Beams

INTRODUCTION

A beam may be defined as a structural element which has one dimension considerably larger than the other two dimensions, namely breadth and depth, and is supported at few points. The distance between two adjacent supports is called span. It is usually loaded normal to its axis. The applied loads make every cross-section to face bending and shearing. The load finally get transferred to supports. The system of forces consisting of applied loads and reactions keep the beam in equilibrium. The reactions depend upon the type of supports and type of loading. In this chapter type of supports, types of beams and types of loading are first explained and then the methods of finding reactions, bending moment and shear forces are illustrated for the following determinate beams:

(*a*) Simply supported beams
(*b*) Cantilever beams and
(*c*) Overhanging beams.

TYPES OF SUPPORTS

Various types of supports and reactions developed are listed below:

Simple Support: If the beam rests simply on a support it is called a simple support. In such case the reaction at the support is at right angles to the support and the beam is free to move in the direction of its axis and also it is free to rotate about the support (Fig. 9.1).

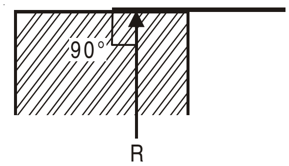

Fig. 9.1

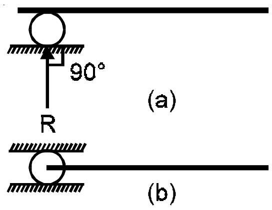

Fig. 9.2

Roller Support: In this case, beam end is supported on rollers. In such cases, reaction is normal to the support since rollers can be treated as frictionless. [Fig. 9.2 (*a*)]. Many mechanical components are having roller supports which roll between guides. In such cases, reaction will be normal to the guides, in both the direction (Fig. 9.2). At roller support beam is free to move along the support. It can rotate about the support also.

Hinged Support: At a hinged end, a beam cannot move in any direction. However, it can rotate about the support (Fig. 9.3). Hence the support will not develop any resisting moment, but it can develop reaction in any direction to keep the end stationary. The reaction R can be split into its horizontal and vertical components for the purpose of analysis.

Thus, in Fig. 9.3,

$$V_A = R \sin \theta$$

$$H_A = R \cos \theta$$

Fixed Support: At such supports, the beam end is not free to translate or rotate. Translation is prevented by developing support reaction in any required direction.

Referring to Fig. 9.4 the support reaction R which is at an angle θ to x axis may be represented by H_A and V_A, where

$$V_A = R \sin \theta$$
$$H_A = R \cos \theta$$

Rotation is prevented by developing support moment M_A as shown in Fig. 9.4. Thus at fixed support A, there are three reactions H_A, V_A and M_A.

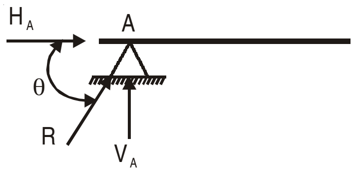

Fig. 9.3

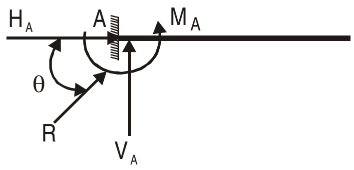

Fig. 9.4

TYPES OF BEAMS

Simply Supported Beam: When both end of a beam are simply supported it is called simply supported beam (Fig. 9.5). Such a beam can support load in the direction normal to its axis.

Fig. 9.5

Beam with One End Hinged and the Other on Rollers: If one end of a beam is hinged and other end is on rollers, the beam can resist load in any direction (*see* Fig. 9.6).

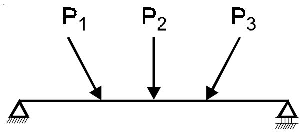

Fig. 9.6

Over-hanging Beam: If a beam is projecting beyond the support. It is called an over-hanging beam (Fig. 9.7). The overhang may be only on one side as in Fig. 9.7(a) or may be on both sides as in Fig. 9.7(b).

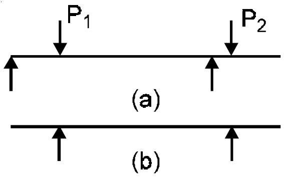

Fig. 9.7

Cantilever Beam: If a beam is fixed at one end and is free at the other end, it is called cantilever beam (Fig. 9.8).

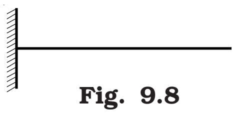

Fig. 9.8

Propped Cantilever: It is a beam with one end fixed and the other end simply supported (Fig. 9.9).

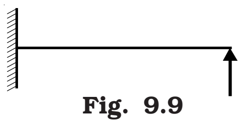

Fig. 9.9

Both Ends Hinged: In these beams both ends will be having hinged supports (Fig. 9.10).

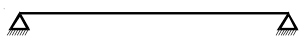

Fig. 9.10

Continuous Beam: A beam is said to be continuous, if it is supported at more than two points (Fig. 9.11).

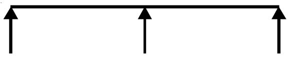

Fig. 9.11

In the case of simply supported beams, beams with one end hinged and the other on rollers, cantilever and over-hanging beams, it is possible to determine the reactions for given loadings by using the equations of equilibrium only. In the other cases, the number of independent equilibrium equations are less than the number of unknown reactions and hence it is not possible to analyse them by using equilibrium equations alone. The beams which can be analysed using only equilibrium equations are known as *Statically Determinate* beams and those which cannot be analysed are known as *Statically Indeterminate* beams. The latter beams can be analysed using the conditions of continuity in deformations in addition to equilibrium equations. Such cases will not be treated in this book.

TYPES OF LOADING

Usual types of loadings on the beams are discussed here.

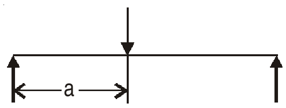

Fig. 9.12

Concentrated Loads: If a load is acting on a beam over a very small length, it is approximated as acting at the mid point of that length and is represented by an arrow as shown in Fig. 9.12.

Uniformly Distributed Load (UDL): Over considerably long distance such load has got uniform intensity. It is represented as shown in Fig. 9.13 (*a*) or as in (*b*). For finding reaction, this load may be assumed as total load acting at the centre of gravity of the loading (middle of the loaded length). For example, in the beam shown in Fig. 9.13, the given load may be replaced by a $20 \times 4 = 80$ kN concentrated load acting at a distance 2 m from the left support.

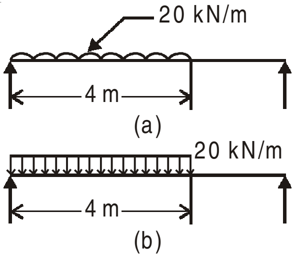

Fig. 9.13

Uniformly Varying Load: The load shown in Fig. 9.14 varies uniformly from *C* to *D*. Its intensity is zero at *C* and is 20 kN/m at *D*. In the load diagram, the ordinate represents the load intensity and the abscissa represents the position of load on the beam.

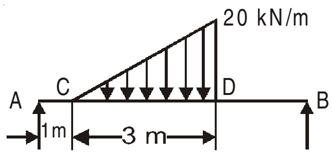

Fig. 9.14

Hence the area of the triangle represents the total load and the centroid of the triangle represents the centre of gravity of the load. Thus, total load in this case is $\frac{1}{2} \times 3 \times 20 = 30$ kN and the centre of gravity of this loading is at $\frac{1}{3} \times 3 = 1$ m from *D*, *i.e.,* $1 + 3 - 1 = 3$ m from *A*. For finding the reactions, we can assume that the given load is equivalent to 30 kN acting at 3 m from *A*.

General Loadings: Figure 9.15 shows a general loading. Here the ordinate represents the intensity of loading and abscissa represents position of the load on the beam. For simplicity in analysis such loadings are replaced by a set of equivalent concentrated loads.

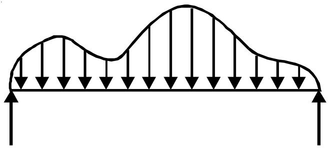

Fig. 9.15

External Moment: A beam may be subjected to external moment at certain points. In Fig. 9.16, the beam is subjected to clockwise moment of 30 kN-m at a distance of 2 m from the left support.

In this chapter the beams subjected to concentrated loads, *udl* and external moments are dealt with.

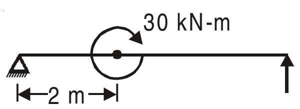

Fig. 9.16

REACTIONS FROM SUPPORTS OF BEAMS

A beam is in equilibrium under the action of the loads and the reactions. Hence the equilibrium may be written for the system of forces consisting of reactions and the loads. Solutions of these equations give the unknown reactions.

Example 9.1. *The beam AB of span 12 m shown in Fig. 9.17 (a) is hinged at A and is on rollers at B. Determine the reactions at A and B for the loading shown in the Figure.*

Solution: At A the reaction can be in any direction. Let this reaction be represented by its components V_A and H_A as shown in Fig. 9.17 (*b*). At B the reaction is in vertical direction only. The beam is in equilibrium under the action of system of forces shown in Fig. 9.17 (*b*).

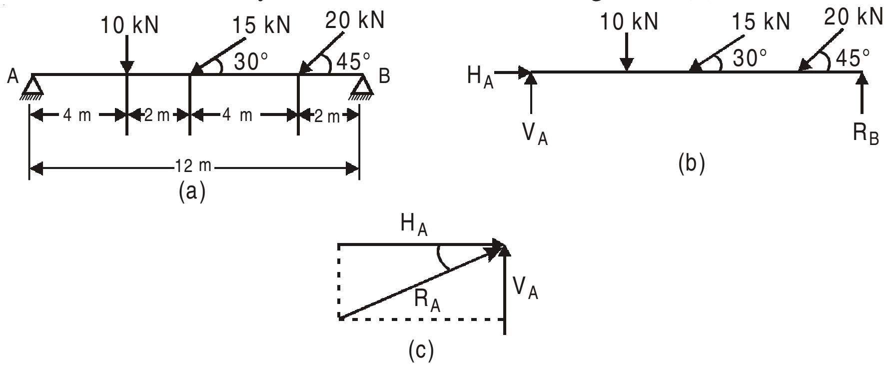

Fig. 9.17

Now, $\Sigma H = 0$, gives

$$H_A - 15 \cos 30° - 20 \cos 45° = 0$$

$$H_A = 27.1325 \text{ kN}$$

$$\Sigma M_A = 0, \text{ gives}$$

$$R_B \times 12 - 10 \times 4 - 15 \sin 30° \times 6 - 20 \sin 45° \times 10 = 0$$

$$\boldsymbol{R_B = 18.8684 \text{ kN.}}$$

$$\Sigma V = 0, \text{ gives}$$

$$V_A + 18.8684 - 10 - 15 \sin 30° - 20 \sin 45° = 0$$

$\therefore$ $$V_A = 12.7737 \text{ kN}$$

$\therefore$ $$R_A = \sqrt{H_A^2 + V_A^2} = \sqrt{27.1325^2 + 12.7737^2}$$

i.e., $$\boldsymbol{R_A = 29.989 \text{ kN.}}$$

and $$\alpha = \tan^{-1}\frac{(12.7737)}{(27.1325)} = \mathbf{25.21°.}$$

[**Note:** For finding moments, inclined loads are split into their vertical and horizontal components. Horizontal components do not produce moment about *A*.]

Example 9.2. *Find the reactions at supports A and B in the beam AB shown in Fig. 9.18 (a).*

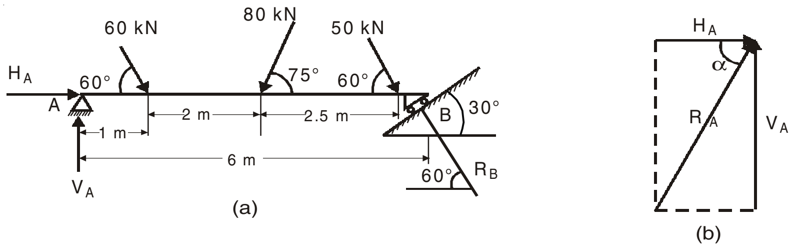

Fig. 9.18

Solution: The reaction at B will be at right angles to the support, *i.e.,* at 60° to horizontal as shown in the figure. Let the components of the reactions at A be H_A and V_A. Then

$$\Sigma M_A = 0 \text{ gives}$$

$$R_B \sin 60° \times 6 - 60 \sin 60° \times 1 - 80 \times \sin 75° \times 3 - 50 \times \sin 60° \times 5.5 = 0$$

$$\therefore \quad \boldsymbol{R_B = 100.4475 \text{ kN.}}$$

$$\Sigma H = 0, \text{ gives}$$

$$H_A + 60 \cos 60° - 80 \cos 75° + 50 \cos 60° - R_B \cos 60° = 0$$

$$H_A = -60 \cos 60° + 80 \cos 75° - 50 \cos 60° + 100.4475 \cos 60°$$

$$= 15.9293 \text{ kN}$$

$$\Sigma V = 0, \text{ gives}$$

$$V_A + R_B \sin 60° - 60 \sin 60° - 80 \sin 75° - 50 \sin 60° = 0$$

$$V_A = -100.4475 \sin 60° + 60 \sin 60° + 80 \sin 75° + 50 \sin 60°$$

$$= 85.5468 \text{ kN}$$

$$\therefore \quad R_A = \sqrt{15.9293^2 + 85.5468^2}$$

i.e., $\boldsymbol{R_A = 87.0172}$ **kN.**

$$\alpha = \tan^{-1} \frac{85.5468}{15.9293}$$

i.e., $\boldsymbol{\alpha = 79.45°}$, as shown in Fig. 9.18($b$).

Example 9.3. *Find the reactions at supports A and B of the loaded beam shown in Fig. 9.19(a).*

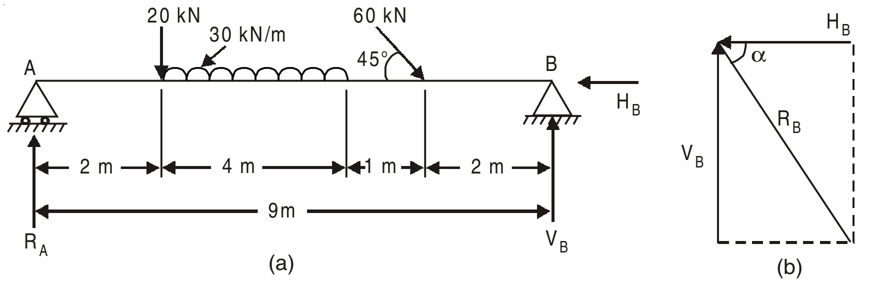

Fig. 9.19

Solution: The reaction at A is vertical. Let H_B and V_B be the components of the reaction at B.

$$\Sigma M_B = 0, \text{ gives}$$

$$R_A \times 9 - 20 \times 7 - 30 \times 4 \times 5 - 60 \sin 45° \times 2 = 0$$

$$\therefore \quad \mathbf{R_A = 91.6503 \text{ kN.}}$$

$$\Sigma H_A = 0, \text{ gives}$$

$$H_B - 60 \cos 45° = 0$$

$$\therefore \quad \mathbf{H_B = 42.4264 \text{ kN.}}$$

$$\Sigma V_A = 0$$

$$91.6503 + V_B - 20 - 30 \times 4 - 60 \sin 45° = 0$$

$$\mathbf{V_B = 90.7761 \text{ kN.}}$$

$$\therefore \quad R_B = \sqrt{42.4264^2 + 90.7761^2}$$

$$\therefore \quad \mathbf{R_B = 100.2013 \text{ kN.}}$$

$$\alpha = \tan^{-1}\frac{90.7761}{42.4264}$$

$\mathbf{\alpha = 64.95}°$, as shown in Fig. 9.19(*b*).

Example 9.4. *The cantilever shown in Fig. 9.20 is fixed at A and is free at B. Determine the reactions when it is loaded as shown in the Figure.*

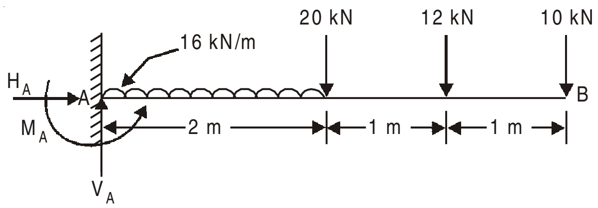

Fig. 9.20

Solution: Let the reactions at A be H_A, V_A and M_A as shown in the figure

Now $\quad \Sigma H = 0$, gives

$$\mathbf{H_A = 0.}$$

$$\Sigma V = 0, \text{ gives}$$

$$V_A - 16 \times 2 - 20 - 12 - 10 = 0$$

$$\therefore \quad \mathbf{V_A = 74 \text{ kN.}}$$

$$\Sigma M = 0, \text{ gives}$$

$$M_A - 16 \times 2 \times 1 - 20 \times 2 - 12 \times 3 - 10 \times 4 = 0$$

$$\therefore \quad \mathbf{M_A = 148 \text{ kN-m.}}$$

Example 9.5. *Compute the reaction developed at support in the cantilever beam shown in Fig. 9.21.*

Solution: Let the vertical reaction be V_A and moment be M_A. There is no horizontal component of reactions, since no load is having horizontal component

$$\Sigma V = 0, \text{ gives}$$

$$V_A - 20 \times 2 - 15 \times 10 = 0$$

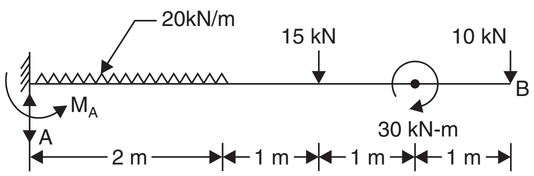

Fig. 9.21

$\therefore$ $\quad$ $\mathbf{V_A = 65\ kN.}$

$$\Sigma M = 0, \text{ gives}$$

$$M_A - 20 \times 2 \times 1 - 15 \times 3 - 30 - 10 \times 5 = 0$$

$$\mathbf{M_A = 165\ kN\text{-}m}$$

Example 9.6. *Determine the reactions at supports A and B of the overhanging beam shown in Fig. 9.22.*

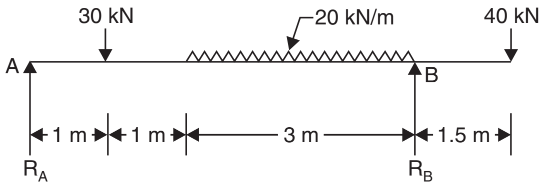

Fig. 9.22

Solution: As supports *A* and *B* are simple supports and loading is only in vertical direction, the reactions R_A and R_B are in vertical directions only.

$$\Sigma M_A = 0, \text{ gives}$$

$$R_B \times 5 - 30 \times 1 - 20 \times 3 \times (2 + 1.5) - 40 \times 6.5 = 0$$

$\therefore$ $\quad$ $\mathbf{R_B = 100\ kN.}$

$$\Sigma V = 0, \text{ gives}$$

$$R_A + R_B - 30 - 20 \times 3 - 40 = 0$$

$\therefore$ $\quad$ $\boldsymbol{R_A} = 130 - R_B = 130 - 100 = \mathbf{30\ kN.}$

Example 9.7. *Find the reactions at supports A and B of the beam shown in Fig. 9.23.*

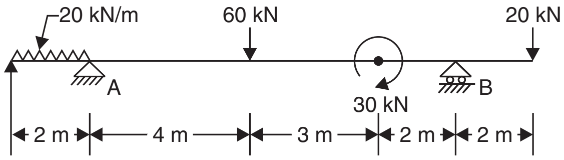

Fig. 9.23

Solution: Let V_A and H_A be the vertical and the horizontal reactions at *A* and V_B be vertical reaction at *B*.

$$\Sigma H = 0, \text{ gives}$$

$$\mathbf{H_A = 0.}$$

$$\Sigma M_A = 0, \text{ gives}$$
$$-20 \times 2 \times 1 + 60 \times 4 + 30 + 20 \times 11 - V_B \times 9 = 0$$
$$\therefore \quad \mathbf{V_B = 50\ kN.}$$
$$\Sigma V = 0, \text{ gives}$$
$$-20 \times 2 + V_A - 60 + V_B - 20 = 0$$
$$V_A = 120 - V_B = 120 - 50$$
$$\mathbf{V_A = 70\ kN.}$$

Example 9.8. *Determine the reactions at A and B of the overhanging beam shown in Fig. 9.24(a).*

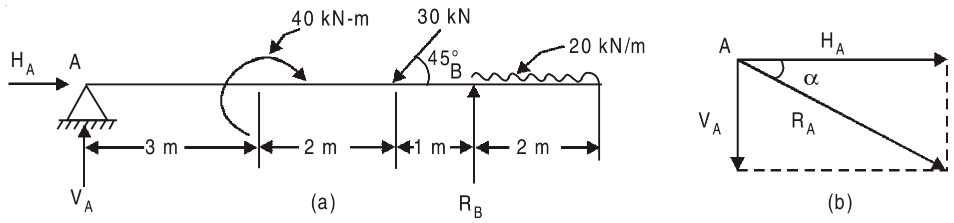

Fig. 9.24

Solution:

$$\sum M_A = 0$$
$$R_B \times 6 - 40 - 30 \sin 45° \times 5 - 20 \times 2 \times 7 = 0$$
$$\boldsymbol{R_B} = \mathbf{71.0110\ kN.}$$
$$\sum H = 0$$
$$H_A = 30 \cos 45° = 21.2132 \text{ kN}$$
$$\sum V = 0$$
$$V_A - 30 \sin 45° + R_B - 20 \times 2 = 0$$
$$V_A = 30 \sin 45° - R_B + 40$$
$$V_A = -9.7978$$

(Negative sign show that the assumed direction of V_A is wrong. In other words, V_A is acting vertically downwards).

$$R_A = \sqrt{V_A^2 + H_A^2}$$
$$\boldsymbol{R_A} = \mathbf{23.3666\ kN.}$$
$$\alpha = \tan^{-1} \frac{V_A}{H_A}$$
$$\boldsymbol{\alpha} = \mathbf{24.79°}, \text{ as shown in Fig. 9.24(b).}$$

Example 9.9. *A beam AB 20 m long supported on two intermediate supports 12 m apart, carries a uniformly distributed load of 6 kN/m and two concentrated loads of 30 kN at left end A and 50 kN at the right end B as shown in Fig. 9.25. How far away should the first support C be located from the end A so that the reactions at both the supports are equal ?*

Solution: Let the left support C be at a distance x metres from A.

Now, $\quad R_C = R_D$ (given)

$$\sum V = 0, \text{ gives}$$
$$R_C + R_D - 30 - 6 \times 20 - 50 = 0$$

or $$2R_C = 30 + 120 + 50 \qquad \text{since } R_C = R_D$$

or $$R_C = 100 \text{ kN}$$

$\therefore$ $$R_D = 100 \text{ kN}$$

$\sum M_A = 0$, gives

$$100x + 100\,(12 + x) - 6 \times 20 \times 10 - 50 \times 20 = 0$$

$$200x = 1000$$

$$\boldsymbol{x = 5 \text{ m.}}$$

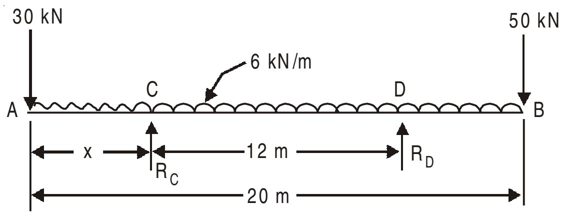

Fig. 9.25