Simple Stresses and Strains

In this chapter general meaning of stress is explained. Expressions for stresses and strains is derived with the following assumptions:

1. For the range of forces applied the material is elastic *i.e.* it can regain its original shape and size, if the applied force is removed.
2. Material is homogeneous *i.e.* every particle of the material possesses identical mechanical properties.
3. Material is isotropic *i.e.* the material possesses identical mechanical property at any point in any direction.

Presenting the typical stress-strain curve for a typical steel, the commonly referred terms like limits of elasticity and proportionality, yield points, ultimate strength and strain hardening are explained.

Linear elastic theory is developed to analyse different types of members subject to axial, shear, thermal and hoop stresses.

MEANING OF STRESS

When a member is subjected to loads it develops resisting forces. To find the resisting forces developed a section plane may be passed through the member and equilibrium of any one part may be considered. Each part is in equilibrium under the action of applied forces and internal resisting forces. The resisting forces may be conveniently split into normal and parallel to the section plane. The resisting force parallel to the plane is called *shearing resistance*. The intensity of resisting force normal to the sectional plane is called *intensity of Normal Stress* (Ref. Fig. 8.1).

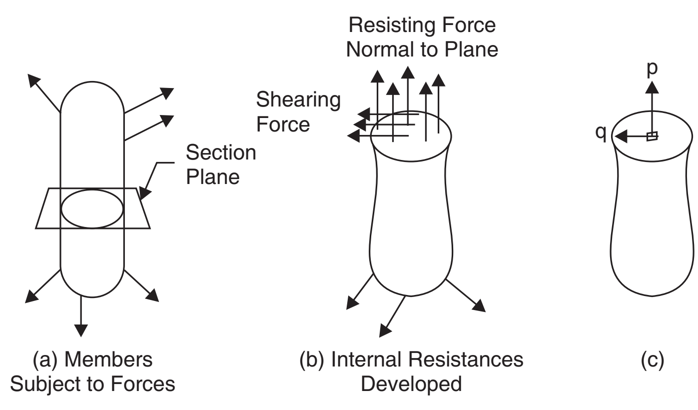


Fig. 8.1

In practice, intensity of stress is called as ''stress'' only. Mathematically

$$\text{Normal Stress} = p = \lim_{\Delta A \to 0} \frac{\Delta R}{\Delta A}$$

$$= \frac{dR}{dA} \quad \text{...(8.1)}$$

where R is normal resisting force.

The intensity of resisting force parallel to the sectional plane is called *Shearing Stress* (q).

$$\text{Shearing Stress} = q = \lim_{\Delta A \to 0} \frac{\Delta Q}{\Delta A} = \frac{dQ}{dA} \quad \text{...(8.2)}$$

where Q is Shearing Resistance.

Thus, *stress at any point may be defined as resistance developed per unit area.* From equations (8.1) and (8.2), it follows that

$$dR = pdA$$

or $$R = \int pdA \quad \text{...(8.3}a\text{)}$$

and $$Q = \int qdA \quad \text{...(8.3}b\text{)}$$

At any cross-section, stress developed may or may not be uniform. In a bar of uniform cross-section subject to axial concentrated loads as shown in Fig. 8.2*a*, the stress is uniform at a section away from the applied loads (Fig. 8.2*b*); but there is variation of stress at the section near the applied loads (Fig. 8.2*c*).

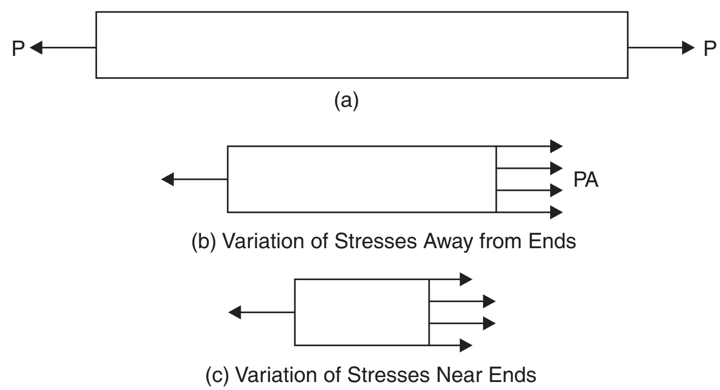


Fig. 8.2

Similarly stress near the hole or at fillets will not be uniform as shown in Figs. 8.3 and 8.4. It is very common that at some points in such regions maximum stress will be as high as 2 to 4 times the average stresses.

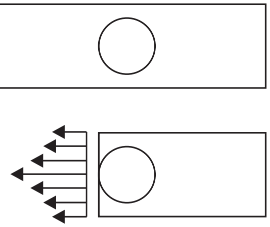


Fig. 8.3. Stresses in a Plate with a Hole

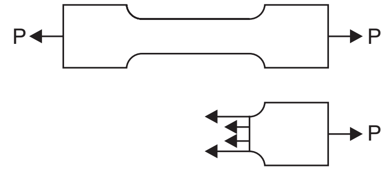


Fig. 8.4

UNIT OF STRESS

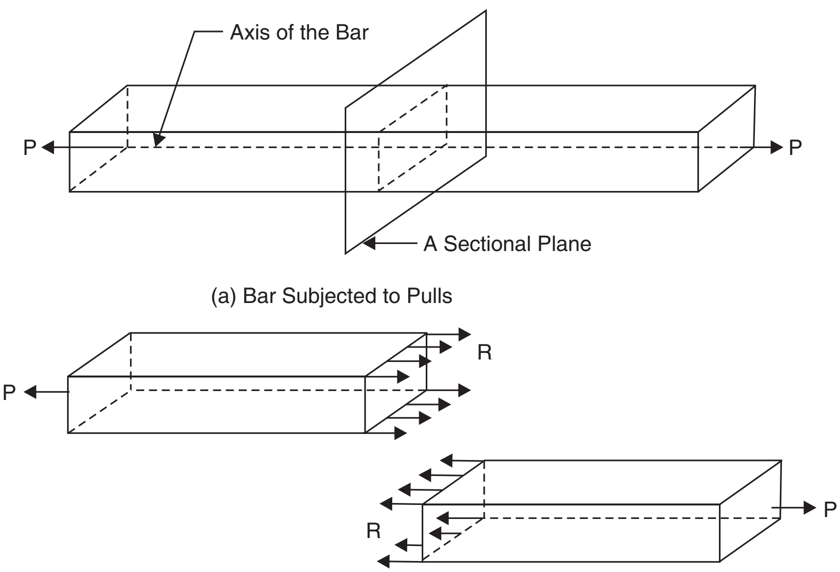
When Newton is taken as unit of force and millimetre as unit of area, unit of stress will be N/mm^2. The other derived units used in practice are kN/mm^2, N/m^2, kN/m^2 or MN/m^2. A stress of one N/m^2 is known as Pascal and is represented by Pa.

Hence, $\quad 1 \text{ MPa} = 1 \text{ MN/m}^2 = 1 \times 10^6 \text{ N/(1000 mm)}^2 = 1 \text{ N/mm}^2.$

Thus one Mega Pascal is equal to 1 N/mm^2. In most of the standard codes published unit of stress has been used as Mega Pascal (MPa or N/mm^2).

AXIAL STRESS

Consider a bar subjected to force P as shown in Fig. 8.5. To maintain the equilibrium the end applied must be the same, say P.
forces



(a) Bar Subjected to Pulls

(b) Resisting Force Developed

Fig. 8.5. Tensile Stresses

The resisting forces acting on a section are shown in Fig. 8.5*b*. Now since the stresses are uniform

$$R = \int p dA = p \int dA = pA \qquad \text{...(8.4)}$$

where A is the cross-sectional area.

Considering the equilibrium of a cut piece of the bar, we get

$$P = R \quad \text{...(8.5)}$$

From equations (8.4) and (8.5), we get

$$P = pA$$

or

$$p = \frac{P}{A} \quad \text{...(8.6)}$$

Thus, in case of axial load 'P' the stress developed is equal to the load per unit area. Under this type of normal stresses the bar is being extended. Such stress which is causing extension of the bar is called tensile stress.

A bar subjected to two equal forces pushing the bar is shown in Fig. 8.6. It causes shortening of the bar. *Such forces which are causing shortening, are known as compressive forces and corresponding stresses as compressive stresses.*

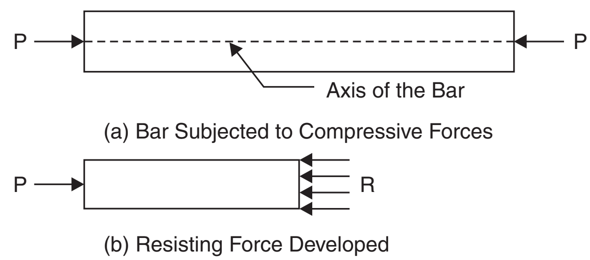


Fig. 8.6. Compressive Stresses

Now $R = \int pdA = p \int dA$ (as stress is assumed uniform)

For equilibrium of the piece of the bar

$$P = R = pA$$

or

$$p = \frac{P}{A} \text{ as in equation 8.6}$$

Thus, whether it is tensile or compressive, the stress developed in a bar subjected to axial forces, is equal to load per unit area.

STRAIN

No material is perfectly rigid. Under the action of forces a rubber undergoes changes in shape and size. This phenomenon is very well known to all since in case of rubber, even for small forces deformations are quite large. Actually all materials including steel, cast iron, brass, concrete, etc. undergo similar deformation when loaded. But the deformations are very small and hence we cannot see them with naked eye. There are instruments like extensometer, electric strain gauges which can measure extension of magnitude 1/100th, 1/1000th of a millimetre. There are machines like universal testing machines in which bars of different materials can be subjected to accurately known forces of magnitude as high as 1000 kN. The studies have shown that the bars extend under tensile force and shorten under compressive forces as shown in Fig. 8.7. *The change in length per unit length is known as linear strain.* Thus,

$$\text{Linear Strain} = \frac{\text{Change in Length}}{\text{Original Length}}$$

$$e = \frac{\Delta}{L} \quad \text{...(8.7)}$$

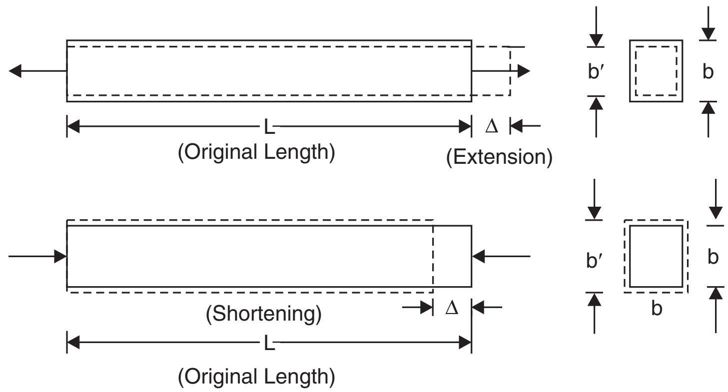


Fig. 8.7

When changes in longitudinal direction is taking place changes in lateral direction also take place. The nature of these changes in lateral direction are exactly opposite to that of changes in longitudinal direction *i.e.*, if extension is taking place in longitudinal direction, the shortening of lateral dimension takes place and if shortening is taking place in longitudinal direction extension takes place in lateral directions (See Fig. 8.7). *The lateral strain may be defined as changes in the lateral dimension per unit lateral dimension.* Thus,

$$\text{Lateral Strain} = \frac{\text{Change in Lateral Dimension}}{\text{Original Lateral Dimension}}$$

$$= \frac{b' - b}{b} = \frac{\delta b}{b} \quad \text{...(8.8)}$$

STRESS-STRAIN RELATION

The stress-strain relation of any material is obtained by conducting tension test in the laboratories on standard specimen. Different materials behave differently and their behaviour in tension and in compression differ slightly.

Behaviour in Tension

Mild steel. Figure 8.8 shows a typical tensile test specimen of mild steel. Its ends are gripped into universal testing machine. Extensometer is fitted to test specimen which measures extension over the length L_1, shown in Fig. 8.8. The length over which extension is mesured is called *gauge length.* The load is applied gradually and at regular interval of loads extension is measured. After certain load, extension increases at faster rate and the capacity of extensometer to measure extension comes to an end and, hence, it is removed before this stage is reached and extension is measured from scale on the universal testing machine. Load is increased gradually till the specimen breaks.

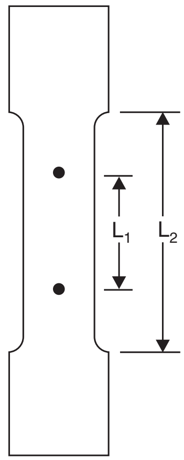


Fig. 8.8. Tension Test Specimen

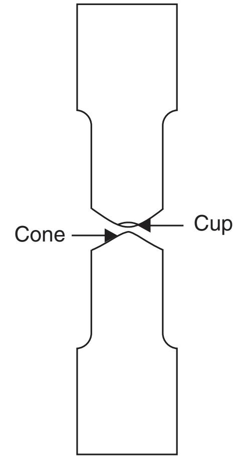


Fig. 8.9. Tension Test Specimen after Breaking

Load divided by original cross-sectional area is called as nominal stress or simply as stress. Strain is obtained by dividing extensometer readings by gauge length of extensometer (L_1) and by dividing scale readings by grip to grip length of the specimen (L_2). Figure 8.10 shows stress *vs* strain diagram for the typical mild steel specimen. The following salient points are observed on stress-strain curve:

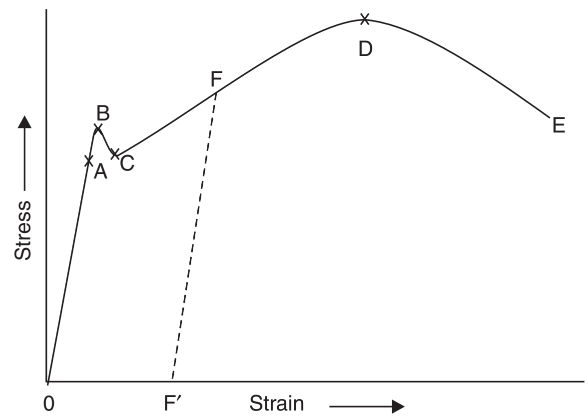


Fig. 8.10

(*a*) ***Limit of Proportionality (A):*** *It is the limiting value of the stress up to which stress is proportional to strain.*

(*b*) ***Elastic Limit:*** *This is the limiting value of stress up to which if the material is stressed and then released (unloaded) strain disappears completely and the original length is regained.* This point is slightly beyond the limit of proportionality.

(*c*) ***Upper Yield Point (B):*** *This is the stress at which, the load starts reducing and the extension increases.* This phenomenon is called yielding of material. At this stage strain is about 0.125 per cent and stress is about 250 N/mm^2.

(*d*) ***Lower Yield Point (C):*** *At this stage the stress remains same but strain increases for some time.*

(*e*) ***Ultimate Stress (D):*** This is the maximum stress the material can resist. This stress is about 370–400 N/mm^2. At this stage cross-sectional area at a particular section starts reducing very fast (Fig. 8.9). This is called neck formation. After this stage load resisted and hence the stress developed starts reducing.

(*f*) ***Breaking Point (E):*** *The stress at which finally the specimen fails is called breaking point.* At this strain is 20 to 25 per cent.

If unloading is made within elastic limit the original length is regained *i.e.*, the stress-strain curve follows down the loading curve shown in Fig. 8.6. If unloading is made after loading the specimen beyond elastic limit, it follows a straight line parallel to the original straight portion as shown by line *FF′* in Fig. 8.10. Thus if it is loaded beyond elastic limit and then unloaded a permanent strain (*OF*) is left in the specimen. This is called *permanent* set.

Stress-strain relation in aluminium and high strength steel. In these elastic materials there is no clear cut yield point. The necking takes place at ultimate stress and eventually the breaking point is lower than the ultimate point. The typical stress-strain diagram is shown in Fig. 8.11. The stress *p* at which if unloading is made there will be 0.2 per cent permanent set is known as 0.2 per cent proof stress and this point is treated as yield point for all practical purposes.

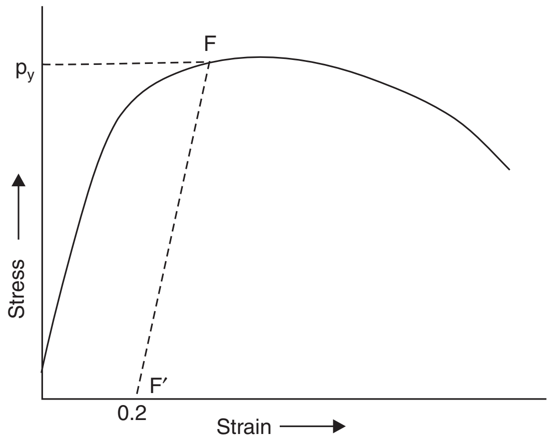


Fig. 8.11. Stress-Strain Relation in Aluminium and High Strength Steel

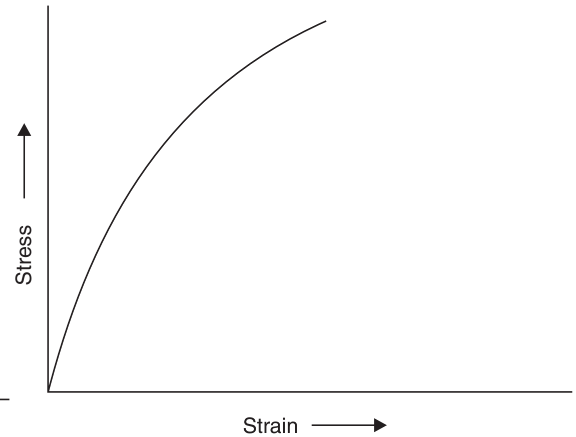


Fig. 8.12. Stress-Strain Relation for Brittle Material

Stress-strain relation in brittle material. The typical stress-strain relation in a brittle material like cast iron, is shown in Fig. 8.12.

In these material, there is no appreciable change in rate of strain. There is no yield point and no necking takes place. Ultimate point and breaking point are one and the same. The strain at failure is very small.

Percentage elongation and percentage reduction in area. Percentage elongation and percentage reduction in area are the two terms used to measure the ductility of material.

(*a*) ***Percentage Elongation:*** *It is defined as the ratio of the final extension at rupture to original length expressed, as percentage*. Thus,

$$\text{Percentage Elongation} = \frac{L' - L}{L} \times 100 \qquad \text{...(8.9)}$$

where L – original length, L'– length at rupture.

The code specify that original length is to be five times the diameter and the portion considered must include neck (whenever it occurs). Usually marking are made on tension rod at every '2.5 *d*' distance and after failure the portion in which necking takes place is considered. In case of ductile material percentage elongation is 20 to 25.

(*b*) ***Percentage Reduction in Area:*** *It is defined as the ratio of maximum changes in the cross-sectional area to original cross-sectional area, expressed as percentage.* Thus,

$$\text{Percentage Reduction in Area} = \frac{A - A'}{A} \times 100 \qquad ...(8.10)$$

where A–original cross-sectional area, A'–minimum cross-sectional area. In case of ductile material, A' is calculated after measuring the diameter at the neck. For this, the two broken pieces of the specimen are to be kept joining each other properly. For steel, the percentage reduction in area is 60 to 70.

Behaviour of Materials under Compression

As there is chance to bucking (laterally bending) of long specimen, for compression tests short specimens are used. Hence, this test involves measurement of smaller changes in length. It results into lesser accuracy. However precise measurements have shown the following results:

(*a*) In case of ductile materials stress-strain curve follows exactly same path as in tensile test up to and even slightly beyond yield point. For larger values the curves diverge. There will not be necking in case of compression tests.

(*b*) For most brittle materials ultimate compresive stress in compression is much larger than in tension. It is because of flows and cracks present in brittle materials which weaken the material in tension but will not affect the strength in compression.

NOMINAL STRESS AND TRUE STRESS

So far our discussion on direct stress is based on the value obtained by dividing the load by original cross-sectional area. That is the reason why the value of stress started dropping after neck is formed in mild steel (or any ductile material) as seen in Fig. 8.10. But actually as material is stressed its cross-sectional area changes. We should divide load by the actual cross-sectional area to get true stress in the material. To distinguish between the two values we introduce the terms nominal stress and true stress and define them as given below:

$$\text{Nominal Stress} = \frac{\text{Load}}{\text{Original Cross-sectional Area}} \qquad ...(8.11a)$$

$$\text{True Stress} = \frac{\text{Load}}{\text{Actual Cross-sectional Area}} \qquad ...(8.11b)$$

So far discussion was based on nominal stress. That is why after neck formation started (after ultimate stress), stress-strain curve started sloping down and the breaking took place at lower stress (nominal). If we consider true stress, it is increasing continuously as strain increases as shown in Fig. 8.13.

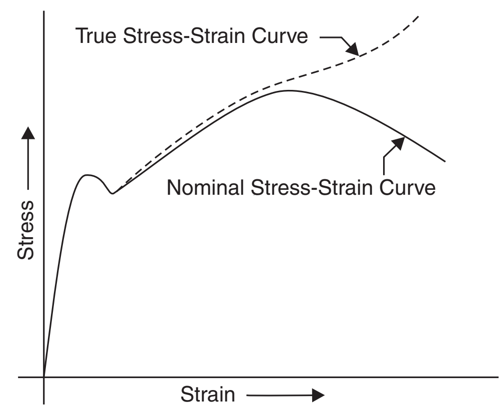


Fig. 8.13. Nominal Stress-Strain Curve and True Stress-Strain Curve for Mild Steel.

FACTOR OF SAFETY

In practice it is not possible to design a mechanical component or structural component permitting stressing up to ultimate stress for the following reasons:

1. Reliability of material may not be 100 per cent. There may be small spots of flaws.
2. The resulting deformation may obstruct the functional performance of the component.
3. The loads taken by designer are only estimated loads. Occasionally there can be overloading. Unexpected impact and temperature loadings may act in the lifetime of the member.
4. There are certain ideal conditions assumed in the analysis (like boundary conditions). Actually ideal conditions will not be available and, therefore, the calculated stresses will not be 100 per cent real stresses.

Hence, *the maximum stress to which any member is designed is much less than the ultimate stress, and this stress is called Working Stress. The ratio of ultimate stress to working stress is called factor of safety*. Thus

$$\text{Factor of Safety} = \frac{\text{Ultimate Stress}}{\text{Working Stress}} \qquad \text{...(8.12)}$$

In case of elastic materials, since excessive deformation create problems in the performance of the member, working stress is taken as a factor of yield stress or that of a 0.2 proof stress (if yield point do not exist).

Factor of safety for various materials depends up on their reliability. The following values are commonly taken in practice:

1. For steel – 1.85
2. For concrete – 3
3. For timber – 4 to 6

HOOKE'S LAW

Robert Hooke, an English mathematician conducted several experiments and concluded that *stress is proportional to strain up to elastic limit.* This is called Hooke's law. Thus Hooke's law is, up to elastic limit

$$p \propto e \qquad \text{...(8.13}a\text{)}$$

where p is stress and e is strain

Hence,

$$p = Ee \qquad \text{...(8.13}b\text{)}$$

where E is the constant of proportionality of the material, known as modulus of elasticity or Young's modulus, named after the English scientist Thomas Young (1773–1829).

However, present day sophisticated experiments have shown that for mild steel the Hooke's law holds good up to the proportionality limit which is very close to the elastic limit. For other materials, as seen in art. 1.5, Hooke's law does not hold good. However, in the range of working stresses, assuming Hooke's law to hold good, the relationship does not deviate considerably from actual behaviour. Accepting Hooke's law to hold good, simplifies the analysis and design procedure considerably. Hence Hooke's law is widely accepted. The analysis procedure accepting Hooke's law is known as Linear Analysis and the design procedure is known as the working stress method.

EXTENSION/SHORTENING OF A BAR

Consider the bars shown in Fig. 8.14

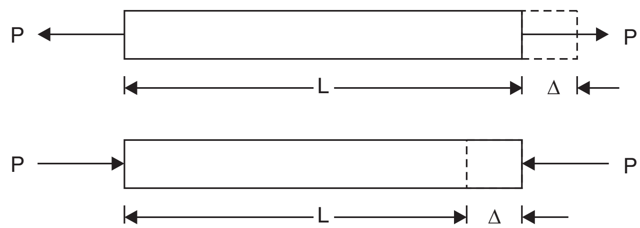


Fig. 8.14

From equation (8.6), Stress $p = \dfrac{P}{A}$

From equation (8.7), Strain, $e = \dfrac{\Delta}{L}$

From Hooke's Law we have,

$$E = \frac{\text{Stress}}{\text{Strain}} = \frac{p}{e} = \frac{P/A}{\Delta/L} = \frac{PL}{A\Delta}$$

or

$$\Delta = \frac{PL}{AE}. \qquad \text{...(8.14)}$$

Example 8.1. *A circular rod of diameter 16 mm and 500 mm long is subjected to a tensile force 40 kN. The modulus of elasticity for steel may be taken as 200 kN/mm². Find stress, strain and elongation of the bar due to applied load.*

***Solution*:**

$$\text{Load } P = 40 \text{ kN} = 40 \times 1000 \text{ N}$$

$$E = 200 \text{ kN/mm}^2 = 200 \times 10^3 \text{ N/mm}^2$$

$$L = 500 \text{ mm}$$

$$\text{Diameter of the rod } d = 16 \text{ mm}$$

Therefore, sectional area

$$A = \frac{\pi d^2}{4} = \frac{\pi}{4} \times 16^2$$

$$= 201.06 \text{ mm}^2$$

$$\textbf{Stress } p = \frac{P}{A} = \frac{40 \times 1000}{201.06} = \mathbf{198.94 \ N/mm^2}$$

$$\textbf{Strain } e = \frac{p}{E} = \frac{198.94}{200 \times 10^3} = \mathbf{0.0009947}$$

$$\textbf{Elongation } \Delta = \frac{PL}{AE} = \frac{4.0 \times 1000 \times 500}{201.06 \times 200 \times 10^3} = \mathbf{0.497 \ mm}$$

Example 8.2. *A Surveyor's steel tape 30 m long has a cross-section of 15 mm × 0.75 mm. With this, line AB is measure as 150 m. If the force applied during measurement is 120 N more than the force applied at the time of calibration, what is the actual length of the line?*

Take modulus of elasticity for steel as 200 kN/mm².

***Solution*:**

$$A = 15 \times 0.75 = 11.25 \text{ mm}^2$$

$$P = 120 \text{ N}, \; L = 30 \text{ m} = 30 \times 1000 \text{ mm}$$

$$E = 200 \text{ kN/mm}^2 = 200 \times 10^3 \text{ N/mm}^2$$

$$\text{Elongation } \Delta = \frac{PL}{AE} = \frac{120 \times 30 \times 1000}{11.25 \times 200 \times 10^3} = 1.600 \text{ mm}$$

Hence, if measured length is 30 m.

Actual length is 30 m + 1.600 mm = 30.001600 m

$$\therefore \quad \textbf{Actual length of line } AB = \frac{150}{30} \times 30.001600 = \mathbf{150.008 \text{ m}}$$

Example 8.3. *A hollow steel tube is to be used to carry an axial compressive load of 160 kN. The yield stress for steel is 250 N/mm². A factor of safety of 1.75 is to be used in the design. The following three class of tubes of external diameter 101.6 mm are available.*

Class	**Thickness**
Light	3.65 mm
Medium	4.05 mm
Heavy	4.85 mm

Which section do you recommend?

***Solution*:** Yield stress = 250 N/mm²

Factor of safety = 1.75

Therefore, permissible stress

$$p = \frac{250}{1.75} = 142.857 \text{ N/mm}^2$$

$$\text{Load } P = 160 \text{ kN} = 160 \times 10^3 \text{ N}$$

but

$$p = \frac{P}{A}$$

i.e.

$$142.857 = \frac{160 \times 10^3}{A}$$

$$\therefore \quad A = \frac{160 \times 10^3}{142.857} = 1120 \text{ mm}^2$$

For hollow section of outer diameter 'D' and inner diameter 'd'

$$A = \frac{\pi}{4}(D^2 - d^2) = 1120$$

$$\frac{\pi}{4}(101.6^2 - d^2) = 1120$$

$$d^2 = 8896.53 \quad \therefore \quad d = 94.32 \text{ mm}$$

$$\therefore \qquad t = \frac{D-d}{2} = \frac{101.6 - 94.32}{2} = 3.63 \text{ mm}$$

Hence, use of light section is recommended.

Example 8.4. *A specimen of steel 20 mm diameter with a gauge length of 200 mm is tested to destruction. It has an extension of 0.25 mm under a load of 80 kN and the load at elastic limit is 102 kN. The maximum load is 130 kN.*

The total extension at fracture is 56 mm and diameter at neck is 15 mm. Find

(*i*) *The stress at elastic limit.*

(*ii*) *Young's modulus.*

(*iii*) *Percentage elongation.*

(*iv*) *Percentage reduction in area.*

(*v*) *Ultimate tensile stress.*

***Solution*:** Diameter $d = 20$ mm

$$\text{Area } A = \frac{\pi d^2}{4} = 314.16 \text{ mm}^2$$

(*i*) **Stress at elastic limit** $= \dfrac{\text{Load at elastic limit}}{\text{Area}}$

$$= \frac{102 \times 10^3}{314.16} = \mathbf{324.675 \ N/mm^2}$$

(*ii*) **Young's modulus** $E = \dfrac{\text{Stress}}{\text{Strain}}$ within elastic limit

$$= \frac{P/A}{\Delta/L} = \frac{80 \times 10^3/314.16}{0.25/200}$$

$$= \mathbf{203718 \ N/mm^2}$$

(*iii*) **Percentage elongation** $= \dfrac{\text{Final extension}}{\text{Original length}}$

$$= \frac{56}{200} \times 100 = \mathbf{28}$$

(*iv*) **Percentage reduction in area**

$$= \frac{\text{Initial area} - \text{Final area}}{\text{Initial area}} \times 100$$

$$= \frac{\frac{\pi}{4} \times 20^2 - \frac{\pi}{4} \times 15^2}{\frac{\pi}{4} \times 20^2} \times 100 = \mathbf{43.75}$$