GENERAL PRINCIPLES

***Mechanics*:** is a branch of physical sciences that is concerned with the state of rest or motion of bodies that are subjected to the action of forces.

In general, this subject can be divided into three branches:

- Rigid-body mechanics,
- Deformable-body mechanics, and
- Fluid mechanics.

Rigid-body mechanics is divided into two areas:

1. **Static:** deals with the equilibrium of bodies, that is, those that are at rest or move with a constant velocity.
2. **Dynamics:** is concerned with the accelerated motion of bodies.

Note: We can consider static special case of dynamics, in which the acceleration is zero.

Fundamental Concepts

Basic Quantities: The basic quantities which used throughout mechanics are:

Length, Time, Mass, and Force

Force: ***force*** is the action of one body to another. A force tends to move a body in the direction of its action.

Note: A force is completely characterized by its magnitude, direction and point of application.

Idealizations: Models or idealization are used in mechanics in order to simplify application of theory.

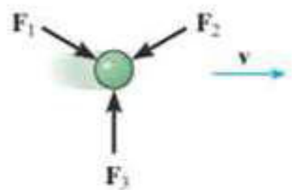
Particle: A **particle** is a body of negligible dimensions. When a body is idealized as a particle, the principles of mechanics reduce to a rather simplified form since the geometry of the body *will not be involved* in the analysis of the problem.

Rigid body: A **rigid body** can be considered as a combination of a large number of particles in which all the particles remain at a fixed distance from one another, both before and after applying load. In most cases the actual deformations occurring in structures, machines, mechanisms, and the like are relatively small, and rigid-body assumption is suitable for analysis.

Concentrated Force: A **concentrated force** represents the effect of a loading which is assumed to act at a point on a body.

Newton's Three Laws of Motion:

First Law: A particle originally at rest, or moving in a straight line with constant velocity, tends to remain in this state provided the particle is *not* subjected to an unbalanced force.

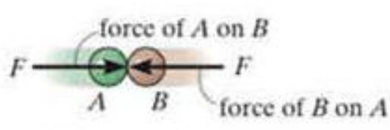


Equilibrium

Second Law: A particle acted upon by an unbalanced Force ***F*** experiences an acceleration ***a*** that has the same direction as the force and magnitude that is directly proportional to the force. If ***F*** is applied to a particle of mass ***m***, this law may be mathematically as

$$F = ma \qquad \text{...(1-1)}$$

Third Law: The forces of action and reaction between particles are equal, in magnitude, opposite in direction and collinear (they lie in the same direction).

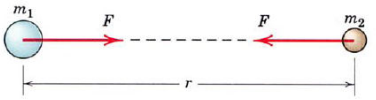


Action – reaction

Newton's Law of Gravitational Attraction:

Newton postulated a law governing the gravitational attraction between any two particles. Stated mathematically:

$$F = G\frac{m_1 m_2}{r^2} \qquad \text{...(1-2)}$$



Where F = force of gravitation between the two particles,

G = universal constant of gravitational according experimental evidence, $G = 66.73(10^{-12})\ m^3/(kg.s^2)$,

m_1, m_2 = mass of each of the two particles, and

r = *distance* between the two particles.

Weight: The **weight** of body is the gravitational force acting on it.

From Eq. 1-2, we can develop an approximate expression for finding the weight $\boldsymbol{W}$ of a particle having a mass $\boldsymbol{m_1 = m}$. If we assume the earth to be a nonrotating sphere of constant density and having a mass $\boldsymbol{m_2 = M_e}$, then if $\boldsymbol{r}$ is the distance between the earth's center and the particle, we have

$$F = G\frac{mM_e}{r^2} \qquad \text{...(1-3)}$$

Letting $\boldsymbol{g = GM_e/r^2}$ yields:

$$W = mg \qquad \text{....(1-4)}$$

By comparison with $\boldsymbol{F = ma}$, we can see that $\boldsymbol{g}$ is the acceleration due to gravity. Since it depends on $\boldsymbol{r}$**,** then weight of a body is *not* an absolute quantity. Instead, its magnitude is determined from where the measurement was made. For most engineering calculations, however, g is determined at sea level and at latitude of 45°, which is considered the "standard location".

Units of Measurement

The four basic quantities (length, time, mass, and force) are not all independent from one another: in fact they are related by Newton's second law of motion, $F = ma$.

SI Units: The **I**nternational **S**ystem (SI) (from French, System International d'units) defines length in meter (m), time in seconds (s), and mass in kilogram. The unit of force, called a Newton (N), is derived from $F = ma$.

Thus **1 Newton** is equal to a force required to give 1 kilogram of mass an acceleration of 1 m/s^2 ($N = kg.m/s^2$). The value of g, in the standard location, was found to be 9.806 65 m/s^2; however for calculations the value $g = 9.81\ m/s^2$ will be used.

U.S. Customary: In the U. S. Customary system of units (or British System of units) (FPS) length is measured in feet (ft), time in seconds, and force in pounds (lb). The unit of mass, called a *slug*, is derived from $F = ma$.

Hence, **1 slug** is equal to the amount of matter accelerated at 1 ft/s^2 when acted upon by a force of 1Ib ($slug = Ib.s^2/ft$). Therefore if measurements are made at the "standard location" where $g = 32.2\ ft/s^2$, then from Eq. 1-3,

$$m = \frac{W}{g} \qquad (g = 32.2\ ft/s^2) \qquad \textbf{...(1-5)}$$

TABLE 1-1 Systems of units

Name	Length	Time	Mass	Force
International System (SI)	meter	second	Kilogram	Newton*
	m	s	Kg	$N = kg.m/s^2$
U. S. Customary system (FPS)	foot	second	Slug	pound
	ft	s	$slug = lb.s^2/ft$	lb

* Derived unit

Conversion of Units:

In the FPS system, 1 ft =12 in. (inches),

5280 ft = 1 mi (mile),

1000 lb = 1 kip (kilo-pound), and

2000 lb = 1 ton.

TABLE 1-2 Conversion Factors

Quantity	Unit of Measurement (FPS)	Equals	Unit of Measurement (SI)
Force	lb		4.448 N
Mass	slug		14.59 kg
Length	ft		0.304 8 m

Prefixes: When a numerical quantity is either very large or very small, the units used to define its size may be modified by using a prefix.

Note: The SI system does not include the multiple deca (10) or the submultiple centi (0.01), which form part of metric system. Except for some volume and area measurements, the use of these prefixes is to be avoided in science and engineering.

TABLE 1-3 Prefixes

	Exponential form	Prefix	SI Symbol
Multiple			
1 000 000 000	10^9	giga	*G*
1 000 000	10^6	mega	*M*
1 000	10^3	kilo	*K*
Submultiple			
0.001	10^{-3}	milli	*m*
0.000 001	10^{-6}	micro	μ
0.000 000 001	10^{-9}	nano	*N*

* The kilogram is the only base unit that is defined with a prefix

Example 1: Convert 2km/h to m/s. How many ft/s is this?

Solution: Since 1 km = 1000 m and 1 h = 3600 s, the factors of conversion are arranged in the following order, so that a cancellation of units can be applied;

$$2km/h = \frac{2km}{h}\left(\frac{1000m}{km}\right)\left(\frac{1h}{2600s}\right)$$
$$= \frac{2000m}{3600s} = 0.556m/s$$

From Table 1-2, 1 ft = 0.304 8m. Thus,

$$0.556m/s = \left(\frac{0.556m}{s}\right)\left(\frac{1\,ft}{0.3048m}\right)$$
$$= 1.82\,ft/s$$

Note: Remember to round off the final answer to three significant figures.

Example 2: Convert the quantities 300 lb.s and 52 slug/ft^3 to appropriate SI units.

Solution: Using Table 1-2, 1 lb =4.448 2 N

$$300\ lb.s = 300\ lb.s * \left(\frac{4.448N}{1\ lb}\right)$$
$$= 1334.5N.s = 1.33kN.s$$

Since 1 slug = 14.593 8 kg and 1 ft = 0.304 8 m, then

$$52slug/ft^3 = \frac{52slug}{ft^3} * \left(\frac{14.59kg}{1slug}\right)\left(\frac{1\,ft}{0.3048m}\right)^3$$
$$= 26.8(10)^3\,kg/m^3$$
$$= 26.8Mg/m^3$$

Example 3: Evaluate each of the following and express with SI units having an appropriate prefix:

(a) (50 mN)(6 GN),

(b) (400 mm)(0.6 MN)^2,

(c) 45mN^3/900Gg.

Solution: First convert each number to base units, perform the indicated operations, and then choose an appropriate prefix.

Part (a)

$$(50\text{ mN})(6\text{ GM}) = [50(10^{-3})\text{N}][6(10^{9})\text{N}]$$

$$= 300(10^{6})\text{ N}^2$$

$$= 300(10^{6})\text{ }\cancel{\text{N}^2}\left(\frac{1kN}{10^3\,\cancel{N}}\right)\left(\frac{1kN}{10^3\,\cancel{N}}\right)$$

$$= 300\text{ kN}^2$$

Part (b)

$$(400\text{ mm})(0.6\text{ MN})^2 = [300(10^{-3})\text{m}][0.6(10^{6})\text{N}]^2$$

$$= [300(10^{-3})\text{m}][0.36(10^{12})\text{N}^2]$$

$$= 144(10^{9})\text{ m.N}^2$$

$$= 144\text{ Gm.N}^2$$

We can also write

$$144(10^{9})\text{ m.N}^2 = 144(10^{9})\text{m.}\cancel{\text{N}^2}\left(\frac{1MN}{10^6\,\cancel{N}}\right)\left(\frac{1MN}{10^6\,\cancel{N}}\right)$$

$$= 0.144\text{ m. GN}^2$$

Part (c)

$$\frac{45MN^3}{900Gg} = \frac{45(10^6\,N)^3}{900(10^6)kg} = 50(10^{9})\text{N}^3\text{/kg}$$

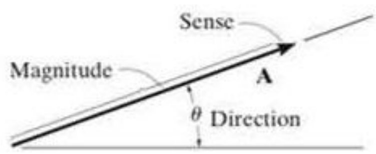
$$= 50(10^{9})\cancel{\text{N}^3}\left(\frac{1kN}{10^3\,\cancel{N}}\right)^3\frac{1}{kg} = 50\text{ kN}^3\text{/kg}$$

Scalars and Vectors

All physical quantities in engineering mechanics are measured using either scalars or vectors.

A **scalar** is a quantity that has magnitude only. Examples of scalar quantities include length, mass, and time. Because scalars possess only magnitudes, they are real numbers that can be positive, negative, or zero.

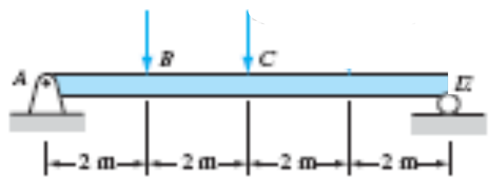
A **vector** is a quantity that possesses magnitude and direction and obeys the parallelogram law for addition. Examples of vectors encountered in static are force, position, and moment. A vector is shown graphically by an arrow. The length of the arrow represents *the magnitude of the vector*, and angle θ between the vector and a fixed axis defines *the direction of its line of action*. The head or tip of the arrow indicates *the sense of direction of the vector*.



Vector quantities can be further divided into:

1. **Free vector** is one whose action is not confined to or associated with a unique line in space. The wind and moment of a couple are examples of free vectors. Their effect does not depend on their position.

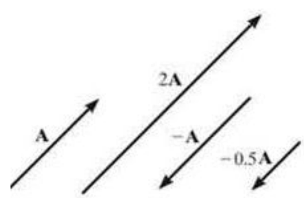
2. **Localized vector** has a definite or specific line of action. Consider the beam shown in figure below. When the load is placed in the position C at the center of beam, the reaction of the supports at A and D on the beam are equal. If the load were moved to position B, the support at A would carry more of the load and the support D carry less. In the other words, the effect of the supports on the beam (the external effect) depend on the position of the load it carries as well as the slope, sense, and magnitude of that load.



Vector Operations

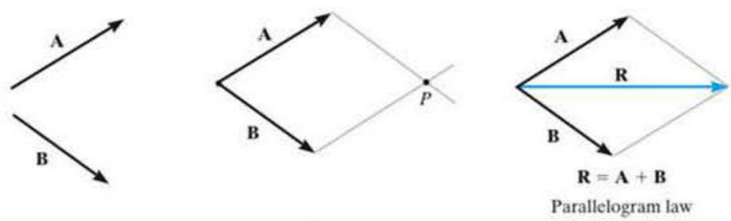
Multiplication and Division of a Vector by a Scalar:

Vector is multiplied by a positive scalar; its magnitude is increased by that amount. When multiplied by a negative scalar it will also change the directional sense of the vector.



Vector Addition: All vector quantities obey the *parallelogram law of addition*. To illustrate the two "*component*" vectors ***A*** and ***B*** figure below are added to form a "*resultant*" vector ***R*** = ***A*** + ***B*** using the following procedure:

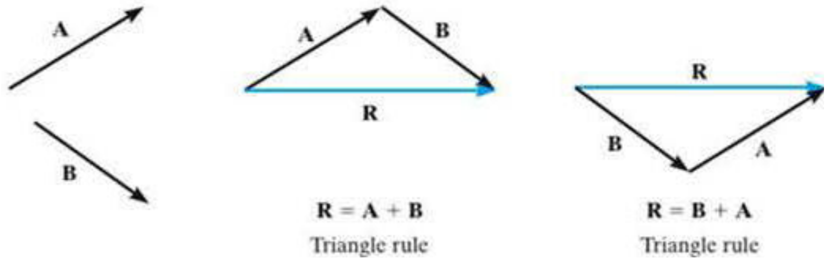
- First joint the tails of the components at a point so that it makes them concurrent
- From the head of ***B***, draw a line parallel to ***A***. Draw another line from the head of ***A*** that is parallel to ***B***. These tow lines intersect at point P to form the adjacent sides of a parallelogram.
- The diagonal of this parallelogram that extends to *P* forms ***R***, which



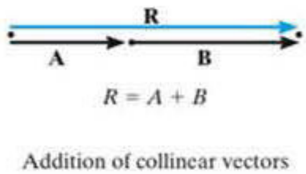
then represents the resultant vector ***R*** = ***A*** + ***B***.

We can also add ***B*** to ***A*** using triangle rule which is special case of the parallelogram law, whereby vector ***B*** is added to vector ***A*** in a "head-to tail" fashion, i.e., by connecting the head of ***A*** to the tail of ***B***. The resultant ***R*** extends from the tail of ***A*** to the head of ***B***. In a similar manner, ***R*** can also obtained by adding ***A*** to ***B***. By comparison, it is seen that vector addition is commutative; in other words, the vectors can be added in either, i.e.

$$\boldsymbol{R} = \boldsymbol{A} + \boldsymbol{B} = \boldsymbol{B} + \boldsymbol{A} \qquad \text{...(1-6)}$$



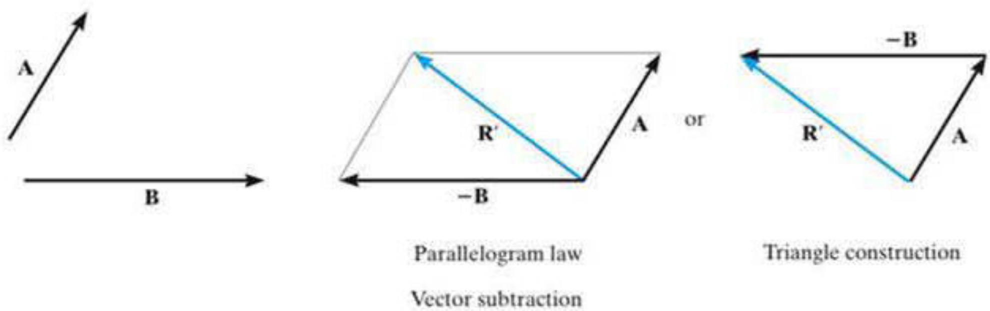
As a special case, if the two vectors ***A*** and ***B*** are collinear, i.e., both have the same line of action, the parallelogram law reduce to an *algebraic* or *scalar addition* $R = A + B$.



Vector Subtraction: The resultant of the difference between two vectors ***A*** and ***B*** of the same type may be expressed as

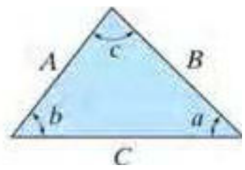
$$\boldsymbol{R} = \boldsymbol{A} - \boldsymbol{B} = \boldsymbol{A} + (-\boldsymbol{B}) \qquad \textbf{...(1-7)}$$

Subtraction is therefore defined as special case of addition, so the rules of vectors addition also apply to vector subtraction.



Cosine Law and Sine law:

The **cosine law** and **sine law** are applicable to compute angles and sides of a triangle



- **Law of cosines**: $C = \sqrt{A^2 + B^2 - 2A.B\cos c}$ **…(1-8)**
- **Law of sines:** $\dfrac{A}{\sin a} = \dfrac{B}{\sin b} = \dfrac{C}{\sin c}$ **…(1-9)**