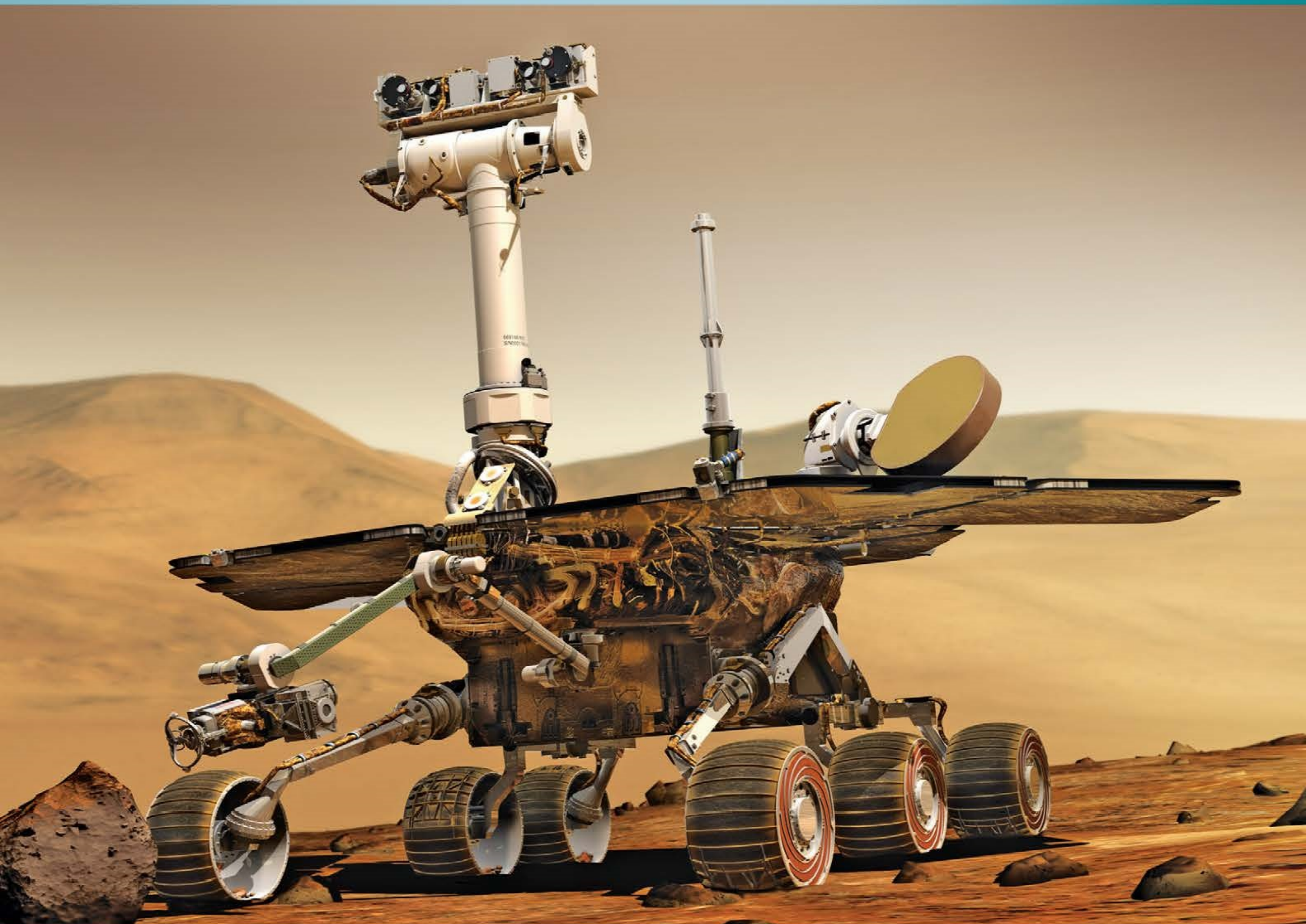
Fundamentals of Electric Circuits



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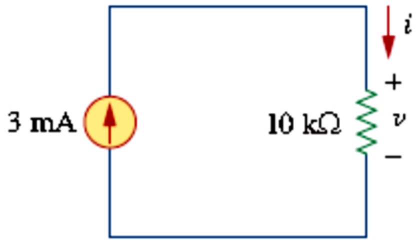
❖ ***Homework***: For the circuit shown in Figure shown below , calculate the:

1- voltage v

2- Conductance G

3- power p.



Answer: 30 V, 100 μS, 90 mW.

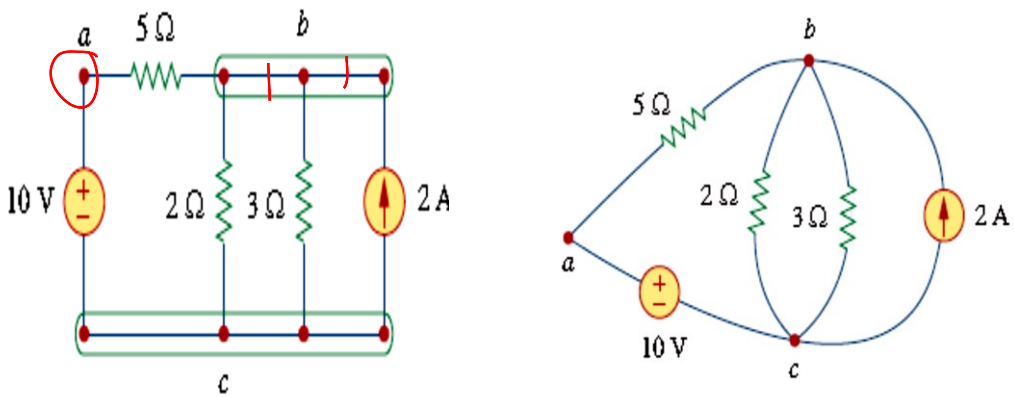
❖ ***Homework***: A resistor absorbs an instantaneous power of ($30 \cos^2 t$) mW when connected to a voltage source ($v = 15 \cos t$)V. Find i and R.

Answer: $2 \cos t$ mA, 7.5 kΩ.

2.2 Nodes, Branches, and Loops

A branch. represents. a single element such as a voltage source or a resistor.
A node is the.point of connection between two or more branches.
A.loop is any closed path in a circuit.

As shown in the figures below :-



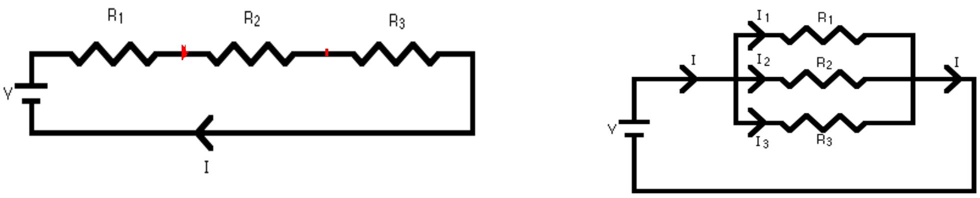
a b

Figure 2.4: (a) Nodes, branches , and loops, (b) the three- node circuit of figure 2.3

A network with ***b*** branches, ***n*** nodes, and ***l*** independent loops will satisfy the fundamental theorem of network topology:

$$b = l + n - 1$$

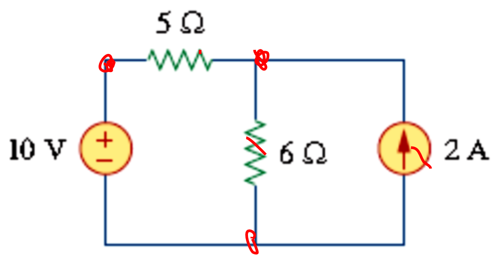
Two or more elements are in ***series*** if they exclusively share a single node and consequently carry the same current, figure 2.5 (a).
Two or more elements are in ***parallel*** if they are connected to the same two nodes and consequently have the same voltage across them, figure 2.5 (b).



a b

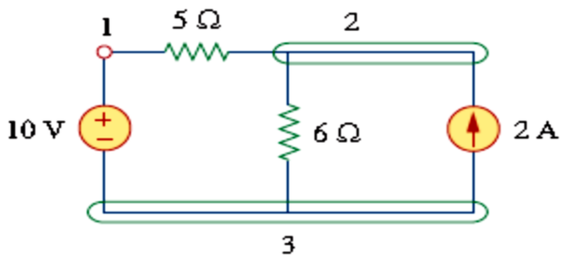
Figure 2.5: (a) series connection, (b) parallel connection

Example: Determine the number of branches and nodes in the circuit shown below. Identify which elements are in series and which are in parallel.

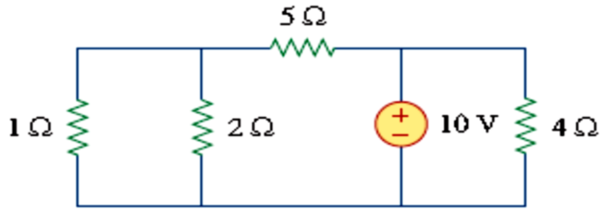


Solution:

Since there are four elements in the circuit, the circuit has four branches: 10 V,5 Ω, 6 Ω and 2 A. The circuit has three nodes as identified in Figure below. The 5 Ω- resistor is in series with the 10-V voltage source because the same current would flow in both. The 6 Ω- resistor is in parallel with the 2-A current source because both are connected to the same nodes 2 and 3.



Homework: How many branches and nodes does the circuit in Figure below have? In addition, identify the elements that are in series and in parallel.



2.3 Kirchhoff's Laws (1847)

There are two laws (Kirchhoff's current law (KCL) and Kirchhoff's voltage law (KVL)).

2.3.1 Kirchhoff's current law (KCL)

States that the algebraic sum of currents entering a node (or a closed boundary) is zero. Mathematically, KCL implies that

$$\sum_{n=1}^{N} i_n = 0$$

Where N is the number of branches connected to the node and i_n is the n*th* current entering (or leaving) the node. **The algebraic sum of the current should equal zero (KCL).**

$$i_T(t) = i_1(t) + i_2(t) + i_3(t) + \ldots\ldots$$

Integrating both sides of Eq.

$$q_T(t) = q_1(t) + q_2(t) + q_3(t) + \ldots\ldots\ldots$$

entering a node may be regarded as positive, while currents leaving the node may be taken as negative or vice versa as shown in figure 2.6.

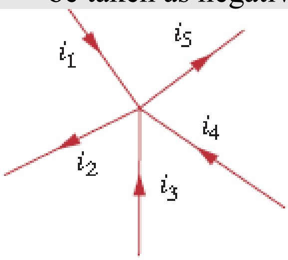
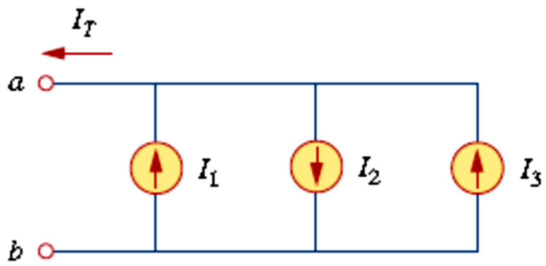


Figure 2.6 KCL

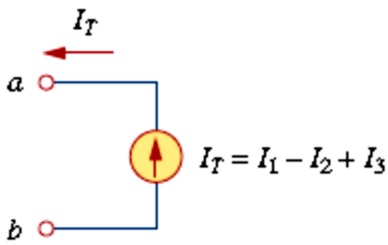
$$i_1 + (-i_2) + i_3 + i_4 + (-i_5) = 0$$

Example: - The combined or equivalent current source can be found by applying KCL to node *a* in the figure shown below.



Solution:-

Based on the KCL, current in the node *a* is equal to:



$$I_T + I_2 = I_1 + I_3$$

$$I_T = I_1 - I_2 + I_3$$

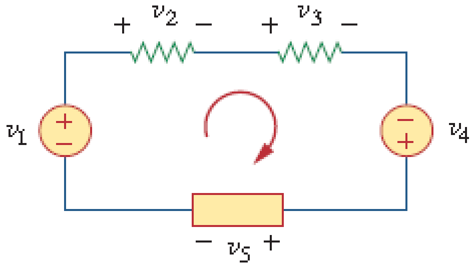
2.3.2 Kirchhoff's voltage law (KVL): states that the algebraic sum of all voltages around a closed path (or loop) is zero.

Expressed mathematically, KVL states that

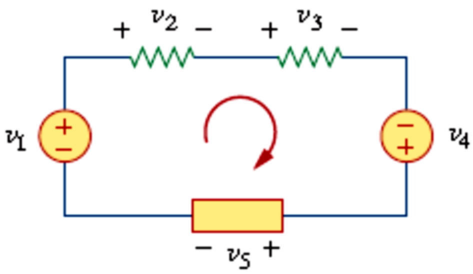
$$\sum_{m=1}^{M} v_m = 0$$

where M is the number of voltages in the loop (or the number of branches in the loop) and v_m is the *mth* voltage

Example: illustrate KVL, consider the circuit in Figure shown below



Solution: We can start with any branch and go around the loop either clockwise or counterclockwise. Suppose we start with the voltage source and go clockwise around the loop as shown, based on the KVL can find the algebraic sum of the voltage is equal zero, as shown in figure 2.7.



Figrue 2.7 KVL

$$-v_1 + v_2 + v_3 - v_4 + v_5 = 0$$

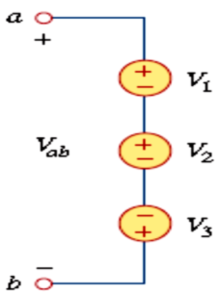
Rearranging terms gives

$$v_2 + v_3 + v_5 = v_1 + v_4$$

Result show: - Sum of voltage drops = Sum.of.voltage rises

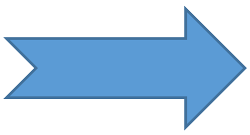
When voltage sources are connected in series, KVL can be applied to obtain the total voltage. (figure 2.8)

To find the voltage in the figure below v_{ab} :-



based on KVL

$$V_{ab} = V_1 + V_2 - V_3$$



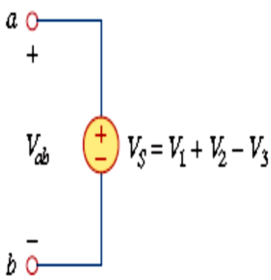
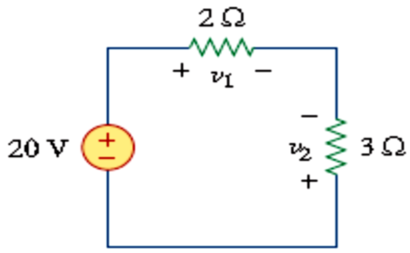
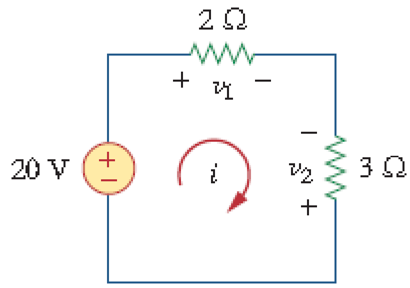


Figure 2.8 : total voltage of KVL

Example: - For the circuit in Figure shown below find voltages v_1 and v_2



Solution :- To find and we apply Ohm's law and Kirchhoff's voltage law. Assume that current i flows through the loop as shown in Figure below.



From Ohm's law,

$$v_1 = 2i, \quad v_2 = 3i \quad \ldots\ldots (1)$$

Applying KVL around the loop gives

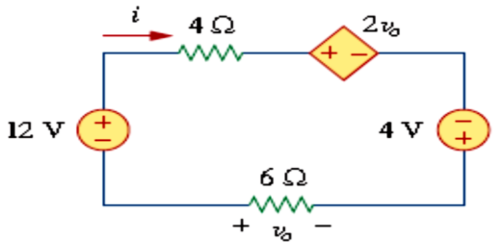
$$-20 + v_1 - v_2 = 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Sub. equ. 1 in the 2

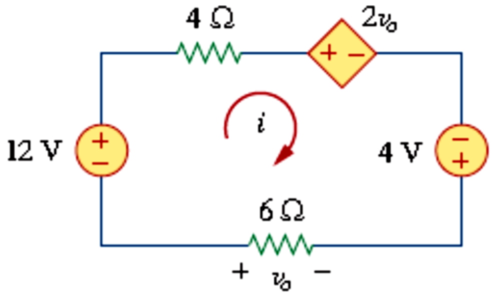
-20 + 2i + 3i = 0 or 5i = 20 thus $i = 4$ A

Sub value of i in equ. 1 (v_1=8V , v_2= -12V)

Example: - Determine v_o and i in the circuit shown in Figure shown below



Solution:



First step should draw the loop to find the KVL , then

$$-12 + 4i + 2v_o - 4 + 6i = 0 \qquad \ldots\ldots\ldots (1)$$

Applying Ohm's law to the 6-Ω resistor gives

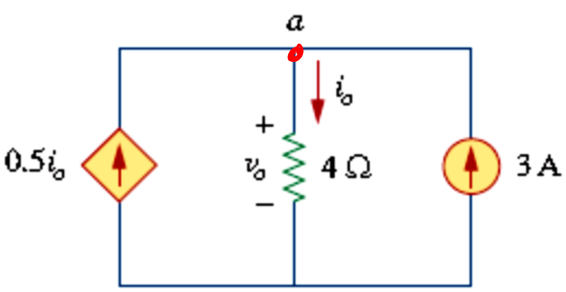
$$v_o = -6i \qquad \ldots\ldots\ldots(2)$$

Sub. equation (2) in equation (1)

$$-16 + 10i - 12i = 0 \quad \Rightarrow \quad i = -8 \text{ A}$$

and $v_o = 48$ V.

Example: - Find current i_o and voltage v_o in the circuit shown in Figure below



Solution:

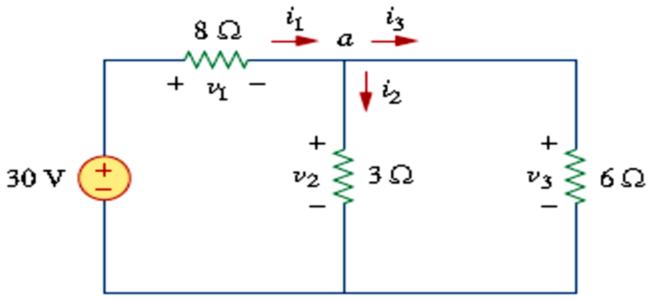
Applying KCL to node a, we obtain

$$3 + 0.5i_o = i_o \quad \Rightarrow \quad i_o = 6 \text{ A}$$

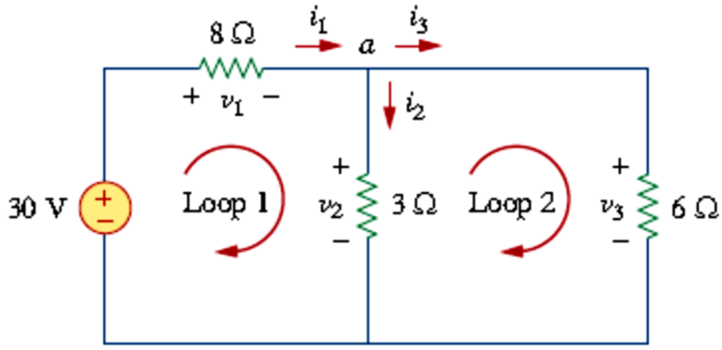
For the 4-Ω resistor, Ohm's law gives

$$v_o = 4i_o = 24 \text{ V}$$

Example: - Find currents and voltages in the circuit shown in Figure below



Solution: based on the KCL, first steps find the direction of the current loop



We apply Ohm's law and Kirchhoff's laws. By Ohm's law, ….(1)

$$v_1 = 8i_1, \quad v_2 = 3i_2, \quad v_3 = 6i_3$$

Since the voltage and current of each resistor are related by Ohm's law as shown, we are really looking for three things: (v_1, v_2, v_3) or (i_1, i_2, i_3). At node a, KCL gives

$$i_1 - i_2 - i_3 = 0 \quad \text{….(2)}$$

Applying KVL to loop 1 as in Figure above

$$-30 + v_1 + v_2 = 0 \quad \text{…(3)}$$

Sub. equation (1) in equ. (3) to find the equation interm of currents

$$-30 + 8i_1 + 3i_2 = 0$$

$$i_1 = \frac{(30 - 3i_2)}{8} \qquad \text{....(4)}$$

Applying KVL to loop 2,

$$-v_2 + v_3 = 0 \qquad \Rightarrow \qquad v_3 = v_2$$

as expected since the two resistors are in parallel. We express v_1 and v_2 in terms of i_1 and i_2 as in Eq. (1)

Thus, equation (4) will be as :-

$$6i_3 = 3i_2 \qquad \Rightarrow \qquad i_3 = \frac{i_2}{2} \qquad \text{.....(5)}$$

Finaly : sub equation (4) and (5) in (2)

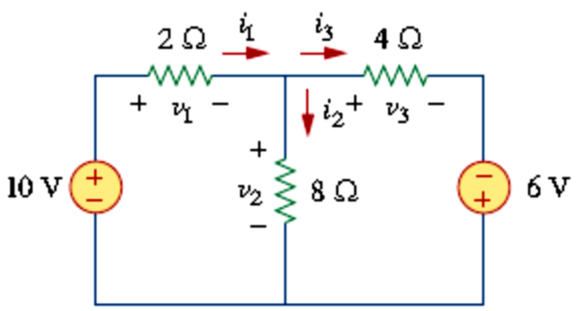
$$\frac{30 - 3i_2}{8} - i_2 - \frac{i_2}{2} = 0$$

i_2= 8A

$$i_1 = 3\text{ A}, \qquad i_3 = 1\text{ A}, \qquad v_1 = 24\text{ V}, \qquad v_2 = 6\text{ V}, \qquad v_3 = 6\text{ V}$$

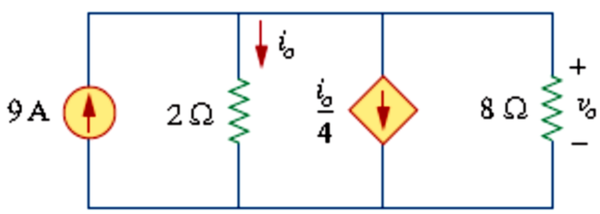
Homework:-

❖ Find the currents and voltages in the circuit shown in Figure below



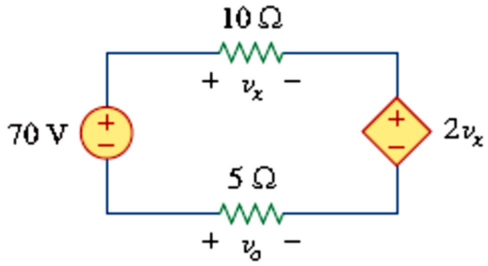
Answer: $v_1 = 6$ V, $v_2 = 4$ V, $v_3 = 10$ V, $i_1 = 3$ A, $i_2 = 500$ mA, $i_3 = 1.25$ A.

❖ Find v_0 and i_0 in the circuit of Figure below.



Answer: 12 V, 6 A.

❖ Find v_x and v_0 in the circuit of Figure below



Answer: 20 V, −10 V.

2.4 Series Resistors and Voltage Division

The two resistors are in series, since the same current *i* flow in both of them, figure (2.9). Applying Ohm's law to each of the resistors, we obtain

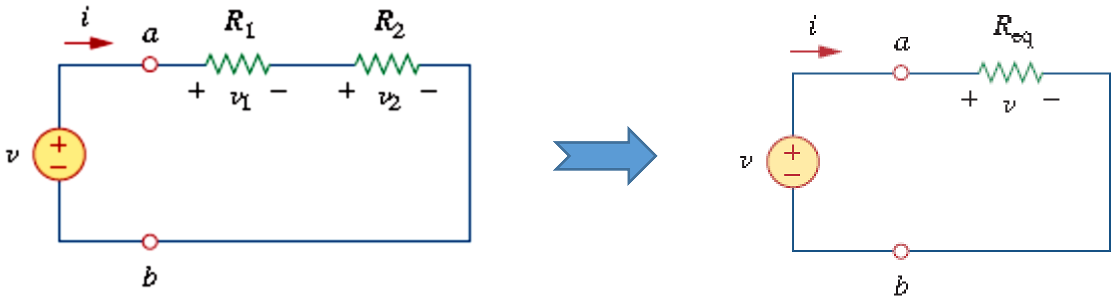


Figure 2.9 A single-loop circuit with two resistors in series

$$v_1 = iR_1, \qquad v_2 = iR_2 \qquad \text{……..(1.5)}$$

If we apply KVL to the loop (moving in the clockwise direction), we have

$$-v + v_1 + v_2 = 0 \qquad \text{……….(2.5)}$$

Cobine the equation (1.5) with (2.5)

$$v = v_1 + v_2 = i(R_1 + R_2) \quad \Rightarrow \quad i = \frac{v}{R_1 + R_2}$$

$$v = iR_{eq} \quad \Rightarrow \quad R_{eq} = R_1 + R_2$$

The equivalent resistance of any number of resistors connected in series is the sum of the individual resistances.
For *N* resistors in series then

$$R_{eq} = R_1 + R_2 + \cdots + R_N = \sum_{n=1}^{N} R_n$$

To determine the voltage across each resistor in Figure 2.9, can obtain as:

$$v_1 = \frac{R_1}{R_1 + R_2} v, \qquad v_2 = \frac{R_2}{R_1 + R_2} v$$

In general, if a voltage divider has *N* resistors (R1,R2,……RN) in series with the source voltage *v*, the *n*th resistor (Rn) will have a voltage drop of

$$v_n = \frac{R_n}{R_1 + R_2 + \cdots + R_N} v$$

2.5 Parallel Resistors and Current Division

Consider the circuit in Figure 2.10, where two resistors are connected in parallel and therefore have the same voltage across them.

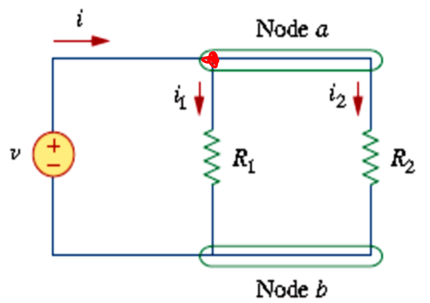


Figure 2.10 : Two resistors in parallel.

From Ohm's law,

$$v = i_1 R_1 = i_2 R_2$$

$$i_1 = \frac{v}{R_1}, \qquad i_2 = \frac{v}{R_2} \qquad \text{…….(1.6)}$$

Applying KCL at node *a* gives the total current *i* as

$$i = i_1 + i_2 \qquad \text{...... ....(2.6)}$$

Sub. eq. (1.6) in (2.6) , we get

$$i = \frac{v}{R_1} + \frac{v}{R_2} = v\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{v}{R_{eq}}$$

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

The **equivalent resistance** of two parallel resistors is equal to the product of their resistances divided by their sum.

The general case of a circuit with *N* resistors in parallel. The equivalent resistance is

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N} \qquad \text{.....(3.6)}$$

Note that R_{eq} is always smaller than the resistance of the smallest resistor in the parallel combination.

If $R_1 = R_2 = \cdots = R_N = R$, then	$R_{eq} = \dfrac{R}{N}$

For example, if four 100Ω resistors are connected in parallel, their equivalent resistance is **25Ω.**
It is often more convenient to use conductance rather than resistance when dealing with resistors in parallel. From Eq. (3.6), the equivalent conductance for N resistors in parallel is**:**

$$G_{eq} = G_1 + G_2 + G_3 + \cdots + G_N$$

where $G_{eq} = 1/R_{eq}$, $G_1 = 1/R_1$, $G_2 = 1/R_2$, $G_3 = 1/R_3$, ..., $G_N = 1/R_N$.

The **equivalent conductance** of resistors connected in parallel is the sum of their individual conductance.

The equivalent conductance of parallel resistors (figure 2. 11) is obtained the same way as the equivalent resistance of series resistors. In the same manner, the equivalent conductance of resistors in series is obtained just the same way as the resistance of resistors in parallel. **Thus the equivalent conductance (figure 2.12) G_{eq} of N resistors in series, as shown in figure below.**

$$\frac{1}{G_{eq}} = \frac{1}{G_1} + \frac{1}{G_2} + \frac{1}{G_3} + \cdots + \frac{1}{G_N}$$

Given the total current i entering node a in Figure 11.2, the equivalent resistor has the same voltage, or

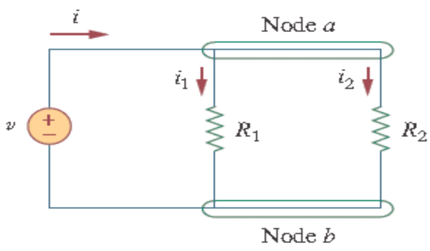


Figure 11.2 parallel circuit

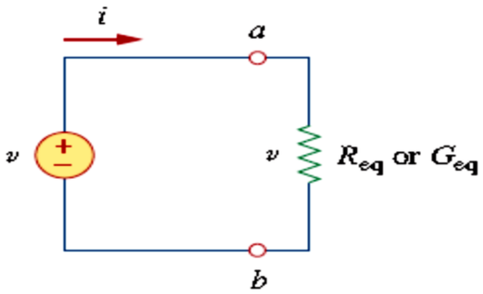


Figure 12.2 equiveline parallel circuit

$$v = iR_{eq} = \frac{iR_1R_2}{R_1 + R_2} \quad \text{............(4.6)}$$

The current division low is utilize for find the current in the one parallel branch figure (2.13) that has many branch. Can find the current division for the circuit shown in figure (2.13) by Combining Eqs. (1.6) and (4.6) to give the results as:

$$i_1 = \frac{R_2\, i}{R_1 + R_2}, \qquad i_2 = \frac{R_1\, i}{R_1 + R_2}$$

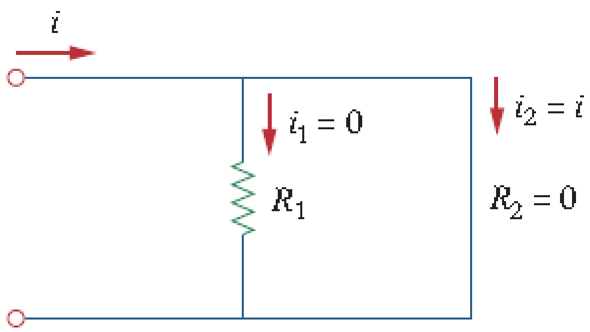
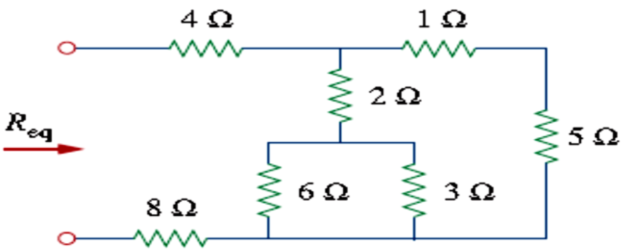


Figure 2.13 parallel circuit (current division)

Example :- Find R_{eq} for the circuit shown in Figure below



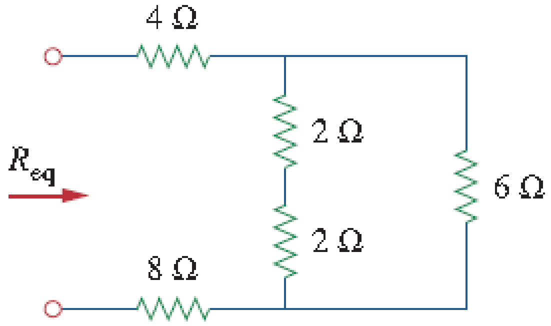
Solution: -
The 6 Ω- and 3 Ω- resistors ***are in parallel***

$$6\ \Omega \parallel 3\Omega = \frac{6 \times 3}{6 + 3} = 2\ \Omega$$

The 1 Ω- and 5 Ω- resistors ***are in series***

$$1\ \Omega + 5\ \Omega = 6\ \Omega$$

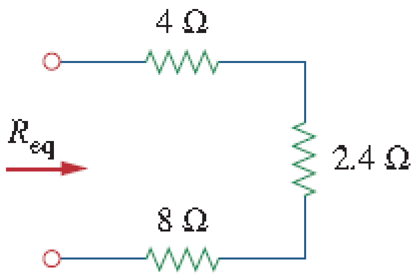
Then the equivalent circuit will be as shown below:-



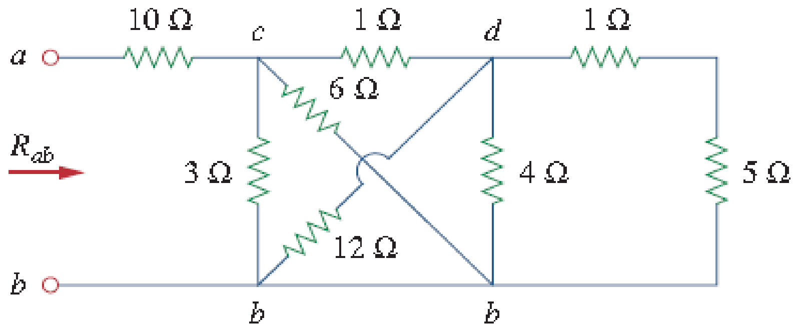
$$2\ \Omega + 2\ \Omega = 4\ \Omega$$

these resistance in series

This 4Ω- resistor result from (2+2=4Ω) is now in parallel with the 6 Ω- resistor**, result shown in figure below**



Example :- Calculate the equivalent resistance R_{ab} in the circuit in Figure shown below: -



Solution:

The 3 Ω- and 6 Ω- resistors are in parallel because they are connected to the same two nodes *c* and *b***.**

$$3\ \Omega \parallel 6\ \Omega = \frac{3 \times 6}{3 + 6} = 2\ \Omega$$

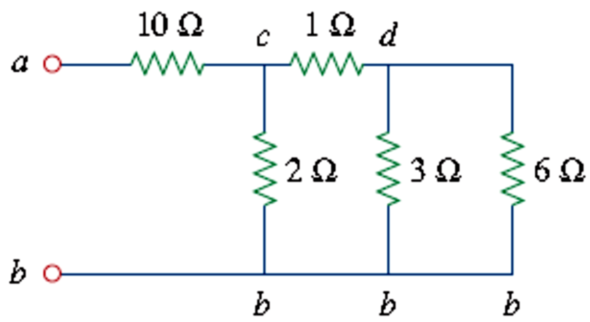
In addition, the 12 Ω- and 4 Ω- resistors are in parallel because they are connected to the same two nodes *d* and *b*.

$$12\ \Omega \parallel 4\ \Omega = \frac{12 \times 4}{12 + 4} = 3\ \Omega$$

Also the 1Ω- and 5Ω- resistors are in series; hence, their equivalent resistance is

$$1\ \Omega + 5\ \Omega = 6\ \Omega$$

The circuit will be as below



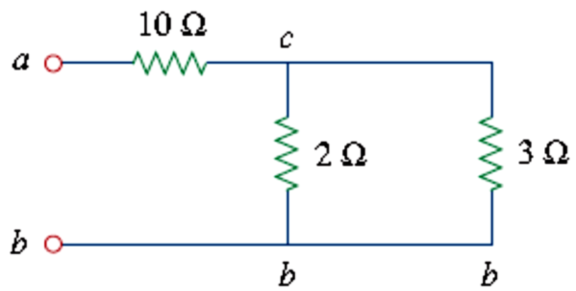
3Ω in parallel with 6 Ω gives 2 Ω , because they are connected in two nodes (*d, b*)

3|| 6= 2Ω

The 2Ω is series with 1 Ω, thus

1+2=3 Ω

Result of that process will give the figure below



Is clear 2||3 :-

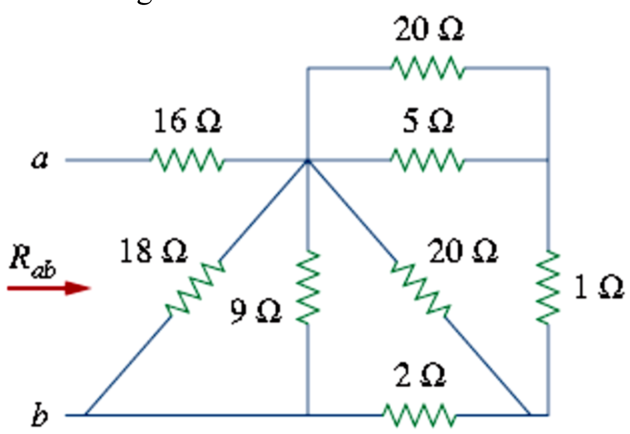
$$2\ \Omega \parallel 3\ \Omega = \frac{2 \times 3}{2 + 3} = 1.2\ \Omega$$

This 1.2 Ω- resistor is in series with the 10 Ω- resistor, so that

$$R_{ab} = 10\ \Omega + 1.2\ \Omega = 11.2\ \Omega$$

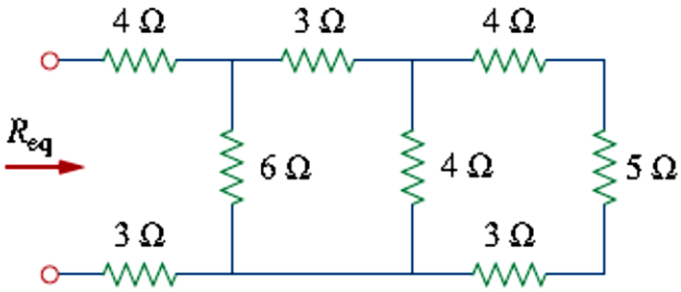
Homework

❖. Find R_{ab} for the circuit in Fig**ure below**



Answer: 19 Ω

❖ Find R_{ab} for the circuit in Fig**ure below**



Answer: 10 Ω

Example :- Find the equivalent conductance G_{eq} for the circuit in Figure below

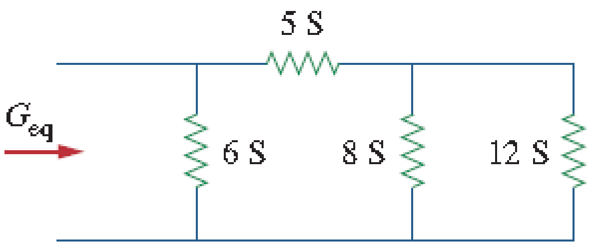
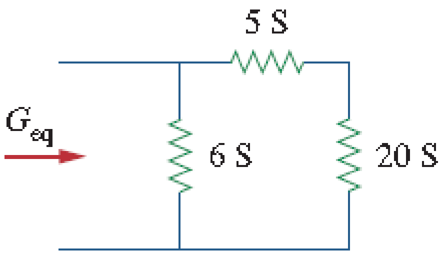


Figure (14-2)

The 8-S and 12-S resistors are in parallel, so their conductance is

$$8\ S + 12\ S = 20\ S$$

This 20 S resistor is now in series with 5 S as shown in Figure below

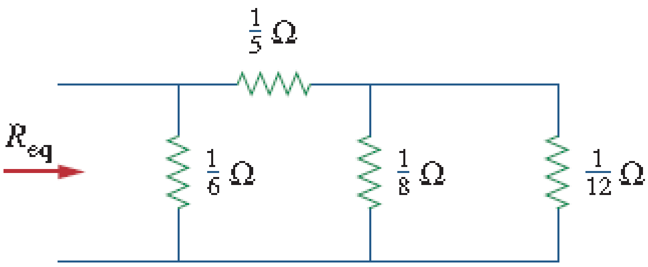


$$\frac{20 \times 5}{20 + 5} = 4\ S$$

The 4S is in parallel with the 6S resistor. Hence

$$G_{eq} = 6\ S + 4\ S = 10\ S$$

the figure below is equivalent to figure (14-2)



and we can prove that as :

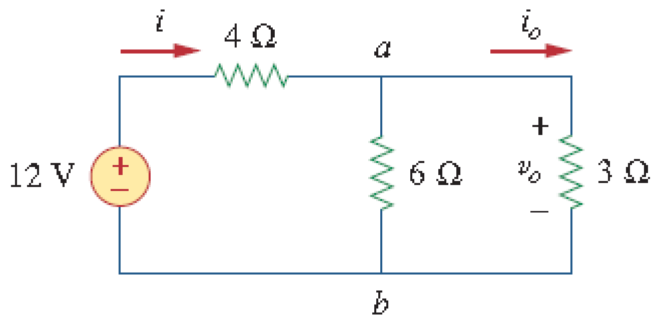
$$R_{eq} = \frac{1}{6} \left\| \left(\frac{1}{5} + \frac{1}{8} \right\| \frac{1}{12} \right) = \frac{1}{6} \left\| \left(\frac{1}{5} + \frac{1}{20} \right) = \frac{1}{6} \right\| \frac{1}{4}$$

$$= \frac{\frac{1}{6} \times \frac{1}{4}}{\frac{1}{6} + \frac{1}{4}} = \frac{1}{10} \ \Omega$$

$$G_{eq} = \frac{1}{R_{eq}} = 10 \text{ S}$$

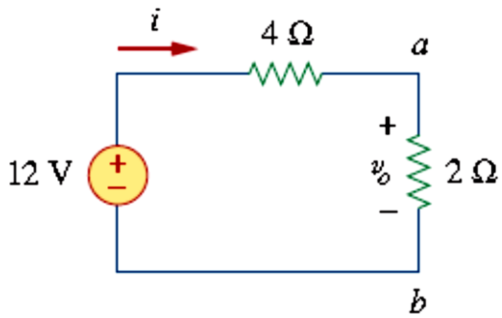
This is the same as we obtained previously

Example : Find i_0 and v_0 in the circuit shown in Figure below . Calculate the power dissipated in the 3Ω resistor.



Solution:-

$$6 \ \Omega \| 3 \ \Omega = \frac{6 \times 3}{6 + 3} = 2 \ \Omega$$



Therefor , the total current is :-

$$i = \frac{12}{4 + 2} = 2 \text{ A}$$

$$v_o = 2i = 2 \text{ X } 2 = 4 \text{ V}$$

Another way can find the v_0 via voltage division

$$v_o = \frac{2}{2+4}(12\text{ V}) = 4\text{ V}$$

v_0 is same on the 6Ω and 3Ω because they are parallel, thus

$$v_o = 3i_o = 4 \quad \Rightarrow \quad i_o = \frac{4}{3}\text{A}$$

Another approach is to apply current division to the circuit in main Figure, now that we know i_0 by writing :-

$$i_o = \frac{6}{6+3}i = \frac{2}{3}(2\text{ A}) = \frac{4}{3}\text{A}$$

The power dissipated in the 3Ω resistor is

$$p_o = v_o i_o = 4\left(\frac{4}{3}\right) = 5.333\text{ W}$$

Example: - For the circuit shown in Figure (a.2) determine-

(a) The voltage v_0
(b) The power supplied by the current source
(c) The power absorbed by each resistor.

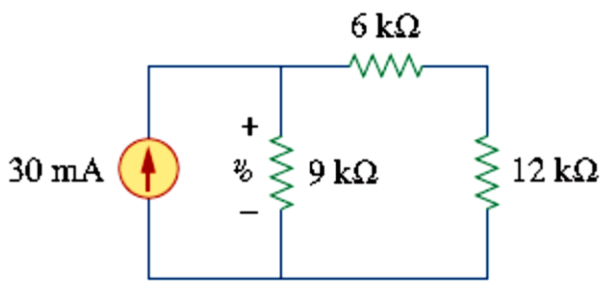


Figure (a.2)

(a) The 6-kΩ and 12-kΩ resistors are in series so that combined value is 18kΩ. Thus, the circuit in Figure (a.2) reduces to that shown in Figure (b.2).

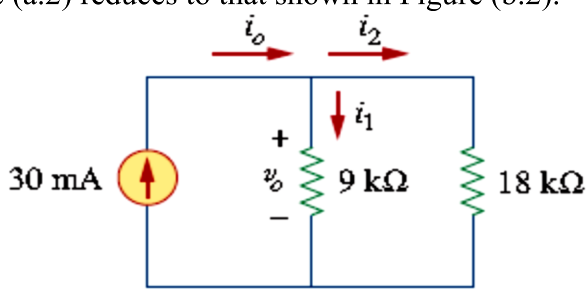


Figure (b.2)

We now apply the current division technique to i_1 find and i_2

$$i_1 = \frac{18{,}000}{9{,}000 + 18{,}000}(30 \text{ mA}) = 20 \text{ mA}$$

$$i_2 = \frac{9{,}000}{9{,}000 + 18{,}000}(30 \text{ mA}) = 10 \text{ mA}$$

Notice that the voltage across the 9-kΩ and 18-kΩ resistors is the same, and $v_o = 9{,}000 i_1 = 18{,}000 i_2 = 180$ V, as expected.
(b) Power supplied by the source is

$$p_o = v_o i_o = 180(30) \text{ mW} = 5.4 \text{ W}$$

(c) Power absorbed by the 12-kΩ resistor is

$$p = iv = i_2(i_2 R) = i_2^2 R = (10 \times 10^{-3})^2 \, (12{,}000) = 1.2 \text{ W}$$

Power absorbed by the 6-kΩ resistor is

$$p = i_2^2 R = (10 \times 10^{-3})^2 \, (6{,}000) = 0.6 \text{ W}$$

Power absorbed by the 9-kΩ resistor is

$$p = \frac{v_o^2}{R} = \frac{(180)^2}{9{,}000} = 3.6 \text{ W}$$

or

$$p = v_o i_1 = 180(20) \text{ mW} = 3.6 \text{ W}$$

Notice that the power supplied (5.4 W) equals the power absorbed (1.2 + 0.6 + 3.6 = 5.4 W). This is one way of checking results.