Atomic Structure and Atomic Models (2):

Electron energy according to Rutherford Model:

In order to have a stable orbit, the two forces must be equal in magnitude and opposite in direction, for hydrogen atom:

$$\frac{e^2}{4\pi\varepsilon_o r^2} = \frac{mv^2}{r}$$

The kinetic energy of the electron is given as:

$$E_k = \frac{mv^2}{2}$$

Where E_k is the Kinetic energy of the electron in joule (J). Substituting (v) produces:

$$E_k = \frac{e^2}{8\pi\varepsilon_o r}$$

On the other side, the electron posses a potential energy (E_p) because it lies at a distance (r) from the nucleus:

$$E_p = \frac{-\,e^2}{4\pi\varepsilon_o r^2}$$

The total energy equals is E_T:

$$E_T = E_k + E_p = \frac{-\,e^2}{8\pi\varepsilon_o r}$$

According to the electromagnetic theory, the electron revolving around the nucleus must continuously radiate energy in the form of electromagnetic radiation. If the electron is radiating energy, the electron must move in smaller and smaller orbits, eventually falling into the nucleus. On the other hand, the frequency of oscillation depends upon the radius of the circular orbit, the energy radiated would be of a gradually changing frequency. This is incompatible with the sharply defined frequencies of spectral lines.

Bohr's Tomic Model

Bohr considered the simplest of all atoms, the hydrogen atom. Bohr assumed that the electron revolves in a circular orbit. In Bohr's model of the atom, the electrons are assumed to be in a definite planetary system (in circular orbits) of fixed energy. These stationary states are called as energy levels. Since there is a definite value of potential energy associated with each orbit. In this model of the atom, Bohr considered that more than one energy level to be possible for any electron of the atom. One can determine the allowed energy levels by the condition that the angular momentum of the electron moving in a circular orbit can take one of values

$$L = nh/2\pi = nh$$

where n is a positive integer known as the quantum number, and its possible values are n = 1, 2, 3,. . . etc. This equation is known as Bohr's quantum condition. The electron in this case is in a stationary state. According to Bohr's model, an electron can either emit or absorb energy when making a transition from one possible orbit to another.

Bohr's Postulates:

- The electron can revolve round the nucleus in certain definite orbits, known as stationary orbits. The electrons are permitted to have only those orbits for which the angular momenta of the planetary electron are integral multiple of h/2π or h. This is Bohr's first postulate.
- When the electron revolves in a stationary orbit, it does not emit electromagnetic radiation as predicted by the electromagnetic theory of light. Radiation occurs only when an electron falls from a higher energy state to a lower energy state. If the transition is from an orbit of higher energy E2 to an orbit of energy E1, then the energy *hv* of the emitted radiation, according to Planck's law, will be:

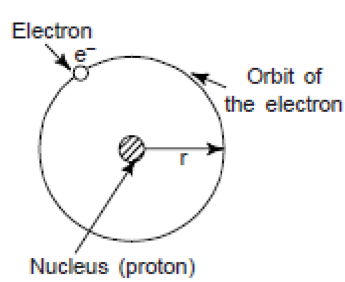
$$hv = hw = E_2 - E_1$$

where v is the frequency of the emitted radiation; w =2πv is the angular frequency. hv, the energy difference ejected from the atom in the form of light radiations of energy called a photon. This reveals the origin of light waves from atom. Obviously, light is not emitted by an electron when moving in one of its stationary orbits, but it ejects light only when it jumps from one orbit to another. This is Bohr's second postulate.

Note:

We must note that the above postulates of Bohr are in direct contradiction to the laws of classical mechanics and to Maxwell's electromagnetic theory. According to Kepler's laws in classical mechanics, all orbits are permissible for the electron revolving round the nucleus. Thus Bohr's first postulate regarding the stationary orbits contradicts the laws of classical mechanics. Again according to the electromagnetic theory of light, a revolving electron must emit electromagnetic radiation because its motion is under centripetal acceleration. Obviously, Bohr's second postulate that no emission or absorption of radiation takes place when the electron revolves in a stationary orbit contradicts the electromagnetic theory of light. We must note that the postulates of Bohr were introduced as ad hoc hypotheses. Bohr, accepted the classical laws of mechanics as the laws governing the motion of electrons. He imposed certain restrictions upon the permissible orbits through the quantum condition. Bohr made no attempts to propound any mechanics to describe the motion of the electron.

- To keep the electron in its orbit around the small heavy central part of the atom called nucleus and prevent it from spiraling toward the nucleus or away from it to escape, Bohr next assumed that the centripetal force (=mv2/r) required for rotation is provided by inward electrostatic force of attraction between the positively charged nucleus and the negatively charged electrons.



Bohr developed the theory of hydrogen atom (which is also applicable to hydrogen like atoms). A hydrogen atom (Z = 1) is the simplest of all atoms consists of a nucleus with one positive charge e (proton) and a single electron of charge –e revolving around it in a circular orbit of radius r. Proton (i.e., nucleus of hydrogen atom) is 1836 times heavier than electron, the nucleus could be assumed at rest.

$$\frac{-e^2}{4\pi\varepsilon_o r^2}$$

In accordance with Newton's third law of motion:

$$\frac{Ze^2}{4\pi\varepsilon_o r^2} = \frac{mv^2}{r}$$

From Bohr's quantum condition:

$$mvr = L = mr^2 w = \frac{nh}{2\pi}$$

Where n = 1, 2, 3, . . . is an integer, each value of n is associated with a different orbit.

$$v = \frac{nh}{2\pi mr} = \frac{nh}{mr}$$

$$v^2 = \frac{Ze^2}{4\pi\varepsilon_o mr}$$

$$r = \frac{4\pi\varepsilon_o n^2 h^2}{mZe^2}$$

We can see that the radii of the stationary orbits are proportional to the square of the principal quantum number n, i.e., they are in the ratio of 1 : 4 : 9 : 16 : . . . The orbit with n = 1 has the smallest radius. For hydrogen (Z = 1), this radius is known as Bohr radius and is given by:

$$a_o = \frac{4\pi\varepsilon_o h^2}{me^2} = 0.529\ x\ 10^{-10}\ m = 0.529\ A$$

Velocity of the revolving electron, i.e., orbital velocity Substituting the value of radius, r from relation (15) in Eq. (13), one obtains:

$$v = \frac{Ze^2}{2\varepsilon_o nh} = \frac{Ze^2}{4\pi\varepsilon_o nh}$$

The electron velocity also depends on the quantum numbers n, i.e., velocity of the electron is inversely proportional n and is the highest in the smallest orbit having n = 1. The velocities become progressively less in the orbits of increasing radii. For hydrogen, the velocity of the electron in the Bohr orbit is:

$$v = \frac{e^2}{4\pi\varepsilon_o h} = 2.18\ x\ 10^6\ m/s$$

This velocity is about 1/137 times the velocity of light c (= 3 x 10^{10} m/s).

Orbital frequency

We have $w = 2\pi v = v/r$

$$v = \frac{v}{2\pi r}$$

Substituting the values of v and r from Eqs. (16) and (15) in the above relation, one obtains:

$$v = \frac{nZ^2e^4}{64\pi^2{\varepsilon_o}^2h^3} = \frac{nZ^2e^4}{4{\varepsilon_o}^2n^3h^3}$$

Obviously, frequency *v* is inversely proportional to n^3.

Energy of the Electron

The energy of an electron revolving around the nucleus in a orbit is of two parts:

(i) Kinetic Energy (E_k) The K.E. is:

$$E_k = \frac{1}{2}mv^2 = \frac{Ze^2}{2\pi\varepsilon_o r}$$

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Substituting the value of r, one obtains:

$$E_k = \frac{mZ^2e^4}{8{\varepsilon_o}^2n^2h^2}$$

Potential Energy E_p

The potential energy is due to the position of nuclear electron in the orbit, i.e. the electron lies in the electric field of the positive nucleus, i.e. E_p = the product of electrical potential, *v* and nuclear charge, e

$$E_p = -eV = \frac{Ze^2}{2\pi\varepsilon_o r}$$

$$V = -\frac{Ze}{4\pi\varepsilon_o r}$$

Total Energy

Total orbital energy E is the sum of,

$$E \quad = E_p + E_k = \frac{1}{2}mv^2 - \frac{Ze^2}{2\pi\varepsilon_o r}$$

$$= \frac{Ze^2}{8\pi\varepsilon_o r} - \frac{Ze^2}{4\pi\varepsilon_o r} = -\frac{Ze^2}{8\pi\varepsilon_o r}$$