2.6 Wye-Delta Transformations

Situations often arise in circuit analysis when the resistors are neither in parallel nor in series, For example , the circuit shown in figure (15-2):-

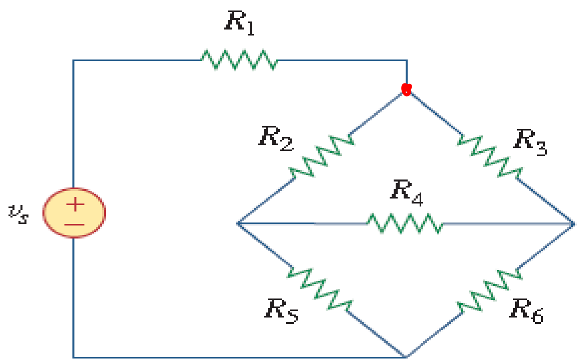


Figure 15-2: The bridge network

In this circuit R1, R2, R3, R4, R5, and R6 are neither in series nor in parallel

- ❖ the wye (Y) or tee (T) or star network shown in Figure (16- 2)

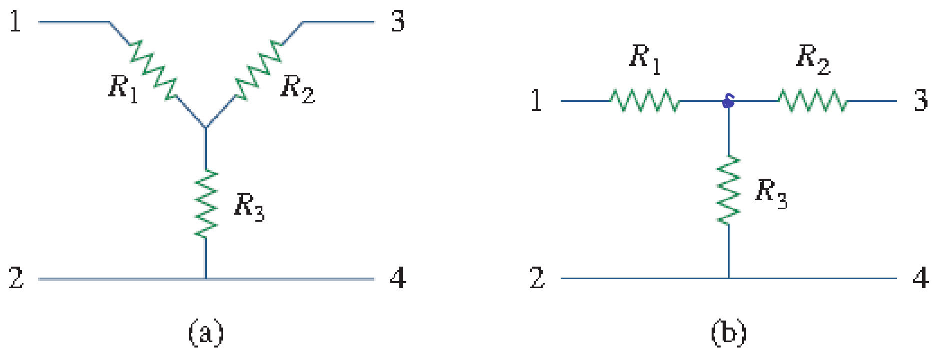


Figure 16-2 : (a) wye (Y), (b) tee (T)

- ❖ the delta (△) or pi (π) network shown in Figure (17-2)

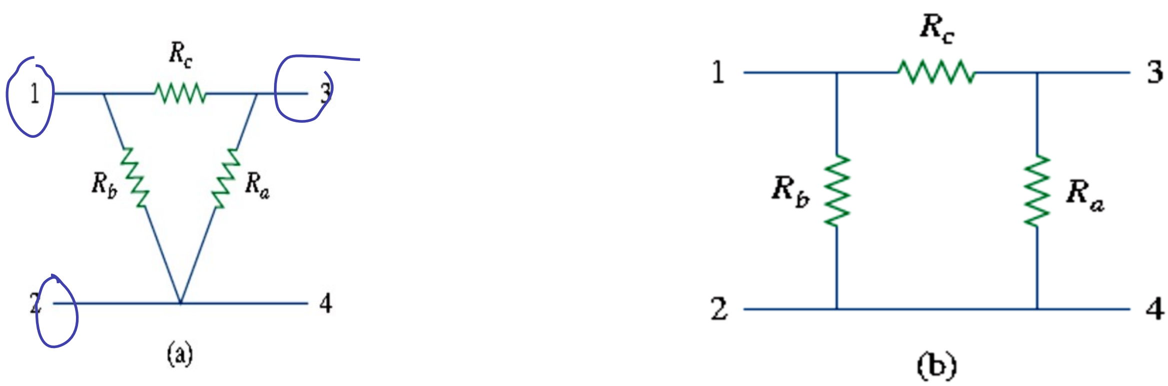


Figure 17-2:- **(a)** delta (△) , **(b)** pi (π)

2.6.1 **Delta to Wye Conversion**

In this case transfer the(△) to (Y) connection , let consider the the circuit shown in the figure (18-2) that is help us to understand the transfer process

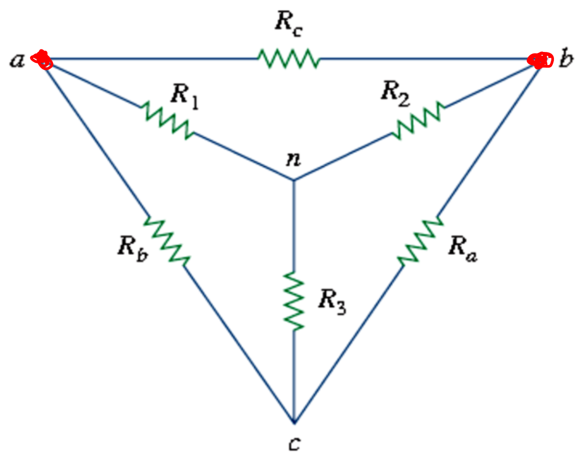


figure (18-2) : Delta to Wye Conversion

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

2.6.2 Wye to Delta Conversion

In this case transfer the(Y) to (△) connection , let consider the same circuit shown in the figure (18-2) that is help us to understand laws of the transformation

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$

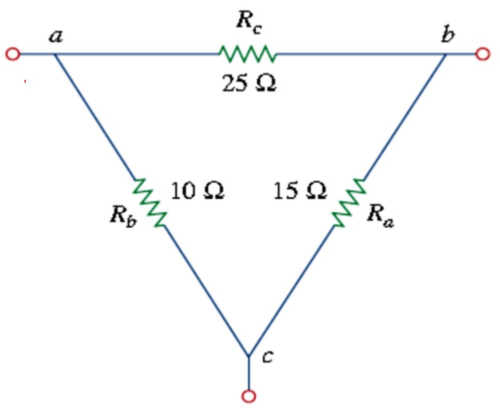
If the all value of the resistances are balanced thus,

$R_1=R_2=R_3=R_Y \qquad R_A=R_B=R_C=R_\wedge$

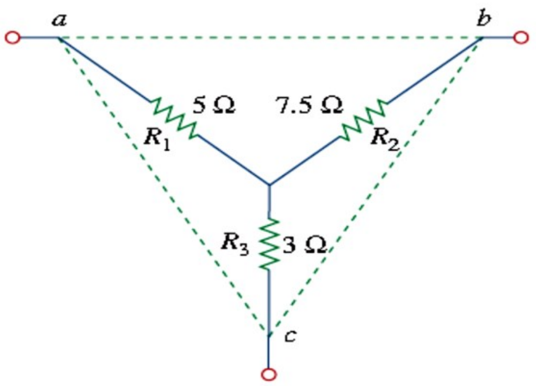
Under these conditions, conversion formulas become

$$R_Y = \frac{R_\Delta}{3} \qquad \text{or} \qquad R_\Delta = 3R_Y$$

Example : Convert the △ network in Figure shown below to an equivalent Y network



Solution :- start to draw the Y- connection as shown below , and apply the (△to Y) laws



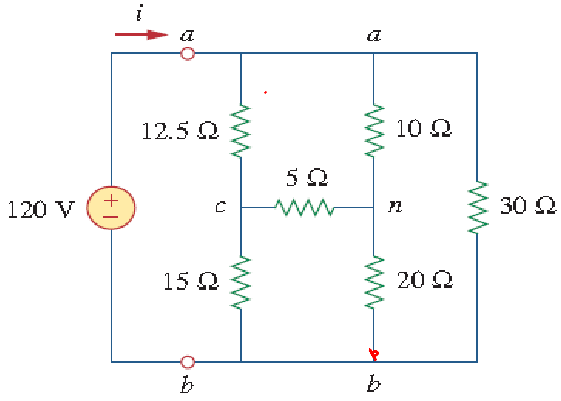
$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c} = \frac{10 \times 25}{15 + 10 + 25} = \frac{250}{50} = 5\,\Omega$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c} = \frac{25 \times 15}{50} = 7.5\,\Omega$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c} = \frac{15 \times 10}{50} = 3\,\Omega$$

The equivalent Y network is shown in Figure that shown above

Example: - Obtain the equivalent resistance R_{ab} for the circuit in Figure shown below and use it to find current *i*.



Solution :- In this circuit, there are two Y networks and three △ networks. Transforming just one of these will simplify the circuit. If we convert the Y network comprising the 5Ω, 10Ω and 20Ω resistors, we may select

$$R_1 = 10\,\Omega, \qquad R_2 = 20\,\Omega, \qquad R_3 = 5\,\Omega$$

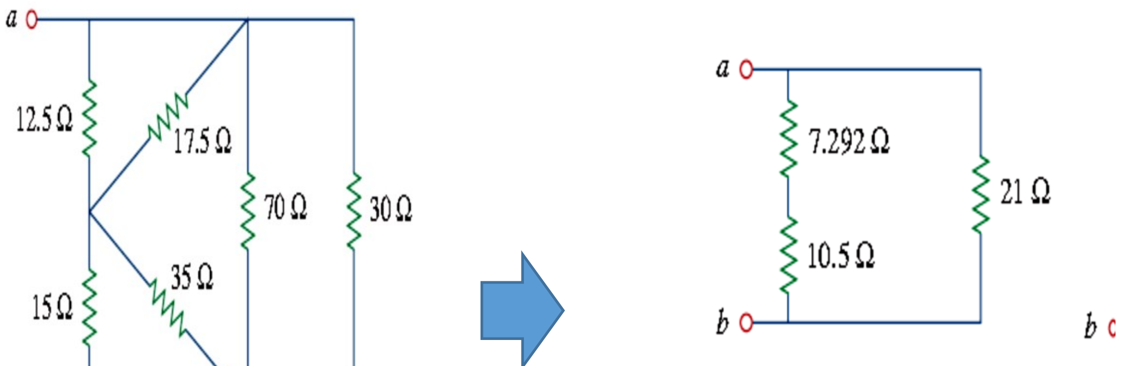
$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1} = \frac{10 \times 20 + 20 \times 5 + 5 \times 10}{10}$$

$$= \frac{350}{10} = 35\ \Omega$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2} = \frac{350}{20} = 17.5\ \Omega$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3} = \frac{350}{5} = 70\ \Omega$$

Thus the result of transfer (Y to △) will be as shown in figure (a) and simplify it to be as shown in figure (b)



(b)

(a)

The process of the simplify is

$$70 \parallel 30 = \frac{70 \times 30}{70 + 30} = 21\ \Omega$$

$$12.5 \parallel 17.5 = \frac{12.5 \times 17.5}{12.5 + 17.5} = 7.292\ \Omega$$

$$15 \parallel 35 = \frac{15 \times 35}{15 + 35} = 10.5\ \Omega$$

Hence, we find

$$R_{ab} = (7.292 + 10.5) \parallel 21 = \frac{17.792 \times 21}{17.792 + 21} = \mathbf{9.632\ \Omega}$$

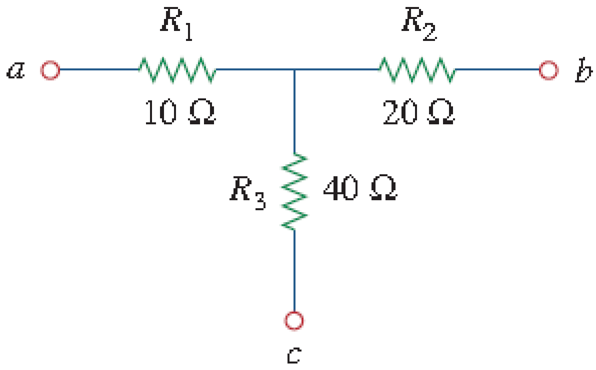
Then

$$i = \frac{v_s}{R_{ab}} = \frac{120}{9.632} = \mathbf{12.458\ A}$$

Homework: solve the above example but use the (△ to Y) in first step

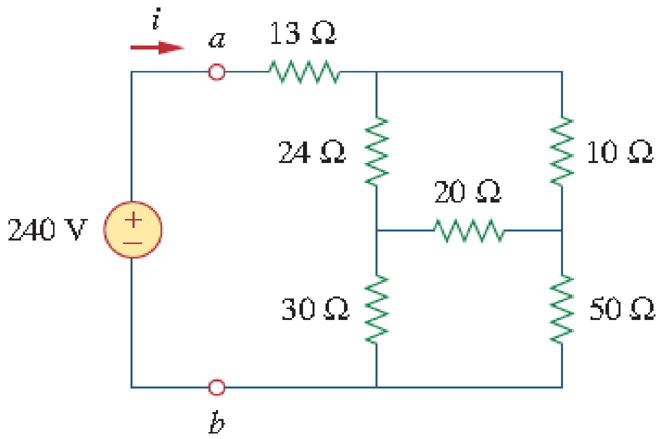
Home work:

- **Transform the wye network in Figure shown below to a delta network**



Answer: R_a =140 Ω, R_b = 70 Ω, R_c =35 Ω

- **Obtain the equivalent resistance R_{ab} for the circuit in Figure shown below and use it to find current *i*.**



Answer : 40 Ω, 6 A.

Summary

1 A resistor is a passive element in which the voltage v across it is directly proportional to the current i through it. That is, a resistor is a device that obeys Ohm's law,

$$v = iR$$

where R is the resistance of the resistor.

2 A short circuit is a resistor (a perfectly, conducting wire) with zero resistance ($R = 0$). An open circuit is a resistor with infinite resistance ($R = \infty$).

3 The conductance G of a resistor is the reciprocal of its resistance:

$$G = \frac{1}{R}$$

4 A branch is a single two-terminal element in an electric circuit. A node is the point of connection between two or more branches. A loop is a closed path in a circuit. The number of branches b, the number of nodes n, and the number of independent loops l in a network are related as

$$b = l + n - 1$$

5 Kirchhoff's current law (KCL) states that the currents at any node algebraically sum to zero. In other words, the sum of the currents entering a node equals the sum of currents leaving the node.

6 Kirchhoff's voltage law (KVL) states that the voltages around a closed path algebraically sum to zero. In other words, the sum of voltage rises equals the sum of voltage drops.

7 When two resistors $R_1 (=1/G_1)$ and $R_2 (=1/G_2)$ are in series, their equivalent resistance R_{eq} and equivalent conductance G_{eq} are

$$R_{eq} = R_1 + R_2, \qquad G_{eq} = \frac{G_1 G_2}{G_1 + G_2}$$

8 When two resistors $R_1 (=1/G_1)$ and $R_2 (=1/G_2)$ are in parallel, their equivalent resistance R_{eq} and equivalent conductance G_{eq} are

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}, \qquad G_{eq} = G_1 + G_2$$

9 The voltage division principle for two resistors in series is

$$v_1 = \frac{R_1}{R_1 + R_2} v, \qquad v_2 = \frac{R_2}{R_1 + R_2} v$$

10 The current division principle for two resistors in parallel is

$$i_1 = \frac{R_2}{R_1 + R_2} i, \qquad i_2 = \frac{R_1}{R_1 + R_2} i$$

11 The formulas for a delta-to-wye transformation are

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}, \qquad R_2 = \frac{R_c R_a}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

CHAPTER THREE

Methods of Analysis

3.1 Nodal Analysis

Nodal analysis provides a general procedure for analyzing circuits using node voltages as the circuit variables. To simplify matters, we shall assume in this section that circuits do not contain voltage sources.

Steps to Determine Node Voltages:

1. Select a node as the reference node. Assign voltages $v_1, v_2, \ldots, v_{n-1}$ to the remaining $n-1$ nodes. The voltages are referenced with respect to the reference node.
2. Apply KCL to each $n-1$ of the nonreference nodes. Use Ohm's law to express the branch currents in terms of node voltages.
3. Solve the resulting simultaneous equations to obtain the unknown node voltages

The reference node should has the on of these samples shown in figure below

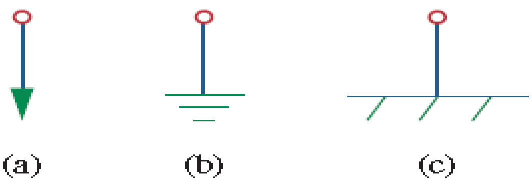


Figure 3.1: type to represent the ground of the circuit

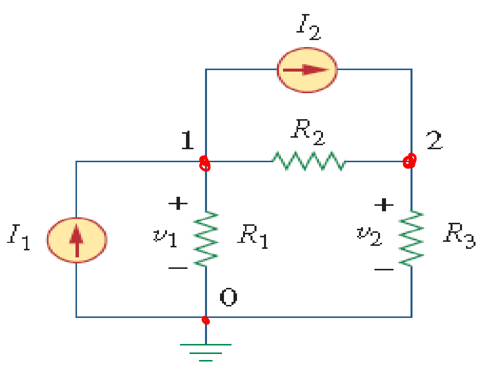
The second important thing should understand this condition with analysis is:-

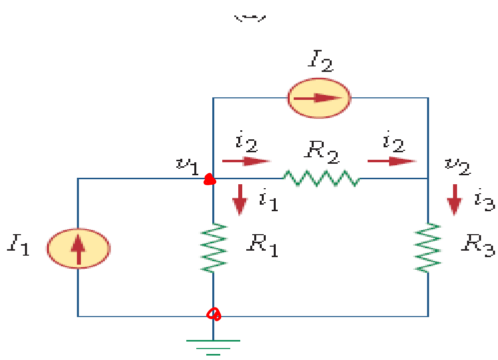
Current flows from a higher potential to a lower potential in a resistor.

We can express this principle as

$$i = \frac{v_{\text{higher}} - v_{\text{lower}}}{R}$$

Let analysis this circuit shown below





This circuit is contain three node , one of them is reference , is node 0

- At node 1, applying KCL gives

$$I_1 = I_2 + i_1 + i_2 \qquad \text{...........(3.1)}$$

- At node 2,

$$I_2 + i_2 = i_3 \qquad \text{........(3.2)}$$

- Voltage analysis of the circuit based on the **a higher potential to a lower potential in a resistor**

$$i_1 = \frac{v_1 - 0}{R_1} \quad \text{or} \quad i_1 = G_1 v_1$$

$$i_2 = \frac{v_1 - v_2}{R_2} \quad \text{or} \quad i_2 = G_2(v_1 - v_2)$$

$$i_3 = \frac{v_2 - 0}{R_3} \quad \text{or} \quad i_3 = G_3 v_2 \qquad \text{.. .... (3.3)}$$

Sub. The equation 3.3 in to 3.1 and 3.2 results

$$I_1 = I_2 + \frac{v_1}{R_1} + \frac{v_1 - v_2}{R_2}$$

$$I_2 + \frac{v_1 - v_2}{R_2} = \frac{v_2}{R_3}$$

❖ Method 2

$$I_1 = I_2 + G_1 v_1 + G_2(v_1 - v_2) \qquad \text{.... (3.4)}$$

$$I_2 + G_2(v_1 - v_2) = G_3 v_2 \qquad \text{......(3.5)}$$

Cramer’s rule, or matrix inversion. To use either of the last two methods, one must cast the simultaneous equations in matrix form. For example, Eqs. (3.4) and (3.5) can be cast in matrix form as:

$$\begin{bmatrix} G_1 + G_2 & -G_2 \\ -G_2 & G_2 + G_3 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} I_1 - I_2 \\ I_2 \end{bmatrix}$$