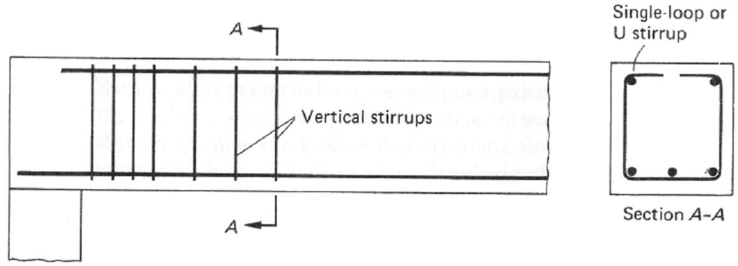
Shear Behavior

Horizontal shear stresses occur along with bending stresses to cause tensile stresses where the concrete cracks. Vertical reinforcement is required to bridge the cracks which are called **shear stirrups** (or **stirrups**).



The maximum shear for design, V_u is the value at a distance of *d* from the face of the support.

Nominal Shear Strength

The shear force that can be resisted is the shear stress × cross section area: $V_c = \upsilon_c \times b_w d$

The shear stress for beams (one way) $\upsilon_c = 2\lambda\sqrt{f'_c}$ so $\phi V_c = \phi 2\lambda\sqrt{f'_c}\, b_w d$

where b_w = the beam width or the minimum width of the stem.
ϕ = 0.75 for shear
λ = modification factor for lightweight concrete

One-way joists are allowed an increase to $1.1 \times V_c$ if the joists are closely spaced.

Stirrups are necessary for strength (as well as crack control): $V_s = \frac{A_v f_{yt} d}{s} \le 8\sqrt{f'_c}\, b_w d\ (max)$

where A_v = area of all vertical legs of stirrup
s = spacing of stirrups
d = effective depth

For shear design:

$$V_u \le \phi V_c + \phi V_s \quad \phi = 0.75 \text{ for shear}$$

Spacing Requirements

Stirrups are required when V_u is greater than $\frac{\phi V_c}{2}$. A minimum is required because shear failure of a beam without stirrups is sudden and brittle and because the loads can vary with respect to the design values.

Table 3-8 ACI Provisions for Shear Design*

		$V_u \le \frac{\phi V_c}{2}$	$\phi V_c \ge V_u > \frac{\phi V_c}{2}$	$V_u > \phi V_c$	
Required area of stirrups, A_v**		none	greater of $\frac{50 b_w s}{f_{yt}}$ and $\frac{0.75\sqrt{f'_c}\, b_w s}{f_{yt}}$		$\frac{(V_u - \phi V_c)s}{\phi f_{yt} d}$
Stirrup spacing, s	Required	—	smaller of $\frac{A_v f_{yt}}{50 b_w}$ and $\frac{A_v f_{yt}}{0.75\sqrt{f'_c}\, b_w}$		$\frac{\phi A_v f_{yt} d}{V_u - \phi V_c}$
	Recommended Minimum†	—	—	4 in.	
	Maximum†† (ACI 11.5.4)	—	$\frac{d}{2}$ or 24 in.	$\frac{d}{2}$ or 24 in. for $(V_u - \phi V_c) \le \phi 4\sqrt{f'_c}\, b_w d$	
				$\frac{d}{4}$ or 12 in. for $(V_u - \phi V_c) > \phi 4\sqrt{f'_c}\, b_w d$	

*Members subjected to shear and flexure only; $\phi V_c = \phi 2\lambda\sqrt{f'_c}\, b_w d$ $\phi = 0.75$ (ACI 11.3.1.1)

**$A_v = 2 \times A_b$ for U stirrups; $f_y \le 60$ ksi (ACI 11.5.2)

†A practical limit for minimum spacing is d/4

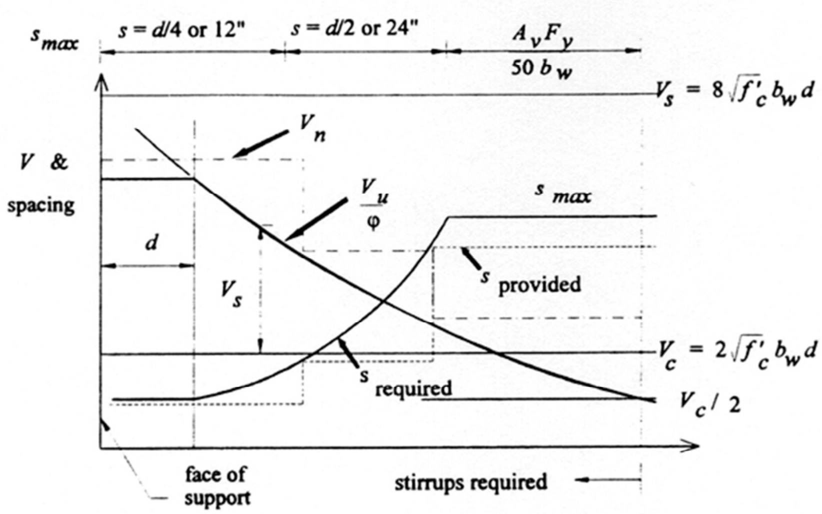
††Maximum spacing based on minimum shear reinforcement (= $A_v f_y / 50 b_w$) must also be considered (ACI 11.5.5.3).

NOTE: section numbers are pre ACI 318-14

Economical spacing of stirrups is considered to be greater than d/4. Common spacings of d/4, d/3 and d/2 are used to determine the values of ϕV_s at which the spacings can be increased.

$$\phi V_s = \frac{\phi A_v f_{yt} d}{s}$$

This figure shows that the size of V_n provided by $V_c + V_s$ (long dashes) exceeds V_u/ϕ in a step-wise function, while the spacing provided (short dashes) is at or less than the required s (limited by the maximum allowed). *(Note that the maximum shear permitted from the stirrups is* $8\sqrt{f'_c}\, b_w d$*)*



The minimum recommended spacing for the first stirrup is 2 inches from the face of the support.

Torsional Shear Reinforcement

On occasion beam members will see twist along the axis caused by an eccentric shape supporting a load, like on an L-shaped spandrel (edge) beam. The torsion results in shearing stresses, and closed stirrups may be needed to resist the stress that the concrete cannot resist.

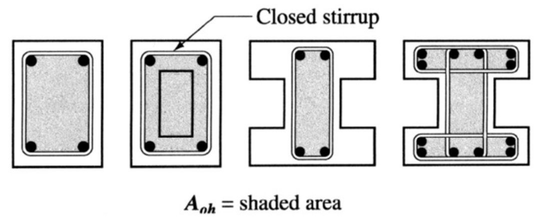


Fig. R11.6.3.6(b)—Definition of $\mathbf{A_{oh}}$

Development Length for Reinforcement

Because the design is based on the reinforcement attaining the yield stress, the reinforcement needs to be properly bonded to the concrete for a finite length (*both sides*) so it won't slip. This is referred to as the development length, l_d. Providing sufficient length to anchor bars that need to reach the yield stress near the end of connections are also specified by hook lengths. *Detailing reinforcement is a tedious job.* The equations for development length must be modified if the bar is epoxy coated or is cast with more than 12 in. of fresh concrete below it. Splices are also necessary to extend the length of reinforcement that come in standard lengths. The equations for splices are not provided here.

<u>Development Length in Tension</u>

With the proper bar to bar spacing and cover, the common development length equations are:

#6 bars and smaller: $l_d = \dfrac{d_b f_y}{25\lambda\sqrt{f_c'}}$ or 12 in. minimum

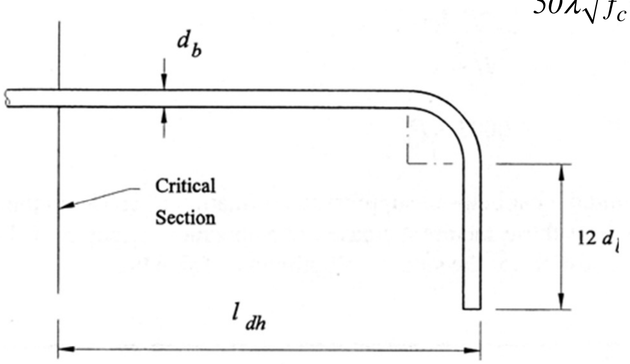
#7 bars and larger: $l_d = \dfrac{d_b f_y}{20\lambda\sqrt{f_c'}}$ or 12 in. minimum

<u>Development Length in Compression</u>

$$l_d = \frac{d_b f_y}{50\lambda\sqrt{f_c'}} \leq 0.0003 f_y d_b \text{ or 8 in. minimum}$$

<u>Hook Bends and Extensions</u>

The minimum hook length is $l_{dh} = \dfrac{d_b f_y}{50\lambda\sqrt{f_c'}}$ but not less than the larger of $8d_b$ and 6 in.



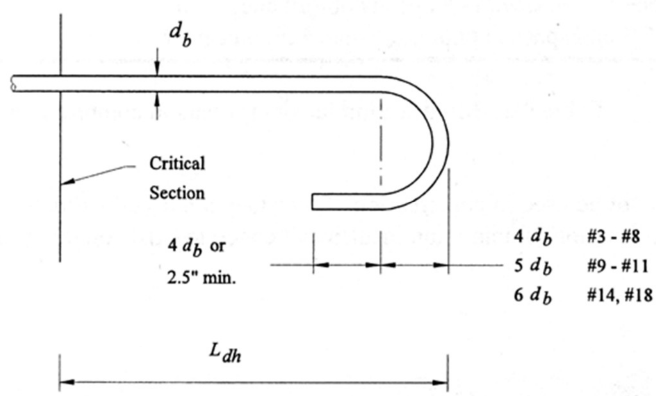


Figure 9-17: Minimum requirements for 90°bar hooks. **Figure 9-18**: Minimum requirements for 180° bar hooks.

Modulus of Elasticity & Deflection

E_c for deflection calculations can be used with the transformed section modulus in the elastic range. After that, the cracked section modulus is calculated and E_c is adjusted.

Code values:

$$E_c = 57{,}000\sqrt{f'_c} \text{ (normal weight)} \qquad E_c = w_c^{1.5} 33\sqrt{f'_c},\ w_c = 90 \text{ lb/ft}^3 - 160 \text{ lb/ft}^3$$

Deflections of beams and one-way slabs need not be computed if the overall member thickness meets the minimum specified by the code, and are shown in Table 7.3.1.1 (see *Slabs*). The span lengths for continuous beams or slabs is taken as the clear span, l_n.