جامعة المثنى

كلية الهندسة / قسم الالكترونيك والاتصالات

المادة : رياضيات

المرحلة الاولى

المحاضر: د.محمد مزعل

المحاضرة الثالثة

Limits

If the values of f(x) tend to get closer and closer to the number L as x gets closer and closer to the number a (from either side of a) but $x \neq a$

1 DEFINITION We write

$$\lim_{x \to a} f(x) = L$$

and say "the limit of $f(x)$, as x approaches a, equals L"

if we can make the values of $f(x)$ arbitrarily close to L (as close to L as we like) by taking x to be sufficiently close to a (on either side of a) but not equal to a.

Example: Find the limit of the function $f(x) = x^2 - x + 2$ as x approaches

Solution:

Based on the function $f(x) = x^2 - x + 2$ will test values of x near

The following table gives values of f(x) for values of x close to 2, but not equal to 2, but not equal to 2, as shown in table below:-

x	f(x)	x	f(x)
1.0	2.000000	3.0	8.000000
1.5	2.750000	2.5	5.750000
1.8	3.440000	2.2	4.640000
1.9	3.710000	2.1	4.310000
1.95	3.852500	2.05	4.152500
1.99	3.970100	2.01	4.030100
1.995	3.985025	2.005	4.015025
1.999	3.997001	2.001	4.003001

The results of the table showed in Figure 1, we see that when x is close to 2 (on either side of 2), $f(x)$ is close to 4. Thus, we can express this by saying "the limit of the function $f(x) = x^2 - x + 2$ as x approaches 2 is equal to 4.

The notation for this is:

$$\lim_{\chi \to 2} (x^2 - \chi + 2) = 4$$

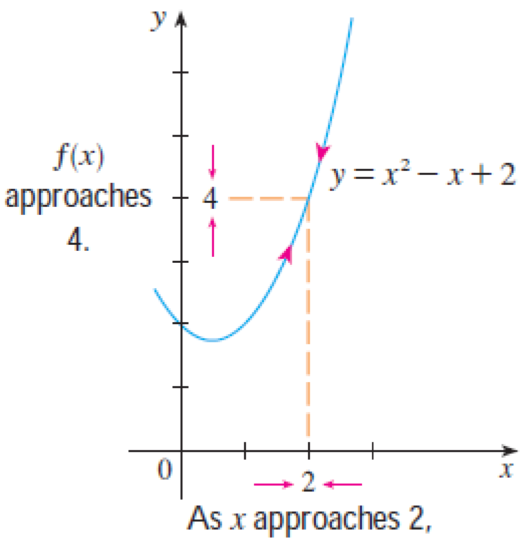

Figure 1

Example: Guess the value of $\lim_{\chi\to 1}\frac{x-1}{x^2-1}$

Solution:

The function $f(x) = (x-1)/(x^2-1)$ is not defined when $= 1$.

Thus will test the values closer to 1, because the definition of $\lim_{\chi\to 1} f(x)$ says that we consider values of x that are close to a but not equal to a,as shown in table below:-

$x < 1$	$f(x)$
0.5	0.666667
0.9	0.526316
0.99	0.502513
0.999	0.500250
0.9999	0.500025

$x > 1$	$f(x)$
1.5	0.400000
1.1	0.476190
1.01	0.497512
1.001	0.499750
1.0001	0.499975

Based on the results in the tables and graph(shown in figure 2), we make the guess that

$$\lim_{\chi\to 1}\frac{x-1}{x^2-1} = 0.5$$

The value of $\lim_{\chi\to 1}\frac{x-1}{x^2-1}$ can be solved to give same results by:

$$\lim_{\chi\to 1}\frac{x-1}{x^2-1} = \lim_{\chi\to 1}\frac{(x-1)}{(x-1)(x+1)}$$

$$\lim_{\chi\to 1}\frac{1}{(x+1)}$$

$$\lim_{\chi\to 1}\frac{1}{(1+1)} = 0.5$$

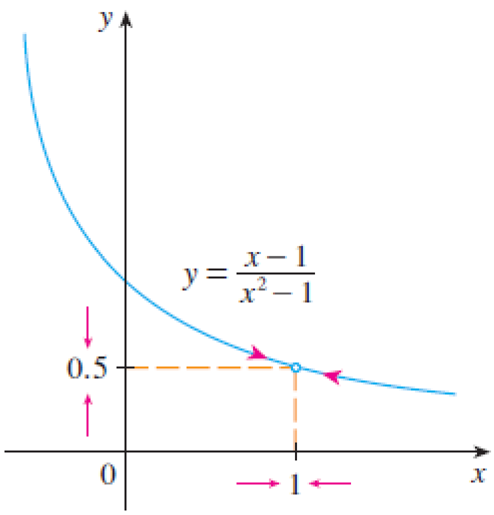

Figure 2