Erp

$$\lim_{x \to \frac{\pi}{2}} (\sec x - \tan x) = \infty - \infty$$

Sol

$$\lim_{x \to \frac{\pi}{2}} \frac{1}{\cos x} - \frac{\sin x}{\cos x}$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{1 - \sin x}{\cos x}$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{1 - \sin x}{\cos x} * \frac{1 + \sin x}{1 + \sin x}$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{1 - \sin^2 x}{\cos x (1 + \sin x)} = \lim_{x \to \frac{\pi}{2}} \frac{\cos^2 x}{\cos x (1 + \sin x)}$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{\cos x}{(1 + \sin x)} = \frac{0}{1+1} = \frac{0}{2} = 0$$

$$\lim_{x \to \frac{\pi}{4}} \tan 2x \cdot \tan\left(\frac{\pi}{4} - x\right) \qquad (\infty \cdot 0)$$

Sol.:

let

$$y = \frac{\pi}{4} - x \qquad x \to \frac{\pi}{4}$$

$$x = \frac{\pi}{4} - y \qquad y \to 0$$

$$= \lim_{y \to 0} \tan 2\left(\frac{\pi}{4} - y\right) \cdot \tan y$$

$$= \lim_{y \to 0} \tan\left(\frac{\pi}{2} - 2y\right) \cdot \tan y$$

$$= \lim_{y \to 0} \frac{\sin\left(\frac{\pi}{2} - 2y\right)}{\cos\left(\frac{\pi}{2} - 2y\right)} \cdot \frac{\sin y}{\cos y}$$

$$= \lim_{y \to 0} \frac{\cos 2y}{\sin 2y} \cdot \frac{\sin y}{\cos y} \cdot \frac{2y}{2y}$$

$$= \lim_{y \to 0} \frac{\cos 2y}{\cos 2y} \cdot \frac{2y}{\sin 2y} \cdot \frac{\sin y}{2y} = \frac{1}{2}$$

$$\lim_{x \to \infty} \frac{x^2 - \sin\frac{1}{x}}{x + \sqrt{2x^3 - 1}}$$

sol:

$$\lim_{x \to \infty} \frac{\frac{x^2}{x^2} \cdot \sin\frac{1}{x}}{\frac{x}{x^2} + \sqrt{\frac{2x^3}{x^4} + \frac{1}{x^3}}}$$

$$= \lim_{x \to \infty} \frac{\sin\frac{1}{x}}{\frac{1}{x} + \sqrt{\frac{2}{x} + \frac{1}{x^4}}}$$

let $y = \frac{1}{x}$ when $x \to \infty$

$x = \frac{1}{y}$ when $y \to 0$

$$= \lim_{y \to 0} \frac{\sin y}{y + \sqrt{2y + y^4}} \cdot$$

$$\lim_{y \to 0} \frac{\sin y}{y + \sqrt{2y + y^4}} \cdot \frac{y - \sqrt{2y + y^4}}{y - \sqrt{2y + y^4}}$$

$$\lim_{y \to 0} \frac{y - \sqrt{2y + y^4} - \sin y}{y^2 - (2y + y^4)}$$

$$= \lim_{y \to 0} \frac{y - \sqrt{2y + y^4} \cdot \sin y}{(y - 2 - y^3)\, y}$$

$$= \lim_{y \to 0} \frac{y \sqrt{2y - y^4}}{y - 2 - y^3} = \frac{0}{-2} = 0$$

Hopital's Rule

١- قاعدة لحل مسائل limit ولها شروط:

أ- لا تستخدم الا عندما يطلب استخدامها.

ب- تستخدم عندما يكون ناتج limit بعد التعويض $\frac{0}{0}$ ، $\frac{\infty}{\infty}$

ملاحظة: اشتقاقها يكون باشتقاق البسط والمقام كلا على حده:

Exp:- use Hopital's Rule

$$\lim_{x \to 0} \frac{1-\cos x}{x+x^2}$$ بالتعويض $\frac{0}{0}$

sol:-

$$\lim_{x \to 0} \frac{0-(-\sin x)}{1+2x}$$

$$= \lim_{x \to 0} \frac{\sin x}{1+2x} = \frac{0}{1+0} = 0$$

Exp use Hopital's Rule

$$\lim_{x \to 0} \frac{x-\sin x}{x^3} = \frac{0}{0}$$

sol:

$$\lim_{x \to 0} \frac{1-\cos x}{3x^2} = \frac{0}{0}$$

$$\lim_{x \to 0} \frac{\sin x}{6x} = \frac{0}{0}$$

$$\lim_{x \to 0} \frac{\cos x}{6} = \frac{1}{6}$$

The continuos Function

A function (f) is said to be continuos at $x=a$ if:-

① F(a) exists or difind

② $\lim\limits_{x \to a^+} f(x)$ exists

③ $\lim\limits_{x \to a^-} f(x) = f(a)$

otherwise the function is not continuos

EXP:

$$F(x) = \begin{cases} x^2+1 & x \geq 0 \\ x & x < 0 \end{cases}$$

is the function continuos at $x=0$?

sol:-

① $F(a) \rightarrow$ exists

$f(0) = (0)^2+1 = 1$ exist

② $\lim\limits_{x \to 0^+} F(x) = (0)^2+1 = 1$

③ $\lim\limits_{x \to 0^-} f(x) = 0$

$\therefore \lim\limits_{x \to 0^+} f(x) \neq \lim\limits_{x \to 0^-} f(x)$

dis continuos function at $x=0$

Exp:-

Is the function $f(x) = \begin{cases} x^2-1 & x<1 \\ 0 & x=1 \\ x & x>1 \end{cases}$

Is continuos at $x=1$

Sol:-

① $F(x) = 0$

$f(1) = 0$

$F(x) = x$

② $\lim\limits_{x \to 1^+} x = 1$ الغاية من جهة اليمين

③ $\lim\limits_{x \to 1^-} x^2-1 = 0$ الغاية من جهة اليسار ,

$\therefore \lim\limits_{x \to 1^+} f(x) \neq \lim\limits_{x \to 1^-} f(x)$

the function dis continuos at $x=1$

Exp:- Given $F(x) = \dfrac{|x|}{x}$ is continous at $x = 0$

$$f(x) = |x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

$$F(x) = \begin{cases} \dfrac{x}{x} & x \geq 0 \\ -\dfrac{x}{x} & x < 0 \end{cases}$$

$$F(x) = \begin{cases} 1 & x \geq 0 \\ -1 & x < 0 \end{cases}$$

① $f(0) = 1$

② $\lim\limits_{x \to 0^+} (1) = 1$

③ $\lim\limits_{x \to 0^-} (-1) = -1$

∴ الغايه من جهه اليمين $\neq$ الغايه من جهه اليسار

the Function discontinuos at $x = 0$

let $y = \begin{cases} 3x^2 - 2x - 4 & -2 \le x < -1 \\ 2x + a & -1 \le x \le 2 \\ bx^2 + 2 & 2 < x \le 4 \end{cases}$

1. Find the value of the constant (a) that will make the function continuos at $x = -1$?

2. Find the value of the constant (b) that will make the function continuos at $x = 2$

Sol-

1. the Function is continuos at $x = -1$

$$\lim_{x \to -1^+} f(x) = \lim_{x \to -1^-} f(x)$$

$$\lim_{x \to -1^+} 2x + a = \boxed{-2 + a}$$

$$\lim_{x \to -1^-} 3x^2 - 2x - 4 = 3 - 2(-1) - 4 = 3 + 2 - 4 = 1$$

$$\therefore -2 + a = 1 \longrightarrow \boxed{a = 3}$$

2. the Function is continuos at $x = 2$

$$y = \begin{cases} 3x^2 - 2x - 4 & -2 \le x < 1 \\ 2x + 3 & -1 \le x \le 2 \\ bx^2 + 2 & 2 < x \le 4 \end{cases}$$

$$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^-} f(x)$$

use $f(x) = 2x + 3 \longrightarrow$ من جهة اليسار

$$\lim_{x \to 2^-} 2x + 3 = 2 \times 2 + 3 = 7$$

use $f(x) = bx^2 + 2$ صبا طرابلسية

$$\lim_{x \to 2^+} bx^2 + 2 = 4b + 2$$

$$\therefore \; 4b + 2 = 7$$

$$\boxed{b = \frac{5}{4}}$$