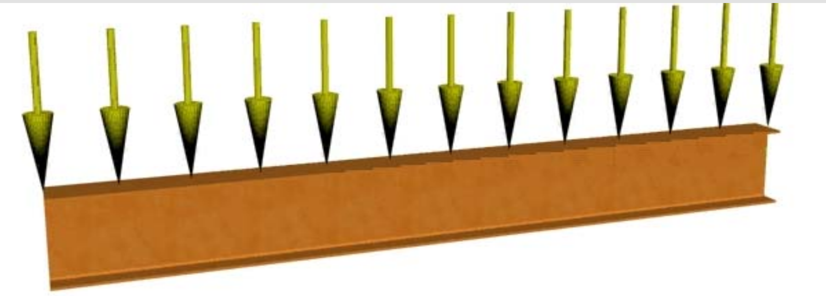
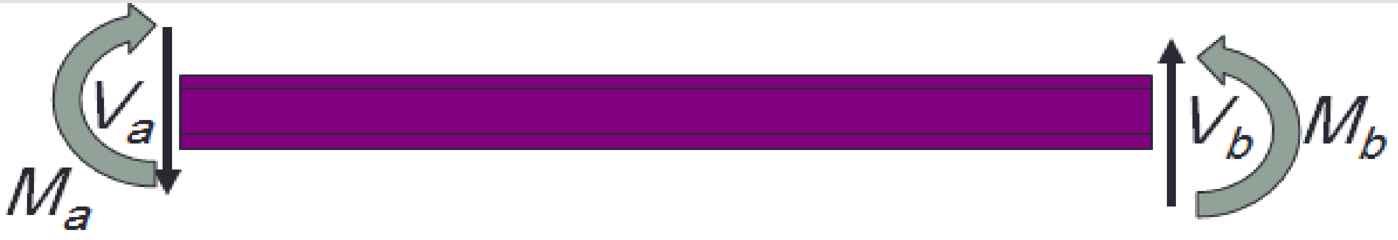
Design of Steel Structures II

Module No. 4

Analysis and Design of Flexural Members

FLEXURAL MEMBERS

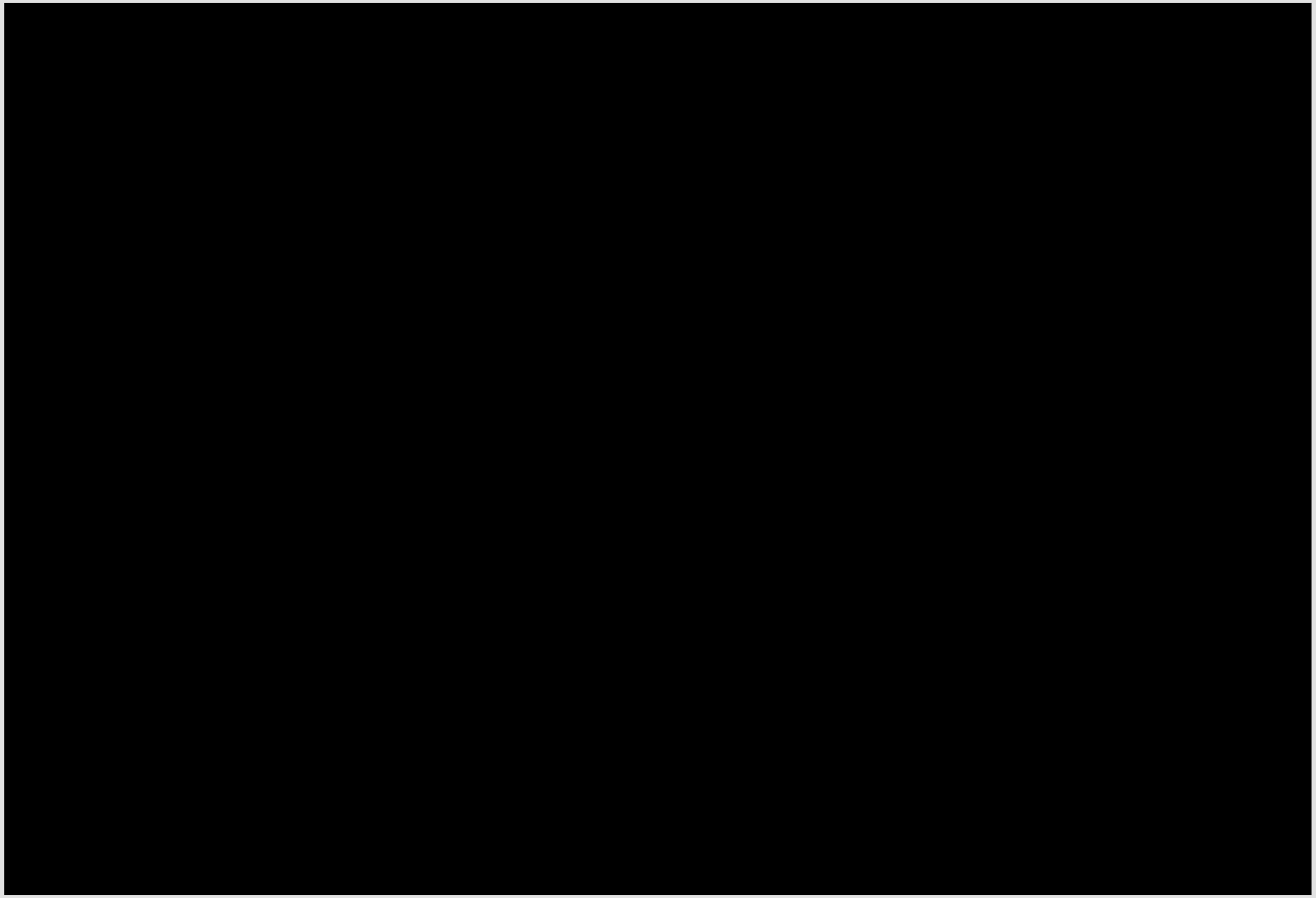




Beams are structural members that support transverse loads and are therefore subjected primarily to flexure, or bending

The beams and girders are erected.
These members frame bays as well as openings such as an elevator shaft, HVAC duct chase, or stair well.

Types of Loads



Girders span between columns.

Beams and Open web bar joists span between girders.

BEAM THEORY

Yield and Plastic Analysis

Yield and Plastic Moments

Moment can be related to stresses, σ, strains, ε, and curvature, φ.

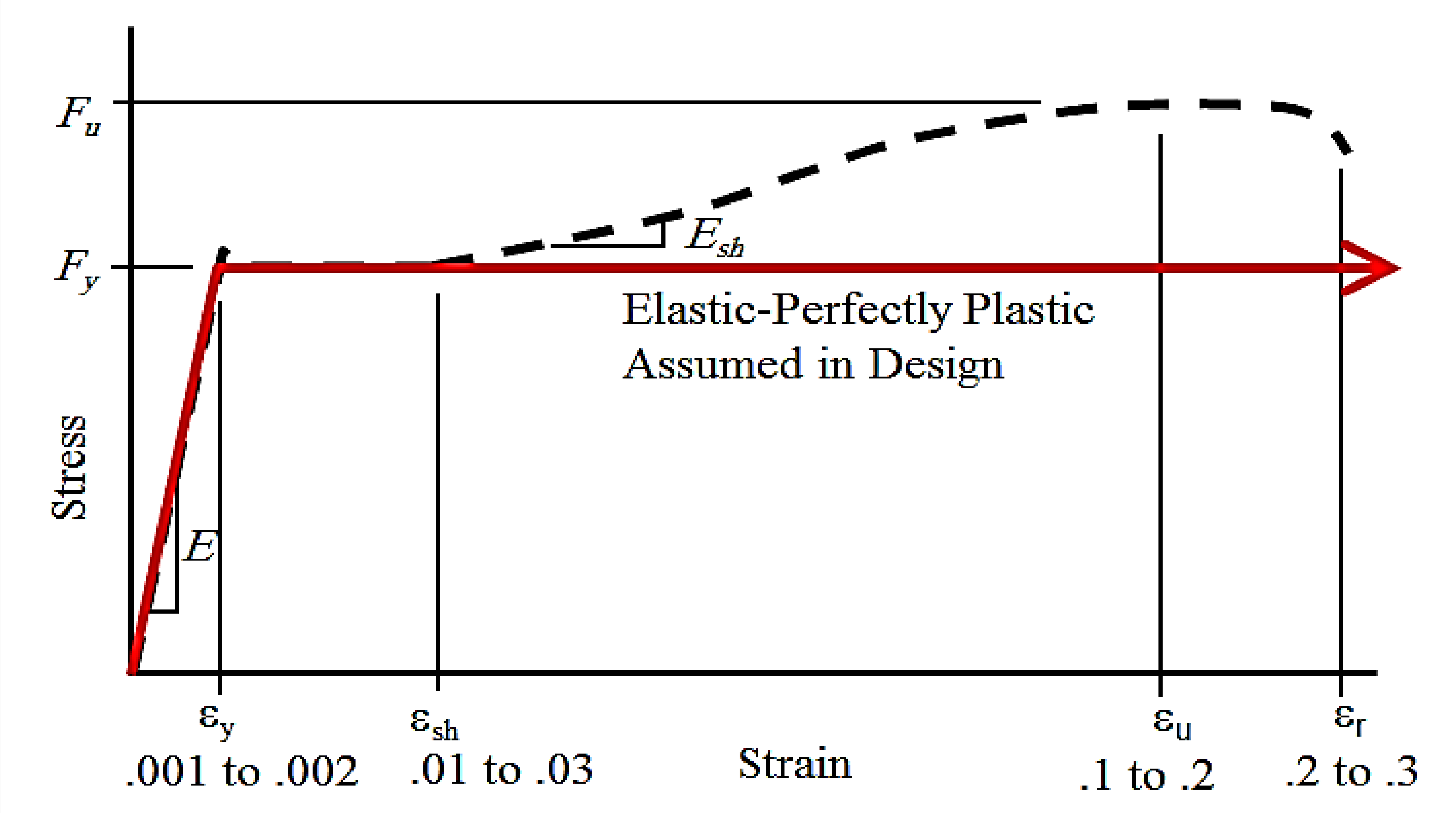
Assumptions:

Stress strain law -

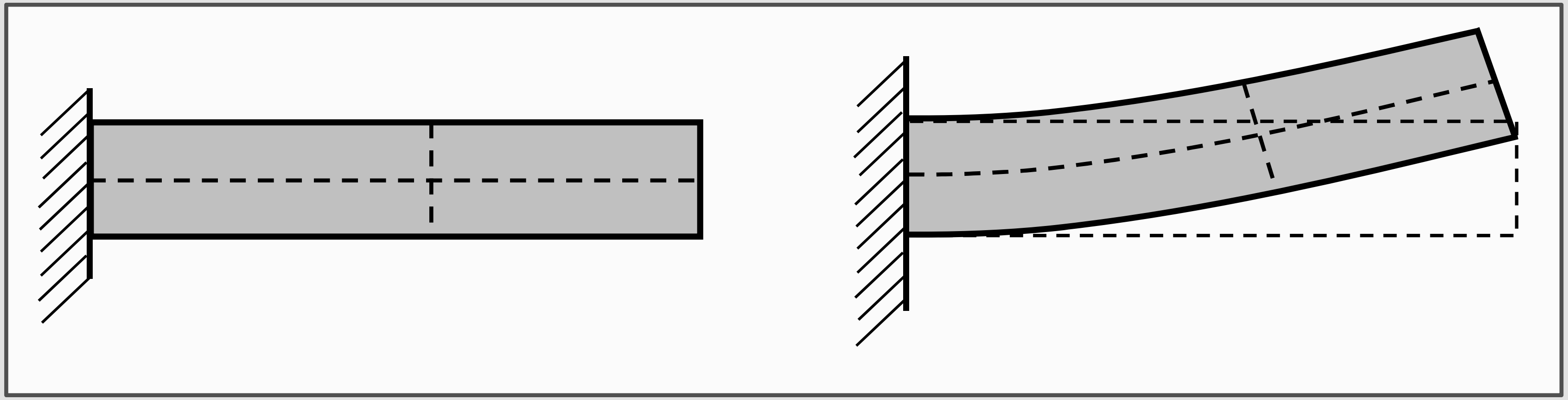
Initially assume linearly elastic, no residual stresses (for elastic only).

Plane sections remain plane -

Strain varies linearly over the height of the cross section (for elastic and inelastic range).



Stress-Strain Law



Plane sections remain plane.

Yield and Plastic Moments

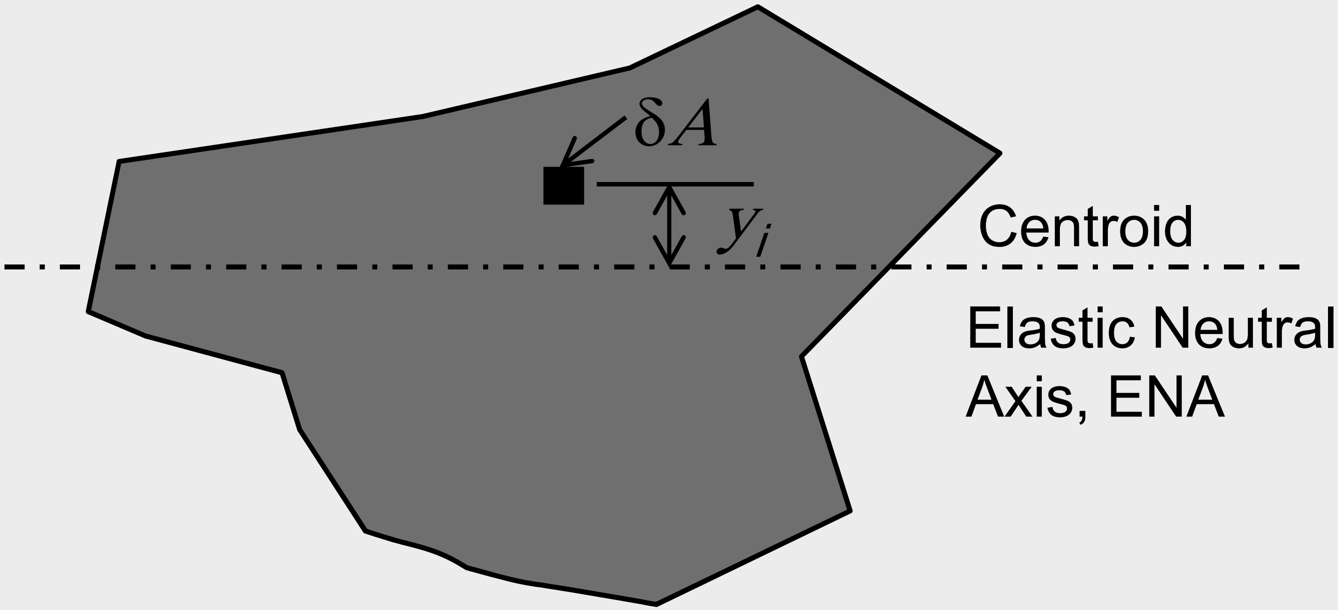
$$P = \int \sigma \delta A = 0$$

$$F_i = \sigma \delta A$$

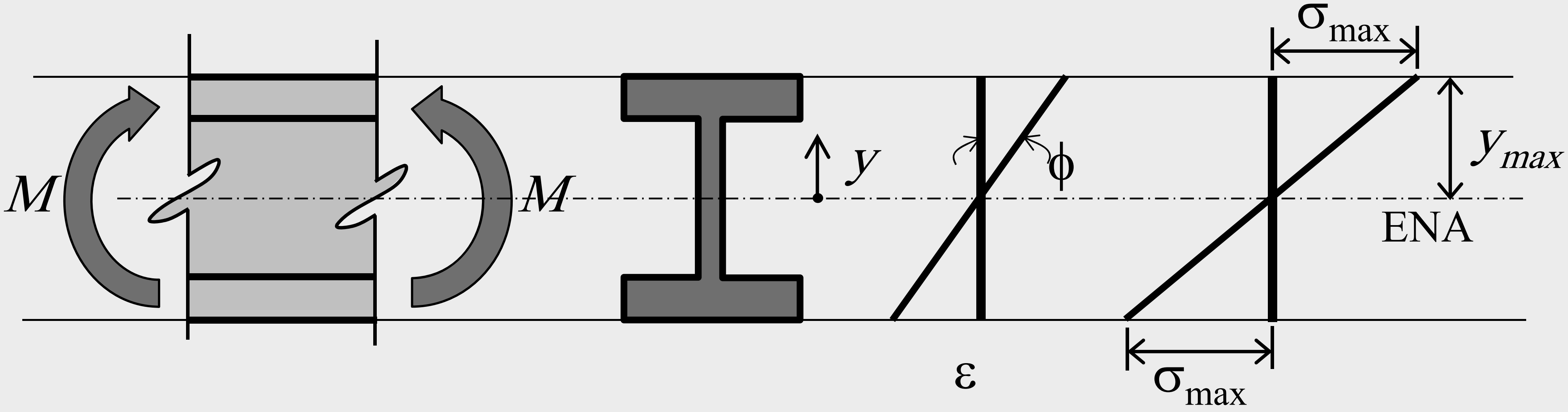
$$\Sigma F_i = 0$$

$$M = \int y \sigma \delta A$$

$$M = \Sigma y_i F_i$$



Yield and Plastic Moments

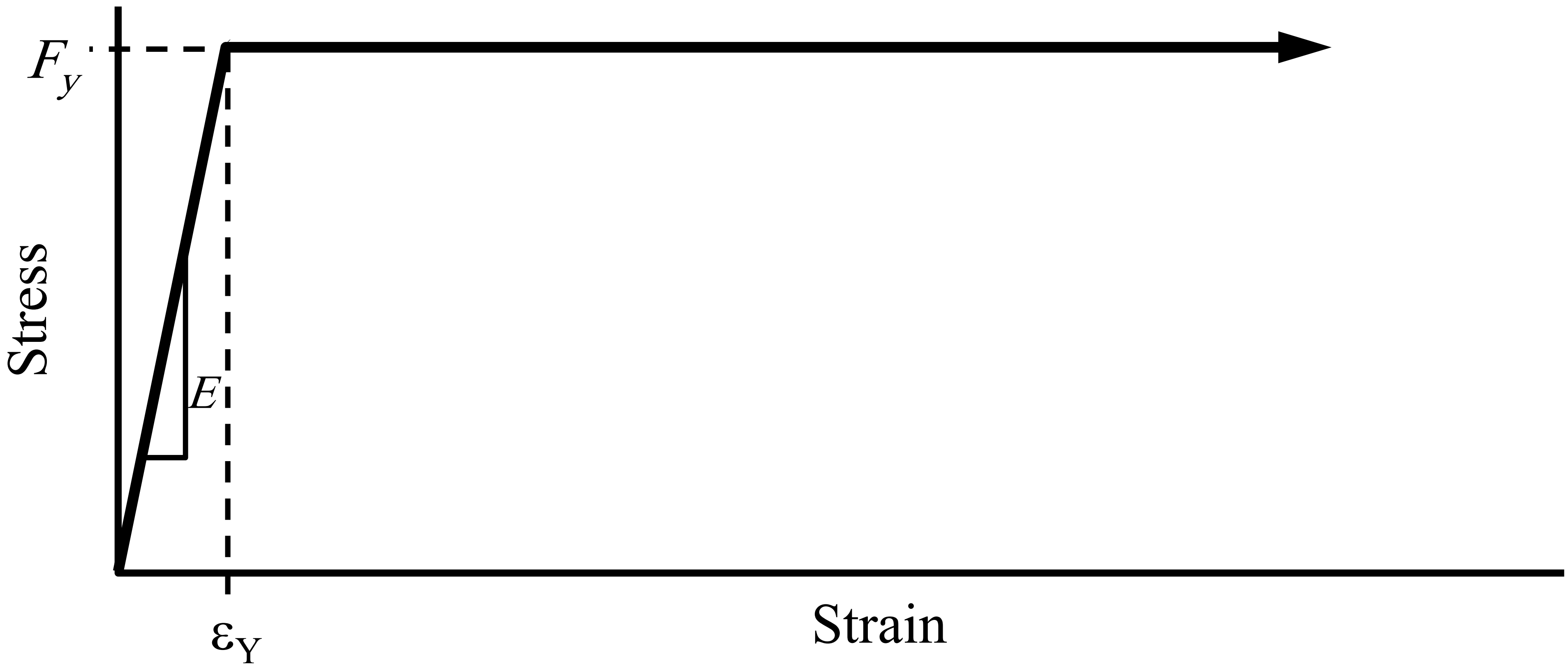


Elastic Behavior:

Strain related to stress by Modulus of Elasticity, E

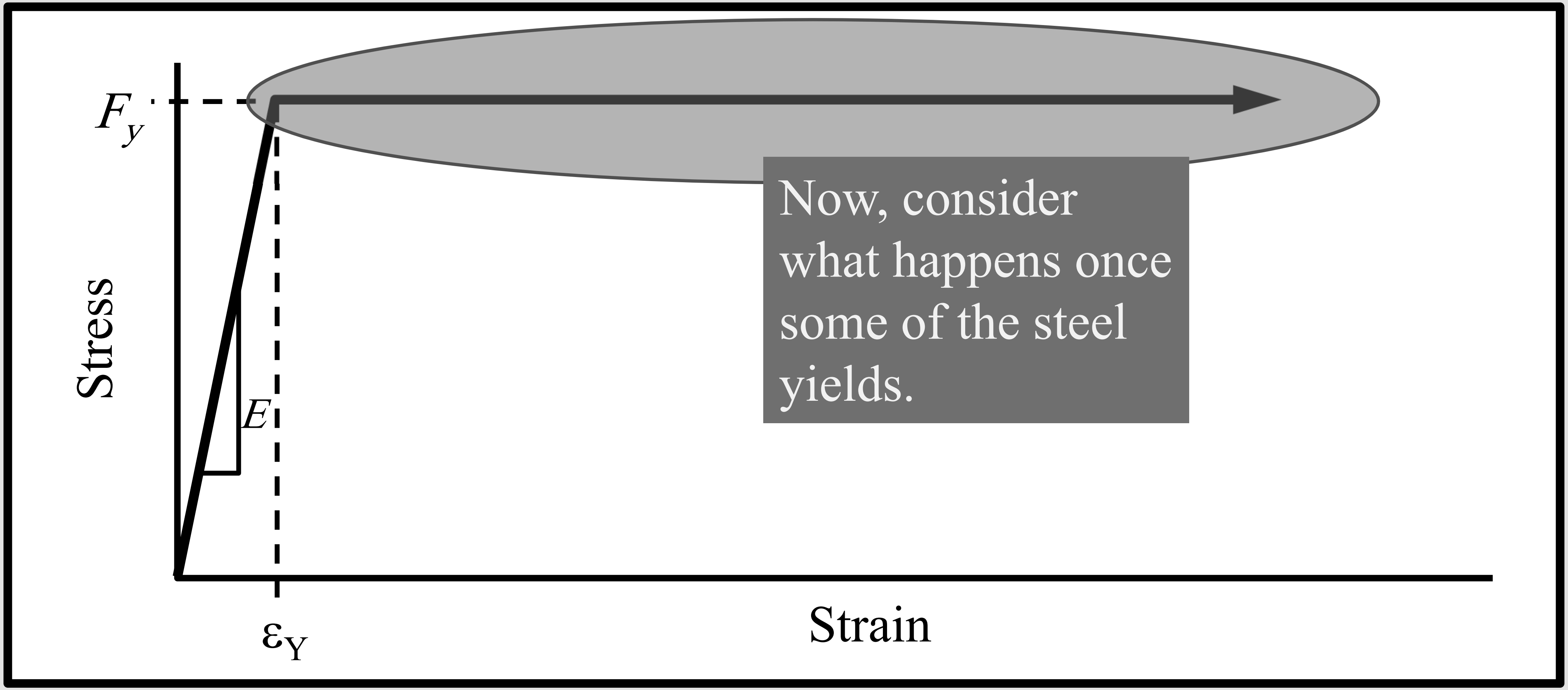
$\sigma = E\varepsilon$

Yield and Plastic Moments



Beyond yield -
Stress is constant,
Strain is ***not*** related to stress by Modulus of Elasticity, E

Yield and Plastic Moments



Beyond yield -
Stress is constant,
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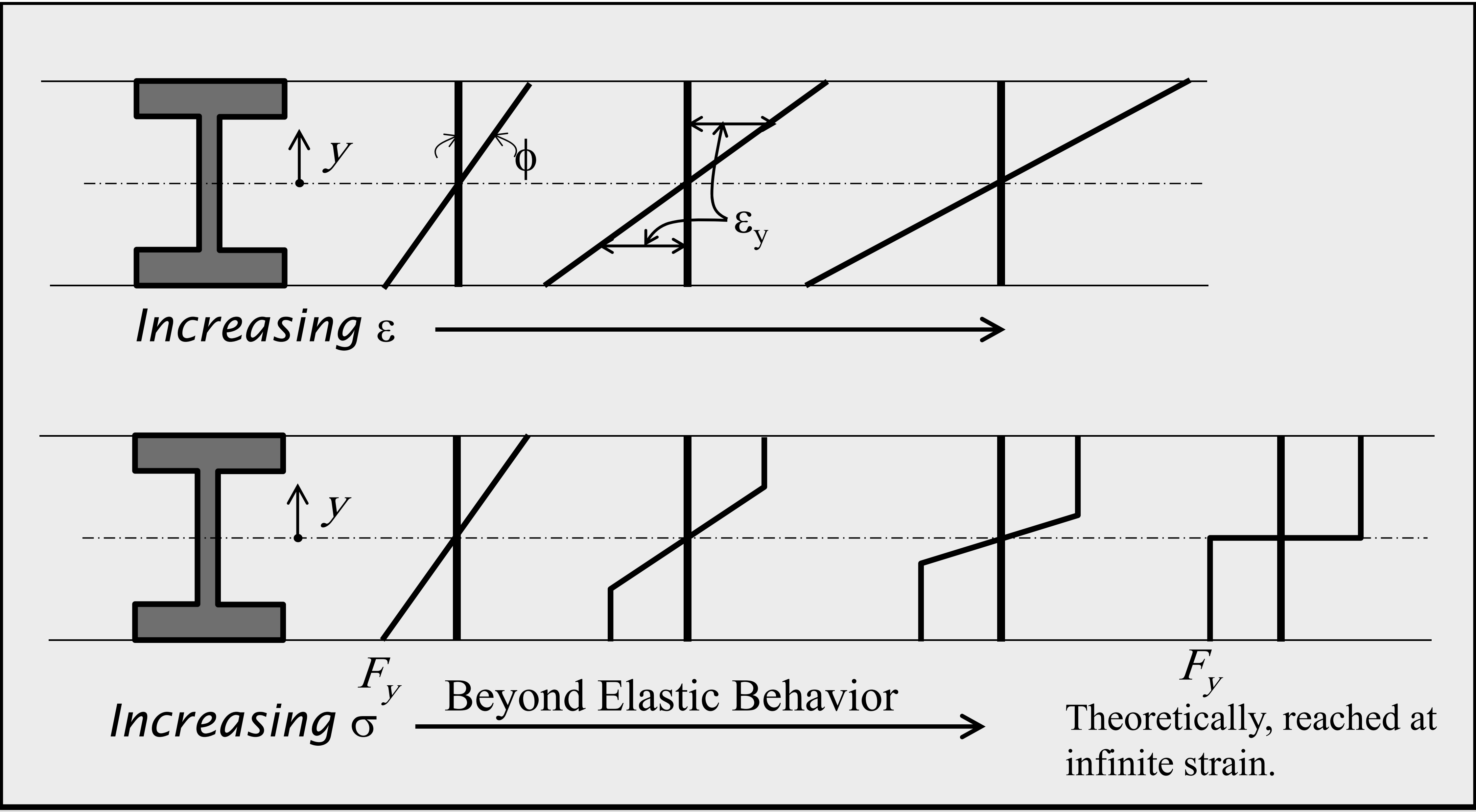
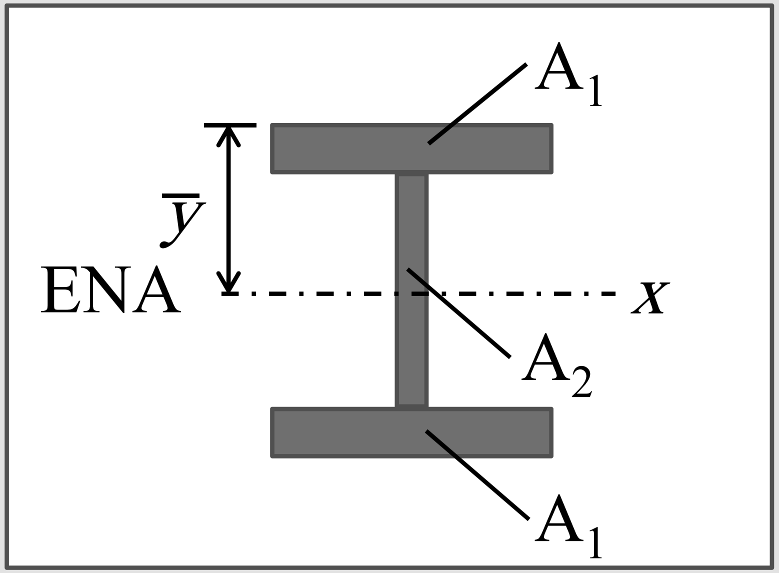
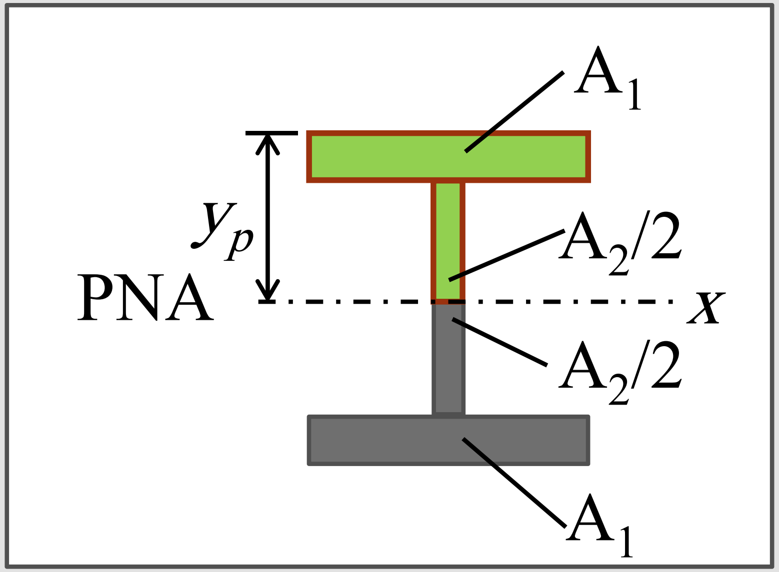


The far right stress diagram, with essentially all of the section at yield stress, corresponds to the PLASTIC MOMENT (M_p). Due to ductile nature of steel, this can be reached in a typical section without fracture of the extreme fibers.

NOTE THAT stress=*Mc/I* IS *NOT* VALID

Yield and Plastic Moments





Elastic Neutral Axis = Centroid

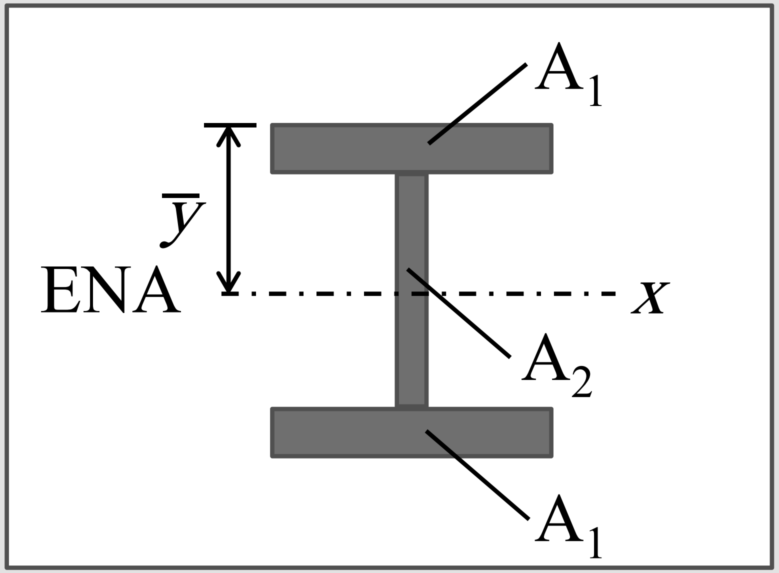
$$\text{ENA} = \frac{\sum A_i \overline{y_i}}{\sum A_i} = \bar{y}$$

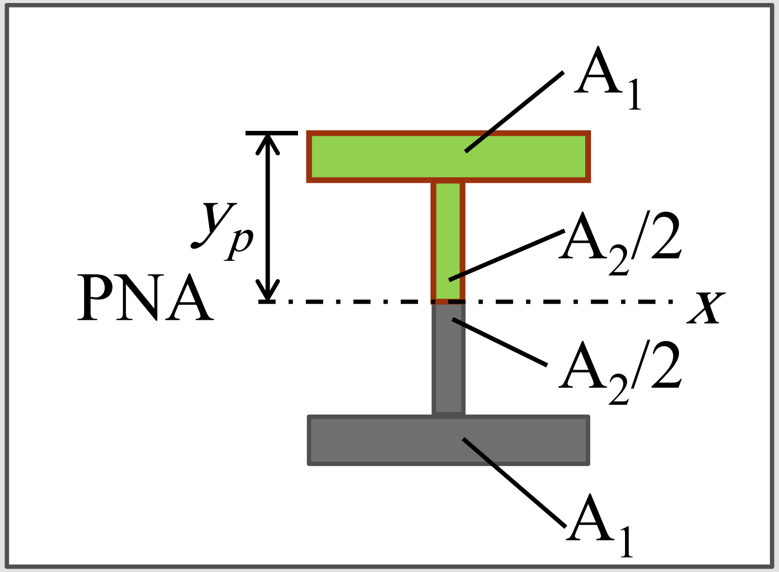
Plastic Neutral Axis –

If homogenous material (similar F_y), PNA divides Equal Areas, $A_1 + A_2/2$.

For symmetric homogeneous sections, PNA = ENA = Centroid

Yield and Plastic Moments





Yield Moment, $M_y = (I_x/c)F_y = S_xF_y$

$S_x = I_x/c$

$c = \bar{y} =$ distance to outer fiber

$I_x =$ Moment of Inertia

$$I_x = \sum \left(bh^3/12 \right) + \sum A\bar{y}^2$$

Plastic Moment, $M_p = Z_xF_y$

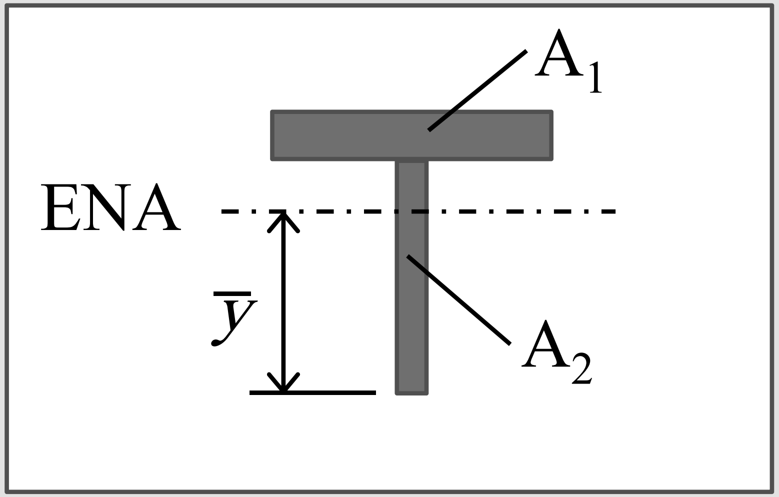
$Z_x = \int_A y \delta A$

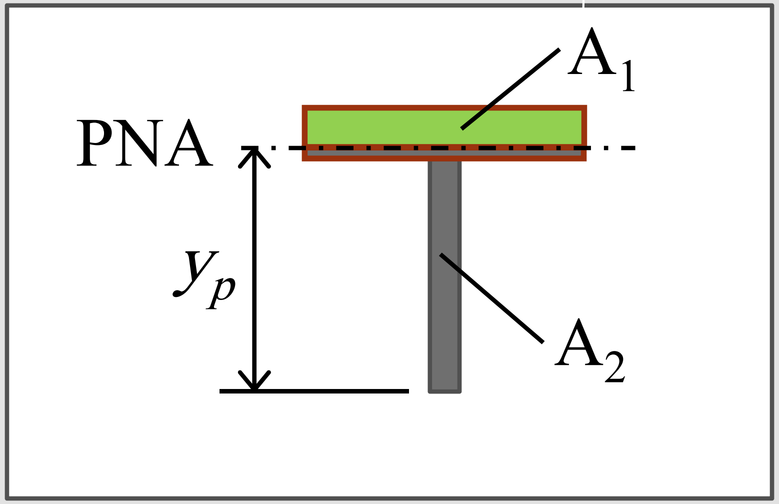
For homogenous materials,

$$Z_x = \Sigma A_i y_i$$

Shape Factor = M_p/M_y

Yield and Plastic Moments





Elastic Neutral Axis = Centroid

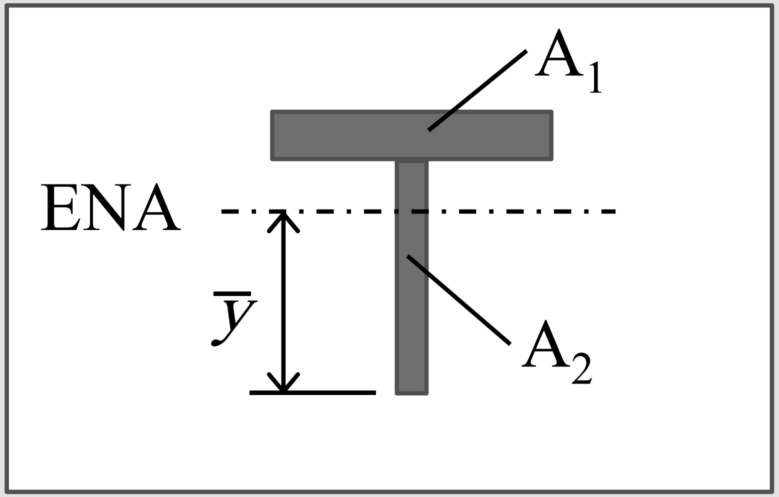
$$ENA = \frac{\sum A_i \overline{y_i}}{\sum A_i} = \bar{y}$$

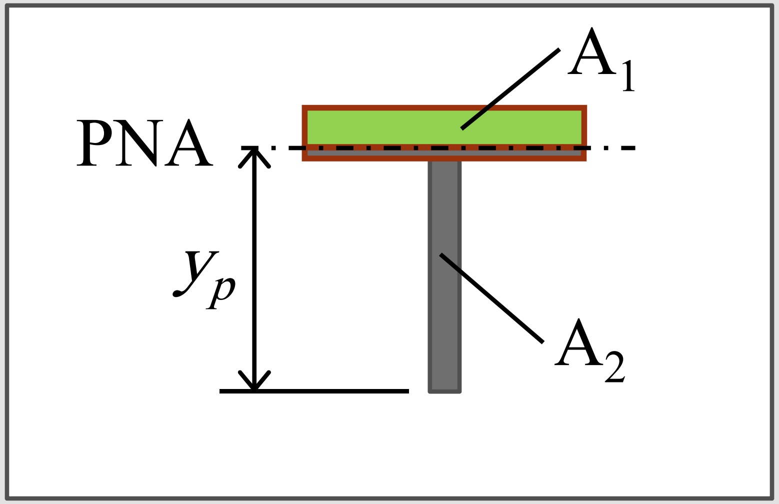
Plastic Neutral Axis ≠ Centroid

PNA divides equal forces in compression and tension.

If all similar grade of steel PNA divides equal areas.

Yield and Plastic Moments





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$S_x = I_x/c$

$c = \bar{y} =$ distance to outer fiber

$I_x =$ Moment of Inertia

Plastic Moment, $M_p = Z_xF_y$

$Z_x = \int_A y\delta A = \Sigma A_i y_i$,

for similar material throughout the section.

Shape Factor $= M_p/M_y$

Yield and Plastic Moments

With residual stresses, first yield actually occurs before M_y.

Therefore, all first yield equations in the specification reference

$$0.7F_yS_x$$

This indicates first yield 30% earlier than M_y.
For 50 ksi steel this indicates an expected residual stress of (50 * 0.3) = 15 ksi.

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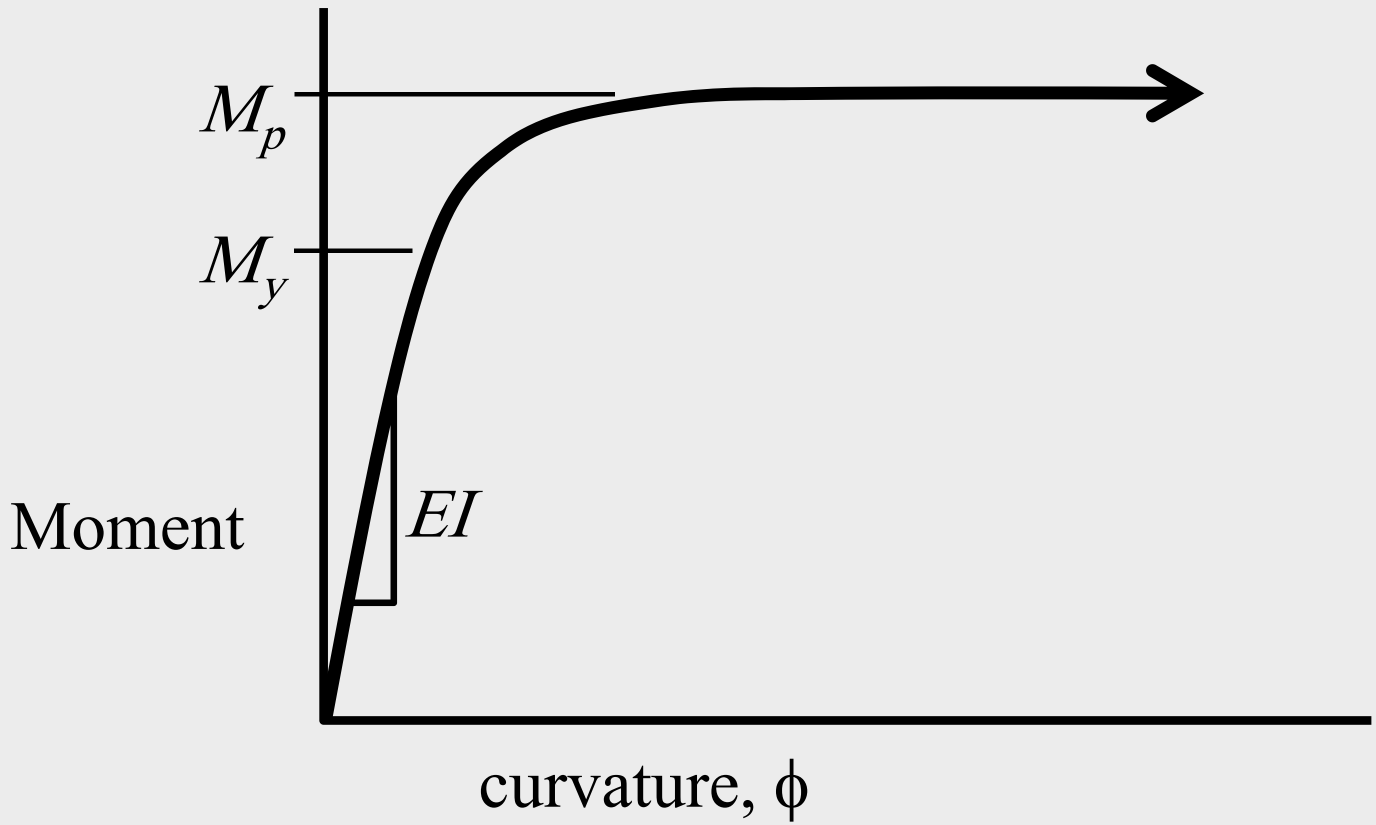
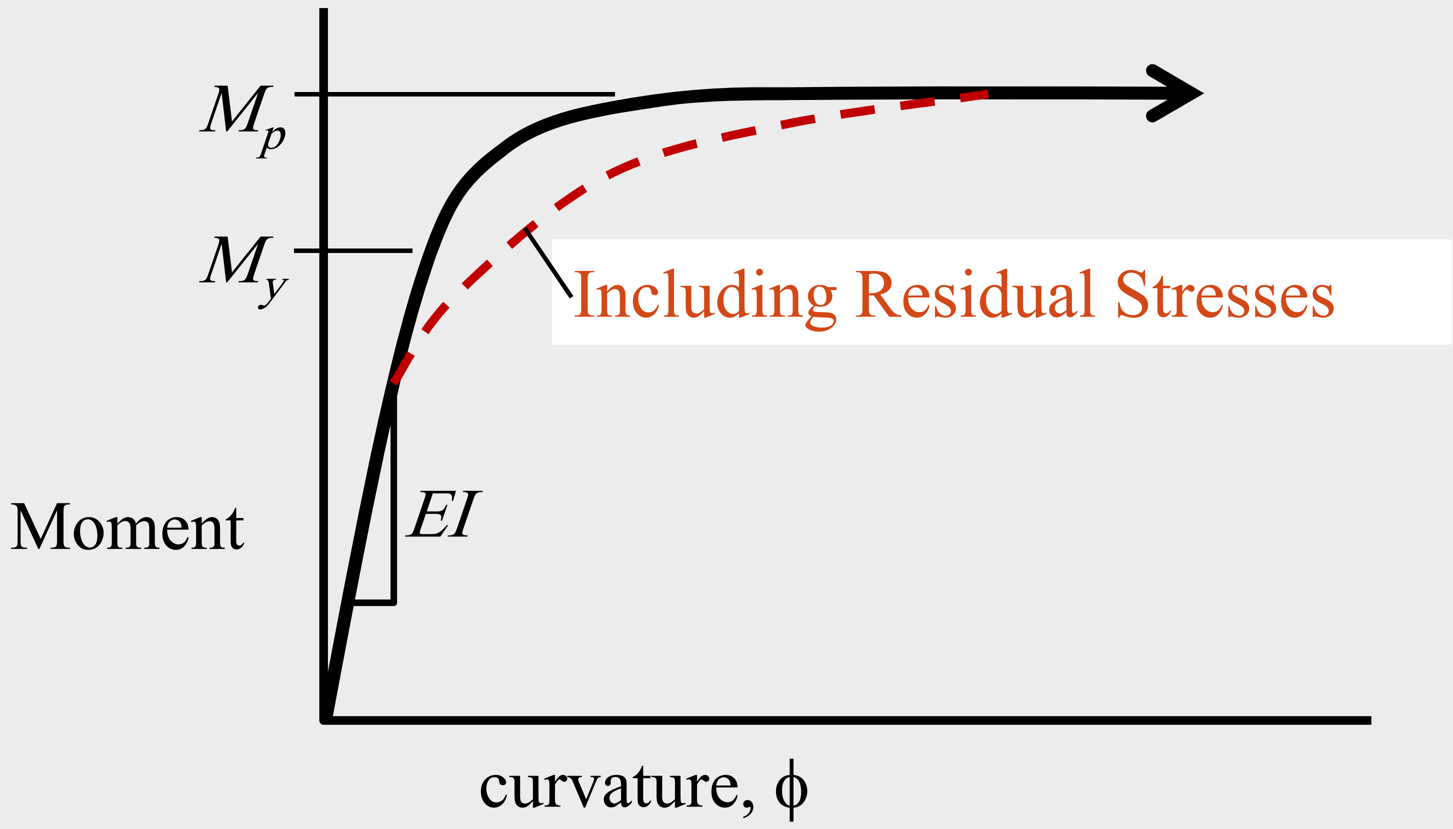
Yield and Plastic Moments

Consider what this does to the Moment-Curvature relationship



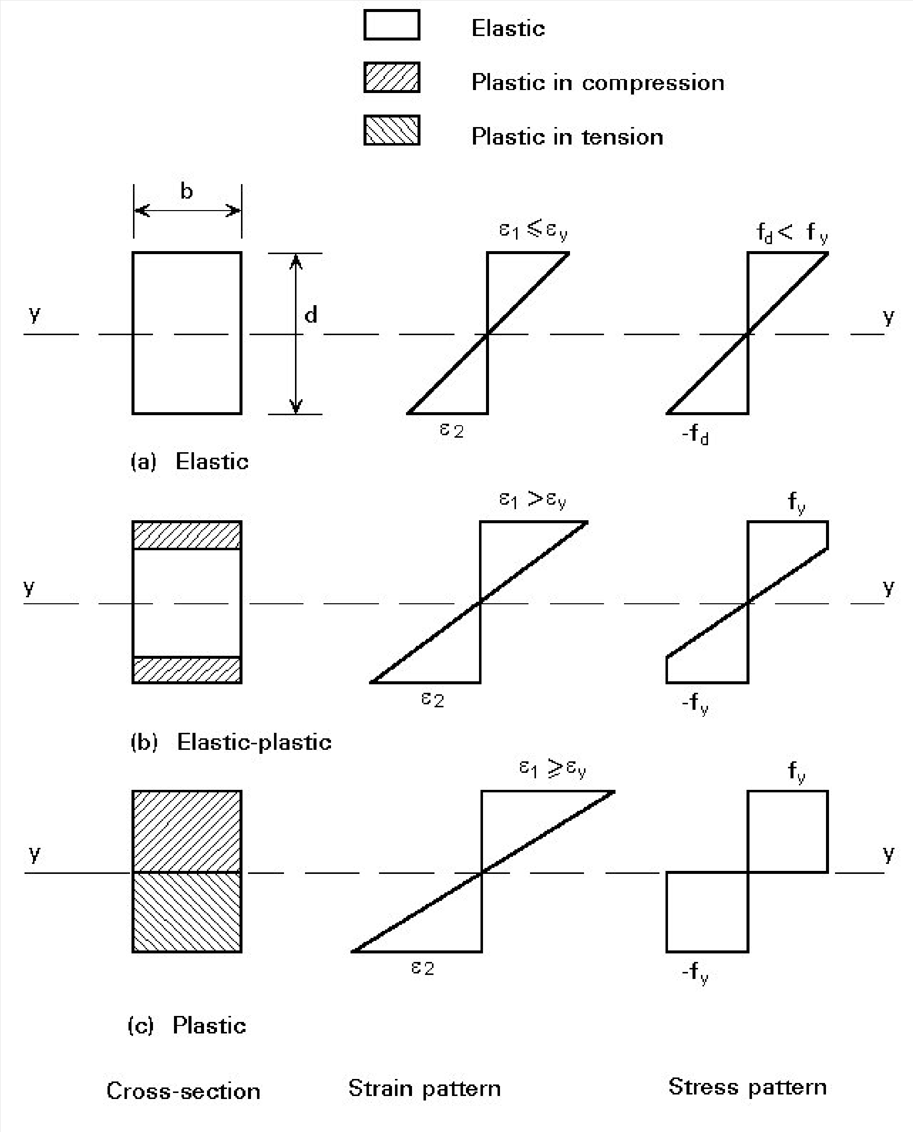
Yield and Plastic Moments

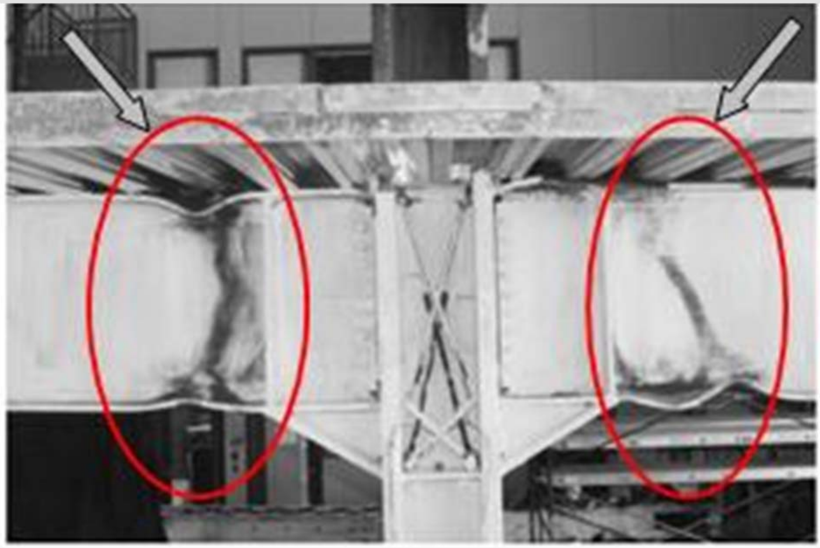
Consider what this does to the Moment-Curvature relationship



Plastic Hinge

Plastic hinge is deformation of a section of a **beam** where **plastic** bending occurs





Beam Stability

(Buckling in Beam)

Beam Stability

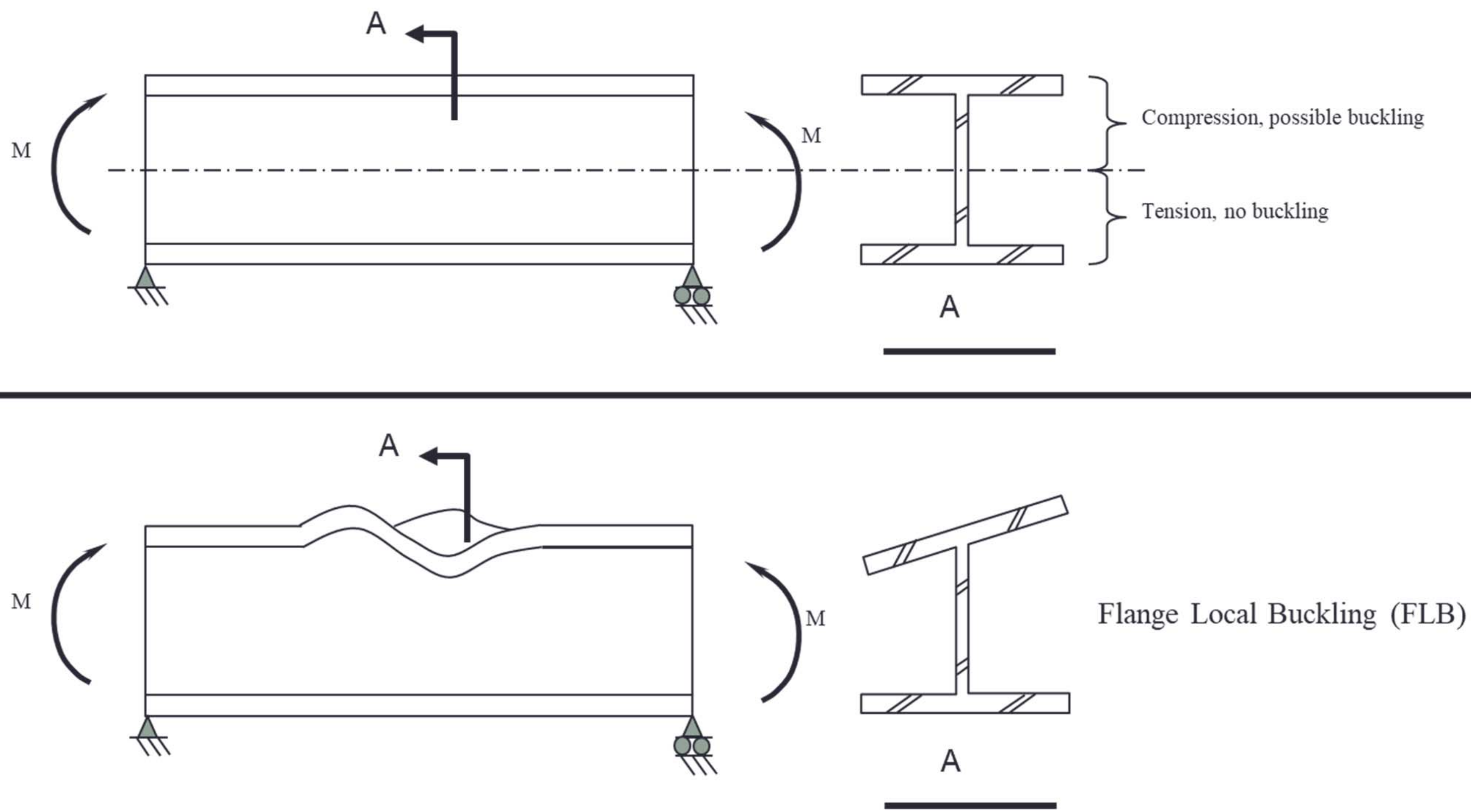
- If a beam can be counted on to remain stable up to the fully plastic condition, the nominal moment strength can be taken as the plastic moment capacity; that is,

$$M_n = M_p$$

- Instability can be in:
  - **a. Local instability**
  - **b. Overall instability**

Local Buckling

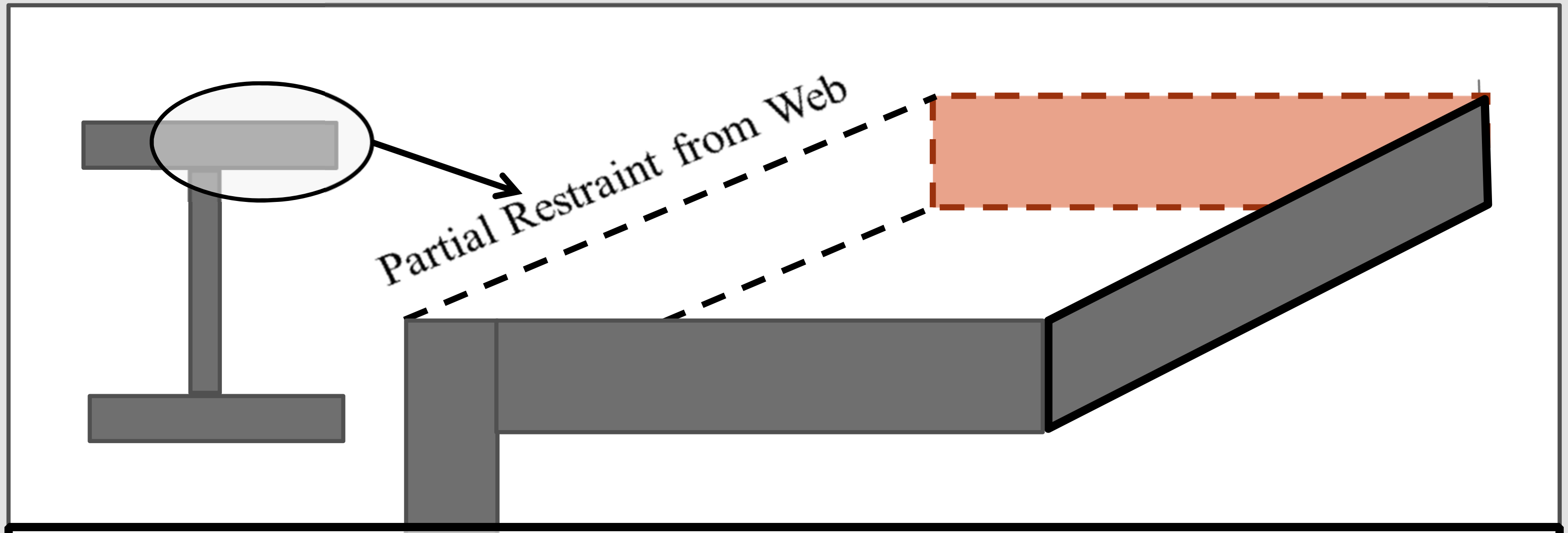
Beam Local Buckling



Compression flange buckling (Flange Local Buckling)

Local Buckling is related to Plate Buckling

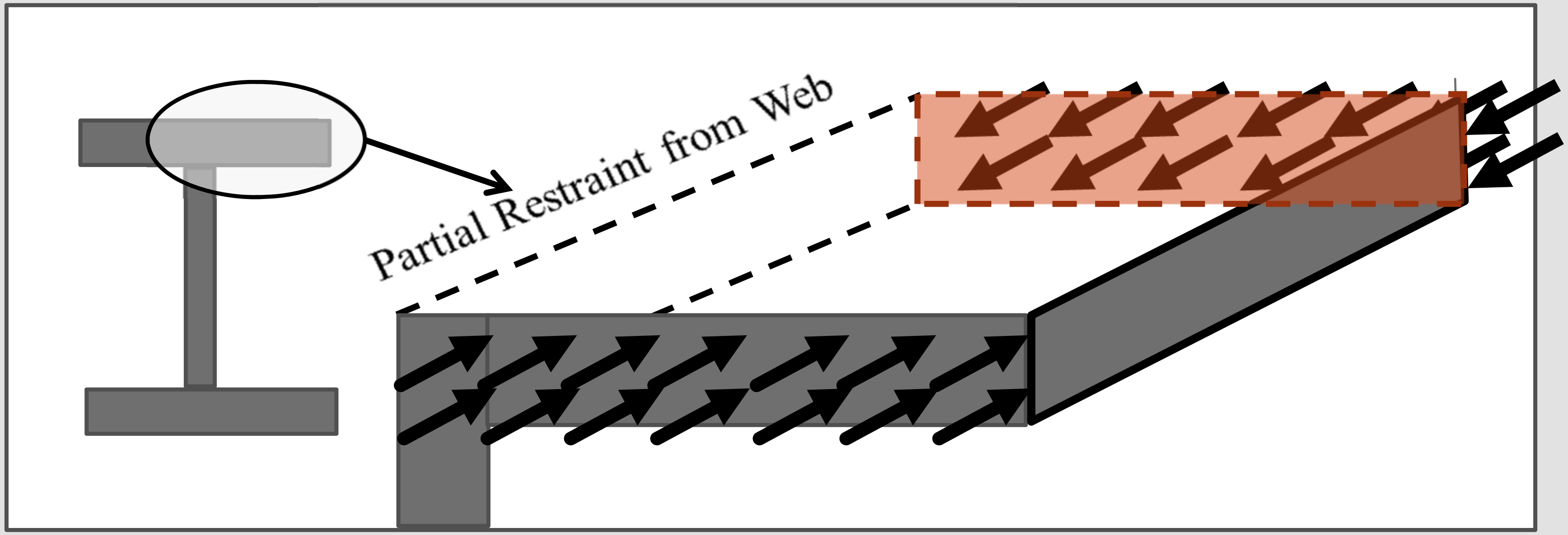
Flange is restrained by the web at one edge.



Failure is localized at areas of high stress (maximum moment) or imperfections.

Local Buckling is related to Plate Buckling

Flange is restrained by the web at one edge.



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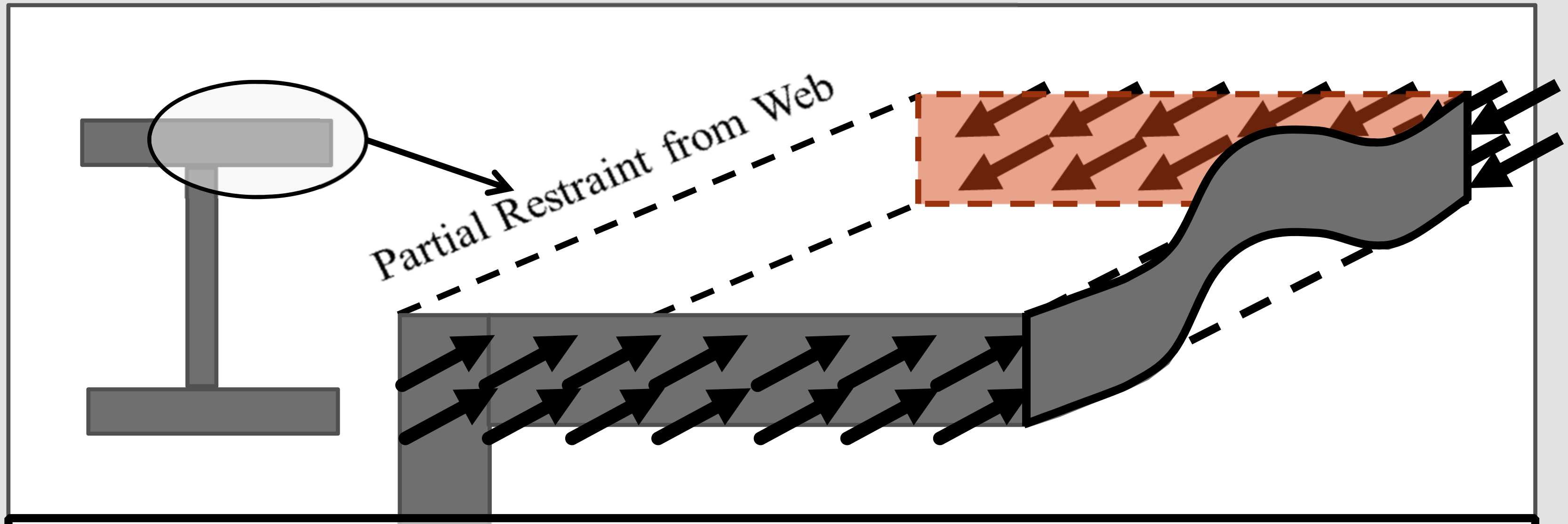

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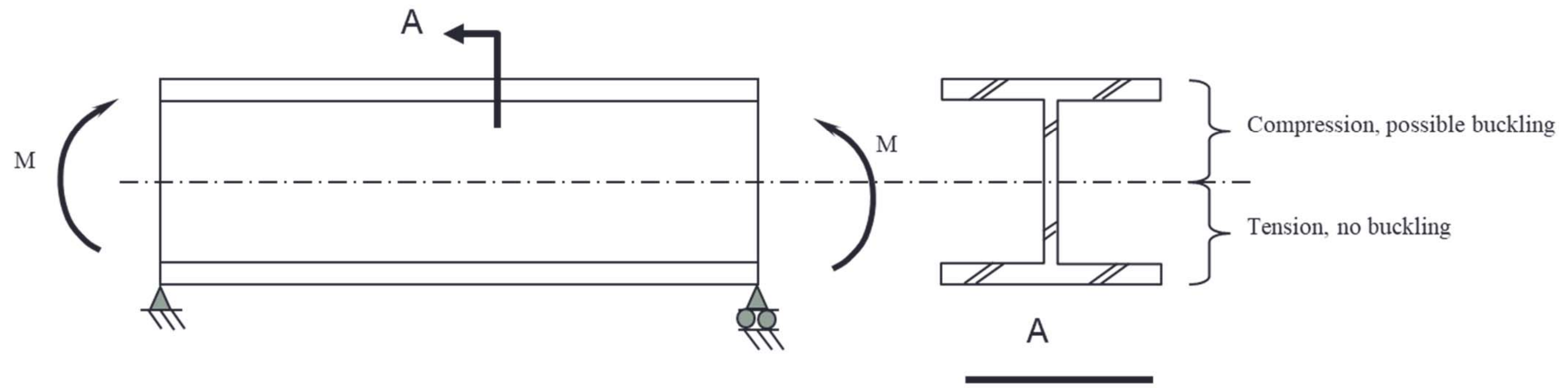
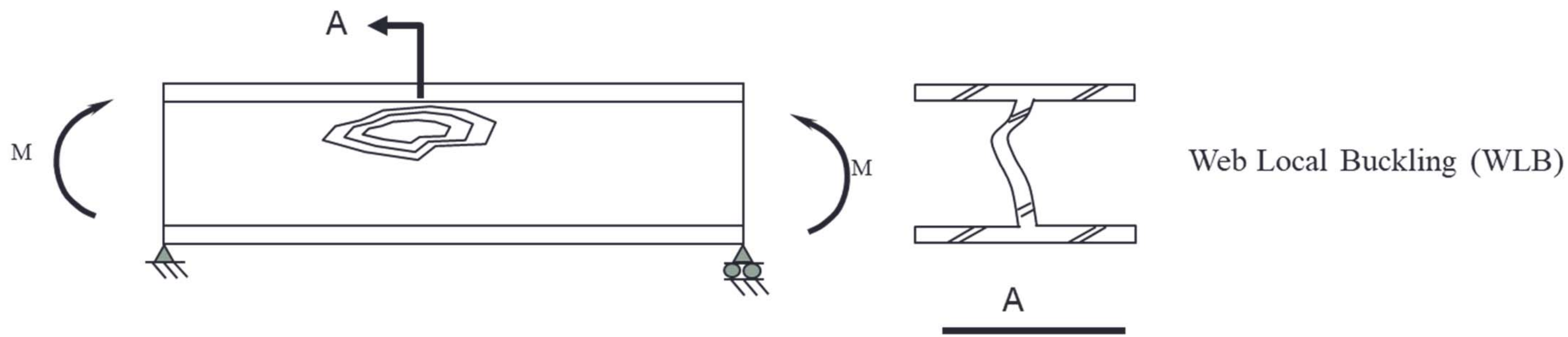
Beam Local Buckling

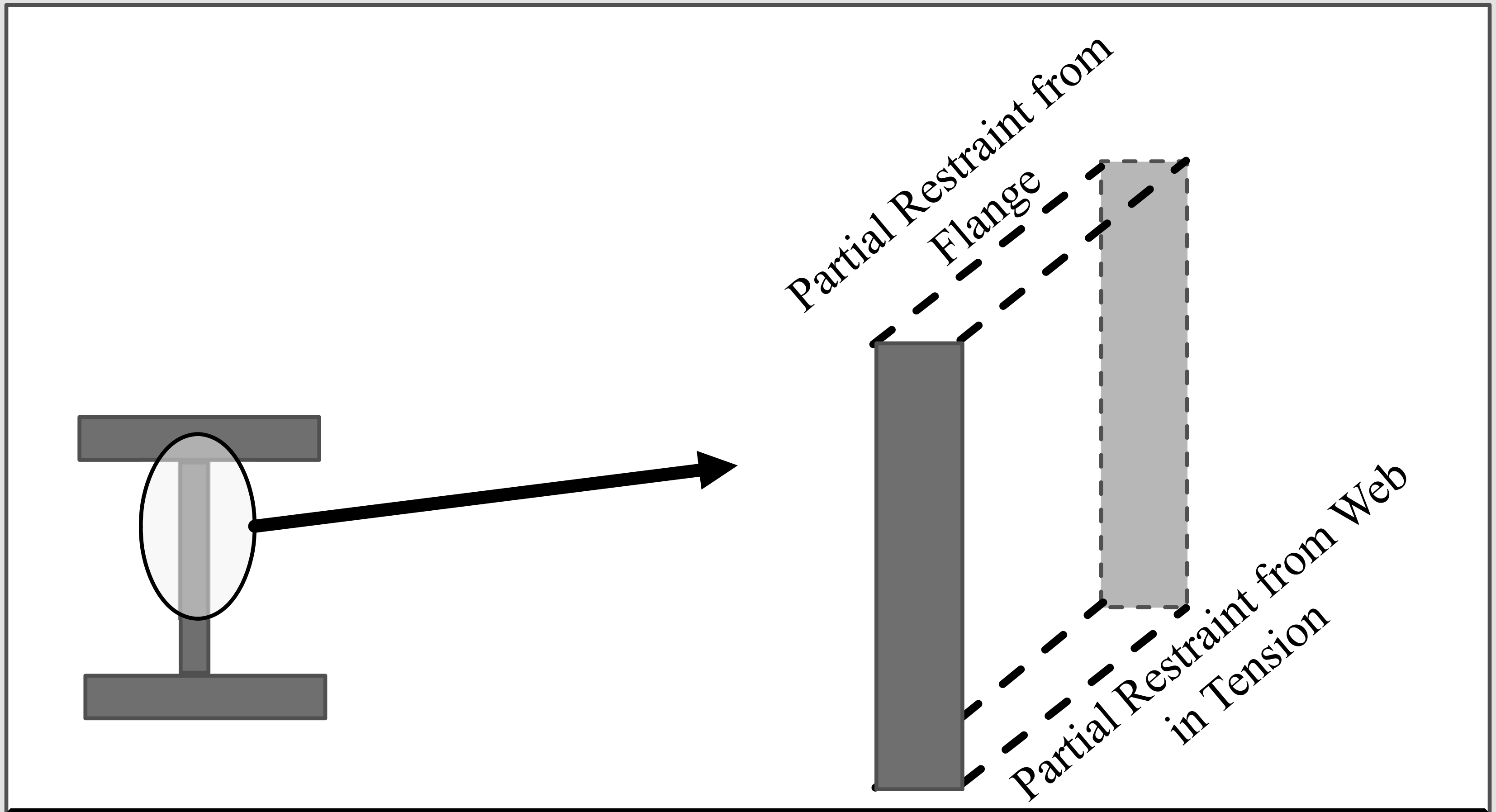




Buckling of the compression part of the web (Web Local Buckling)

Local Buckling is related to Plate Buckling

Web is restrained by the flange at one edge, web in tension at other.



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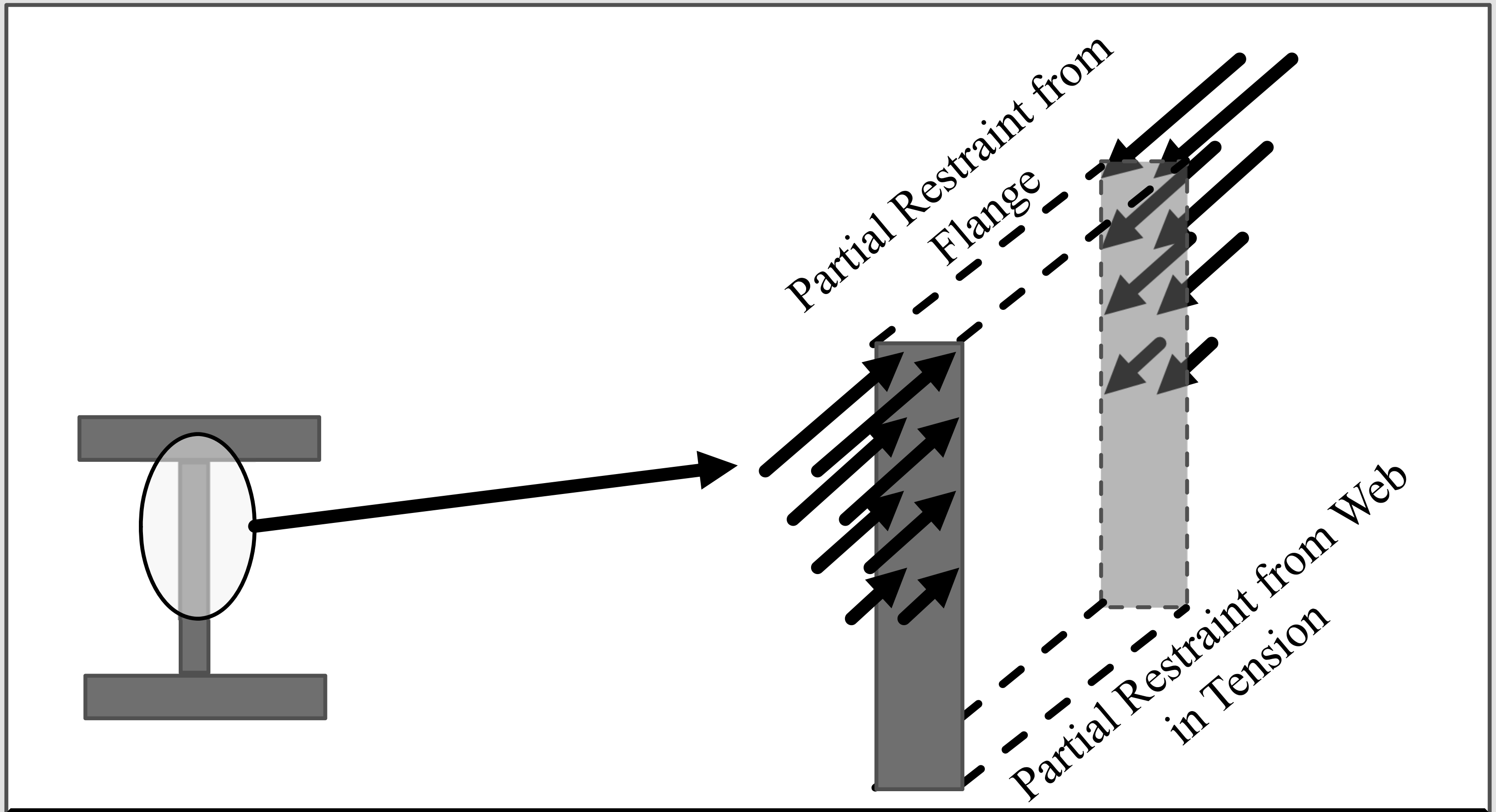

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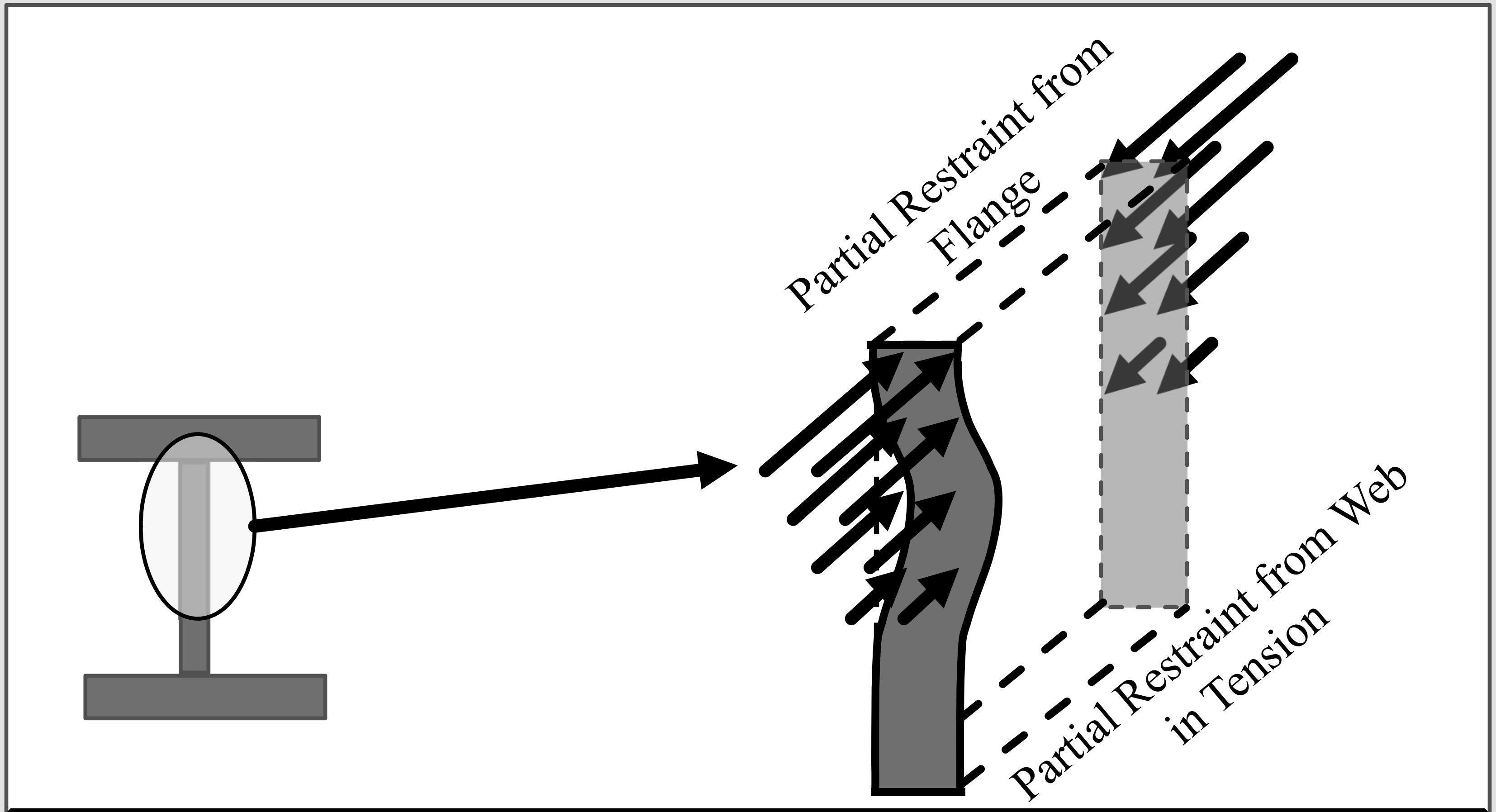
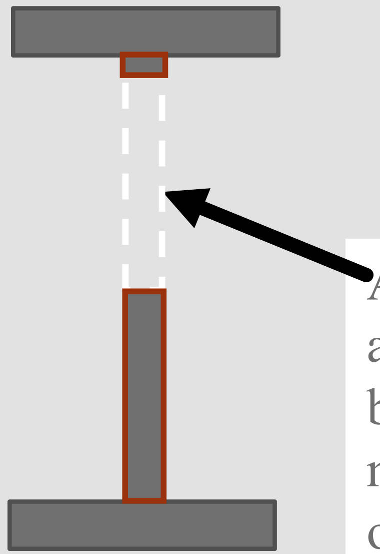


Failure is localized at areas of high stress (maximum moment) or imperfections.

If a web buckles, this is not necessarily a final failure mode. Significant post-buckling strength of the entire section may be possible (see advanced topics).

One can conceptually visualize that a cross section could be analyzed as if the buckled portion of the web is “missing” from the cross section.



Advanced analysis assumes that buckled sections are not effective, but overall section may still have additional strength in bending and shear.

Local Web Buckling Concerns

Bending in the plane of the web;

Reduces the ability of the web to carry its share of the bending moment (even in elastic range).

Support in vertical plane;

Vertical stiffness of the web may be compromised to resist compression flange downward motion.

Shear buckling;

Shear strength may be reduced.

Beam Cross-Sectional Integrity

- Whether the beam can sustain a moment large enough to bring it to the fully plastic condition also depends on whether the ***cross-sectional integrity is maintained.***

- This integrity will be lost if one of the compression elements of the cross section buckles.

- AISC classifies cross-sectional shapes depending on the values of the width-to-thickness ratios as:
  - a. Compact Section
  - b. Non-compact Section
  - c. Slender Section

The classification of shapes is found in Section B4 of the Specification, in Table B4.1

λ = width-to-thickness ratio

λ_p = upper limit for compact category

λ_r = upper limit for noncompact category

Then

if $\lambda \leq \lambda_p$ and the flange is continuously connected to the web, the shape is compact;

if $\lambda_p < \lambda \leq \lambda_r$, the shape is noncompact; and

if $\lambda > \lambda_r$, the shape is slender.

Beam Cross-Sectional Category

The classification of shapes is found in Section B4 of the Specification, in Table B4.1

λ = width-to-thickness ratio

λ_p = upper limit for compact category

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Then

if $\lambda \leq \lambda_p$ and the flange is continuously connected to the web, the shape is compact;

if $\lambda_p < \lambda \leq \lambda_r$, the shape is noncompact; and

if $\lambda > \lambda_r$, the shape is slender.

- ***The category is based on the worst width-to-thickness ratio of the cross section.***

- ***For example, if the web is compact and the flange is noncompact, the shape is classified as noncompact***

Beam Cross-Sectional Category

Summary For hot-rolled I shapes in flexure

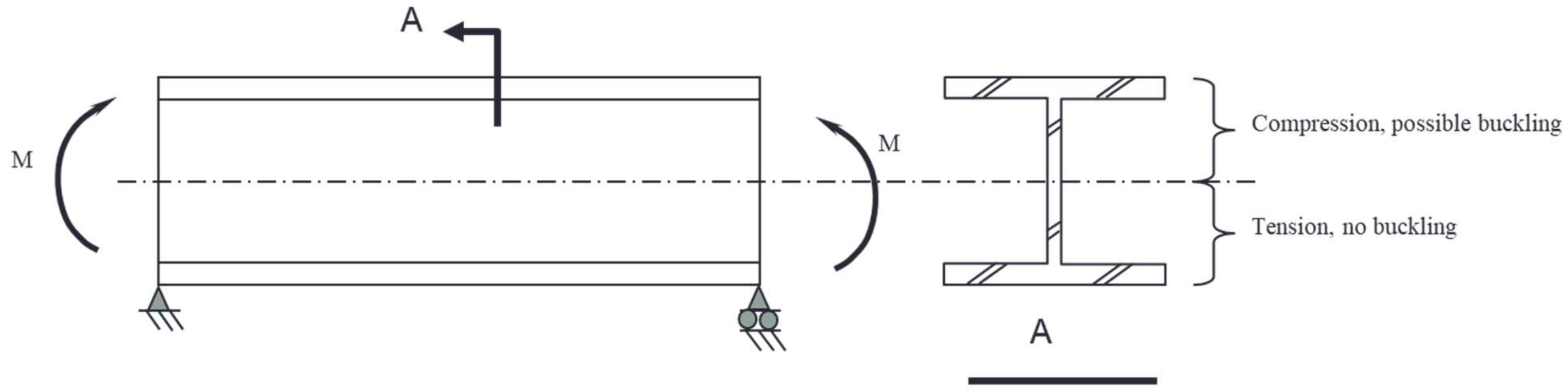
Element	λ	λ_p	λ_r
Flange	$\dfrac{b_f}{2t_f}$	$0.38\sqrt{\dfrac{E}{F_y}}$	$1.0\sqrt{\dfrac{E}{F_y}}$
Web	$\dfrac{h}{t_w}$	$3.76\sqrt{\dfrac{E}{F_y}}$	$5.70\sqrt{\dfrac{E}{F_y}}$

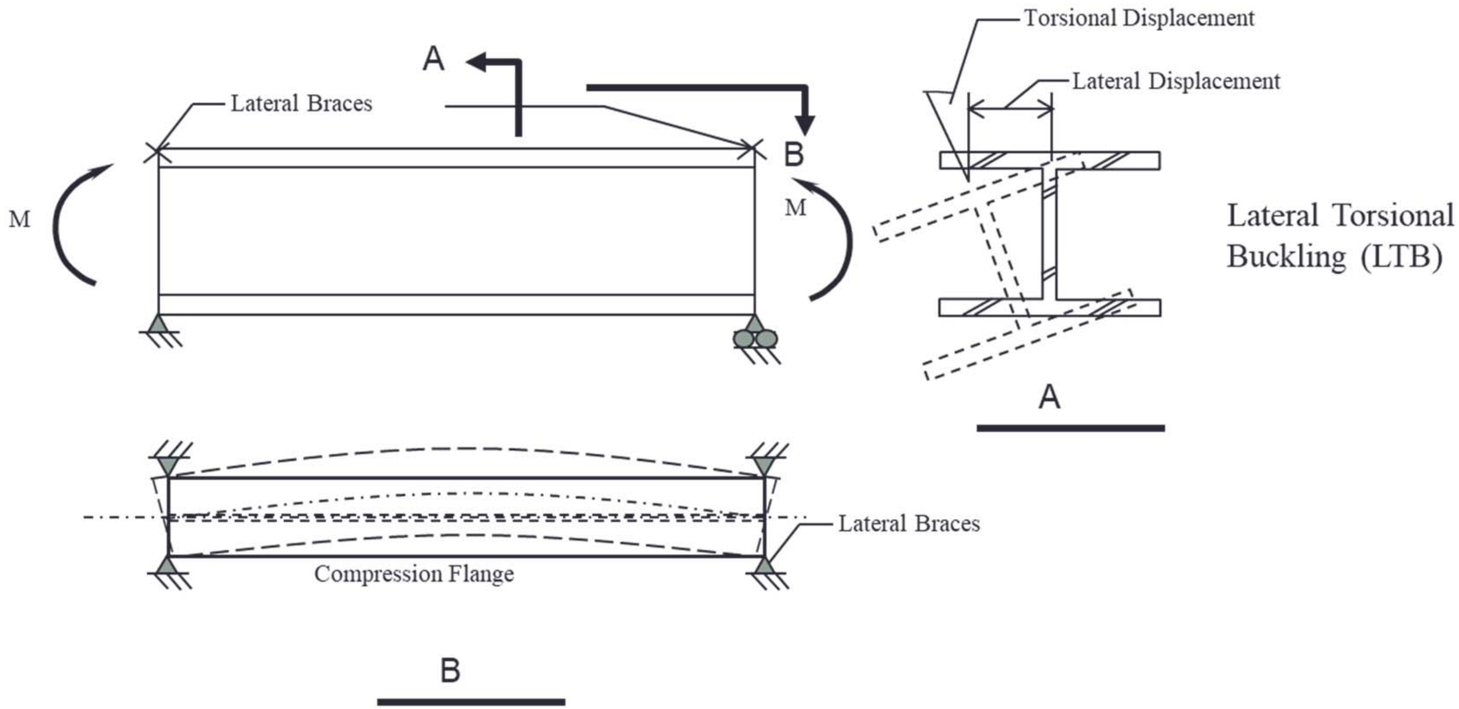
For I shapes:

- The ratio for the projecting flange (*unstiffened* element) is $b_f / 2t_f$
- The ratio for the web (*stiffened* element) is h / tw

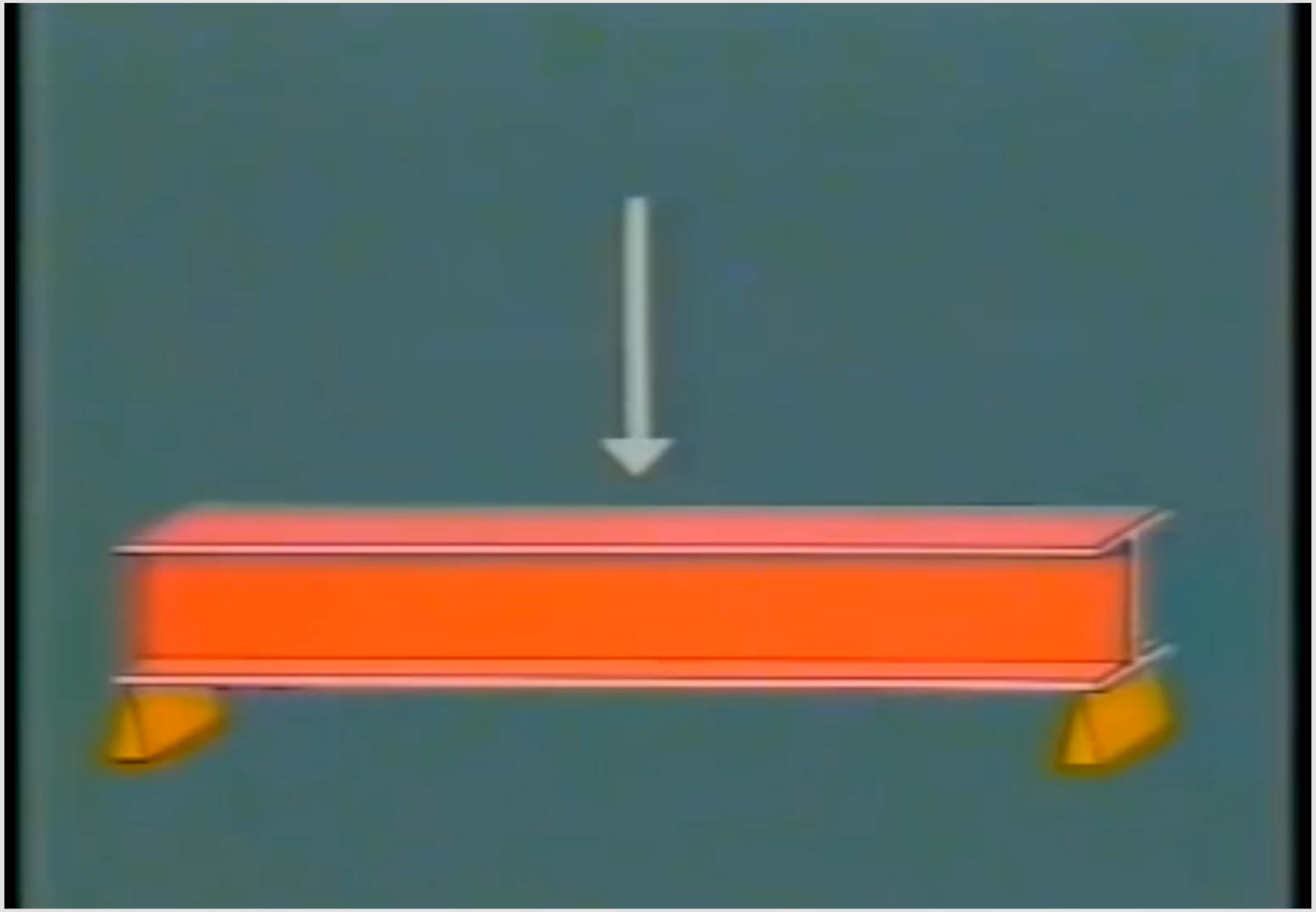
Beam Over all Buckling

Beam Over all Buckling

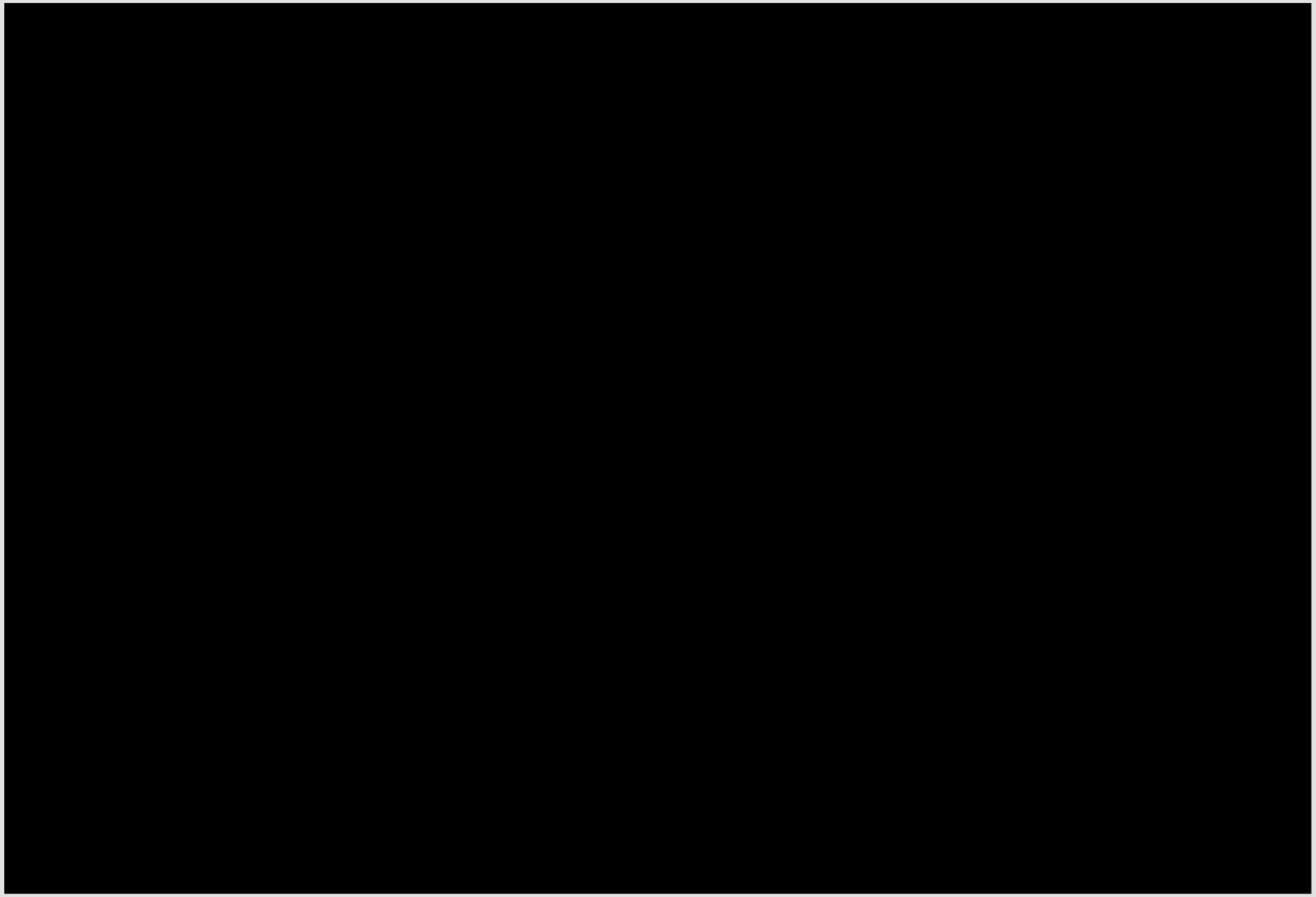




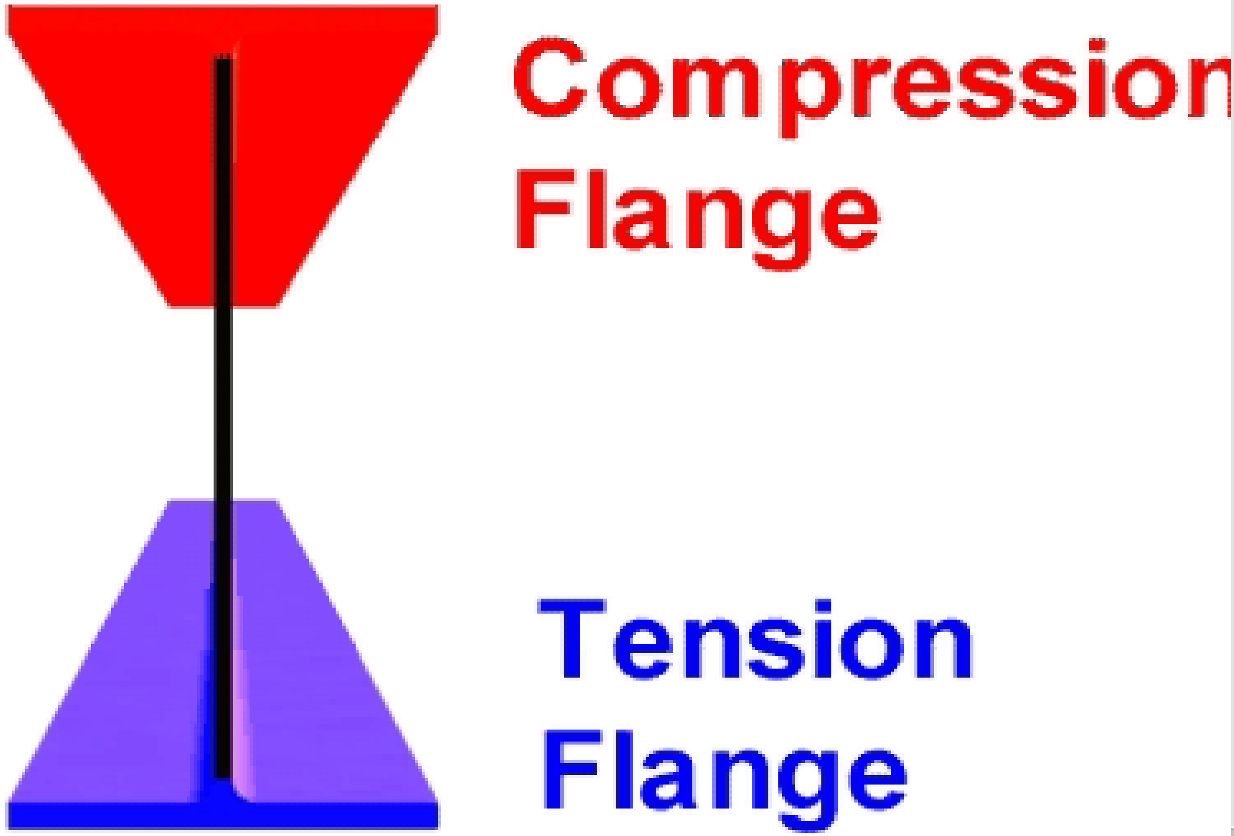
Over all Buckling of Beam



Over all Buckling of Beam



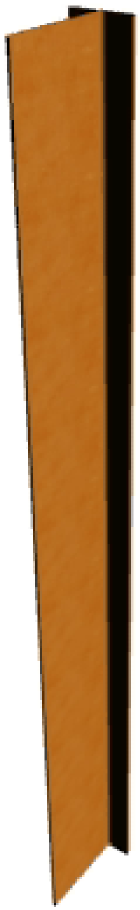
A compression flange by itself would tend to buckle much like a column

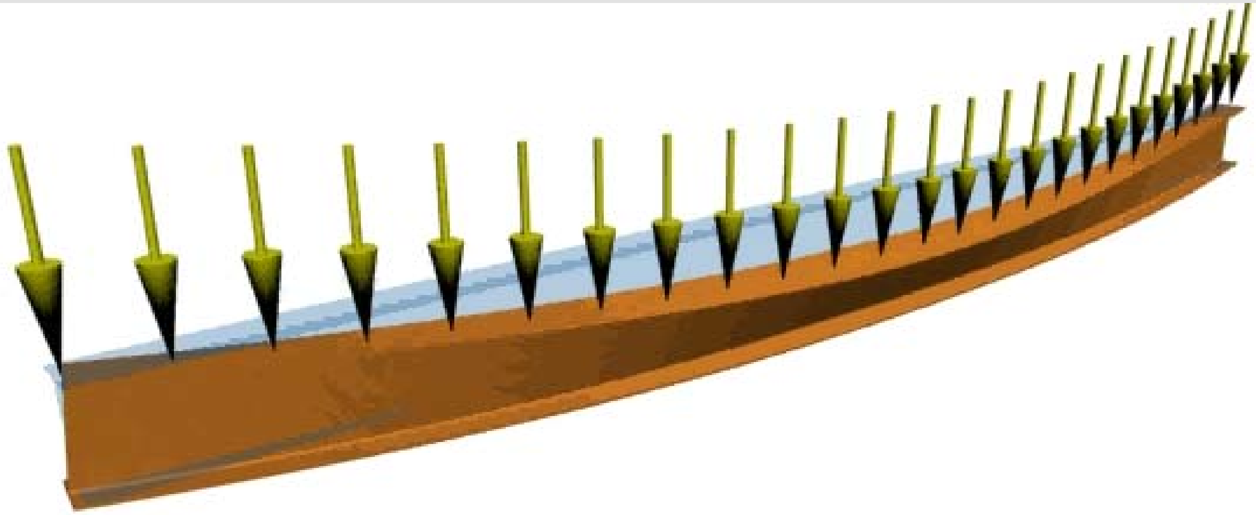


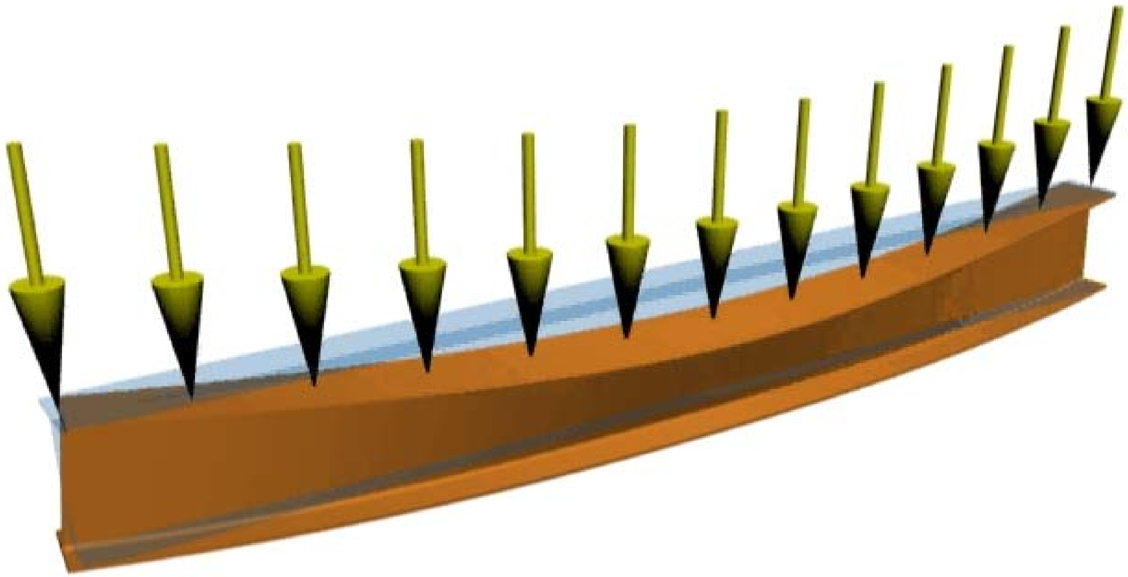
The web, however prevents the flange from buckling about its minor axis

But if there is enough compression with no lateral support, the flange and web will buckle together

The tension flange, which is stable, restrains the compression flange and web causing the beam to twist. The result is Lateral Torsional Buckling (LTB)







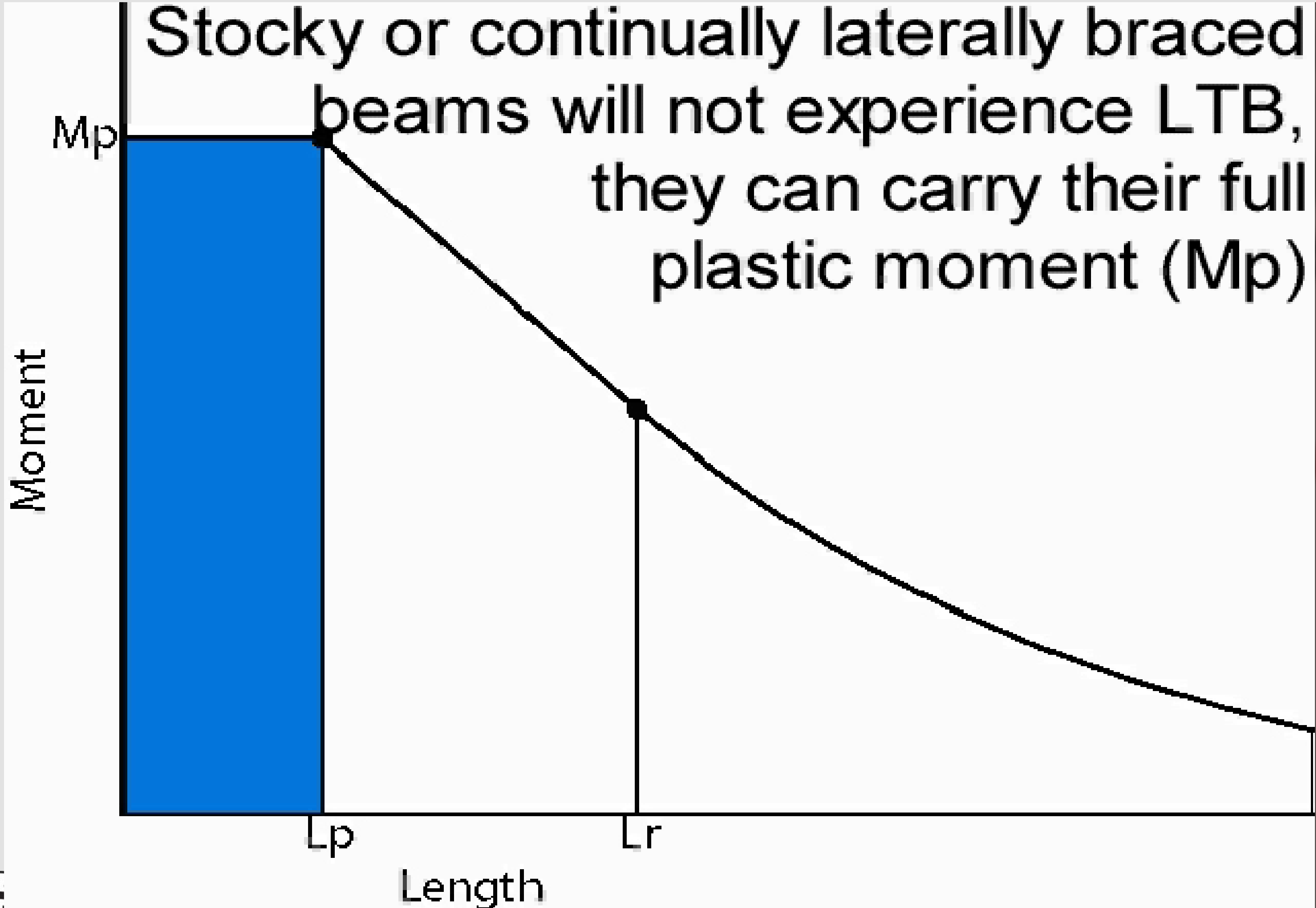
Lateral Torsional Buckling
(LTB)

Lateral Torsional Buckling

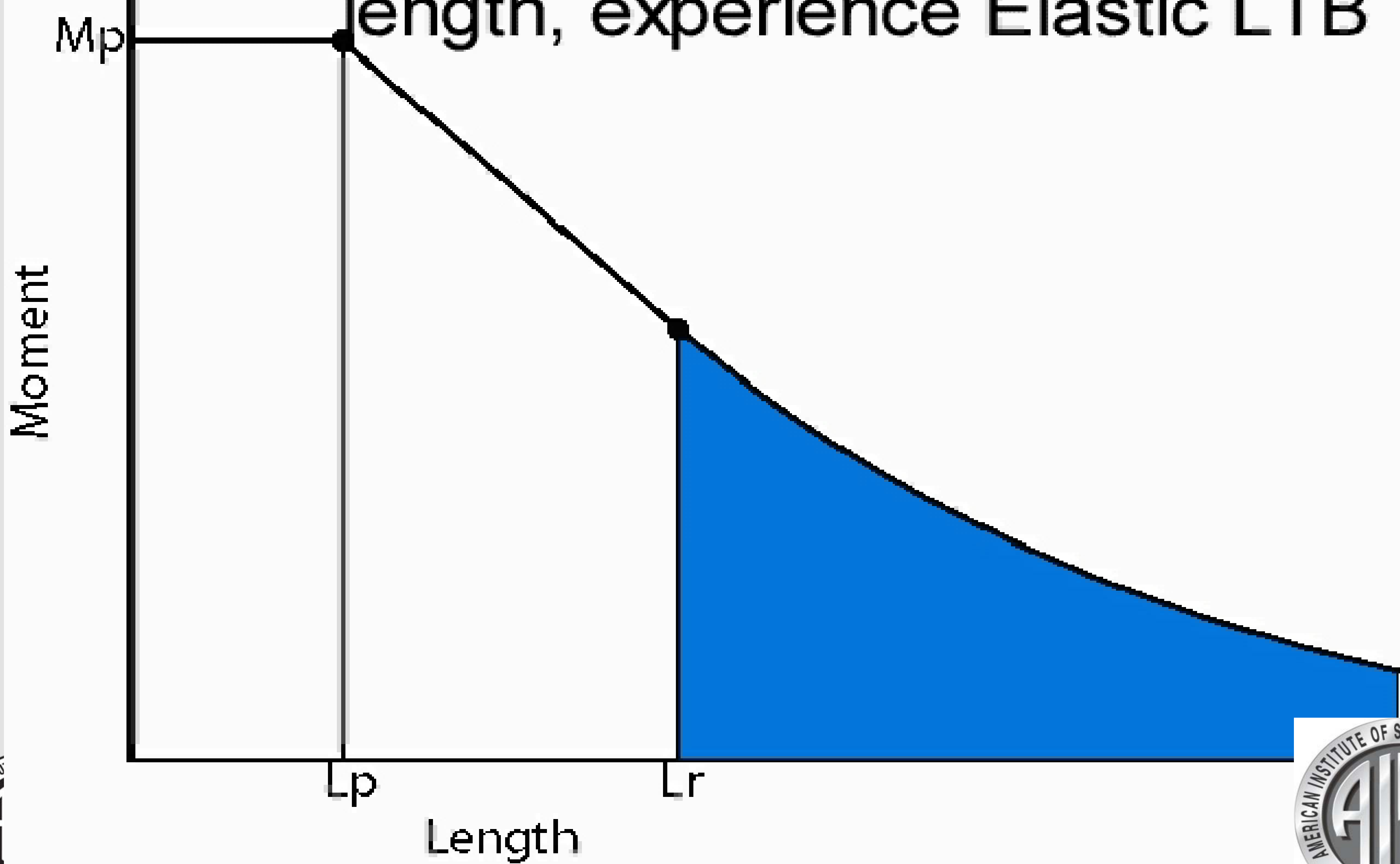
LTB occurs along the length of the section.

Compression flange tries to buckle as a column.
Tension flange tries to stay in place.

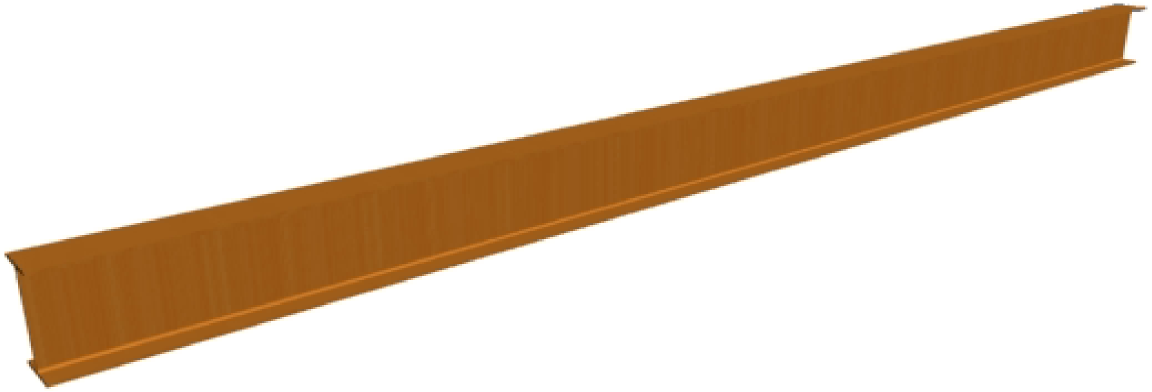
Result is lateral movement of the compression flange and torsional twist of the cross section.

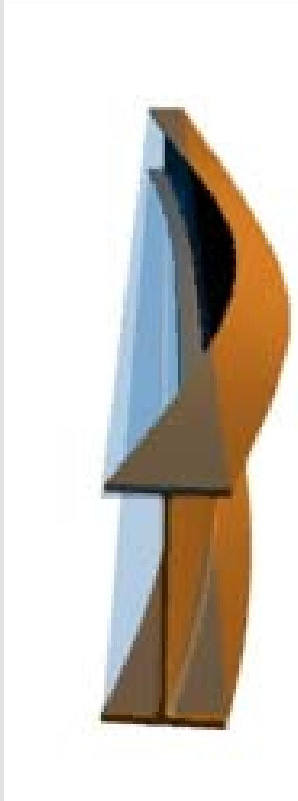

Stocky or continually laterally braced beams will not experience LTB, they can carry their full plastic moment (Mp)
Mp
Moment
Lp
Lr
Length

Slender beams with a long unbraced length, experience Elastic LTB

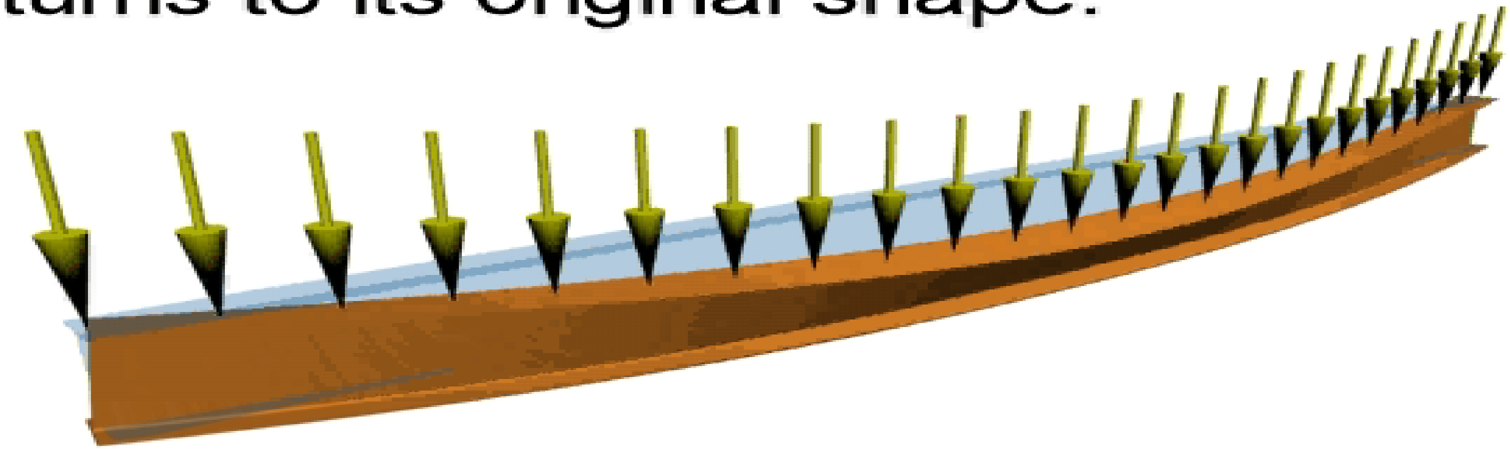


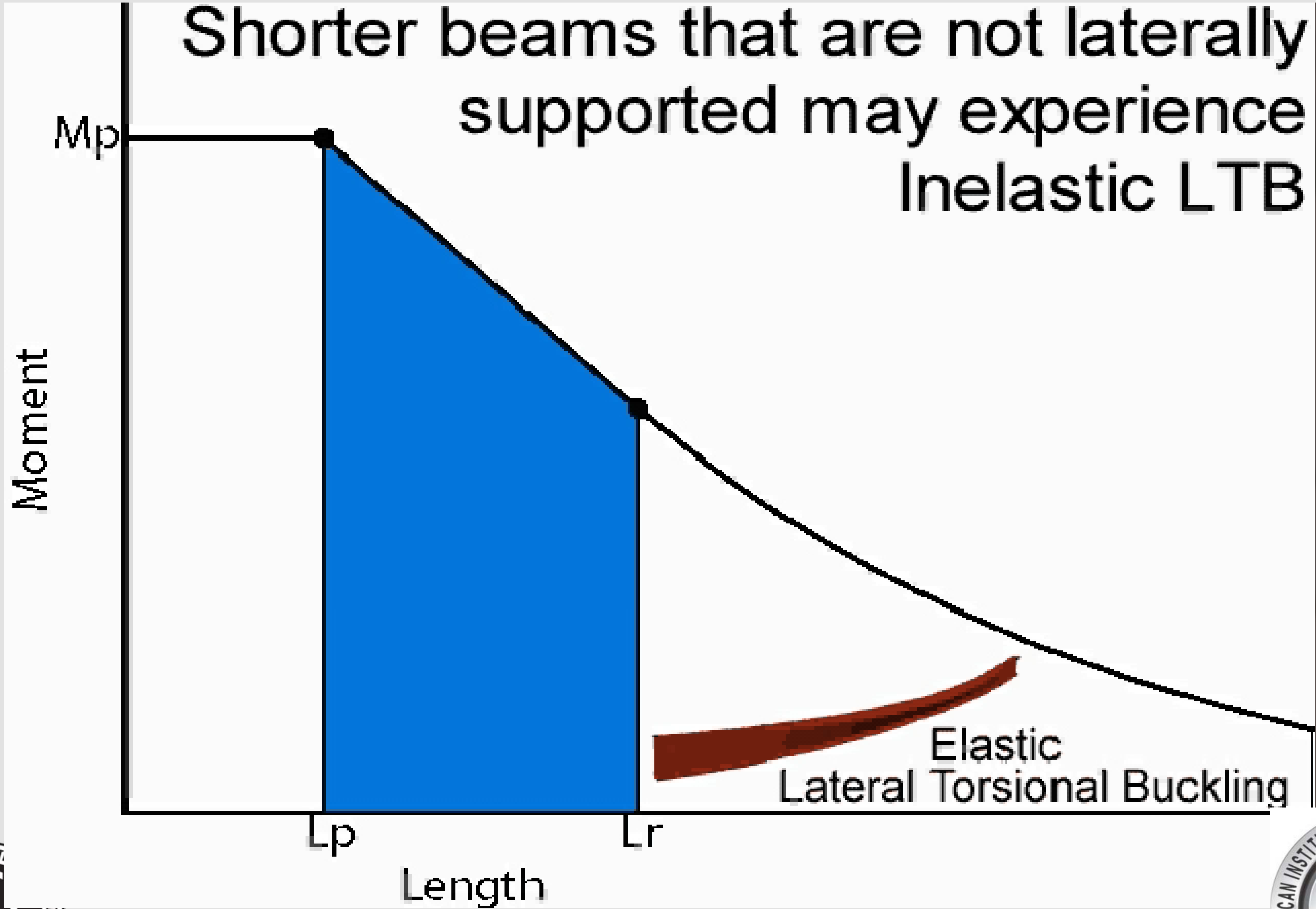
Elastic LTB occurs when the member is slender enough to deflect without yielding.



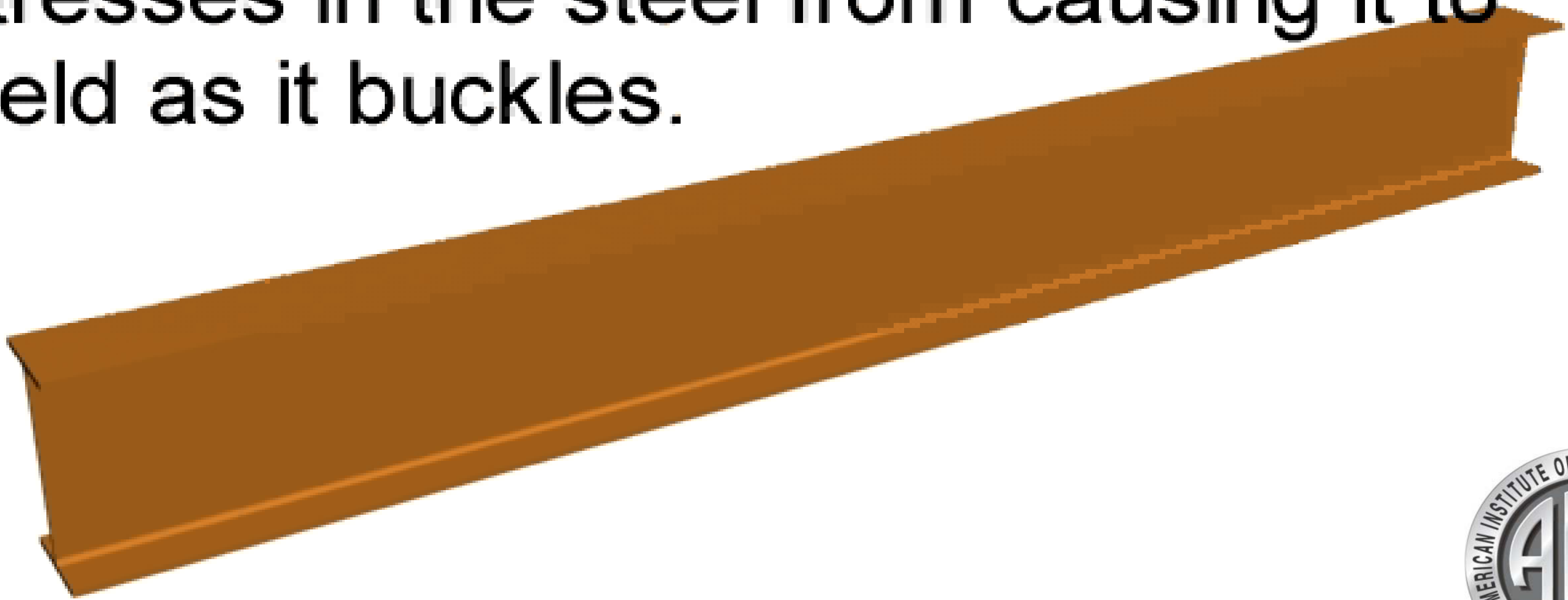


This type of buckling is elastic. When the load is removed, the member returns to its original shape.

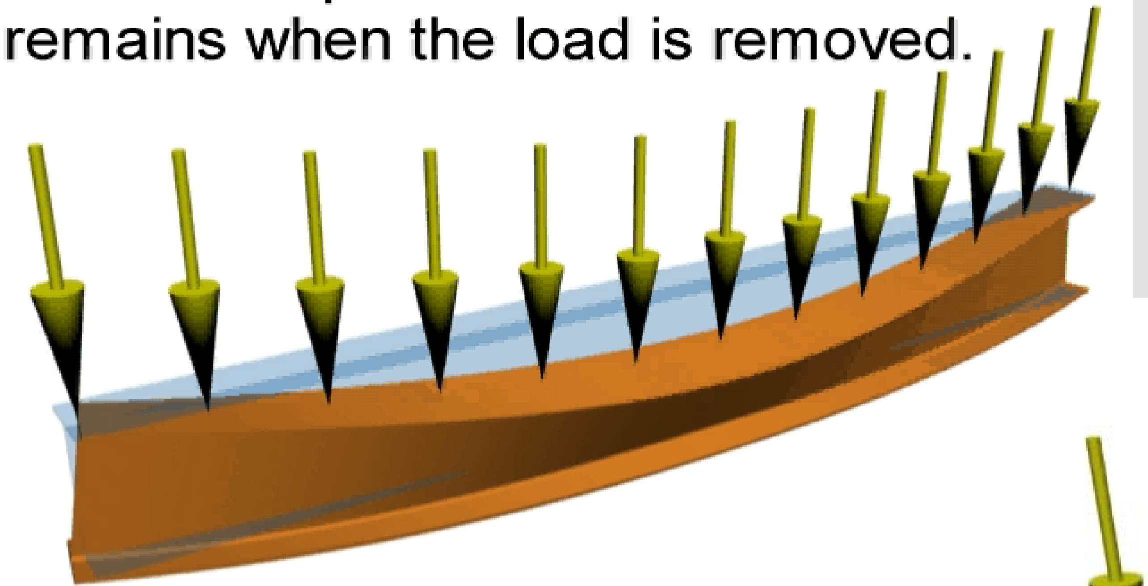


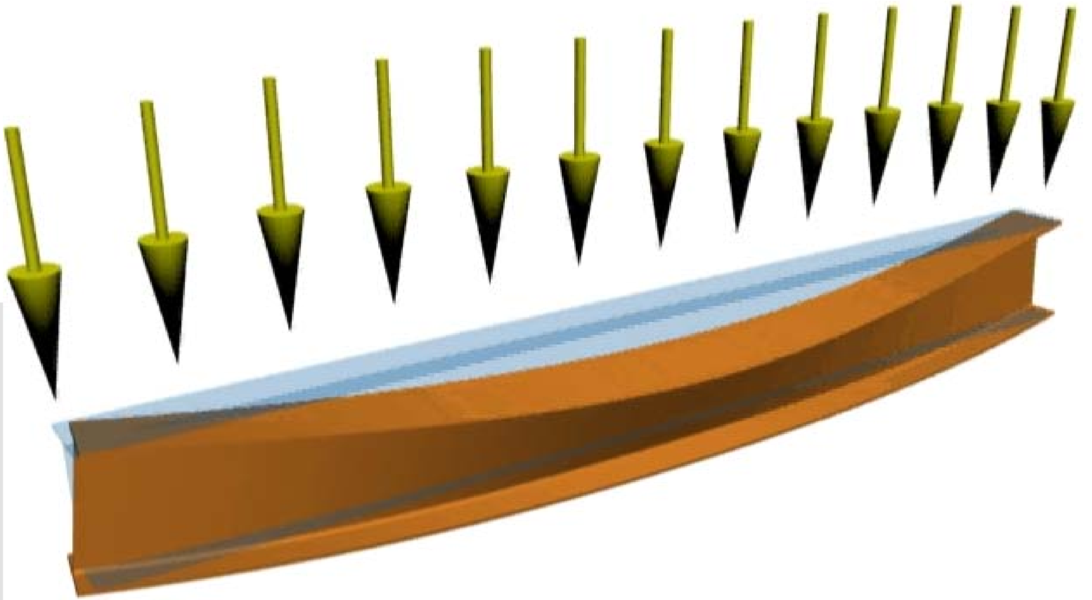

Shorter beams that are not laterally supported may experience Inelastic LTB
Mp
Moment
Elastic
Lateral Torsional Buckling
Lp
Lr
Length

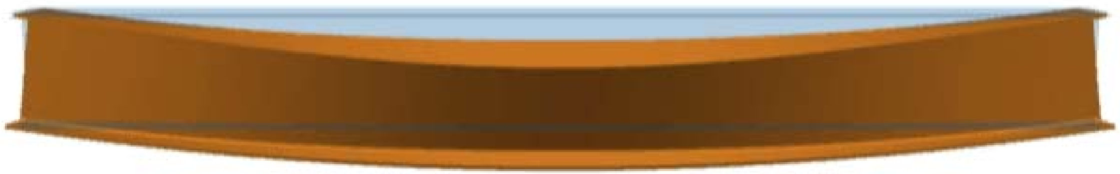
Inelastic LTB is similar to elastic LTB, but the shorter unbraced length allows for a larger capacity and does not allow enough deflection to prevent the residual stresses in the steel from causing it to yield as it buckles.



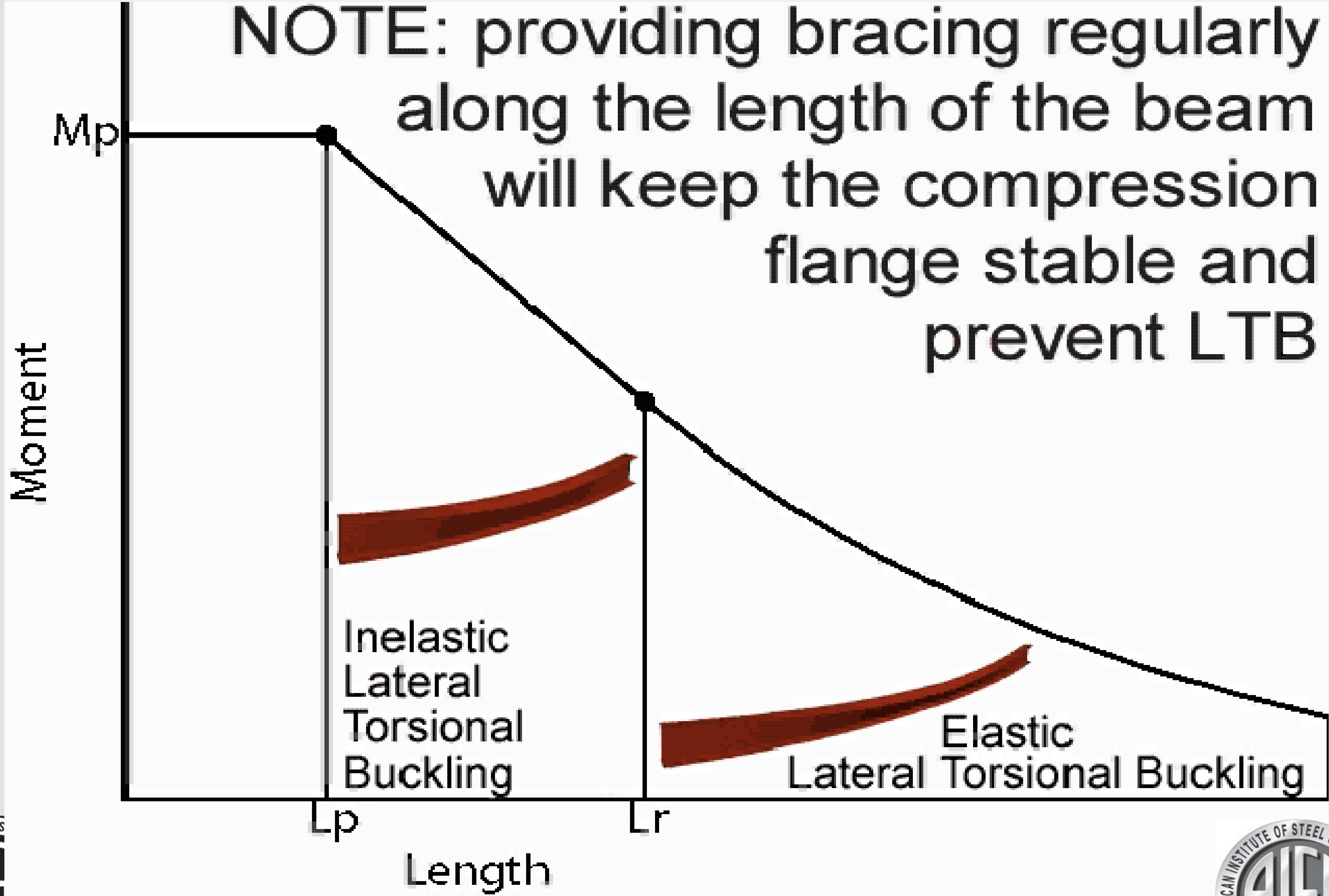
This type of buckling is inelastic. The non-elastic portion of the deformation remains when the load is removed.

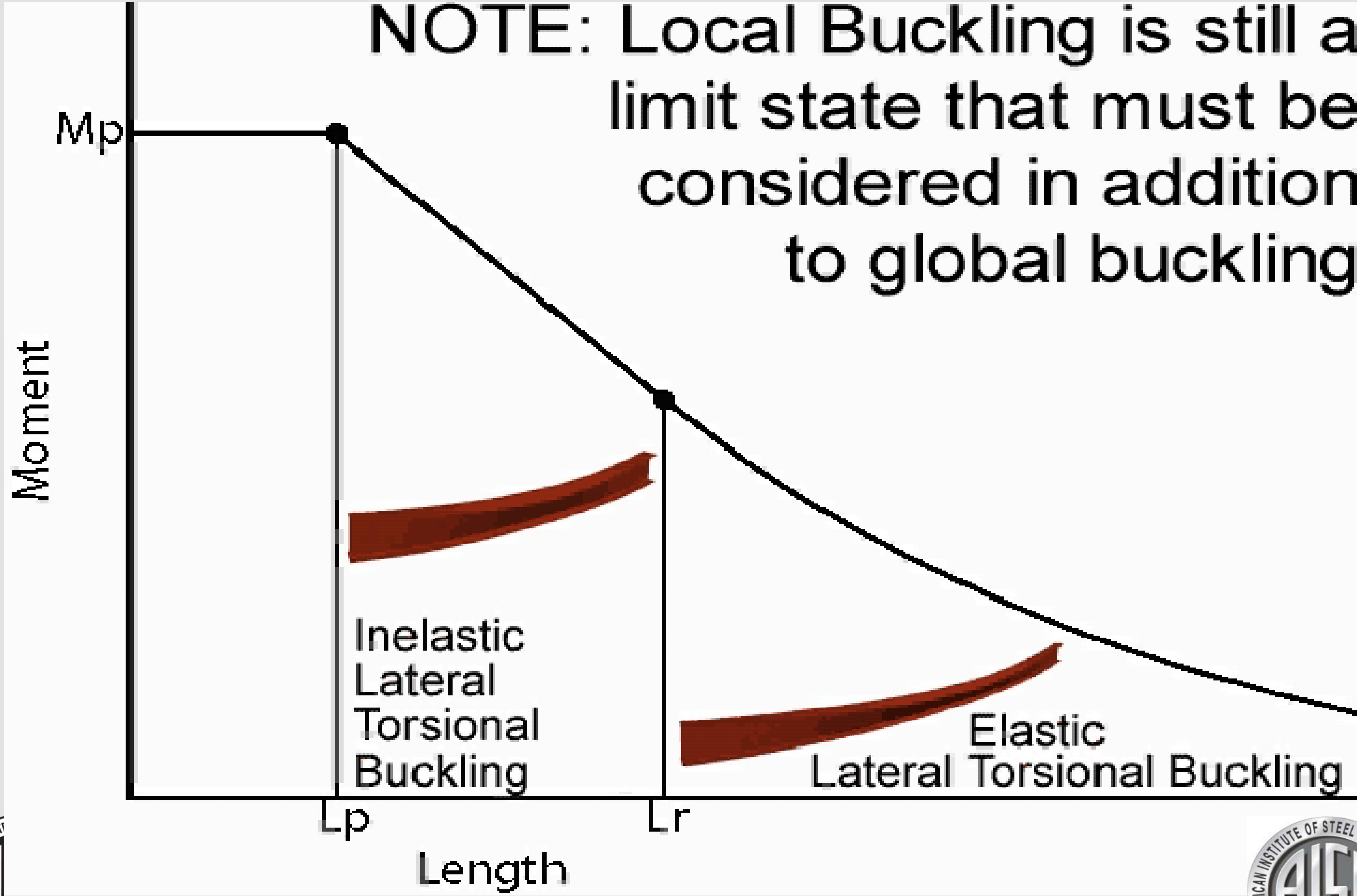






NOTE: providing bracing regularly along the length of the beam will keep the compression flange stable and prevent LTB




NOTE: Local Buckling is still a limit state that must be considered in addition to global buckling
Mp
Moment
Inelastic Lateral Torsional Buckling
Elastic Lateral Torsional Buckling
Lp
Lr
Length

LATERAL SUPPORT

Fully unsupported beams.
The unbraced length is equal to the entire length of the member.

These beams are continually supported with the deck that is mechanically attached with screws or welds.

These girders are fully unsupported temporarily during construction.

Once the joists are placed in position and attached, the unbraced length is reduced to the joist spacing.

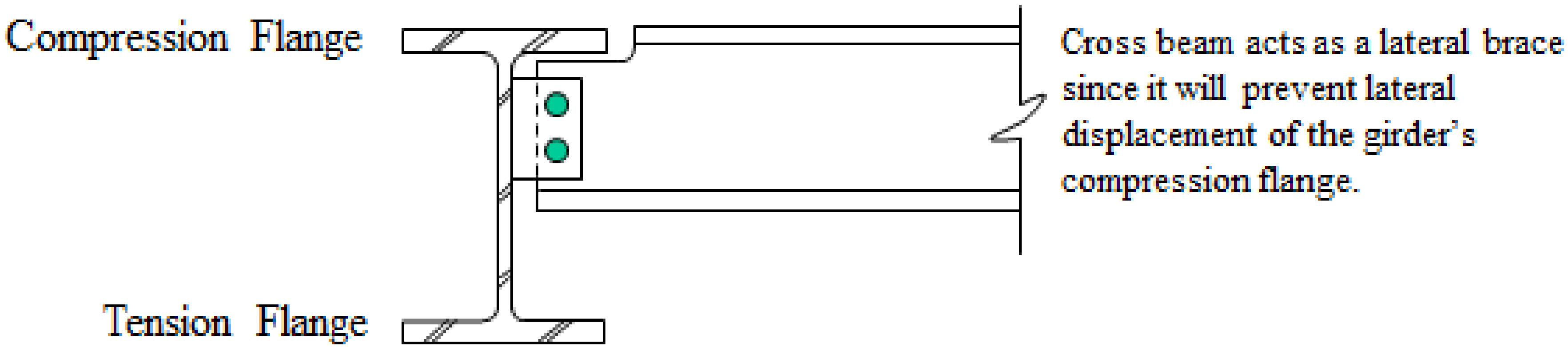
Here, beams are used as blocking and spaced to provide the necessary lateral support.

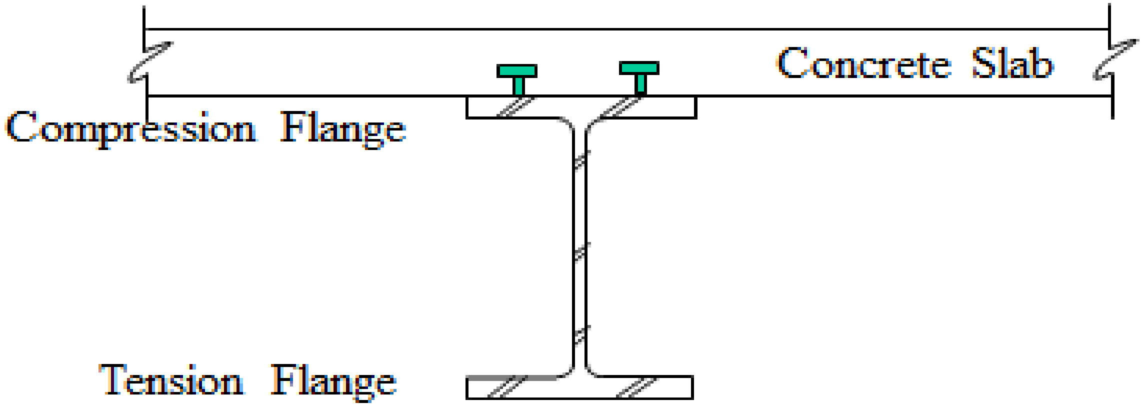
Lateral bracing must be used for any member loaded in bending. It could be curved or in any orientation.

LATERAL SUPPORT

Beam Lateral Bracing Examples

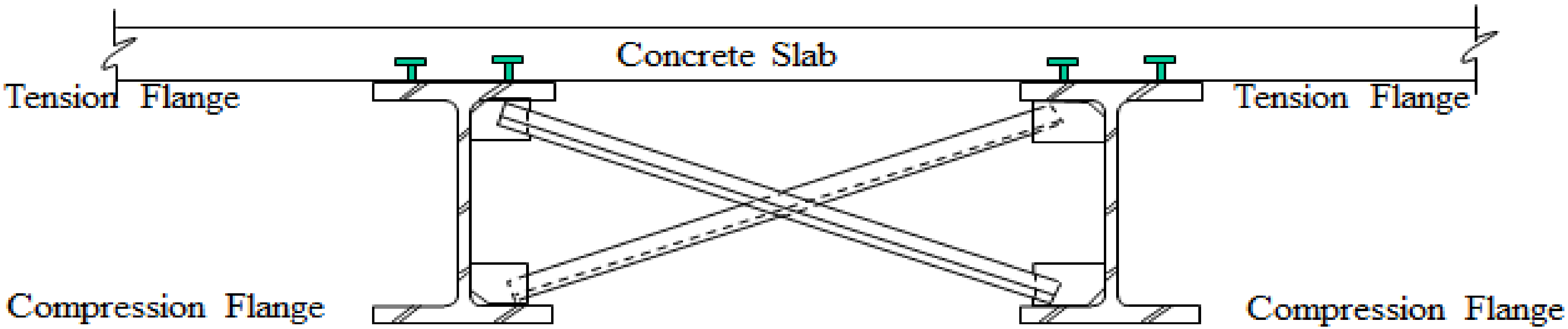
Brace must either prevent lateral displacement of the compression flange, or twist of the cross section





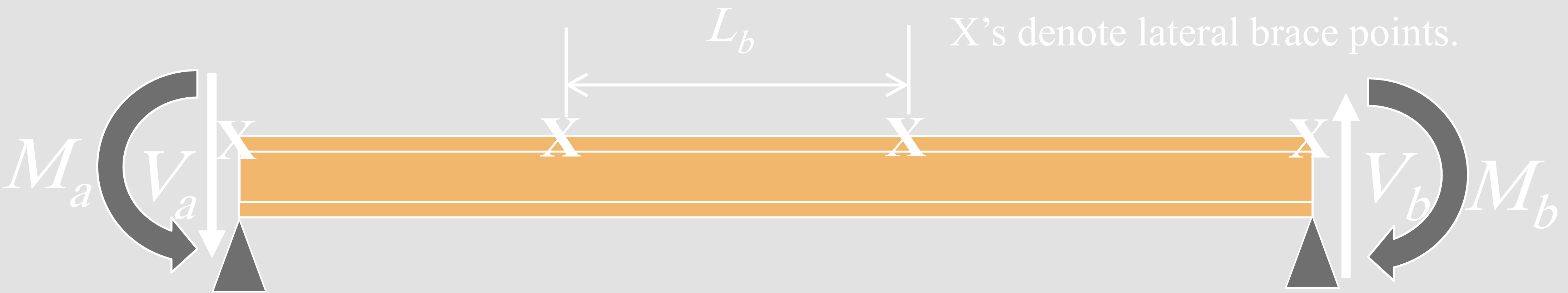
Continuous concrete floor slab provides continuous bracing for the compression flange, $L_b=0$, no LTB.

Note that if the bottom flange was in compression there would be no lateral bracing provided.



Lateral Displacement of the compression flange is prevented by the diagonal members (typically angles)

Lateral Torsional Buckling



L_b is referred to as the unbraced length.

Braces restrain EITHER:

Lateral movement of compression flange or

Twisting in torsion.

Lateral Torsional Buckling

FACTORS IN LTB STRENGTH

L_b - the length between beam lateral bracing points.

C_b - measure of how much of flange is at full compression within L_b.

F_y and residual stresses (1st yield).

Beam section properties - J, C_w, r_y, S_x, and Z_x.

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Lateral Torsional Buckling

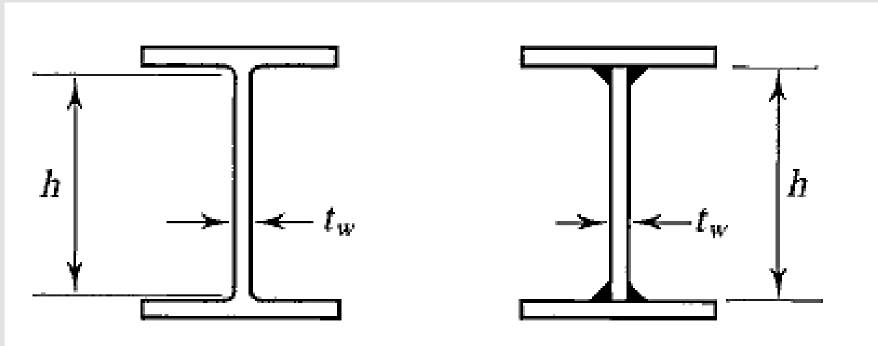
The following sections have inherent restraint against LTB for typical shapes and sizes.

W shape bent about its minor axis.
Box section about either axis.
HSS section about any axis.

For these cases LTB typically does not occur.

SECTIONS

Commonly used cross-sectional shapes include:



- W
- S
- M shapes.
- Channel shapes are sometimes used, as are beams built up from plates, in the form of I or box shapes.

For reasons already discussed , doubly symmetric shapes such as the standard rolled W, M, and S shapes are the most

SECTIONS

- ✓ Hot-rolled shapes, for all the standard sections in the *Manual*, the webs are compact.
- ✓ Some have noncompact flanges
- ✓ None have slender flanges.

GIRDERS

- **Girders are large flexural members that are composed of plate elements—in particular, those with noncompact or slender webs.**

- **With shapes built up from plates, both flanges and webs can be compact, noncompact, or slender.**

- **These built-up shapes usually are used when the bending moments are larger than standard hot-rolled shapes can resist, usually because of a large span.**

- **These girders are invariably very deep, resulting in noncompact or slender webs.**

BEAM
AISC Requirements for Beam

- **Beam Members:**
  - Chapter F: Flexural Strength
  - Chapter G: Shear Strength
  - Part 3: Design Charts and Tables
  - Chapter B: Local Buckling Classification

Flexural Strength

Flexural Strength Limit States:

- Plastic Moment Strength
- Lateral Torsional Buckling
- Local Buckling (Flange or web)

Flexural Strength Requirements:

$M_u \leq \phi M_n$ LRFD ($M_a \leq M_n/\Omega$) ASD

$V_u \leq \phi V_n$ LRFD ($V_a \leq V_n/\Omega$) ASD

Serviceability Requirements:

(under service loads):

Beam deflections

Floor vibrations

Flexural Strength

- **Local Buckling:**
  - Criteria in Table B4.1
  - Strength in Chapter F: Flexure
  - Strength in Chapter G: Shear

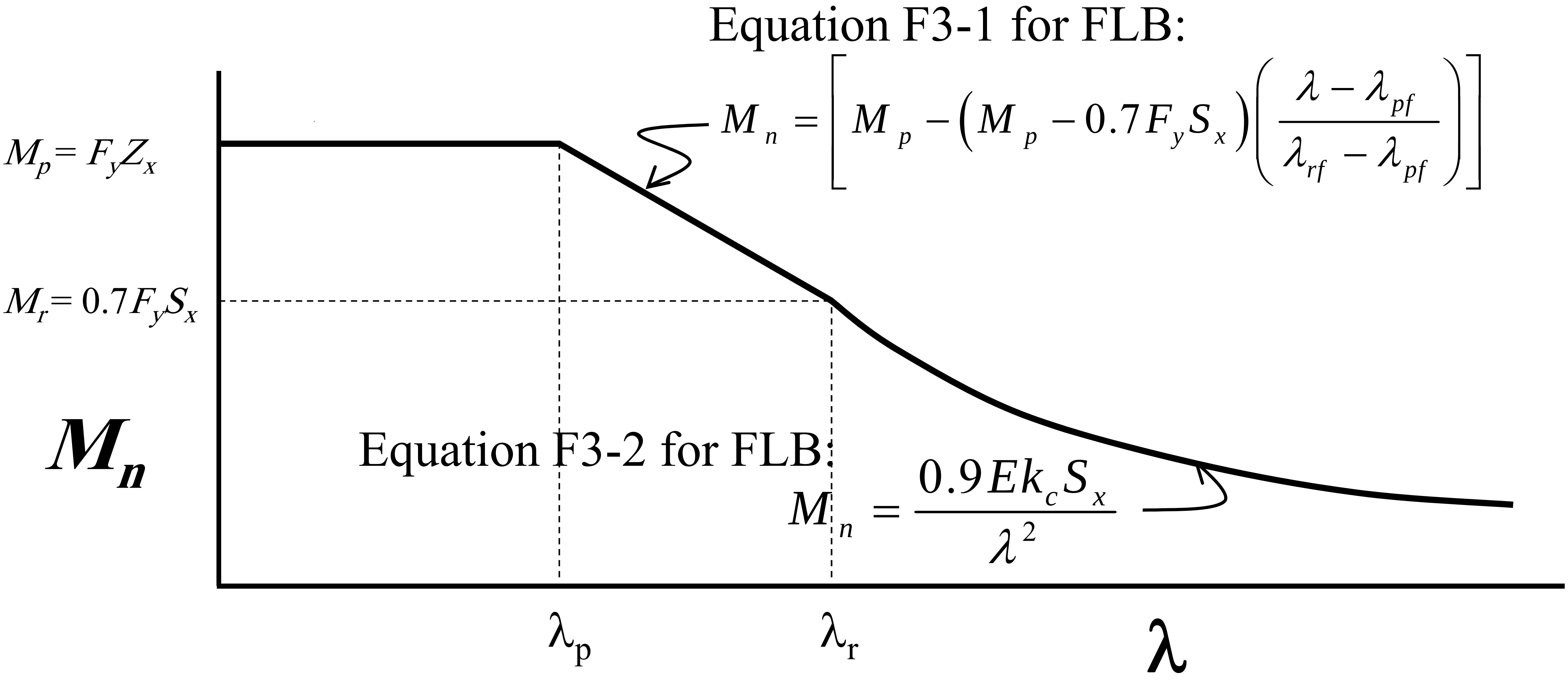
Chapter F: Flexural Strength

$$\Phi_b = 0.90 \ (\Omega_b = 1.67)$$

Specification assumes that the following failure modes have minimal interaction and can be checked independently from each other:

- Lateral Torsional Buckling(LTB)
- Flange Local Buckling (FLB)
- Shear

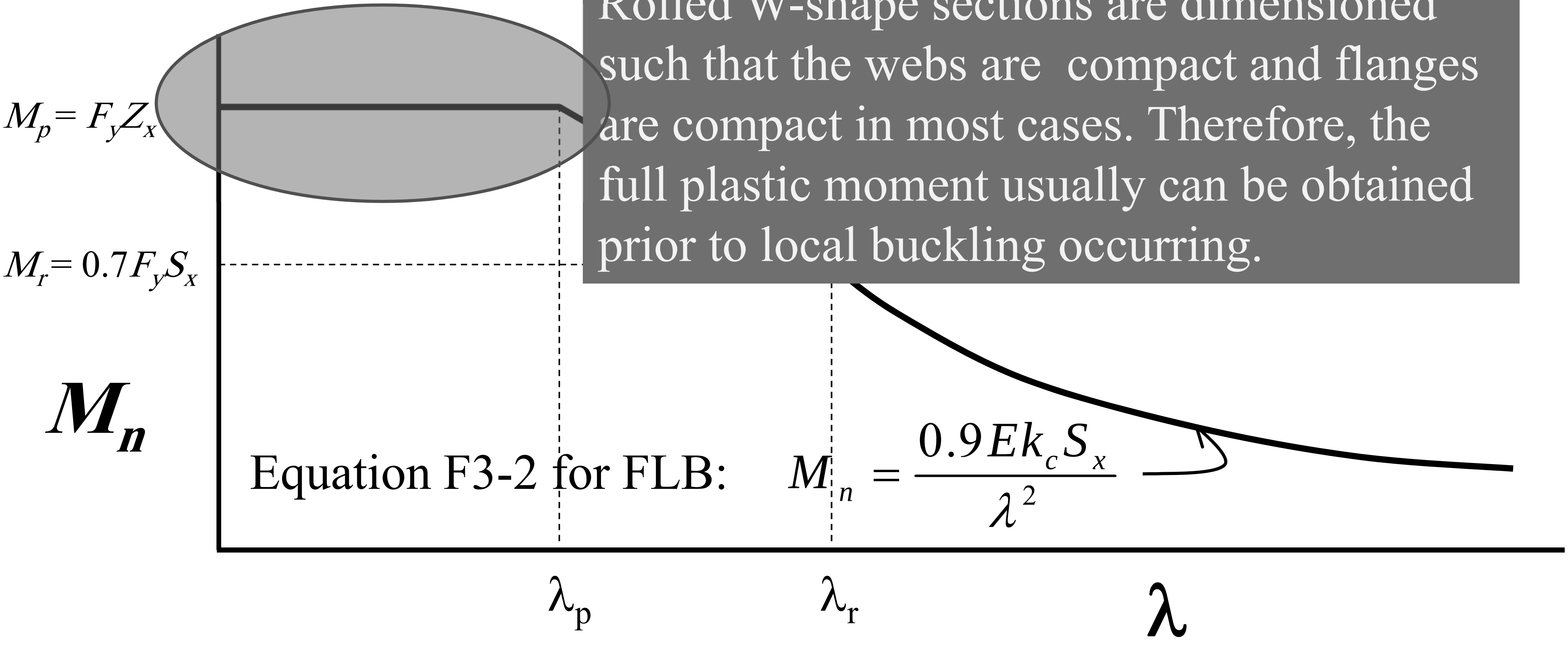
Local Buckling Criteria
Doubly Symmetric I-Shaped Members



Note: WLB not shown.

Local Buckling Criteria
Doubly Symmetric I-Shaped Members

Rolled W-shape sections are dimensioned such that the webs are compact and flanges are compact in most cases. Therefore, the full plastic moment usually can be obtained prior to local buckling occurring.



Note: WLB not shown.

Flexural Strength

Local Buckling Criteria

Slenderness of the flange and web, λ, are used as criteria to determine whether buckling would control in the elastic or inelastic range, otherwise the plastic moment can be obtained before local buckling occurs.

Criteria λ_p and λ_r are based on plate buckling theory.

For W-Shapes

FLB, $\lambda = b_f/2t_f$ $\quad \lambda_{pf} = 0.38\sqrt{\dfrac{E}{F_y'}}$ $\quad \lambda_{rf} = 1.0\sqrt{\dfrac{E}{F_y}}$

WLB, $\lambda = h/t_w$ $\quad \lambda_{pw} = 3.76\sqrt{\dfrac{E}{F_y'}}$ $\quad \lambda_{rw} = 5.70\sqrt{\dfrac{E}{F_y}}$

Flexural Strength

Local Buckling

$\lambda \leq \lambda_p$ **"compact"**

M_p is reached and maintained before local buckling.

$$\phi M_n = \phi M_p$$

$\lambda_p \leq \lambda \leq \lambda_r$ **"non-compact"**

Local buckling occurs in the inelastic range.

$$\phi 0.7 M_y \leq \phi M_n < \phi M_p$$

$\lambda > \lambda_r$ **"slender element"**

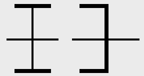
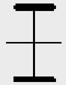
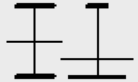
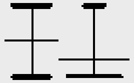
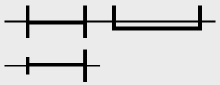
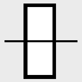
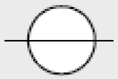
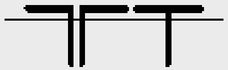
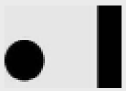
Local buckling occurs in the elastic range.

$$\phi M_n < \phi 0.7 M_y$$

- F2. Doubly Symmetric Compact I-Shaped Members and Channels Bend About Their Major Axis
  - 1. Yielding
  - 2. Lateral-Torsional Buckling
- F3. Doubly Symmetric I-Shaped Members with Compact Webs and Noncompact or Slender Flanges Bent About Their Major Axis
  - 1. Lateral-Torsional Buckling
  - 2. Compression Flange Local Buckling
- F4. Other I-Shaped Members with Compact or Noncompact Webs, Bent About Their Major Axis
  - 1. Compression Flange Yielding
  - 2. Lateral-Torsional Buckling
  - 3. Compression Flange Local Buckling
  - 4. Tension Flange Yielding
- F5. Doubly Symmetric and Singly Symmetric I-Shaped Members with Slender Webs Bent About Their Major Axis
  - 1. Compression Flange Yielding
  - 2. Lateral-Torsional Buckling
  - 3. Compression Flange Local Buckling
  - 4. Tension Flange Yielding
- F6. I-Shaped Members and Channels Bent About Their Minor Axis
  - 1. Yielding
  - 2. Flange Local Buckling

- F7. Square and Rectangular HSS and Box-Shaped Members
  - 1. Yielding
  - 2. Flange Local Buckling
  - 3. Web Local Buckling
- F8. Round HSS
  - 1. Yielding
  - 2. Local Buckling
- F9. Tees and Double Angles Loaded in the Plane of Symmetry
  - 1. Yielding
  - 2. Lateral-Torsional Buckling
  - 3. Flange Local Buckling of Tees
- F10. Single Angles
  - 1. Yielding
  - 2. Lateral-Torsional Buckling
  - 3. Leg Local Buckling
- F11. Rectangular Bars and Rounds
  - 1. Yielding
  - 2. Lateral-Torsional Buckling
- F12. Unsymmetrical Shapes
  - 1. Yielding
  - 2. Lateral-Torsional Buckling
  - 3. Local Buckling

TABLE USER NOTE F1.1
Selection Table for the Application of Chapter F Sections

Section in Chapter F	Cross Section	Flange Slenderness	Web Slenderness	Limit States
F2		C	C	Y, LTB
F3		NC, S	C	LTB, FLB
F4		C, NC, S	C, NC	CFY, LTB, FLB, TFY
F5		C, NC, S	S	CFY, LTB, FLB, TFY
F6		C, NC, S	N/A	Y, FLB
F7		C, NC, S	C, NC, S	Y, FLB, WLB, LTB
F8		N/A	N/A	Y, LB
F9		C, NC, S	N/A	Y, LTB, FLB, WLB
F10		N/A	N/A	Y, LTB, LLB
F11		N/A	N/A	Y, LTB
F12	Unsymmetrical shapes, other than single angles	N/A	N/A	All limit states

Y = yielding, CFY = compression flange yielding, LTB = lateral-torsional buckling, FLB = flange local buckling, WLB = web local buckling, TFY = tension flange yielding, LLB = leg local buckling, LB = local buckling, C = compact, NC = noncompact, S = slender, N/A = not applicable

Flexural Strength

- The following slides assume:
  - Compact sections
  - Doubly symmetric members and channels
  - Major axis Bending
  - Section F2

Flexural Strength

When members are compact:

Only consider LTB as a potential failure mode prior to reaching the plastic moment.

LTB depends on unbraced length, L_b, and can occur in the elastic or inelastic range.

If the section is also fully braced against LTB,

$M_n = M_p = F_y Z_x$ Equation F2-1

When LTB is a possible failure mode:

$M_p = F_y Z_x$ Equation F2-1

$M_r = 0.7 F_y S_x$

$$L_p = 1.76 r_y \sqrt{\frac{E}{F_y}}$$ Equation F2-5

$$L_r = 1.95 r_{ts} \frac{E}{0.7F_y} \sqrt{\frac{J_c}{S_x h_o} + \sqrt{\left(\frac{J_c}{S_x h_0}\right)^2 + 6.76\left(\frac{.7F_y}{E}\right)^2}}$$ Equation F2-6

$$r_{ts}^2 = \frac{\sqrt{I_y C_w}}{S_x}$$ Equation F2-7

$$r_y = \sqrt{\frac{I_y}{A}}$$

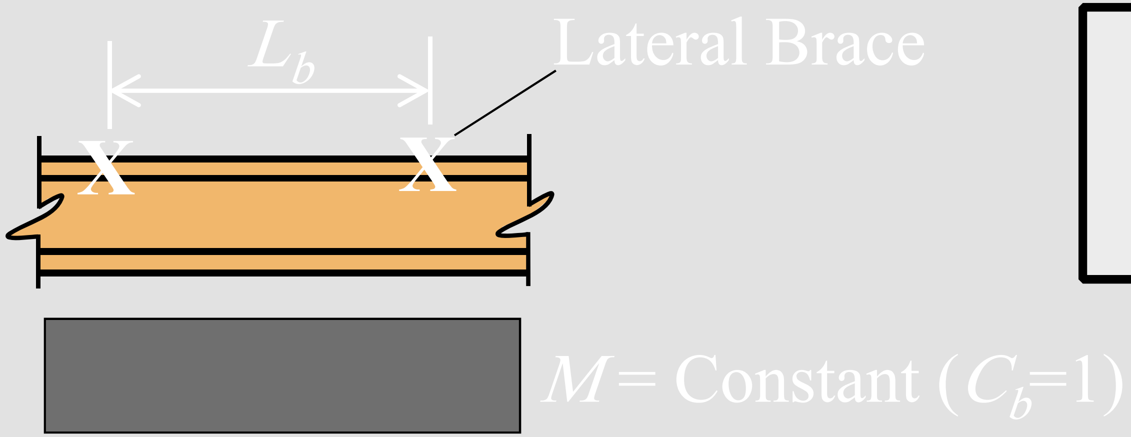
For W shapes

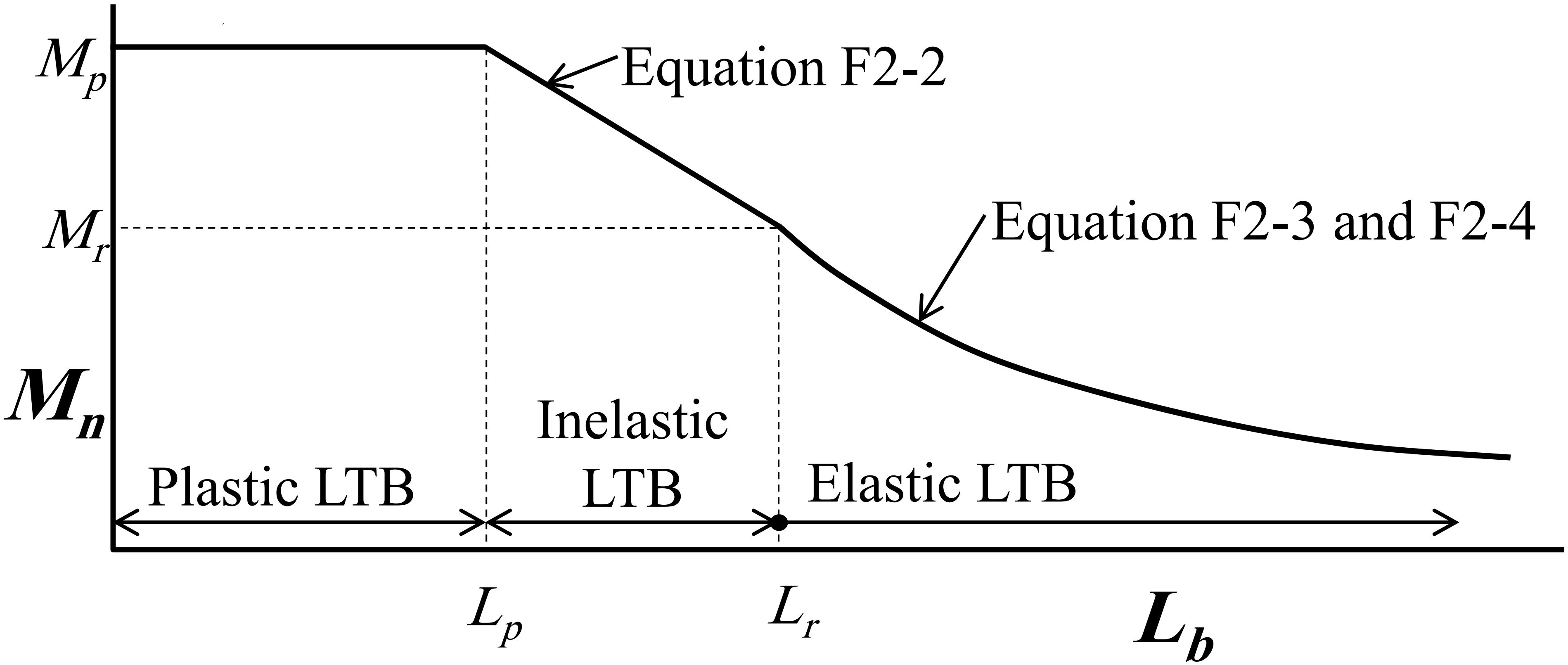
$c = 1$ (Equation F2-8a)

h_o = distance between flange centroids

Values of ϕM_p, ϕM_r, L_p and L_r are tabulated in Table 3-2

Lateral Torsional Buckling Strength for Compact W-Shape Sections





If $L_b \leq L_p$,

$$M_n = M_p$$

If $L_p < L_b \leq L_r$,

$$M_n = C_b \left[M_p - \left(M_p - .7 F_y S_x \right) \left(\frac{L_b - L_p}{L_r - L_p} \right) \right] \leq M_p$$ Equation F2-2

Note that this is a straight line.

If $L_b > L_r$,

$$M_n = F_{cr} S_x \leq M_p$$ Equation F2-3

Where

$$F_{cr} = \frac{C_b \pi^2 E}{\left(\frac{L_b}{r_{ts}} \right)^2} \sqrt{1 + 0.078 \frac{J_c}{S_x h_0} \left(\frac{L_b}{r_{ts}} \right)^2}$$ Equation F2-4

Assume C_b=1 for now

Flexural Strength

Plots of ϕM_n versus L_b for $C_b = 1.0$ are tabulated, Table 3-10

Results are included only for:

- W sections typical for beams
- $F_y = 50$ ksi
- $C_b = 1$

Flexural Strength

To compute M_n for any moment diagram,

$$M_n = C_b(M_{n(Cb1)}) \le M_p$$
$$\phi M_n = C_b(\phi M_{n(Cb1)}) \le \phi M_p$$

$(M_{n(Cb1)}) = M_n$, assuming $C_b = 1$

C_b, Equation F1-1

$$C_b = \frac{12.5 M_{\max}}{2.5 M_{\max} + 3M_A + 4M_B + 3M_C}$$

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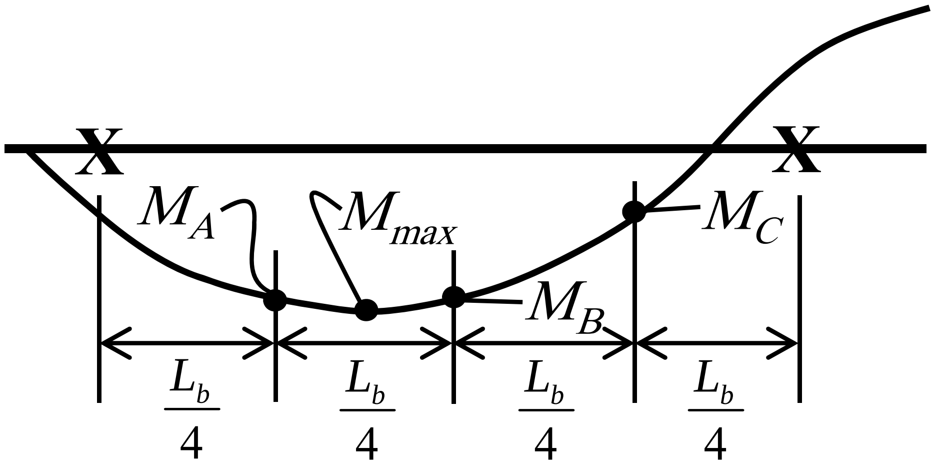
Flexural Strength



Shown is the section of the moment diagram between lateral braces.

M_{max} = absolute value of maximum moment in unbraced section

M_A = absolute value of moment at quarter point of unbraced section

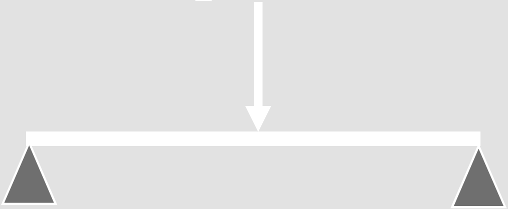
M_B = absolute value of moment at centerline of unbraced section

M_C = absolute value of moment at three-quarter point of unbraced section

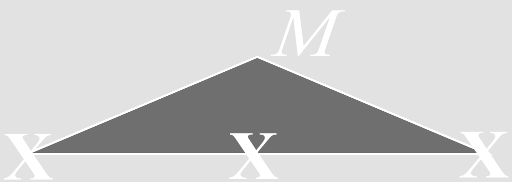
Flexural Strength

Consider a simple beam with differing lateral brace locations.

Example



$$C_b = \frac{12.5M}{2.5M + 3\left(M/2\right) + 4M + 3\left(M/2\right)} = \frac{12.5}{9.5} = 1.31$$

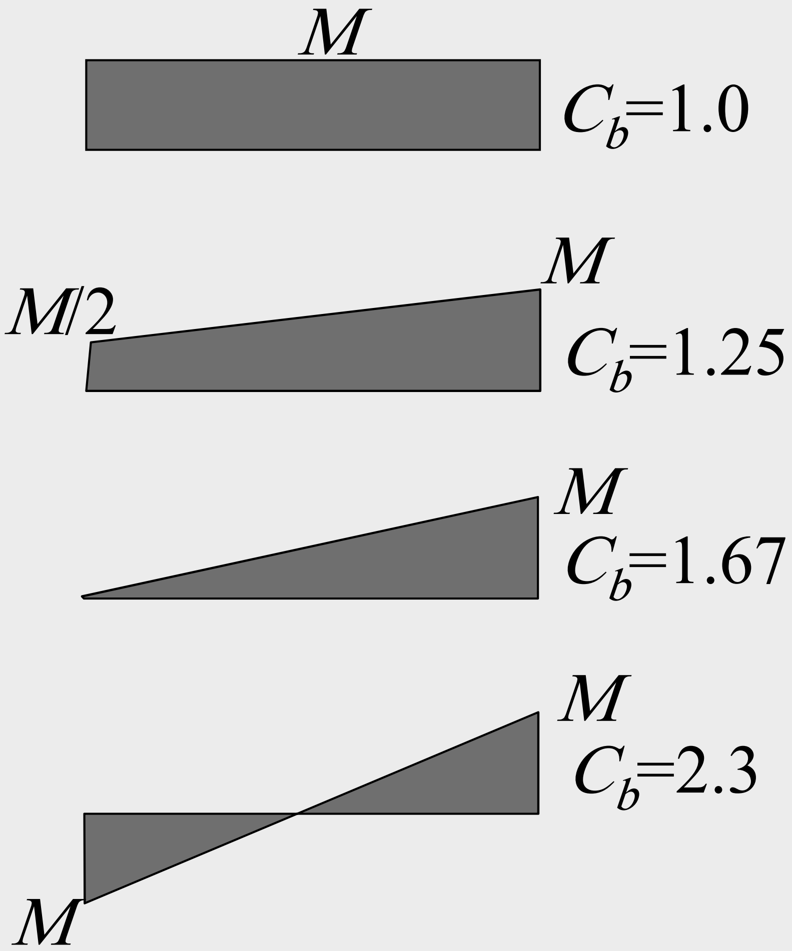


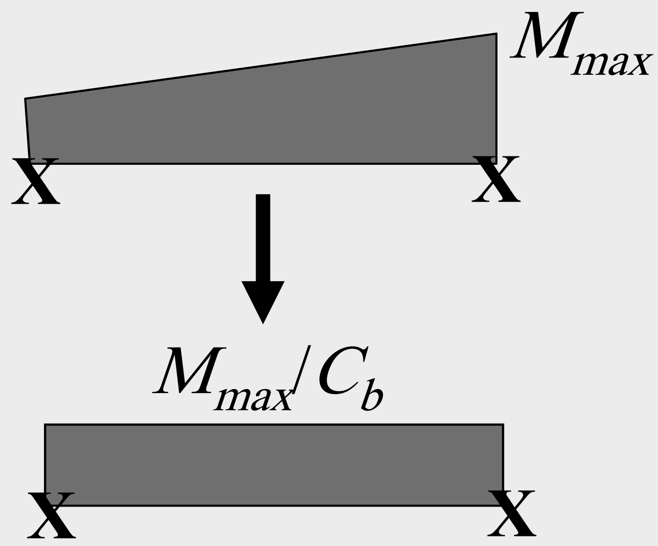
$$C_b = \frac{12.5M}{2.5M + 3\left(M/4\right) + 4\left(M/2\right) + 3\left(3M/4\right)} = \frac{12.5}{7.5} = 1.67$$

X – lateral brace location

Note that the moment diagram is unchanged by lateral brace locations.

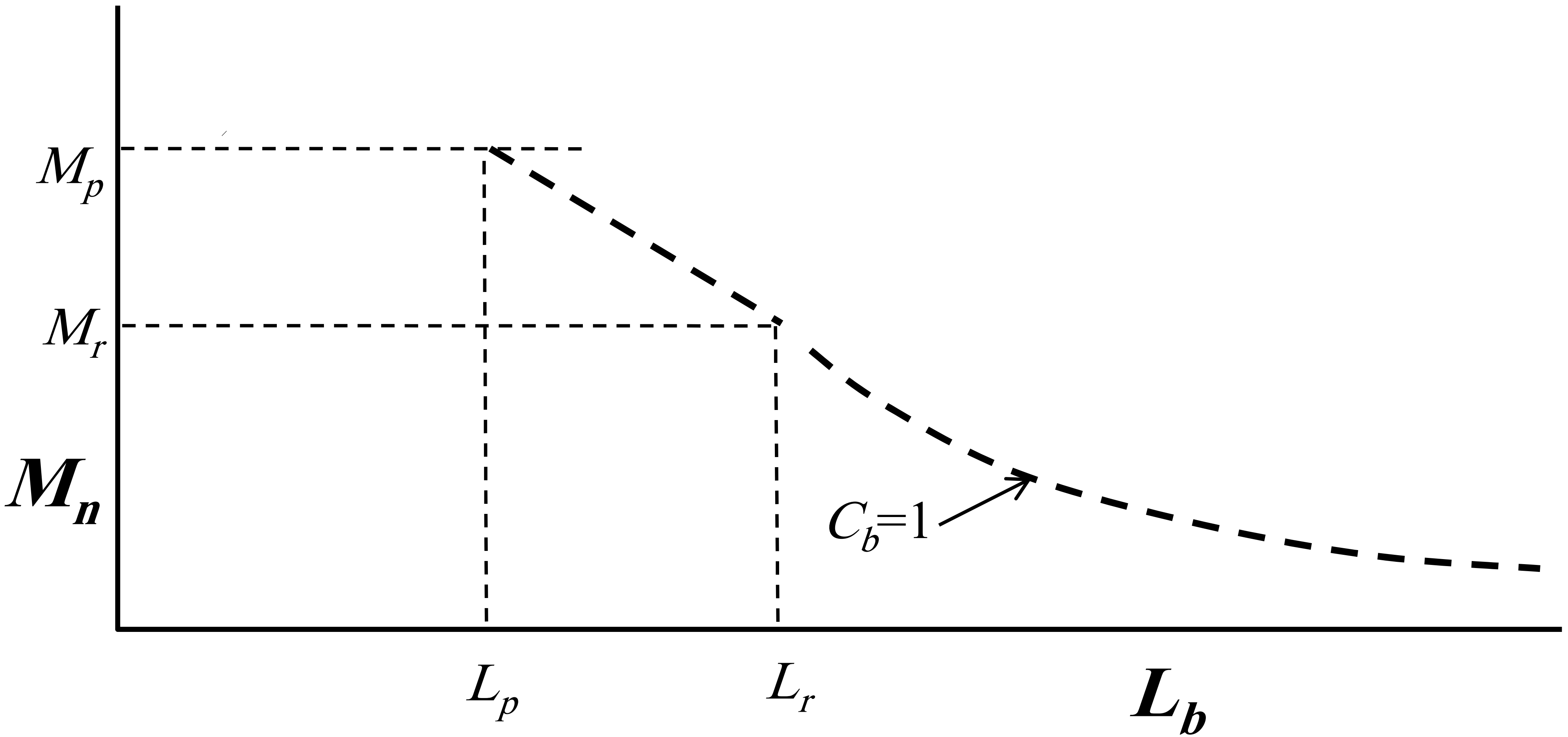
Flexural Strength





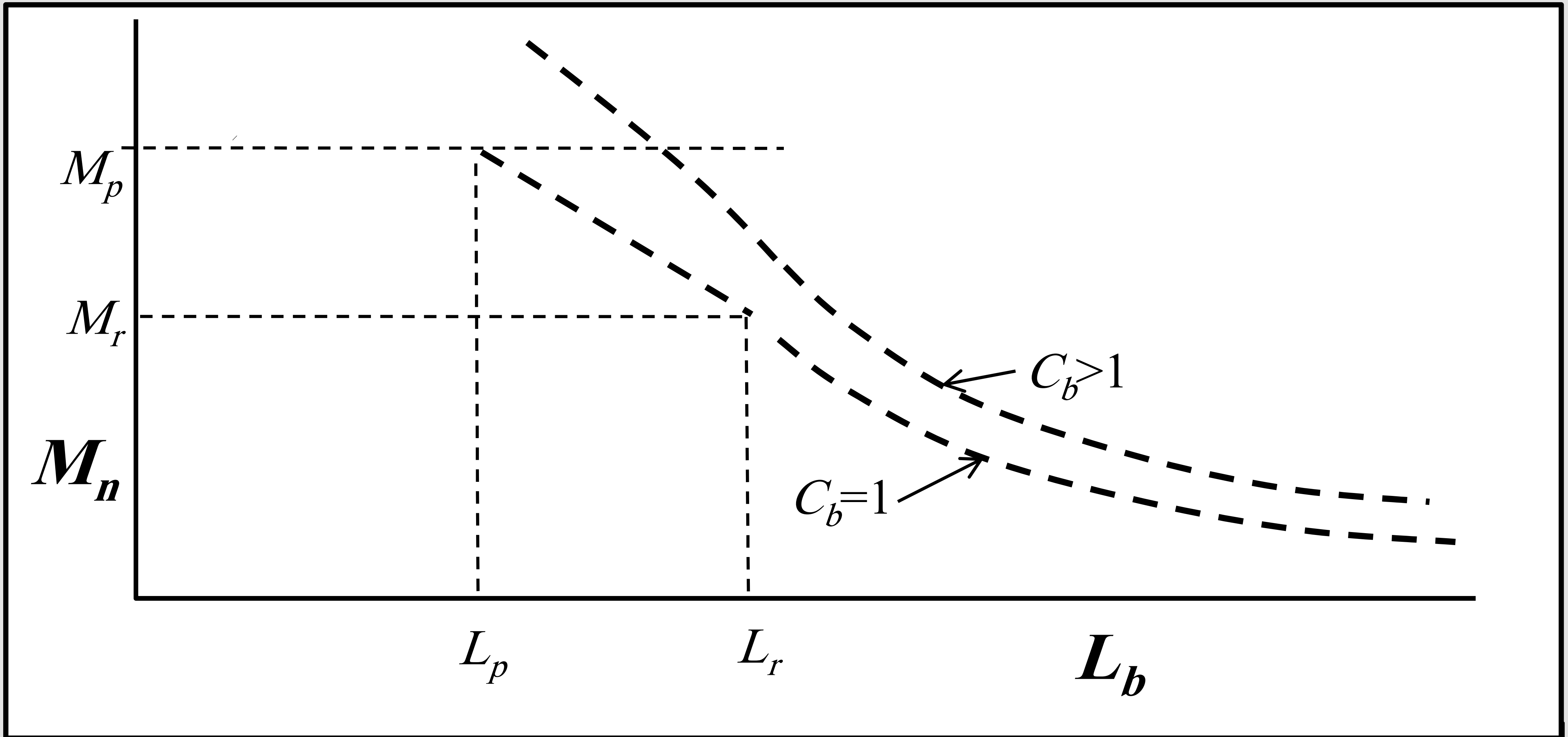
C_b approximates an equivalent beam of constant moment.

Flexural Strength



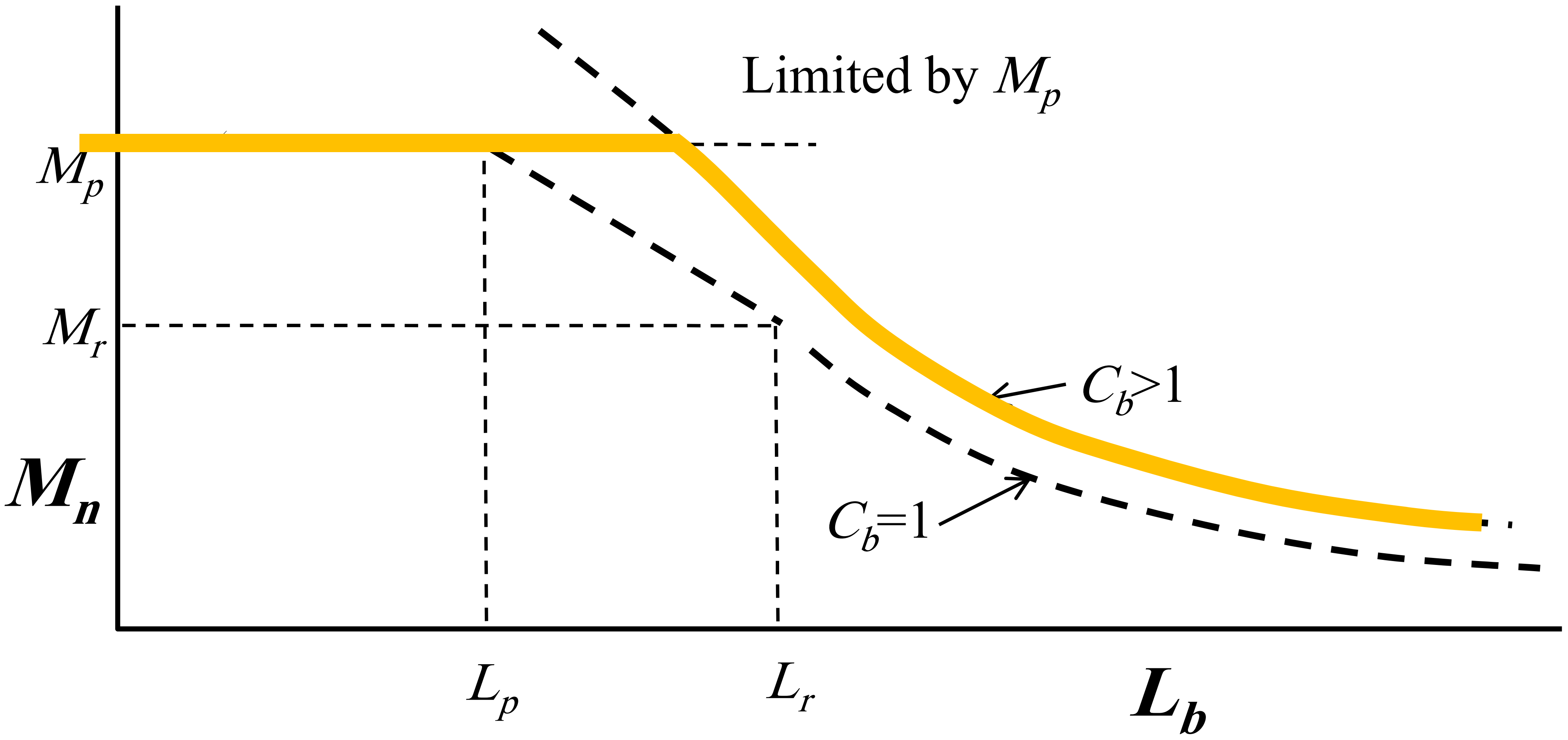
Lateral Torsional Buckling
Strength for Compact W-Shape Sections
Effect of C_b

Flexural Strength



Lateral Torsional Buckling
Strength for Compact W-Shape Sections
Effect of C_b

Flexural Strength



Lateral Torsional Buckling
Strength for Compact W-Shape Sections
Effect of C_b

Nominal Flexural Strength

Compact Sections

Nominal Flexural Strength of Compact Sections

Covered in F2

The nominal bending strength for compact sections can be summarized as follows:

For $L_b \leq L_p$,

$$M_n = M_p \qquad \text{(AISC Equation F2-1)}$$

For $L_p < L_b \leq L_r$,

$$M_n = C_b\left[M_p - (M_p - 0.7F_yS_x)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \leq \boldsymbol{M_p} \qquad \text{(AISC Equation F2-2)}$$

For $L_b > L_r$,

$$M_n = F_{cr}S_x \leq M_p \qquad \text{(AISC Equation F2-3)}$$

where

$$F_{cr} = \frac{C_b\pi^2 E}{(L_b/r_{ts})^2}\sqrt{1 + 0.078\frac{Jc}{S_xh_0}\left(\frac{L_b}{r_{ts}}\right)^2} \qquad \text{(AISC Equation F2-4)}$$

Nominal Flexural Strength

NonCompact Sections

Nominal Flexural Strength of NonCompact Sections

Covered in F3

- Most standard ***W, M, S, and C shapes*** are compact.
- A few are noncompact because of the flange width-to-thickness ratio,
- None are slender.
- In general, a noncompact beam may fail by:
  - Lateral-torsional buckling (Elastic or Inelastic)
  - Flange local buckling (Elastic or Inelastic)
  - Web local buckling (Elastic or Inelastic)
- **The strength corresponding to each of these three limit states must be computed, and the smallest value will control.**

Nominal Flexural Strength of NonCompact Sections

Covered in F3

The nominal flexural strength, M_n, shall be the lower value obtained according to the *limit states* of *lateral-torsional buckling* and compression flange *local buckling*.

1. Lateral-Torsional Buckling

For *lateral-torsional buckling*, the provisions of Section F2.2 shall apply.

2. Compression Flange Local Buckling

(a) For sections with noncompact flanges

$$M_n = \left[M_p - (M_p - 0.7F_yS_x)\left(\frac{\lambda - \lambda_{pf}}{\lambda_{rf} - \lambda_{pf}}\right)\right] \quad \text{(F3-1)}$$

(b) For sections with slender flanges

$$M_n = \frac{0.9Ek_cS_x}{\lambda^2} \quad \text{(F3-2)}$$

where

$\lambda = \dfrac{b_f}{2t_f}$

$\lambda_{pf} = \lambda_p$ is the limiting slenderness for a compact flange, Table B4.1

$\lambda_{rf} = \lambda_r$ is the limiting slenderness for a noncompact flange, Table B4.1

$k_c = \dfrac{4}{\sqrt{h/t_w}}$ and shall not be taken less than 0.35 nor greater than 0.76 for calculation purposes

Nominal Flexural Strength of NonCompact Sections

- Noncompact shapes are identified by an "*f*" footnote in the dimensions and properties tables and Z_x table
- Values of L_p and L_r are also given in Z_x table
- The webs of all hot-rolled shapes in the *Manual* are compact
- Hence, the noncompact shapes are subject only to the limit states of lateral-torsional buckling and flange local buckling
- Built-up welded shapes, can have noncompact or slender webs as well as noncompact or slender flanges. These cases are covered in AISC Sections F4 and F5.

Nominal Flexural Strength

Summary from Segui 5th Edition

1. Determine whether the shape is compact.
2. If the shape is compact, check for lateral-torsional buckling as follows.

   If $L_b \leq L_p$, there is no LTB, and $M_n = M_p$

   If $L_p < L_b \leq L_r$, there is inelastic LTB, and

$$M_n = C_b\left[M_p - (M_p - 0.7F_yS_x)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \leq \boldsymbol{M}_p$$

   If $L_b > L_r$, there is elastic LTB, and

$$M_n = F_{cr}S_x \leq M_p$$

where

$$F_{cr} = \frac{C_b\pi^2 E}{(L_b/r_{ts})^2}\sqrt{1 + 0.078\frac{Jc}{S_xh_0}\left(\frac{L_b}{r_{ts}}\right)^2}$$

Nominal Flexural Strength of NonCompact Sections

3. If the shape is noncompact because of the flange, the nominal strength will be the smaller of the strengths corresponding to flange local buckling and lateral-torsional buckling.

   a. Flange local buckling:

      If $\lambda \le \lambda_p$, there is no FLB

      If $\lambda_p < \lambda \le \lambda_r$, the flange is noncompact, and

$$M_n = M_p - (M_p - 0.7F_yS_x)\left(\frac{\lambda - \lambda_p}{\lambda_r - \lambda_p}\right)$$

   b. Lateral-torsional buckling:

      If $L_b \le L_p$, there is no LTB

      If $L_p < L_b \le L_r$, there is inelastic LTB, and

$$M_n = C_b\left[M_p - (M_p - 0.7F_yS_x)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le \boldsymbol{M}_p$$

      If $L_b > L_r$, there is elastic LTB, and

$$M_n = F_{cr}S_x \le M_p$$

      where

$$F_{cr} = \frac{C_b\pi^2E}{(L_b/r_{ts})^2}\sqrt{1 + 0.078\frac{Jc}{S_xh_0}\left(\frac{L_b}{r_{ts}}\right)^2}$$

Shear Strength in Beam

Shear Strength in Beam

Shear limit states for beams

<u>Shear Yielding of the web:</u>
Failure by excessive deformation.

<u>Shear Buckling of the web:</u>
Slender webs (large d/t_w) may buckle prior to yielding.

Shear Strength in Beam

Failure modes:

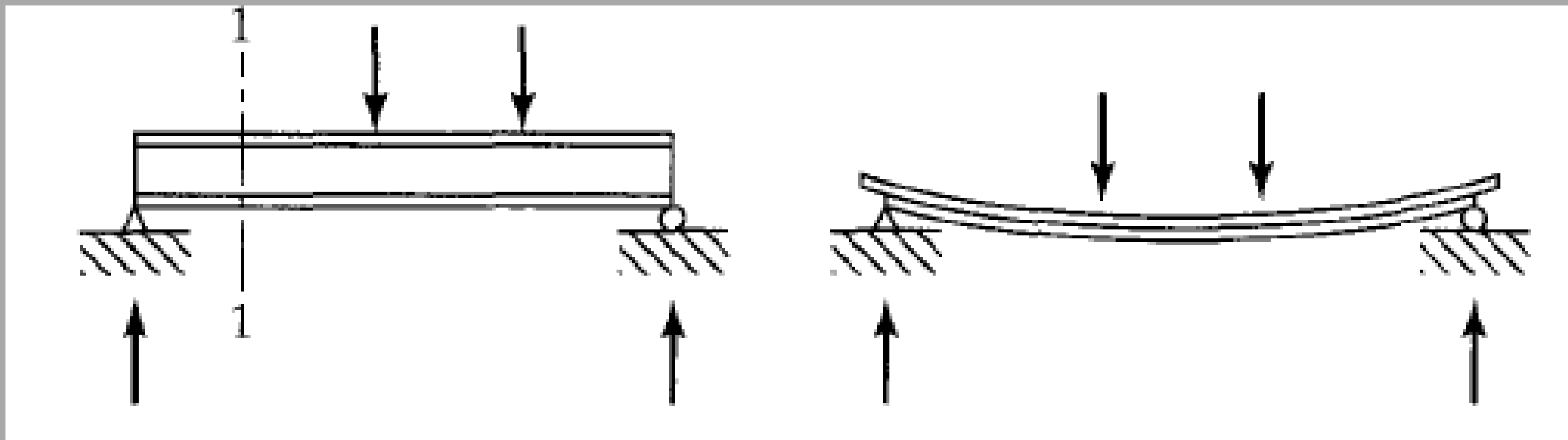
Shear Yielding

Elastic Shear Buckling

Inelastic Shear Buckling

Shear in Beam

- As the member bends, shear stresses occur because of the changes in length of its longitudinal fibers.
- For positive bending the lower fibers are stretched and the upper fibers are shortened
- somewhere in between there is a neutral axis where the fibers do not change in length.
- Owing to these varying deformations, a particular fiber has a tendency to slip on the fiber above or below it.



- *This presentation may be entirely misleading in seeming to completely separate horizontal and vertical shears.*
- *In reality, horizontal and vertical shears at any point are the same, as long as the critical section at which the shear stress is evaluated is taken parallel to the axis of symmetry. Furthermore, one cannot occur without the other.*

Shear Stress, $\tau = (VQ)/(Ib)$

τ = shear stress at any height on the cross section

V = total shear force on the cross section

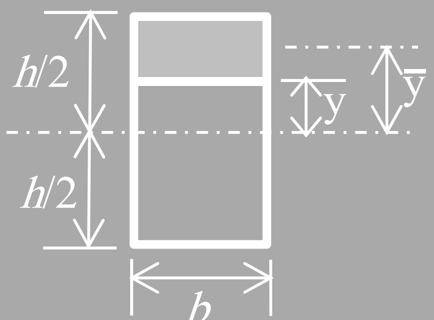
Q = first moment about the centroidal axis of the area between the extreme fiber and where τ is evaluated

I = moment of inertia of the entire cross section

b = width of the section at the location where τ is evaluated

Shear Strength in Beam

Shear in a solid rectangular section



Compute τ at level y of the section:

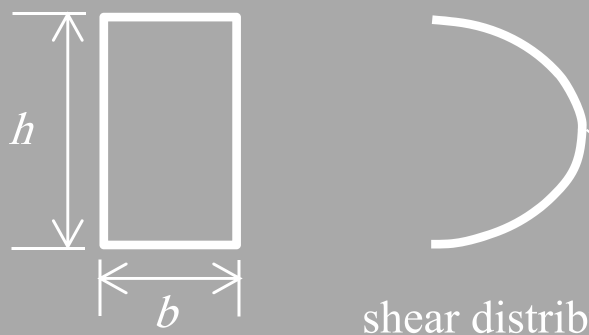
$$\tau = \frac{VQ}{Ib}$$

Where: $$Q = A\bar{y} = b\left(\frac{h}{2} - y\right)\left(y + \frac{h/2 - y}{2}\right) = \frac{bh^2}{8} - \frac{by^2}{2}$$

$$I = \frac{bh^3}{12}$$

$$\tau = \frac{V\left(\frac{b}{2}\right)\left(\frac{h^2}{4} - y^2\right)}{\frac{b^2h^3}{12}} = \frac{6V\left(\frac{h^2}{4} - y^2\right)}{bh^3} = 1.5\frac{V}{bh}\left(1 - \left(\frac{2y}{h}\right)^2\right)$$

where $y = 0$



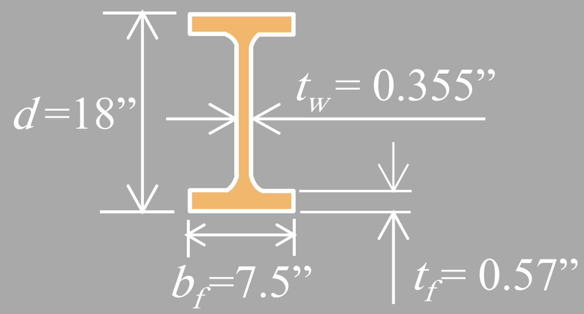
$$\tau_{max} = 1.5\frac{V}{bh}$$

shear distribution

This is a parabolic distribution of shear, as shown.

Shear in a W-Shape

Example: Compute shear strength of W18x50 if V = 100 kips



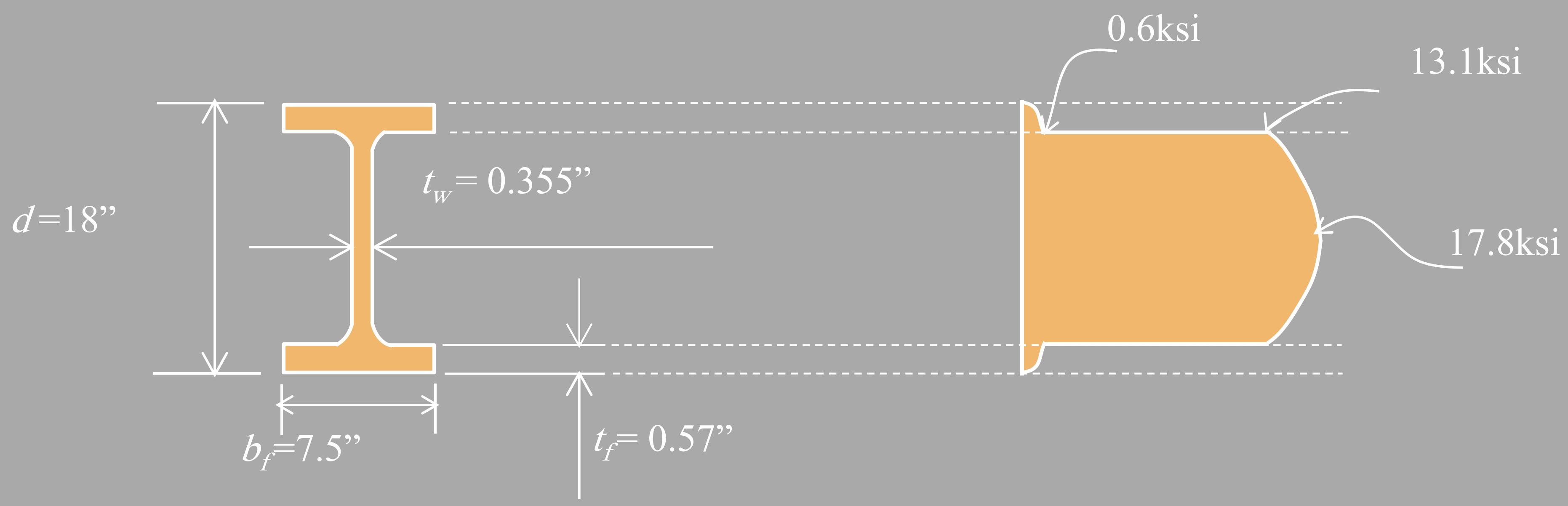
Solution:

$$\tau = \frac{VQ}{It}$$

$$\tau_{f\,\max} = \frac{100(7.5)(0.57)\left(9 - \frac{0.57}{2}\right)}{800(7.5)} = 0.6ksi$$

$$\tau_{w\min} = \frac{100(7.5)(0.57)\left(9 - \frac{0.57}{2}\right)}{800(0.355)} = 13.1ksi$$

$$\tau_{w\max} = \frac{100\left[(7.5)(0.57)(8.715) + (9 - 0.57)(0.355)\left(\frac{9 - 0.57}{2}\right)\right]}{800(0.355)} = 17.6ksi$$



τ is very low in the flanges.

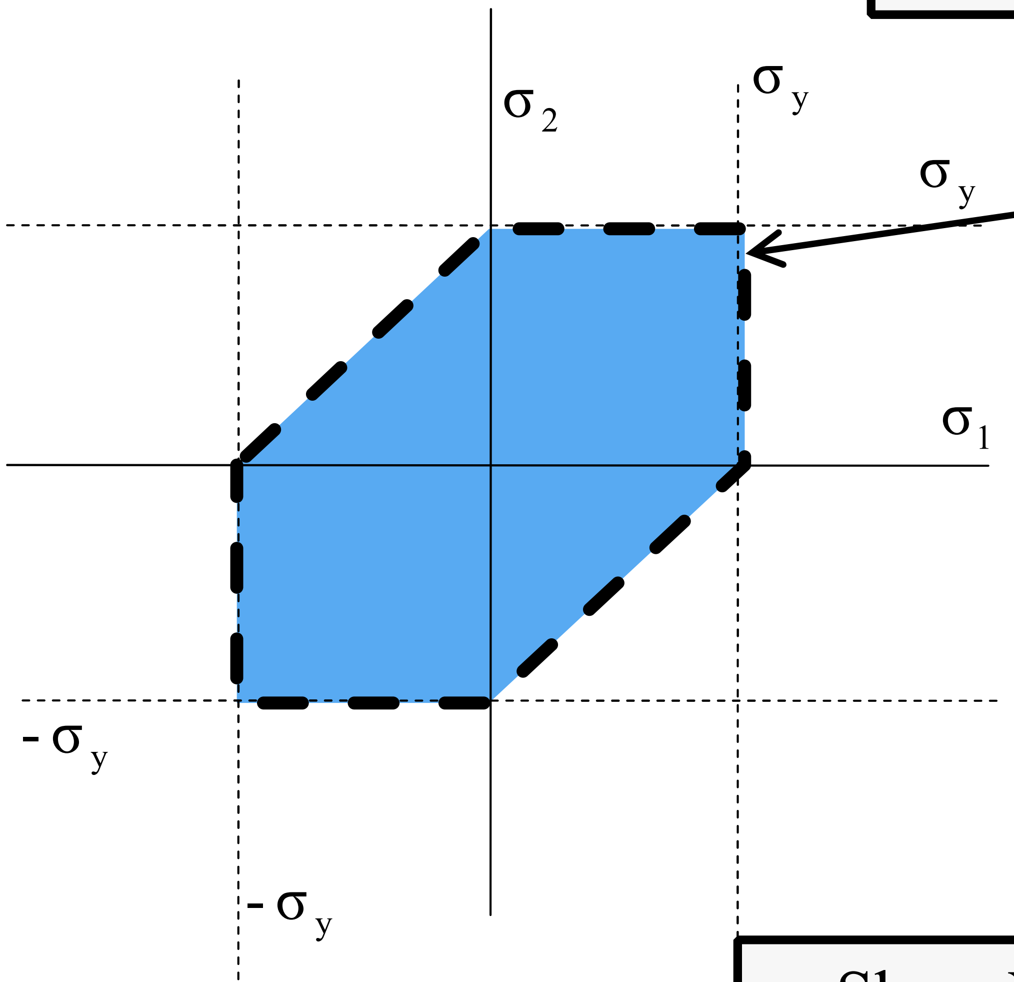
$V_{flange} \approx 1/2(0.6)7.5(0.57)) = 1.3$ kips

$V_{web} = 100 - 2(1.3) = 97.3$ kips

For a W-shape most of the shear is carried by the web. Therefore, in the AISC *Specification,* the shear strength of a W-shape section is calculated based on an effective area equal to the overall depth of the section times the web thickness.

Shear Strength

Shear Yield Criteria



Yield defined by Mohr's Circle

$$|\sigma_1| \leq \sigma_y$$

$$|\sigma_2| \leq \sigma_y$$

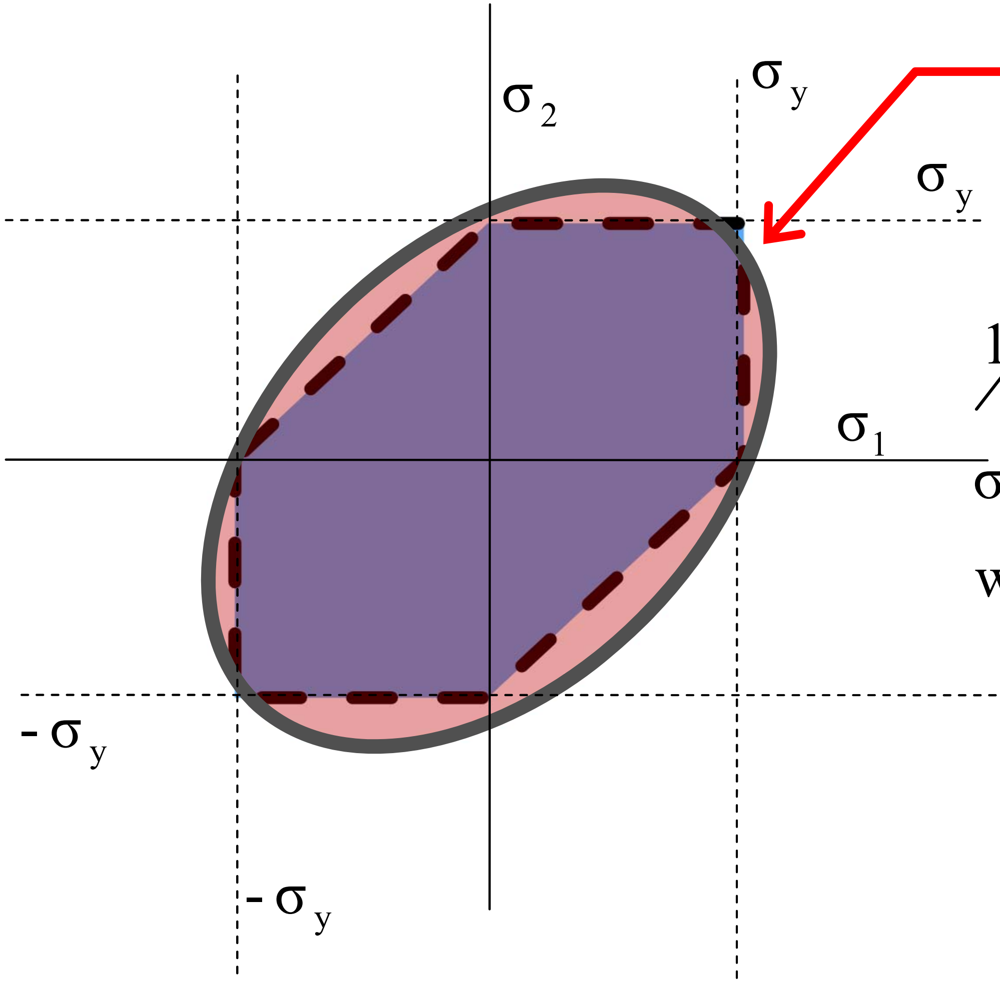
$$|\sigma_1 - \sigma_2| \leq \sigma_y$$

<u>Shear Yielding of the web:</u>

Failure by excessive deformation.

Shear Strength

Shear Yield Criteria



Von Mises Yield defined by maximum distortion strain energy criteria (applicable to ductile materials):

$$\tfrac{1}{2}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right]\le\sigma_y^{\,2}$$

$$\sigma_1^{\,2}-\sigma_1\sigma_2+\sigma_2^{\,2}\le\sigma_y^{\,2}$$

when $\sigma_3 = 0$

For F_y = constant for load directions

$$\tau_{max}\le\frac{F_y}{\sqrt{3}}=0.577F_y$$

<u>**AISC Specification uses 0.6 F_y**</u>

Shear Strength

Von Mises Failure Criterion
(Shear Yielding)

When average web shear stress $V/A_{web} = 0.6F_y$

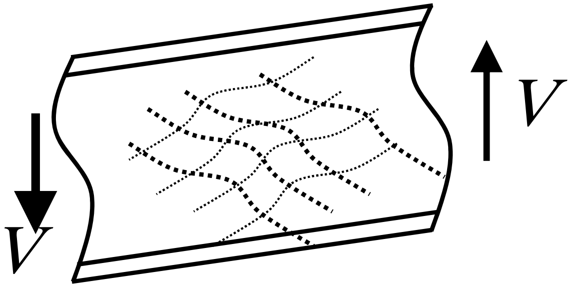
$$V = 0.6F_yA_{web}$$

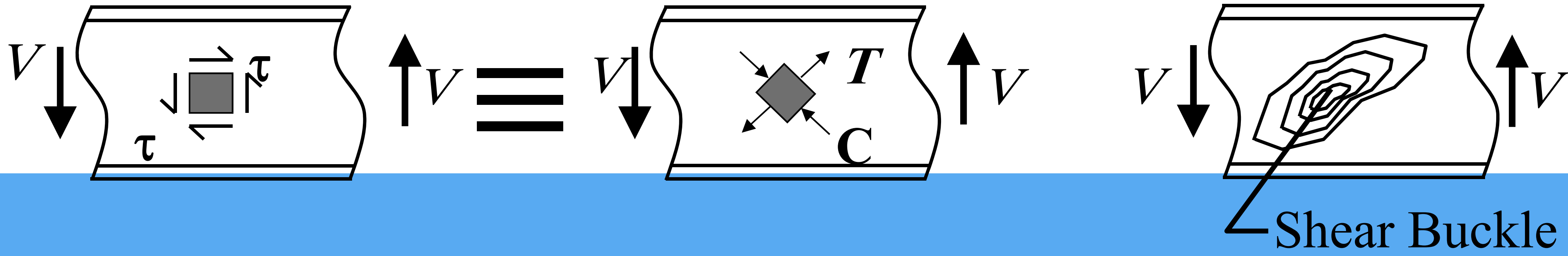
Shear Strength

Shear Buckling

<u>Shear Buckling of the web:</u>

Slender webs (large d/t_w) may buckle prior to yielding.



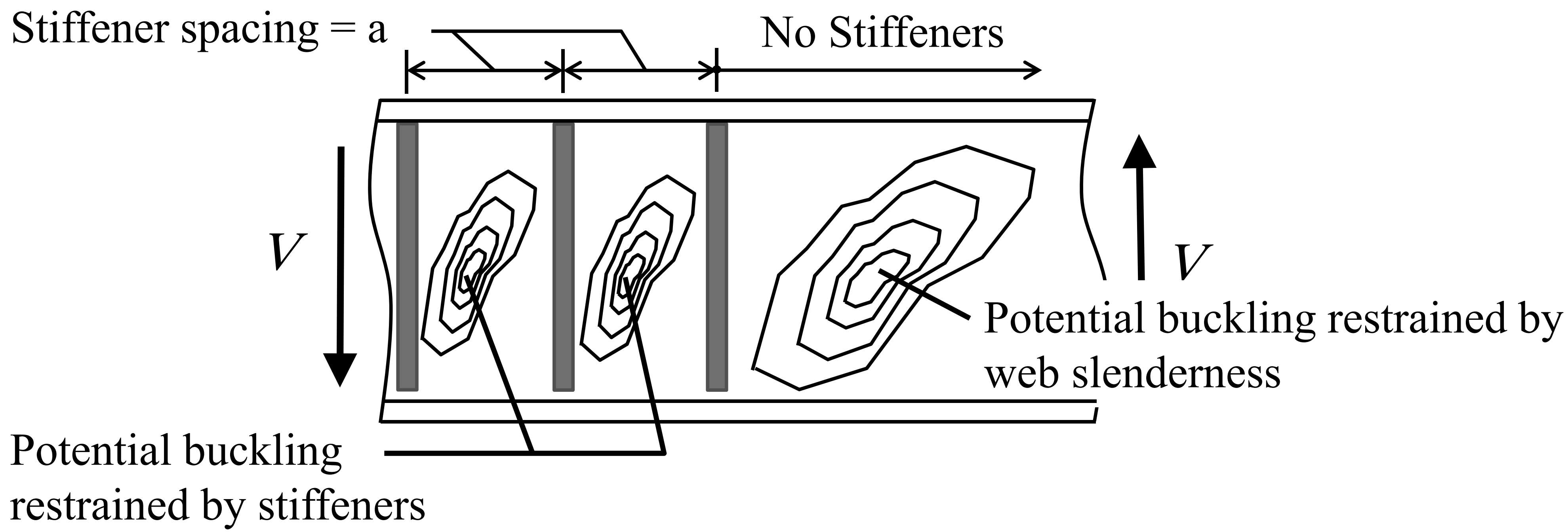


Shear buckling occurs due to diagonal compressive stresses.

Extent of shear buckling depends on h/t_w of the web (web slenderness).

Shear Strength

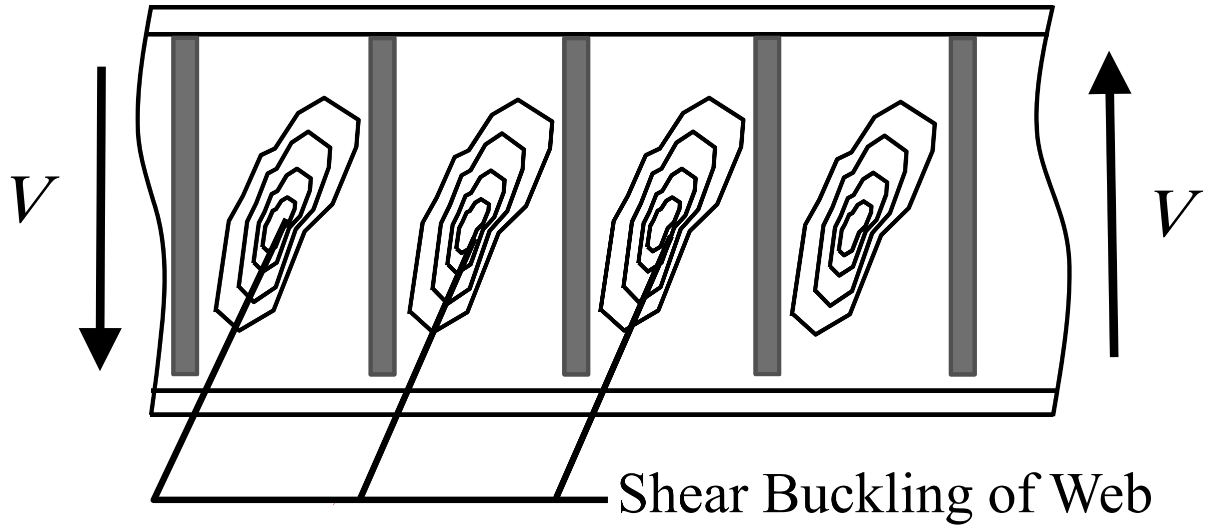
Shear Buckling



If shear buckling controls a beam section, the plate section which buckles can be "stiffened" with stiffeners. These are typically vertical plates welded to the web (and flange) to limit the area that can buckle. Horizontal stiffener plates are also possible, but less common.

Shear Strength

Shear Buckling

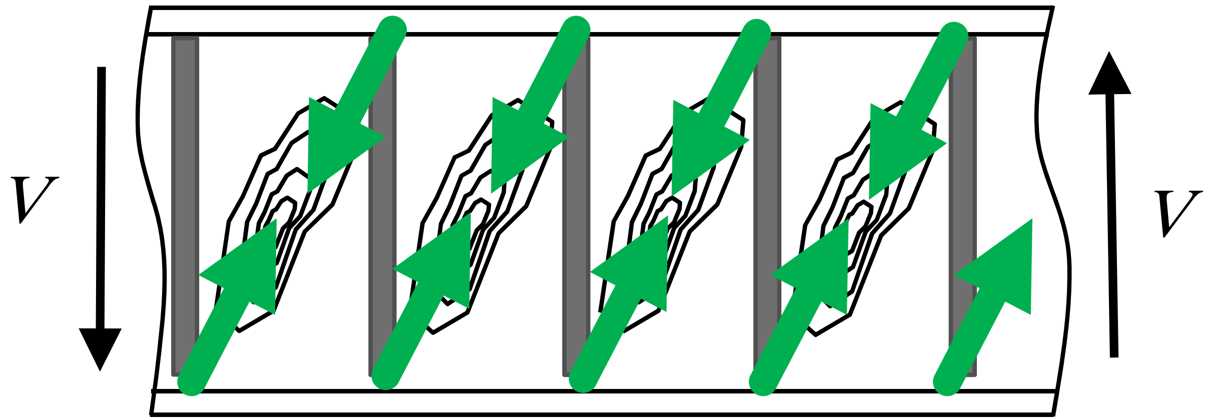


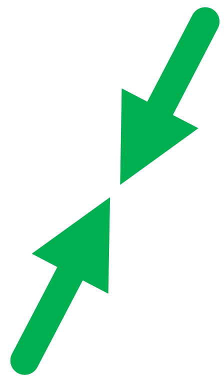
When the web is slender, it is more susceptible to web shear buckling. However, there is additional shear strength beyond when the web buckles.
Web shear buckling is therefore not the final limit state.
The strength of a truss mechanism controls shear strength called "Tension Field Action."

Tension Field Action is a post buckling mechanism that is relied on to increase the overall shear strength. For very slender webs this post-buckling strength is significant.
This strength is often relied upon in built up beam sections. Stiffeners must be properly designed to carry the compression forces – in addition to out of plane stiffness criteria for typical shear stiffener design.

Shear Strength

Shear Buckling



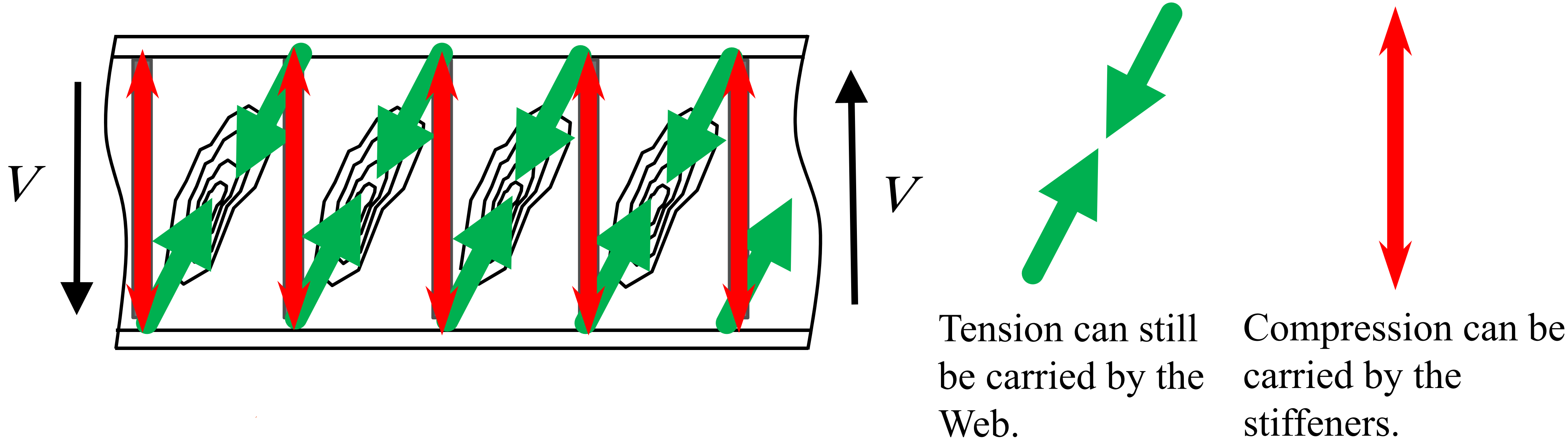


Tension can still be carried by the Web.

When the web is slender, it is more susceptible to web shear buckling. However, there is additional shear strength beyond when the web buckles.
Web shear buckling is therefore not the final limit state.
The strength of a truss mechanism controls shear strength called "Tension Field Action."

Shear Strength

Shear Buckling



When the web is slender, it is more susceptible to web shear buckling. However, there is additional shear strength beyond when the web buckles.

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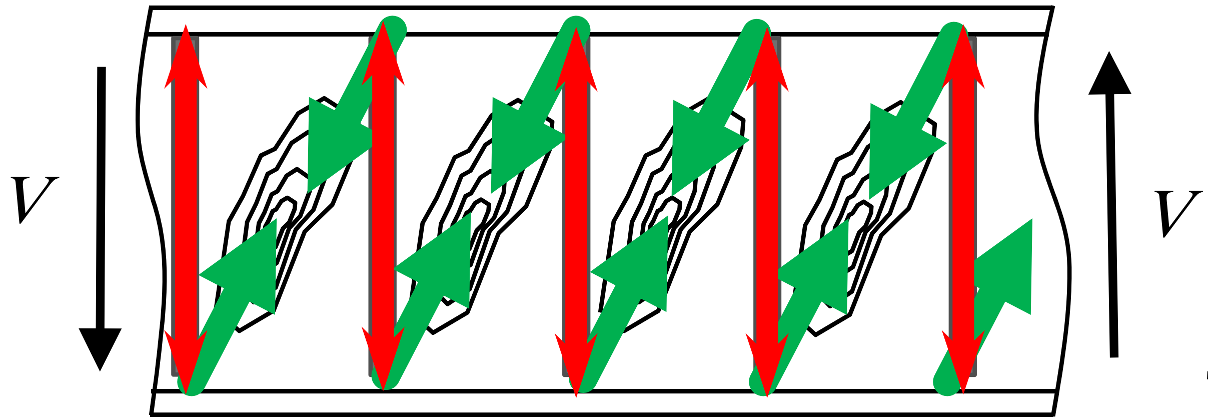
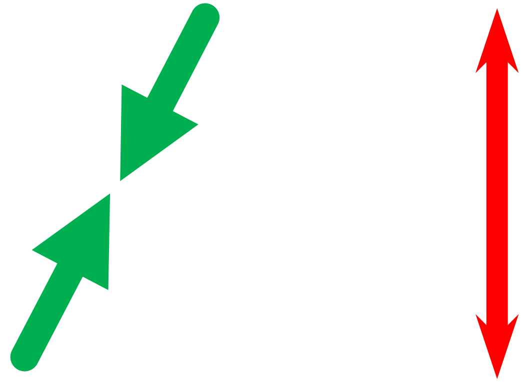
The strength of a truss mechanism controls shear strength called “Tension Field Action.”

Shear Strength

Shear Buckling





For Tension Field Action to be effective the truss forces must be resisted at each node point.

Therefore , end panels are not effective, nor are widely spaced stiffeners, nor panels that are not well restrained around their perimeter.

Shear Strength

Shear stresses generally are low in the flange area (where moment stresses are highest).

For design, simplifying assumptions are made:

1) Shear and Moment stresses are independent.
2) Web carries the entire shear force.
3) Shear stress is simply the average web value.

i.e. $\tau_{web(avg)} = V/A_{web} = V/dt_w$

AISC Requirements for Shear Strength
Chapter G

Nominal Shear Strength

$$V_n = 0.6 F_y A_w C_v$$ Equation G2-1

$0.6F_y$ = Shear yield strength per Von Mises Failure Criteria

A_w = area of web = dt_w

C_v = reduction factor for shear buckling

Shear Strength

C_v depends on slenderness of web and locations of shear stiffeners. It is a function of k_v.

$$k_v = 5 + \frac{5}{\left(a/h\right)^2} \qquad \text{Equation G2-6}$$

a = clear distance between transverse stiffeners

h = clear distance between flanges minus fillet on a rolled shape

$k_v = 5$ if no stiffeners are present, if $a/h > 3.0$, or $a/h > \left[\frac{260}{h/t_w}\right]^2$

Shear Strength

For a rolled I-shaped member

If $h/t_w \leq 2.24\sqrt{E/F_y}$

Then $\phi_v = 1.00$ ($\Omega = 1.50$)

$V_n = 0.6F_yA_{web}$ (shear yielding) ($C_v = 1.0$)

Otherwise, for other doubly symmetric shapes

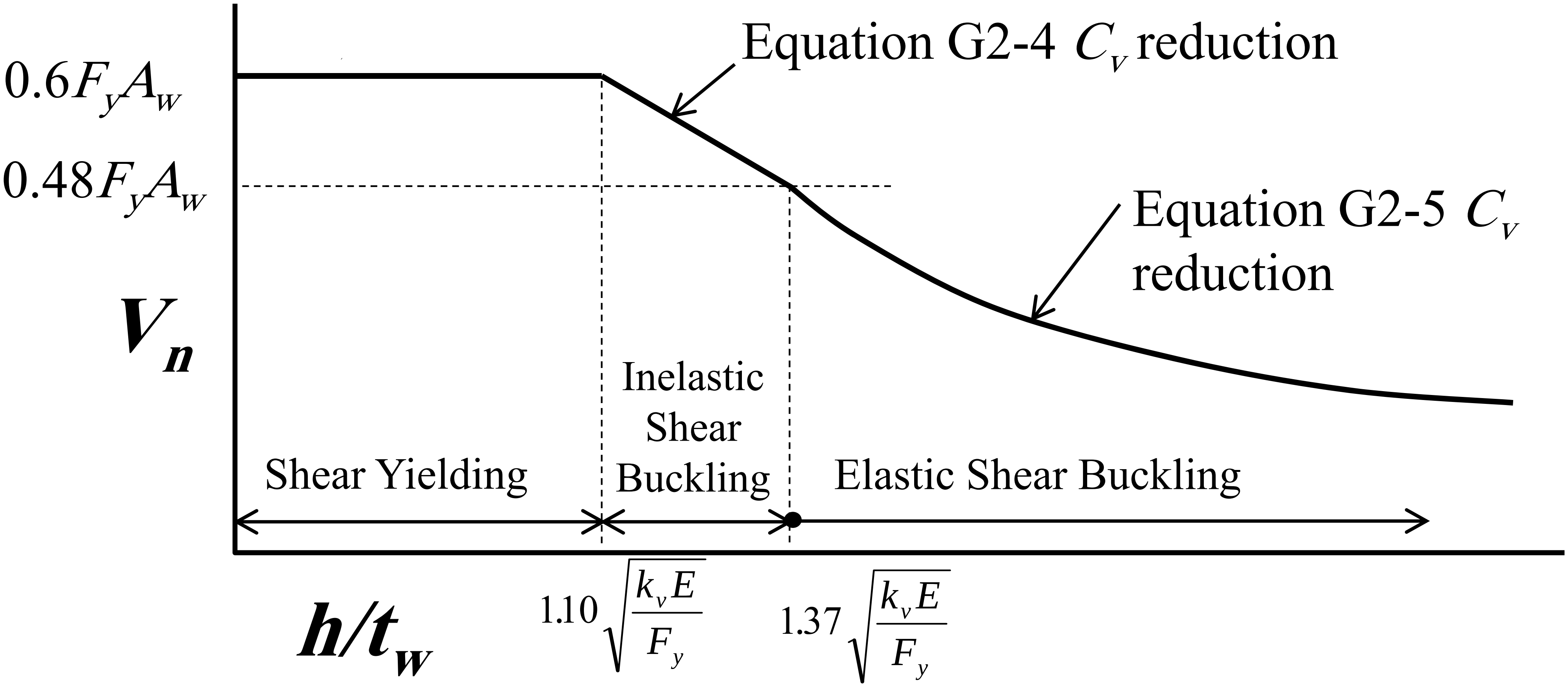
$\phi_v = 0.9$ ($\Omega = 1.67$)

If $h/t_w \le 1.10\sqrt{k_v E/F_y}$ then $C_v = 1$ Equation G2-3

If $1.10\sqrt{k_v E/F_y} \le h/t_w \le 1.37\sqrt{k_v E/F_y}$ then $C_v = \dfrac{1.10\sqrt{k_v E/F_y}}{h/t_w}$ Equation G2-4

If $h/t_w > 1.37\sqrt{k_v E/F_y}$ then $C_v = \dfrac{1.51 k_v E}{\left(h/t_w\right)^2 F_y}$ Equation G2-5

Shear Strength



AISC Requirements for Shear Strength

Reference: Segui, 5th

$$V_n = 0.6F_yA_wC_v \qquad \text{(AISC Equation G2-1)}$$

where

A_w = area of the web dt_w

d = overall depth of the beam

C_v = ratio of critical web stress to shear yield stress

The value of C_v depends on whether the limit state is web yielding, web inelastic buckling, or web elastic buckling.

Case 1: For hot-rolled I shapes with

$$\frac{h}{t_w} \leq 2.24\sqrt{\frac{E}{F_y}}$$

The limit state is shear yielding, and

$$C_v = 1.0 \qquad \text{(AISC Equation G2-2)}$$

$$\phi_v = 1.00$$

$$\Omega_v = 1.50$$

Most W shapes with $F_y \leq 50$ ksi fall into this category (see User Note in AISC G2.1[a]).

AISC Requirements for Shear Strength

Case 2: For all other doubly and singly symmetric shapes,

$\phi_v = 0.90$

$\Omega_v = 1.67$

and C_v is determined as follows:

For $\frac{h}{t_w} \leq 1.10\sqrt{\frac{k_v E}{F_y}}$, there is no web instability, and

$C_v = 1.0$ (AISC Equation G2-3)

(This corresponds to Equation 5.8 for shear yielding.)

For $1.10\sqrt{\frac{k_v E}{F_y}} < \frac{h}{t_w} \leq 1.37\sqrt{\frac{k_v E}{F_y}}$, inelastic web buckling can occur, and

$$C_v = \frac{1.10\sqrt{\frac{k_v E}{F_y}}}{h/t_w}$$ (AISC Equation G2-4)

For $\frac{h}{t_w} > 1.37\sqrt{\frac{k_v E}{F_y}}$, the limit state is elastic web buckling, and

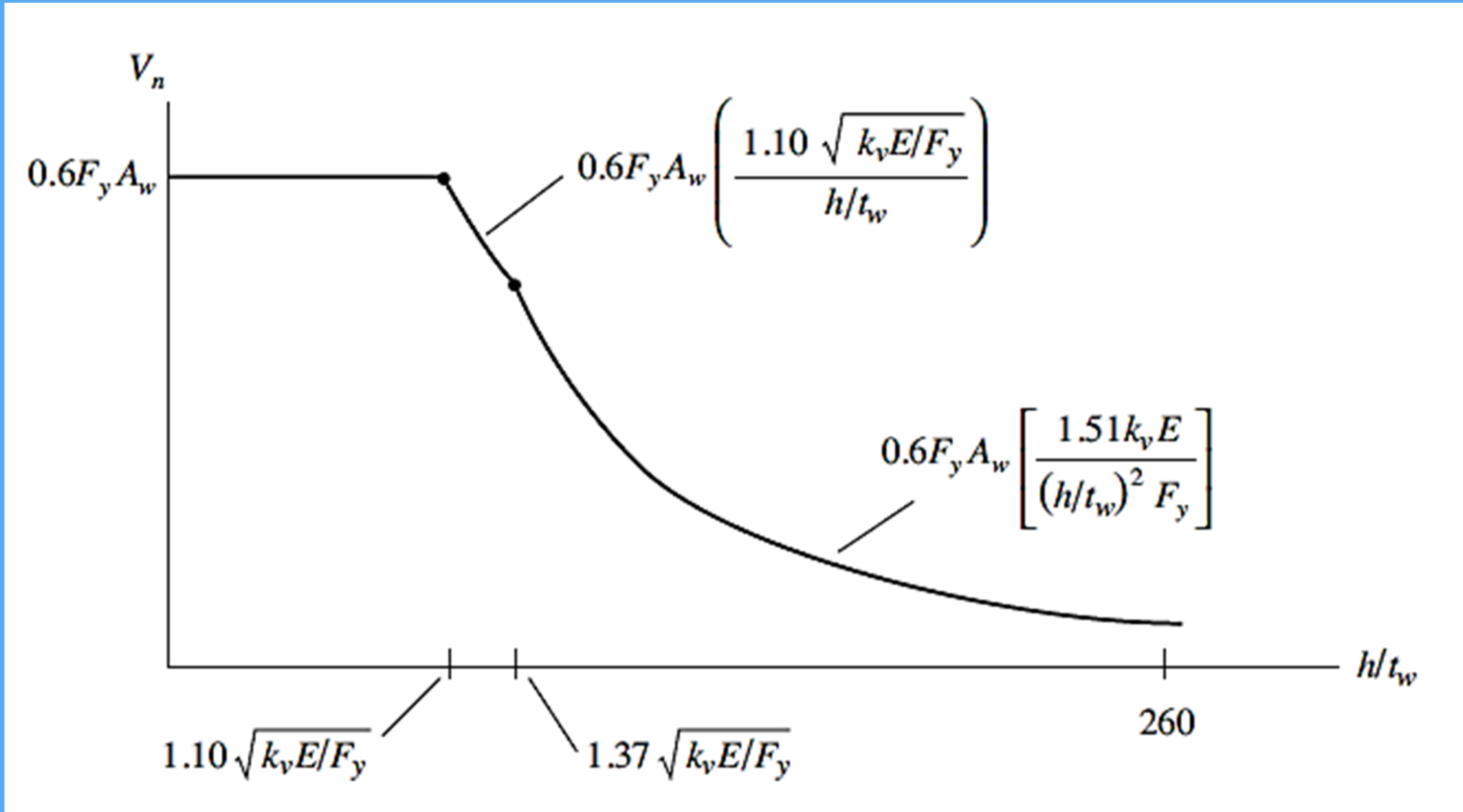
$$C_v = \frac{1.51 k_v E}{(h/t_w)^2 F_y}$$ (AISC Equation G2-5)

where

$k_v = 5$

Reference:
Segui, 5th

AISC Requirements for Shear Strength



Reference:
Segui, 5th

AISC Requirements for Shear Strength

Note:

$kv = 5$

- This value of k_v is for unstiffened webs with $h/t_w < 260$.
- Although section G2.1 of the Specification does not give $h/t_w = 260$ as an upper limit, no value of k_v is given when $ht_w \geq 260$.
- In addition, AISC F13.2, "Proportioning Limits for I-Shaped Members" states that h/t_w in unstiffened girders shall not exceed 260.

Reference:
Segui, 5th

AISC Requirements for Shear Strength

Situations where Shear might be excessive

Generally, shear is not a problem in steel beams, because the webs of rolled shapes are capable of resisting rather large shearing forces. most common situations where shear might be excessive:

- ❑ Should large concentrated loads be placed near beam supports, they will cause large internal forces without corresponding increases in bending moments. A fairly common example of this type of loading occurs in tall buildings where, on a particular floor, the upper columns are offset with respect to the columns below. The loads from the upper columns applied to the beams on the floor level in question will be quite large if there are many stories above.

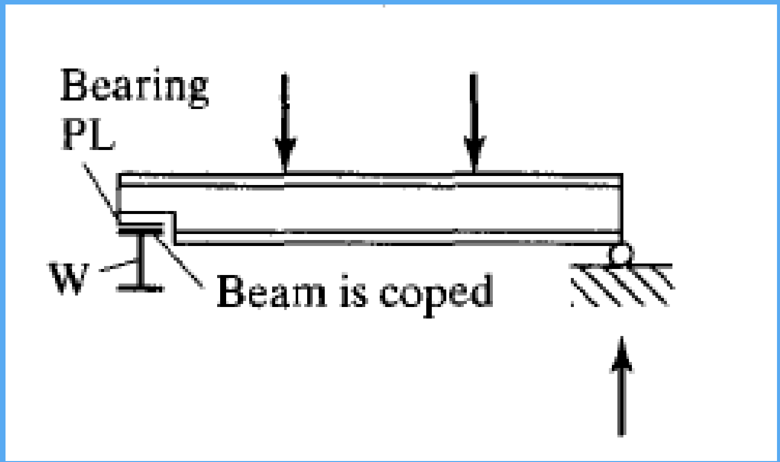
- ❑ Probably the most common shear problem occurs where two members (as a beam and a column) are rigidly connected together so that their webs lie in a common plane. This situation frequently occurs at the junction of columns and beams (or rafters) in rigid frame structures.

Reference:
Mcormac, 5th

AISC Requirements for Shear Strength

Situations where Shear might be excessive

- Where beams are notched or coped, shear can be a problem. For this case, shear forces must be calculated for the remaining beam depth. A similar discussion can be made where holes are cut in beam webs for ductwork or other items.



- Theoretically, very heavily loaded short beams can have excessive shears, but practically, this does not occur too often unless large concentrated loads be placed near beam supports

Reference:
Mcormac, 5th

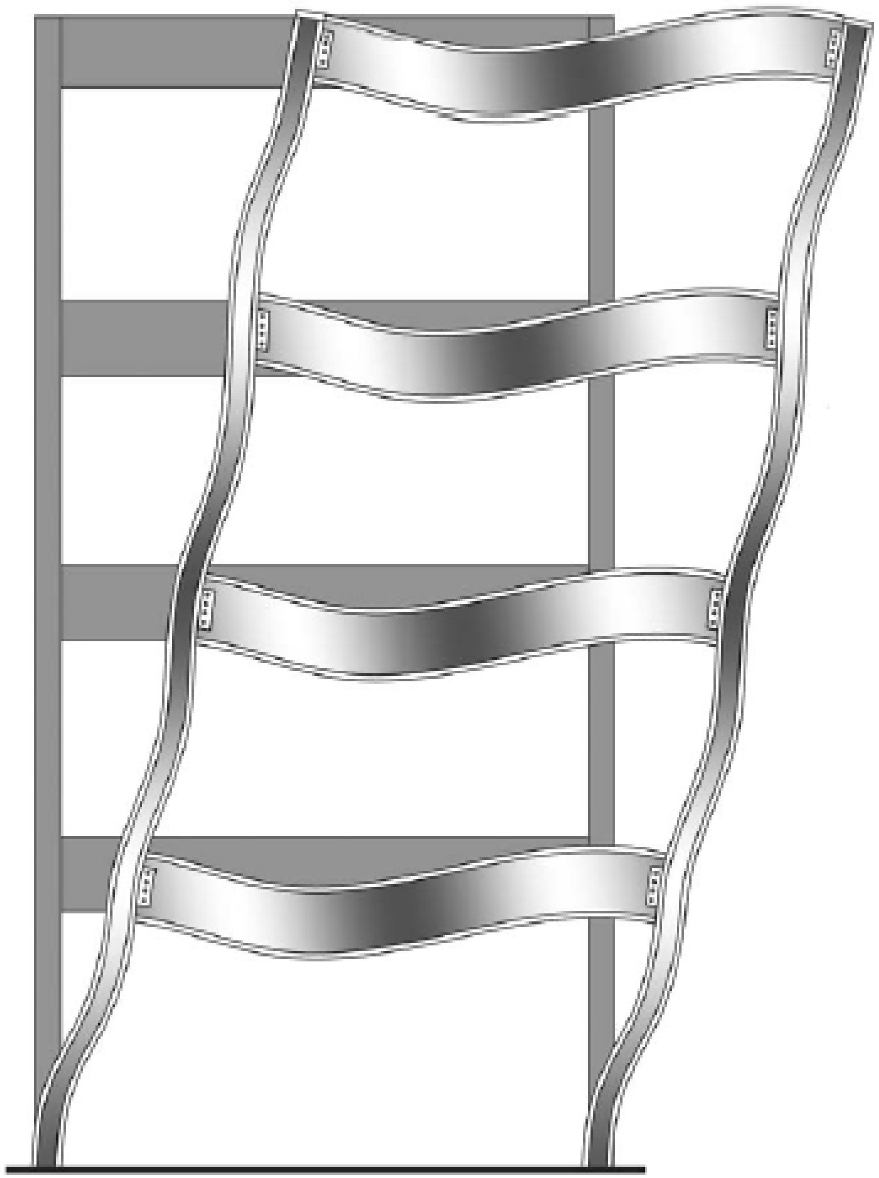
AISC Requirements for Shear Strength

Situations where Shear might be excessive

- Shear may very well be a problem even for ordinary loadings when very thin webs are used, as in plate girders or in light-gage cold-formed steel members.

Reference:
Mcormac, 5th

Beam Deflections



In addition to being safe, a structure must be serviceable.

A serviceable structure is one that performs satisfactorily, not causing any discomfort or perceptions of unsafety for the occupants or users of the structure.

Serviceability is defined in the AISC Specification as "a state in which the function of a building, its appearance, maintainability, durability, and comfort of its occupants are preserved under normal usage".

Why deflections of steel beams are limited to certain maximum values?

1. Excessive deflections may damage other materials attached to or supported by the beam in question. Plaster cracks caused by large ceiling joist deflections are one example.

2. The appearance of structures is often damaged by excessive deflections.

3. Extreme deflections do not inspire confidence in the persons using a structure, although the structure may be completely safe from a strength standpoint.

4. It may be necessary for several different beams supporting the same loads to deflect equal amounts.

Beam Deflections

Typical limitation based on Service Live Load Deflection

Typical criteria:

Max. Deflection, $\delta = L/240$, $L/360$, $L/500$, or $L/1000$

L = Span Length

For situations where precise and delicate machinery is supported, maximum deflections may be limited to 111500 or 112000 of the span lengths.

Note that deflections are "serviceability" concern, so No Load factors should be used in calculations.

Methods for Calculating Beam Deflections

These methods include:

1. the moment area,
2. conjugate beam,
3. virtual-work procedures.

From these methods, various expressions can be determined, such as the following common one for the center line deflection of a uniformly loaded simple beam:

$$\Delta_{\text{¢}} = \frac{5wL^4}{384EI}$$

Beam Deflections: Deflection Limit

Code: ICC 2015

TABLE 5.4 Deflection Limits

Type of member	Max. live load defl.	Max. dead + live load defl.	Max. snow or wind load defl.
Roof beam:			
Supporting plaster ceiling	$L/360$	$L/240$	$L/360$
Supporting nonplaster ceiling	$L/240$	$L/180$	$L/240$
Not supporting a ceiling	$L/180$	$L/120$	$L/180$
Floor beam	$L/360$	$L/240$	—

Beam Deflections: Maximum Deflections

Code: 2015 مدونة البناء العراقية 305 لسنة

الجدول 6−2/2: محددات الهطول

المحدد	استعمال العتبة
L/360	أرضيات ساندة لأعمال جصية أو بياض أو أي مواد قصيفة أخرى
L/240	أرضيات غير ساندة لمواد قصيفة
L/180	سقوف غير ساندة لمواد قصيفة

حيث أن (L) يمثل فضاء العتبة الكلي.

Beam Deflections: Maximum Deflections

Code: IBC Table 1604.3

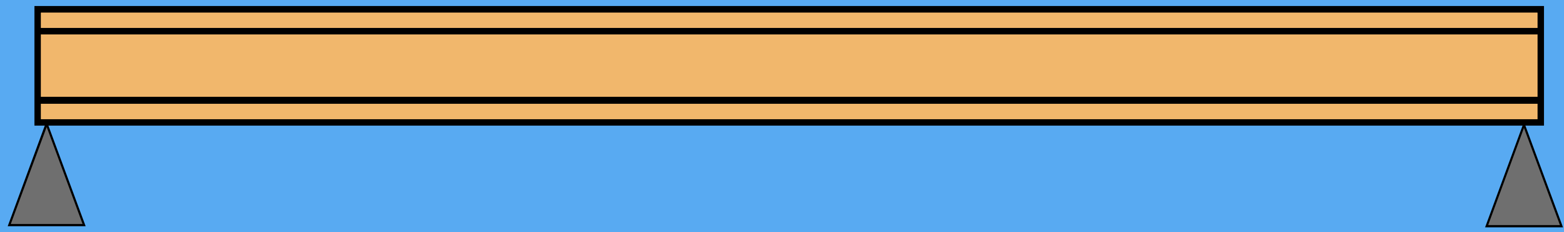
Construction	*L*	*S* or *W*	*D* + *L**
Roof members:			
Supporting plaster ceiling	1/360	1/360	1/240
Supporting nonplaster ceiling	1/240	1/240	1/180
Not supporting ceiling	1/180	1/180	1/120
Floor members	1/360	–	1/240
Exterior walls and interior partitions:			
With brittle finishes	–	1/240	–
With flexible finishes	–	1/120	–

Beam Deflections: Maximum Deflections

AISC Requirements, L3

1. L/360 for floors subjected to reduced live load
2. L/240 for roof members.
3. In long-span floors, it may be necessary to impose a limit on the maximum deflection, independent of span length, to prevent damage to adjacent nonstructural elements. Damage to nonload-bearing partitions may occur when deflections exceed 3/8 in.

Beam Deflections: Camber



Beam without Camber

Camber: curvature applied to a beam during fabrication to allow it to be relatively straight (no deflection) when dead loads are applied (self weight, floor slab) – this allows for a flat floor of consistent depth in a completed structure. Note that this is not an exact process, so camber calculations to the nearest ½ inch are usually sufficient.

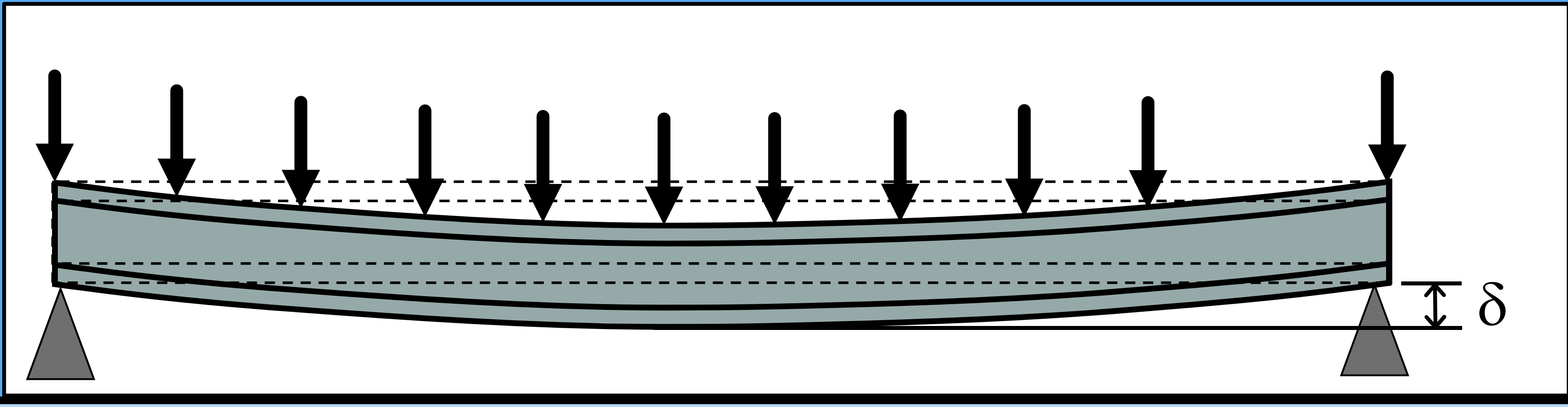
Beam Deflections: Camber

Calculate deflection in beams from expected service dead load.

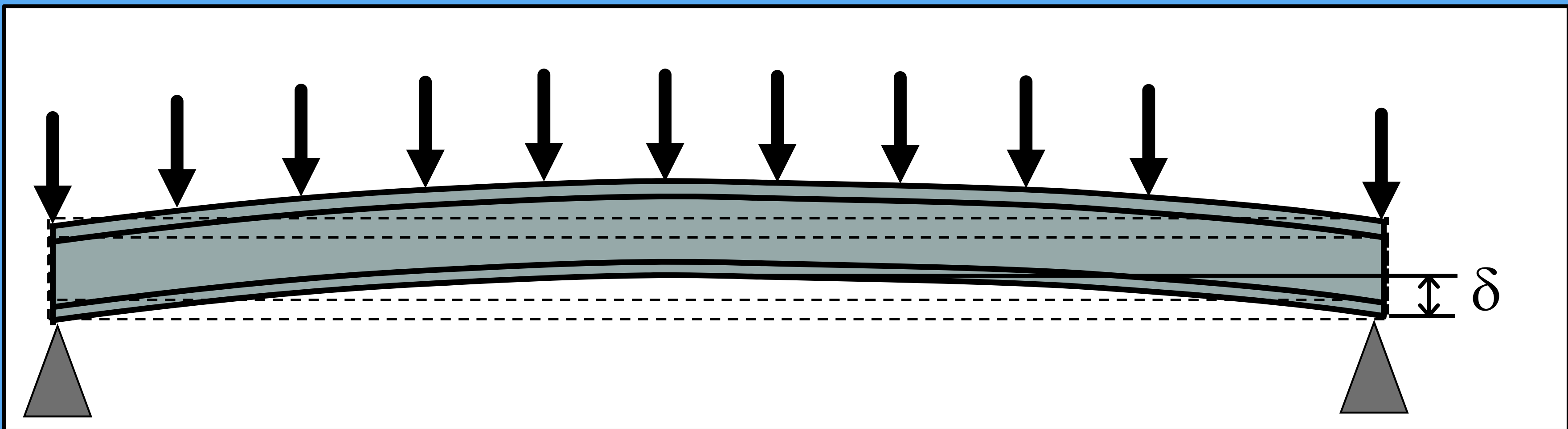
Provide deformation in beam equal to a percentage of the dead load deflection and opposite in direction. It is important not to over-camber.

Result is a straight beam after construction.

Specified on construction drawings.



Results in deflection in floor under Dead Load.
This can affect thickness of slab and fit of non-structural components.



Beam with Camber
Cambered beam counteracts service dead load deflection.`

Beam Deflections: AISC Formulas, Page 3-7

The maximum vertical deflection Δ, in., can be calculated using the equations given in Tables 3–22 and 3–23. Alternatively, for common cases of simple-span beams and I-shaped members and channels, the following equation can be used:

$$\Delta = ML^2 / (C_1 I_x)$$

where

M = maximum service-load moment, kip-ft
L = span length, ft
I_x = moment of inertia, in.^{4}
C_1 = loading constant (see Figure 3–2) which includes the numerical constants appropriate for the given loading pattern, E, which has units of ksi, and a ft-to-in. conversion factor of 1,728 in.^{3}/ft^3.

Beam Deflections: AISC Formulas, Page 3-7

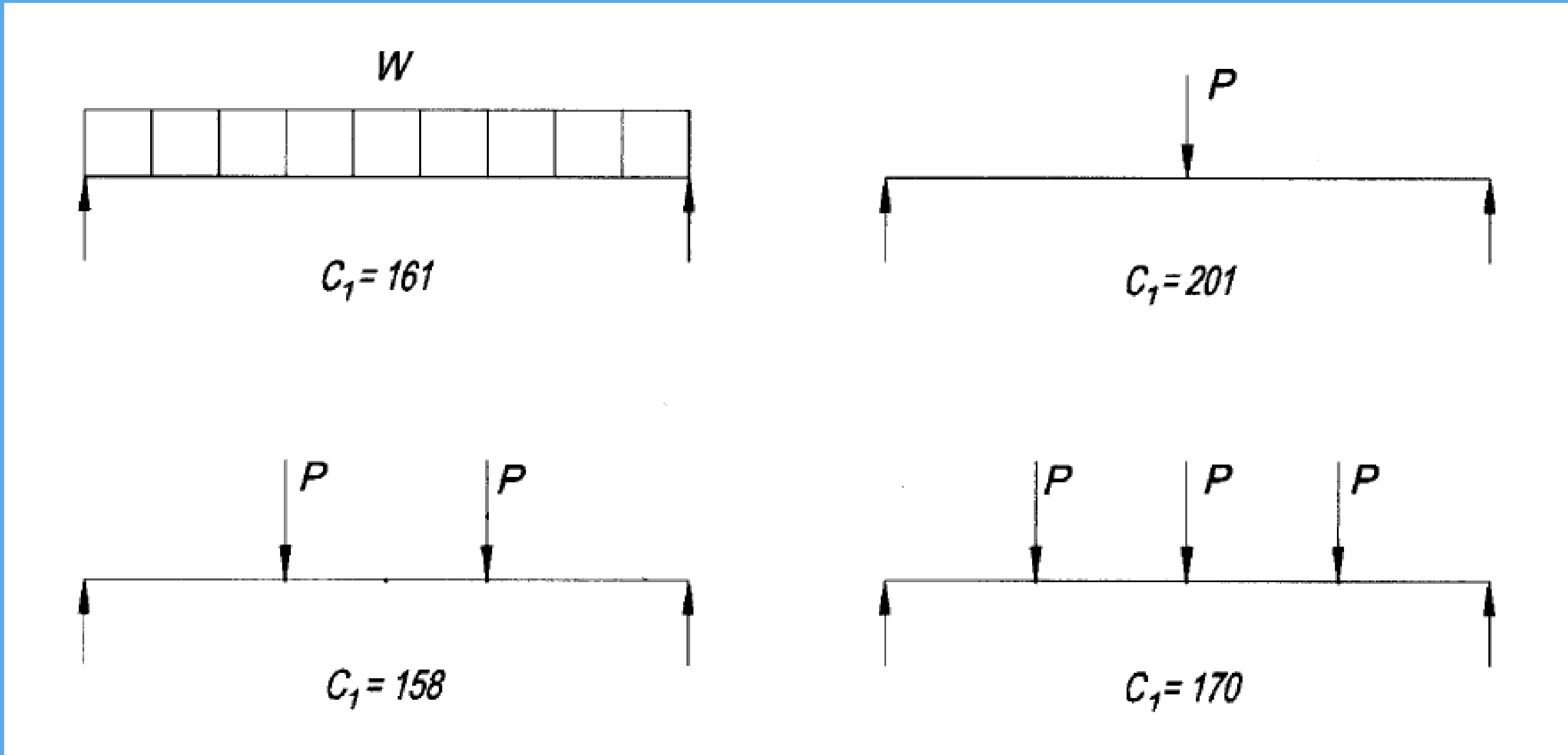


Table 3–22c

Continuous Beams

Moments and Shear Coefficients – Equal Spans, Equally Loaded

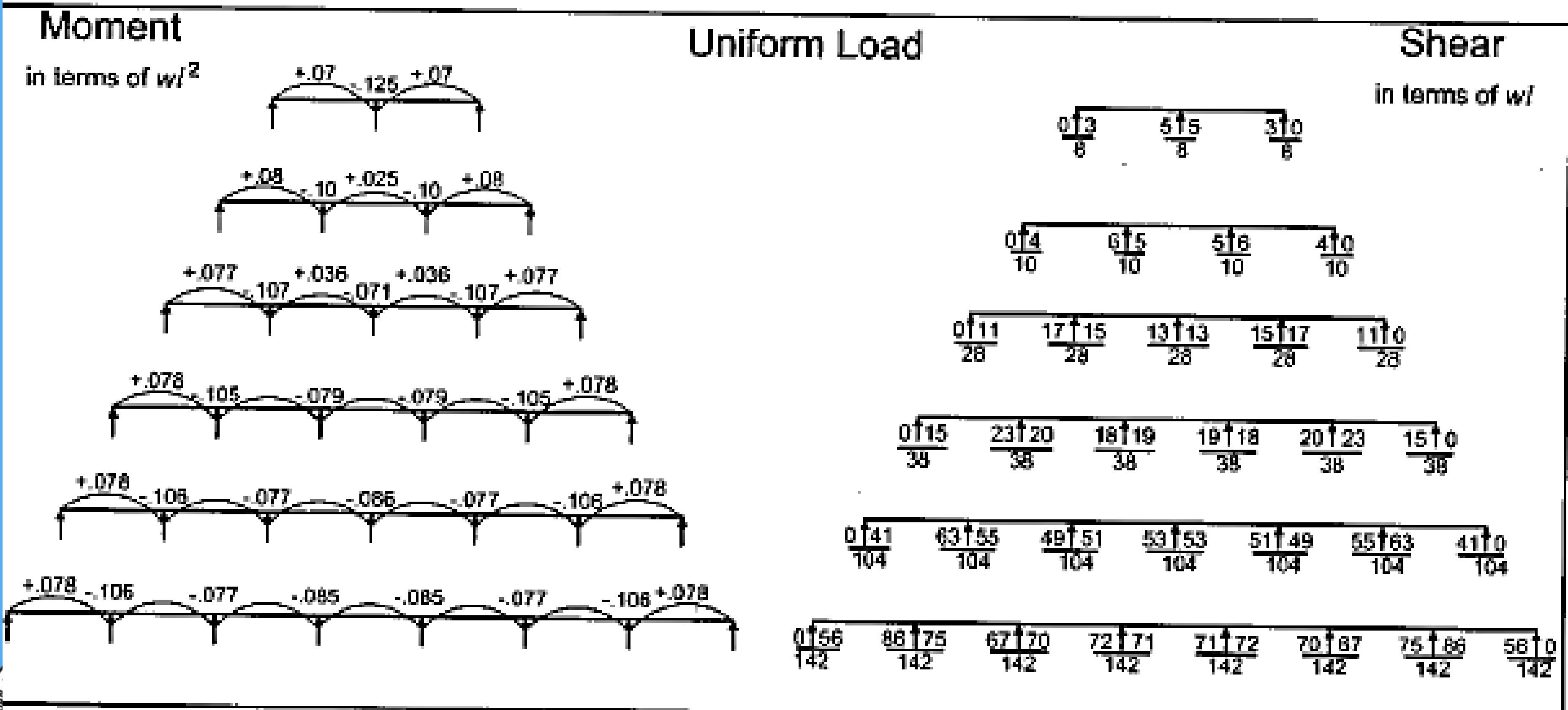
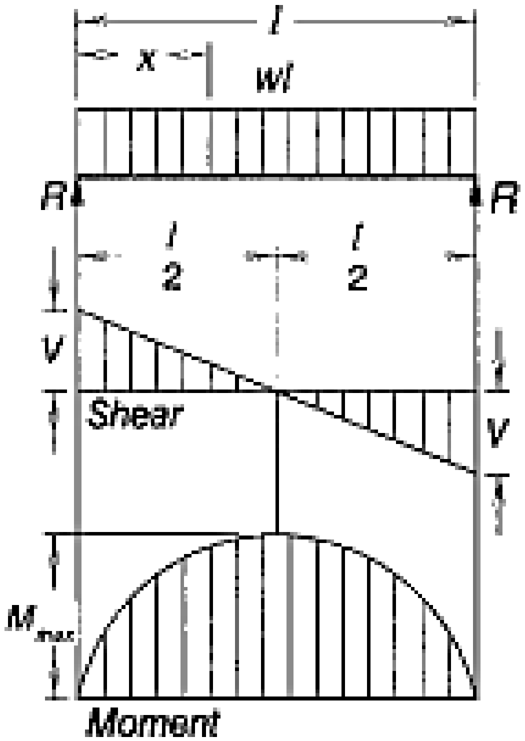


Table 3–23
Shears, Moments, and Deflections

1. SIMPLE BEAM — UNIFORMLY DISTRIBUTED LOAD



Total Equiv. Uniform Load .... $= wl$

$R = V$ .... $= \frac{wl}{2}$

V_x .... $= w\left(\frac{l}{2} - x\right)$

M_{max} (at center) .... $= \frac{wl^2}{8}$

M_x .... $= \frac{wx}{2}(l - x)$

Δ_{max} (at center) .... $= \frac{5wl^4}{384EI}$

Δ_x .... $= \frac{wx}{24EI}\left(l^3 - 2lx^2 + x^3\right)$

Design of Beam

Design of Beam

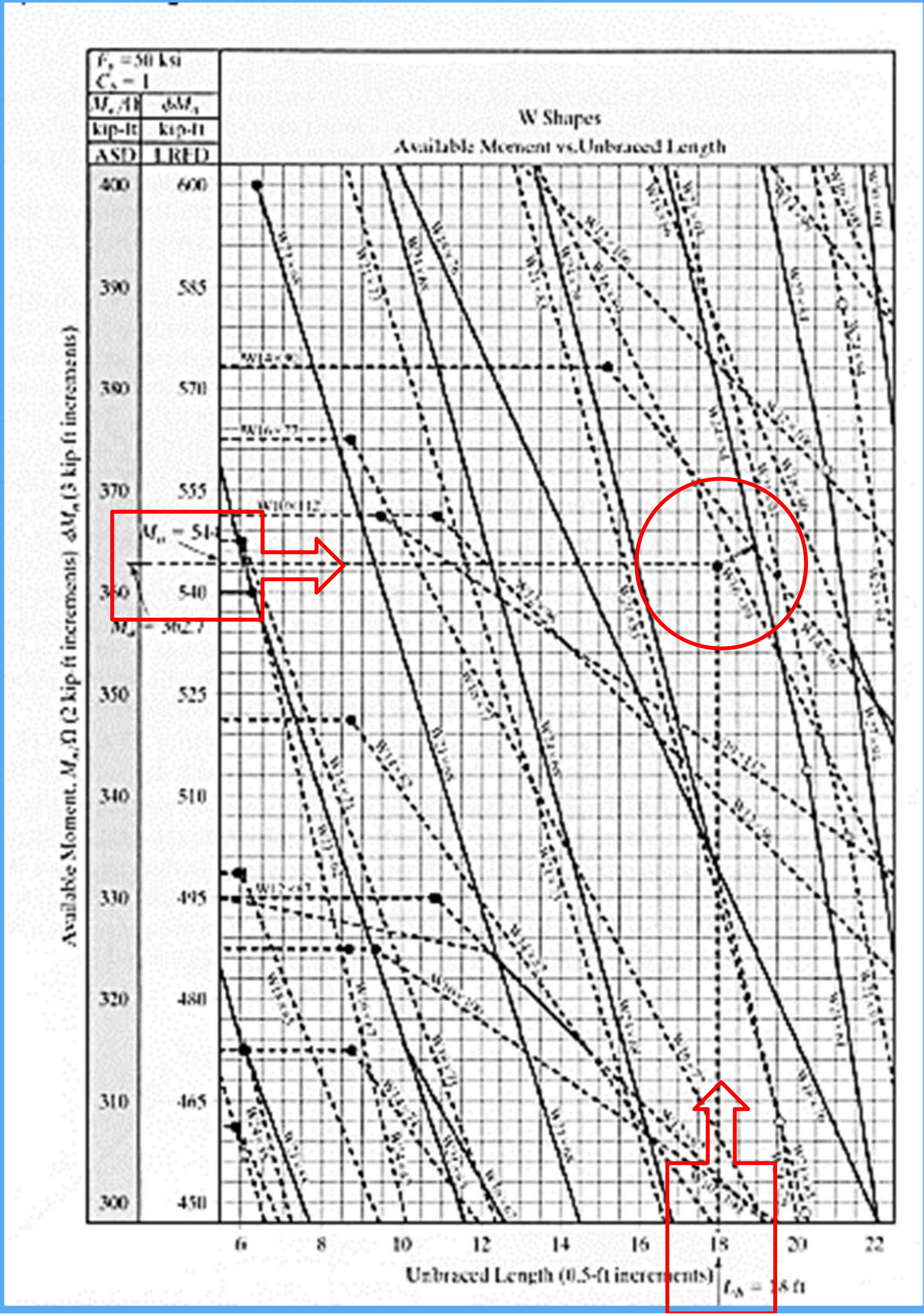
Selection of a cross-sectional shape that will have enough strength and that will meet serviceability requirements.

For any flexural member, flexure is almost always more critical than shear, so the usual practice is to design for flexure and then check shear.

Design of Beam

1. Compute the required moment strength, weight may be ignored initially and checked after a shape has been selected.

2. Select a shape that satisfies this strength requirement. Use the beam design charts in Part 3 of the Manual.


W Shapes
Available Moment vs. Unbraced Length
ASD
LRFD
kip-ft
Available Moment, M_n/Ω (2 kip-ft increments) ϕM_n (3 kip-ft increments)
Unbraced Length (0.5-ft increments)
M_u = 544
L_b = 18 ft

Beam Subjected to Biaxial Bending

Beam Subjected to Biaxial Bending

Biaxial Bending

Bending about two axes

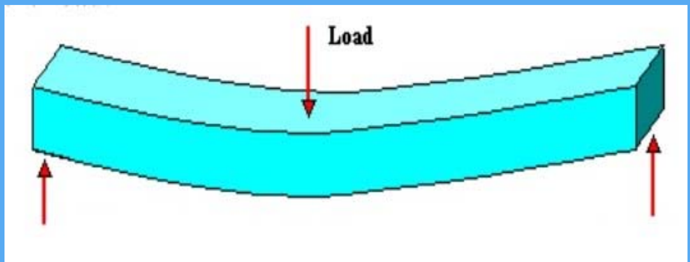
- Biaxial bending is produced in a member when bending moments are applied simultaneously about both principal axes.

- Biaxial bending Occurs when a beam is subjected to a loading condition that produces bending about both the major (strong) axis and the minor (weak) axis.

Beam Subjected to Biaxial Bending

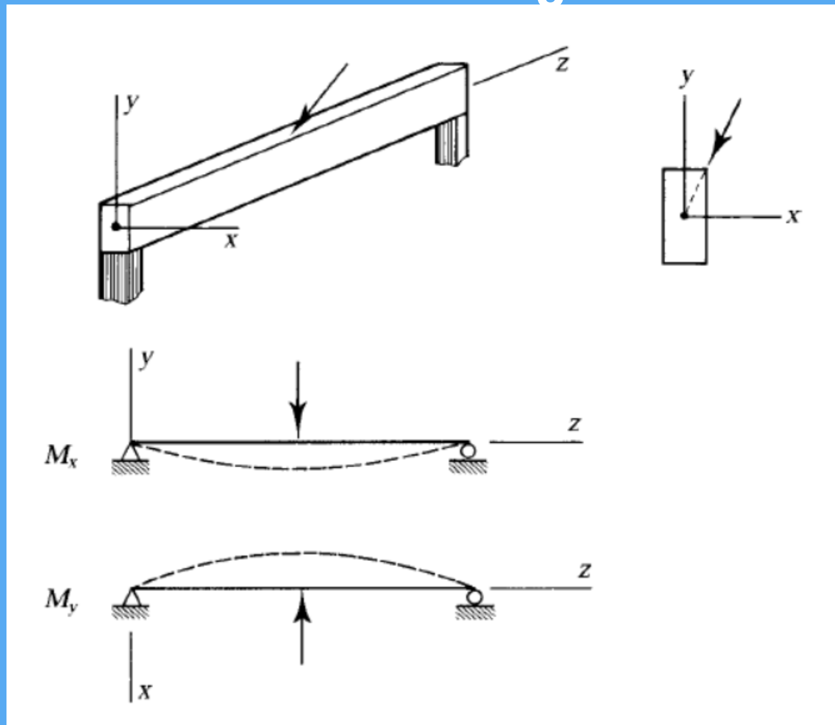
Uniaxial bending vs Biaxial bending

Uniaxial bending



- Where a single concentrated load acts normal to the longitudinal axis of the beam.
- Load passes through shear center

Biaxial bending



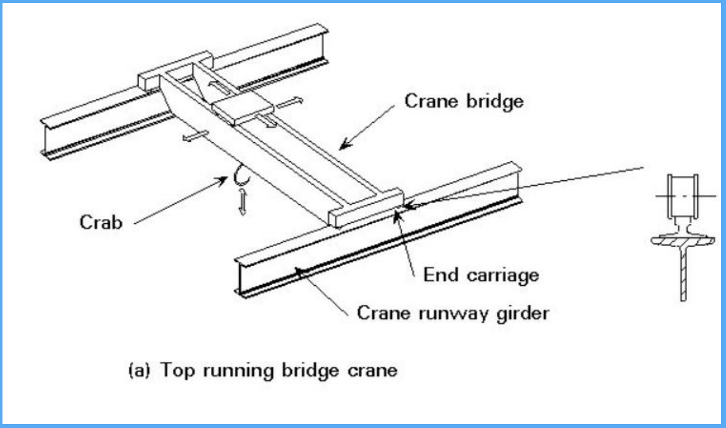
Where a single concentrated load acts normal to the longitudinal axis of the beam but is inclined with respect to each of the principal axes of the cross section.

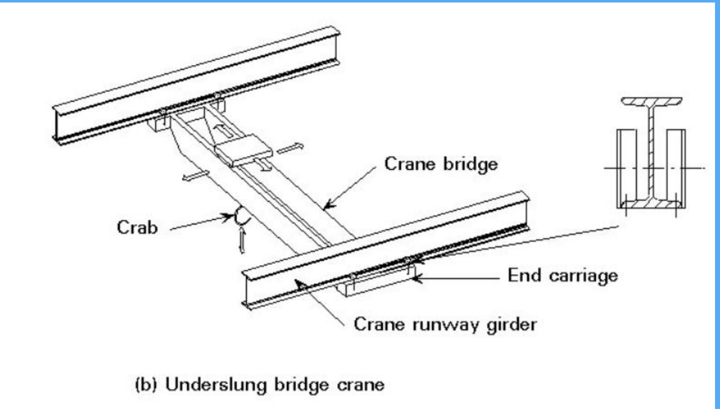
In Biaxial Bending, Load may passes through the shear center on not through the shear center.

Beam Subjected to Biaxial Bending

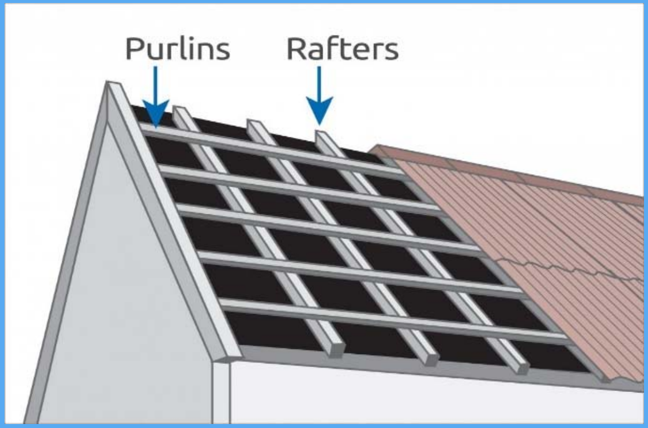
Applications of Biaxial bending

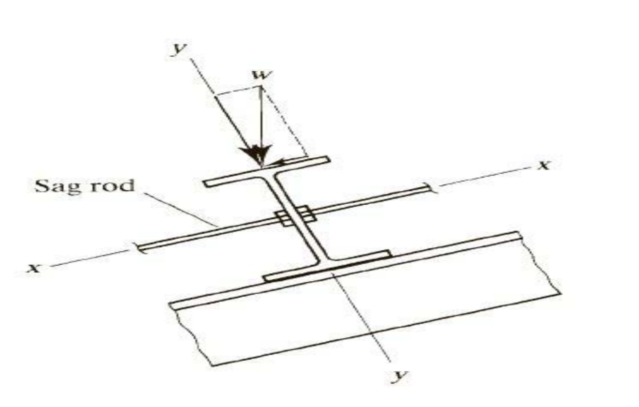
Overhead crane runway girders



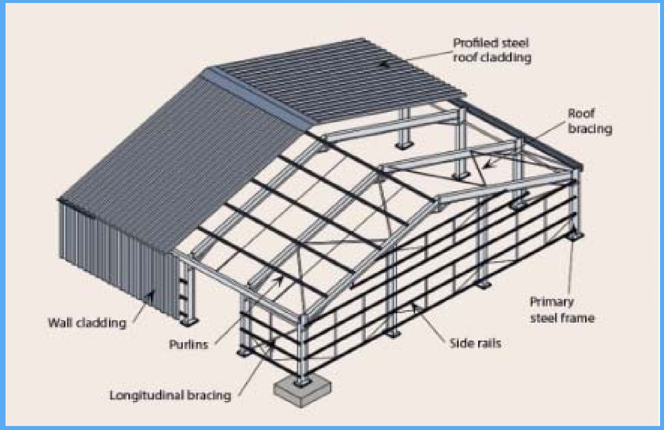


Roof purlins





side sheeting rails

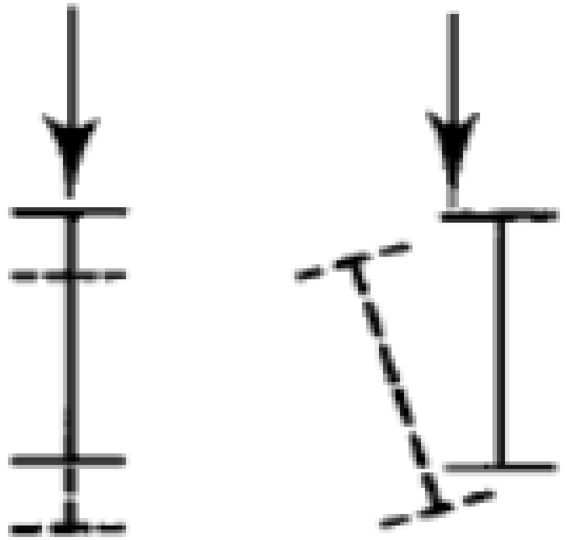


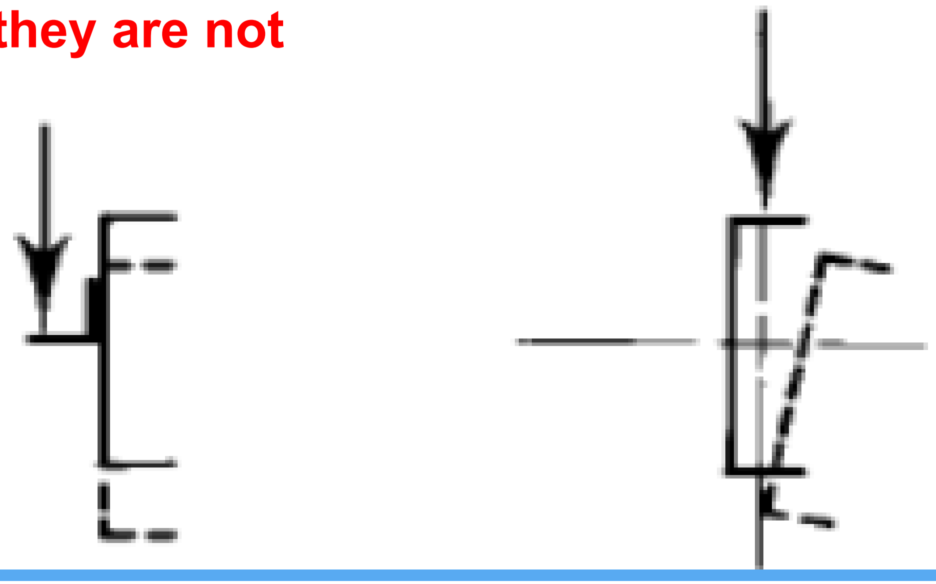
Beam Subjected to Biaxial Bending

Shear Center

- The shear center is that point through which the loads must act if there is to be no twisting, or torsion, of the beam.
- The shear center is defined as the point on the cross section of a beam through which the resultant of the transverse loads must pass so that the stresses in the beam may be calculated only from the theories of pure bending and transverse shear.
- Should the resultant pass through this point. it is unnecessary to analyze the beam for torsional moments.

The deflected position of two beams when loads are applied through the shear center and when they are not

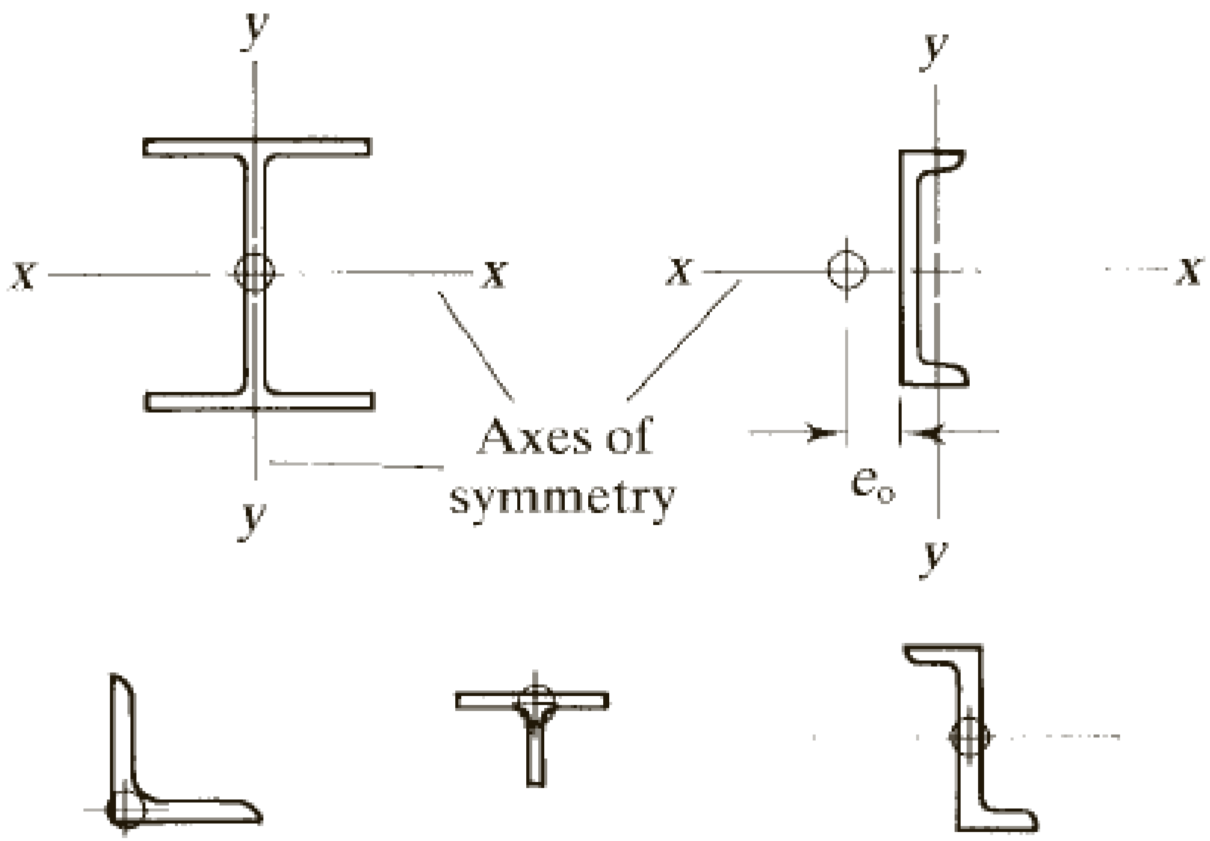




Beam Subjected to Biaxial Bending

Location of Shear Center

The location of the shear center can be determined from elementary mechanics of materials by equating the internal resisting torsional moment, derived from the shear flow on the cross section, to the external torque.



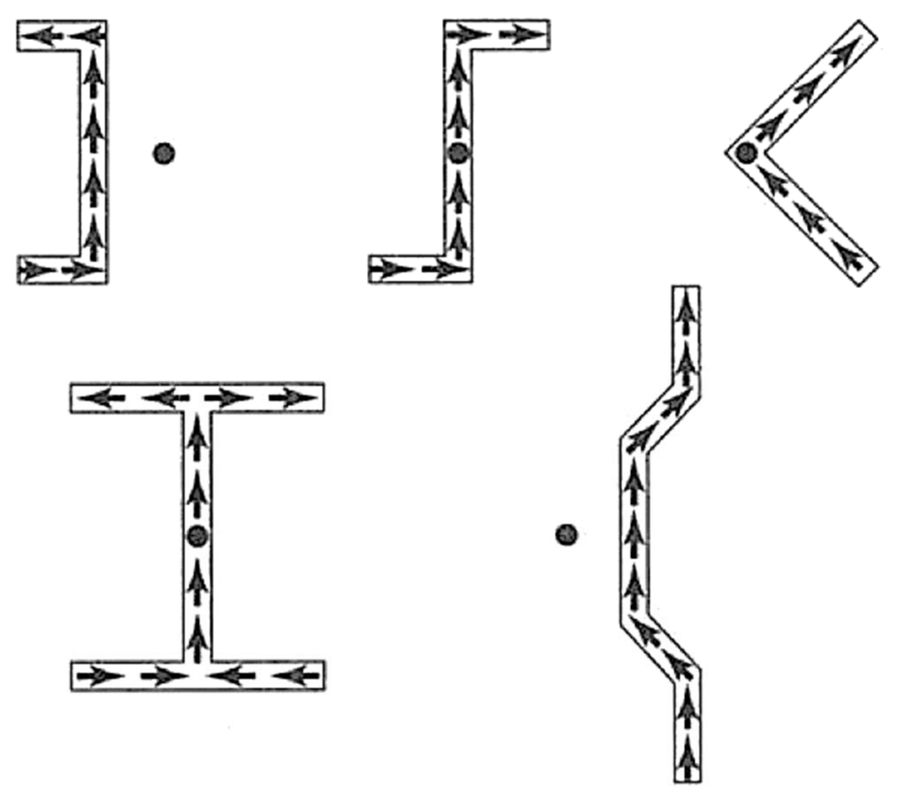
- shear center is indicated by a circle.
- The value of e_o, which Locates the shear center for channel shapes, is tabulated in the *Manual*.
- The shear center is always located on an axis of symmetry; thus the shear center will be at the centroid of a cross section with two axes of symmetry

Beam Subjected to Biaxial Bending

Location of Shear Center

- The shear center is o f particular importance for beams whose cross sections are composed of thin parts that provide considerable bending resistance but little resistance to torsion.

- Many common structural members. such as the W. S, and C sections, angles, and various beams made up of thin plates fall into this class.



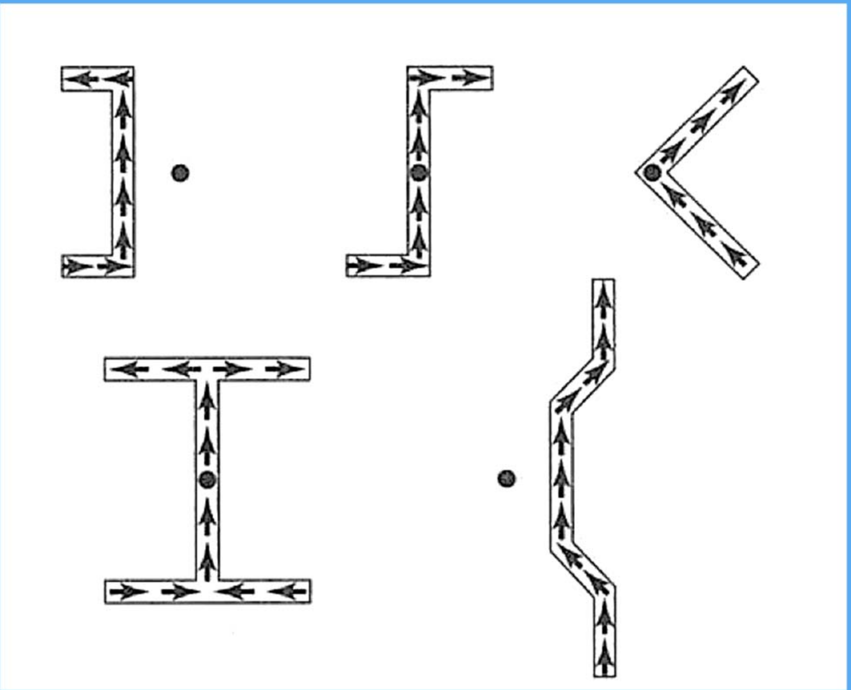
- To avoid torsion in some beams the lines of action of the applied loads and beam reactions should not pass through the centroids of the sections.

- Sections such as these are relatively weak in torsion, and for them and similar shapes, the location of the resultant of the external loads can be a very serious mailer.

Beam Subjected to Biaxial Bending

Thin Wall Members and Shear Flow

- The term *shear flow* is often used when reference is made to thin-wall members, although there is really no flowing involved.
- *Shear flow* refers to the shear per inch of the cross section and equals the unit shearing stress times the thickness of the member.



The unit shearing stress:

$F_v = VQ/bI$,

and

the shear flow *qv* can be determined by:

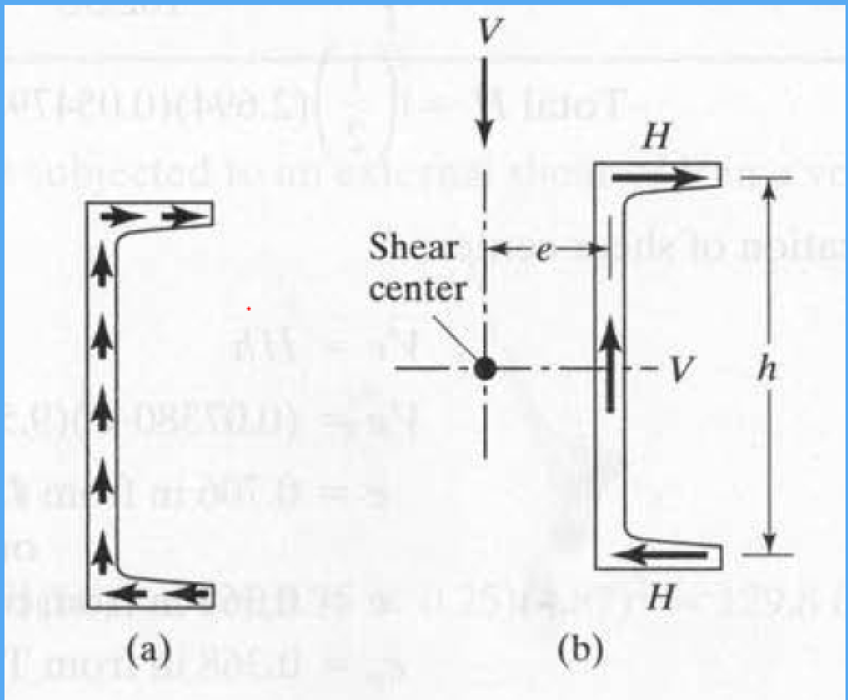
$q_v = VQ/I$

If the shearing stress is assumed to be constant across the thick ness of the section.) The shear flow acts parallel to the sides of each element of a member.

Beam Subjected to Biaxial Bending

Thin Wall Members and Shear Flow

- The term *shear flow* is often used when reference is made to thin-wall members, although there is really no flowing involved.
- *Shear flow* refers to the shear per inch of the cross section and equals the unit shearing stress times the thickness of the member.
- The shear flow is shown with the small arrows in Figure a, and the values are totaled for each component of the shape and labeled H and V in part (b).
- The two *H* values are in equilibrium horizontally, and the internal *V* value balances the external shear at the section.



- Although the horizontal and vertical forces are in equilibrium, the same cannot be said for the moment forces, unless the lines of action of the resultant of the external forces pass through a certain point called the *shear center.*
- The horizontal *H* forces in part (b) of the figure can be seen to form a couple. The moment produced by this couple must be opposed by an equal and opposite moment, which can be produced only by the two *V* values.
- The location of the shear center is a problem in equilibrium; therefore, moments should be taken about a point that eliminates the largest number of forces possible. With this information, the following equation can be written from which the shear center can be located:

$Ve = H\,h$ (moments taken about c.g. of web)

Beam Subjected to Biaxial Bending

Case I: Loads Applied Through the Shear Center

When the applied loads about both principal axes act through the shear center, twisting effects do not have to be considered.

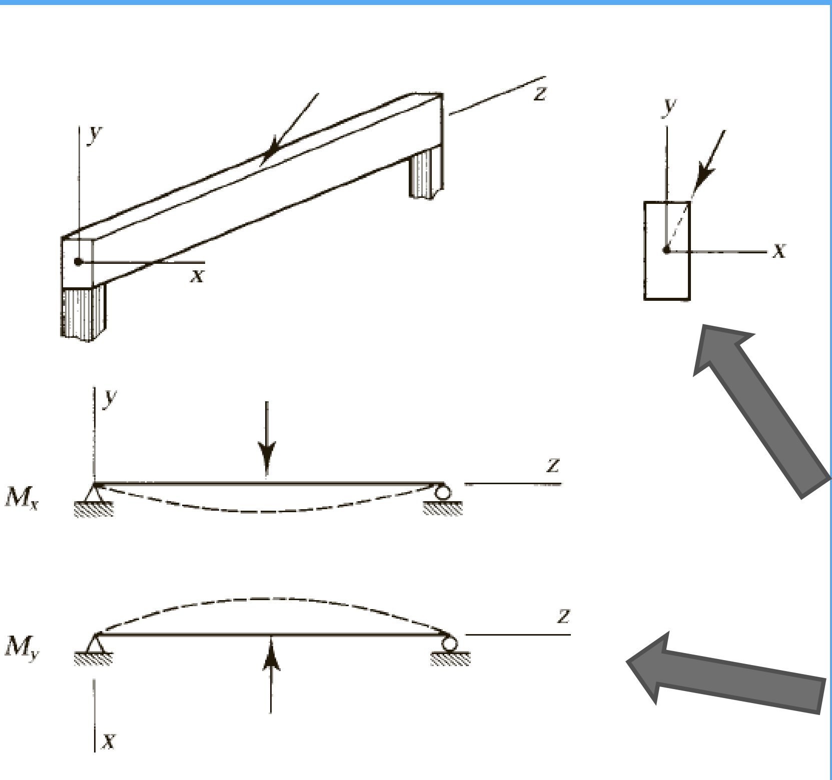
For this situation, AISC 360 Commentary. Sec. H1.1 states that biaxial bending may be considered a special case of AISC 360 Eq. (H1-1b) with the axial load term equated to

$$\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}} \le 1.0$$

For LRFD

$$\frac{M_{ax}}{M_{nx}/\Omega_b} + \frac{M_{ay}}{M_{ny}/\Omega_b} \le 1.0$$

For ASD



where
Mux = factored-load moment about the *x* axis
Mnx = nominal moment strength for the *x*-axis
Muy = factored-load moment about the *y* axis
Mny = nominal moment strength for the *y* axis
Max = service load moment about the x axis
May = service load moment about the y axis

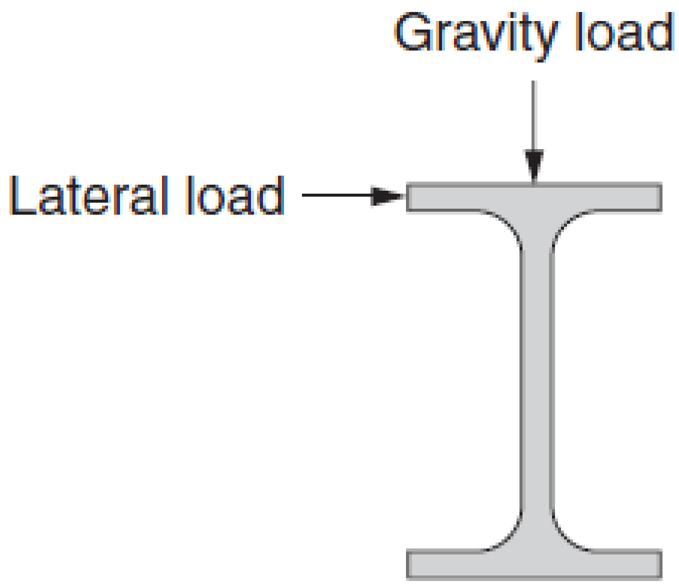
- The load can be resolved into rectangular components in the x- and y-directions
- Each component producing bending about a different axis.

Beam Subjected to Biaxial Bending

Case I: Loads Applied Through the Shear Center

- In most cases, the lateral load is applied to the compression flange of the member as shown below.

- In this situation, the design method proposed by Fisher, is appropriate. It is assumed that the lateral load is resisted solely by the compression flange and torsional effects are neglected.



For I-shapes, the plastic section modulus of one flange about the *y*-axis, is given by:

$Zyi = Zy/2$

where *Zy* is plastic section modulus of the full section about the *y*-axis.

The nominal moment capacity of one flange about the *y*-axis is:

$Mny = FyZyi$

Beam Subjected to Biaxial Bending

The rule:

Weak-Axis Bending Strength

Any shape bent about its weak axis cannot buckle in the other direction, so lateral-torsional buckling is not a limit state

Hence,

If the shape is compact, then the strength is given by:

$$M_{ny} = M_{py} = F_y Z_y \leq 1.6 F_y S_y \qquad \text{(AISC Equation F6-1)}$$

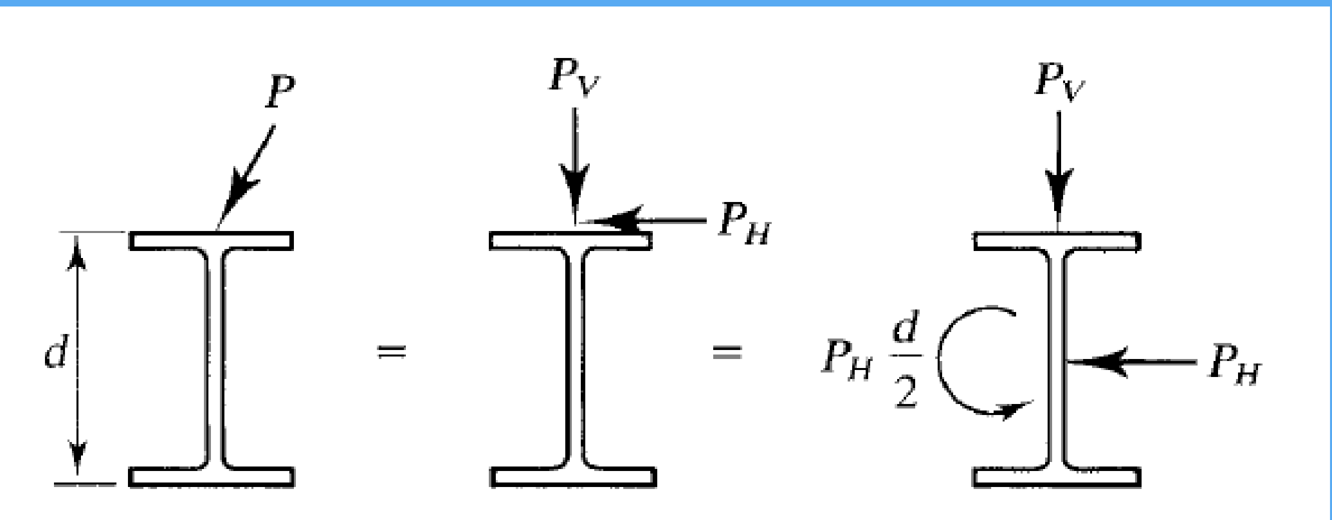
If the shape is non-compact
because of the flange width-to-thickness ratio, the strength will be given by:

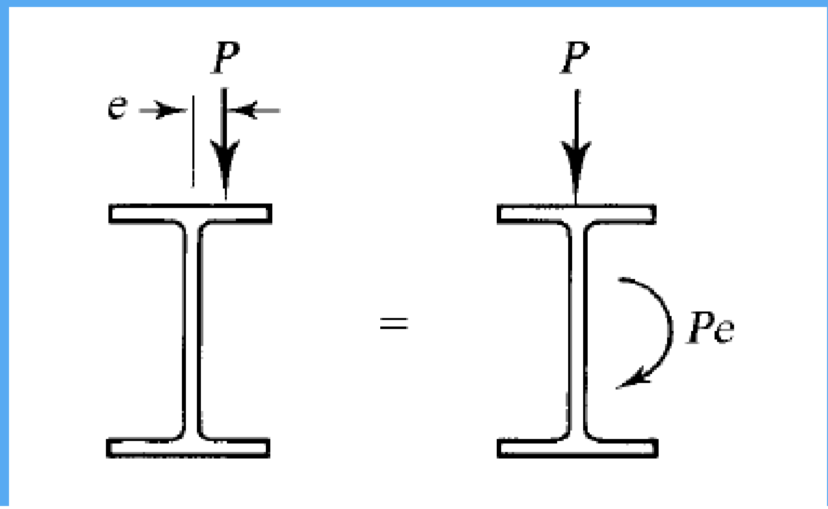
$$M_{ny} = M_{py} - (M_{py} - 0.7 F_y S_y)\left(\frac{\lambda - \lambda_p}{\lambda_r - \lambda_p}\right) \qquad \text{(AISC Equation F6-2)}$$

Beam Subjected to Biaxial Bending

Case II: Loads Not Applied Through the Shear Center

- When loads are not applied through the shear center of a cross section, the result is flexure plus torsion
- If possible, the structure or connection geometry should be modified to remove the eccentricity
- The problem of torsion in rolled shapes is a complex one, and we resort to approximate, methods for dealing with it.



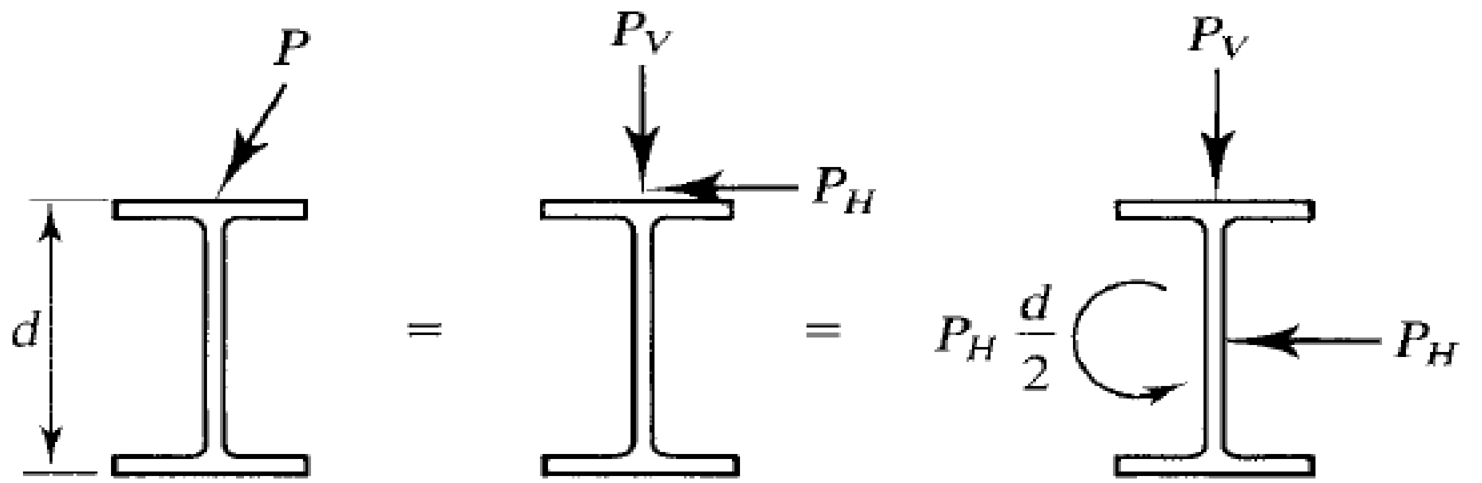


Beam Subjected to Biaxial Bending

Case II: Loads Not Applied Through the Shear Center

A typical loading condition that gives rise to torsion

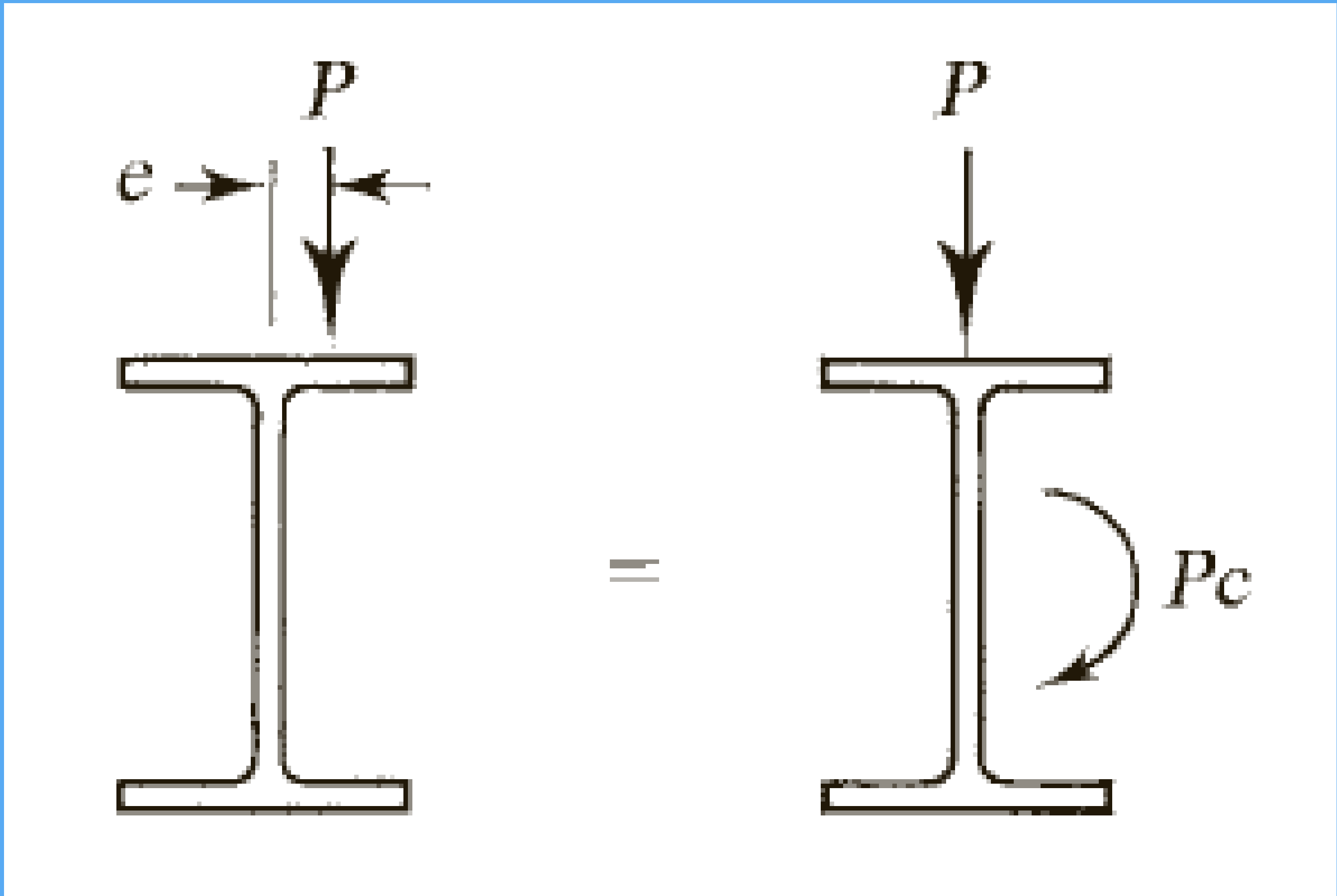
- The resultant load is applied to the center of the top flange, but its line of action does not pass through the shear center of the section.
- As far as equilibrium is concerned, the force can be moved to the shear center provided that a couple is added.
- The equivalent system thus obtained will consist of the given force acting through the shear center plus a twisting moment.



Beam Subjected to Biaxial Bending

Case II: Loads Not Applied Through the Shear Center

A typical loading condition that gives rise to torsion



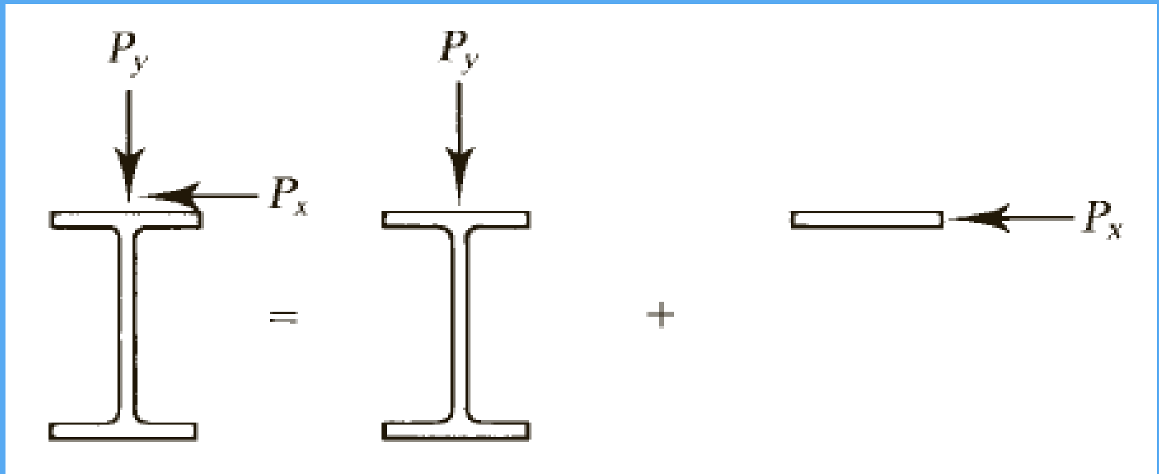
There is only one component of load to contend with, but the concept is the same

Beam Subjected to Biaxial Bending

Case II: Loads Not Applied Through the Shear Center

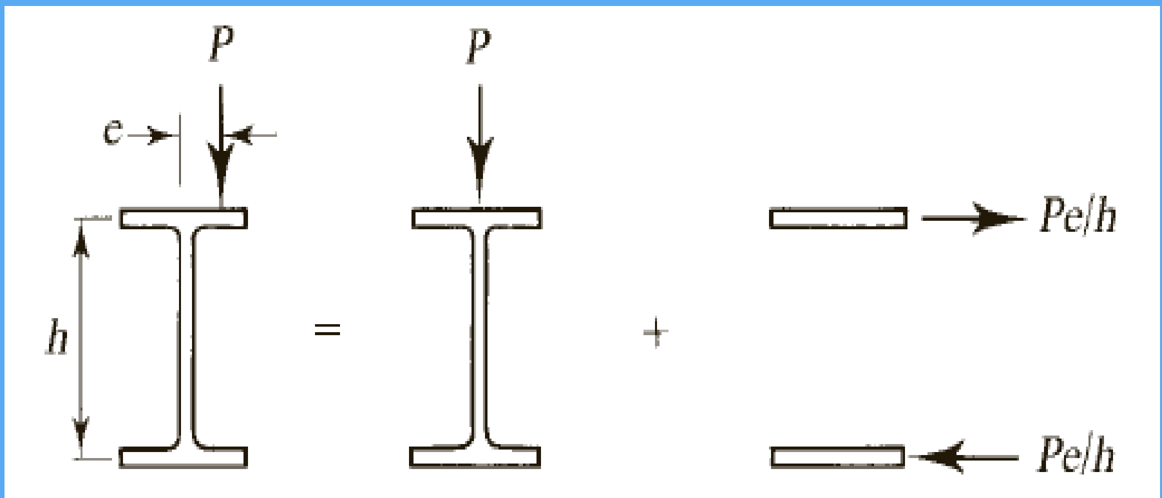
Approximate method

The problem is reduced to a case of bending of two shapes, each one loaded through its shear center.



The top flange is assumed to provide the total resistance to the horizontal component of the load

only about half the cross section is considered to be effective with respect to its y axis; therefore, when considering the strength of a single flange, use half the tabulated value of Z_y for the shape



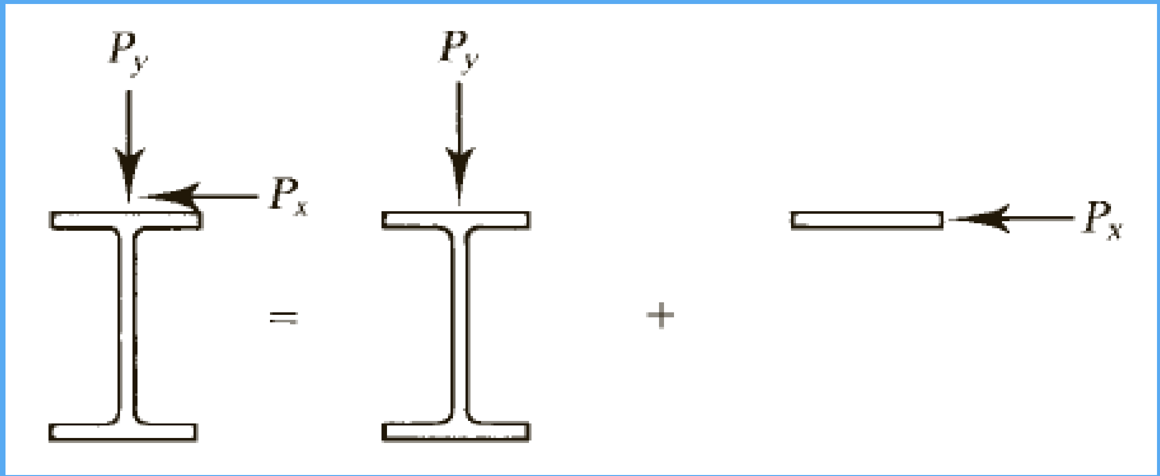
- The twisting moment Pe is resisted by a couple consisting of equal forces acting at each flange
- As an approximation, each flange can be considered to resist each of these forces independently

Beam Subjected to Biaxial Bending

Case II: Loads Not Applied Through the Shear Center

Approximate method

The problem is reduced to a case of bending of two shapes, each one loaded through its shear center.



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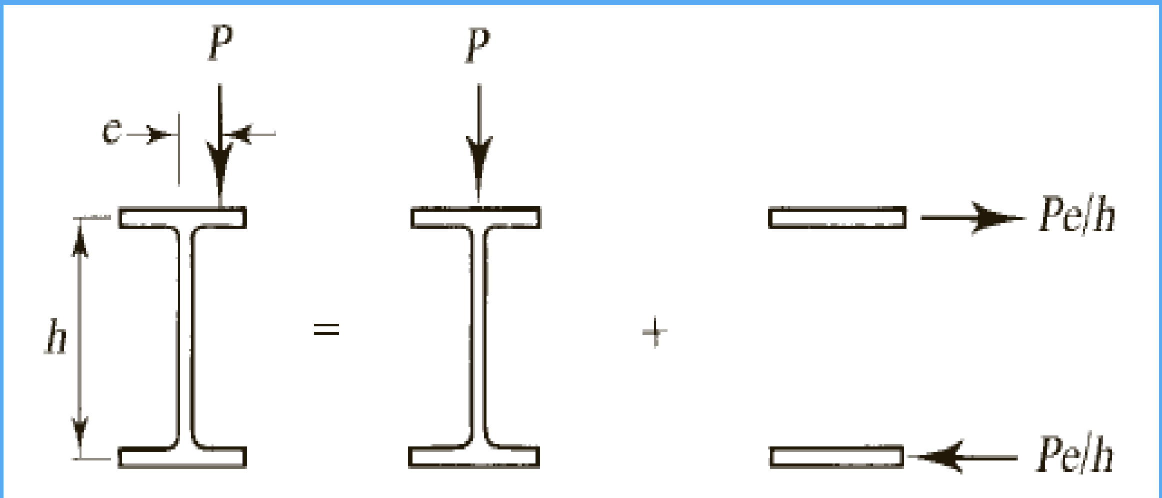

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Beam Subjected to Biaxial Bending

SOLVED EXAMPLES

Beam Bearing Plates

Beam Bearing Plates

Beam Bearing plate is the plate used to distribute a concentrated load (usually compressive) to the supporting material (structure).

Two types of beam bearing plates are considered:

Plate that transmits the beam reaction to a support such as a concrete wall

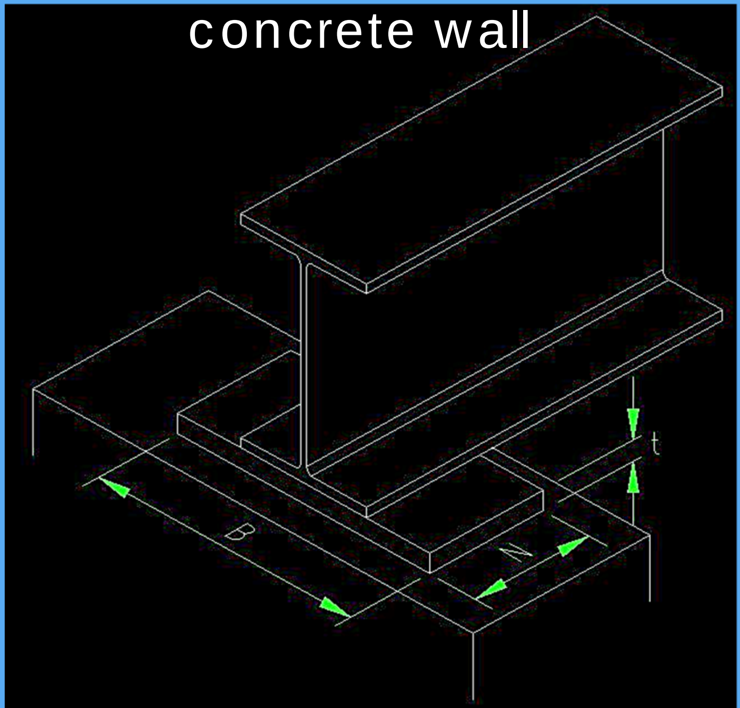
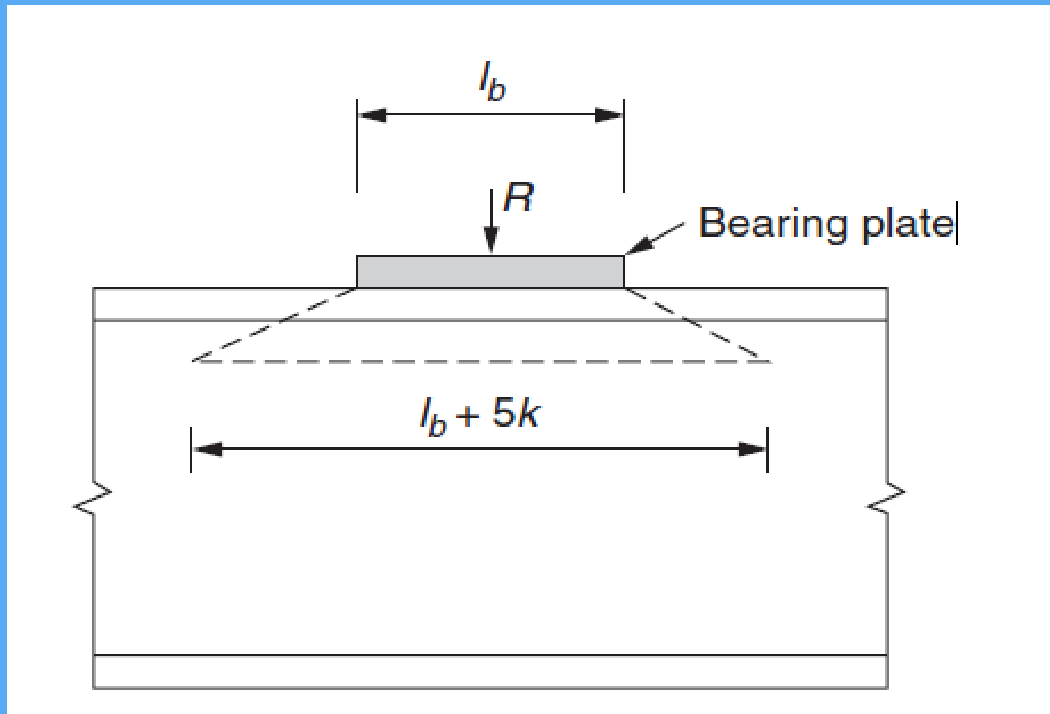


Plate that transmits a load to the top flange of a beam.



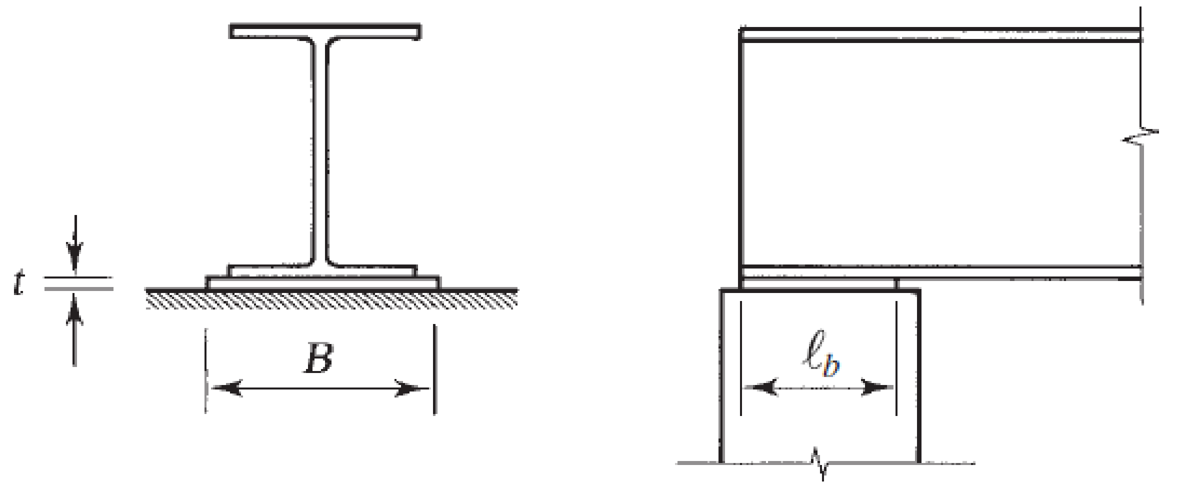
Beam Bearing Plates

The design of the bearing plate consists of three steps:

1. **Determine dimension lb (bearing length) so that web yielding and web crippling are prevented.**

2. Determine dimension *B* so that the area *B* × *lb* is sufficient to prevent the supporting material (usually concrete) from being crushed in bearing.

3. Determine the thickness *t* so that the plate has sufficient bending strength.

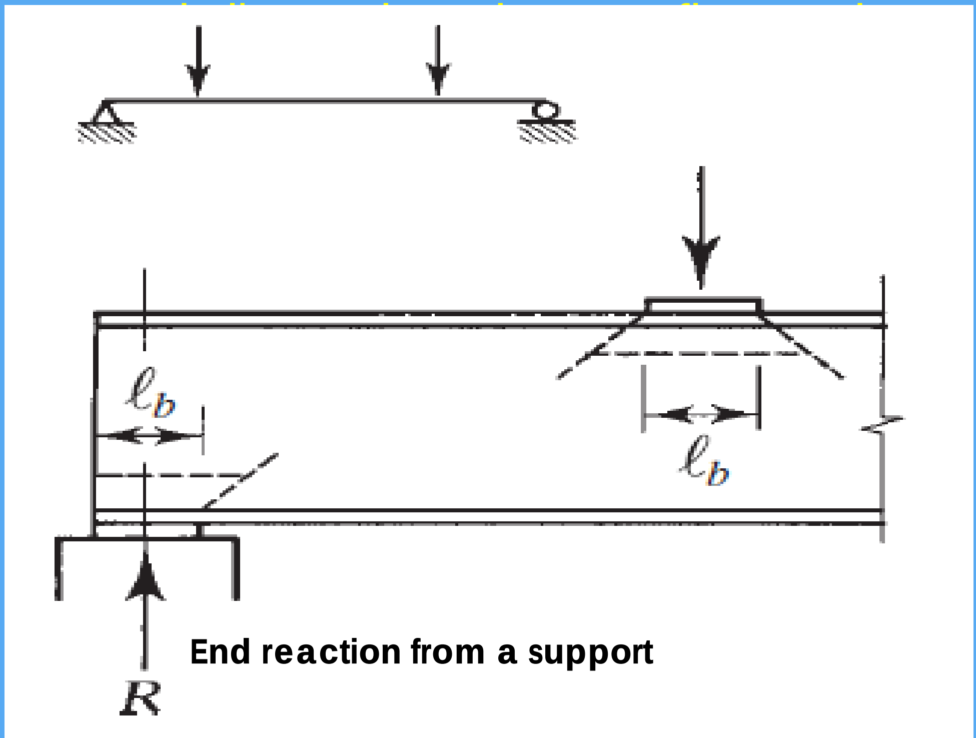


Web Yielding

- The support reaction at the end of a girder is transmitted through the flange to the
girder web. The failure stress in the web equals the yield stress Fy.

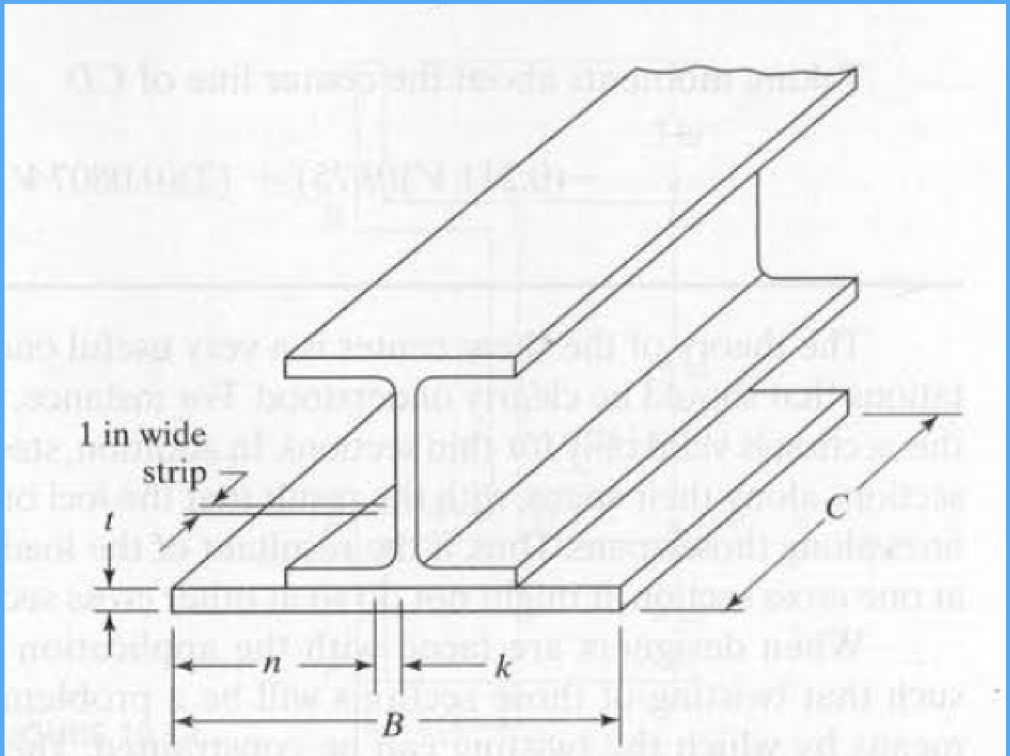
- Is the compressive crushing of a beam web caused by the application of a compressive force to the flange directly above or below the web.

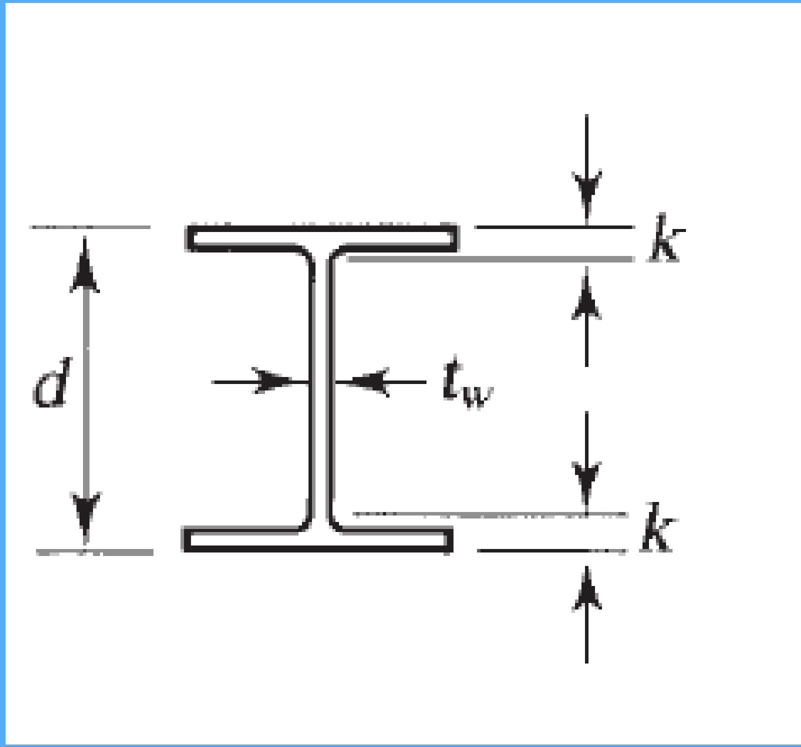
- This force could be an end reaction from a support or it could be a load
column or another beam



Web Yielding

- Yielding occurs when the compressive stress on a horizontal section through the web reaches the yield point.
- When the load is transmitted through a plate, web yielding is assumed to take place on the nearest section of width *tw*
- In a rolled shape, this section will be at the toe of the fillet, a distance *k* from the outside face of the flange. (this dimension is tabulated in the dimensions and properties tables in the *Manual*)





Web Yielding At Support

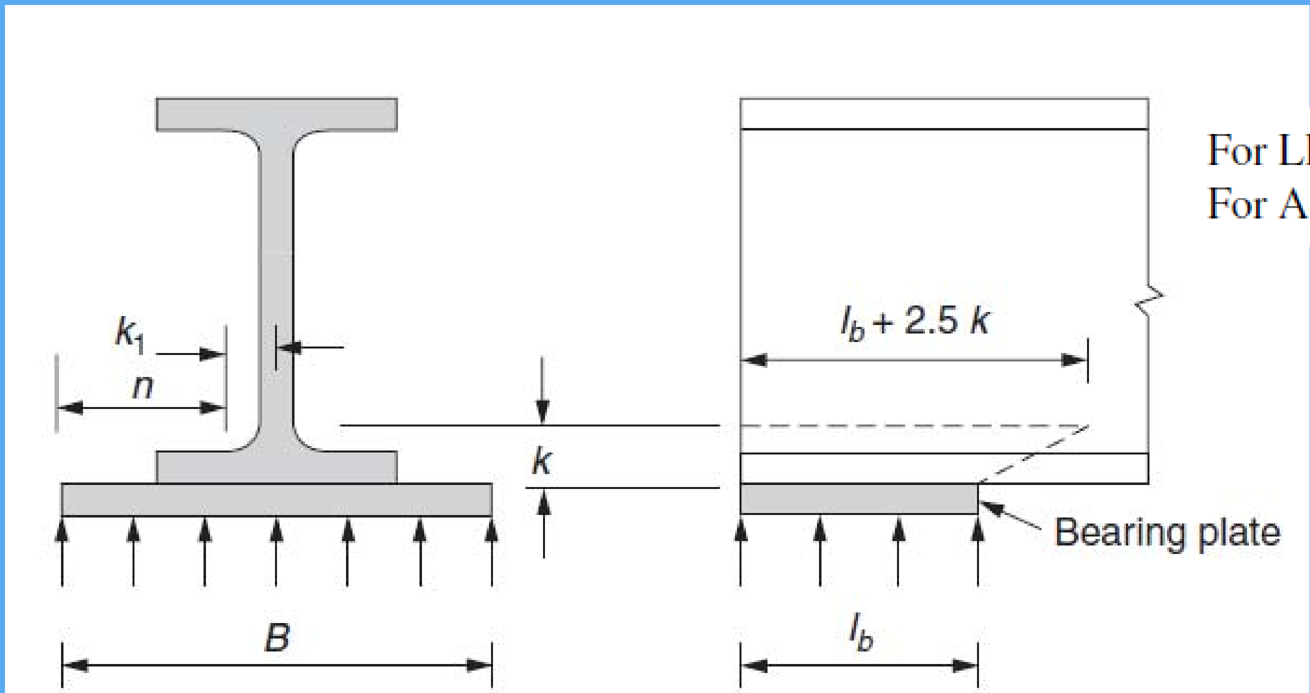
If the load is assumed to distribute itself at a slope of 1 : 2.5, the area at the support subject to yielding is:

$$t_w (2.5k + l_b)$$

Multiplying this area by the yield stress gives the nominal strength for web yielding at the support:

$R_n = F_y t_w (2.5k + l_b)$ (AISC Equation J10-3)

The bearing length l_b at the support should not be less than k



For LRFD, the design strength is ϕR_n, where $\phi = 1.0$.
For ASD, the allowable strength is R_n/Ω, where $\Omega = 1.50$.

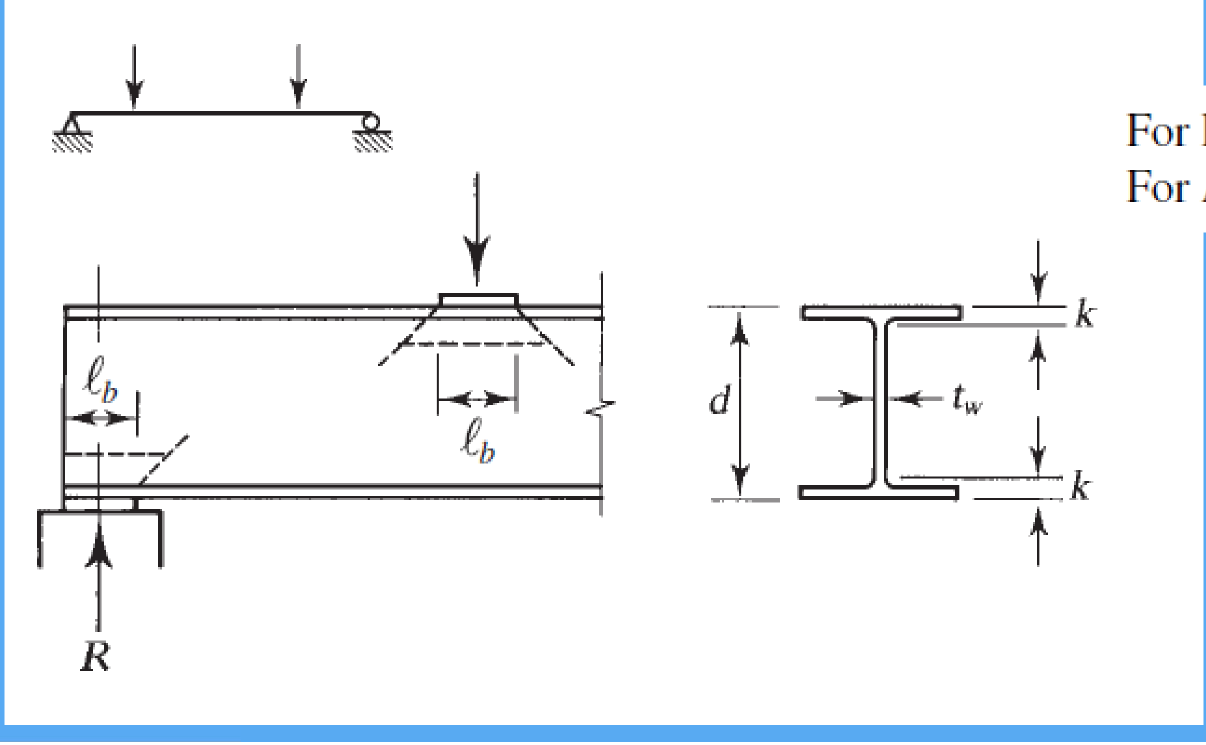
Web Yielding At Interior Girder

At the interior load, the length of the section subject to yielding is:

$$2(2.5k) + l_b = 5k + l_b$$

and the nominal strength is:

$$R_n = F_y t_w (5k + l_b) \qquad \text{(AISC Equation J10-2)}$$



For LRFD, the design strength is ϕR_n, where $\phi = 1.0$.
For ASD, the allowable strength is R_n/Ω, where $\Omega = 1.50$.

Web Yielding-AISC Specifications

J10. FLANGES AND WEBS WITH CONCENTRATED FORCES

2. Web Local Yielding

This section applies to *single-concentrated forces* and both components of *double-concentrated forces*.

The *available strength* for the *limit state* of web *local yielding* shall be determined as follows:

$$\phi = 1.00 \text{ (LRFD)} \qquad \Omega = 1.50 \text{ (ASD)}$$

The *nominal strength*, R_n, shall be determined as follows:

(a) When the concentrated *force* to be resisted is applied at a distance from the member end that is greater than the depth of the member d,

$$R_n = (5k + N)F_{yw}t_w \qquad \text{(J10-2)}$$

Web Yielding-AISC Specifications

J10. FLANGES AND WEBS WITH CONCENTRATED FORCES

(b) When the concentrated force to be resisted is applied at a distance from the member end that is less than or equal to the depth of the member d,

$$R_n = (2.5k + N)F_{yw}t_w \quad \text{(J10-3)}$$

where

k = distance from outer face of the flange to the web toe of the fillet, in. (mm)

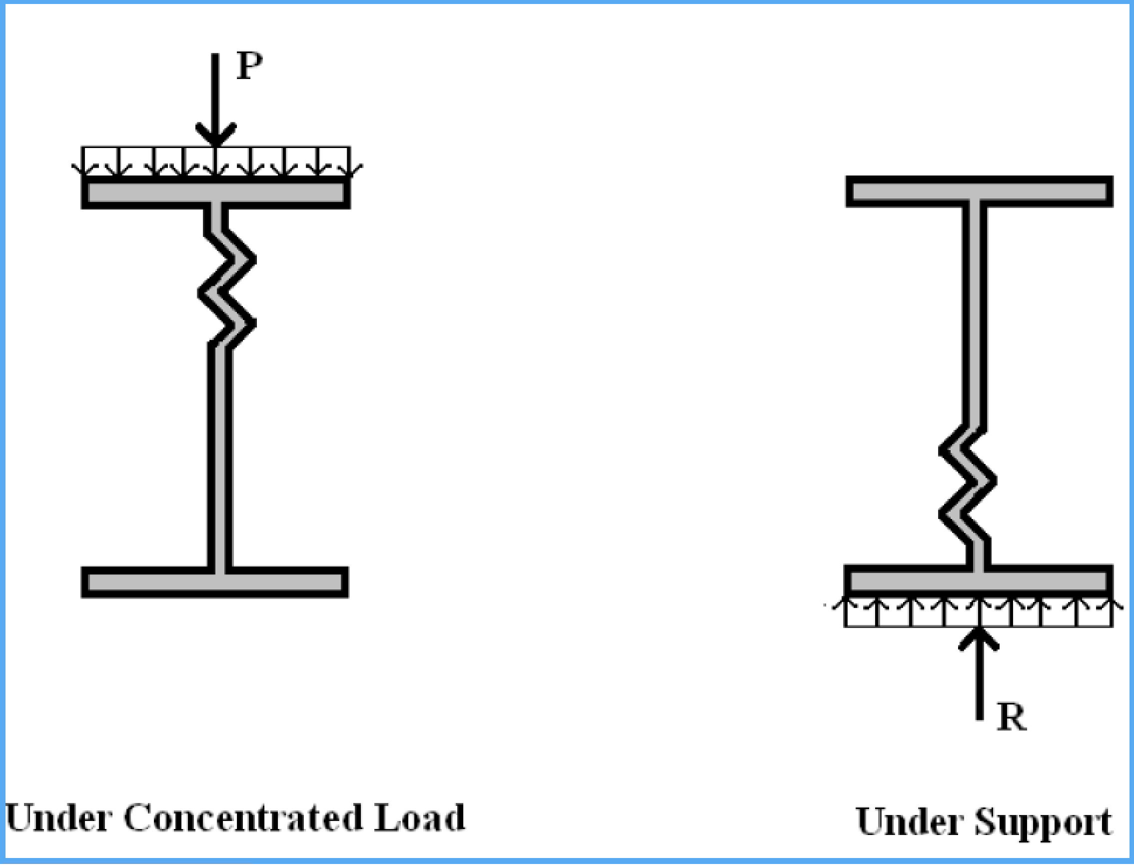
F_{yw} = *specified minimum yield stress* of the web, ksi (MPa)

N = length of bearing (not less than k for end *beam* reactions), in. (mm)

t_w = web thickness, in. (mm)

Web Crippling

A concentrated load applied to a girder flange produces a compressive stress in the web. If the compressive stress is excessive, local buckling of the web may occur near the junction of the flange and the web. This is known as web crippling and is more critical at the ends of a girder than in the interior.



Web crippling is buckling of the web caused by the compressive force delivered through the flange.

Web Crippling-AISC Specifications

(a)The nominal web crippling strength of a beam, with a concentrated load applied at a distance of not less than d/2 from the end of the beam, is given by AISC 360 Eq. (J10-4) as:

(a) When the concentrated compressive *force* to be resisted is applied at a distance from the member end that is greater than or equal to $d/2$:

$$R_n = 0.80 t_w^2 \left[1 + 3\left(\frac{N}{d}\right)\left(\frac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\frac{E F_{yw} t_f}{t_w}} \qquad \text{(J10-4)}$$

$$\phi = 0.75\ \text{(LRFD)} \qquad \Omega = 2.00\ \text{(ASD)}$$

Web Crippling-AISC Specifications

(b) When the concentrated compressive force resiste to be d is applied at a distance from the member end that is less than *d*/2:

(b) When the concentrated compressive force to be resisted is applied at a distance from the member end that is less than $d/2$:

(i) For $N/d \le 0.2$

$$R_n = 0.40t_w^2\left[1 + 3\left(\frac{N}{d}\right)\left(\frac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\frac{EF_{yw}t_f}{t_w}} \qquad \text{(J10-5a)}$$

(ii) For $N/d > 0.2$

$$R_n = 0.40t_w^2\left[1 + \left(\frac{4N}{d} - 0.2\right)\left(\frac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\frac{EF_{yw}t_f}{t_w}} \qquad \text{(J10-5b)}$$

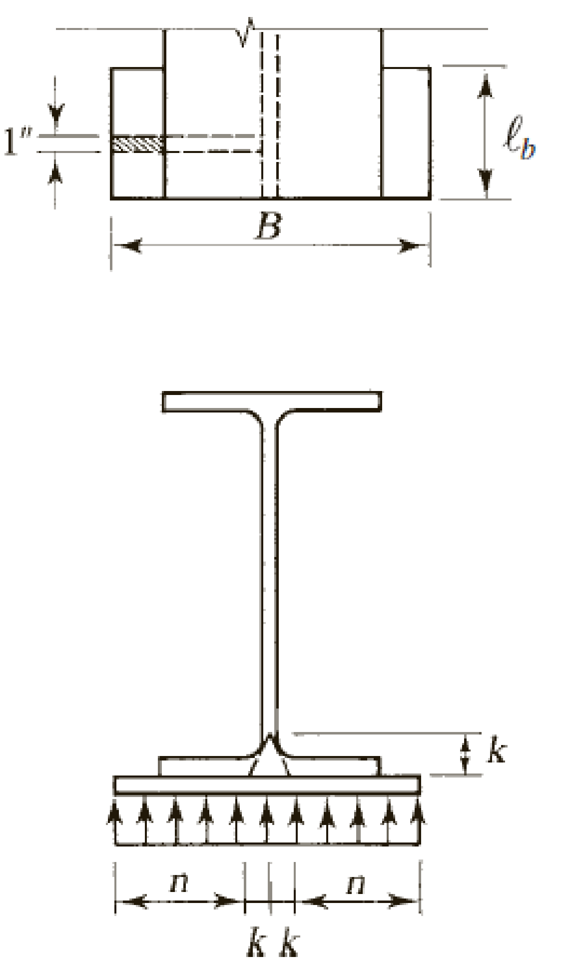
where

d = overall depth of the member, in. (mm)
t_f = flange thickness, in. (mm)

$\phi = 0.75$ (LRFD) $\qquad \Omega = 2.00$ (ASD)

Plate Thickness

- The average bearing pressure is treated as a uniform load on the bottom of the plate, which is assumed to be supported at the top over a central width of 2*k* and length l*b*.
- The plate is then considered to bend about an axis parallel to the beam span.



- The plate is treated as a cantilever of span length

n = (B – 2k)/2 and a width of lb

$$t \geq \sqrt{\frac{2.2 R_u n^2}{B \ell_b F_y}}$$

LRFD

where Ru is the factored-load beam reaction

$$t \geq \sqrt{\frac{3.34 R_a n^2}{B \ell_b F_y}}$$

ASD

where Ra is the service-load beam reaction

Bearing on Concrete

The nominal bearing capacity of the concrete support, when the bearing plate covers the full area of the support, is given by AISC 360 Eq. (J8-1) as

(a) On the full area of a concrete support:

$$P_p = 0.85 f_c' A_1 \qquad \text{(J8-1)}$$

(b) On less than the full area of a concrete support:

$$P_p = 0.85 f_c' A_1 \sqrt{A_2 / A_1} \leq 1.7 f_c' A_1 \qquad \text{(J8-2)}$$

$\phi_c = 0.65$ (LRFD) $\quad \Omega_c = 2.31$ (ASD)

where

A_1 = area of steel concentrically bearing on a concrete support, in.^{2} (mm^2)

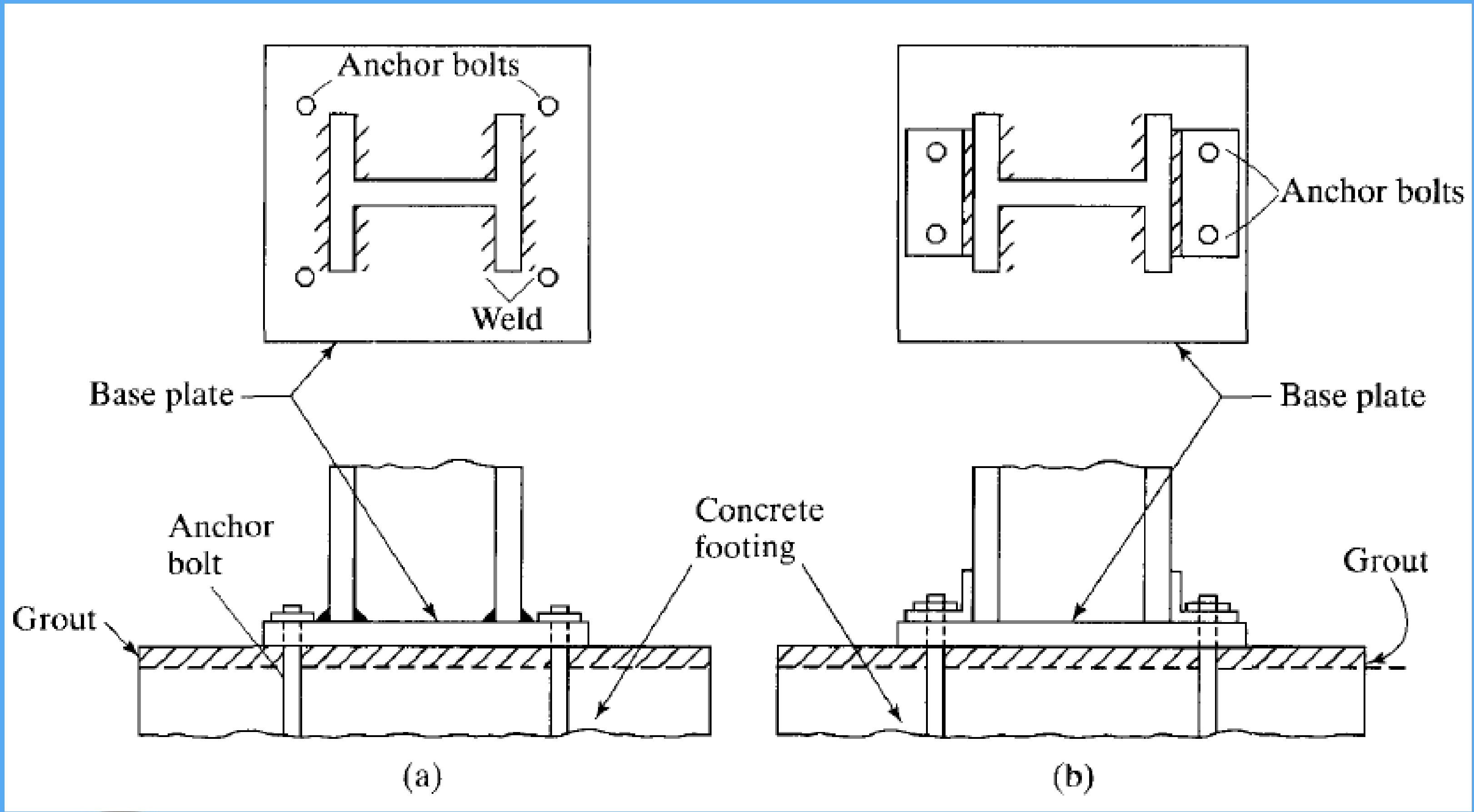
A_2 = maximum area of the portion of the supporting surface that is geometrically similar to and concentric with the loaded area, in.^{2} (mm^2)

f_c' = specified compressive strength of concrete, ksi (MPa)

Column Base Plates

Column Base Plates

- Loads from steel columns are transferred through a steel base plate to a fairly large area of the footing below.
- Footing spreads the load over an even larger area so that the underlying soil will not be overstressed.



OSHA regulations for the safe erection of structural steel requires use of no less than four anchor bolts for each column.
These bolts will preferably be placed at base plate

Column Base Plates

- Anchor Bolts are positioned and placed in the concrete before it cures.
- Plastic caps are placed over the anchor bolts to protect laborers.

Column Base Plates

Perimeter Column

Interior Column

Concrete block-outs are made so that the base plate and anchor bolts are recessed below floor level for safety and aesthetics.

Column Base Plates

- Base plate holes are located and drilled in the shop.
- The base plates were shop-welded to their respective columns and shipped to the site.
- Slotted holes are allow for difference in standard field tolerances between concrete and steel.

Column Base Plates

Base plate and column installation is complete. The column is leveled by adjusting the anchor bolt nuts below the plate.

Grout will be placed under and around the base plate to smoothly transfer axial forces to the concrete below.

Interior and perimeter column installations are complete. After inspection, the block outs will be filled with concrete to floor height for a uniform appearance and safety.

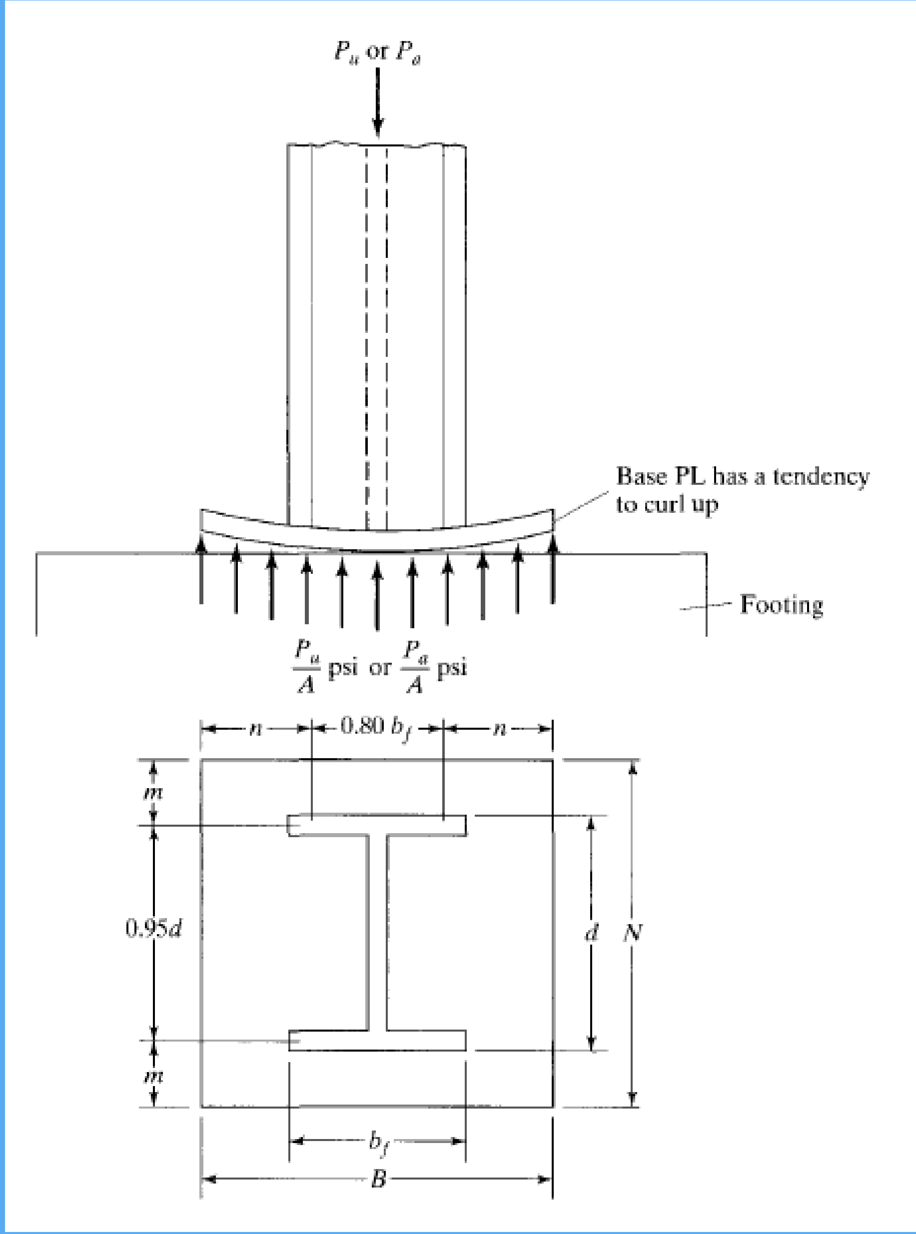
Column Base Plates

Column Base Plates vs. Beam Bearing Plates

The major difference is that bending in beam bearing plates is in one direction, whereas column base plates are subjected to two-way bending.

Moreover, web crippling and web yielding are not factors in column base plates design.

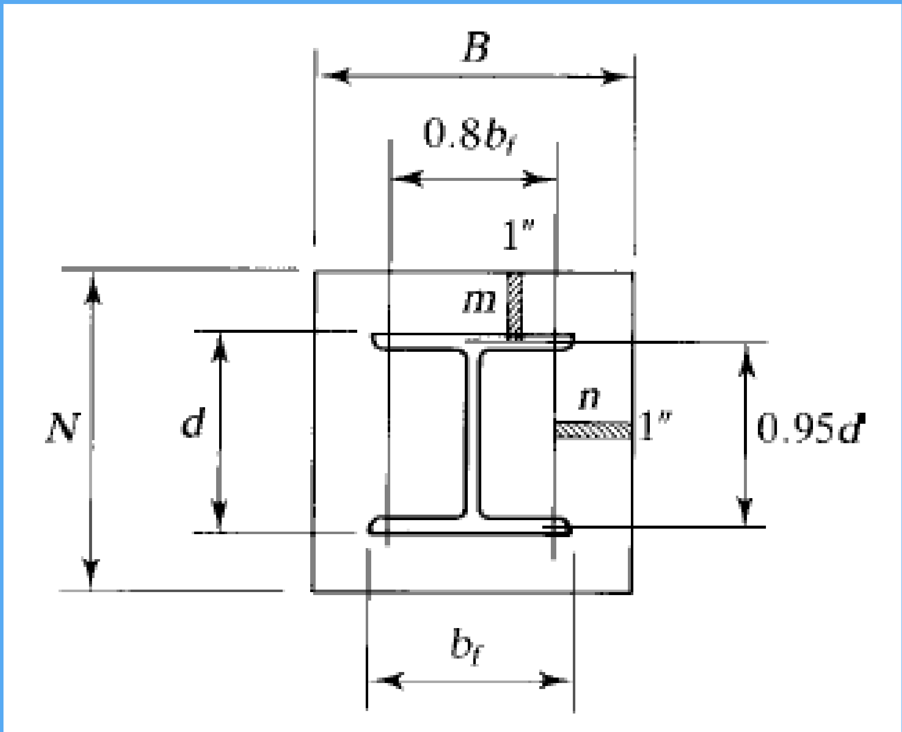
The AISC Specification docs not stipulate a particular method for designing column base plates.

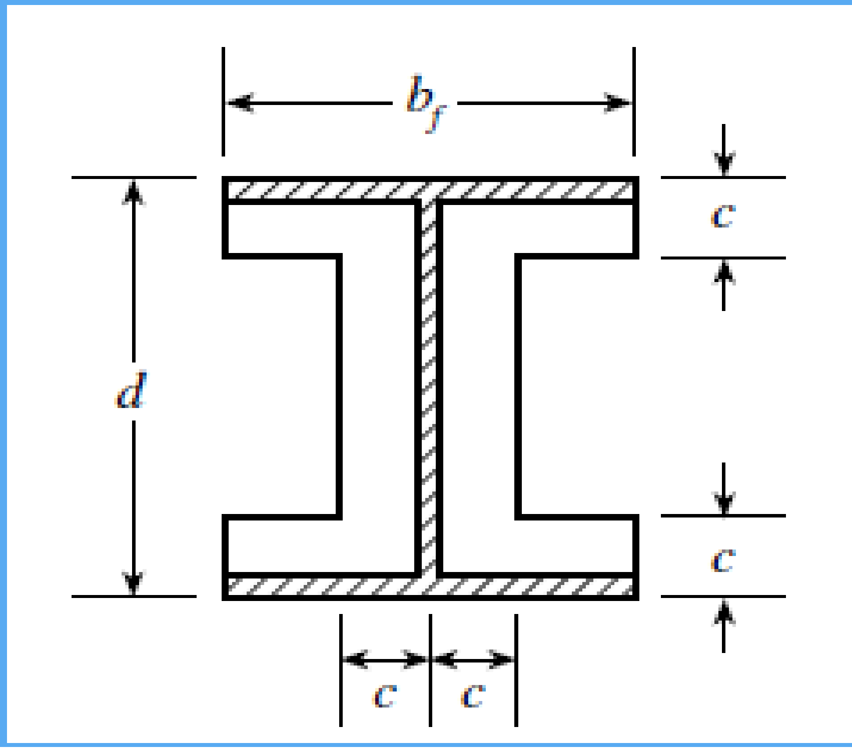


Column Base Plates

Large Plate vs. Small Plate

- Large Plates are larger than the column dimensions.
- Small plates are those whose dimensions are approximately the same as the column dimensions

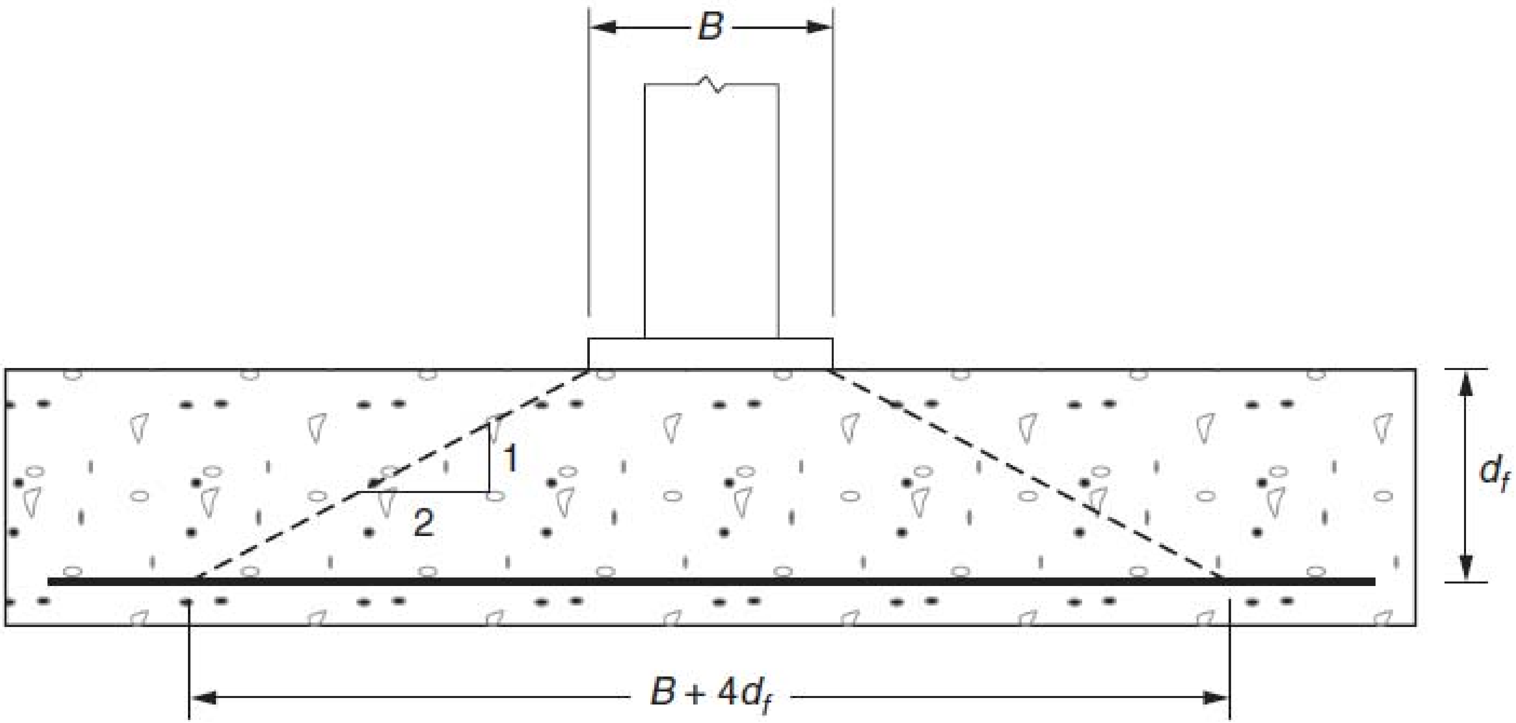




Column Base Plates

Design of Plate

- **Base plates are provided to columns to ensure that the column load is distributed to the concrete footing without exceeding the capacity of the concrete.**
- **The design of column base plates requires consideration of bearing pressure on the supporting material and bending of the plate.**



The column load is assumed dispersed in the footing at a slope of 2 in 1.

Column Base Plates

The nominal bearing strength of the concrete is given as:

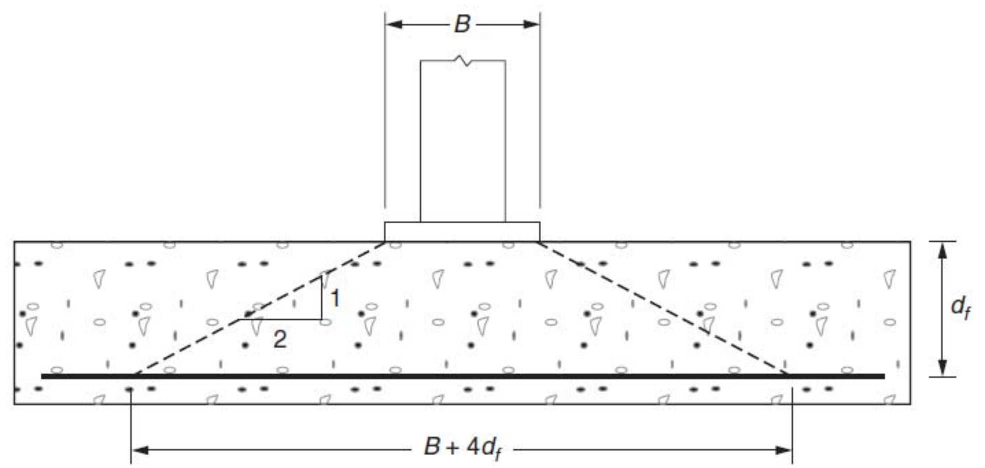
(a) On the full area of a concrete support:

$$P_p = 0.85 f_c' A_1 \qquad \text{(J8-1)}$$

(b) On less than the full area of a concrete support:

$$P_p = 0.85 f_c' A_1 \sqrt{A_2 / A_1} \leq 1.7 f_c' A_1 \qquad \text{(J8-2)}$$

$$\phi_c = 0.65 \text{ (LRFD)} \qquad \Omega_c = 2.31 \text{ (ASD)}$$



- **Where:**
- **f'c = compressive strength of footing concrete**
- **A1 = area of the base plate = N × B**
- **A2 = area of the base of the pyramid, with side slopes of 1:2, formed within the footing by the base plate= (B + 4df)(N + 4df)**
- **df = effective depth of the concrete footing**
- **N = length of base plate**
- **B = width of base plate**

Column Base Plates

Plate Area

(Cantilever method)

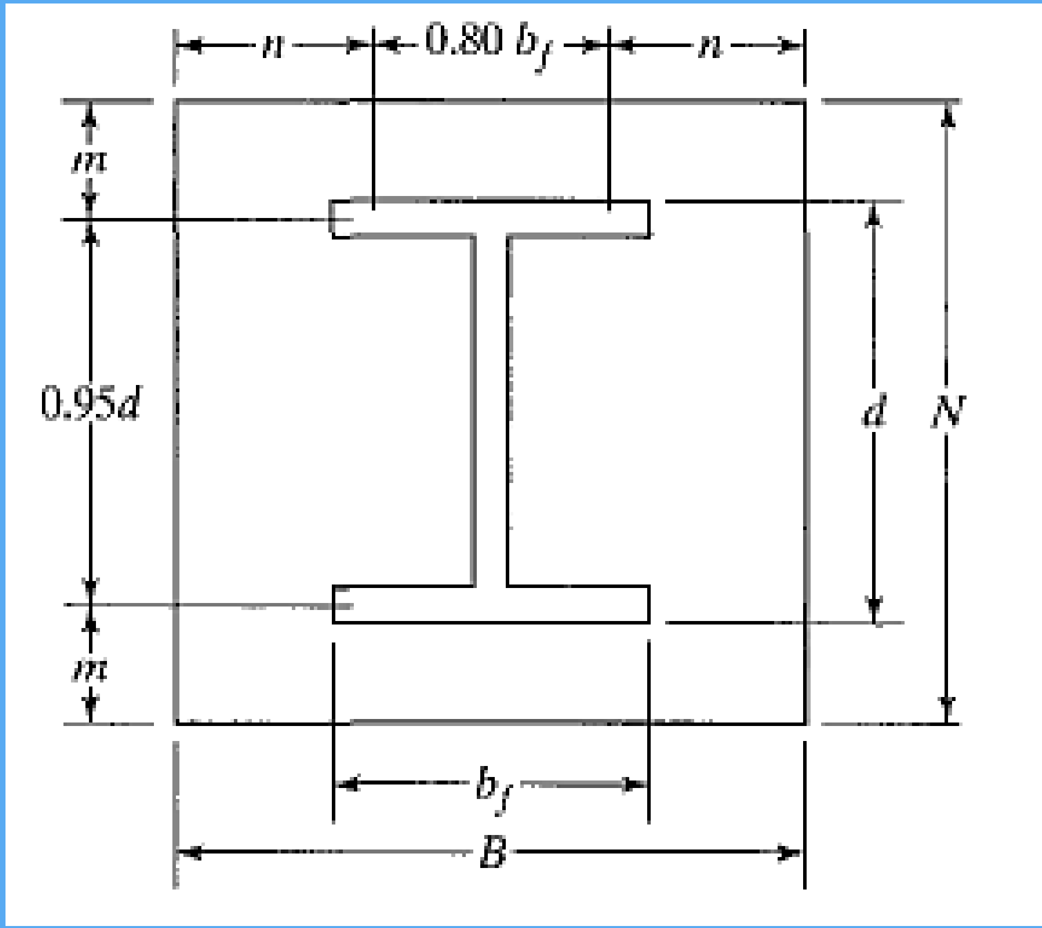
$$N \approx \sqrt{A_1} + \Delta$$

$$A_1 = \text{area of plate} = BN$$

$$\Delta = 0.5\,(0.95\,d - 0.80\,b_f)$$

$$N = \sqrt{A_1} + \Delta$$

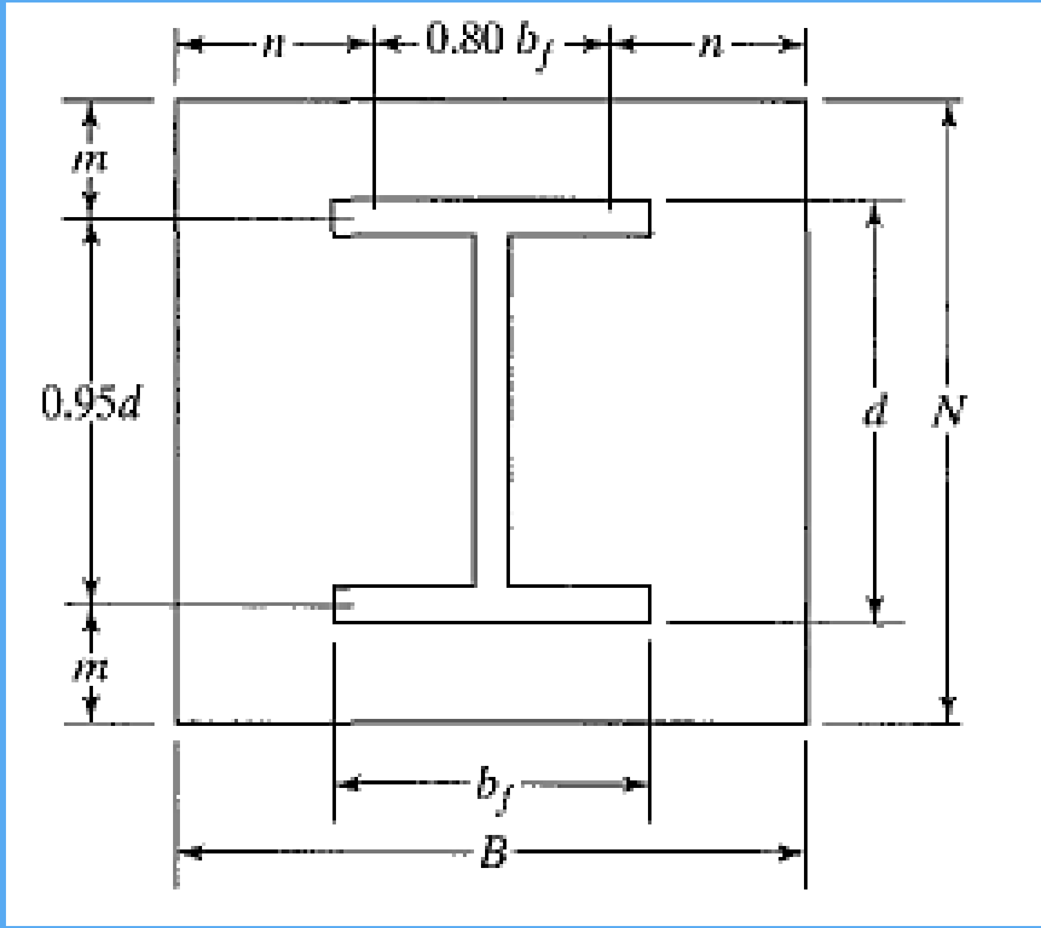
$$B \approx \frac{A_1}{N}$$



Column Base Plates

Plate Thickness

$$t \geq \sqrt{\frac{2P_u \ell^2}{0.90BNF_y}}$$



where l is the larger of *m, n and n'*

m= (N-0.95d)/2

n= (B-0.8bf)/2

n'= $\frac{1}{4}\sqrt{d.b_f}$

Column Base Plates

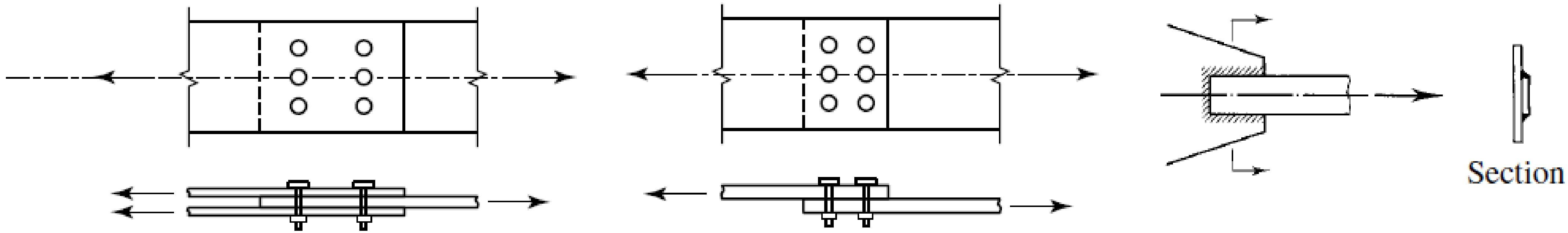
Solved Examples

Connections

Major categories of connections

Simple Connections

- The line of action of the resultant force to be resisted passes through the center of gravity of the connection.
- Each part of the connection is assumed to resist an equal share of the load



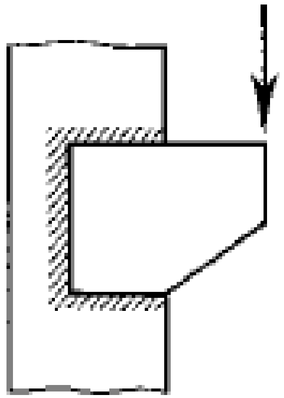
The load capacity of the connection can be found by multiplying the capacity of each fastener or inch of weld by the total number of fasteners or the total length of weld

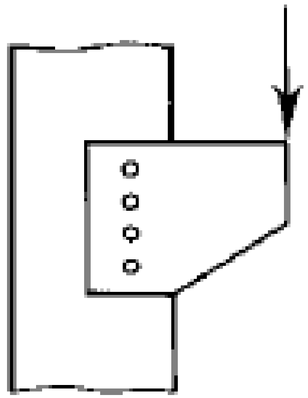
Connections

Major categories of connections

Eccentric Connections

- The line of action of the load does not act through the center of gravity of the connection.
- The load is not resisted equally by each fastener or each segment of weld
- The determination of the distribution of the load is the complicating factor in the design of this type of connection.

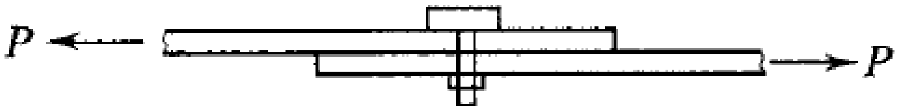


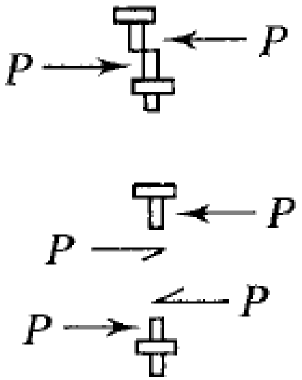


Connections

BOLTED SHEAR CONNECTIONS FAILURE MODES:

1. ***Failure of the fastener***
2. Failure of the parts being connected

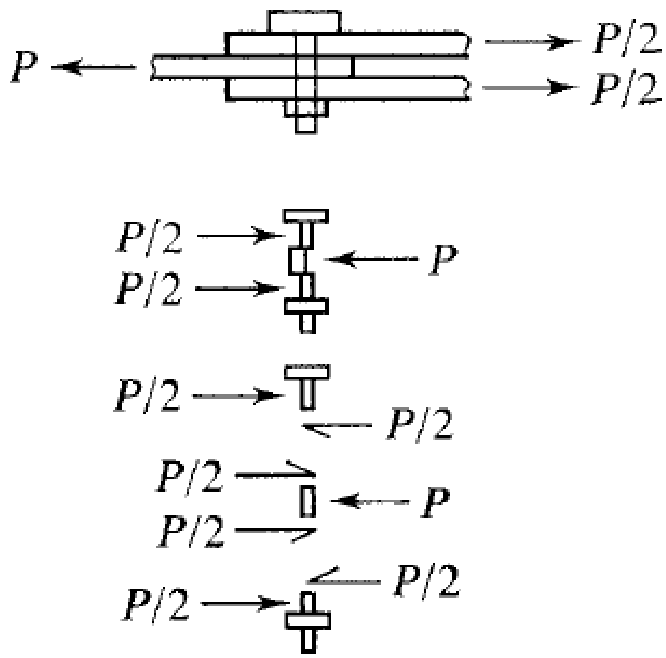




$$f_v = \frac{P}{A} = \frac{P}{\pi d^2/4}$$

$$P = f_v A$$

(a) Single Shear



(b) Double Shear

$$f_v = \frac{P}{A} = \frac{P}{\pi d^2/4}$$

$$P = 2f_v A$$

Where

fv is the shearing stress on the cross-sectional area of the bolt.

A is the cross-sectional area

Connections

2. Failure of the parts being connected

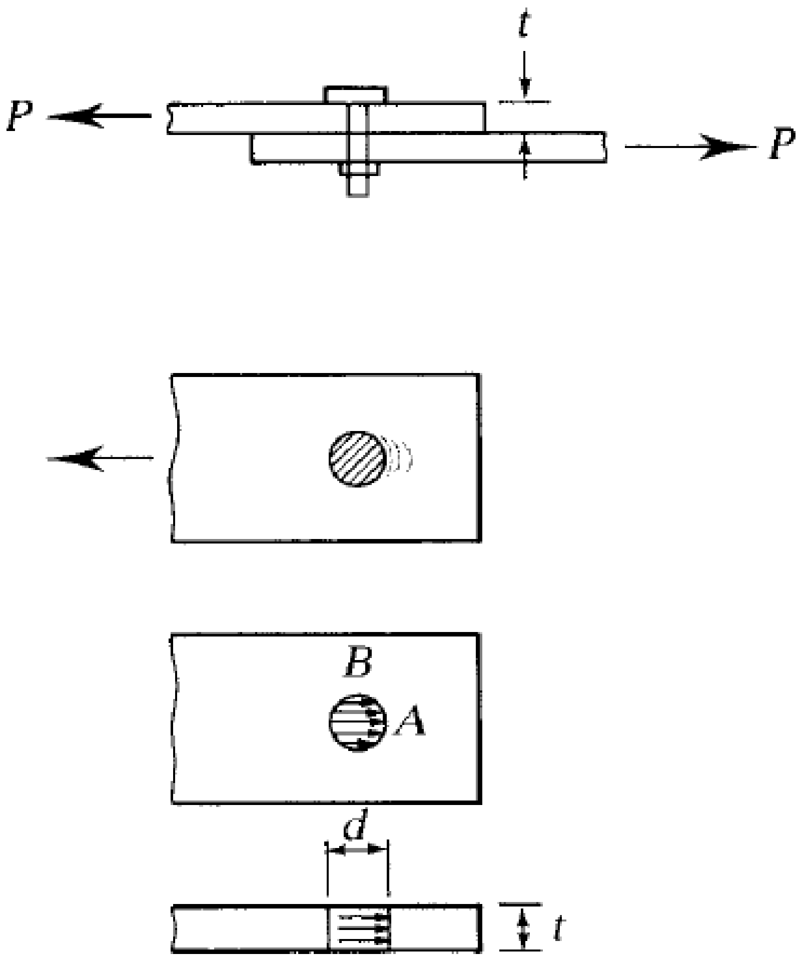
Failure resulting from excessive tension, shear, or bending in the parts being connected

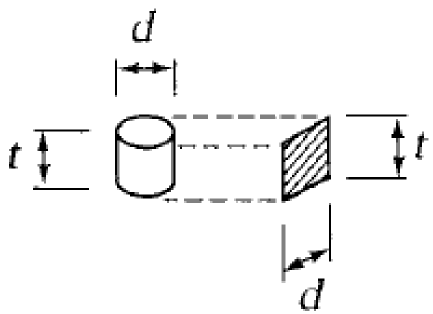
- If a tension member is being connected, tension on both the gross area and effective net area must be investigated.
- Block shear might also need to be considered.
- Block shear must also be examined in beam-to-column connections in which the top flange of the beam is coped

Failure of the connected part because of bearing exerted by the fasteners

- Hole is slightly larger than the fastener
- Fastener is assumed to be placed loosely in the hole
- Contact between the fastener and the connected part will exist over approximately half the circumference of the fastener when a load is applied

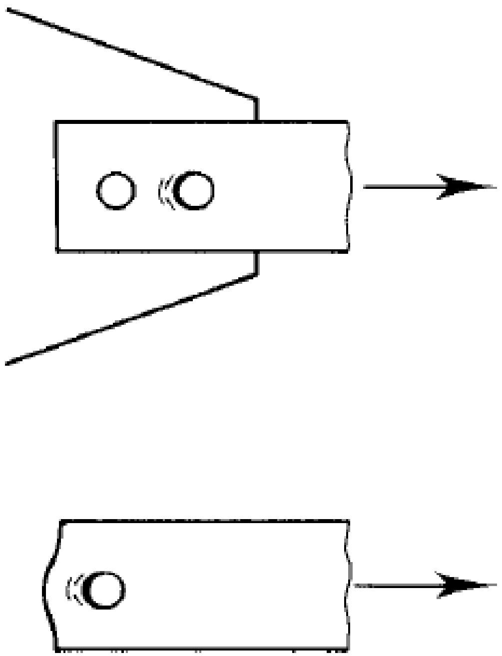
Connections





- The stress will vary from a maximum at *A* to zero at *B*
- For simplicity, an average stress, computed as the applied force divided by the projected area of contact, is used

Connections



The bearing stress would be computed as

$$f_p = P/(dt).$$

The bearing load is therefore

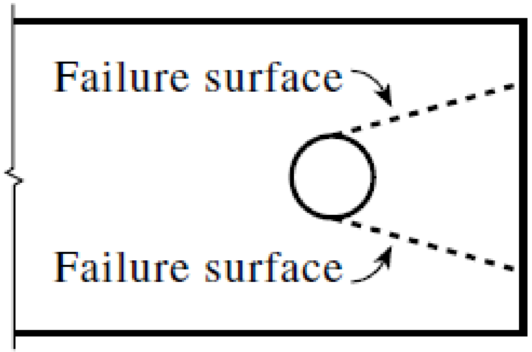
$$P = f_p dt$$

The bolt spacing and edge distance will have an effect on the bearing strength

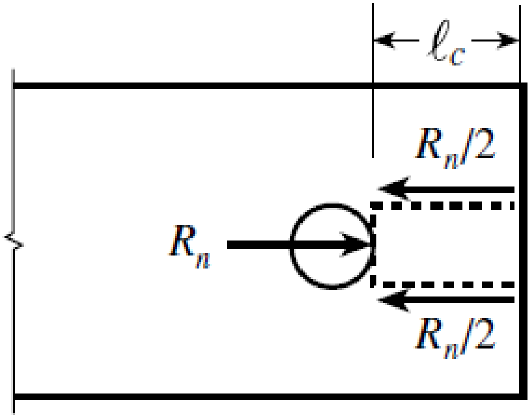
Connections

BEARING STRENGTH, SPACING, AND EDGE-DISTANCE REQUIREMENTS

A possible failure mode resulting from excessive bearing is shear tear-out at the end of a connected element



If the failure surface is idealized



$$\frac{R_n}{2} = 0.6 F_u \ell_c t$$

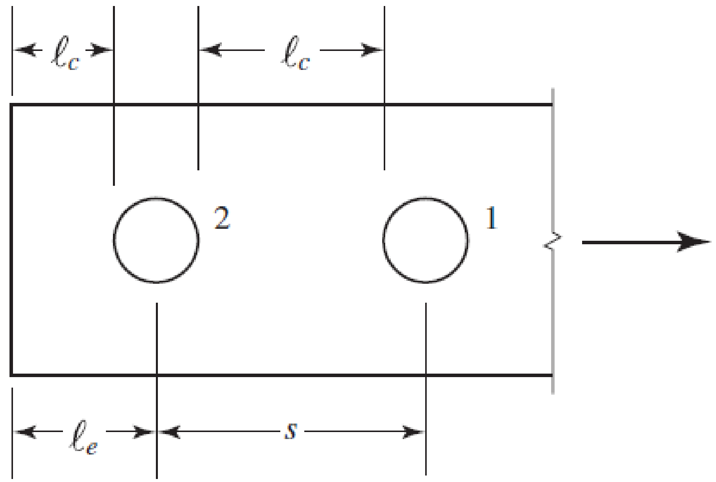
where

$0.6F_u$ = shear fracture stress of the connected part

ℓ_c = distance from edge of hole to edge of connected part

t = thickness of connected part

Connections



The total strength is

$$R_n = 2(0.6F_u\ell_c t) = 1.2F_u\ell_c t$$

The nominal bearing strength of a single bolt

$$R_n = 1.2\ell_c tF_u \leq 2.4dtF_u \qquad \text{(AISC Equation J3-6a)}$$

where

ℓ_c = clear distance, in the direction parallel to the applied load, from the edge of the bolt hole to the edge of the adjacent hole or to the edge of the material

t = thickness of the connected part

F_u = ultimate tensile stress of the connected part (*not* the bolt)

$\phi R_n = 0.75R_n$ LRFD

$\dfrac{R_n}{\Omega} = \dfrac{R_n}{2.00}$ ASD

Connections

Summary of Bearing Strength, Spacing, and Edge-Distance Requirements (Standard Holes)

a. Bearing strength:

$$R_n = 1.2\ell_c t F_u \leq 2.4 d t F_u \qquad \text{(AISC Equation J3-6a)}$$

b. Minimum spacing and edge distance: In any direction, both in the line of force and transverse to the line of force,

$s \geq 2\tfrac{2}{3}d$ (preferably $3d$)

$\ell_e \geq$ value from AISC Table J3.4

- For single- and double-angle shapes, the usual gage distances given in Table 1-7A in Part 1 of the Manual.
- **Bearing strength is independent of the type of fastener**

TABLE 12.3 Minimum Edge Distance[a] from Center of Standard Hole[b] to Edge of Connected Part, inches

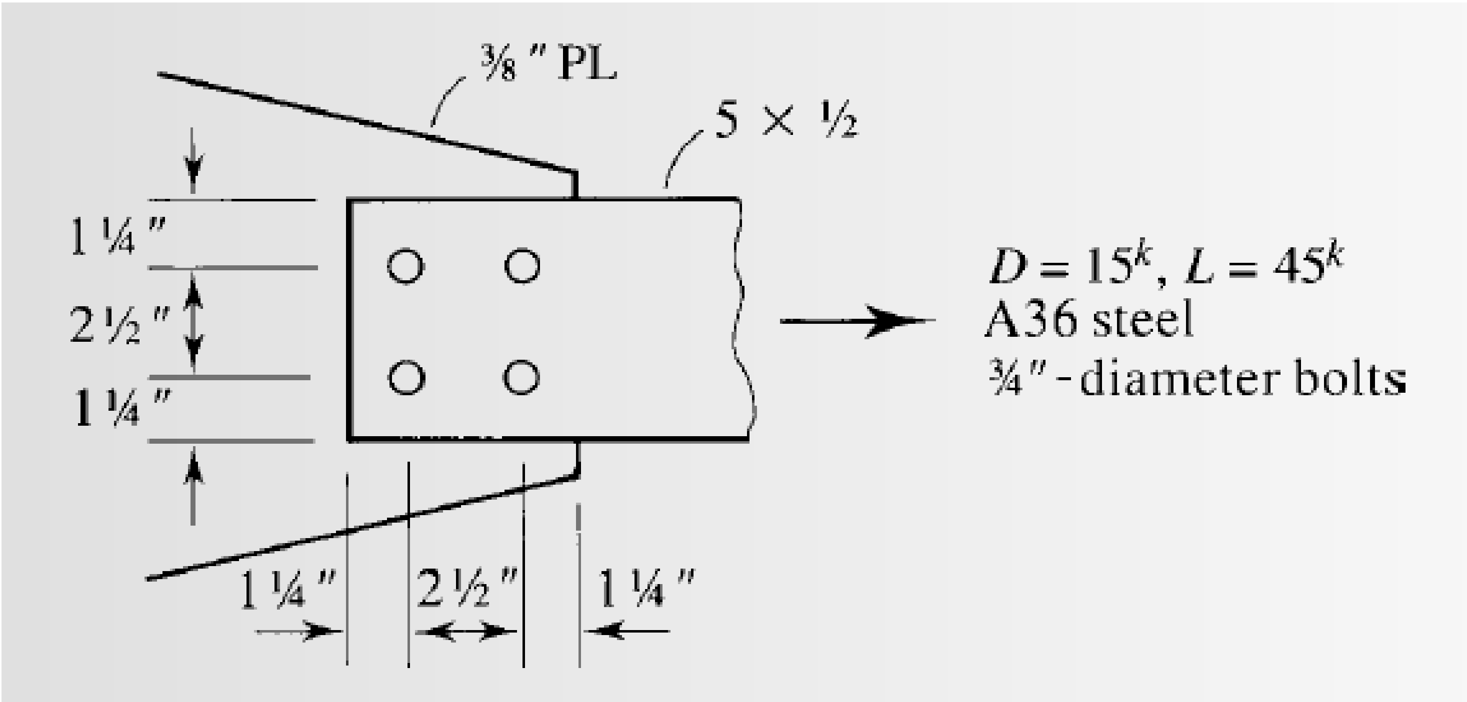
Bolt Diameter (in)	Minimum Edge Distance (in)
1/2	3/4
5/8	7/8
3/4	1
7/8	1 1/8
1	1 1/4
1 1/8	1 1/2
1 1/4	1 5/8
Over 1 1/4	1 1/4 × Diameter

[a] If necessary, lesser edge distances are permitted provided the appropriate provisions from Sections J3.10 and J4 are satisfied, but edge distances less than one bolt diameter are not permitted without approval from the engineer of record.
[b] For oversized or slotted holes, see Table J3.5.

AISC Specification Tables J3.4 Manual

Connections

Example:
Check bolt spacing, edge distances, and bearing for the connection shown in Figure



From AISC Table J3.4, the minimum edge distance in any direction is 1 inch.

$$\text{Actual edge distance} = 1\frac{1}{4} \text{ in.} > 1 \text{ in.} \quad \text{(OK)}$$

For computation of the bearing strength, use a hole diameter of

$$h = d + \frac{1}{16} = \frac{3}{4} + \frac{1}{16} = \frac{13}{16} \text{ in.}$$

Check bearing on both the tension member and the gusset plate. For the tension member and the holes nearest the edge of the member,

$$\ell_c = \ell_e - \frac{h}{2} = 1.25 - \frac{13/16}{2} = 0.8438 \text{ in.}$$

$$R_n = 1.2\ell_c t F_u \leq 2.4 d t F_u$$

$$1.2\ell_c t F_u = 1.2(0.8438)\left(\frac{1}{2}\right)(58) = 29.36 \text{ kips}$$

Check upper limit:

$$2.4 d t F_u = 2.4\left(\frac{3}{4}\right)\left(\frac{1}{2}\right)(58) = 52.20 \text{ kips}$$

$$29.36 \text{ kips} < 52.20 \text{ kips} \qquad \text{Use } R_n = 29.36 \text{ kips/bolt.}$$

(This result means that ℓ_c is small enough so that it must be accounted for.)

For the other holes,

$$\ell_c = s - h = 2.5 - \frac{13}{16} = 1.688 \text{ in.}$$

$$R_n = 1.2\ell_c t F_u \leq 2.4 d t F_u$$

$$1.2\ell_c t F_u = 1.2(1.688)\left(\frac{1}{2}\right)(58) = 58.74 \text{ kips}$$

Upper limit (the upper limit is independent of ℓ_c and is the same for all bolts):

$$2.4 d t F_u = 52.20 \text{ kips} < 58.74 \text{ kips} \qquad \therefore \ \text{Use } R_n = 52.20 \text{ kips/bolt.}$$

(This result means that ℓ_c is large enough so that it does not need to be accounted for. Hole deformation controls.)

The bearing strength for the tension member is

$$R_n = 2(29.36) + 2(52.20) = 163.1 \text{ kips}$$

For the gusset plate and the holes nearest the edge of the plate,

$$\ell_c = \ell_e - \frac{h}{2} = 1.25 - \frac{13/16}{2} = 0.8438 \text{ in.}$$

$$R_n = 1.2\ell_c tF_u \le 2.4dtF_u$$

$$1.2\ell_c tF_u = 1.2(0.8438)\left(\frac{3}{8}\right)(58) = 22.02 \text{ kips}$$

$$\text{Upper limit} = 2.4dtF_u = 2.4\left(\frac{3}{4}\right)\left(\frac{3}{8}\right)(58)$$

$$= 39.15 \text{ kips} > 22.02 \text{ kips} \qquad \text{Use } R_n = 22.02 \text{ kips/bolt.}$$

For the other holes,

$$\ell_c = s - h = 2.5 - \frac{13}{16} = 1.688 \text{ in.}$$

$$R_n = 1.2\ell_c tF_u \le 2.4dtF_u$$

$$1.2\ell_c tF_u = 1.2(1.688)\left(\frac{3}{8}\right)(58) = 44.06 \text{ kips}$$

$$\text{Upper limit} = 2.4dtF_u = 39.15 \text{ kips} < 44.06 \text{ kips} \qquad \therefore \text{ Use } R_n = 39.15 \text{ kips/bo}$$

The bearing strength for the gusset plate is

$$R_n = 2(22.02) + 2(39.15) = 122.3 \text{ kips}$$

The gusset plate controls. The nominal bearing strength for the connection is theref

$$R_n = 122.3 \text{ kips}$$

LRFD SOLUTION The design strength is $\phi R_n = 0.75(122.3) = 91.7$ kips.

The required strength is

$$R_u = 1.2D + 1.6L = 1.2(15) + 1.6(45) = 90.0 \text{ kips} < 91.7 \text{ kips} \qquad \text{(OK)}$$

ASD SOLUTION The allowable strength is $\frac{R_n}{\Omega} = \frac{122.3}{2.00} = 61.2$ kips.

The required strength is

$$R_a = D + L = 15 + 45 = 60 \text{ kips} < 61.2 \text{ kips} \qquad \text{(OK)}$$

ANSWER Bearing strength, spacing, and edge-distance requirements are satisfied.

Connections

SHEAR STRENGTH

the shear load on a bolt is

$$P = f_v A_b$$

Where

- fv is the shearing stress on the cross-sectional area of the bolt
- Ab is the cross-sectional area

When the stress is at its limit, the shear load is the nominal strength, given by

$$R_n = F_{nv} A_b$$

where

F_{nv} = nominal shear strength (expressed as a stress)

A_b = cross-sectional area of the unthreaded part of the bolt (also known as the *nominal bolt area* or *nominal body area*)

Connections

High-strength bolts

High-strength bolts are available in two groups, defined by the strength of the bolts in those groups.

Group A: ASTM A325, F1852, A352, A354 Grade BC, and A449.

Group B: ASTM A490, F2280, and A354 Grade BD.

TABLE 12.5 Nominal Strength of Fasteners and Threaded Parts, ksi (MPa)

Description of Fasteners	Nominal Tensile Strength, F_{nt}, ksi (MPa)[a]	Nominal Shear Strength in Bearing-Type Connections, F_{nv}, ksi (MPa)[b]
A307 bolts	45 (310)	27 (188)[c][d]
Group A (A325 type) bolts, when threads are not excluded from shear planes	90 (620)	54 (372)
Group A (A325 type) bolts, when threads are excluded from shear planes	90 (620)	68 (457)
Group B (A490 type) bolts, when threads are not excluded from shear planes	113 (780)	68 (457)
Group B (A490 type) bolts, when threads are excluded from shear planes	113 (780)	84 (579)
Threaded parts meeting the requirements of Section A3.4 of the Manual, when threads are not excluded from shear planes	$0.75F_u$	$0.450F_u$
Threaded parts meeting the requirements of Section A3.4 of the Manual, when threads are excluded from shear planes	$0.75F_u$	$0.563F_u$

[a] For high-strength bolts subjected to tensile fatigue loading, see Appendix 3.
[b] For end loaded connections with a fastener pattern length greater than 38 in (965 mm), F_{nv} shall be reduced to 83.3 percent of the tabulated values. Fastener pattern length is the maximum distance parallel to the line of force between the centerline of the bolts connecting two parts with one faying surface.
[c] For A307 bolts, the tabulated values shall be reduced by 1 percent for each $^1/_{16}$ in (2 mm) over 5 diameters of length in the grip.
[d] Threads permitted in shear planes.

Source: American Institute of Steel Construction, *Manual of Steel Construction*, 14th ed. (Chicago: AISC, 2011), Table J3.2, p. 16.1-120.

High-strength bolts

ASTM A325 (from Group A) and A490 (from Group B) are the traditional high strength bolts and are covered in the Specification for Structural Joints Using High- Strength Bolts (RCSC, 2009), which is the basis for the AISC provisions for high-strength bolts.

A490 bolts have a higher ultimate tensile strength than A325 bolts and are assigned a higher nominal strength. They were introduced long after A325 bolts had been in general use, primarily for use with high-strength steels

The other bolts listed in Groups A and B have the same strengths, but have special distinguishing characteristics. For example, F1852 and F2280 bolts have special twist-off ends that simplify installation when a special bolt pretension is required.

we will use the designations Group A and Group B. For example, instead of referring to an ASTM A325 bolt, we will call it a Group A bolt.

The usual selection process is to determine the number of Group A bolts needed in a connection, and if too many are required, use Group B bolts.