Electric Circuits

First course

Electronic and communication department

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chapter one

Basic Concepts

SYSTEMS OF UNITS

Table 1.1: The SI prefixes

Multiplier	Prefix	Symbol
10^{18}	exa	E
10^{15}	peta	P
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{2}	hecto	h
10	deka	da
10^{-1}	deci	d
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f
10^{-18}	atto	a

Table 1.2: Six basic SI units and one derived unit relevant to this text.

Quantity	Basic unit	Symbol
Length	meter	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Thermodynamic temperature	kelvin	K
Luminous intensity	candela	cd

1.1 Charge and Current

The concept of electric charge is the underlying principle for explaining all electrical phenomena. Also, the most basic quantity in an electric circuit is the *electric charge.*

Charge is an electrical property of the atomic particles of which matter consists, measured in coulombs (C).

The following points should be noted about electric charge:

1. The coulomb is a large unit for charges. In 1 C of charge, there are 1/ (1.602 X 10^{-19}) = 6.24×10^{18} electrons. Thus realistic or laboratory values of charges are on the order of pC, nC, or μC^{1}.
2. According to experimental observations, the only charges that occur in nature are integral multiples of the electronic charge e= 1.602 X 10^{-19} C.

electronic charge e= 1.602 X 10^{-19} C

3. The *law of conservation of charge* states that charge can neither be created nor destroyed, only transferred. Thus, the algebraic sum of the electric charges in a system does not change.

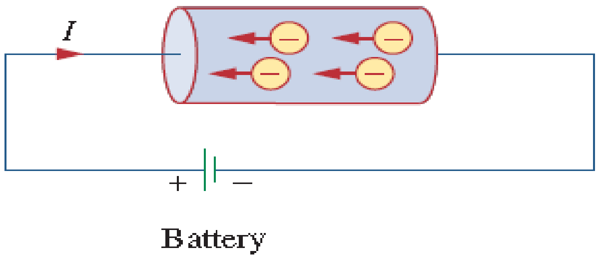


Figure 1.1 Electric current due to flow of electronic charge in a conductor

Electric current is the time rate of change of charge, measured in amperes (A).

Mathematically, the relationship between current i, charge q, and time t is

$$\boxed{i \triangleq \frac{dq}{dt}} \quad \ldots\ldots\ldots (1.1)$$

where current is measured in amperes (A), and

1 ampere = 1 coulomb/second

The charge transferred between time t_0 and t is obtained by integrating both sides of Eq. (1.1). We obtain

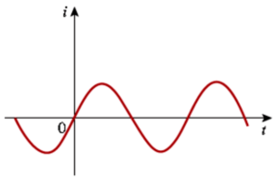
$$Q \triangleq \int_{t_0}^{t} i \, dt$$

The way we define current as i in Eq. above

If the current does not change with time, but remains constant, we call it a direct current (dc).

A direct current (dc) is a current that remains constant with time

A time-varying current is represented by the symbol i. A common form of time-varying current is the sinusoidal current or *alternating current* (ac).



An **alternating current (ac)** is a current that varies sinusoidally with time.

Example

The total charge entering a terminal is given by $q = 5t \sin 4\pi t$ mC. Calculate the current at $t = 0.5$ s.

Solution:

$$i = \frac{dq}{dt} = \frac{d}{dt}(5t \sin 4\pi t) \text{ mC/s} = (5 \sin 4\pi t + 20\pi t \cos 4\pi t) \text{ mA}$$

At $t = 0.5$,

$$i = 5 \sin 2\pi + 10\pi \cos 2\pi = 0 + 10\pi = 31.42 \text{ mA}$$

Practice Problem

H.W

If in Example 1.2, $q = (10 - 10e^{-2t})$ mC, find the current at $t = 0.5$ s.

Answer: 7.36 mA.

Example

Determine the total charge entering a terminal between $t = 1$ s and $t = 2$ s if the current passing the terminal is $i = (3t^2 - t)$ A.

Solution:

$$q = \int_{t=1}^{2} i\,dt = \int_{1}^{2} (3t^2 - t)\,dt$$

$$= \left(t^3 - \frac{t^2}{2}\right)\Bigg|_1^2 = (8 - 2) - \left(1 - \frac{1}{2}\right) = 5.5\text{ C}$$

Practice Problem

The current flowing through an element is

$$i = \begin{cases} 2\text{ A}, & 0 < t < 1 \\ 2t^2\text{ A}, & t > 1 \end{cases}$$

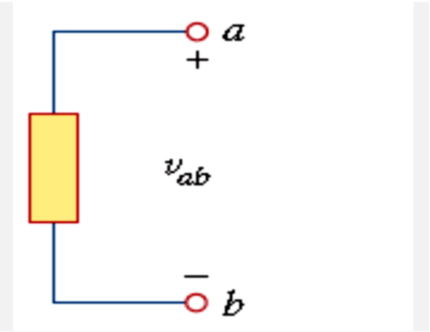
Calculate the charge entering the element from $t = 0$ to $t = 2$ s.

Answer: 6.667 C.

1.2 VOLTAGE

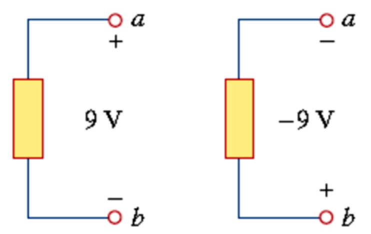
Voltage (or potential difference) is the energy required to move a unit charge through an element, measured in ***volts (V).***

$$v_{ab} = \frac{dw}{dq}$$



Where energy in joules (J) and q is charge in ***coulombs (C).*** The voltage v_{ab} or simply v is measured in ***volts (V)***

$$\boxed{1\text{ volt} = 1\text{ joule/coulomb} = 1\text{ newton meter/coulomb}}$$

The plus and minus signs are used to define reference direction or voltage polarity. The v_{ab} can be interpreted in two ways: (1) Point *a* is at a potential of v_{ab} , where, volts of point *a* higher than point *b*, or (2) the potential at point *a* with respect to point, *b* is v_{ab}. It follows logically that in general	

$$\boxed{v_{ab} = -v_{ba}}$$

1.3 POWER AND ENERGY

Power is the time rate of expending or absorbing energy, measured in watts. We write the relation as shown below :-

$$\boxed{p = \frac{dw}{dt}}$$

where *p* is power in watts (W), *w* is energy in joules (J), and *t* is time in seconds (s). From Eqs. (1), and (2), it follows that

$$p = \frac{dw}{dq}\frac{dq}{dt} = vi$$

passive sign convention. By the passive sign convention, current enters through the positive polarity of the voltage. In this case, $p = +vi \;\; vi > 0$ or implies that the element is absorbing power as in figure 1.2(a). However, if $= -vi \quad or \quad vi < 0$, as in Figure 1.2(b), the element is releasing or supplying power.

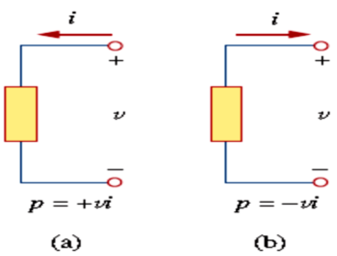


Figure 1.2

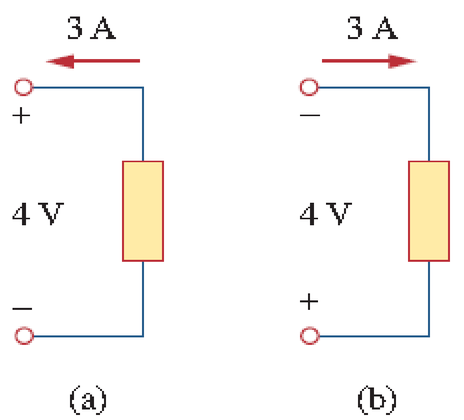


Figure 1.3

Two cases of an element with a supplying power of 12 W: (a) p= -4 x3= -12W, (b): p = -4 x 3 = -12 W.

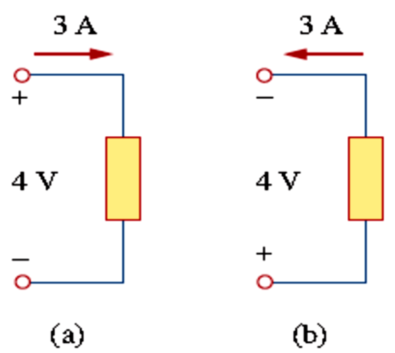


Figure 1.4

Two cases of an element with a supplying power of 12 W: (a) p= 4 x3= 12W, (b): p = 4 x 3 = 12 W.

In fact, the law of conservation of energy must be obeyed in any electric circuit. For this reason, the algebraic sum of power in a circuit, at any instant of time, must be zero:

$$\sum p = 0$$

This again confirms the fact that the total power supplied to the circuit must balance The total power absorbed. The energy absorbed or supplied by an element from time t_0 to time t is

$$w = \int_{t_0}^{t} p\,dt = \int_{t_0}^{t} vi\,dt$$

Energy is the capacity to do work, measured in joules (J).

The electric power utility companies measure energy in watt-hours (Wh), where

1 Wh = 3,600 J

Example :- An energy source forces a constant current of 2 A for 10 s to flow through a light bulb. If 2.3 kJ is given off in the form of light and heat energy, calculate the voltage drop across the bulb.

Solution:

The total charge is

$$\Delta q = i\,\Delta t = 2 \times 10 = 20\,\text{C}$$

The voltage drop is

$$v = \frac{\Delta w}{\Delta q} = \frac{2.3 \times 10^3}{20} = 115\ \text{V}$$

Example: Find the power delivered to an element at *3ms* if the current entering

$$i = 5 \cos 60\pi\, t \ \text{A}$$

and the voltage is: (a) $v = 3i$, (b) $v = 3\, di/dt$.

Solution:

(a) The voltage is $v = 3i = 15 \cos 60\pi t$; hence, the power is

$$p = vi = 75 \cos^2 60\pi t\ \text{W}$$

At $t = 3$ ms,

$$p = 75 \cos^2 (60\pi \times 3 \times 10^{-3}) = 75 \cos^2 0.18\pi = 53.48\ \text{W}$$

(b) We find the voltage and the power as

$$v = 3\frac{di}{dt} = 3(-60\pi)5 \sin 60\pi t = -900\pi \sin 60\pi t\ \text{V}$$

$$p = vi = -4500\pi \sin 60\pi t \cos 60\pi t\ \text{W}$$

At $t = 3$ ms,

$$\begin{aligned} p &= -4500\pi \sin 0.18\pi \cos 0.18\pi\ \text{W} \\ &= -14137.167 \sin 32.4^\circ \cos 32.4^\circ = -6.396\ \text{kW} \end{aligned}$$

❖ Homework

Find the power delivered to the element in Example above at t= 5ms if the current remains the same but the voltage is: (a) v= 2i V, (b) v= (10 +5 $\int_0^t i\,dt$)V.

Answer: (a) 17.27 W, (b) 29.7 W.

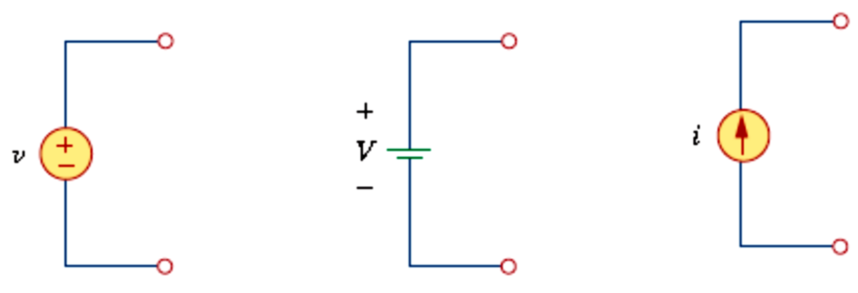
1.4 Circuit Elements

There are two types of elements found in electric circuits: ***passive*** elements and ***active*** elements. An active element is capable of generating energy while a passive element is not. Examples of passive elements are resistors, capacitors, and inductors. Typical active elements include generators, batteries, and operational amplifiers.

	passive	⟶	resistors	capacitors,	inductors
Circuit Elements					
	active	⟶	generators	batteries	operational amplifiers

	Voltage or current	
Dependent		**Independent**

An ideal **independent** source is an active element that provides a specified voltage or current that is completely independent of other circuit elements.



a **b** **c**

Figure 1.5 : Symbols for independent voltage sources: (a) used for constant or time-varying voltage,(b) used for constant voltage (dc), (c) symbol for independent current source.

An **ideal dependent** (or controlled) source is an active element in which the source quantity is controlled by another voltage or current.

Figure 1.6: Symbols for: (a) dependent voltage source, (b) dependent current source.

Dependent sources are usually designated by diamond-shaped symbols there are four possible types of dependent sources, namely:

1. A voltage-controlled voltage source (VCVS).
2. A current-controlled voltage source (CCVS).
3. A voltage-controlled current source (VCCS).
4. A current-controlled current source (CCCS

Chapter two

Basic Laws

Chapter two

Basic Laws

2.1 OHM 'S LAW

Materials in general have a characteristic behavior of resisting the flow of electric charge. This physical property, or ability to resist current, is known as *resistance* and is represented by the symbol (R). The resistance of any material with a uniform cross-sectional area A depends on (A) and its length (L). Where, the resistivity (ρ) of the material in ohm-meters, as shown in Fig. 2.1(a). The circuit symbol for the resistor is shown in Fig. 2.1(b), where R stands for the resistance of the resistor. The resistor is the simplest passive element. We can represent resistance (as measured in the laboratory), in mathematical form

$$R = \rho \frac{L}{A}$$

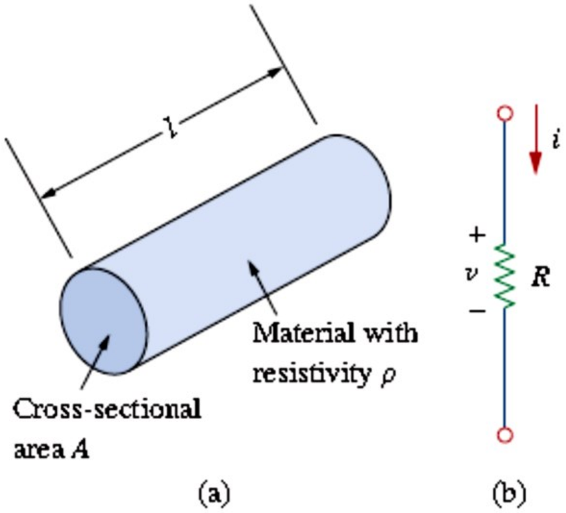


Figure 2.1: (a) Resistor, (b) Circuit symbol for resistance.

Table 2.1 presents the values for some common materials and shows which materials are used for conductors, insulators, and semiconductors

Table 2: resistivity for common materials

Resistivities of common materials.

Material	Resistivity ($\Omega \cdot m$)	Usage
Silver	1.64×10^{-8}	Conductor
Copper	1.72×10^{-8}	Conductor
Aluminum	2.8×10^{-8}	Conductor
Gold	2.45×10^{-8}	Conductor
Carbon	4×10^{-5}	Semiconductor
Germanium	47×10^{-2}	Semiconductor
Silicon	6.4×10^{2}	Semiconductor
Paper	10^{10}	Insulator
Mica	5×10^{11}	Insulator
Glass	10^{12}	Insulator
Teflon	3×10^{12}	Insulator

Ohm's law states that the voltage *v* across a resistor is directly proportional to the current *i* flowing through the resistor. (**Georg Simon Ohm-1826**)

$$v \propto i$$

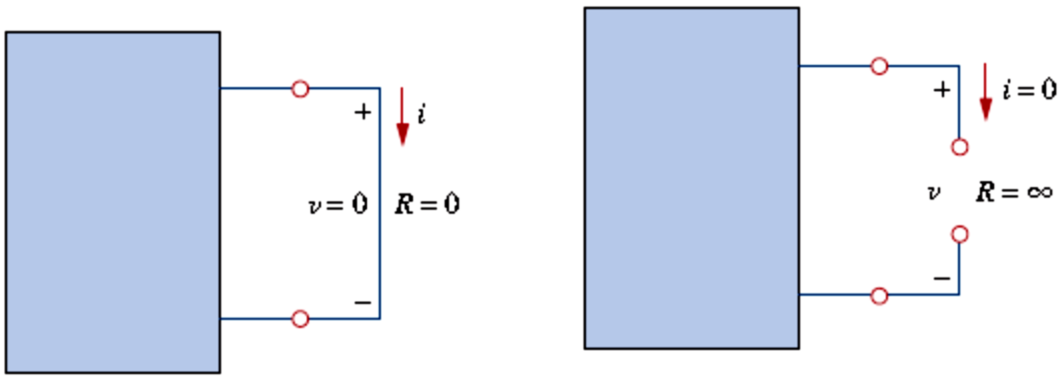
$$\boxed{v = iR}$$

The *resistance R* of an element denotes its ability to resist the flow of electric current; it is measured in ohms (Ω).thus, we can find the resistance value from Ohm law as shown below

$$R = \frac{v}{i}$$

Based on the Ohm law the resistance has two cases shown as:

A ***short circuit*** is a circuit element with resistance approaching **zero**.
An ***open circuit*** is a circuit element with resistance approaching **infinity.**



a b

Figure 2.2 (a) Short circuit (R=0, v=0), (b) Open circuit (R= ∞, i=0).

A useful quantity in circuit analysis is the reciprocal of resistance *R*, known as ***conductance*** and denoted by *G*:

$$\boxed{G = \frac{1}{R} = \frac{i}{v}}$$

The uint of the conductance is the invesr of the (Ω) is (S) or (℧)

$$1\,\text{S} = 1\,℧ = 1\,\text{A/V}$$

The power dissipated by a resistor can be expressed in terms of *R*.

$$\boxed{p = vi = i^2R = \frac{v^2}{R}}$$

The power dissipated by a resistor may also be expressed in terms of G as

$$p = vi = v^2G = \frac{i^2}{G}$$

A LINEAR RESISTOR, It has a constant resistance and thus its current-voltage characteristic is represented the slop , as illustrated in

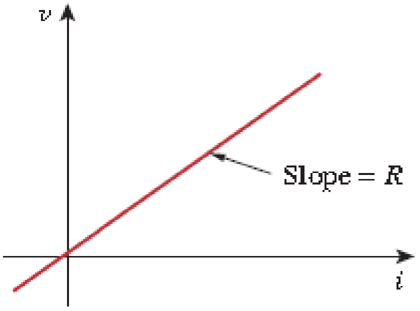
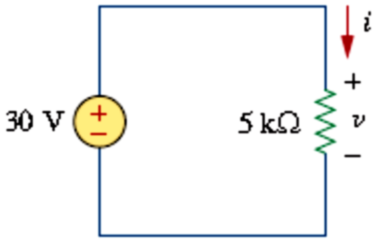


Figure 2.3 rlation between v & i with linear resistance

Example :- In the circuit shown in Figure shown below ,

1- calculate the current i,
2- the conductance G,
3- power P.



Solution:

$$i = \frac{v}{R} = \frac{30}{5 \times 10^3} = 6\ \text{mA}$$

The conductance is

$$G = \frac{1}{R} = \frac{1}{5 \times 10^3} = 0.2\ \text{mS}$$

We can calculate the power in various ways

$$p = vi = 30(6 \times 10^{-3}) = 180\ \text{mW}$$

or

$$p = i^2R = (6 \times 10^{-3})^2 5 \times 10^3 = 180\ \text{mW}$$

or

$$p = v^2G = (30)^2 0.2 \times 10^{-3} = 180\ \text{mW}$$

Example:- A voltage source of (20 sin πt) V is connected across a 5-k resistor. Find the current through the resistor and the power dissipated.

Solution:

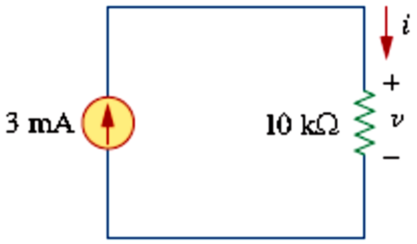
$$i = \frac{v}{R} = \frac{20 \sin \pi t}{5 \times 10^3} = 4 \sin \pi t \text{ mA}$$

Hence,

$$p = vi = 80 \sin^2 \pi t \text{ mW}$$

❖ ***Homework***: For the circuit shown in Figure shown below , calculate the:

1- voltage v
2- Conductance G
3- power p.



Answer: 30 V, 100 μS, 90 mW.

❖ ***Homework***: A resistor absorbs an instantaneous power of (30 $\cos^2 t$) mW when connected to a voltage source (v = 15 cos t)V. Find i and R.

Answer: 2 cos t mA, 7.5 kΩ.

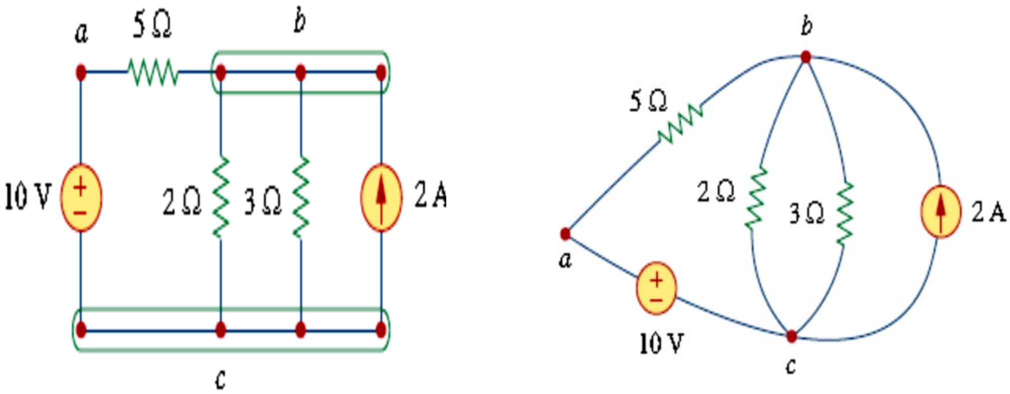
2.2 Nodes, Branches, and Loops

A branch represents a single element such as a voltage source or a resistor.

A node is the point of connection between two or more branches.

A loop is any closed path in a circuit.

As shown in the figures below:-



a b

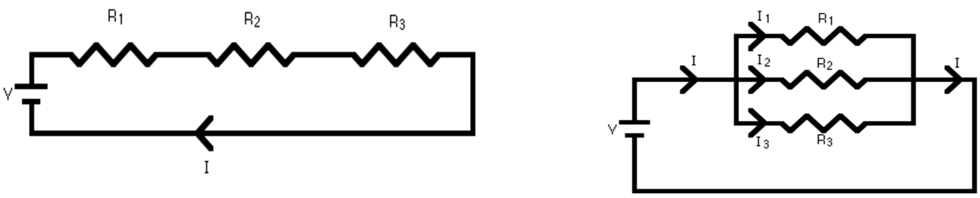
Figure 2.4: (a) Nodes, branches , and loops, (b) the three- node circuit of figure 2.3

A network with ***b*** branches, ***n*** nodes, and ***l*** independent loops will satisfy the fundamental theorem of network topology:

$$\boxed{b = l + n - 1}$$

Two or more elements are in ***series*** if they exclusively share a single node and consequently carry the same current, figure 2.5 (a).

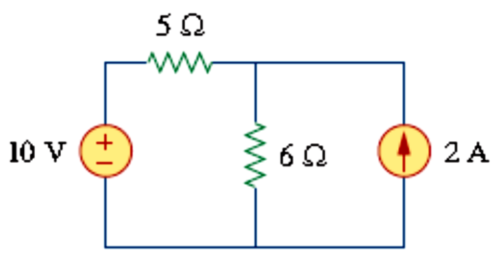
Two or more elements are in ***parallel*** if they are connected to the same two nodes and consequently have the same voltage across them, figure 2.5 (b).



a b

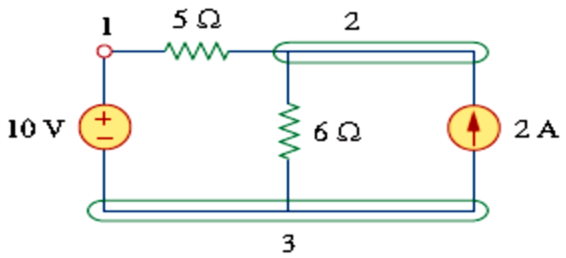
Figure 2.5: (a) series connection, (b) parallel connection

Example: Determine the number of branches and nodes in the circuit shown below. Identify which elements are in series and which are in parallel.



Solution:

Since there are four elements in the circuit, the circuit has four branches: 10 V,5 **Ω**, 6 **Ω** and 2 A. The circuit has three nodes as identified in Figure below. The 5 **Ω**- resistor is in series with the 10-V voltage source because the same current would flow in both. The 6 **Ω**- resistor is in parallel with the 2-A current source because both are connected to the same nodes 2 and 3.



Homework: How many branches and nodes does the circuit in Figure below have? In addition, identify the elements that are in series and in parallel.