Unsteady-State Conduction (Transient Conduction)

If a solid body is subjected to a change in environment, some time must elapse before an equilibrium temperature condition will prevail in the body. We refer to the equilibrium condition as the steady state. The transient heating or cooling process that takes place in the interim period before equilibrium is the unsteady state.

In the preceding course, we considered heat conduction under steady conditions, for which the temperature of a body at any point does not change with time. This certainly simplified the analysis, especially when the temperature varied in one direction only, and we were able to obtain analytical solutions. In this lecture, we consider the variation of temperature with time as well as position.

To analyze a transient heat-transfer problem, we could proceed by solving the general heat-conduction equation:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

For one-diementional unsteady state heat conduction (no heat generation):

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

Where the quantity $\alpha = k/\rho c_p$ is called the thermal diffusivity of the material (m^2/s).

Lumped Heat Capacity System

The lumped-heat-capacity method of analysis is used in which no temperature gradient exists, so the temperature vary with time only ($T = f(t)$). This means that the internal resistance of the body (conduction) is negligible in comparison with the external resistance (convection).

For a hot body in a cold fluid, the energy balance is:

$$In - out = Accumulation$$

$$In = 0,$$

$$out = hAs(T - T_\infty)$$

$$Acc = mcp\frac{dT}{dt} \quad \Rightarrow$$

$$-hAs(T - T_\infty) = mcp\frac{dT}{dt} = \rho Vcp\frac{dT}{dt}$$

Where:

A = surface area for convection ($4\pi r^2$ for sphere).

V = volume ($4/3\ \pi r^3$ for sphere).

ρ = density of body.

cp = specific heat of the body.

$\therefore$ $$\frac{dT}{dt}+\frac{hA_s}{\rho Vcp}(T-T_\infty)=0$$

But $\theta = T - T_\infty$, $\quad \frac{1}{\tau}=\frac{hA_s}{\rho Vcp}$, $\quad \tau = time\ constant$

$\Rightarrow$ $$\frac{d\theta}{dt}+\frac{hA}{\rho Vcp}\theta=0$$

The initial condition is

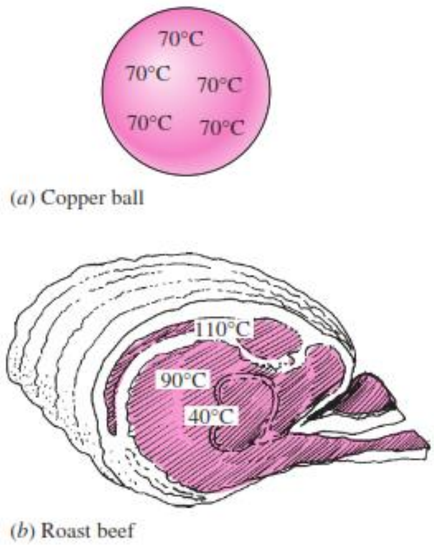
$$t=0, \quad T=T_i, \quad \theta=\theta_i$$

The solution is

$$\frac{\theta}{\theta_i}=\frac{T-T_\infty}{T_i-T_\infty}=e^{-(\frac{hA}{\rho Vcp})t} \longrightarrow \frac{\theta}{\theta_i}=e^{-t/\tau}$$

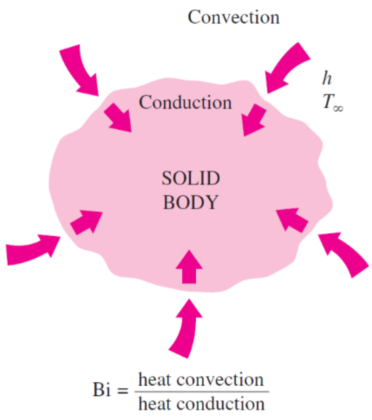
The above system is called capacitance system.

Consider a small hot copper ball coming out of an oven. Measurements indicate that the temperature of the copper ball changes with time, but it does not change much with position at any given time. Thus the temperature of the ball remains uniform at all times, and we can talk about the temperature of the ball with no reference to a specific location. Now let us go to the other extreme and consider a large roast in an oven. If you have done any roasting, you must have noticed that the temperature distribution within the roast is not even close to being uniform. You can easily verify this by taking the roast out before it is completely done and cutting it in half. You will see that the outer parts of the roast are well done while the center part is barely warm. Thus, lumped system analysis is not applicable in this case.



A small copper ball can be modeled as a lumped system, but a roast beef cannot.

Applicability of Lumped Heat Capacity System:



The criteria used for a system to be LHC system is:

$$\frac{hLc}{k} \leq 0.1$$

Where hL_c/k is called **Biot number** and it represents the ratio of the convection at the surface to conduction within the body.

Lc is the characteristic length.

$L_c = V/A_s$

So that lumped system analysis is applicable if

Bi ≤ 0.1

Where:

L_c = 1/2 (thick) = L for plate

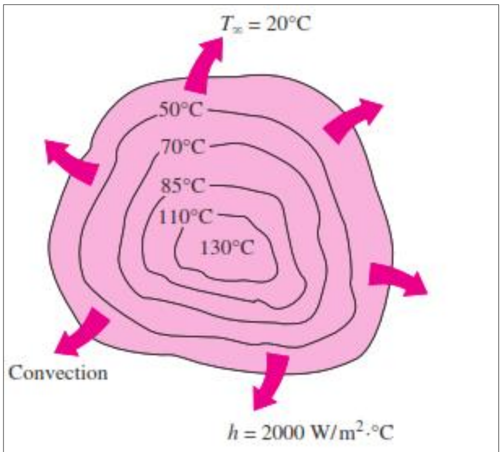
L_c = r/2 for cylinder

L_c = r/3 for sphere

Consider heat transfer from a hot body to its cooler surroundings (Fig. aside). Heat will be transferred from the body to the surrounding fluid as a result of a temperature difference. But this energy will come from the region near the surface, and thus the temperature of the body near the surface will drop.

This creates a temperature gradient between the inner and outer regions of the body and initiates heat flow by conduction from the interior of the body toward the outer surface.

When the convection heat transfer coefficient ***h*** and thus convection heat transfer from the body are high, the temperature of the body near the surface will drop quickly



When the convection coefficient h is high and k is low, large temperature differences occur between the inner and outer regions of a large solid.

This will create a larger temperature difference between the inner and outer regions unless the body is able to transfer heat from the inner to the outer regions just as fast.

Thus, the magnitude of the maximum temperature difference within the body depends strongly on the ability of a body to conduct heat toward its surface relative to the ability of the surrounding medium to convect this heat away from the surface. The Biot number is a measure of the relative magnitudes of these two competing effects.

Example: A steel ball [c = 0.46 kJ/kg · ◦C, k = 35 W/m · ◦C] 5.0 cm in diameter and initially at a uniform temperature of 450◦C is suddenly placed in a controlled environment in which the temperature is maintained at 100◦C. The convection heat-transfer coefficient is 10 W/m2 · ◦C. Calculate the time required for the ball to attain a temperature of 150◦C. (ρsteel= 7800 kg/m3)

Solution:

We anticipate that the lumped-capacity method will apply because of the low value of h and high value of k. We can check by using Bi:

Lc=D/6=0.05/6=0.0083 m

$$Bi = \frac{hLc}{k} = \frac{(10)(0.0083)}{(35)} = 0.0023 < 0.1$$

So we can use the equation of LHC system:

$$T = 150°\mathrm{C} \qquad \rho = 7800\mathrm{kg/m^3}$$
$$T_\infty = 100°\mathrm{C} \qquad h = 10\mathrm{W/m^2} \cdot° \mathrm{C}$$
$$T_i = 450°\mathrm{C} \qquad c = 460\mathrm{J/kg} \cdot° \mathrm{C}$$

$$\frac{\rho cV}{hAs} = \frac{(7800)(460)(0.0083)}{10} = 2990\ s$$

$$\theta = \theta_i e^{-t/\tau}$$

$$\theta = T - T_\infty = 150 - 100 = 50\ °\mathrm{C}$$

$$\theta_i = Ti - T_\infty = 450 - 100 = 350\ °\mathrm{C}$$

$$50 = 350\ e^{-t/2990} \quad \Longrightarrow$$

t$\approx 5818\ s = 1.62\ h$

Example: An ingot (K=40 W/m.°C) has a cylindrical shape with 10 cm diameter and 30 cm long passes through a heat treatment furnace .The ingot must reach a temperature of 800 °C before if comes out of the furnace. The furnace gas is at 1250 °C and ingot initial temperatures is 90 °C. What is the time with which the ingot should stay in the furnace to attain the required temperature? The convective surface heat transfer coefficient is 100 W/m^2. °C. Take the thermal diffusivity of ingot=1.16 m^2/sec, (ρ=7800 kg/m3, C=460 J/kg. °C).

$$L_c = \frac{D}{4} = \frac{r}{2} = \frac{0.05}{2} = 0.025 \text{ m}$$

$$Bi = \frac{hL_c}{K} = \frac{100\,(0.025)}{40} = 0.062 < 0.1 \longrightarrow \text{LHC system}$$

$$\tau = \frac{\rho C V}{h A_s} = \frac{(7800)(460)(0.025)}{100} = 897 \text{ sec}$$

$$\theta/\theta_i = e^{-t/\tau}$$

$$\frac{\theta}{\theta_i} = \frac{800-1250}{90-1250} = e^{-t/897}$$

$$0.388 = e^{-t/897} \Rightarrow t = 849.4 \text{ sec}$$

H.W: A thermocouple junction, which may be approximated by a sphere, is to be used for temperature measurement in a gas stream. The convection heat transfer coefficient between the junction surface and the gas is known to be h = 400 $W/m^2.K$, and the junction thermophysical properties are k = 20 W/m.K, Cp = 400 J/kg.K, and ρ = 8500 kg/m^3 . Determine the junction diameter needed for the thermocouple to have a time constant of ***1*** s. If the junction is at 25°C and is placed in a gas stream that is at 200°C, how long will it take for the junction to reach 199°C?

THE OVERALL HEAT TRANSFER COEFFICIENT

A heat exchanger typically involves two flowing fluids separated by a solid wall. Heat is first transferred from the hot fluid to the wall by convection, through the wall by conduction, and from the wall to the cold fluid again by convection.

The thermal resistance network associated with this heat transfer process involves two convection and one conduction resistances.

Here the subscripts ***i*** and ***o*** represent the inner and outer surfaces of the inner tube

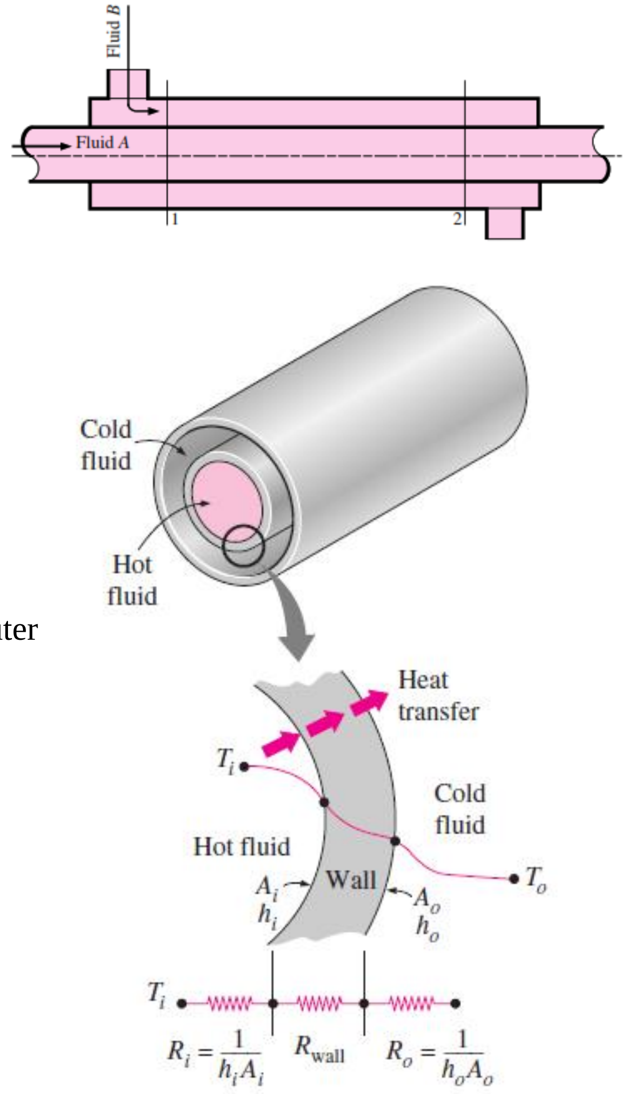


FIGURE 13–7
Thermal resistance network associated with heat transfer in a double-pipe heat exchanger.

$$q = UA\,\Delta T_{\text{overall}}$$

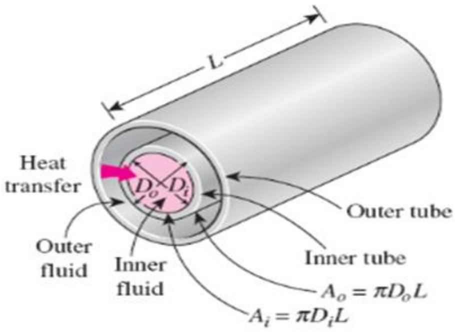
Where U: is the overall heat transfer coefficient.

For plane wall:

$$U = \frac{1}{1/h_1 + \Delta x/k + 1/h_2}$$

For cylindrical wall:

The overall heat-transfer coefficient may be based on either the inside or the outside area of the tube. Accordingly,



$$\dot{Q} = \frac{\Delta T}{R} = UA\Delta T = U_i A_i \Delta T = U_o A_o \Delta T$$

U the overall heat transfer coefficient, W/m²·°C

$$\frac{1}{U_i A_i} = \frac{1}{U_o A_o} = R = \frac{1}{h_i A_i} + R_{\text{wall}} + \frac{1}{h_o A_o}$$

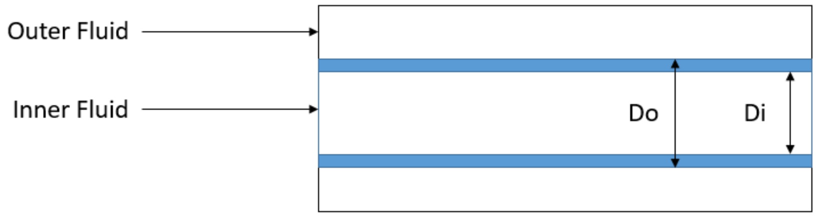
$$U_i A_i = U_o A_o, \text{ but } U_i \neq U_o$$

$$U_i = \frac{1}{\frac{1}{h_i} + \frac{A_i \ln(r_o/r_i)}{2\pi kL} + \frac{A_i}{A_o}\frac{1}{h_o}}$$

$$U_o = \frac{1}{\frac{A_o}{A_i}\frac{1}{h_i} + \frac{A_o \ln(r_o/r_i)}{2\pi kL} + \frac{1}{h_o}}$$

Fouling Factor

The performance of heat exchangers usually deteriorates with time as a result of accumulation of deposits on heat transfer surfaces. The layer of deposits represents additional resistance to heat transfer, which is represented by a *fouling factor* R_f.



R_f must be included along with the other thermal resistances making up the overall heat-transfer coefficient, thus For an unfinned shell-and-tube heat exchanger, it can be expressed as:

$$\frac{1}{UA_s} = \frac{1}{U_iA_i} = \frac{1}{U_oA_o} = R = \frac{1}{h_iA_i} + \frac{R_{f,i}}{A_i} + \frac{\ln(D_o/D_i)}{2\pi kL} + \frac{R_{f,o}}{A_o} + \frac{1}{h_oA_o}$$

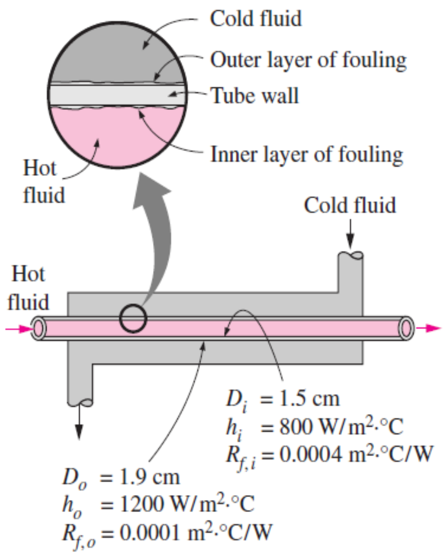
where $A_i = \pi D_i L$ and $A_o = \pi D_o L$ are the areas of inner and outer surfaces, and $R_{f,i}$ and $R_{f,o}$ are the fouling factors at those surfaces.

Table 10-2 | Table of selected fouling factors, according to Reference 2.

Type of fluid	Fouling factor, $h \cdot ft^2 \cdot °F/Btu$	$m^2 \cdot °C/W$
Seawater, below 125°F	0.0005	0.00009
Above 125°F	0.001	0.002
Treated boiler feedwater above 125°F	0.001	0.0002
Fuel oil	0.005	0.0009
Quenching oil	0.004	0.0007
Alcohol vapors	0.0005	0.00009
Steam, non-oil-bearing	0.0005	0.00009
Industrial air	0.002	0.0004
Refrigerating liquid	0.001	0.0002

EXAMPLE **Effect of Fouling on the Overall Heat Transfer Coefficient**

A double-pipe (shell-and-tube) heat exchanger is constructed of a stainless steel ($k = 15.1$ W/m · °C) inner tube of inner diameter $D_i = 1.5$ cm and outer diameter $D_o = 1.9$ cm and an outer shell of inner diameter 3.2 cm. The convection heat transfer coefficient is given to be $h_i = 800$ W/m² · °C on the inner surface of the tube and $h_o = 1200$ W/m² · °C on the outer surface. For a fouling factor of $R_{f,\,i} = 0.0004$ m² · °C/W on the tube side and $R_{f,\,o} = 0.0001$ m² · °C/W on the shell side, determine (*a*) the thermal resistance of the heat exchanger per unit length and (*b*) the overall heat transfer coefficients, U_i and U_o based on the inner and outer surface areas of the tube, respectively.



SOLUTION The heat transfer coefficients and the fouling factors on the tube and shell sides of a heat exchanger are given. The thermal resistance and the overall heat transfer coefficients based on the inner and outer areas are to be determined.

Assumptions The heat transfer coefficients and the fouling factors are constant and uniform.

Analysis (*a*) The schematic of the heat exchanger is given in Figure 13–11. The thermal resistance for an unfinned shell-and-tube heat exchanger with fouling on both heat transfer surfaces is given by Eq. 13-8 as

$$R = \frac{1}{UA_s} = \frac{1}{U_iA_i} = \frac{1}{U_oA_o} = \frac{1}{h_iA_i} + \frac{R_{f,i}}{A_i} + \frac{\ln(D_o/D_i)}{2\pi kL} + \frac{R_{f,o}}{A_o} + \frac{1}{h_oA_o}$$

where

$$A_i = \pi D_i L = \pi(0.015 \text{ m})(1 \text{ m}) = 0.0471 \text{ m}^2$$
$$A_o = \pi D_o L = \pi(0.019 \text{ m})(1 \text{ m}) = 0.0597 \text{ m}^2$$

Substituting, the total thermal resistance is determined to be

$$R = \frac{1}{(800 \text{ W/m}^2 \cdot {}^\circ\text{C})(0.0471 \text{ m}^2)} + \frac{0.0004 \text{ m}^2 \cdot {}^\circ\text{C/W}}{0.0471 \text{ m}^2}$$
$$+ \frac{\ln (0.019/0.015)}{2\pi(15.1 \text{ W/m} \cdot {}^\circ\text{C})(1 \text{ m})}$$
$$+ \frac{0.0001 \text{ m}^2 \cdot {}^\circ\text{C/W}}{0.0597 \text{ m}^2} + \frac{1}{(1200 \text{ W/m}^2 \cdot {}^\circ\text{C})(0.0597 \text{ m}^2)}$$
$$= (0.02654 + 0.00849 + 0.0025 + 0.00168 + 0.01396){}^\circ\text{C/W}$$
$$= \mathbf{0.0532{}^\circ C/W}$$

Note that about 19 percent of the total thermal resistance in this case is due to fouling and about 5 percent of it is due to the steel tube separating the two fluids. The rest (76 percent) is due to the convection resistances on the two sides of the inner tube.

(*b*) Knowing the total thermal resistance and the heat transfer surface areas, the overall heat transfer coefficient based on the inner and outer surfaces of the tube are determined again from Eq. 13-8 to be

$$U_i = \frac{1}{RA_i} = \frac{1}{(0.0532 \ {}^\circ\text{C/W})(0.0471 \text{ m}^2)} = \mathbf{399 \ W/m^2 \cdot {}^\circ C}$$

and

$$U_o = \frac{1}{RA_o} = \frac{1}{(0.0532 \ {}^\circ\text{C/W})(0.0597 \text{ m}^2)} = \mathbf{315 \ W/m^2 \cdot {}^\circ C}$$

Discussion Note that the two overall heat transfer coefficients differ significantly (by 27 percent) in this case because of the considerable difference between the heat transfer surface areas on the inner and the outer sides of the tube. For tubes of negligible thickness, the difference between the two overall heat transfer coefficients would be negligible.

ANALYSIS OF HEAT EXCHANGERS

Heat exchangers are commonly used in practice, and an engineer often finds himself or herself in a position to select a heat exchanger that will achieve a *specified temperature change* in a fluid stream of known mass flow rate, or to *predict the outlet temperatures* of the hot and cold fluid streams in a specified heat exchanger.

We will discuss the two methods used in the analysis of heat exchangers, the *log mean temperature difference (or LMTD) method* and *the effectiveness–NTU method*.

But first we present some general considerations. Heat exchangers usually operate for long periods of time with no change in their operating conditions. Therefore, they can be modelled as steady-flow devices. As such,

- the mass flow rate of each fluid remains constant.
- the fluid properties such as temperature and velocity at any inlet or outlet remain the same.
- Axial heat conduction along the tube is usually insignificant and can be considered negligible.
- The specific heat of a fluid, in general, changes with temperature. But, in a specified temperature range, it can be treated as a constant.
- Finally, the outer surface of the heat exchanger is assumed to be perfectly insulated, so that there is no heat loss to the surrounding medium, and any heat transfer occurs between the two fluids only.

THE LOG MEAN TEMPERATURE DIFFERENCE (LMTD) METHOD

If q is the total rate of heat transfer between the hot and cold fluids and there is negligible heat transfer between the exchanger and its surroundings:

$$q = \dot{m}_c C_{pc}\left(T_{c,\text{out}} - T_{c,\text{in}}\right)$$

$$q = \dot{m}_h C_{ph}(T_{h,\text{in}} - T_{h,\text{out}})$$

$$\begin{aligned} \dot{m}_c, \dot{m}_h &= \text{mass flow rates} \\ C_{pc}, C_{ph} &= \text{specific heats} \\ T_{c,\text{ out}}, T_{h,\text{ out}} &= \text{outlet temperatures} \\ T_{c,\text{ in}}, T_{h,\text{ in}} &= \text{inlet temperatures} \end{aligned}$$

Where the subscripts *c* and *h* stand for cold and hot fluids, respectively.

The heat transfer rate can be calculated by:

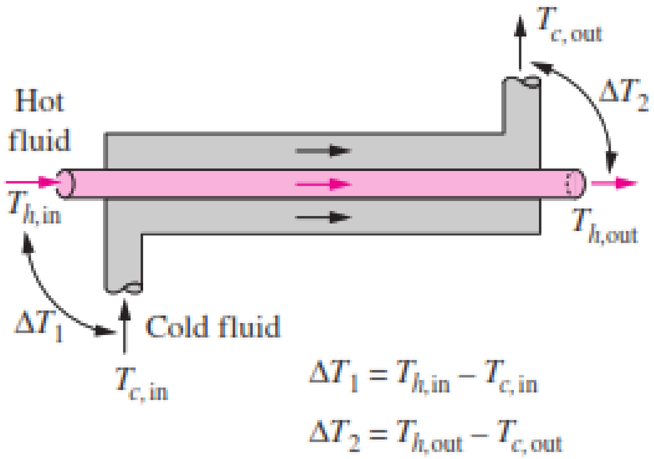
$$q = UA_s \Delta T_{\text{lm}}$$

Where:

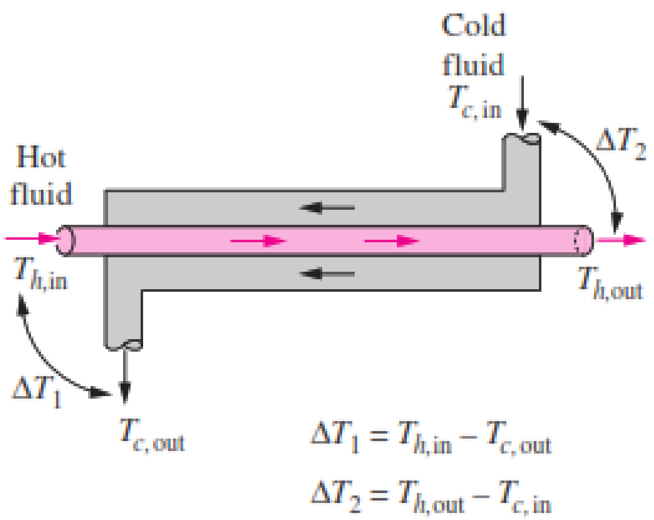
ΔT_{lm} : is called the log mean temperature difference (LMTD).

$$\Delta T_{\text{lm}} = \frac{\Delta T_1 - \Delta T_2}{\ln\left(\frac{\Delta T_1}{\Delta T_2}\right)}$$

Here ΔT_1 and ΔT_2 represent the temperature difference between the two fluids at the two ends (inlet and outlet) of the heat exchanger.

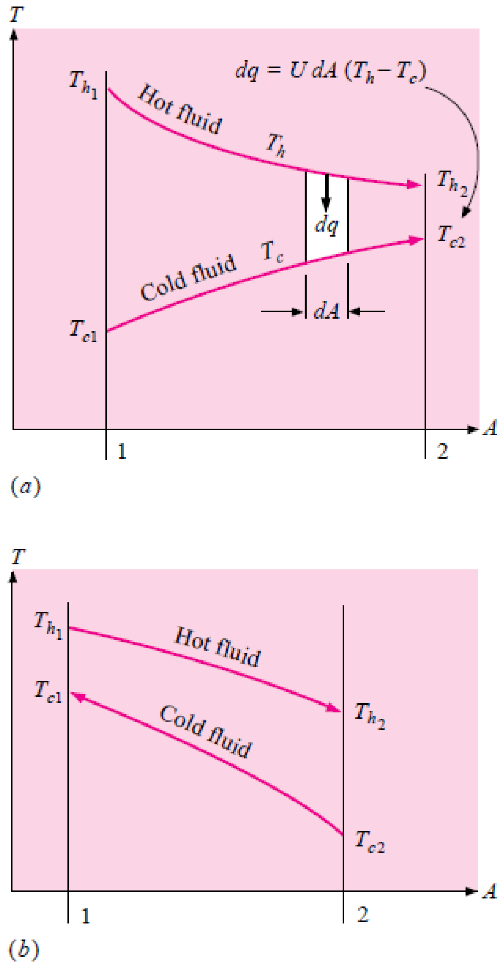


(*a*) Parallel-flow heat exchangers



(*b*) Counter-flow heat exchangers

Figure 10-7 | Temperature profiles for (*a*) parallel flow and (*b*) counterflow in double-pipe heat exchanger.



The variation of temperatures of hot and cold fluids in a counter-flow heat exchanger is given in figure below. Note that the hot and cold fluids enter the heat exchanger from opposite ends, and the outlet temperature of the cold fluid in this case may exceed the outlet temperature of the hot fluid.

However, **the outlet temperature of the cold fluid can never exceed the inlet temperature of the hot fluid**, since this would be a violation of the second law of thermodynamics.

In a counter-flow heat exchanger, the temperature difference between the hot and the cold fluids will remain constant along the heat exchanger when the *heat capacity rates* of the two fluids are *equal* (that is, ΔT = constant when $C_h = C_c$ or $\dot{m}_h C_{ph} = \dot{m}_c C_{pc}$). Then we have $\Delta T_1 = \Delta T_2$, and the last log mean temperature difference relation gives $\Delta T_{lm} = \frac{0}{0}$, which is indeterminate. It can be shown by the application of l'Hôpital's rule that in this case we have $\Delta T_{lm} = \Delta T_1 = \Delta T_2$, as expected.

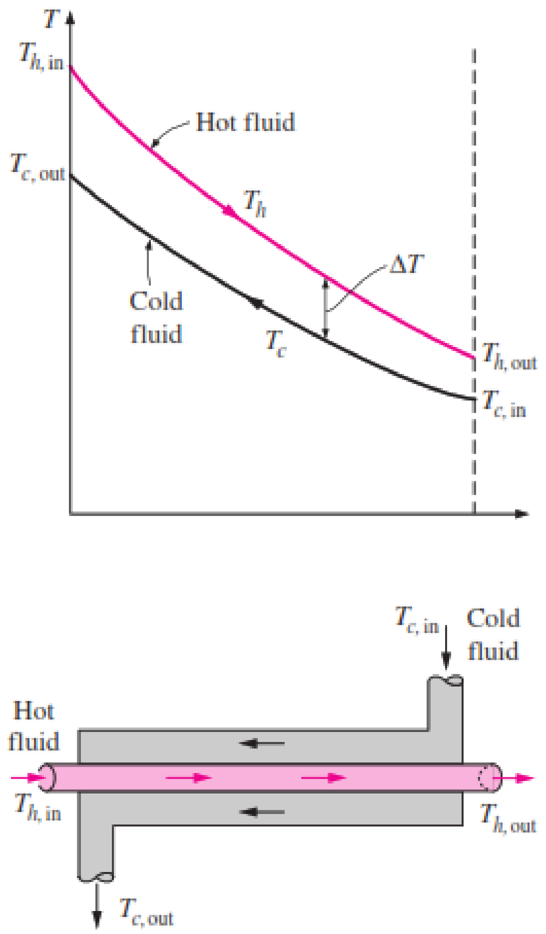


FIGURE 13–16
The variation of the fluid temperatures in a counter-flow double-pipe heat exchanger.

For specified inlet and outlet temperatures, the log mean temperature difference for a counter-flow heat exchanger is always ***greater*** than that for a parallel-flow heat exchanger. That is, $\Delta T_{\text{lm,CF}} > \Delta T_{\text{lm,PF}}$ and thus a smaller surface area (and thus a smaller heat exchanger) is needed to achieve a specified heat transfer rate in a counter-flow heat exchanger.

Therefore, it is common practice to use counter-flow arrangements in heat exchangers.

EXAMPLE 10-4 Calculation of Heat-Exchanger Size from Known Temperatures

Water at the rate of 68 kg/min is heated from 35 to 75°C by an oil having a specific heat of 1.9 kJ/kg · °C. The fluids are used in a counterflow double-pipe heat exchanger, and the oil enters the exchanger at 110°C and leaves at 75°C. The overall heat-transfer coefficient is 320 W/m^2 · °C. Calculate the heat-exchanger area.

Solution

The total heat transfer is determined from the energy absorbed by the water:

$$q = \dot{m}_w c_w \Delta T_w = (68)(4180)(75 - 35) = 11.37 \text{ MJ/min} \qquad [a]$$

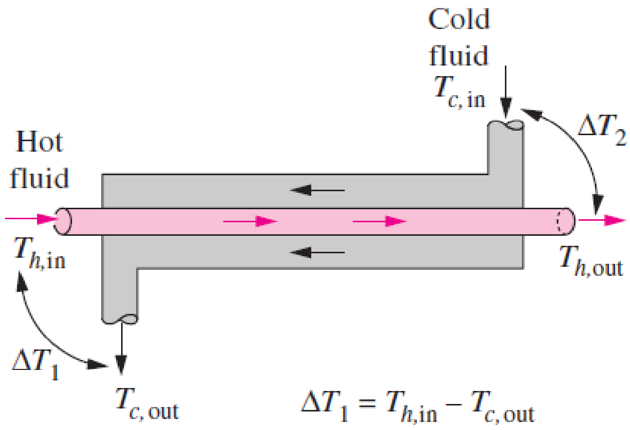
$$= 189.5 \text{ kW} \qquad [6.47 \times 10^5 \text{ Btu/h}]$$

Since all the fluid temperatures are known, the LMTD can be calculated by using the temperature scheme in Figure 10-7*b*:

$$\Delta T_m = \frac{(110 - 75) - (75 - 35)}{\ln[(110 - 75)/(75 - 35)]} = 37.44°\text{C} \qquad [b]$$

Then, since $q = UA\,\Delta T_m$,

$$A = \frac{1.895 \times 10^5}{(320)(37.44)} = 15.82 \text{ m}^2 \qquad [170 \text{ ft}^2]$$



$\Delta T_1 = T_{h,\text{in}} - T_{c,\text{out}}$

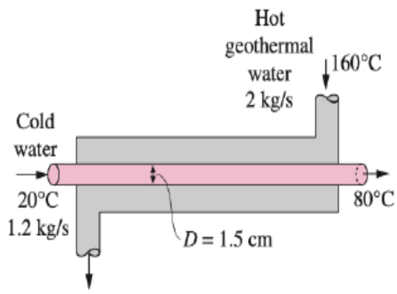
$\Delta T_2 = T_{h,\text{out}} - T_{c,\text{in}}$

Heating Water in a Counter-Flow Heat Exchanger

A counter-flow double-pipe heat exchanger is to heat water from 20°C to 80°C at a rate of 1.2 kg/s. The heating is to be accomplished by geothermal water available at 160°C at a mass flow rate of 2 kg/s. The inner tube is thin-walled and has a diameter of 1.5 cm. If the overall heat transfer coefficient of the heat exchanger is 640 W/m^2 · °C, determine the length of the heat exchanger re- quired to achieve the desired heating.

Solution:

The rate of heat transfer in the heat exchanger can be determined from



$$\dot{Q} = [\dot{m}C_p(T_{\text{out}} - T_{\text{in}})]_{\text{water}} = (1.2 \text{ kg/s})(4.18 \text{ kJ/kg} \cdot °\text{C})(80 - 20)°\text{C} = 301 \text{ kW}$$

Noting that all of this heat is supplied by the geothermal water, the outlet temperature of the geothermal water is determined to be

$$\dot{Q} = [\dot{m}C_p(T_{\text{in}} - T_{\text{out}})]_{\text{geothermal}} \longrightarrow T_{\text{out}} = T_{\text{in}} - \frac{\dot{Q}}{\dot{m}C_p} = 160°\text{C} - \frac{301 \text{ kW}}{(2 \text{ kg/s})(4.31 \text{ kJ/kg} \cdot °\text{C})} = 125°\text{C}$$

Knowing the inlet and outlet temperatures of both fluids, the logarithmic mean temperature difference for this counter-flow heat exchanger

$$\Delta T_1 = T_{h,\text{in}} - T_{c,\text{out}} = (160 - 80)°\text{C} = 80°\text{C}$$
$$\Delta T_2 = T_{h,\text{out}} - T_{c,\text{in}} = (125 - 20)°\text{C} = 105°\text{C}$$

Then the surface area of the heat exchanger is determined to be

$$\dot{Q} = UA_s\,\Delta T_{\text{lm}} \longrightarrow A_s = \frac{\dot{Q}}{U\,\Delta T_{\text{lm}}} = \frac{301{,}000 \text{ W}}{(640 \text{ W/m}^2 \cdot °\text{C})(92.0°\text{C})} = 5.11 \text{ m}^2$$

To provide this much heat transfer surface area, the length of the tube must be

$$A_s = \pi DL \longrightarrow L = \frac{A_s}{\pi D} = \frac{5.11 \text{ m}^2}{\pi(0.015 \text{ m})} = \mathbf{108 \text{ m}}$$

H.W: It is desired to heat 230 kg/h of water [c_p =4.175 kJ/kg. ◦C] from 35 to 93◦C with oil [c_p =2.1 kJ/kg .◦C] having an initial temperature of 175◦C. The mass flow of oil is also 230 kg/h. Two double-pipe heat exchangers are available:

exchanger 1: U =570 W/m^2 · ◦C A=0.47 m^2

exchanger 2: U =370 W/m^2 · ◦C A=0.94 m^2

Which exchanger should be used?

ملاحظة مهمة :

When Tc,out > Th,out counter flow must be used

If LMTD = 0 °C use AMTD

So, AMTD $= \dfrac{\theta_1 + \theta_2}{2}$ [AMTD: Arithmetic mean temperature difference]

What happens to LMTD expression in case $\Delta T_1 = \Delta T_2$? How to overcome this problem?

$$\text{LMTD} = \frac{\Delta T_1 - \Delta T_2}{\log \dfrac{\Delta T_1}{\Delta T_2}}$$

when $\Delta T_1 - \Delta T_2 = 0$, then-

$$\text{LMTD} = \frac{0}{\log 1} = \frac{0}{0} \text{ i.e. LMTD is indeterminate.}$$

This type of problem can be solved by employing '*L*' Hospital's rule and LMTD becomes average of inlet and outlet temperature differences.

$$\text{LMTD} = \frac{\Delta T_1 + \Delta T_2}{2}$$

Heat-Exchanger Design Considerations

In the process and power industries, or related activities, many heat exchangers are purchased as off-the-shelf items, and a selection is made on the basis of cost and specifications furnished by the various manufacturers. In more specialized applications, such as the aerospace and electronics industries, a particular design is frequently called for. Where a heat exchanger forms a part of an overall machine or device to be manufactured, a standard item may be purchased; or if cost considerations and manufacturing quantities warrant, the heat exchanger may be specially designed for the application. Whether the heat exchanger is selected as an off-the-shelf item or designed especially for the application, the following factors are almost always considered:

i. Heat Transfer Rate

This is the most important quantity in the selection of a heat exchanger. A heat exchanger should be capable of transferring heat at the specified rate in order to achieve the desired temperature change of the fluid at the specified mass flow rate.

ii. Cost

Budgetary limitations usually play an important role in the selection of heat exchangers, except for some specialized cases where “money is no object.” An off-the-shelf heat exchanger has a definite cost advantage over those made to order. However, in some cases, none of the existing heat exchangers will do, and it may be necessary to undertake the expensive and time-consuming task of designing and manufacturing a heat exchanger from scratch to suit the needs. This is often the case when the heat exchanger is an integral part of the overall device to be manufactured. The ***operation*** and ***maintenance*** costs of the heat exchanger are also important considerations in assessing the overall cost.

iii. Pumping Power

In a heat exchanger, both fluids are usually forced to flow by pumps or fans that consume electrical power. The annual cost of electricity associated with the operation of the pumps and fans can be determined from:

$$\begin{aligned}\text{Operating cost} = &\text{(Pumping power, kW)} \times \text{(Hours of operation, h)}\\ &\times \text{(Price of electricity, \$/kWh)}\end{aligned}$$

Where the pumping power is the total electrical power consumed by the motors of the pumps and fans.

For example, a heat exchanger that involves a 1-hp pump and a 1/3-hp fan (*1 hp = 0.746 kW*) operating 8 h a day and 5 days a week will consume 2017 kWh of electricity per year, which will cost $161.4 at an electricity cost of 8 cents/kWh. (*1 cent= $ 0.01*)

iv. Size and Weight

Normally, the smaller and the lighter the heat exchanger, the better it is. This is especially the case in the automotive and aerospace industries, where size and weight requirements are most stringent.

v. Type

The type of heat exchanger to be selected depends primarily on the type of fluids involved, the size and weight limitations, and the presence of any phase change processes.

vi. Materials

The materials used in the construction of the heat exchanger may be an important consideration in the selection of heat exchangers. In the case of corrosive fluids, we may have to select expensive corrosion-resistant materials such as stainless steel or even titanium.

HEAT EXCHANGERS

Heat exchangers are devices that used to promote the exchange of heat between two fluids that are at different temperatures while keeping them from mixing with each other.

Heat exchangers are commonly used in practice in a wide range of applications, from heating and air-conditioning systems in a household, to chemical processing and power production in large plants.

Heat exchangers are often given specific names to reflect the specific application for which they are used. For example, a *condenser* is a heat exchanger in which one of the fluids is cooled and condenses as it flows through the heat exchanger.

A *boiler* is another heat exchanger in which one of the fluids absorbs heat and vaporizes. A space *radiator* is a heat exchanger that transfers heat from the hot fluid to the surrounding space by radiation.

Types of Heat Exchanger

Heat exchangers are typically classified according to flow arrangement and types of construction.

1. Double pipe (or concentric tube) - Parallel and counter flow.

Either the hot or cold fluid occupying the annular space and the other fluid occupying the inside of the inner pipe. Heat Exchangers can be classified depending on the basic of the *fluid paths* into:

- **Parallel flow**: Both the hot and cold fluids enter the heat exchanger at the same end and move in the *same direction*.
- **Counter flow**: Hot and cold fluids enter the heat exchanger at opposite ends and flow in *opposite directions.*

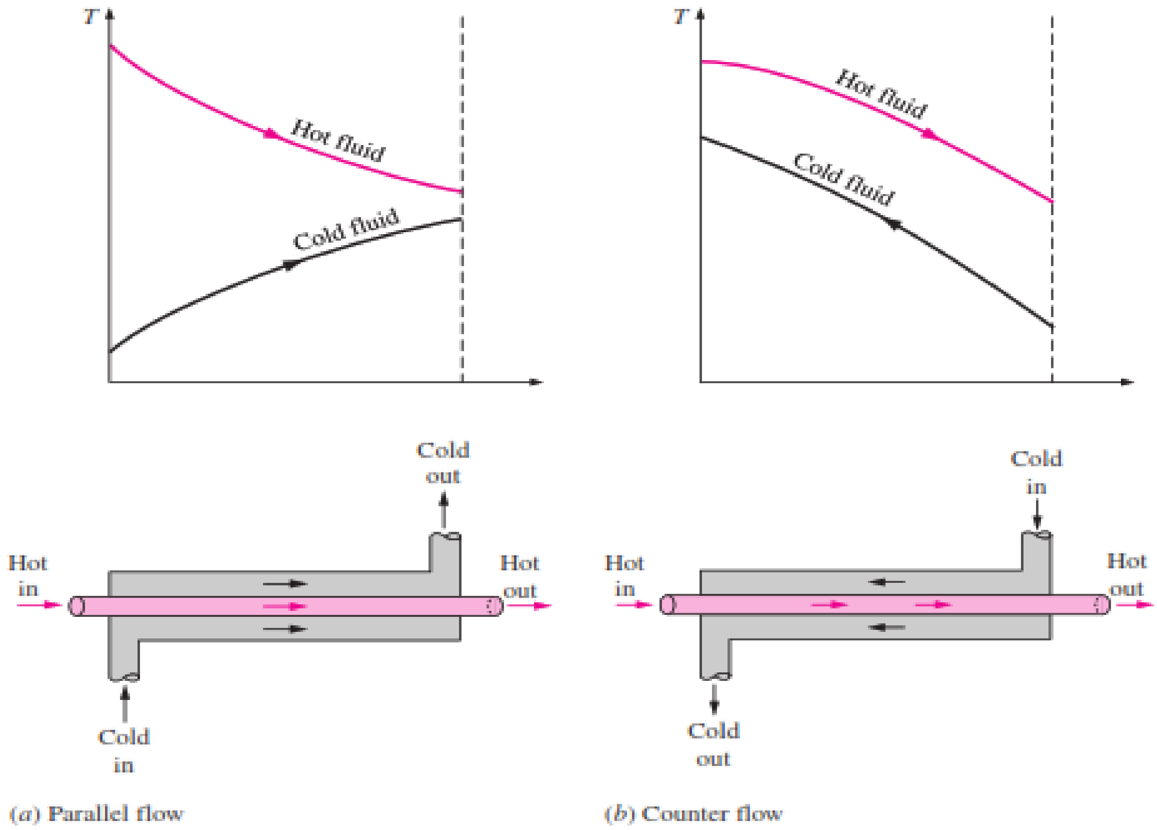
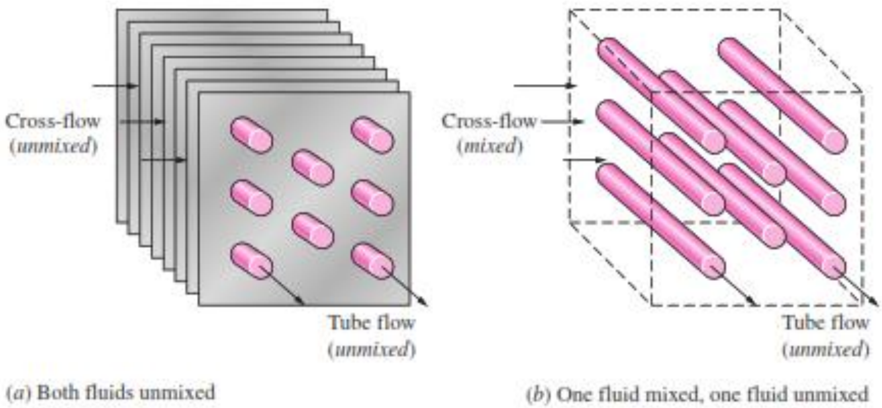


Fig: Scheme of flow regimes and associated temperature profiles in a double-pipe heat exchanger

2. **Cross flow:** the two fluids usually move <ins>*perpendicular*</ins> *to* each other (mixed and unmixed).



Different flow configurations in cross-flow heat exchangers.

3. Compact heat exchangers

Another type of heat exchanger, which is specifically designed to realize a large heat transfer surface area per unit volume. The ratio of the heat transfer surface area of a heat exchanger to its volume is called the **area density (β)**.

A heat exchanger with $\beta > 700\ m^2/m^3$ (or $200\ ft^2/ft^3$) is classified as being compact. Examples of compact heat exchangers are car radiators ($\beta \approx 1000\ m^2/m^3$), glass ceramic gas turbine heat exchangers ($\beta \approx 6000 m^2/m^3$), and the human lung ($\beta \approx 20,000\ m^2/m^3$).

Compact heat exchangers enable us to achieve high heat transfer rates between two fluids in a small volume, and they are commonly used in applications with strict limitations on the weight and volume of heat exchangers (see Fig. below).

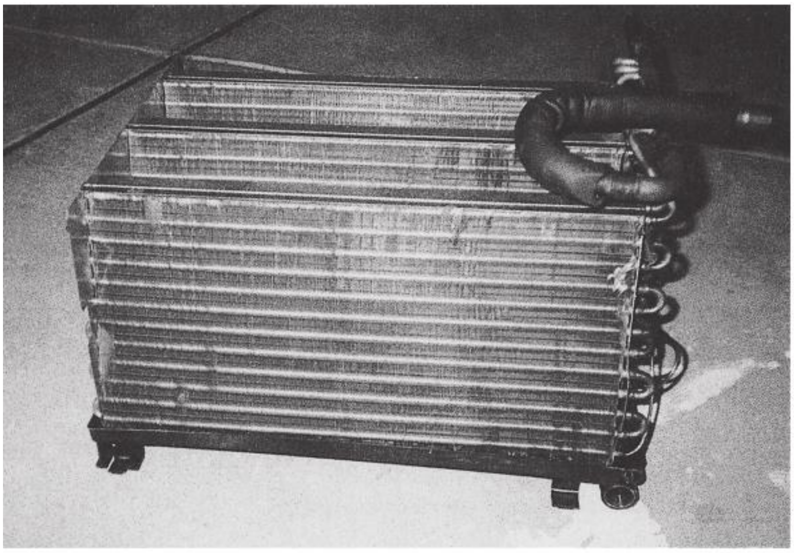


FIGURE 13–2
A gas-to-liquid compact heat exchanger for a residential air-conditioning system.

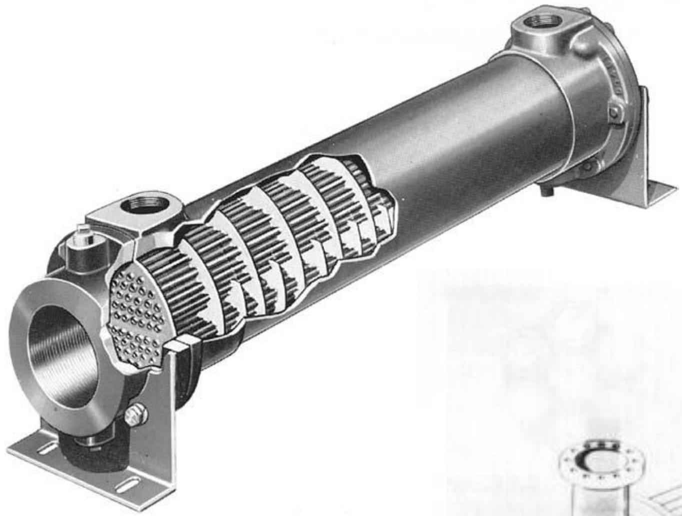
The large surface area in compact heat exchangers is obtained by attaching closely spaced *thin plate* or *corrugated fins* to the walls separating the two fluids. Compact heat exchangers are commonly used in gas-to-gas and gas-to-liquid (or liquid-to-gas) heat exchangers to counteract the low heat transfer coefficient associated with gas flow with increased surface area. In a car radiator, which is a water-to-air compact heat exchanger, for example, it is no surprise that fins are attached to the air side of the tube surface.

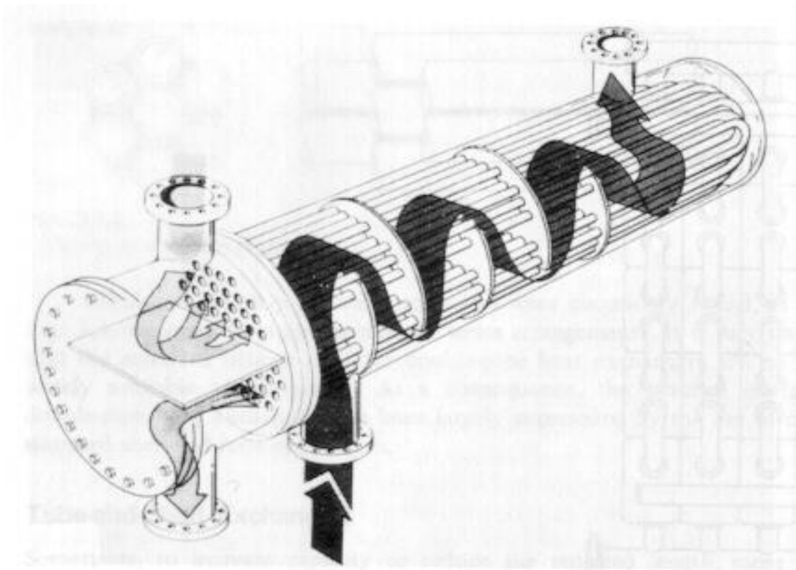
In compact heat exchangers, the two fluids usually move perpendicular to each other, and such flow configuration is called cross-flow.

4. Shell-and-tube Heat Exchanger

A type of heat exchanger widely used in the chemical-process industries is that of the shell-and-tube arrangement, One fluid flows on the inside of the tubes, while the other fluid is forced through the shell and over the outside of the tubes.

Shell-and-tube heat exchangers contain a large number of tubes (sometimes several hundred) packed in a shell with their axes parallel to that of the shell. Heat transfer takes place as one fluid flows inside the tubes while the other fluid flows outside the tubes through the shell. *Baffles* are commonly placed in the shell to force the shell-side fluid to flow across the shell to enhance heat transfer and to maintain uniform spacing between the tubes. Despite their widespread use, shell and-tube heat exchangers are not suitable for use in automotive and aircraft applications because of their relatively large size and weight. Note that the tubes in a shell-and-tube heat exchanger open to some large flow areas called headers at both ends of the shell, where the tube-side fluid accumulates before entering the tubes and after leaving them.





Shell-and-tube heat exchangers are further classified according to the number of shell and tube passes involved. Heat exchangers in which all the tubes make one U-turn in the shell, for example, are called one-shell-pass and two tube-passes heat exchangers. Likewise, a heat exchanger that involves two passes in the shell and four passes in the tubes is called a two-shell-passes and four-tube-passes heat exchanger.

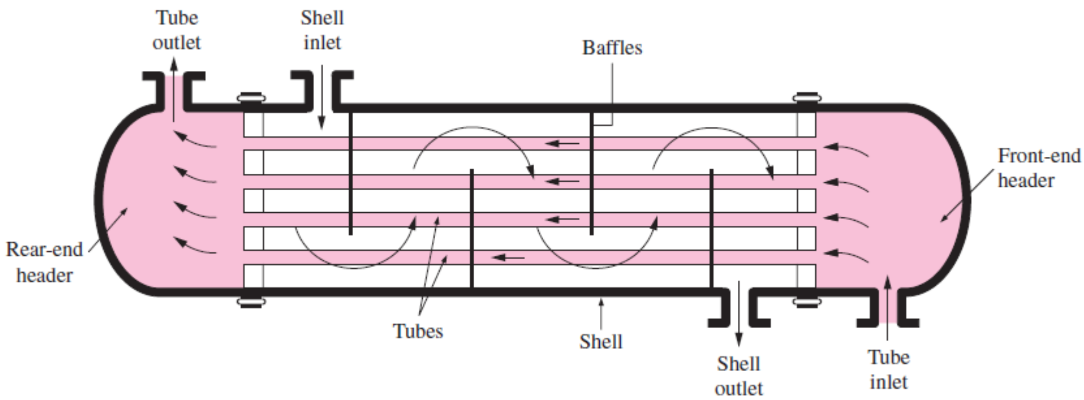


Fig: The schematic of a shell-and-tube heat exchanger (one-shell pass and one tube pass)

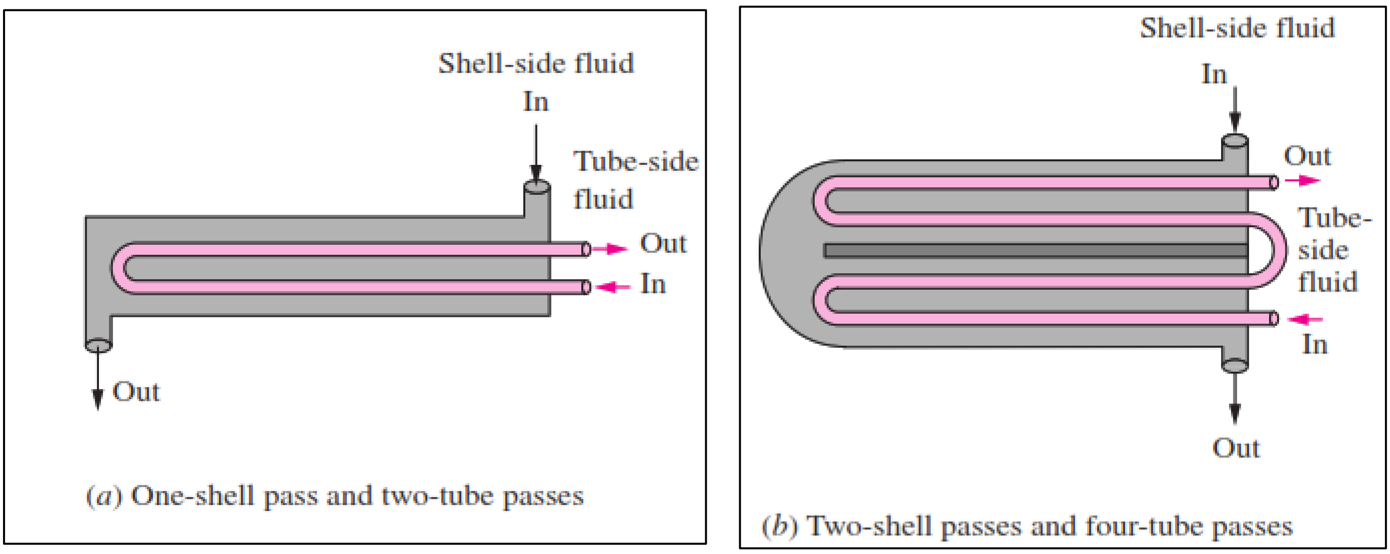
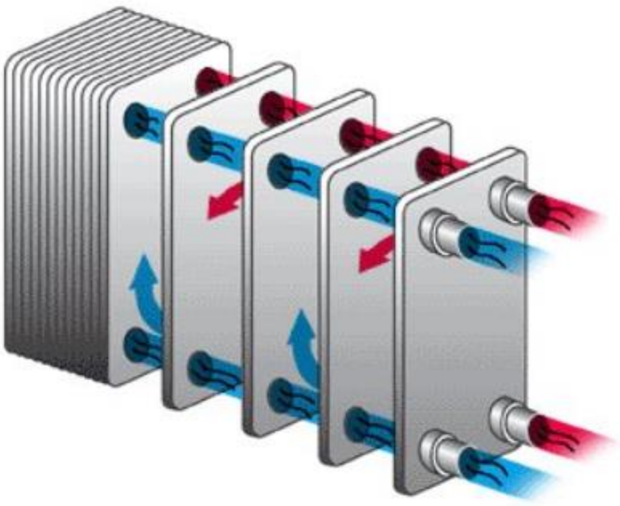
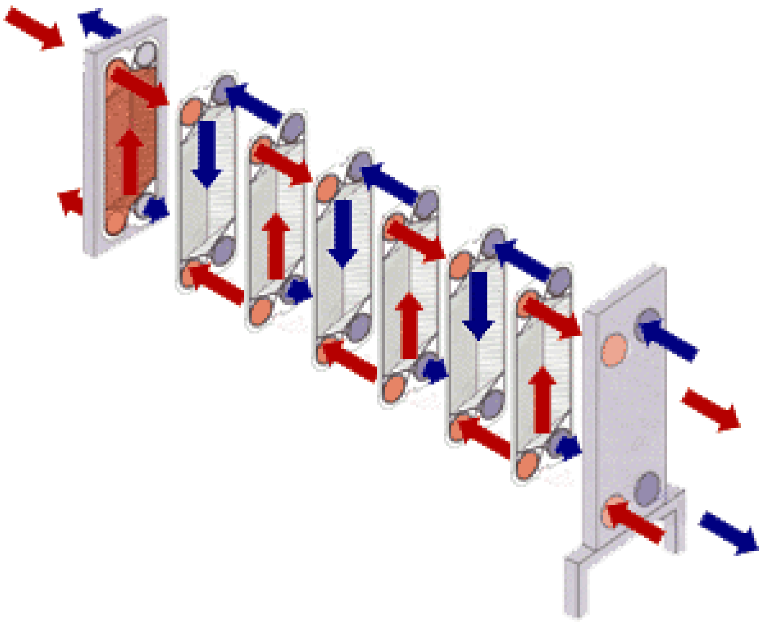


Fig: Multipass flow arrangements in shell and-tube heat exchangers

5. Plate heat exchanger



A plate heat exchanger is a unit which transfers heat continuously from one media to another media without adding energy to the process. The basic concept of a plate and frame heat exchanger is two liquids flowing on either side of a thin corrugated metal plate so heat may be easily transferred between the two. (By conduction across the plate)



The plates are compressed by means of tie bolts between a stationary frame part (called the head) and a movable frame part (called the follower).
The plate heat exchanger efficiency requires less floor space compared to other types of heat transfer equipment and is lighter in weight.

Also, plate heat exchangers can grow with increasing demand for heat transfer by simply mounting more plates. They are well suited for liquid-to-liquid heat exchange applications, provided that the hot and cold fluid streams are at about the same pressure.

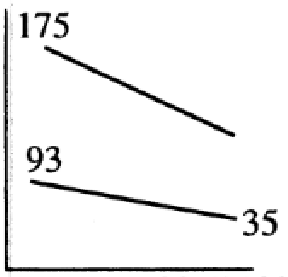
H.W: It is desired to heat 230 kg/h of water [c_p =4.175 kJ/kg. ◦C] from 35 to 93◦C with oil [c_p =2.1 kJ/kg .◦C] having an initial temperature of 175◦C. The mass flow of oil is also 230 kg/h. Two double-pipe heat exchangers are available:

exchanger 1: U =570 W/m^2 · ◦C A=0.47 m^2

exchanger 2: U =370 W/m^2 · ◦C A=0.94 m^2

Which exchanger should be used?

10-23



$$m_o c_o = \frac{(230)(2100)}{3600} = 134.17 \qquad m_w c_w = \frac{(230)(4175)}{3600} = 266.7$$

$$\Delta T_o = (93-35)\left(\frac{266.7}{134.17}\right) = 115.3 \qquad T_{o\ \text{out}} = 175 - 115.3 = 59.7°\text{C}$$

$$\Delta T_m = \frac{82-24.7}{\ln\left(\frac{82}{24.7}\right)} = 47.75°\text{C} \qquad q = (266.7)(93-35) = 15{,}469\ \text{W}$$

<u>Ex. 1</u> $A = \dfrac{15{,}469}{(570)(47.75)} = 0.568\ \text{m}^2$ not large enough

<u>Ex. 2</u> $A = \dfrac{15{,}469}{(370)(47.75)} = 0.876\ \text{m}^2$ 0.94 m^2 is large enough

Eample 10.3. *The flow rates of hot and cold water streams running through a parallel flow heat exchanger are 0.2 kg/s and 0.5 kg/s respectively. The inlet temperatures on the hot and cold sides are 75°C and 20°C respectively. The exit temperature of hot water is 45°C. If the individual heat transfer coefficients on both sides are 650 W/m²°C, calculate the area of the heat exchanger.*

Solution. *Given* : $\dot{m}_h = 0.2$ kg/s; $\dot{m}_c = 0.5$ kg/s; $t_{h1} = 75°$ C; $t_{h2} = 45°$C; $t_{c1} = 20°$C; $h_i = h_o =$ 650 W/m²°C.

The area of heat exchanger, *A* :

The heat exchanger is shown diagrammatically in Fig. 10.14.

The heat transfer rate, $$Q = \dot{m}_h \times c_{ph} \times (t_{h1} - t_{h2})$$

$$= 0.2 \times 4.187 \times (75 - 45) = 25.122 \text{ kJ/s}$$

Heat lost by hot water = Heat gained by cold water

$$\dot{m}_h \times c_{ph} \times (t_{h1} - t_{h2}) = \dot{m}_c \times c_{pc} \times (t_{c2} - t_{c1})$$

$$0.2 \times 4.187 \times (75 - 45) = 0.5 \times 4.187 \times (t_{c2} - 20)$$

$$\therefore \quad t_{c2} = 32°\text{C}$$

Logarithmic mean temperature difference (*LMTD*) is given by

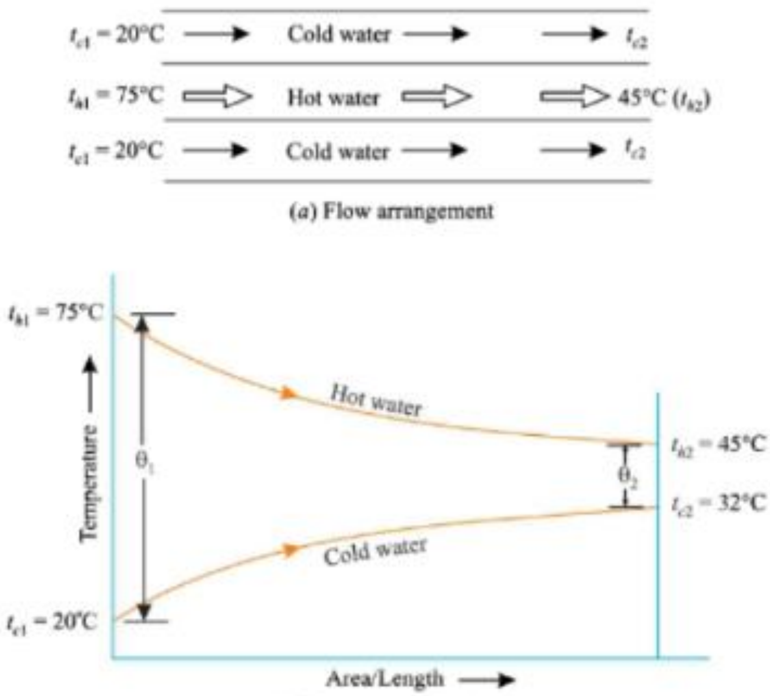
$$\theta_m = \frac{\theta_1 - \theta_2}{\ln(\theta_1/\theta_2)} \qquad \text{...[Eqn. (10.9)]}$$

or,

$$\theta_m = \frac{(t_{h1} - t_{c1}) - (t_{h2} - t_{c2})}{\ln[(t_{h1} - t_{c1})/(t_{h2} - t_{c2})]}$$

$$= \frac{(75 - 20) - (45 - 32)}{\ln[(75 - 20)/(45 - 32)]}$$

$$= \frac{55 - 13}{\ln(55/13)} = 29.12°\text{C}$$



(*a*) Flow arrangement

(*b*) Temperature distribution

Fig. 10.14. Parallel flow heat exchanger.

Overall heat transfer coefficient U is calculated from the relation,

$$\frac{1}{U} = \frac{1}{h_i} + \frac{1}{h_o}$$

$$= \frac{1}{650} + \frac{1}{650} = \frac{1}{325}$$

$\therefore$ $$U = 325 \text{ W/m}^2\text{°C}$$

Also, $$Q = U A \theta_m$$

or, $$A = \frac{Q}{U \theta_m} = \frac{25.122 \times 1000}{325 \times 29.12} = \mathbf{2.66\ m^2} \qquad \textbf{(Ans.)}$$

Multipass and Cross-Flow Heat Exchangers (Use of a Correction Factor)

If a heat exchanger other than the double-pipe type is used, the heat transfer is calculated by using a correction factor applied to the LMTD for a counterflow double-pipe arrangement with the same hot and cold fluid temperatures. The heat-transfer equation then takes the form:

$$q=UA\,F\Delta T_{lm,CF}$$

Where:

F: is the correction factor, which depends on the geometry of the heat exchanger and the inlet and outlet temperatures of the hot and cold fluid streams.

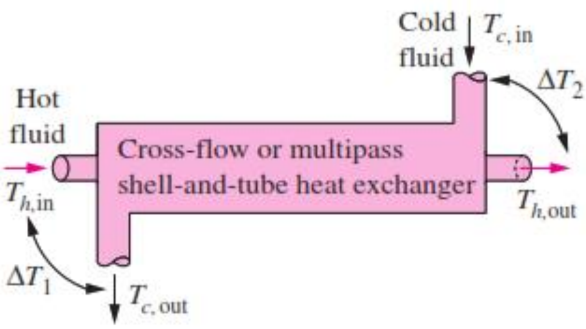
$\Delta T_{lm,CF}$: is the log mean temperature difference for the case of a counter-flow heat exchanger with the same inlet and outlet temperatures and is determined from below:

$$\Delta T_{lm,CF} = \frac{\Delta T_1 - \Delta T_2}{\ln\left(\frac{\Delta T_1}{\Delta T_2}\right)}$$

By taking

$\Delta T_1 = T_{h,in} - T_{c,out}$; and

$\Delta T_2 = T_{h,out} - T_{c,in}$



Heat transfer rate:

$$\dot{Q} = UA_s F \Delta T_{lm,CF}$$

where $\Delta T_{lm,CF} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)}$

$\Delta T_1 = T_{h,in} - T_{c,out}$

$\Delta T_2 = T_{h,out} - T_{c,in}$

and $F = \ldots$ (Fig. 13–18)

The determination of the heat transfer rate for cross-flow and multipass shell-and-tube heat exchangers using the correction factor.

Values of the correction factor are plotted in Figures below for several different types of heat exchangers versus two temperature ratios ***P*** and ***R*** defined as.

$$P = \frac{t_2 - t_1}{T_1 - t_1}$$

and

$$R = \frac{T_1 - T_2}{t_2 - t_1}$$

Where the subscripts ***1*** and ***2*** represent the inlet and outlet, respectively. Not that for a shell-and-tube heat exchanger, ***T*** and ***t*** represent the shell- and tube-side temperatures, respectively, as shown in the correction factor charts. It makes no difference whether the hot or the cold fluid flows through the shell or the tube. The determination of the correction factor ***F*** requires the availability of the inlet and the outlet temperatures for both the cold and hot fluids.

When a phase change is involved, as n condensation or boiling (evaporation), the fluid normally remains at essentially constant temperature and the relations are simplified. For this condition, P or R becomes zero and we obtain:

F =1.0   for boiling or condensation

EXAMPLE 13–5 Heating of Glycerin in a Multipass Heat Exchanger

A 2-shell passes and 4-tube passes heat exchanger is used to heat glycerin from 20°C to 50°C by hot water, which enters the thin-walled 2-cm-diameter tubes at 80°C and leaves at 40°C (Fig. 13–21). The total length of the tubes in the heat exchanger is 60 m. The convection heat transfer coefficient is 25 W/m² · °C on the glycerin (shell) side and 160 W/m² · °C on the water (tube) side. Determine the rate of heat transfer in the heat exchanger (*a*) before any fouling occurs and (*b*) after fouling with a fouling factor of 0.0006 m² · °C/W occurs on the outer surfaces of the tubes.

SOLUTION Glycerin is heated in a 2-shell passes and 4-tube passes heat exchanger by hot water. The rate of heat transfer for the cases of fouling and no fouling are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** Heat transfer coefficients and fouling factors are constant and uniform. **5** The thermal resistance of the inner tube is negligible since the tube is thin-walled and highly conductive.

Analysis The tubes are said to be thin-walled, and thus it is reasonable to assume the inner and outer surface areas of the tubes to be equal. Then the heat transfer surface area becomes

$$A_s = \pi DL = \pi(0.02 \text{ m})(60 \text{ m}) = 3.77 \text{ m}^2$$

The rate of heat transfer in this heat exchanger can be determined from

$$\dot{Q} = UA_s F \Delta T_{\text{lm, }CF}$$

where F is the correction factor and $\Delta T_{\text{lm, }CF}$ is the log mean temperature difference for the counter-flow arrangement. These two quantities are determined from

$$\Delta T_1 = T_{h,\text{ in}} - T_{c,\text{ out}} = (80 - 50)°\text{C} = 30°\text{C}$$
$$\Delta T_2 = T_{h,\text{ out}} - T_{c,\text{ in}} = (40 - 20)°\text{C} = 20°\text{C}$$
$$\Delta T_{\text{lm, }CF} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} = \frac{30 - 20}{\ln(30/20)} = 24.7°\text{C}$$

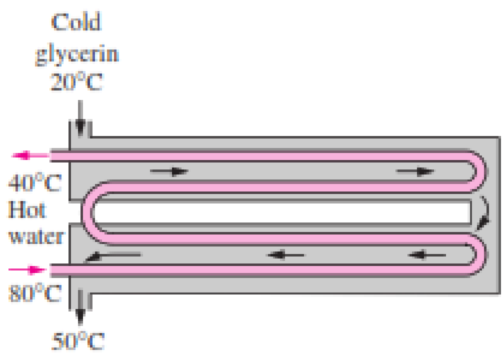


FIGURE 13–21
Schematic for Example 13–5.

and

$$\left.\begin{aligned} P &= \frac{t_2 - t_1}{T_1 - t_1} = \frac{40 - 80}{20 - 80} = 0.67 \\ R &= \frac{T_1 - T_2}{t_2 - t_1} = \frac{20 - 50}{40 - 80} = 0.75 \end{aligned}\right\} F = 0.91 \qquad \text{(Fig. 13–18}b\text{)}$$

(*a*) In the case of no fouling, the overall heat transfer coefficient U is determined from

$$U = \frac{1}{\dfrac{1}{h_i} + \dfrac{1}{h_o}} = \frac{1}{\dfrac{1}{160\ \text{W/m}^2 \cdot {}^\circ\text{C}} + \dfrac{1}{25\ \text{W/m}^2 \cdot {}^\circ\text{C}}} = 21.6\ \text{W/m}^2 \cdot {}^\circ\text{C}$$

Then the rate of heat transfer becomes

$$\dot{Q} = UA_s F\,\Delta T_{\text{lm},\,CF} = (21.6\ \text{W/m}^2 \cdot {}^\circ\text{C})(3.77\text{m}^2)(0.91)(24.7^\circ\text{C}) = \mathbf{1830\ W}$$

(*b*) When there is fouling on one of the surfaces, the overall heat transfer coefficient U is

$$U = \frac{1}{\dfrac{1}{h_i} + \dfrac{1}{h_o} + R_f} = \frac{1}{\dfrac{1}{160\ \text{W/m}^2 \cdot {}^\circ\text{C}} + \dfrac{1}{25\ \text{W/m}^2 \cdot {}^\circ\text{C}} + 0.0006\ \text{m}^2 \cdot {}^\circ\text{C/W}}$$
$$= 21.3\ \text{W/m}^2 \cdot {}^\circ\text{C}$$

The rate of heat transfer in this case becomes

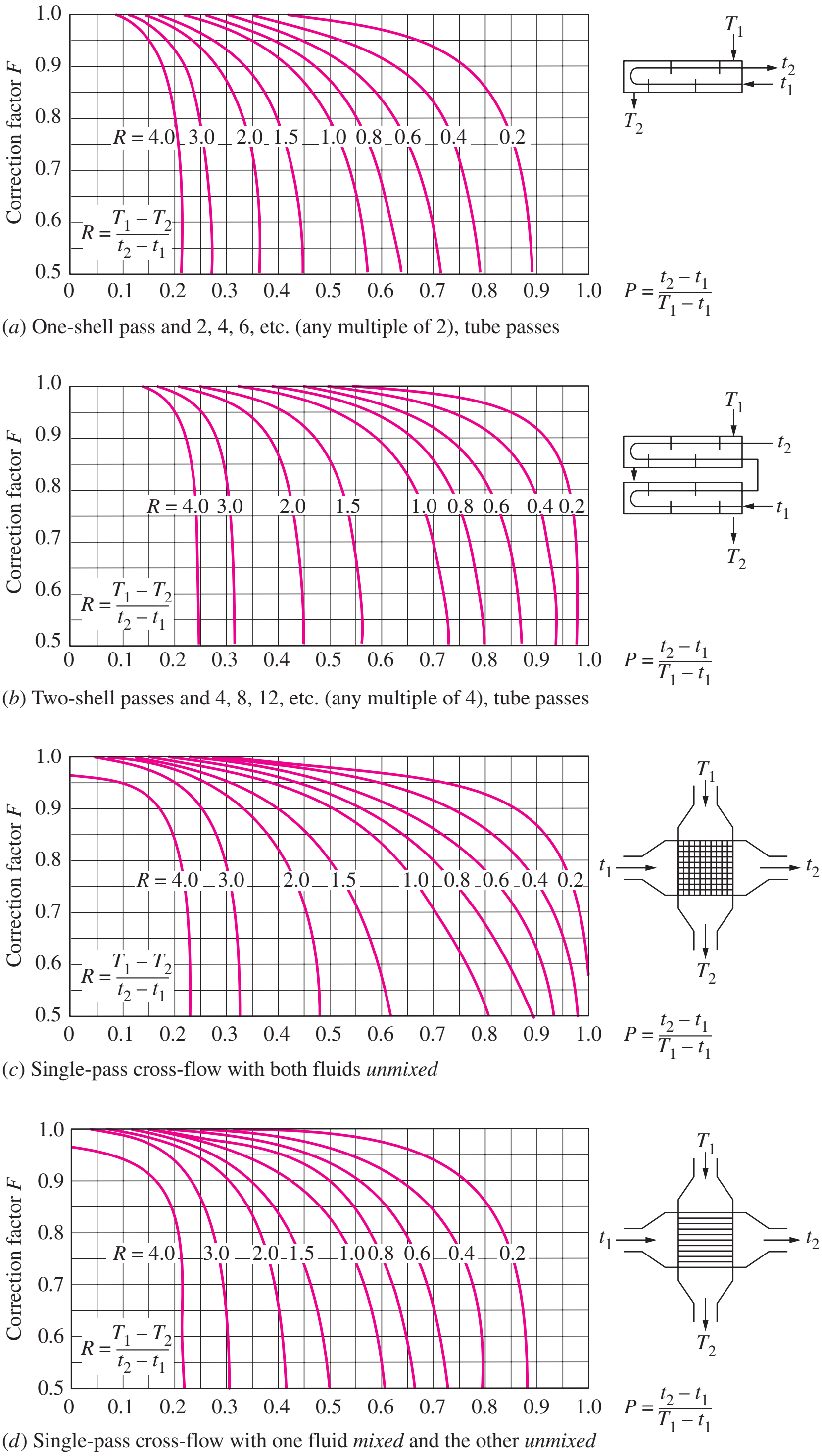
$$\dot{Q} = UA_s F\,\Delta T_{\text{lm},\,CF} = (21.3\ \text{W/m}^2 \cdot {}^\circ\text{C})(3.77\ \text{m}^2)(0.91)(24.7^\circ\text{C}) = \mathbf{1805\ W}$$

Discussion Note that the rate of heat transfer decreases as a result of fouling, as expected. The decrease is not dramatic, however, because of the relatively low convection heat transfer coefficients involved.

Remember that:

$$\frac{1}{UA_s} = \frac{1}{U_iA_i} = \frac{1}{U_oA_o} = R = \frac{1}{h_iA_i} + \frac{R_{f,i}}{A_i} + \frac{\ln(D_o/D_i)}{2\pi kL} + \frac{R_{f,o}}{A_o} + \frac{1}{h_oA_o}$$

where $A_i = \pi D_i L$ and $A_o = \pi D_o L$ are the areas of inner and outer surfaces, and $R_{f,i}$ and $R_{f,o}$ are the fouling factors at those surfaces.



(*a*) One-shell pass and 2, 4, 6, etc. (any multiple of 2), tube passes

(*b*) Two-shell passes and 4, 8, 12, etc. (any multiple of 4), tube passes

(*c*) Single-pass cross-flow with both fluids *unmixed*

(*d*) Single-pass cross-flow with one fluid *mixed* and the other *unmixed*

FIGURE 13–18
Correction factor F charts for common shell-and-tube and cross-flow heat exchangers (from Bowman, Mueller, and Nagle, Ref. 2).

Heat Exchanger Effectiveness (NTU method)

If more than one of the inlet and outlet temperature of the heat exchanger is unknown, LMTD may be obtained by trial and errors solution. Another approach introduce the definition of heat exchanger effectiveness (Є), which is a dimensionless with ranging between 0 to1.

$$Є = \frac{q_{act}}{q_{\max}}$$

Where, q_{max} is the maximum possible heat transfer for the exchanger. The maximum value could be attained if one of the fluids were to undergo a temperature change equal to the maximum temperature difference present in the exchanger, which is the difference in the entering temperatures for the hot and cold fluids.

Let C = mCp

$$q_{act} = C_h(Th_i - Th_o) = Cc(Tc_o - Tc_i)$$

The maximum possible heat transfer when the fluid of small C undergoes the maximum temperature difference available

$$q_{\max} = C_{\min}(Th_i - Tc_i)$$

$$q_{act} = \varepsilon\, C_{\min}(Th_i - Tc_i)$$

For parallel flow H.E with combining the last three equations, we get two expressions for effectiveness

$$Є = \frac{C_h(Th_i - Th_o)}{C_{\min}(Th_i - Tc_i)} = \frac{Cc(Tc_o - Tc_i)}{C_{\min}(Th_i - Tc_i)}$$

For $C_h < Cc$: $$Є_h = \frac{Th_i - Th_o}{Th_i - Tc_i}$$

$$\varepsilon = \frac{\Delta T\ (\text{min fluid})}{\Delta T\ (\text{max Hx})}$$

For $C_h > Cc$: $$Є_c = \frac{Tc_o - Tc_i}{Th_i - Tc_i}$$

Using the following equation,

$$\ln\frac{Th_o - Tc_o}{Th_i - Tc_i} = -UA\left(\frac{1}{C_h} + \frac{1}{Cc}\right)$$

We get

$$\epsilon = \frac{1 - \exp[(-UA/C_{\min})(1 + C_{\min}/C_{\max})]}{1 + C_{\min}/C_{\max}}$$

The terms *UA/Cmin* is called the number of transfer units (NTU) since it is indicative of the size of the heat exchanger, i.e

$$NTU = \frac{UA}{C_{\min}}$$

Note, in a boiler or condenser, *Cmin/Cmax* → 0 and all the heat-exchanger effectiveness relations approach a single simple equation,

$$\epsilon = 1 - e^{-\mathrm{NTU}}$$

$$\begin{aligned} C_{\min} &= \dot{m}_c c_c \text{ for a condenser (condensing fluid is } \textit{losing} \text{ heat)} \\ &= \dot{m}_h c_h \text{ for a boiler (boiling fluid is } \textit{gaining} \text{ heat)} \end{aligned}$$

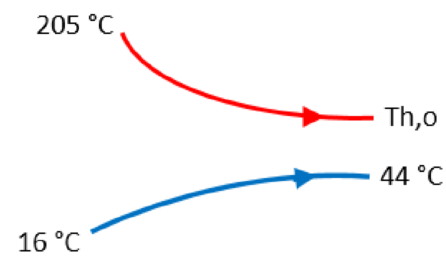
where
C= ˙mc is defined as the capacity rate.

Cmin is the smaller heat capacity ratio and Cmax is the larger one, and it makes no difference whether Cmin belongs to the hot or cold fluid.

Note that NTU is proportional to As . Therefore, for specified values of U and Cmin, the value of NTU is a measure of the heat transfer surface area As. Thus, the larger the NTU, the larger the heat exchanger.

Example: A double pipe parallel flow H.E. use oil (cp = 1.88 kJ/kg.K) at an initial temperature of 205°C to heat water, flowing at 225kg/hr from 16°C to 44°C. The oil flow rate is 270 kg/hr. a) what is the heat transfer area required for an overall heat transfer coefficient of 340 W/m^2.K. b) Determine the number of transfer unit (NTU). c) Calculate the effectiveness of the H.E.

Solution:



$(mcp\Delta T)_{oil} = (mcp\Delta T)_{water}$

$cp_{water} = 4.18kJ/kg.K$

$\therefore\ 270\times 1.88\times(205 - Th_o) = 225\times 4.18\times(44-16)$

$\Rightarrow Th_o = 153°\mathrm{C}$

$\therefore\ \Delta T_1 = 205-16 = 189°\mathrm{C}, \Delta T_2 = 153-44 = 109°\mathrm{C},$

$$\therefore\ \Delta TLM = \frac{189-109}{\ln\frac{189}{109}} = 145.4°\mathrm{C}$$

a) $A = q/U\cdot \Delta TLM = m_w cp_w \Delta T_w = 0.148m^2$

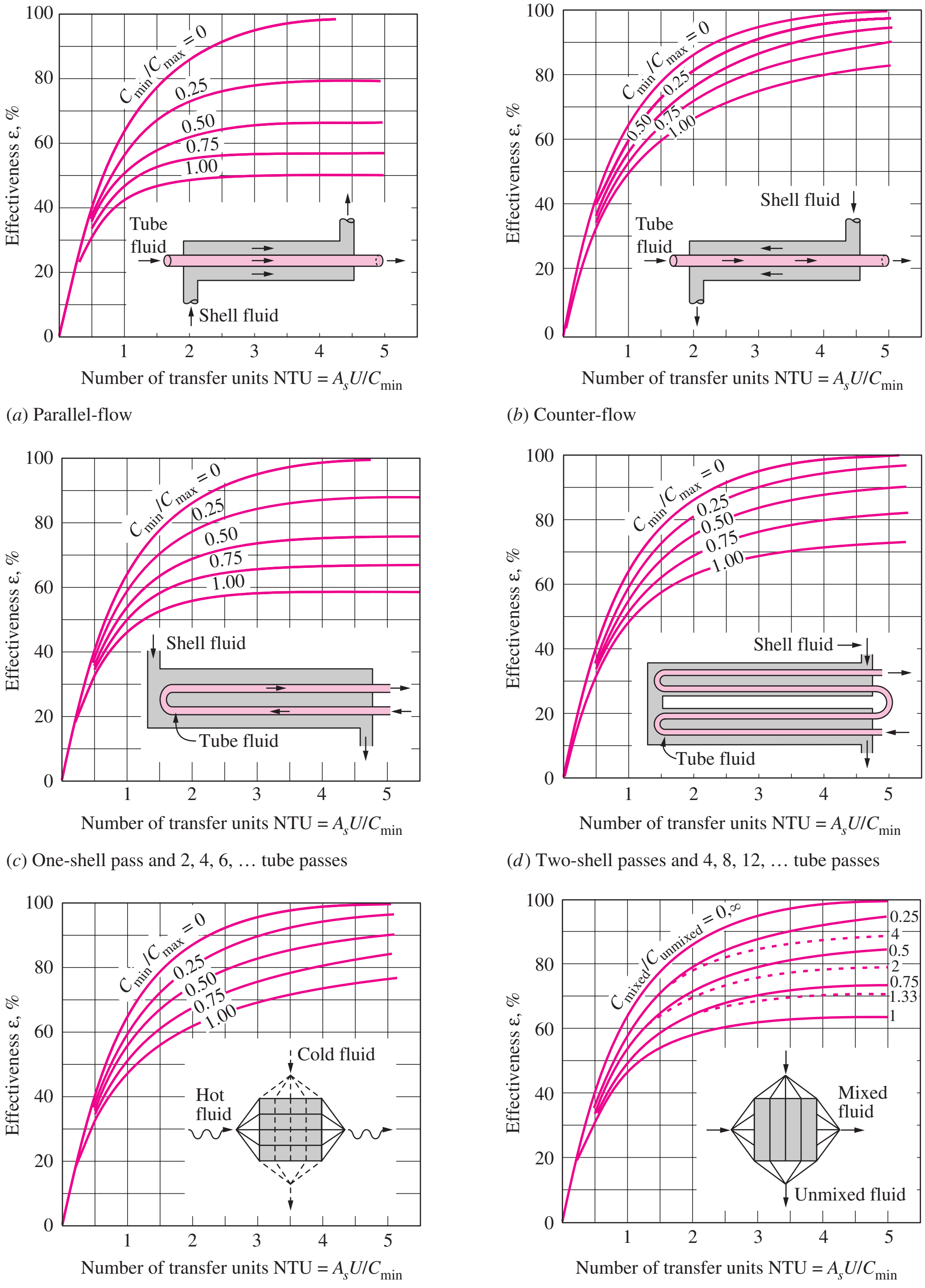
$b)\ (mcp)_{water} = 225\times 4.18 = 9.405\times 10^5\ J/hr.K,\ (mcp)_{oil} = 270\times 1.88 = 5.076\times 10^5\ J/hr.K$

$\therefore\ Cmin = 5.076\times 10^5\ J/hr.K = 141W/K$

$NTU = UA/C\min = 340\times 0.148/141 = 0.36$

c) $$\epsilon = \frac{1-\exp[(-UA/C_{\min})(1+C_{\min}/C_{\max})]}{1+C_{\min}/C_{\max}} = 28\%$$

try to use other methods to calculate the effectiveness



(*a*) Parallel-flow

(*b*) Counter-flow

(*c*) One-shell pass and 2, 4, 6, … tube passes

(*d*) Two-shell passes and 4, 8, 12, … tube passes

(*e*) Cross-flow with both fluids unmixed

(*f*) Cross-flow with one fluid mixed and the other unmixed

FIGURE 13–26

Effectiveness for heat exchangers (from Kays and London, Ref. 5).

(4) A stainless-steel rod (ρ=7817 kg/m3, C=460 J/kg. °C) 6.4 mm in diameter is initially at a uniform temperature of 150◦C and is suddenly immersed in a liquid at 25◦C with h = 120 W/m2°C. Using the lumped-capacity method of analysis, calculate the time necessary for the rod temperature to reach 120◦C.

Sol:

Given information:

$T = 120°\text{C} \quad \rho = 7817\ \text{kg/m}^3$
$T_\infty = 25\ °\text{C} \quad h = 120\text{W/m}^2 \cdot° \text{C}$
$T_i = 150°\text{C} \quad c = 460\text{J/kg} \cdot° \text{C}$

Lc=D/4=0.0064/4=0.0016 m

By using the equation of LHC system:

$\theta = \theta_i e^{-t/\tau}$

$$\tau = \frac{\rho c V}{h A s} = \frac{(7817)(460)(0.0016)}{120} = 47.94\ s$$

$\theta = \theta_i e^{-t/\tau}$

$\theta = T - T_\infty = 120 - 25 = 95\ °\text{C}$

$\theta_i = Ti - T_\infty = 150 - 25 = 125\ °\text{C}$

$95 = 125\ e^{-t/47.94} \Longrightarrow$

t=13.16 s

(11) A sphere has a thermal diffusivity of 9.5 ×10^{-7} m^2/s, a diameter of 2.5 cm, and a thermal conductivity of 1.52 W/m.◦C. The sphere is initially at a uniform temperature of 25 ◦C and is suddenly subjected to a convection environment at 200 ◦C. The convection heat-transfer coefficient is 110 W/m^2◦C. Calculate the temperatures at the center and at a radius of 6.4 mm after a time of 3 min. **Ans: T_o= 179 ◦C , $T_{(6.4)}$= 181.3◦C**

Given information:

k = 1.52 W/m · ◦C

α = 9.5 × 10^{-7} m^2/s

Ti=25◦C

T∞=200◦C

h = 110 W/m^2 · ◦C

t = 3 min = 180 s

r_0 = 1.25 cm=0.0125 m

r = 0.064 cm

Then:

To calculate (T_o):

$$\left.\begin{aligned} \frac{\alpha t}{r_0^2} &= \frac{(9.5\times10^{-7})(180)}{(0.0125)^2} = 1.094 \\ \frac{k}{hr_0} &= \frac{1.52}{(110)(0.0125)} = 1.105 \end{aligned}\right\}$$

From figure (7)

$$\frac{\theta_0}{\theta_i} = 0.12$$

$$\theta_0 = \frac{\theta_0}{\theta_i} * \theta_i = 0.12 * (25 - 200) = -21\ °C$$

θ_0= T_o-T∞→T_o=-21+200=179 $°C$

Now you complete the solution:

To calculate (T):

$$\left.\begin{aligned} \frac{r}{r_0} &= \text{—} = \\ \frac{k}{hr_0} &= \end{aligned}\right\}$$

From figure ( )

$$\frac{\theta}{\theta_o} =$$

$$\theta = \frac{\theta}{\theta_o} \times \theta o = \ldots\ldots \times \ldots\ldots \quad = \quad ℃$$

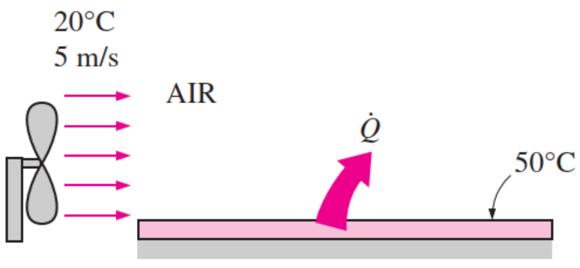
$$\theta = T - T_\infty \rightarrow T =$$

Principles of Convection

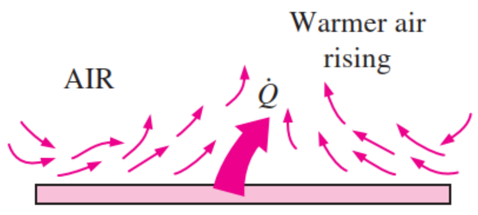
We have considered conduction, which is the mechanism of heat transfer through a solid or a quiescent fluid. We now consider convection, which is the mechanism of heat transfer through a fluid in the presence of fluid motion.

Conduction and convection are similar in that both mechanisms require the presence of a material medium. But they are different in that convection requires the presence of fluid motion. The higher the fluid velocity, the higher the rate of heat transfer.

Convection is classified as natural (or free) and forced convection, depending on how the fluid motion is initiated. In forced convection, the fluid is forced to flow over a surface or in a pipe by external means such as a pump or a fan. In natural convection, any fluid motion is caused by natural means such as the buoyancy effect, which manifests itself as the rise of warmer fluid and the fall of the cooler fluid.



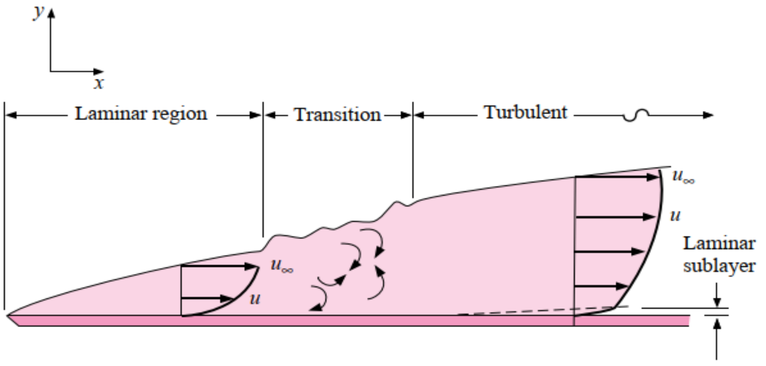
(*a*) Forced convection



(*b*) Free convection

VISCOUS FLOW

Figure 5-1 | Sketch showing different boundary-layer flow regimes on a flat plate.



- For a flow over a flat plate as shown in Figure above, different flow region develops by the influence of viscous forces.
- The viscous forces are described in terms of a shear stress τ between the fluid layers.
- The region of flow that develops from the leading edge of the plate in which the effects of viscosity are observed is called the ***boundary layer***.

The development of ***boundary layer*** from the leading edge passes three stages:

1- Laminar: flow of the fluid in smooth streamlines.

2- Transition: small disturbances in flow.

3- Turbulent: random flow which is characterized by eddies.

For flat plate the transition from laminar to turbulent flow occurs when:

$$\frac{u_\infty x}{v} = \frac{\rho u_\infty x}{\mu} > 5 \times 10^5$$

Where

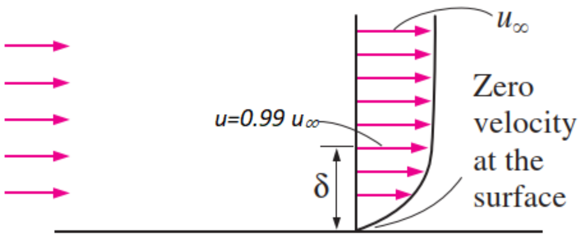
u_∞ = free-stream velocity, m/s

x = distance from leading edge, m

$v = \mu/\rho$ = kinematic viscosity, m^2/s

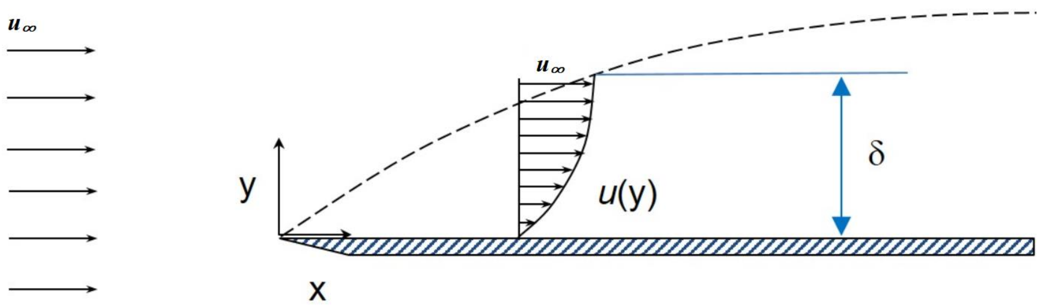
μ= dynamic viscosity, ($N.s/m^2$ or kg/m.s)

The boundary layer thickness, δ, is typically defined as the distance y from the surface to the position at which u= 0.99 u_∞.



This region of the flow above the plate bounded by δ in which the effects of the viscous shearing forces caused by fluid viscosity, it is called the velocity boundary layer.

Laminar Boundary Layer on a Flat Plate



Consider the boundary-layer flow system shown in Figure. The free-stream velocity outside the boundary layer is u_∞ , and the boundary-layer thickness is δ.

The thickness of the boundary layer at any distance on the flat plate can be estimated by using the following equation:

$$\frac{\delta}{x} = \frac{4.64}{\mathrm{Re}_x^{\frac{1}{2}}}$$

Where:

$$\mathrm{Re}_x = \frac{u_\infty x}{v} = \frac{\rho u_\infty x}{\mu}$$

Example: Air at 27◦C and 1 atm flows over a flat plate at a speed of 2 m/s. Calculate the boundary-layer thickness at distances of 20 cm and 40 cm from the leading edge of the plate. The viscosity of air at 27◦C is 1.85×10^{-5} kg/m.s.

Solution:

The density of air is calculated from

$$\rho = \frac{p}{RT} = \frac{1.0132 \times 10^5}{(287)(300)} = 1.177 \text{ kg/m}^3 \quad [0.073 \text{ lb}_m/\text{ft}^3]$$

The Reynolds number is calculated as

$$\text{At } x = 20 \text{ cm:} \qquad \text{Re} = \frac{(1.177)(2.0)(0.2)}{1.85 \times 10^{-5}} = 25{,}448$$

$$\text{At } x = 40 \text{ cm:} \qquad \text{Re} = \frac{(1.177)(2.0)(0.4)}{1.85 \times 10^{-5}} = 50{,}897$$

The boundary-layer thickness is calculated from Equation (5-21):

$$\text{At } x = 20 \text{ cm:} \qquad \delta = \frac{(4.64)(0.2)}{(25{,}448)^{1/2}} = 0.00582 \text{ m} \quad [0.24 \text{ in}]$$

$$\text{At } x = 40 \text{ cm:} \qquad \delta = \frac{(4.64)(0.4)}{(50{,}897)^{1/2}} = 0.00823 \text{ m} \quad [0.4 \text{ in}]$$

H.W: Air flows over a flat plate at a constant velocity of 20 m/s and ambient conditions of 1 atm and 60 °C. Calculate the boundary-layer thickness at a 3 cm distance from the leading edge of the plate.

Tables of Properties

Table A-5 | Properties of air at atmospheric pressure.†

The values of μ, k, c_p, and Pr are not strongly pressure-dependent and may be used over a fairly wide range of pressures

T, K	ρ kg/m^3	c_p kJ/kg · °C	$\mu \times 10^5$ kg/m · s	$\nu \times 10^6$ m^2/s	k W/m · °C	$\alpha \times 10^4$ m^2/s	Pr
100	3.6010	1.0266	0.6924	1.923	0.009246	0.02501	0.770
150	2.3675	1.0099	1.0283	4.343	0.013735	0.05745	0.753
200	1.7684	1.0061	1.3289	7.490	0.01809	0.10165	0.739
250	1.4128	1.0053	1.5990	11.31	0.02227	0.15675	0.722
300	1.1774	1.0057	1.8462	15.69	0.02624	0.22160	0.708
350	0.9980	1.0090	2.075	20.76	0.03003	0.2983	0.697
400	0.8826	1.0140	2.286	25.90	0.03365	0.3760	0.689
450	0.7833	1.0207	2.484	31.71	0.03707	0.4222	0.683
500	0.7048	1.0295	2.671	37.90	0.04038	0.5564	0.680
550	0.6423	1.0392	2.848	44.34	0.04360	0.6532	0.680
600	0.5879	1.0551	3.018	51.34	0.04659	0.7512	0.680
650	0.5430	1.0635	3.177	58.51	0.04953	0.8578	0.682
700	0.5030	1.0752	3.332	66.25	0.05230	0.9672	0.684
750	0.4709	1.0856	3.481	73.91	0.05509	1.0774	0.686
800	0.4405	1.0978	3.625	82.29	0.05779	1.1951	0.689
850	0.4149	1.1095	3.765	90.75	0.06028	1.3097	0.692
900	0.3925	1.1212	3.899	99.3	0.06279	1.4271	0.696
950	0.3716	1.1321	4.023	108.2	0.06525	1.5510	0.699
1000	0.3524	1.1417	4.152	117.8	0.06752	1.6779	0.702
1100	0.3204	1.160	4.44	138.6	0.0732	1.969	0.704
1200	0.2947	1.179	4.69	159.1	0.0782	2.251	0.707
1300	0.2707	1.197	4.93	182.1	0.0837	2.583	0.705
1400	0.2515	1.214	5.17	205.5	0.0891	2.920	0.705
1500	0.2355	1.230	5.40	229.1	0.0946	3.262	0.705
1600	0.2211	1.248	5.63	254.5	0.100	3.609	0.705
1700	0.2082	1.267	5.85	280.5	0.105	3.977	0.705
1800	0.1970	1.287	6.07	308.1	0.111	4.379	0.704
1900	0.1858	1.309	6.29	338.5	0.117	4.811	0.704
2000	0.1762	1.338	6.50	369.0	0.124	5.260	0.702
2100	0.1682	1.372	6.72	399.6	0.131	5.715	0.700
2200	0.1602	1.419	6.93	432.6	0.139	6.120	0.707
2300	0.1538	1.482	7.14	464.0	0.149	6.540	0.710
2400	0.1458	1.574	7.35	504.0	0.161	7.020	0.718
2500	0.1394	1.688	7.57	543.5	0.175	7.441	0.730

†From *Natl. Bur. Stand. (U.S.) Circ.* 564, 1955.

THE RELATION BETWEEN FLUID FRICTION AND HEAT TRANSFER

We have already seen that the temperature and flow fields are related. Now we seek an expression whereby the frictional resistance may be directly related to heat transfer.

The shear stress at the wall may be expressed in terms of a friction coefficient C_f:

$$\tau_w = C_f \frac{\rho u_\infty^2}{2}$$

The exact solution of the boundary-layer equations yields:

$$\frac{C_{fx}}{2} = 0.332\mathrm{Re}_x^{-1/2}$$

And:

$$\mathrm{Nu}_x = 0.332\mathrm{Pr}^{1/3}\mathrm{Re}_x^{1/2}$$

The above equation may be rewritten in the following form:

$$\frac{\mathrm{Nu}_x}{\mathrm{Re}_x\,\mathrm{Pr}} = \frac{h_x}{\rho c_p u_\infty} = 0.332\,\mathrm{Pr}^{-\frac{2}{3}}\,\mathrm{Re}_x^{-\frac{1}{2}}$$

The group on the left is called the **Stanton number**,

$$\mathrm{St}_x = \frac{h_x}{\rho c_p u_\infty}$$

so that

$$\mathrm{St}_x\mathrm{Pr}^{\frac{2}{3}} = 0.332\mathrm{Re}_x^{-\frac{1}{2}}$$

And

$$\mathrm{St}_x\mathrm{Pr}^{2/3} = \frac{C_{fx}}{2}$$

The last equation called the Reynolds-Colburn analogy, expresses the relation between fluid friction and heat transfer for laminar flow on a flat plate.

Example: For the flow system in the previous example which includes: Air at 27°C and 1 atm flow over a flat plate at a speed of 2 m/s. Assume that the plate is heated over its entire length to a temperature of 60◦C. compute the drag force exerted on the first 40 cm of the plate using the analogy between fluid friction and heat transfer. Assume unit depth in the z direction.

Solution:

All fluid properties are calculated at the film temperature:

$$T_f = \frac{27+60}{2} = 43.5°\text{C} = 316.5\text{K}$$

We use Equation (5-56) to compute the friction coefficient and then calculate the drag force. An average friction coefficient is desired, so

$$\overline{\text{St}}\,\text{Pr}^{2/3} = \frac{\overline{C}_f}{2} \qquad \textbf{[}\boldsymbol{a}\textbf{]}$$

The density at 316.5 K is

$$\rho = \frac{p}{RT} = \frac{1.0132\times10^5}{(287)(316.5)} = 1.115\ \text{kg/m}^3$$

For the 40-cm length

$$\overline{\text{St}} = \frac{\bar{h}}{\rho c_p u_\infty} = \frac{8.698}{(1.115)(1006)(2)} = 3.88\times10^{-3}$$

Then from Equation (*a*)

$$\frac{\overline{C}_f}{2} = (3.88\times10^{-3})(0.7)^{2/3} = 3.06\times10^{-3}$$

The average shear stress at the wall is computed from Equation (5-52):

$$\begin{aligned}\bar{\tau}_w &= \overline{C}_f\,\rho\frac{u_\infty^2}{2}\\ &= (3.06\times10^{-3})(1.115)(2)^2\\ &= 0.0136\ \text{N/m}^2\end{aligned}$$

The drag force is the product of this shear stress and the area,

$$D = (0.0136)(0.4) = 5.44\ \text{mN} \quad [1.23\times10^{-3}\ \text{lb}_f]$$

H.W

1) Air at 90◦C and 1 atm flows over a flat plate at a velocity of 30 m/s. How thick is the boundary layer at a distance of 2.5 cm from the leading edge of the plate?

   Ans: 6.21×10^{-4} m

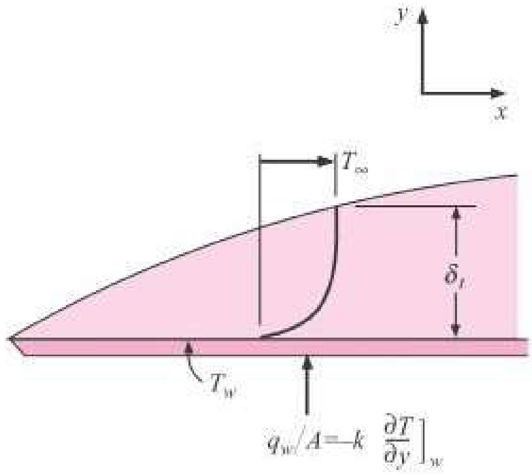
2) Air at 1 atm and 30◦C flows over a 15-cm-square plate at a velocity of 10 m/s. Calculate the maximum boundary layer thickness.

   Ans: 0.00225 m

3) Helium at 1 atm and 300 K is used to cool a l-m-square plate maintained at 500 K. The flow velocity is 50 m/s. Calculate the total heat loss from the plate. What is the boundary-layer thickness as the flow leaves the plate?

   Ans: 10.52 kW, 0.0094 m

The thermal Boundary Layer



A thermal boundary layer is defined as that region where temperature gradients are present in the flow. These temperature gradients would result from a heat-exchange process between the fluid and the wall.

The temperature of the wall is Tw, the temperature of the fluid outside the thermal boundary layer is $T\infty$, and the thickness of the thermal boundary layer is designated as δt. At the wall, the velocity is zero, and the heat transfer into the fluid takes place by conduction. Thus the local heat flux per unit area, q'', is

$$\frac{q}{A} = q'' = -k \left.\frac{\partial T}{\partial y}\right]_{\text{wall}}$$

From Newton's law of cooling

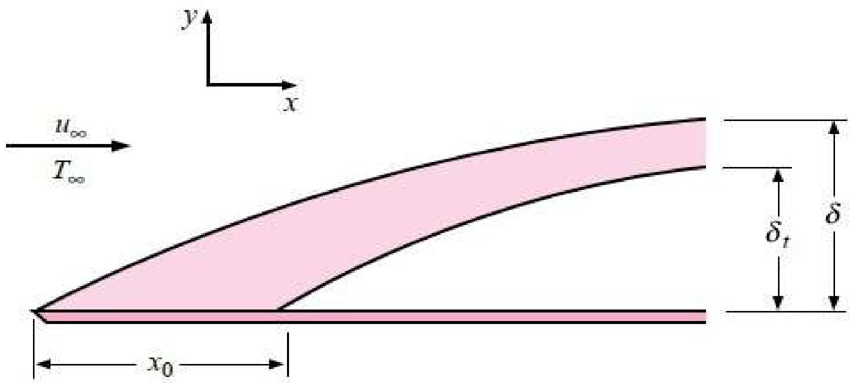
$$q'' = h(T_w - T_\infty)$$

Where h is the convection heat-transfer coefficient. Combining these equations, we have

$$h = \frac{-k(\partial T/\partial y)_{\text{wall}}}{T_w - T_\infty}$$

$\left.\frac{\partial T}{\partial y}\right]_{\text{wall}}$: is the temperature gradient at the wall thickness of thermal boundary layer.

Hydrodynamic and thermal boundary layers on a flat plate. Heating starts at $x = x_0$.



The thickness boundary layer can be calculated by using the following equation

$$\frac{\delta_t}{\delta} = \frac{1}{1.026}\mathrm{Pr}^{-1/3}\left[1-\left(\frac{x_0}{x}\right)^{3/4}\right]^{1/3}$$

Where
Pr = Prandtl number

$$\mathrm{Pr} = \frac{\nu}{\alpha} = \frac{\mu/\rho}{k/\rho c_p} = \frac{c_p\mu}{k}$$

x_0= is the distance from the leading edge where the heating begins.

When the plate is heated over the entire length, $x_o = 0$, then

$$\frac{\delta_t}{\delta} = \frac{1}{1.026}\mathrm{Pr}^{-1/3}$$

Example: Calculate the ratio of thermal boundary layer thickness to hydrodynamic boundary layer thickness for the following liquids:

Air at 100 KPa and 20 °C; water at 20°C; liquid ammonia at 20 °C.

Solution:

By applying this eq:

$$\frac{\delta_t}{\delta} = \frac{1}{1.026}\mathrm{Pr}^{-1/3}$$

And by extracting Pr the from fluid properties tables:

Subs	Pr	$\frac{\delta_t}{\delta}$
Air	0.709	1.093
H_2O	7	0.509
NH_3	0.887	1.014

Example: For air at 15°C flowing over a flat plate at a free stream velocity of 6 m/s. Determine the velocity boundary layer and thermal boundary layer thickness at a distance of 0.5 m from the leading edge.

Solution:

$$Re = \frac{\rho u_\infty x}{\mu} = \frac{(1.23)(6)(0.5)}{1.8 \times 10^{-5}} = 205{,}000$$

$$\delta = \frac{(4.64)(0.5)}{(205{,}000)^{1/2}} = 0.005124 \text{ m}$$

Pr = 0.711 from Table A-5

Then:

$$\delta_t = \frac{\delta}{1.026}\mathrm{Pr}^{-1/3} = \frac{0.005124}{1.026}(0.711)^{-1/3} = 0.005595\, m$$

Table A-6 | Properties of gases at atmospheric pressure† (*Continued*).

Values of μ, k, c_p, and Pr are not strongly pressure-dependent for He, H_2, O_2, and N_2 and may be used over a fairly wide range of pressures

T, K	ρ kg/m^3	c_p kJ/kg · °C	μ, kg/m · s	ν, m^2/s	k W/m · °C	α, m^2/s	Pr
Carbon dioxide							
220	2.4733	0.783	11.105×10^{-6}	4.490×10^{-6}	0.010805	0.05920×10^{-4}	0.818
250	2.1657	0.804	12.590	5.813	0.012884	0.07401	0.793
300	1.7973	0.871	14.958	8.321	0.016572	0.10588	0.770
350	1.5362	0.900	17.205	11.19	0.02047	0.14808	0.755
400	1.3424	0.942	19.32	14.39	0.02461	0.19463	0.738
450	1.1918	0.980	21.34	17.90	0.02897	0.24813	0.721
500	1.0732	1.013	23.26	21.67	0.03352	0.3084	0.702
550	0.9739	1.047	25.08	25.74	0.03821	0.3750	0.685
600	0.8938	1.076	26.83	30.02	0.04311	0.4483	0.668
Ammonia, NH_3							
273	0.7929	2.177	9.353×10^{-6}	1.18×10^{-5}	0.0220	0.1308×10^{-4}	0.90
323	0.6487	2.177	11.035	1.70	0.0270	0.1920	0.88
373	0.5590	2.236	12.886	2.30	0.0327	0.2619	0.87
423	0.4934	2.315	14.672	2.97	0.0391	0.3432	0.87
473	0.4405	2.395	16.49	3.74	0.0467	0.4421	0.84
Water vapor							
380	0.5863	2.060	12.71×10^{-6}	2.16×10^{-5}	0.0246	0.2036×10^{-4}	1.060
400	0.5542	2.014	13.44	2.42	0.0261	0.2338	1.040
450	0.4902	1.980	15.25	3.11	0.0299	0.307	1.010
500	0.4405	1.985	17.04	3.86	0.0339	0.387	0.996
550	0.4005	1.997	18.84	4.70	0.0379	0.475	0.991
600	0.3652	2.026	20.67	5.66	0.0422	0.573	0.986
650	0.3380	2.056	22.47	6.64	0.0464	0.666	0.995
700	0.3140	2.085	24.26	7.72	0.0505	0.772	1.000
750	0.2931	2.119	26.04	8.88	0.0549	0.883	1.005
800	0.2739	2.152	27.86	10.20	0.0592	1.001	1.010
850	0.2579	2.186	29.69	11.52	0.0637	1.130	1.019

†Adapted to SI units from E. R. G. Eckert and R. M. Drake, *Heat and Mass Transfer,* 2nd ed. New York: McGraw-Hill, 1959.

Table A-9 | Properties of water (saturated liquid).†

Note: $\mathrm{Gr}_x \mathrm{Pr} = \left(\frac{g\beta\rho^2 c_p}{\mu k}\right) x^3 \Delta T$

°F	°C	c_p kJ/kg · °C	ρ kg/m^3	μ kg/m · s	k W/m · °C	Pr	$\frac{g\beta\rho^2 c_p}{\mu k}$ 1/m^3 · °C
32	0	4.225	999.8	1.79×10^{-3}	0.566	13.25	
40	4.44	4.208	999.8	1.55	0.575	11.35	1.91×10^{9}
50	10	4.195	999.2	1.31	0.585	9.40	6.34×10^{9}
60	15.56	4.186	998.6	1.12	0.595	7.88	1.08×10^{10}
70	21.11	4.179	997.4	9.8×10^{-4}	0.604	6.78	1.46×10^{10}
80	26.67	4.179	995.8	8.6	0.614	5.85	1.91×10^{10}
90	32.22	4.174	994.9	7.65	0.623	5.12	2.48×10^{10}
100	37.78	4.174	993.0	6.82	0.630	4.53	3.3×10^{10}
110	43.33	4.174	990.6	6.16	0.637	4.04	4.19×10^{10}
120	48.89	4.174	988.8	5.62	0.644	3.64	4.89×10^{10}
130	54.44	4.179	985.7	5.13	0.649	3.30	5.66×10^{10}
140	60	4.179	983.3	4.71	0.654	3.01	6.48×10^{10}
150	65.55	4.183	980.3	4.3	0.659	2.73	7.62×10^{10}
160	71.11	4.186	977.3	4.01	0.665	2.53	8.84×10^{10}
170	76.67	4.191	973.7	3.72	0.668	2.33	9.85×10^{10}
180	82.22	4.195	970.2	3.47	0.673	2.16	1.09×10^{11}
190	87.78	4.199	966.7	3.27	0.675	2.03	
200	93.33	4.204	963.2	3.06	0.678	1.90	
220	104.4	4.216	955.1	2.67	0.684	1.66	
240	115.6	4.229	946.7	2.44	0.685	1.51	
260	126.7	4.250	937.2	2.19	0.685	1.36	
280	137.8	4.271	928.1	1.98	0.685	1.24	
300	148.9	4.296	918.0	1.86	0.684	1.17	
350	176.7	4.371	890.4	1.57	0.677	1.02	
400	204.4	4.467	859.4	1.36	0.665	1.00	
450	232.2	4.585	825.7	1.20	0.646	0.85	
500	260	4.731	785.2	1.07	0.616	0.83	
550	287.7	5.024	735.5	9.51×10^{-5}			
600	315.6	5.703	678.7	8.68			

†Adapted to SI units from A. I. Brown and S. M. Marco, *Introduction to Heat Transfer,* 3rd ed. New York: McGraw-Hill, 1958.

Heat Transfer Coefficient

The heat transfer coefficient at any x position on the flat plate can be found by the equation

$$h_x = 0.332k\,\mathrm{Pr}^{1/3}\left(\frac{u_\infty}{\nu x}\right)^{1/2}\left[1-\left(\frac{x_0}{x}\right)^{3/4}\right]^{-1/3}$$

The last equation can be written in dimensionless form as

$$\mathrm{Nu}_x = 0.332\mathrm{Pr}^{1/3}\,\mathrm{Re}_x^{1/2}\left[1-\left(\frac{x_0}{x}\right)^{3/4}\right]^{-1/3}$$

For a plate heated over its entire length, $x_0 = 0$ and

$$\mathrm{Nu}_x = 0.332\mathrm{Pr}^{1/3}\,\mathrm{Re}_x^{1/2}$$

Where

Nu_x= Nusselt number = $\dfrac{h_x\, x}{k}$

The above equation expresses the local values of the heat-transfer coefficient in terms of the distance from the leading edge of the plate and the fluid properties. For the case where x0 = 0 the average heat-transfer coefficient and Nusselt number may be obtained by integrating over the length of the plate:

$$\bar{h} = 2h_x$$

Where

$\bar{h}$ is the average heat transfer coefficient.

$\therefore$

$$\overline{\mathrm{Nu}}_L = \frac{\bar{h}L}{k} = 2\,\mathrm{Nu}_x$$

or

$$\overline{\mathrm{Nu}}_L = \frac{\bar{h}L}{k} = 0.664\,\mathrm{Re}_L^{1/2}\mathrm{Pr}^{1/3}$$

where

$$\mathrm{Re}_L = \frac{\rho u_\infty L}{\mu}$$

The above relations are used for laminar flow constant wall temperature. Note that, all the physical properties are evaluated at film temperature T_f, which is

$$T_f = \frac{T_w + T_\infty}{2}$$

Example: Air at 27◦C and 1 atm flows over a flat plate at a speed of 2 m/s, assume that the plate is heated over its entire length to a temperature of 60 ◦ C. Calculate the heat transferred in (a) the first 20 cm of the plate and (b) the first 40 cm of the plate.

Solution:

$$T_f = \frac{27+60}{2} = 43.5°\text{C} = 316.5\ \text{K} \quad [110.3°\text{F}]$$

From Appendix A the properties are

$$\nu = 17.36 \times 10^{-6}\ \text{m}^2/\text{s} \quad [1.87 \times 10^{-4}\ \text{ft}^2/\text{s}]$$
$$k = 0.02749\ \text{W/m} \cdot °\text{C} \quad [0.0159\ \text{Btu/h} \cdot \text{ft} \cdot °\text{F}]$$
$$\text{Pr} = 0.7$$
$$c_p = 1.006\ \text{kJ/kg} \cdot °\text{C} \quad [0.24\ \text{Btu/lb}_m \cdot °\text{F}]$$

At $x = 20$ cm

$$\text{Re}_x = \frac{u_\infty x}{\nu} = \frac{(2)(0.2)}{17.36 \times 10^{-6}} = 23{,}041$$
$$\text{Nu}_x = \frac{h_x x}{k} = 0.332\text{Re}_x^{1/2}\text{Pr}^{1/3}$$
$$= (0.332)(23{,}041)^{1/2}(0.7)^{1/3} = 44.74$$
$$h_x = \text{Nu}_x\left(\frac{k}{x}\right) = \frac{(44.74)(0.02749)}{0.2}$$
$$= 6.15\ \text{W/m}^2 \cdot °\text{C} \quad [1.083\ \text{Btu/h} \cdot \text{ft}^2 \cdot °\text{F}]$$

The average value of the heat-transfer coefficient is twice this value, or

$$\bar{h} = (2)(6.15) = 12.3\ \text{W/m}^2 \cdot °\text{C} \quad [2.17\ \text{Btu/h} \cdot \text{ft}^2 \cdot °\text{F}]$$

The heat flow is

$$q = \bar{h}A(T_w - T_\infty)$$

If we assume unit depth in the z direction,

$$q = (12.3)(0.2)(60-27) = 81.18\ \text{W} \quad [277\ \text{Btu/h}]$$

At $x = 40$ cm

$$\text{Re}_x = \frac{u_\infty x}{\nu} = \frac{(2)(0.4)}{17.36 \times 10^{-6}} = 46{,}082$$
$$\text{Nu}_x = (0.332)(46{,}082)^{1/2}(0.7)^{1/3} = 63.28$$
$$h_x = \frac{(63.28)(0.02749)}{0.4} = 4.349\ \text{W/m}^2 \cdot °\text{C}$$
$$\bar{h} = (2)(4.349) = 8.698\ \text{W/m}^2 \cdot °\text{C} \quad [1.53\ \text{Btu/h} \cdot \text{ft}^2 \cdot °\text{F}]$$
$$q = (8.698)(0.4)(60-27) = 114.8\ \text{W} \quad [392\ \text{Btu/h}]$$

Heisler Charts

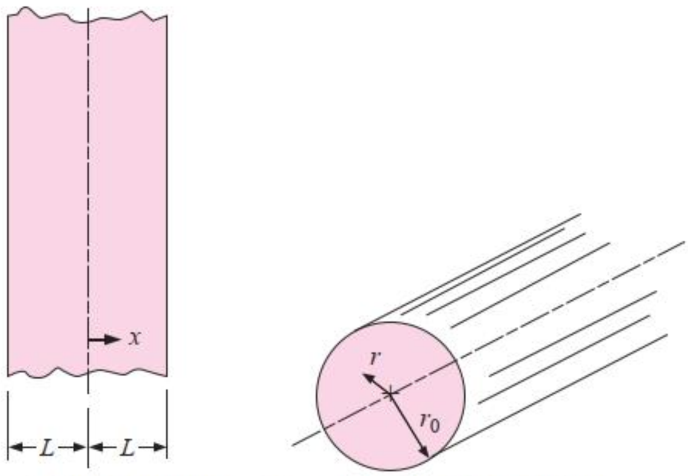
A group of curves are used with unsteady-state case when Biot no. is greater than 0.1. The most cases that to be treated are:

1- Infinite plate (plate where thickness is very small in comparison to other dimension).

2- Infinite cylinder (where the diameter is very small compared to length)

3- Sphere.

Nomenclature for one-dimensional solids suddenly subjected to convection environment at T_∞: (*a*) infinite plate of thickness $2L$; (*b*) infinite cylinder of radius r_0; (*c*) sphere of radius r_0.



T_0 = centerline temperature
(*a*)

T_0 = centerline axis temperature
(*b*)

T_0 = center temperature
(*c*)

In these charts we note the definitions:

$$\mathrm{T\infty} = \text{Environment temperature}$$
$$\mathrm{Ti} = \text{Initial temperature of the solid}(\mathrm{t} = 0)$$
$$\theta = T(x,t) - T_\infty \text{or} T(r,t) - T_\infty$$
$$\theta_i = T_i - T_\infty$$
$$\theta_0 = T_0 - T_\infty$$

<u>There are three charts associated with each geometry:</u>

The first chart is to determine the temperature T_o at the center of the geometry at a given time (t).

$$\frac{\theta_0}{\theta_i} = (T_0 - T_\infty)/(T_i - T_\infty)$$

The second chart is to determine the temperature at other locations at the same time in terms of T_o.

$$\theta/\theta_0 = (T - T_\infty)/(T_0 - T_\infty)$$

The third chart is to determine the total amount of heat transfer up to the time t (Q/Q_o).

Where:

$$Q_0 = \rho c V(T_i - T_\infty) = \rho c V \theta_i$$

- If a centreline temperature (T_o) is desired, only one chart is required to obtain a value for θ_0 and then T_0, i.e Chart (1) for infinite plate, Chart (4) for infinite cylinder and Chart (7) for sphere.
- To determine an off-center temperature, two charts are required to calculate the product:

$$\frac{\theta}{\theta_i} = \frac{\theta_0}{\theta_i}\frac{\theta}{\theta_0}$$

For example, Charts (1) and (2) would be employed to calculate an off-center temperature for an infinite plate.

-The heat losses for the infinite plate, infinite cylinder, and sphere are given in Charts 3, 6 and 9, respectively, where Q_0 represents the initial internal energy content of the body in reference to the environment temperature, and Q is the actual heat lost by the body in time t.

Note that: (x,r) is measured from the center towards the surface.

The Biot and Fourier Numbers

The temperature profiles and heat flows may all be expressed in terms of two dimensionless parameters called the Biot and Fourier numbers:

$$\text{Biot number} \qquad = \text{Bi} = \frac{hs}{k}$$

$$\text{Fourier number} \quad = \text{Fo} = \frac{\alpha t}{s^2} = \frac{kt}{\rho c s^2}$$

In these parameters ***s*** for the plate it is the half-thickness, whereas for the cylinder and sphere it is the radius.

The value of Fo no. means that the period of time is required to heat or cool the body. A low value of (Fo) indicates that a long period of time is required to cool or heat the body.

Applicability of the Heisler Charts

The Heisler charts is applicable in the case of Fo no. is greater than 0.2.

$$(\text{Fo}) = \frac{\alpha t}{s^2} > 0.2$$

Example: A large plate of aluminum (k = 215 W/m · ◦C) 5.0 cm thick and initially at 200◦C is suddenly exposed to the convection environment of 70◦C with a heat-transfer coefficient of 525 W/m^2 · ◦C. Calculate the temperature at a depth of 1.25 cm from one of the faces 1 min after the plate has been exposed to the environment. How much energy has been removed per unit area from the plate in this time?

$\alpha = 8.4 \times 10^{-5}$ m^2/s, ρ=2700 kg/m3, c_p=900 j/kg.°C

Solution:

The Heisler charts of Figures (1) and (2) may be used for solution of this problem. We first calculate the center temperature of the plate, using Figure (1), and then use Figure (2) to calculate the temperature at the specified x position.

Given information:

k = 215 W/m · ◦C

ρ=2700 kg/m3

c_p=900 j/kg.°C

$\alpha = 8.4 \times 10{-}5\ m^2/s$

Ti=200◦C

T∞=70◦C

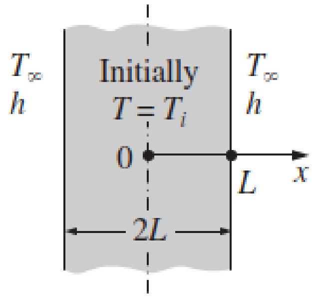
$h = 525\ W/m^2 \cdot {\circ}C$

2L = 5.0 cm then L = 2.5 cm =0.025 m

x = 2.5 − 1.25 = 1.25 cm

t = 1 min = 60 s

Then



$$\frac{\alpha t}{L^2} = \frac{(8.4 \times 10^{-5})(60)}{(0.025)^2} = 8.064$$

$$\frac{k}{hL} = \frac{215}{(525)(0.025)} = 16.38$$

From figure (1)

$$\frac{\theta_0}{\theta_i} = 0.61$$

$$\frac{x}{L} = \frac{1.25}{2.5} = 0.5$$

$$\frac{k}{hL} = 16.38$$

From figure (2)

$$\frac{\theta}{\theta_0} = 0.98$$

$$\frac{\theta}{\theta_i} = \frac{\theta_0}{\theta_i} \times \frac{\theta}{\theta_0} = 0.61 * 0.98 = 0.5918$$

$$\frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty} = \frac{T - 70}{200 - 70} \rightarrow \boldsymbol{T = 147.7\ {}^\circ C}$$

To find the amount of heat transfer by using Figure (3):

$$\frac{h^2\alpha t}{k^2} = \frac{(525)^2(8.4\times 10^{-5})(60)}{(215)^2} = 0.03$$

$$\frac{hL}{k} = 0.061$$

From figure (3)

$$\frac{Q}{Q_0} = 0.41$$

$$Q_0 = mc\theta_i = \rho V c\theta_i = \rho A(2L)c\theta_i$$

For unit area:

$$\frac{Q_0}{A} = \frac{\rho c A(2L)\theta_i}{A} = \rho c(2L)\theta_i$$
$$= (2700)(900)(0.05)(130)$$
$$= 15.8\times 10^6 \mathrm{J/m^2}$$

so that the heat removed per unit surface area is:

$$\frac{Q}{A} = (15.8\times 10^6)(0.41) = \mathbf{6.48\times 10^6\,\frac{J}{m^2}}$$

Example: A long aluminum cylinder (k = 215 W/m · ◦C) 5.0 cm in diameter and initially at 200◦C is suddenly exposed to a convection environment at 70◦C and h = 525 W/m^2 · ◦C. Calculate the centreline temperature; and temperature at a radius of 1.25 cm and the heat lost per unit length 1 min after the cylinder is exposed to the environment.

$\alpha = 8.4 \times 10{-}5\ m^2/s$, ρ=2700 kg/m^3, c_p=900 j/kg.°C

Solution:

Given information:

k = 215 W/m · ◦C

ρ=2700 kg/m3

c_p=900 j/kg.°C

$\alpha = 8.4 \times 10{-}5\ m^2/s$

Ti=200◦C

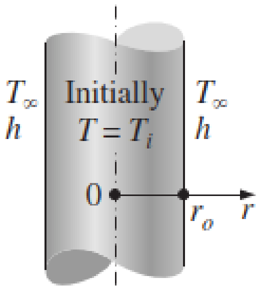
$T_\infty = 70 \circ C$

$h = 525\ W/m^2 \cdot \circ C$

$t = 1\ min = 60\ s$

$r_0 = 2.5\ cm = 0.025\ m$

$r = 1.25\ cm$



Then:

To calculate (T_o):

$$\frac{\alpha t}{r_0^2} = \frac{(8.4 \times 10^{-5})(60)}{(0.025)^2} = 8.064$$

$$\frac{k}{hr_0} = \frac{215}{(525)(0.025)} = 16.38$$

From figure (4)

$$\frac{\theta_0}{\theta_i} = 0.38$$

$$\theta_0 = \frac{\theta_0}{\theta_i} * \theta_i = 0.38(200 - 70) = 49.4\ °C$$

$\theta_0 = T_o - T_\infty \rightarrow T_o = 49.4 + 70 = 119.4\ °C$

To calculate (T):

$$\frac{r}{r_0} = \frac{1.25}{2.5} = 0.5$$

$$\frac{k}{hr_0} = 16.38$$

From figure (5)

$$\frac{\theta}{\theta_o} = 0.98$$

$$\theta = 0.98 \times \theta o = 0.98 \times 49.4 = 48.4\ ℃$$

$$\theta = T - T_\infty \rightarrow T = 70 + 48.4 = 118.4°C$$

To compute the heat lost:

$$\frac{h^2 \alpha t}{k^2} = \frac{(525)^2(8.4 \times 10^{-5})(60)}{(215)^2} = 0.03$$

$$\frac{hr_0}{k} = \frac{(525)(0.025)}{215} = 0.061$$

From figure (6)

$$\frac{Q}{Q_0} = 0.65$$

$Q_0 = mc\theta_i = \rho Vc\theta_i = \rho cL\pi r_0^2\theta_i$

For unit length:

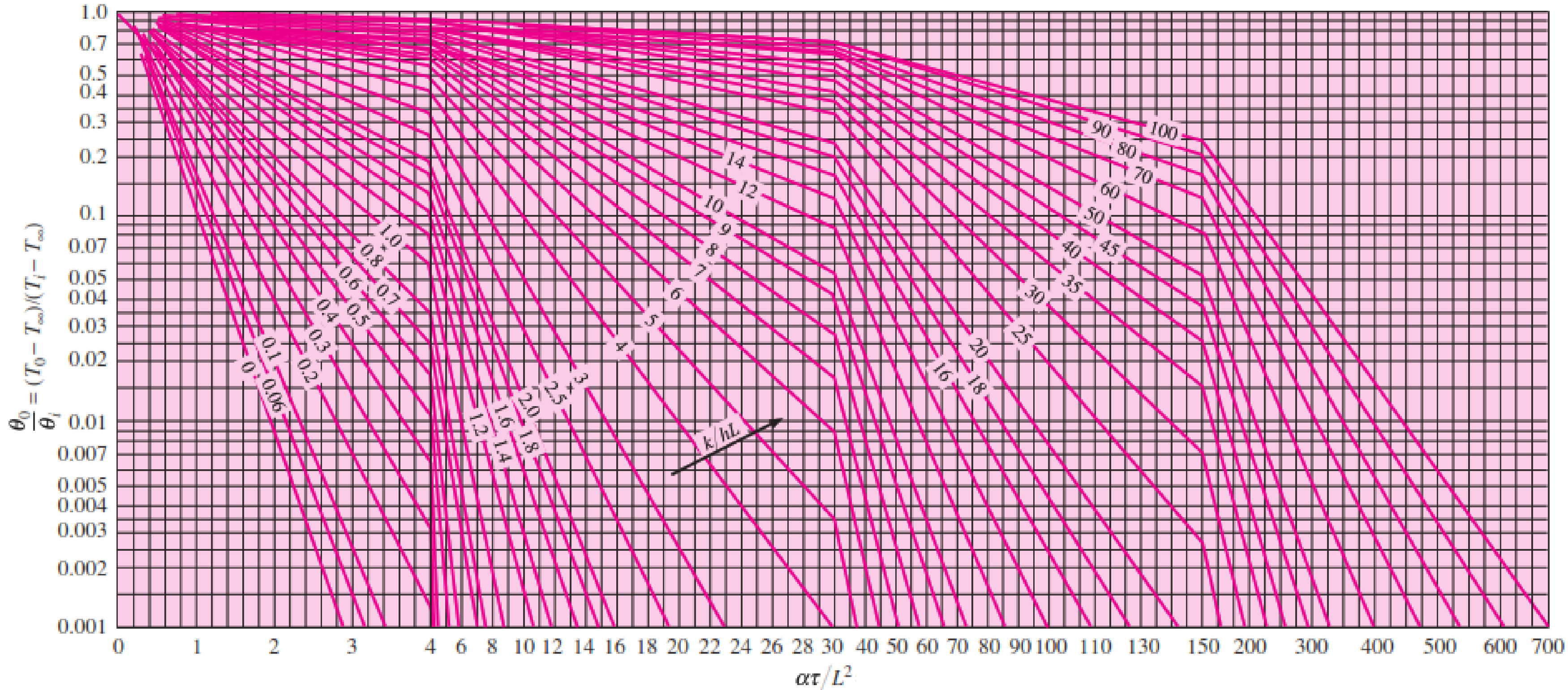
$$\frac{Q_0}{L} = \frac{\rho cL\pi r_0^2\theta_i}{L} = \rho c\pi r_0^2\theta_i = (2700)(900)\pi(0.025)^2(130) = 6.203 \times 10^5 \mathrm{J/m}$$

and the actual heat lost per unit length is:

$$\frac{Q}{L} = (6.203 \times 10^5)(0.65) = 4.032 \times 10^5 \mathrm{J/m}$$

Chart (1)

Midplane temperature for an infinite plate of thickness $2L$: (a) full scale.



Temperature as a function of center temperature in an infinite plate of thickness $2L$, from Reference 2.

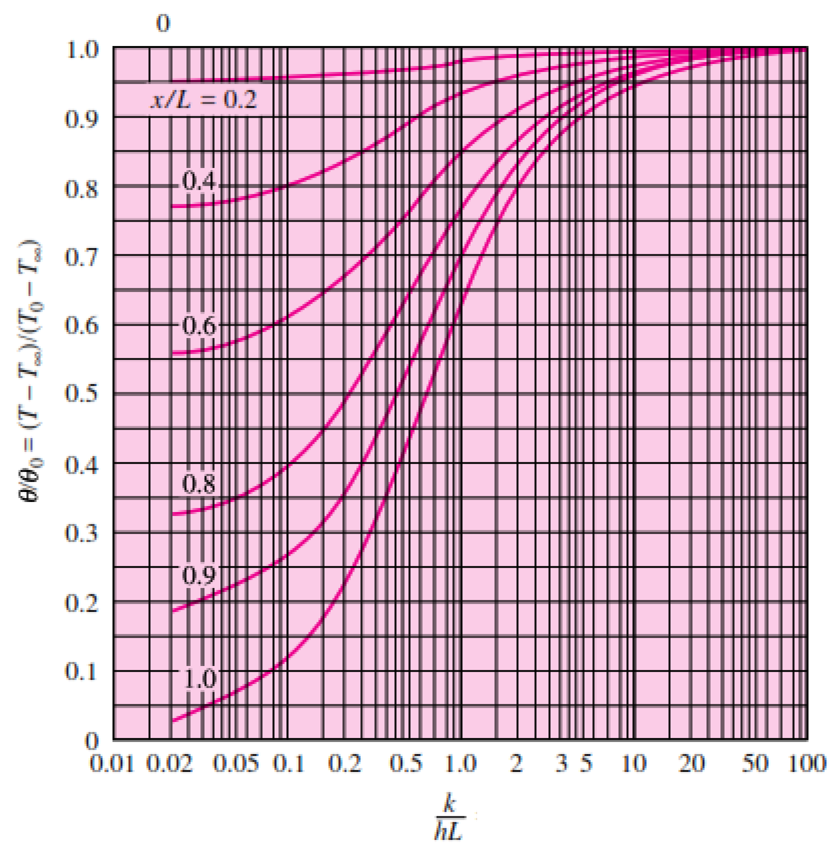


Chart (2)

Dimensionless heat loss Q/Q_0 of an infinite plane of thickness $2L$ with time, from Reference 6.

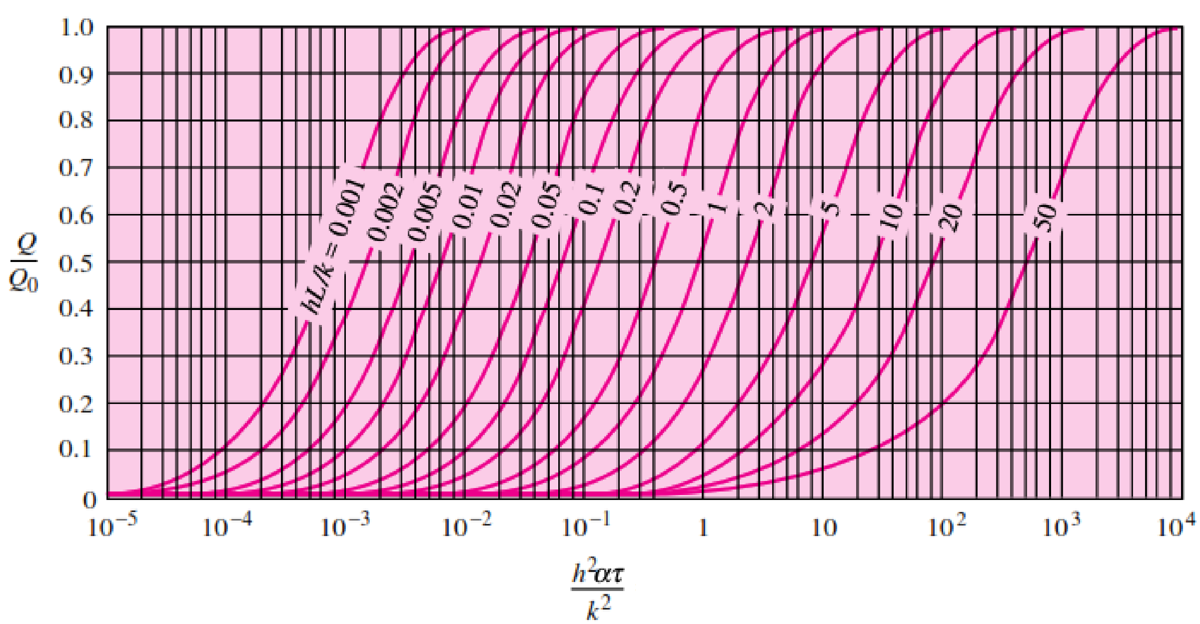
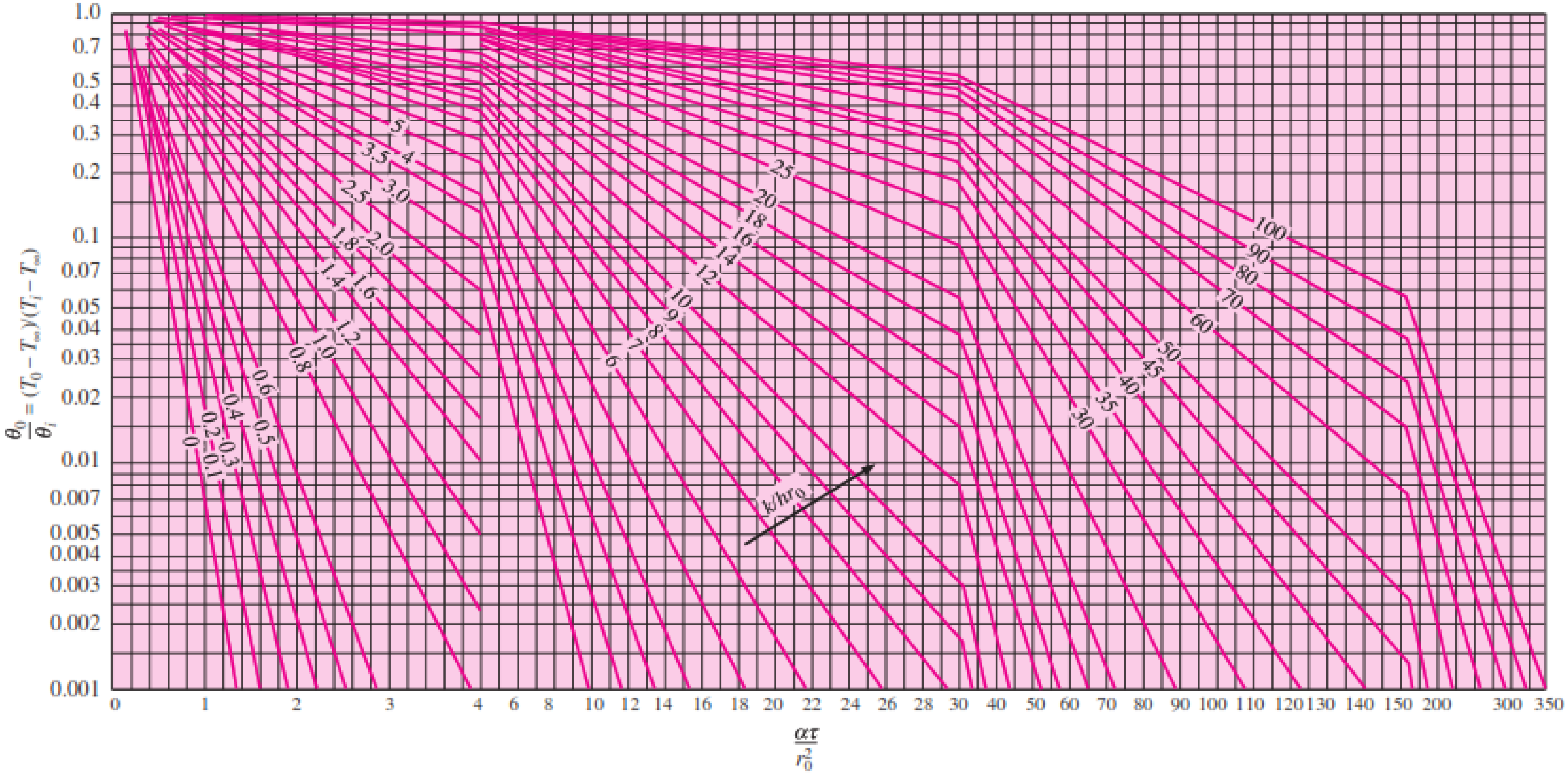


Chart (3)

Chart (4)

Axis temperature for an infinite cylinder of radius r_0: (*a*) full scale.



Temperature as a function of axis temperature in an infinite cylinder of radius r_0, from Reference 2.

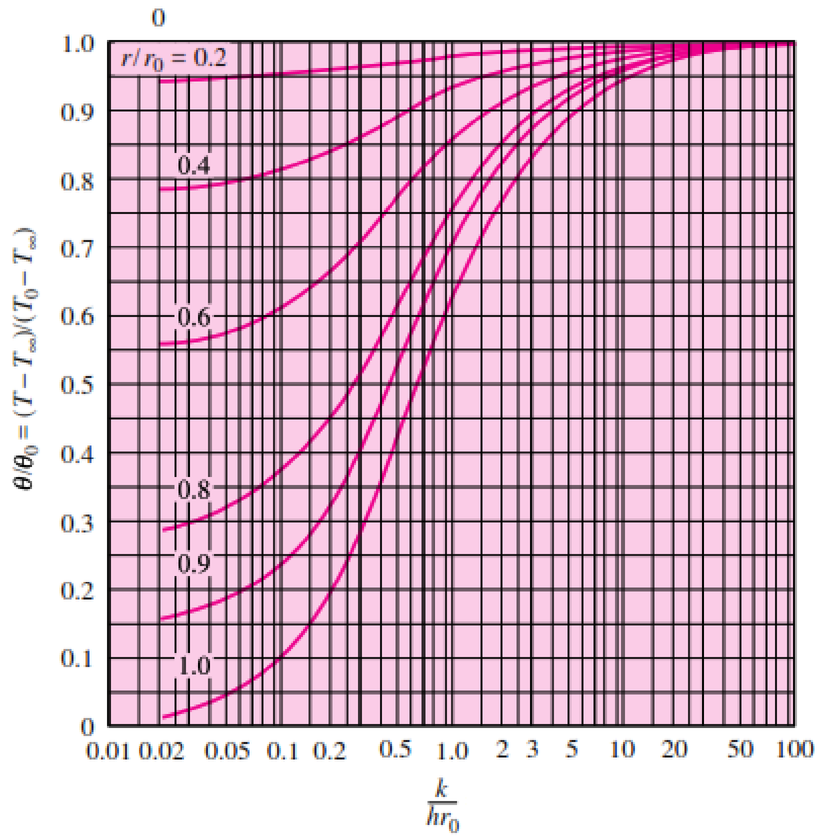


Chart (5)

Dimensionlesss heat loss Q/Q_0 of an infinite cylinder of radius r_0 with time, from Reference 6.

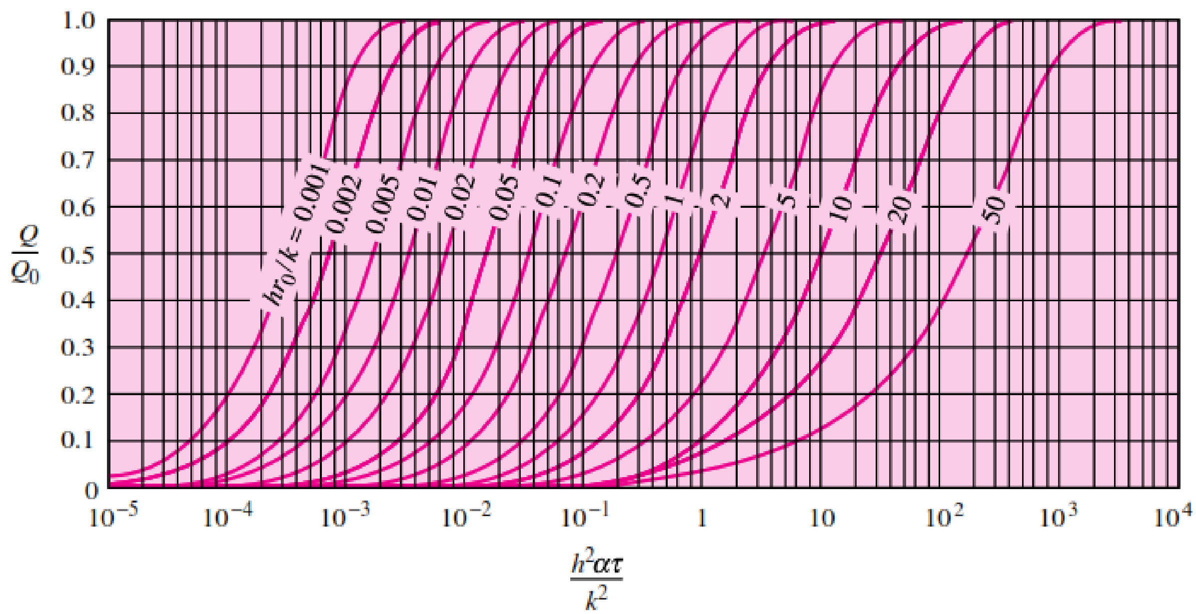
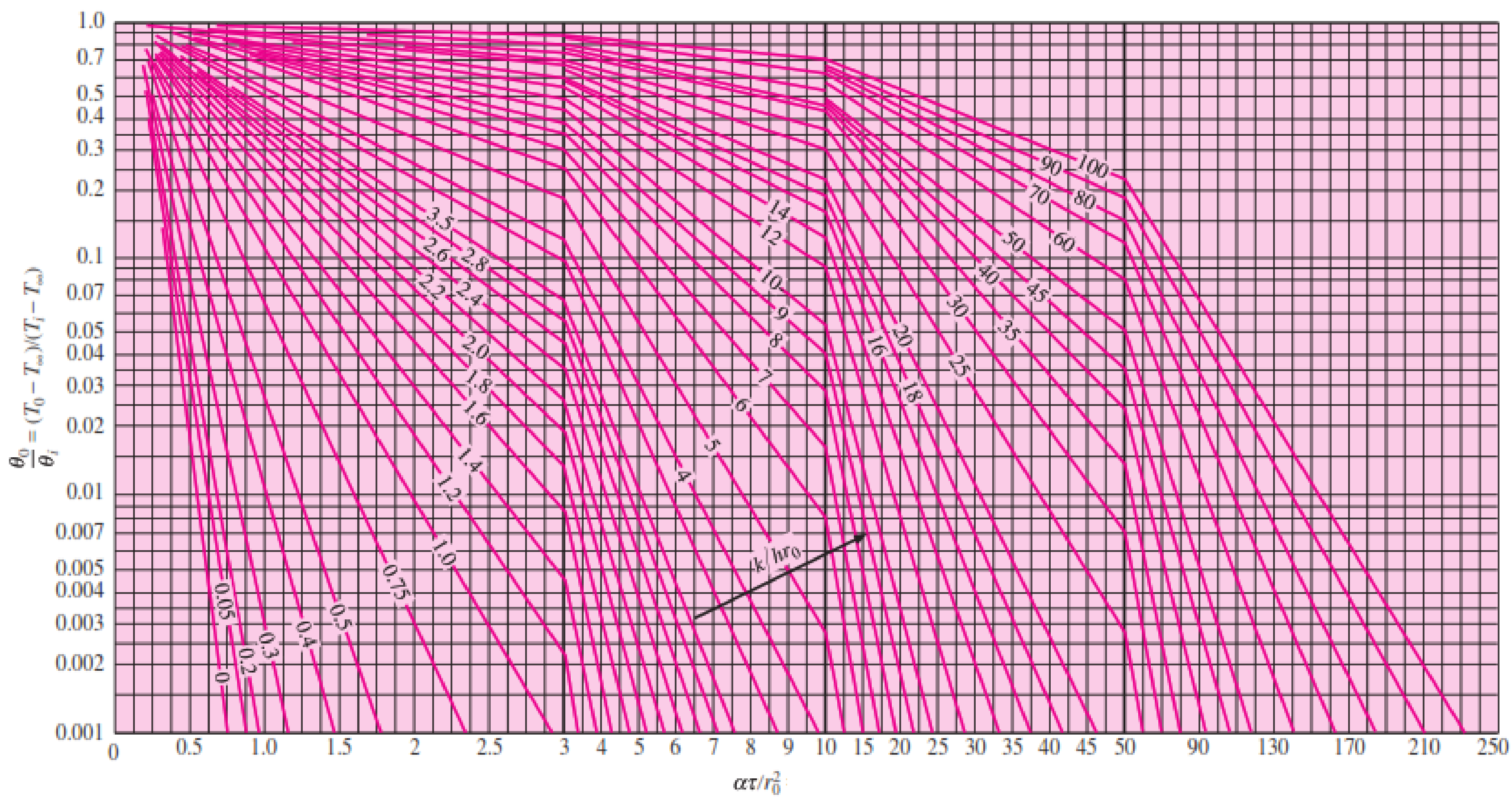


Chart (6)

Chart (7)

Center temperature for a sphere of radius r_0: (*a*) full scale.



Temperature as a function of center temperature for a sphere of radius r_0, from Reference 2.

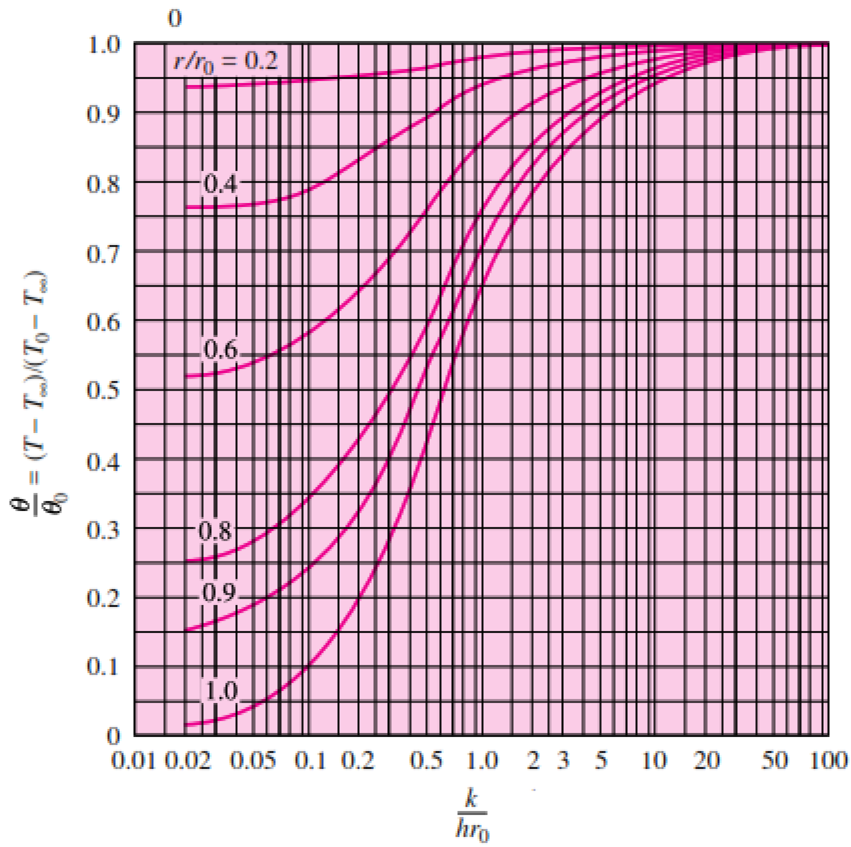


Chart (8)

Dimensionless heat loss Q/Q_0 of a sphere of radius r_0 with time, from Reference 6.

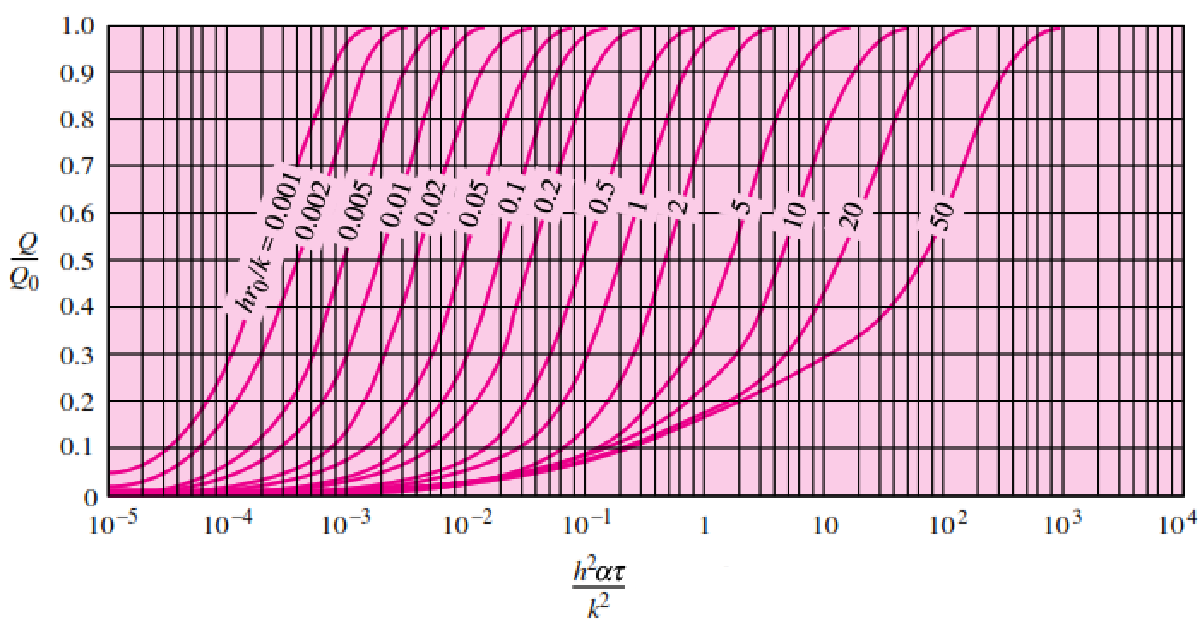


Chart (9)