## Engineering of Structures

## Rectangular components of a force

Usually a force has two components in the $X$-axis and $Y$-axis, and they are perpendicular to each other.
$F_{x}$ is the component of F in the X -axis,


Or

$$
\begin{aligned}
& F_{x}=F \cos \theta \\
& F_{x}=F \sin \delta
\end{aligned}
$$

$F_{y}$ is the component of F in the Y -axis,
$F_{y}=F \sin \theta$
Or
$F_{y}=F \cos \delta$


## Rectangular components of a force

$$
\frac{F_{x}}{F}=\frac{b}{c}
$$

Then

$$
F_{x}=\frac{b}{c} * F
$$

Similarly

$$
\frac{-F_{y}}{F}=\frac{a}{c}
$$

Then


$$
F_{y}=-\frac{a}{c} * F
$$

## Example 1: Calculate the horizontal and vertical components of $F$ ?



## Example 2: Determine the vertical and horizontal of the tension force of the cable $=300 \mathrm{KN}$ ?



## Coplanar force resultants

$$
\begin{gathered}
F_{R x}=\sum F_{x} \\
F_{R x}=F_{1 x}-F_{2 x}+F_{3 x} \\
F_{R y}=\sum F_{y} \\
F_{R y}=F_{1 y}+F_{2 y}-F_{3 y} \\
\boldsymbol{F}_{R}=\sqrt{F_{R x}^{2}+F_{R y}^{2}} \\
\underset{\tan ^{-1}}{F_{R y}}=\cos ^{-1} \frac{F_{R x}}{F_{R}}=\sin ^{-1} \frac{F_{R y}}{F_{R}} \\
\mathbf{F}_{R y}
\end{gathered}
$$




## Example 3: Determine the components of the two forces acting on the boom.

$$
\begin{aligned}
F_{1 x} & =-200 \sin 30^{\circ} \mathrm{N}=-100 \mathrm{~N}=100 \mathrm{~N} \leftarrow \\
F_{1 y} & =200 \cos 30^{\circ} \mathrm{N}=173 \mathrm{~N}=173 \mathrm{~N} \uparrow \\
\frac{F_{2 x}}{260 \mathrm{~N}} & =\frac{12}{13} \quad F_{2 x}=260 \mathrm{~N}\left(\frac{12}{13}\right)=240 \mathrm{~N}
\end{aligned}
$$

Similarly,

$$
\begin{aligned}
& \quad F_{2 y}=260 \mathrm{~N}\left(\frac{5}{13}\right)=100 \mathrm{~N} \\
& F_{2 x}=240 \mathrm{~N}=240 \mathrm{~N} \rightarrow \\
& F_{2 y}=-100 \mathrm{~N}=100 \mathrm{~N} \downarrow
\end{aligned}
$$



## Example 4: Determine the

 magnitude and direction of the resultant force.$\xrightarrow{ \pm} F_{R x}=\Sigma F_{x} ; \quad F_{R x}=600 \cos 30^{\circ} \mathrm{N}-400 \sin 45^{\circ} \mathrm{N}$

$$
=236.8 \mathrm{~N} \rightarrow
$$

$+\uparrow F_{R y}=\Sigma F_{y} ; \quad F_{R y}=600 \sin 30^{\circ} \mathrm{N}+400 \cos 45^{\circ} \mathrm{N}$ $=582.8 \mathrm{~N} \uparrow$
The resultant force, shown in Fig. 2-18c, has a magnitude of

$$
\begin{aligned}
F_{R} & =\sqrt{(236.8 \mathrm{~N})^{2}+(582.8 \mathrm{~N})^{2}} \\
& =629 \mathrm{~N}
\end{aligned}
$$

From the vector addition,

$$
\theta=\tan ^{-1}\left(\frac{582.8 \mathrm{~N}}{236.8 \mathrm{~N}}\right)=67.9^{\circ}
$$


(a)

(b)

Ans.

Ans.
(c)

## Example 5: The end of the boom is subjected to three forces. Calculate the magnitude and direction of the resultant force.

$$
\begin{array}{ll}
\text { 」 } F_{R x}=\Sigma F_{x} ; \quad F_{R x} & =-400 \mathrm{~N}+250 \sin 45^{\circ} \mathrm{N}-200\left(\frac{4}{5}\right) \mathrm{N} \\
& =-383.2 \mathrm{~N}=383.2 \mathrm{~N} \leftarrow \\
+\uparrow F_{R y}=\Sigma F_{y} ; \quad F_{R y} & =250 \cos 45^{\circ} \mathrm{N}+200\left(\frac{3}{5}\right) \mathrm{N} \\
& \\
& =296.8 \mathrm{~N} \uparrow
\end{array}
$$

The resultant force, shown in Fig. 2-20c, has a magnitude of

$$
\begin{aligned}
F_{R} & =\sqrt{(-383.2 \mathrm{~N})^{2}+(296.8 \mathrm{~N})^{2}} \\
& =485 \mathrm{~N}
\end{aligned}
$$

From the vector addition in Fig. 2-20c, the direction angle $\theta$ is

$$
\theta=\tan ^{-1}\left(\frac{296.8}{383.2}\right)=37.8^{\circ}
$$



Ans



Example 6: Determine the magnitude and direction of F1, knowing that the resultant is 600 N and its direction measured $\theta=30$ clockwise from the positive $x$-axis.

$$
\begin{gathered}
\boldsymbol{F}_{\boldsymbol{R} x}=\boldsymbol{F}_{x 1}+\boldsymbol{F}_{x 2}-\boldsymbol{F}_{x 3} \\
\boldsymbol{F}_{\boldsymbol{R}} * \cos 30=\boldsymbol{F}_{1} * \cos \emptyset+\boldsymbol{F}_{2} * \cos \mathbf{6 0}-\boldsymbol{F}_{3} * \frac{3}{5} \\
600 * \mathbf{0 . 8 6 6}=\boldsymbol{F}_{\mathbf{1}} * \cos \emptyset+\mathbf{5 0 0} * \mathbf{0 . 5 - 4 5 0 * 0 . 6} \\
\boldsymbol{F}_{1} * \cos \emptyset=539.6 \\
\boldsymbol{F}_{1}=\frac{539.6}{\cos \emptyset} \quad \text { equation }(\mathbf{1})
\end{gathered}
$$


and,

$$
F_{R y}=F_{y 1}-F_{y 2}-F_{y 3}
$$



$$
\begin{gathered}
\boldsymbol{F}_{\boldsymbol{R}} * \sin 30=\boldsymbol{F}_{1} * \sin \emptyset-\boldsymbol{F}_{2} * \sin 60-\boldsymbol{F}_{3} * \frac{4}{5} \\
-600 * 0.5=\boldsymbol{F}_{1} * \sin \emptyset-500 * 0.866-450 * 0.8 \\
\boldsymbol{F}_{1}=\frac{493}{\sin \emptyset} \quad \boldsymbol{F}_{1} * \sin \emptyset=493
\end{gathered}
$$



$$
\frac{493}{\sin \emptyset}=\frac{539.6}{\cos \emptyset} \rightarrow \emptyset=\tan ^{-1} \frac{493}{539.6}=42.42 \rightarrow F_{1}=731 \mathrm{~N}
$$

HW: Resolve each acting force into its $x$ and $y$ components and find the magnitude and direction of the resultant force.


HW: Determine the magnitude and direction of F2, knowing that the resultant force is acting along the positive x -axis.


