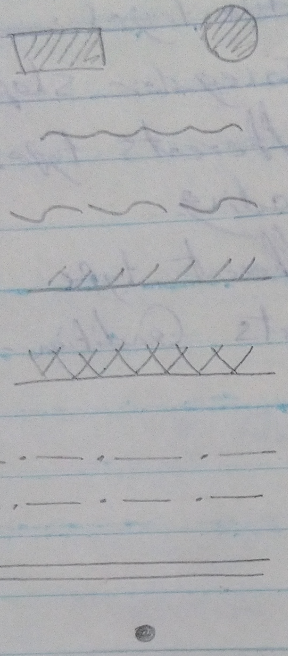


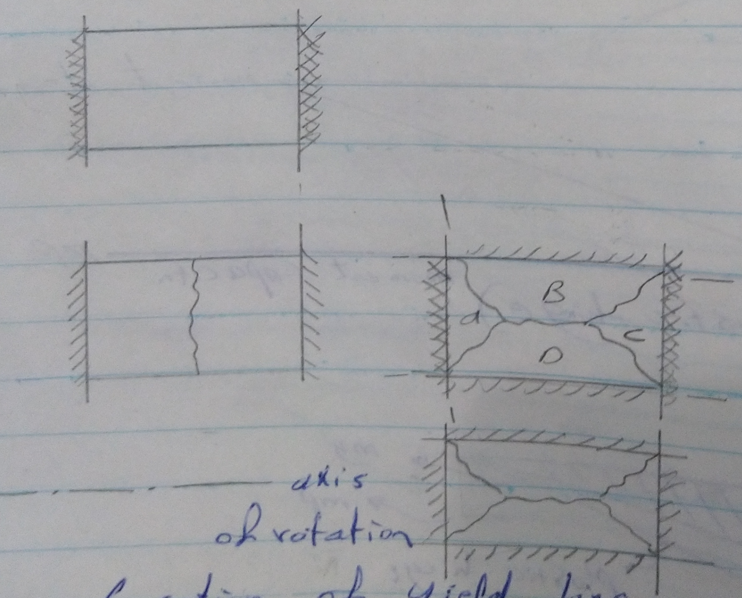
Braced
(a)

Unbraced
(b)

FIGURE 19.6 Effective length factors. (From ACI Code Commentary.) (*a*) Braced frames. (*b*) Unbraced frames. ψ = ratio of $\Sigma EI/\ell_c$ of compression members to $\Sigma EI/\ell$ of flexural members in a plane at one end of a compression member; k = effective length factor.

Symboles:—





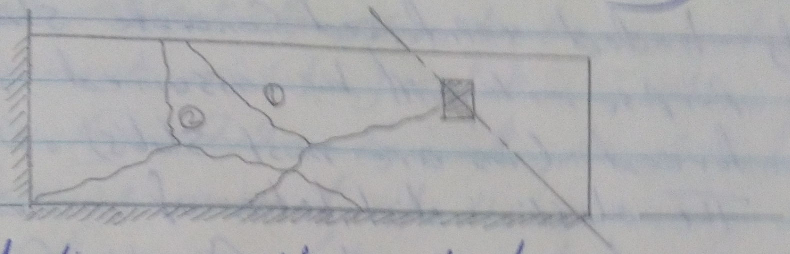
Location of yield line :—

The following & guide lines for drawing yeid lines & located axes of rotation :—

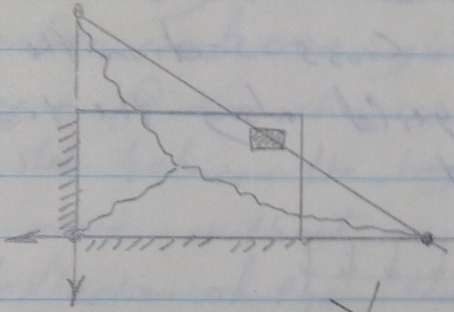
1 - yeild liner generally straight.

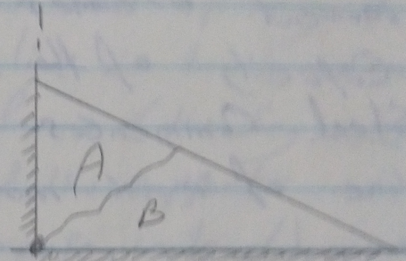
2 - axis of rotation generally lie along line of support

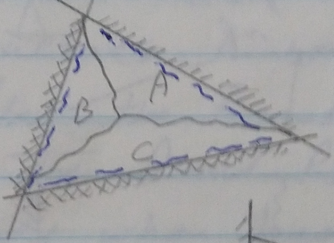
3- Axises of rotation pass over any Column.

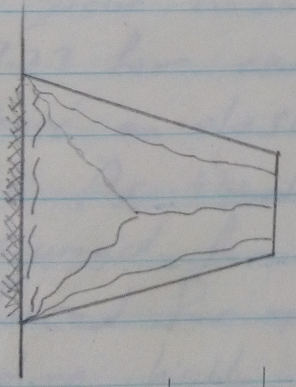


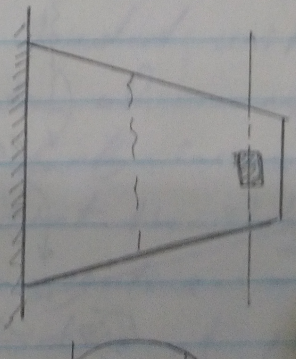
4- A yeild line or it's extend pass through The Intersection of the axis of rotation of Adjsent slab segment.

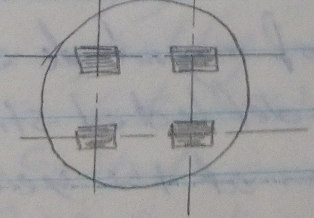


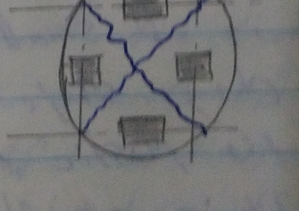


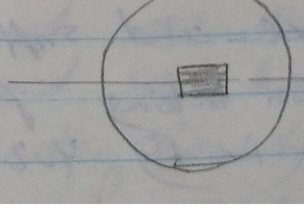




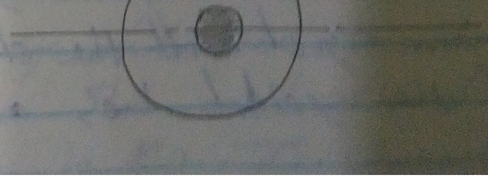


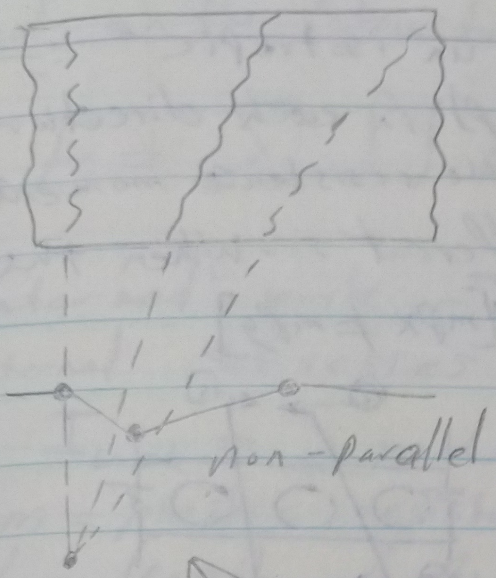


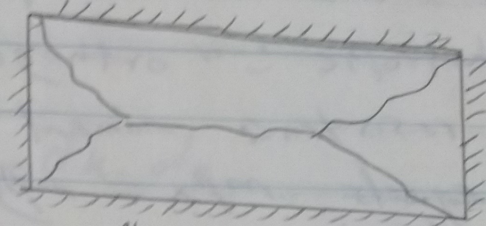




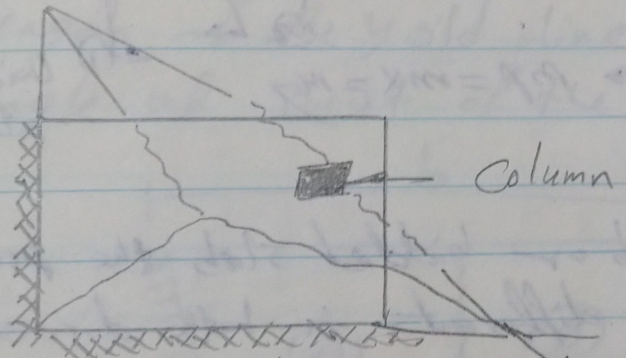
uniform distributed load.



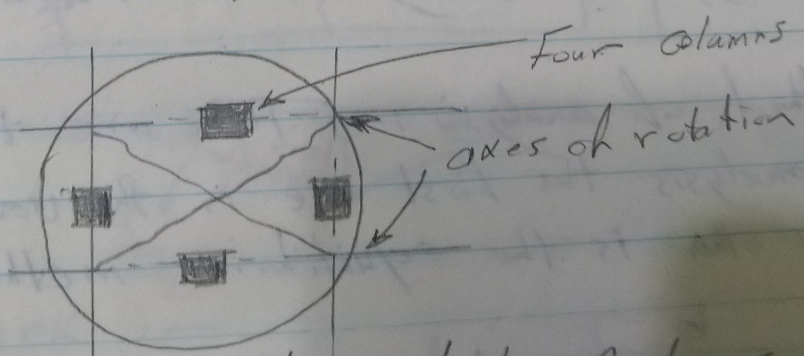




simple-supports all side



Fixed support two sides



Typical Yield-line Parterns

- The "hinge" which forms at the yeild line rotates at assentially aconstant moment (This moment-rotation) relation is shown graph-ically in Fig 7.9a - For all practical purposes the restraining moment at the yeilding hinge can be taken equal to the ultimate moment: that is mp = mu.

For determining slab such as that of Fig 4-22 the formation of one yield line is tantomoment of failure. A «mechanism» forms (the segments of the slab between the hinge & the supports are able to move without an increase in load), and gross deflection of the structure results.

Indeterminate structures, however, can maintain equilibruim even after the formation of one or more yield lines The fixed-fixed slab of fig 4.23 for example when loaded uniformly, will have an elastic distribution of moments as in (b). As the load is gradually increased (it is assumed for simplicity that the slab is equally reinforced for positive & negative moment) the more highly stressed sections at the supports commence yielding. Rotation of the end tangents occures, but restraining moments of substan-tially constant amount Mp. Continue to act the support lines. The load can be increased amount ~~Mp~~ still further, untile the moment at mid span becomes equal to the ultimate moment capacity of the slab, and athird yield line [illegible] as shown in (C). This converts the Structure in to amechanism & results in Collapse.

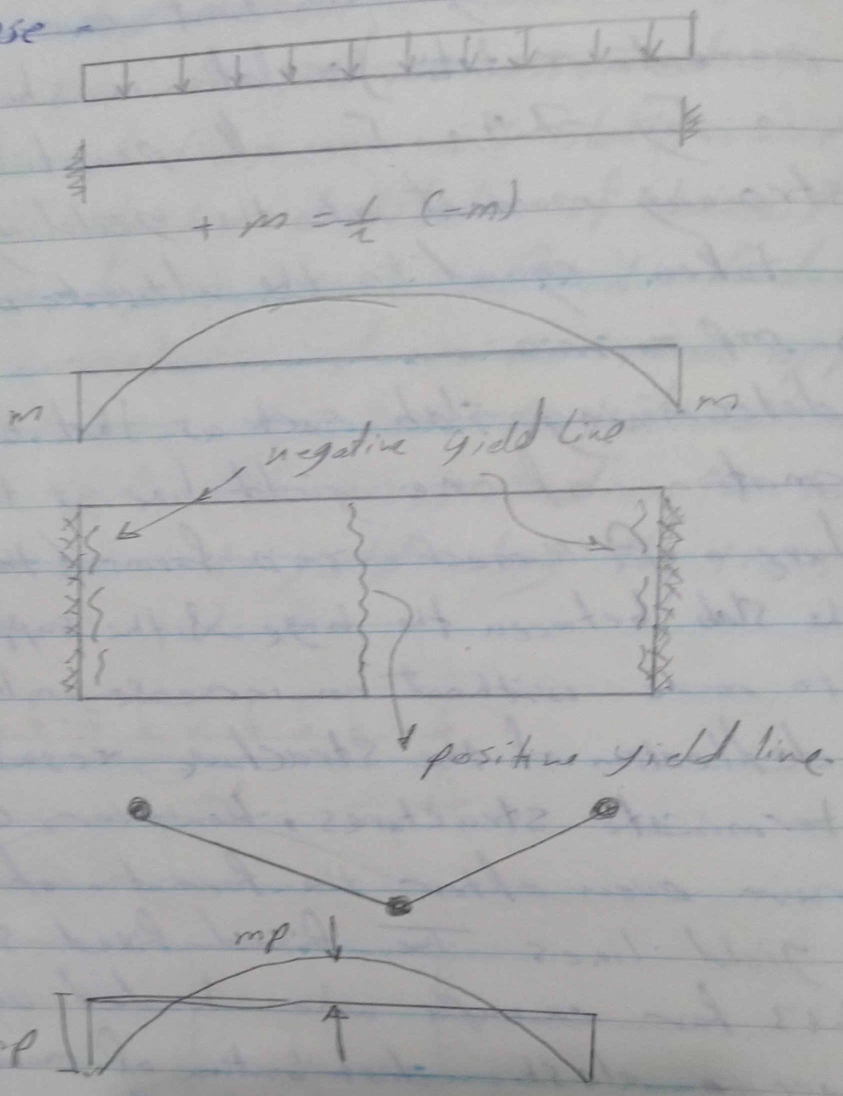


Fig 4-23 Fixed-end uniformly loaded slab.

The moment digram just before failure is shown in fig 4-23 d. Not that the ratio of elastic positive to negative moments of 1:2 is no longer obtained. Owing to inelastic deformation, the ratio of these moments just before Collapse is 1:1 for this Particular structure. This is known as inelastic redistributi of moments.

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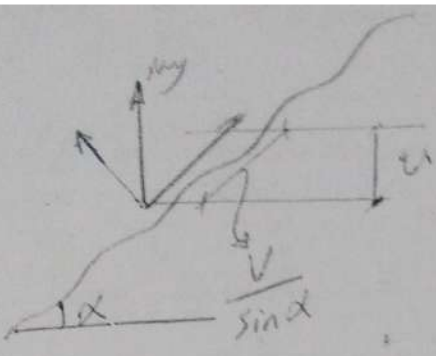
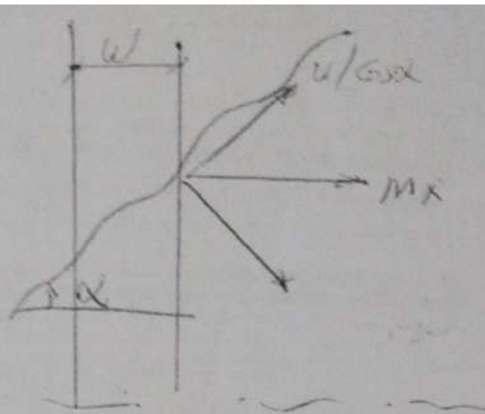
whether or not a concrete structure can sustain rotation and deformation such as to ensure full moment redistribution depends at least in part on the reinforcement ratio. Lightly reinforced members may fail by concr crushing before much rotation occures. For more complete discussion of this impoltant aspect of plastic or limit analysis of conc structrues

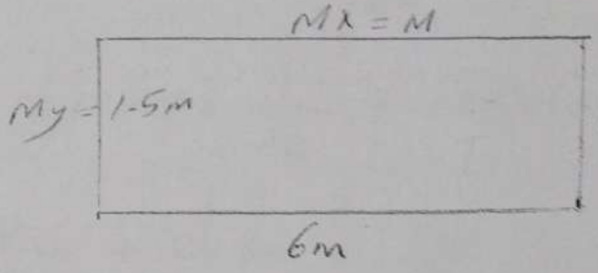
Location of yield lines:—

The location & orientation of the yield line were evident in the case of simple slab of fig 4.22 similary. for the one-way indeterminate slab, the yield lines were easily established. For other cases it is helpfull to have a set guid lines were easily established for other cases it is helpful to have aguide lines for drawing yield lines & location axes of rotation. When aslab is on the verge of Collapes owing to the existance of asufficient number of real or plastic hinges to form amechanism axes of rotation will be located along the lines of support or over point supports such as Columes. The slab segments Can be Considered to rotate as arigid bodies in space above these axes of rotation. The yield line between any two adjacent slab segment is a straight line - being the intersection of two assentially plane surface. Since the yield Cas aline of intersection of





- طريقة حل أخرى بالنسبة للشكل الأول يعتبر un – Isot ، لأجل جعله Isot نعاكس سوف نساوي Mx مع My وتتم ذلك من خلال قسمة البعد على العزم الأكبر ، وبذلك يقلل هذا البعد الذي يكون في البعد الأفقي عمودي



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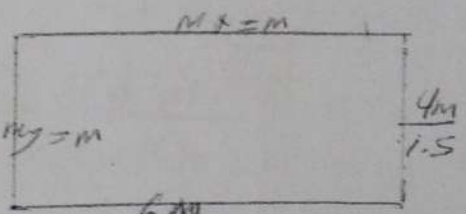
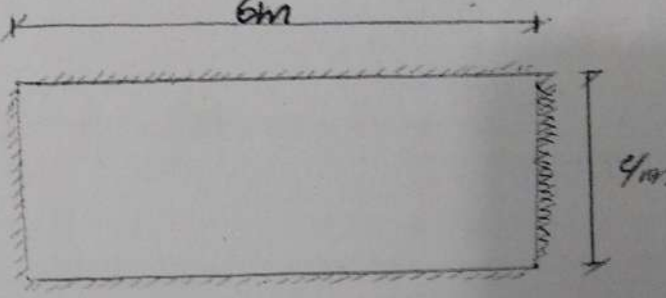


Ex

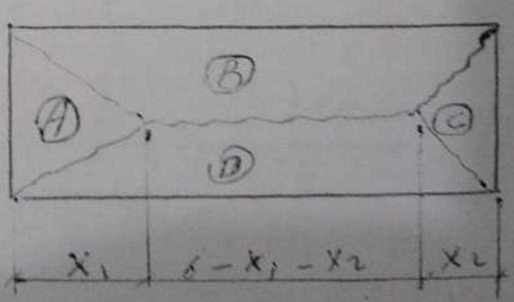
① $M_{\ominus} = 1.25\, M_{\oplus}$

2) $M_{\oplus} = M_{\oplus}$



ملاحظة- أ) الزاوية تساوي 45° عندما يكون الطول = العرض

ب) الخط الوسطي يقع دائماً في المنتصف لأنه غير إنشائي بولب simple supp



$$WE(A) = w \times \frac{x_1 \cdot 4}{2} \times \frac{1}{3} = \frac{2}{3} w \cdot x_1$$

$$WE(B \& D) = w \cdot \frac{x_1 \cdot 2}{2} \times \frac{1}{3} + w \cdot \frac{x_2 \cdot 2}{2} \times \frac{1}{3} + (6 - x_1 - x_2) \times 2 \times \frac{w}{2}$$

$$= \frac{w\, x_1}{3} + \frac{w \cdot x_2}{3} + 6w - x_1 w - x_2 w$$

$$= -\frac{2}{3} w x_1 - \frac{2}{3} w x_2 + 6w$$

$$W.E(C) = w \times \frac{x_2 - 4}{2} \times \frac{1}{3} = \frac{2}{3} w x_2$$

$$\Sigma WE = \frac{2}{3} w x_1 + 2\left[-\frac{2}{3} w x_1 - \frac{2}{3} w x_2 + 6w\right] + \frac{2}{3} w x_2$$

$$= \frac{2}{3} w x_1 - \frac{4}{3} w x_1 - \frac{4}{3} w x_2 + 12w + \frac{2}{3} w x_2$$

$$= -\frac{2}{3} w x_1 - \frac{2}{3} w x_2 + 12 w$$

$$WI(A) = m - 4 \times \frac{1}{x_1} = \frac{4m}{x_1}$$

$$WI(B) = m \times 6 \times \frac{1}{2} = 3m$$

$$WI(C) = m \times 4 \times \frac{1}{x_2} + 1.25 m \times 4 \times \frac{1}{x_2} = \frac{9m}{x_2}$$

$$\Sigma WI = \frac{4m}{x_1} + 2 \times 3m + \frac{9m}{x_2} = \frac{4m}{x_1} + 6m + \frac{9m}{x_2}$$

let $\Sigma W_{ex} = \Sigma W_{int}$

$$-\frac{2}{3} w x_1 - \frac{2}{3} w x_2 + 12w = \frac{4m}{x_1} + 6m + \frac{9m}{x_2}$$

$$w = \frac{\frac{4m}{x_1} + 6m + \frac{9m}{x_2}}{-\frac{2}{3} x_1 - \frac{2}{3} x_2 + 12}$$

$$\frac{dw}{dx_1} = \frac{\left(-\frac{2}{3} x_1 - \frac{2}{3} x_2 + 12\right)\left(\frac{-4m}{x_1^2}\right) - \left(\frac{4m}{x_1} + 6m + \frac{9m}{x_2}\right)\left(-\frac{2}{3}\right)}{\left(-\frac{2}{3} x_1 - \frac{2}{3} x_2 + 12\right)^2} = \text{Zero}$$

$$\therefore \frac{8m}{3x_1} + \frac{8}{3} \frac{m x_2}{x_1^2} - \frac{48m}{x_1^2} + \frac{8m}{\cancel{3} x_1} + 4m + 6\frac{m}{x_2} = 0$$

$$\frac{16}{3} \frac{m}{x_1} + \frac{8}{3} \frac{m x_2}{x_1^2} - \frac{48m}{x_1^2} + 4m + 6\frac{m}{x_2} = 0$$

$$\frac{16}{3} m x_1 + \frac{8}{3} m x_2 - 48m + 4m x_1^2 + \frac{6m x_1^2}{x_2} = 0 \quad ①$$

$\underline{\underline{3}}$

$$\frac{dw}{dx_2}=\frac{\left(-\frac{2}{3}x_1-\frac{2}{3}x_2+12\right)\left(\frac{-9m}{x_2^2}\right)-\left(\frac{4m}{x_1}+6m+\frac{9m}{x_2}\right)\left(-\frac{2}{3}\right)}{\left(-\frac{2}{3}x_1-\frac{2}{3}x_2+12\right)^2}=0$$

$$\frac{6mX1}{X_2^2}+\frac{6m}{X_2}-108\frac{m}{X_2^2}+\frac{8}{3}\frac{m}{X_1}+4m+\frac{6m}{X_2}=0$$

$$\frac{6mX1}{X_2^2}+\frac{12m}{X_2}-108\frac{m}{X_2^2}+\frac{8}{3}\frac{m}{X_1}+4m=0$$

$$6mX_1^2+12mX_1X_2-108mX_1+\frac{8}{3}mX_2^2+4mX_1X_2^2=0$$

by solving the two equations find x_1, x_2 and find w then find the moment

نفرض بأن تكون 1.25M ونستخدم المثال

H.w — rectangular panel

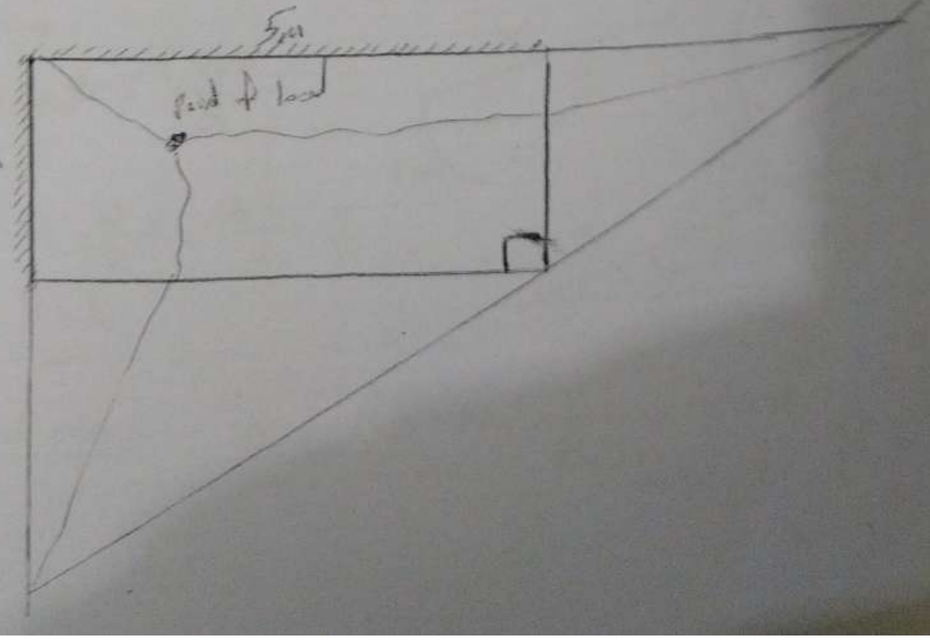
١- نفرض طول لانه يكون fix ، hing او اي شي

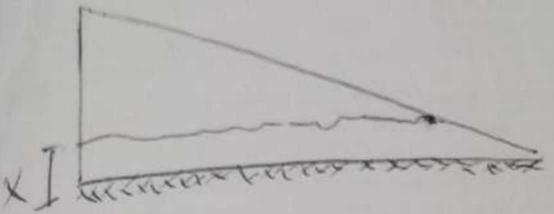
٢- نفرض w_d ، w_l

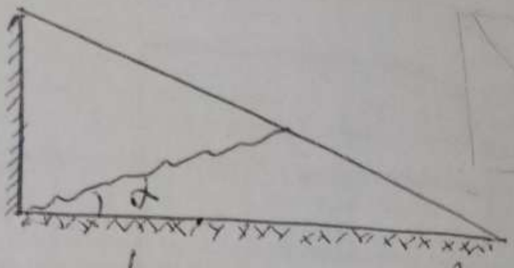
٣- سمك السقف 20cm ، $f_c' = 25$ MPa ، $f_y = 350$ MPa

٤- تصمم بطريقتين ① method three (winter page 252)

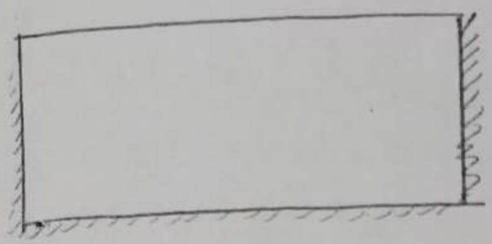
② بطريقة ٢ yield line theory

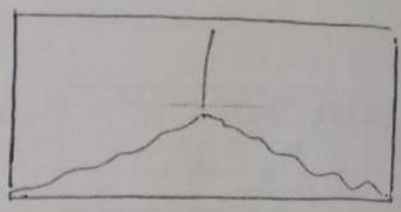






$$\frac{dw}{d\alpha} = 0 \Rightarrow \alpha = ?$$

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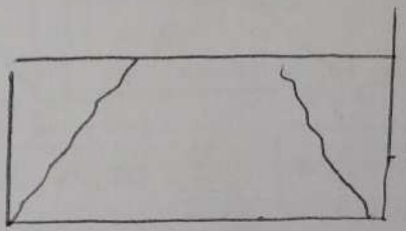
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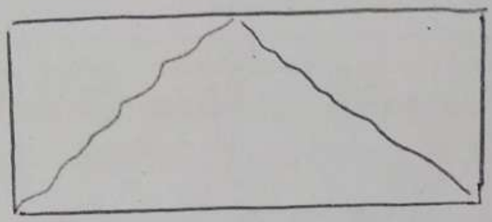

③

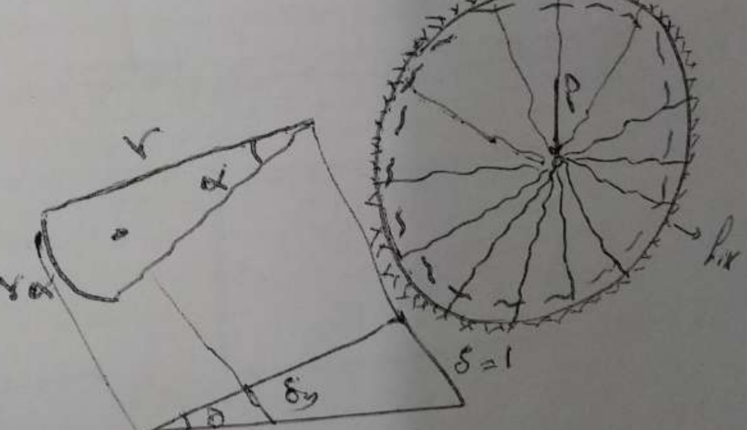
Fan Pattern

Ext. work, -

$A_{st} \cdot W \cdot \delta_g$

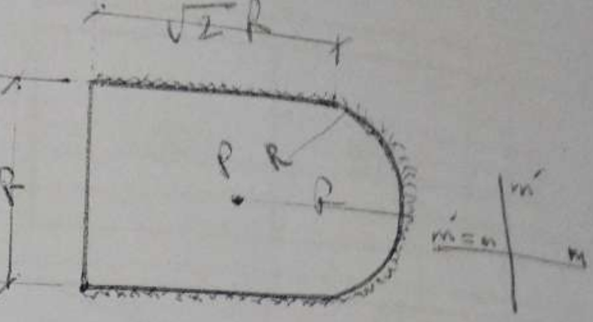
number of segments $= \dfrac{2\pi}{\alpha}$

$$= \sum w \cdot R \cdot \frac{R\alpha}{2} \times \frac{2\pi}{\alpha} \cdot \delta_g + P \times 1$$



Internal work, - $= \left[R\alpha \cdot m \times \dfrac{1}{R} + R\alpha m' \dfrac{1}{R} \right] \dfrac{2\pi}{\alpha}$

Ex:- neglect the value of uniform load



Sol

① Ext work

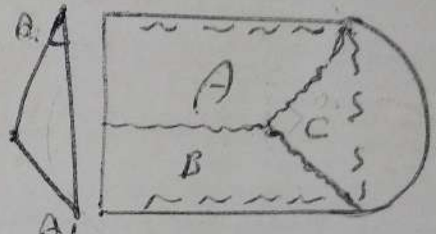
$P \times 1$

① الاحتمال الأول للفشل

② Internal work :- For A & B

حيث نفترض في الاحتمال الأول انه الفشل في (C) يحدث في منتصف الدائرة ابتداء { neg. M

ويحصل كذلك الجزء .



$$-\text{For } Ⓐ \& Ⓑ = \left[\sqrt{2}Rm \times \frac{2}{\sqrt{2}R} + \sqrt{2}Rm\frac{2}{\sqrt{2}R}\right] \times 2 = 8m$$

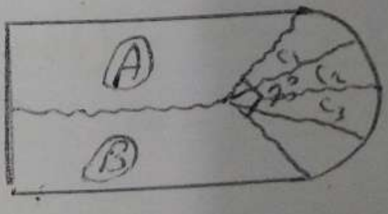
$$-\text{For } Ⓒ \quad \sqrt{2}Rm\frac{2}{\sqrt{2}R} + \sqrt{2}Rm\frac{2}{\sqrt{2}R} = 4m$$

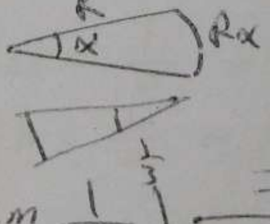
$$\sum \text{int.} w = 12\ m \qquad \sum Ext = \sum int$$

$$P \times 1 = 12m \longrightarrow P = 12m \longrightarrow m = \frac{P}{12}$$

② الاحتمال الثاني للفشل حيث نفترض الفشل يحدث عند طرف الدائرة بالجوانب

- $\sum Ext.w = P \times 1$
- $Int.w$ for A & B $= 8m$
- $Int.w$ for Ⓒ

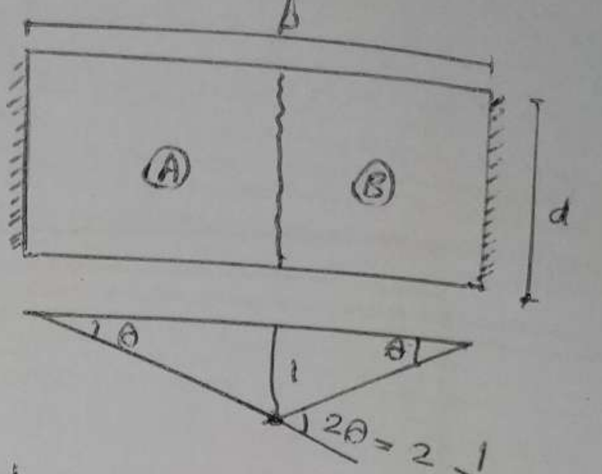




$$\left(R\alpha.m.\frac{1}{R} + R\alpha.m\frac{1}{R}\right)\frac{\frac{\pi}{2}}{\alpha} = \pi m \ , \ \sum ex = \sum int$$

$$8m + \pi m = P \times 1 \longrightarrow P = \pi m + 8m = m(\pi + 8) \longrightarrow \text{Control}$$

لأنه يعطي أقل load وأعلى m



$2\theta = 2\frac{1}{\frac{b}{2}} = \frac{4}{b}$

<u>Sol</u> 1) Ext-work :—

Segment Ⓐ $= W * a \frac{b}{2} * \frac{1}{2}$

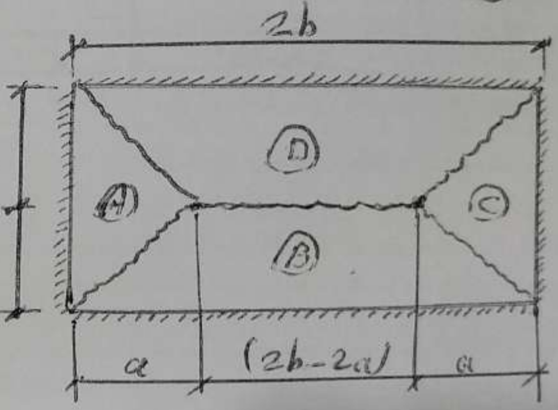
" Ⓑ $= w * a \frac{b}{2} * \frac{1}{2}$

$\sum Ext.w = \frac{Wa - b}{2}$

2) Int-work :—

For A & B $= M * a \cdot \frac{1}{\frac{b}{2}} + M.a * \frac{1}{\frac{b}{2}} = M a \cdot \frac{4}{b}$

$\sum Ex = \sum int \longrightarrow \frac{w.a.b}{2} = m.a.\frac{4}{b} \Longrightarrow w = \frac{8m}{b^2}$



1) Ext.w

For A & C $= W.\frac{2a.a}{2} * \frac{1}{3} = \frac{Wa^2}{3}$

For B & D

$= \frac{W.a^2}{2} * 2 * \frac{1}{3} + (2b-2a).w.a.\frac{1}{2}$

$\Rightarrow \frac{Wa^2}{3} + wba - wa^2$

$\sum Ext.w = 2\frac{wa^2}{3} + 2\left(w\frac{a^2}{3} + wab - wa^2\right)$

$= 2\frac{wa^2}{3} + 2\left(-\frac{2}{3}wa^2 + wab\right) = \frac{2wa^2}{3} - \frac{4}{3}wa^2 + 2wab$

$= -\frac{2}{3}wa^2 + 2wab$

2) Int-work

$-$ For Ⓐ $= m * 2a * \frac{1}{a} =$ For Ⓒ $= 2m$

$-$ For Ⓑ $= m * 2b * \frac{1}{a} =$ For Ⓓ $= \frac{2mb}{a}$

$\sum int.w = 4M + 2 * \frac{2mb}{a} = 4m + \frac{4mb}{a}$

$\sum Ex = \sum int \Rightarrow 2wab - \frac{2}{3}wa^2 = 4m + 4m\frac{b}{a}$

$$w = \frac{4m + 4m\frac{b}{a}}{2ab - \frac{2}{3}a^2}$$

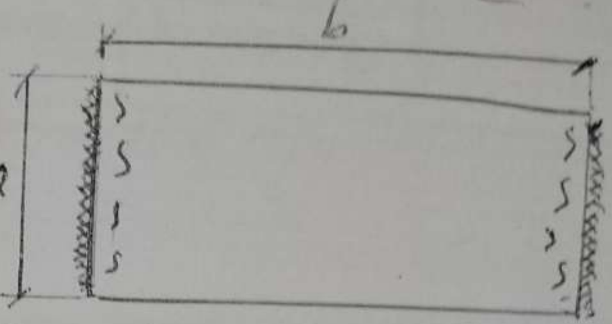
إذا كانت a و b مش ثابتة سوف نشتق
لو ما نعرف x/١ ، ثابت نجيب $\frac{dw}{da}$ و $\frac{dw}{db}$

assume isotropic

$-mx = -my = m$

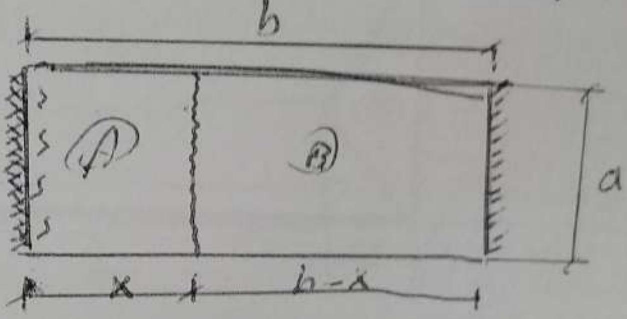
$-Mx = +my = m$

② $M = M$, $+M = M$



1) Ext work

- For A $= w \cdot a \cdot x \cdot \frac{1}{2}$
- For B $= w \cdot a \cdot (b-x)\frac{1}{2}$



$\Sigma Ex = \frac{1}{2}wa(x+b-x) = \frac{1}{2}wab$

2) Internal work

- For Ⓐ $2m \cdot a\frac{1}{x} = \frac{2m \cdot a}{x}$
- For Ⓑ $m \cdot a \cdot \frac{1}{(b-x)} = \frac{m \cdot a}{b-x}$

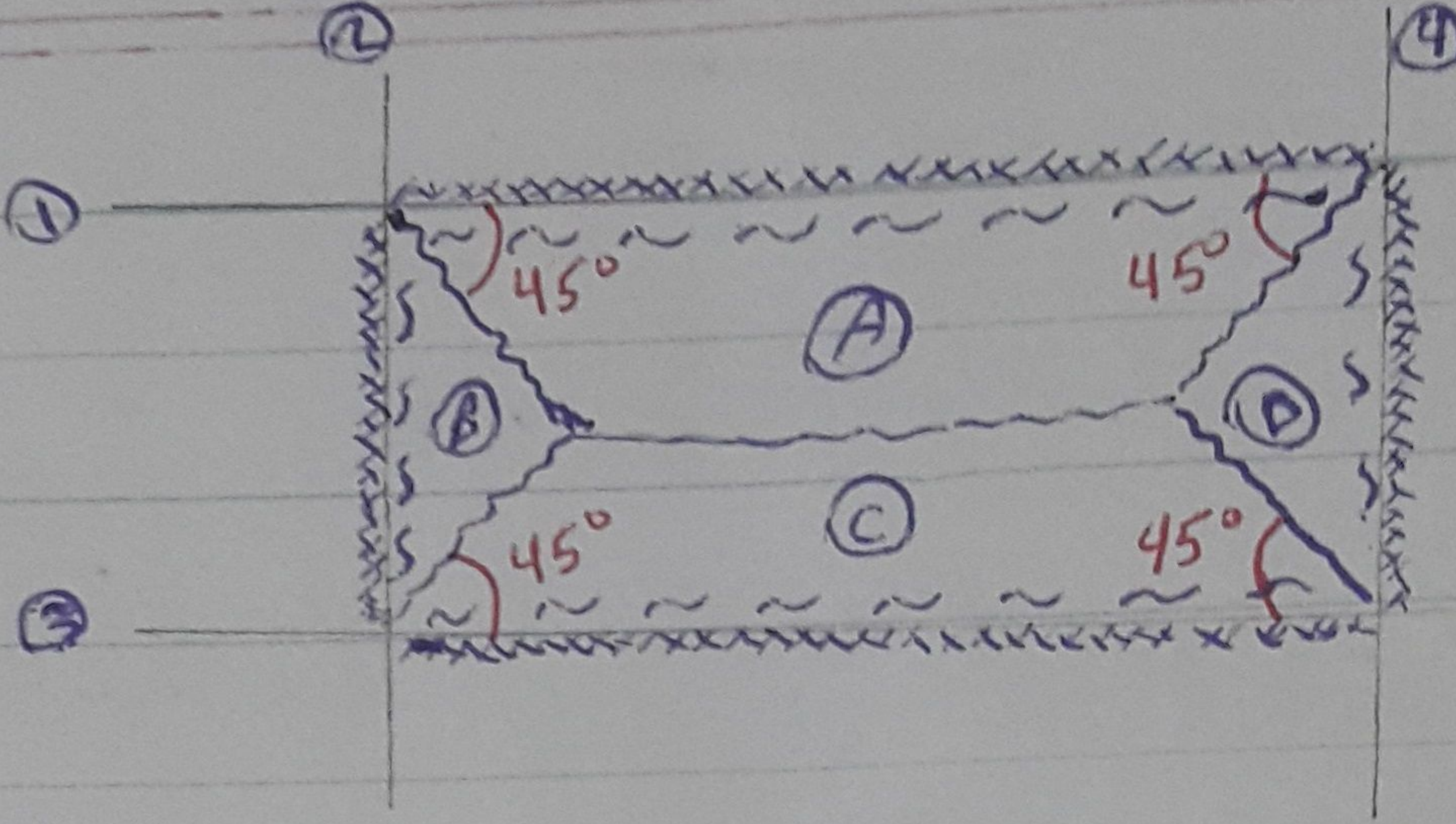
$\Sigma int\text{-}w = m \cdot a\left(\frac{2}{x} + \frac{1}{b-x}\right) = ma\left(\frac{2b-2x+x}{x(b-x)}\right)$

$\Sigma int\text{-}w = ma\left(\frac{2b-x}{xb-x^2}\right)$ let $\Sigma Ext = \Sigma int$

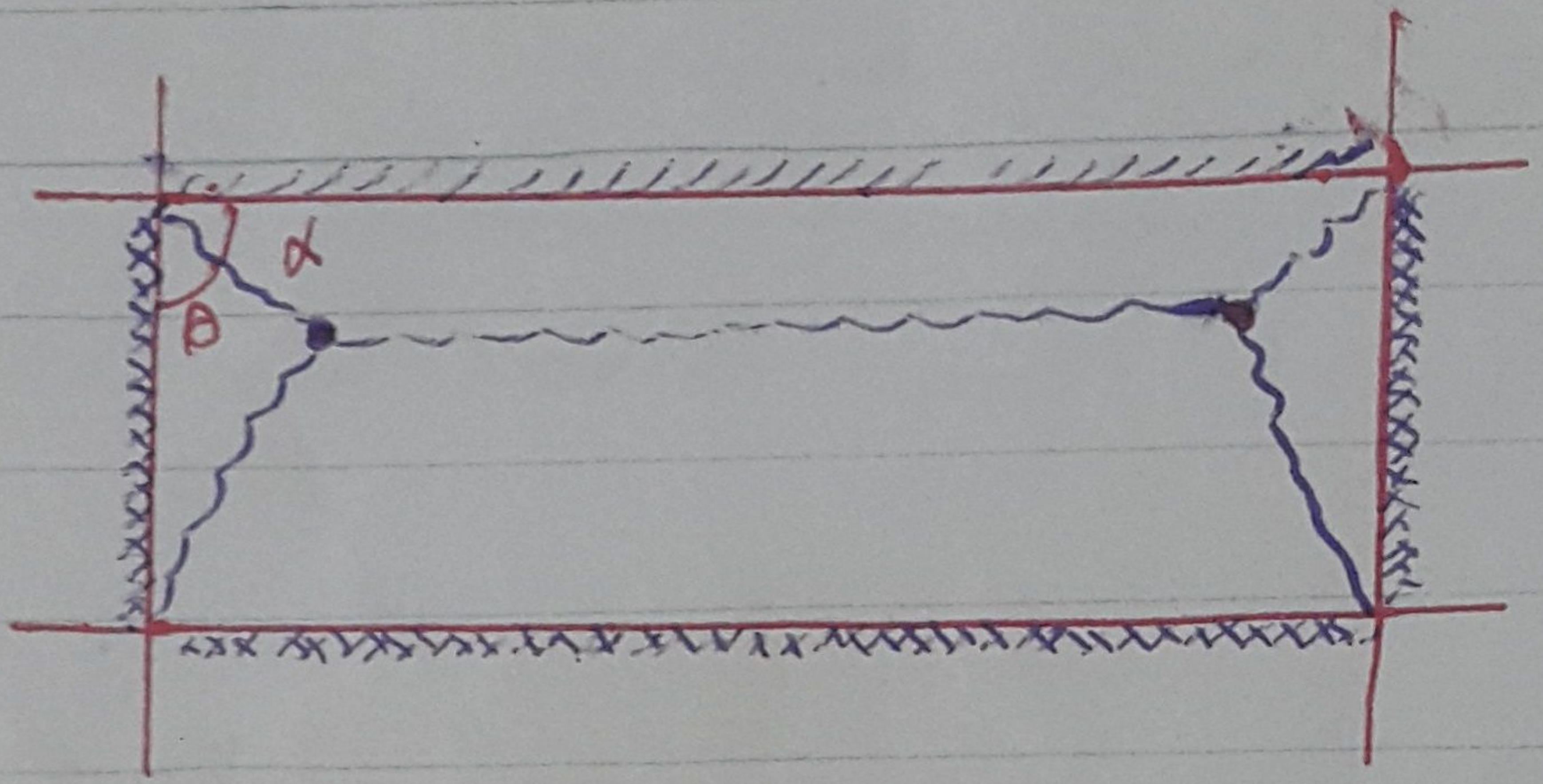
$$\frac{1}{2}wab = m \cdot a\left(\frac{2b-x}{xb-x^2}\right) = \frac{2mab - max}{xb-x^2}$$

$$w = \frac{2mab - max}{\frac{1}{2}axb^2 - \frac{1}{2}abx^2} = \frac{\frac{a}{2}(4mb + 2mx)}{\frac{a}{2}(xb^2 - bx^2)} = \frac{4mb-2mx}{xb^2-bx^2}$$

$$\frac{dw}{dx} = \frac{(xb^2-bx^2)(-2m) - (4mb-2mx)(b^2-2bx)}{(xb^2-bx^2)^2} = 0$$



- هنا نضع دور الاجزاء A ، B ، C ، D يساوي دور axes of rotation

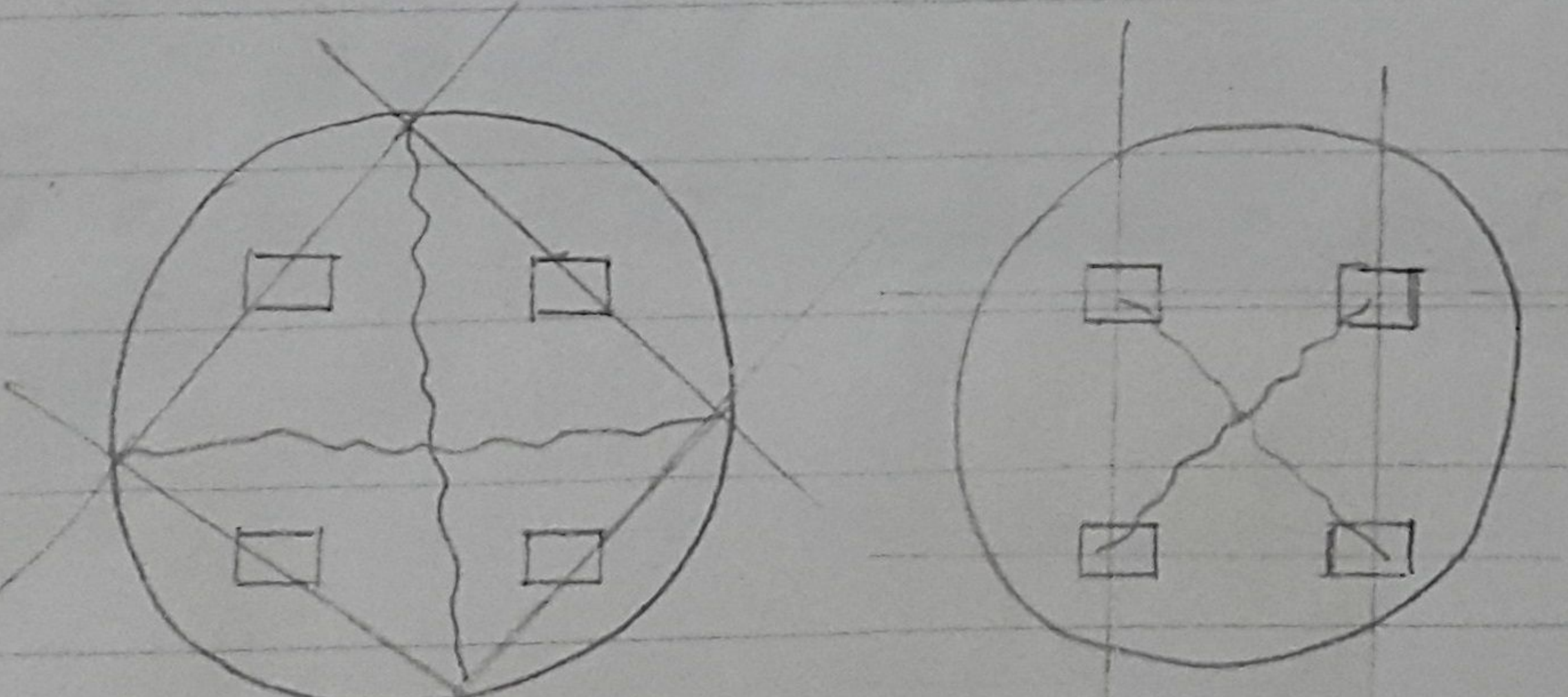


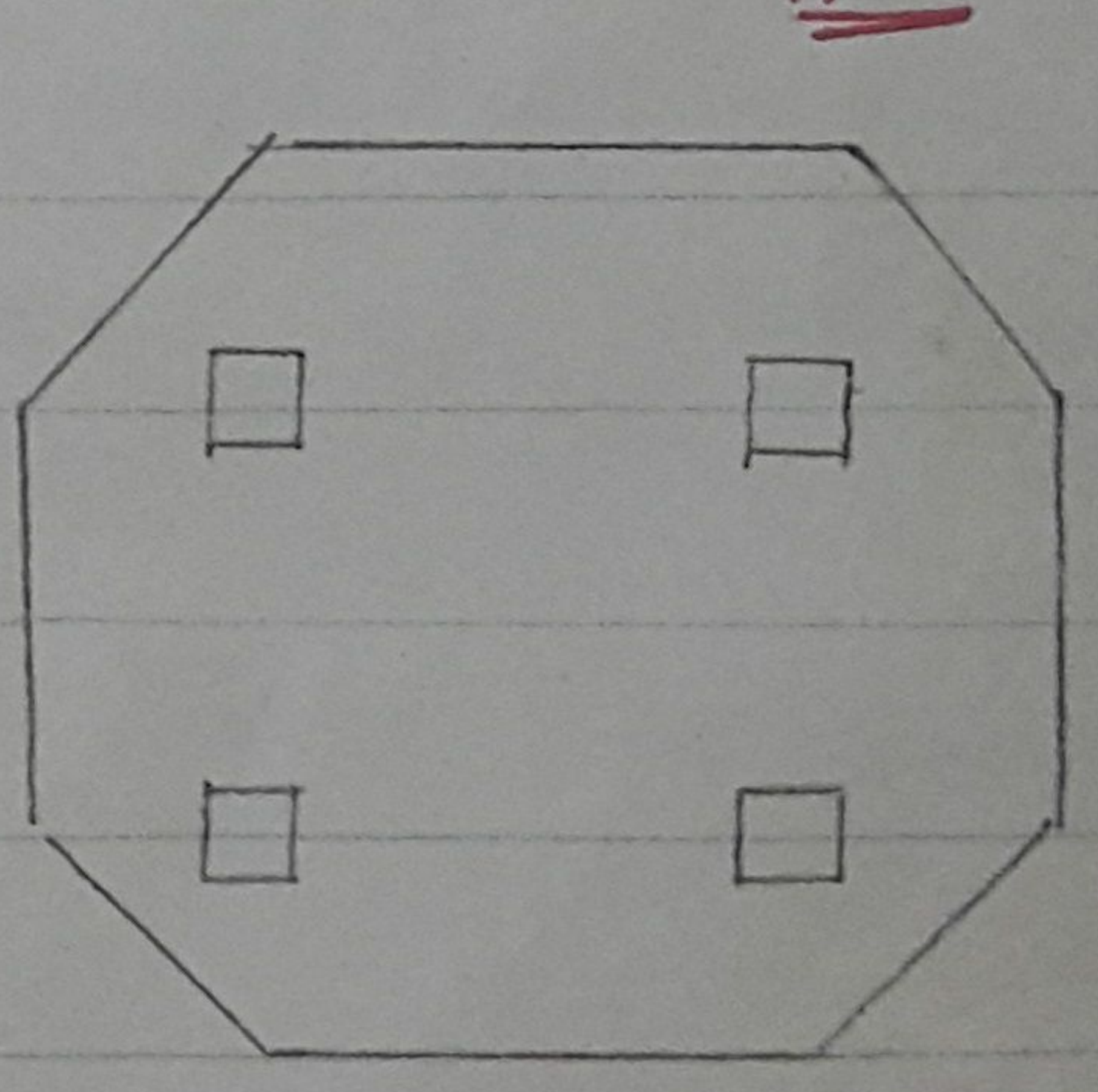
في هذه الحالة فانه نقطة التقاطع ترتفع قليلاً الى اليمين hinge وبذلك الزاوية لا تساوي 45°

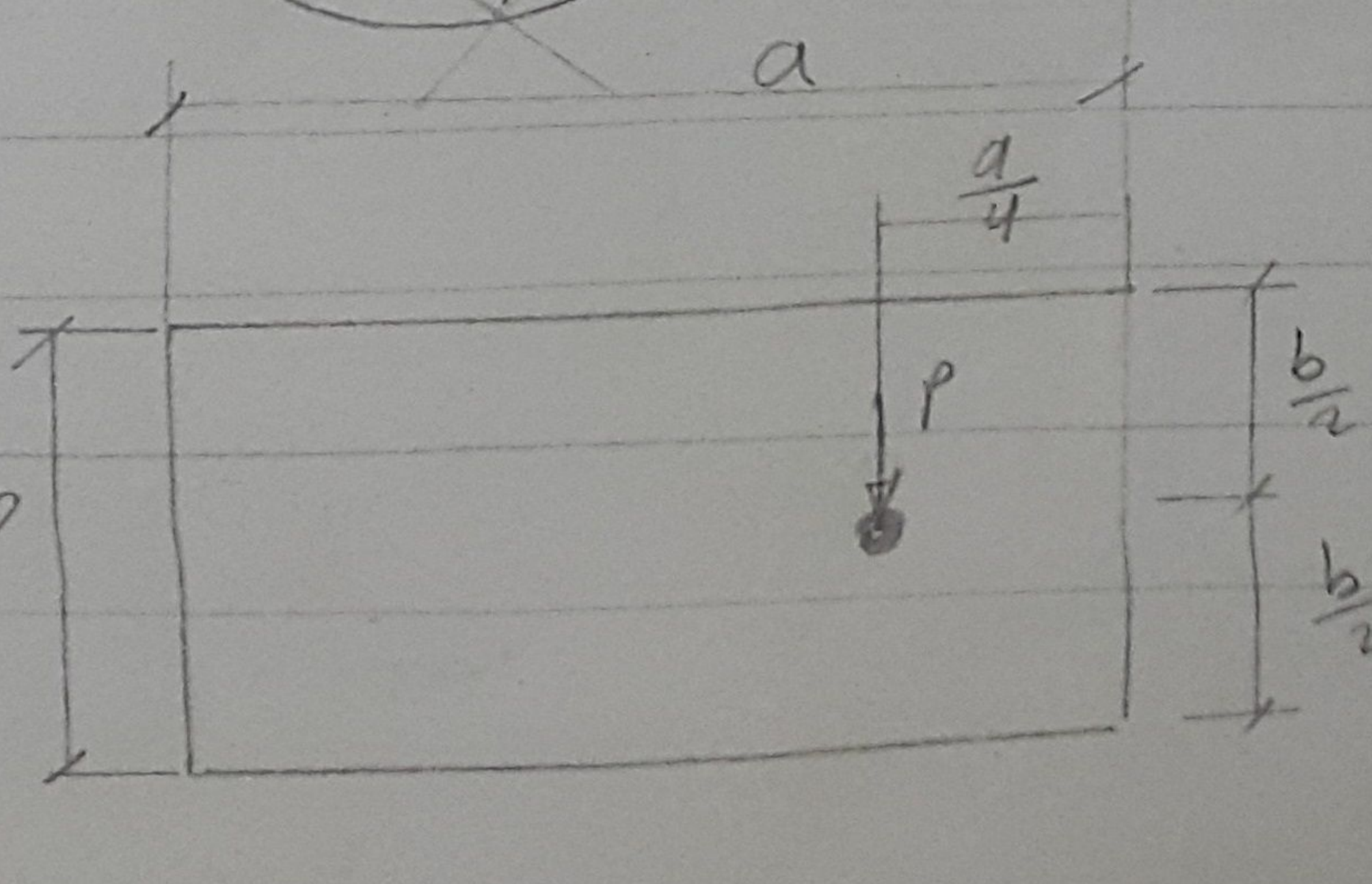
عند ما تكون الاحمال كبيرة فانه نوجد معادلة $m = f(x, y, \theta)$ ثم نشتق

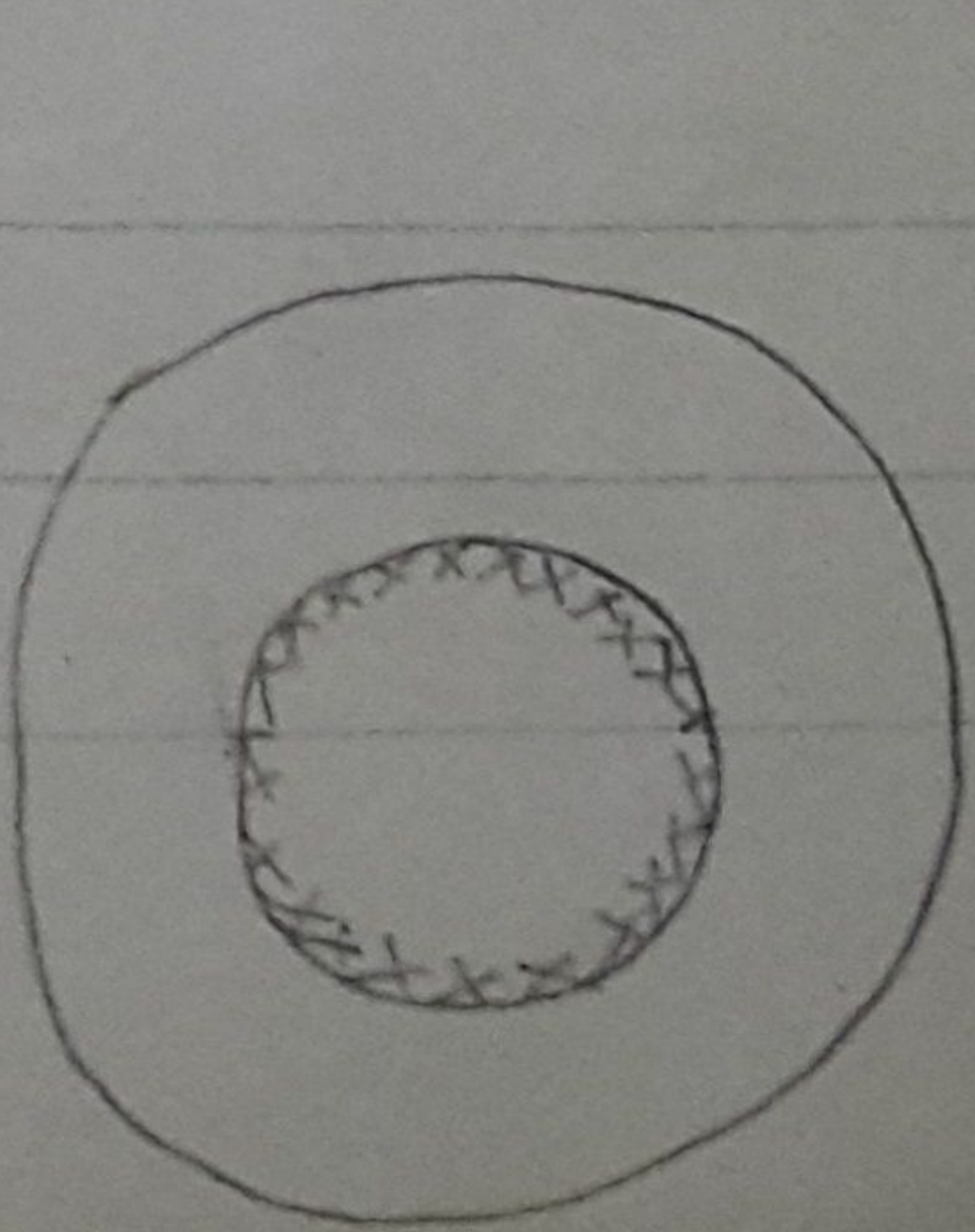
$$\frac{dm}{dx} = 0 \quad , \quad \frac{dm}{dy} = 0 \quad , \quad \frac{dm}{d\theta} = 0$$

H-W









زمنا بحمل $W = W_1$ يحصل ~ ~ ~ ~ ~

<- ، ، $W = W_2$ (زيادة الحمل) يعطي ~ ~ ~ ~ + ~ ~ ~ ~ + ~ ~ ~

يحصل الفشل ،

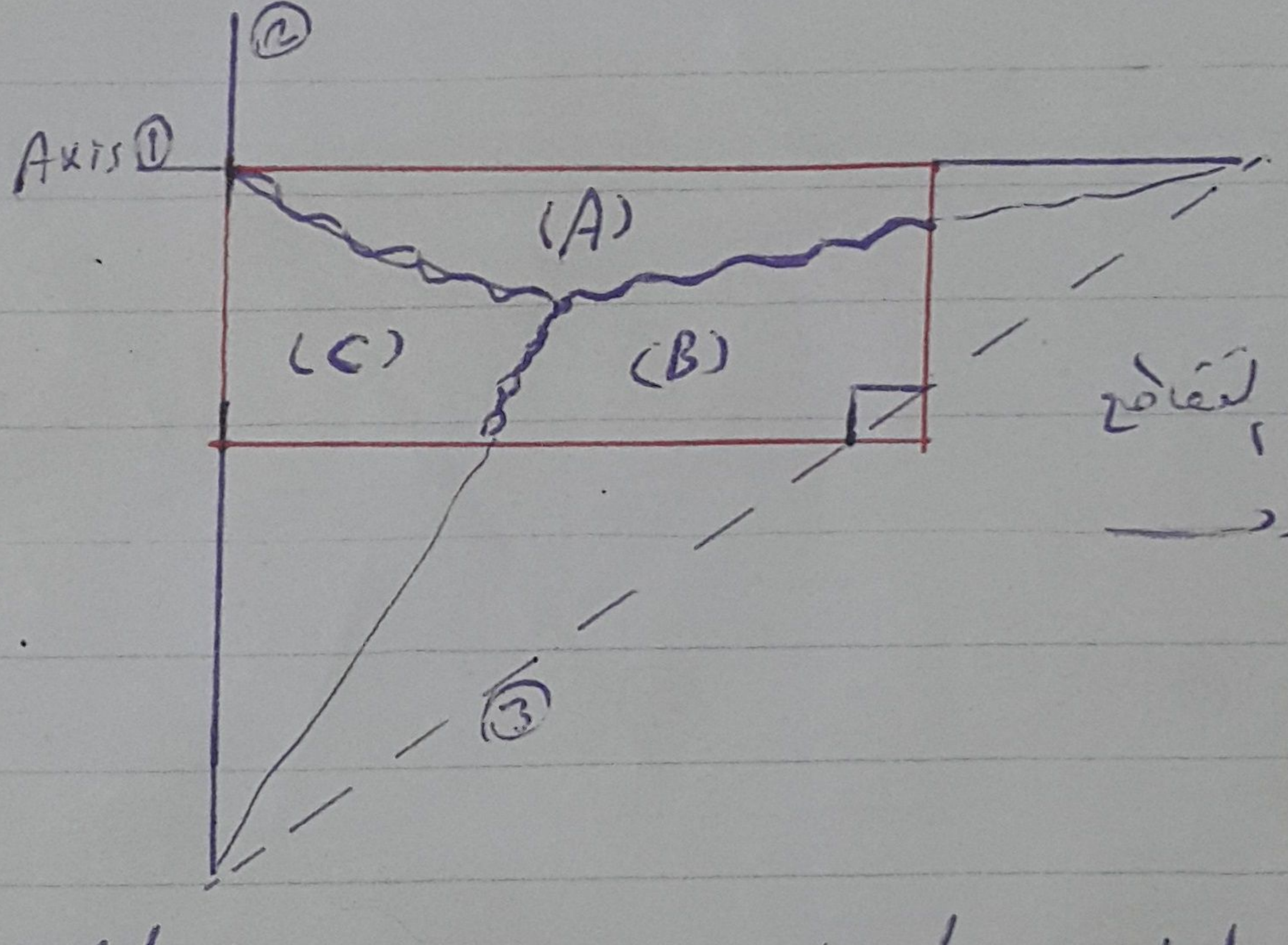
لهذا في الواقع هذه يجب تحقيق بالنسبة لـ ٧٠٪ لأجل حدوث الفشل ،

Yield Line theory :- النقاط الأساسية لأجل رسم

1- The yield line generally is a straight line

2- Axes of rotation generally lie in support line.

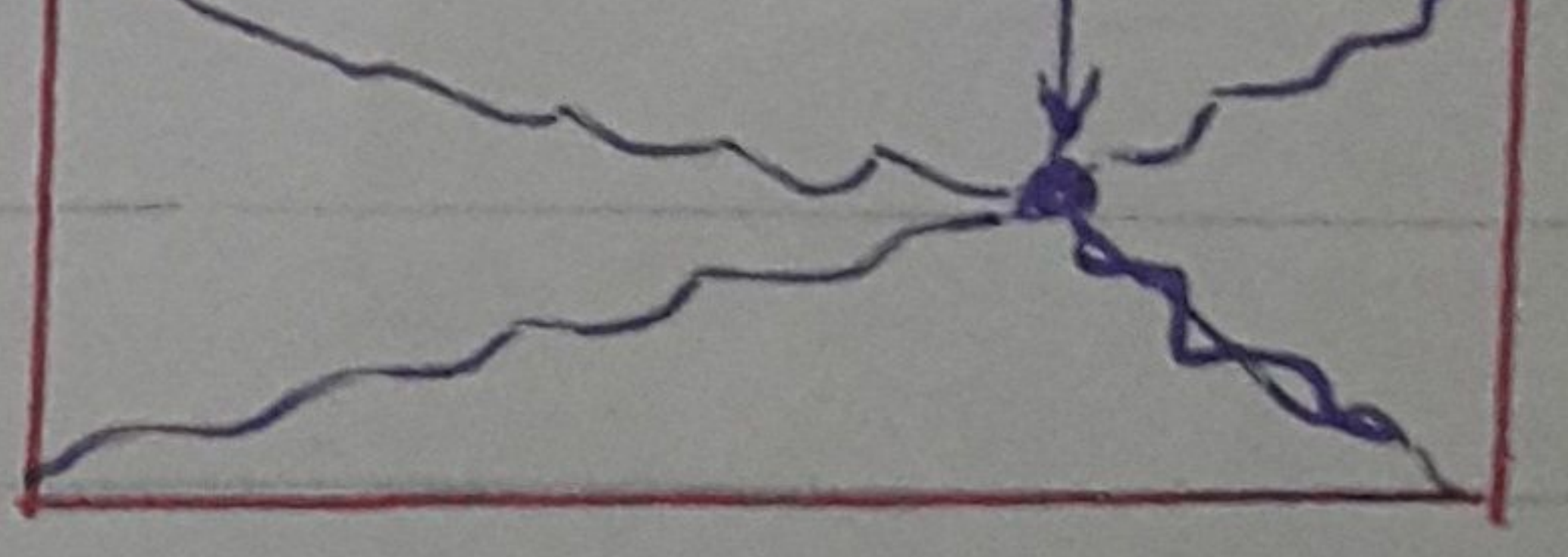
3- The axes of rotation must be passing through the Columns.

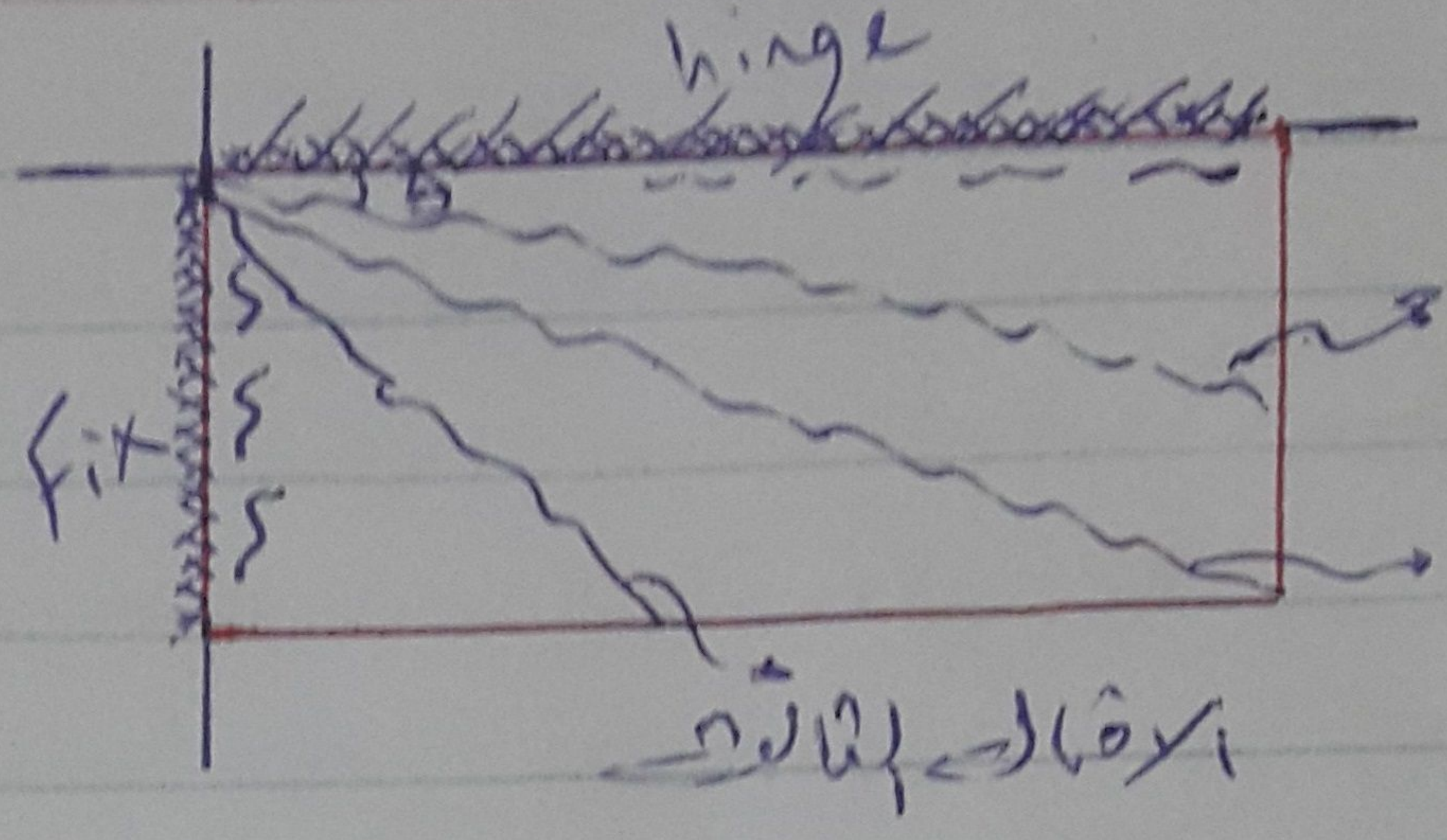


ملاحظة :- موضع نقطة التقاطع داخل البلاطة غير محدد

4- The yield line or its extend must be passing through t intersecting point of two point axes of rotation

5- In-case of Concentrated load. Yield line must be interse at the point of loading.



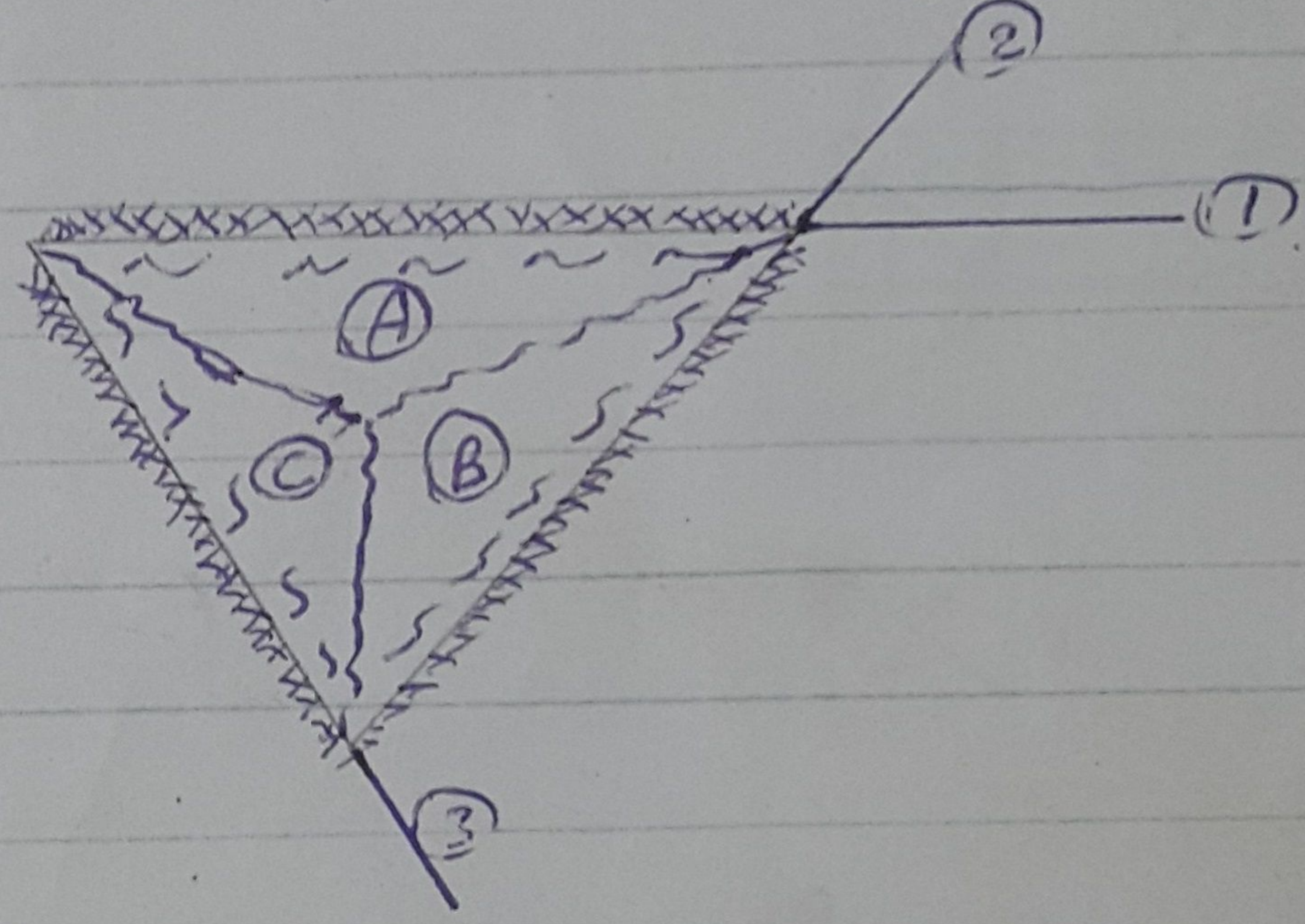


خطوات الحل

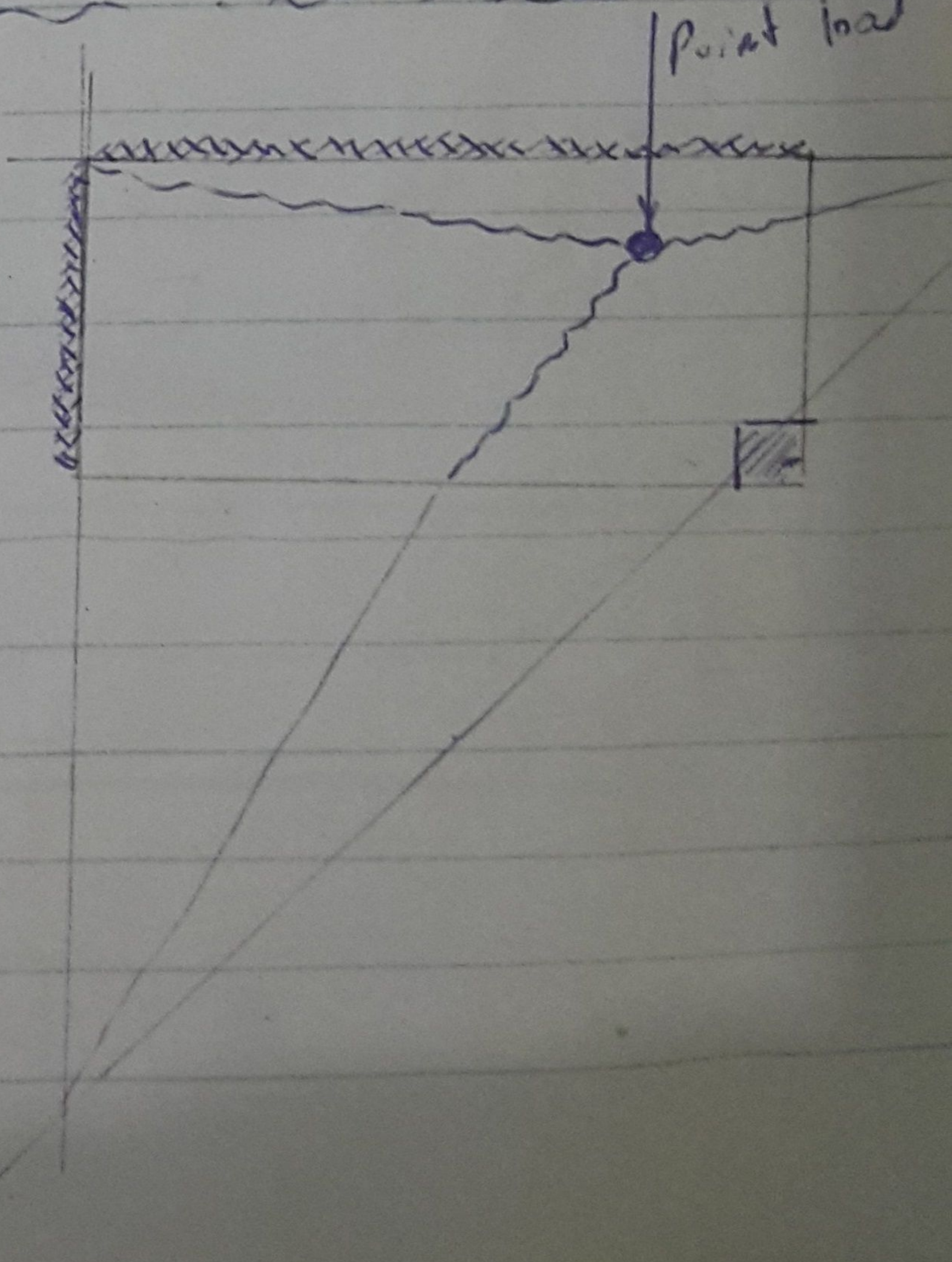
نحدد زاوية θ لكل حالة من الحالات

نوجد معادلة $M = f(\theta)$

نشتق المعادلة ونساويها للصفر ونستنتج قيمة θ



في هذه الحالة يوجد لدينا point load
لذلك فانه يجب الخطوط ان تمر
تقطع الـ point load.



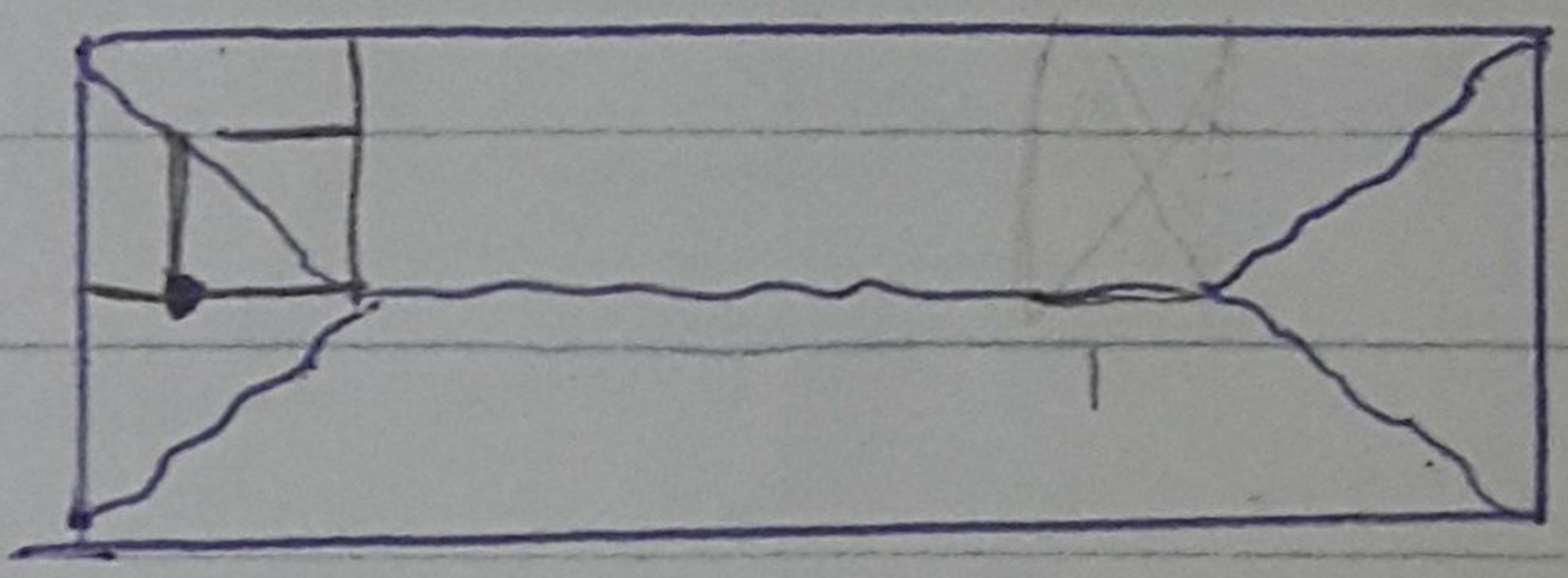
There are two methods for analysis:—

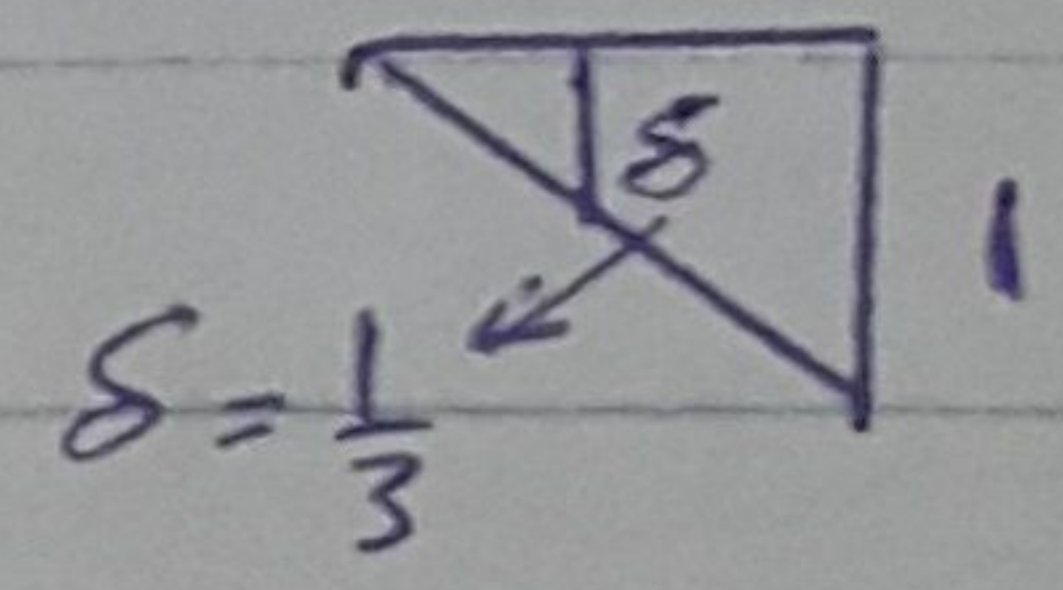
1- Equilibrium method 2- Vertual $\overset{\text{work}}{}$ method

Vertual wark method:— the external wark Caused by applied load must equal the wark done by the resisting moment.

ناتج ضرب كل الحمل $\sum W_{ex} = \sum W_I$ → ناتج ضرب مقاومة العزوم

$W_{ex} = P * \delta$ (Concentrated) $\sum W_E = \int W * \delta * A$ (uniform في حالة

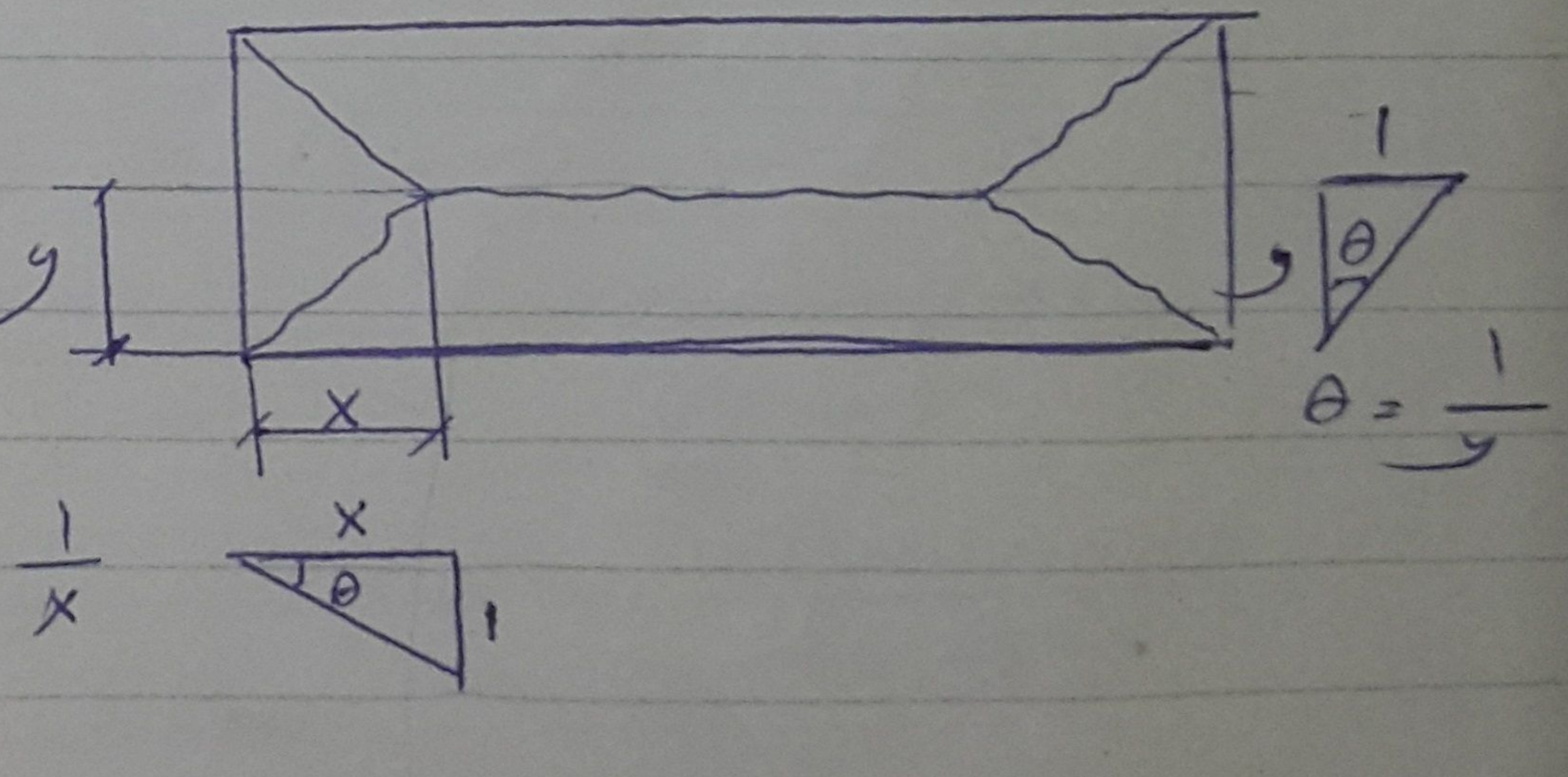




Internal work:— وهو ناتج ضرب العزم لكل وحدة أطوال في الـ yield line مضروب في طوله الـ yield-line مضروب في مقدار الدوران للجزء الذي تتعامل معه

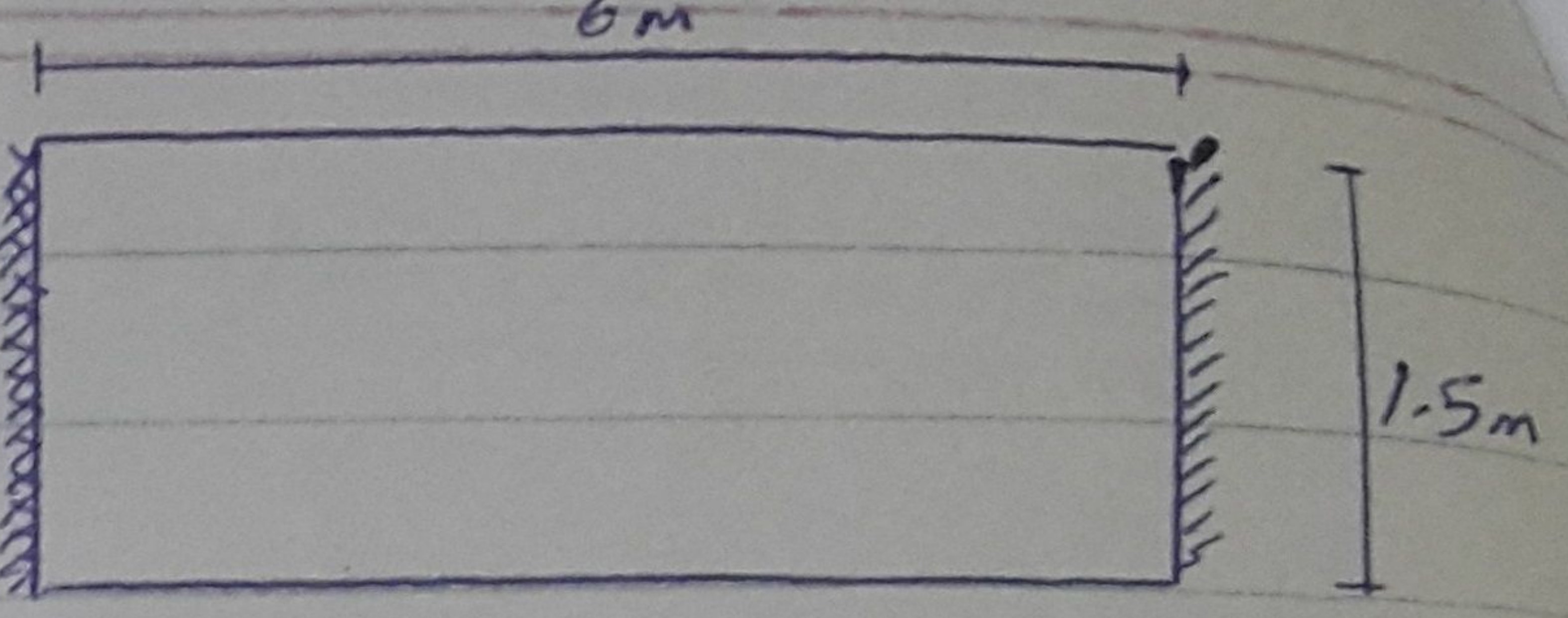
$W_I = m \cdot l \cdot \theta$ → rotation of segment.

l:— yield line length (مسقط الـ yield على axes of rotation)



$\theta = \frac{1}{y}$

$\theta = \frac{1}{x}$

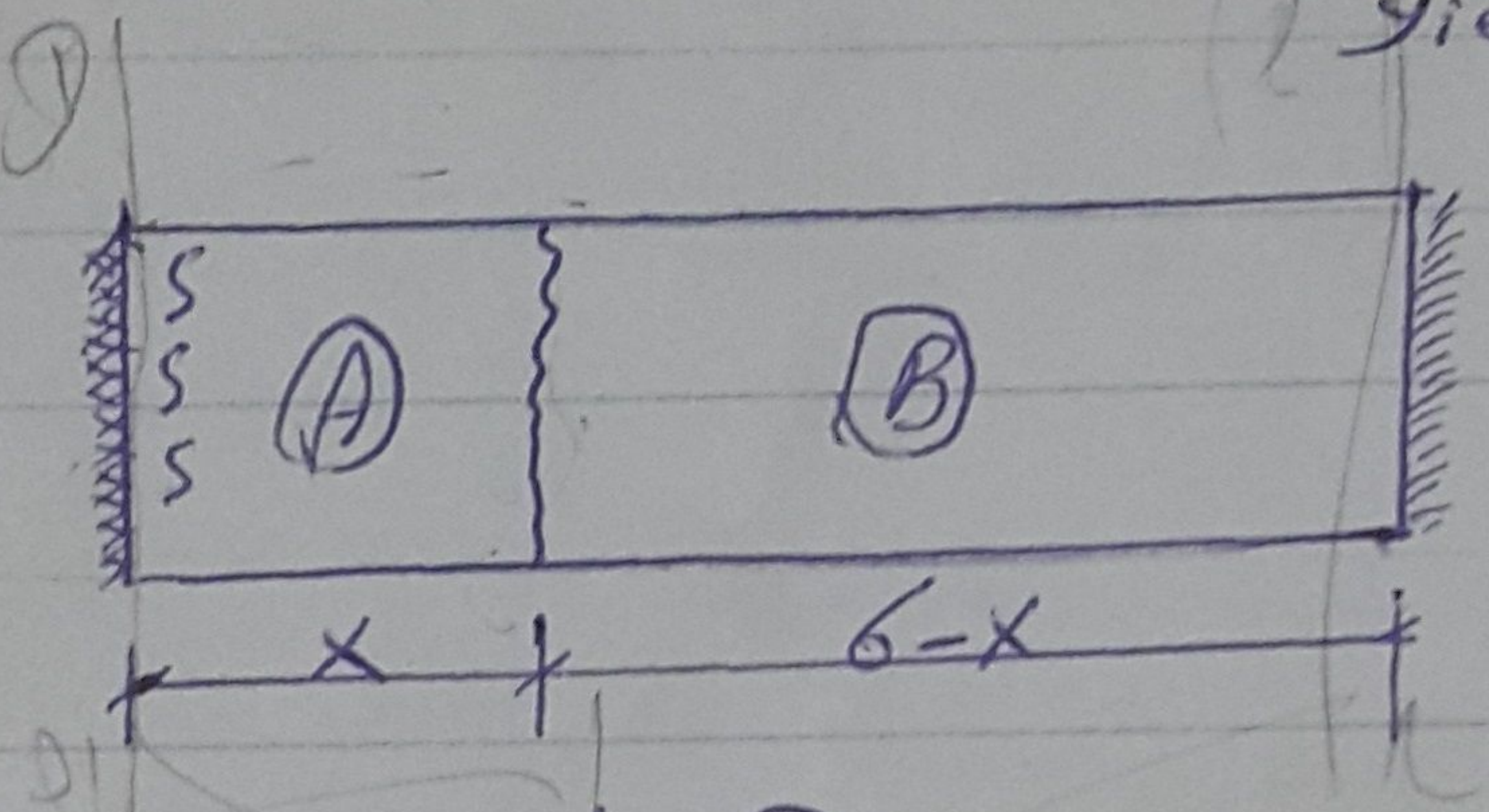


Ex.. $M\ominus = M\oplus = m$
find the largest value of (w) which is can be applied on the above system.

sol

١- يجب معرفة فيما إذا كانت one-way او two-way ، وهو يعتبر one-way

٢- رسم الـ yield line



1) For segment Ⓐ

$$WE = w \times X \cdot 1.5 \times \frac{1}{2} = 0.75\,wX$$

$$WI = 2m \times 1.5 \times \frac{1}{X} = \frac{3m}{X}$$

ملاحظة: عندما نكبر في W أكثر من هذه القيمة تساوي القيمة المطلوبة، ولكن بعد ذلك
لدينا مشكلة أنه تشقق إنشائي وليس فقط شقوق functional إنما
عندما يسير في العمل -

2) For segment Ⓑ

$$WE = w\,(6-x)\,1.5 \times \frac{1}{2} = 0.75\,w(6-x)$$

$$WI = m \times 1.5 \times \frac{1}{(6-x)} = \frac{1.5\,M}{(6-x)}$$

$$\Sigma WE = 0.75\,wX + 0.75\,w\,(6-x) = 0.75\,wX + 4.5\,w - 0.75\,wx$$

$$= 4.5\,w$$

$$\Sigma W_I = \frac{3M}{X} + \frac{1.5M}{(6-X)} = \frac{3m(6-X) + 1.5M \cdot X}{X(6-X)}$$

$$\Sigma W_I = \frac{18M - 3MX + 1.5M \cdot X}{6X - x^2} = \frac{18m - 1.5mX}{6X - X^2}$$

by equating $\Sigma WE = \Sigma W_I$ $\quad 4.5\,w = \dfrac{18M - 1.5MX}{6X - X^2}$

$$27\,wX - 4.5\,wX^2 = 18M - 1.5MX$$

$$\rightarrow \quad w = \frac{18m - 1.5MX}{27X - 4.5X^2}$$

$$\frac{dw}{dx} = \frac{(27x - 4.5x^2) * (-1.5m) - (18m - 1.5mx)(27 - 9x)}{(27x - 4.5x^2)^2}$$

$\frac{dw}{dx} = 0 \Rightarrow -40.5\, m \cdot x + 6.75x^2 m - 486m + 162mx + 40.5mx$ [illegible]

$6.75mx^2 - 162mx + 486m = 0$

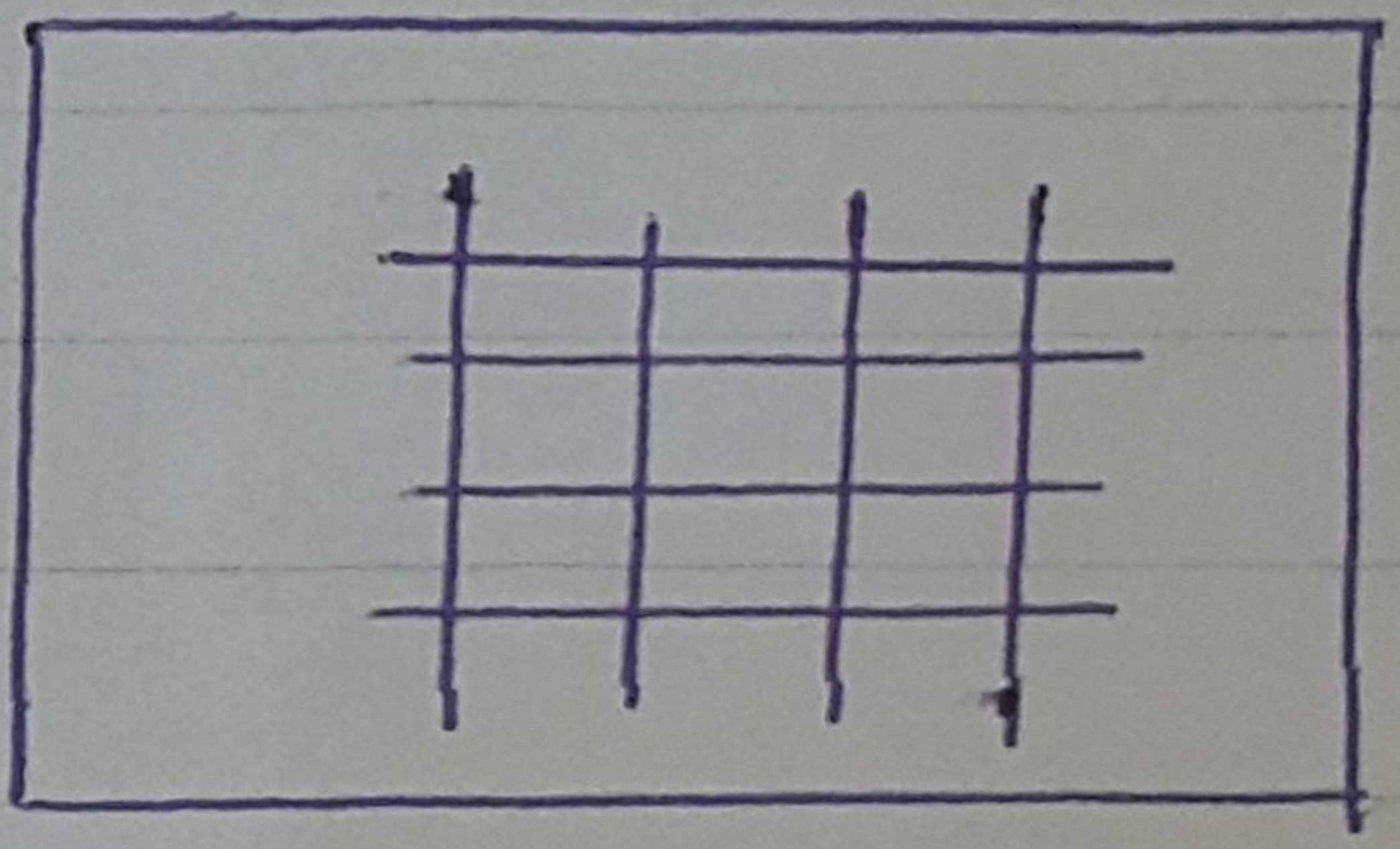
~~$-6.75x^2m - 324m = 0$~~ $\Rightarrow x = 5.4m$

- find $m = f(w)$ بوحدات $\frac{kN.m}{m}$
- from $m \rightarrow$ find A_s

Isotropic & un-Isotropic

Isotropic :- تكون مقاومة العزم متساوية بالاتجاهين (x) (y) ايضاً

$mx = my$



un-Isotropic $mx \neq my$

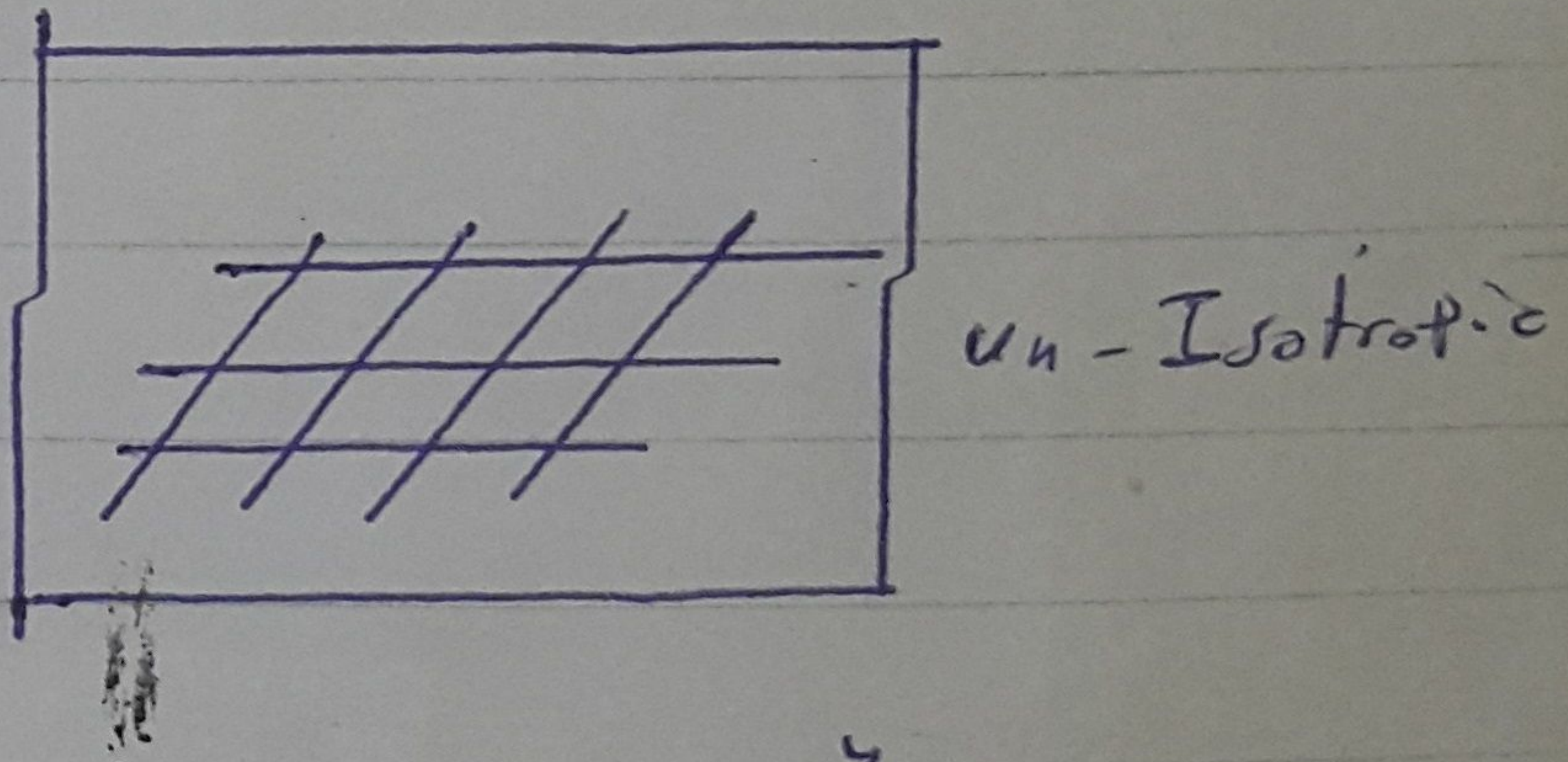
(m) (m)

+x -x

+y -y } Isotropic

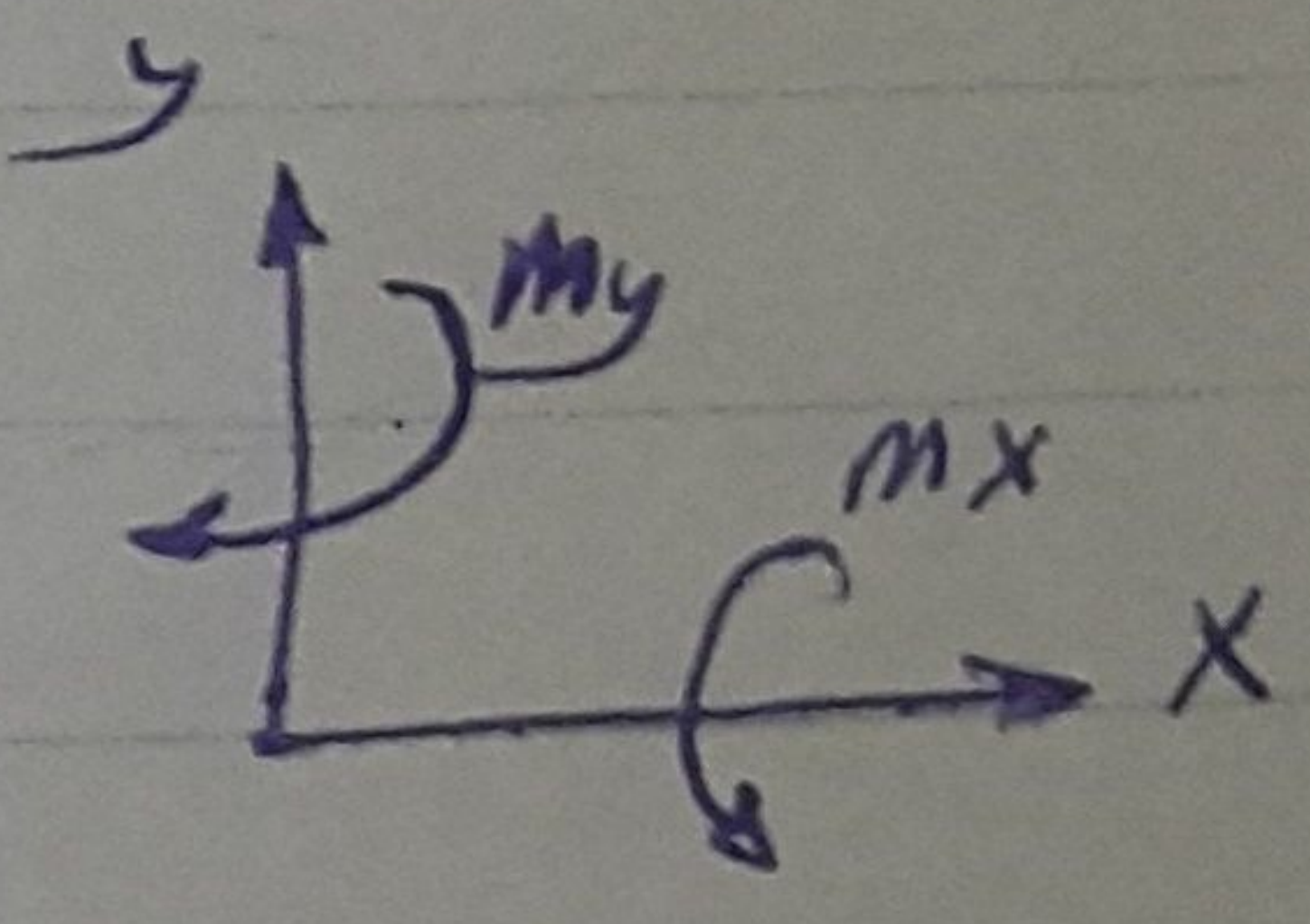
⇓ ⇓

متساوين متساوين

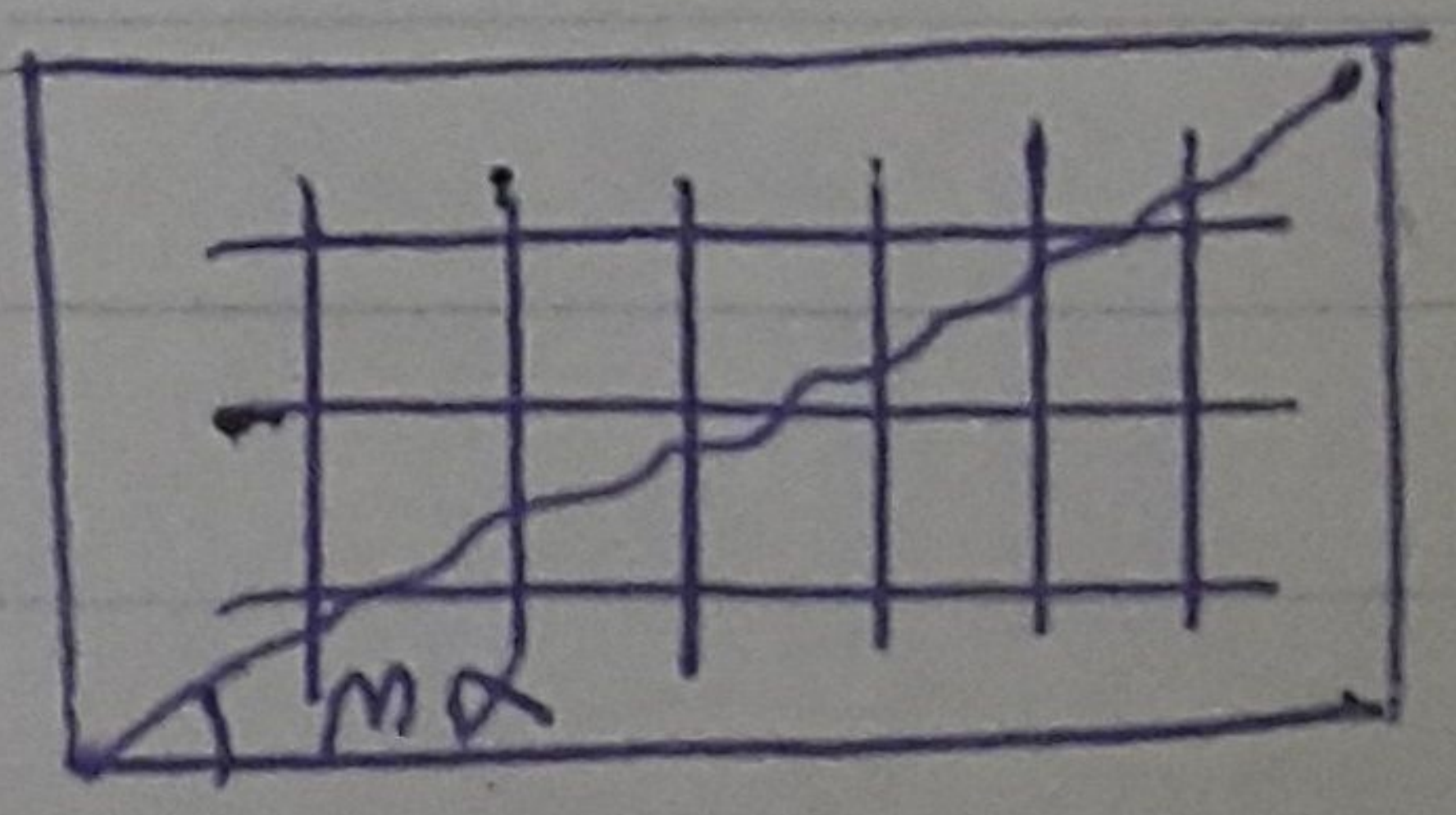


	x		y
M⊕	1.5	M⊕	1.5
M⊖	4.5	M⊖	4.5

} ⇒ Isotropic



- un-Isotropic $mx \neq my$ ←



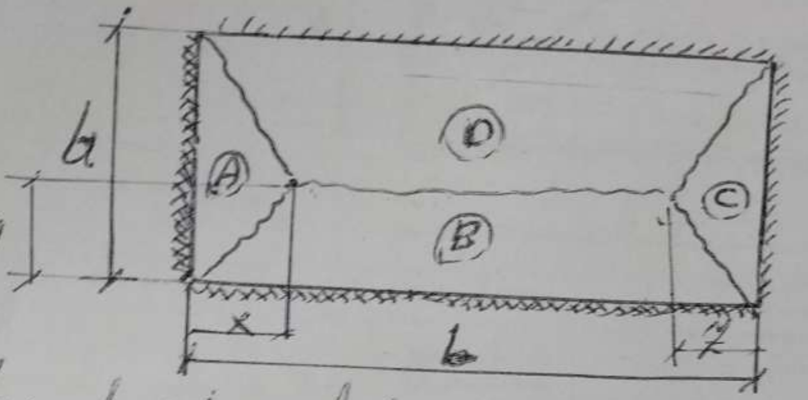
mα في حالة زاوية معينة الفا

$$m\alpha = Mx \cos^2\alpha + my \sin^2\alpha$$

$X = 0.585\,b > 0.5x$

$$\therefore\ W = \frac{2m}{L}\left[\frac{2b - 0.585b}{0.585b(b - 0.585b)}\right]$$

$$W = \frac{2mB}{b}[5.8 = 8) = 11.6\ mfa$$



Find the largest value of (m)

① Ext. w

- For Ⓐ $w.\frac{x.b}{2} \times \frac{1}{3}$
- For Ⓑ $w\left[\frac{x.y}{2}.\frac{1}{3} + (L - x - Z)y \times \frac{1}{2} + \frac{Z.y}{2} . \frac{1}{3}\right]$
- For Ⓓ $w\left[\frac{(b-y)x}{2} \times \frac{1}{3} + \frac{(b-y)Z}{2} . \frac{1}{3} + (L - Z - x)(b-y)\frac{1}{2}\right]$
- For Ⓒ $w\left[\frac{Z.b}{2} \times \frac{1}{3}\right]$

2) Int. w

- For $A = m.b.\frac{1}{x} + m.b.\frac{1}{x} = 2\frac{mb}{x}$
- For $B = m.L\frac{1}{y} + \frac{mL}{y} = \frac{2mL}{y}$
- For $D = m\,L.\frac{1}{(b-y)} = \frac{m.L}{b-y}$
- For $C = m.b.\frac{1}{Z} = \frac{mb}{Z}$, Ex = Int

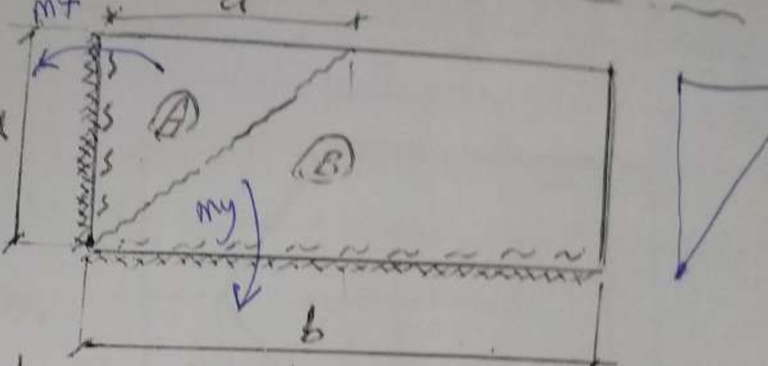
find $\frac{dM}{dx} = 0$ — ①, $\frac{dm}{dy} = 0$ — ②, $\frac{dm}{dx} = 0$ — ③

by solving the three equations we will find x, y, x → w, mp

[illegible] 2 أصغر w ، الـ m

Ex: $M_x = 0.5 M_y = m$

Neg moment = 2m

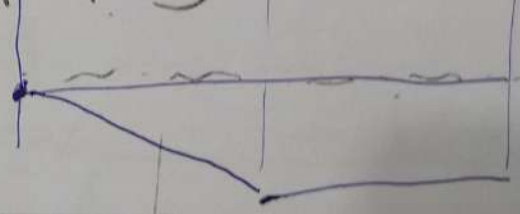


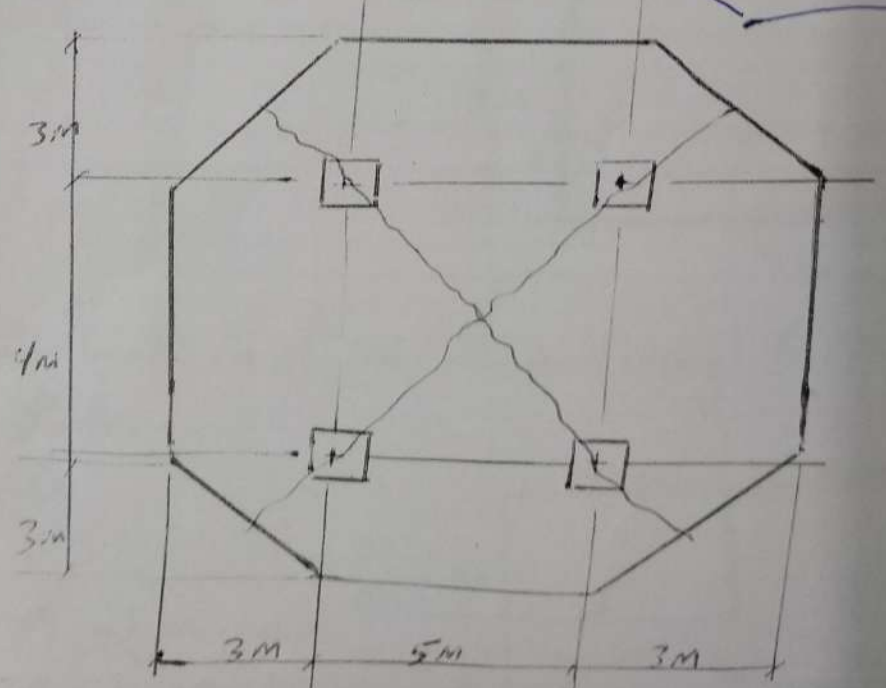
- For interior
- For Ⓐ

$$2\underset{\text{neg}}{m} \cdot a \cdot \frac{1}{a} + \overset{\text{Pos} = mK}{m} \cdot a \cdot \frac{1}{a} = 3m$$

- For Ⓑ

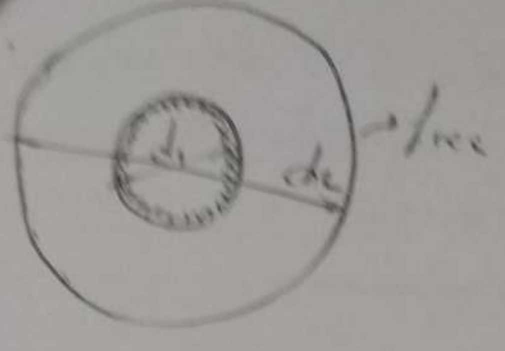
$$2\underset{\text{neg}}{m} \cdot b \cdot \frac{1}{a} + \underset{\text{posit} = my}{2m} \cdot a \times \frac{1}{a}$$

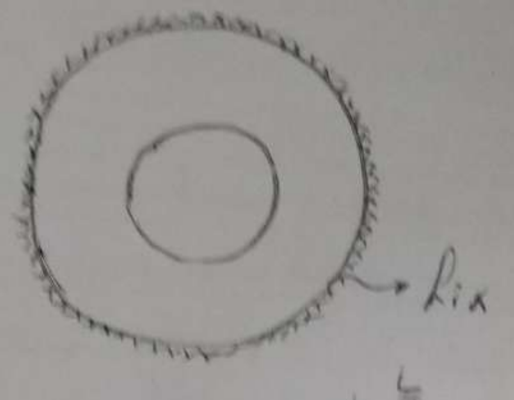


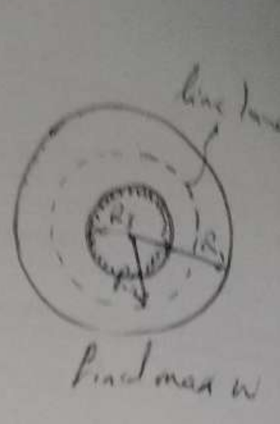


W = 12 KN/m² → find m

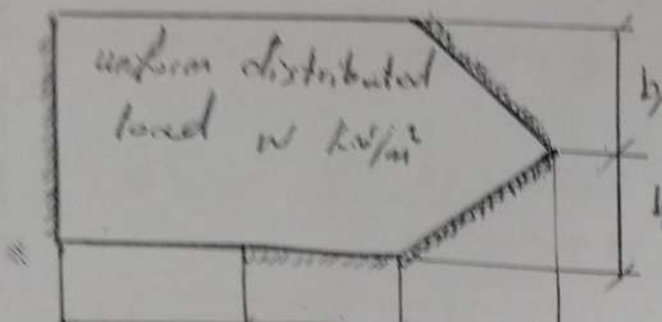
isotropic افتراضات



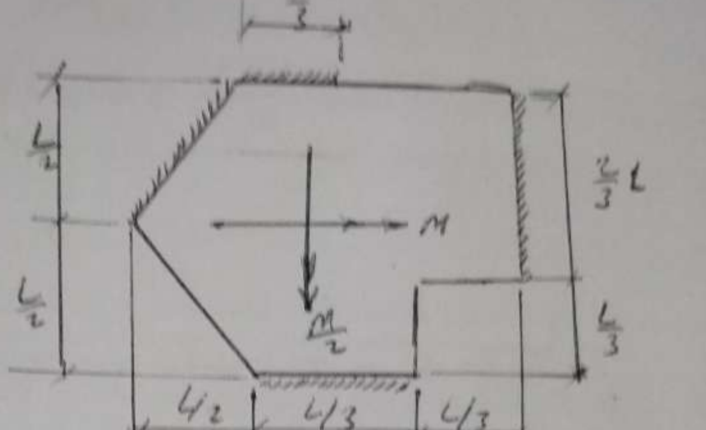




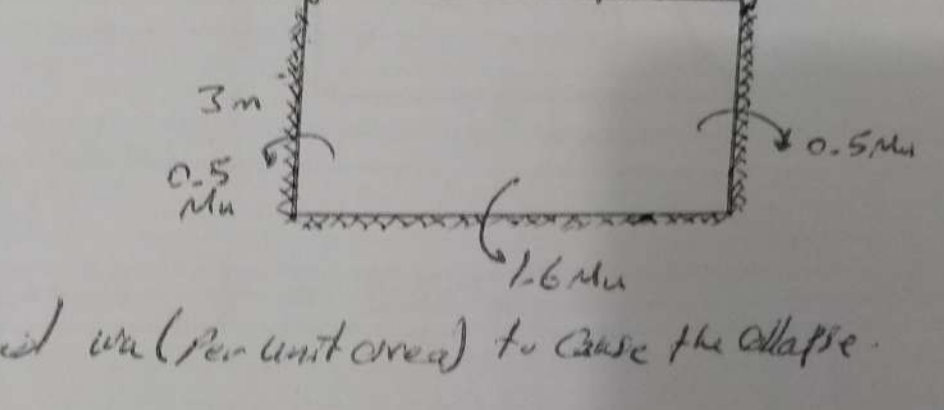
find max w



(find max w)

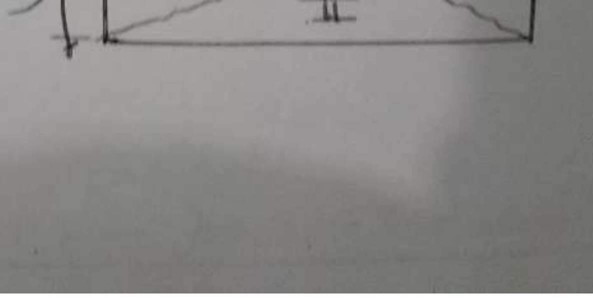


$L = 9m$, $m = 28$ kN.m/m , $w_u = ?$



- Find max. ultimate load w_u (Per unit area) to cause the collapse yield line pattern.

① Case ①



Ext work

Part I = $2.5(3-y) . w_u \times 0.5$
$+ 2.5y\ (0.5) \times w_u \times \frac{1}{3}$

Part II = $\frac{5y}{2} \times \frac{1}{3} \times w_u =$

Total Ex = 2 I + II = $7.5 W_u - 0.833 W_u \cdot y$

Internal work :-

Ⓘ → $3 (M + 0.5M) \left(\frac{1}{2.5}\right) = 1.8M$

Ⓘⓘ → $5 (m + 1.6m) \left(\frac{1}{y}\right) = \frac{158.6}{y}$

Total internal = $\underbrace{43.92}_{I \times 2} + \underbrace{\frac{158.6}{y}}_{II}$, $\Sigma Ex = \Sigma int$

$$7.5 W_u - 0.833 W_u \cdot y = 43.92 + \frac{158.6}{y}$$

$$W_u = \frac{43.92y + 158.6}{7.5y - 0.833y^2} , \quad \frac{dw_u}{dy} = 0$$

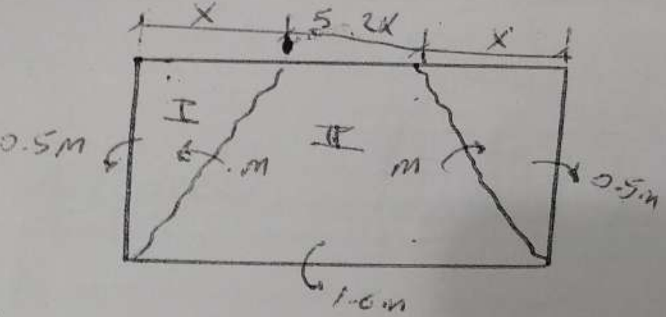
$$36.69 y^2 + 264.39 y - 1189.5 = 0 \rightarrow y_1 = 3.137$$

$y_2 = -10.858$ (impossible)

2) Case two

1 — Ext work

I — $\frac{3X}{2} \times \frac{1}{3} W_u = 0.5 W_u X$



$\Sigma Ex.w = 7.5 W_u - 2X W_u$

2 - Internal w , Ⓘ

$$1.5m \cdot 3 \times \frac{1}{X} = \frac{4.5m}{X} = \frac{54.9}{X}$$

Ⓘⓘ $2.6M \cdot X \frac{1}{3} \times 2 + 1.6m (5 - 2X) \frac{1}{3}$

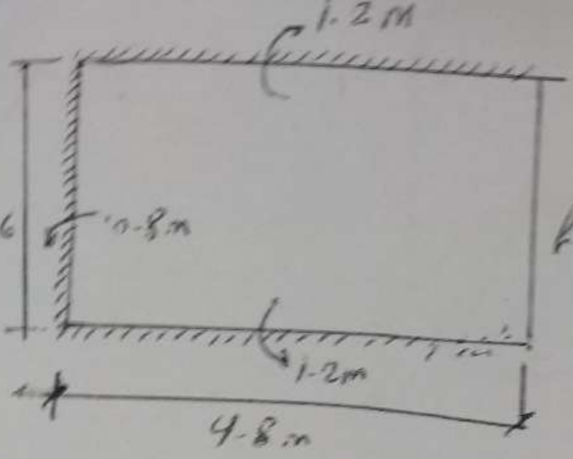
$= 8.13X + 32.533$ Int = Ext

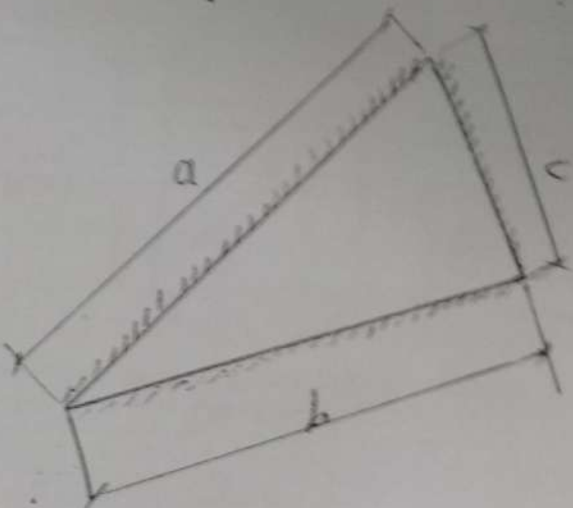
$$7.5 W_u - 2X W_u = \frac{109.8}{X} + 8.13X + 32.533$$

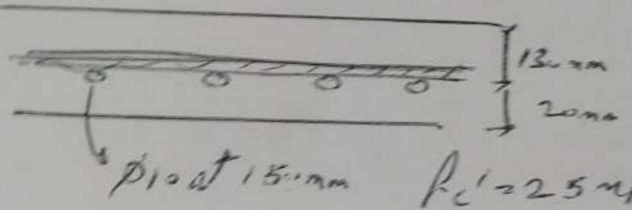
$$W_u = \frac{109.8 + 8.13X^2 + 32.533X}{7.5X - X^2} , \quad \frac{dw}{dx} = 0$$

$93.512 x^2 + 219.6x - 823.5 = 0 \Rightarrow X_1 = 2.017m$, $X_2 = -4.366m$ impossible

$w_u = 18.853$ kN/m², for $x = 2.017$







$f_c' = 25$ mPa, $f_y = 350$ mPa

Ex

M +ve moment

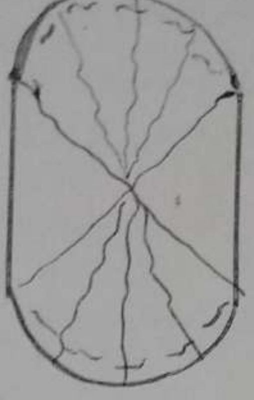
m' −ve moment

W_o dead load (per unit area) $\gamma \cdot t$

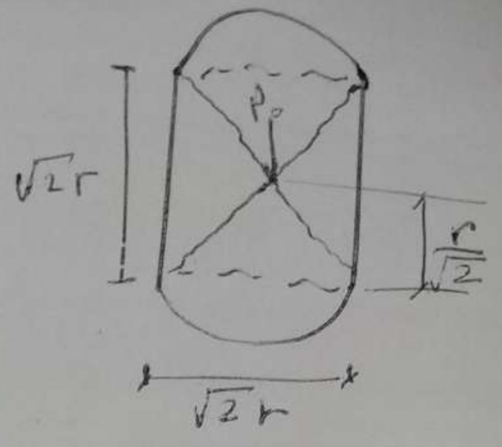
− External work = Internal

$$P_o \cdot 1 + W_o \cdot \frac{1}{2} r\alpha \cdot r \left(\frac{1}{3}\right) \frac{2\pi}{\alpha} = (m+m') r\alpha \left(\frac{1}{r}\right) \frac{2\pi}{\alpha}$$

$$P_o = 2\pi(m+m') - \frac{W_o \cdot \pi r^2}{3}$$



Case ②



Case ①

1) Case ① Ex = Int

$$P_o \times 1 + w_d \times \frac{1}{2}\sqrt{2}\,r \cdot \frac{r}{\sqrt{2}}\left(\frac{1}{3}\right)\times 4 = (m+\bar{m})\sqrt{2}\,r\left(\frac{\sqrt{2}}{r}\right)\times 4$$

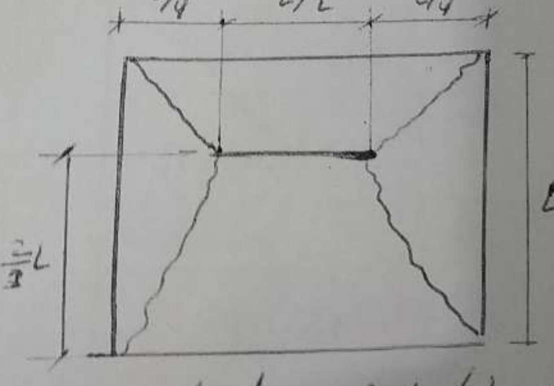
$$P_o = 8(m+\bar{m}) - \frac{2}{3} w_o \cdot r^2$$

2) Case ②

$$P_o \times 1 + \left[w_d \cdot \frac{1}{2}\sqrt{2}\,r \cdot \frac{r}{\sqrt{2}} \cdot \frac{1}{3}\right]\times 2 + \left(w_d \cdot \frac{r}{2} \cdot \alpha r \cdot \frac{1}{3} \times \frac{\pi}{\alpha}\right)$$

$$= (m+\bar{m})\left(\sqrt{2}\,r \cdot \frac{\sqrt{2}}{r}\times 2\right) + (m+\bar{m})\, r\alpha \,\frac{1}{r} \cdot \frac{\pi}{\alpha}$$

$$\therefore P_o = (4+\pi)(m+\bar{m}) - w_d\, r^2\left(\frac{\pi}{6}+\frac{1}{3}\right)$$ Controll

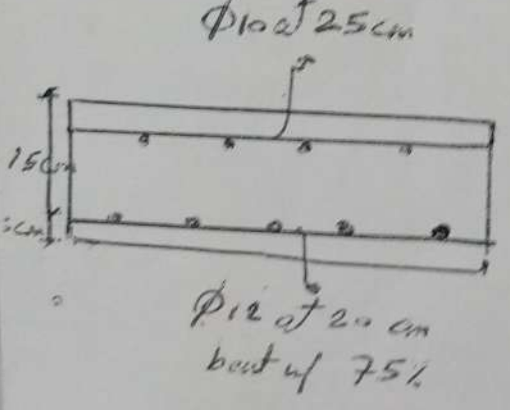


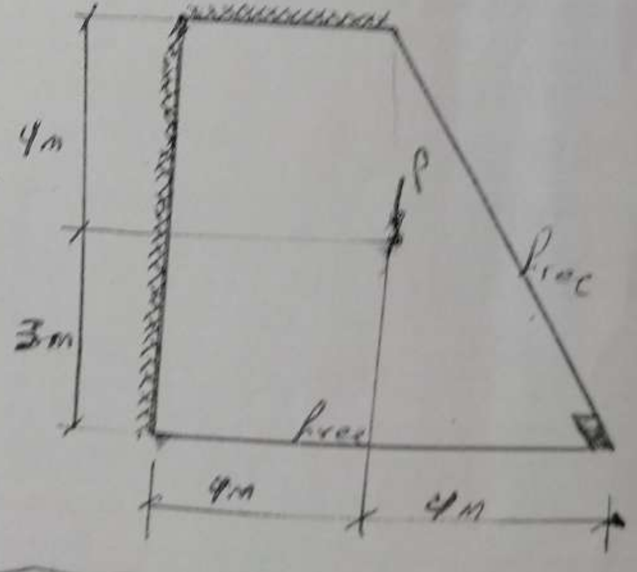
$$P \cdot \frac{L}{2}\times 1 + w_d \cdot \frac{L}{2} \cdot \frac{2}{3}L \times \frac{1}{2} + w_d \cdot \frac{L}{4} \cdot \frac{2}{3}L\left(\frac{1}{3}\right) + w_d \frac{L}{2} \cdot \frac{1}{3}L \cdot \frac{1}{2}$$
$$+ w_d \cdot \frac{L}{4} \cdot \frac{1}{3}L\left(\frac{1}{3}\right) + w_d \cdot \frac{L}{2}\left(\frac{L}{4}\right)\times\left(\frac{1}{3}\right)\times 2 =$$

$$(m+m')L\left(\frac{1}{\frac{2}{3}L}\right)+(m+m')L\left(\frac{1}{\frac{L}{3}}\right)+(m+m')L\left(\frac{1}{\frac{L}{5}}\right)(2$$

$$\therefore\ P=(m+m')\frac{25}{L}-W.d.L\left(\frac{15}{18}\right)$$

Final examination :- An - isotropically reinforced slab supported
as shown in Fig carrying a single concentrated load (P).
Determine the max force (P) which cause the collapse (neglect slab
weight), $f_c' = 30$ MPa, $f_y = 350$ MPa.





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