Stresses in Beams

INTRODUCTION

In the preceding chapter we saw how the loads acting on a beam create internal actions (or *stress resultants*) in the form of shear forces and bending moments. In this chapter we go one step further and investigate the *stresses* and *strains* associated with those shear forces and bending moments. Knowing the stresses and strains, we will be able to analyze and design beams subjected to a variety of loading conditions.

The loads acting on a beam cause the beam to bend (or *flex*), thereby deforming its axis into a curve. As an example, consider a cantilever beam AB subjected to a load P at the free end (Fig. 5-1a). The initially straight axis is bent into a curve (Fig. 5-1b), called the **deflection curve** of the beam.

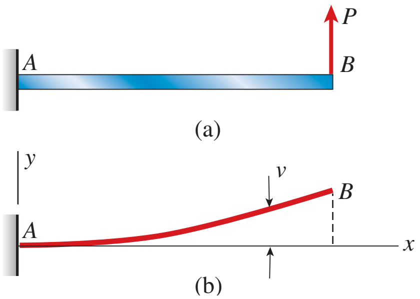


FIG. 5-1 Bending of a cantilever beam: (a) beam with load, and (b) deflection curve

For reference purposes, we construct a system of **coordinate axes** (Fig. 5-1b) with the origin located at a suitable point on the longitudinal axis of the beam. In this illustration, we place the origin at the fixed support. The positive x axis is directed to the right, and the positive y axis is directed upward. The z axis, not shown in the figure, is directed outward (that is, toward the viewer), so that the three axes form a right-handed coordinate system.

The beams considered in this chapter (like those discussed in Chapter 4) are assumed to be symmetric about the xy plane, which means that the y axis is an axis of symmetry of the cross section. In addition, all loads must act in the xy plane. As a consequence, the bending deflections occur in this same plane, known as the **plane of bending**. Thus, the deflection curve shown in Fig. 5-1b is a plane curve lying in the plane of bending.

The **deflection** of the beam at any point along its axis is the *displacement* of that point from its original position, measured in the y direction. We denote the deflection by the letter v to distinguish it from the coordinate y itself (see Fig. 5-1b).*

PURE BENDING AND NONUNIFORM BENDING

When analyzing beams, it is often necessary to distinguish between pure bending and nonuniform bending. **Pure bending** refers to flexure of a beam under a constant bending moment. Therefore, pure bending occurs only in regions of a beam where the shear force is zero (because $V = dM/dx$; see Eq. 4-6). In contrast, **nonuniform bending** refers to flexure in the presence of shear forces, which means that the bending moment changes as we move along the axis of the beam.

As an example of pure bending, consider a simple beam AB loaded by two couples M_1 having the same magnitude but acting in opposite directions (Fig. 5-2a). These loads produce a constant bending moment $M = M_1$ throughout the length of the beam, as shown by the bending

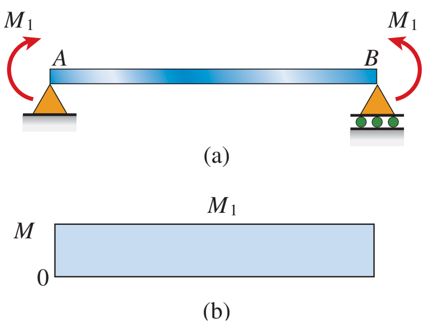


FIG. 5-2 Simple beam in pure bending ($M = M_1$)

*In applied mechanics, the traditional symbols for displacements in the x, y, and z directions are u, v, and w, respectively.

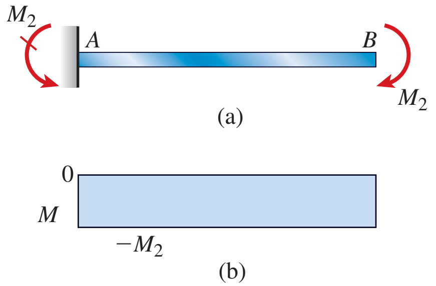


FIG. 5-3 Cantilever beam in pure bending ($M = -M_2$)

moment diagram in part (b) of the figure. Note that the shear force V is zero at all cross sections of the beam.

Another illustration of pure bending is given in Fig. 5-3a, where the cantilever beam AB is subjected to a clockwise couple M_2 at the free end. There are no shear forces in this beam, and the bending moment M is constant throughout its length. The bending moment is negative ($M = -M_2$), as shown by the bending moment diagram in part (b) of Fig. 5-3.

The symmetrically loaded simple beam of Fig. 5-4a is an example of a beam that is partly in pure bending and partly in nonuniform bending, as seen from the shear-force and bending-moment diagrams (Figs. 5-4b and c). The central region of the beam is in pure bending because the shear force is zero and the bending moment is constant. The parts of the beam near the ends are in nonuniform bending because shear forces are present and the bending moments vary.

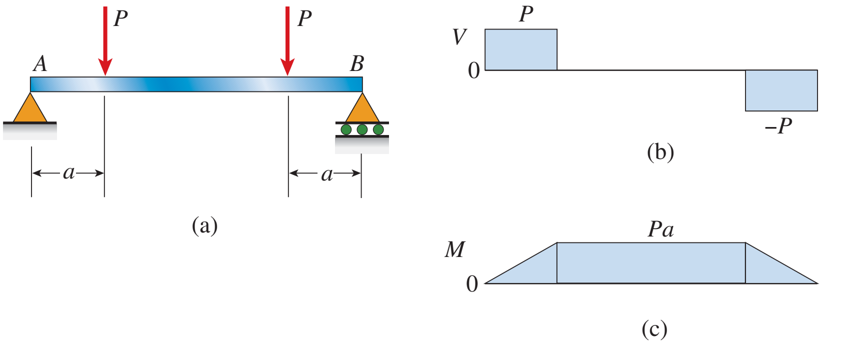


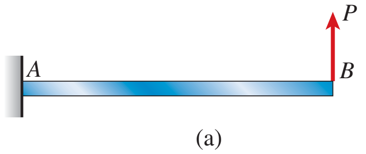
FIG. 5-4 Simple beam with central region in pure bending and end regions in nonuniform bending

In the following two sections we will investigate the strains and stresses in beams subjected only to pure bending. Fortunately, we can often use the results obtained for pure bending even when shear forces are present, as explained later (see the last paragraph in Section 5.8).

CURVATURE OF A BEAM

When loads are applied to a beam, its longitudinal axis is deformed into a curve, as illustrated previously in Fig. 5-1. The resulting strains and stresses in the beam are directly related to the **curvature** of the deflection curve.

To illustrate the concept of curvature, consider again a cantilever beam subjected to a load P acting at the free end (see Fig. 5-5a on the next page). The deflection curve of this beam is shown in Fig. 5-5b. For purposes of analysis, we identify two points m_1 and m_2 on the deflection curve. Point m_1 is selected at an arbitrary distance x from the y axis and point m_2 is located a small distance ds further along the curve. At each of these points we draw a line normal to the *tangent* to the deflection curve,



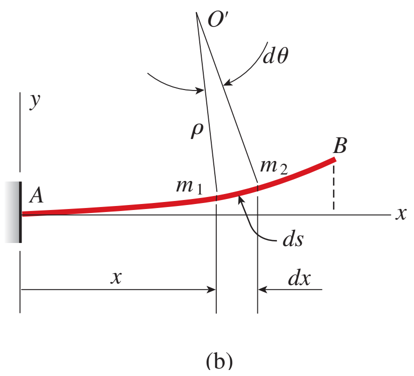


FIG. 5-5 Curvature of a bent beam: (a) beam with load, and (b) deflection curve

that is, normal to the curve itself. These normals intersect at point O', which is the **center of curvature** of the deflection curve. Because most beams have very small deflections and nearly flat deflection curves, point O' is usually located much farther from the beam than is indicated in the figure.

The distance m_1O' from the curve to the center of curvature is called the **radius of curvature** ρ (Greek letter rho), and the **curvature** κ (Greek letter kappa) is defined as the reciprocal of the radius of curvature. Thus,

$$\boxed{\kappa = \frac{1}{\rho}} \tag{5-1}$$

Curvature is a measure of how sharply a beam is bent. If the load on a beam is small, the beam will be nearly straight, the radius of curvature will be very large, and the curvature will be very small. If the load is increased, the amount of bending will increase—the radius of curvature will become smaller, and the curvature will become larger.

From the geometry of triangle $O'm_1m_2$ (Fig. 5-5b) we obtain

$$\rho \, d\theta = ds \tag{a}$$

in which $d\theta$ (measured in radians) is the infinitesimal angle between the normals and ds is the infinitesimal distance along the curve between points m_1 and m_2. Combining Eq. (a) with Eq. (5-1), we get

$$\boxed{\kappa = \frac{1}{\rho} = \frac{d\theta}{ds}} \tag{5-2}$$

This equation for **curvature** is derived in textbooks on calculus and holds for any curve, regardless of the amount of curvature. If the curvature is *constant* throughout the length of a curve, the radius of curvature will also be constant and the curve will be an arc of a circle.

The deflections of a beam are usually very small compared to its length (consider, for instance, the deflections of the structural frame of an automobile or a beam in a building). Small deflections mean that the deflection curve is nearly flat. Consequently, the distance ds along the curve may be set equal to its horizontal projection dx (see Fig. 5-5b). Under these special conditions of **small deflections**, the equation for the curvature becomes

$$\boxed{\kappa = \frac{1}{\rho} = \frac{d\theta}{dx}} \tag{5-3}$$

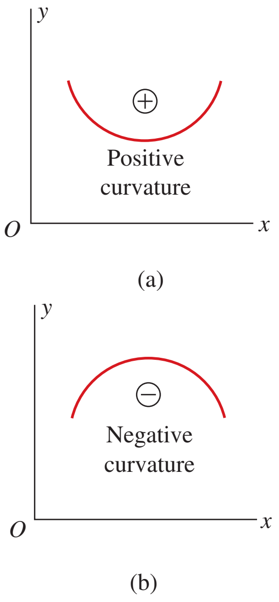


FIG. 5-6 Sign convention for curvature

Both the curvature and the radius of curvature are functions of the distance x measured along the x axis. It follows that the position O' of the center of curvature also depends upon the distance x.

In Section 5.5 we will see that the curvature at a particular point on the axis of a beam depends upon the bending moment at that point and upon the properties of the beam itself (shape of cross section and type of material). Therefore, if the beam is prismatic and the material is homogeneous, the curvature will vary only with the bending moment. Consequently, a beam in *pure bending* will have constant curvature and a beam in *nonuniform bending* will have varying curvature.

The **sign convention for curvature** depends upon the orientation of the coordinate axes. If the x axis is positive to the right and the y axis is positive upward, as shown in Fig. 5-6, then the curvature is positive when the beam is bent concave upward and the center of curvature is above the beam. Conversely, the curvature is negative when the beam is bent concave downward and the center of curvature is below the beam.

In the next section we will see how the longitudinal strains in a bent beam are determined from its curvature, and in Chapter 9 we will see how curvature is related to the deflections of beams.

NORMAL STRESSES IN BEAMS (LINEARLY ELASTIC MATERIALS)

In the preceding section we investigated the longitudinal strains ϵ_x in a beam in pure bending (see Eq. 5-4 and Fig. 5-7). Since longitudinal elements of a beam are subjected only to tension or compression, we can use the **stress-strain curve** for the material to determine the stresses from the strains. The stresses act over the entire cross section of the beam and vary in intensity depending upon the shape of the stress-strain diagram and the dimensions of the cross section. Since the x direction is longitudinal (Fig. 5-7a), we use the symbol σ_x to denote these stresses.

The most common stress-strain relationship encountered in engineering is the equation for a **linearly elastic material**. For such materials we substitute Hooke's law for uniaxial stress ($\sigma = E\epsilon$) into Eq. (5-4) and obtain

$$\sigma_x = E\epsilon_x = -\frac{Ey}{\rho} = -E\kappa y \tag{5-7}$$

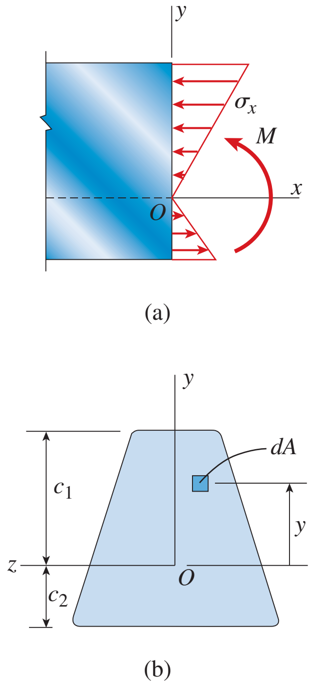


FIG. 5-9 Normal stresses in a beam of linearly elastic material: (a) side view of beam showing distribution of normal stresses, and (b) cross section of beam showing the z axis as the neutral axis of the cross section

This equation shows that the normal stresses acting on the cross section vary linearly with the distance y from the neutral surface. This stress distribution is pictured in Fig. 5-9a for the case in which the bending moment M is positive and the beam bends with positive curvature.

When the curvature is positive, the stresses σ_x are negative (compression) above the neutral surface and positive (tension) below it. In the figure, compressive stresses are indicated by arrows pointing *toward* the cross section and tensile stresses are indicated by arrows pointing *away* from the cross section.

In order for Eq. (5-7) to be of practical value, we must locate the origin of coordinates so that we can determine the distance y. In other words, we must locate the neutral axis of the cross section. We also need to obtain a relationship between the curvature and the bending moment—so that we can substitute into Eq. (5-7) and obtain an equation relating the stresses to the bending moment. These two objectives can be accomplished by determining the resultant of the stresses σ_x acting on the cross section.

In general, the **resultant of the normal stresses** consists of two stress resultants: (1) a force acting in the x direction, and (2) a bending couple acting about the z axis. However, the axial force is zero when a beam is in pure bending. Therefore, we can write the following equations of statics: (1) The resultant force in the x direction is equal to zero, and (2) the resultant moment is equal to the bending moment M. The first equation gives the location of the neutral axis and the second gives the moment-curvature relationship.

Location of Neutral Axis

To obtain the first equation of statics, we consider an element of area dA in the cross section (Fig. 5-9b). The element is located at distance y from

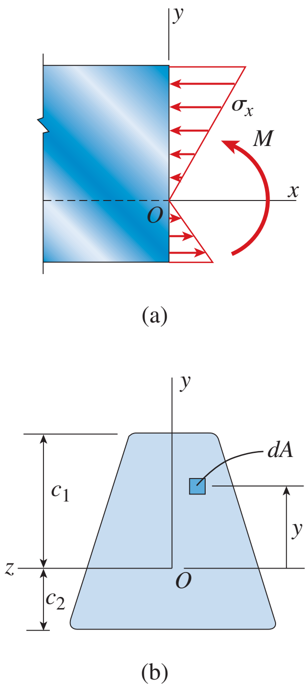


FIG. 5-9 (Repeated)

the neutral axis, and therefore the stress σ_x acting on the element is given by Eq. (5-7). The *force* acting on the element is equal to $\sigma_x dA$ and is compressive when y is positive. Because there is no resultant force acting on the cross section, the integral of $\sigma_x dA$ over the area A of the entire cross section must vanish; thus, the *first equation of statics* is

$$\int_A \sigma_x dA = -\int_A E\kappa y\, dA = 0 \qquad \text{(a)}$$

Because the curvature κ and modulus of elasticity E are nonzero constants at any given cross section of a bent beam, they are not involved in the integration over the cross-sectional area. Therefore, we can drop them from the equation and obtain

$$\boxed{\int_A y\, dA = 0} \qquad \text{(5-8)}$$

This equation states that the first moment of the area of the cross section, evaluated with respect to the z axis, is zero. In other words, the z axis must pass through the centroid of the cross section.*

Since the z axis is also the neutral axis, we have arrived at the following important conclusion: *The neutral axis passes through the centroid of the cross-sectional area when the material follows Hooke's law and there is no axial force acting on the cross section.* This observation makes it relatively simple to determine the position of the neutral axis.

As explained in Section 5.1, our discussion is limited to beams for which the y axis is an axis of symmetry. Consequently, the y axis also passes through the centroid. Therefore, we have the following additional conclusion: *The origin O of coordinates* (Fig. 5-9b) *is located at the centroid of the cross-sectional area.*

Because the y axis is an axis of symmetry of the cross section, it follows that the y axis is a *principal axis* (see Chapter 12, Section 12.9, for a discussion of principal axes). Since the z axis is perpendicular to the y axis, it too is a principal axis. Thus, when a beam of linearly elastic material is subjected to pure bending, *the y and z axes are principal centroidal axes.*

Moment-Curvature Relationship

The *second equation of statics* expresses the fact that the moment resultant of the normal stresses σ_x acting over the cross section is equal to the bending moment M (Fig. 5-9a). The element of force $\sigma_x dA$ acting on the element of area dA (Fig. 5-9b) is in the positive direction of the x axis when σ_x is positive and in the negative direction when σ_x is negative. Since the element dA is located above the neutral axis, a positive stress

*Centroids and first moments of areas are discussed in Chapter 12, Sections 12.2 and 12.3.

σ_x acting on that element produces an element of moment equal to $\sigma_x y\, dA$. This element of moment acts opposite in direction to the positive bending moment M shown in Fig. 5-9a. Therefore, the elemental moment is

$$dM = -\sigma_x y\, dA$$

The integral of all such elemental moments over the entire cross-sectional area A must equal the bending moment:

$$M = -\int_A \sigma_x y\, dA \tag{b}$$

or, upon substituting for σ_x from Eq. (5-7),

$$M = \int_A \kappa E y^2\, dA = \kappa E \int_A y^2\, dA \tag{5-9}$$

This equation relates the curvature of the beam to the bending moment M.

Since the integral in the preceding equation is a property of the cross-sectional area, it is convenient to rewrite the equation as follows:

$$M = \kappa E I \tag{5-10}$$

in which

$$\boxed{I = \int_A y^2\, dA} \tag{5-11}$$

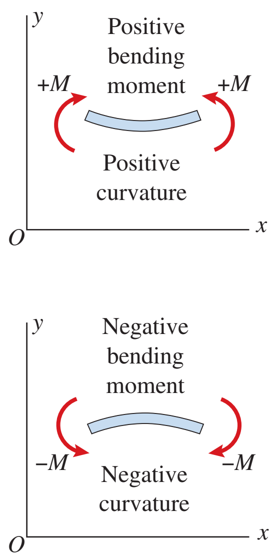


FIG. 5-10 Relationships between signs of bending moments and signs of curvatures

This integral is the **moment of inertia** of the cross-sectional area with respect to the z axis (that is, with respect to the neutral axis). Moments of inertia are always positive and have dimensions of length to the fourth power; for instance, typical USCS units are in.^{4} and typical SI units are mm^4 when performing beam calculations.*

Equation (5-10) can now be rearranged to express the *curvature* in terms of the bending moment in the beam:

$$\boxed{\kappa = \frac{1}{\rho} = \frac{M}{EI}} \tag{5-12}$$

Known as the **moment-curvature equation**, Eq. (5-12) shows that the curvature is directly proportional to the bending moment M and inversely proportional to the quantity EI, which is called the **flexural rigidity** of the beam. Flexural rigidity is a measure of the resistance of a beam to bending, that is, the larger the flexural rigidity, the smaller the curvature for a given bending moment.

Comparing the **sign convention** for bending moments (Fig. 4-5) with that for curvature (Fig. 5-6), we see that *a positive bending moment produces positive curvature and a negative bending moment produces negative curvature* (see Fig. 5-10).

*Moments of inertia of areas are discussed in Chapter 12, Section 12.4.

Flexure Formula

Now that we have located the neutral axis and derived the moment-curvature relationship, we can determine the stresses in terms of the bending moment. Substituting the expression for curvature (Eq. 5-12) into the expression for the stress σ_x (Eq. 5-7), we get

$$\sigma_x = -\frac{My}{I} \qquad (5\text{-}13)$$

This equation, called the **flexure formula**, shows that the stresses are directly proportional to the bending moment M and inversely proportional to the moment of inertia I of the cross section. Also, the stresses vary linearly with the distance y from the neutral axis, as previously observed. Stresses calculated from the flexure formula are called **bending stresses** or **flexural stresses**.

If the bending moment in the beam is positive, the bending stresses will be positive (tension) over the part of the cross section where y is negative, that is, over the lower part of the beam. The stresses in the upper part of the beam will be negative (compression). If the bending moment is negative, the stresses will be reversed. These relationships are shown in Fig. 5-11.

Maximum Stresses at a Cross Section

The maximum tensile and compressive bending stresses acting at any given cross section occur at points located farthest from the neutral axis. Let us denote by c_1 and c_2 the distances from the neutral axis to the extreme elements in the positive and negative y directions, respectively (see Fig. 5-9b and Fig. 5-11). Then the corresponding **maximum normal stresses** σ_1 and σ_2 (from the flexure formula) are

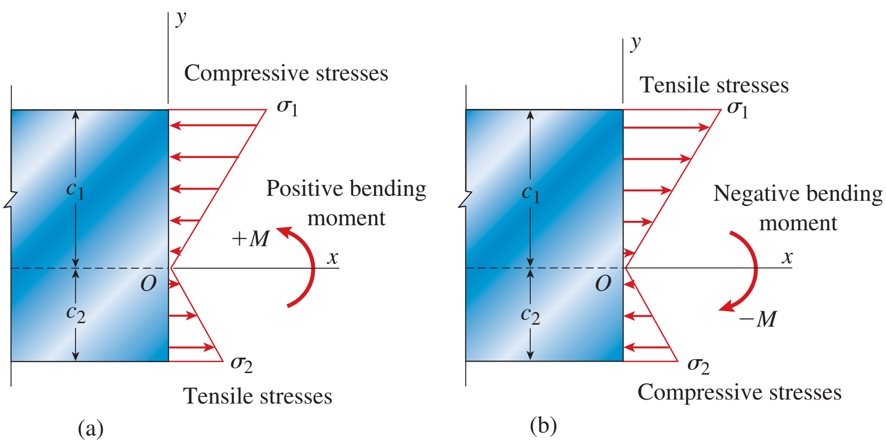


FIG. 5-11 Relationships between signs of bending moments and directions of normal stresses: (a) positive bending moment, and (b) negative bending moment

Example

A simple beam AB of span length $L = 22$ ft (Fig. 5-14a) supports a uniform load of intensity $q = 1.5$ k/ft and a concentrated load $P = 12$ k. The uniform load includes an allowance for the weight of the beam. The concentrated load acts at a point 9.0 ft from the left-hand end of the beam. The beam is constructed of glued laminated wood and has a cross section of width $b = 8.75$ in. and height $h = 27$ in. (Fig. 5-14b).

Determine the maximum tensile and compressive stresses in the beam due to bending.

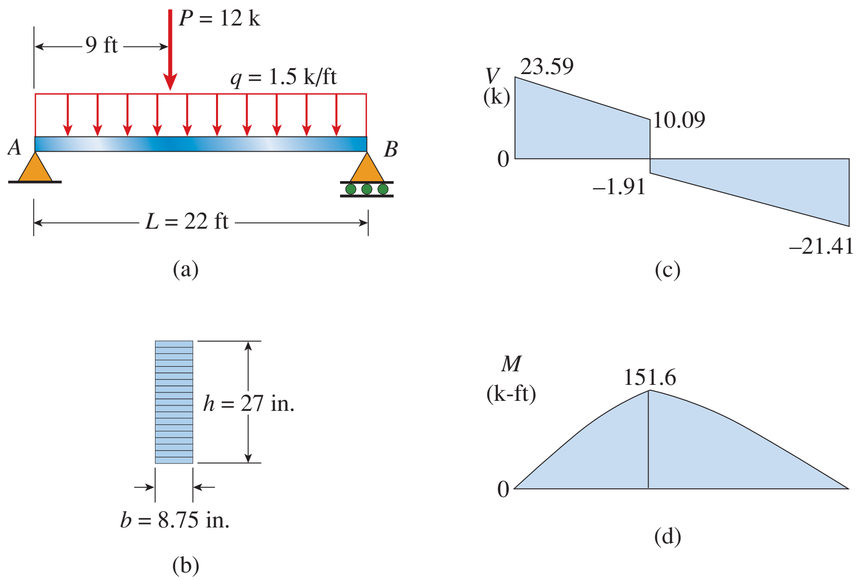


FIG. 5-14 Example 5-3. Stresses in a simple beam

Solution

Reactions, shear forces, and bending moments. We begin the analysis by calculating the reactions at supports A and B, using the techniques described in Chapter 4. The results are

$$R_A = 23.59 \text{ k} \qquad R_B = 21.41 \text{ k}$$

Knowing the reactions, we can construct the shear-force diagram, shown in Fig. 5-14c. Note that the shear force changes from positive to negative under the concentrated load P, which is at a distance of 9 ft from the left-hand support.

continued

Next, we draw the bending-moment diagram (Fig. 5-14d) and determine the maximum bending moment, which occurs under the concentrated load where the shear force changes sign. The maximum moment is

$$M_{\max} = 151.6 \text{ k-ft}$$

The maximum bending stresses in the beam occur at the cross section of maximum moment.

Section modulus. The section modulus of the cross-sectional area is calculated from Eq. (5-18b), as follows:

$$S = \frac{bh^2}{6} = \frac{1}{6}(8.75 \text{ in.})(27 \text{ in.})^2 = 1063 \text{ in.}^3$$

Maximum stresses. The maximum tensile and compressive stresses σ_t and σ_c, respectively, are obtained from Eq. (5-16a):

$$\sigma_t = \sigma_2 = \frac{M_{\max}}{S} = \frac{(151.6 \text{ k-ft})(12 \text{ in./ft})}{1063 \text{ in.}^3} = 1710 \text{ psi}$$

$$\sigma_c = \sigma_1 = -\frac{M_{\max}}{S} = -1710 \text{ psi}$$

Because the bending moment is positive, the maximum tensile stress occurs at the bottom of the beam and the maximum compressive stress occurs at the top.

Example 5

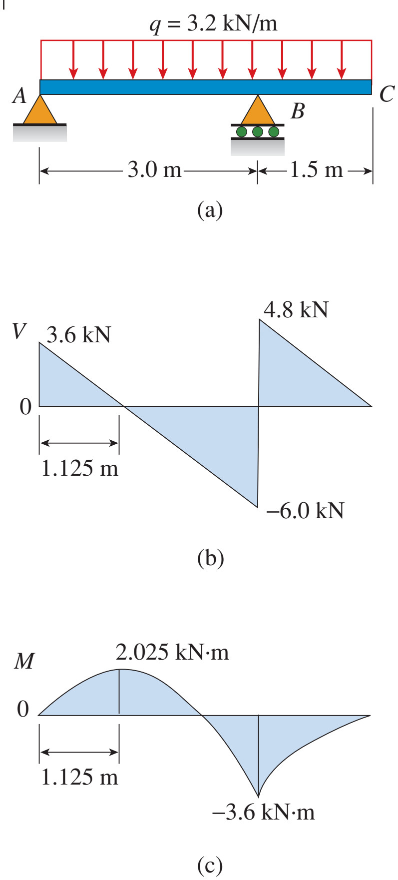


FIG. 5-15 Example 5-4. Stresses in a beam with an overhang

The beam *ABC* shown in Fig 5-15a has simple supports at *A* and *B* and an overhang from *B* to *C*. The length of the span is 3.0 m and the length of the overhang is 1.5 m. A uniform load of intensity $q = 3.2$ kN/m acts throughout the entire length of the beam (4.5 m).

The beam has a cross section of channel shape with width $b = 300$ mm and height $h = 80$ mm (Fig. 5-16a). The web thickness is $t = 12$ mm, and the average thickness of the sloping flanges is the same. For the purpose of calculating the properties of the cross section, assume that the cross section consists of three rectangles, as shown in Fig. 5-16b.

Determine the maximum tensile and compressive stresses in the beam due to the uniform load.

Solution

Reactions, shear forces, and bending moments. We begin the analysis of this beam by calculating the reactions at supports *A* and *B*, using the techniques described in Chapter 4. The results are

$$R_A = 3.6 \text{ kN} \qquad R_B = 10.8 \text{ kN}$$

From these values, we construct the shear-force diagram (Fig. 5-15b). Note that the shear force changes sign and is equal to zero at two locations: (1) at a distance of 1.125 m from the left-hand support, and (2) at the right-hand reaction.

Next, we draw the bending-moment diagram, shown in Fig. 5-15c. Both the maximum positive and maximum negative bending moments occur at the cross sections where the shear force changes sign. These maximum moments are

$$M_{\text{pos}} = 2.025 \text{ kN·m} \qquad M_{\text{neg}} = -3.6 \text{ kN·m}$$

respectively.

Neutral axis of the cross section (Fig. 5-16b). The origin *O* of the *yz* coordinates is placed at the centroid of the cross-sectional area, and therefore the *z* axis becomes the neutral axis of the cross section. The centroid is located by using the techniques described in Chapter 12, Section 12.3, as follows.

First, we divide the area into three rectangles (A_1, A_2, and A_3). Second, we establish a reference axis *Z-Z* across the upper edge of the cross section, and we let y_1 and y_2 be the distances from the *Z-Z* axis to the centroids of areas A_1 and

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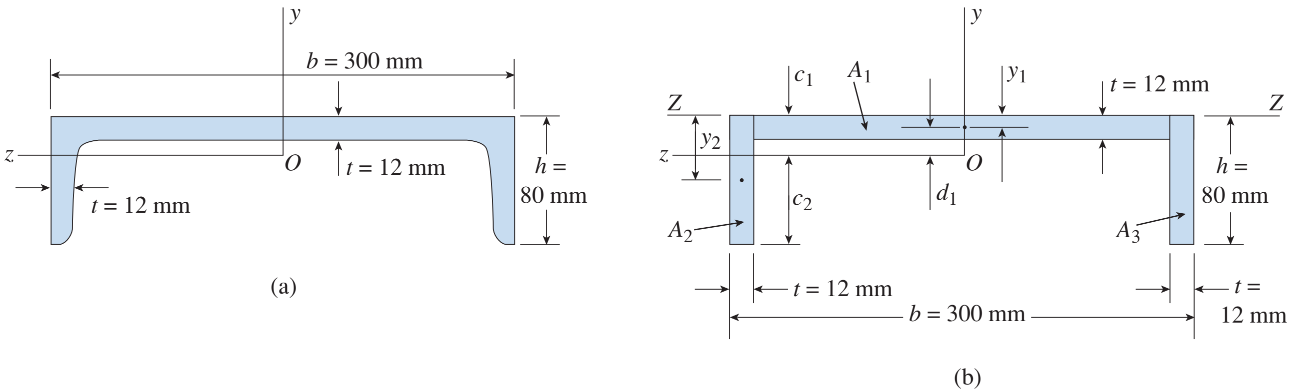

FIG. 5-16 Cross section of beam discussed in Example 5-4. (a) Actual shape, and (b) idealized shape for use in analysis (the thickness of the beam is exaggerated for clarity)

A_2, respectively. Then the calculations for locating the centroid of the entire channel section (distances c_1 and c_2) are as follows:

Area 1: $y_1 = t/2 = 6\text{ mm}$
$A_1 = (b - 2t)(t) = (276\text{ mm})(12\text{ mm}) = 3312\text{ mm}^2$

Area 2: $y_2 = h/2 = 40\text{ mm}$
$A_2 = ht = (80\text{ mm})(12\text{ mm}) = 960\text{ mm}^2$

Area 3: $y_3 = y_2 \qquad A_3 = A_2$

$$c_1 = \frac{\sum y_i A_i}{\sum A_i} = \frac{y_1 A_1 + 2y_2 A_2}{A_1 + 2A_2}$$

$$= \frac{(6\text{ mm})(3312\text{ mm}^2) + 2(40\text{ mm})(960\text{ mm}^2)}{3312\text{ mm}^2 + 2(960\text{ mm}^2)} = 18.48\text{ mm}$$

$$c_2 = h - c_1 = 80\text{ mm} - 18.48\text{ mm} = 61.52\text{ mm}$$

Thus, the position of the neutral axis (the z axis) is determined.

Moment of inertia. In order to calculate the stresses from the flexure formula, we must determine the moment of inertia of the cross-sectional area with respect to the neutral axis. These calculations require the use of the parallel-axis theorem (see Chapter 12, Section 12.5).

Beginning with area A_1, we obtain its moment of inertia $(I_z)_1$ about the z axis from the equation

$$(I_z)_1 = (I_c)_1 + A_1 d_1^2 \qquad \text{(c)}$$

In this equation, $(I_c)_1$ is the moment of inertia of area A_1 about its own centroidal axis:

$$(I_c)_1 = \frac{1}{12}(b-2t)(t)^3 = \frac{1}{12}(276\text{ mm})(12\text{ mm})^3 = 39{,}744\text{ mm}^4$$

and d_1 is the distance from the centroidal axis of area A_1 to the z axis:

$$d_1 = c_1 - t/2 = 18.48\text{ mm} - 6\text{ mm} = 12.48\text{ mm}$$

Therefore, the moment of inertia of area A_1 about the z axis (from Eq. c) is

$$(I_z)_1 = 39{,}744 \text{ mm}^4 + (3312 \text{ mm}^2)(12.48 \text{ mm}^2) = 555{,}600 \text{ mm}^4$$

Proceeding in the same manner for areas A_2 and A_3, we get

$$(I_z)_2 = (I_z)_3 = 956{,}600 \text{ mm}^4$$

Thus, the centroidal moment of inertia I_z of the entire cross-sectional area is

$$I_z = (I_z)_1 + (I_z)_2 + (I_z)_3 = 2.469 \times 10^6 \text{ mm}^4$$

Section moduli. The section moduli for the top and bottom of the beam, respectively, are

$$S_1 = \frac{I_z}{c_1} = 133{,}600 \text{ mm}^3 \qquad S_2 = \frac{I_z}{c_2} = 40{,}100 \text{ mm}^3$$

(see Eqs. 5-15a and b). With the cross-sectional properties determined, we can now proceed to calculate the maximum stresses from Eqs. (5-14a and b).

Maximum stresses. At the cross section of maximum positive bending moment, the largest tensile stress occurs at the bottom of the beam (σ_2) and the largest compressive stress occurs at the top (σ_1). Thus, from Eqs. (5-14b) and (5-14a), respectively, we get

$$\sigma_t = \sigma_2 = \frac{M_{\text{pos}}}{S_2} = \frac{2.025 \text{ kN·m}}{40{,}100 \text{ mm}^3} = 50.5 \text{ MPa}$$

$$\sigma_c = \sigma_1 = -\frac{M_{\text{pos}}}{S_1} = -\frac{2.025 \text{ kN·m}}{133{,}600 \text{ mm}^3} = -15.2 \text{ MPa}$$

Similarly, the largest stresses at the section of maximum negative moment are

$$\sigma_t = \sigma_1 = -\frac{M_{\text{neg}}}{S_1} = -\frac{-3.6 \text{ kN·m}}{133{,}600 \text{ mm}^3} = 26.9 \text{ MPa}$$

$$\sigma_c = \sigma_2 = \frac{M_{\text{neg}}}{S_2} = \frac{-3.6 \text{ kN·m}}{40{,}100 \text{ mm}^3} = -89.8 \text{ MPa}$$

A comparison of these four stresses shows that the largest tensile stress in the beam is 50.5 MPa and occurs at the bottom of the beam at the cross section of maximum positive bending moment; thus,

$$(\sigma_t)_{\max} = 50.5 \text{ MPa}$$

The largest compressive stress is -89.8 MPa and occurs at the bottom of the beam at the section of maximum negative moment:

$$(\sigma_c)_{\max} = -89.8 \text{ MPa}$$

Thus, we have determined the maximum bending stresses due to the uniform load acting on the beam.