Lecture2

المادة: رياضيات
المرحلة : الاولى
المحاضر: د. محمد مزعل راشد

وزارة التعليم العالي والبحث العلمي
جامعة المثنى
كلية الهندسة
قسم الاتصالات والالكترونيك

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Absolute Value

The absolute value of a number .t, denoted by |.i |, is deflned by the formula

$$|x| \qquad -x, \qquad x < 0.$$

.4lisolute \\tlue Plo}iel ties

1. $|-a| = |a|$

2. |a/›| = |a||/›

A numbcr and its additixc i‹a›crsc or ncpatis'e hax'e the sa«c absolute s'alue.

4. $|a + b|$ w. $|n| + |f›$

The absolute x'alue of a product is the product of thc absolutc values.

Thc absolule x'aluc of a quotient is thc quoticnt of the absolutc x'alucs.

The triangle inequalit;'. The absolute value of the sum of two ntlllllbers is less than or equal to the sum of their absolute values.

EXANPLE 2 Finding Absolute Values

3| 3. |0| 0. |—5| —(—5) 5. |—|«|| |n| n

Geoivlctrically, the absolutc s'aluc of.v is the distance from ,v to 0 on the real number line. Sincc distanccs ure alw'ays positit'c or 0, wee sce that |.v| T 0 for es'ery real number.v. and |.v| 0 if and only if.v 0. Also,

|.›‘ — yr' | the dlstance between v and v

on the real llne (Figure 1.2).

Siitce the symbol always denotes the *tionuc•gativc•* squarc root of a. an alternate dDfil2ltion of |.v | lS

$$|x| = \sqrt{x^2}.$$

It is iiaaportant to reiaaeinbcr that | a |. Do not wrlte *a* n unless you already know that *a* at 0.

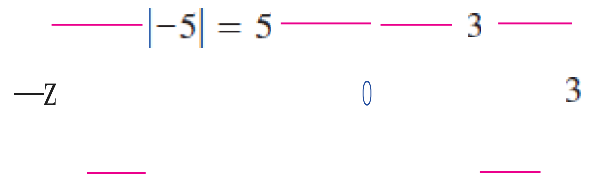


FIGURE 1.2 Absolute values give distances between points on the number

EXAMPLE 3 Illustrating the Triangle Inequality

—3 + 5| = | 2 | = 2 < | —3 | + |5] 8

3 + 5| = |8| | 3| + |5

—3 — 5| = | —8] = 8 = | —3 | + | —5

4. \ *a* + *b*\ :::S \ *a*\ -i- *b* \

.1l›solute S"alue» anJ Inter‹’nls

IfO 1s any pos1tive number, then

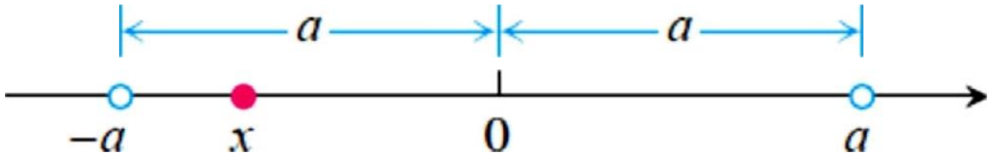
.$. |.v| n if and only if .v la

6. |*x*|€ a if and only if —a€ z •t n

7. | .v | ?> a if and Dnly if .v ?> ri or .v € —p

8. |.v | m a if and only if —a « x «p

9. | .v | m a if and only if .v m ri or .v m —n



FI6URE 1.3 |x | < a means x lies between —*a* and *a*.

The inequality]x | < *a* says that the distance from a to 0 is less than the positive number *a*. This means that a must lie between —*a* and *a*, as we can see from Figure 1.3.

The following statements are all consequences of the definition of absolute value and are often helpful when solving equations or inequalities involving absolute values.

EXAMPLE 4 Solving an Equation with Absolute Values

Solve the equalion | 2x — 3| = 7.

Solution By Property 5, 2x — 3 = A7, so there are two possibilities:

2x — 3 = 7	2x — 3 —7	Equivalent equations without absolute values
2x = l0	2x = —4	Solve as usual.
a=5	x —2	

The solutions of | 2x — 3| = 7 are x = 5 and a: = —2.

.11Jsolute \ alue» anJ Inter1'aI»

If n is any positive number, then

5. |.r| = n if and only if .v = -kn
6. |.r| n if and only if —n z n
7. |.r| n if and only if .r n or .v —n
g. ;.‹; n if and only if —n x n
9. |.r| n if and only if .r n or .v —n

EXAMPLE 5 Solving an Inequality Involving Absolute Va\ues

Solve the inequality $5 — \frac{2}{x} < l.$

Solution We have

$$5 — \frac{2}{x} < 1 \qquad -1 < 5 - \frac{2}{x} < 1 \qquad \text{Property 6}$$

$$\Leftrightarrow —6 < —\frac{2}{x} < —4 \qquad \text{Subtract 5.}$$

$$\Leftrightarrow 3 > \frac{1}{x} > 2 \qquad \text{Multiply by ——.}$$

$$\Leftrightarrow \frac{1}{3} < x < \frac{1}{2"} \qquad \text{Take reciprocals.}$$

EXAMPLE 6 Solve the inequality and show the solution set on the real line:

(a) |2x — 3| » l   (@) | 2x — 3| n 1

Solution

(a) |2x — 3| « l

—l « 2x — 3 « l   Property 8

2 « 2x « 4   Add 3.

l % x Z 2   Divide by 2.

The solution set is the closed interval [l, 2] (Figure l.4a).

(@) | 2x — 3| n 1

2x — 3 n l   or   2x — 3 I —1   Property 9

3/2 1/2 " 3/2 1/2   Divide by 2.

x m 2   or   x E- l   Add .

The solution set ∞ l] U [2, '^›) (Figure l.4b).

Absolute1'alues and Intervals

If *a* is any positive number, then

5.	|.T| = n	if and only if .v = -L *a*	
6.	|.T| *a*	if and only if —*a* z n	
7.	|.T| *a*	if and only If .v *a* or .r —n	
8.	|.T| O	if and only if —n x n	
9.	|.v| n	if and only If .v n or .r —n	

x

1   2

(a)

1   2

(b)

FIGURE 1.4 The solution sets (a) [l, 2] and (b) (— co, l] U [2, os) in Example 6.fi(

Lines, Circles, and Parabolas

This section reviews coordinates, lines, distance, circles, and parabolas in the plane. The notion of increment is also discussed.

- **Cartesian Coordinates in the Plane**

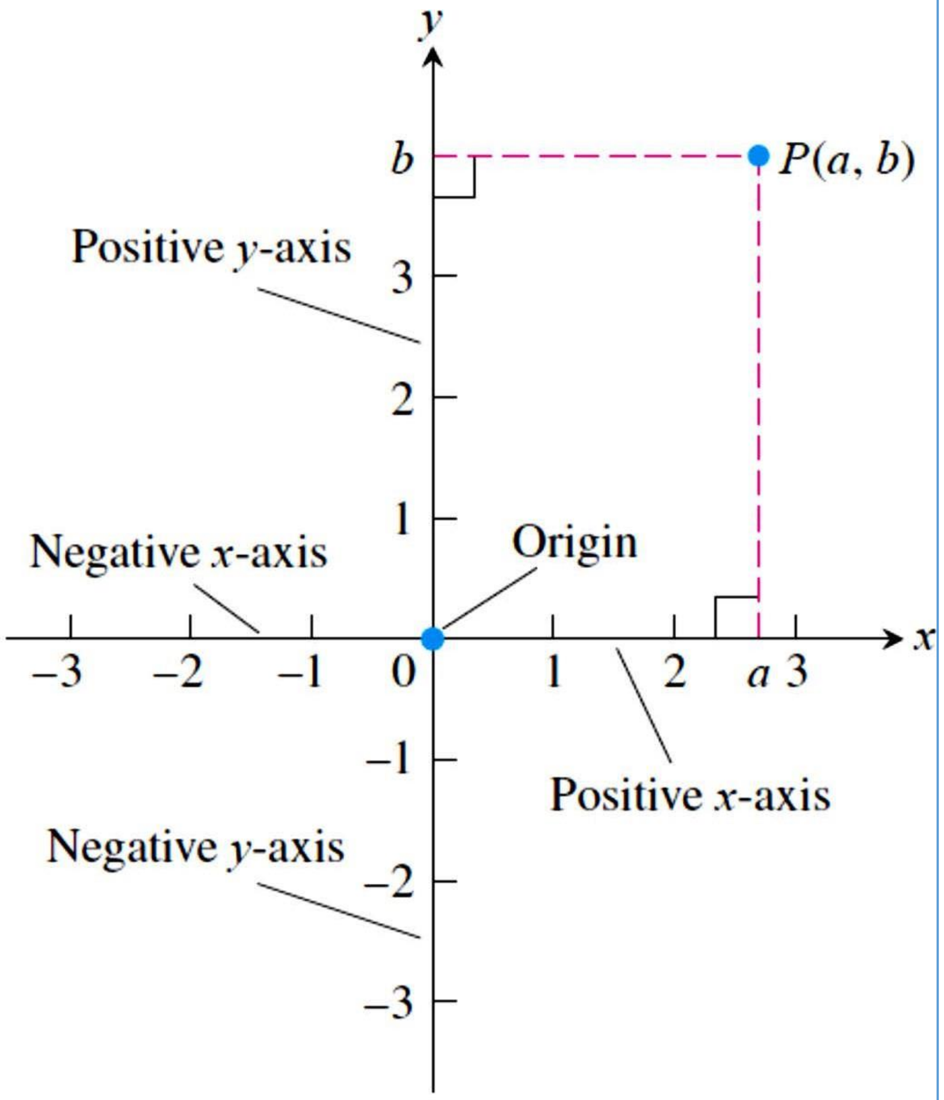


FIGURE 1.5 Cartesian coordinates in the plane are based on two perpendicular axes intersecting at the origin.

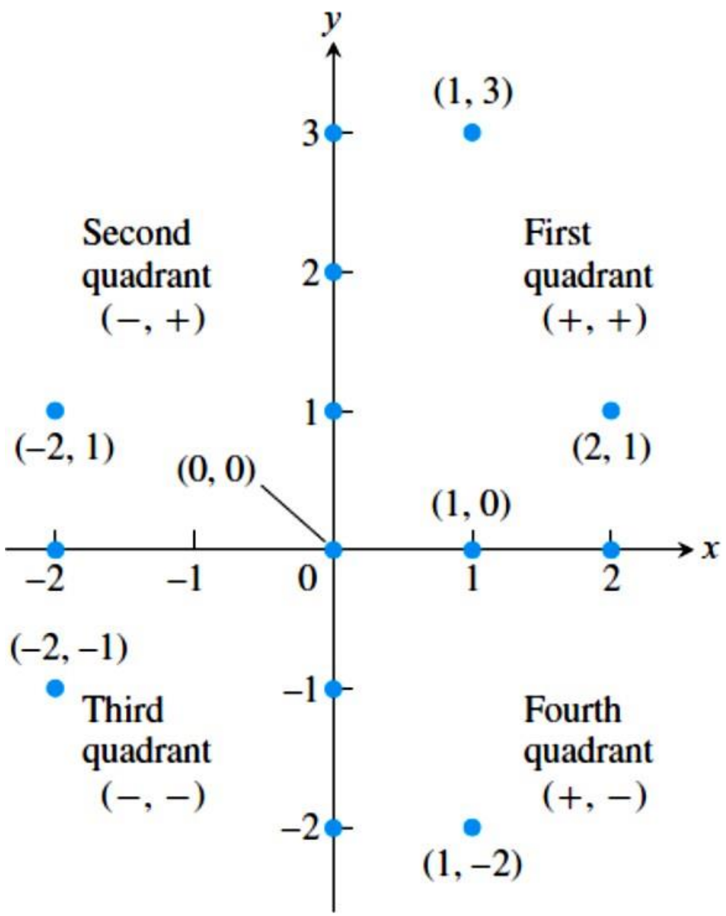


FIGURE 1.6 Points labeled in the xy-coordinate or Cartesian plane. The points on the axes all have coordinate pairs but are usually labeled with single real numbers, (so $(1, 0)$ on the x-axis is labeled as 1). Notice the coordinate sign patterns of the quadrants.

Increments and Straight Lines

Whcn a particle moves front one point in the plane to another. the net changes in its coordinates are called *inkr‹•iit‹•iit.‹.* They arc calculated by subtracting the coordinates of the starting polnt from the coordinates of the ending point. If.v changes front x to ›2, the increment in .v is

$$\Delta x = x_2 - x_1 .$$

EXAMPLE 1 In going front the point *A!4.* —3) to the point *B(2,* 5) the increments in the .v- and i -coordinates are

A.v = 2 — 4 = —2, AJ = 5 — (—3) = 8.

From C(S. 6 to *D(* 5, 1) the coordinate increments are

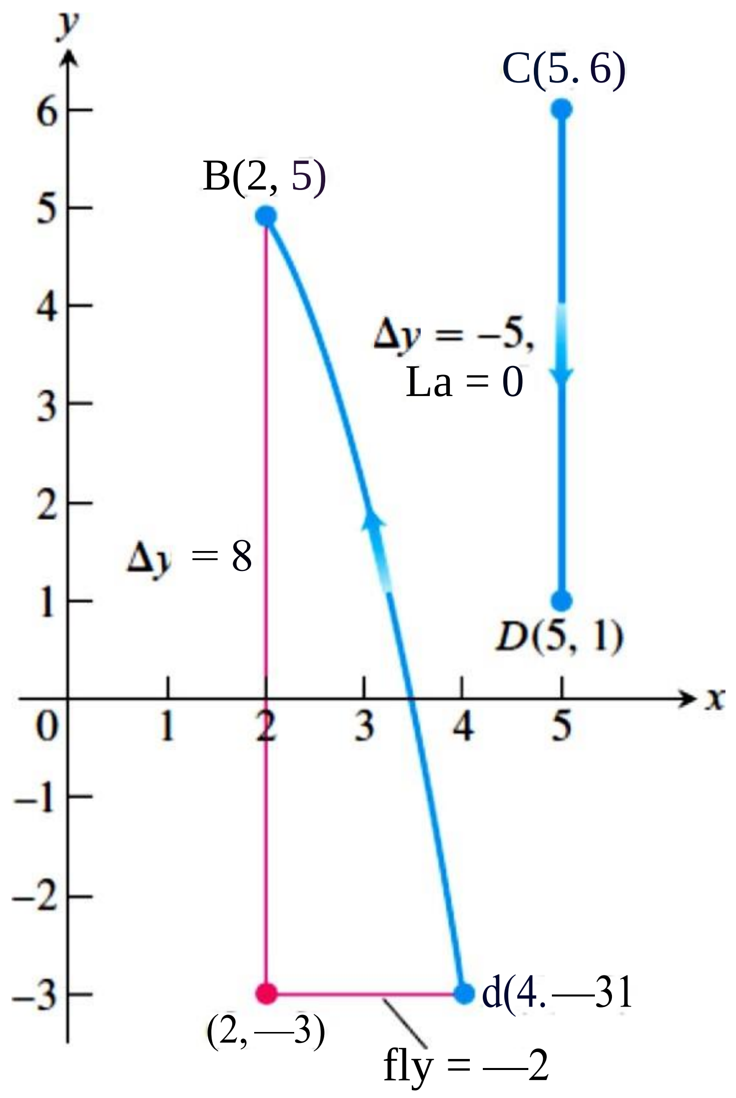
ñ.v = 5 — 1 = 0. ñJ = 1 — 6 = —5.

See Figure 1.7. ■

Given two points P1t-'i. ›) and P2(·'2- ,› 2) in the plane. we call the increments 3› = -›2 v; and A i = ,i 2 i the run and the rise. respectively. between *P*1 and P2 . Two such points always determine a unique straight line (usually called simply a line) passing through them both. We call the line P l P 2

Any nonvertical line in the plane has the property that the ratio

$$m \quad \frac{\text{rise}}{\text{run}} \quad \frac{\Delta y}{\Delta x} \quad \frac{y_2 - y_1}{x_2 - x_1}$$



FItURE 1.7 C'OOf itlatC lliCrcmcnts ‹xay be positis'e, ltep•atls'e. or zero l Exaltaple 11.

DEFINITION Slope

The constant

' rise _.1' .22 .l'1

run ,¿,> >2 i

is the slope of the nonvertical line *P;* P 2 -

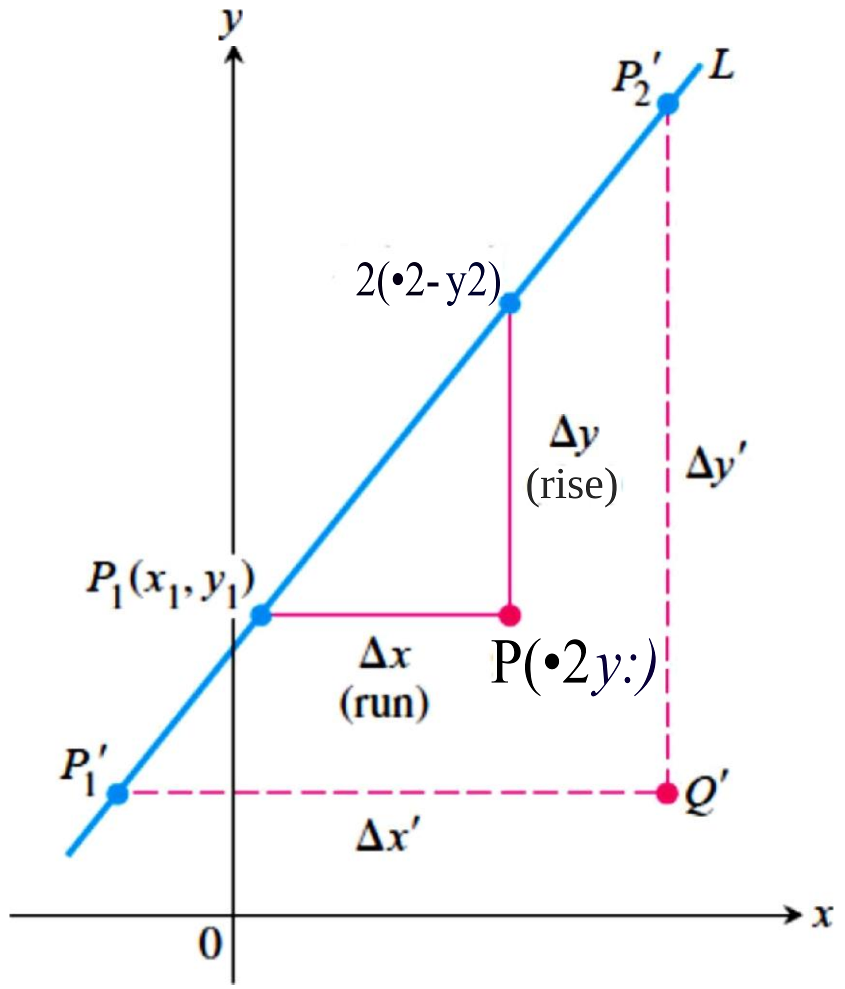
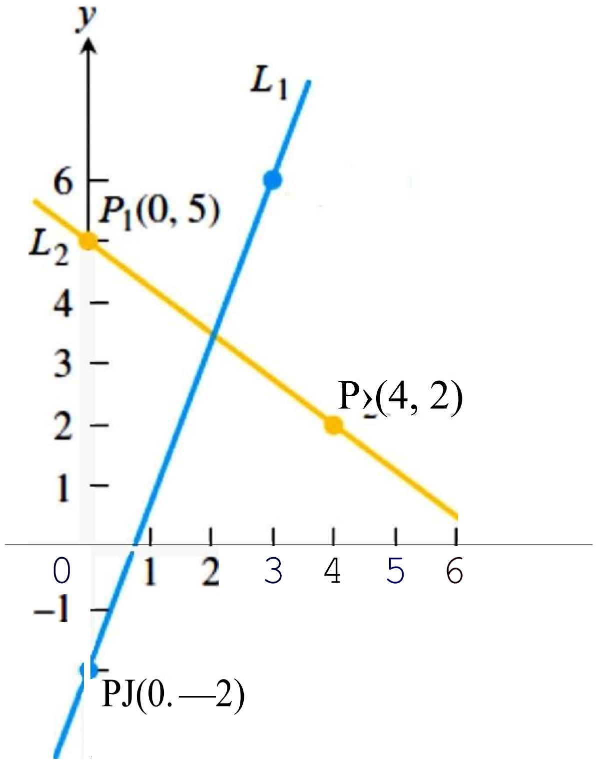


FIGURE 1.8 Triangles *P; QP2* and *P; Q'P2'* are similar. so the mtio of their sides has the same value for any two points on the line. This common value is the line's slope.

The slope tells us the direction (uphill. downhill) and steepness of a line. A line with positive slope rises uphill to the right; one with negative slope falls downhill to the right (Figure 1.9). The grealer the absolute value of the slope, the morc rapid the risc or fall. Thc slope of a vertical line is i‹›tr/‹-/i›i‹-‹/. Sincc the run ñ.v is zero for a veñical line. we cannot evaluate the slope ratio ›ir.



FIGUR£ 1.9 Tlte slope of L is

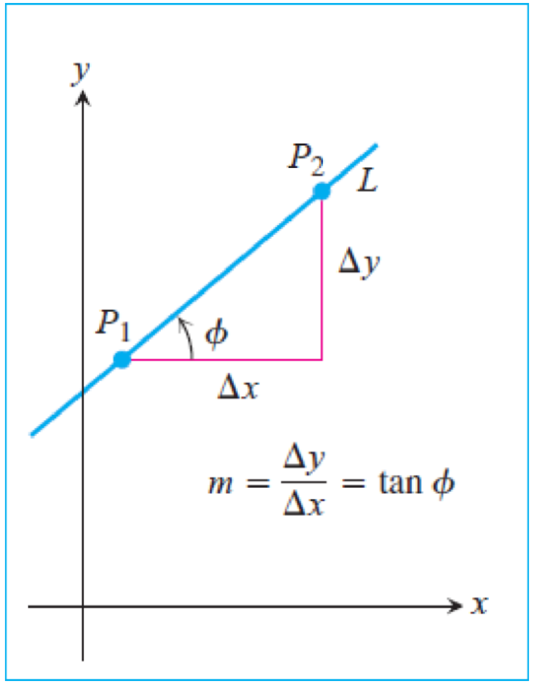
$$m — \frac{\Delta y}{A.x} — \frac{6-(-2)}{3 — 0} — \frac{8}{3}$$

That 1s, t iitereises 8 titbits es’ery iiille x iilcreaaes I merits. The slope of L2 lS

$$m — \frac{\Delta y}{lx} — \frac{2-5}{4 — 0} — \frac{-3}{4}$$

Th.at is. v decreases I units exery tune x i›lcrcaa-es 4 titbits.

The direction and steepness o£ a line can also be measured with an angle. The angle of inclination of a line that crosses the .v-axis is the smallest counterclocku'ise angle from the .v-axis to the line (Figure 1.10). The inclination of a horizontal line is 0°. The inclination of a vertical line is 90°. If Q tthe Greek lener phi) i5 the inclination of a line, then o « 9 < i o



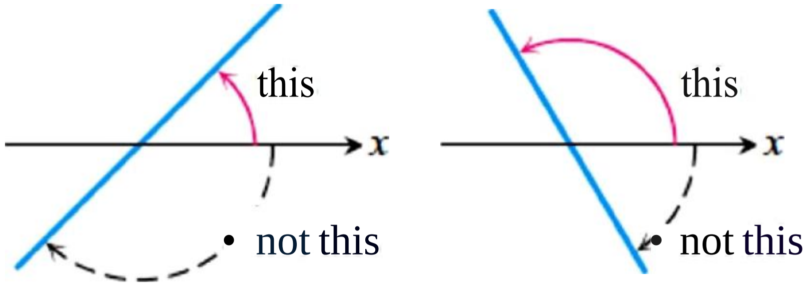
The relationship between the slope ›i› of a nons'ertical line and the line's anple of inclination 9 is shown in Fipure 1.1 1 :

» tan Q.

$\underline{y - y_1}$

so thal

*••(• — -‹t/ or i ' v, + /›‹(.• — .›,)



I URE 1.1 Anp!cs ofinclilmtioii arc 1 as&rcd col&tc block disc from 1 & *x*-axis.

The equation

)' ' I' } + f)f (. k •fi I)

is the point-slope equation of the line that passes through the point (i . i) and has slope iii.

EXAHPLE 2 Write an equation for the line through the point (2, 3) with slope —3/2.

Solution We substitute x = 2 y —3, and m = —3/2 into the point-slope equation and obtain

$$y \quad 3 \quad \frac{3}{2}(z \quad 2), \qquad \text{or} \qquad y \quad -\frac{3}{2}x + 6.$$

When x 0, y 6 so the line intersects the yr-axis at y 6. ■

EXANPLE 3 A Line Through Two PoinU

Write an equation for the line through (—2, —1) and (3, 4).

Solution The line's slope is

$$m \quad \frac{-1 - 4}{-2 - 3} \_ \frac{-5}{-5} = 1$$

We can use this slope with either of the two given points in the point-slope equation:

With (x„y,) = (—2, —1)	Witb (x„y,) = (3, 4)
r = — 1 + 1 • (z — (—2))	y = 4 + -1 (x — 3)
r — 1 + z + 2	v 4 + x — 3
y x + 1	+ i

Same result

Either way, y = x -1- I is an equation for the line (Figure 1.13). ■

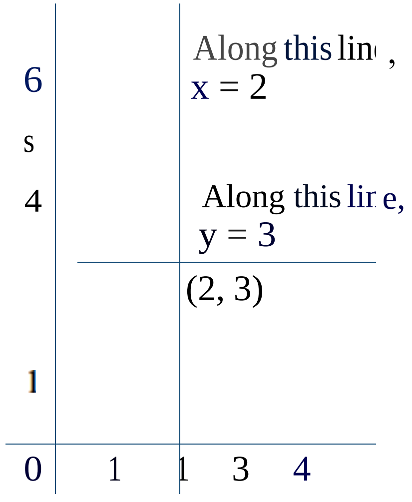


FIGURE 1.12 The standard equations for the vertical and horizontal lines through (2, 3) are = 2 and y = 3.

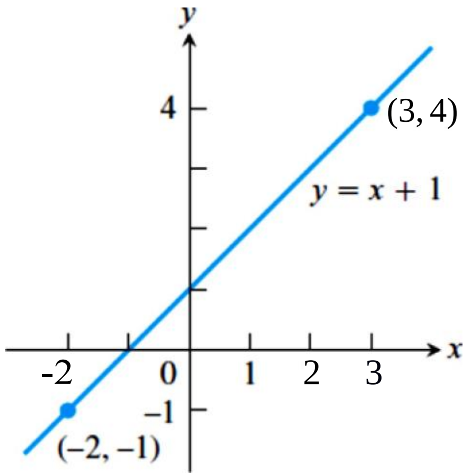


FIGURE 1.13 The line in Example 3.