EXP:-

Graph the fuction

$y = -x^2$

① Domain

$y = -x^2$ DF = R $\equiv$ $(-\infty, +\infty)$

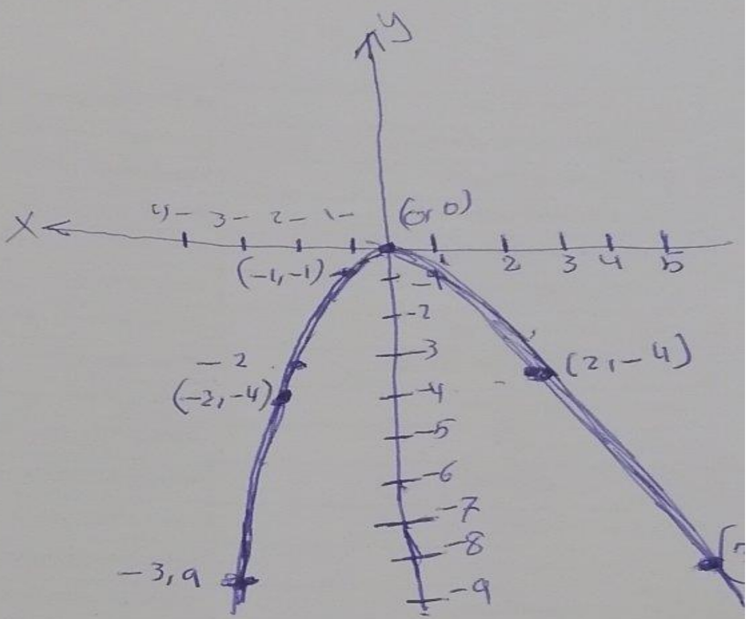
② Range

$y = -x^2 \rightarrow x^2 = -y \Rightarrow x = \pm\sqrt{-y}$

$-y \geq 0 \Rightarrow y \leq 0$ interval $(-\infty, 0]$

③ Graph

x	$y = -x^2$
0	0
1	-1
2	-4
3	-9
-1	-1
-2	-4
-3	-9



Vertex

$x = \frac{-b}{2a}$

$y = ax^2 + bx + c$

$x = \frac{-0}{2 \times 0} = 0$

$y = -x^2 + 0 - 0$

$y = 0$

(0,0) vetex

Exp:- Graph the fuction (2)

$y = |x-1| + 2$

$$|x-1| = \begin{cases} +(x-1) & x-1 \geq 0 \quad x \geq 1 \\ -(x-1) & x-1 < 0 \quad x < 1 \end{cases}$$

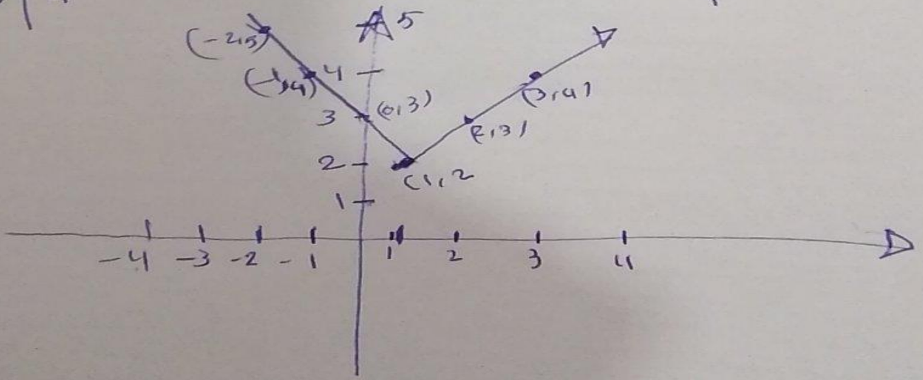
$$y = \begin{cases} +(x-1)+2 \\ -(x-1)+2 \end{cases} = \begin{cases} x+1 & x \geq 1 \\ 3-x & x < 1 \end{cases}$$

$y = x+1 \quad x \geq 1$

x	y
1	2
2	3
3	4

$y = 3 - x \quad x < 1$

x	y
0	3
−1	4
−2	5



Exp :

$|y| - |x| = 1$

sol :-

$|y| = 1 + |x|$

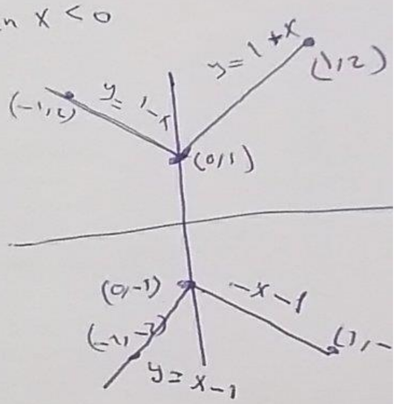
b, $y = \begin{cases} 1+x & \text{when } x \geq 0 \\ 1-x & \text{when } x < 0 \end{cases}$

$y = 1 + x$ when $x \geq 0$

x	y
0	1
1	2
2	3

$y = 1 - x$ when $x < 0$

x	y
-1	2
-2	3
-3	4



or $-y = 1 + |x| \Longrightarrow y = -|x| - 1$

$y = \begin{cases} -x-1 & \text{when } x \geq 0 \\ x-1 & \text{when } x < 0 \end{cases}$

$y = -x - 1$ when $x \geq 0$

x	y
0	-1
1	-2
2	-3

$y = x - 1$ when $x < 0$

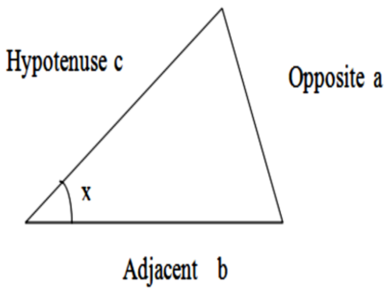
x	y
-1	-2
-2	-3

1.8.4 Trigonometric Functions

The six basic trigonometric functions are

Sine $\sin x = a/c$
Cosine $\cos x = b/c$
Tangent $\tan x = a/b = \sin x/\cos x$

Cotangent $\cot x = b/a = \cos x/\sin x$
Secant $\sec x = c/b = 1/\cos x$
Cosecant $\csc x = c/a = 1/\sin x$



- **Identifies**

Trigonometric Identities – part 1 www.GIMathS.Com

Reciprocal Identities

$\sin\theta = \frac{1}{\csc\theta}$	$\csc\theta = \frac{1}{\sin\theta}$
$\cos\theta = \frac{1}{\sec\theta}$	$\sec\theta = \frac{1}{\cos\theta}$
$\tan\theta = \frac{1}{\cot\theta}$	$\cot\theta = \frac{1}{\tan\theta}$

Half Angle Identities

$$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{2}}$$
$$\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos\theta}{2}}$$
$$\tan\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$

Double Angle Identities

$$\sin(2\theta) = 2\sin\theta\cos\theta$$
$$\cos(2\theta) = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$$
$$\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$$

Pythagoras Identities

$$\sin^2\theta + \cos^2\theta = 1$$
$$1 + \tan^2\theta = \sec^2\theta$$
$$1 + \cot^2\theta = \csc^2\theta$$

Even/Odd Identities

$$\sin(-\theta) = -\sin\theta$$
$$\cos(-\theta) = \cos\theta$$
$$\tan(-\theta) = -\tan\theta$$
$$\csc(-\theta) = -\csc\theta$$
$$\sec(-\theta) = \sec\theta$$
$$\cot(-\theta) = -\cot\theta$$

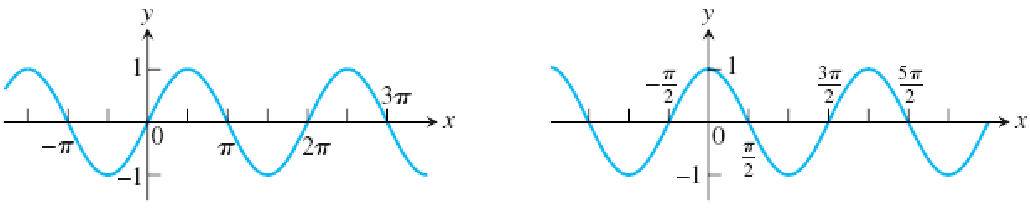
Sum to Product Identities

$$\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$
$$\sin\alpha - \sin\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$
$$\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$
$$\cos\alpha - \cos\beta = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

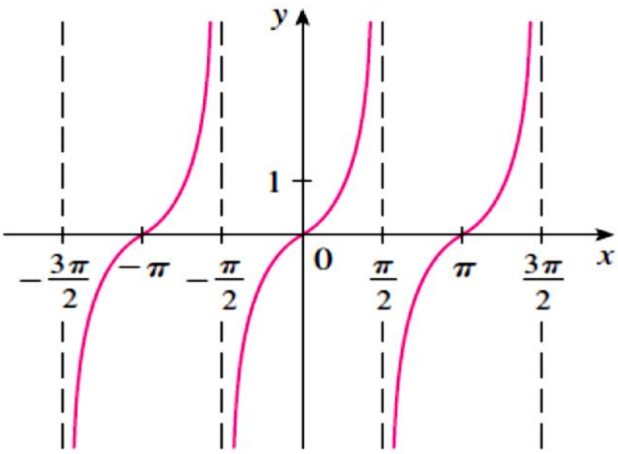
Product to Sum Identities

$$\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]$$
$$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha-\beta) + \cos(\alpha+\beta)]$$
$$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$$
$$\cos\alpha\sin\beta = \frac{1}{2}[\sin(\alpha+\beta) - \sin(\alpha-\beta)]$$

- The graphs of the sine, cosine and tangent functions are shown in Figure 1.20.



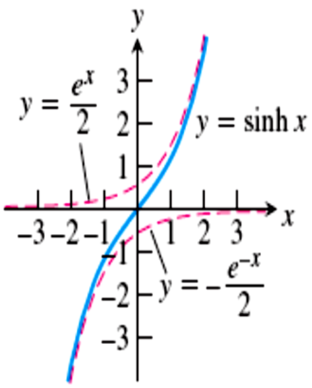
(a) $f(x) = \sin x$ (b) $f(x) = \cos x$



$y = \tan x$

Figure 1 .20

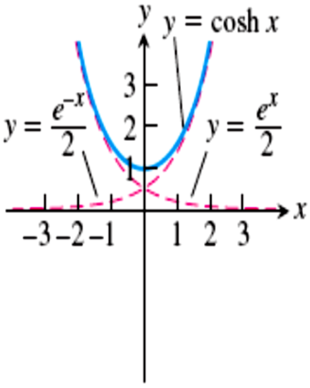
❖ Hyperbolic Functions



(a)

Hyperbolic sine:

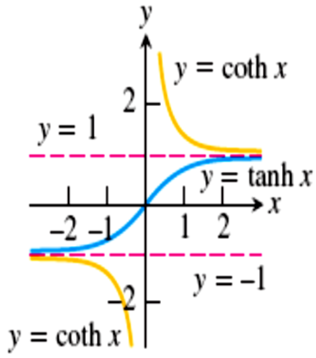
$$\sinh x = \frac{e^x - e^{-x}}{2}$$



(b)

Hyperbolic cosine:

$$\cosh x = \frac{e^x + e^{-x}}{2}$$



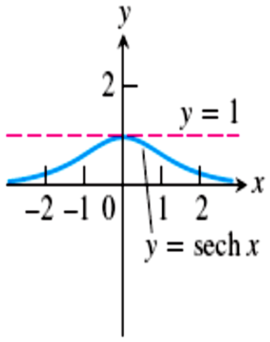
(c)

Hyperbolic tangent:

$$\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

Hyperbolic cotangent:

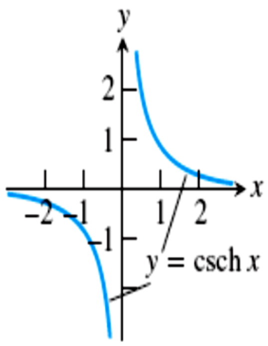
$$\coth x = \frac{\cosh x}{\sinh x} = \frac{e^x + e^{-x}}{e^x - e^{-x}}$$



(d)

Hyperbolic secant:

$$\operatorname{sech} x = \frac{1}{\cosh x} = \frac{2}{e^x + e^{-x}}$$



(e)

Hyperbolic cosecant:

$$\operatorname{csch} x = \frac{1}{\sinh x} = \frac{2}{e^x - e^{-x}}$$

1.8.5 Exponential Functions

Exponential functions formula is

$$f(x) = a^x, \quad \text{where } a > 0 \text{ and } a \neq 1, \; a = \text{constant value}$$

- Domain $(-\infty, \infty)$ and range $(0, \infty)$.
- In fact, natural exponential function (e^x) , where e is constant number, e =2.71828
- An exponential function never assumes the value 0 , as shown in Figure 1.21.

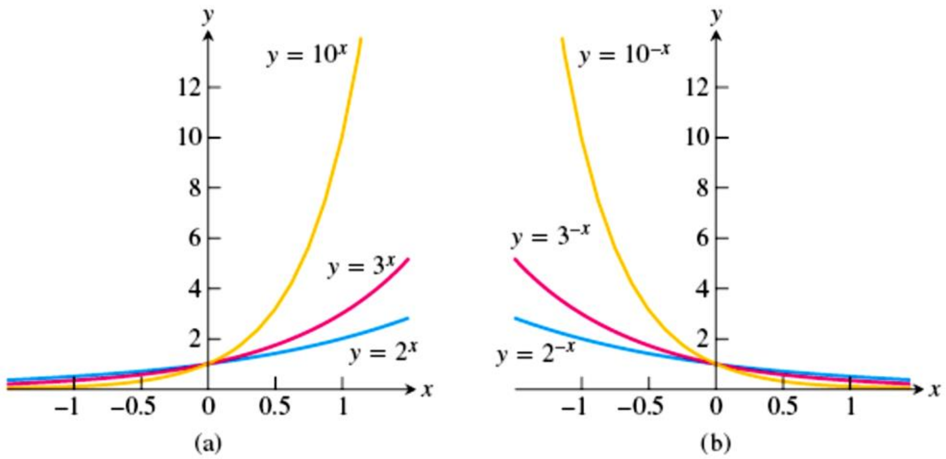
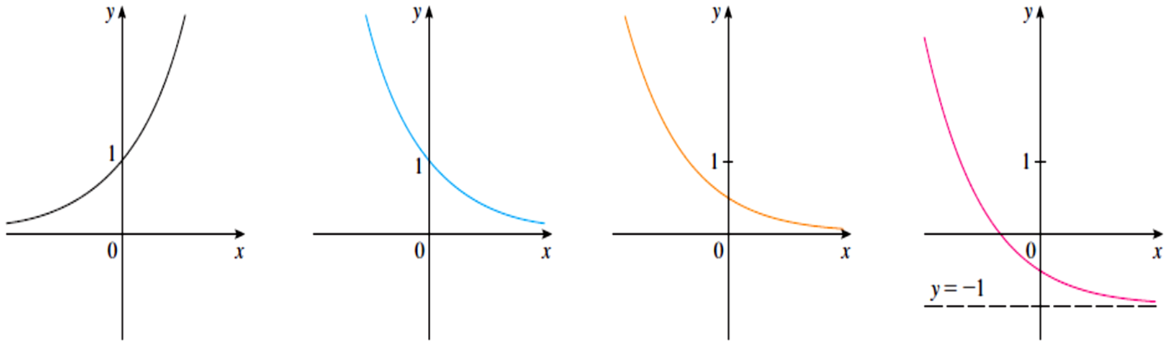


Figure 1 .21

Illustrative example: Graph the function $y = \frac{1}{2}e^{-x} - 1$ and state the domain and range.

Solution We start with the graph of $y = e^x$ from Figure below, and reflect about the y-axis to get the graph of $y = e^{-x}$ in . Figure (b). (Notice that the graph crosses the y-axis with a slope of -1). Then we compress the graph vertically by a factor of 2 to obtain the graph of $y = \frac{1}{2}e^{-x}$ in Figure (c). . Finally, we shift the graph downward one unit to get the desired graph in Figure (d). The domain is $\mathbf{R}$ and the range is $(-1, \infty)$.



(a) $y = e^x$ (b) $y = e^{-x}$ (c) $y = \frac{1}{2}e^{-x}$ (d) $y = \frac{1}{2}e^{-x} - 1$

Rules for Exponents

If $a > 0$ and $b > 0$, the following rules hold true for all real numbers x and y.

1. $a^x \cdot a^y = a^{x+y}$
2. $\dfrac{a^x}{a^y} = a^{x-y}$
3. $(a^x)^y = (a^y)^x = a^{xy}$
4. $a^x \cdot b^x = (ab)^x$
5. $\dfrac{a^x}{b^x} = \left(\dfrac{a}{b}\right)^x$

Example : We illustrate the flowing functions using the rules for exponents

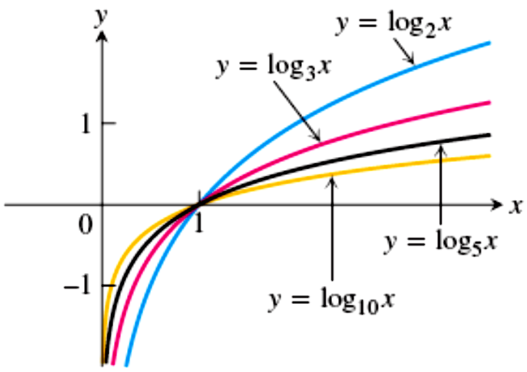
1. $3^{1.1} \cdot 3^{0.7} = 3^{1.1+0.7} = 3^{1.8}$
2. $\dfrac{\left(\sqrt{10}\right)^3}{\sqrt{10}} = \left(\sqrt{10}\right)^{3-1} = \left(\sqrt{10}\right)^2 = 10$
3. $\left(5^{\sqrt{2}}\right)^{\sqrt{2}} = 5^{\sqrt{2}\cdot\sqrt{2}} = 5^2 = 25$
4. $7^{\pi} \cdot 8^{\pi} = (56)^{\pi}$
5. $\left(\dfrac{4}{9}\right)^{1/2} = \dfrac{4^{1/2}}{9^{1/2}} = \dfrac{2}{3}$

1.8.6 Logarithmic Functions

The functions describe as

$f(x) = \log_a x$ where $a \neq 1$,is a positive constant

- It is the *inverse functions* of the exponential functions
- In each case the domain is $(0, \infty)$ and the range is $(-\infty, \infty)$. as shown in Figure 1.19.



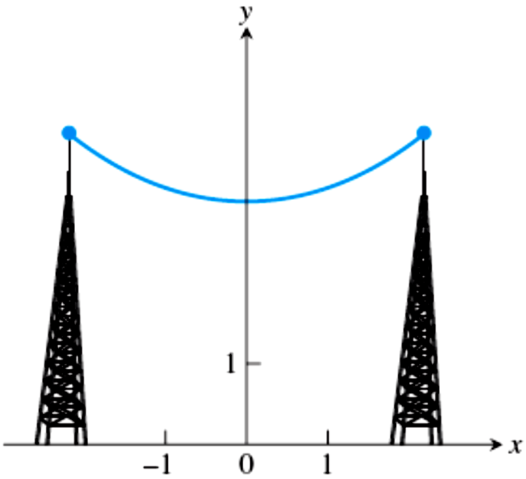


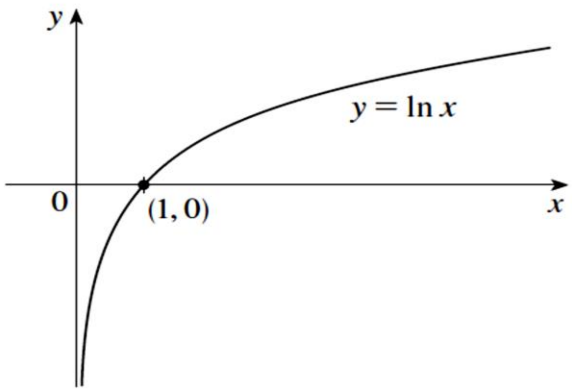
Figure 1 .19

❖ **NATURAL LOGARITHMS**

The logarithm with base is called the **natural logarithm** and has a special notation:

$$\log_e x = \ln x$$

$$\ln e = 1$$



x	$y = lnx$
0	∞
1	0
2	0.693
3	1.098
4	1.386
5	1.609
−1	∞
0.9	−0.105
0.5	−0.693
0.2	−1.609
0. 1	−2.302

Example: Classify the following functions as one of the types of functions that we have discussed.

(a) $f(x) = 5^x$ (b) $g(x) = x^5$

(c) $h(x) = \dfrac{1 + x}{1 - \sqrt{x}}$ (d) $u(t) = 1 - t + 5t^4$

(a) $f(x) = 5^x$ is an exponential function. (The x is the exponent.)

(b) $g(x) = x^5$ is a power function. (The x is the base.) We could also consider it to be a polynomial of degree 5.

(c) $h(x) = \dfrac{1 + x}{1 - \sqrt{x}}$ is an algebraic function.

(d) $u(t) = 1 - t + 5t^4$ is a polynomial of degree 4.

1.8.7 Algebra of functions

Let f is a function of x then we get f(x) and g is a function of x also we get g(x)

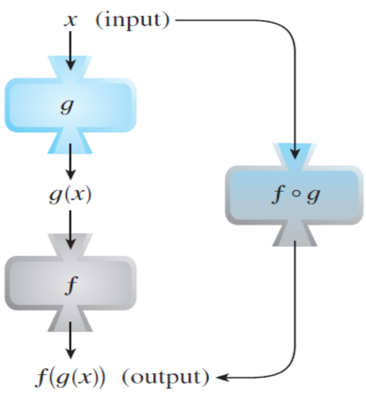
Df is the domain of f(x)
Dg is the domain of g(x)
Then:

f+g = f(x) + g(x) and Df ∩ Dg

f – g = f(x) - g(x)

f.g = f(x) . g(x)

and the domain is as same before



if f/g then Df ∩ Dg but g(x) ≠ 0

if g/f then Dg ∩ Df but f(x) ≠ 0

and Df_og = {x: x ∈ Dg, g(x) ∈ Df}

where $f_og(x) = f(g(x))$ also called the composition of f and g

Example: Find f_og and g_of if $\boldsymbol{f}(x)= \sqrt{1-x}$ and $g(x)= \sqrt{5+x}$
Solution:-

$(f_og)x= f(g(x)) = f(\sqrt{5+x}) = \sqrt{1-\sqrt{5+x}}$
(1-x) ≥ 0 then x ≤ 1 Df: x≤1
5+x ≥ 0 then x ≥ -5 Dg: x≥-5
$D\ f_og = \{x: x \geq -5, \sqrt{5+x} \leq 1\} = \{x: -5 \leq x \leq -4\}$

Example: If $f_{(x)} = \sqrt{x}$ and $g_{(x)} = \sqrt{1-x}$
Find:
$f+g$, f-g, g-f, f_og, f/g, g/f then graph f_og and also $f+g$. Solution
$f_{(x)} = \sqrt{x}$ domain $x \geq 0$ $g_{(x)} = \sqrt{1-x}$ domain $x \leq 1$
$f+g = (f+g)x = \sqrt{x} + \sqrt{1-x}$ domain $0 \leq x \leq 1$ or [0,1]
f-g = $\sqrt{x} - \sqrt{1-x}$ domain $0 \leq x \leq 1$
g-f = $\sqrt{1-x} - \sqrt{x}$ domain $0 \leq x \leq 1$
$f_og = f(x)g(x) = f(g(x)) = f(\sqrt{1-x}) = \sqrt{\sqrt{1-x}} = 4\sqrt{1-x}$ domain $(-\infty, 1]$ (why?)

$f/g = f(x)/g(x) = \sqrt{\frac{x}{1-x}}$ domain $(-\infty, 1]$

$g/f = g(x)/f(x) = \sqrt{\frac{1-x}{x}}$ domain (0,1]

This chapter based on many reference [1][2][3]

reference :

[1] G. B. Thomas, *calculus*, Twelfth Ed. .
[2] "Chapter 1, calculus," *Tikrit Univ. Coll. Eng. Civ. Eng. Dep.*, pp. 1–54, 2008.
[3] "University of Anbar College of Engineering Department(s): Dams & Water Resources Eng. Electrical Eng .," pp. 1–113, 2018.