Example 6

A simply supported beam 25 ft long carries a service dead load of 2 k/ft, an estimated self weight of 500 lb/ft and a live load of 3 k/ft. Design an appropriate beam (for flexure only). Use grade 60 steel and concrete strength of 3000 psi.

SOLUTION:

Find the design moment, M_u, from the factored load combination of 1.2D + 1.6L. If self weight is estimated, and the selected size has a larger self weight, the design moment must be adjusted for the extra load.

w_u = 1.2(2 k/ft + 0.5 k/ft) + 1.6(3 k/ft) = 7.8 k/ft So, $M_u = \dfrac{w_u l^2}{8} = \dfrac{7.8\,{}^{k}\!/_{ft}(25ft)^2}{8}$ 609.4 k-ft

M_n required = $M_u/\phi = \dfrac{609.4^{k-ft}}{0.9}$ = 677.1 k-ft

To use the design chart aid, we can find $R_n = \dfrac{M_n}{bd^2}$, and estimate that h is roughly 1.5-2 times the size of b, and h = 1.1d (rule of thumb): d = h/1.1 = (2b)/1.1, so d ≈ 1.8b or b ≈ 0.55d.

We can find R_n at the maximum reinforcement ratio for our materials off of the chart at about 700 psi with $\rho_{max\text{-}0.005}$ = 0.0135. Let's substitute b for a function of d:

R_n = 700 psi = $\dfrac{677.1^{k-ft}(1000^{lb/k})}{(0.55d)(d)^2}\cdot(12\,{}^{in}\!/_{ft})$ Rearranging and solving for d = 27.6 inches

That would make b 15.2 in. (from 0.55d). Let's try 15. So,
h ≅ d + 1.5 (clear cover) +½(1" diameter bar guess) +3/8 in (stirrup diameter) = 27.6 +2.375 = 29.975 in.

Choosing a depth of 30 inches, d ≅ 30 - 1.5 (clear cover) - ½(1" diameter bar guess) -3/8 in (stirrup diameter) = 27.625 in.

Now calculating an updated $R_n = \dfrac{677{,}100^{\,lb-ft}}{(15in)(27.625in)^2}\cdot(12\,{}^{in}\!/_{ft}) = 710psi$ This is larger than R_n for the 0.005 strain limit!

We can't just use $\rho_{max\text{-}.005}$. The way to reduce R_n is to increase b or d or both. Let's try increasing h to 31 in., then R_n = 661 psi with d = 28.625 in.. That puts us under $\rho_{max\text{-}0.005}$. We'd have to remember to keep UNDER the area of steel calculated, which is hard to do.

From the chart, ρ ≈ 0.013, less than the $\rho_{max\text{-}0.005}$ of 0.0135, so the estimated area required, A_s, can be found:
A_s = ρbd = (0.013)(15in)(29.625in) = 5.8 in^2

The number of bars for this area can be found from handy charts. Our charts say there can be 3 – 6 bars that fit when ¾" aggregate is used. We'll assume 1 inch spacing between bars. The actual limit is the maximum of 1 in, the bar diameter or 1.33 times the maximum aggregate size.

Try A_s = 6.0 in^2 from 6#9 bars. Check the width: 15 – 3 (1.5 in cover each side) – 0.75 (two #3 stirrup legs) – 6*1.128 – 5*1.128 in. = -1.16 in NOT OK.

Try A_s = 5.08 in^2 from 4#10 bars. Check the width: 15 – 3 (1.5 in cover each side) – 0.75 (two #3 stirrup legs) – 4*1.27 – 3*1.27 in. = 2.36 OK.

d(actually) = 31 in. – 1.5 in (cover) – ½ (1.27 in bar diameter) – 3/8 in. (stirrup diameter) = 28.49 in.

Find the moment capacity of the beam as designed, ϕM_n

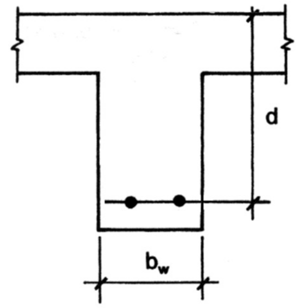
$a = A_s f_y/0.85f'_c b$ = 5.08 in^2 (60 ksi)/[0.85(3 ksi)15 in] = 8.0 in

$\phi M_n = \phi A_s f_y(d-a/2) = 0.9(5.08in^2)(60ksi)(28.49in - \dfrac{8.0in}{2})\cdot(\dfrac{1}{12\,{}^{in}\!/_{ft}}) =$ 559.8 k-ft < 609 k-ft needed!! (NO GOOD)

More steel isn't likely to increase the capacity much unless we are close. It looks like we need more steel **and** lever arm. Try h = 32 in. AND b = 16 in., then M_u^* (with the added self weight of 33.3 lb/ft) = 680.2 k-ft, ρ ≈ 0.012, As = 0.012(16in)(29.42in)=5.66 in^2. 6#9's won't fit, but 4#11's will: ρ = 0.0132 ✓, a = 9.18 in, and ϕM_n = 697.2 k-ft which is finally larger than 680.2 k-ft **OK**

Example 7

Example 3. A T-section is to be used for a beam to resist positive moment. The following data are given: beam span is 18 ft [5.49 m], beams are 9 ft [2.74 m] center to center, slab thickness is 4 in. [0.102 m], beam stem dimensions are $b_w = 15$ in. [0.381 m] and $d = 22$ in. [0.559 m], $f'_c = 4$ ksi [27.6 MPa], $f_y = 60$ ksi [414 MPa]. Find the required area of steel and select the reinforcing bars for a dead load moment of 125 kip-ft [170 kN-m] plus a live load moment of 100 kip-ft [136 kN-m].



Example 8

Design a T-beam for a floor with a 4 in slab supported by 22-ft-span-length beams cast monolithically with the slab. The beams are 8 ft on center and have a web width of 12 in. and a total depth of 22 in.; f'_c = 3000 psi and f_y = 60 ksi. Service loads are 125 psf and 200 psf dead load which does not include the weight of the floor system.

SOLUTION:

1. Establish the design moment:

$$\text{slab weight} = \frac{96(4)}{144}(0.150) = 0.400 \text{ kip/ft}$$

$$\text{stem weight} = \frac{12(18)}{144}(0.150) = \underline{0.225}$$

$$\text{total} = 0.625 \text{ kip/ft}$$

$$\text{service DL} = 8(0.200) = 1.60 \text{ kips/ft}$$

$$\text{service LL} = 8(0.125) = 1.00 \text{ kip/ft}$$

Calculate the factored load and moment:

$$w_u = 1.2(0.625 + 1.60) + 1.6(1.00) = 4.27 \text{ kip/ft}$$

$$M_u = \frac{w_u \ell^2}{8} = \frac{4.27(22)^2}{8} = 258 \text{ ft-kips}$$

2. Assume an effective depth $d = h - 3$ in.:

$$d = 22 - 3 = 19 \text{ in.}$$

3. Determine the effective flange width:

$$\tfrac{1}{4} \text{ span length} = 0.25(22)(12) = 66 \text{ in.}$$

$$b_w + 16h_f = 12 + 16(4) = 76 \text{ in.}$$

$$\text{beam spacing} = 96 \text{ in.}$$

Use an effective flange width b = 66 in.

4. Determine whether the beam behaves as a true T-beam or as a rectangular beam by computing the practical moment strength ϕM_{nf} with the full effective flange assumed to be in compression. This assumes that the bottom of the compressive stress block coincides with the bottom of the flange, as shown in Figure 3-10. Thus

$$\phi M_{nf} = \phi(0.85 f'_c) b h_f \left(d - \frac{h_f}{2}\right)$$

$$= 0.9(0.85)(3)(66)\frac{4(19 - 4/2)}{12} = 858 \text{ ft-kips}$$

5. Since 858 ft-kips > 258 ft-kips, the total effective flange need not be completely utilized in compression (i.e., $a < h_f$), and the T-beam behaves as a wide rectangular beam with a width b of 66 in.

6. Design as a rectangular beam with b and d as known values (see Section 2-15):

$$\text{required } R_n = \frac{M_u}{\phi b d^2} = \frac{258(12)}{0.9(66)(19)^2} = 0.1444 \text{ ksi}$$

7. From Table A-8, select the required steel ratio to provide a R_n of 0.1444 ksi

required ρ =0.0024

8. Calculate the required steel area:

$$\text{required } A_s = \rho b d$$

$$= 0.0024(66)(19) = 3.01 \text{ in.}^2$$

9. Select the steel bars. Use 3#9 (A_s = 3.00 in.^{2})

minimum b_w = 7.125 in **(O.K.)**

Check the effective depth d:

$$d = 22 - 1.5 - 0.38 - \frac{1.125}{2} = 19.56 \text{ in.}$$

19.49 in. > 19 in. (O.K.)

10. Check $A_{s,\min}$. From Table A-5:

$$A_{s,\min} = 0.0033 b_w d$$

$$= 0.0033(12)(19) = 0.75 \text{ in.}^2$$

$$0.75 \text{ in.}^2 < 3.00 \text{ in}^2$$

11. Check $A_{s,\max}$:

$$A_{s,\max} = 0.0135(66)(19)$$

$$= 16.93 \text{ in.}^2 > 3.00 \text{ in.}^2 \quad \text{(O.K)}$$

12. Verify the moment capacity:
(Is $M_u \le \phi M_n$)

a = (3.00)(60)/[0.85(3)(66)] = 1.07 in.

$$\phi M_n = 0.9(3.00)(60)(19.56 - \frac{1.07}{2})\,\tfrac{1}{12}$$

= 256.9 ft-kips (Not O.K)

Choose more steel, A_s = 3.16 in^2 from 4-#8's

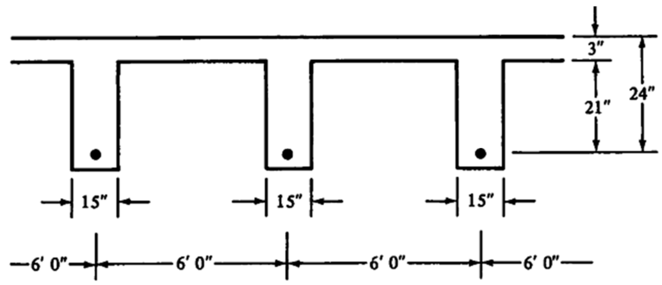
d = 19.62 in, a = 1.13 in

ϕM_n = 271.0 ft-kips, which is OK

13. Sketch the design

Example 9

Design a T-beam for the floor system shown for which b_w and d are given. M_D = 200 ft-k, M_L = 425 ft-k, f'_c = 3000 psi and f_y = 60 ksi, and simple span = 18 ft.



SOLUTION

Effective Flange Width

(a) $\frac{1}{4} \times 18' = 4'6'' = \underline{\underline{54''}}$

(b) $15'' + (2)(8)(3) = 63''$

(c) $6'0'' = 72''$

Moments Assuming $\phi = 0.90$

$$M_u = (1.2)(200) + (1.6)(425) = 920 \text{ ft-k}$$

$$M_n = \frac{M_u}{0.90} = \frac{920}{0.90} = 1022 \text{ ft-k}$$

First assume $a \leq h_f$ (which is very often the case. Then the design would proceed like that of a rectangular beam with a width equal to the effective width of the T beam flange.

$$\frac{M_u}{\phi b d^2} = \frac{920(12{,}000)}{(0.9)(54)(24)^2} = 394.4 \text{ psi}$$

from Table A.12, $\rho = 0.0072$

$$a = \frac{\rho f_y d}{0.85 f'_c} = \frac{0.0072(60)(24)}{(0.85)(3)} = 4.06 \text{ in.} > h_f = 3 \text{ in.}$$

The beam acts like a T beam, not a rectangular beam, and if the values for ρ and a above are not correct. If the value of a had been $\leq h_f$, the value of A_s would have been simply $\rho bd = 0.0072(54)(24) = 9.33$ in^2. Now break the beam up into two parts (Figure 5.7) and design it as a T beam.

Assuming $\phi = 0.90$

$$A_{sf} = \frac{(0.85)(3)(54 - 15)(3)}{60} = 4.97 \text{ in.}^2$$

$$M_{uf} = (0.9)(4.97)(60)(24 - \tfrac{3}{2}) = 6039 \text{ in.-k} = 503 \text{ ft-k}$$

$$M_{uw} = 920 - 503 = 417 \text{ ft-k}$$

Designing a rectangular beam with b_w = 15 in. and d = 24 in. to resist 417 k-ft

$$\frac{M_{uw}}{\phi b_w d^2} = \frac{(12)(417)(1000)}{(0.9)(15)(24)^2} = 643.5$$

$$\rho_w = 0.0126 \text{ from Appendix Table A.12}$$

$$A_{sw} = (0.0126)(15)(24) = 4.54 \text{ in.}^2$$

$$A_s = 4.97 + 4.54 = \underline{\underline{9.51 \text{ in.}^2}}$$

Check minimum reinforcing:

$$A_{s\ \min} = \frac{3\sqrt{f'_c}}{f_y} b_w d = \frac{3\sqrt{3000}(15)(24)}{60{,}000} = 0.986 \text{ in}^2$$

but not less than

$$A_{s\ \min} = \frac{200 b_w d}{f_y} = \frac{200(15)(24)}{60{,}000} = \underline{\underline{1.2 in^2}}$$

Only 2 rows fit, so try 8-#10 bars, A_s = 10.16 in^2
for equilibrium: $T = C_w + C_f$

$T = A_s f_y$ = (10.16)(60) = 609.6 k
$C_f = 0.85f'_c(b-b_w)h_f$ and $C_w = 0.85f'_c a b_w$
$C_w = T - C_f$ = 609.6 k – (0.85)(3)(54-15)3 = 311.25 k
a = 311.25/(0.85*3*15) = 8.14 in

Check strain (ε_t) and ϕ:
$c = a/\beta_1$ = 8.14 in/0.85 = 9.58

$$\varepsilon_t = \left(\frac{d-c}{c}\right)(0.003) = \left(\frac{24 - 9.58}{9.58}\right)(0.003) = 0.0045 \not> 0.005!$$

We could try 10-#9 bars at 10 in^2, T =600 k, C_w = 301.65 k,
a = 7.89, ε_t = 0.0061; ϕ = 0.9!

Finally check the capacity:

$$M_n = C_w(d - \frac{a}{2}) + C_f(d - \frac{h_f}{2})$$
= [301.65(24-7.89/2) + 298.35(24-3/2)]1ft/12in
=1063.5 k-ft

So: ϕM_n = 0.9(1063.5) = 957.2 k-ft ≥ <u>920 k-ft</u> (OK)

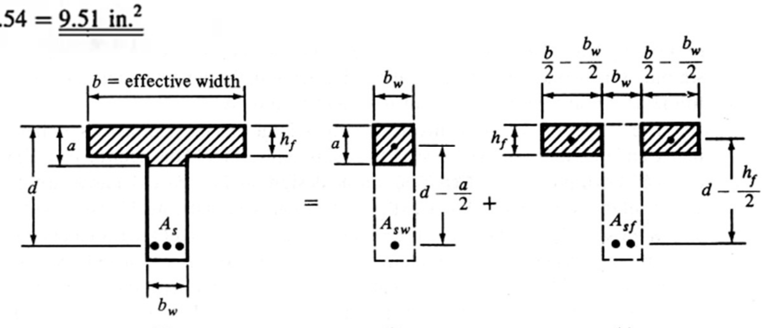


Figure 5.7 Separation of T beam into rectangular parts.