Equilibrium and Determinacy

Equilibrium Equation

- For general 3D equilibrium:

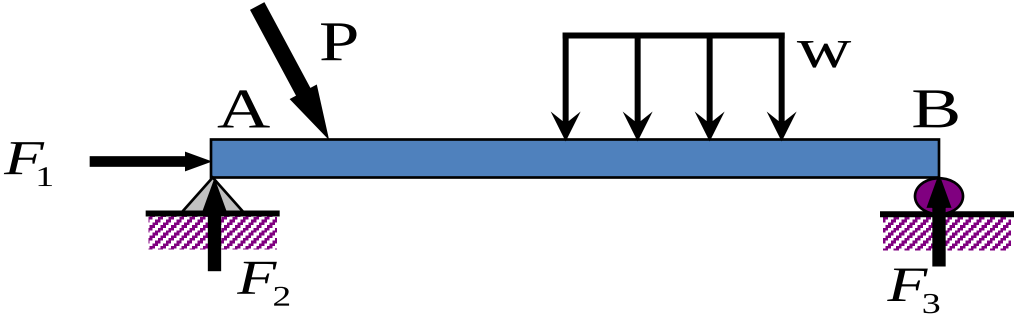
$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum F_z = 0$$

$$\sum M_x = 0 \qquad \sum M_y = 0 \qquad \sum M_z = 0$$

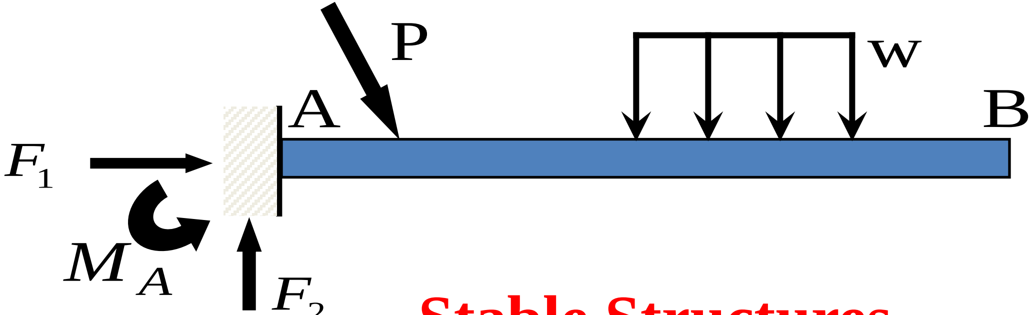
- For 2D structures, it can be reduced to:

$$\sum F_x = 0$$
$$\sum F_y = 0$$
$$\sum M_o = 0$$

Equilibrium and Determinacy

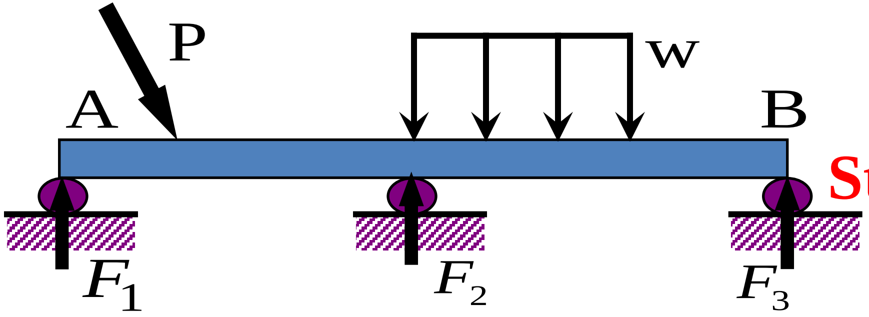


3 EQs → 3 unknown reactions



Stable Structures

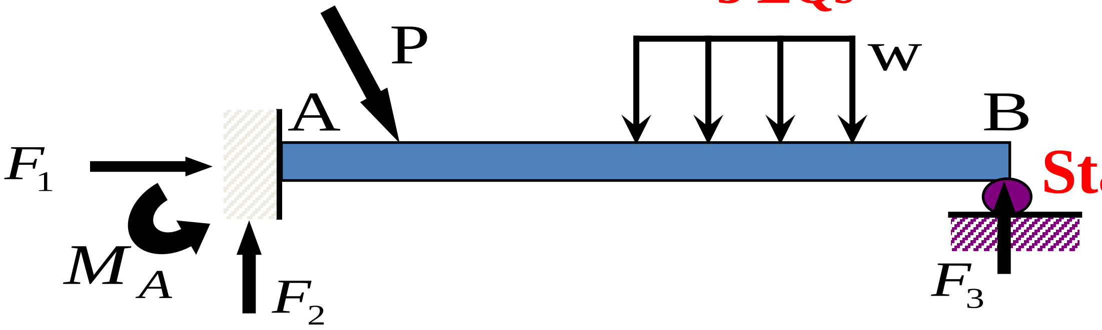
Equilibrium and Determinacy



Stable Structures?

3 EQs → 3 unknown reactions

Not properly supported



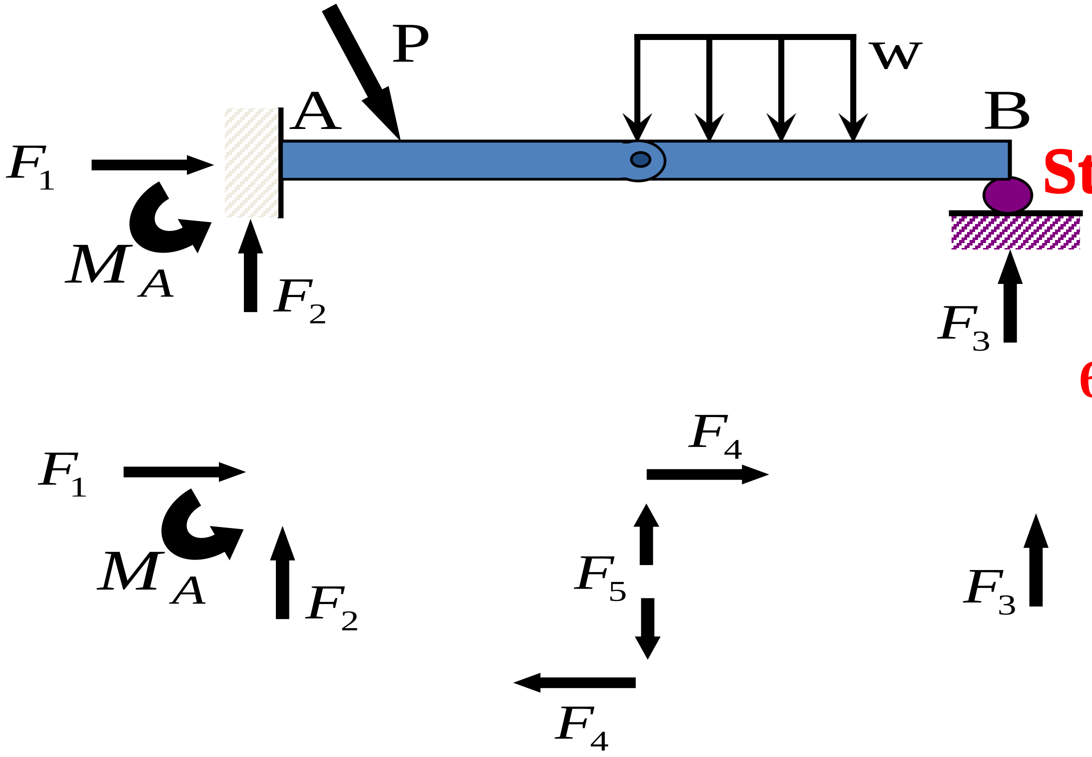
Stable Structures?

3 EQs → 4 unknown reactions

Indeterminate stable

1 degree indeterminacy

Equilibrium and Determinacy



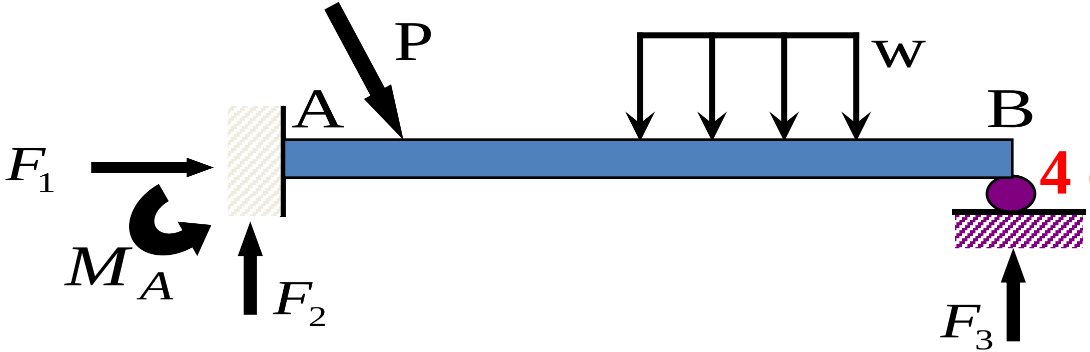
Stable Structures ?

6 equilibrium conditions

=

6 unknown forces

Equilibrium and Determinacy



4 equilibrium conditions

4 unknown forces

Equilibrium and Determinacy

- Equilibrium and Determinacy
  - If the reaction forces can be determined from the equilibrium EQs → STATICALLY DETERMINATE STRUCTURE
  - No. of unknown forces > equilibrium EQs → STATICALLY INDETERMINATE
  - Can be viewed globally or locally (via free body diagram)

Equilibrium and Determinacy

- Determinacy and Indeterminacy
  - For a 2D structure

No. of components

$$r = 3n, \quad \text{statically determinate}$$
$$r > 3n, \quad \text{statically indeterminate}$$

No. of unknown forces

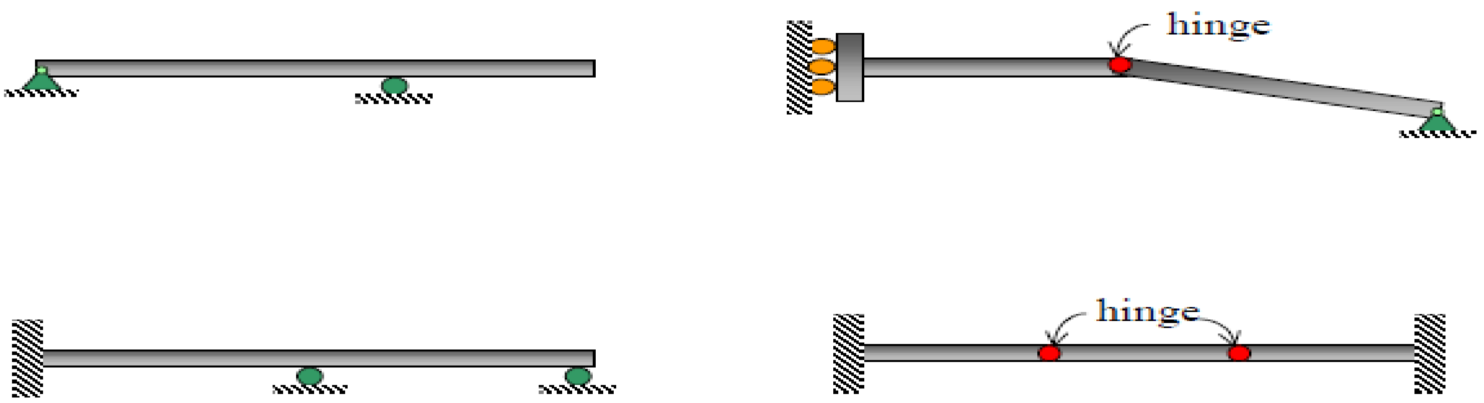
$r - 3n$: degree of indeterminacy

  - The additional EQs needed to solve for the unknown forces are referred to as compatibility EQs

Discuss the Determinacy

<u>Example 2-1</u>

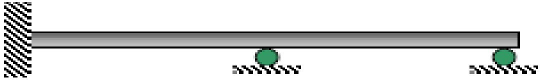
Classify each of the beams shown below as statically determinate or statically indeterminate. If statically indeterminate, report the number of degrees of indeterminacy. The beams are subjected to external loadings that are assumed to be known and can act anywhere on the beams.

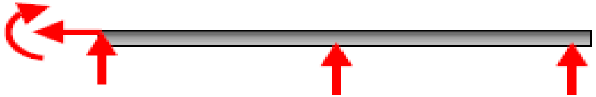


Discuss the Determinacy

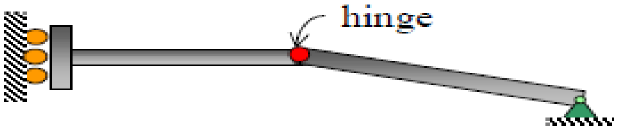
SOLUTION

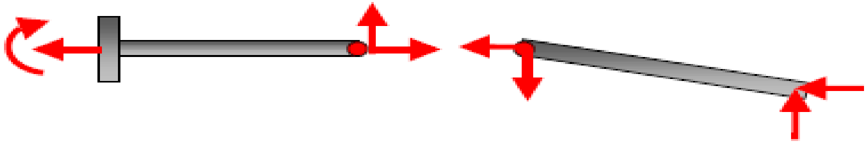
$r = 3,\ n = 1,\ 3 = 3(1)$ Statically **determinate**



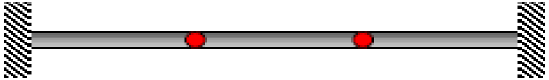


$r = 5,\ n = 1,\ 5 - 3(1) = 2$ Statically **indeterminate** to the second degree





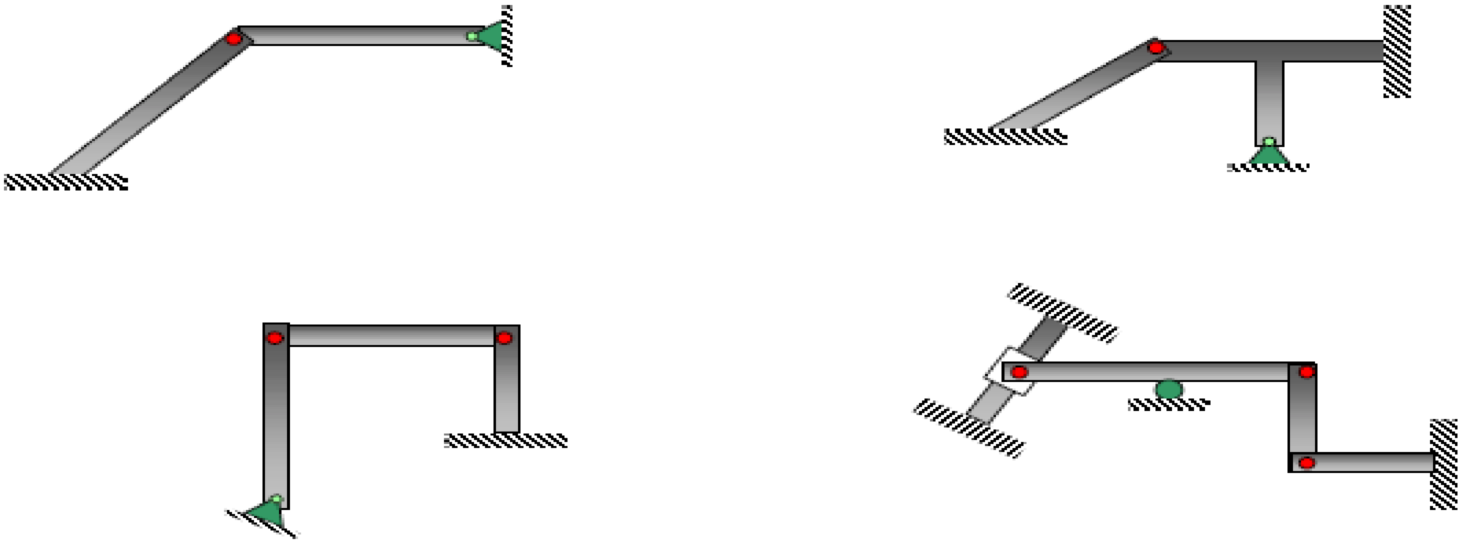
$r = 6,\ n = 2,\ 6 = 3(2)$ Statically **determinate**



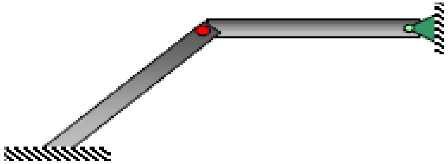
$r = 10,\ n = 3,\ 10 - 3(3) = 1$ Statically **indeterminate** to the first degree

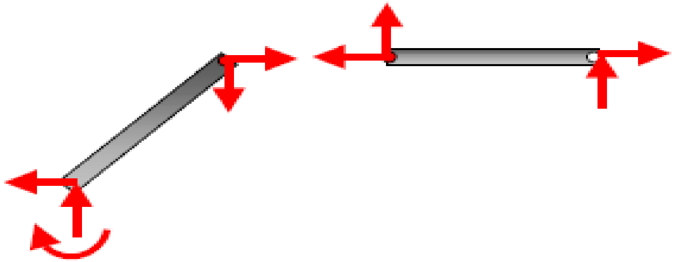
Example 2-2

Classify each of the pin-connected structures shown in figure below as statically determinate or statically indeterminate. If statically are subjected to arbitrary external loadings that are assumed to be known and can act anywhere on the structures.



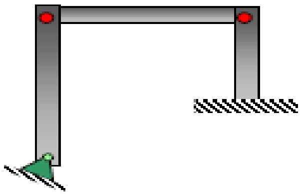
SOLUTION

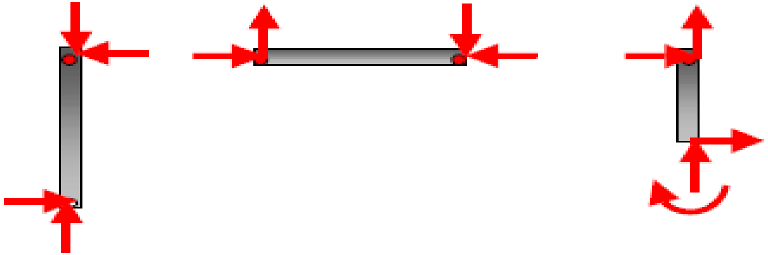




$r = 7$, $n = 2$, $7 - 3(2) = 1$

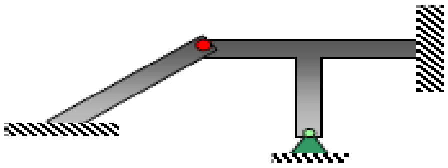
Statically indeterminate to the first degree

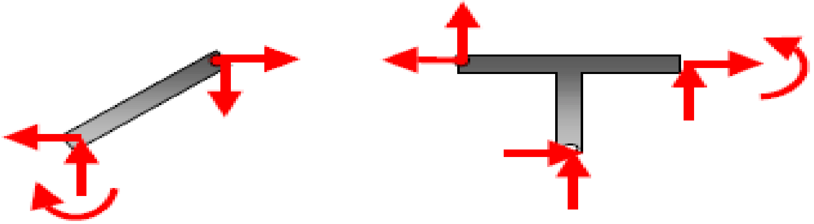




$r = 9$, $n = 3$, $9 = 3(3)$

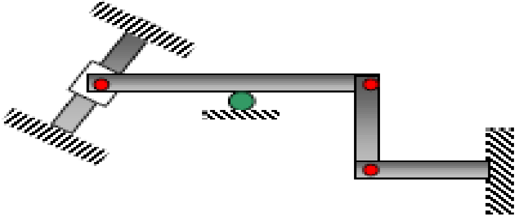
Statically determinate

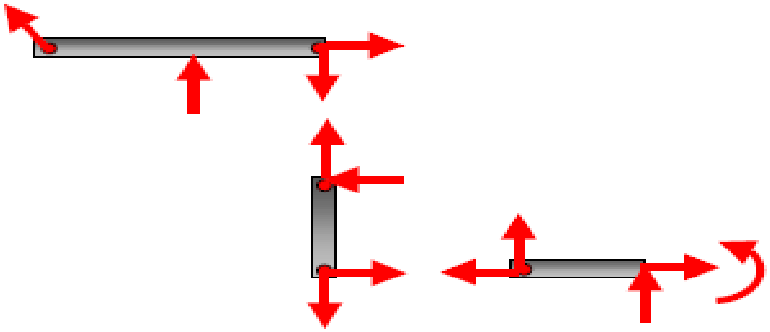




$r = 10$, $n = 2$, $10 - 6 = 4$ degree

Statically **indeterminate** to the fourth



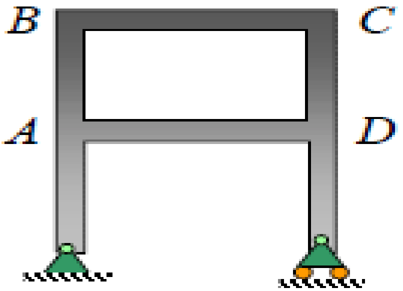


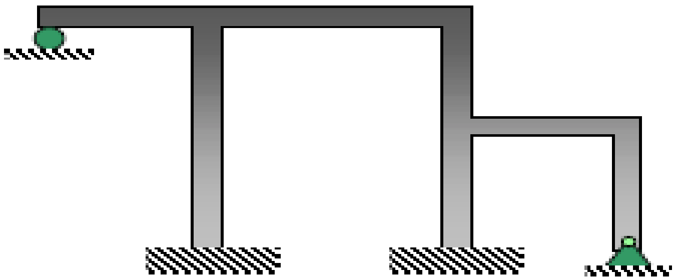
$r = 9$, $n = 3$, $9 = 3(3)$

Statically **determinate**

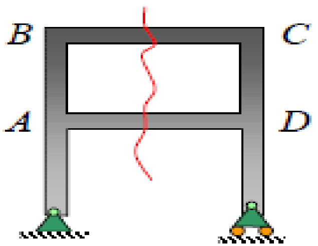
Example 2-3

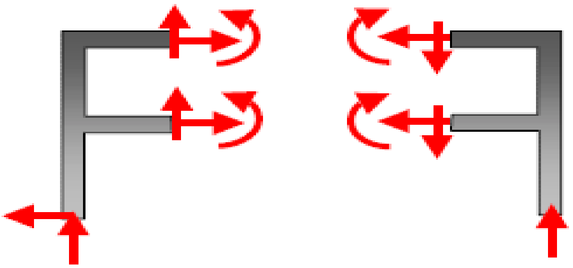
Classify each of the frames shown in figure below as statically determinate or statically indeterminate. If statically indeterminate, report the number of degrees of indeterminacy. The frames are subjected to external loadings that are assumed to be known and can act anywhere on the frames.





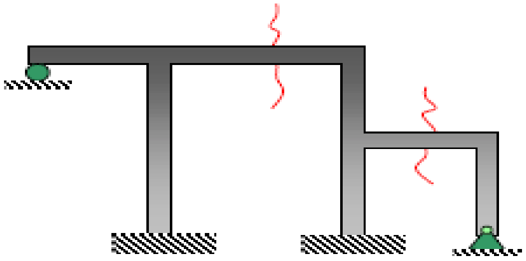
SOLUTION

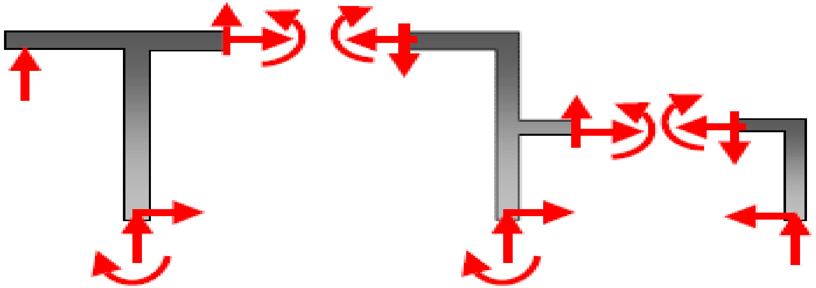




$r = 9,\ n = 2,\ 9 - 6 = 3$

Statically indeterminate to the third degree





$r = 15,\ n = 3,\ 15 - 9 = 6$

Statically indeterminate to the sixth degree