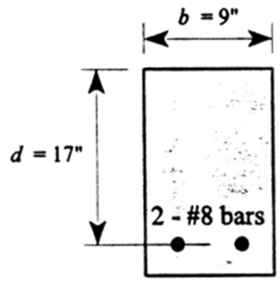
Example 1

Determine the design moment capacity for the reinforced concrete cross section shown Assume f'_c = 3000 psi and Grade 60 reinforcing steel.



Example 2

(a) Determine the ultimate moment capacity of a beam with dimensions b = 10 in. and $d_{\text{effective}}$ = 15 in. and that has three No. 9 bars (3.0 in.^{2}) of tension-reinforcing steel. Assume that h = 18 in., F_y = 40 ksi, and f'_c = 5 *ksi*. (b) Assume also that the section is used as a cantilever beam 10 ft long, where the service loads are dead load = 400 lb/ft and live load = 300 lb/ft. Is the beam adequate in bending? Calculate the ultimate moment capacity of the beam first.

Solution:

(a)

$$a = A_sF_y/0.85f'_cb = (3)(40{,}000)/(0.85)(5000)(10) = 2.82 \text{ in.}$$

$$\phi M_n = \phi A_sF_y[d - a/2] = 0.9(3)(40{,}000)[15 - (2.82)/(2)] = 1{,}466{,}640 \text{ in.-lb}$$

Check for overreinforcement, $c = 0.375 \cdot 15 = 5.625$. Depth of stress block $a = 0.80 \cdot 5.625$ in. = 4.5 in. $A_{s,\max}$ = (0.85)(5ksi)(4.5in.)(10in.)/(40ksi) = 4.78 in.^{2} The beam is not over reinforced Check for minimum steel: $A_{s,\min} = \dfrac{3\sqrt{f'_c}}{F_y}bd$ $= 0.80\ in^2$, so beam is sufficiently reinforced.

(b)

$$U = 1.2D + 1.6L = 1.2(400) + 1.6(300) = 960 \text{ lb/ft}$$

$$M_u = w_uL^2/2 = (960)(10^2)/2 = 48{,}000 \text{ ft-lb} = 576{,}000 \text{ in.-lb}$$

lb-in lb-in

Since $M_u = 576{,}000 < \phi M_n = 1{,}466{,}640$, the beam is adequate in bending.

EXAMPLE

Determine the ultimate moment capacity of a beam of dimensions b = 250 mm and d = 350 mm and that has 300 mm^2 of reinforcing steel. Assume that F_y = 400 MPa and f'_c = 25 MPa.

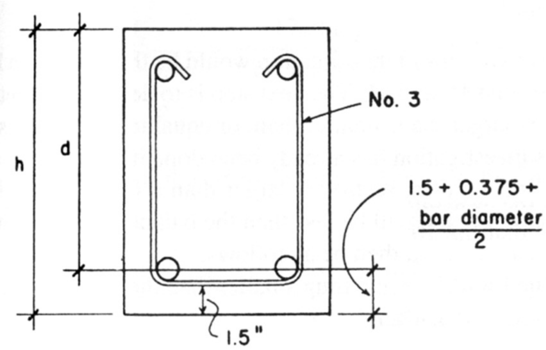
Solution:

$$a = \frac{A_sF_y}{0.85f'_cb} = \frac{(300)(400)}{(0.85)(25)(250)} = 22.6 \text{ mm}$$

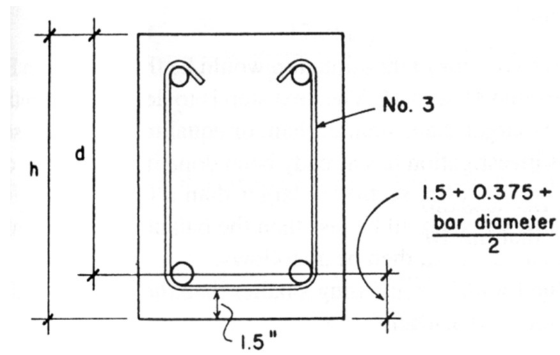
$$\phi M_n = \phi A_sF_y\left(d - \frac{a}{2}\right) = 0.9(300)(400)\left(350 - \frac{22.6}{2}\right) = 36.5 \text{ kN}\cdot\text{m}$$

<u>Example 3</u>

Example 1. The service load bending moments on a beam are 58 kip-ft [78.6 kN-m] for dead load and 38 kip-ft [51.5 kN-m] for live load. The beam is 10 in. [254 mm] wide, f'_c is 3000 psi [27.6 MPa], and f_y is 60 ksi [414 MPa]. Determine the depth of the beam and the tensile reinforcing required.



Example 3 *(continued)*



Example 4

A simply supported beam 20 ft long carries a service dead load of 300 lb/ft and a live load of 500 lb/ft. Design an appropriate beam (for flexure only). Use grade 40 steel and concrete strength of 5000 psi.

SOLUTION:

Find the design moment, M_u, from the factored load combination of 1.2D + 1.6L. It is good practice to guess a beam size to include self weight in the dead load, because "service" means dead load of everything except the beam itself.

Guess a size of 10 in x 12 in. Self weight for normal weight concrete is the density of 150 lb/ft³ multiplied by the cross section area: self weight = $150\ {}^{lb}\!/\!_{ft^3}(10in)(12in)\cdot(\frac{1ft}{12in})^2$ = 125 lb/ft

w_u = 1.2(300 lb/ft + 125 lb/ft) + 1.6(500 lb/ft) = 1310 lb/ft

The maximum moment for a simply supported beam is $\frac{wl^2}{8}$: $M_u = \frac{w_u l^2}{8} = \frac{1310\ {}^{lb}\!/\!_{ft}(20ft)^2}{8}$ 65,500 lb-ft

M_n required = M_u/ϕ = $\frac{65{,}500^{lb-ft}}{0.9}$ = 72,778 lb-ft

To use the design chart aid, find $R_n = \frac{M_n}{bd^2}$, estimating that d is about 1.75 inches less than h:

d = 12in – 1.75 in – ~~(0.375)~~ = 10.25 in (NOTE: If there are stirrups, you must also subtract the diameter of the stirrup bar.)

$R_n = \frac{72{,}778^{lb-ft}}{(10in)(1025in)^2}\cdot(12\ {}^{in}\!/\!_{ft})$ = 831 psi

ρ corresponds to approximately 0.023 (which is less than that for 0.005 strain of 0.0319) , so the estimated area required, A_s, can be found:

$A_s = \rho bd$ = (0.023)(10in)(10.25in) = 2.36 in²

The number of bars for this area can be found from handy charts.

(Whether the number of bars actually fit for the width with cover and space between bars <u>must also be considered</u>. If you are at ρ_{max} <u>do not</u> choose an area bigger than the maximum!)

Try A_s = 2.37 in² from 3#8 bars

d = 12 in – 1.5 in (cover) – ½ (8/8in diameter bar) = 10 in

Check ρ = 2.37 in²/(10 in)(10 in) = 0.0237 which is less than $\rho_{max-0.005}$ = 0.0319 OK (We cannot have an over reinforced beam!!)

Find the moment capacity of the beam as designed, ϕM_n

> $a = A_s f_y / 0.85 f'_c b$ = 2.37 in² (40 ksi)/[0.85(5 ksi)10 in] = 2.23 in
>
> $\phi M_n = \phi A_s f_y (d-a/2)$ = $0.9(2.37in^2)(40ksi)(10in - \frac{2.23in}{2})\cdot(\frac{1}{12\ {}^{in}\!/\!_{ft}})$ = 63.2 k-ft $\not>$ 65.5 k-ft needed (not OK)

So, we can increase d to 13 in, and ϕM_n = 70.3 k-ft (OK). Or increase A_s to 2 # 10's (2.54 in²), for a = 2.39 in and ϕM_n of 67.1 k-ft (OK). <u>*Don't exceed ρ_{max} or $\rho_{max-0.005}$ if you want to use ϕ=0.9*</u>

Example 5

A simply supported beam 20 ft long carries a service dead load of 425 lb/ft (including self weight) and a live load of 500 lb/ft. Design an appropriate beam (for flexure only). Use grade 40 steel and concrete strength of 5000 psi.

SOLUTION:

Find the design moment, M_u, from the factored load combination of 1.2D + 1.6L. *If self weight is not included in the service loads,* you need to guess a beam size to include self weight in the dead load, because "service" means dead load of everything except the beam itself.

w_u = 1.2(425 lb/ft) + 1.6(500 lb/ft) = 1310 lb/ft

The maximum moment for a simply supported beam is $\frac{wl^2}{8}$: $M_u = \frac{w_u l^2}{8} = \frac{1310^{lb}/_{ft}(20ft)^2}{8}$ 65,500 lb-ft

M_n required = M_u/ϕ = $\frac{65{,}500^{lb-ft}}{0.9}$ = 72,778 lb-ft

To use the design chart aid, we can find $R_n = M_n/bd^2$, and estimate that h is roughly 1.5-2 times the size of b, and h = 1.1d (rule of thumb): d = h/1.1 = (2b)/1.1, so $d \approx 1.8b$ or $b \approx 0.55d$.

We can find R_n at the maximum reinforcement ratio for our materials, keeping in mind ρ_{max} at a strain = 0.005 is 0.0319 off of the chart at about 1070 psi, with ρ_{max} = 0.037. Let's substitute b for a function of d:

$R_n = 1070\text{ psi} = \frac{72{,}778^{lb-ft}}{(0.55d)(d)^2}\cdot(12^{in}/_{ft})$ Rearranging and solving for d = 11.4 inches

That would make b a little over 6 inches, which is impractical. 10 in is commonly the smallest width.

So if h is commonly 1.5 to 2 times the width, b, h ranges from 14 to 20 inches. (10x1.5=15 and 10x2 = 20)

Choosing a depth of 14 inches, d ≅ 14 - 1.5 (clear cover) - ½(1" diameter bar guess) -3/8 in (stirrup diameter) = 11.625 in.

Now calculating an updated $R_n = \frac{72{,}778\ \text{lb}-\text{ft}}{(10\text{in})(11.625\text{in})^2}\cdot(12\,\text{in}/_{\text{ft}}) = 646.2\text{psi}$

ρ now is 0.020 (under the limit at 0.005 strain of 0.0319), so the estimated area required, A_s, can be found:

$A_s = \rho bd$ = (0.020)(10in)(11.625in) = 1.98 in^2

The number of bars for this area can be found from handy charts.
(Whether the number of bars actually fit for the width with cover and space between bars <u>must also be considered</u>. If you are at $\rho_{max-0.005}$ <u>do not</u> choose an area bigger than the maximum!)

Try A_s = 2.37 in^2 from 3#8 bars. (or 2.0 in^2 from 2 #9 bars. 4#7 bars don't fit...)

d(actually) = 14 in. – 1.5 in (cover) – ½ (8/8 in bar diameter) – 3/8 in. (stirrup diameter) = 11.625 in.

Check ρ = 2.37 in^2/(10 in)(11.625 in) = 0.0203 which is less than $\rho_{max-0.005}$ = 0.0319 OK (We cannot have an over reinforced beam!!)

Find the moment capacity of the beam as designed, ϕM_n

$a = A_s f_y/0.85f'_c b$ = 2.37 in^2 (40 ksi)/[0.85(5 ksi)10 in] = 2.23 in

$\phi M_n = \phi A_s f_y(d-a/2) = 0.9(2.37\text{in}^2)(40\text{ksi})(11.625\text{in} - \frac{2.23\text{in}}{2})\cdot(\frac{1}{12^{\text{in}}/_{\text{ft}}}) =$ 74.7 k-ft > 65.5 k-ft needed

OK! <u>Note</u>**:** *If the section doesn't work, you need to increase d or A_s as long as you don't exceed $\rho_{max-0.005}$*