SHORT-CUT PROCEDURE

To save time, one can calculate the values only at salient points and draw shear force and bending moment diagrams, noting the following points:

(*a*) Wherever concentrated load is acting (including support reaction), there will be sudden change in *SF* value to the extent of that force at that point, in the direction of that force (while proceeding from left to right).

(*b*) If an external moment is acting at a point on the beam, there will be sudden change in the value of bending moment to the extent of that external bending moment. While proceeding from left, it will be increase in value, if the moment is clockwise. There will be drop in bending moment value if the external bending moment is anticlockwise. Exactly opposite phenomenon will be observed while proceeding from right to left.

(*c*) Since $\frac{dF}{dx} = w$ and $\frac{dM}{dx} = F$, the shear force curve will be one degree higher than that of load curve and bending moment curve will be one degree higher than that of shear force curve. The nature of variation, for various loads is listed in Table 9.1.

Table 9.1 Nature of SF and BM Variation

Load	*SF*	*BM*
No load	Constant	Linear
Uniformly distributed load	Linear	Parabolic
Uniformly varying load	Parabolic	Cubic

The following points also may be noted while drawing shear force and bending moment diagrams:

(*a*) The bending moment is maximum when shear force is zero. The location and the value of maximum moment should always be indicated in bending moment diagram.

(*b*) The point of contraflexure is an important point in a *BMD*, hence if it exists, its location should be indicated.

This method of drawing *SFD* and *BMD is illustrated with two problems below:*

Example 9.15. *Determine the reactions and construct the shear force and bending moment diagrams for the beam shown in Fig. 9.42(a). Mark the salient points and the values at those points.*

Solution: $\sum M_A = 0 \rightarrow$

$$R_B \times 6 + 120 - 60 \times 4 - 60 \times 7 = 0$$

$\therefore$ $R_B = 90$ kN.

$$\Sigma V = 0 \rightarrow R_A = 60 + 60 - 90 = 30 \text{ kN.}$$

SFD:

In portion *ADE*, $F = 30$ kN.

In portion *EB*, $F = 30 - 60 = -30$ kN.

In portion *BC*, $F = 60$ kN.

SFD is as shown in Fig. 3.45(*b*).

BMD:

At *A*, $M_A = 0$

Just to the left of *D*, $M = 30 \times 2 = 60$ kN-m.

Just to the right of *D*, $M = 30 \times 2 - 120 = -60$ kN-m.

At *E*, $M = -60 \times 3 + 90 \times 2 = 0$

At *B*, $M = -60 \times 1 = -60$ kN-m.

At *C*, $M = 0$

BMD is as shown in Fig. 9.42(*c*).

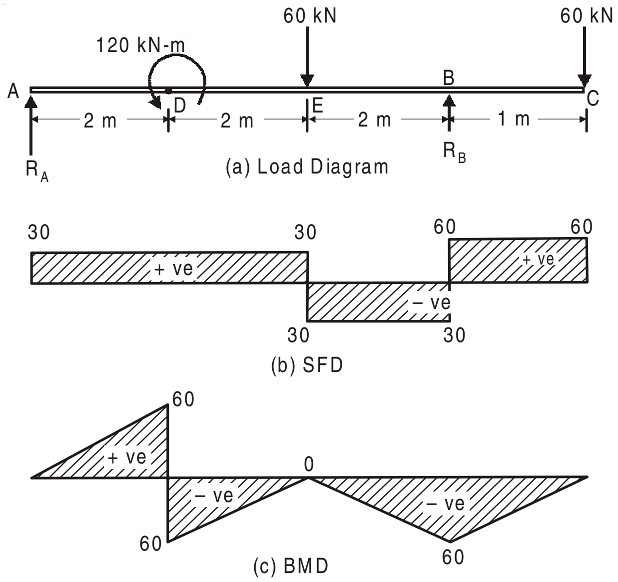

Fig. 9.42

Example 9.16. *Draw the bending moment and shear force diagram for the beam loaded as shown in Fig. 9.43. (a). Mark the values at the salient points Determine the point of contraflexure also.*

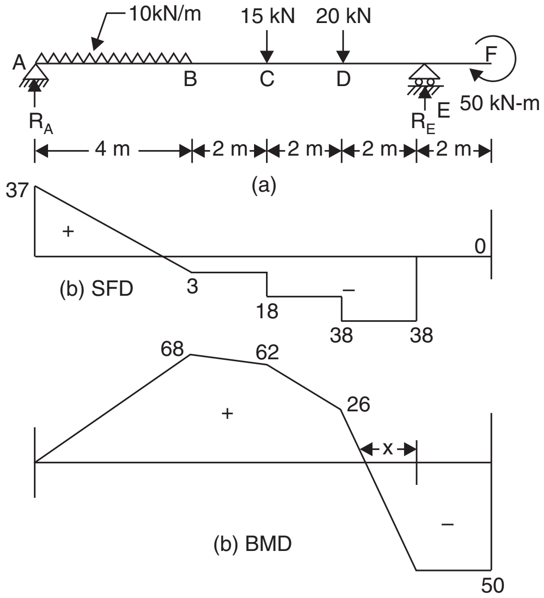

Fig. 9.43

Solution: $\Sigma M_A = 0$, gives

$$R_E \times 10 - 10 \times 4 \times 2 - 15 \times 6 - 20 \times 8 - 50 = 0$$

$$\therefore \quad R_E = 38 \text{ kN}$$

$$\Sigma V = 0, \text{ gives}$$

$$R_A + R_E - 10 \times 4 - 15 - 20 = 0$$

$$R_A = 75 - R_E = 75 - 38 = 37 \text{ kN}$$

SFD:

Values at salient points are:

$$F_A = R_A = 37 \text{ kN}$$

In portion AB, $F_B = 37 - 10 \times 4 = -3$ kN

In portion BC, $F = 37 - 40 = 3$ kN

In portion CD, $F = 37 - 40 - 15 = -18$ kN

In portion DE, $F = 37 - 40 - 15 - 20 = -38$ kN

In portion EF, $F = 0$.

$\therefore$ SFD is as shown in Fig. 9.43 (*b*)

BMD:

At A, $M_A = 0$

At B, $M_B = 37 \times 4 - 10 \times 4 \times 2 = 68$ kN-m

At C, $M_C = 37 \times 6 - 10 \times 4 \times 4 = 62$ kN-m

At D, calculating from right hand side $M_D = -50 + 38 \times 2 = 26$ kN-m

At E, $M_E = -50$ kN-m

Point of contraflexure is between D and E. Let it be at distance x from support E. Then

$$-50 + 38x = 0$$

$\therefore$ **x = 1.32 m from E as shown in Fig. 9.43 (c)**

BMD is as shown in Fig. 9.43 (c)

IMPORTANT FORMULAE

1. Shear force at a section of the beam = Σ all forces to the left or right of the section.

2. Bending moment at a section of the beam = Σ moments of all forces to the left or to the right of the section, moment centre being the section.

3. $\dfrac{dF}{dx} = w$ and $\dfrac{dM}{dx} = -F$

THEORY QUESTIONS

1. Explain the following types of supports to beams:

(i) Simple support (ii) Hinged support

(iii) Fixed support

2. Bring out the difference between statically determinate and indeterminate beams.

3. Derive the relationship between,

(i) Shear force and load intensity (ii) Bending moment and shear force

PROBLEMS FOR EXERCISE

1. Determine the reaction at A and B on the overhanging beam shown in Fig. 9.44.

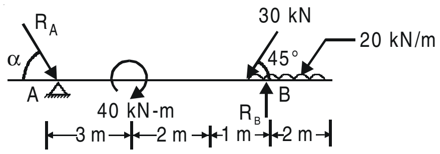

Fig. 9.44

[**Ans.** R_A = 23.3666 kN; a = 24.79°; R_B = 71.011 kN]

2. An overhanging beam is on rollers at A and is hinged at B and is loaded as shown in Fig. 9.45. Determine the reactions at A and B.

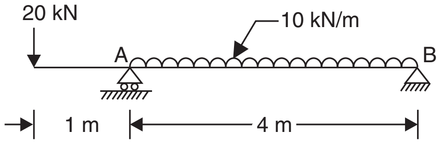

Fig. 9.45

[**Ans.** R_A = 45 kN; R_B = 15 kN]

3. In all the problems given below draw *SFD* and *BMD* indicating values at salient point. (See Fig. 9.46 to 9.50).

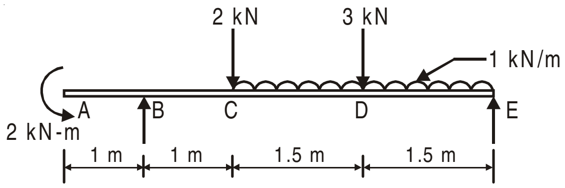

Fig. 9.46

[**Ans.** F_B = 4.25, F_E = 3.75, M_{max} = 4.5 kN-m at *D*
Point of contraflexure at 1.47 m from *A*]

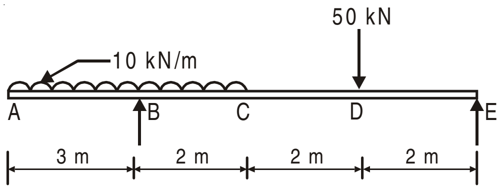

Fig. 9.47

[**Ans.** F_B = 70.83, F_E = 29.17, M_{max} = 45 kN-m at *B*
Point of contraflexure at 4.313 m from *A*]

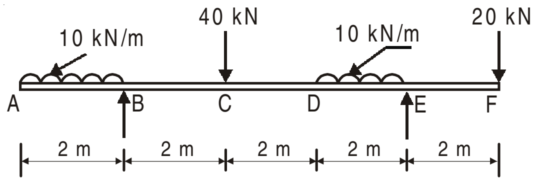

Fig. 9.48

[**Ans.** R_B = 46.67 kN, R_E = 53.33 kN, M_B = –20 kN-m
M_E = – 40 kN-m, M_C = 33.33 kN-m,
Point of contraflexure : 2.75 m from *A* and 3.57 m from *F*]

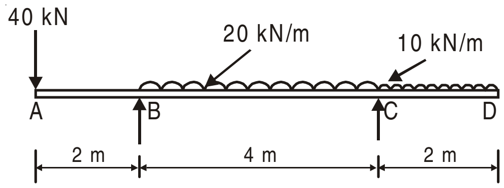

Fig. 9.49

[**Ans.** R_B = 95, R_C = 45, M_{max} = –4.375 at 4.75 m from *A*
No +ve moment anywhere]

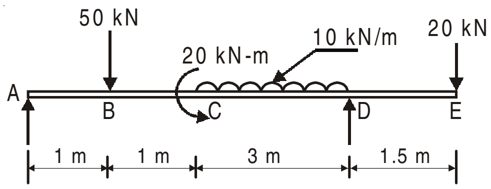

Fig. 9.50

[**Ans.** R_A = 47, R_D = 53, M_B = 47, M_C = 44 to 24, M_D = 30
Point of contraflexure 2.59 m from *F*]

4. Determine load *P* such that reactions at supports *A* and *B* are equal in the beam shown in Fig. 9.51. Draw *SF* and *BM* diagram marking the values at salient point.

[**Ans.** P = 20 kN, $R_A = R_B$ = 50 kN, M_{max} = 62.5 kN-m at 2.5 m from *A*,
Point of contraflexure 1.667 m from *C*]

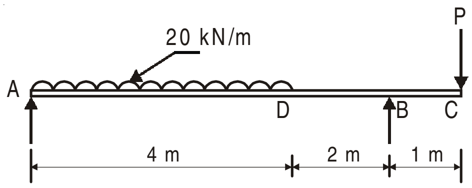

Fig. 9.51

5. A bar of length '*l*' is supported at *A* and *B* which are at distances '*a*' from the ends as shown in Fig. 9.52. Find the distance '*a*' such that maximum moment is least. Using the above results find the most economical length of railway sleeper if the rails are 1.6 m apart.

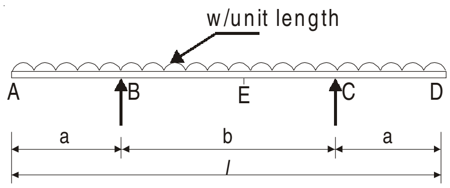

Fig. 9.52

[**Hint:** Moment at *B* = Moment at central point *E*]
[**Ans.** *a* = 0.2071
Economical length of railway sleeper = 2.73 m]