Reinforced Concrete Design

Notation:

a = depth of the effective compression block in a concrete beam
A = name for area
A_g = gross area, equal to the total area ignoring any reinforcement
A_s = area of steel reinforcement in concrete beam design
A'_s = area of steel compression reinforcement in concrete beam design
A_{st} = area of steel reinforcement in concrete column design
A_v = area of concrete shear stirrup reinforcement
ACI = American Concrete Institute
b = width, often cross-sectional
b_E = effective width of the flange of a concrete T beam cross section
b_f = width of the flange
b_w = width of the stem (web) of a concrete T beam cross section
c = distance from the top to the neutral axis of a concrete beam *(see x)*
cc = shorthand for clear cover
C = name for centroid
= name for a compression force
C_c = compressive force in the compression steel in a doubly reinforced concrete beam
C_s = compressive force in the concrete of a doubly reinforced concrete beam
d = effective depth from the top of a reinforced concrete beam to the centroid of the tensile steel
d' = effective depth from the top of a reinforced concrete beam to the centroid of the compression steel
d_b = bar diameter of a reinforcing bar
D = shorthand for dead load
DL = shorthand for dead load
E = modulus of elasticity or Young's modulus
= shorthand for earthquake load
E_c = modulus of elasticity of concrete
E_s = modulus of elasticity of steel
f = symbol for stress
f_c = compressive stress
f'_c = concrete design compressive stress
f_{pu} = tensile strength of the prestressing reinforcement
f_s = stress in the steel reinforcement for concrete design
f'_s = compressive stress in the compression reinforcement for concrete beam design
f_y = yield stress or strength
f_{yt} = yield stress or strength of transverse reinforcement
F = shorthand for fluid load
F_y = yield strength
G = relative stiffness of columns to beams in a rigid connection, as is Ψ
h = cross-section depth
H = shorthand for lateral pressure load
h_f = depth of a flange in a T section
$I_{transformed}$ = moment of inertia of a multi-material section transformed to one material
k = effective length factor for columns
ℓ_b = length of beam in rigid joint
ℓ_c = length of column in rigid joint
l_d = development length for reinforcing steel
l_{dh} = development length for hooks
l_n = clear span from face of support to face of support in concrete design
L = name for length or span length, as is l
= shorthand for live load
L_r = shorthand for live roof load
LL = shorthand for live load
M_n = nominal flexure strength with the steel reinforcement at the yield stress and concrete at the concrete design strength for reinforced concrete beam design
M_u = maximum moment from factored loads for LRFD beam design

n = modulus of elasticity transformation coefficient for steel to concrete
n.a. = shorthand for neutral axis (N.A.)
pH = chemical alkalinity
P = name for load or axial force vector
P_o = maximum axial force with no concurrent bending moment in a reinforced concrete column
P_n = nominal column load capacity in concrete design
P_u = factored column load calculated from load factors in concrete design
R = shorthand for rain or ice load
= radius of curvature in beam deflection relationships (*see* ρ *)*
R_n = concrete beam design ratio = M_u/bd^2
s = spacing of stirrups in reinforced concrete beams
S = shorthand for snow load
t = name for thickness
T = name for a tension force
= shorthand for thermal load
U = factored design value
V_c = shear force capacity in concrete
V_s = shear force capacity in steel shear stirrups
V_u = shear at a distance of *d* away from the face of support for reinforced concrete beam design
w_c = unit weight of concrete
w_{DL} = load per unit length on a beam from dead load
w_{LL} = load per unit length on a beam from live load
$w_{self\,wt}$ = name for distributed load from self weight of member
w_u = load per unit length on a beam from load factors
W = shorthand for wind load
x = horizontal distance
= distance from the top to the neutral axis of a concrete beam *(see c)*
y = vertical distance
β_1 = coefficient for determining stress block height, *a,* based on concrete strength, f'_c
Δ = elastic beam deflection
ε = strain
ε_t = strain in the steel
ε_y = strain at the yield stress
λ = modification factor for lightweight concrete
ϕ = resistance factor
ϕ_c = resistance factor for compression
γ = density or unit weight
ρ = radius of curvature in beam deflection relationships (see R)
= reinforcement ratio in concrete beam design = A_s/bd
$\rho_{balanced}$ = balanced reinforcement ratio in concrete beam design
υ_c = shear strength in concrete design

Reinforced Concrete Design

Structural design standards for reinforced concrete are established by the *Building Code and Commentary (ACI 318-14)* published by the American Concrete Institute International, and uses strength design (also known as *limit state* design).

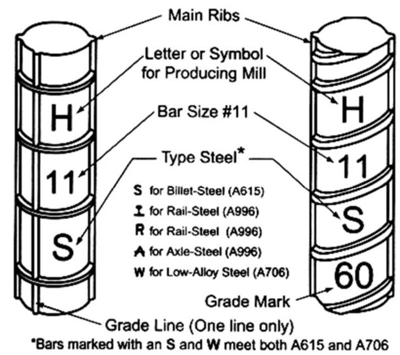
f'_c = concrete compressive design strength at 28 days (units of psi when used in equations)

Materials

Concrete is a mixture of cement, coarse aggregate, fine aggregate, and water. The cement hydrates with the water to form a binder. The result is a hardened mass with "filler" and pores. There are various types of cement for low heat, rapid set, and other properties. Other minerals or cementitious materials (like fly ash) may be added.

ASTM designations are

Type I:	Ordinary portland cement (OPC)
Type II:	Moderate heat of hydration and sulfate resistance
Type III:	High early strength (rapid hardening)
Type IV:	Low heat of hydration
Type V:	Sulfate resistant



GRADE 60

The proper proportions, by volume, of the mix constituents determine strength, which is related to the water to cement ratio (w/c). It also determines other properties, such as workability of fresh concrete. Admixtures, such as retardants, accelerators, or superplasticizers, which aid flow without adding more water, may be added. Vibration may also be used to get the mix to flow into forms and fill completely.

Slump is the measurement of the height loss from a compacted cone of fresh concrete. It can be an indicator of the workability.

Proper mix design is necessary for durability. The pH of fresh cement is enough to prevent reinforcing steel from oxidizing (rusting). If, however, cracks allow corrosive elements in water to penetrate to the steel, a corrosion cell will be created, the steel will rust, expand and cause further cracking. Adequate cover of the steel by the concrete is important.

Deformed reinforcing bars come in grades 40, 60 & 75 (for 40 ksi, 60 ksi and 75 ksi yield strengths). Sizes are given as # of 1/8" up to #8 bars. For #9 and larger, the number is a nominal size (while the actual size is larger).

Reinforced concrete is a composite material, and the average density is considered to be $150\ lb/ft^3$. It has the properties that it will creep (deformation with long term load) and shrink (a result of hydration) that must be considered.

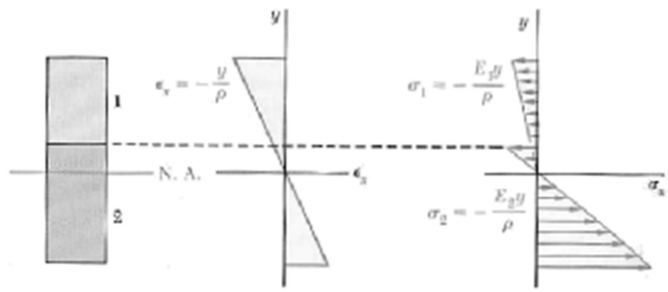
Construction

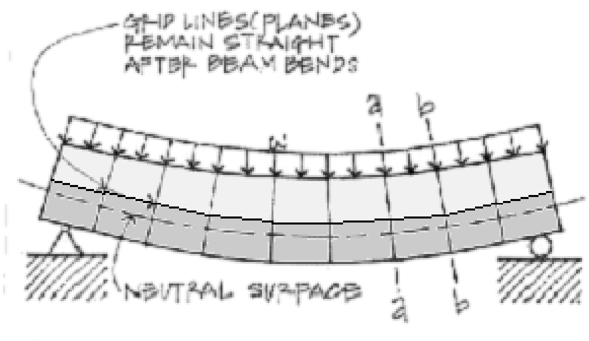
Because fresh concrete is a viscous suspension, it is cast or placed and *not poured*. Formwork must be able to withstand the hydraulic pressure. *Vibration* may be used to get the mix to flow around reinforcing bars or into tight locations, but excess vibration will cause segregation, honeycombing, and excessive *bleed* water which will reduce the water available for hydration and the strength, subsequently.

After casting, the surface must be worked. *Screeding* removes the excess from the top of the forms and gets a rough level. *Floating* is the process of working the aggregate under the surface and to "float" some paste to the surface. *Troweling* takes place when the mix has hydrated to the point of supporting weight and the surface is smoothed further and consolidated. *Curing* is allowing the hydration process to proceed with adequate moisture. Black tarps and curing compounds are commonly used. *Finishing* is the process of adding a texture, commonly by using a broom, after the concrete has begun to set.

<u>Behavior</u>

Plane sections of composite materials can still be assumed to be plane (strain is linear), *but* the stress distribution *is not* the same in both materials because the *modulus of elasticity* is different. (f=E·ε)





$$f_1 = E_1\varepsilon = -\frac{E_1 y}{R} \qquad f_2 = E_2\varepsilon = -\frac{E_2 y}{R}$$

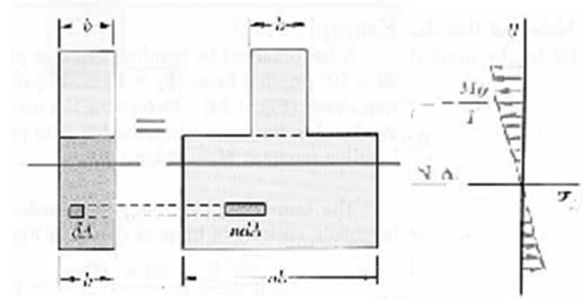
where R (or ρ) is the radius of curvature

In order to determine the stress, we can define n as the ratio of the elastic moduli: $n = \frac{E_2}{E_1}$

n is used to <u>transform</u> the <u>width</u> of the second material such that it sees the equivalent element stress.

Transformed Section y and I

In order to determine stresses in all types of material in the beam, we transform the materials into a single material, and calculate the location of the neutral axis and modulus of inertia for that material.



ex: When material 1 above is concrete and material 2 is steel

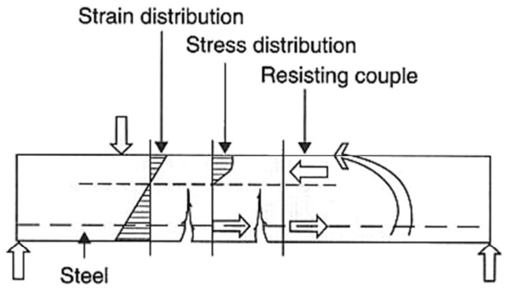
to transform steel into concrete $n = \frac{E_2}{E_1} = \frac{E_{steel}}{E_{concrete}}$

to find the neutral axis of the equivalent concrete member we transform the width of the steel by multiplying by n

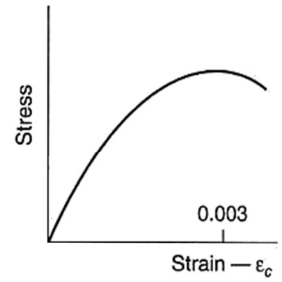
to find the moment of inertia of the equivalent concrete member, $I_{transformed}$, use the new geometry resulting from transforming the width of the steel

concrete stress: $f_{concrete} = -\frac{My}{I_{\text{transformed}}}$ steel stress: $f_{steel} = -\frac{Myn}{I_{transformed}}$

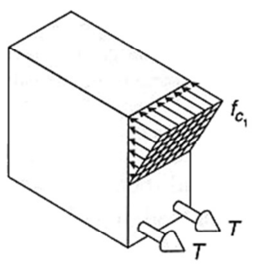
<u>Reinforced Concrete Beam Members</u>



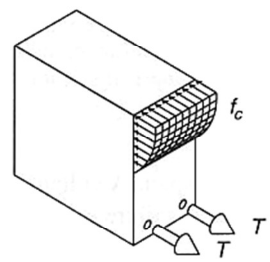
Stresses in the concrete above the neutral axis are compressive and nonlinearly distributed. In the tension zone below the neutral axis, the concrete is assumed to be cracked and the tensile force present to be taken up by reinforcing steel.



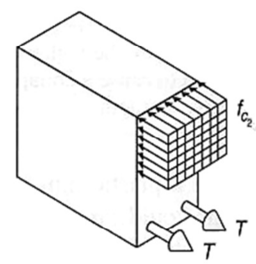
Typical stress-strain curve for concrete.



Working stress analysis. (Concrete stress distribution is assumed to be linear. Service loads are used in calculations.)



Actual stress distribution near ultimate strength (nonlinear).



Ultimate strength analysis. (A rectangular stress block is used to idealize the actual stress distribution. Calculations are based on ultimate loads and failure stresses.)

<u>Strength Design for Beams</u>

Strength design method is similar to LRFD. There is a *nominal* strength that is reduced by a factor ϕ which must exceed the factored design stress. For beams, the concrete only works in compression over a rectangular "stress" block above the n.a. from elastic calculation, and the steel is exposed and reaches the yield stress, F_y

For stress analysis in reinforced concrete beams

- the steel is transformed to concrete
- any concrete in tension is assumed to be cracked and to have <u>no strength</u>
- the steel can be in tension, and is placed in the bottom of a beam that has positive bending moment
-

The neutral axis is where there is no stress and no strain. The concrete above the n.a. is in compression. The concrete below the n.a. is considered ineffective. The steel below the n.a. is in tension.

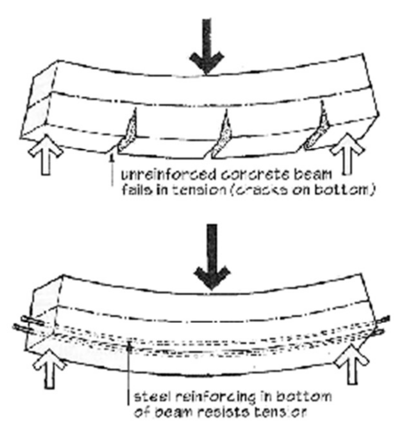
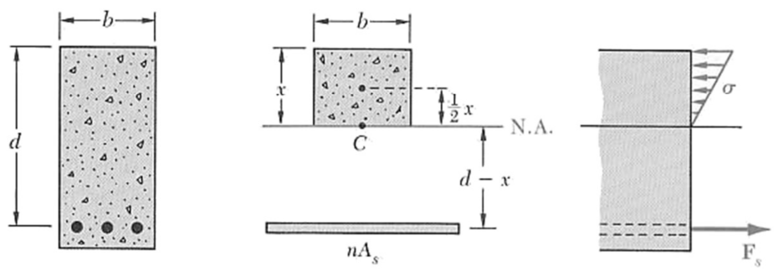


Figure 8.5: Bending in a concrete beam without and with steel reinforcing.

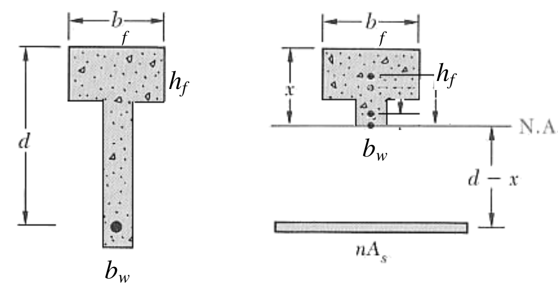
Because the n.a. is defined by the moment areas, we can solve for x (or c) knowing that d is the distance from the top of the concrete section to the centroid of the steel:

$$bx \cdot \frac{x}{2} - nA_s(\, d - x \,) = 0$$



x can be solved for when the equation is rearranged into the generic format with a, b & c in the binomial equation: $ax^2 + bx + c = 0$ by $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

T-sections



If the n.a. is *above* the bottom of a flange in a T section, x is found as for a rectangular section.

If the n.a. is *below* the bottom of a flange in a T section, x is found by including the flange and the stem of the web (b_w) in the moment area calculation:

$$b_f h_f\left(x - {h_f}/{2}\right) + \left(x - h_f\right) b_w \frac{\left(x - h_f\right)}{2} - nA_s(d - x) = 0$$

Load Combinations (Alternative values are allowed)

1.4*D*

1.2*D* + 1.6*L* +0.5(L_r or *S* or *R*)

1.2*D* + 1.6(L_r or *S* or *R*) + (1.0*L* or 0.5*W*)

1.2*D* + 1.0*W* +1.0*L* + 0.5(L_r or *S* or *R*)

1.2*D* + 1.0*E* + 1.0*L* + 0.2*S*

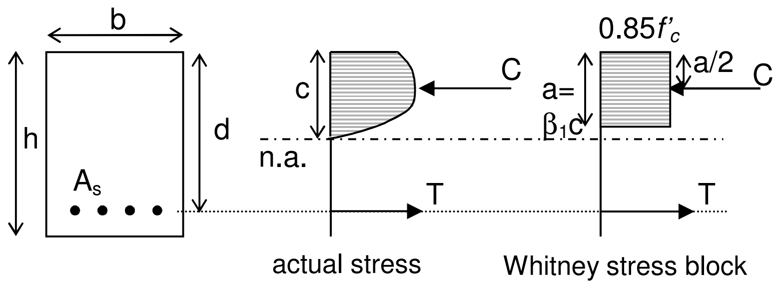
0.9*D* + 1.0*W*

0.9*D* + 1.0*E*

ASTM STANDARD REINFORCING BARS

Bar size, no.	Nominal diameter, in.	Nominal area, in.^{2}	Nominal weight, lb/ft
3	0.375	0.11	0.376
4	0.500	0.20	0.668
5	0.625	0.31	1.043
6	0.750	0.44	1.502
7	0.875	0.60	2.044
8	1.000	0.79	2.670
9	1.128	1.00	3.400
10	1.270	1.27	4.303
11	1.410	1.56	5.313
14	1.693	2.25	7.650
18	2.257	4.00	13.600

Internal Equilibrium



C = compression in concrete = stress x area = $0.85 f'_c ba$

T = tension in steel = stress x area = $A_s f_y$

$C = T$ and $M_n = T(d\text{-}a/2)$

where
- f'_c = concrete compression strength
- a = height of stress block
- β_1 = factor based on f'_c $\qquad \beta_1 = 0.85 - \left(\frac{f'_c - 4000}{1000}\right)(0.05) \geq 0.65$
- c = location to the neutral axis
- b = width of stress block
- f_y = steel yield strength
- A_s = area of steel reinforcement
- d = effective depth of section = depth to n.a. of reinforcement

With C=T, $A_s f_y = 0.85 f'_c ba$ so a can be determined with $a = \frac{A_s f_y}{0.85 f'_c b} = \beta_1 c$

Criteria for Beam Design

For flexure design:

$M_u \leq \phi M_n$ $\quad$ *ϕ = 0.9 for flexure (when the section is <u>tension</u> controlled)*

so for design, M_u *can be set to* $\phi M_n = \phi T(d\text{-}a/2) = \phi A_s f_y (d\text{-}a/2)$

Reinforcement Ratio

The amount of steel reinforcement is *limited*. Too much reinforcement, or **over-reinforcing** will not allow the steel to yield before the concrete crushes and there is a sudden failure. A beam with the proper amount of steel to allow it to yield at failure is said to be **under reinforced**.

The reinforcement ratio is just a fraction: $\rho = \frac{A_s}{bd}$ (or p). The amount of reinforcement is limited to that which results in a concrete strain of 0.003 and a minimum tensile strain of 0.004.

When the strain in the reinforcement is 0.005 or greater, the section is **tension controlled**. (For smaller strains the resistance factor reduces to 0.65 because the stress is less than the yield stress in the steel.) Previous codes limited the amount to $0.75\rho_{balanced}$ where $\rho_{balanced}$ was determined from the amount of steel that would make the concrete start to crush at the exact same time that the steel would yield based on strain (ε_y) of 0.002.

The strain in tension can be determined from $\varepsilon_t = \frac{d-c}{c}(0.003)$. At yield, $\varepsilon_y = \frac{f_y}{E_s}$.

The resistance factor expressions for **transition** and **compression controlled** sections are:

$$\phi = 0.75 + (\varepsilon_t - \varepsilon_y)\frac{0.15}{(0.005 - \varepsilon_y)} \quad \textit{for spiral members} \qquad \text{(not less than 0.75)}$$

$$\phi = 0.65 + (\varepsilon_t - \varepsilon_y)\frac{0.25}{(0.005 - \varepsilon_y)} \quad \textit{for other members} \qquad \text{(not less than 0.65)}$$

Maximum Reinforcement Ratio ρ for Singly Reinforced Rectangular Beams (tensile strain = 0.005) for which ϕ is permitted to be 0.9

f_y	f'_c = 3000 psi β_1 = 0.85	f'_c = 3500 psi β_1 = 0.85	f'_c = 4000 psi β_1 = 0.85	f'_c = 5000 psi β_1 = 0.80	f'_c = 6000 psi β_1 = 0.75
40,000 psi	0.0203	0.0237	0.0271	0.0319	0.0359
50,000 psi	0.0163	0.0190	0.0217	0.0255	0.0287
60,000 psi	0.0135	0.0158	0.0181	0.0213	0.0239
f_y	f'_c = 20 MPa β_1 = 0.85	f'_c = 25 MPa β_1 = 0.85	f'_c = 30 MPa β_1 = 0.85	f'_c = 35 MPa β_1 = 0.81	f'_c = 40 MPa β_1 = 0.77
300 MPa	0.0181	0.0226	0.0271	0.0301	0.0327
350 MPa	0.0155	0.0194	0.0232	0.0258	0.0281
400 MPa	0.0135	0.0169	0.0203	0.0226	0.0245
500 MPa	0.0108	0.0135	0.0163	0.0181	0.0196

Flexure Design of Reinforcement

One method is to "wisely" estimate a height of the stress block, a, and solve for A_s, and calculate a new value for a using M_u.

1. guess a (less than n.a.)
2. $A_s = \dfrac{0.85 f'_c ba}{f_y}$
3. solve for a from

   setting $M_u = \phi A_s f_y (d - a/2)$:

   $$a = 2\left(d - \frac{M_u}{\phi A_s f_y}\right)$$
4. repeat from 2. until a found from step 3 matches a used in step 2.

Design Chart Method:

1. calculate $R_n = \dfrac{M_n}{bd^2}$ $\left(R_n = \dfrac{M_u}{\phi bd^2}\right)$
2. find curve for f'_c and f_y to get ρ
3. calculate A_s and a, where:

   $$A_s = \rho bd \;\; and \;\; a = \frac{A_s f_y}{0.85 f'_c b}$$

Any method can simplify the size of d using h = 1.1d

Maximum Reinforcement

Based on the limiting strain of 0.005 in the steel, $c = 0.375d$ so

$a = \beta_1(0.375d)$ to find $A_{s\text{-}max}$

<u>(β_1 is shown in the table above)</u>

from <u>Reinforced Concrete, 7th,</u>
Wang, Salmon, Pincheira, Wiley & Sons, 2007

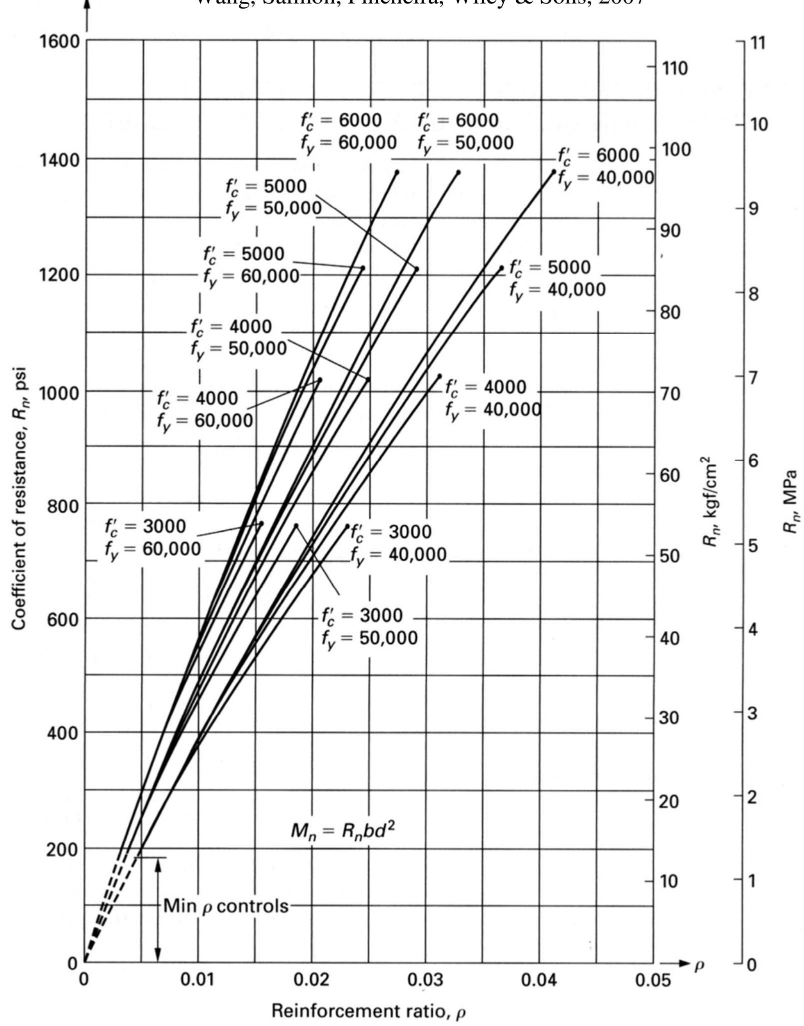


Figure 3.8.1 Strength curves (R_n vs ρ) for singly reinforced rectangular sections. Upper limit of curves is at ρ_{max}. (tensile strain of 0.004)

Minimum Reinforcement

Minimum reinforcement is provided even if the concrete can resist the tension. This is a means to control cracking.

Minimum required: $A_s = \dfrac{3\sqrt{f_c'}}{f_y}(b_w d)$ but not less than: $A_s = \dfrac{200}{f_y}(b_w d)$

where f_c' is in psi, and λ is for material:

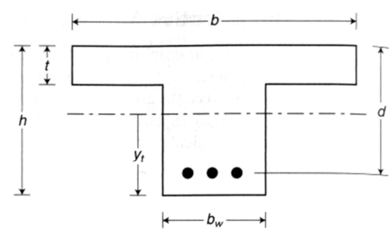
This can be translated to $\rho_{\min} = \dfrac{3\sqrt{f_c'}}{f_y}$ but not less than $\dfrac{200}{f_y}$

Lightweight Concrete

Lightweight concrete has strength properties that are different from normalweight concretes, and a modification factor, λ, must be multiplied to the strength value of $\sqrt{f_c'}$. for concrete for some specifications (ex. shear). Depending on the aggregate and the lightweight concrete, the value of λ ranges from 075 to 0.85, 0.85 or 0.85 to 1.0. λ is 1.0 for normalweight concrete.

Cover for Reinforcement

Cover of concrete over/under the reinforcement must be provided to protect the steel from corrosion. For indoor exposure, 1.5 inch is typical for beams and columns, 0.75 inch is typical for slabs, and for concrete cast against soil, 3 inch minimum is required.



Bar Spacing

Minimum bar spacings are specified to allow proper consolidation of concrete around the reinforcement. The minimum spacing is the maximum of 1 in, a bar diameter, or 1.33 times the maximum aggregate size.

T-beams and T-sections (pan joists)

Beams cast with slabs have an effective width, b_E, that sees compression stress in a wide flange beam or joist in a slab system with positive bending.

For *interior* T-sections, b_E is the smallest of

L/4, b_w *+ 16t, or center to center of beams*

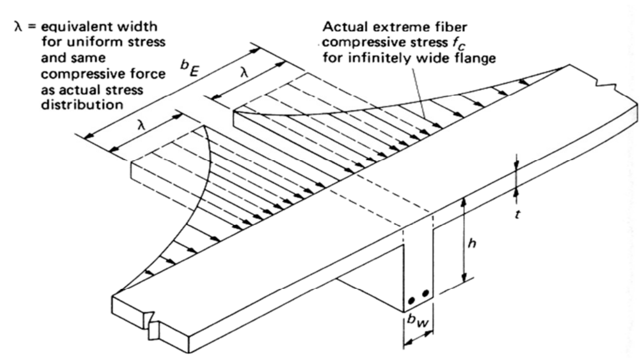


Figure 9.3.1 Actual and equivalent stress distribution over flange width.

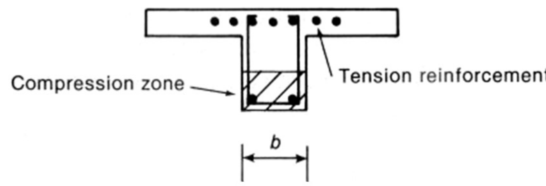
For *exterior* T-sections, b_E is the smallest of

b_w *+ L/12,* b_w *+ 6t, or* b_w *+ ½(clear distance to next beam)*

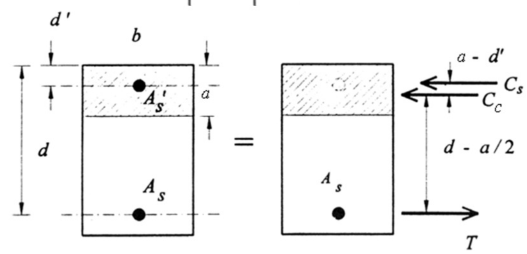
When the **web** is in tension the minimum reinforcement required is the same as for rectangular sections with the web width (b_w) in place of b. $M_n = C_w(d\text{-}a/2)+C_f(d\text{-}h_f/2)$ (h_f is height of flange or t)

When the **flange** is in tension (negative bending), the minimum reinforcement required is the greater value of $A_s = \dfrac{6\sqrt{f_c'}}{f_y}(b_w d)$ or $A_s = \dfrac{3\sqrt{f_c'}}{f_y}(b_f d)$

where f_c' is in psi, b_w is the beam width, and b_f is the effective flange width



Compression Reinforcement



If a section is *doubly reinforced*, it means there is steel in the beam seeing compression. The force in the compression steel that *may not be yielding* is

$$C_s = A_s'(f_s' - 0.85f_c')$$

The total compression that balances the tension is now:

$$T = C_c + C_s.$$

And the moment taken about the centroid of the compression stress is $M_n = T(d\text{-}a/2)+C_s(a\text{-}d')$
where A_s' is the area of compression reinforcement, and d' is the effective depth to the centroid of the compression reinforcement

Because the compression steel may not be yielding, the neutral axis x must be found from the force equilibrium relationships, and the stress can be found based on strain to see if it has yielded.