Engineering of Structures

Equilibrium of a particle

When the resultant of all the acting forces is equal to ZERO, the particle is in equilibrium. This means:

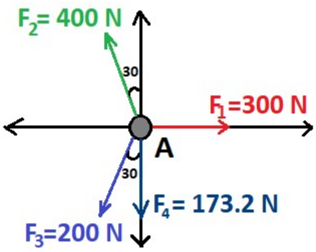
$$R = \sum F = 0 \;\rightarrow\; \sum F_x = 0 \;\rightarrow\; \sum F_y = 0$$

For example:

Prove that the particle **A** is in equilibrium.

$$\sum F_x = 300 - 400 \sin 30 - 200 \sin 30 = 0$$

$$\sum F_y = 400 \cos 30 - 200 \cos 30 - 173.2 = 0$$



A is in equilibrium

Example 1: Determine the tension in cables BA and BC necessary to support 60 Kg cylinder.

$\xrightarrow{+} \Sigma F_x = 0;$ $\quad T_C \cos 45^\circ - \left(\frac{4}{5}\right)T_A = 0 \quad (1)$

$+\uparrow \Sigma F_y = 0;$ $\quad T_C \sin 45^\circ + \left(\frac{3}{5}\right)T_A - 60(9.81)\text{ N} = 0 \quad (2)$

Equation (1) can be written as $T_A = 0.8839T_C$. Substituting this into Eq. (2) yields

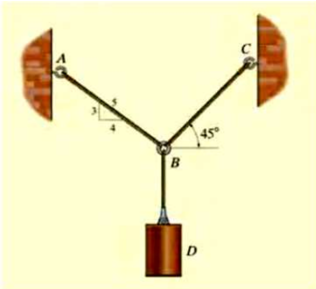
$$T_C \sin 45^\circ + \left(\tfrac{3}{5}\right)(0.8839T_C) - 60(9.81)\text{ N} = 0$$

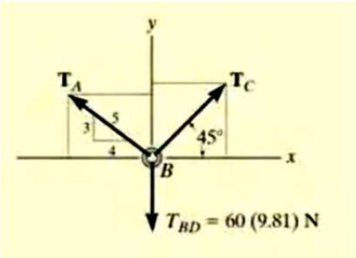
So that

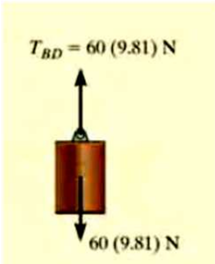
$$T_C = 475.66\text{ N} = 476\text{ N} \qquad \textit{Ans.}$$

Substituting this result into either Eq. (1) or Eq. (2), we get

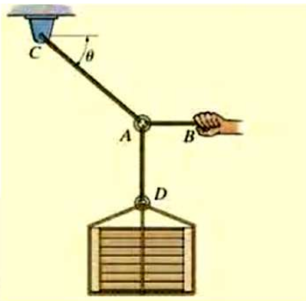
$$T_A = 420\text{ N} \qquad \textit{Ans.}$$







Example 2: The 200 Kg crate is supported by the ropes AB and AC. Each rope is withstand a maximum force of 10 KN before it breaks. If AB always remains horizontal, determine the smallest θ to which the crate can be supported before one of the ropes breaks.



$F_D = 200\ (9.81)\text{ N} = 1962\text{ N} < 10\text{ kN}.$

$\xrightarrow{+} \Sigma F_x = 0;$ $\quad -F_C \cos\theta + F_B = 0;\ F_C = \dfrac{F_B}{\cos\theta} \quad (1)$

$+\uparrow \Sigma F_y = 0;$ $\quad F_C \sin\theta - 1962\text{ N} = 0 \quad (2)$

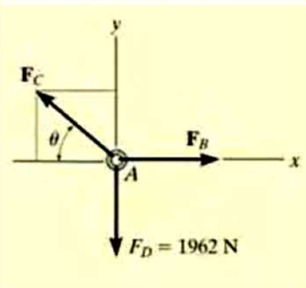
rope AC will reach the maximum tensile force of 10 kN

Substituting $F_C = 10$ kN into Eq. (2), we get

$$[10(10^3)\text{ N}]\sin\theta - 1962\text{ N} = 0$$

$$\theta = \sin^{-1}(0.1962) = 11.31^\circ = 11.3^\circ$$

$$10(10^3)\text{ N} = \frac{F_B}{\cos 11.31^\circ} \qquad F_B = 9.81\text{ kN}$$



Example 3: Determine the required length of cord AC so that the 8 Kg lamp can be suspended in the position shown. The undeformed length of spring AB is 0.4 m and the spring has stiffness of 300 N/m

The lamp has a weight $W = 8(9.81) = 78.5\text{ N}$

$\xrightarrow{+} \Sigma F_x = 0; \qquad T_{AB} - T_{AC}\cos 30^\circ = 0$

$+\uparrow \Sigma F_y = 0; \qquad T_{AC}\sin 30^\circ - 78.5\text{ N} = 0$

Solving, we obtain

$$T_{AC} = 157.0\text{ N}$$

$$T_{AB} = 135.9\text{ N}$$

The stretch of spring AB is therefore

$T_{AB} = k_{AB}s_{AB}; \qquad 135.9\text{ N} = 300\text{ N/m}(s_{AB})$

$$s_{AB} = 0.453\text{ m}$$

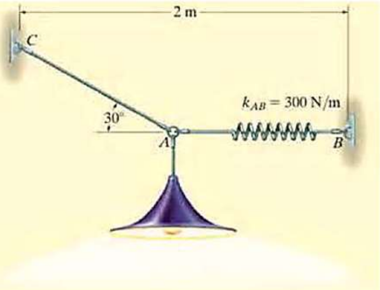
so the stretched length is

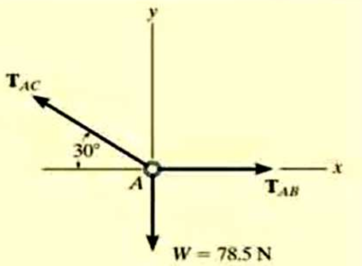
$$l_{AB} = l'_{AB} + s_{AB}$$

$$l_{AB} = 0.4\text{ m} + 0.453\text{ m} = 0.853\text{ m}$$

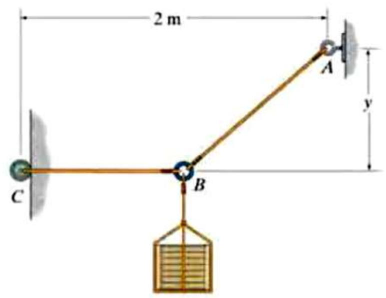
The horizontal distance from C to B, requires ,

$2\text{ m} = l_{AC}\cos 30^\circ + 0.853\text{ m} \qquad l_{AC} = 1.32\text{ m}$



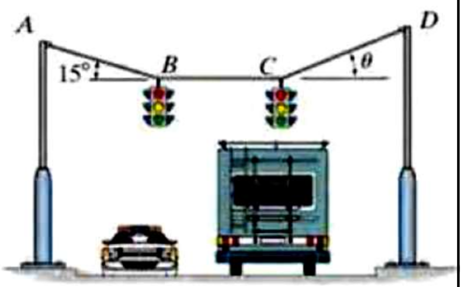


HW1: If the 1.5 m long cord AB can withstand a maximum force of 3500 N. Calculate the force in cord BC and the distance y so that the 200 kg crate can be supported.



$F_{BC} = 2.90\text{ kN}, y = 0.841\text{ m}$

HW2: Determine the tension in cables AB, BC and CD, necessary to support the 10 kg and 15 kg traffic light at B and C, respectively. Also, find the angle θ.



$T_{AB} = 379\text{ N}$ *Ans.*

$T_{BC} = 366\text{ N}$ *Ans.*

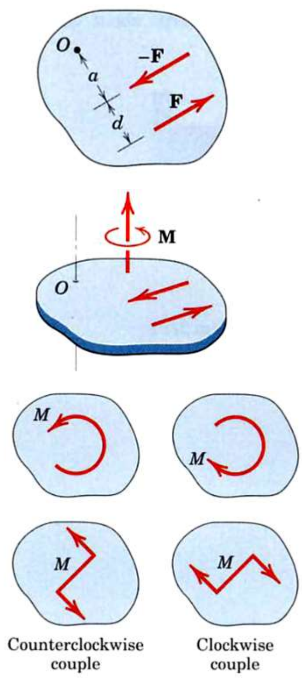
$T_{CD} = 395\text{ N}$ *Ans.*

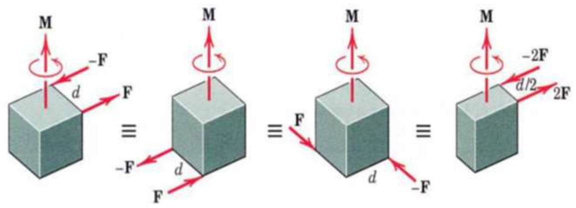
$\theta = 21.9^\circ$ *Ans.*

Couples

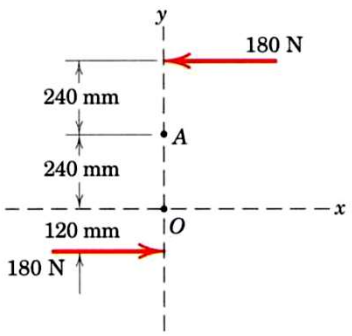
Couple can be defined as the moment produced by two equal, opposite and non-collinear forces.

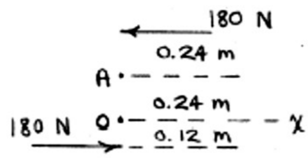
$$M_o = F(a + d) - F(a)$$
$$M_o = F(a) + F(d) - F(a)$$
$$M_o = F(d)$$





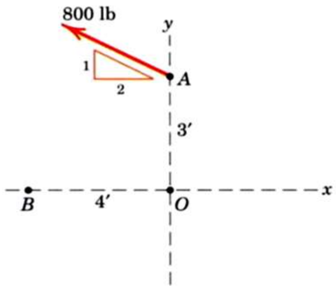
Example 1: Compute the combined moment of two 180 N forces about point O and point A.

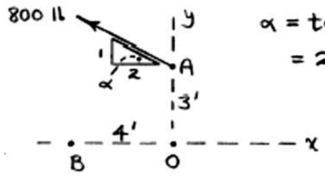




$$\curvearrowleft + \; M = M_O = M_A = Fd$$
$$= 180\,(0.24 + 0.24 + 0.12)$$
$$= \underline{108 \text{ N}\cdot\text{m CCW}}$$

Example 2: Replace the 800 Ib force acting at point A by a force couple system at point O and point B.





$\alpha = \tan^{-1}\frac{1}{2}$

$= 26.6°$

At O: $F = 800$ lb $26.6°$

$M_O = 800\frac{2}{\sqrt{5}}(3) = 2150$ lb-ft CCW

At B: $F = 800$ lb $26.6°$

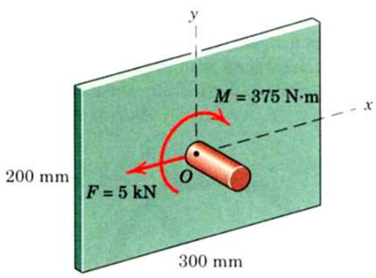
$M_B = 800\frac{2}{\sqrt{5}}(3) + 800\frac{1}{\sqrt{5}}(4) = 3580$ lb-ft CCW

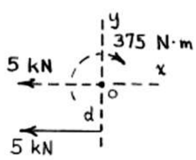
Example 3: The force-couple system was applied to the small shaft at the centre of rectangular plate. Replace this system by a single force and specify the coordinate of the point at the y-axis through which the line of action of this resultant force passes.

$M = Fd$: $375 = 5000d$

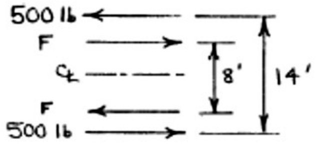
$d = 0.075$ m

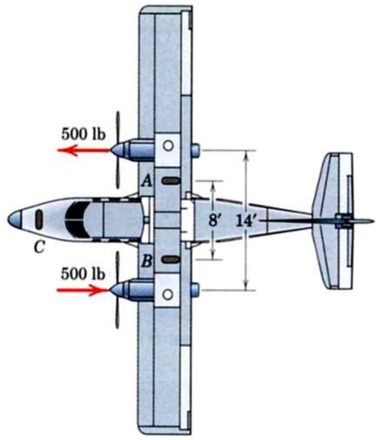
$\therefore$ $y = -75$ mm





Example 4: What force F must be exerted by the ground on each of the main braked wheels at A and B to counteract the turning effect of the two propeller thrusts?

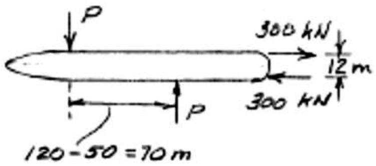


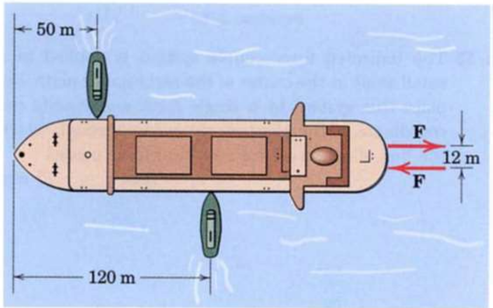


$$\circlearrowleft + \ \Sigma M = 500(14) - F(8) = 0$$

$$F = 875 \text{ lb}$$

Example 5: What thrust P must each tug exert on the ship to counteract the turning effect of the two ship's propellers F? Knowing that F = 300 KN

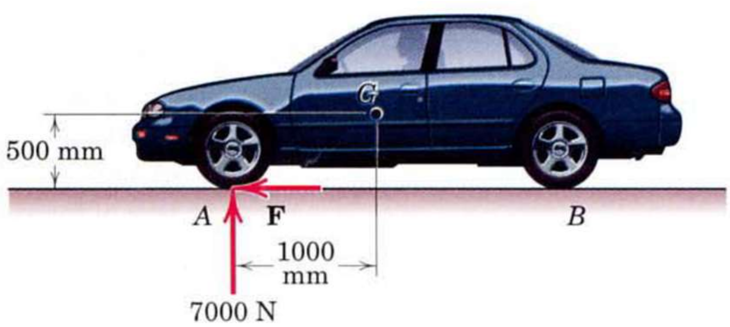




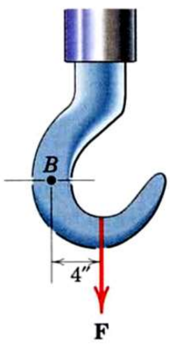
$$70P = 300(12)$$

$$P = 51.4 \text{ kN}$$

The combined drive wheels of a front-wheel-drive automobile are acted on by 7000-N normal reaction force and a friction force **F**, both of which are exerted by the road surface. If it is known that the resultant of these two forces makes a 15° angle with the vertical, determine the equivalent force–couple system at the car mass center *G*. Treat this as a two-dimensional problem.

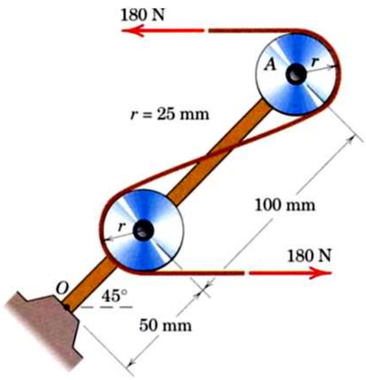


HW1: Calculate the magnitude of the acting force F. Knowing that the applied couple at point B is 4000 Ib.ft



Ans. F = 12,000 lb

HW2: Determine the couple force at point O.



Ans. M *= 21.7 N.m CCW*